Online Appendix to Fiscal Consolidation in an Open Economy with Sovereign Premia and without Monetary Policy Independence*

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Appendix 1. Households

This appendix presents and solves the problem of the household in some detail. There are $i = 1, 2, \dots, N$ identical domestic households that act competitively.

Household's Problem

Each household i maximizes expected lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_{i,t}, n_{i,t}, g_t\right), \tag{1}$$

where $c_{i,t}$ is i's consumption bundle, $n_{i,t}$ is i's hours of work, g_t is per capita public spending, $0 < \beta < 1$ is the time preference rate, and E_0 is the rational expectations operator conditional on the information set.

The consumption bundle, $c_{i,t}$, is defined as

$$c_{i,t} = \frac{\left(c_{i,t}^H\right)^{\nu}\left(c_{i,t}^F\right)^{1-\nu}}{\nu^{\nu}(1-\nu)^{1-\nu}}, \tag{2}$$

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where $c_{i,t}^H$ and $c_{i,t}^F$ denote the composite domestic good and the composite foreign good, respectively, and ν is the degree of preference for the domestic good.

We define the two composites, $c_{i,t}^H$ and $c_{i,t}^F$, as

$$c_{i,t}^H = \left[\sum_{h=1}^{N} [c_{i,t}^H(h)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{3}$$

$$c_{i,t}^F = \left[\sum_{f=1}^{N} [c_{i,t}^F(f)]^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \tag{4}$$

where $\phi > 0$ is the elasticity of substitution across goods produced in the domestic country.

The period budget constraint of each i expressed in real terms is (see, e.g., Benigno and Thoenissen 2008, for similar modeling)

$$\begin{aligned}
&(1+\tau_t^c)\left[\frac{P_t^H}{P_t}c_{i,t}^H + \frac{P_t^F}{P_t}c_{i,t}^F\right] + \frac{P_t^H}{P_t}x_{i,t} + b_{i,t} + \frac{S_t P_t^*}{P_t}f_{i,t}^h \\
&\quad + \frac{\phi^h}{2}\left(\frac{S_t P_t^*}{P_t}f_{i,t}^h - \frac{SP^*}{P}f^h\right)^2 \\
&= \left(1-\tau_t^k\right)\left[r_t^k \frac{P_t^H}{P_t}k_{i,t-1} + \widetilde{\omega}_{i,t}\right] + (1-\tau_t^n)\, w_t n_{i,t} \\
&\quad + R_{t-1}\frac{P_{t-1}}{P_t}b_{i,t-1} + Q_{t-1}\frac{S_t P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}f_{i,t-1}^h - \tau_{i,t}^l,
\end{aligned} \tag{5}$$

where P_t is the consumer price index (CPI), P_t^H is the price index of home tradables, P_t^F is the price index of foreign tradables (expressed in domestic currency), $x_{i,t}$ is i's domestic investment, $b_{i,t}$ is i's end-of-period real domestic government bonds, $f_{i,t}^h$ is i's end-of-period real internationally traded assets denominated in foreign currency, r_t^k is the real return to inherited domestic capital, $k_{i,t-1}$, $\widetilde{\omega}_{i,t}$ is i's real dividends received by domestic firms, w_t is the real wage rate, $R_{t-1} \geq 1$ is the gross nominal return to domestic government bonds between $t-1$ and t, $Q_{t-1} \geq 1$ is the gross nominal return to international assets between $t-1$ and t, $\tau_{i,t}^l$ are real lump-sum taxes if positive (or transfers if negative) to each household, and

τ_t^c, τ_t^k, τ_t^n are tax rates on consumption, capital income, and labor income, respectively. The parameter $\phi^h \geq 0$ captures adjustment costs related to private foreign assets, where variables without time subscripts denote steady-state values.

Each household i also faces the budget constraints

$$P_t c_{i,t} = P_t^H c_{i,t}^H + P_t^F c_{i,t}^F \tag{6}$$

$$P_t^H c_{i,t}^H = \sum_{h=1}^{N} P_t^H(h) c_{i,t}^H(h) \tag{7}$$

$$P_t^F c_{i,t}^F = \sum_{f=1}^{N} P_t^F(f) c_{i,t}^F(f), \tag{8}$$

where $P_t^H(h)$ is the price of each variety h produced at home and $P_t^F(f)$ is the price of each variety f produced abroad (expressed in domestic currency).

Finally, the law of motion of physical capital for household i is

$$k_{i,t} = (1-\delta)k_{i,t-1} + x_{i,t} - \frac{\xi}{2}\left(\frac{k_{i,t}}{k_{i,t-1}} - 1\right)^2 k_{i,t-1}, \tag{9}$$

where $0 < \delta < 1$ is the depreciation rate of capital and $\xi \geq 0$ is a parameter capturing adjustment costs related to physical capital.

For our numerical solutions, we use the usual functional form:

$$u_{i,t}\left(c_{i,t}, n_{i,t}, m_{i,t}, g_t\right) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_{i,t}^{1+\eta}}{1+\eta} + \chi_g \frac{g_t^{1-\zeta}}{1-\zeta}, \tag{10}$$

where χ_n, χ_g, σ, η, ζ are preference parameters.

Household's Optimality Conditions

Each household i acts competitively, taking prices and policy as given. Following the literature, to solve the household's problem, we follow a two-step procedure. We first suppose that the household determines its desired consumption of composite goods, $c_{i,t}^H$ and $c_{i,t}^F$, and, in turn, chooses how to distribute its purchases of individual varieties, $c_{i,t}^H(h)$ and $c_{i,t}^F(f)$.

The first-order conditions of each i include the budget constraints written above and also

$$\frac{\partial u_{i,t}}{\partial c_{i,t}}\frac{\partial c_{i,t}}{\partial c_{i,t}^{H}}\frac{P_t}{P_t^H\left(1+\tau_t^c\right)} = \beta E_t\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^{H}}\frac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)}R_t\frac{P_t}{P_{t+1}} \tag{11}$$

$$\frac{\partial u_{i,t}}{\partial c_{i,t}}\frac{\partial c_{i,t}}{\partial c_{i,t}^{H}}\frac{P_t}{P_t^H\left(1+\tau_t^c\right)}\frac{S_tP_t^*}{P_t}\left[1+\phi^h\left(\frac{S_tP_t^*}{P_t}f_t^h-\frac{SP^*}{P}f^h\right)\right] = \beta E_t\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\frac{\partial c_{i,t+1}}{\partial c_{t+1}^{H}}\frac{P_{t+1}}{P_{t+1}^H\left(1+\tau_{t+1}^c\right)}Q_t\frac{S_{t+1}P_{t+1}^*}{P_{t+1}}\frac{P_t^*}{P_{t+1}^*} \tag{12}$$

$$\frac{\partial u_{i,t}}{\partial c_{i,t}}\frac{\partial c_{i,t}}{\partial c_{i,t}^{H}}\frac{1}{\left(1+\tau_t^c\right)}\left\{1+\xi\left(\frac{k_{i,t}}{k_{i,t-1}}-1\right)\right\} = \beta E_t\frac{\partial u_{i,t+1}}{\partial c_{i,t+1}}\frac{\partial c_{i,t+1}}{\partial c_{i,t+1}^{H}}\frac{1}{\left(1+\tau_{t+1}^c\right)} \times\left\{\begin{array}{c}(1-\delta)-\frac{\xi}{2}\left(\frac{k_{i,t+1}}{k_{i,t}}-1\right)^2+\\ \xi\left(\frac{k_{i,t+1}}{k_{i,t}}-1\right)\frac{k_{i,t+1}}{k_{i,t}}+\left(1-\tau_{t+1}^k\right)r_{t+1}^k\end{array}\right\} \tag{13}$$

$$-\frac{\partial u_{i,t}}{\partial n_{i,t}} = \frac{\partial u_{i,t}}{\partial c_{i,t}}\frac{\partial c_{i,t}}{\partial c_{i,t}^{H}}\frac{P_t}{P_t^H}\frac{\left(1-\tau_t^n\right)w_t}{\left(1+\tau_t^c\right)} \tag{14}$$

$$\frac{c_{i,t}^H}{c_{i,t}^F} = \frac{\nu}{1-\nu}\frac{P_t^F}{P_t^H} \tag{15}$$

$$c_{i,t}^H(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi}c_{i,t}^H \tag{16}$$

$$c_{i,t}^F(f) = \left[\frac{P_t^F(f)}{P_t^F}\right]^{-\phi}c_{i,t}^F. \tag{17}$$

Equations (11)–(13) are, respectively, the Euler equations for domestic bonds, foreign assets, and domestic capital, and (14) is the optimality condition for work hours. Finally, (15) shows the optimal allocation between domestic and foreign goods, while (16) and (17) show the optimal demand for each variety of domestic and foreign goods, respectively; these conditions are used in the firm's optimization problem below.

Implications for Price Bundles

Equations (15), (16), and (17), combined with the household's budget constraints, imply that the three price indexes are

$$P_t = (P_t^H)^{\nu}(P_t^F)^{1-\nu} \tag{18}$$

$$P_t^H = \left[\sum_{h=1}^{N}[P_t^H(h)]^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{19}$$

$$P_t^F = \left[\sum_{f=1}^{N}[P_t^F(f)]^{1-\phi}\right]^{\frac{1}{1-\phi}}. \tag{20}$$

Appendix 2. Firms

This appendix presents and solves the problem of the firm in some detail. There are $h = 1, 2, \ldots, N$ domestic firms. Each firm h produces a differentiated good of variety h under monopolistic competition in its own product market facing Calvo-type nominal price fixities.

Demand for the Firm's Product

Demand for firm h's product, $y_t^H(h)$, comes from domestic households' consumption and investment, $C_t^H(h)$ and $X_t(h)$, where $C_t^H(h) \equiv \sum_{i=1}^{N} c_{i,t}^H(h)$ and $X_t(h) \equiv \sum_{i=1}^{N} x_{i,t}(h)$, from the domestic government, $G_t(h)$, and from foreign households' consumption, $C_t^{F*}(h) \equiv \sum_{i=1}^{N*} c_{i,t}^{F*}(h)$. Thus, the demand for each domestic firm's product is

$$y_t^H(h) = C_t^H(h) + X_t(h) + G_t(h) + C_t^{F*}(h). \tag{21}$$

Using demand functions like those derived by solving the household's problem above, we have

$$c_{i,t}^H(h) = \left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi} c_{i,t}^H \tag{22}$$

$$x_{i,t}\left(h\right)=\left[\frac{P_t^H\left(h\right)}{P_t^H}\right]^{-\phi}x_{i,t} \tag{23}$$

$$G_t\left(h\right)=\left[\frac{P_t^H\left(h\right)}{P_t^H}\right]^{-\phi}G_t \tag{24}$$

$$c_{i,t}^{F*}(h)=\left[\frac{P_t^{F*}(h)}{P_t^{F*}}\right]^{-\phi}c_{i,t}^{F*}, \tag{25}$$

where, using the law of one price, we have in (25)

$$\frac{P_t^{F*}(h)}{P_t^{F*}}=\frac{\frac{P_t^H(h)}{S_t}}{\frac{P_t^H}{S_t}}=\frac{P_t^H\left(h\right)}{P_t^H}. \tag{26}$$

Since, at the economy level, aggregate demand is

$$Y_t^H=C_t^H+X_t+G_t+C_t^{F*}, \tag{27}$$

the above equations imply that the demand for each firm's product is

$$y_t^H\left(h\right)=\left[\frac{P_t^H\left(h\right)}{P_t^H}\right]^{-\phi}Y_t^H. \tag{28}$$

The Firm's Problem

The real profit of each firm h is (see, e.g., Benigno and Thoenissen 2008 for similar modeling)

$$\widetilde{\omega}_t(h)=\frac{P_t^H(h)}{P_t}y_t^H(h)-\frac{P_t^H}{P_t}r_t^k k_{t-1}(h)-w_t n_t(h). \tag{29}$$

The production technology is

$$y_t^H(h)=A_t[k_{t-1}(h)]^{\alpha}[n_t(h)]^{1-\alpha}, \tag{30}$$

where A_t is an exogenous stochastic total factor productivity process whose motion is defined below.

Profit maximization is also subject to the demand for its product as derived above:

$$y_t^H(h)=\left[\frac{P_t^H(h)}{P_t^H}\right]^{-\phi}Y_t^H. \tag{31}$$

In addition, firms choose their prices facing a Calvo-type nominal fixity. In each period, firm h faces an exogenous probability θ of not being able to reset its price. A firm h, which is able to reset its price, chooses its price $P_t^{\#}(h)$ to maximize the sum of discounted expected nominal profits for the next k periods in which it may have to keep its price fixed. This is modeled next.

Firm's Optimality Conditions

To solve the firm's problem, we follow a two-step procedure. We first solve a cost-minimization problem, where each firm h minimizes its cost by choosing factor inputs given technology and prices. The solution will give a minimum nominal cost function, which is a function of factor prices and output produced by the firm. In turn, given this cost function, each firm, which is able to reset its price, solves a maximization problem by choosing its price.

The solution to the cost-minimization problem gives the input demand functions:

$$w_t = mc_t(h)(1-a)\frac{y_t(h)}{n_t(h)} \tag{32}$$

$$\frac{P_t^H}{P_t}r_t^k = mc_t(h)a\frac{y_t(h)}{k_{t-1}(h)}, \tag{33}$$

where $mc_t(h) = \frac{\Psi_t'(h)}{P_t}$ denotes the real marginal cost.

Then, the firm chooses its price to maximize nominal profits written as (see also, e.g., Rabanal 2009 for similar modeling)

$$E_t\sum_{k=0}^{\infty}\theta^k\Xi_{t,t+k}\left\{P_t^{\#}(h)\,y_{t+k}^H(h) - \Psi_{t+k}\left(y_{t+k}^H(h)\right)\right\},$$

where $\Xi_{t,t+k}$ is a discount factor taken as given by the firm, $y_{t+k}^H(h) = \left[\frac{P_t^{\#}(h)}{P_{t+k}^H}\right]^{-\phi} Y_{t+k}^H$ and $\Psi_t(h)$ denotes the minimum nominal cost function for producing $y_t^H(h)$ at t so that $\Psi_t'(h)$ is the associated marginal cost.

The first-order condition gives

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H} \right]^{-\phi} Y_{t+k}^H \left\{ P_t^{\#}(h) - \frac{\phi}{\phi - 1} \Psi'_{t+k}(h) \right\} = 0. \tag{34}$$

Dividing by P_t^H, we have

$$E_t \sum_{k=0}^{\infty} \theta^k \left[\Xi_{t,t+k} \left[\frac{P_t^{\#}(h)}{P_{t+k}^H} \right]^{-\phi} Y_{t+k}^H \left\{ \frac{P_t^{\#}(h)}{P_t^H} - \frac{\phi}{\phi - 1} mc_{t+k}(h) \frac{P_{t+k}}{P_t^H} \right\} \right] = 0. \tag{35}$$

Summing up, the behavior of each firm h is summarized by (32), (33), and (35). A recursive expression of the equation above is presented below.

Notice, finally, that each firm h, which can reset its price in period t, solves an identical problem, so $P_t^{\#}(h) = P_t^{\#}$ is independent of h, and each firm h, which cannot reset its price, just sets its previous period price $P_t^H(h) = P_{t-1}^H(h)$. Thus, the evolution of the aggregate price level is given by

$$\left(P_t^H\right)^{1-\phi} = \theta \left(P_{t-1}^H\right)^{1-\phi} + (1-\theta) \left(P_t^{\#}\right)^{1-\phi}. \tag{36}$$

Appendix 3. Government Budget Constraint

This appendix presents in some detail the government budget constraint and the menu of fiscal policy instruments. Recall that we have assumed a cashless economy for simplicity.

Then, the period budget constraint of the government written in aggregate and nominal quantities is

$$\begin{aligned} B_t + S_t F_t^g = {} & P_t \frac{\phi^g}{2} \left(\frac{S_t F_t^g}{P_t} - \frac{S F^g}{P} \right)^2 + R_{t-1} B_{t-1} + Q_{t-1} S_t F_{t-1}^g \\ & + P_t^H G_t - \tau_t^c (P_t^H C_t^H + P_t^F C_t^F) \\ & - \tau_t^k (r_t^k P_t^H K_{t-1} + P_t \widetilde{\Omega}_t) - \tau_t^n W_t \widetilde{N}_t - T_t^l, \end{aligned} \tag{37}$$

where B_t is the end-of-period nominal domestic public debt and F_t^g is the end-of-period nominal foreign public debt expressed in foreign currency so it is multiplied by the exchange rate, S_t. The parameter $\phi^g \geq 0$ captures adjustment costs related to public foreign debt. The rest of the notation is as above. Note that $B_t = \sum_{i=1}^{N} B_{i,t}$, $C_t^H = \sum_{i=1}^{N} c_{i,t}^H$, $C_t^F = \sum_{i=1}^{N} c_{i,t}^F$, $K_{t-1} = \sum_{i=1}^{N} k_{i,t-1}$, $\widetilde{\Omega}_t \equiv \sum_{i=1}^{N} \widetilde{\omega}_{i,t}$, $\widetilde{N}_t \equiv \sum_{i=1}^{N} n_{i,t}$, and T_t^l denotes the nomimal value of lump-sum taxes.

Let $D_t = B_t + S_t F_t^g$ denote the total nominal public debt at the end of the period. This can be held both by domestic private agents, $\lambda_t D_t$, where in equilibrium $B_t \equiv \lambda_t D_t$, and by foreign private agents, $S_t F_t^g \equiv (1 - \lambda_t) D_t$, where $0 \leq \lambda_t \leq 1$. Then, dividing by the price level and the number of households, the budget constraint in real and per capita terms is

$$\begin{aligned} d_t = {} & \frac{\phi^g}{2}[(1-\lambda_t) d_t - (1-\lambda) d]^2 + R_{t-1}\frac{P_{t-1}}{P_t}\lambda_{t-1} d_{t-1} \\ & + Q_{t-1}\frac{S_t P_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\frac{P_{t-1}}{P_{t-1}^* S_{t-1}}(1-\lambda_{t-1}) d_{t-1} \\ & + \frac{P_t^H}{P_t} g_t - \tau_t^c\left(\frac{P_t^H}{P_t} c_t^H + \frac{P_t^F}{P_t} c_t^F\right) \\ & - \tau_t^k\left(r_t^k \frac{P_t^H}{P_t} k_{t-1} + \widetilde{\omega}_t\right) - \tau_t^n w_t n_t - \tau_t^l, \end{aligned} \tag{38}$$

where, as said in the main text, in each period, one of the policy instruments (τ_t^c, τ_t^k, τ_t^n, g_t, τ_t^l, λ_t, d_t) follows residually to satisfy the government budget constraint.

Appendix 4. Decentralized Equilibrium (DE)

This appendix presents in some detail the DE system. Following the related literature, we work in steps.

Equilibrium Equations

The DE is summarized by the following equations (quantities are in per capita terms):

$$\frac{\partial u_t}{\partial c_t}\frac{\partial c_t}{\partial c_t^H}\frac{1}{(1+\tau_t^c)}\frac{P_t}{P_t^H} = \beta E_t \frac{\partial u_{t+1}}{\partial c_{t+1}}\frac{\partial c_{t+1}}{\partial c_{t+1}^H}\frac{1}{\left(1+\tau_{t+1}^c\right)}\frac{P_{t+1}}{P_{t+1}^H}R_t\frac{P_t}{P_{t+1}} \tag{39}$$

$$\frac{\partial u_t}{\partial c_t}\frac{\partial c_t}{\partial c_t^H}\frac{1}{(1+\tau_t^c)}\frac{P_t}{P_t^H}\frac{S_t P_t^*}{P_t}\left(1+\phi^p\left(\frac{S_t P_t^*}{P_t}f_t^h - \frac{SP^*}{P}f^h\right)\right)$$
$$= \beta\frac{\partial u_{t+1}}{\partial c_{t+1}}\frac{\partial c_{t+1}}{\partial c_{t+1}^H}\frac{1}{\left(1+\tau_{t+1}^c\right)}\frac{P_{t+1}}{P_{t+1}^H}Q_t\frac{S_{t+1}P_{t+1}^*}{P_{t+1}}\frac{P_t^*}{P_{t+1}^*} \tag{40}$$

$$\frac{\partial u_t}{\partial c_t}\frac{\partial c_t}{\partial c_t^H}\frac{1}{(1+\tau_t^c)}\left\{1+\xi\left(\frac{k_t}{k_{t-1}}-1\right)\right\}$$
$$= \beta E_t\frac{\partial u_{t+1}}{\partial c_{t+1}}\frac{\partial c_{t+1}}{\partial c_{t+1}^H}\frac{1}{\left(1+\tau_{t+1}^c\right)}$$
$$\times\left\{\begin{array}{c}(1-\delta)-\frac{\xi}{2}\left(\frac{k_{t+1}}{k_t}-1\right)^2 \\ +\xi\left(\frac{k_{t+1}}{k_t}-1\right)\frac{k_{t+1}}{k_t}+\left(1-\tau_{t+1}^k\right)r_{t+1}^k\end{array}\right\} \tag{41}$$

$$-\frac{\partial u_t}{\partial n_t} = \frac{\partial u_t}{\partial c_t}\frac{\partial c_t}{\partial c_t^H}\frac{P_t}{P_t^H}\frac{(1-\tau_t^n)\,w_t}{(1+\tau_t^c)} \tag{42}$$

$$\frac{c_t^H}{c_t^F} = \frac{\nu}{1-\nu}\frac{P_t^F}{P_t^H} \tag{43}$$

$$k_t = (1-\delta)k_{t-1} + x_t - \frac{\xi}{2}\left(\frac{k_t}{k_{t-1}}-1\right)^2 k_{t-1} \tag{44}$$

$$c_t \equiv \frac{\left(c_t^H\right)^\nu\left(c_t^F\right)^{1-\nu}}{\nu^\nu\left(1-\nu\right)^{1-\nu}} \tag{45}$$

$$w_t = mc_t(1-a)A_t k_{t-1}^a n_t^{-a} \tag{46}$$

$$\frac{P_t^H}{P_t}r_t^k = mc_t a A_t k_{t-1}^{a-1} n_t^{1-a} \tag{47}$$

$$\widetilde{\omega}_t = \frac{P_t^H}{P_t}y_t^H - \frac{P_t^H}{P_t}r_t^k k_{t-1} - w_t n_t \tag{48}$$

$$E_t\sum_{k=0}^{\infty}\theta^k\left[\Xi_{t,t+k}\left[\frac{P_t^\#}{P_{t+k}^H}\right]^{-\phi}y_{t+k}^H\left\{\frac{P_t^\#}{P_t}-\frac{\phi}{\phi-1}mc_{t+k}\frac{P_{t+k}}{P_t}\right\}\right]=0 \tag{49}$$

$$y_t^H = \frac{1}{\left[\frac{\widetilde{P}_t^H}{P_t^H}\right]^{-\phi}} A_t k_{t-1}^a n_t^{1-a} \tag{50}$$

$$\begin{aligned} d_t = {} & R_{t-1}\frac{P_{t-1}}{P_t}\lambda_{t-1}d_{t-1} + Q_{t-1}\frac{S_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*}\frac{P_{t-1}}{P_{t-1}^*S_{t-1}}\left(1-\lambda_{t-1}\right)d_{t-1} \\ & + \frac{P_t^H}{P_t}g_t - \tau_t^c\left(\frac{P_t^H}{P_t}c_t^H + \frac{P_t^F}{P_t}c_t^F\right) - \tau_t^k\left(r_t^k\frac{P_t^H}{P_t}k_{t-1} + \widetilde{\omega}_t\right) \\ & - \tau_t^n w_t n_t - \tau_t^l + \frac{\phi^g}{2}\left[\left(1-\lambda_t\right)d_t - \left(1-\lambda\right)d\right]^2 \end{aligned} \tag{51}$$

$$y_t^H = c_t^H + x_t + g_t + c_t^{F*} \tag{52}$$

$$\begin{aligned} & - \frac{P_t^H}{P_t}c_t^{F*} + \frac{P_t^F}{P_t}c_t^F + \frac{\phi^g}{2}\left[\left(1-\lambda_t\right)d_t - \left(1-\lambda\right)d\right]^2 \\ & + \frac{\phi^p}{2}\left(\frac{S_tP_t^*}{P_t}f_t^h - \frac{SP^*}{P}f^h\right)^2 Q_{t-1}\frac{S_tP_t^*}{P_t}\frac{P_{t-1}^*}{P_t^*} \\ & \times\left(\frac{\left(1-\lambda_{t-1}\right)d_{t-1}}{\frac{S_{t-1}P_{t-1}^*}{P_{t-1}}} - f_{t-1}^h\right) = \left(1-\lambda_t\right)d_t - \frac{S_tP_t^*}{P_t}f_t^h \end{aligned} \tag{53}$$

$$\left(P_t^H\right)^{1-\phi} = \theta P_{t-1}^{1-\phi} + \left(1-\theta\right)\left(P_t^{\#}\right)^{1-\phi} \tag{54}$$

$$P_t = \left(P_t^H\right)^{\nu}\left(P_t^F\right)^{1-\nu} \tag{55}$$

$$P_t^F = S_tP_t^{H*} \tag{56}$$

$$P_t^* = \left(P_t^{H*}\right)^{\nu^*}\left(\frac{P_t^H}{S_t}\right)^{1-\nu^*} \tag{57}$$

$$\left(\widetilde{P}_t^H\right)^{-\phi} = \theta\left(\widetilde{P}_{t-1}^H\right)^{-\phi} + \left(1-\theta\right)\left(P_t^{\#}\right)^{-\phi} \tag{58}$$

$$Q_t = Q_t^* + \psi\left(e^{\frac{d_t}{\frac{P_t^H}{P_t}Y_t^H} - \overline{d}} - 1\right) \tag{59}$$

$$l_t \equiv \frac{R_t\lambda_t d_t + Q_t\frac{S_{t+1}}{S_t}(1-\lambda_t)d_t}{\frac{P_t^H}{P_t}y_t^H}, \tag{60}$$

where $f_t^{*g} \equiv \frac{(1-\lambda_t)d_t}{\frac{S_t P_t^*}{P_t}}$, $\Xi_{t,t+k} \equiv \beta^k \frac{c_{t+k}^{-\sigma}}{c_t^{-\sigma}} \frac{P_t}{P_{t+k}} \frac{\tau_t^c}{\tau_{t+k}^c}$, $y_t^H = \left[\sum_{h=1}^N [y_t^H(h)]^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}$, and $\widetilde{P}_t^H \equiv \left(\sum_{h=1}^N [P_t(h)]^{-\phi}\right)^{-\frac{1}{\phi}}$. Thus, $\left(\frac{\widetilde{P}_t^H}{P_t^H}\right)^{-\phi}$ is a measure of price dispersion.

We thus have twenty-two equations in twenty-two variables, $\{y_t^H$, c_t, c_t^H, c_t^F, n_t, x_t, k_t, f_t^h, P_t^F, P_t, P_t^H, $P_t^{\#}$, $\widetilde{P}_t^H$, w_t, mc_t, $\widetilde{\omega}_t$, r_t^k, Q_t, d_t, P_t^*, R_t, $l_t\}_{t=0}^{\infty}$. This is given the independently set monetary and fiscal policy instruments, $\{S_t, \tau_t^c, \tau_t^k, \tau_t^n, g_t, \tau_t^l, \lambda_t\}_{t=0}^{\infty}$; the rest-of-the-world variables, $\{c_t^{F*}, Q_t^*, P_t^{H*}\}_{t=0}^{\infty}$; technology, $\{A_t\}_{t=0}^{\infty}$; and initial conditions for the state variables.

In what follows, we shall transform the above equilibrium conditions. In particular, following the related literature, we rewrite them—first, by expressing price levels in inflation rates, secondly, by writing the firm's optimality conditions in recursive form, and, thirdly, by introducing a new equation that helps us to compute expected discounted lifetime utility. Finally, we will present the final transformed system that is solved numerically.

Variables Expressed in Ratios

We first express prices in rate form. We define seven new variables, which are the gross domestic CPI inflation rate, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$; the gross foreign CPI inflation rate, $\Pi_t^* \equiv \frac{P_t^*}{P_{t-1}^*}$; the gross domestic goods inflation rate, $\Pi_t^H \equiv \frac{P_t^H}{P_{t-1}^H}$; the auxiliary variable, $\Theta_t \equiv \frac{P_t^{\#}}{P_t^H}$; the price dispersion index, $\Delta_t \equiv \left[\frac{\widetilde{P}_t^H}{P_t^H}\right]^{-\phi}$; the gross rate of exchange rate depreciation, $\epsilon_t \equiv \frac{S_t}{S_{t-1}}$; and the terms of trade, $TT_t \equiv \frac{P_t^F}{P_t^H} = \frac{S_t P_t^{*H}}{P_t^H}$.[1] In what follows, we use $\Pi_t, \Pi_t^*, \Pi_t^H, \Theta_t, \Delta_t, \epsilon_t, TT_t$ instead of $P_t, P_t^*, P_t^H, P_t^{\#}, \widetilde{P}_t, S_t, P_t^F$, respectively.

Also, for convenience and comparison with the data, we express fiscal and public finance variables as shares of nominal output,

[1]Thus, $\frac{TT_t}{TT_{t-1}} = \frac{\frac{S_t}{S_{t-1}} \frac{P_t^{*H}}{P_{t-1}^{*H}}}{\frac{P_t^H}{P_{t-1}^H}} = \frac{\epsilon_t \Pi_t^{*H}}{\Pi_t^H}$.

$P_t^H y_t^H$. In particular, using the definitions above, real government spending, g_t, can be written as $g_t = s_t^g y_t^H$, while real government transfers, τ_t^l, can be written as $\tau_t^l = s_t^l y_t^H TT_t^{\nu-1}$, where s_t^g and s_t^l denote, respectively, the output shares of government spending and government transfers.

Equation (49) Expressed in Recursive Form

We now replace equation (49), from the firm's optimization problem, with an equivalent equation in recursive form. In particular, following Schmitt-Grohé and Uribe (2007), we look for a recursive representation of the form

$$E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[\frac{P_t^{\#}}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H \left\{ P_t^{\#} - \frac{\phi}{(\phi-1)} mc_{t+k} P_{t+k} \right\} = 0. \tag{61}$$

We define two auxiliary endogenous variables:

$$z_t^1 \equiv E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[\frac{P_t^{\#}}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H \frac{P_t^{\#}}{P_t} \tag{62}$$

$$z_t^2 \equiv E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} \left[\frac{P_t^{\#}}{P_{t+k}^H} \right]^{-\phi} y_{t+k}^H mc_{t+k} \frac{P_{t+k}}{P_t}. \tag{63}$$

Using these two auxiliary variables, z_t^1 and z_t^2, as well as equation (61), we come up with two new equations that enter the dynamic system and allow a recursive representation of (61).

Thus, in what follows, we replace equation (49) with

$$z_t^1 = \frac{\phi}{(\phi-1)} z_t^2, \tag{64}$$

where we also have the two new equations (and the two new endogenous variables, z_t^1 and z_t^2)

$$z_t^1 = \Theta_t^{1-\phi} y_t TT_t^{\nu-1} + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}} \right)^{1-\phi} \left(\frac{1}{\Pi_{t+1}^H} \right)^{1-\phi} z_{t+1}^1 \tag{65}$$

$$z_t^2 = \Theta_t^{-\phi} y_t mc_t + \beta\theta E_t \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{-\phi} \left(\frac{1}{\Pi_{t+1}^H}\right)^{-\phi} z_{t+1}^2. \tag{66}$$

Lifetime Utility Written as a First-Order Difference Equation

Since we want to compute social welfare, we follow Schmitt-Grohé and Uribe (2007) by defining a new endogenous variable, V_t, whose motion is given by

$$V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_t^{1+\eta}}{1+\eta} + \chi_g \frac{\left(s_t^g y_t^H\right)^{1-\zeta}}{1-\zeta} + \beta E_t V_{t+1}, \tag{67}$$

where V_t is household's expected discounted lifetime utility at time t.

Thus, in what follows, we add equation (67) and the new variable V_t to the equilibrium system. Note that, in the welfare numerical solutions reported, we add a constant number, 2, to each period's utility; this makes the welfare numbers easier to read.

Equilibrium Equations Transformed

Using the above, the transformed DE system is summarized by

$$V_t = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_n \frac{n_t^{1+\eta}}{1+\eta} + \chi_g \frac{\left(s_t^g y_t^H\right)^{1-\zeta}}{1-\zeta} + \beta E_t V_{t+1} \tag{68}$$

$$\beta E_t \; c_{t+1}^{-\sigma} \frac{1}{(1+\tau_{t+1}^c)} R_t \frac{1}{\Pi_{t+1}} = c_t^{-\sigma} \frac{1}{(1+\tau_t^c)} \tag{69}$$

$$\begin{aligned} &\beta E_t c_{t+1}^{-\sigma} \frac{1}{(1+\tau_{t+1}^c)} Q_t TT_{t+1}^{v^*+\nu-1} \frac{1}{\Pi_{t+1}^*} \\ &\quad = c_t^{-\sigma} \frac{1}{(1+\tau_t^c)} TT_t^{v^*+\nu-1} \left[1 + \phi^p \left(TT_t^{\nu^*+\nu-1} f_t^h - TT^{\nu^*+\nu-1} f^h\right)\right] \end{aligned} \tag{70}$$

$$\begin{aligned} &\beta c_{t+1}^{-\sigma} TT_{t+1}^{\nu-1} \frac{1}{(1+\tau_{t+1}^c)} \left\{1 - \delta - \frac{\xi}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 \right. \\ &\quad \left. + \xi \left(\frac{k_{t+1}}{k_t} - 1\right) \frac{k_{t+1}}{k_t} + \left(1 - \tau_{t+1}^k\right) r_{t+1}^k \right\} \\ &\quad = c_t^{-\sigma} TT_t^{\nu-1} \frac{1}{(1+\tau_t^c)} \left[1 + \xi\left(\frac{k_t}{k_{t-1}} - 1\right)\right] \end{aligned} \tag{71}$$

$$\chi_n n_t^{\eta} = c_t^{-\sigma} \frac{(1-\tau_t^n)\, w_t}{(1+\tau_t^c)} \tag{72}$$

$$\frac{c_t^H}{c_t^F} = \frac{\nu}{1-\nu} TT_t \tag{73}$$

$$k_t = (1-\delta)k_{t-1} + x_t - \frac{\xi}{2}\left(\frac{k_t}{k_{t-1}} - 1\right)^2 k_{t-1} \tag{74}$$

$$c_t \equiv \frac{\left(c_t^H\right)^{\nu}\left(c_t^F\right)^{1-\nu}}{(\nu)^{\nu}(1-\nu)^{1-\nu}} \tag{75}$$

$$w_t = mc_t(1-a)A_t k_{t-1}^a n_t^{-a} \tag{76}$$

$$\frac{1}{TT_t^{1-v}} r_t^k = mc_t a A_t k_{t-1}^{a-1} n_t^{1-a} \tag{77}$$

$$\widetilde{\omega}_t = \frac{1}{TT_t^{1-v}} y_t^H - \frac{1}{TT_t^{1-v}} r_t^k k_{t-1} - w_t n_t \tag{78}$$

$$z_t^1 = \frac{\phi}{(\phi-1)} z_t^2 \tag{79}$$

$$y_t^H = \frac{1}{\Delta_t} A_t k_{t-1}^a n_t^{1-a} \tag{80}$$

$$\begin{aligned} d_t = {} & \frac{\phi^g}{2}[(1-\lambda_t)d_t - (1-\lambda)d]^2 + R_{t-1}\frac{1}{\Pi_t}\lambda_{t-1}d_{t-1} \\ & + Q_{t-1}TT_t^{v+v^*-1}\frac{1}{\Pi_t^*}\frac{1}{TT_{t-1}^{v+v^*-1}}(1-\lambda_{t-1})d_{t-1} + TT_t^{\nu-1}s_t^g y_t^H \\ & - \tau_t^c\left(\frac{1}{TT_t^{1-v}}c_t^H + TT_t^v c_t^F\right) - \tau_t^k\left(r_t^k\frac{1}{TT_t^{1-v}}k_{t-1} + \widetilde{\omega}_t\right) \\ & - \tau_t^n w_t n_t - TT_t^{\nu-1}s_t^l y_t^H \end{aligned} \tag{81}$$

$$y_t^H = c_t^H + x_t + s_t^g y_t^H + c_t^{F*} \tag{82}$$

$$\begin{aligned} (1-\lambda_t)d_t - TT_t^{\nu^*+\nu-1}f_t^h = {} & -TT_t^{\nu-1}c_t^{F*} + TT_t^{\nu}c_t^F \\ & + Q_{t-1}TT_t^{\nu^*+\nu-1}\frac{1}{\Pi_t^*}\left(\frac{1}{TT_{t-1}^{v+v^*-1}}(1-\lambda_{t-1})d_{t-1} - f_{t-1}^h\right) \end{aligned}$$

$$+\frac{\phi^p}{2}\left(TT_t^{\nu^*+\nu-1}f_t^h - TT^{\nu^*+\nu-1}f^h\right)^2$$
$$+\frac{\phi^g}{2}\left((1-\lambda_t)d_t-(1-\lambda)d\right)^2 \tag{83}$$
$$\left(\Pi_t^H\right)^{1-\phi}=\theta+(1-\theta)\left(\Theta_t\Pi_t^H\right)^{1-\phi} \tag{84}$$
$$\frac{\Pi_t}{\Pi_t^H}=\left(\frac{TT_t}{TT_{t-1}}\right)^{1-\nu} \tag{85}$$
$$\frac{TT_t}{TT_{t-1}}=\frac{\epsilon_t\Pi_t^{H*}}{\Pi_t^H} \tag{86}$$
$$\frac{\Pi_t^*}{\Pi_t^{H*}}=\left(\frac{TT_{t-1}}{TT_t}\right)^{1-\nu^*} \tag{87}$$
$$\Delta_t=\theta\Delta_{t-1}\left(\Pi_t^H\right)^{\phi}+(1-\theta)\left(\Theta_t\right)^{-\phi} \tag{88}$$
$$Q_t=Q_t^*+\psi\left(e^{\left(\frac{d_t}{TT_t^{\nu-1}y_t^H}-\bar{d}\right)}-1\right) \tag{89}$$
$$z_t^1=\Theta_t^{1-\phi}y_tTT_t^{\nu-1}+\beta\theta E_t\ \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{1-\phi}\left(\frac{1}{\Pi_{t+1}^H}\right)^{1-\phi}z_{t+1}^1 \tag{90}$$
$$z_t^2=\Theta_t^{-\phi}y_tmc_t+\beta\theta E_t\ \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}}\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\left(\frac{\Theta_t}{\Theta_{t+1}}\right)^{-\phi}\left(\frac{1}{\Pi_{t+1}^H}\right)^{-\phi}z_{t+1}^2 \tag{91}$$
$$l_t=\frac{R_ts_td_t+Q_t\epsilon_{t+1}\left(1-\lambda_t\right)d_t}{TT_t^{\nu-1}y_t^H}. \tag{92}$$

We thus have twenty-five equations in twenty-five variables, $\{V_t, y_t^H, c_t,\ c_t^H, c_t^F, n_t, x_t, k_t, f_t^h,\ TT_t, \Pi_t, \Pi_t^H, \Theta_t, \Delta_t, w_t, mc_t, \widetilde{\omega}_t, r_t^k, Q_t, d_t, \Pi_t^*, z_t^1, z_t^2, R_t,\ l_t\}_{t=0}^{\infty}$. This is given the independently set policy instruments, $\{\epsilon_t, \tau_t^c,\ \tau_t^k, \tau_t^n, s_t^g, s_t^l,\ \lambda_t\}_{t=0}^{\infty}$; the rest-of-the-world variables, $\{c_t^{F*}, Q_t^*, \Pi_t^{H*}\}_{t=0}^{\infty}$; technology, $\{A_t\}_{t=0}^{\infty}$; and initial conditions for the state variables.

Equations and Unknowns (Given Feedback Policy Coefficients)

We can now define the final equilibrium system. It consists of the twenty-five equations of the transformed DE presented in the previous subsection, the four policy rules in subsection 2.7 in the main text, and the equation for domestic exports in subsection 2.8 in the main text. Thus, we have a system of thirty equations. By using two auxilliary variables, we transform it to a first order,[2] so we end up with thirty-two equations in thirty-two variables, $\{y_t^H$, c_t, c_t^H, c_t^F, x_t, n_t, f_t^h, d_t, k_t, $\widetilde{\omega}_t$, mc_t, Π_t, Π_t^H, Π_t^*, Θ_t, Δ_t, TT_t, w_t, r_t^k, Q_t, l_t, z_t^1, z_t^2, V_t, R_t; τ_t^c, s_t^g, τ_t^k, τ_t^n; $klead_t$, $TTlag_t$, $c_t^{F*}\}_{t=0}^{\infty}$. This is given the exogenous variables, $\{Q_t^*, \Pi_t^{H*}, A_t, \epsilon_t,\ s_t^l,\ \lambda_t\}_{t=0}^{\infty}$, and initial conditions for the state variables. The thirty-two endogenous variables are distinguished in twenty-four control variables, $\{y_t^H, c_t, c_t^H, c_t^F, x_t, n_t, \widetilde{\omega}_t, mc_t, \Pi_t, \Pi_t^H, \Pi_t^*, \Theta_t, TT_t, w_t, r_t^k, z_t^1, z_t^2, V_t, klead_t, c_t^{F*}, \tau_t^c,\ s_t^g,\ \tau_t^k,\ \tau_t^n\}_{t=0}^{\infty}$, and eight state variables, $\{f_{t-1}^h, d_{t-1}, k_{t-1}, \Delta_{t-1}, Q_{t-1}, R_{t-1}, l_{t-1}, TTlag_t\}_{t=0}^{\infty}$. All this is given the values of feedback policy coefficients in the policy rules, which are chosen optimally in the computational part of the paper.

Status Quo Steady State

In the steady state of the above model economy, if $\epsilon_t = 1$, our equations imply $\Pi_t^H = \Pi_t^{H*} = \Pi^* = \Pi$. If we also set $\Pi^{H*} = 1$ (this is the exogenous component of foreign inflation), then $\Pi_t^H = \Pi_t^{H*} = \Pi^* = \Pi = 1$; this in turn implies $\Delta = \Theta = 1$. Also, in order to solve the model numerically, we need a value for the exogenous exports, c^{F*}; we assume $c^{F*} = 1.01c^F$, as it is the case in the data. Finally, since from the Euler for bonds, $1 = \frac{\beta R}{\Pi}$, we residually get $R = \frac{1}{\beta}$. (Note that these conditions will also hold in all steady-state solutions throughout the paper.)

The steady-state system is (in the labor supply condition, we use $\frac{\nu c}{c^H} TT^{1-v} = 1$)

$$1 = \beta[1 - \delta + \left(1 - \tau^k\right) r^k] \tag{93}$$

$$1 = \beta Q \tag{94}$$

$$\chi_n n^\eta = c^{-\sigma} \frac{\nu c}{c^H} TT^{1-v} \frac{(1 - \tau^n)}{(1 + \tau^c)} w \tag{95}$$

[2]In particular, when we use the Schmitt-Grohé and Uribe (2004) MATLAB routines, we add two auxiliary endogenous variables, *klead* and *TTlag*, to reduce the dynamic system into a first-order one.

$$\frac{c^H}{c^F} = \frac{\nu}{1-\nu} TT \tag{96}$$

$$x = \delta k \tag{97}$$

$$c = \frac{\left(c^H\right)^{\nu} \left(c^F\right)^{1-\nu}}{(\nu)^{\nu} (1-\nu)^{1-\nu}} \tag{98}$$

$$w = mc(1-a)Ak^a n^{-a} \tag{99}$$

$$r^k = TT^{1-v} mcaAk^{a-1} n^{1-a} \tag{100}$$

$$\widetilde{\omega} = \frac{1}{TT^{1-v}} y^H - \frac{1}{TT^{1-v}} r^k k - wn \tag{101}$$

$$y^H = Ak^a n^{1-a} \tag{102}$$

$$\begin{aligned} d = Qd + TT^{\nu-1} s^g y^H - \tau^c \left(\frac{1}{TT^{1-v}} c^H + TT^v c^F \right) \\ - \tau^k \left(r^k \frac{1}{TT^{1-v}} k + \widetilde{\omega} \right) - \tau^n wn - TT^{\nu-1} s^l y^H \end{aligned} \tag{103}$$

$$y^H = c^H + x + s^g y^H + c^{F*} \tag{104}$$

$$\begin{aligned} (1-\lambda)d - TT^{\nu^*+\nu-1} f^h = -TT^{\nu-1} c^{F*} + TT^{\nu} c^F \\ + QTT^{\nu^*+\nu-1} \left(\frac{1}{TT^{v+v^*-1}} (1-\lambda)d - f^h \right) \end{aligned} \tag{105}$$

$$Q = Q^* + \psi \left(e^{\left(\frac{d}{TT^{\nu-1} y^H} - \overline{d} \right)} - 1 \right) \tag{106}$$

$$z^1 = \frac{\phi}{(\phi - 1)} z^2 \tag{107}$$

$$z^1 = TT^{\nu-1} y^H + \beta\theta z^1 \tag{108}$$

$$z^2 = y^H mc + \beta\theta z^2. \tag{109}$$

We thus have seventeen equations in seventeen variables, c, c^H, c^F, k, x, n, y^H_t, TT, $mc, \widetilde{\omega}, d, z^1, z^2$, Q, f^h, r^k, w. The numerical solution of this system is in table 2 in the main text.

Appendix 5. The Reformed Economy

This appendix presents the reformed economy as defined in subsection 4.1 in the main text. The equations describing the reformed economy are as in the previous appendix except that now we set $Q = Q^*$, so that the public debt ratio is determined by the no premium condition, $\frac{d}{TT^{\nu-1}y^H} \equiv \overline{d}$ or $d \equiv \overline{d}TT^{\nu-1}y^H$. Note also that since $Q = Q^*$, we set $\beta = \frac{1}{Q^*}$ in the parameterization stage.

In what follows, we present steady-state solutions of this economy. We will start with the case in which the country's net foreign debt position is unrestricted so that $\widetilde{f} \equiv \frac{(1-\lambda)TT^{1-\nu}d - TT^{\nu^*}f^h}{y^H}$ is endogenously determined. In turn, we will study the case in which we set the country's net foreign debt position equal to zero (meaning that the trade is balanced) so that $\widetilde{f} = 0$ in the new reformed steady state.

Steady State of the Reformed Economy with Unrestricted Foreign Debt

Now $Q = Q^*$ so that $d \equiv \overline{d}TT^{\nu-1}y^H$. The steady-state system is

$$1 = \beta[1 - \delta + (1 - \tau^k)r^k] \tag{110}$$

$$\chi_n n^\eta = c^{-\sigma} \frac{\nu c}{c^H} TT^{1-\nu} \frac{(1 - \tau_t^n)}{(1 + \tau_t^c)} w \tag{111}$$

$$\frac{c^H}{c^F} = \frac{\nu}{1 - \nu} TT \tag{112}$$

$$x = \delta k \tag{113}$$

$$c = \frac{\left(c^H\right)^\nu \left(c^F\right)^{1-\nu}}{(\nu)^\nu (1 - \nu)^{1-\nu}} \tag{114}$$

$$y^H = Ak^a n^{1-a} \tag{115}$$

$$y^H = c^H + x + s^g y^H + c^{F*} \tag{116}$$

$$r^k = TT^{1-v} mcaAk^{a-1} n^{1-a} \tag{117}$$

$$w = mc(1 - a)Ak^a n^{-a} \tag{118}$$

$$\widetilde{\omega} = \frac{1}{TT^{1-v}} y^H - \frac{1}{TT^{1-v}} r^k k - wn \tag{119}$$

$$z^1 = \frac{\phi}{(\phi - 1)} z^2 \tag{120}$$

Table A1. Steady-State Solution of the Reformed Economy with Unrestricted $\widetilde{f}$

Variables	Description	Steady-State Solution
u	Period Utility	0.9458
y^H	Output	0.858759
TT	Terms of Trade	0.838459
$Q - Q^*$	Interest Rate Premium	0
$TT^{1-\nu}\frac{c}{y^H}$	Consumption as Share of GDP	0.5522
$\frac{k}{y^H}$	Physical Capital as Share of GDP	4.2291
$\frac{TT^{\nu^*} f^h}{y^H}$	Private Foreign Assets as Share of GDP	−4.5742
$\frac{TT^{1-\nu} d}{y^H}$	Total Public Debt as Share of GDP	0.9
$\widetilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{\nu^*} f^h}{y^H}$	Total Foreign Debt as Share of GDP	4.8982

$$z^1 = yTT^{\nu-1} + \beta\theta z^1 \tag{121}$$

$$z^2 = ymc + \beta\theta z^2 \tag{122}$$

$$d \equiv \overline{d}TT^{\nu-1}y^H \tag{123}$$

$$d = Q^*d + TT^{\nu-1}s^g y^H - \tau^c\left(\frac{1}{TT^{1-v}}c^H + TT^v c^F\right) - \tau^k\left(\frac{1}{TT^{1-v}}r^k k + \widetilde{\omega}\right) - \tau^n wn - TT^{\nu-1}s^l y_t^H \tag{124}$$

$$(1-\lambda)d - TT^{\nu^*+\nu-1}f^h = -TT^{\nu-1}c^{F*} + TT^{\nu}c^F + QTT^{\nu^*+\nu-1}\left(\frac{1}{TT^{v+v^*-1}}(1-\lambda)d - f^h\right). \tag{125}$$

We thus have sixteen equations in c, c^H, c^F, k, x, n, y_t^H, TT, $mc, \widetilde{\omega}, d, z^1, z^2$, r^k, w, f^h. The numerical solution of this system is presented in table A1. Notice that, for the reasons explained in the text, private foreign debt, $-f^h > 0$, and hence the country's net foreign debt-to-GDP ratio, $\widetilde{f}$, are extremely high in this case.

Table A2. Steady-State Solution of the Reformed Economy with $\widetilde{f} = 0$ when the Residual Fiscal Instrument Is s^g

Variables	Description	Steady-State Solution
u	Period Utility	0.9125
r^k	Real Return to Physical Capital	0.0749
w	Real Wage Rate	1.2442
n	Hours Worked	0.3403
y^H	Output	0.8237
TT	Terms of Trade	1.01
s^g	Capital Tax Rate	0.2306
$Q - Q^*$	Interest Rate Premium	0
$TT^{1-\nu}\frac{c}{y^H}$	Consumption as Share of GDP	0.6002
$\frac{k}{y^H}$	Physical Capital as Share of GDP	4.2291
$TT^{\nu^*}\frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.3240
$\frac{TT^{1-\nu}d}{y^H}$	Total Public Debt as Share of GDP	0.9
$\widetilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu}-TT^{\nu^*}f^h}{y^H}$	Total Foreign Debt as Share of GDP	0

Steady State of the Reformed Economy with Restricted Foreign Debt

Now $Q = Q^*$, so that $d \equiv \overline{d}TT^{\nu-1}y^H$ as above, and, in addition, $\widetilde{f} \equiv \frac{(1-\lambda)TT^{1-\nu}d-TT^{\nu^*}f^h}{y^H} = 0$, so that $(1-\lambda)\,dTT^{1-\nu} - TT^{\nu^*}f^h = 0$ or $f^h \equiv \frac{(1-\lambda)dTT^{1-\nu}}{TT^{\nu^*}}$. In this case, the equation for the balance of payments (125) simplifies to

$$TTc^F = c^{F*}. \tag{126}$$

Therefore, using this equation in the place of (125), we have sixteen equations in c, c^H, c^F, k, x, n, y_t^H, TT, mc, $\widetilde{\omega}, d, z^1, z^2$, r^k, w, and one of the tax-spending policy instruments, s^g, τ^c, τ^k, τ^n. Note that in turn f^h follows residually from $f^h \equiv \frac{(1-\lambda)dTT^{1-\nu}}{TT^{\nu^*}}$. The numerical solution of this system under each public financing instrument

Table A3. Steady-State Solution of the Reformed Economy with $\widetilde{f} = 0$ when the Residual Fiscal Instrument Is τ^c

Variables	Description	Steady-State Solution
u	Period Utility	0.9227
r^k	Real Return to Physical Capital	0.0749
w	Real Wage Rate	1.2442
n	Hours Worked	0.3403
y^H	Output	0.8237
TT	Terms of Trade	1.01
τ^c	Capital Tax Rate	0.1593
$Q - Q^*$	Interest Rate Premium	0
$TT^{1-\nu} \frac{c}{y^H}$	Consumption as Share of GDP	0.6086
$\frac{k}{y^H}$	Physical Capital as Share of GDP	4.2291
$TT^{\nu^*} \frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.3240
$\frac{TT^{1-\nu} d}{y^H}$	Total Public Debt as Share of GDP	0.9
$\widetilde{f} \equiv \frac{(1-\lambda) d TT^{1-\nu} - TT^{\nu^*} f^h}{y^H}$	Total Foreign Debt as Share of GDP	0

is presented in tables A2–A5, while a summary of these solutions appears in table 3 in the main text.

Restricted Changes in Fiscal Policy Instruments

We now repeat the main computations restricting the magnitude of feedback coefficients in the policy rules so that all tax-spending policy instruments cannot change by more than 10 percentage points from their averages in the data (we report that the results also do not change when we allow for 5 percentage points only).

Restricted results for optimal feedback policy coefficients, the implied statistics, and welfare over various time horizons are reported in tables A6, A7, and A8, which correspond to tables 4, 5, and 6, respectively, in the main text.

Inspection of the new tables, and comparison to their counterparts in the main text, implies that the main qualitative results do

Table A4. Steady-State Solution of the Reformed Economy with $\widetilde{f} = 0$ when the Residual Fiscal Instrument Is τ^k

Variables	Description	Steady-State Solution
u	Period Utility	0.9311
r^k	Real Return to Physical Capital	0.0729
w	Real Wage Rate	1.2649
n	Hours Worked	0.3393
y^H	Output	0.8348
TT	Terms of Trade	1.01
τ^k	Capital Tax Rate	0.2930
$Q - Q^*$	Interest Rate Premium	0
$TT^{1-\nu} \frac{c}{y^H}$	Consumption as Share of GDP	0.6040
$\frac{k}{y^H}$	Physical Capital as Share of GDP	4.3448
$TT^{\nu^*} \frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.3240
$\frac{TT^{1-\nu} d}{y^H}$	Total Public Debt as Share of GDP	0.9
$\widetilde{f} \equiv \frac{(1-\lambda) d TT^{1-\nu} - TT^{\nu^*} f^h}{y^H}$	Total Foreign Debt as Share of GDP	0

not change. Namely, debt consolidation is again preferable to non-debt consolidation after the first ten years. Also, although obviously feedback policy coefficients are now smaller, the best fiscal policy mix again implies that we should earmark public spending for the reduction of public debt and, at the same time, cut taxes to mitigate the recessionary effects of debt consolidation. The only difference is that now, since cuts in income taxes are restricted, we should also cut consumption taxes.

Impulse Response Functions (IRFs) with One Fiscal Instrument at a Time

Figures A1 and A2 compare debt consolidation results when the fiscal authorities can use all instruments jointly to the case in which they are restricted to use one instrument at a time only. Results are always with optimized rules. To save on space, we focus on results for the public debt-to-GDP ratio and private consumption.

Table A5. Steady-State Solution of the Reformed Economy with $\widetilde{f} = 0$ when the Residual Fiscal Instrument Is τ^n

Variables	Description	Steady-State Solution
u	Period Utility	0.9290
r^k	Real Return to Physical Capital	0.0749
w	Real Wage Rate	1.2442
n	Hours Worked	0.3435
y^H	Output	0.8314
TT	Terms of Trade	1.01
τ^n	Capital Tax Rate	0.4018
$Q - Q^*$	Interest Rate Premium	0
$TT^{1-\nu} \frac{c}{y^H}$	Consumption as Share of GDP	0.6086
$\frac{k}{y^H}$	Physical Capital as Share of GDP	4.2291
$TT^{\nu^*} \frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.3240
$\frac{TT^{1-\nu} d}{y^H}$	Total Public Debt as Share of GDP	0.9
$\widetilde{f} \equiv \frac{(1-\lambda) d TT^{1-\nu} - TT^{\nu^*} f^h}{y^H}$	Total Foreign Debt as Share of GDP	0

Table A6. Optimal Reaction to Debt and Output with Debt Consolidation (restricted optimal fiscal policy mix)

Fiscal Instruments	Optimal Reaction to Debt	Optimal Reaction to Output
s_t^g	$\gamma_l^g = 0.4$	$\gamma_y^g = 0.0011$
τ_t^c	$\gamma_l^c = 0.0005$	$\gamma_y^c = 0.4865$
τ_t^k	$\gamma_l^k = 0.0026$	$\gamma_y^k = 0.9496$
τ_t^n	$\gamma_l^n = 0.0023$	$\gamma_y^n = 0.95$

Note: $V_0 = 79.9336$.

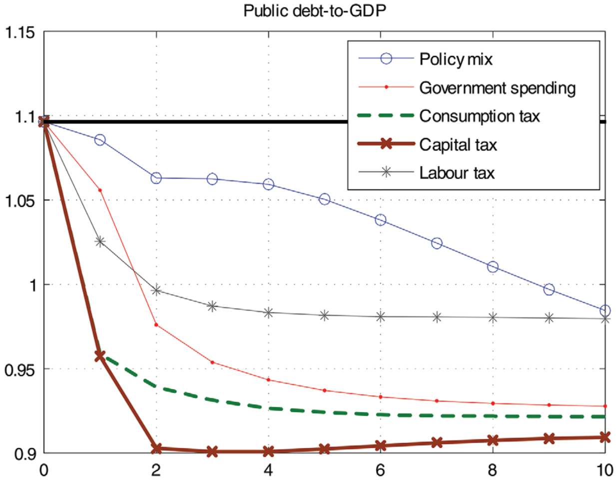
Table A7. Statistics Implied by Table A6

	Elasticity to Liabilities	Elasticity to Output	Min/Max	Five Periods Average	Ten Periods Average	Twenty Periods Average	Data Average
s_t^g	−2.016%	−0.0035%	0.1310/0.2273	0.1473	0.1576	0.1760	0.2222
τ_t^c	0.0032%	1.97%	0.1352/0.1799	0.1492	0.1545	0.1614	0.1756
τ_t^k	0.0093%	2.16%	0.2143/0.3014	0.2418	0.2520	0.2654	0.3118
τ_t^n	0.0062%	1.61%	0.3423/0.4295	0.3698	0.38	0.3934	0.421

Table A8. Welfare over Different Time Horizons with, and without, Debt Consolidation (restricted optimal fiscal policy mix)

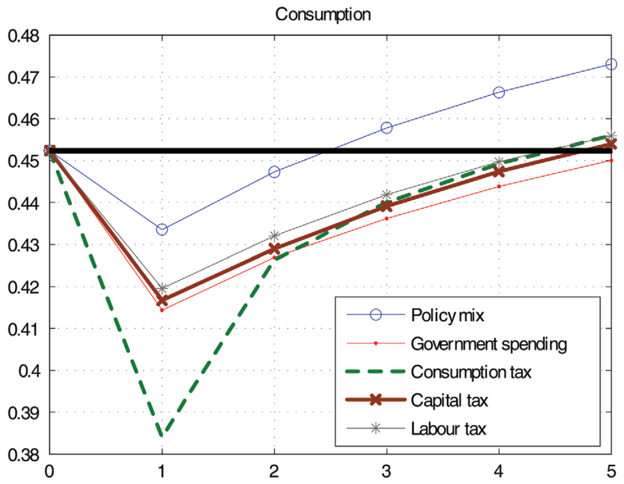
	Two Periods	Four Periods	Ten Periods	Twenty Periods	Thirty Periods
With Consolid.	1.4862	3.0164	7.7221	15.3294	22.2939
Without Consolid.	(1.6251)	(3.1791)	(7.4481)	(13.4127)	(18.1904)
Welfare Gain/Loss	−0.0458	−0.0327	0.0267	0.1075	0.1693

Figure A1. IRFs of Public Debt to GDP (comparison of alternative fiscal policies)



Note: IRFs are in levels and converge to the reformed steady state, while the solid horizontal line indicates the point of departure (status quo value).

Figure A2. IRFs of Consumption (comparison of alternative fiscal policies)



Note: IRFs are in levels and converge to the reformed steady state, while the solid horizontal line indicates the point of departure (status quo value).

Appendix 6. Robustness to the Public Debt Threshold Parameter

The Case with a Lower Public Debt Threshold

We now assume $\overline{d} \equiv 0.8$. This implies $\psi \equiv 0.0319$ to hit the data. Then, we have the solution shown in tables A9–A11.

Table A9. Status Quo Steady-State Solution

Variables	Description	Steady-State Solution	Data
u	Period Utility	0.8217	—
r^k	Real Return to Physical Capital	0.0908	—
w	Real Wage Rate	1.1140	—
n	Hours Worked	0.3313	0.2183
y^H	Output	0.7124	—
TT	Terms of Trade	0.9945	—
$Q - Q^*$	Interest Rate Premium	0	0.011
$TT^{1-\nu}\frac{c}{y^H}$	Consumption as Share of GDP	0.6334	0.5961
$\frac{k}{y^H}$	Physical Capital as Share of GDP	3.4872	—
$TT^{\nu^*}\frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.1749	0.1039
$\frac{TT^{1-\nu}d}{y^H}$	Total Public Debt as Share of GDP	1.0965	1.0828
$\widetilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{\nu^*}f^h}{y^H}$	Total Foreign Debt as Share of GDP	0.2194	0.2109

Table A10. Reformed Steady-State Solution

Variables	Description	Steady-State Solution
u	Period Utility	0.9326
r^k	Real Return to Physical Capital	0.0727
w	Real Wage Rate	1.2673
n	Hours Worked	0.3394
y^H	Output	0.8367
TT	Terms of Trade	1.01
τ^k	Capital Tax Rate	0.2908
$Q - Q^*$	Interest Rate Premium	0
$TT^{1-\nu}\frac{c}{y^H}$	Consumption as Share of GDP	0.6035
$\frac{k}{y^H}$	Physical Capital as Share of GDP	4.3583
$TT^{\nu^*}\frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.2880
$\frac{TT^{1-\nu}d}{y^H}$	Total Public Debt as Share of GDP	0.8
$\widetilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{\nu^*}f^h}{y^H}$	Total Foreign Debt as Share of GDP	0

Table A11. Optimal Reaction to Debt and Output with Debt Consolidation (optimal fiscal policy mix)

Fiscal Instruments	Optimal Reaction to Debt	Optimal Reaction to Output
s_t^g	$\gamma_l^g = 0.4338$	$\gamma_y^g = 0$
τ_t^c	$\gamma_l^c = 0$	$\gamma_y^c = 0.0279$
τ_t^k	$\gamma_l^k = 0$	$\gamma_y^k = 2.2569$
τ_t^n	$\gamma_l^n = 0.0001$	$\gamma_y^n = 2.136$

Note: $V_0 = 80.1038$.

The Case with a Higher Public Debt Threshold

Next, we assume $\overline{d} \equiv 1$. This implies $\psi \equiv 0.108$ to hit the data. Then, we have the solution shown in tables A12–A14.

Table A12. Status Quo Steady-State Solution

Variables	Description	Steady-State Solution	Data
u	Period Utility	0.8217	—
r^k	Real Return to Physical Capital	0.0908092	—
w	Real Wage Rate	1.11373	—
n	Hours Worked	0.331273	0.2183
y^H	Output	0.712311	—
TT	Terms of Trade	0.995009	—
$Q - Q^*$	Interest Rate Premium	0.011	0.011
$TT^{1-\nu}\frac{c}{y^H}$	Consumption as Share of GDP	0.6335	0.5961
$\frac{k}{y^H}$	Physical Capital as Share of GDP	3.4872	—
$TT^{\nu^*}\frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.1827	0.1039
$\frac{TT^{1-\nu}d}{y^H}$	Total Public Debt as Share of GDP	1.0965	1.0828
$\widetilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu}-TT^{\nu^*}f^h}{y^H}$	Total Foreign Debt as Share of GDP	0.2122	0.2109

Table A13. Reformed Steady-State Solution

Variables	Description	Steady-State Solution
u	Period Utility	0.9296
r^k	Real Return to Physical Capital	0.0731
w	Real Wage Rate	1.2625
n	Hours Worked	0.3391
y^H	Output	0.8328
TT	Terms of Trade	1.01
τ^k	Capital Tax Rate	0.2951
$Q - Q^*$	Interest Rate Premium	0
$TT^{1-\nu}\frac{c}{y^H}$	Consumption as Share of GDP	0.6045
$\frac{k}{y^H}$	Physical Capital as Share of GDP	4.3314
$TT^{\nu^*}\frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.3600
$\frac{TT^{1-\nu}d}{y^H}$	Total Public Debt as Share of GDP	1
$\widetilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu}-TT^{\nu^*}f^h}{y^H}$	Total Foreign Debt as Share of GDP	0

Table A14. Optimal Reaction to Debt and Output with Debt Consolidation (optimal fiscal policy mix)

Fiscal Instruments	Optimal Reaction to Debt	Optimal Reaction to Output
s_t^g	$\gamma_l^g = 0.4588$	$\gamma_y^g = 0.0017$
τ_t^c	$\gamma_l^c = 0$	$\gamma_y^c = 0.0018$
τ_t^k	$\gamma_l^k = 0.0014$	$\gamma_y^k = 2.2569$
τ_t^n	$\gamma_l^n = 0.1306$	$\gamma_y^n = 1.5629$

Note: $V_0 = 79.8482$.

Appendix 7. Robustness to the Net Foreign Debt Position in the Reformed Steady State

The Case with 0.1 Net Foreign Debt Ratio in the Reformed Steady State

When we set $\widetilde{f} \equiv \frac{(1-\lambda)TT^{1-\nu}d - TT^{\nu^*}f^h}{y^H} = 0.1$ in the reformed steady state, the results are as shown in tables A15 and A16.

Table A15. Steady-State Solution of the Reformed Economy with $\widetilde{f} = 0.1$

Variables	Description	Steady-State Solution
u	Period Utility	0.9315
y^H	Output	0.835228
TT	Terms of Trade	1.00615
$Q - Q^*$	Interest Rate Premium	0
$TT^{1-\nu}\frac{c}{y^H}$	Consumption as Share of GDP	0.6029
$\frac{k}{y^H}$	Physical Capital as Share of GDP	4.3425
$TT^{\nu^*}\frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.2240
$\frac{TT^{1-\nu}d}{y^H}$	Total Public Debt as Share of GDP	0.9
$\widetilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{\nu^*}f^h}{y^H}$	Total Foreign Debt as Share of GDP	0.1

Table A16. Optimal Reaction to Debt and Output with Debt Consolidation (optimal fiscal policy mix)

Fiscal Instruments	Optimal Reaction to Debt	Optimal Reaction to Output
s_t^g	$\gamma_l^g = 0.6725$	$\gamma_y^g = 0.0020$
τ_t^c	$\gamma_l^c = 0$	$\gamma_y^c = 0.7435$
τ_t^k	$\gamma_l^k = 0.0011$	$\gamma_y^k = 2.2572$
τ_t^n	$\gamma_l^n = 0.0015$	$\gamma_y^n = 2.14$

Note: $V_0 = 80.2538$.

The Case with 0.2109 Net Foreign Debt Ratio in the Reformed Steady State

When we set $\widetilde{f} \equiv \frac{(1-\lambda)TT^{1-\nu}d - TT^{\nu^*}f^h}{y^H} = 0.2109$ (as in the data) in the reformed steady state, the results are as shown in tables A17 and A18.

Table A17. Steady-State Solution of the Reformed Economy with $\widetilde{f} = 0.2109$

Variables	Description	Steady-State Solution
u	Period Utility	0.9320
y^H	Output	0.8358
TT	Terms of Trade	1.002
$Q - Q^*$	Interest Rate Premium	0
$TT^{1-\nu}\frac{c}{y^H}$	Consumption as Share of GDP	0.6017
$\frac{k}{y^H}$	Physical Capital as Share of GDP	4.3397
$TT^{\nu^*}\frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.1050
$\frac{TT^{1-\nu}d}{y^H}$	Total Public Debt as Share of GDP	0.9
$\widetilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{\nu^*}f^h}{y^H}$	Total Foreign Debt as Share of GDP	0.2109

Table A18. Optimal Reaction to Debt and Output with Debt Consolidation (optimal fiscal policy mix)

Fiscal Instruments	Optimal Reaction to Debt	Optimal Reaction to Output
s_t^g	$\gamma_l^g = 0.5558$	$\gamma_y^g = 0.0114$
τ_t^c	$\gamma_l^c = 0.0094$	$\gamma_y^c = 1.3091$
τ_t^k	$\gamma_l^k = 0.0159$	$\gamma_y^k = 1.3759$
τ_t^n	$\gamma_l^n = 0$	$\gamma_y^n = 2.14$
Note: $V_0 = 80.3931$.		

Appendix 8. Robustness to World Interest Rate Shocks

Using the new specification presented in the main text, the results are as shown in tables A19 and A20.

Table A19. Optimal Reaction to Debt and Output with Debt Consolidation (optimal fiscal policy mix)

Fiscal Instruments	Optimal Reaction to Debt	Optimal Reaction to Output
s_t^g	$\gamma_l^g = 0.2068$	$\gamma_y^g = 0$
τ_t^c	$\gamma_l^c = 0.0002$	$\gamma_y^c = 0.2097$
τ_t^k	$\gamma_l^k = 0$	$\gamma_y^k = 2.2569$
τ_t^n	$\gamma_l^n = 0$	$\gamma_y^n = 2.1360$

Note: $V_0 = 79.5457$.

Table A20. Welfare over Different Time Horizons with, and without, Debt Consolidation (optimal fiscal policy mix)

	Two Periods	**Four Periods**	**Ten Periods**	**Twenty Periods**	**Thirty Periods**
With Consolid.	1.4589	2.8359	7.0660	14.4129	21.9026
Without Consolid.[a]	(1.5236)	(2.9533)	(6.8521)	(12.6868)	(17.8657)
Welfare Gain/Loss	−0.0216	−0.0237	0.0208	0.0963	0.1662

[a]The values of the optimized feedback policy coefficients when we start from, and return to, the same status quo steady state are $\gamma_l^g = 0.0109, \gamma_l^c = 0.5206, \gamma_l^k = 0, \gamma_l^n = 0.3032, \gamma_y^g = 0.0129, \gamma_y^c = 0, \gamma_y^k = 2.2564$, and $\gamma_y^n = 0.0236$.

Appendix 9. Ramsey Policy Problem

The Ramsey government chooses the paths of policy instruments to maximize the household's expected discounted lifetime utility, V_t, subject to the equations of the DE. As shown in appendix 4 above, the DE system contains twenty-four equations (we do not include the equation for V_t).

As said in the main text, we solve for a "timeless" Ramsey equilibrium, meaning that the resulting equilibrium conditions are time invariant (see also, e.g., Schmitt-Grohé and Uribe 2005). If we follow the dual approach to the Ramsey policy problem, the government chooses the paths of $\{y_t^H, c_t, c_t^H, c_t^F, n_t, x_t, k_t, f_t^h, TT_t, \Pi_t, \Pi_t^H, \Theta_t, \Delta_t, w_t, mc_t, \widetilde{\omega}_t, r_t^k, Q_t, d_t, \Pi_t^*, z_t^1, z_t^2, R_t, l_t\}_{t=0}^{\infty}$, which are the twenty-four endogenous variables of the DE system, plus the paths of the three tax rates, $\{\tau_t^c, \tau_t^k, \tau_t^n\}_{t=0}^{\infty}$. Notice that public debt, $\{d_t\}_{t=0}^{\infty}$, is also among the choice variables. Also notice that, in solving this optimization problem, we take as given the rate of exchange rate depreciation, $\{\epsilon_t\}_{t=0}^{\infty}$; public spending on goods/services, $\{s_t^g\}_{t=0}^{\infty}$; lump-sum government transfers, $\{s_t^l\}_{t=0}^{\infty}$; and the share of public debt issued for the domestic market, $\{\lambda_t\}_{t=0}^{\infty}$. We treat these variables as exogenous for different reasons: in a currency union model, $\epsilon_t = 1$ all the time; we take $\{s_t^g\}_{t=0}^{\infty}$ as given for simplicity; we take $\{s_t^l\}_{t=0}^{\infty}$ as given because, if the Ramsey government can choose a lump-sum policy instrument, then its problem becomes trivial; finally, we set λ_t exogenously and equal to its data average value (as we have done so far anyway) because, if it is chosen by the Ramsey government, its optimality condition will be identical to the household's optimality condition for financial assets/debt in a Ramsey steady state without sovereign interest rate premia, so the Ramsey steady-state system will be indeterminate (in particular, in the steady state, the government's optimality condition for λ_t is reduced to $1 = \beta Q$, which is also the household's optimality condition for foreign assets/debt). For the very same reason (namely, the optimality conditions of the household and the government with respect to f_t^h are both reduced to $1 = \beta Q$ in the Ramsey steady state without sovereign interest rate premia), we set f_t^h so as to satisfy $\widetilde{f} = 0$ in the reformed steady state (as we have done so far anyway in the reformed steady state). Thus, from $\widetilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{\nu^*} f^h}{y^H} = 0$, it follows $f^h = (1-\lambda)\,\overline{d}TT^{\nu-1}y^H$.

Table A21. Ramsey Steady-State Solution (unrestricted)

Variables	Description	Steady-State Solution
u	Period Utility	1.2215
y^H	Output	1.56176
TT	Terms of Trade	1.01
τ^c	"Optimal" Consumption Tax Rate	26.5726
τ^k	"Optimal" Capital Tax Rate	0.00272056
τ^n	"Optimal" Consumption Tax Rate	−26.4976
$Q - Q^*$	Interest Rate Premium	0
$TT^{1-\nu}\frac{c}{y^H}$	Consumption as Share of GDP	0.5327
$\frac{k}{y^H}$	Physical Capital as Share of GDP	6.1284
$TT^{\nu^*}\frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.3240
$\frac{TT^{1-\nu}d}{y^H}$	Total Public Debt as Share of GDP	0.9
$\widetilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu}-TT^{\nu^*}f^h}{y^H}$	Total Foreign Debt as Share of GDP	0

Working in this way, and including Ramsey multipliers, we end up with a well-defined system consisting of fifty-one equations in fifty-one endogenous variables (the system is available upon request from the authors). The numerical solution of this system in the steady state is presented in table A21 (we do not report solutions for Ramsey multipliers). In this solution, as said before, $Q = Q^* = R = \frac{1}{\beta}$, and $f^h = (1-\lambda)\,\overline{d}TT^{\nu-1}y^H$. (Note that nominal variables, like R, do not affect the real allocation, and hence utility, at steady state.)

Notice, as also said in the text, and as is well established in the Ramsey literature with optimally chosen consumption taxes, that the solution in table A21 is characterized by an extremely large consumption tax rate, $\tau^c = 26.5726$, and an equally large labor

Table A22. Ramsey Steady-State Solution (restricted $\tau_t^n = 0$)

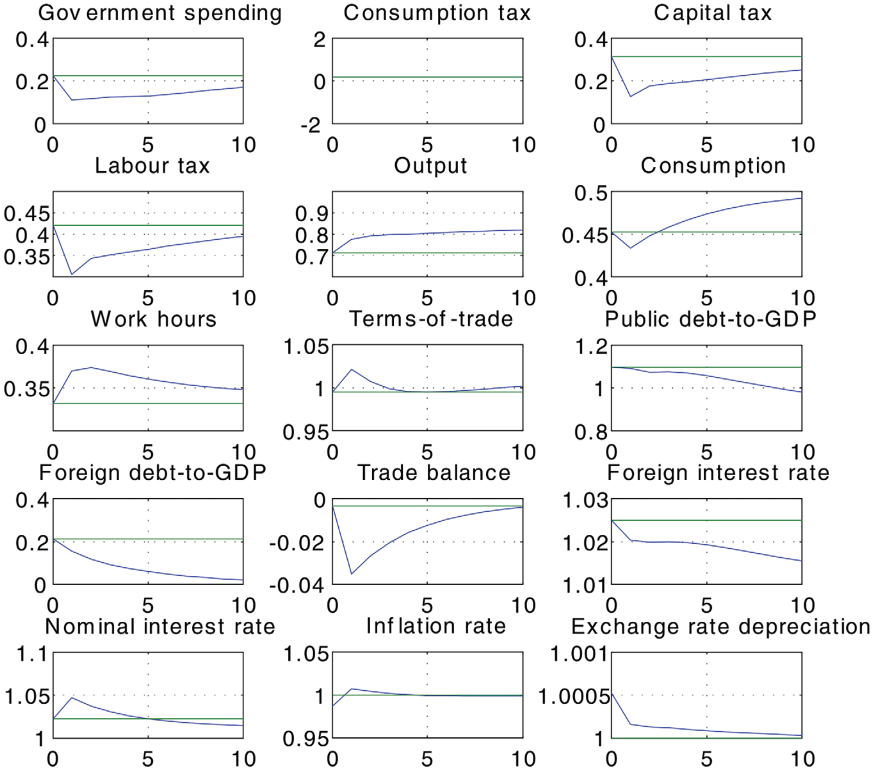
Variables	Description	Steady-State Solution
u	Period Utility	1.0950
y^H	Output	1.11622
TT	Terms of Trade	1.01
τ^k	"Optimal" Capital Tax Rate	0.0500465
τ^c	"Optimal" Consumption Tax Rate	0.810199
$Q - Q^*$	Interest Rate Premium	0
$TT^{1-\nu}\frac{c}{y^H}$	Consumption as Share of GDP	0.5443
$\frac{k}{y^H}$	Physical Capital as Share of GDP	5.8387
$TT^{\nu^*}\frac{f^h}{y^H}$	Private Foreign Assets as Share of GDP	0.3240
$\frac{TT^{1-\nu}d}{y^H}$	Total Public Debt as Share of GDP	0.9
$\widetilde{f} \equiv \frac{(1-\lambda)dTT^{1-\nu} - TT^{\nu^*}f^h}{y^H}$	Total Foreign Debt as Share of GDP	0

subsidy, $\tau^n = -26.4976 < 0$. Hence, following the same literature, we rule out a subsidy to labor by setting $\tau_t^n = 0$ at all t. This gives the solution in table A22 (this table is also presented in the main text, but we repeat it here for the reader's convenience).

Appendix 10. Flexible Exchange Rates

The IRFs under flexible exchange rates are shown in figure A3 (the zero lower bound for the nominal interest rate is not violated).

Figure A3. IRFs under Flexible Exchange Rates



Note: IRFs are in levels and converge to the reformed steady state, while the solid horizontal line indicates the point of departure (status quo value).

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