Online Appendix to Lower-Bound Beliefs and Long-Term Interest Rates

Christian Grisse,[a] Signe Krogstrup,[b] and Silvio Schumacher[a]
[a]Swiss National Bank
[b]International Monetary Fund

Proof of Proposition 1

The proof closely follows Ruge-Murcia (2006), with a slight modification to allow for a non-zero lower bound. Define $\delta_t \equiv \varepsilon_t/\sigma$ such that $\delta_t \sim N(0,1)$ and

$$r^*_{t+1} = \mathbb{E}(r^*_{t+1}) + \sigma\delta_{t+1},$$

where from (2)

$$\mathbb{E}(r^*_{t+1}) = \alpha + \psi r_t.$$

Then, $r_{t+1} = r^*_{t+1}$ if $r^*_{t+1} > \bar{r}$ or, equivalently, if

$$\delta_{t+1} > \frac{\bar{r} - \mathbb{E}(r^*_{t+1})}{\sigma} \equiv c_{t+1}.$$

Note that c_{t+1} is observed in t. Thus, we have

$$\mathbb{E}(r_{t+1}) = \mathbb{E}(r_{t+1} \mid \delta_{t+1} > c_{t+1})Pr(\delta_{t+1} > c_{t+1}) + \bar{r}Pr(\delta_{t+1} \leq c_{t+1}),$$

where

$$Pr(\delta_{t+1} > c_{t+1}) = 1 - \Phi(c_{t+1})$$

and

$$\begin{aligned}\mathbb{E}(r_{t+1} \mid \delta_{t+1} > c_{t+1}) &= \mathbb{E}(r^*_{t+1}) + \sigma\mathbb{E}(\delta_{t+1} \mid \delta_{t+1} > c_{t+1}) \\ &= (\bar{r} - \sigma c_{t+1}) + \frac{\sigma\phi(c_{t+1})}{1 - \Phi(c_{t+1})},\end{aligned}$$

where the second line uses a result in Maddala (1983, p. 366). Substituting back into the expression for $\mathbb{E}(r_{t+1})$ and simplifying, we get

$$\mathbb{E}(r_{t+1}) = \bar{r} - \sigma c_{t+1}(1 - \Phi(c_{t+1})) + \sigma\phi(c_{t+1}),$$

which is the expression in the text.

Table 1. Data Sources and Description

Variable	Source	Datastream Code	Units	Description
Monetary Policy Surprises	Bloomberg and National Central Banks	N/A	bps	Announced policy rate change, less the median from the Bloomberg survey. The survey is conducted among financial analysts, typically on the Friday before the central bank decision.
Interest Rate Swap Yields	Datastream	CCmmY03*	%	Yield on interest rate swap with a variable leg of three months, based on prices as of 5:00 p.m. CET London.
Central Bank Policy Rates				
Reserve Bank of Australia	Datastream	RBACASH	%	Official cash rate
Bank of Canada	Datastream	CN14309	%	Key interest rate (target overnight rate)
Czech National Bank	Datastream	PRREPOR	%	Two-week repo rate
Danmarks Nationalbank	Datastream	DKCTDEP	%	Certificates of deposit
Central Bank of Hungary	Datastream	HNBBASE	%	Base rate
Bank of Japan	Datastream	JPCALLT	%	Uncollateralized call rate
Reserve Bank of New Zealand	Datastream	NZRBCSH	%	Official cash rate
Norges Bank	Datastream	NWFOLIN	%	Sight deposit rate
National Bank of Poland	Datastream	POPRATE	%	Reference rate (seven-day bill rate)
Sveriges Riksbank	Datastream	SDREPOR	%	Repo
Swiss National Bank	Datastream	SWSNBTI	%	Three-month LIBOR target rate
Bank of England	Datastream	LCBBASE	%	Base rate
U.S. Federal Reserve	Datastream	USFDTRG	%	Federal funds target rate
European Central Bank	Datastream	EURODEP	%	Overnight deposit rate

Note: *CC = country code, mm = maturity.

Table 2. Panel Regression Results: Robustness Excluding Lower-Policy-Rate Country Observations

	Specification (16)				Specification (17)			
Maturities	**2**	**5**	**7**	**10**	**2**	**5**	**7**	**10**
Δr_t	0.00	−0.06	−0.09	−0.11	0.01	−0.06	−0.07	−0.09
	(0.05)	(0.08)	(0.08)	(0.09)	(0.05)	(0.08)	(0.08)	(0.09)
$r_{i,t-1}$	−1.46***	−1.03*	−0.93*	−0.77				
	(0.44)	(0.57)	(0.55)	(0.56)				
$\Delta r_t \times r_{i,t-1}$	−0.05***	−0.04*	−0.03*	−0.02				
	(0.01)	(0.02)	(0.02)	(0.02)				
$R_{i,t-1}$					−1.40***	−0.86	−0.81	−0.76
					(0.53)	(0.63)	(0.59)	(0.57)
$\Delta r_t \times R_{i,t-1}$					−0.05***	−0.03*	−0.03**	−0.02*
					(0.01)	(0.02)	(0.01)	(0.01)
Constant	−0.90	−2.52*	−3.04*	−3.41*	−0.72	−2.28	−2.59	−2.66
	(0.81)	(1.38)	(1.58)	(1.83)	(0.87)	(1.49)	(1.73)	(2.02)
No. Obs.	137	137	137	137	137	137	137	137
R^2	0.12	0.10	0.11	0.10	0.10	0.08	0.10	0.09

Notes: Observations for countries where the policy rate is already lower than the policy rate reached in the announcing country are excluded. Cluster-robust standard errors are in parentheses. Statistical significance is indicated by *** if $p < 0.01$, ** if $p < 0.05$, and * if $p < 0.1$. $\Delta R_{i,t}$ for the announcing country has been excluded.

Table 3. Panel Regression Results: Robustness Excluding Australia and New Zealand

	Specification (16)				Specification (17)			
Maturities	**2**	**5**	**7**	**10**	**2**	**5**	**7**	**10**
Δr_t	−0.00	−0.05	−0.07	−0.09	0.01	−0.02	−0.02	−0.02
	(0.05)	(0.07)	(0.08)	(0.09)	(0.05)	(0.07)	(0.07)	(0.08)
$r_{i,t-1}$	−1.21***	−1.90***	−2.17***	−2.30***				
	(0.44)	(0.56)	(0.55)	(0.59)				
$\Delta r_t \times r_{i,t-1}$	−0.06***	−0.07***	−0.08***	−0.08***				
	(0.02)	(0.03)	(0.03)	(0.03)				
$R_{i,t-1}$					−1.11**	−1.65***	−1.90***	−2.02***
					(0.49)	(0.55)	(0.49)	(0.51)
$\Delta r_t \times R_{i,t-1}$					−0.06***	−0.07***	−0.08**	−0.07***
					(0.02)	(0.02)	(0.02)	(0.02)
Constant	−1.45**	−2.33*	−2.62*	−2.82*	−1.26*	−1.59	−1.34	−0.87
	(0.73)	(1.20)	(1.41)	(1.66)	(0.76)	(1.14)	(1.35)	(1.62)
No. Obs.	131	131	131	131	131	131	131	131
R^2	0.06	0.10	0.13	0.13	0.04	0.08	0.11	0.10

Notes: Cluster-robust standard errors are in parentheses. Statistical significance is indicated by *** if $p < 0.01$, ** if $p < 0.05$, and * if $p < 0.1$. $\Delta R_{i,t}$ for the announcing country has been excluded.

Table 4. Panel Regression Results: Robustness Excluding Australia, New Zealand, Poland, and Hungary

	Specification (16)				Specification (17)			
Maturities	**2**	**5**	**7**	**10**	**2**	**5**	**7**	**10**
Δr_t	−0.00	−0.04	−0.06	−0.08	0.03*	0.01	0.01	0.01
	(0.04)	(0.06)	(0.07)	(0.08)	(0.02)	(0.03)	(0.02)	(0.03)
$r_{i,t-1}$	−1.37	−2.56***	−2.64***	−2.62**				
	(0.99)	(0.97)	(0.94)	(1.01)				
$\Delta r_t \times r_{i,t-1}$	−0.10*	−0.14**	−0.13**	−0.12*				
	(0.06)	(0.06)	(0.06)	(0.06)				
$R_{i,t-1}$					−1.12	−1.84	−1.92	−2.02*
					(1.22)	(1.25)	(1.23)	(1.21)
$\Delta r_t \times R_{i,t-1}$					−0.10	−0.10	−0.10	−0.09
					(0.08)	(0.08)	(0.08)	(0.08)
Constant	−1.38**	−2.23*	−2.57*	−2.79*	−1.11*	−1.33	−1.20	−0.75
	(0.67)	(1.16)	(1.38)	(1.63)	(0.57)	(0.85)	(0.97)	(1.10)
No. Obs.	107	107	107	107	107	107	107	107
R^2	0.05	0.08	0.08	0.08	0.04	0.07	0.07	0.07

Notes: Cluster-robust standard errors are in parentheses. Statistical significance is indicated by *** if $p < 0.01$, ** if $p < 0.05$, and * if $p < 0.1$. $\Delta R_{i,t}$ for the announcing country has been excluded.

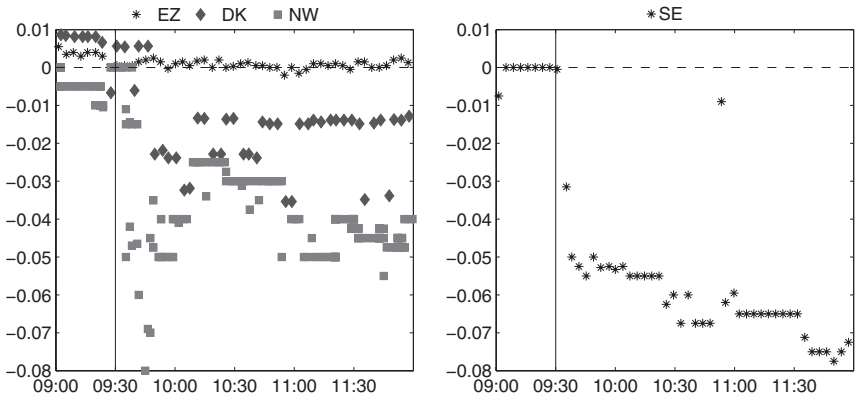
Table 5. Panel Regression Results: Robustness to Omitted-Variable Lower Bound

	Specification (16)				Specification (17)			
Maturities	**2**	**5**	**7**	**10**	**2**	**5**	**7**	**10**
Δr_t	0.02	−0.04	−0.07	−0.09	0.02	−0.03	−0.05	−0.06
	(0.04)	(0.07)	(0.08)	(0.09)	(0.04)	(0.07)	(0.08)	(0.08)
$\tilde{r}_{i,t-1}$	−1.18***	−1.07**	−1.04**	−0.94**				
	(0.38)	(0.45)	(0.42)	(0.42)				
$\Delta r_t \times \tilde{r}_{i,t-1}$	−0.05***	−0.04**	−0.04***	−0.03**				
	(0.01)	(0.01)	(0.01)	(0.01)				
$\tilde{R}_{i,t-1}$					−1.10**	−0.93*	−0.95**	−0.96**
					(0.44)	(0.52)	(0.48)	(0.46)
$\Delta r_t \times \tilde{R}_{i,t-1}$					−0.05***	−0.03**	−0.03***	−0.03***
					(0.01)	(0.01)	(0.01)	(0.01)
Constant	−0.77	−1.84	−2.25	−2.56	−0.68	−1.58	−1.68	−1.55
	(0.82)	(1.28)	(1.47)	(1.71)	(0.91)	(1.42)	(1.62)	(1.85)
No. Obs.	155	155	155	155	155	155	155	155
R^2	0.10	0.11	0.12	0.12	0.09	0.10	0.11	0.10

Notes: Cluster-robust standard errors are in parentheses. Statistical significance is indicated by *** if $p < 0.01$, ** if $p < 0.05$, and * if $p < 0.1$. $\Delta R_{i,t}$ for the announcing country has been excluded. $\tilde{r}_{i,t-1}$ denotes the difference between $r_{i,t-1}$ and the minimum of policy rates observed across the countries in the sample and up to t. A similar definition holds for $\tilde{R}_{i,t-1}$.

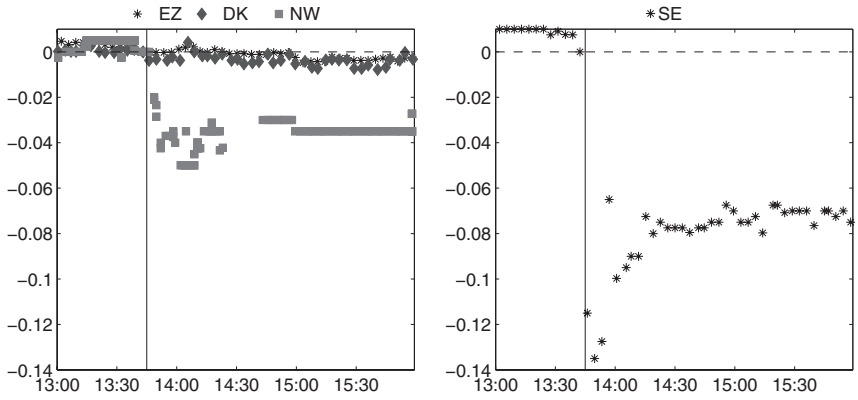
Appendix Figures

Figure 1. Response of Two-Year Yields on Interest Rate Swaps around the Riksbank's Policy Announcement of February 12, 2015



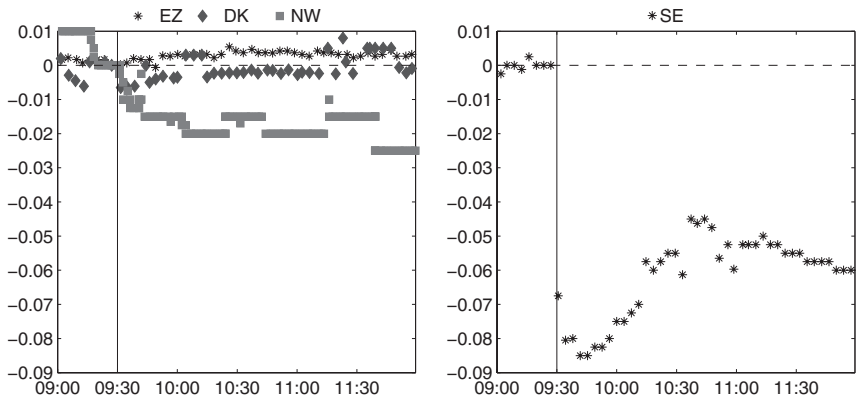
Notes: The cut was unexpected and included in our sample. Vertical lines mark the time of the announcement. Yields are normalized by adding a constant so that the average of the last two observations before the announcement is zero. Yields are based on transaction prices from Bloomberg.

Figure 2. Response of Two-Year Yields on Interest Rate Swaps around the Riksbank's Policy Announcement of March 18, 2015



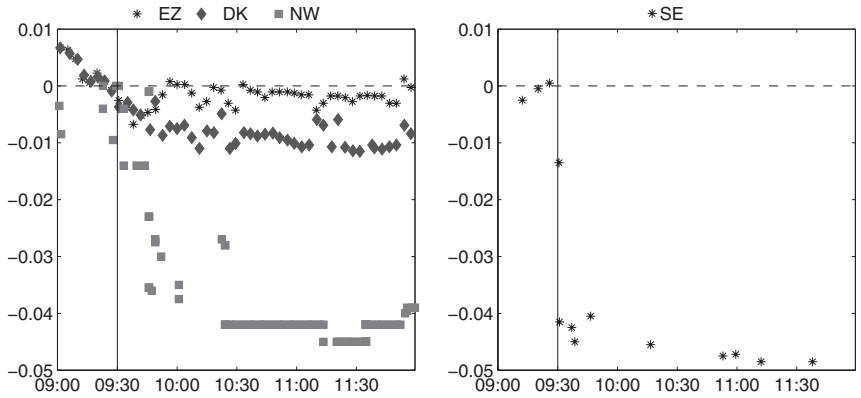
Notes: The cut was unexpected and included in our sample. Vertical lines mark the time of the announcement. Yields are normalized by adding a constant so that the average of the last two observations before the announcement is zero. Yields are based on transaction prices from Bloomberg.

Figure 3. Response of Two-Year Yields on Interest Rate Swaps around the Riksbank's Policy Announcement of July 2, 2015



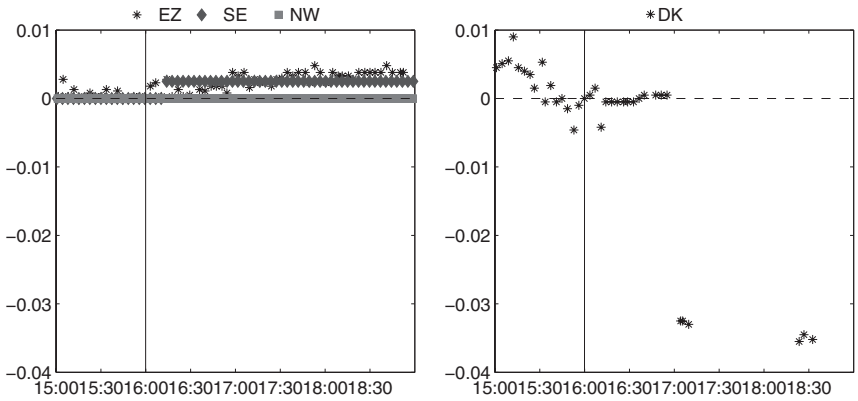
Notes: The cut was unexpected and included in our sample. Vertical lines mark the time of the announcement. Yields are normalized by adding a constant so that the average of the last two observations before the announcement is zero. Yields are based on transaction prices from Bloomberg.

Figure 4. Response of Two-Year Yields on Interest Rate Swaps around the Riksbank's Policy Announcement of February 11, 2016



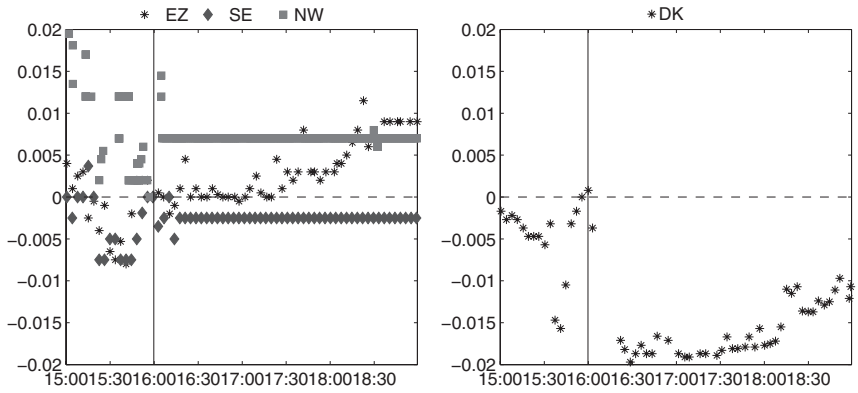
Notes: The cut was partly unexpected and the unexpected component is included in our sample. Vertical lines mark the time of the announcement. Yields are normalized by adding a constant so that the average of the last two observations before the announcement is zero. Yields are based on transaction prices from Bloomberg.

Figure 5. Response of Two-Year Yields on Interest Rate Swaps around the Danmarks Nationalbank's Policy Announcement of January 19, 2015



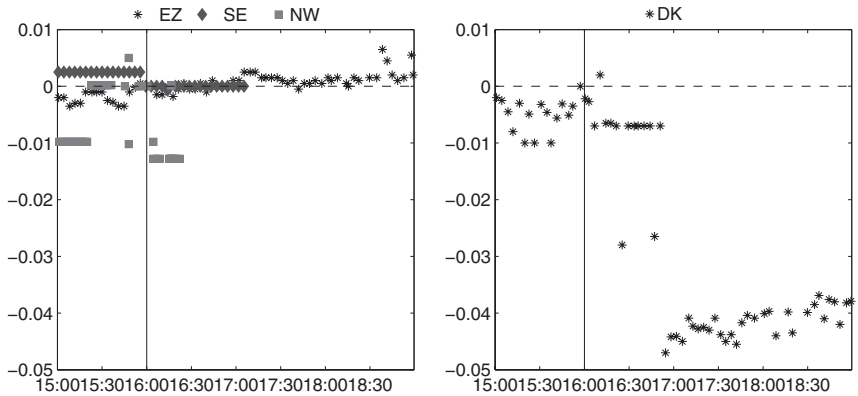
Notes: The cut was unexpected and included in our sample. Vertical lines mark the time of the announcement. Yields are normalized by adding a constant so that the average of the last two observations before the announcement is zero. Yields are based on transaction prices from Bloomberg.

Figure 6. Response of Two-Year Yields on Interest Rate Swaps around the Danmarks Nationalbank's Policy Announcement of January 22, 2015



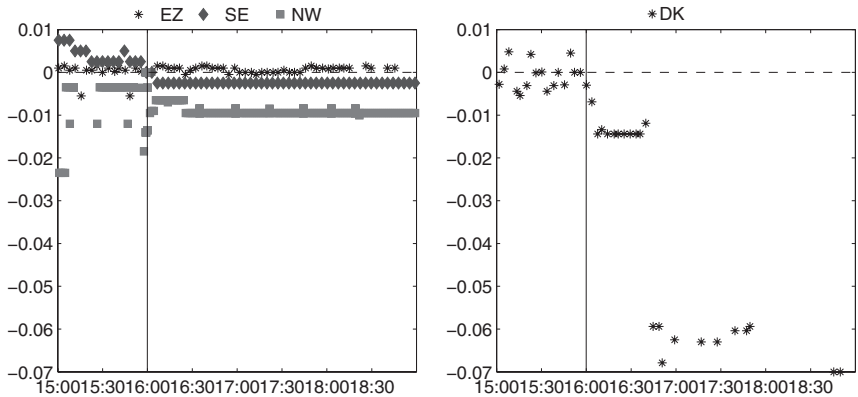
Notes: The cut was unexpected and included in our sample. Vertical lines mark the time of the announcement. Yields are normalized by adding a constant so that the average of the last two observations before the announcement is zero. Yields are based on transaction prices from Bloomberg.

Figure 7. Response of Two-Year Yields on Interest Rate Swaps around the Danmarks Nationalbank's Policy Announcement of January 29, 2015



Notes: The cut was unexpected and included in our sample. Vertical lines mark the time of the announcement. Yields are normalized by adding a constant so that the average of the last two observations before the announcement is zero. Yields are based on transaction prices from Bloomberg.

Figure 8. Response of Two-Year Yields on Interest Rate Swaps around the Danmarks Nationalbank's Policy Announcement of February 5, 2015



Notes: The cut was unexpected and included in our sample. Vertical lines mark the time of the announcement. Yields are normalized by adding a constant so that the average of the last two observations before the announcement is zero. Yields are based on transaction prices from Bloomberg.

References

Maddala, G. S. 1983. *Limited-Dependent and Qualitative Variables in Econometrics*. Econometric Society Monograph No. 3. Cambridge: Cambridge University Press.