Online Appendices to Aggregate Dynamics after a Shock to Monetary Policy in Developing Countries

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Appendix A. Appendix to the Empirical Section

In this section, we first perform some robustness checks to test the validity of our results under alternative identification schemes. Next, we look at the outcomes from another identification strategy where the monetary policy equation is not disciplined.

A.1 Robustness Checks

A.1.1 The Alternative Specification of the Monetary Policy Equation

The first robustness check we consider is an alternative specification of the systematic portion of monetary policy in developing economies. This alternative specification is the same as the benchmark specification except that monetary authorities are assumed to contemporaneously react to consumer price inflation, the rate of nominal depreciation, and commodity price inflation rather than to *the levels* of these variables, as assumed in the benchmark specification. Indeed, we replace restriction 2 in the benchmark specification with restriction A.

RESTRICTION A. *A monetary-policy-related interest rate is the key instrument of monetary policy in developing countries that adopted an inflation-targeting regime. The monetary authorities in these economies contemporaneously increase the policy rate if output, consumer price inflation, the rate of nominal depreciation, and commodity price inflation measured in the national currencies (*$P_t^{com.} \times \mathcal{E}_{it} - P_{t-1}^{com.} \times \mathcal{E}_{it-1}$*) increase. Yet, they can only react to innovations in monetary aggregates with a one-quarter lag.*

In this alternative specification, we consider a monetary policy equation in which output is in levels, whereas nominal variables are in rates of change in the spirit of Taylor (1993). Under restriction A, one can rewrite equation (6) in the main text as follows:

$$R_{\mathcal{H},it} = -\,a^{-1}_{Y0,c_{51}}\left\{a_{Y0,c_{11}}\mathcal{Y}_{it} + a_{Y0,c_{21}}P_{it} + a_{Y0,c_{31}}\mathcal{E}_{it} + a_{Y0,c_{41}}M1_{it} - a_{X0,c_{11}}P^{com.}_{it}\right\} + a^{-1}_{Y0,c_{51}}\left\{a_{Y1,c_{21}}P_{it-1} + a_{Y1,c_{31}}\mathcal{E}_{it-1} + a_{X1,c_{11}}P^{com.}_{it-1}\right\} + G'_{it}A_G + a^{-1}_{Y0,c_{51}}\varepsilon_{1,it}$$

$$
\begin{aligned}
&= -a^{-1}_{Y0,c_{51}}\left\{a_{Y0,c_{11}}\mathcal{Y}_{it} + a_{Y0,c_{21}}P_{it}\right. \\
&\quad + [a_{Y0,c_{31}} + a_{X0,c_{11}}]\mathcal{E}_{it} + a_{Y0,c_{41}}M1_{it} \\
&\quad \left. - a_{X0,c_{11}}\left(P^{com.}_{it} + \mathcal{E}_{it}\right)\right\} + a^{-1}_{Y0,c_{51}}\left\{a_{Y1,c_{21}}P_{it-1} + a_{Y1,c_{31}}\mathcal{E}_{it-1}\right. \\
&\quad \left. + a_{X1,c_{11}}P^{com.}_{it-1}\right\} + G'_{it}A_G + a^{-1}_{Y0,c_{51}}\varepsilon_{1,it} \\
&= -a^{-1}_{Y0,c_{51}}\left\{a_{Y0,c_{11}}\mathcal{Y}_{it} + a_{Y0,c_{21}}\Delta P_{it} + [a_{Y0,c_{31}} + a_{X0,c_{11}}]\Delta\mathcal{E}_{it}\right. \\
&\quad \left. + a_{Y0,c_{41}}M1_{it} - a_{X0,c_{11}}\left(\Delta P^{com.}_{it} + \Delta\mathcal{E}_{it}\right)\right\} \\
&\quad + a^{-1}_{Y0,c_{51}}\left\{[\mathbf{a_{Y1,c_{21}}} - \mathbf{a_{Y0,c_{21}}}]P_{it-1} + [\mathbf{a_{Y1,c_{31}}} - \mathbf{a_{Y0,c_{31}}}]\mathcal{E}_{it-1}\right. \\
&\quad \left. + [\mathbf{a_{X1,c_{11}}} + \mathbf{a_{X0,c_{11}}}]P^{com.}_{it-1}\right\} + G'_{it}A_G + a^{-1}_{Y0,c_{51}}\varepsilon_{1,it}, \qquad \text{(A1)}
\end{aligned}
$$

where $a_{Y1,c_{k1}}$ and $a_{X1,c_{k1}}$ denote the k^{th} element of the first column of A_{Y1} and A_{X1}, respectively. G_{it}, on the other hand, is a matrix of variables that enter into the monetary policy equation but that are unconstrained by restriction A. G_{it} may contain the lags of both the endogenous and exogenous variables in addition to the current exogenous variables.

We define the matrices $S_{1A_{Y0}}, S_{1A_{X0}}, S_{1L_0}, Z_{1A_{Y0}}, Z_{1A_{Y1}}, Z_{1A_{X0}}$, and $Z_{1A_{X1}}$, which characterize the sign and zero restrictions from restriction 1 in the benchmark identification and restriction A above, as

$$
S_{1A_{Y0}} = \begin{vmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}, \quad S_{1A_{X0}} = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{vmatrix},
$$

$$
S_{1L_0} = \begin{vmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} \qquad \text{(A2)}
$$

$$Z_{1A_{Y0}} = \begin{vmatrix} 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{vmatrix}, \; Z_{1A_{Y1}} = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix},$$

$$Z_{1A_{X0}} = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}, \; Z_{1A_{X1}} = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}. \quad \text{(A3)}$$

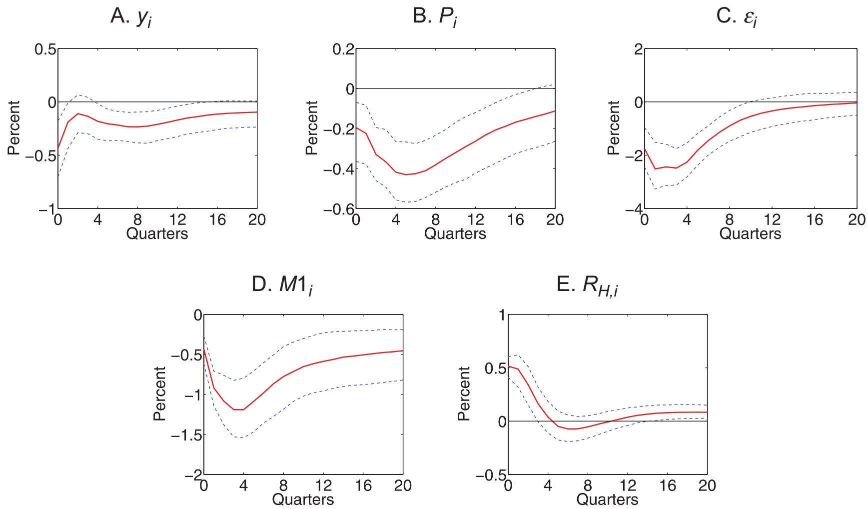
Restriction 1 in the main text and restriction A above can be summarized with the following matrices:

$$S_1 = \begin{vmatrix} S_{1A_{Y0}} & 0_{5\times n_Y} & S_{1A_{X0}} & 0_{5\times n_X} & 0_{5\times n_Y} \\ 0_{4\times n_Y} & 0_{4\times n_Y} & 0_{4\times n_X} & 0_{4\times n_X} & S_{1L_0} \end{vmatrix} \begin{vmatrix} A_{Y0} \\ A_{Y1} \\ A_{X0} \\ A_{X1} \\ L_0 \end{vmatrix} e_1 > 0 \quad \text{(A4)}$$

$$Z_1 = \begin{vmatrix} Z_{1A_{Y0}} & Z_{1A_{Y1}} & Z_{1A_{X0}} & Z_{1A_{X1}} & 0_{4\times n_Y} \end{vmatrix} \begin{vmatrix} A_{Y0} \\ A_{Y1} \\ A_{X0} \\ A_{X1} \\ L_0 \end{vmatrix} e_1 = 0. \quad \text{(A5)}$$

S_1 in (A4) is a matrix of 9×1. Its first five elements characterize the sign restrictions from the systematic component of monetary policy on output, consumer price inflation, the rate of nominal depreciation, the policy rate, and commodity price inflation measured in national currencies, respectively. The remaining four elements are the sign restrictions on the contemporaneous impulse response functions (IRFs) to a monetary policy shock on all of the endogenous variables except output, about which we remain agnostic. Z_1 in (A5) is a matrix of 4×1. Its first three elements are the zero restrictions from the systematic component of monetary policy on the lags of output, consumer prices, and the nominal exchange rate, whereas its

Figure A1. IRFs to Monetary Policy Shocks (Restrictions 1 and A)



Notes: Our calculations are based on the IMF's International Finance Statistics. The solid lines indicate the estimated median impulse responses. The area between the dashed lines shows the 68 percent confidence interval estimated using algorithms in Arias, Rubio-Ramirez, and Waggoner (2014).

last element is the zero restriction implying that monetary authorities under inflation targeting supply reserves completely elastically if innovations in $M1$ occur.[1]

Next, we discuss the dynamic effects from a tightening shock to monetary policy under the alternative specification. It is clear from figure A1 that the outcomes under this specification are largely similar to those under the benchmark specification, as displayed in figure 1 in the main text. Yet, there are two noteworthy differences. First, the effect of the shock on output is more persistent in the former

[1]It is notable that besides interest rate rules, Arias, Caldara, and Rubio-Ramirez (2015) also consider money rules in which short-term interest rates can only respond contemporaneously to innovations in monetary aggregates. Since these rules are inconsistent with our findings in subsection 2.1.3 of the main text, we do not consider them in our paper.

than in the latter. Second, the contemporaneous response of the policy rate is larger in the former compared with that in the latter.

A.1.2 A Weaker Assumption on the IRFs

Second, we maintain restriction 2 in the benchmark specification but replace restriction 1 with a weaker restriction that a tightening shock contemporaneously results in an increase in the policy rate.

RESTRICTION B. *A tightening monetary policy shock contemporaneously results in an increase in the policy rate.*

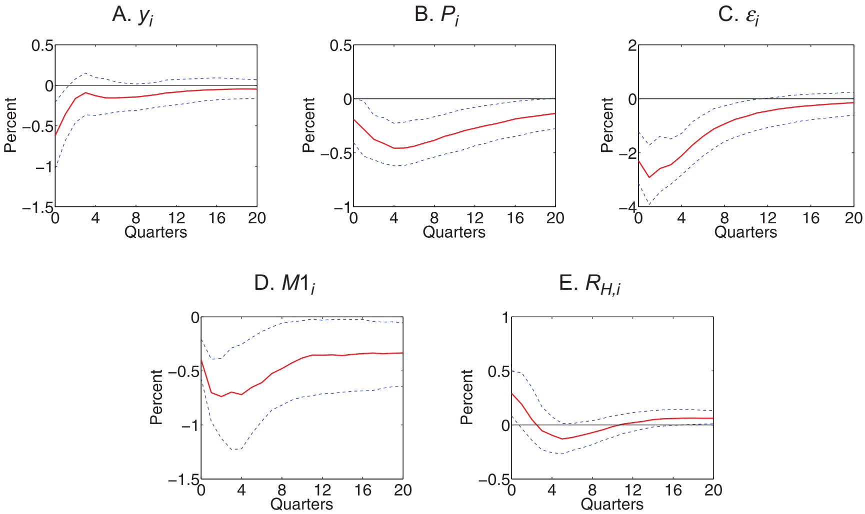
Restriction B is weaker than restriction 1 in the benchmark specification since the latter restricts the contemporaneous response of the price level, the nominal exchange rate, and $M1$ in addition to that of the policy rate. Restriction B implies we remain agnostic not only about output but also about the price level, the nominal exchange rate, and $M1$. The matrices characterizing the sign and zero restrictions in this case are similar to (9) and (10) in the main text and can be written analogously. Figure A2 displays the IRFs from the specification where restrictions 2 and B are imposed. The findings from this specification are almost identical to those from the benchmark specification except that one can barely reject the hypothesis that the contemporaneous response of the price level is zero in the former.

Next, we discuss the IRFs from the alternative specification in which we impose both restrictions A and B. These restrictions imply that the authorities implement a Taylor-type rule specified in restriction A and that we do not constrain the IRFs of any variable except the contemporaneous response of the policy rate. The IRFs diplayed in figure A3 are broadly similar to those displayed in figure A1.

A.1.3 Results from a Larger Sample of Developing Countries

Finally, we study monetary policy shocks in a larger set of developing countries than previously studied in the main text by using the *money-market rate* as a measure of monetary policy if the monetary-policy-related interest rate series are unavailable for developing countries under an inflation-targeting regime. The countries

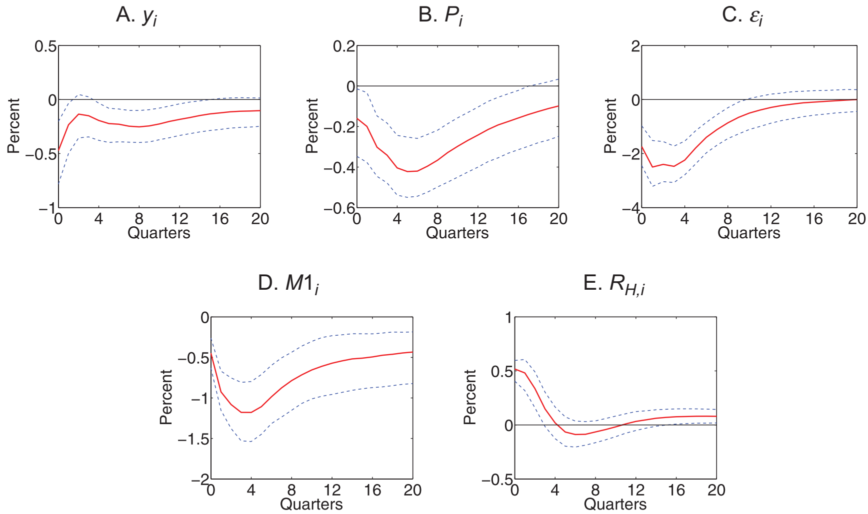
Figure A2. IRFs to Monetary Policy Shocks (Restrictions B and 2)



Notes: Our calculations are based on the IMF's International Finance Statistics. The solid lines indicate the estimated median impulse responses. The area between the dashed lines shows the 68 percent confidence interval estimated using algorithms in Arias, Rubio-Ramirez, and Waggoner (2014).

included in this sample and their adoption dates of inflation targeting are reported in table A1. Our sample is quarterly and contains the post-inflation-targeting period for each country until 2013:Q1. In addition to the countries previously studied in the main text, this sample includes Poland, Romania, and the Republic of Serbia, where we use the interbank money-market rate as a measure of monetary policy. For the remaining countries, the monetary-policy-related interest rate is used. However, this identification comes with a caveat that needs to be explained. The interbank money-market rate in some countries, such as Poland, persistently deviated from the policy rate over the period we consider, suggesting authorities have a limited influence on the short-term rates in these countries (see Lu 2012). This implies the money-market rate may be a poor instrument of monetary policy in these economies. If this is the case, the

Figure A3. IRFs to Monetary Policy Shocks (Restrictions A and B)



Notes: Our calculations are based on the IMF's International Finance Statistics. The solid lines indicate the estimated median impulse responses. The area between the dashed lines shows the 68 percent confidence interval estimated using algorithms in Arias, Rubio-Ramirez, and Waggoner (2014).

results from this sample can be misleading, and care should therefore be taken when interpreting the results presented below.

Figure A4 illustrates the IRFs for this larger sample of countries from the specification where restrictions 1 and 2 are imposed.[2] They look quite similar to the IRFs from the benchmark specification, as displayed in figure 1 in the main text. The only noteworthy difference is that the fall in output is long lasting in the former compared with that in the latter.

Finally, figure A5 shows the IRFs of the variables for the larger sample from the identification where restrictions 1 and A are imposed. They look similar to the IRFs displayed in figure A1.

[2] These restrictions are the same restrictions that we impose in the benchmark specification.

Table A1. Adoption Dates of Inflation Targeting in Developing Economies

Country	Effective IT Adoption Date
Brazil	1999:Q2
Chile	1999:Q3
Colombia	1999:Q3
Guatemala	2005:Q1
Indonesia	2005:Q3
Mexico	2001:Q1
Poland*	1998:Q4
Romania*	2005:Q3
Serbia*	2006:Q3
South Africa	2000:Q1
Turkey	2006:Q1

Source: Roger (2009).
Notes: Where available, we use monetary-policy-related interest rate series to identify monetary policy shocks in our sample of countries. If not, we use money-market rate series. The countries for which we use the money-market rate as a measure of policy rate are indicated with an asterisk.

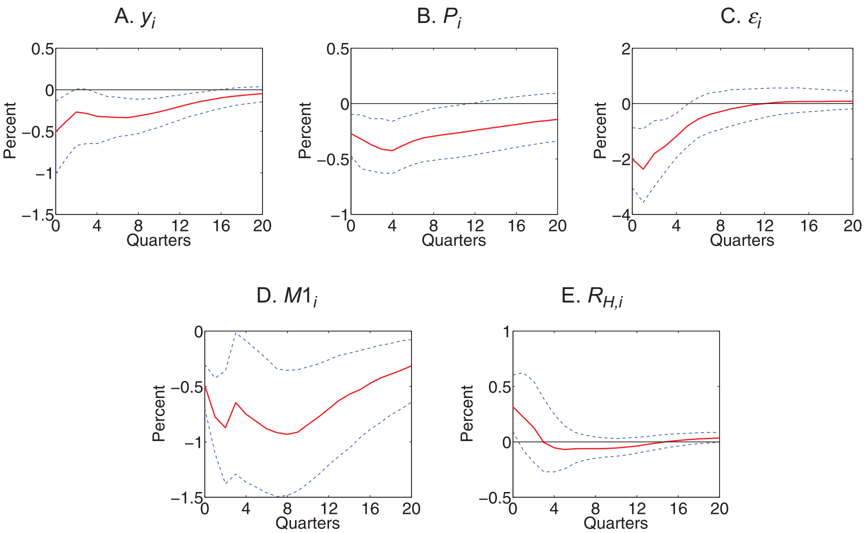
A.2 When the Systematic Component of Monetary Policy Is Not Disciplined

This subsection discusses the results from the identification of policy shocks in which the systematic component of monetary policy is not disciplined. Indeed, the identification strategy we consider in this section is the same as the benchmark identification except that we drop restriction 2. Consequently, there are only sign restrictions on the IRFs in this identification, which can be stated as

$$S_1 = S_{1L_0} \times L_0 \times e_1 > 0, \tag{A6}$$

where S_1 characterizes the sign restrictions on the price level, the nominal exchange rate, $M1$, and the policy rate (see the main text for the definitions of S_{1L_0}, L_0, and e_1). We estimate the median responses and 68 percent confidence intervals using the algorithm in Rubio-Ramirez, Waggoner, and Zha (2010). Figure A6 shows the IRFs from this identification. The findings from this specification are

Figure A4. IRFs to Monetary Policy Shocks (Restrictions 1 and 2, Large Sample)

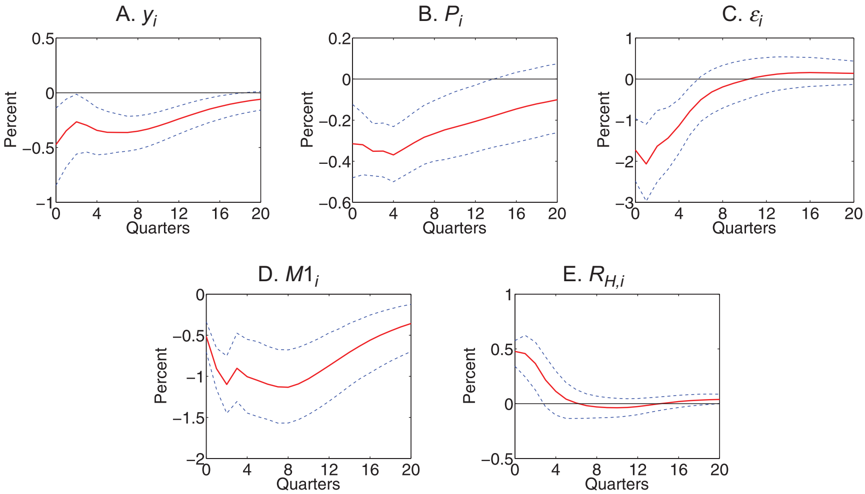


Notes: Our calculations are based on the IMF's International Finance Statistics. The solid lines indicate the estimated median impulse responses. The area between the dashed lines shows the 68 percent confidence interval estimated using algorithms in Arias, Rubio-Ramirez, and Waggoner (2014).

different than those from the benchmark identification in two important ways. First, output no longer shows a sizable decline after the shock. Second, the interest rate shows a persistent increase. This suggests that if monetary policy is exercised in developing countries in such a way that the policy rate is contemporaneously increased if output, the price level, the nominal exchange rate, or the commodity price level increase, then inference about output dynamics from the specification where the systematic component is not disciplined would be misleading. Indeed, while one may infer that output remains almost unresponsive to the shock in this specification, one should conclude that output falls following the shock according to the benchmark specification.[3]

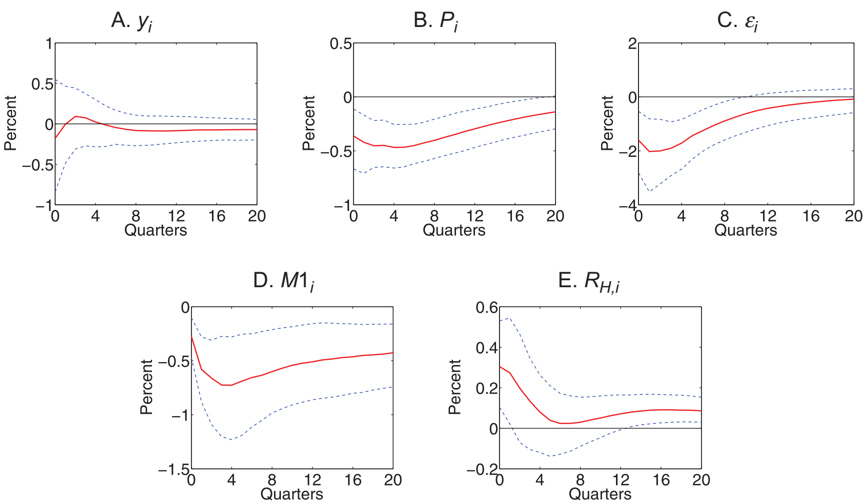
[3]Similarly, Arias, Caldara, and Rubio-Ramirez (2015) note that disciplining the systematic portion of monetary policy substantially changes inference regarding output dynamics after a monetary policy shock for the United States.

Figure A5. IRFs to Monetary Policy Shocks (Restrictions 1 and A, Large Sample)



Notes: Our calculations are based on the IMF's International Finance Statistics. The solid lines indicate the estimated median impulse responses. The area between the dashed lines shows the 68 percent confidence interval estimated using algorithms in Arias, Rubio-Ramirez, and Waggoner (2014).

Figure A6. IRFs to Monetary Policy Shocks (When the Systematic Portion of Monetary Policy Is Not Disciplined)



Notes: Our calculations are based on the IMF's International Finance Statistics. The solid lines indicate the estimated median impulse responses. The area between the dashed lines shows the 68 percent confidence interval estimated using algorithms in Rubio-Ramirez, Waggoner, and Zha (2010).

Appendix B. Log-Linear Approximation of the DSGE Model in the Text

The model presented in this appendix is an elaborate version of the model presented in the text. Indeed, unlike the latter, the former may contain multiple sectors that differ in their frequency of price changes.

B.1 Notation

Table B1 lists the notation used in the model.

Before describing the model, it is useful to mention further remarks about notation. First, $\bar{\mathcal{V}}$ and $\hat{\mathcal{V}}$ denote the flexible-price steady-state value of a generic variable $\mathcal{V}$ and its log-deviation from this steady-state value, respectively. Second, subscript $\mathcal{H}$ or $\mathcal{F}$ stands for whether a good is produced by a home firm or a foreign firm; subscript ¢(¢*) indicates the goods invoiced in the *home currency* (the *foreign currency*); and subscripts k, j, and t denote sectors, variety and period, respectively. Third, superscript * represents the variables originating from the foreign country. Variables without superscript *, on the other hand, originate from the home country. For instance, we denote demand for the home-export variety j priced in the *foreign currency* by

$$Y^*_{\mathcal{H},¢^*,k,j,t}, \tag{B1}$$

where $\mathcal{H}$ indicates that the firm is a home firm and ¢* stands for the *foreign currency* with which the firm sets the price. Subscripts k, j, and t represent sectors, variety, and period, respectively, and superscript * denotes demand from the foreign country. Another example of our notation is demand for the foreign-export variety j priced in the *home currency* denoted by

$$Y_{\mathcal{F},¢,k,j,t}, \tag{B2}$$

where $\mathcal{F}$ indicates that the firm is a foreign firm and ¢ stands for the *home currency* with which the firm sets the price. Subscripts k, j, and t represent sectors, variety, and period, respectively, and demand from the home country is indicated by the absence of an asterisk.

Table B1. Notation

Variable or Coefficient	Explanation
$a(u)$	Real costs in units of the final good due to increasing capacity utilization
α_k	The probability that prices remain unchanged in sector k in the home country
α_k^*	The probability that prices remain unchanged in sector k in the foreign country
B_{t+1}	Home-household holdings of a risk-free home-country bond denominated in the *home currency*
B_{t+1}^*	Foreign-household holdings of the risk-free home-country bond denominated in the *home currency*
β	Discount factor
C_t	Home final good consumption
C_t^*	Foreign final good consumption
χ	Labor share in GDP
D_{t+1}^*	Foreign-household holdings of a risk-free *foreign-country* bond denominated in the *foreign currency*
δ	Depreciation rate of capital
$\mathcal{E}_t$	Nominal exchange rate
e_t^r	Monetary policy shocks in the home country
$\varepsilon_t^{r^*}$	Monetary policy shocks in the foreign country
η	Elasticity of substitution between different sector goods
f_k	Expenditure share of sector k
I_t	Home-country investment
I_t^*	Foreign-country investment
K_t	Home-country capital stock
K_t^*	Foreign-country capital stock
$K_{\mathcal{F},k,j,t}^*$	Capital demanded by the foreign firm that supplies the foreign country with variety j
$K_{\mathcal{F},¢,k,j,t}$	Capital demanded by the foreign-export firm that supplies the home country with variety j and sets prices in the *home currency*
$K_{\mathcal{F},¢^*,k,j,t}$	Capital demanded by the foreign-export firm that supplies the home country with variety j and sets prices in the *foreign currency*
$K_{\mathcal{H},k,j,t}$	Capital demanded by the home firm that supplies the home country with variety j

(*continued*)

Table B1. (Continued)

Variable or Coefficient	Explanation
$K^*_{\mathcal{H},¢,k,j,t}$	Capital demanded by the home-export firm that supplies the foreign country with variety j and sets prices in the *home currency*
$K^*_{\mathcal{H},¢^*,k,j,t}$	Capital demanded by the home-export firm that supplies the foreign country with variety j and sets prices in the *foreign currency*
λ_t	The Lagrange multiplier from the home-household budget constraint
λ^*_t	The Lagrange multiplier from the foreign-household budget constraint
M_t	Demand for money in the home country
M^*_t	Demand for money in the foreign country
M^a_t	Economy-wide per capita stock of money in the home country
${M^*_t}^a$	Economy-wide per capita stock of money in the foreign country
μ_t	The shadow price of K_{t+1} that shows what the price of K_{t+1} would be if there were a market for K_{t+1} at period t
μ^*_t	The shadow price of K^*_{t+1} that shows what the price of K_{t+1} would be if there were a market for K^*_{t+1} at period t
n_t	The hours worked by members of the home households whose wage contracts are signed in period $t-1$
n^*_t	The hours worked by members of the foreign households whose wage contracts are signed in period $t-1$
$\tilde{n}_t$	The hours worked by members of the home households whose wage contracts are signed in period t
$\tilde{n}^*_t$	The hours worked by members of the foreign households whose wage contracts are signed in period t
N_t	The total composite-labor demand in the home country
N^*_t	The total composite-labor demand in the foreign country
$N^*_{\mathcal{F},k,j,t}$	The composite labor demanded by the foreign firm that supplies the foreign country with variety j
$N_{\mathcal{F},¢,k,j,t}$	The composite labor demanded by the foreign-export firm that supplies the home country with variety j and sets prices in the *home currency*

(*continued*)

Table B1. (Continued)

Variable or Coefficient	Explanation
$N_{\mathcal{F},¢^*,k,j,t}$	The composite labor demanded by the foreign-export firm that supplies the home country with variety j and sets prices in the *foreign currency*
$N_{\mathcal{H},k,j,t}$	The composite labor demanded by the home firm that supplies the home country with variety j
$N^*_{\mathcal{H},¢,k,j,t}$	The composite labor demanded by the home-export firm that supplies the foreign country with variety j and sets prices in the *home currency*
$N^*_{\mathcal{H},¢^*,k,j,t}$	The composite labor demanded by the home-export firm that supplies the foreign country with variety j and sets prices in the *foreign currency*
$\omega_{¢}$	Share of home exports invoiced in the *home currency*
$\omega^*_{¢^*}$	Share of home imports invoiced in the *foreign currency*
P_t	Price of the final good in the home country
P^*_t	Price of the final good in the foreign country
$P_{k,t}$	Price of the sector k good in the home country
$P^*_{k,t}$	Price of the sector k good in the foreign country
$P_{\mathcal{F},k,t}$	Price of the imported good in sector k in the home country
$P^*_{\mathcal{F},k,t}$	Price set by the foreign firms that supply the foreign country with the sector k good
$P_{\mathcal{F},¢,k,t}$	Price set in the *home currency* by the foreign-export firms in sector k
$P_{\mathcal{F},¢^*,k,t}$	Price set in the *foreign currency* by the foreign-export firms in sector k
$P^*_{\mathcal{F},k,j,t}$	Price set by the foreign firm supplying the foreign country with variety j
$P_{\mathcal{F},¢,k,j,t}$	Price set in the *home currency* by the foreign-export firm that supplies variety j
$P_{\mathcal{F},¢^*,k,j,t}$	Price set in the *foreign currency* by the foreign-export firm that supplies variety j
$P_{\mathcal{H},k,t}$	Price set by the home firms supplying the home country with the sector k good
$P^*_{\mathcal{H},k,t}$	Price set by the foreign-import firms in sector k
$P^*_{\mathcal{H},¢,k,t}$	Price set in the *home currency* by the home-export firms in sector k

(*continued*)

Table B1. (Continued)

Variable or Coefficient	Explanation
$P^*_{\mathcal{H},¢^*,k,t}$	Price set in the *foreign currency* by the home-export firms in sector k
$P_{\mathcal{H},k,j,t}$	Price set by the home firm that supplies the home country with variety j in sector k
$P^*_{\mathcal{H},¢,k,j,t}$	Price set in the *home currency* by the home-export firm that supplies variety j
$P^*_{\mathcal{H},¢^*,k,j,t}$	Price set in the *foreign currency* by the home-export firm that supplies variety j
$\phi\left(\frac{I}{K}\right)K$	The additional capital stock created by new investment in the current period and which is available for use in the next period in the home country
$\phi\left(\frac{I^*}{K^*}\right)K^*$	The additional capital stock created by new investment in the current period and which is available for use in the next period in the foreign country
Π_{t+s}	The profits of home-household-owned firms
Π^*_{t+s}	The profits of foreign-household-owned firms
$\Pi^*_{\mathcal{F},k,j,t}$	The profits of the foreign firm supplying the foreign country with variety j
$\Pi_{\mathcal{F},¢,k,j,t}$	The profits of the foreign-export firm producing variety j and setting prices in the *home currency*
$\Pi_{\mathcal{F},¢^*,k,j,t}$	The profits of the foreign-export firm producing variety j and setting prices in the *foreign currency*
$\Pi_{\mathcal{H},k,j,t}$	The profits of the home firm supplying variety j domestically
$\Pi^*_{\mathcal{H},¢,k,j,t}$	The profits of the home-export firm producing variety j and setting prices in the *home currency*
$\Pi^*_{\mathcal{H},¢^*,k,j,t}$	The profits of the home-export firm producing variety j and setting prices in the *foreign currency*
ψ	Share of imports in home-country output
$\frac{\psi}{\tau}$	Share of home exports in foreign-country output
Q_t	Real exchange rate
r^k_t	The real rental rate of capital in the home country
$r^{*^k}_t$	The real rental rate of capital in the foreign country
R^*_t	The nominal interest rate on the foreign risk-free bond
R^k_t	The nominal rental rate of capital in the home country
$R^{*^k}_t$	The nominal rental rate of capital in the foreign country

(*continued*)

Table B1. (Continued)

Variable or Coefficient	Explanation
$R_{\mathcal{F},t}$	The nominal interest rate that the foreign country faces on international borrowing and lending
$R_{\mathcal{H},t}$	The nominal interest rate that the home country faces on international borrowing and lending
ρ	The elasticity of substitution between the domestic and imported goods
ρ_e	Persistence of a monetary policy shock in the home country
Φ_p	The coefficient on the price level in the home monetary policy equation
$\Phi_{\mathcal{Y}}$	The coefficient on output in the home monetary policy equation
s_C	% share of final consumption expenditure in home-country output
$s^*_{C^*}$	% share of final consumption expenditure in foreign-country output
s_I	% share of investment in GDP of the home country
$s^*_{I^*}$	% share of investment in GDP of the foreign country
σ_a	The *reciprocal* of the elasticity of capacity utilization with respect to the rental rate of capital
σ_c	The inverse of the elasticity of intertemporal substitution
σ_n	The inverse of the Frisch elasticity of labor supply
σ_m	$\sigma_m(\bar{R}_{\mathcal{H},t}-1)$ stands for the *reciprocal* of the interest semi-elasticity of money demand
$\sigma_\phi = \frac{\phi'' \frac{\bar{I}}{\bar{K}}}{\phi'}$	The elasticity of the adjustment cost technology for investment with respect to $\frac{I_t}{K_t}$. Note that since $\phi'' < 0$, σ_ϕ has to be negative.
τ	The size of the foreign country relative to that of the home country
θ_p	Price elasticity of varieties' demand
θ_w	Wage elasticity of labor demand
u_t	Home capacity utilization rate
u^*_t	Foreign capacity utilization rate
W_t	Home aggregate wage
W^*_t	Foreign aggregate wage

(*continued*)

Table B1. (Continued)

Variable or Coefficient	Explanation
x_t	Home wages set by the owners of differentiated labor in period t
x_t^*	Foreign wages set by the owners of differentiated labor in period t
Y_t	Output from final-good firms in the home country
Y_t^*	Output from final-good firms in the foreign country
$Y_{k,t}$	Output from sector k in the home country
$Y_{k,t}^*$	Output from sector k in the foreign country
$Y_{\mathcal{F},k,t}$	Output from the home-import firms that supply the home country with sector k good
$Y_{\mathcal{F},k,t}^*$	Output from the foreign firms that supply the foreign country with sector k good
$Y_{\mathcal{F},¢,k,t}$	Output from foreign-export firms in sector k that set prices in the *home currency*
$Y_{\mathcal{F},¢^*,k,t}$	Output from foreign-export firms in sector k that set prices in the *foreign currency*
$Y_{\mathcal{F},k,j,t}^*$	Demand for variety j of the firms producing the intermediate good in the foreign country in sector k
$Y_{\mathcal{F},¢,k,j,t}$	Demand for the foreign-export variety j whose price is set in the *home currency*
$Y_{\mathcal{F},¢^*,k,j,t}$	Demand for the foreign-export variety j whose price is set in the *foreign currency*
$Y_{\mathcal{H},k,t}$	Output from home firms that supply domestically in sector k
$Y_{\mathcal{H},k,t}^*$	Output from foreign-import firms in sector k
$Y_{\mathcal{H},¢,k,t}^*$	Output from the home-export firms that set prices in the *home currency* in sector k
$Y_{\mathcal{H},¢^*,k,t}^*$	Output from the home-export firms that set prices in the *foreign currency* in sector k
$Y_{\mathcal{H},k,j,t}$	Demand for variety j of the firms producing the domestic sector k good in the home country
$Y_{\mathcal{H},¢,k,j,t}^*$	Demand for the home-export variety j whose price is set in the *home currency*
$Y_{\mathcal{H},¢^*,k,j,t}^*$	Demand for the home-export variety j whose price is set in the *foreign currency*

B.2 The Home-Household Budget Constraint

Home households aim to maximize the following discounted utility function:

$$E_t \sum_{s=0}^{\infty} \beta^{t+s} \left(\frac{C_{t+s}^{1-\sigma_c} - 1}{1-\sigma_c} - \frac{\tilde{n}_{t+s,i}^{1+\sigma_n}}{1+\sigma_n} - \frac{n_{t+s,i}^{1+\sigma_n}}{1+\sigma_n} + \frac{\left(\frac{M_{t+s}}{P_{t+s}}\right)^{1-\sigma_m} - 1}{1-\sigma_m} \right)$$

subject to the flow budget constraint given by

$$\begin{aligned} & P_{t+s}\left(C_{t+s} + I_{t+s} + a(u_{t+s})K_{t+s} + B_{t+s+1}\right) + M_t \\ & \quad = x_{t+s,i}\tilde{n}_{t+s,i} + x_{t+s-1,i}n_{t+s,i} + M_{t-1} + M_t^a - M_{t-1}^a \\ & \qquad + R_{t+s}^k u_{t+s} K_{t+s} + R_{\mathcal{H},t+s-1} P_{t+s-1} B_{t+s} + \Pi_{t+s}, \end{aligned} \tag{B3}$$

where $M_t^a - M_{t-1}^a$ denotes seigniorage revenue, which is transferred back to home households by the home monetary authority. The law of motion for capital is given by

$$K_{t+s+1} = (1-\delta)K_{t+s} + \phi\left(\frac{I_{t+s}}{K_{t+s}}\right)K_{t+s} \tag{B4}$$

$$\begin{aligned} \mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^{t+s} \Bigg[& \frac{C_{t+s}^{1-\sigma_c} - 1}{1-\sigma_c} - \frac{\tilde{n}_{t+s,i}^{1+\sigma_n}}{1+\sigma_n} - \frac{n_{t+s,i}^{1+\sigma_n}}{1+\sigma_n} + \frac{\left(\frac{M_{t+s}}{P_{t+s}}\right)^{1-\sigma_m} - 1}{1-\sigma_m} \\ & - \lambda_{t+s}\left\{P_{t+s}\left(C_{t+s} + I_{t+s} + a(u_{t+s})K_{t+s} + B_{t+s+1}\right)\right. \\ & + M_t - x_{t+s,i}\tilde{n}_{t+s,i} - x_{t+s-1,i}n_{t+s,i} - M_{t-1} \\ & \left. - M_t^a + M_{t-1}^a - R_{t+s}^k u_{t+s}K_{t+s} - R_{\mathcal{H},t+s-1}P_{t+s-1}B_{t+s} - \Pi_{t+s}\right\} \\ & - \mu_{t+s}\left(K_{t+s+1} - (1-\delta)K_{t+s} - \phi\left(\frac{I_{t+s}}{K_{t+s}}\right)K_{t+s}\right)\Bigg]. \end{aligned} \tag{B5}$$

B.2.1 With respect to C_t

$$\begin{aligned} C_t^{-\sigma_c} &= \lambda_t P_t \\ -\sigma_c \hat{C}_t &= \hat{\lambda}_t + \hat{P}_t \end{aligned} \tag{B6}$$

$$-\sigma_c \hat{C}_t - \hat{P}_t = \hat{\lambda}_t \tag{B7}$$

$$\bar{C}^{-\sigma_c} = \bar{\lambda}_t \bar{P}_t = \bar{\lambda}_{t+1} \bar{P}_{t+1} \tag{B8}$$

Define $\bar{\pi}$ as

$$log\bar{P}_{t+1} - log\bar{P}_t \tag{B9}$$

$$\bar{\pi} = log\frac{\bar{P}_{t+1}}{\bar{P}_t} \Rightarrow \frac{\bar{P}_{t+1}}{\bar{P}_t} = e^{\bar{\pi}} \tag{B10}$$

$$\begin{aligned} E_t C_{t+1}^{-\sigma_c} &= E_t \lambda_{t+1} P_{t+1} \\ \bar{C}^{-\sigma_c} - \sigma_c \bar{C}^{-\sigma_c} E_t \hat{C}_{t+1} &= \bar{\lambda}_{t+1} \bar{P}_{t+1} + \bar{\lambda}_{t+1} \bar{P}_{t+1} E_t \hat{\lambda}_{t+1} \\ &\quad + \bar{\lambda}_{t+1} \bar{P}_{t+1} E_t \hat{P}_{t+1} \end{aligned} \tag{B11}$$

$$-\sigma_c E_t \hat{C}_{t+1} - E_t \hat{P}_{t+1} = E_t \hat{\lambda}_{t+1} \tag{B12}$$

$$\sigma_c E_t \left(\hat{C}_{t+1} - \hat{C}_t\right) + E_t\left(\hat{P}_{t+1} - \hat{P}_t\right) = E_t\left(\hat{\lambda}_t - \hat{\lambda}_{t+1}\right). \tag{B13}$$

From (B16), one can rewrite (B13) as

$$\sigma_c E_t \left(\hat{C}_{t+1} - \hat{C}_t\right) + E_t\left(\hat{P}_{t+1} - \hat{P}_t\right) = \hat{R}_{\mathcal{H},t}. \tag{B14}$$

B.2.2 With Respect to B_{t+1}

$$\beta^t \lambda_t P_t = \beta^{t+1} E_t \lambda_{t+1} R_{\mathcal{H},t} P_t$$

$$\lambda_t P_t = \beta R_{\mathcal{H},t} E_t \lambda_{t+1} P_{t+1} \frac{P_t}{P_{t+1}}$$

$$\bar{R}_{\mathcal{H}} = \frac{1}{\beta} \frac{\bar{P}_{t+1}}{\bar{P}_t} = \frac{1}{\beta} e^{\bar{\pi}} \tag{B15}$$

$$\bar{\lambda}_t \bar{P}_t \left(\hat{\lambda}_t + \hat{P}_t\right) = \beta \bar{R}_{\mathcal{H}} \bar{\lambda}_{t+1} \bar{P}_{t+1} \frac{\bar{P}_t}{\bar{P}_{t+1}} \left[\hat{R}_{\mathcal{H},t} + E_t \left(\hat{\lambda}_{t+1} + \hat{P}_t\right)\right]$$

$$\left(\hat{\lambda}_t + \hat{P}_t\right) = \hat{R}_{\mathcal{H},t} + E_t \left(\hat{\lambda}_{t+1} + \hat{P}_t\right)$$

$$E_t\left(\hat{\lambda}_t - \hat{\lambda}_{t+1}\right) = \hat{R}_{\mathcal{H},t} \tag{B16}$$

B.2.3 With Respect to M_t

$$\frac{M_t^{-\sigma_m}}{P_t^{1-\sigma_m}} = \lambda_t - \beta E_t \lambda_{t+1}$$

$$\frac{M_t^{-\sigma_m}}{P_t^{-\sigma_m}} \frac{1}{\lambda_t P_t} = 1 - \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \tag{B17}$$

Using (B6) and (B15), (B17) can be rewritten as

$$\begin{aligned}
\frac{M_t^{-\sigma_m}}{P_t^{-\sigma_m}} \frac{1}{C_t^{-\sigma_c}} &= 1 - \frac{1}{R_{\mathcal{H},t}} \frac{\bar{M}_t^{-\sigma_m}}{\bar{P}_t^{-\sigma_m}} \frac{1}{\bar{C}^{-\sigma_c}} - \sigma_m \frac{\bar{M}_t^{-\sigma_m}}{\bar{P}_t^{-\sigma_m}} \frac{1}{\bar{C}^{-\sigma_c}} \hat{M}_t \\
&\quad + \sigma_m \frac{\bar{M}_t^{-\sigma_m}}{\bar{P}_t^{-\sigma_m}} \frac{1}{\bar{C}^{-\sigma_c}} \hat{P}_t + \sigma_c \frac{\bar{M}_t^{-\sigma_m}}{\bar{P}_t^{-\sigma_m}} \frac{1}{\bar{C}^{-\sigma_c}} \hat{C}_t \\
&= 1 - \frac{1}{\bar{R}_{\mathcal{H},t}} + \frac{1}{\bar{R}_{\mathcal{H},t}} \hat{R}_{\mathcal{H},t} - \sigma_m \left(\hat{M}_t - \hat{P}_t \right) + \sigma_c \hat{C}_t \\
&= \frac{1}{\frac{1}{\beta} e^{\bar{\pi}} - 1} \hat{R}_{\mathcal{H},t}.
\end{aligned} \tag{B18}$$

B.2.4 With Respect to I_t

$$\lambda_t P_t = \mu_t \frac{1}{K_t} \phi' \left(\frac{I_t}{K_t} \right) K_t \tag{B19}$$

$$\lambda_t P_t = \mu_t \phi' \left(\frac{I_t}{K_t} \right) \tag{B20}$$

$$C_t^{-\sigma_c} = \mu_t \phi' \left(\frac{I_t}{K_t} \right) \tag{B21}$$

Define $\mathcal{I}_t$ as

$$\frac{I_t}{K_t} \tag{B22}$$

$$C_t^{-\sigma_c} = \mu_t \phi' \left(\mathcal{I}_t \right)$$

$$-\sigma_c \hat{C}_t = \hat{\mu}_t + \sigma_\phi \hat{\mathcal{I}}_t, \tag{B23}$$

where

$$\sigma_\phi = \frac{\phi''(\bar{\mathcal{I}})\bar{\mathcal{I}}}{\phi'(\bar{\mathcal{I}})}$$

$$-\sigma_c \hat{C}_t = \hat{\mu}_t + \sigma_\phi \left(\hat{I}_t - \hat{K}_t\right). \tag{B24}$$

B.2.5 With Respect to u_t

$$-\lambda_t \left(P_t a'(u_t) K_t - R_t^k K_t\right) = 0 \tag{B25}$$

Letting r_t^k denote the real rental rate of capital, we define r_t^k as $\frac{R_t^k}{P_t}$

$$\bar{r}^k = a'(\bar{u})$$
$$a'(u_t) = r_t^k$$
$$\hat{r}_t^k = \sigma_a \hat{u}_t, \tag{B26}$$

where

$$\sigma_a = \frac{a''\bar{u}}{a'}.$$

B.2.6 With Respect to K_{t+1}

$$\begin{aligned} &-\beta^t \mu_t + \beta^{t+1} E_t \Big[-\lambda_{t+1} P_{t+1} a(u_{t+1}) + \lambda_{t+1} R_{t+1}^k u_{t+1} + \mu_{t+1} \\ &\quad \times \left((1-\delta) - \phi' \left(\frac{I_{t+1}}{K_{t+1}}\right) \frac{I_{t+1}}{K_{t+1}} + \phi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\Big] = 0 \end{aligned} \tag{B27}$$

Using (B20), (B27) can be written as

$$\begin{aligned} \mu_t = \beta E_t \Big[&-\mu_{t+1} \phi' \left(\frac{I_{t+1}}{K_{t+1}}\right) a(u_{t+1}) + \mu_{t+1} \phi' \left(\frac{I_{t+1}}{K_{t+1}}\right) r_{t+1}^k u_{t+1} \\ &+ \mu_{t+1} \left((1-\delta) - \phi' \left(\frac{I_{t+1}}{K_{t+1}}\right) \frac{I_{t+1}}{K_{t+1}} + \phi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\Big]. \end{aligned} \tag{B28}$$

At the steady state, the following assumptions are made:

$$\left\{\bar{u}=1, a(\bar{u})=0, \phi\left(\frac{\bar{I}}{\bar{K}}\right)=\delta, \frac{\bar{I}}{\bar{K}}=\delta, \phi^{'}\left(\frac{\bar{I}}{\bar{K}}\right)=1\right\}. \tag{B29}$$

Consequently,

$$\bar{r}^k=\frac{1}{\beta}-(1-\delta) \tag{B30}$$

$$\begin{aligned}\mu_t=\beta E_t\Big[&\mu_{t+1}\left\{-\phi'\left(\mathcal{I}_{t+1}\right)a(u_{t+1})+\phi'\left(\mathcal{I}_{t+1}\right)r^k_{t+1}u_{t+1}\right.\\&\left.+\left((1-\delta)-\phi'\left(\mathcal{I}_{t+1}\right)\mathcal{I}_{t+1}+\phi\left(\mathcal{I}_{t+1}\right)\right)\right\}\Big]\end{aligned} \tag{B31}$$

$$\begin{aligned}\hat{\mu}_t=E_t\Big[&\hat{\mu}_{t+1}-\beta\phi''\left(\bar{\mathcal{I}}\right)a(\bar{u})\bar{\mathcal{I}}\;\hat{\mathcal{I}}_{t+1}-\beta\phi'\left(\bar{\mathcal{I}}\right)a'(\bar{u})\bar{u}\;\hat{u}_{t+1}\\&+\beta\phi''\left(\bar{\mathcal{I}}\right)\bar{r}^k\bar{u}\bar{\mathcal{I}}\;\hat{\mathcal{I}}_{t+1}+\beta\phi'\left(\bar{\mathcal{I}}\right)\bar{r}^k\bar{u}\;\hat{r}^k_{t+1}+\beta\phi'\left(\bar{\mathcal{I}}\right)\bar{r}^k\bar{u}\;\hat{u}_{t+1}\\&-\beta\phi''\left(\bar{\mathcal{I}}\right)\bar{\mathcal{I}}\bar{\mathcal{I}}\;\hat{\mathcal{I}}_{t+1}-\beta\phi'\left(\bar{\mathcal{I}}\right)\bar{\mathcal{I}}\;\hat{\mathcal{I}}_{t+1}+\beta\phi'\left(\bar{\mathcal{I}}\right)\bar{\mathcal{I}}\;\hat{\mathcal{I}}_{t+1}\Big]\end{aligned} \tag{B32}$$

$$\begin{aligned}\hat{\mu}_t&=E_t\left(\hat{\mu}_{t+1}+\beta\phi''\left(\bar{\mathcal{I}}\right)\left(\bar{r}^k\delta-\delta^2\right)\;\hat{\mathcal{I}}_{t+1}+\beta\bar{r}^k\;\hat{r}^k_{t+1}\right)\\&=E_t\left(\hat{\mu}_{t+1}+\beta\sigma_\phi\left(\frac{1-\beta}{\beta}\right)\;\hat{\mathcal{I}}_{t+1}+\beta\left(\frac{1-\beta(1-\delta)}{\beta}\right)\hat{r}^k_{t+1}\right)\end{aligned} \tag{B33}$$

$$\hat{\mu}_t=E_t\Big(\hat{\mu}_{t+1}+\sigma_\phi\left(1-\beta\right)\;\left(\hat{I}_{t+1}-\hat{K}_{t+1}\right)+\left\{1-\beta(1-\delta)\right\}\hat{r}^k_{t+1}\Big). \tag{B34}$$

B.3 The Law of Motion for Capital in the Home Country

$$K_{t+1}=(1-\delta)K_t+\phi\left(\mathcal{I}_t\right)K_t \tag{B35}$$

$$\bar{K}=(1-\delta)\bar{K}+\phi(\bar{\mathcal{I}})\bar{K}\Rightarrow\phi(\bar{\mathcal{I}})=\delta \tag{B36}$$

$$\bar{K}\hat{K}_{t+1}=(1-\delta)\bar{K}\hat{K}_t+\phi(\bar{\mathcal{I}})\bar{K}\hat{K}_t+\bar{K}\phi'(\bar{\mathcal{I}})\bar{\mathcal{I}}\hat{\mathcal{I}}_t$$

$$\hat{K}_{t+1}=(1-\delta)\hat{K}_t+\delta\hat{K}_t+\delta\hat{\mathcal{I}}_t \tag{B37}$$

$$\hat{K}_{t+1}=(1-\delta)\hat{K}_t+\delta\hat{I}_t \tag{B38}$$

B.4 The Aggregate Resource Constraint in the Home Country

$$C_t + I_t + a(u_t)K_t = Y_t \tag{B39}$$

$$\bar{C} + \bar{I} + a(\bar{u})\bar{K} = \bar{Y}$$

$$\bar{C}\hat{C}_t + \bar{I}\hat{I}_t + a'(\bar{u})\bar{K}\hat{u}_t + a(\bar{u})\bar{K}\hat{K}_t = \bar{Y}\hat{Y}_t \tag{B40}$$

$$s_C\hat{C}_t + s_I\hat{I}_t + \frac{\bar{K}}{\bar{Y}}a'(\bar{u})\bar{u}\hat{u}_t = \hat{Y}_t$$

$$\bar{I} = \delta\bar{K} \Rightarrow \frac{\bar{I}}{\bar{Y}} = \delta\frac{\bar{K}}{\bar{Y}} \Rightarrow \frac{s_I}{\delta} = \frac{\bar{K}}{\bar{Y}}$$

$$a'(\bar{u}) = \bar{r}^k = \frac{1}{\beta} - (1-\delta) \tag{B41}$$

$$s_C\hat{C}_t + s_I\hat{I}_t + \frac{s_I}{\delta}\left(\frac{1}{\beta} - (1-\delta)\right)\hat{u}_t = \hat{Y}_t \tag{B42}$$

B.5 The Wage-Setting Equation in the Home Country

Regarding wage setting in the home country, we work with both predetermined- and flexible-wage arrangements. We first describe predetermined wage setting.

B.5.1 Predetermined Wages

B.5.1.1 The Problem of Employment Offices in the Home Country

Our model of staggered wage setting is a modified version of the Huang and Liu (2002) model. Indeed, while households contain one member in the Huang and Liu (2002) model, they contain two members in our model, the wife and the husband. There is a continuum of employment offices with a mass of one in the home economy. Employment offices combine the differentiated hours of work supplied by household members ($\tilde{n}_{t,i}$ and $n_{t,i}$) into a composite labor of (N_t) and sell it to the firms. The employment offices use the following technology to form the composite of labor:

$$N_t = \left(\int_0^1 \tilde{n}_{t,i}^{(\theta_w-1)/\theta_w} di + \int_0^1 n_{t,i}^{(\theta_w-1)/\theta_x} di\right)^{\theta_w/(\theta_w-1)}, \tag{B43}$$

where $\tilde{n}_{t,i}$ and $n_{t,i}$ are the hours worked by the members of the home household whose wage contracts are signed in period t and $t-1$, respectively. The representative employment office solves the following problem:

$$\max_{n_{t+s,i},n_{t+s,i}} W_{t+s}N_{t+s} - \int_0^1 x_{t+s,i}\tilde{n}_{t+s,i}di - \int_0^1 x_{t+s-1,i}n_{t+s,i}di,$$

where, because of the assumption of a continuum of employment offices, individual offices do not have an effect on the aggregate wage (W_t) and the wages set by the owners of the differentiated labors in period t and $t-1$ ($x_{t,i}$ and $x_{t-1,i}$). Employment offices' demand for differentiated labor of workers whose wages are set in period t and $t-1$ is given by

$$N_{t+s}^{(\theta_w-1)/\theta_w} = \left(\int_0^1 \tilde{n}_{t+s,i}^{(\theta_w-1)/\theta_w}di + \int_0^1 n_{t+s,i}^{(\theta_w-1)/\theta_w}di\right) \tag{B44}$$

$$W_{t+s}\left(\int_0^1 \tilde{n}_{t+s,i}^{(\theta_w-1)/\theta_w}di + \int_0^1 n_{t+s,i}^{(\theta_w-1)/\theta_w}di\right)^{\theta_w/(\theta_w-1)-1} \tilde{n}_{t+s,i}^{-\frac{1}{\theta_w}} = x_{t+s,i} \tag{B45}$$

$$\tilde{n}_{t+s,i} = \left(\frac{x_{t+s,i}}{W_{t+s}}\right)^{-\theta_w} N_{t+s} \tag{B46}$$

$$n_{t+s,i} = \left(\frac{x_{t+s-1,i}}{W_{t+s}}\right)^{-\theta_w} N_{t+s}. \tag{B47}$$

Because the problem of each household is identical, they will set the same wage. This, together with the assumption that employment offices operate in a competitive market, yields

$$x_{t+s,i}\tilde{n}_{t+s,i} + x_{t+s-1,i}n_{t+s,i} = W_{t+s}N_{t+s} \tag{B48}$$

$$W_{t+s} = \left(\int_0^1 x_{t+s,i}^{1-\theta_w}di + \int_0^1 x_{t+s-1,i}^{1-\theta_w}di\right)^{\frac{1}{1-\theta_w}}. \tag{B49}$$

B.5.1.2 Wage Setting for Differentiated Labor in the Home Country

In period t, *before observing the shock*, only one of the two members of home households sets his/her wage, which remains fixed in period t and period $t+1$.

$$\max_{x_{t,i}} E_{t-1}\left[\left(-\frac{\tilde{n}_{t,i}^{1+\sigma_n}}{1+\sigma_n}+\lambda_t x_{t,i}\tilde{n}_{t,i}\right)+\beta\left(-\frac{n_{t+1,i}^{1+\sigma_n}}{1+\sigma_n}+\lambda_{t+1}x_{t,i}n_{t+1,i}\right)\right] \tag{B50}$$

$$\tilde{n}_{t,i}=\left(\frac{x_{t,i}}{W_t}\right)^{-\theta_w}N_t \;\Rightarrow \tag{B51}$$

$$\frac{\tilde{n}_{t,i}^{1+\sigma_n}}{1+\sigma_n}=\frac{\left(\left(\frac{x_{t,i}}{W_t}\right)^{-\theta_w}N_t\right)^{1+\sigma_n}}{1+\sigma_n}=\frac{W_t^{\theta_w(1+\sigma_n)}N_t^{1+\sigma_n}x_{t,i}^{-\theta_w(1+\sigma_n)}}{1+\sigma_n} \tag{B52}$$

$$E_{t-1}\lambda_t x_{t,i}\tilde{n}_{t,i}=E_{t-1}\lambda_t W_t^{\theta_w}N_t x_{t,i}^{1-\theta_w} \tag{B53}$$

$$E_{t-1}\lambda_{t+1}x_{t,i}n_{t+1,i}=E_{t-1}\lambda_{t+1}W_{t+1}^{\theta_w}N_{t+1}x_{t,i}^{1-\theta_w} \tag{B54}$$

$$E_{t-1}\frac{n_{t+1,i}^{1+\sigma_n}}{1+\sigma_n}=E_{t-1}\frac{W_{t+1}^{\theta_w(1+\sigma_n)}N_{t+1}^{1+\sigma_n}x_{t,i}^{-\theta_w(1+\sigma_n)}}{1+\sigma_n} \tag{B55}$$

The first-order condition can be written as

$$\begin{aligned}&E_{t-1}\theta_w W_t^{\theta_w(1+\sigma_n)}N_t^{1+\sigma_n}x_{t,i}^{-\theta_w(1+\sigma_n)-1}+E_{t-1}(1-\theta_w)\lambda_t W_t^{\theta_w}N_t x_{t,i}^{-\theta_w}\\&\quad+\beta E_{t-1}\left(\theta_w W_{t+1}^{\theta_w(1+\sigma_n)}N_{t+1}^{1+\sigma_n}x_{t,i}^{-\theta_w(1+\sigma_n)-1}\right)\\&\quad+\beta E_{t-1}\left((1-\theta_w)\lambda_{t+1}W_{t+1}^{\theta_w}N_{t+1}x_{t,i}^{-\theta_w}\right)=0\end{aligned} \tag{B56}$$

$$\begin{aligned}&\theta_w E_{t-1}W_t^{\theta_w(1+\sigma_n)}N_t^{1+\sigma_n}x_{t,i}^{-\theta_w(1+\sigma_n)-1}+(1-\theta_w)E_{t-1}\lambda_t W_t^{\theta_w}N_t x_{t,i}^{-\theta_w}\\&\quad+\beta\theta_w E_{t-1}W_{t+1}^{\theta_w(1+\sigma_n)}N_{t+1}^{1+\sigma_n}x_{t,i}^{-\theta_w(1+\sigma_n)-1}\\&\quad+\beta(1-\theta_w)E_{t-1}\lambda_{t+1}W_{t+1}^{\theta_w}N_{t+1}x_{t,i}^{-\theta_w}=0\end{aligned} \tag{B57}$$

$$\begin{aligned}&\theta_w E_{t-1}W_t^{\theta_w+\theta_w\sigma_n}N_t^{1+\sigma_n}x_{t,i}^{-\theta_w-\theta_w\sigma_n-1}+(1-\theta_w)E_{t-1}\lambda_t W_t^{\theta_w}N_t x_{t,i}^{-\theta_w}\\&\quad+\beta\theta_w E_{t-1}W_{t+1}^{\theta_w+\theta_w\sigma_n}N_{t+1}^{1+\sigma_n}x_{t,i}^{-\theta_w-\theta_w\sigma_n-1}\\&\quad+(1-\theta_w)\beta E_{t-1}\lambda_{t+1}W_{t+1}^{\theta_w}N_{t+1}x_{t,i}^{-\theta_w}=0.\end{aligned} \tag{B58}$$

Since $x_{t,i}$ is predetermined, its value is known for certain in period $t-1$. Hence, (B58) can be rewritten as

$$\begin{aligned}&\theta_w E_{t-1} W_t^{\theta_w+\theta_w\sigma_n} N_t^{1+\sigma_n} x_{t,i}^{-\theta_w\sigma_n-1} + (1-\theta_w) E_{t-1}\lambda_t W_t^{\theta_w} N_t \\ &\quad + \beta\theta_w E_{t-1} W_{t+1}^{\theta_w+\theta_w\sigma_n} N_{t+1}^{1+\sigma_n} x_{t,i}^{-\theta_w\sigma_n-1} \\ &\quad + (1-\theta_w)\beta E_{t-1}\lambda_{t+1} W_{t+1}^{\theta_w} N_{t+1} = 0. \qquad \text{(B59)}\end{aligned}$$

The log-linear approximation of (B59) yields

$$\begin{aligned}&\left\{(\theta_w+\theta_w\sigma_n)E_{t-1}\hat{W}_t + (1+\sigma_n)E_{t-1}\hat{N}_t - (1+\theta_w\sigma_n)E_{t-1}\hat{x}_{t,i}\right\} \\ &\times \theta_w \bar{W}_t^{\theta_w\sigma_n} \bar{N}^{1+\sigma_n} \bar{x}_{t,i}^{-\theta_w\sigma_n-1} + \left\{E_{t-1}\hat{\lambda}_t + \theta_w E_{t-1}\hat{W}_t + E_{t-1}\hat{N}_t\right\} \\ &\times (1-\theta_w)\bar{\lambda}_t \bar{W}_t^{\theta_w} \bar{N} \Big\{\beta(\theta_w+\theta_w\sigma_n)E_{t-1}\hat{W}_{t+1} + \beta(1+\sigma_n) \\ &\times E_{t-1}\hat{N}_{t+1} - \beta(1+\theta_w\sigma_n)E_{t-1}\hat{x}_{t,i}\Big\}\theta_w \bar{W}_{t+1}^{\theta_w\sigma_n} \bar{N}^{1+\sigma_n} \bar{x}_{t,i}^{-\theta_w\sigma_n-1} \\ &+ \left\{\beta E_{t-1}\hat{\lambda}_{t+1} + \beta\theta_w E_{t-1}\hat{W}_{t+1} + \beta E_{t-1}\hat{N}_{t+1}\right\} \\ &\times (1-\theta_w)\bar{\lambda}_{t+1} \bar{W}_{t+1}^{\theta_w} \bar{N} = 0. \qquad \text{(B60)}\end{aligned}$$

At the steady state, all wages are flexible. Hence, the following equation has to hold:

$$\theta_w \bar{W}_t^{\theta_w+\theta_w\sigma_n} \bar{N}^{1+\sigma_n} \bar{x}_{t,i}^{-\theta_w\sigma_n-1} = -(1-\theta_w)\bar{\lambda}_t \bar{W}_t^{\theta_w} \bar{N} \qquad \text{(B61)}$$

$$\theta_w \bar{W}_{t+1}^{\theta_w+\theta_w\sigma_n} \bar{N}^{1+\sigma_n} \bar{x}_{t+1,i}^{-\theta_w\sigma_n-1} = -(1-\theta_w)\bar{\lambda}_{t+1} \bar{W}_{t+1}^{\theta_w} \bar{N} \qquad \text{(B62)}$$

$$\theta_w \bar{W}_t^{\theta_w\sigma_n} \bar{N}^{1+\sigma_n} \bar{x}_{t,i}^{-\theta_w\sigma_n-1} = -(1-\theta_w)\bar{\lambda}_t \bar{N}$$

$$\theta_w \left(\frac{\bar{W}_t}{\bar{P}_t}\right)^{\theta_w\sigma_n} \bar{N}^{1+\sigma_n} \left(\frac{\bar{x}_{t,i}}{\bar{P}_t}\right)^{-\theta_w\sigma_n-1} = -(1-\theta_w)\bar{\lambda}_t \bar{P}_t \bar{N}. \qquad \text{(B63)}$$

From (B8), $\bar{\lambda}_t \bar{P}_t$ must be constant over time, suggesting that the left-hand side of (B63) is also constant over time, which occurs if both $\frac{\bar{W}_t}{\bar{P}_t}$ and $\frac{\bar{x}_t}{\bar{P}_t}$ are constant. Using this and (B10), one can show that

$$\bar{\lambda}_t \bar{P}_t = \bar{\lambda}_{t+1} \bar{P}_{t+1} \Rightarrow \bar{\lambda}_{t+1} = \bar{\lambda}_t \frac{\bar{P}_t}{\bar{P}_{t+1}} = \bar{\lambda}_t \frac{1}{\frac{\bar{P}_{t+1}}{\bar{P}_t}} \Rightarrow \bar{\lambda}_{t+1} = \frac{\bar{\lambda}_t}{e^{\bar{\pi}}}$$

$$\frac{\bar{W}_{t+1}}{\bar{P}_{t+1}} = \frac{\bar{W}_t}{\bar{P}_t} \Rightarrow \bar{W}_{t+1} = \bar{W}_t \frac{\bar{P}_{t+1}}{\bar{P}_t} \Rightarrow \bar{W}_{t+1} = \bar{W}_t e^{\bar{\pi}}$$

$$\bar{x}_{t+1,i} = \bar{x}_{t,i} \frac{\bar{P}_{t+1}}{\bar{P}_t} \Rightarrow \bar{x}_{t+1,i} = \bar{x}_{t,i} e^{\bar{\pi}}. \tag{B64}$$

One can use (B62) and (B64) to show that

$$\begin{aligned}
&\theta_w \bar{W}_{t+1}^{\theta_w+\theta_w\sigma_n} \bar{N}^{1+\sigma_n} \bar{x}_{t,i}^{-\theta_w\sigma_n-1} \\
&= \theta_w \bar{W}_{t+1}^{\theta_w+\theta_w\sigma_n} \bar{N}^{1+\sigma_n} \bar{x}_{t+1,i}^{-\theta_w\sigma_n-1} \left(\frac{\bar{x}_{t,i}}{\bar{x}_{t+1,i}}\right)^{-\theta_w\sigma_n-1} \\
&= -(1-\theta_w)\bar{\lambda}_{t+1} \bar{W}_{t+1}^{\theta_w} \bar{N} \left(\frac{\bar{x}_{t,i}}{\bar{x}_{t+1,i}}\right)^{-\theta_w\sigma_n-1} \\
&= -(1-\theta_w)\left(\frac{\bar{\lambda}_t}{e^{\bar{\pi}}}\right)\left(\bar{W}_t e^{\bar{\pi}}\right)^{\theta_w} \bar{N} e^{\bar{\pi}(\theta_w\sigma_n+1)} \\
&= -(1-\theta_w)\bar{\lambda}_t \bar{W}_t^{\theta_w} \bar{N} e^{\bar{\pi}\theta_w(1+\sigma_n)}.
\end{aligned} \tag{B65}$$

Similarly, using (B64), one can write that

$$\begin{aligned}
(1-\theta_w)\bar{\lambda}_{t+1}\bar{W}_{t+1}^{\theta_w}\bar{N} &= (1-\theta_w)\left(\frac{\bar{\lambda}_t}{e^{\bar{\pi}}}\right)\left(\bar{W}_t e^{\bar{\pi}}\right)^{\theta_w} \bar{N} \\
&= (1-\theta_w)\bar{\lambda}_t \bar{W}_t^{\theta_w} \bar{N} e^{\bar{\pi}(\theta_w-1)}.
\end{aligned} \tag{B66}$$

Using (B61), (B65), and (B66), (B60) can be rewritten as

$$\begin{aligned}
&\Bigg[-\Big\{(\theta_w+\theta_w\sigma_n)E_{t-1}\hat{W}_t + (1+\sigma_n)E_{t-1}\hat{N}_t - (1+\theta_w\sigma_n)\hat{x}_{t,i}\Big\} \\
&\times (1-\theta_w)\bar{\lambda}_t \bar{W}_t^{\theta_w}\bar{N} + \Big\{E_{t-1}\hat{\lambda}_t + \theta_w E_{t-1}\hat{W}_t + E_{t-1}\hat{N}_t\Big\} \\
&\times (1-\theta_w)\bar{\lambda}_t \bar{W}_t^{\theta_w}\bar{N} - e^{\bar{\pi}\theta_w(\sigma_n+1)}\Big\{\beta(\theta_w+\theta_w\sigma_n)E_{t-1}\hat{W}_{t+1} \\
&+ \beta(1+\sigma_n)E_{t-1}\hat{N}_{t+1} - \beta(1+\theta_w\sigma_n)\hat{x}_{t,i}\Big\}(1-\theta_w)\bar{\lambda}_t \bar{W}_t^{\theta_w}\bar{N} \\
&+ e^{\bar{\pi}(\theta_w-1)}\Big\{\beta E_{t-1}\hat{\lambda}_{t+1} + \beta\theta_w E_{t-1}\hat{W}_{t+1} + \beta E_{t-1}\hat{N}_{t+1}\Big\} \\
&\times (1-\theta_w)\bar{\lambda}_t \bar{W}_t^{\theta_w}\bar{N}\Bigg] = 0
\end{aligned} \tag{B67}$$

$$\Bigg[-(\theta_w+\theta_w\sigma_n)E_{t-1}\hat{W}_t-(1+\sigma_n)E_{t-1}\hat{N}_t+(1+\theta_w\sigma_n)\hat{x}_{t,i}$$
$$+E_{t-1}\hat{\lambda}_t+\theta_w E_{t-1}\hat{W}_t+E_{t-1}\hat{N}_t-e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta(\theta_w+\theta_w\sigma_n)$$
$$\times E_{t-1}\hat{W}_{t+1}-e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta(1+\sigma_n)E_{t-1}\hat{N}_{t+1}+e^{\bar{\pi}\theta_w(\sigma_n+1)}$$
$$\times\beta(1+\theta_w\sigma_n)\hat{x}_{t,i}+e^{\bar{\pi}(\theta_w-1)}\beta E_{t-1}\hat{\lambda}_{t+1}$$
$$+\ e^{\bar{\pi}(\theta_w-1)}\beta\theta_w E_{t-1}\hat{W}_{t+1}+e^{\bar{\pi}(\theta_w-1)}\beta E_{t-1}\hat{N}_{t+1}\Bigg]=0 \qquad \text{(B68)}$$

$$\Bigg[-\theta_w\sigma_n E_{t-1}\hat{W}_t-\sigma_n E_{t-1}\hat{N}_t+\left\{1+e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)$$
$$\times E_{t-1}\hat{x}_{t,i}+E_{t-1}\hat{\lambda}_t+e^{\bar{\pi}(\theta_w-1)}\beta E_{t-1}\hat{\lambda}_{t+1}\Big\{e^{\bar{\pi}(\theta_w-1)}\beta\theta_w$$
$$-e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta(\theta_w+\theta_w\sigma_n)\Big\}E_{t-1}\hat{W}_{t+1}$$
$$+\left\{e^{\bar{\pi}(\theta_w-1)}\beta-e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta(1+\sigma_n)\right\}E_{t-1}\hat{N}_{t+1}\Bigg]=0. \qquad \text{(B69)}$$

Using (B7), (B12), and the law of iterated expectations, one can rewrite (B69) as

$$\Bigg[-\theta_w\sigma_n E_{t-1}\hat{W}_t-\sigma_n E_{t-1}\hat{N}_t+\left\{1+e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)\hat{x}_{t,i}$$
$$+E_{t-1}\left(-\sigma_c\hat{C}_t-\hat{P}_t\right)+e^{\bar{\pi}(\theta_w-1)}\beta E_{t-1}\left(-\sigma_c\hat{C}_{t+1}-\hat{P}_{t+1}\right)$$
$$-\left\{e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta(\theta_w+\theta_w\sigma_n)-e^{\bar{\pi}(\theta_w-1)}\beta\theta_w\right\}E_{t-1}\hat{W}_{t+1}$$
$$-\left\{e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta(1+\sigma_n)-e^{\bar{\pi}(\theta_w-1)}\beta\right\}E_{t-1}\hat{N}_{t+1}\Bigg]=0 \qquad \text{(B70)}$$

$$\hat{x}_{t,i}=\frac{\theta_w\sigma_n}{\left\{1+e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_{t-1}\hat{W}_t$$
$$+\frac{\sigma_n}{\left\{1+e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_{t-1}\hat{N}_t$$

$$+ \frac{\sigma_c}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_{t-1}\hat{C}_t$$

$$+ \frac{1}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_{t-1}\hat{P}_t$$

$$+ \frac{\left\{e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta(\theta_w + \theta_w\sigma_n) - e^{\bar{\pi}(\theta_w-1)}\beta\theta_w\right\}}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_{t-1}\hat{W}_{t+1}$$

$$+ \frac{\left\{e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta(1 + \sigma_n) - e^{\bar{\pi}(\theta_w-1)}\beta\right\}}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_{t-1}\hat{N}_{t+1}$$

$$+ \frac{\sigma_c e^{\bar{\pi}(\theta_w-1)}\beta}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_{t-1}\hat{C}_{t+1}$$

$$+ \frac{e^{\bar{\pi}(\theta_w-1)}\beta}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_{t-1}\hat{P}_{t+1} \tag{B71}$$

$$E_t\hat{x}_{t+1} = \frac{\theta_w\sigma_n}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_t\hat{W}_{t+1}$$

$$+ \frac{\sigma_n}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_t\hat{N}_{t+1}$$

$$+ \frac{\sigma_c}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_t\hat{C}_{t+1}$$

$$+ \frac{1}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_t\hat{P}_{t+1}$$

$$+ \frac{\left\{e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta(\theta_w + \theta_w\sigma_n) - e^{\bar{\pi}(\theta_w-1)}\beta\theta_w\right\}}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_t\hat{W}_{t+2}$$

$$+ \frac{\left\{e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta(1 + \sigma_n) - e^{\bar{\pi}(\theta_w-1)}\beta\right\}}{\left\{1 + e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)} E_t\hat{N}_{t+2}$$

$$+\frac{\sigma_c e^{\bar{\pi}(\theta_w-1)}\beta}{\left\{1+e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_t\hat{C}_{t+2}$$

$$+\frac{e^{\bar{\pi}(\theta_w-1)}\beta}{\left\{1+e^{\bar{\pi}\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_t\hat{P}_{t+2} \tag{B72}$$

$$W_t=\left(\int_0^1 x_{t,i}^{1-\theta_w}di+\int_0^1 x_{t-1,i}^{1-\theta_w}di\right)^{\frac{1}{1-\theta_w}}$$

$$\bar{W}_t^{1-\theta_w}=\bar{x}_t^{1-\theta_w}+\bar{x}_{t-1}^{1-\theta_w}$$

$$=\left(\frac{\bar{P}_t}{\bar{P}_{t-1}}\bar{x}_{t-1}\right)^{1-\theta_w}+\bar{x}_{t-1}^{1-\theta_w}$$

$$=\left\{e^{\bar{\pi}(1-\theta_w)}+1\right\}\bar{x}_{t-1}^{1-\theta_w}$$

$$\left(\frac{\bar{x}_{t-1}}{\bar{W}_t}\right)^{1-\theta_w}=\frac{1}{e^{\bar{\pi}(1-\theta_w)}+1}$$

$$\left(\frac{\bar{x}_t}{\bar{W}_t}\right)^{1-\theta_w}=\frac{e^{\bar{\pi}(1-\theta_w)}}{e^{\bar{\pi}(1-\theta_w)}+1}$$

$$(1-\theta_w)\bar{W}_t^{1-\theta_w}\hat{W}_t=(1-\theta_w)\bar{x}_t^{1-\theta_w}\hat{x}_t+(1-\theta_w)\bar{x}_{t-1}^{1-\theta_w}\hat{x}_{t-1}$$

$$\hat{W}_t=\left(\frac{\bar{x}_t}{\bar{W}_t}\right)^{1-\theta_w}\hat{x}_t+\left(\frac{\bar{x}_{t-1}}{\bar{W}_t}\right)^{1-\theta_w}\hat{x}_{t-1} \tag{B73}$$

$$\hat{W}_t=\frac{e^{\bar{\pi}(1-\theta_w)}}{e^{\bar{\pi}(1-\theta_w)}+1}\hat{x}_t+\frac{1}{e^{\bar{\pi}(1-\theta_w)}+1}\hat{x}_{t-1}. \tag{B74}$$

B.5.2 Flexible Wages

When wages are flexible, the discounted utility function that home households aim to maximize is given by the following function:

$$E_t\sum_{s=0}^{\infty}\beta^{t+s}\left(\frac{C_{t+s}^{1-\sigma_c}-1}{1-\sigma_c}-\frac{N_t^{1+\sigma_n}}{1+\sigma_n}+\frac{\left(\frac{M_{t+s}}{P_{t+s}}\right)^{1-\sigma_m}-1}{1-\sigma_m}\right).$$

Under flexible wage adjustment, home households' flow budget constraint is given by

$$\begin{aligned} &P_{t+s}\left(C_{t+s}+I_{t+s}+a(u_{t+s})K_{t+s}\right)+\mathcal{E}_{t+s}B_{t+s+1}+M_t \\ &\quad = M_{t-1}+M_t^a-M_{t-1}^a+W_tN_t+R_{t+s}^k u_{t+s}K_{t+s} \\ &\qquad + R_{\mathcal{H},t+s-1}\mathcal{E}_{t+s}B_{t+s}+\Pi_{t+s}. \end{aligned} \tag{B75}$$

The first-order condition with respect to N_t can be written as

$$N_t^{\sigma_n}=\lambda_t W_t. \tag{B76}$$

Using (B6), (B76) can be restated as

$$N_t^{\sigma_n}=C_t^{-\sigma_c}\frac{W_t}{P_t}. \tag{B77}$$

Log-linearizing (B77) yields

$$\sigma_n\hat{N}_t+\sigma_c\hat{C}_t=\hat{W}_t-\hat{P}_t. \tag{B78}$$

B.6 The Foreign-Household Budget Constraint

Foreign households' optimization problem and flow budget constraint can be written as

$$\begin{aligned} &\max_{C_t^*,u_t^*,I_t^*,B_{t+1}^*,D_{t+1}^*,M_t^*} E_t\sum_{s=0}^{\infty}\beta^{t+s} \\ &\quad\times\left(\frac{{C_{t+s}^*}^{1-\sigma_c}-1}{1-\sigma_c}-\frac{{\tilde{n}_{t+s,i}^*}^{1+\sigma_n}}{1+\sigma_n}-\frac{{n_{t+s,i}^*}^{1+\sigma_n}}{1+\sigma_n}+\frac{\left(\frac{M_{t+s}^*}{P_{t+s}^*}\right)^{1-\sigma_m}-1}{1-\sigma_m}\right) \end{aligned} \tag{B79}$$

$$\begin{aligned} &\frac{P_{t+s}}{\mathcal{E}_{t+s}}B_{t+s+1}^*+P_{t+s}^*\left(C_{t+s}^*+I_{t+s}^*+a(u_{t+s}^*)K_{t+s}^*+D_{t+s+1}^*\right)+M_t^* \\ &\quad = M_{t-1}^*+{M_t^*}^a-{M_{t-1}^*}^a+x_{t+s,i}^*\tilde{n}_{t+s,i}^*+x_{t+s-1,i}^*n_{t+s,i}^* \\ &\qquad + {R_{t+s}^*}^k u_{t+s}^*K_{t+s}^*+R_{t+s-1}^*P_{t+s-1}^*D_{t+s}^*+\frac{R_{\mathcal{F},t+s-1}P_{t+s-1}}{\mathcal{E}_{t+s}}B_{t+s}^* \\ &\qquad + \Pi_{t+s}^*, \end{aligned} \tag{B80}$$

where the variables denoted with the superscript * represent the foreign counterparts of the home variables. In addition to the one-period risk-free home-country bond, foreign households also have access to the one-period foreign-country bond, their holdings of which are denoted by D^*_{t+1}. $\mathcal{E}_t$ stands for the nominal exchange rate between the currency of the home country (¢) and the foreign country (¢*). ${M^*_t}^a - {M^*_{t-1}}^a$ denotes seigniorage revenue in the foreign country, which is transferred back to foreign households by the foreign monetary authority. The law of motion for capital is given by

$$K^*_{t+s+1} = (1-\delta)K^*_{t+s} + \phi\left(\frac{I^*_{t+s}}{K^*_{t+s}}\right)K^*_{t+s} \tag{B81}$$

$$\begin{aligned}\mathcal{L}^* = E_t\sum_{s=0}^{\infty}\beta^{t+s}\Bigg[&\left(\frac{{C^*_{t+s}}^{1-\sigma_c}-1}{1-\sigma_c} - \frac{{\tilde{n}^*_{t+s,i}}^{1+\sigma_n}}{1+\sigma_n} - \frac{{n^*_{t+s,i}}^{1+\sigma_n}}{1+\sigma_n} + \frac{\left(\frac{M^*_{t+s}}{P^*_{t+s}}\right)^{1-\sigma_m}-1}{1-\sigma_m}\right)\\
&-\lambda^*_{t+s}\Bigg\{P^*_{t+s}\left(C^*_{t+s}+I^*_{t+s}+a(u^*_{t+s})K^*_{t+s}+D^*_{t+s+1}\right)+\frac{P_{t+s}}{\mathcal{E}_{t+s}}B^*_{t+s+1}\\
&+M^*_t - x^*_{t+s,i}\tilde{n}^*_{t+s,i} - x^*_{t+s-1,i}n^*_{t+s,i} - M^*_{t-1}\\
&-{M^*_t}^a + {M^*_{t-1}}^a - {R^*_{t+s}}^k u^*_{t+s}K^*_{t+s} - R^*_{t+s-1}P^*_{t+s-1}D^*_{t+s}\\
&-\frac{R_{\mathcal{F},t+s-1}P_{t+s-1}}{\mathcal{E}_{t+s}}B^*_{t+s} - \Pi^*_{t+s}\Bigg\}\\
&-\mu_{t+s}\left(K^*_{t+s+1}-(1-\delta)K^*_{t+s}-\phi\left(\frac{I^*_{t+s}}{K^*_{t+s}}\right)K^*_{t+s}\right)\Bigg].\end{aligned} \tag{B82}$$

B.6.1 With Respect to C^*_t

$${C_t^*}^{-\sigma_c} = \lambda^*_t {P_t}^*$$

$$-\sigma_c{\hat{C}_t}^* = \hat{\lambda}^*_t + {\hat{P}_t}^* \tag{B83}$$

$$-\sigma_c\hat{C}^*_t - \hat{P}^*_t = \hat{\lambda}^*_t \tag{B84}$$

$${\bar{C}^*}^{-\sigma_c} = \bar{\lambda}^*_t\bar{P}^*_t = \bar{\lambda}^*_{t+1}\bar{P}^*_{t+1} \tag{B85}$$

Define $\bar{\pi}^*$ as $log\bar{P}^*_{t+1} - log\bar{P}^*_t$.

$$\bar{\pi}^* = log\frac{\bar{P}^*_{t+1}}{\bar{P}^*_t} \Rightarrow \frac{\bar{P}^*_{t+1}}{\bar{P}^*_t} = e^{\bar{\pi}^*} \tag{B86}$$

$$E_t C^{*\,-\sigma_c}_{t+1} = E_t \lambda^*_{t+1} P^*_{t+1}$$

$$\bar{C}^{*\,-\sigma_c} - \sigma_c \bar{C}^{*\,-\sigma_c} E_t \hat{C}^*_{t+1} = \bar{\lambda}^*_{t+1}\bar{P}^*_{t+1} + \bar{\lambda}^*_{t+1}\bar{P}^*_{t+1} E_t \hat{\lambda}^*_{t+1} + \bar{\lambda}^*_{t+1}\bar{P}^*_{t+1} E_t \hat{P}^*_{t+1} \tag{B87}$$

$$-\sigma_c E_t \hat{C}^*_{t+1} - E_t \hat{P}^*_{t+1} = E_t \hat{\lambda}^*_{t+1} \tag{B88}$$

$$\sigma_c E_t \left(\hat{C}^*_{t+1} - \hat{C}^*_t\right) + E_t\left(\hat{P}^*_{t+1} - \hat{P}^*_t\right) = E_t\left(\hat{\lambda}^*_t - \hat{\lambda}^*_{t+1}\right) \tag{B89}$$

From (B97), one can rewrite (B89) as

$$\sigma_c E_t \left(\hat{C}^*_{t+1} - \hat{C}^*_t\right) + E_t\left(\hat{P}^*_{t+1} - \hat{P}^*_t\right) = \hat{R}_{\mathcal{F},t} + E_t\left(\hat{\mathcal{E}}^*_t - \hat{\mathcal{E}}^*_{t+1}\right) \tag{B90}$$

$$\sigma_c E_t \left(\hat{C}^*_{t+1} - \hat{C}^*_t\right) + E_t\left(\hat{P}^*_{t+1} - \hat{P}^*_t\right) + E_t\left(\hat{\mathcal{E}}^*_{t+1} - \hat{\mathcal{E}}^*_t\right) = \hat{R}_{\mathcal{F},t}. \tag{B91}$$

B.6.2 With Respect to D^*_{t+1}

$$\beta^t \lambda^*_t P^*_t = \beta^{t+1} E_t \lambda^*_{t+1} R^*_t P^*_t$$

$$\lambda^*_t P^*_t = \beta R^*_t E_t \lambda^*_{t+1} P^*_{t+1} \frac{P^*_t}{P^*_{t+1}}$$

$$\bar{R}^* = \frac{1}{\beta}\frac{\bar{P}^*_{t+1}}{\bar{P}^*_t} = \frac{1}{\beta} e^{\bar{\pi}^*} \tag{B92}$$

$$\bar{\lambda}^*_t \bar{P}^*_t \left(\hat{\lambda}^*_t + \hat{P}^*_t\right) = \beta \bar{R}^* \bar{\lambda}^*_{t+1} \bar{P}^*_{t+1} \frac{\bar{P}^*_t}{\bar{P}^*_{t+1}} \left[\hat{R}^*_t + E_t\left(\hat{\lambda}^*_{t+1} + \hat{P}^*_t\right)\right]$$

$$\left(\hat{\lambda}^*_t + \hat{P}^*_t\right) = \hat{R}^*_t + E_t\left(\hat{\lambda}^*_{t+1} + \hat{P}^*_t\right)$$

$$E_t\left(\hat{\lambda}^*_t - \hat{\lambda}^*_{t+1}\right) = \hat{R}^*_t \tag{B93}$$

B.6.3 With Respect to B^*_{t+1}

$$\beta^t \lambda^*_t \frac{P_t}{\mathcal{E}_t} = \beta^{t+1} E_t \lambda^*_{t+1} \frac{R_{\mathcal{F},t} P_t}{\mathcal{E}_{t+1}}$$

$$\lambda^*_t P^*_t \frac{1}{P^*_t} \frac{P_t}{\mathcal{E}_t} = \beta R_{\mathcal{F},t} E_t \lambda^*_{t+1} P^*_{t+1} \frac{1}{P^*_{t+1}} \frac{P_t}{\mathcal{E}_{t+1}} \tag{B94}$$

Using (B110), one can write the following equation:

$$\bar{R}_{\mathcal{F}} = \frac{1}{\beta} \frac{\bar{P}^*_{t+1}}{\bar{P}^*_t} \frac{\bar{\mathcal{E}}^*_{t+1}}{\bar{\mathcal{E}}^*_t} = \frac{1}{\beta} e^{\bar{\pi}^*} e^{\bar{\pi}-\bar{\pi}^*} = \frac{1}{\beta} e^{\bar{\pi}} \tag{B95}$$

$$\bar{\lambda}^*_t \bar{P}^*_t \frac{1}{\bar{P}^*_t} \frac{\bar{P}_t}{\bar{\mathcal{E}}_t} \left(\hat{\lambda}^*_t + \hat{P}^*_t - \hat{P}^*_t + \hat{P}_t - \hat{\mathcal{E}}_t \right)$$

$$= \beta \bar{R}_{\mathcal{F}} \bar{\lambda}^*_{t+1} \bar{P}^*_{t+1} \frac{1}{\bar{P}^*_{t+1}} \frac{\bar{P}_t}{\bar{\mathcal{E}}_{t+1}}$$

$$\times \left[\hat{R}_{\mathcal{F},t} + E_t \left(\hat{\lambda}^*_{t+1} + \hat{P}^*_{t+1} - \hat{P}^*_{t+1} + \hat{P}_t - \hat{\mathcal{E}}_{t+1} \right) \right]$$

$$\left(\hat{\lambda}^*_t - \hat{\mathcal{E}}_t \right) = \hat{R}_{\mathcal{F},t} + E_t \left(\hat{\lambda}^*_{t+1} - \hat{\mathcal{E}}_{t+1} \right) \tag{B96}$$

$$E_t \left(\hat{\lambda}^*_t - \hat{\lambda}^*_{t+1} \right) = \hat{R}_{\mathcal{F},t} + E_t \left(\hat{\mathcal{E}}^*_t - \hat{\mathcal{E}}^*_{t+1} \right). \tag{B97}$$

B.6.4 With Respect to M^*_t

Similar to (B18), one can show that demand for money in the foreign country is given by

$$-\sigma_m \left(\hat{M}^*_t - \hat{P}^*_t \right) + \sigma_c \hat{C}^*_t = \frac{1}{\frac{1}{\beta} e^{\bar{\pi}^*} - 1} \hat{R}^*_t. \tag{B98}$$

B.6.5 With Respect to I^*_t

Similar to (B24), one can show that

$$-\sigma_c \hat{C}^*_t = \hat{\mu}^*_t + \sigma_\phi \left(\hat{I}^*_t - \hat{K}^*_t \right). \tag{B99}$$

B.6.6 With Respect to u_t^*

$$-\lambda_t^* \left(P_t^* a'(u_t^*) K_t^* - R_t^{*^k} K_t \right) = 0 \tag{B100}$$

Letting $r_t^{*^k}$ denote the real rental rate of capital in the foreign country, we define $r_t^{*^k}$ as $\frac{R_t^{*^k}}{P_t}$

$$\begin{aligned} \bar{r}^{*^k} &= a'(\bar{u}^*) \\ a'(u_t^*) &= r_t^{*^k} \\ \hat{r}_t^{*^k} &= \sigma_a \hat{u}_t^*. \end{aligned} \tag{B101}$$

B.6.7 With Respect to K_{t+1}^*

Similar to (B34), one can show that

$$\hat{\mu}_t^* = E_t \left(\hat{\mu}_{t+1}^* + \sigma_\phi (1-\beta) \left(\hat{I}_{t+1}^* - \hat{K}_{t+1}^* \right) + \{1 - \beta(1-\delta)\} \hat{r}_{t+1}^{*^k} \right). \tag{B102}$$

B.7 The Law of Motion for Capital in the Foreign Country

Similar to (B38), the law of motion for capital in the foreign country evolves as

$$\hat{K}_{t+1}^* = (1-\delta)\hat{K}_t^* + \delta \hat{I}_t^*. \tag{B103}$$

B.8 The Aggregate Resource Constraint in the Foreign Country

Similar to (B42), one can show that

$$s_C^* \hat{C}_t^* + s_I^* \hat{I}_t^* + \frac{\bar{s}_I^*}{\delta} \left(\frac{1}{\beta} - (1-\delta) \right) \hat{u}_t^* = \hat{Y}_t^*. \tag{B104}$$

B.9 The Wage-Setting Equation in the Foreign Country

As in the case for the home country, we work with both predetermined- and flexible-wage arrangements in the foreign country. We first describe predetermined wage setting.

B.9.1 Predetermined Wages

B.9.1.1 The Wage Setting for Differentiated Labor in the Foreign Country

Similar to (B71), (B72), and (B74), one can show that

$$
\begin{aligned}
\hat{x}^*_{t,i} = {} & \frac{\theta_w \sigma_n}{\left\{1 + e^{\bar{\pi}^* \theta_w (\sigma_n + 1)} \beta\right\}(1 + \theta_w \sigma_n)} E_{t-1} \hat{W}^*_t \\
& + \frac{\sigma_n}{\left\{1 + e^{\bar{\pi}^* \theta_w (\sigma_n + 1)} \beta\right\}(1 + \theta_w \sigma_n)} E_{t-1} \hat{N}^*_t \\
& + \frac{\sigma_c}{\left\{1 + e^{\bar{\pi}^* \theta_w (\sigma_n + 1)} \beta\right\}(1 + \theta_w \sigma_n)} E_{t-1} \hat{C}^*_t \\
& + \frac{1}{\left\{1 + e^{\bar{\pi}^* \theta_w (\sigma_n + 1)} \beta\right\}(1 + \theta_w \sigma_n)} E_{t-1} \hat{P}^*_t \\
& + \frac{\left\{e^{\bar{\pi}^* \theta_w (\sigma_n + 1)} \beta(\theta_w + \theta_w \sigma_n) - e^{\bar{\pi}^* (\theta_w - 1)} \beta \theta_w\right\}}{\left\{1 + e^{\bar{\pi}^* \theta_w (\sigma_n + 1)} \beta\right\}(1 + \theta_w \sigma_n)} E_{t-1} \hat{W}^*_{t+1} \\
& + \frac{\left\{e^{\bar{\pi}^* \theta_w (\sigma_n + 1)} \beta(1 + \sigma_n) - e^{\bar{\pi}^* (\theta_w - 1)} \beta\right\}}{\left\{1 + e^{\bar{\pi}^* \theta_w (\sigma_n + 1)} \beta\right\}(1 + \theta_w \sigma_n)} E_{t-1} \hat{N}^*_{t+1} \\
& + \frac{\sigma_c e^{\bar{\pi}^* (\theta_w - 1)} \beta}{\left\{1 + e^{\bar{\pi}^* \theta_w (\sigma_n + 1)} \beta\right\}(1 + \theta_w \sigma_n)} E_{t-1} \hat{C}^*_{t+1} \\
& + \frac{e^{\bar{\pi}^* (\theta_w - 1)} \beta}{\left\{1 + e^{\bar{\pi}^* \theta_w (\sigma_n + 1)} \beta\right\}(1 + \theta_w \sigma_n)} E_{t-1} \hat{P}^*_{t+1} \qquad \text{(B105)}
\end{aligned}
$$

$$
\begin{aligned}
E_t\hat{x}^*_{t+1} = & \frac{\theta_w\sigma_n}{\left\{1+e^{\bar{\pi}^*\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_t\hat{W}^*_{t+1} \\
& + \frac{\sigma_n}{\left\{1+e^{\bar{\pi}^*\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_t\hat{N}^*_{t+1} \\
& + \frac{\sigma_c}{\left\{1+e^{\bar{\pi}^*\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_t\hat{C}^*_{t+1} \\
& + \frac{1}{\left\{1+e^{\bar{\pi}^*\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_t\hat{P}^*_{t+1} \\
& + \frac{\left\{e^{\bar{\pi}^*\theta_w(\sigma_n+1)}\beta(\theta_w+\theta_w\sigma_n) - e^{\bar{\pi}^*(\theta_w-1)}\beta\theta_w\right\}}{\left\{1+e^{\bar{\pi}^*\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_t\hat{W}^*_{t+2} \\
& + \frac{\left\{e^{\bar{\pi}^*\theta_w(\sigma_n+1)}\beta(1+\sigma_n) - e^{\bar{\pi}^*(\theta_w-1)}\beta\right\}}{\left\{1+e^{\bar{\pi}^*\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_t\hat{N}^*_{t+2} \\
& + \frac{\sigma_c e^{\bar{\pi}^*(\theta_w-1)}\beta}{\left\{1+e^{\bar{\pi}^*\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_t\hat{C}^*_{t+2} \\
& + \frac{e^{\bar{\pi}^*(\theta_w-1)}\beta}{\left\{1+e^{\bar{\pi}^*\theta_w(\sigma_n+1)}\beta\right\}(1+\theta_w\sigma_n)}E_t\hat{P}^*_{t+2} \qquad \text{(B106)}
\end{aligned}
$$

$$
\hat{W}^*_t = \frac{e^{\bar{\pi}^*(1-\theta_w)}}{e^{\bar{\pi}^*(1-\theta_w)}+1}\hat{x}^*_t + \frac{1}{e^{\bar{\pi}^*(1-\theta_w)}+1}\hat{x}^*_{t-1}. \qquad \text{(B107)}
$$

B.9.2 Flexible Wages

Similar to (B78), one can show the foreign-wage equation under flexible wages can be written as

$$
\sigma_n\hat{N}^*_t + \sigma_c\hat{C}^*_t = \hat{W}^*_t - \hat{P}^*_t. \qquad \text{(B108)}
$$

B.10 The Equation for the Real Exchange Rate

B.10.1 Debt-Elastic Interest Rate

It is notable that the nominal return pertinent to the holdings of the risk-free home-country bond in the foreign country—denoted by $R^B_{\mathcal{F},t-1}$ in (B80) may differ from that in the home country—denoted by $R^B_{\mathcal{H},t-1}$ in (B3). Following Devereux and Smith (2005), we assume that countries face a debt-elastic interest rate. Let the net position of the home country in the risk-free home-country bond be given as $\mathcal{B}_t$. The debtor country has to pay a higher interest rate than the lender country due to upward-sloping bond supply in international financial markets. The differential between $R^B_{\mathcal{F},t-1}$ and $R^B_{\mathcal{H},t-1}$ depends on the net home-country bond holdings of the countries in the following way:

$$R^B_{\mathcal{H},t} = \Theta\left(\mathcal{B}_{t+1} - \bar{\mathcal{B}}\right) R^B_{\mathcal{F},t}, \tag{B109}$$

where $\Theta\left(\mathcal{B}_{t+1} - \bar{\mathcal{B}}\right)$ satisfies $\Theta(0) = 1$ and $\Theta'(.) < 0$. Since there is a continuum of households in both countries, home-country bond holdings of any individual household (B_{t+1}) have only a negligible effect on the net position of countries' bond holdings ($\mathcal{B}_{t+1}$). Thus, households *do not internalize* the interest rate faced by their country.[4]

The following assumptions have been made: $\Theta(0) = 1$, $\Theta'(0) < 0$. In our notation, a hat over a variable denotes the variable's log-deviation from its steady-state value. However, since the log-linearization of the home-budget constraint involves the term $\frac{B_{t+s} - \bar{B}}{\bar{Y}}$, $\hat{B}_{t+s}$ is instead defined as $\hat{B}_{t+s} = \frac{B_{t+s} - \bar{B}}{\bar{Y}}$.

[4]Assuming a debt-elastic differential in the two countries' interest rates is a standard way to circumvent the problem of multiple steady states in imperfect financial markets. Without such an assumption, stationarity of the model would not be ensured, as when a shock is introduced into the model, the model oscillates between different steady states without ever reaching a stable equilibrium. For a more complete description, see Schmitt-Grohe and Uribe (2003) and Boileau and Normandin (2008), who describe the problem of multiple steady states in the small and large open-economy models with imperfect financial markets, respectively. They also evaluate different methods to circumvent this problem.

From (B15), (B95), and (B109), one can write that

$$\bar{R}_{\mathcal{H}} = \Theta(0)\bar{R}_{\mathcal{F}} \Rightarrow \bar{R}_{\mathcal{H}} = \bar{R}_{\mathcal{F}} \Rightarrow \frac{1}{\beta}e^{\bar{\pi}} = \frac{1}{\beta}\frac{\bar{P}^*_{t+1}}{\bar{P}^*_t}\frac{\bar{\mathcal{E}}_{t+1}}{\bar{\mathcal{E}}_t}$$
$$\Rightarrow \frac{\bar{\mathcal{E}}_{t+1}}{\bar{\mathcal{E}}_t} = e^{\bar{\pi}-\bar{\pi}^*} \Rightarrow log\bar{\mathcal{E}}_{t+1} - log\bar{\mathcal{E}}_t = \bar{\pi} - \bar{\pi}^*. \quad \text{(B110)}$$

The log-linearization of (B109) yields

$$\hat{R}_{\mathcal{H},t} = \hat{R}_{\mathcal{F},t} + \Theta'(0)\bar{Y}\hat{B}_{t+1}. \quad \text{(B111)}$$

B.10.2 The Equation for the Real Exchange Rate

$$\sigma_c E_t\left(\hat{C}^*_{t+1} - \hat{C}^*_t\right) + E_t\left(\hat{P}^*_{t+1} - \hat{P}^*_t\right) + E_t\left(\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t\right) = \hat{R}_{\mathcal{F},t}$$
$$\sigma_c E_t\left(\hat{C}_{t+1} - \hat{C}_t\right) + E_t\left(\hat{P}_{t+1} - \hat{P}_t\right) = \hat{R}_{\mathcal{H},t}$$
$$\hat{R}_{\mathcal{H},t} = \hat{R}_{\mathcal{F},t} + \Theta'(0)\bar{Y}\hat{B}_{t+1}$$
$$\sigma_c E_t\left(\hat{C}_{t+1} - \hat{C}_t\right) + E_t\left(\hat{P}_{t+1} - \hat{P}_t\right)$$
$$= \sigma_c E_t\left(\hat{C}^*_{t+1} - \hat{C}^*_t\right) + E_t\left(\hat{P}^*_{t+1} - \hat{P}^*_t\right) + E_t\left(\hat{\mathcal{E}}_{t+1} - \hat{\mathcal{E}}_t\right)$$
$$+ \Theta'(0)\bar{Y}\hat{B}_{t+1} \quad \text{(B112)}$$
$$\sigma_c\left\{E_t\left(\hat{C}_{t+1} - \hat{C}_t\right) - E_t\left(\hat{C}^*_{t+1} - \hat{C}^*_t\right)\right\}$$
$$= E_t\hat{Q}_{t+1} - \hat{Q}_t + \Theta'(0)\bar{Y}\hat{B}_{t+1} \quad \text{(B113)}$$

B.11 The Change in the Nominal Exchange Rate

$$\hat{\mathcal{E}}_t = \mathcal{Q}_t + \hat{P}_t - \hat{P}^*_t \quad \text{(B114)}$$

B.12 Log-Linearizing Home Households' Budget Constraint

Home households' budget constraint is given by

$$P_{t+s}\left\{C_{t+s} + I_{t+s} + a(u_{t+s})K_{t+s} + B_{t+s+1}\right\} + M_t$$
$$= x_{t+s,i}\tilde{n}_{t+s,i} + x_{t+s-1,i}n_{t+s,i} + M_{t-1} + M^a_t - M^a_{t-1}$$
$$+ R^k_{t+s}u_{t+s}K_{t+s} + R_{\mathcal{H},t+s-1}P_{t+s-1}B_{t+s} + \Pi_{t+s}. \quad \text{(B115)}$$

Firms' demand for the composite labor is given by the right-hand side of (B116):

$$\begin{aligned} x_{t+s,i}\tilde{n}_{t+s,i} + x_{t+s-1,i}n_{t+s,i} &= W_{t+s}N_{t+s} \\ &= W_{t+s}\left[\sum_{k=1}^{K}\int_0^1 N_{\mathcal{H},k,j,t+s} + \sum_{k=1}^{K}\int_0^1 N^*_{\mathcal{H},\cent,k,j,t+s}\right. \\ &\quad \left. + \sum_{k=1}^{K}\int_0^1 N^*_{\mathcal{H},\cent^*,k,j,t+s}\right]. \end{aligned} \tag{B116}$$

Firms' demand for capital is given by the right-hand side of (B117):

$$\begin{aligned} R^k_{t+s}u_{t+s}K_{t+s} = R^k_{t+s}&\left[\sum_{k=1}^{K}\int_0^1 K_{\mathcal{H},k,j,t+s} + \sum_{k=1}^{K}\int_0^1 K^*_{\mathcal{H},\cent,k,j,t+s}\right. \\ &\left. + \sum_{k=1}^{K}\int_0^1 K^*_{\mathcal{H},\cent^*,k,j,t+s}\right]. \end{aligned} \tag{B117}$$

The profits of the home-household-owned firms are given by (B118):

$$\begin{aligned} \Pi_{t+s} = &\sum_{k=1}^{K}\int_0^1 \Pi_{\mathcal{H},k,j,t+s}dj + \sum_{k=1}^{K}\int_0^1 \Pi^*_{\mathcal{H},\cent,k,j,t+s}dj \\ &+ \sum_{k=1}^{K}\int_0^1 \Pi^*_{\mathcal{H},\cent^*,k,j,t+s}dj, \end{aligned} \tag{B118}$$

where

$$\begin{aligned} \Pi_{\mathcal{H},k,j,t+s} &= P_{\mathcal{H},k,j,t+s}Y_{\mathcal{H},k,j,t+s} - W_{t+s}N_{\mathcal{H},k,j,t+s} \\ &\quad - R^k_{t+s}K_{\mathcal{H},k,j,t+s} \\ \Pi^*_{\mathcal{H},\cent,k,j,t+s} &= P^*_{\mathcal{H},\cent,k,j,t+s}Y^*_{\mathcal{H},\cent,k,j,t+s} - W_{t+s}N^*_{\mathcal{H},\cent,k,j,t+s} \\ &\quad - R^k_{t+s}K^*_{\mathcal{H},\cent,k,j,t+s} \\ \Pi^*_{\mathcal{H},\cent^*,k,j,t+s} &= \mathcal{E}_{t+s}P^*_{\mathcal{H},\cent^*,k,j,t+s}Y^*_{\mathcal{H},\cent^*,k,j,t+s} - W_{t+s}N^*_{\mathcal{H},\cent^*,k,j,t+s} \\ &\quad - R^k_{t+s}K^*_{\mathcal{H},\cent^*,k,j,t+s}. \end{aligned}$$

Using (B116), (B117), (B118), and the fact that domestic and export goods are competitively produced, one can write that

$$\begin{aligned}
&W_{t+s}N_{t+s} + R^k_{t+s}u_{t+s}K_{t+s} + \Pi_{t+s} \\
&\quad = \sum_{k=1}^{K}\int_0^1 P_{\mathcal{H},k,j,t+s}Y_{\mathcal{H},k,j,t+s}dj + \sum_{k=1}^{K}\int_0^1 P^*_{\mathcal{H},\cent,k,j,t+s}Y^*_{\mathcal{H},\cent,k,j,t+s}dj \\
&\qquad + \sum_{k=1}^{K}\int_0^1 \mathcal{E}_{t+s}P^*_{\mathcal{H},\cent^*,k,j,t+s}Y^*_{\mathcal{H},\cent^*,k,j,t+s}dj \\
&\quad = \sum_{k=1}^{K} P_{\mathcal{H},k,t+s}Y_{\mathcal{H},k,t+s} + \sum_{k=1}^{K} P^*_{\mathcal{H},\cent,k,t+s}Y^*_{\mathcal{H},\cent,k,t+s} \\
&\qquad + \sum_{k=1}^{K} \mathcal{E}_{t+s}P^*_{\mathcal{H},\cent^*,k,t+s}Y^*_{\mathcal{H},\cent^*,k,t+s}. \qquad \text{(B119)}
\end{aligned}$$

In equilibrium, $M_t = M_t^a$. Using this, (B39), and (B119), one can rewrite (B115) as

$$\begin{aligned}
&P_{t+s}Y_{t+s} + P_{t+s}B_{t+s+1} \\
&\quad = \sum_{k=1}^{K} P_{\mathcal{H},k,t+s}Y_{\mathcal{H},k,t+s} + \sum_{k=1}^{K} P^*_{\mathcal{H},\cent,k,t+s}Y^*_{\mathcal{H},\cent,k,t+s} \\
&\qquad + \sum_{k=1}^{K} \mathcal{E}_{t+s}P^*_{\mathcal{H},\cent^*,k,t+s}Y^*_{\mathcal{H},\cent^*,k,t+s} + R_{\mathcal{H},t+s-1}P_{t+s-1}B_{t+s} \\
&Y_{t+s} + B_{t+s+1} \\
&\quad = \sum_{k=1}^{K} \frac{P_{\mathcal{H},k,t+s}}{P_{t+s}}Y_{\mathcal{H},k,t+s} + \sum_{k=1}^{K} Q_{t+s}\left(\frac{\frac{1}{\mathcal{E}_{t+s}}P^*_{\mathcal{H},\cent,k,t+s}}{P^*_{t+s}}\right)Y^*_{\mathcal{H},\cent,k,t+s} \\
&\quad + \sum_{k=1}^{K} Q_{t+s}\left(\frac{P^*_{\mathcal{H},\cent^*,k,t+s}}{P^*_{t+s}}\right)Y^*_{\mathcal{H},\cent^*,k,t+s} + R_{\mathcal{H},t+s-1}\frac{P_{t+s-1}}{P_{t+s}}B_{t+s}. \qquad \text{(B120)}
\end{aligned}$$

We define the relative price of the domestic sector k good and home-export goods as

$$P^{rel.}_{\mathcal{H},k,t+s} = \frac{P_{\mathcal{H},k,t+s}}{P_{t+s}}$$
$$P^{*^{rel.}}_{\mathcal{H},\not{c},k,t+s} = \frac{\frac{1}{\mathcal{E}_{t+s}} P^*_{\mathcal{H},\not{c},k,t+s}}{P^*_{t+s}} \qquad \text{(B121)}$$
$$P^{*^{rel.}}_{\mathcal{H},\not{c}^*,k,t+s} = \frac{P^*_{\mathcal{H},\not{c}^*,k,t+s}}{P^*_{t+s}}.$$

Let $\hat{B}_{t+s}$ be defined as $\hat{B}_{t+s} = \frac{B_{t+s} - \bar{B}}{\bar{Y}}$, where we assume $\bar{B} = 0$. Then, log-linearizing (B120) yields

$$\begin{aligned}\hat{\mathbf{Y}}_{\mathbf{t}} + \hat{\mathbf{B}}_{\mathbf{t+1}} = {} & \sum_{\mathbf{k=1}}^{\mathbf{K}} \mathbf{f_k}(\mathbf{1} - \psi)\left(\hat{\mathbf{P}}^{\mathbf{rel.}}_{\mathcal{H},\mathbf{k},\mathbf{t}} + \hat{\mathbf{Y}}_{\mathcal{H},\mathbf{k},\mathbf{t}}\right) + \psi \hat{\mathbf{Q}}_{\mathbf{t}} \\ & + \sum_{k=1}^{K} f_k \psi \omega_{\not{c}} \left(\hat{P}^{*^{rel.}}_{\mathcal{H},\not{c},k,t} + \hat{Y}^*_{\mathcal{H},\not{c},k,t}\right) \\ & + \sum_{k=1}^{K} f_k \psi (1 - \omega_{\not{c}}) \left(\hat{P}^{*^{rel.}}_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right) + \frac{1}{\beta}\hat{B}_t. \end{aligned} \qquad \text{(B122)}$$

Using (B121), one can rewrite (B122) as

$$\begin{aligned}\hat{\mathbf{Y}}_{\mathbf{t}} + \hat{\mathbf{B}}_{\mathbf{t+1}} = {} & \sum_{k=1}^{K} f_k (1 - \psi)\left(\hat{P}_{\mathcal{H},k,t} + \hat{Y}_{\mathcal{H},k,t}\right) \\ & - (1 - \psi)\hat{P}_t + \psi \hat{Q}_t + \sum_{k=1}^{K} f_k \psi \omega_{\not{c}} \hat{P}^*_{\mathcal{H},\not{c},k,t} \\ & + \sum_{k=1}^{K} f_k \psi \omega_{\not{c}} \hat{Y}^*_{\mathcal{H},\not{c},k,t} \\ & + \sum_{k=1}^{K} f_k \psi (1 - \omega_{\not{c}}) \left(\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right) \\ & - \psi \omega_{\not{c}} \hat{\mathcal{E}}_t - \psi \hat{P}^*_t + \frac{1}{\beta}\hat{B}_t. \end{aligned} \qquad \text{(B123)}$$

B.13 Log-Linearizing Foreign Households' Budget Constraint

Foreign households' budget constraint is given by

$$\begin{aligned} & P^*_{t+s}\left(C^*_{t+s} + I^*_{t+s} + a(u^*_{t+s})K^*_{t+s} + D^*_{t+s+1}\right) + \frac{P_{t+s}}{\mathcal{E}_{t+s}}B^*_{t+s+1} + M^*_t \\ & \quad = M^*_t + M^{*^a}_t - M^{*^a}_{t-1} + x^*_{t+s,i}\tilde{n}^*_{t+s,i} + x^*_{t+s-1,i}n^*_{t+s,i} \\ & \qquad + R^{*^k}_{t+s}u^*_{t+s}K^*_{t+s} + R^*_{t+s-1}P^*_{t+s-1}D^*_{t+s} + \frac{R_{\mathcal{F},t+s-1}P_{t+s-1}}{\mathcal{E}_{t+s}}B^*_{t+s} \\ & \qquad + \Pi^*_{t+s}. \end{aligned} \tag{B124}$$

Similar to (B120), one can show that

$$\begin{aligned} & Y^*_{t+s} + D^*_{t+s+1} + \frac{1}{Q_{t+s}}B^*_{t+s+1} \\ & \quad = \sum_{k=1}^{K}\frac{1}{Q_{t+s}}\left(\frac{P_{\mathcal{F},¢,k,t+s}}{P_{t+s}}\right)Y_{\mathcal{F},¢,k,t+s} \\ & \qquad + \sum_{k=1}^{K}\frac{1}{Q_{t+s}}\left(\frac{\mathcal{E}_{t+s}P_{\mathcal{F},¢^*,k,t+s}}{P_{t+s}}\right)Y_{\mathcal{F},¢^*,k,t+s} \\ & \qquad + \sum_{k=1}^{K}\frac{P^*_{\mathcal{F},k,t+s}}{P^*_{t+s}}Y^*_{\mathcal{F},k,t+s} + R^*_{t+s-1}\frac{P^*_{t+s-1}}{P^*_{t+s}}D^*_{t+s} \\ & \qquad + \frac{1}{Q_{t+s}}\frac{R_{\mathcal{F},t+s-1}P_{t+s-1}}{P_{t+s}}B^*_{t+s}. \end{aligned} \tag{B125}$$

Since the home-country and foreign-country risk-free bonds are supplied at zero net supply, $\bar{D} = 0$ and $B^*_{t+s+1} = -B_{t+s+1}$. In addition, we assume $\bar{Q} = 1$. Using these, (B125) can be log-linearized as

$$\begin{aligned} & \hat{\mathbf{Y}}^*_{\mathbf{t}} + \hat{\mathbf{D}}^*_{\mathbf{t+1}} - \frac{\mathbf{1}}{\tau}\hat{\mathbf{B}}_{\mathbf{t+1}} \\ & \quad = \sum_{\mathbf{k=1}}^{\mathbf{K}}\mathbf{f_k}\frac{\psi}{\tau}(\mathbf{1} - \omega^*_{¢^*})\left(\hat{\mathbf{P}}^{\mathbf{rel.}}_{\mathcal{F},¢,\mathbf{k},\mathbf{t}} + \hat{\mathbf{Y}}_{\mathcal{F},¢,\mathbf{k},\mathbf{t}}\right) \\ & \qquad + \sum_{k=1}^{K}f_k\frac{\psi}{\tau}\omega^*_{¢^*}\left(\hat{P}^{rel.}_{\mathcal{F},¢^*,k,t} + \hat{Y}_{\mathcal{F},¢^*,k,t}\right) \end{aligned}$$

$$+\sum_{k=1}^{K} f_k \left(1-\frac{\psi}{\tau}\right)\left(\hat{P}^{*^{rel.}}_{\mathcal{F},k,t}+\hat{Y}^{*}_{\mathcal{F},k,t}\right)$$
$$-\frac{\psi}{\tau}\hat{Q}_t+\frac{1}{\beta}\hat{D}^*_t-\frac{1}{\tau}\frac{1}{\beta}\hat{B}_t, \tag{B126}$$

where $\hat{D}^*_{t+s} = \frac{D^*_{t+s}-\bar{D}^*}{\bar{Y}^*}$. $P^{*^{rel.}}_{\mathcal{F},k,t+s}$, $P^{rel.}_{\mathcal{F},¢,k,t+s}$, and $P^{rel.}_{\mathcal{F},¢^*,k,t+s}$ denote the relative price of the foreign good supplied to the foreign country, that of the home-currency-invoiced foreign-export goods and that of the foreign-currency-invoiced foreign-export goods in sector k, respectively, and are defined as

$$P^{*^{rel.}}_{\mathcal{F},k,t+s}=\frac{P^*_{\mathcal{F},k,t+s}}{P^*_{t+s}}$$
$$P^{rel.}_{\mathcal{F},¢,k,t+s}=\frac{P^*_{\mathcal{F},¢,k,t+s}}{P_{t+s}}$$
$$P^{rel.}_{\mathcal{F},¢^*,k,t+s}=\frac{\mathcal{E}_{t+s}P_{\mathcal{F},¢^*,k,t+s}}{P_{t+s}}. \tag{B127}$$

B.14 Firms' Objective in the Home Country

Figure B1 illustrates how firms producing different goods are related to each other in the home country and their production technologies.

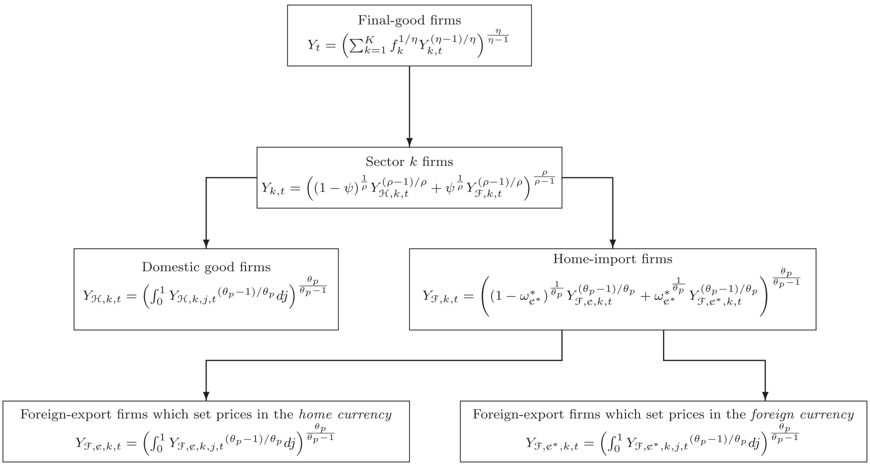
Next, we separately describe how each good is produced.

B.14.1 Firms Producing Final Goods in the Home Country

Non-traded final goods in the home country are produced by a continuum of perfectly competitive firms. Firms produce final goods using the following technology, which involves combining goods from different sectors:

$$Y_t=\left(\sum_{k=1}^{K} f_k^{1/\eta} Y_{k,t}^{(\eta-1)/\eta}\right)^{\frac{\eta}{\eta-1}}.$$

Figure B1. How Are Different Firms Related in the Home Country?



Each period, firms producing these goods solve the following problem:

$$\max P_t Y_t - \sum_{k=1}^{K} P_{t,k} Y_{t,k}$$

$$\text{subject to } Y_t = \left(\sum_{k=1}^{K} f_k^{1/\eta} Y_{k,t}^{(\eta-1)/\eta}\right)^{\frac{\eta}{\eta-1}}. \tag{B128}$$

From this problem, one can show that

$$\frac{\partial Y_t}{\partial Y_{k,t}} = \left(\frac{\eta}{\eta-1}\right)\left(\sum_{k=1}^{K} f_k^{1/\eta} Y_{k,t}^{(\eta-1)/\eta}\right)^{\frac{\eta}{\eta-1}-1}$$

$$f_k^{1/\eta}\left(\frac{\eta-1}{\eta}\right) Y_{k,t}^{\frac{\eta-1}{\eta}-1} = f_k^{1/\eta}\left(\frac{Y_t}{Y_{k,t}}\right)^{\frac{1}{\eta}}$$

$$P_t f_k^{1/\eta}\left(\frac{Y_t}{Y_{k,t}}\right)^{\frac{1}{\eta}} = P_{k,t}. \tag{B129}$$

From (B129), one can write demand for the sector k good as

$$Y_{k,t} = f_k \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} Y_t. \tag{B130}$$

In addition, since final goods are produced by perfectly competitive firms, the profits of firms producing these goods are equal to zero, yielding

$$P_t Y_t = \sum_{k=1}^{K} P_{t,k} Y_{t,k} = \sum_{k=1}^{K} P_{t,k} f_k \left(\frac{P_{k,t}}{P_t} \right)^{-\eta} Y_t. \tag{B131}$$

From (B131), it is easy to show that the consumer price index in the home country is a weighted average of sectoral prices:

$$P_t = \left(\sum_{k=1}^{K} f_k P_{k,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}. \tag{B132}$$

B.14.2 Firms Producing the Sector k Good in the Home Country

Sectoral goods are produced by an infinitely large number of perfectly competitive firms. The home firms producing sectoral goods combine domestic goods ($Y_{\mathcal{H},k,t}$) and imported goods ($Y_{\mathcal{F},k,t}$) to produce sectoral output ($Y_{k,t}$) with the following technology:

$$Y_{k,t} = \left((1-\psi)^{\frac{1}{\rho}} Y_{\mathcal{H},k,t}^{(\rho-1)/\rho} + \psi^{\frac{1}{\rho}} Y_{\mathcal{F},k,t}^{(\rho-1)/\rho} \right)^{\frac{\rho}{\rho-1}}. \tag{B133}$$

Each period, firms producing sector k output solve the following problem:

$$\max P_{k,t} Y_{k,t} - P_{\mathcal{H},k,t} Y_{\mathcal{H},k,t} - P_{\mathcal{F},k,t} Y_{\mathcal{F},k,t}$$

$$\text{subject to } Y_{k,t} = \left((1-\psi)^{\frac{1}{\rho}} Y_{\mathcal{H},k,t}^{(\rho-1)/\rho} + \psi^{\frac{1}{\rho}} Y_{\mathcal{F},k,t}^{(\rho-1)/\rho} \right)^{\frac{\rho}{\rho-1}}. \tag{B134}$$

From this problem, one can show that

$$\frac{\partial Y_{k,t}}{\partial Y_{\mathcal{H},k,t}} = \left(\frac{\rho}{\rho-1} \right) \left((1-\psi)^{\frac{1}{\rho}} Y_{\mathcal{H},k,t}^{(\rho-1)/\rho} + \psi^{\frac{1}{\rho}} Y_{\mathcal{F},k,t}^{(\rho-1)/\rho} \right)^{\frac{\rho}{\rho-1}-1}$$

$$\times (1-\psi)^{\frac{1}{\rho}} \left(\frac{\rho-1}{\rho} \right) Y_{\mathcal{H},k,t}^{\frac{\rho-1}{\rho}-1} = (1-\psi)^{\frac{1}{\rho}} \left(\frac{Y_{k,t}}{Y_{\mathcal{H},k,t}} \right)^{\frac{1}{\rho}}$$

$$\frac{\partial Y_{k,t}}{\partial Y_{\mathcal{F},k,t}} = \left(\frac{\rho}{\rho-1}\right)\left((1-\psi)^{\frac{1}{\rho}} Y_{\mathcal{H},k,t}^{(\rho-1)/\rho} + \psi^{\frac{1}{\rho}} Y_{\mathcal{F},k,t}^{(\rho-1)/\rho}\right)^{\frac{\rho}{\rho-1}-1} \psi^{\frac{1}{\rho}}$$
$$\times \left(\frac{\rho-1}{\rho}\right) Y_{\mathcal{F},k,t}^{\frac{\rho-1}{\rho}-1} = \psi^{\frac{1}{\rho}} \left(\frac{Y_{k,t}}{Y_{\mathcal{F},k,t}}\right)^{\frac{1}{\rho}}. \tag{B135}$$

From (B135), one can write demand for domestic and home-import goods from sector k as

$$P_{k,t}(1-\psi)^{\frac{1}{\rho}} \left(\frac{Y_{k,t}}{Y_{\mathcal{H},k,t}}\right)^{\frac{1}{\rho}} = P_{\mathcal{H},k,t} \Rightarrow Y_{\mathcal{H},k,t} = (1-\psi)\left(\frac{P_{\mathcal{H},k,t}}{P_{k,t}}\right)^{-\rho} Y_{k,t}$$
$$P_{k,t}\psi^{\frac{1}{\rho}} \left(\frac{Y_{k,t}}{Y_{\mathcal{F},k,t}}\right)^{\frac{1}{\rho}} = P_{\mathcal{F},k,t} \Rightarrow Y_{\mathcal{F},k,t} = \psi \left(\frac{P_{\mathcal{F},k,t}}{P_{k,t}}\right)^{-\rho} Y_{k,t}. \tag{B136}$$

Using (B130), one can rewrite (B136) as

$$Y_{\mathcal{H},k,t} = f_k(1-\psi)\left(\frac{P_{\mathcal{H},k,t}}{P_{k,t}}\right)^{-\rho}\left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Y_t$$
$$Y_{\mathcal{F},k,t} = f_k\psi\left(\frac{P_{\mathcal{F},k,t}}{P_{k,t}}\right)^{-\rho}\left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Y_t. \tag{B137}$$

In addition, since sector k output is produced by perfectly competitive firms, the profits of firms producing the sector k good are equal to zero:

$$P_{k,t}Y_{k,t} = P_{\mathcal{H},k,t}Y_{\mathcal{H},k,t} + P_{\mathcal{F},k,t}Y_{\mathcal{F},k,t}$$
$$= P_{\mathcal{H},k,t}(1-\psi)\left(\frac{P_{\mathcal{H},k,t}}{P_{k,t}}\right)^{-\rho} Y_{k,t} + P_{\mathcal{F},k,t}\psi\left(\frac{P_{\mathcal{F},k,t}}{P_{k,t}}\right)^{-\rho} Y_{k,t}. \tag{B138}$$

From (B138), it follows that the sector k price index is a weighted average of domestic and home-import goods' prices given by

$$P_{k,t} = \left((1-\psi)P_{\mathcal{H},k,t}^{1-\rho} + \psi P_{\mathcal{F},k,t}^{1-\rho}\right)^{\frac{1}{1-\rho}}. \tag{B139}$$

B.14.3 Home Firms Producing Domestically in Sector k

The intermediate domestic goods in the home country are composite goods composed of a variety of goods produced by firms engaging in monopolistic competition. The production technology used in the production of intermediate domestic goods is given as

$$Y_{\mathcal{H},k,t} = \left(\int_0^1 Y_{\mathcal{H},k,j,t}{}^{(\theta_p - 1)/\theta_p} dj \right)^{\frac{\theta_p}{\theta_p - 1}}. \tag{B140}$$

Firms producing sector k output operate in a perfectly competitive market and solve the following problem each period:

$$\begin{aligned} &\max P_{\mathcal{H},k,t} Y_{\mathcal{H},k,t} - \int_0^1 P_{\mathcal{H},k,j,t} Y_{\mathcal{H},k,j,t} dj \\ &\text{subject to } Y_{\mathcal{H},k,t} = \left(\int_0^1 Y_{\mathcal{H},k,j,t}{}^{(\theta_p - 1)/\theta_p} dj \right)^{\frac{\theta_p}{\theta_p - 1}}. \end{aligned} \tag{B141}$$

From the optimization problem of these firms, it is easy to see that

$$\begin{aligned} \frac{\partial Y_{\mathcal{H},k,t}}{\partial Y_{\mathcal{H},k,j,t}} &= \frac{\theta_p}{\theta_p - 1} \left(\int_0^1 Y_{\mathcal{H},k,j,t}{}^{(\theta_p - 1)/\theta_p} dj \right)^{\frac{\theta_p}{\theta_p - 1} - 1} \\ &\quad \times \left(\frac{\theta_p - 1}{\theta_p} \right) Y_{\mathcal{H},k,j,t}{}^{\frac{\theta_p - 1}{\theta_p} - 1} = \left(\frac{Y_{\mathcal{H},k,t}}{Y_{\mathcal{H},k,j,t}} \right)^{\frac{1}{\theta_p}}. \end{aligned} \tag{B142}$$

Using (B142), demand for variety j from sector k firms that supply domestically can be written as

$$P_{\mathcal{H},k,t} \left(\frac{Y_{\mathcal{H},k,t}}{Y_{\mathcal{H},k,j,t}} \right)^{\frac{1}{\theta_p}} = P_{\mathcal{H},k,j,t} \Rightarrow Y_{\mathcal{H},k,j,t} = \left(\frac{P_{\mathcal{H},k,j,t}}{P_{\mathcal{H},k,t}} \right)^{-\theta_p} Y_{\mathcal{H},k,t}. \tag{B143}$$

Since the domestic sector k good is produced in a perfectly competitive market, firms' profits are equal to zero:

$$P_{\mathcal{H},k,t}Y_{\mathcal{H},k,t} = \int_0^1 P_{\mathcal{H},k,j,t}Y_{\mathcal{H},k,j,t}dj$$
$$= \int_0^1 P_{\mathcal{H},k,j,t}\left(\frac{P_{\mathcal{H},k,j,t}}{P_{\mathcal{H},k,t}}\right)^{-\theta_p} Y_{\mathcal{H},k,t}dj. \quad \text{(B144)}$$

From (B144), the price index for the domestic good in sector k is given as a geometric average of price indexes for varieties of goods:

$$P_{\mathcal{H},k,t} = \left(\int_0^1 {P_{\mathcal{H},k,j,t}}^{1-\theta_p} dj\right)^{\frac{1}{1-\theta_p}}. \quad \text{(B145)}$$

B.14.4 Sector k Home-Export Firms that Set Prices in the Home Currency

The home-currency-priced home-export good is a composite good composed of a variety of goods produced by firms engaging in monopolistic competition. The production technology used in the production of the home-export good is given as

$$Y^*_{\mathcal{H},¢,k,t} = \left(\int_0^1 {Y^*_{\mathcal{H},¢,k,j,t}}^{(\theta_p-1)/\theta_p} dj\right)^{\frac{\theta_p}{\theta_p-1}}. \quad \text{(B146)}$$

The composite home-export good whose price is set in the *home currency* is produced in a perfectly competitive market. Each period, firms producing this good solve the following problem:

$$\max P^*_{\mathcal{H},¢,k,t}Y^*_{\mathcal{H},¢,k,t} - \int_0^1 P^*_{\mathcal{H},¢,k,j,t}Y^*_{\mathcal{H},¢,k,j,t}dj$$
$$\text{subject to } Y^*_{\mathcal{H},¢,k,t} = \left(\int_0^1 {Y^*_{\mathcal{H},¢,k,j,t}}^{(\theta_p-1)/\theta_p} dj\right)^{\frac{\theta_p}{\theta_p-1}}. \quad \text{(B147)}$$

From the optimization problem of these firms, it is easy to see that

$$\frac{\partial Y^*_{\mathcal{H},¢,k,t}}{\partial Y^*_{\mathcal{H},¢,k,j,t}} = \left(\frac{\theta_p}{\theta_p-1}\right)\left(\int_0^1 {Y^*_{\mathcal{H},¢,k,j,t}}^{(\theta_p-1)/\theta_p} dj\right)^{\frac{\theta_p}{\theta_p-1}-1}$$
$$\times \left(\frac{\theta_p-1}{\theta_p}\right){Y^*_{\mathcal{H},¢,k,j,t}}^{\frac{\theta_p-1}{\theta_p}-1} = \left(\frac{Y^*_{\mathcal{H},¢,k,t}}{Y^*_{\mathcal{H},¢,k,j,t}}\right)^{\frac{1}{\theta_p}}. \quad \text{(B148)}$$

Using (B148), demand for home-export variety j from the firms that produce the composite home-export good is given by

$$P^*_{\mathcal{H},\cent,k,t}\left(\frac{Y^*_{\mathcal{H},\cent,k,t}}{Y^*_{\mathcal{H},\cent,k,j,t}}\right)^{\frac{1}{\theta_p}} = P^*_{\mathcal{H},\cent,k,j,t} \Rightarrow Y^*_{\mathcal{H},\cent,k,j,t} = \left(\frac{P^*_{\mathcal{H},\cent,k,j,t}}{P^*_{\mathcal{H},\cent,k,t}}\right)^{-\theta_p} Y^*_{\mathcal{H},\cent,k,t}. \quad \text{(B149)}$$

Since the composite home-export good is produced in a perfectly competitive market, profits of firms in this market are equal to zero:

$$P^*_{\mathcal{H},\cent,k,t}Y^*_{\mathcal{H},\cent,k,t} = \int_0^1 P^*_{\mathcal{H},\cent,k,j,t}Y^*_{\mathcal{H},\cent,k,j,t}dj = \int_0^1 P^*_{\mathcal{H},\cent,k,j,t}\left(\frac{P^*_{\mathcal{H},\cent,k,j,t}}{P^*_{\mathcal{H},\cent,k,t}}\right)^{-\theta_p} Y^*_{\mathcal{H},\cent,k,t}dj. \quad \text{(B150)}$$

From (B150), the price index for the home-export good whose price is set in the *home currency* is given as a geometric average of price indexes for varieties of goods:

$$P^*_{\mathcal{H},\cent,k,t} = \left(\int_0^1 {P^*_{\mathcal{H},\cent,k,j,t}}^{1-\theta_p}dj\right)^{\frac{1}{1-\theta_p}}. \quad \text{(B151)}$$

B.14.5 Sector k Home-Export Firms that Set Prices in the Foreign Currency

The foreign-currency-priced home-export good is a composite good composed of a variety of goods produced by firms engaging in monopolistic competition. The production technology of these firms is given as

$$Y^*_{\mathcal{H},\cent^*,k,t} = \left(\int_0^1 {Y^*_{\mathcal{H},\cent^*,k,j,t}}^{(\theta_p-1)/\theta_p}dj\right)^{\frac{\theta_p}{\theta_p-1}}. \quad \text{(B152)}$$

The composite home-export good whose price is set in the *foreign currency* is produced in a perfectly competitive market. Each period, firms producing this good solve the following problem:

$$\max \mathcal{E}_t \Big(P^*_{\mathcal{H},\cent^*,k,t} Y^*_{\mathcal{H},\cent^*,k,t} - \int_0^1 P^*_{\mathcal{H},\cent^*,k,j,t} Y^*_{\mathcal{H},\cent^*,k,j,t} dj \Big)$$

$$\text{subject to } Y^*_{\mathcal{H},\cent^*,k,t} = \left(\int_0^1 {Y^*_{\mathcal{H},\cent^*,k,j,t}}^{(\theta_p - 1)/\theta_p} dj \right)^{\frac{\theta_p}{\theta_p - 1}}. \quad \text{(B153)}$$

From the optimization problem of these firms, it is easy to see that

$$\frac{\partial Y^*_{\mathcal{H},\cent^*,k,t}}{\partial Y^*_{\mathcal{H},\cent^*,k,j,t}} = \left(\frac{\theta_p}{\theta_p - 1} \right) \left(\int_0^1 {Y^*_{\mathcal{H},\cent^*,k,j,t}}^{(\theta_p - 1)/\theta_p} dj \right)^{\frac{\theta_p}{\theta_p - 1} - 1}$$

$$\times \left(\frac{\theta_p - 1}{\theta_p} \right) {Y^*_{\mathcal{H},\cent^*,k,j,t}}^{\frac{\theta_p - 1}{\theta_p} - 1} = \left(\frac{Y^*_{\mathcal{H},\cent^*,k,t}}{Y^*_{\mathcal{H},\cent^*,k,j,t}} \right)^{\frac{1}{\theta_p}}. \quad \text{(B154)}$$

Using (B154), demand for home-export variety j from the firms that produce the composite home-export good and set prices in the *foreign currency* is given by

$$P^*_{\mathcal{H},\cent^*,k,t} \left(\frac{Y^*_{\mathcal{H},\cent^*,k,t}}{Y^*_{\mathcal{H},\cent^*,k,j,t}} \right)^{\frac{1}{\theta_p}} = P^*_{\mathcal{H},\cent^*,k,j,t} \Rightarrow Y^*_{\mathcal{H},\cent^*,k,j,t}$$

$$= \left(\frac{P^*_{\mathcal{H},\cent^*,k,j,t}}{P^*_{\mathcal{H},\cent^*,k,t}} \right)^{-\theta_p} Y^*_{\mathcal{H},\cent^*,k,t}. \quad \text{(B155)}$$

Since the composite home-export good is produced in a perfectly competitive market, firms' profits are equal to zero:

$$P^*_{\mathcal{H},\cent^*,k,t} Y^*_{\mathcal{H},\cent^*,k,t} = \int_0^1 P^*_{\mathcal{H},\cent^*,k,j,t} Y^*_{\mathcal{H},\cent^*,k,j,t} dj$$

$$= \int_0^1 P^*_{\mathcal{H},\cent^*,k,j,t} \left(\frac{P^*_{\mathcal{H},\cent^*,k,j,t}}{P^*_{\mathcal{H},\cent^*,k,t}} \right)^{-\theta_p} Y^*_{\mathcal{H},\cent^*,k,t} dj. \quad \text{(B156)}$$

From (B156), the price index for the home-export good whose price is set in the *foreign currency* is given as a geometric average of price indexes for varieties of goods:

$$P^*_{\mathcal{H},\text{¢}^*,k,t} = \left(\int_0^1 {P^*_{\mathcal{H},\text{¢}^*,k,j,t}}^{1-\theta_p} dj \right)^{\frac{1}{1-\theta_p}}. \tag{B157}$$

B.14.6 Home-Import Firms Composing Foreign-Export Goods Priced in the Home and Foreign Currencies

The home-import goods in sector k ($Y_{\mathcal{F},k,t}$) are produced by perfectly competitive home firms. Producing these goods involves combining intermediate foreign goods which are invoiced in different currencies. Indeed, while some intermediate goods are invoiced in the *home currency* (¢), others are invoiced in the *foreign currency* (¢*). In producing the home-import good in sector k, the home-import firms combine output from the foreign firms that set prices in the home and foreign currencies (denoted by $Y_{\mathcal{F},\text{¢},k,t}$ and $Y_{\mathcal{F},\text{¢}^*,k,t}$, respectively) with the following technology:

$$Y_{\mathcal{F},k,t} = \left((1-\omega^*_{\text{¢}^*})^{\frac{1}{\theta_p}} Y_{\mathcal{F},\text{¢},k,t}^{(\theta_p-1)/\theta_p} + {\omega^*_{\text{¢}^*}}^{\frac{1}{\theta_p}} Y_{\mathcal{F},\text{¢}^*,k,t}^{(\theta_p-1)/\theta_p} \right)^{\frac{\theta_p}{\theta_p-1}}. \tag{B158}$$

Each period, home-import firms solve the following problem:

$$\max P_{\mathcal{F},k,t} Y_{\mathcal{F},k,t} - P_{\mathcal{F},\text{¢},k,t} Y_{\mathcal{F},\text{¢},k,t} - \mathcal{E}_t P_{\mathcal{F},\text{¢}^*,k,t} Y_{\mathcal{F},\text{¢}^*,k,t}$$

$$\text{subject to } Y_{\mathcal{F},k,t} = \left((1-\omega^*_{\text{¢}^*})^{\frac{1}{\theta_p}} Y_{\mathcal{F},\text{¢},k,t}^{(\theta_p-1)/\theta_p} + {\omega^*_{\text{¢}^*}}^{\frac{1}{\theta_p}} Y_{\mathcal{F},\text{¢}^*,k,t}^{(\theta_p-1)/\theta_p} \right)^{\frac{\theta_p}{\theta_p-1}}. \tag{B159}$$

From (B159), it is easy to see that

$$\begin{aligned} \frac{\partial Y_{\mathcal{F},k,t}}{\partial Y_{\mathcal{F},\text{¢},k,t}} &= \left(\frac{\theta_p}{\theta_p - 1} \right) \left((1-\omega^*_{\text{¢}^*})^{\frac{1}{\theta_p}} Y_{\mathcal{F},\text{¢},k,t}^{(\theta_p-1)/\theta_p} + {\omega^*_{\text{¢}^*}}^{\frac{1}{\theta_p}} Y_{\mathcal{F},\text{¢}^*,k,t}^{(\theta_p-1)/\theta_p} \right)^{\frac{\theta_p}{\theta_p-1}-1} \\ &\quad \times (1-\omega^*_{\text{¢}^*})^{\frac{1}{\theta_p}} \left(\frac{\theta_p - 1}{\theta_p} \right) Y_{\mathcal{F},\text{¢},k,t}^{\frac{\theta_p-1}{\theta_p}-1} \\ &= (1-\omega^*_{\text{¢}^*})^{\frac{1}{\theta_p}} \left(\frac{Y_{\mathcal{F},k,t}}{Y_{\mathcal{F},\text{¢},k,t}} \right)^{\frac{1}{\theta_p}} \end{aligned}$$

$$\frac{\partial Y_{\mathcal{F},k,t}}{\partial Y_{\mathcal{F},¢^*,k,t}} = \left(\frac{\theta_p}{\theta_p - 1}\right)\left((1-\omega^*_{¢^*})^{\frac{1}{\theta_p}} Y^{(\theta_p-1)/\theta_p}_{\mathcal{F},¢,k,t} + \omega^{*\frac{1}{\theta_p}}_{¢^*} Y^{(\theta_p-1)/\theta_p}_{\mathcal{F},¢^*,k,t}\right)^{\frac{\theta_p}{\theta_p-1}-1}$$
$$\times\, \omega^{*\frac{1}{\theta_p}}_{¢^*}\left(\frac{\theta_p-1}{\theta_p}\right) Y^{\frac{\theta_p-1}{\theta_p}-1}_{\mathcal{F},¢^*,k,t} = \omega^{*\frac{1}{\theta_p}}_{¢^*}\left(\frac{Y_{\mathcal{F},k,t}}{Y_{\mathcal{F},¢^*,k,t}}\right)^{\frac{1}{\theta_p}}. \quad \text{(B160)}$$

Using (B160), one can write demand for home- and foreign-currency-priced home-import goods as

$$P_{\mathcal{F},k,t}(1-\omega^*_{¢^*})^{\frac{1}{\theta_p}}\left(\frac{Y_{\mathcal{F},k,t}}{Y_{\mathcal{F},¢,k,t}}\right)^{\frac{1}{\theta_p}}$$
$$= P_{\mathcal{F},¢,k,t} \Rightarrow Y_{\mathcal{F},¢,k,t} = (1-\omega^*_{¢^*})\left(\frac{P_{\mathcal{F},¢,k,t}}{P_{\mathcal{F},k,t}}\right)^{-\theta_p} Y_{\mathcal{F},k,t}$$
$$P_{\mathcal{F},k,t}\omega^{*\frac{1}{\theta_p}}_{¢^*}\left(\frac{Y_{\mathcal{F},k,t}}{Y_{\mathcal{F},¢^*,k,t}}\right)^{\frac{1}{\theta_p}}$$
$$= \mathcal{E}_t P_{\mathcal{F},¢^*,k,t} \Rightarrow Y_{\mathcal{F},¢^*,k,t} = \omega^*_{¢^*}\left(\frac{\mathcal{E}_t P_{\mathcal{F},¢^*,k,t}}{P_{\mathcal{F},k,t}}\right)^{-\theta_p} Y_{\mathcal{F},k,t}. \quad \text{(B161)}$$

Using (B137), (B161) can be rewritten as

$$Y_{\mathcal{F},¢,k,t} = f_k(1-\omega^*_{¢^*})\psi\left(\frac{P_{\mathcal{F},¢,k,t}}{P_{\mathcal{F},k,t}}\right)^{-\theta_p}\left(\frac{P_{\mathcal{F},k,t}}{P_{k,t}}\right)^{-\rho}\left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Y_t$$
$$Y_{\mathcal{F},¢^*,k,t} = f_k\omega^*_{¢^*}\psi\left(\frac{\mathcal{E}_t P_{\mathcal{F},¢^*,k,t}}{P_{\mathcal{F},k,t}}\right)^{-\theta_p}\left(\frac{P_{\mathcal{F},k,t}}{P_{k,t}}\right)^{-\rho}\left(\frac{P_{k,t}}{P_t}\right)^{-\eta} Y_t. \quad \text{(B162)}$$

Since home-import goods are produced in a perfectly competitive market, home-import firms' profits are equal to zero:

$$P_{\mathcal{F},k,t}Y_{\mathcal{F},k,t} = P_{\mathcal{F},¢,k,t}Y_{\mathcal{F},¢,k,t} + \mathcal{E}_t P_{\mathcal{F},¢^*,k,t}Y_{\mathcal{F},¢^*,k,t}$$
$$= P_{\mathcal{F},¢,k,t}(1-\omega^*_{¢^*})\left(\frac{P_{\mathcal{F},¢,k,t}}{P_{\mathcal{F},k,t}}\right)^{-\theta_p} Y_{\mathcal{F},k,t}$$
$$+ \mathcal{E}_t P_{\mathcal{F},¢^*,k,t}\omega^*_{¢^*}\left(\frac{\mathcal{E}_t P_{\mathcal{F},¢^*,k,t}}{P_{\mathcal{F},k,t}}\right)^{-\theta_p} Y_{\mathcal{F},k,t}. \quad \text{(B163)}$$

From (B163), one can write the home-import price index as a geometric average of home- and foreign-currency-priced foreign goods:

$$P_{\mathcal{F},k,t} = \left((1-\omega^*_{\cent^*})P^{1-\theta_p}_{\mathcal{F},\cent,k,t} + \omega^*_{\cent^*}\left(\mathcal{E}_t P_{\mathcal{F},\cent^*,k,t}\right)^{1-\theta_p}\right)^{\frac{1}{1-\theta_p}}. \quad \text{(B164)}$$

B.14.7 Cost-Minimization Problem for the Firm Supplying the Home Country with Variety j

All firms in the economy use the same production function given by

$$Y_{\mathcal{H},k,j,t} = K^{1-\chi}_{\mathcal{H},k,j,t} N^{\chi}_{\mathcal{H},k,j,t}. \quad \text{(B165)}$$

Each period, they aim to minimize their cost function:

$$\min W_t N_{\mathcal{H},k,j,t} + R^k_t K_{\mathcal{H},k,j,t}, \quad \text{(B166)}$$

subject to

$$Y_{\mathcal{H},k,j,t} = K^{1-\chi}_{\mathcal{H},k,j,t} N^{\chi}_{\mathcal{H},k,j,t}. \quad \text{(B167)}$$

Let κ denote the Lagrange multiplier from the constraint (B167). Solving the problem of firms' cost minimization leads to the following equations:

$$\begin{aligned} W_t &= \kappa_t \chi K^{1-\chi}_{\mathcal{H},k,j,t} N^{\chi-1}_{\mathcal{H},k,j,t} \\ W_t &= \kappa_t \chi \left(\frac{K_{\mathcal{H},k,j,t}}{N_{\mathcal{H},k,j,t}}\right)^{1-\chi} \\ R^k_t &= \kappa_t (1-\chi)\left(\frac{K_{\mathcal{H},k,j,t}}{N_{\mathcal{H},k,j,t}}\right)^{-\chi}. \end{aligned} \quad \text{(B168)}$$

$$\begin{aligned} \frac{W_t}{R^k_t} &= \frac{\chi}{1-\chi}\frac{K_{\mathcal{H},k,j,t}}{N_{\mathcal{H},k,j,t}} \\ \frac{K_{\mathcal{H},k,j,t}}{N_{\mathcal{H},k,j,t}} &= \frac{1-\chi}{\chi}\frac{W_t}{R^k_t}. \end{aligned} \quad \text{(B169)}$$

Using (B167), one can write that

$$Y_{\mathcal{H},k,j,t} = K_{\mathcal{H},k,j,t}\left(\frac{K_{\mathcal{H},k,j,t}}{N_{\mathcal{H},k,j,t}}\right)^{-\chi}$$
$$K_{\mathcal{H},k,j,t} = Y_{\mathcal{H},k,j,t}\left(\frac{K_{\mathcal{H},k,j,t}}{N_{\mathcal{H},k,j,t}}\right)^{\chi}$$
$$K_{\mathcal{H},k,j,t} = Y_{\mathcal{H},k,j,t}\left(\frac{1-\chi}{\chi}\frac{W_t}{R_t^k}\right)^{\chi}. \tag{B170}$$

Similarly, one can show that

$$Y_{\mathcal{H},k,j,t} = N_{\mathcal{H},k,j,t}\left(\frac{K_{\mathcal{H},k,j,t}}{N_{\mathcal{H},k,j,t}}\right)^{1-\chi}$$
$$N_{\mathcal{H},k,j,t} = Y_{\mathcal{H},k,j,t}\left(\frac{K_{\mathcal{H},k,j,t}}{N_{\mathcal{H},k,j,t}}\right)^{-1+\chi}$$
$$N_{\mathcal{H},k,j,t} = Y_{\mathcal{H},k,j,t}\left(\frac{1-\chi}{\chi}\frac{W_t}{R_t^k}\right)^{-1+\chi}. \tag{B171}$$

Inserting (B170) and (B171) into the cost function yields

$$\begin{aligned}
&W_t N_{\mathcal{H},k,j,t} + R_t^k K_{\mathcal{H},k,j,t}\\
&\quad = W_t Y_{\mathcal{H},k,j,t}\left(\frac{1-\chi}{\chi}\frac{W_t}{R_t^k}\right)^{-1+\chi} + R_t^k Y_{\mathcal{H},k,j,t}\left(\frac{1-\chi}{\chi}\frac{W_t}{R_t^k}\right)^{\chi}\\
&\quad = Y_{\mathcal{H},k,j,t}\left[W_t^{\chi} {R_t^k}^{1-\chi}\left(\left(\frac{\chi}{1-\chi}\right)^{1-\chi} + \left(\frac{1-\chi}{\chi}\right)^{\chi}\right)\right]\\
&\quad = \left(\frac{1}{1-\chi}\right)^{1-\chi}\left(\frac{1}{\chi}\right)^{\chi} W_t^{\chi} {R_t^k}^{1-\chi} Y_{\mathcal{H},k,j,t}.
\end{aligned} \tag{B172}$$

B.14.8 The Objective of the Home Firm Supplying the Home Country with Variety j

When a price-change signal is received, the expected profit of the firm producing domestic variety j can be written as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t}\alpha_k^s\left(X_{\mathcal{H},k,j,t}Y_{\mathcal{H},k,j,t+s} - W_{t+s}N_{\mathcal{H},k,j,t+s} - R_{t+s}^k K_{\mathcal{H},k,j,t+s}\right), \tag{B173}$$

where $X_{\mathcal{H},k,j,t}$ denotes the price set by the firm, which remains unchanged with a probability of α_k in each period. In (B173), the stochastic discount factor the firm uses in discounting future profits is assumed to be equal to that of home households $\left(\beta^s \frac{\lambda_{t+s}}{\lambda_t}\right)$. Using (B172), (B173) can be rewritten as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^s \left(X_{\mathcal{H},k,j,t} Y_{\mathcal{H},k,j,t+s} - \left(\frac{1}{1-\chi}\right)^{1-\chi} \right.$$
$$\left. \times \left(\frac{1}{\chi}\right)^{\chi} W_{t+s}^{\chi} {R_{t+s}^{k}}^{1-\chi} Y_{\mathcal{H},k,j,t+s} \right). \tag{B174}$$

Using (B143), $Y_{\mathcal{H},k,j,t+s}$ can be written as

$$Y_{\mathcal{H},k,j,t+s} = \left(\frac{X_{\mathcal{H},k,j,t}}{P_{\mathcal{H},k,t+s}}\right)^{-\theta_p} Y_{\mathcal{H},k,t+s}$$
$$= X_{\mathcal{H},k,j,t}^{-\theta_p} \left(\frac{1}{P_{\mathcal{H},k,t+s}}\right)^{-\theta_p} Y_{\mathcal{H},k,t+s}. \tag{B175}$$

Inserting (B175) into (B174) gives

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^s \left(X_{\mathcal{H},k,j,t}^{1-\theta_p} \left(\frac{1}{P_{\mathcal{H},k,t+s}}\right)^{-\theta_p} Y_{\mathcal{H},k,t+s} - \left(\frac{1}{1-\chi}\right)^{1-\chi} \right.$$
$$\left. \times \left(\frac{1}{\chi}\right)^{\chi} W_{t+s}^{\chi} {R_{t+s}^{k}}^{1-\chi} X_{\mathcal{H},k,j,t}^{-\theta_p} \left(\frac{1}{P_{\mathcal{H},k,t+s}}\right)^{-\theta_p} Y_{\mathcal{H},k,t+s} \right). \tag{B176}$$

When a firm receives a price-change signal, it maximizes (B176) by choosing $X_{\mathcal{H},k,j,t}$. One can show that the first-order condition from this problem can be written as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^s \left[X_{\mathcal{H},k,j,t}^{-\theta_p} - \frac{\theta_p}{\theta_p - 1} \left(\frac{1}{1-\chi}\right)^{1-\chi} \left(\frac{1}{\chi}\right)^{\chi} \right.$$
$$\left. \times W_{t+s}^{\chi} {R_{t+s}^{k}}^{1-\chi} X_{\mathcal{H},k,j,t}^{-1-\theta_p} \right] \left(\frac{1}{P_{\mathcal{H},k,t+s}}\right)^{-\theta_p} Y_{\mathcal{H},k,t+s} = 0. \tag{B177}$$

Therefore, one can write

$$X_{\mathcal{H},k,j,t} = \frac{\theta_p}{\theta_p - 1}\left(\frac{1}{1-\chi}\right)^{1-\chi}\left(\frac{1}{\chi}\right)^{\chi} \times \frac{E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^s W_{t+s}^{\chi} R_{t+s}^{k}{}^{1-\chi} \left(\frac{1}{P_{\mathcal{H},k,t+s}}\right)^{-\theta_p} Y_{\mathcal{H},k,t+s}}{E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^s \left(\frac{1}{P_{\mathcal{H},k,t+s}}\right)^{-\theta_p} Y_{\mathcal{H},k,t+s}}. \tag{B178}$$

To log-linearize (B178), first rewrite it as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^s \left(\frac{1}{P_{\mathcal{H},k,t+s}}\right)^{-\theta_p} Y_{\mathcal{H},k,t+s} X_{\mathcal{H},k,j,t} = \frac{\theta_p}{\theta_p - 1}\left(\frac{1}{1-\chi}\right)^{1-\chi}\left(\frac{1}{\chi}\right)^{\chi} E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^s W_{t+s}^{\chi} R_{t+s}^{k}{}^{1-\chi} \times \left(\frac{1}{P_{\mathcal{H},k,t+s}}\right)^{-\theta_p} Y_{\mathcal{H},k,t+s}. \tag{B179}$$

At the flexible-price steady state, the following equation must hold:

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t} \alpha_k^s \left(\frac{1}{\bar{P}_{\mathcal{H},k,t+s}}\right)^{-\theta_p} \bar{Y}_{\mathcal{H},k} \bar{X}_{\mathcal{H},k,j,t} = \frac{\theta_p}{\theta_p - 1}\left(\frac{1}{1-\chi}\right)^{1-\chi}\left(\frac{1}{\chi}\right)^{\chi} E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t} \alpha_k^s \bar{W}_{t+s}^{\chi} \bar{R}_{t+s}^{k\,1-\chi} \times \left(\frac{1}{\bar{P}_{\mathcal{H},k,t+s}}\right)^{-\theta_p} \bar{Y}_{\mathcal{H},k}. \tag{B180}$$

Using (B8) and that all prices rise by the same rate at the flexible-price steady state as assumed in (B10), one can rewrite the left-hand side of (B180) as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t} \alpha_k^s \left(\frac{1}{\bar{P}_{\mathcal{H},k,t+s}}\right)^{-\theta_p} \bar{Y}_{\mathcal{H},k} \bar{X}_{\mathcal{H},k,j,t}$$

$$= E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{P}_t}{\bar{P}_{t+s}} \alpha_k^s \left(\frac{1}{\bar{P}_{\mathcal{H},k,t+s}} \right)^{-\theta_p} \bar{Y}_{\mathcal{H},k} \bar{X}_{\mathcal{H},k,j,t}$$

$$= E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{P}_t}{\bar{P}_t e^{\bar{\pi} s}} \alpha_k^s \left(\frac{1}{\bar{P}_t e^{\bar{\pi} s}} \right)^{-\theta_p} \bar{Y}_{\mathcal{H},k} \bar{X}_{\mathcal{H},k,j,t}$$

$$= E_t \sum_{s=0}^{\infty} \left(\beta \alpha_k e^{\bar{\pi}(\theta_p - 1)} \right)^s \bar{P}_t^{\theta_p} \bar{Y}_{\mathcal{H},k} \bar{X}_{\mathcal{H},k,j,t}$$

$$= \frac{1}{1 - \beta \alpha_k e^{\bar{\pi}(\theta_p - 1)}} \bar{P}_t^{\theta_p} \bar{Y}_{\mathcal{H},k} \bar{X}_{\mathcal{H},k,j,t}. \tag{B181}$$

Next, we aim to simplify the term excluding the constant in the right-hand side of (B180). To do this, first note that since real wages and the real rental rate of capital are constant at the steady state, their nominal values must grow by steady-state inflation:

$$\bar{W}_{t+s}^{\chi} \bar{R}_{t+s}^{k^{1-\chi}} = \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} e^{\bar{\pi} s}. \tag{B182}$$

Using (B8), (B182), and the assumption that all prices rise by the same rate at the flexible-price steady state, one can rewrite the right-hand side of (B180) as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t} \alpha_k^s \bar{W}_{t+s}^{\chi} \bar{R}_{t+s}^{k^{1-\chi}} \left(\frac{1}{\bar{P}_{\mathcal{H},k,t+s}} \right)^{-\theta_p} \bar{Y}_{\mathcal{H},k}$$

$$= E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{P}_t}{\bar{P}_t e^{\bar{\pi} s}} \alpha_k^s \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} e^{\bar{\pi} s} \left(\frac{1}{\bar{P}_t e^{\bar{\pi} s}} \right)^{-\theta_p} \bar{Y}_{\mathcal{H},k}$$

$$= E_t \sum_{s=0}^{\infty} \left(\beta \alpha_k e^{\bar{\pi} \theta_p} \right)^s \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} \bar{P}_t^{\theta_p} \bar{Y}_{\mathcal{H},k}$$

$$= \frac{1}{1 - \beta \alpha_k e^{\bar{\pi} \theta_p}} \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} \bar{P}_t^{\theta_p} \bar{Y}_{\mathcal{H},k}. \tag{B183}$$

From (B180), (B181), and (B183), it is easy to see that

$$\frac{1}{1 - \beta \alpha_k e^{\bar{\pi}(\theta_p - 1)}} \bar{P}_t^{\theta_p} \bar{Y}_{\mathcal{H},k} \bar{X}_{\mathcal{H},k,j,t}$$

$$= \frac{\theta_p}{\theta_p - 1} \left(\frac{1}{1 - \chi} \right)^{1-\chi} \left(\frac{1}{\chi} \right)^{\chi} \frac{1}{1 - \beta \alpha_k e^{\bar{\pi} \theta_p}} \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} \bar{P}_t^{\theta_p} \bar{Y}_{\mathcal{H},k}$$

$$\times \frac{1-\beta\alpha_k e^{\bar{\pi}\theta_p}}{1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}}\bar{P}_t^{\theta_p}\bar{Y}_{\mathcal{H},k}\bar{X}_{\mathcal{H},k,j,t}$$

$$= \frac{\theta_p}{\theta_p-1}\left(\frac{1}{1-\chi}\right)^{1-\chi}\left(\frac{1}{\chi}\right)^{\chi}\bar{W}_t^{\chi}\bar{R}_t^{k^{1-\chi}}\bar{P}_t^{\theta_p}\bar{Y}_{\mathcal{H},k}. \qquad \text{(B184)}$$

Now, note that taking a log-linear approximation around positive steady-state inflation of the left-hand side of (B179) yields

$$E_t\sum_{s=0}^{\infty}\beta^s\frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t}\alpha_k^s\left(\frac{1}{\bar{P}_{\mathcal{H},k,t+s}}\right)^{-\theta_p}\bar{Y}_{\mathcal{H},k}\bar{X}_{\mathcal{H},k,j,t}$$

$$\times\left[\hat{\lambda}_{t+s}-\hat{\lambda}_t+\theta_p\hat{P}_{\mathcal{H},k,t+s}+\hat{Y}_{\mathcal{H},k,t+s}+\hat{X}_{\mathcal{H},k,j,t}\right]$$

$$= E_t\sum_{s=0}^{\infty}\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^s\bar{P}_t^{\theta_p}\bar{Y}_{\mathcal{H},k}\bar{X}_{\mathcal{H},k,j,t}$$

$$\times\left[\hat{\lambda}_{t+s}-\hat{\lambda}_t+\theta_p\hat{P}_{\mathcal{H},k,t+s}+\hat{Y}_{\mathcal{H},k,t+s}+\hat{X}_{\mathcal{H},k,j,t}\right]. \qquad \text{(B185)}$$

Similarly, taking a log-linear approximation around positive steady-state inflation of the right-hand side of (B179) yields

$$\frac{\theta_p}{\theta_p-1}\left(\frac{1}{1-\chi}\right)^{1-\chi}\left(\frac{1}{\chi}\right)^{\chi}\times$$

$$\times E_t\sum_{s=0}^{\infty}\beta^s\frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t}\alpha_k^s\bar{W}_{t+s}^{\chi}\bar{R}_{t+s}^{k^{1-\chi}}\left(\frac{1}{\bar{P}_{\mathcal{H},k,t+s}}\right)^{-\theta_p}\bar{Y}_{\mathcal{H},k}$$

$$\times\left[\hat{\lambda}_{t+s}-\hat{\lambda}_t+\chi\hat{W}_{t+s}+(1-\chi)\hat{R}_{t+s}^k+\theta_p\hat{P}_{\mathcal{H},k,t+s}+\hat{Y}_{\mathcal{H},k,t+s}\right]$$

$$= \frac{\theta_p}{\theta_p-1}\left(\frac{1}{1-\chi}\right)^{1-\chi}\left(\frac{1}{\chi}\right)^{\chi}\times$$

$$E_t\sum_{s=0}^{\infty}\left(\beta\alpha_k e^{\bar{\pi}\theta_p}\right)^s\bar{W}_t^{\chi}\bar{R}_t^{k^{1-\chi}}\bar{P}_t^{\theta_p}\bar{Y}_{\mathcal{H},k}$$

$$\times\left[\hat{\lambda}_{t+s}-\hat{\lambda}_t+\chi\hat{W}_{t+s}+(1-\chi)\hat{R}_{t+s}^k+\theta_p\hat{P}_{\mathcal{H},k,t+s}+\hat{Y}_{\mathcal{H},k,t+s}\right]$$

$$= \frac{1-\beta\alpha_k e^{\bar{\pi}\theta_p}}{1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}}\bar{P}_t^{\theta_p}\bar{Y}_{\mathcal{H},k}\bar{X}_{\mathcal{H},k,j,t}$$

$$\times E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k e^{\bar{\pi}\theta_p}\right)^s \Big[\hat{\lambda}_{t+s} - \hat{\lambda}_t + \chi \hat{W}_{t+s} + (1-\chi)\hat{R}^k_{t+s}$$
$$+ \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\Big]. \qquad \text{(B186)}$$

Using (B185) and (B186), it is easy to show that

$$E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^s \Big[\hat{\lambda}_{t+s} - \hat{\lambda}_t + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s} + \hat{X}_{\mathcal{H},k,j,t}\Big]$$
$$= E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k e^{\bar{\pi}\theta_p}\right)^s \frac{1-\beta\alpha_k e^{\bar{\pi}\theta_p}}{1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}} \Big[\hat{\lambda}_{t+s} - \hat{\lambda}_t + \chi \hat{W}_{t+s}$$
$$+ (1-\chi)\hat{R}^k_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\Big]. \qquad \text{(B187)}$$

Rearranging terms in (B187) gives

$$\frac{1}{1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}}\left(\hat{X}_{\mathcal{H},k,j,t} - \hat{\lambda}_t\right) + E_t \sum_{s=0}^{\infty} \left\{\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^s\right\}$$
$$\times \Big[\hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\Big]$$
$$= E_t \sum_{s=0}^{\infty} \left\{\left(\beta\alpha_k e^{\bar{\pi}\theta_p}\right)^s \frac{1-\beta\alpha_k e^{\bar{\pi}\theta_p}}{1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right\}$$
$$\times \Big[\chi \hat{W}_{t+s} + (1-\chi)\hat{R}^k_{t+s} + \hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\Big]$$
$$- \frac{1}{1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}}\hat{\lambda}_t$$
$$\Downarrow$$
$$\hat{X}_{\mathcal{H},k,j,t} + E_t \sum_{s=0}^{\infty} \left\{\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^s \left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\right\}$$
$$\times \Big[\hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\Big]$$

$$= E_t \sum_{s=0}^{\infty} \left\{ \left(\beta \alpha_k e^{\bar{\pi}\theta_p}\right)^s \left(1 - \beta \alpha_k e^{\bar{\pi}\theta_p}\right) \right\} \times \left[\chi \hat{W}_{t+s} + (1-\chi)\hat{R}^k_{t+s} + \hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\right]. \tag{B188}$$

To simplify (B188), rewrite it as

$$\hat{X}_{\mathcal{H},k,j,t} + E_t \sum_{s=0}^{\infty} \left\{ \left(\beta \alpha_k e^{\bar{\pi}(\theta_p - 1)}\right)^s \left(1 - \beta \alpha_k e^{\bar{\pi}(\theta_p-1)}\right) \right\} \times \left[\hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\right] = \left(1 - \beta \alpha_k e^{\bar{\pi}\theta_p}\right) \times \left[\chi \hat{W}_t + (1-\chi)\hat{R}^k_t + \hat{\lambda}_t + \theta_p \hat{P}_{\mathcal{H},k,t} + \hat{Y}_{\mathcal{H},k,t}\right] + E_t \sum_{s=1}^{\infty} \left\{ \left(\beta \alpha_k e^{\bar{\pi}\theta_p}\right)^s \left(1 - \beta \alpha_k e^{\bar{\pi}\theta_p}\right) \right\} \times \left[\chi \hat{W}_{t+s} + (1-\chi)\hat{R}^k_{t+s} + \hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\right]. \tag{B189}$$

Next, we restate (B189) as

$$\hat{X}_{\mathcal{H},k,j,t} + E_t \sum_{s=0}^{\infty} \left\{ \left(\beta \alpha_k e^{\bar{\pi}(\theta_p - 1)}\right)^s \left(1 - \beta \alpha_k e^{\bar{\pi}(\theta_p-1)}\right) \right\} \times \left[\hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\right] = \left(1 - \beta \alpha_k e^{\bar{\pi}\theta_p}\right) \times \left[\chi \hat{W}_t + (1-\chi)\hat{R}^k_t + \hat{\lambda}_t + \theta_p \hat{P}_{\mathcal{H},k,t} + \hat{Y}_{\mathcal{H},k,t}\right] + \left(\beta \alpha_k e^{\bar{\pi}\theta_p}\right) E_t \sum_{s=0}^{\infty} \left\{ \left(\beta \alpha_k e^{\bar{\pi}\theta_p}\right)^s \left(1 - \beta \alpha_k e^{\bar{\pi}\theta_p}\right) \right\} \times \left[\chi \hat{W}_{t+s+1} + (1-\chi)\hat{R}^k_{t+s+1} + \hat{\lambda}_{t+s+1} + \theta_p \hat{P}_{\mathcal{H},k,t+s+1} + \hat{Y}_{\mathcal{H},k,t+s+1}\right]. \tag{B190}$$

Using (B188), (B190) can be rewritten as

$$\hat{X}_{\mathcal{H},k,j,t} + E_t \sum_{s=0}^{\infty} \left\{ \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^s \left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right) \right\} \times \left[\hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\right] = \left(1 - \beta\alpha_k e^{\bar{\pi}\theta_p}\right) \times \left[\chi\hat{W}_t + (1-\chi)\hat{R}_t^k + \hat{\lambda}_t + \theta_p \hat{P}_{\mathcal{H},k,t} + \hat{Y}_{\mathcal{H},k,t}\right] + \beta\alpha_k e^{\bar{\pi}\theta_p} \left\{ E_t \hat{X}_{\mathcal{H},k,j,t+1} + E_t \sum_{s=0}^{\infty} \left\{ \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^s \times \left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right) \right\} \left[\hat{\lambda}_{t+s+1} + \theta_p \hat{P}_{\mathcal{H},k,t+s+1} + \hat{Y}_{\mathcal{H},k,t+s+1}\right] \right\}. \tag{B191}$$

In equations (B192), (B193), and (B194), equations (B191), (B192), and (B193) are rewritten, respectively, as

$$\hat{X}_{\mathcal{H},k,j,t} - \beta\alpha_k e^{\bar{\pi}\theta_p} E_t \hat{X}_{\mathcal{H},k,j,t+1} + E_t \sum_{s=0}^{\infty} \left\{ \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^s \left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right) \right\} \times \left[\hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\right] = \left(1 - \beta\alpha_k e^{\bar{\pi}\theta_p}\right) \left[\chi\hat{W}_t + (1-\chi)\hat{R}_t^k + \hat{\lambda}_t + \theta_p \hat{P}_{\mathcal{H},k,t} + \hat{Y}_{\mathcal{H},k,t}\right] + \beta\alpha_k e^{\bar{\pi}\theta_p} \left\{ E_t \sum_{s=0}^{\infty} \left\{ \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^s \left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right) \right\} \times \left[\hat{\lambda}_{t+s+1} + \theta_p \hat{P}_{\mathcal{H},k,t+s+1} + \hat{Y}_{\mathcal{H},k,t+s+1}\right] \right\} \tag{B192}$$

$$\hat{X}_{\mathcal{H},k,j,t} - \beta\alpha_k e^{\bar{\pi}\theta_p} E_t \hat{X}_{\mathcal{H},k,j,t+1} + E_t \sum_{s=0}^{\infty} \left\{ \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^s \left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right) \right\} \times \left[\hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\right]$$

$$= \left(1 - \beta\alpha_k e^{\bar{\pi}\theta_p}\right)\left[\chi \hat{W}_t + (1-\chi)\hat{R}_t^k + \hat{\lambda}_t + \theta_p \hat{P}_{\mathcal{H},k,t} + \hat{Y}_{\mathcal{H},k,t}\right]$$
$$+ e^{\bar{\pi}}\left\{E_t \sum_{s=0}^{\infty}\left\{\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^{s+1}\left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\right\}\right.$$
$$\left.\times\left[\hat{\lambda}_{t+s+1} + \theta_p \hat{P}_{\mathcal{H},k,t+s+1} + \hat{Y}_{\mathcal{H},k,t+s+1}\right]\right\} \quad \text{(B193)}$$

$$\hat{X}_{\mathcal{H},k,j,t} - \beta\alpha_k e^{\bar{\pi}\theta_p} E_t \hat{X}_{\mathcal{H},k,j,t+1}$$
$$+ E_t \sum_{s=0}^{\infty}\left\{\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^{s}\left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\right\}$$
$$\times\left[\hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\right]$$
$$= \left(1 - \beta\alpha_k e^{\bar{\pi}\theta_p}\right)\left[\chi \hat{W}_t + (1-\chi)\hat{R}_t^k + \hat{\lambda}_t + \theta_p \hat{P}_{\mathcal{H},k,t} + \hat{Y}_{\mathcal{H},k,t}\right]$$
$$+ e^{\bar{\pi}}\left\{E_t \sum_{s=1}^{\infty}\left\{\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^{s}\left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\right\}\right.$$
$$\left.\times\left[\hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\right]\right\} \quad \text{(B194)}$$

$$\hat{X}_{\mathcal{H},k,j,t} - \beta\alpha_k e^{\bar{\pi}\theta_p} E_t \hat{X}_{\mathcal{H},k,j,t+1} - \left(1 - \beta\alpha_k e^{\bar{\pi}\theta_p}\right)\left[\chi \hat{W}_t + (1-\chi)\hat{R}_t^k\right]$$
$$+ \left(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left[\hat{\lambda}_t + \theta_p \hat{P}_{\mathcal{H},k,t} + \hat{Y}_{\mathcal{H},k,t}\right]$$
$$= E_t \sum_{s=1}^{\infty}\left\{\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^{s}\left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}} - 1\right)\right\}$$
$$\times\left[\hat{\lambda}_{t+s} + \theta_p \hat{P}_{\mathcal{H},k,t+s} + \hat{Y}_{\mathcal{H},k,t+s}\right] \quad \text{(B195)}$$

$$\hat{X}_{\mathcal{H},k,j,t} - \beta\alpha_k e^{\bar{\pi}\theta_p} E_t \hat{X}_{\mathcal{H},k,j,t+1} - \left(1 - \beta\alpha_k e^{\bar{\pi}\theta_p}\right)\left[\chi \hat{W}_t + (1-\chi)\hat{R}_t^k\right]$$
$$+ \left(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left[\hat{\lambda}_t + \theta_p \hat{P}_{\mathcal{H},k,t} + \hat{Y}_{\mathcal{H},k,t}\right]$$

$$
\begin{aligned}
&= \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right) \\
&\quad \times E_t\left[\hat{\lambda}_{t+1}+\theta_p\hat{P}_{\mathcal{H},k,t+1}+\hat{Y}_{\mathcal{H},k,t+1}\right] \\
&\quad + E_t\sum_{s=2}^{\infty}\left\{\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^s\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right)\right\} \\
&\quad \times \left[\hat{\lambda}_{t+s}+\theta_p\hat{P}_{\mathcal{H},k,t+s}+\hat{Y}_{\mathcal{H},k,t+s}\right] \qquad \text{(B196)}
\end{aligned}
$$

$$
\begin{aligned}
&\hat{X}_{\mathcal{H},k,j,t}-\beta\alpha_k e^{\bar{\pi}\theta_p}E_t\hat{X}_{\mathcal{H},k,j,t+1}-\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\left[\chi\hat{W}_t+(1-\chi)\hat{R}_t^k\right] \\
&\quad + \left(\beta\alpha_k e^{\bar{\pi}\theta_p}-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left[\hat{\lambda}_t+\theta_p\hat{P}_{\mathcal{H},k,t}+\hat{Y}_{\mathcal{H},k,t}\right] \\
&= \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right) \\
&\quad \times E_t\left[\hat{\lambda}_{t+1}+\theta_p\hat{P}_{\mathcal{H},k,t+1}+\hat{Y}_{\mathcal{H},k,t+1}\right] \\
&\quad + \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)E_t\sum_{s=1}^{\infty}\left\{\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)^s\right. \\
&\quad \times \left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right) \\
&\quad \left. \times \left[\hat{\lambda}_{t+s+1}+\theta_p\hat{P}_{\mathcal{H},k,t+s+1}+\hat{Y}_{\mathcal{H},k,t+s+1}\right]\right\}. \qquad \text{(B197)}
\end{aligned}
$$

Using (B195), one can restate (B197) as

$$
\begin{aligned}
&\hat{X}_{\mathcal{H},k,j,t}-\beta\alpha_k e^{\bar{\pi}\theta_p}E_t\hat{X}_{\mathcal{H},k,j,t+1} \\
&= \left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\left[\chi\hat{W}_t+(1-\chi)\hat{R}_t^k\right] \\
&\quad - \left(\beta\alpha_k e^{\bar{\pi}\theta_p}-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left[\hat{\lambda}_t+\theta_p\hat{P}_{\mathcal{H},k,t}+\hat{Y}_{\mathcal{H},k,t}\right] \\
&\quad + \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right)E_t \\
&\quad \times \left[\hat{\lambda}_{t+1}+\theta_p\hat{P}_{\mathcal{H},k,t+1}+\hat{Y}_{\mathcal{H},k,t+1}\right] \\
&\quad + \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left[E_t\hat{X}_{\mathcal{H},k,j,t+1}-\beta\alpha_k e^{\bar{\pi}\theta_p}E_t\hat{X}_{\mathcal{H},k,j,t+2}\right]
\end{aligned}
$$

$$
\begin{aligned}
&- \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\left[\chi E_t\hat{W}_{t+1}+(1-\chi)E_t\hat{R}^k_{t+1}\right] \\
&+ \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(\beta\alpha_k e^{\bar{\pi}\theta_p}-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right) \\
&\times \left[E_t\hat{\lambda}_{t+1}+\theta_p E_t\hat{P}_{\mathcal{H},k,t+1}+E_t\hat{Y}_{\mathcal{H},k,t+1}\right]. \qquad \text{(B198)}
\end{aligned}
$$

In each period, the fraction of the firms receiving a price-change signal is $1-\alpha_k$. Since firms are identical, the firms receiving this signal set their prices to the same price. This, together with the randomization assumption in the Calvo model and (B145), yields

$$
\begin{aligned}
P^{1-\theta_p}_{\mathcal{H},k,t} &= (1-\alpha_k)X^{1-\theta_p}_{\mathcal{H},k,t}+\alpha_k P^{1-\theta_p}_{\mathcal{H},k,t-1} \\
1 &= (1-\alpha_k)\left(\frac{\bar{X}_{\mathcal{H},k,t}}{\bar{P}_{\mathcal{H},k,t}}\right)^{1-\theta_p}+\alpha_k\left(\frac{\bar{P}_{\mathcal{H},k,t-1}}{\bar{P}_{\mathcal{H},k,t}}\right)^{1-\theta_p} \\
&\times\left(\frac{1-\alpha_k e^{-\pi(1-\theta_p)}}{1-\alpha_k}\right)=\left(\frac{\bar{X}_{\mathcal{H},k,t}}{\bar{P}_{\mathcal{H},k,t}}\right)^{1-\theta_p}. \qquad \text{(B199)}
\end{aligned}
$$

Log-linearizing (B199) yields

$$
\begin{aligned}
\hat{P}_{\mathcal{H},k,t} &= (1-\alpha_k)\left(\frac{1-\alpha_k e^{-\bar{\pi}(1-\theta_p)}}{1-\alpha_k}\right)\hat{X}_{\mathcal{H},k,t}+\alpha_k e^{-\bar{\pi}(1-\theta_p)}\hat{P}_{\mathcal{H},k,t-1} \\
\hat{P}_{\mathcal{H},k,t} &= \left(1-\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\hat{X}_{\mathcal{H},k,t}+\alpha_k e^{\bar{\pi}(\theta_p-1)}\hat{P}_{\mathcal{H},k,t-1} \\
\hat{X}_{\mathcal{H},k,t} &= \left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\left[\hat{P}_{\mathcal{H},k,t}-\alpha_k e^{\bar{\pi}(\theta_p-1)}\hat{P}_{\mathcal{H},k,t-1}\right]. \qquad \text{(B200)}
\end{aligned}
$$

Using (B200), one can rewrite (B198) as

$$
\begin{aligned}
&\left\{\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\left[\hat{P}_{\mathcal{H},k,t}-\alpha_k e^{\bar{\pi}(\theta_p-1)}\hat{P}_{\mathcal{H},k,t-1}\right]\right\} \\
&\quad -\beta\alpha_k e^{\bar{\pi}\theta_p}\left\{\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\left[E_t\hat{P}_{\mathcal{H},k,t+1}-\alpha_k e^{\bar{\pi}(\theta_p-1)}\hat{P}_{\mathcal{H},k,t}\right]\right\} \\
&= \left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\left[\chi\hat{W}_t+(1-\chi)\hat{R}^k_t\right]
\end{aligned}
$$

$$
\begin{aligned}
&- \left(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left[\hat{\lambda}_t + \theta_p \hat{P}_{\mathcal{H},k,t} + \hat{Y}_{\mathcal{H},k,t}\right] \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}} - 1\right) \\
&\times E_t\left[\hat{\lambda}_{t+1} + \theta_p \hat{P}_{\mathcal{H},k,t+1} + \hat{Y}_{\mathcal{H},k,t+1}\right] \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left\{\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\right. \\
&\left.\times \left[E_t \hat{P}_{\mathcal{H},k,t+1} - \alpha_k e^{\bar{\pi}(\theta_p-1)} \hat{P}_{\mathcal{H},k,t}\right]\right\} \\
&- \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\beta\alpha_k e^{\bar{\pi}\theta_p}\left\{\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\right. \\
&\left.\times \left[E_t \hat{P}_{\mathcal{H},k,t+2} - \alpha_k e^{\bar{\pi}(\theta_p-1)} E_t \hat{P}_{\mathcal{H},k,t+1}\right]\right\} \\
&- \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1 - \beta\alpha_k e^{\bar{\pi}\theta_p}\right)\left[\chi E_t \hat{W}_{t+1} + (1-\chi) E_t \hat{R}^k_{t+1}\right] \\
&+ \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right) \\
&\times \left[E_t \hat{\lambda}_{t+1} + \theta_p E_t \hat{P}_{\mathcal{H},k,t+1} + E_t \hat{Y}_{\mathcal{H},k,t+1}\right]. \qquad \text{(B201)}
\end{aligned}
$$

Next, we restate (B201) as

$$
\begin{aligned}
&\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\hat{P}_{\mathcal{H},k,t} - \left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)}\hat{P}_{\mathcal{H},k,t-1} \\
&\quad - \beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right) E_t \hat{P}_{\mathcal{H},k,t+1} \\
&\quad + \beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)}\hat{P}_{\mathcal{H},k,t} \\
&= \left(1 - \beta\alpha_k e^{\bar{\pi}\theta_p}\right)\chi \hat{W}_t + \left(1 - \beta\alpha_k e^{\bar{\pi}\theta_p}\right)(1-\chi)\hat{R}^k_t \\
&\quad - \left(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\hat{\lambda}_t \\
&\quad - \left(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\theta_p \hat{P}_{\mathcal{H},k,t}
\end{aligned}
$$

$$
\begin{aligned}
&- \left(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\hat{Y}_{\mathcal{H},k,t} \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}} - 1\right)E_t\hat{\lambda}_{t+1} \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}} - 1\right)\theta_p E_t\hat{P}_{\mathcal{H},k,t+1} \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(1 - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}} - 1\right)E_t\hat{Y}_{\mathcal{H},k,t+1} \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)E_t\hat{P}_{\mathcal{H},k,t+1} \\
&- \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)}\hat{P}_{\mathcal{H},k,t} \\
&- \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)E_t\hat{P}_{\mathcal{H},k,t+2} \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)}E_t\hat{P}_{\mathcal{H},k,t+1} \\
&- \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(1 - \beta\alpha_k e^{\bar{\pi}\theta_p}\right)\chi E_t\hat{W}_{t+1} \\
&- \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(1 - \beta\alpha_k e^{\bar{\pi}\theta_p}\right)(1-\chi)E_t\hat{R}^k_{t+1} \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)E_t\hat{\lambda}_{t+1} \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\theta_p E_t\hat{P}_{\mathcal{H},k,t+1} \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)E_t\hat{Y}_{\mathcal{H},k,t+1} \qquad \text{(B202)}
\end{aligned}
$$

$$
\begin{aligned}
&\Bigg\{\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right) + \beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)} \\
&\quad + \left(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\theta_p \\
&\quad + \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)}\Bigg\}\hat{P}_{\mathcal{H},k,t} \\
&= \left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)}\hat{P}_{\mathcal{H},k,t-1}
\end{aligned}
$$

$$+\left\{\beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\right.$$

$$+\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right)\theta_p$$

$$+\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)$$

$$+\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)}$$

$$\left.+\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(\beta\alpha_k e^{\bar{\pi}\theta_p}-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\theta_p\right\}E_t\hat{P}_{\mathcal{H},k,t+1}$$

$$+\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\chi\hat{W}_t$$

$$+\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)(1-\chi)\hat{R}^k_t-\left(\beta\alpha_k e^{\bar{\pi}\theta_p}-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\hat{\lambda}_t$$

$$-\left(\beta\alpha_k e^{\bar{\pi}\theta_p}-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\hat{Y}_{\mathcal{H},k,t}$$

$$+\left\{\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right)\right.$$

$$\left.+\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(\beta\alpha_k e^{\bar{\pi}\theta_p}-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\right\}E_t\hat{\lambda}_{t+1}$$

$$+\left\{\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right)\right.$$

$$\left.+\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(\beta\alpha_k e^{\bar{\pi}\theta_p}-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\right\}E_t\hat{Y}_{\mathcal{H},k,t+1}$$

$$-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)E_t\hat{P}_{\mathcal{H},k,t+2}$$

$$-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\chi E_t\hat{W}_{t+1}$$

$$-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)(1-\chi)E_t\hat{R}^k_{t+1}. \tag{B203}$$

B.14.9 The Objective of the Home-Export Firm Producing Variety j

In our model, we make an important assumption that the invoice currency also determines the degree of price rigidity faced by the firms. Indeed, if prices are set in the home (foreign) currency, they remain fixed with the probability of not receiving a price-change signal in the home (foreign) country. That is, if a sector k firm sets its price in the home (foreign) currency, the probability of not receiving a price-change signal in each period is α_k (α_k^*). We first describe the problem of the home-export firm that receives a price-change signal in period t and sets prices in the *home currency.*

B.14.9.1 The Objective of the Home-Export Firm Producing Variety j and Setting Prices in the Home Currency

When a price-change signal is received, the expected profit of the home-export firm producing variety j can be written as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^s \Big(X^*_{\mathcal{H},¢,k,j,t} Y^*_{\mathcal{H},¢,k,j,t} - W_{t+s} N^*_{\mathcal{H},¢,k,j,t} - R^k_{t+s} K^*_{\mathcal{H},¢,k,j,t} \Big), \tag{B204}$$

where the price set by the home-export firm in the *home currency* is denoted by $X^*_{\mathcal{H},¢,k,j,t}$, which remains unchanged with a probability of α_k in each period. Similar to (B178) and (B203), one can show that (B205) and (B206) must hold:

$$X^*_{\mathcal{H},¢,k,j,t} = \frac{\theta_p}{\theta_p - 1} \left(\frac{1}{1-\chi} \right)^{1-\chi} \left(\frac{1}{\chi} \right)^{\chi} \times \frac{E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^s W^{\chi}_{t+s} {R^k_{t+s}}^{1-\chi} \left(\frac{1}{P^*_{\mathcal{H},¢,k,t+s}} \right)^{-\theta_p} Y^*_{\mathcal{H},¢,k,t+s}}{E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^s \left(\frac{1}{P^*_{\mathcal{H},¢,k,t+s}} \right)^{-\theta_p} Y^*_{\mathcal{H},¢,k,t+s}} \tag{B205}$$

$$\Bigg\{ \left(\frac{1}{1 - \alpha_k e^{\bar{\pi}(\theta_p - 1)}} \right) + \beta \alpha_k e^{\bar{\pi}\theta_p} \left(\frac{1}{1 - \alpha_k e^{\bar{\pi}(\theta_p - 1)}} \right) \alpha_k e^{\bar{\pi}(\theta_p - 1)} + \left(\beta \alpha_k e^{\bar{\pi}\theta_p} - \beta \alpha_k e^{\bar{\pi}(\theta_p - 1)} \right) \theta_p$$

$$
\begin{aligned}
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right) \alpha_k e^{\bar{\pi}(\theta_p-1)} \Bigg\} \hat{P}^*_{\mathcal{H},\not{c},k,t} \\
= &\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right) \alpha_k e^{\bar{\pi}(\theta_p-1)} \hat{P}^*_{\mathcal{H},\not{c},k,t-1} \\
&+ \Bigg\{ \beta\alpha_k e^{\bar{\pi}\theta_p} \left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right) \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \Big(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\Big) \Big(e^{\bar{\pi}}-1\Big)\theta_p \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right) \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \beta\alpha_k e^{\bar{\pi}\theta_p} \left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right) \alpha_k e^{\bar{\pi}(\theta_p-1)} \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \Big(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\Big)\theta_p \Bigg\} E_t \hat{P}^*_{\mathcal{H},\not{c},k,t+1} \\
&+ \Big(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\Big)\chi \hat{W}_t + \Big(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\Big)(1-\chi)\hat{R}^k_t \\
&- \Big(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\Big)\hat{\lambda}_t \\
&- \Big(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\Big)\hat{Y}^*_{\mathcal{H},\not{c},k,t} \\
&+ \Bigg\{ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \Big(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\Big) \Big(e^{\bar{\pi}}-1\Big) \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \Big(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\Big) \Bigg\} E_t \hat{\lambda}_{t+1} \\
&+ \Bigg\{ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \Big(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\Big) \Big(e^{\bar{\pi}}-1\Big) \\
&+ \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \Big(\beta\alpha_k e^{\bar{\pi}\theta_p} - \beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\Big) \Bigg\} E_t \hat{Y}^*_{\mathcal{H},\not{c},k,t+1} \\
&- \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \beta\alpha_k e^{\bar{\pi}\theta_p} \left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right) E_t \hat{P}^*_{\mathcal{H},\not{c},k,t+2} \\
&- \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \Big(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\Big)\chi E_t \hat{W}_{t+1} \\
&- \beta\alpha_k e^{\bar{\pi}(\theta_p-1)} \Big(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\Big)(1-\chi) E_t \hat{R}^k_{t+1}. \qquad \text{(B206)}
\end{aligned}
$$

B.14.9.2 The Objective of the Home-Export Firm Producing Variety j and Setting Prices in the Foreign Currency

When a price-change signal is received, the expected profit of the home-export firm that produces variety j and sets its price in the *foreign currency* can be written as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^{*^s} \left(\mathcal{E}_{t+s} X^*_{\mathcal{H},\not{c}^*,k,j,t} Y^*_{\mathcal{H},\not{c}^*,k,j,t+s} \right.$$
$$\left. - W_{t+s} N^*_{\mathcal{H},\not{c}^*,k,j,t+s} - R^k_{t+s} K^*_{\mathcal{H},\not{c}^*,k,j,t+s} \right), \qquad \text{(B207)}$$

where the price set by the home-export firm in the *foreign currency* is denoted by$X^*_{\mathcal{H},\not{c},k,j,t}$, which remains unchanged with a probability of α^*_k in each period. Using (B172), one can rewrite (B207) as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^{*^s} \left(\mathcal{E}_{t+s} X^*_{\mathcal{H},\not{c}^*,k,j,t} Y^*_{\mathcal{H},\not{c}^*,k,j,t+s} \right.$$
$$\left. - \left(\frac{1}{1-\chi} \right)^{1-\chi} \left(\frac{1}{\chi} \right)^{\chi} W^{\chi}_{t+s} {R^k_{t+s}}^{1-\chi} Y^*_{\mathcal{H},\not{c}^*,k,j,t+s} \right). \qquad \text{(B208)}$$

Using (B155), $Y^*_{\mathcal{H},\not{c}^*,k,j,t+s}$ can be written as

$$Y^*_{\mathcal{H},\not{c}^*,k,j,t+s} = \left(\frac{X^*_{\mathcal{H},\not{c}^*,k,j,t}}{P^*_{\mathcal{H},\not{c}^*,k,t+s}} \right)^{-\theta_p} Y^*_{\mathcal{H},\not{c}^*,k,t+s}$$
$$= {X^*_{\mathcal{H},\not{c}^*,k,j,t}}^{-\theta_p} \left(\frac{1}{P^*_{\mathcal{H},\not{c}^*,k,t+s}} \right)^{-\theta_p} Y^*_{\mathcal{H},\not{c}^*,k,t+s}. \qquad \text{(B209)}$$

Inserting (B209) into (B208) gives

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \alpha_k^{*^s} \left[\mathcal{E}_{t+s} {X^*_{\mathcal{H},\not{c}^*,k,j,t}}^{1-\theta_p} \left(\frac{1}{P^*_{\mathcal{H},\not{c}^*,k,t+s}} \right)^{-\theta_p} Y^*_{\mathcal{H},\not{c}^*,k,t+s} \right.$$
$$- \left(\frac{1}{1-\chi} \right)^{1-\chi} \left(\frac{1}{\chi} \right)^{\chi} W^{\chi}_{t+s} {R^k_{t+s}}^{1-\chi} {X^*_{\mathcal{H},\not{c}^*,k,j,t}}^{-\theta_p}$$
$$\left. \times \left(\frac{1}{P^*_{\mathcal{H},\not{c}^*,k,t+s}} \right)^{-\theta_p} Y^*_{\mathcal{H},\not{c}^*,k,t+s} \right]. \qquad \text{(B210)}$$

When the firm receives a price-change signal, it maximizes (B210) by choosing $X^*_{\mathcal{H},\not{c}^*,k,j,t}$. One can show that the first-order condition from this problem can be written as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} {\alpha_k^*}^s \left[\mathcal{E}_{t+s} {X^*_{\mathcal{H},\not{c}^*,k,j,t}}^{-\theta_p} - \left(\frac{\theta_p}{\theta_p - 1}\right) \left(\frac{1}{1-\chi}\right)^{1-\chi} \left(\frac{1}{\chi}\right)^{\chi} \right.$$
$$\left. \times W^{\chi}_{t+s} {R^k_{t+s}}^{1-\chi} {X^*_{\mathcal{H},\not{c}^*,k,j,t}}^{-\theta_p - 1} \right] \left(\frac{1}{P^*_{\mathcal{H},\not{c}^*,k,t+s}}\right)^{-\theta_p} Y^*_{\mathcal{H},\not{c}^*,k,t+s} = 0. \tag{B211}$$

One can therefore write that

$$X^*_{\mathcal{H},\not{c}^*,k,j,t} = \frac{\theta_p}{\theta_p - 1} \left(\frac{1}{1-\chi}\right)^{1-\chi} \left(\frac{1}{\chi}\right)^{\chi}$$
$$\times \frac{E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} {\alpha_k^*}^s W^{\chi}_{t+s} {R^k_{t+s}}^{1-\chi} \left(\frac{1}{P^*_{\mathcal{H},\not{c}^*,k,t+s}}\right)^{-\theta_p} Y^*_{\mathcal{H},\not{c}^*,k,t+s}}{E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} {\alpha_k^*}^s \mathcal{E}_{t+s} \left(\frac{1}{P^*_{\mathcal{H},\not{c}^*,k,t+s}}\right)^{-\theta_p} Y^*_{\mathcal{H},\not{c}^*,k,t+s}}. \tag{B212}$$

To log-linearize (B212), first rewrite it as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} {\alpha_k^*}^s \mathcal{E}_{t+s} \left(\frac{1}{P^*_{\mathcal{H},\not{c}^*,k,t+s}}\right)^{-\theta_p} Y^*_{\mathcal{H},\not{c}^*,k,t+s} X^*_{\mathcal{H},\not{c}^*,k,j,t}$$
$$= \frac{\theta_p}{\theta_p - 1} \left(\frac{1}{1-\chi}\right)^{1-\chi} \left(\frac{1}{\chi}\right)^{\chi} E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} {\alpha_k^*}^s W^{\chi}_{t+s} {R^k_{t+s}}^{1-\chi}$$
$$\times \left(\frac{1}{P^*_{\mathcal{H},\not{c}^*,k,t+s}}\right)^{-\theta_p} Y^*_{\mathcal{H},\not{c}^*,k,t+s}. \tag{B213}$$

At the flexible-price steady state, the following equation must hold:

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t} {\alpha_k^*}^s \bar{\mathcal{E}}_{t+s} \left(\frac{1}{\bar{P}^*_{\mathcal{H},\not{c}^*,k,t+s}}\right)^{-\theta_p} \bar{Y}^*_{\mathcal{H},\not{c}^*,k} \bar{X}^*_{\mathcal{H},\not{c}^*,k,j,t}$$

$$= \frac{\theta_p}{\theta_p - 1}\left(\frac{1}{1-\chi}\right)^{1-\chi}\left(\frac{1}{\chi}\right)^{\chi} E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t} \alpha_k^{*^s} \bar{W}_{t+s}^{\chi} \bar{R}_{t+s}^{k^{1-\chi}}$$
$$\times \left(\frac{1}{\bar{P}^*_{\mathcal{H},\cent^*,k,t+s}}\right)^{-\theta_p} \bar{Y}^*_{\mathcal{H},\cent^*,k}. \quad \text{(B214)}$$

Using (B8), (B110), and that all foreign-currency prices increase by the same rate at the flexible-price steady state as assumed in (B86), one can rewrite the left-hand side of (B214) as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t} \alpha_k^{*^s} \bar{\mathcal{E}}_{t+s} \left(\frac{1}{\bar{P}^*_{\mathcal{H},\cent^*,k,t+s}}\right)^{-\theta_p} \bar{Y}^*_{\mathcal{H},\cent^*,k} \bar{X}^*_{\mathcal{H},\cent^*,k,j,t}$$
$$= E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{P}_t}{\bar{P}_{t+s}} \alpha_k^{*^s} \bar{\mathcal{E}}_{t+s} \left(\frac{1}{\bar{P}^*_{\mathcal{H},\cent^*,k,t+s}}\right)^{-\theta_p} \bar{Y}^*_{\mathcal{H},\cent^*,k} \bar{X}^*_{\mathcal{H},\cent^*,k,j,t}$$
$$= E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{P}_t}{\bar{P}_t e^{\bar{\pi} s}} \alpha_k^{*^s} \bar{\mathcal{E}}_t e^{(\bar{\pi}-\bar{\pi}^*)s}$$
$$\times \left(\frac{1}{\bar{P}^*_{\mathcal{H},\cent^*,k,t} e^{\bar{\pi}^* s}}\right)^{-\theta_p} \bar{Y}^*_{\mathcal{H},\cent^*,k} \bar{X}^*_{\mathcal{H},\cent^*,k,j,t}$$
$$= E_t \sum_{s=0}^{\infty} \left(\beta \alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}\right)^s \bar{\mathcal{E}}_t \bar{P}^{*^{\theta_p}}_{\mathcal{H},\cent^*,k,t} \bar{Y}^*_{\mathcal{H},\cent^*,k} \bar{X}^*_{\mathcal{H},\cent^*,k,j,t}$$
$$= \frac{1}{1 - \beta \alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}} \bar{\mathcal{E}}_t \bar{P}^{*^{\theta_p}}_{\mathcal{H},\cent^*,k,t} \bar{Y}^*_{\mathcal{H},\cent^*,k} \bar{X}^*_{\mathcal{H},\cent^*,k,j,t}. \quad \text{(B215)}$$

Next, we aim to simplify the term excluding the constant in the right-hand side of (B214). To do this, first note that since real wages and the real rental rate of capital are constant at the steady state, their nominal values must grow by steady-state inflation:

$$\bar{W}_{t+s}^{\chi} \bar{R}_{t+s}^{k^{1-\chi}} = \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} e^{\bar{\pi} s}. \quad \text{(B216)}$$

Using (B8), (B216), and the assumption that all home-currency (foreign-currency) prices rise by home (foreign) inflation at the flexible-price steady state, one can rewrite the right-hand side of (B214) as

$$
\begin{aligned}
&E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t} \alpha_k^{*^s} \bar{W}_{t+s}^{\chi} \bar{R}_{t+s}^{k^{1-\chi}} \left(\frac{1}{\bar{P}^*_{\mathcal{H},\not{c}^*,k,t+s}} \right)^{-\theta_p} \bar{Y}^*_{\mathcal{H},\not{c}^*,k} \\
&= E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{P}_t}{\bar{P}_{t+s}} \alpha_k^{*^s} \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} e^{\bar{\pi} s} \left(\frac{1}{\bar{P}^*_{\mathcal{H},\not{c}^*,k,t} e^{\bar{\pi}^* s}} \right)^{-\theta_p} \bar{Y}^*_{\mathcal{H},\not{c}^*,k} \\
&= E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{P}_t}{\bar{P}_t e^{\bar{\pi} s}} \alpha_k^{*^s} \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} e^{\bar{\pi} s} \left(\frac{1}{\bar{P}^*_{\mathcal{H},\not{c}^*,k,t} e^{\bar{\pi}^* s}} \right)^{-\theta_p} \bar{Y}^*_{\mathcal{H},\not{c}^*,k} \\
&= E_t \sum_{s=0}^{\infty} \left(\beta \alpha_k^* e^{\bar{\pi}^* \theta_p} \right)^s \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} \bar{P}^{*^{\theta_p}}_{\mathcal{H},\not{c}^*,k,t} \bar{Y}^*_{\mathcal{H},\not{c}^*,k} \\
&= \frac{1}{1 - \beta \alpha_k^* e^{\bar{\pi}^* \theta_p}} \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} \bar{P}^{*^{\theta_p}}_{\mathcal{H},\not{c}^*,k,t} \bar{Y}^*_{\mathcal{H},\not{c}^*,k}. \qquad \text{(B217)}
\end{aligned}
$$

Using (B214), (B215), and (B217), it is easy to see that

$$
\begin{aligned}
&\frac{1}{1 - \beta \alpha_k^* e^{\bar{\pi}^* (\theta_p - 1)}} \bar{\mathcal{E}}_t \bar{P}^{*^{\theta_p}}_{\mathcal{H},\not{c}^*,k,t} \bar{Y}^*_{\mathcal{H},\not{c}^*,k} \bar{X}^*_{\mathcal{H},\not{c}^*,k,j,t} \\
&\quad = \frac{\theta_p}{\theta_p - 1} \left(\frac{1}{1-\chi} \right)^{1-\chi} \left(\frac{1}{\chi} \right)^{\chi} \\
&\qquad \times \frac{1}{1 - \beta \alpha_k^* e^{\bar{\pi}^* \theta_p}} \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} \bar{P}^{*^{\theta_p}}_{\mathcal{H},\not{c}^*,k,t} \bar{Y}^*_{\mathcal{H},\not{c}^*,k} \\
&\frac{1 - \beta \alpha_k^* e^{\bar{\pi}^* \theta_p}}{1 - \beta \alpha_k^* e^{\bar{\pi}^* (\theta_p - 1)}} \bar{\mathcal{E}}_t \bar{P}^{*^{\theta_p}}_{\mathcal{H},\not{c}^*,k,t} \bar{Y}^*_{\mathcal{H},\not{c}^*,k} \bar{X}^*_{\mathcal{H},\not{c}^*,k,j,t} \\
&\quad = \frac{\theta_p}{\theta_p - 1} \left(\frac{1}{1-\chi} \right)^{1-\chi} \left(\frac{1}{\chi} \right)^{\chi} \bar{W}_t^{\chi} \bar{R}_t^{k^{1-\chi}} \bar{P}^{*^{\theta_p}}_{\mathcal{H},\not{c}^*,k,t} \bar{Y}^*_{\mathcal{H},\not{c}^*,k}. \qquad \text{(B218)}
\end{aligned}
$$

Using (B215), a log-linear approximation around positive steady-state inflation of the left-hand side of (B213) can be written as

$$
E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t} \alpha_k^{*^s} \bar{\mathcal{E}}_{t+s} \left(\frac{1}{\bar{P}^*_{\mathcal{H},\not{c}^*,k,t+s}} \right)^{-\theta_p} \bar{Y}^*_{\mathcal{H},\not{c}^*,k} \bar{X}^*_{\mathcal{H},\not{c}^*,k,j,t}
$$

$$= \bar{\mathcal{E}}_t \bar{P}^{*^{\theta_p}}_{\mathcal{H},\cent^*,k,t} \bar{Y}^*_{\mathcal{H},\cent^*,k} \bar{X}^*_{\mathcal{H},\cent^*,k,j,t} E_t \sum_{s=0}^{\infty} \left(\beta \alpha^*_k e^{\bar{\pi}^*(\theta_p - 1)}\right)^s \times \left[\hat{\lambda}_{t+s} - \hat{\lambda}_t + \hat{\mathcal{E}}_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\cent^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\cent^*,k} + \hat{X}^*_{\mathcal{H},\cent^*,k,j,t}\right]. \quad \text{(B219)}$$

Similarly, using (B217), one can write a log-linear approximation around positive steady-state inflation of the right-hand side of (B213) as

$$\left(\frac{\theta_p}{\theta_p - 1}\right) \cdot \left(\frac{1}{1-\chi}\right)^{1-\chi} \cdot \left(\frac{1}{\chi}\right)^{\chi} .E_t \sum_{s=0}^{\infty} \beta^s \frac{\bar{\lambda}_{t+s}}{\bar{\lambda}_t} \alpha^{*^s}_k$$

$$\bar{W}^{\chi}_{t+s} \bar{R}^{k^{1-\chi}}_{t+s} \left(\frac{1}{\bar{P}^*_{\mathcal{H},\cent^*,k,t+s}}\right)^{-\theta_p} \bar{Y}^*_{\mathcal{H},\cent^*,k}$$

$$\times \left[\hat{\lambda}_{t+s} - \hat{\lambda}_t + \chi \hat{W}_{t+s} + (1-\chi)\hat{R}^k_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\cent^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\cent^*,k,t+s}\right]$$

$$= \left(\frac{\theta_p}{\theta_p - 1}\right) \cdot \left(\frac{1}{1-\chi}\right)^{1-\chi} \cdot \left(\frac{1}{\chi}\right)^{\chi} \bar{W}^{\chi}_t \bar{R}^{k^{1-\chi}}_t \bar{P}^{*^{\theta_p}}_{\mathcal{H},\cent^*,k,t} \bar{Y}^*_{\mathcal{H},\cent^*,k}$$

$$\times E_t \sum_{s=0}^{\infty} \left(\beta \alpha^*_k e^{\bar{\pi}^* \theta_p}\right)^s \left[\hat{\lambda}_{t+s} - \hat{\lambda}_t + \chi \hat{W}_{t+s} + (1-\chi)\hat{R}^k_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\cent^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\cent^*,k,t+s}\right]$$

$$= \frac{1 - \beta \alpha^*_k e^{\bar{\pi}^* \theta_p}}{1 - \beta \alpha^*_k e^{\bar{\pi}^*(\theta_p - 1)}} \bar{\mathcal{E}}_t \bar{P}^{*^{\theta_p}}_{\mathcal{H},\cent^*,k,t} \bar{Y}^*_{\mathcal{H},\cent^*,k} \bar{X}^*_{\mathcal{H},\cent^*,k,j,t}$$

$$\times E_t \sum_{s=0}^{\infty} \left(\beta \alpha^*_k e^{\bar{\pi}^* \theta_p}\right)^s \left[\hat{\lambda}_{t+s} - \hat{\lambda}_t + \chi \hat{W}_{t+s} + (1-\chi)\hat{R}^k_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\cent^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\cent^*,k,t+s}\right]. \quad \text{(B220)}$$

Using (B219) and (B220), it is easy to show that

$$E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s \Big[\hat{\lambda}_{t+s} - \hat{\lambda}_t + \hat{\mathcal{E}}_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\cent^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\cent^*,k,t+s} + \hat{X}^*_{\mathcal{H},\cent^*,k,j,t}\Big]$$
$$= \frac{1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}}{1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}} E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)^s \Big[\hat{\lambda}_{t+s} - \hat{\lambda}_t + \chi \hat{W}_{t+s} + (1-\chi)\hat{R}^k_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\cent^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\cent^*,k,t+s}\Big]. \quad \text{(B221)}$$

Rearranging terms in (B221) gives

$$\frac{1}{1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}} \Big[\hat{X}^*_{\mathcal{H},\cent^*,k,j,t} - \hat{\lambda}_t\Big] + E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s \times \Big[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\cent^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\cent^*,k,t+s}\Big]$$
$$= \frac{1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}}{1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}} E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)^s \times \Big[\hat{\lambda}_{t+s} + \chi \hat{W}_{t+s} + (1-\chi)\hat{R}^k_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\cent^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\cent^*,k,t+s}\Big] - \frac{1}{1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}} \hat{\lambda}_t. \quad \text{(B222)}$$

The terms involving λ_t in (B222) cancel. Consequently, one may write (B222) as

$$\hat{X}^*_{\mathcal{H},\cent^*,k,j,t} + E_t \sum_{s=0}^{\infty} \left\{ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right) \right\} \times \Big[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\cent^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\cent^*,k,t+s}\Big]$$
$$= E_t \sum_{s=0}^{\infty} \left\{ \left(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)^s \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right) \right\} \times \Big[\hat{\lambda}_{t+s} + \chi \hat{W}_{t+s} + (1-\chi)\hat{R}^k_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\cent^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\cent^*,k,t+s}\Big]. \quad \text{(B223)}$$

Next, we restate (B223) as

$$\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t} + E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)$$
$$\times \left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s}\right]$$
$$= \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\left[\hat{\lambda}_t + \chi\hat{W}_t + (1-\chi)\hat{R}_t^k + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right]$$
$$+ E_t \sum_{s=1}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)^s \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)$$
$$\times \left[\hat{\lambda}_{t+s} + \chi\hat{W}_{t+s} + (1-\chi)\hat{R}_{t+s}^k + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s}\right]. \tag{B224}$$

By restating the last term in (B224), one can also write (B224) as

$$\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t} + E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)$$
$$\times \left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s}\right]$$
$$= \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\left[\hat{\lambda}_t + \chi\hat{W}_t + (1-\chi)\hat{R}_t^k\right.$$
$$\left. + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right]$$
$$+ \beta\alpha_k^* e^{\bar{\pi}^*\theta_p} E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)^s \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)$$
$$\times \left[\hat{\lambda}_{t+s+1} + \chi\hat{W}_{t+s+1} + (1-\chi)\hat{R}_{t+s+1}^k\right.$$
$$\left. + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s+1}\right]. \tag{B225}$$

Using (B223), (B225) can be rewritten as

$$\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t} + E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)$$
$$\times \left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s}\right]$$

$$= \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\left[\hat{\lambda}_t + \chi\hat{W}_t + (1-\chi)\hat{R}_t^k + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right]$$
$$+ \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\left\{E_t\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t+1} + E_t\sum_{s=0}^{\infty}\left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s\right.$$
$$\times\left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)$$
$$\left.\times\left[\hat{\lambda}_{t+s+1} + \hat{\mathcal{E}}_{t+s+1} + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s+1}\right]\right\}. \tag{B226}$$

Distributing $\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}$ in the last term of (B226) gives

$$\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t} + E_t\sum_{s=0}^{\infty}\left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s\left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)$$
$$\times\left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s}\right]$$
$$= \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\left[\hat{\lambda}_t + \chi\hat{W}_t + (1-\chi)\hat{R}_t^k + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right]$$
$$+ \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}E_t\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t+1} + \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}E_t\sum_{s=0}^{\infty}\left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s$$
$$\times\left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)$$
$$\times\left[\hat{\lambda}_{t+s+1} + \hat{\mathcal{E}}_{t+s+1} + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s+1}\right]. \tag{B227}$$

Since $\beta\alpha_k^* e^{\bar{\pi}^*\theta_p} = e^{\bar{\pi}^*}\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}$, one can rewrite (B227) as

$$\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t} + E_t\sum_{s=0}^{\infty}\left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s\left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)$$
$$\times\left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s}\right]$$
$$= \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\left[\hat{\lambda}_t + \chi\hat{W}_t + (1-\chi)\hat{R}_t^k + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right]$$
$$+ \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t+1} + e^{\bar{\pi}^*}E_t\sum_{s=0}^{\infty}\left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^{s+1}$$

$$\times \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}\right)$$
$$\times \left[\hat{\lambda}_{t+s+1} + \hat{\mathcal{E}}_{t+s+1} + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s+1}\right]. \quad \text{(B228)}$$

Rearranging (B228) yields

$$\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t} - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p} \hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t+1} + E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}\right)^s$$
$$\times \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}\right)$$
$$\times \left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s}\right]$$
$$= \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right) \left[\hat{\lambda}_t + \chi \hat{W}_t + (1-\chi)\hat{R}_t^k\right.$$
$$\left. + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s}\right]$$
$$+ e^{\bar{\pi}^*} E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}\right)^{s+1} \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}\right)$$
$$\times \left[\hat{\lambda}_{t+s+1} + \hat{\mathcal{E}}_{t+s+1} + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s+1}\right]. \quad \text{(B229)}$$

Now, note that

$$E_t \sum_{s=0}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}\right)^s \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}\right)$$
$$\times \left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s}\right]$$
$$= \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}\right) \left[\hat{\lambda}_t + \hat{\mathcal{E}}_t + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right]$$
$$+ E_t \sum_{s=1}^{\infty} \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}\right)^s \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}\right)$$
$$\times \left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s}\right]. \quad \text{(B230)}$$

Using (B230), one can restate (B229) as

$$\begin{aligned}
&\hat{X}^*_{\mathcal{H},¢^*,k,j,t} - \beta\alpha^*_k e^{\bar{\pi}^*\theta_p}\hat{X}^*_{\mathcal{H},¢^*,k,j,t+1} + \left(1 - \beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\\
&\quad \times \left[\hat{\lambda}_t + \hat{\mathcal{E}}_t + \theta_p\hat{P}^*_{\mathcal{H},¢^*,k,t} + \hat{Y}^*_{\mathcal{H},¢^*,k,t}\right]\\
&\quad + E_t\sum_{s=1}^{\infty}\left(\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)^s\left(1 - \beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\\
&\quad \times \left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p\hat{P}^*_{\mathcal{H},¢^*,k,t+s} + \hat{Y}^*_{\mathcal{H},¢^*,k,t+s}\right]\\
&= \left(1 - \beta\alpha^*_k e^{\bar{\pi}^*\theta_p}\right)\left[\hat{\lambda}_t + \chi\hat{W}_t + (1-\chi)\hat{R}^k_t\right.\\
&\quad \left. + \theta_p\hat{P}^*_{\mathcal{H},¢^*,k,t} + \hat{Y}^*_{\mathcal{H},¢^*,k,t}\right]\\
&\quad + e^{\bar{\pi}^*}E_t\sum_{s=1}^{\infty}\left(\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)^s\left(1 - \beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\\
&\quad \times \left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p\hat{P}^*_{\mathcal{H},¢^*,k,t+s} + \hat{Y}^*_{\mathcal{H},¢^*,k,t+s}\right]. \qquad \text{(B231)}
\end{aligned}$$

Rearranging (B231) yields

$$\begin{aligned}
&\hat{X}^*_{\mathcal{H},¢^*,k,j,t} - \beta\alpha^*_k e^{\bar{\pi}^*\theta_p}\hat{X}^*_{\mathcal{H},¢^*,k,j,t+1} + \left(1 - \beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\\
&\quad \times \left[\hat{\lambda}_t + \hat{\mathcal{E}}_t + \theta_p\hat{P}^*_{\mathcal{H},¢^*,k,t} + \hat{Y}^*_{\mathcal{H},¢^*,k,t}\right] - \left(1 - \beta\alpha^*_k e^{\bar{\pi}^*\theta_p}\right)\\
&\quad \times \left[\hat{\lambda}_t + \chi\hat{W}_t + (1-\chi)\hat{R}^k_t + \theta_p\hat{P}^*_{\mathcal{H},¢^*,k,t} + \hat{Y}^*_{\mathcal{H},¢^*,k,t}\right]\\
&= E_t\sum_{s=1}^{\infty}\left(\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)^s\left(1 - \beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*} - 1\right)\\
&\quad \times \left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p\hat{P}^*_{\mathcal{H},¢^*,k,t+s} + \hat{Y}^*_{\mathcal{H},¢^*,k,t+s}\right]. \qquad \text{(B232)}
\end{aligned}$$

It is easy to see that

$$\begin{aligned}
&E_t\sum_{s=1}^{\infty}\left(\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)^s\left(1 - \beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*} - 1\right)\\
&\quad \times \left[\hat{\lambda}_{t+s} + \hat{\mathcal{E}}_{t+s} + \theta_p\hat{P}^*_{\mathcal{H},¢^*,k,t+s} + \hat{Y}^*_{\mathcal{H},¢^*,k,t+s}\right]
\end{aligned}$$

$$
\begin{aligned}
&= \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)E_t \\
&\quad \times \left[\hat{\lambda}_{t+1}+\hat{\mathcal{E}}_{t+1}+\theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1}+\hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right] \\
&\quad + E_t\sum_{s=2}^{\infty}\left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right) \\
&\quad \times \left[\hat{\lambda}_{t+s}+\hat{\mathcal{E}}_{t+s}+\theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s}+\hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s}\right] \\
&= \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)E_t \\
&\quad \times \left[\hat{\lambda}_{t+1}+\hat{\mathcal{E}}_{t+1}+\theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1}+\hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right] \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)E_t\sum_{s=1}^{\infty}\left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s \\
&\quad \times \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right) \\
&\quad \times \left[\hat{\lambda}_{t+s+1}+\hat{\mathcal{E}}_{t+s+1}+\theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s+1}+\hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s+1}\right].
\end{aligned} \tag{B233}
$$

Using (B233), one can rewrite (B232) as

$$
\begin{aligned}
&\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t}-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t+1}+\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right) \\
&\quad \times \left[\hat{\lambda}_t+\hat{\mathcal{E}}_t+\theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t}+\hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right]-\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right) \\
&\quad \times \left[\hat{\lambda}_t+\chi\hat{W}_t+(1-\chi)\hat{R}^k_t+\theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t}+\hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right] \\
&= \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)E_t \\
&\quad \times \left[\hat{\lambda}_{t+1}+\hat{\mathcal{E}}_{t+1}+\theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1}+\hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right] \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)E_t\sum_{s=1}^{\infty}\left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)^s \\
&\quad \times \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right) \\
&\quad \times \left[\hat{\lambda}_{t+s+1}+\hat{\mathcal{E}}_{t+s+1}+\theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+s+1}+\hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+s+1}\right].
\end{aligned} \tag{B234}
$$

Using (B232), one can restate (B234) as

$$
\begin{aligned}
&\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t} - \beta\alpha^*_k e^{\bar{\pi}^*\theta_p}\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t+1} \\
&\quad + \left(1-\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\left[\hat{\lambda}_t + \hat{\mathcal{E}}_t + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right] \\
&\quad - \left(1-\beta\alpha^*_k e^{\bar{\pi}^*\theta_p}\right)\left[\hat{\lambda}_t + \chi\hat{W}_t + (1-\chi)\hat{R}^k_t \right.\\
&\quad \left. + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right] \\
&= \left(\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)E_t \\
&\quad \times \left[\hat{\lambda}_{t+1} + \hat{\mathcal{E}}_{t+1} + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right] \\
&\quad + \left(\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\left\{E_t\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t+1} - \beta\alpha^*_k e^{\bar{\pi}^*\theta_p}E_t\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t+2}\right. \\
&\quad + \left(1-\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right) \\
&\quad \times E_t\left[\hat{\lambda}_{t+1} + \hat{\mathcal{E}}_{t+1} + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right] \\
&\quad - \left(1-\beta\alpha^*_k e^{\bar{\pi}^*\theta_p}\right)E_t\left[\hat{\lambda}_{t+1} + \chi\hat{W}_{t+1} + (1-\chi)\hat{R}^k_{t+1}\right. \\
&\quad \left.\left. + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right]\right\}. \qquad \text{(B235)}
\end{aligned}
$$

By distributing $\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}$ in the last term of (B235) and rearranging terms, one can write that

$$
\begin{aligned}
&\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t} - \beta\alpha^*_k e^{\bar{\pi}^*\theta_p}\hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t+1} \\
&= \left(1-\beta\alpha^*_k e^{\bar{\pi}^*\theta_p}\right)\left[\hat{\lambda}_t + \chi\hat{W}_t + (1-\chi)\hat{R}^k_t \right.\\
&\quad \left. + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right] \\
&\quad - \left(1-\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\left[\hat{\lambda}_t + \hat{\mathcal{E}}_t + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right] \\
&\quad + \left(\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)E_t
\end{aligned}
$$

$$
\begin{aligned}
&\times \left[\hat{\lambda}_{t+1} + \hat{\mathcal{E}}_{t+1} + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right] \\
&+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left[E_t \hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t+1} - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p} E_t \hat{X}^*_{\mathcal{H},\not{c}^*,k,j,t+2}\right] \\
&+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1 - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right) E_t \\
&\times \left[\hat{\lambda}_{t+1} + \hat{\mathcal{E}}_{t+1} + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right] \\
&- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1 - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right) E_t \\
&\times \left[\hat{\lambda}_{t+1} + \chi \hat{W}_{t+1} + (1-\chi)\hat{R}^k_{t+1} + \theta_p \hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right].
\end{aligned}
\tag{B236}
$$

In each period, the fraction of the firms receiving a price-change signal is $1-\alpha_k^*$. Since firms are identical, firms receiving this signal set the same price. This, together with the randomization assumption in the Calvo model and (B157), yields

$$
\begin{aligned}
&P^{*^{1-\theta_p}}_{\mathcal{H},\not{c}^*,k,t} = (1-\alpha_k^*) X^{*^{1-\theta_p}}_{\mathcal{H},\not{c}^*,k,j,t} + \alpha_k^* P^{*^{1-\theta_p}}_{\mathcal{H},\not{c}^*,k,t-1} \\
&1 = (1-\alpha_k^*)\left(\frac{\bar{X}^*_{\mathcal{H},\not{c}^*,k,t}}{\bar{P}^*_{\mathcal{H},\not{c}^*,k,t}}\right)^{1-\theta_p} + \alpha_k^* \left(\frac{\bar{P}^*_{\mathcal{H},\not{c}^*,k,t-1}}{\bar{P}^*_{\mathcal{H},\not{c}^*,k,t}}\right)^{1-\theta_p} \\
&\times \left(\frac{1-\alpha_k^* e^{-\pi^*(1-\theta_p)}}{1-\alpha_k^*}\right) = \left(\frac{\bar{X}^*_{\mathcal{H},\not{c}^*,k,t}}{\bar{P}^*_{\mathcal{H},\not{c}^*,k,t}}\right)^{1-\theta_p}.
\end{aligned}
\tag{B237}
$$

Log-linearizing (B237) yields

$$
\begin{aligned}
\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} &= (1-\alpha_k^*)\left(\frac{1-\alpha_k^* e^{-\bar{\pi}^*(1-\theta_p)}}{1-\alpha_k^*}\right)\hat{X}^*_{\mathcal{H},\not{c}^*,k,t} \\
&\quad + \alpha_k^* e^{-\bar{\pi}^*(1-\theta_p)} \hat{P}^*_{\mathcal{H},\not{c}^*,k,t-1} \\
\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} &= \left(1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\hat{X}^*_{\mathcal{H},\not{c}^*,k,t} + \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \hat{P}^*_{\mathcal{H},\not{c}^*,k,t-1} \\
\hat{X}^*_{\mathcal{H},\not{c}^*,k,t} &= \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)\left[\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} - \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \hat{P}^*_{\mathcal{H},\not{c}^*,k,t-1}\right].
\end{aligned}
\tag{B238}
$$

Using (B238), one can rewrite (B236) as

$$
\begin{aligned}
&\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)\left[\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} - \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\hat{P}^*_{\mathcal{H},\not{c}^*,k,t-1}\right] \\
&\quad - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \\
&\quad \times \left[E_t\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} - \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\hat{P}^*_{\mathcal{H},\not{c}^*,k,t}\right] \\
&= \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\left[\hat{\lambda}_t + \chi\hat{W}_t + (1-\chi)\hat{R}_t^k + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right] \\
&\quad - \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left[\hat{\lambda}_t + \hat{\mathcal{E}}_t + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t}\right] \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)E_t \\
&\quad \times \left[\hat{\lambda}_{t+1} + \hat{\mathcal{E}}_{t+1} + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right] \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \\
&\quad \times \left[E_t\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} - \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\hat{P}^*_{\mathcal{H},\not{c}^*,k,t}\right] \\
&\quad - \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \\
&\quad \times \left[E_t\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+2} - \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}E_t\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1}\right] \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)E_t \\
&\quad \times \left[\hat{\lambda}_{t+1} + \hat{\mathcal{E}}_{t+1} + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right] \\
&\quad - \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)E_t \\
&\quad \times \left[\hat{\lambda}_{t+1} + \chi\hat{W}_{t+1} + (1-\chi)\hat{R}_{t+1}^k + \theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} + \hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1}\right].
\end{aligned}
\tag{B239}
$$

Next, we restate (B239) as

$$
\begin{aligned}
&\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} \\
&\quad - \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\hat{P}^*_{\mathcal{H},\not{c}^*,k,t-1} \\
&\quad - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)E_t\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} \\
&\quad + \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} \\
&= \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\hat{\lambda}_t + \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\chi\hat{W}_t \\
&\quad + \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)(1-\chi)\hat{R}^k_t \\
&\quad + \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} + \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\hat{Y}^*_{\mathcal{H},\not{c}^*,k,t} \\
&\quad - \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\hat{\lambda}_t - \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\hat{\mathcal{E}}_t \\
&\quad - \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\theta_p\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} - \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\hat{Y}^*_{\mathcal{H},\not{c}^*,k,t} \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)E_t\hat{\lambda}_{t+1} \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)E_t\hat{\mathcal{E}}_{t+1} \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)\theta_p E_t\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)E_t\hat{Y}^*_{\mathcal{H},\not{c}^*,k,t+1} \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)E_t\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+1} \\
&\quad - \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\hat{P}^*_{\mathcal{H},\not{c}^*,k,t} \\
&\quad - \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)E_t\hat{P}^*_{\mathcal{H},\not{c}^*,k,t+2} \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}
\end{aligned}
$$

$$
\begin{aligned}
&\times \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} E_t \hat{P}^*_{\mathcal{H},\cancel{c}^*,k,t+1} \\
&+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right) E_t \hat{\lambda}_{t+1} \\
&+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right) E_t \hat{\mathcal{E}}_{t+1} \\
&+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right) \theta_p E_t \hat{P}^*_{\mathcal{H},\cancel{c}^*,k,t+1} \\
&+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right) E_t \hat{Y}^*_{\mathcal{H},\cancel{c}^*,k,t+1} \\
&- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right) E_t \hat{\lambda}_{t+1} \\
&- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right) \chi E_t \hat{W}_{t+1} \\
&- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right) (1-\chi) E_t \hat{R}^k_{t+1} \\
&- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right) \theta_p E_t \hat{P}^*_{\mathcal{H},\cancel{c}^*,k,t+1} \\
&- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right) E_t \hat{Y}^*_{\mathcal{H},\cancel{c}^*,k,t+1}. \qquad \text{(B240)}
\end{aligned}
$$

Rearranging (B240) yields

$$
\begin{aligned}
&\Bigg\{ \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) + \beta\alpha_k^* e^{\bar{\pi}^*\theta_p} \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \\
&\quad - \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\theta_p + \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\theta_p \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right) \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \Bigg\} \hat{P}^*_{\mathcal{H},\cancel{c}^*,k,t} \\
&= \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \hat{P}^*_{\mathcal{H},\cancel{c}^*,k,t-1} \\
&\quad + \Bigg\{ \beta\alpha_k^* e^{\bar{\pi}^*\theta_p} \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) + \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right) \\
&\quad \times \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)\theta_p
\end{aligned}
$$

$$+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}$$

$$+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\theta_p$$

$$- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\theta_p$$

$$+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)\Bigg\}E_t\hat{P}^*_{\mathcal{H},\cent^*,k,t+1}$$

$$+ \left\{\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)-\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\right\}\hat{\lambda}_t$$

$$+ \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\chi\hat{W}_t + \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)(1-\chi)\hat{R}^k_t$$

$$- \left\{\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)-\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\right\}\hat{Y}^*_{\mathcal{H},\cent^*,k,t}$$

$$- \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\hat{\mathcal{E}}_t$$

$$+ \Bigg\{\left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)$$

$$+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)$$

$$- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\Bigg\}E_t\hat{\lambda}_{t+1}$$

$$+ \Bigg\{\left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)$$

$$+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\Bigg\}E_t\hat{\mathcal{E}}_{t+1}$$

$$+ \Bigg\{\left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)$$

$$+ \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)$$

$$
\begin{aligned}
&- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\Bigg\} E_t \hat{Y}^*_{\mathcal{H},\mathcal{C}^*,k,t+1} \\
&- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) E_t \hat{P}^*_{\mathcal{H},\mathcal{C}^*,k,t+2} \\
&- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\chi E_t \hat{W}_{t+1} \\
&- \left(\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)(1-\chi) E_t \hat{R}^k_{t+1}. \qquad \text{(B241)}
\end{aligned}
$$

B.15 Firms' Objective in the Foreign Country

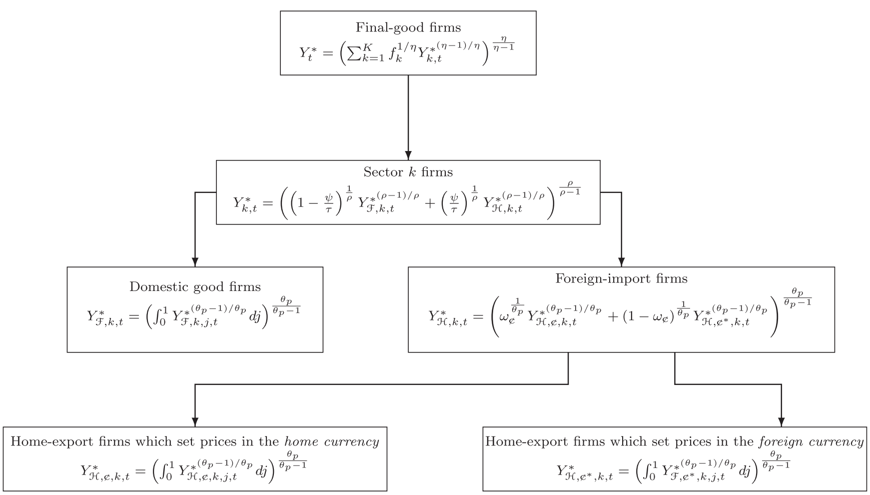
Figure B2 illustrates how firms producing different goods are related to each other in the foreign country and their production technologies.

Next, we separately describe how each good is produced.

B.15.1 Firms Producing the Final Goods in the Foreign Country

The problem of final good producers in the foreign country is analogous to that in the home country. Similar to (B130) and (B132),

Figure B2. How Are Different Firms Related in the Foreign Country?



one can show that demand for sector k good and consumer price index in the foreign country are given by (B242) and (B243), respectively:

$$Y^*_{k,t} = f_k \left(\frac{P^*_{k,t}}{P^*_t} \right)^{-\eta} Y^*_t \tag{B242}$$

$$P^*_t = \left(\sum_{k=1}^{K} f_k P^{*\,1-\eta}_{k,t} \right)^{\frac{1}{1-\eta}}. \tag{B243}$$

B.15.2 Firms Producing Sector k Good in the Foreign Country

Sectoral goods are produced by an infinitely large number of perfectly competitive firms. The foreign firms producing sectoral goods combine goods produced in the foreign country ($Y^*_{\mathcal{F},k,t}$) and those imported from the home country ($Y^*_{\mathcal{H},k,t}$) to produce sectoral output ($Y^*_{k,t}$) with the following technology:

$$Y^*_{k,t} = \left(\left(1 - \frac{\psi}{\tau}\right)^{\frac{1}{\rho}} Y^{*\,(\rho-1)/\rho}_{\mathcal{F},k,t} + \left(\frac{\psi}{\tau}\right)^{\frac{1}{\rho}} Y^{*\,(\rho-1)/\rho}_{\mathcal{H},k,t} \right)^{\frac{\rho}{\rho-1}}. \tag{B244}$$

It is clear from (B244) that the steady-state import share in output of the foreign country is only $\frac{1}{\tau}$ as big as that of the home country. This assumption is convenient since it allows us to study small and large open economies within the same model. Indeed, for a large economy, one can take $\tau = 1$. For a small economy, on the other hand, one can assume τ is *arbitrarily large,* as the size of its trading partners is much larger than its own size.

Similar to (B137), output from the foreign firms that supply the foreign country with the sector k good and output from the foreign-import firms in sector k can be written as

$$Y^*_{\mathcal{F},k,t} = f_k \left(1 - \frac{\psi}{\tau}\right) \left(\frac{P^*_{\mathcal{F},k,t}}{P^*_{k,t}} \right)^{-\rho} \left(\frac{P^*_{k,t}}{P^*_t} \right)^{-\eta} Y^*_t$$

$$Y^*_{\mathcal{H},k,t} = f_k \frac{\psi}{\tau} \left(\frac{P^*_{\mathcal{H},k,t}}{P^*_{k,t}} \right)^{-\rho} \left(\frac{P^*_{k,t}}{P^*_t} \right)^{-\eta} Y^*_t. \tag{B245}$$

Also, similar to (B139), the sector k price index in the foreign country can be written as

$$P^*_{k,t} = \left((1 - \frac{\psi}{\tau}) P^{*\,1-\rho}_{\mathcal{F},k,t} + \frac{\psi}{\tau} P^{*\,1-\rho}_{\mathcal{H},k,t}\right)^{\frac{1}{1-\rho}}. \tag{B246}$$

B.15.3 Foreign Firms Supplying the Foreign Country with the Sector k Good

The problem of foreign firms that supply the foreign country with the sector k good is analogous to that in the home country. Similar to (B143), demand for variety j from foreign firms that supply the foreign country with the sector k good can be written as

$$Y^*_{\mathcal{F},k,j,t} = \left(\frac{P^*_{\mathcal{F},k,j,t}}{P^*_{\mathcal{F},k,t}}\right)^{-\theta_p} Y^*_{\mathcal{F},k,t}. \tag{B247}$$

Similar to (B145), one can show that the price index for the sector k good produced by the foreign firms that supply the foreign country is given as a geometric average of price indexes for varieties of goods:

$$P^*_{\mathcal{F},k,t} = \left(\int_0^1 P^{*\,1-\theta_p}_{\mathcal{F},k,j,t}\, dj\right)^{\frac{1}{1-\theta_p}}. \tag{B248}$$

B.15.4 Foreign-Export Firms Supplying the Home Country with the Sector k Good and Setting Prices in the Home Currency

The objective of the foreign-export firms setting prices in the *home currency* is analogous to that of home-export firms setting prices in the *home currency*. Similar to (B149), one can show that demand for foreign-export variety j from firms that produce the composite foreign-export good is given by

$$Y_{\mathcal{F},¢,k,j,t} = \left(\frac{P_{\mathcal{F},¢,k,j,t}}{P_{\mathcal{F},¢,k,t}}\right)^{-\theta_p} Y_{\mathcal{F},¢,k,t}. \tag{B249}$$

Similar to (B151), one can show that the price index for the foreign-export good is given by

$$P_{\mathcal{F},¢,k,t} = \left(\int_0^1 {P_{\mathcal{F},¢,k,j,t}}^{1-\theta_p} dj \right)^{\frac{1}{1-\theta_p}}. \tag{B250}$$

B.15.5 Foreign-Export Firms Supplying the Home Country with the Sector k Good and Setting Prices in the Foreign Currency

The objective of foreign-export firms setting prices in the *foreign currency* is analogous to that of home-export firms setting prices in the *foreign currency.* Similar to (B155), one can show that demand for foreign-export variety j from firms which produce the composite foreign-export good is given by

$$Y_{\mathcal{F},¢^*,k,j,t} = \left(\frac{P_{\mathcal{F},¢^*,k,j,t}}{P_{\mathcal{F},¢^*,k,t}} \right)^{-\theta_p} Y_{\mathcal{F},¢^*,k,t}. \tag{B251}$$

Also, similar to (B157), one can show that the price index for the foreign-currency-invoiced foreign-export good is given as a geometric average of price indexes of varieties:

$$P_{\mathcal{F},¢^*,k,t} = \left(\int_0^1 {P_{\mathcal{F},¢^*,k,j,t}}^{1-\theta_p} dj \right)^{\frac{1}{1-\theta_p}}. \tag{B252}$$

B.15.6 Foreign-Import Firms Composing Home-Export Goods Priced in the Home and Foreign Currencies

The foreign-import goods in sector k (denoted by $Y^*_{\mathcal{H},k,t}$) are produced by perfectly competitive foreign firms. Producing these goods involves combining intermediate home-export goods which are invoiced in different currencies. Indeed, while some intermediate goods are invoiced in the *home currency* (¢), others are invoiced in the *foreign currency* ($¢^*$). In producing the foreign-import good in sector k, the foreign-import firm combines output from the home-export firms that set prices in the home and foreign currencies (denoted by $Y^*_{\mathcal{H},\cent,k,t}$ and $Y^*_{\mathcal{H},\cent^*,k,t}$, respectively) with the following technology:

$$Y^*_{\mathcal{H},k,t} = \left(\omega_{\cent}^{\frac{1}{\theta_p}} Y^{*(\theta_p-1)/\theta_p}_{\mathcal{H},\cent,k,t} + (1-\omega_{\cent})^{\frac{1}{\theta_p}} Y^{*(\theta_p-1)/\theta_p}_{\mathcal{H},\cent^*,k,t} \right)^{\frac{\theta_p}{\theta_p-1}}. \quad \text{(B253)}$$

Each period, home-import firms solve the following problem:

$$\max P^*_{\mathcal{H},k,t} Y^*_{\mathcal{H},k,t} - \frac{1}{\mathcal{E}_t} P^*_{\mathcal{H},\cent,k,t} Y^*_{\mathcal{H},\cent,k,t} - P^*_{\mathcal{H},\cent^*,k,t} Y^*_{\mathcal{F},\cent^*,k,t}$$

$$\text{subject to } Y^*_{\mathcal{H},k,t} = \left(\omega_{\cent}^{\frac{1}{\theta_p}} Y^{*(\theta_p-1)/\theta_p}_{\mathcal{H},\cent,k,t} + (1-\omega_{\cent})^{\frac{1}{\theta_p}} Y^{*(\theta_p-1)/\theta_p}_{\mathcal{H},\cent^*,k,t} \right)^{\frac{\theta_p}{\theta_p-1}}. \quad \text{(B254)}$$

Similar to (B161), one can write demand for the home- and foreign-currency-priced home-export goods from foreign-import firms as

$$Y^*_{\mathcal{H},\cent,k,t} = \omega_{\cent} \left(\frac{\frac{1}{\mathcal{E}_t} P^*_{\mathcal{H},\cent,k,t}}{P^*_{\mathcal{H},k,t}} \right)^{-\theta_p} Y^*_{\mathcal{H},k,t}$$

$$Y_{\mathcal{H},\cent^*,k,t} = (1-\omega_{\cent}) \left(\frac{P^*_{\mathcal{H},\cent^*,k,t}}{P^*_{\mathcal{H},k,t}} \right)^{-\theta_p} Y^*_{\mathcal{H},k,t}. \quad \text{(B255)}$$

Using (B245), (B255) can be rewritten as

$$Y^*_{\mathcal{H},\cent,k,t} = f_k \omega_{\cent} \frac{\psi}{\tau} \left(\frac{\frac{1}{\mathcal{E}_t} P^*_{\mathcal{H},\cent,k,t}}{P^*_{\mathcal{H},k,t}} \right)^{-\theta_p} \left(\frac{P^*_{\mathcal{H},k,t}}{P^*_{k,t}} \right)^{-\rho} \left(\frac{P^*_{k,t}}{P^*_t} \right)^{-\eta} Y^*_t$$

$$Y^*_{\mathcal{H},\cent^*,k,t} = f_k (1-\omega_{\cent}) \frac{\psi}{\tau} \left(\frac{P^*_{\mathcal{H},\cent^*,k,t}}{P^*_{\mathcal{H},k,t}} \right)^{-\theta_p} \left(\frac{P^*_{\mathcal{H},k,t}}{P^*_{k,t}} \right)^{-\rho} \left(\frac{P^*_{k,t}}{P^*_t} \right)^{-\eta} Y^*_t. \quad \text{(B256)}$$

Similar to (B164), one can write the foreign-import price index as a geometric average of home- and foreign-currency-priced home-export goods:

$$P^*_{\mathcal{H},k,t} = \left(\omega_{\cent} \left(\frac{1}{\mathcal{E}_t} P^*_{\mathcal{H},\cent,k,t} \right)^{1-\theta_p} + (1-\omega_{\cent}) P^{*\,1-\theta_p}_{\mathcal{H},\cent^*,k,t} \right)^{\frac{1}{1-\theta_p}}. \quad \text{(B257)}$$

B.15.7 The Objective of the Foreign Firm Supplying the Foreign Country with Foreign Variety j

When a price-change signal is received, the expected profit of the foreign firm that supplies the foreign country with foreign variety j can be written as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda^*_{t+s}}{\lambda^*_t} {\alpha^*_k}^s \Big(X^*_{\mathcal{F},k,j,t} Y^*_{\mathcal{F},k,j,t+s} - W^*_{t+s} N^*_{\mathcal{F},k,j,t+s} - {R^*_{t+s}}^k K^*_{\mathcal{F},k,j,t+s} \Big), \tag{B258}$$

where the price set by the foreign firm is denoted by $X^*_{\mathcal{F},k,j,t}$, which remains in effect with a probability of α^*_k in each period. Similar to (B178), one can show $X^*_{\mathcal{F},k,j,t}$ is given by

$$X^*_{\mathcal{F},k,j,t} = \frac{\theta_p}{\theta_p - 1} \left(\frac{1}{1-\chi}\right)^{1-\chi} \left(\frac{1}{\chi}\right)^{\chi} \times \frac{E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda^*_{t+s}}{\lambda^*_t} {\alpha^*_k}^s {W^*_{t+s}}^{\chi} {R^*_{t+s}}^{k\,1-\chi} \left(\frac{1}{P^*_{\mathcal{F},k,t+s}}\right)^{-\theta_p} Y^*_{\mathcal{F},k,t+s}}{E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda^*_{t+s}}{\lambda^*_t} {\alpha^*_k}^s \left(\frac{1}{P^*_{\mathcal{F},k,t+s}}\right)^{-\theta_p} Y^*_{\mathcal{F},k,t+s}}. \tag{B259}$$

Also, similar to (B203), one can show that the price equation for the sector k good produced by foreign firms in the foreign country is given by

$$\Bigg\{ \left(\frac{1}{1-\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}}\right) + \beta\alpha^*_k e^{\bar{\pi}^*\theta_p} \left(\frac{1}{1-\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha^*_k e^{\bar{\pi}^*(\theta_p-1)} + \left(\beta\alpha^*_k e^{\bar{\pi}^*\theta_p} - \beta\alpha^*_k e^{\bar{\pi}^*(\theta_p-1)}\right)\theta_p$$

$$
\begin{aligned}
&+ \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \Bigg\} \hat{P}^*_{\mathcal{F},k,t} \\
= &\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \hat{P}^*_{\mathcal{F},k,t-1} \\
&+ \Bigg\{ \beta\alpha_k^* e^{\bar{\pi}^*\theta_p} \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \\
&+ \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \Big(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\Big)\Big(e^{\bar{\pi}^*}-1\Big)\theta_p \\
&+ \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \\
&+ \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \beta\alpha_k^* e^{\bar{\pi}^*\theta_p} \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \\
&+ \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \Big(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p} - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\Big)\theta_p \Bigg\} E_t \hat{P}^*_{\mathcal{F},k,t+1} \\
&+ \Big(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\Big)\chi \hat{W}^*_t \\
&+ \Big(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\Big)(1-\chi) \hat{R}^{*^k}_t \\
&- \Big(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p} - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\Big)\hat{\lambda}^*_t \\
&- \Big(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p} - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\Big)\hat{Y}^*_{\mathcal{F},k,t} \\
&+ \Bigg\{ \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \Big(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\Big)\Big(e^{\bar{\pi}^*}-1\Big) \\
&+ \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \Big(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p} - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\Big) \Bigg\} E_t \hat{\lambda}^*_{t+1} \\
&+ \Bigg\{ \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \Big(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\Big)\Big(e^{\bar{\pi}^*}-1\Big) \\
&+ \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \Big(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p} - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\Big) \Bigg\} E_t \hat{Y}^*_{\mathcal{F},k,t+1}
\end{aligned}
$$

$$- \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)} \beta\alpha_k^* e^{\bar{\pi}^*\theta_p} \left(\frac{1}{1 - \alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)}} \right) E_t \hat{P}^*_{\mathcal{F},k,t+2}$$
$$- \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)} \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right) \chi E_t \hat{W}^*_{t+1}$$
$$- \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p - 1)} \left(1 - \beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right) (1 - \chi) E_t \hat{R}^{*^k}_{t+1}. \quad \text{(B260)}$$

B.15.8 The Objective of the Foreign-Export Firm Producing Variety j

Similar to our assumption regarding home-export firms, we assume that the invoice currency of foreign-export firms determines the degree of price rigidity faced by these firms. We first describe the problem of the foreign-export firm that sets prices in the *foreign currency.*

B.15.8.1 The Objective of the Foreign-Export Firm Producing Variety j and Setting Prices in the Foreign Currency

When a price-change signal is received, the expected profit of the firm that produces foreign-export variety j and sets prices in the *foreign currency* can be written as

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda^*_{t+s}}{\lambda^*_t} \alpha_k^{*^s} \Big(X_{\mathcal{F},¢^*,k,j,t} Y_{\mathcal{F},¢^*,k,j,t} - W^*_{t+s} N_{\mathcal{F},¢^*,k,j,t} - R^{*^k}_{t+s} K_{\mathcal{F},¢^*,k,j,t} \Big), \quad \text{(B261)}$$

where the price set by the foreign-export firm in the *foreign currency* is denoted by $X_{\mathcal{F},¢^*,k,j,t}$, which remains in effect with a probability of α_k^* in each period. Similar to (B259) and (B260), one can show that (B262) and (B263) must hold:

$$X_{\mathcal{F},¢^*,k,j,t} = \frac{\theta_p}{\theta_p - 1} \left(\frac{1}{1-\chi} \right)^{1-\chi} \left(\frac{1}{\chi} \right)^{\chi} \times \frac{E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda^*_{t+s}}{\lambda^*_t} \alpha_k^{*^s} W^{*^\chi}_{t+s} R^{*^k \, 1-\chi}_{t+s} \left(\frac{1}{P_{\mathcal{F},¢^*,k,t+s}} \right)^{-\theta_p} Y_{\mathcal{F},¢^*,k,t+s}}{E_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda^*_{t+s}}{\lambda^*_t} \alpha_k^{*^s} \left(\frac{1}{P_{\mathcal{F},¢^*,k,t+s}} \right)^{-\theta_p} Y_{\mathcal{F},¢^*,k,t+s}} \quad \text{(B262)}$$

$$
\begin{aligned}
&\Bigg\{ \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) + \beta\alpha_k^* e^{\bar{\pi}^*\theta_p} \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \\
&\quad + \left(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p} - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\theta_p \\
&\quad + \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \Bigg\} \hat{P}_{\mathcal{F},\cent^*,k,t} \\
&\;= \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \hat{P}_{\mathcal{F},\cent^*,k,t-1} \\
&\quad + \Bigg\{ \beta\alpha_k^* e^{\bar{\pi}^*\theta_p} \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \\
&\quad + \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)\theta_p \\
&\quad + \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \\
&\quad + \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \beta\alpha_k^* e^{\bar{\pi}^*\theta_p} \left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right) \alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \\
&\quad + \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \left(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p} - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\theta_p \Bigg\} E_t \hat{P}_{\mathcal{F},\cent^*,k,t+1} \\
&\quad + \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\chi \hat{W}_t^* \\
&\quad + \left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)(1-\chi) \hat{R}_t^{*^k} \\
&- \left(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p} - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\hat{\lambda}_t^* \\
&\quad - \left(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p} - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\hat{Y}_{\mathcal{F},\cent^*,k,t} \\
&\quad + \Bigg\{ \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right) \\
&\quad + \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)} \left(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p} - \beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right) \Bigg\} E_t \hat{\lambda}_{t+1}^*
\end{aligned}
$$

$$+\Bigg\{\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\left(1-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\left(e^{\bar{\pi}^*}-1\right)$$

$$+\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\left(\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\right)\Bigg\}E_t\hat{Y}_{\mathcal{F},¢^*,k,t+1}$$

$$-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\left(\frac{1}{1-\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}}\right)E_t\hat{P}_{\mathcal{F},¢,k,t+2}$$

$$-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)\chi E_t\hat{W}^*_{t+1}$$

$$-\beta\alpha_k^* e^{\bar{\pi}^*(\theta_p-1)}\left(1-\beta\alpha_k^* e^{\bar{\pi}^*\theta_p}\right)(1-\chi)E_t\hat{R}^{*^k}_{t+1}. \tag{B263}$$

B.15.8.2 The Objective of the Foreign-Export Firm Producing Variety j and Setting Prices in the Home Currency

When a price-change signal is received, the expected profit of the foreign-export firm that produces variety j and sets price in the *home currency* can be written as

$$E_t\sum_{s=0}^{\infty}\beta^s\frac{\lambda^*_{t+s}}{\lambda^*_t}\alpha_k^s\Bigg(\frac{1}{\mathcal{E}_{t+s}}X_{\mathcal{F},¢,k,j,t}Y_{\mathcal{F},¢,k,j,t+s}-W^*_{t+s}N_{\mathcal{F},¢,k,j,t+s}$$

$$-R^{*^k}_{t+s}K_{\mathcal{F},¢,k,j,t+s}\Bigg), \tag{B264}$$

where the price set by the foreign-export firm in the *home currency* is denoted by $X_{\mathcal{F},¢,k,j,t}$, which remains in effect with a probability of α_k in each period. Similar to (B241), one can show that the price equation for the home-currency-invoiced foreign-export good can be written as (B265):

$$\Bigg\{\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)+\beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)}$$

$$-\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\theta_p+\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\theta_p+\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)$$

$$\times\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)}\Bigg\}\hat{P}_{\mathcal{F},¢,k,t}$$

$$
\begin{aligned}
&= \left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)}\hat{P}_{\mathcal{F},¢,k,t-1} \\
&\quad + \left\{\beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right) + \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\right. \\
&\quad \times \left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right)\theta_p \\
&\quad + \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\alpha_k e^{\bar{\pi}(\theta_p-1)} \\
&\quad + \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\theta_p \\
&\quad - \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\theta_p \\
&\quad \left. + \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)\right\}E_t\hat{P}_{\mathcal{F},¢,k,t+1} \\
&\quad + \left\{\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right) - \left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\right\}\hat{\lambda}_t^* \\
&\quad + \left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\chi\hat{W}_t^* + \left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)(1-\chi)\hat{R}_t^{*^k} \\
&\quad - \left\{\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right) - \left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\right\}\hat{Y}_{\mathcal{F},¢,k,t} \\
&\quad + \left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\hat{\mathcal{E}}_t \\
&\quad + \left\{\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right)\right. \\
&\quad + \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right) \\
&\quad \left. - \left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\right\}E_t\hat{\lambda}_{t+1}^*
\end{aligned}
$$

$$
\begin{aligned}
&-\left\{\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right)\right.\\
&\left.+\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\right\}E_t\hat{\mathcal{E}}_{t+1}\\
&+\left\{\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(e^{\bar{\pi}}-1\right)\right.\\
&+\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\\
&\left.-\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\right\}E_t\hat{Y}_{\mathcal{F},\cent,k,t+1}\\
&-\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\beta\alpha_k e^{\bar{\pi}\theta_p}\left(\frac{1}{1-\alpha_k e^{\bar{\pi}(\theta_p-1)}}\right)E_t\hat{P}_{\mathcal{F},\cent,k,t+2}\\
&-\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)\chi E_t\hat{W}^*_{t+1}\\
&-\left(\beta\alpha_k e^{\bar{\pi}(\theta_p-1)}\right)\left(1-\beta\alpha_k e^{\bar{\pi}\theta_p}\right)(1-\chi)E_t\hat{R}^{*^k}_{t+1}. \qquad \text{(B265)}
\end{aligned}
$$

B.16 Labor Market Equilibrium

B.16.1 Labor Demand from the Home Firms that Supply Domestically in Sector k

From (B140), one can show that

$$
\begin{aligned}
\bar{Y}_{\mathcal{H},k} &= \bar{Y}_{\mathcal{H},k,j}\\
\bar{Y}_{\mathcal{H},k}^{\frac{\theta_p}{\theta_p-1}}+\frac{\theta_p}{\theta_p-1}\bar{Y}_{\mathcal{H},k}^{\frac{\theta_p}{\theta_p-1}}\hat{Y}_{\mathcal{H},k,t} &= \bar{Y}_{\mathcal{H},k,j}^{\frac{\theta_p}{\theta_p-1}}+\frac{\theta_p}{\theta_p-1}\bar{Y}_{\mathcal{H},k,j}^{\frac{\theta_p}{\theta_p-1}}\\
&\quad\times\int_0^1\hat{Y}_{\mathcal{H},k,j,t}dj\\
\hat{Y}_{\mathcal{H},k,t} &= \int_0^1\hat{Y}_{\mathcal{H},k,j,t}dj. \qquad \text{(B266)}
\end{aligned}
$$

As shown in (B171), labor demand from home firms that produce domestic variety j in sector k can be written as

$$N_{\mathcal{H},k,j,t} = \left(\frac{1-\chi}{\chi}\frac{W_t}{R_t^k}\right)^{-1+\chi} Y_{\mathcal{H},k,j,t}. \tag{B267}$$

Log-linearizing (B267) yields

$$\begin{aligned} \bar{N}_{\mathcal{H},k,j} &= \left(\frac{\chi}{1-\chi}\frac{\bar{R}_t^k}{\bar{W}_t}\right)^{1-\chi} \bar{Y}_{\mathcal{H},k,j} \\ \hat{N}_{\mathcal{H},k,j,t} &= (1-\chi)\left(\hat{R}_t^k - \hat{W}_t\right) + \hat{Y}_{\mathcal{H},k,j,t}. \end{aligned} \tag{B268}$$

Integrating (B268) over all varieties and using (B266) give

$$\int_0^1 \hat{N}_{\mathcal{H},k,j,t} dj = (1-\chi)\left(\hat{R}_t^k - \hat{W}_t\right) + \hat{Y}_{\mathcal{H},k,t}. \tag{B269}$$

B.16.2 Labor Demand from Home-Export Firms that Set Prices in the Home Currency

Similar to (B266), one can show that

$$\hat{Y}^*_{\mathcal{H},¢,k,t} = \int_0^1 \hat{Y}^*_{\mathcal{H},¢,k,j,t} dj. \tag{B270}$$

Similar to (B171), one can show that labor demand from home firms that produce home-export variety j in sector k and set prices in the *home currency* can be written as

$$N^*_{\mathcal{H},¢,k,j,t} = \left(\frac{\chi}{1-\chi}\frac{R_t^k}{W_t}\right)^{1-\chi} Y^*_{\mathcal{H},¢,k,j,t}. \tag{B271}$$

Log-linearizing (B271) yields

$$\hat{N}^*_{\mathcal{H},¢,k,j,t} = (1-\chi)\left(\hat{R}_t^k - \hat{W}_t\right) + \hat{Y}^*_{\mathcal{H},¢,k,j,t}. \tag{B272}$$

Integrating (B272) over all varieties and using (B270) give

$$\int_0^1 \hat{N}^*_{\mathcal{H},¢,k,j,t} dj = (1-\chi)\left(\hat{R}_t^k - \hat{W}_t\right) + \hat{Y}^*_{\mathcal{H},¢,k,t}. \tag{B273}$$

B.16.3 Labor Demand from Home-Export Firms that Set Prices in the Foreign Currency

Similar to (B266), one can show that

$$\hat{Y}^*_{\mathcal{H},¢^*,k,t} = \int_0^1 \hat{Y}^*_{\mathcal{H},¢^*,k,j,t} dj. \tag{B274}$$

Similar to (B171), one can show that labor demand from home firms that produce home-export variety j in sector k and set prices in the *foreign currency* can be written as

$$N^*_{\mathcal{H},¢^*,k,j,t} = \left(\frac{\chi}{1-\chi}\frac{R^k_t}{W_t}\right)^{1-\chi} Y^*_{\mathcal{H},¢^*,k,j,t}. \tag{B275}$$

Log-linearizing (B275) yields

$$\hat{N}^*_{\mathcal{H},¢^*,k,j,t} = (1-\chi)\left(\hat{R}^k_t - \hat{W}_t\right) + \hat{Y}^*_{\mathcal{H},¢^*,k,j,t}. \tag{B276}$$

Integrating (B276) over all varieties and using (B274) give

$$\int_0^1 \hat{N}^*_{\mathcal{H},¢^*,k,j,t} dj = (1-\chi)\left(\hat{R}^k_t - \hat{W}_t\right) + \hat{Y}^*_{\mathcal{H},¢^*,k,t}. \tag{B277}$$

B.16.4 Labor Market Equilibrium in the Home Country

In equilibrium, labor demand equals labor supply in the home country:

$$\sum_{k=1}^K \int_0^1 N_{\mathcal{H},k,j,t} dj + \sum_{k=1}^K \int_0^1 N^*_{\mathcal{H},¢,k,j,t} dj + \sum_{k=1}^K \int_0^1 N^*_{\mathcal{H},¢^*,k,j,t} dj = N_t. \tag{B278}$$

From (B278), at a flexible-price steady state, (B279) must hold:

$$\sum_{k=1}^K \int_0^1 \bar{N}_{\mathcal{H},k,j} dj + \sum_{k=1}^K \int_0^1 \bar{N}^*_{\mathcal{H},¢,k,j} dj + \sum_{k=1}^K \int_0^1 \bar{N}^*_{\mathcal{H},¢^*,k,j} dj = \bar{N}. \tag{B279}$$

Using (B137), (B256), (B266), and $\bar{Y} = \frac{\bar{Y}^*}{\tau}$, one can show that $\bar{N}_{\mathcal{H},k,j}$, $\bar{N}^*_{\mathcal{H},¢,k,j}$, and $\bar{N}^*_{\mathcal{H},¢^*,k,j}$ can be written as

$$\bar{N}_{\mathcal{H},k,j} = \left(\frac{\chi}{1-\chi}\frac{\bar{R}^k_t}{\bar{W}_t}\right)^{1-\chi} f_k(1-\psi)\bar{Y} \tag{B280}$$

$$\bar{N}^*_{\mathcal{H},¢,k,j} = \left(\frac{\chi}{1-\chi}\frac{\bar{R}^k_t}{\bar{W}_t}\right)^{1-\chi} f_k\omega_¢\psi\bar{Y} \tag{B281}$$

$$\bar{N}^*_{\mathcal{H},¢^*,k,j} = \left(\frac{\chi}{1-\chi}\frac{\bar{R}^k_t}{\bar{W}_t}\right)^{1-\chi} f_k(1-\omega_¢)\psi\bar{Y}. \tag{B282}$$

Inserting (B280), (B281), and (B282) into (B279) yields

$$\bar{N} = \left(\frac{\chi}{1-\chi}\frac{\bar{R}^k_t}{\bar{W}_t}\right)^{1-\chi} \bar{Y}\left[\sum_{k=1}^{K} f_k\left\{(1-\psi)+\psi\omega_¢+\psi(1-\omega_¢)\right\}\right]$$
$$= \left(\frac{\chi}{1-\chi}\frac{\bar{R}^k_t}{\bar{W}_t}\right)^{1-\chi} \bar{Y}. \tag{B283}$$

Using (B280), (B281), (B282), and (B283), a log-linear approximation of (B279) can be written as

$$\sum_{k=1}^{K} f_k(1-\psi)\int_0^1 \hat{N}_{\mathcal{H},k,j,t}dj + \sum_{k=1}^{K} f_k\psi\omega_¢\int_0^1 \hat{N}^*_{\mathcal{H},¢,k,j,t}dj$$
$$+\sum_{k=1}^{K} f_k\psi(1-\omega_¢)\int_0^1 \hat{N}^*_{\mathcal{H},¢^*,k,j,t}dj = \hat{N}_t. \tag{B284}$$

Lastly, using (B269), (B273), and (B277), one can rewrite (B284) as

$$\mathbf{(1-\chi)\left(\hat{R}^k_t - \hat{W}_t\right) + \sum_{k=1}^{K} f_k(1-\psi)\hat{Y}_{\mathcal{H},k,t} + \sum_{k=1}^{K} f_k\psi\omega_¢\hat{Y}^*_{\mathcal{H},¢,k,t}}$$
$$\mathbf{+\sum_{k=1}^{K} f_k\psi(1-\omega_¢)\hat{Y}^*_{\mathcal{H},¢^*,k,t} = \hat{N}_t.} \tag{B285}$$

B.16.5 Labor Market Equilibrium in the Foreign Country

Similar to (B285), from the labor market equilibrium condition in the foreign country, one can write that

$$\begin{aligned}(\mathbf{1}-\chi)\left(\hat{\mathbf{R}}_{\mathbf{t}}^{\mathbf{k}^*}-\hat{\mathbf{W}}_{\mathbf{t}}^*\right)+\sum_{\mathbf{k}=\mathbf{1}}^{\mathbf{K}}\mathbf{f}_{\mathbf{k}}(\mathbf{1}-\frac{\psi}{\tau})\hat{\mathbf{Y}}_{\mathcal{F},\mathbf{k},\mathbf{t}}^*\\+\sum_{\mathbf{k}=\mathbf{1}}^{\mathbf{K}}\mathbf{f}_{\mathbf{k}}\frac{\psi}{\tau}(\mathbf{1}-\omega_{\not{c}^*}^*)\hat{\mathbf{Y}}_{\mathcal{F},\not{c},\mathbf{k},\mathbf{t}}+\sum_{\mathbf{k}=\mathbf{1}}^{\mathbf{K}}\mathbf{f}_{\mathbf{k}}\frac{\psi}{\tau}\omega_{\not{c}^*}^*\hat{\mathbf{Y}}_{\mathcal{F},\not{c}^*,\mathbf{k},\mathbf{t}}=\hat{\mathbf{N}}_{\mathbf{t}}^*.\end{aligned} \tag{B286}$$

B.17 Capital Market Equilibrium

B.17.1 Capital Market Equilibrium in the Home Country

For capital markets to be in equilibrium, capital demanded by home firms must be equal to capital supplied by households in the home country. Since the degree of capital utilization affects capital supply in a period, one can write that

$$\begin{aligned}\sum_{k=1}^{K}\int_0^1 K_{\mathcal{H},k,j,t}dj+\sum_{k=1}^{K}\int_0^1 K_{\mathcal{H},\not{c},k,j,t}^*dj\\+\sum_{k=1}^{K}\int_0^1 K_{\mathcal{H},\not{c}^*,k,j,t}^*dj=u_tK_t.\end{aligned} \tag{B287}$$

From (B287), one can show the following equation holds:

$$\begin{aligned}\chi\left(\hat{\mathbf{W}}_{\mathbf{t}}-\hat{\mathbf{R}}_{\mathbf{t}}^{\mathbf{k}}\right)+\sum_{\mathbf{k}=\mathbf{1}}^{\mathbf{K}}\mathbf{f}_{\mathbf{k}}(\mathbf{1}-\psi)\hat{\mathbf{Y}}_{\mathcal{H},\mathbf{k},\mathbf{t}}+\sum_{\mathbf{k}=\mathbf{1}}^{\mathbf{K}}\mathbf{f}_{\mathbf{k}}\psi\omega_{\not{c}}\hat{\mathbf{Y}}_{\mathcal{H},\not{c},\mathbf{k},\mathbf{t}}^*\\+\sum_{\mathbf{k}=\mathbf{1}}^{\mathbf{K}}\mathbf{f}_{\mathbf{k}}\psi(\mathbf{1}-\omega_{\not{c}})\hat{\mathbf{Y}}_{\mathcal{H},\not{c}^*,\mathbf{k},\mathbf{t}}^*=\hat{\mathbf{u}}_{\mathbf{t}}+\hat{\mathbf{K}}_{\mathbf{t}}.\end{aligned} \tag{B288}$$

B.17.2 Capital Market Equilibrium in the Foreign Country

Similar to (B288), it is easy to show that

$$\chi\left(\hat{\mathbf{W}}_{\mathbf{t}}^{*}-\hat{\mathbf{R}}_{\mathbf{t}}^{\mathbf{k}^{*}}\right)+\sum_{\mathbf{k}=\mathbf{1}}^{\mathbf{K}}\mathbf{f_k}(\mathbf{1}-\frac{\psi}{\tau})\hat{\mathbf{Y}}_{\mathcal{F},\mathbf{k},\mathbf{t}}^{*}+\sum_{\mathbf{k}=\mathbf{1}}^{\mathbf{K}}\mathbf{f_k}\frac{\psi}{\tau}(\mathbf{1}-\omega_{\not{c}^{*}}^{*})\hat{\mathbf{Y}}_{\mathcal{F},\not{c},\mathbf{k},\mathbf{t}}$$
$$+\sum_{\mathbf{k}=\mathbf{1}}^{\mathbf{K}}\mathbf{f_k}\frac{\psi}{\tau}\omega_{\not{c}^{*}}^{*}\hat{\mathbf{Y}}_{\mathcal{F},\not{c}^{*},\mathbf{k},\mathbf{t}}=\hat{\mathbf{u}}_{\mathbf{t}}+\hat{\mathbf{K}}_{\mathbf{t}}^{*}. \tag{B289}$$

B.18 Monetary Policy Representation

B.18.1 An Interest Rate Rule in the Home Country

We assume monetary policy in the home country is represented by the following interest rate rule:

$$\hat{R}_{\mathcal{H},t}=\Phi_{\mathcal{Y}}\hat{\mathcal{Y}}_t+\Phi_P\hat{P}_t+e_t^r. \tag{B290}$$

where monetary policy shocks in the home country are denoted by e_t^r, which follows an exogenous AR(1) process given by

$$e_t^r=\rho_e e_{t-1}^r+\varepsilon_t^r, \tag{B291}$$

where ε_t^r is Gaussian with mean zero and variance equal to $\sigma_{\varepsilon^r}^2$.

B.18.2 An Interest Rate Rule in the Foreign Country

We assume monetary policy in the foreign country is represented by the following interest rate rule:

$$\hat{R}_t^{*}=\ 1.5\times\left(\hat{P}_t^{*}-\hat{P}_{t-1}^{*}\right)+\ 0.5\times\hat{\mathcal{Y}}_t^{*}+\varepsilon_t^{r^{*}}, \tag{B292}$$

where $\varepsilon_t^{r^*}$ is Gaussian with mean zero and variance equal to $\sigma_{\varepsilon^{r*}}^2$. The coefficients in the foreign interest rate rule are based on the coefficients in the simple monetary policy equation considered in Taylor (1993), who notes this simple rule gives a largely accurate description of the monetary-policy-related interest rate in the United States between 1987 and 1992.

Appendix C. The Real Wage Dynamics

In this section, we study real wage dynamics in our model under both staggered and flexible wage setting. In table C1, we present the estimates of the correlation between real wages and real GDP in developing economies reported in Agnor, McDermott, and Prasad (2000) and Li (2011) along with those in our model under both staggered and flexible wages. The correlation under flexible wages is almost perfect, which contrasts with a moderate correlation of 0.49 or lower in the data. The correlation of 0.46 under staggered wages, on the other hand, is similar to the empirical estimates reported in table C1. This suggests that our staggered wage-setting assumption is useful to account for the estimates of correlation between real wages and output in developing economies.

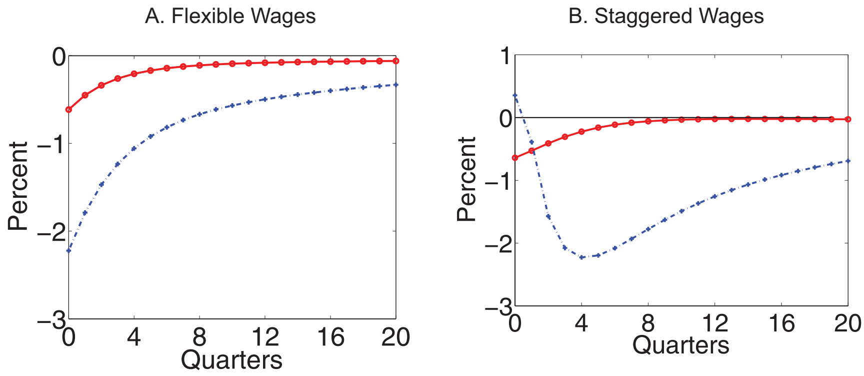
We also study the co-movements of the real wage and output in our model under flexible and staggered wages in figure C1. Under flexible wages, workers' ability to quickly respond to shocks results in the real wage falling strongly in tandem with output following a tightening shock to monetary policy. Consequently, the movements in the real wage closely follow output dynamics under flexible wages.

Table C1. Correlation of the Real Wage with Output in Developing Economies

		$\rho_{w,y}$
	Data	
Agnor, McDermott, and Prasad (2000)	HP	0.49
	BP	0.27
Li (2011)	HP	0.41
	Model	
Flexible Wages		0.99
Staggered Wages		0.46

Notes: HP and BP refer to estimates of the quarterly correlation between real wages and real GDP which are filtered with the Hodrick-Prescott and band-pass filters, respectively. In both Agnor, McDermott, and Prasad (2000) and Li (2011), the reported correlations denote the simple mean of the correlations in the developing economies contained in their sample.

Figure C1. Model-Based Impulse Responses of w and $\mathcal{Y}$ to e_t^r



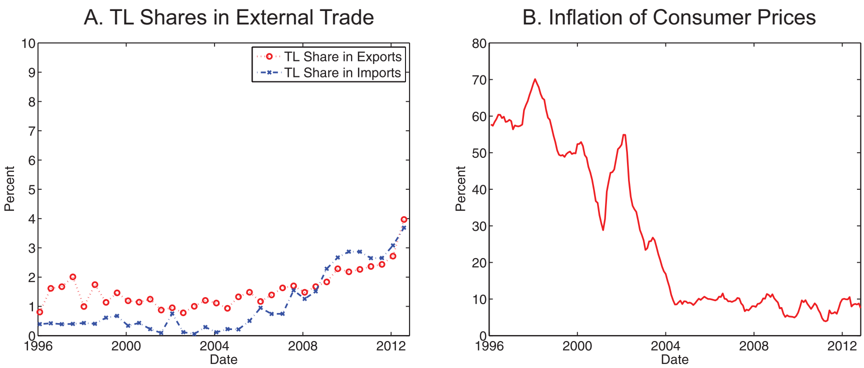
Note: The solid lines with circles and the dashed lines with pluses indicate the model-based impulse response functions of output and the real wage, respectively.

Yet, while output strongly falls, the real wage increases under staggered wages, as wages are predetermined and prices fall after the shock. In the accompanying quarter, since half of all workers obtain a chance to reset their wages, the real wage falls, which is consistent with the finding in Li (2011) that real wages lag business cycles by one quarter in developing economies.

Appendix D. Asymmetry in Currency Invoicing in International Trade between Developing and Advanced Economies

It is a well-known fact that there is an asymmetry between developing and advanced economies in regards to the currency in which exports and imports are denominated. Indeed, while exports and imports are largely denominated in *foreign currencies* in developing economies, they are largely denominated in *national currencies* in advanced economies. For example, Gopinath and Rigobon (2008) report that 97 percent of exports and 90 percent of imports in the United States are priced in the U.S. dollar. To exemplify the pricing practices of exporters and importers in developing economies, we look at exports and imports by currency at Turkish trading ports.

Figure D1. Inflation of Consumer Prices and the Turkish Lira Share of Exports (Imports) in Total Exports (Imports) in Turkey at the Dock



Notes: Our calculations are based on data from the Turkish Statistical Institute. In panel A, the dotted lines with circles and the dashed lines with multiplication signs indicate the share of TL-denominated exports in total Turkish exports at the dock and the share of TL-denominated imports in total Turkish imports at the dock, respectively.

In figure D1, we illustrate the share of exports (imports) priced in the Turkish lira (TL) in total exports (imports) as well as consumer price inflation between 1996 and 2012. Inflation is measured as the percentage change in CPI over the last twelve months. It is notable that the remarkable success in reducing inflation has produced only a modest rise in the shares of TL-denominated exports and imports in the recent years. Indeed, they have stayed at very low levels, below 5 percent, the period under consideration.

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