The Role of Expectations in Inflation Dynamics*

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A growing body of literature examines alternatives to the rational expectations hypothesis in applied macroeconomics. This paper continues this strand of research, examining the role of survey expectations in the inflation process. It reports three principal findings: (i) short-run inflation expectations appear to have a significant role in explaining U.S. inflation over the past twenty to twenty-five years; (ii) long-run expectations generally do not appear to have a direct influence on U.S. inflation over the same period, although they enter indirectly as a key determinant of the short-run expectations (the restrictions implied by "trend inflation" models of inflation are generally rejected in the data); and (iii) the paper develops a first pass at a structural model that incorporates the features discussed above, employing a "survey operator," and assesses its performance in explaining inflation in the post-war period.

JEL Codes: E31, E32.

1. Introduction

Once one assumes that firms set prices that remain fixed or "sticky" for many periods—and "assume" is the appropriate word, for the deeper foundations that would motivate price setters to hold nominal prices fixed for extended periods remain elusive in most applied models—expectations take on a prominent role in price modeling. The intuition is straightforward: If prices must remain fixed for some

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period, then the economic conditions that are expected to prevail during that period must be taken into account if prices are to be set optimally. To date, the bulk of research in inflation has used the rational expectations assumption to generate the required expectations (there are important and notable exceptions: see Roberts 1997, Nunes 2010, and Adam and Padula 2011).

The rational expectations assumption imposes significant restrictions on price-setting models with sticky prices, as has been well documented. In particular, the canonical formulations of price setting under the Calvo (1983), Rotemberg (1983), or Taylor (1980) assumptions can imply a very flexible rate of inflation, in the sense that inflation can jump in response to shocks to marginal cost. Equivalently, any persistence in observed inflation must be attributed to the persistence of the driving process. The data for most (and perhaps all) of the post-war period is difficult to reconcile with such a model, as it appears that inflation behaves as if it both inherits the persistence of the driving process and adds some of its own so-called intrinsic inflation persistence (Fuhrer 2006, 2011).

John Roberts' work in the 1990s suggested that it might be the characterization of expectations, rather than the mechanics of price setting, that leads to such "sticky" inflation behavior. By substituting survey expectations (semi-annual observations from the Livingston Survey), Roberts found considerable success in explaining the behavior of inflation through the early 1990s. However, subsequent work on dynamic stochastic general equilibrium (DSGE) models largely continued with the rational expectations assumption, adopting instead a variety of augmentations to the Calvo et al. model that allowed the models to replicate the key dynamic features of inflation, such as indexation (Christiano, Eichenbaum, and Evans 2005), rule-of-thumb pricing (Galí and Gertler 1999), or serial correlation in the shocks to inflation (Rotemberg 1997; Smets and Wouters 2003; Christiano, Eichenbaum, and Evans 2005).

Some recent work returns to the role of survey expectations as a measure of the expectations employed by price setters (see Del Negro and Eusepi 2010, Nunes 2010, and Adam and Padula 2011). These authors arrive at somewhat contradictory conclusions about such expectations measures, with Nunes (2010) finding little empirical role for survey expectations, while Adam and Padula (2011) find survey expectations to be an important determinant of inflation for UK data.

This paper continues this strand of research, examining the role of survey expectations in the inflation process. It reports three principal findings: (i) short-run inflation expectations appear to have a significant role in explaining U.S. inflation over the past twenty to twenty-five years (this result is explored in more detail in the companion paper Fuhrer 2011); (ii) long-run expectations generally do not appear to have a direct influence on U.S. inflation over the same period, although they enter indirectly as a key determinant of the short-run expectations (the restrictions implied by "trend inflation" models of inflation are generally rejected in the data); (iii) the paper develops a first pass at a structural model that incorporates the features discussed above, employing a "survey operator," and assesses its performance in explaining inflation in the post-war period.

2. Survey and Rational Expectations in Reduced-Form and Structural Models of Inflation

This paper employs quarterly data for inflation, output, unemployment, real marginal cost (the conventional labor share measure), and survey expectations of inflation from the Survey of Professional Forecasters (SPF). The paper uses both short-term (four-quarter) and long-term (the ten-year average inflation rate) inflation expectations from the SPF survey.[1] The ten-year expectations are taken to be a proxy for long-term or "trend" inflation expectations. In this paper we use the Board of Governor's estimate of the ten-year expectations as implemented in their FRB/US model.[2] The paper also uses the SPF's one- to four-quarter-ahead expectations for unemployment.

[1]Most of the empirical work in this area uses the SPF four-quarter inflation expectation as a proxy for the one-quarter-ahead inflation expectation in standard Phillips curves. The one-quarter-ahead SPF expectation, which is available at the Federal Reserve Bank of Philadelphia's web site, would seem more natural. The estimates presented below have been duplicated using this expectational proxy, and the results are essentially the same.

[2]The model variable is "PTR." See the documentation at www.federalreserve.gov/pubs/feds/1996/199642/199642pap.pdf.

Table 1. Reduced-Form Phillips-Curve Estimates, OLS Estimation

$$\pi_t = a\pi_{t-1} + b\pi_t^{SPF,4q} + c\pi_t^{10-year} + d\tilde{U}_t$$

Variable	Coefficient	Standard Error
Lagged Inflation (Sum), a	0.074	0.10
SPF Four-Quarter Expectation, b	0.58	0.24
Ten-Year Expectation (PTR), c	0.34	0.12
Unemployment Gap, d	−0.21	0.067
$R^2 = 0.73$		

2.1 Reduced-Form Models

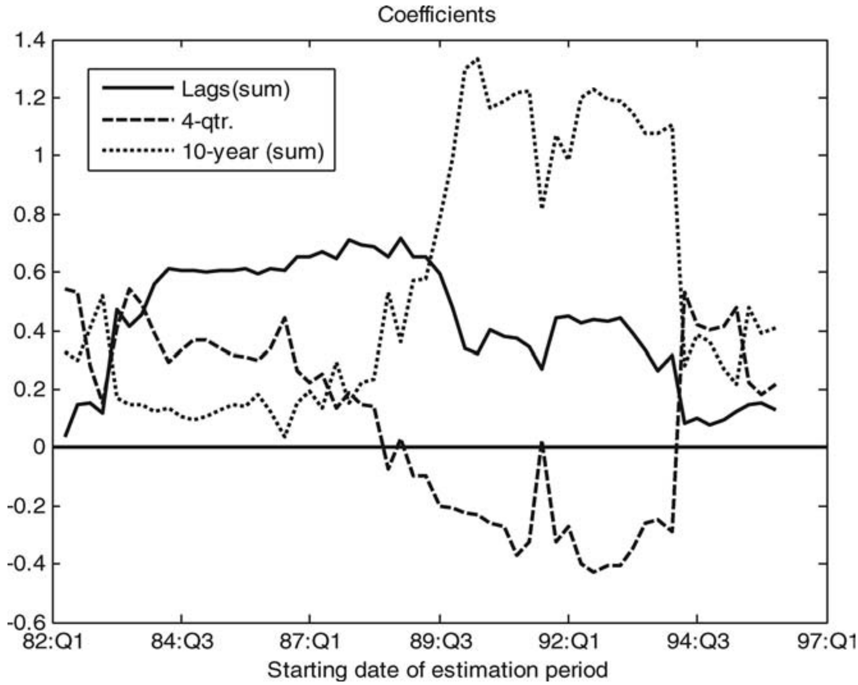
Ordinary least squares (OLS) regressions can serve to motivate the role of expectations in canonical inflation equations. The first regression takes the core inflation rate (here proxied by the consumer price index excluding food and energy) as the dependent variable and regresses it on lagged inflation (two quarterly lags are included), the four-quarter SPF inflation expectation collected at time t, the long-run SPF inflation expectation dated time t, and the unemployment gap, where the NAIRU is as estimated by the Congressional Budget Office (CBO).[3] The estimation period is 1982:Q1 to 2010:Q3, and the results are summarized in table 1.

These unconstrained results suggest a strong and significant correlation between core inflation and the four-quarter expectation, little dependence on lagged inflation, and a modest correlation with the ten-year expectation. The unemployment gap enters quite significantly, with a coefficient that is sizable by recent standards.

But underlying this full-sample estimate lies an array of results for subsamples within the thirty-year span. As figures 1 and 2 suggest, lagged inflation is strongly correlated with core inflation in the early part of the sample, and the four-quarter and ten-year

[3]We employ the four-quarter SPF expectation because others in the literature commonly use this as a proxy for next period's inflation. Results that use next quarter's expectation yield very similar results.

Figure 1. Rolling Regression Coefficient Estimates for Reduced-Form Inflation Models



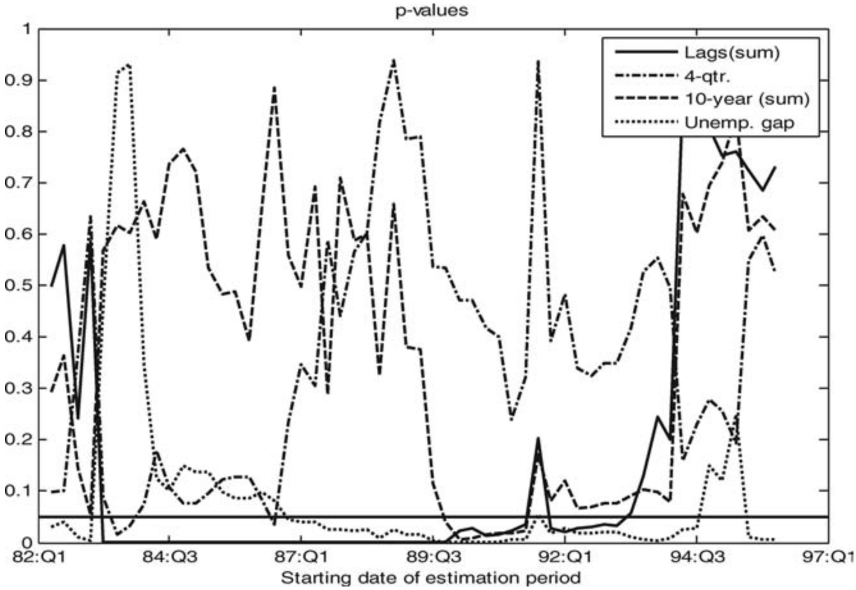
Notes: The dependent variable is core inflation. The estimation sample is 1982:Q1–2010:Q3, with a sixty-quarter window.

expectations measures trade off in size and significance over the sample. The p-value for the unemployment gap lags is typically below 0.05, although this is not the case early in the sample. These results suggest some combination of multicollinearity and subsample instability in this reduced-form regression.[4]

More progress can be made in making sense of this inflation data by imposing a modest amount of structure. To that end, we estimate

[4]The time-varying simple correlation between the two survey expectations measures is fairly steady at 0.9 across most of the sample, falling to 0.4 for the most recent fifteen years.

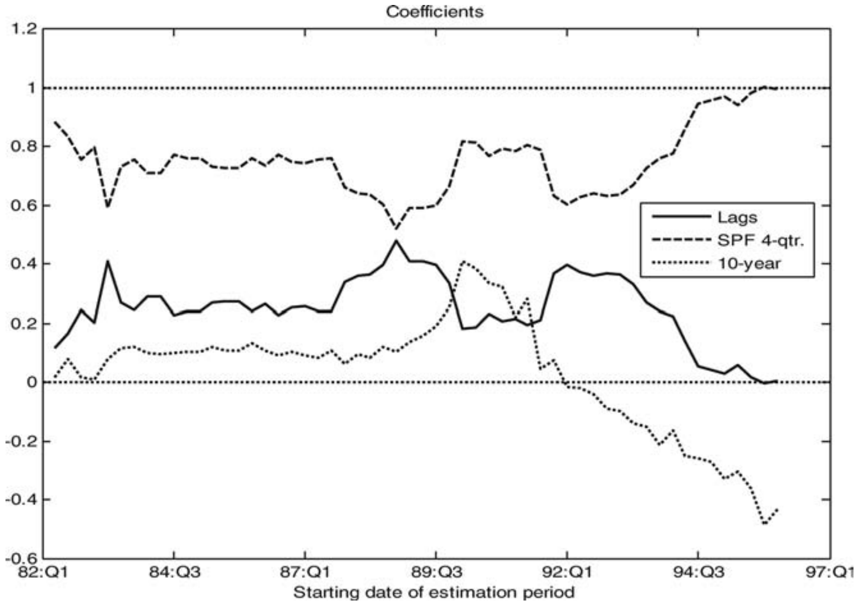
Figure 2. ρ-values for Coefficient Estimates for Reduced-Form Inflation Models in Figure 1



Phillips-curve regressions that impose two restrictions: (i) the sum of the coefficients on lagged inflation and the four-quarter survey expectation is constrained to one, as these measures normally serve as proxies for expected inflation in the next period, and (ii) the long-run survey expectation is not included in that restriction, as we consider it more likely serves as a proxy for "trend inflation" in the sense of Cogley and Sbordone (2008). If that is a reasonable assumption, then the estimated coefficient(s) on this long-run trend inflation proxy should be zero. To see this, consider a trend inflation model that writes the Phillips curve in terms of inflation gap, or deviations of all inflation terms from trend inflation. Denoting the trend inflation proxy by $\bar{\pi}_t$, the Phillips curve as modified from table 1 above takes the form

$$\hat{\pi}_t = a\hat{\pi}_{t-1} + (1-a)\hat{\pi}_t^{SPF,4q} + d\tilde{U}_t$$
$$\hat{\pi}_t \equiv \pi_t - \bar{\pi}_t; \hat{\pi}_{t-1} \equiv \pi_{t-1} - \bar{\pi}_{t-1}; \hat{\pi}_t^{SPF,4q} \equiv \pi_t^{SPF,4q} - \bar{\pi}_t. \qquad (1)$$

Figure 3. Rolling Regression Coefficient Estimates of Equation (2)



Notes: The dependent variable is core inflation. The estimation sample is 1982:Q1–2010:Q3, with a sixty-quarter window.

We can estimate this equation in unconstrained form, allowing us to test the restrictions implied by the trend inflation model:

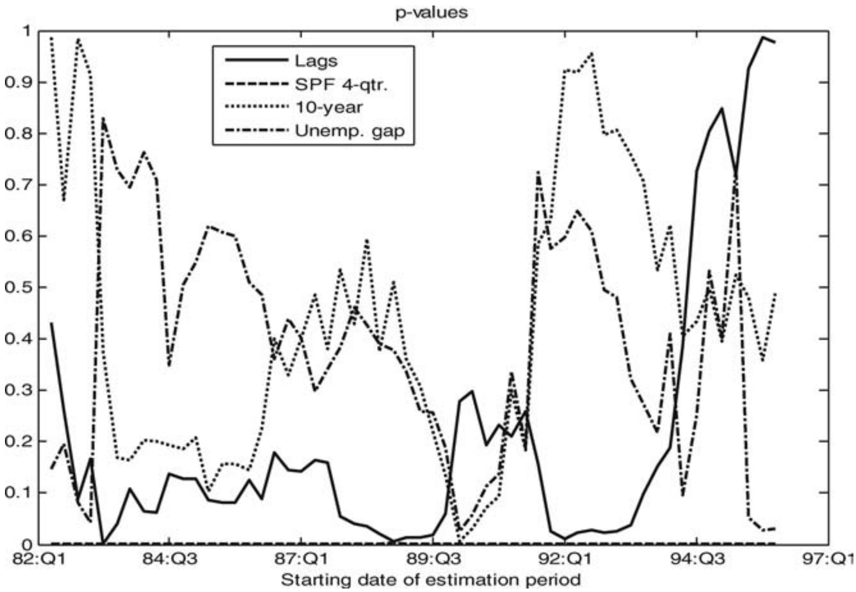
$$\pi_t = a\pi_{t-1} + (1-a)\pi_t^{SPF,4q} + b_1\bar{\pi}_t + b_2\bar{\pi}_{t-1} + d\tilde{U}_t. \qquad (2)$$

If the constraints are satisfied, then one should not be able to reject the constraint $b_1 + b_2 = 0$.[5]

Figures 3 and 4 display the results from estimating equation (2) on a rolling sample of sixty-quarter windows from 1982 to the present. Here we obtain more consistent results across the sample. The four-quarter expectation is estimated extremely significantly with a coefficient that varies between 0.6 and 1 across the entire

[5]Rearranging the inflation gap equation (1), one can see that the coefficients on $\bar{\pi}_t$ and $\bar{\pi}_{t-1}$ are a and $-a$, respectively.

Figure 4. ρ-values for Coefficient Estimates of Equation (2) in Figure 3



sample; lagged inflation is estimated with a modest coefficient that is marginally significant for part of the sample; and the ten-year expectation varies in sign across the sample, and in some cases would not reject the hypothesis that the sum of coefficients is zero, consistent with the trend inflation model. However, one can almost never reject the hypothesis that both lags are jointly insignificant, contributing nothing to this simple Phillips regression. Interestingly, the p-value for the unemployment gap rarely falls below 0.05, which could imply that the four-quarter expectation captures the dependence of inflation on a gap variable. We will return to the implications of this result below. The pattern will play out in tests of more structural versions of the inflation equation later in the paper.

These same results are replicated for a different measure of trend inflation (the Cogley-Sbordone filtered vector autoregression [VAR] estimate), for headline CPI, and for the GDP deflator. In addition, the timing of the four-quarter survey variable is shifted back one quarter, in part because the appropriate expectations date (t or

$t-1$) is not uniform in the literature, and in part because one might worry about simultaneity bias in this simple regression.[6] The results are qualitatively similar in almost all respects: In all cases, the four-quarter SPF expectation enters significantly throughout the sample; the lags enter with, at best, modest significance and size; and the long-run inflation proxies are rarely if ever significant contributors to the regression. One exception is the GDP deflator, which shows a significant effect of the ten-year SPF expectation in the last fifteen years or so of the sample. However, the coefficient estimates over this part of the sample center on -1, which is difficult to reconcile with any reasonable inflation model.

2.2 Models with Explicit Rational Expectations

In Fuhrer, Olivei, and Tootell (2010), we test the nested model that allows survey expectations, rational expectations, and lagged inflation to enter a New Keynesian Phillips curve (NKPC), and examine the relative contributions of each to inflation dynamics for recent decades of U.S. inflation data. Here we replicate the results of that paper and augment the estimation with two GMM estimators to test robustness of the result to methodological differences.

The test equation is

$$\begin{aligned}\pi_t - \bar{\pi}_t &= a(\pi_{t-1} - \bar{\pi}_{t-1}) + b(E_t\pi_{t+1} - \bar{\pi}_{t+1}) \\ &\quad + (1-a-b)\left(\pi_t^{SPF,4q} - \bar{\pi}_{t+1}\right) + cs_t, \qquad (3)\end{aligned}$$

where the inflation variables are as defined above; s is a proxy for real marginal cost (the labor share), which is more commonly used in rational expectations models of inflation; the unit sum constraint is again imposed across the variables that proxy for one-period-ahead expectations; and E_t denotes the model-consistent expectation. Following the example of the preceding section, we also examine less-constrained versions of the equation that allow the trend inflation variables to enter in an unconstrained fashion (as in the previous

[6]In fact, the SPF surveys are collected mid-quarter, at which point forecasters would have, at most, one month of CPI data (depending on the precise date). In any event, the maximum-likelihood estimates presented later on, which take account of any simultaneity in the data, suggest that this should be of little concern in these simple OLS estimates.

section), so as to allow testing of the restrictions imposed by the trend inflation model.[7]

For the maximum-likelihood (ML) estimates of equation (3), we include vector autoregressive equations that allow us to form rational expectations of inflation (and trend inflation) without imposing further structure on the model.[8] The trend inflation variable $\bar{\pi}_t$ is assumed to follow a random walk. For the GMM estimates, we use an instrument set that includes two lags each of the survey expectations variables, real marginal cost, core inflation, and the output gap as defined by the CBO's estimate of potential output. Note that we use two variants of GMM—a conventional iterated estimator with a standard weight matrix and an "optimal instruments" GMM estimator (Fuhrer and Olivei 2004) that imposes more structure on the instruments that are chosen to form the unobserved expectations in the model. The latter is shown to have superior finite-sample properties, a characteristic shared with the maximum-likelihood estimator and demonstrated for an inventory model application in Fuhrer, Moore, and Schuh (1995).

Table 2 summarizes the estimation and test results for the three estimators, and includes the p-value for the test of the restrictions imposed by the trend inflation model.[9] The estimates tell a *nearly*

[7]A set of results that use the unemployment gap are presented in table 4 in the appendix. They are qualitatively similar to the results that employ real marginal cost. In all but the simple GMM estimator, the four-quarter expectation plays a dominant role as the inflation proxy. In the simple GMM case, the Stock-Yogo test for instrument relevance suggests that the instruments do a poor job spanning the relevant endogenous variables. The restrictions imposed by imposing trend inflation, measured either by the ten-year expectation or the Cogley-Sbordone measure, are either rejected outright or the trend variable can be excluded from the specification entirely without any damage to the specification. Files Phil_est_ml_U.m and Phil_est_gmm_U.m produce the parallel sets of results.

[8]The influence of the long-term expectation proxy on four-quarter inflation is extremely strong, whereas it enters insignificantly in the other VAR equations. As a consequence, we excluded the long-term expectation from all the reduced-form equations except for the four-quarter expectation. In addition, the four-quarter expectation enters insignificantly in the output gap and marginal cost equations, so it is excluded from these equations.

[9]The maximum-likelihood estimates take the estimated coefficients in the VAR equations as fixed at their OLS estimates. This restriction is taken for convenience (to minimize the number of estimated parameters) and does not qualitatively affect the other estimated parameters in the model.

Table 2. Estimation Results for Equation (3), Various Estimators, 1990:Q1–2010:Q3

Maximum Likelihood Trend Inflation = SPF Ten-Year Expectation			
Coefficient	Estimate	Standard Error[a]	t-statistic
Lagged Inflation	0.25	0.10	2.5
Rational Expectation	0.001	—	—
Survey Expectation	0.75	0.25	3.0
Test of Trend Inflation Restrictions			
Test of Trend Inflation Restrictions: 0.44 Test for Exclusion of Ten-Year Expectation and RE from Model: 0.44			
Maximum Likelihood Trend Inflation = Cogley-Sbordone Trend Inflation Measure			
Coefficient	Estimate	Standard Error[a]	t-statistic
Lagged Inflation	0.36	0.11	3.47
Rational Expectation	0.001	—	—
Survey Expectation	0.73	0.17	3.81
Test of Trend Inflation Restrictions			
Test of Trend Inflation Restrictions: 0.016 Test for Exclusion of Cogley-Sbordone Trend Inflation Measure and RE from Model: 0.99			
Robustness Check: ML Estimate Including 1980s Trend Inflation = SPF Ten-Year Expectations			
Coefficient	Estimate	Standard Error[a]	t-statistic
Lagged Inflation	0.10	0.063	1.6
Rational Expectation	0.029	0.14	0.22
Survey Expectation	0.87	0.30	2.9
Optimal Instruments GMM Trend Inflation Imposed			
Coefficient	Estimate	Standard Error[a]	t-statistic
Lagged Inflation	0.25	0.093	2.6
Rational Expectation	0.11	0.21	0.54
Survey Expectation	0.57	0.28	2.1

(continued)

Table 2. (Continued)

Conventional GMM, Trend Inflation Imposed			
Coefficient	Estimate	Standard Error[a]	t-statistic
Lagged Inflation	−0.043	0.13	−0.33
Rational Expectation	0.77	0.33	2.3
Survey Expectation	0.22	0.56	0.40

[a]BHHH standard errors.

consistent story: The exception is the conventional GMM estimate, which implies a much higher weight on rational expectations than the others, as has been found in previous papers. The Stock and Yogo (2005) test for weak instruments for this case returns a value of 1.15, well below the 5 percent critical value for the test. Thus the most likely explanation for the discrepancy between the simple GMM estimates and the others is that the instruments are weak and cannot identify both lagged and expected inflation effects on current inflation. A similar set of results are reported in Fuhrer and Olivei (2004). We report the simple GMM estimate for completeness, but the bulk of the evidence points toward a prominent role for the survey expectations and very little for the model-consistent or rational expectation.[10]

Overall, the estimates from these methods are broadly consistent with the simple OLS estimates presented above. They suggest that

- the one-year expectation plays a prominent role in determining inflation in the United States;
- the long-run expectations do not affect inflation directly, either as constrained along the lines of a trend inflation model or unconstrained;
- the restrictions imposed by the trend inflation model are rejected, either because the coefficient restrictions are directly

[10]This finding may reconcile our results—OLS, maximum-likelihood, and optimal instruments estimator—with those of Nunes (2010), who finds a less significant role for survey expectations using a conventional GMM estimator.

rejected or because one cannot reject the hypothesis that either trend inflation proxy should be dropped from the model; and
- the rational expectations term develops a coefficient that is insignificantly different from zero in almost all cases.

Note that the GMM results echo those in Nunes (2010), but these appear to be an artifact of the estimator. An alternative method of moments estimator produces results that conform more closely to the maximum-likelihood and OLS estimates.

3. The Relationship between Survey and "Rational" Expectations

The companion paper to this article examines a number of exercises along the lines of Del Negro and Eusepi (2010), comparing the expectations implied by DSGE models with survey expectations. We will not replicate these exercises here, but we draw some tentative conclusions from these exercises in matching constrained and unconstrained model-consistent inflation expectations to the survey measures. The results generally echo those found in Del Negro and Eusepi (2010):

- One can compute a model-based inflation expectation that is fairly close to the four-quarter survey expectation. However, to do so, one needs to dramatically alter the parameters of a standard structural model in ways that cause it to behave quite poorly with respect to the other variables in the model.
- One can achieve a reasonably close fit as well with an unconstrained VAR, at least for the short-run expectation. This suggests that, with regard to the short-run expectation, the difficulty in replicating the survey data lies in the restrictions imposed by the structural model, the most prominent of which are the restrictions imposed by the rational expectations assumption.
- With regard to long-run expectations, it is difficult to match closely the long-run survey expectations with either constrained or unconstrained models. This is true even when allowing for a time-varying inflation trend that is taken as

Table 3. Estimates of Reduced-Form Equation for Four-Quarter SPF Expectations

$$\pi_t^{4,e} = a\pi_{t-1} + b\pi_t^{40,e} - c\tilde{U}_{t-1}$$

	Coefficient	***p*-value**
Unconstrained		
Lagged Inflation (Four Quarterly Lags)	0.40	0.00
Ten-Year SPF Expectation (PTR)	0.40	0.00
Unemployment Gap	−0.072	0.00
Constrained (Deviations Form, $a + b = 1$)		
Lagged Inflation (Four Quarterly Lags)	0.42	0.00
Ten-Year SPF Expectation (PTR)	0.58	0.00
Unemployment Gap	−0.14	0.00
p-value for Unit Sum Constraint: 0.000		

exogenous to the model. This difficulty represents a significant unresolved challenge.

Overall, this suggests that we have some way to go before we can model the kind of expectations that appear empirically relevant for the determination of inflation. The paper moves part way in this direction in the next and final section.

4. Endogenizing Short-Term Expectations

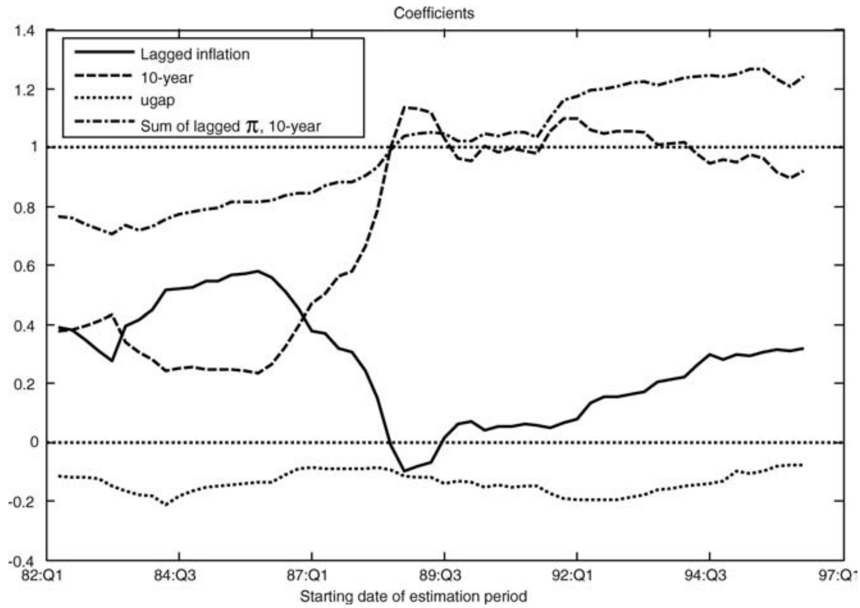
We now revert to the somewhat reduced-form modeling techniques of the earlier sections of this paper to investigate key determinants of the short-term expectations that we have found to be of value in explaining inflation. As a starting point, consider the reduced-form regression linking the four-quarter and ten-year expectations, lagged inflation, and the unemployment gap as summarized in table 3. For notational convenience, the term π_{t-1} represents the sum of the four quarterly lags included in the regression, the term $\tilde{U}_{t-1}$ represents the sum of the two quarterly lags of the unemployment gap included in the regression, and $\pi_t^{40,e}$ represents the current and single lag of the ten-year (forty-quarter) SPF inflation expectation. We include the contemporaneous and lagged values of the latter in order to compare this specification with the next, which imposes that the four-quarter expectation is properly thought of as a deviation from this proxy for the trend in inflation. The constrained version of the equation may be written

$$\pi_t^{4,e} = a\pi_{t-1} + (1-a)\pi_t^{40,e} - c\tilde{U}_{t-1}$$
$$\pi_t^{4,e} = \pi_t^{40,e} + a\left(\pi_{t-1} - \pi_t^{40,e}\right) - c\tilde{U}_{t-1}. \quad (4)$$

In all cases, the sum of the estimated coefficients is significantly different from zero; obviously the test for the null that all the coefficients are jointly zero is rejected with overwhelming confidence. The lower panel of the table considers the same regression with the constraint that the ten-year SPF inflation expectation enter as a trend variable, i.e., that the coefficients on lagged and ten-year expected inflation sum to one. While this constraint boosts the contribution of the unemployment gap term, the unit sum constraint is rejected overwhelmingly.

Like the reduced-form Phillips curves estimated above, there is evidence of significant time variation in these simple relationships. Figures 5–8 summarize the evidence for the unconstrained and constrained versions of the regressions, using a rolling estimation window of sixty quarters. These figures suggest a less tidy picture for the four-quarter expectations than was obtained for the Phillips curves. While all three variables generally contribute to the "fit" of the short-run expectation, lagged inflation loses its significance toward the middle of the sample, and the unit sum constraint is often rejected. The unconstrained regressions show why the test tends to reject: The sum of the lags and the ten-year expectation fall significantly short of one in the first part of the sample, and they significantly exceed one in the latter third of the sample. Interestingly, the unemployment gap is significant throughout, in contrast to the Phillips-curve regressions above. This suggests that the four-quarter expectation influences inflation in part because it captures the dependence of inflation on the unemployment gap, an influence that was clear in the unconstrained Phillips curves presented above.

Figure 5. Unconstrained Regressions



4.1 A More Structural Model of Inflation with Survey Expectations

We build on the evidence so far and construct a quasi-structural model of inflation with the following elements. We begin with the survey-based New Keynesian Phillips curve, allowing for the presence of indexation,

$$\begin{aligned}\pi_t &= a\pi_{t-1} + (1-a)\pi_t^{SPF,4} - b\tilde{U}_t \\ \pi_t &= a\pi_{t-1} + (1-a)S_t\pi_{t+1} - b\tilde{U}_t, \end{aligned} \tag{5}$$

where we have rewritten the equation in the second line to introduce a "survey expectations" operator. Solving equation (5) for the unobserved expectation in period $t+1$ is straightforward under rational expectations. Endogenizing the survey expectations requires additional assumptions about how the survey expectations are formed.

Figure 6. ρ-values for Unconstrained Coefficient Estimates in Figure 5

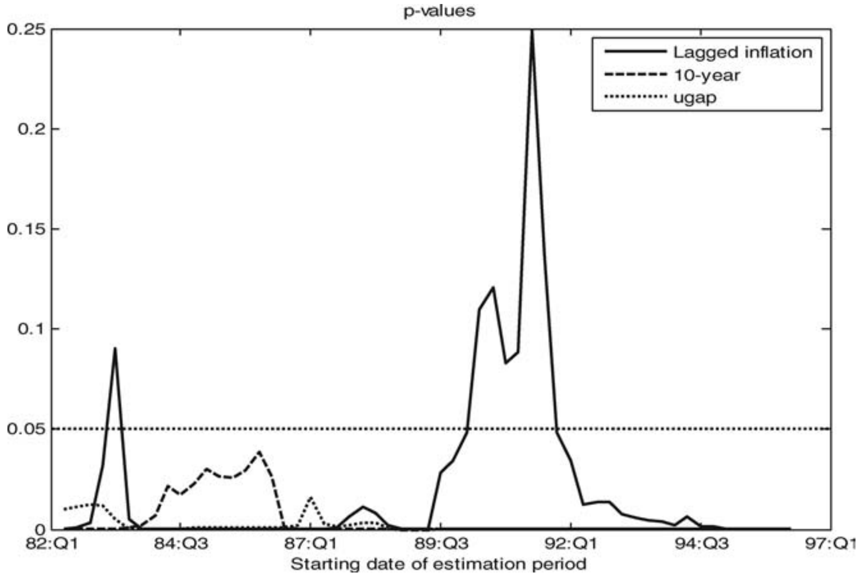


Figure 7. Constrained Regressions

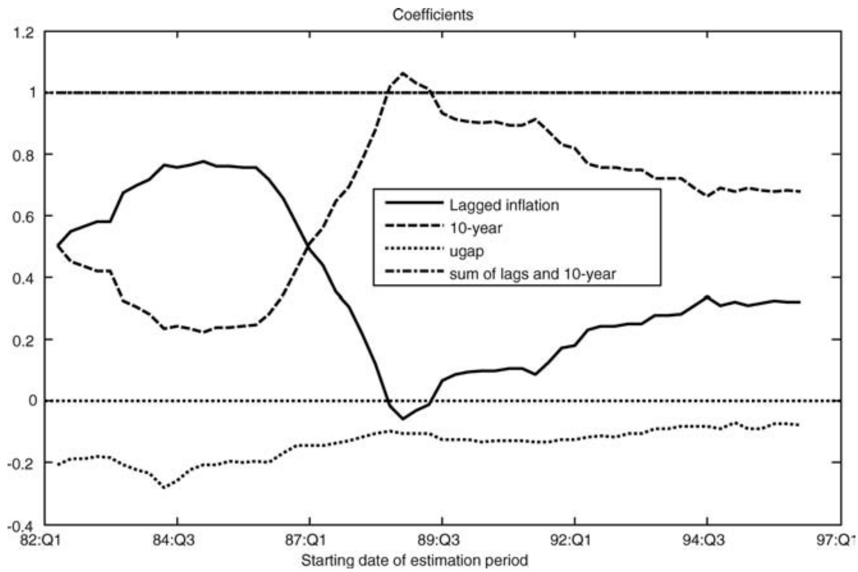
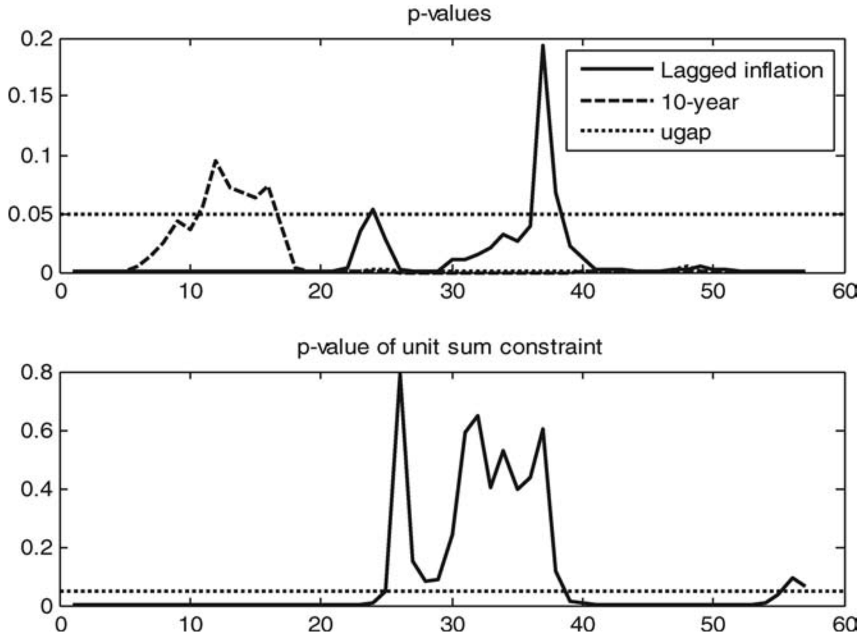


Figure 8. ρ-values for Constrained Regression Coefficients in Figure 7



We will assume that, while clearly different from the rational expectations operator, the S operator maintains the following properties:

- It can be iterated forward consistently.
- It exhibits the simple property that the t-period expectation of t and previous periods' variables equal the realization of these variables.
- In addition, we will assume that as the horizon of the expectation increases, the expectations formed by the S operator converge to those formed by the E operator. In the weak form, this assumes that survey expectations properly estimate the long-run equilibrium value of the variable. In the somewhat stronger form, this assumes that survey expectations capture the later stages of the reversion to equilibrium in the same way

as rational expectations. As a practical matter, this assumption is also motivated by the limitations of the survey data set: Forecasts are available for the current and next four quarters, and for the current and next year.[11]

Taking these properties on board, one can then write the equation for the survey-based expectation of inflation next period as

$$S_t\pi_{t+1} = aS_t\pi_t - b\sum_{i=1}^{\infty}\omega_i S_t\tilde{U}_{t+i}. \tag{6}$$

This expression is convenient, as we observe the SPF survey of future unemployment rates for four quarters beyond the survey date and for the current year and the next year. If in addition we assume that $S_t\pi_t = \pi_t$, we can then approximate (8) by adding the rational expectation of unemployment gaps beyond the observable survey horizon:

$$S_t\pi_{t+1} = a\pi_t - b\sum_{i=1}^{8}\omega_i S_t\tilde{U}_{t+i} - b\sum_{i=9}^{\infty}\delta_i E_t\tilde{U}_{t+i}. \tag{7}$$

This assumes that the rational expectation of the unemployment gap for the horizon beyond the next several quarters is a reasonable proxy for the survey expectation. In many cases, this will be reasonable, as the unemployment gap will be expected to return to its equilibrium (zero) at this horizon. For many initial conditions, this approximation simply assumes that the survey expectations get the mean of the forecasted variable right. For cases in which the unemployment gap has been more dramatically perturbed from its long-run value, we assume that the survey and rational expectations converge at these longer horizons. This approximation is implemented by adding the auxiliary equation that defines the model's expectations of unemployment past the survey horizon as

$$\tilde{U}_t^+ = \delta\tilde{U}_{t+9} + (1-\delta)E_t\tilde{U}_{t+1}^+, \tag{8}$$

[11]In the last several years of the data set, the two-year-ahead forecast is published, but due to its very short span of availability, we cannot use it for this paper.

and equation (7) becomes

$$S_t\pi_{t+1} = a\pi_t - b\sum_{i=1}^{8}\omega_i S_t\tilde{U}_{t+i} - bU_t^+. \tag{9}$$

These assumptions do not completely "close" the model, as the determination of unemployment, required to form the model-consistent forecasts of unemployment that enter U_t^+, has not yet been specified. For this paper, we take an agnostic view on the determination of unemployment, modeling it with a reduced-form VAR equation in three observables (lags of the funds rate, inflation, and unemployment). The SPF unemployment gap forecasts are assumed to revert toward zero at a rate that is determined by a second-order autoregression.

In the empirical implementation of equation (9), we use quarterly values of the current-year and next-year forecast of the unemployment rate, from which the CBO's estimate of the NAIRU for the corresponding year is subtracted to form an unemployment gap. In addition, in recognition of the important role played by trend inflation in the preliminary regressions reported in figures 5–8, we express the inflation variables as deviations from trend inflation:

$$S_t\pi_{t+1} = a\pi_t + (1-a)\bar{\pi}_t - b\sum_{i=0}^{1}\omega_i S_t\tilde{U}_{t+i}^Y - cU_t^+. \tag{10}$$

Preliminary estimates suggest that it may be difficult to identify the ω_i independently, so we condense the equation to

$$S_t\pi_{t+1} = a\pi_t + (1-a)\bar{\pi}_t - b\big(S_t\tilde{U}_t^Y + S_t\tilde{U}_{t+1}^Y\big) + cU_t^+. \tag{11}$$

Finally, allowing for the possibility that survey expectations are formed in a way consistent with a more explicit "hybrid" model of inflation, we introduce two lags of the unemployment gap into the expectation equation. In other words,

$$\begin{aligned} S_t\pi_{t+1} = {} & a\pi_t + (1-a)\bar{\pi}_t - b_L(\tilde{U}_{t-1} + \tilde{U}_{t-2}) \\ & - b\big(S_t\tilde{U}_t^Y + S_t\tilde{U}_{t+1}^Y\big) + cU_t^+. \end{aligned} \tag{12}$$

For reference, the OLS estimate of this equation for the sample 1982–2010, excluding the model-consistent expectations of the unemployment gap, yields (HAC standard errors in parentheses)[12]

$$\underset{}{S_t\pi_{t+1}} = \underset{(0.049)}{0.22\pi_t} + \underset{(0.049)}{0.78\bar{\pi}_t} - \underset{(0.10)}{0.33\tilde{U}_t^Y} + \underset{(0.11)}{0.17\tilde{U}_{t+1}^Y}.$$

The model comprises equations (5) and (12), the definition of U_t^+ from equation (8), and VAR equations to forecast the unemployment gap.[13] The sample begins in 1982:Q1, the first quarter for which the expectations variables (accounting for lags) are available. We employ a sixty-quarter estimation window, although the results are not particularly sensitive to the window size. Figure 9 presents rolling-sample ML estimates of the key equations (5) and (11), while figure 10 displays the p-values for the Wald tests of the null hypotheses that the indicated model parameter (or parameters jointly) are zero.[14] The test uses the BHHH estimate of the inverse of the covariance matrix of the parameters, which we denote Ω. Denoting the estimated and constrained parameter vectors by $\hat{\beta}$ and β^C, the test W is

$$W = (\hat{\beta} - \beta^C)'\Omega(\hat{\beta} - \beta^C).$$

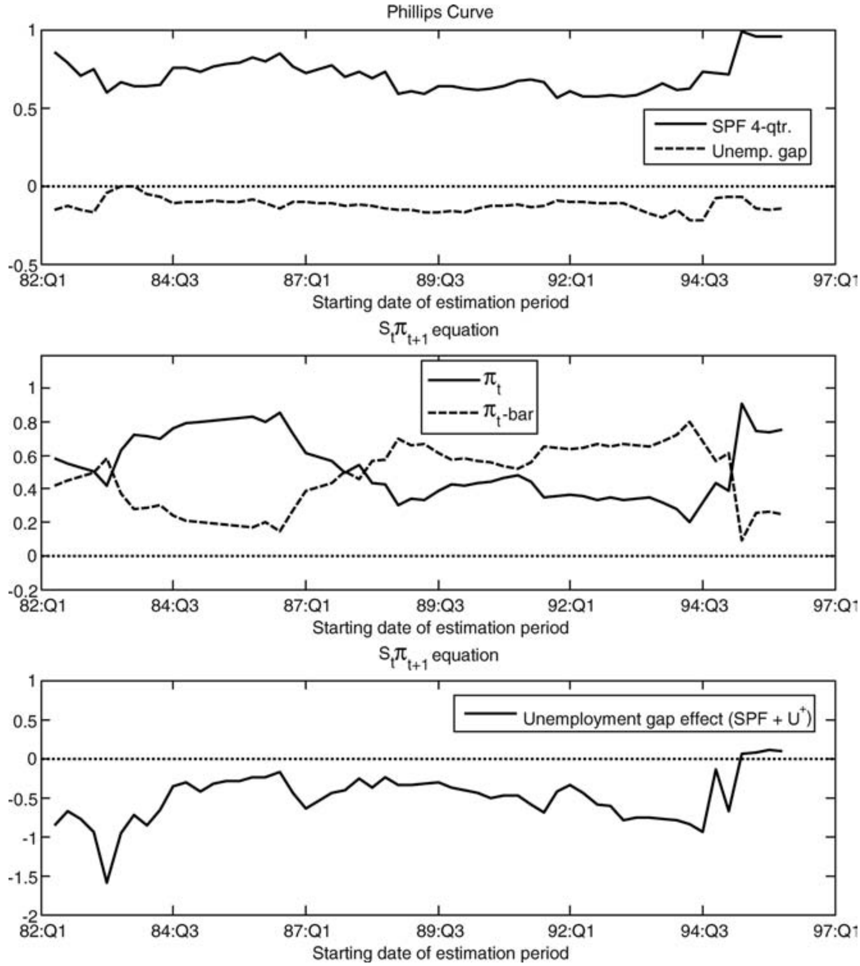
The ML estimates for the survey expectations Phillips curve are reasonably stable across time. As in all the results presented above, the estimated coefficient on the one-year SPF expectation varies between 0.5 and 1, with an average value of about 0.7. This implies a modest coefficient of about one-third for lagged inflation in the Phillips curve. As indicated in figure 10, the p-value for the Wald test that its value is zero rejects overwhelmingly throughout the sample. The unemployment gap, the dashed line in the top panels

[12]The unit sum constraint is rejected over the full sample, as it is in the preliminary regressions above.

[13]These equations include two quarterly lags of the unemployment gap, the core inflation rate, and the federal funds rate.

[14]The parameters of the VAR equation and the second-order autoregressions are also estimated for each sample in the rolling window, jointly with the other structural parameters in the Phillips curve and the inflation expectations equation.

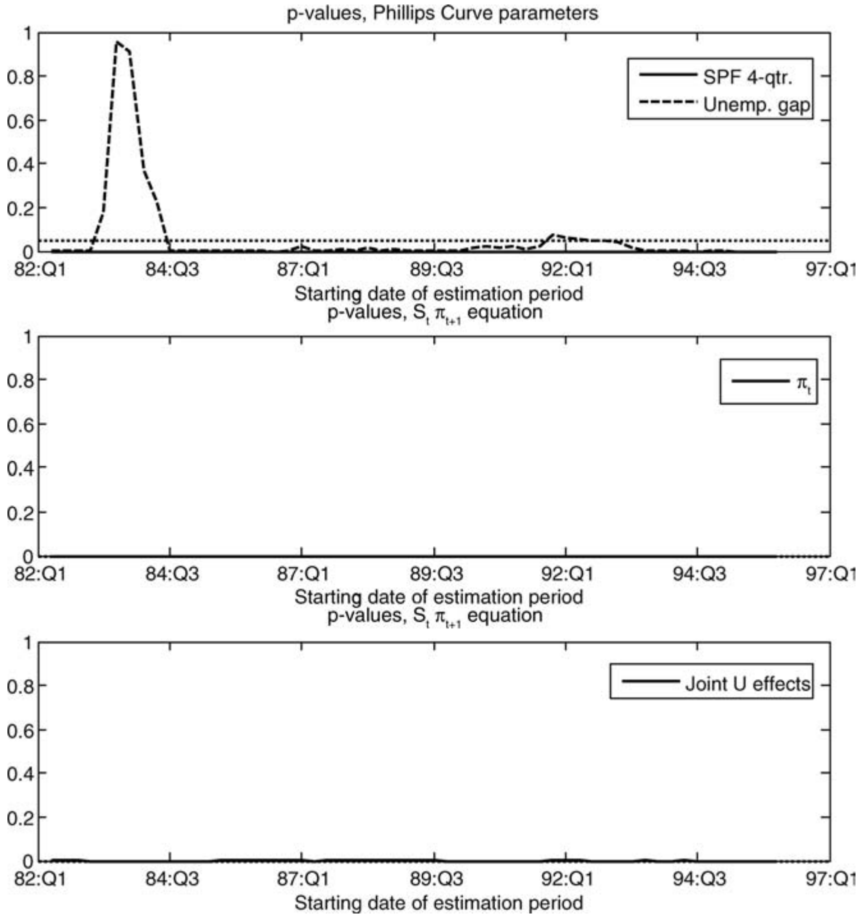
Figure 9. Rolling ML Estimates, Window = Sixty Quarters



of figures 9 and 10, generally enters with the correct sign and quite significantly, except for the early part of the sample. The average coefficient across all rolling samples is −0.12.

Turning to the equation that explains the short-term inflation expectation, figure 9 shows that current inflation and the trend inflation proxy enter with roughly equal weights, although the coefficients

Figure 10. ρ-values for Wald Tests of Parameter Restrictions, Test for Null that Indicated Parameter(s) = 0



fluctuate somewhat over the sample (recall that they are constrained to sum to one). The coefficient is estimated precisely, as indicated by the middle panel in figure 10. The influence of unemployment, the third panel in figure 9, is generally significant and of the expected sign. The average unemployment effect across all estimates, summing the SPF forecasts and the "model-consistent" forecasts, is −0.37. However, two caveats are worthy of note: First, the overall effect of unemployment (the sum of the SPF unemployment gap forecasts and the U^+ term) turns *positive* in several of the samples toward the end of the estimation period, which is difficult to interpret. Second, the influence of the SPF unemployment forecasts (not shown) varies in sign and significance over the estimation samples. This could reflect the presence of "speed-limit" (change in expected unemployment) effects on inflation expectations, but it also suggests that some caution should be exercised in interpreting the results.

5. Conclusions

As suggested in the introduction, these results support a renewed emphasis on non-rational expectations measures in inflation modeling. The reduced-form and moderately constrained estimates suggest a strong role for survey expectations, particularly the one-year expectation (although these results are replicated for one-quarter-ahead expectations as well). The estimated contribution of a structural model's rational expectation is almost always dominated by the contribution from survey expectations. The restrictions implied by the "trend inflation" model are almost uniformly rejected.

The attempt to articulate a model that endogenizes the survey expectations provides some interesting results. While long-run expectations may not enter the Phillips curve directly, they still serve as an anchor for the short-run survey expectations in all the estimates presented. A more careful modeling of the survey expectations that imposes more structural expectational assumptions is partly successful, as it finds *some* role for expected unemployment gaps, in a manner consistent with conventional theories. However, the empirical results in this regard are not uniform across all samples. The sign of the unemployment gap effect sometimes flips to positive, and the survey's unemployment forecasts do not always enter significantly.

While the results presented in this paper are neither conclusive nor free of problems, they do suggest that the use of survey expectations may present an important direction for inflation and DSGE modelers. While the simplicity of working with rational expectations is sacrificed to an extent, it is possible to use survey data on inflation expectations while maintaining a reasonable blend of theoretical and empirical rigor.

Appendix

Table 4 shows the maximum-likelihood estimates for table 2, using the unemployment rate.

Alternative Estimates of Equation (7)

An alternative way to estimate equation (7) is to compute the U^+ term as the weighted forecasts from an unconstrained vector autoregression. Table 5 presents results from the following exercise:

- For a given estimation range, estimate the coefficients for a three-variable VAR in the core inflation rate, the unemployment gap, and the federal funds rate.
- Use these coefficients to compute the weighted sum of unemployment gap forecasts (U^+).
- Forming this weighted sum requires a value for δ in equation (4). We iterate over values of δ between 0 and 1.
- Now estimate equation (7) via OLS, using the estimated values of U^+ for given δ, and choose the value of δ that minimizes the sum of squared residuals for the regression.

Table 4. Estimation Results for Equation (3), Various Estimators, 1990:Q1–2010:Q3 (Unemployment Rate Substitutes for Real Marginal Cost)

Maximum Likelihood Trend Inflation = SPF Ten-Year Expectation			
Coefficient	Estimate	Standard Error[a]	t-statistic
Lagged Inflation	0.30	0.11	2.6
Rational Expectation	0.00	—	—
Survey Expectation	0.70	0.19	3.7
Test of Trend Inflation Restrictions			
Test of Trend Inflation Restrictions: 0.00 Test for Exclusion of Ten-Year Expectation and RE from Model: 0.027			

(continued)

Table 4. (Continued)

Maximum Likelihood Trend Inflation = Cogley-Sbordone Trend Inflation Measure			
Coefficient	Estimate	Standard Error[a]	t-statistic
Lagged Inflation	0.48	0.17	4.9
Rational Expectation	0.00	0.094	0.01
Survey Expectation	0.52	0.098	3.1
Test of Trend Inflation Restrictions			
Test of Trend Inflation Restrictions: 0.37 Test for Exclusion of Cogley-Sbordone Trend Inflation Measure and RE from Model: 0.99			
Robustness Check: ML Estimate Including 1980s Trend Inflation = SPF Ten-Year Expectations			
Coefficient	Estimate	Standard Error[a]	t-statistic
Lagged Inflation	0.022	0.071	0.31
Rational Expectation	0.00	—	—
Survey Expectation	0.98	0.25	3.9
Optimal Instruments GMM Trend Inflation Imposed			
Coefficient	Estimate	Standard Error[a]	t-statistic
Lagged Inflation	0.80	0.080	9.9
Rational Expectation	0.001	0.095	0.01
Survey Expectation	0.98	0.30	3.2
Conventional GMM, Trend Inflation Imposed			
Coefficient	Estimate	Standard Error[a]	t-statistic
Lagged Inflation	−0.045	0.19	−.24
Rational Expectation	0.96	0.42	2.3
Survey Expectation	0.094	0.81	0.12

Stock-Yogo Test for Instrument Relevance: 1.079
[a]BHHH standard errors.
Phil_est_ml_U.m, Phil_est_gmm_U.m

Table 5. Estimates of Structural Equation for Inflation Expectations, Various Subsamples

Variable	Coeff.	Stand. Error	Coeff.	Stand. Error	Coeff.	Stand. Error	Coeff.	Stand. Error	Coeff.	Stand. Error
	1982:Q1–2010:Q3		1982:Q1–1996:Q4		1987:Q1–2001:Q4		1992:Q1–2006:Q4		1996:Q1–2010:Q3	
π_t	0.24	0.089	0.27	0.095	0.23	0.10	−0.064	0.058	−0.052	0.053
$\bar{\pi}_t$	0.76	0.11	0.73	0.15	0.77	0.19	1.06	0.075	1.05	0.053
$S_t\tilde{U}_t^Y$	−0.20	0.14	−0.61	0.17	−0.18	0.21	−0.26	0.12	−0.062	0.062
$S_t\tilde{U}_{t+1}^Y$	0.12	0.14	0.70	0.25	0.11	0.25	−0.070	0.11	−0.30	0.050
U^+	−0.25	0.32	−0.37	0.65	−0.18	0.32	0.52	0.25	0.72	0.16
δ	0.995		0.18		0.995		0.32		0.69	

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