A Bivariate Model of Federal Reserve and ECB Main Policy Rates

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This paper studies when and by how much the Federal Reserve and the European Central Bank change their target interest rates. I develop a new non-linear bivariate framework, which allows for elaborate dynamics and potential interdependence between the two countries, as opposed to linear feedback rules, such as a Taylor rule, and I use a novel real-time data set. Although the data sample is inherently small, through a Bayesian estimation approach, I find some evidence in favor of timing synchronization between central banks and against the hypothesis of follower behaviors. Results for the magnitude model support zero correlation in the size of the target rate changes. Institutional factors and inflation represent relevant variables for both timing and magnitude decisions, while output plays a secondary role.

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1. Introduction

This paper focuses on the Federal Reserve (Fed) and the European Central Bank (ECB) interest rate feedback rules. In particular, it

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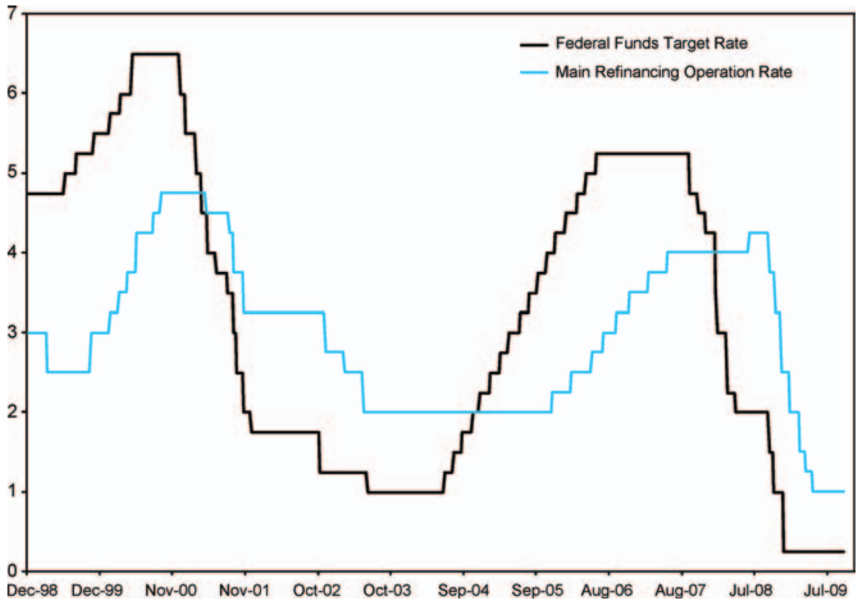
develops an econometric model that empirically analyzes when a target rate change is adopted (timing), as well as by how much the rate is changed (magnitude change). While central banks' behavior has typically been described with the use of univariate linear interest rate feedback rules, such as Taylor rules, I exploit a non-linear bivariate framework, which allows for elaborate dynamics and for potential interactions between the two central banks.

This study is appealing for several reasons. The most important way the Fed and the ECB achieve their monetary policy goals is by setting, respectively, the federal funds target rate (FFTR) and the main refinancing operation (MRO) rate.[1] These policy rates are important because they signal the stance of monetary policy, affect investment decisions, and often have considerable impact on financial markets. Understanding the way the two central banks set their target rates and identifying the variables taken into account in the process is of great interest. Since it started to operate at the beginning of 1999, the ECB, together with the Fed, has been meticulously scrutinized in the way it conducts monetary policy. Bartolini and Prati (2003) analyze the Fed and the ECB with particular attention to institutional structures, policy frameworks, and operational procedures. Cecchetti and O'Sullivan (2003) compare the central banks' approaches to the execution of monetary policy. U.S. and European Monetary Union (EMU) policy rates followed a roughly similar pattern over the period January 1999 to December 2009; see figure 1. The EMU rate fluctuates over a narrower range than the U.S. rate; both rates are characterized by frequent changes in the first half of the sample and sporadic changes in the middle part of the sample; and the FFTR displays frequent changes also in the last part of the sample. In addition, the size and sign of interest rate changes display some similarities. My analysis addresses these issues by studying

[1] In particular, the Federal Open Market Committee (FOMC) implements its monetary policy decisions by changing its target for the federal funds rate (FFR), which is the rate at which depository institutions borrow and lend reserves to and from each other overnight. Although the Federal Reserve does not control the FFR directly, it can do so indirectly by varying the supply of reserves available to be traded in the market. On the other hand, the key policy rate set by the Governing Council of the ECB is the rate applied to main refinancing operations, which provide the bulk of liquidity to the financial system.

Figure 1. Evolution of the Federal Funds Target Rate (FFTR) and of the Main Refinancing Operation (MRO) Rate from the Beginning of 1999 until the End of 2009



when interest rate changes are implemented and by examining the size and sign of those changes, with the idea that the two decisions could carry distinct information and might be triggered by different variables.[2]

Open questions in the monetary policy debate are whether in an open economy interest rate feedback rules should include the exchange rate, in addition to inflation and output, and, more generally, how optimal interest rate feedback rules should be designed within an international framework; see, among others, Benigno (2002), Clarida, Galí, and Gertler (2002), Pappa (2004), and Corsetti and Pesenti (2005). While the theoretical literature has focused on optimality

[2]In a traditional approach (e.g., VAR or Taylor rule), the same variables affect both timing and magnitude because the two decisions are normally analyzed together. That is, when there is a jump/change in the rate (magnitude decision), the event is considered to occur (timing decision).

issues, this paper introduces a new methodology to provide evidence about the interaction between the Fed and the ECB. It does not address issues related to cooperation or potential gains from cooperation. This paper only provides stylized facts about Fed and ECB interest rate feedback rules, exploring the possibility that interdependence could play a role in describing timing and magnitude of interest rate changes.

Moreover, it is not clear that conventional linear interest rate feedback rules are sufficient to explain the complexity of central banks' behavior, especially in the presence of potential interdependences. I therefore investigate the possibility that a non-linear model could better describe interest rate decisions.

Methodologically, one novelty of the paper is to provide a bivariate autoregressive conditional hazard (BACH) model to study the timing of interest rate changes and a conditional bivariate ordered (CBO) probit model to analyze the magnitude of interest rate changes. The BACH model extends the autoregressive conditional hazard (ACH) model of Hamilton and Jordà (2002) in order to account for interdependence between the two central banks. The timing/duration framework is based on the autoregressive conditional duration (ACD) model developed by Engle and Russell (1998, 2002, 2005) and Engle (2000). Bergin and Jordà (2004) analyze empirical evidence of monetary policy interdependence within a set of fourteen OECD countries, making use of the Hamilton and Jordà (2002) model, but they do not analyze the EMU. They study interdependence by investigating whether the probability of a change in the domestic target rate at time t depends on a similar decision by a "leading country" at time $t-1$ (United States, Germany, or Japan). This implies a hierarchy between banks and assumes that the leading central bank's decision is known by the other central bank. Moreover, their setup does not allow them to recover a joint hazard probability for the two central banks. My model differs from Bergin and Jordà (2004) because it is a truly bivariate model which converts the marginal hazards to a joint hazard probability through the use of a conditional discrete copula-type representation. This allows me to treat the Fed and ECB symmetrically and to study interdependence in the form of decision synchronization and follower behaviors. The CBO probit model represents a special case of a bivariate ordered probit model, where I rescale the probability mass to condition on the timing decision. It differs from Bergin and Jordà (2004) because, by rescaling, I am able to study the exact magnitude of the change (basis-points change) as opposed to merely the direction of change (strong increase, increase, decrease, strong decrease).

According to the Taylor-type rule literature, past interest rates, inflation, output gaps, and exchange rate movements are relevant factors in choosing the level of interest rate changes. I analyze whether these variables are important in determining when the Fed and ECB change their interest rates, as well as whether they play a role in explaining the magnitude of the change. Moreover, I study whether the Fed and the ECB synchronize their policies, whether one follows the other, and whether there exists a contemporaneous correlation in the magnitude of their interest rate changes. Timing synchronization of policies is analyzed with the odds ratio, which indicates how much the odds of one bank changing its target rate move when the other bank changes its target. Follower behavior is studied with dummy variables that capture the effect of one country's interest rate decision on the subsequent decisions of the other country. A test on the coefficient of this dummy variable can be interpreted as a test of one country (Granger) causing the other country's interest rate decision, and hence one country following the other country's decision. The correlation between interest rate changes captures the correlation which is left unexplained after traditional explanatory variables have been considered. This paper shows whether the traditional variables that have commonly been used in the literature are sufficient in explaining timing and magnitude changes of policy rates, and whether the interdependence could be a factor in explaining monetary policies. However, it does not provide a complete answer to the underlying problem about what is in fact the source of the interdependence and whether interdependence is optimal.

Another novel feature of the paper is the empirical application with the use of a real-time data set and the Bayesian estimation. Persuaded that the available information set that central banks observe is of great importance to the decisions they make, I construct and use a real-time data set that includes output and inflation measures, exchange rates, and data on target rates and duration between changes. This real-time approach to monetary policy has been studied, among others, by Orphanides (2001), who demonstrates that real-time policy recommendations differ from those derived with ex post revised data.[3] Bayesian estimation is not new to monetary policy studies (see Sims and Zha 1998, Schorfheide 2000, and Cogley and Sargent 2002), but, to the best of my knowledge, it has never been applied to ACD- or ACH-type models. The methods used in the paper generally require fairly large samples to produce results. Because of the youth of the ECB and the type of data, the sample used in the paper is small. The Bayesian framework is particularly well suited to this small-sample problem, because it allows me to incorporate pre-sample information to better evaluate the available information. I use ten years of data for the Fed and the Bundesbank to elicit the prior, following the view that the German central bank is, among the European central banks, the one that most closely resembles the ECB. Although the Bayesian approach facilitates the estimation, it does not completely eliminate the small-sample problem.

Estimation results for the timing model support the hypothesis that institutional factors, such as scheduled meetings of the FOMC and the Governing Council, as well as inflation rates, are important variables in determining timing decisions. I find some evidence of timing synchronization between the two central banks. On the other hand, follower behaviors are not supported. However, although there is strong evidence against the Fed following the ECB timing strategy, the evidence is slim against the reverse scenario of the ECB following the Fed's timing decisions. Estimation results for the magnitude model illustrate the importance of inflation rates as explanatory variables for both countries. Unemployment and exchange rate dynamics turn out to have a secondary role, confirming the idea that the ECB's primary objective is to maintain price stability. I find evidence supporting zero correlation between the magnitude shocks. The zero correlation suggests that although there is *synchronization* in the timing of interest rate changes, its own macroeconomic conditions are sufficient to explain the magnitude of each central bank's target rate change.

The paper is organized as follows. The next section describes the model. Section 3 describes the data. Section 4 describes the

[3] It could be interesting to compare baseline results based on real-time data with results obtained by estimating the model using revised data. However, this goes beyond the scope of this paper. Many papers in the literature have in fact already addressed this issue.

Bayesian implementation and presents empirical results. Section 5 presents results for posterior predictive checks. Section 6 concludes.

2. The Model

For simplicity I refer to the EMU and the United States as countries e and f (for ECB and Fed). The basic idea is to separate timing and magnitude of interest rate changes, and to derive a model capable of accounting for the specific features of both decisions. I describe timing by binary variables that take the value one when the target interest rate is changed. Consequently, magnitude variables take non-zero values only when the timing binary variable is one.

More precisely, let x_t^i be a binary variable that takes values $\{0, 1\}$ according to whether the target rate of country $i \in \{e, f\}$ has changed at calendar time t:

$$x_t^i = \begin{cases} 1 & \text{target rate of country } i \text{ is changed} \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

and let y_t^i be the interest rate change that takes place whenever event x^i occurs (i.e., whenever $x^i = 1$).

I am interested in studying whether the two central banks decide to change their target rate (x) and by how much (y); hence I want to study the joint probability $f(x_t^e, x_t^f, y_t^e, y_t^f|\mathcal{F}_{t-1})$, where $\mathcal{F}_{t-1}$ is the information set available at time $t-1$. The joint probability can be rewritten as the product of the marginal distribution of (x^e, x^f) and the conditional distribution of $(y^e, y^f|\ x^e, x^f)$:

$$\begin{aligned} &f\left(x_t^e, x_t^f, y_t^e, y_t^f|\mathcal{F}_{t-1}; \theta\right) \\ &\quad = g\left(x_t^e, x_t^f|\mathcal{F}_{t-1}; \theta_1\right) \cdot q\left(y_t^e, y_t^f|x_t^e, x_t^f, \mathcal{F}_{t-1}; \theta_2\right) \end{aligned} \tag{2}$$

so that the resulting log-likelihood can be decomposed into two parts,

$$\mathcal{L}(\theta_1, \theta_2) = \mathcal{L}_1(\theta_1) + \mathcal{L}_2(\theta_2), \tag{3}$$

where

$$\mathcal{L}_1(\theta_1) = \sum_{t=1}^{T} \log g\left(x_t^e, x_t^f|\mathcal{F}_{t-1}; \theta_1\right) \quad \text{Timing Model} \tag{4}$$

$$\mathcal{L}_2(\theta_2) = \sum_{t=1}^{T} \log q\big(y_t^e, y_t^f | x_t^e, x_t^f, \mathcal{F}_{t-1}; \theta_2\big) \quad \text{Level Model.} \tag{5}$$

As pointed out by Engle (2000), if θ_1 and θ_2 have no parameters in common and are variation free, then the maximization of $\mathcal{L}(\theta_1, \theta_2)$ is equivalent to maximizing $\mathcal{L}_1(\theta_1)$ and $\mathcal{L}_2(\theta_2)$ separately. The parameters (θ_1, θ_2) are variation free, as in Engle, Hendry, and Richard (1983), if θ_1 and θ_2 are not subject to cross-restrictions, meaning that the range of admissible values for θ_1 does not vary with θ_2, and vice versa. In a maximum-likelihood environment, this means that the separate maximization of $\mathcal{L}_1(\theta_1)$ and $\mathcal{L}_2(\theta_2)$ is equivalent to maximizing $\mathcal{L}(\theta_1, \theta_2)$. The same decomposition would work in a Bayesian framework with independent priors since

$$\max\left\{\frac{\mathcal{L}_1(Y|\theta_1)\mathcal{L}_2(Y|\theta_2)\Pr(\theta_1,\theta_2)}{\Pr(Y)}\right\} = \max\left\{\frac{\mathcal{L}_1(Y|\theta_1)\Pr(\theta_1)}{\Pr(Y)}\right\}\max\left\{\frac{\mathcal{L}_2(Y|\theta_2)\Pr(\theta_2)}{\Pr(Y)}\right\}$$

if the priors are independent and (θ_1, θ_2) are variation free. I follow this strategy and I refer to the first part of the likelihood as the *timing model* and to the second part as the *level model.* The former is characterized as a bivariate autoregressive conditional hazard (BACH) model, while the latter is a conditional bivariate ordered (CBO) probit model.

I describe both models below.

2.1 Bivariate Autoregressive Conditional Hazard (BACH) Model

The *timing model* hinges on the joint probability of type e and f events occurring, where type i event occurring means that country i, $i \in \{e, f\}$, has decided to change its target rate. Marginal probability distributions for individual interest rate decisions have been modeled in the literature with autoregressive conditional hazard (ACH) models; see Hamilton and Jordà (2002). The ACH model is derived from the autoregressive conditional duration (ACD) model proposed by Engle and Russell (1998) and Engle (2000). The ACD model is developed in event time[4] and aims to explain the duration of spells between events (between two consecutive trades or quotes, for example). It is called autoregressive conditional duration because the conditional expectation of the duration depends upon past durations. Within this duration framework, bivariate models have been studied, but none of them is suitable to the present framework. Engle and Lunde (2003), for example, model the joint likelihood function for trade and quote arrivals, but they include the possibility that an intervening trade censors the time between a trade and the subsequent quote.[5] Thus their model does not serve my purpose. Moreover, unlike them, I adopt calendar time because it readily allows me to incorporate updated explanatory variables.

I develop a bivariate model which converts the marginal distribution information, modeled following Hamilton and Jordà (2002), into a joint distribution, by using a conditional discrete copula-type representation.

2.1.1 Marginal Hazard Rates

Define $N^e(t)$ and $N^f(t)$ to be, respectively, the cumulative number of country e and f events as of time t, i.e., the number of target rate changes of country e and f as of time t. Following Hamilton and Jordà (2002), I rewrite the $ACD(g, m)$ model from Engle and Russel (1998) as

$$\psi^i_{N^i(t)} = \sum_{j=1}^{m^i} \alpha^i_j \; u^i_{N^i(t)-j} + \sum_{j=1}^{g^i} \beta^i_j \; \psi^i_{N^i(t)-j}$$
$$i = e, f, \tag{6}$$

where α^i and β^i are country-specific parameters, $\psi^i_{N^i(t)}$ is the expected duration for country i at calendar time t when $N^i(t)$ events have occurred, and $u^i_{N^i(t)-j}$ is the duration for country i when

[4]Event time is defined by a sequence $\{t_0, t_1, .., t_n, ...\}$ with $t_0 < t_1 < \ldots < t_n < \ldots$ representing the arrival time of an event. Calendar time is simply $t = 0, 1, 2, 3, \ldots, n, \ldots$

[5]In particular, they analyze the elapsed times between two consecutive trades and between a trade and a quote.

$N^i(t) - j$ events have occurred; i.e., $u^i_{N^i(t)-j}$ is the time elapsed between event $N^i(t) - j - 1$ and event $N^i(t) - j$. Viewed as a function of time, $\psi^i_{N^i(t)}$ is a step function that changes only when a new event occurs, i.e., when $N^i(t) \neq N^i(t-1)$.[6]

Define the hazard rate h^i_t as the probability of a country i event occurring at time t (the probability that the central bank of country i decides to change its target rate), given the information available up until time $t - 1$, i.e., $\Pr(x^i_t = 1|\mathcal{F}_{t-1})$. The country i marginal hazard rate can be written as

$$\begin{aligned} h^i_{t|t-1} &= \Pr[N^i(t) \neq N^i(t-1)|\mathcal{F}_{t-1}] = \Pr\left[x^i_t = 1\middle|\mathcal{F}_{t-1}\right] \\ &= \frac{1}{\psi^i_{N^i(t-1)} + \delta'_i z_{t-1}}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} \psi^i_{N^i(t-1)} &= \alpha^i\ u^i_{N^i(t-1)-1} + \beta^i \psi^i_{N^i(t-1)-1} \\ g &= m = 1. \end{aligned} \tag{8}$$

z_{t-1} is a vector of variables known at $t-1$ and δ_i is a parameter vector. I need to ensure that $h^i_{t|t-1} \in (0, 1)$. To do so I follow Hamilton and Jordà (2002) and I use a smooth function λ so that

$$h^i_{t|t-1} = \frac{1}{\lambda\left(\psi^i_{N^i(t-1)} + \delta'_i z_{t-1}\right)},$$

where

$$\lambda(\nu) = \begin{array}{cc} 1.0001 & \nu \leq 1 \\ 1.001 + 2\Delta_0(\nu-1)^2/\left[\Delta_0^2 + (\nu-1)^2\right] & 1 < \nu < 1 + \Delta_0 \\ 0.0001 + \nu & \nu \geq 1 + \Delta_0 \end{array}$$

and $\Delta_0 = 0.1$.

Using the above marginal distributions for the Bernoulli variables $x^i,\ i = e, f$, I want to construct a joint distribution for (x^e, x^f). The following section gives some theoretical background about the

[6]Equation (6) does not include a constant, which is instead included in the z vector.

discrete copula-type representation that will allow me to recover the joint hazard of countries e and f.

2.1.2 Conditional Discrete Copula-Type Representation and Joint Hazard Rates

When marginal distributions are continuous, a joint distribution can be constructed from the marginal distributions with the use of a copula. The beauty of a copula is that for bivariate (multivariate) distributions, the univariate marginals and the dependence structure can be separated, with the copula containing all the dependence information. Since the marginals considered here are discrete, problems arise since copulas are not unique in this case. The way I solve the problem follows Tajar, Denuit, and Lambert (2001). As the copula contains the dependence information, Tajar, Denuit, and Lambert (2001) disentangle the dependence structure from the Bernoulli marginals.

In addition, I extend the existing results to allow for conditioning variables. For the purpose of exposition, I will assume below that $\mathcal{F}$ represents the conditioning set (it might contain one or more variables).

Let (x^e, x^f) be random Bernoulli variables for which the marginal conditional distributions $\Pr[x_t^e|\mathcal{F}]$ and $\Pr[x_t^f|\mathcal{F}]$ are known. Associate $(x^e, x^f)|\mathcal{F}$ to a random couple $(u^e, u^f)|\mathcal{F}$ with discrete uniform marginals such that

$$\Pr[u^e = 0|\mathcal{F}] = \Pr[u^e = 1|\mathcal{F}] = \Pr[u^f = 0|\mathcal{F}] = \Pr[u^f = 1|\mathcal{F}] = \frac{1}{2}. \tag{9}$$

Therefore, the joint distributions of $(x^e, x^f)|\mathcal{F}$ and $(u^e, u^f)|\mathcal{F}$ can be written as follows:

$x^e_{\|\mathcal{F}} \diagdown x^f_{\|\mathcal{F}}$	0	1	
0	$h_{00\|\mathcal{F}}$	$h_{01\|\mathcal{F}}$	$1 - h^e_{\|\mathcal{F}}$
1	$h_{10\|\mathcal{F}}$	$h_{11\|\mathcal{F}}$	$h^e_{\|\mathcal{F}}$
	$1 - h^f_{\|\mathcal{F}}$	$h^f_{\|\mathcal{F}}$	

(10)

$$\begin{array}{|c|c|c|c|} \hline u^e_{|\mathcal{F}} \diagdown u^f_{|\mathcal{F}} & 0 & 1 & \\ \hline 0 & \gamma_{00|\mathcal{F}} & \gamma_{01|\mathcal{F}} & 1/2 \\ \hline 1 & \gamma_{10|\mathcal{F}} & \gamma_{11|\mathcal{F}} & 1/2 \\ \hline & 1/2 & 1/2 & \\ \hline \end{array}. \tag{11}$$

The joint probabilities of $(x^e, x^f)|\mathcal{F}$ and $(u^e, u^f)|\mathcal{F}$ are such that

$$h_{lk|\mathcal{F}} = \left(p^e_{l|\mathcal{F}}\right) \cdot \left(p^f_{k|\mathcal{F}}\right) \cdot \left(\gamma_{lk|\mathcal{F}}\right), \; l, k = 0, 1, \tag{12}$$

where $p^e_{l|\mathcal{F}}$ and $p^f_{k|\mathcal{F}}$ depend only on the marginals, while $\gamma_{lk|\mathcal{F}}$ contains the dependence information.

I compute the p's from the marginals as

$$p^e_{0|\mathcal{F}} = \frac{\left(1 - h^e_{|\mathcal{F}}\right)}{nf_1}, \quad p^e_{1|\mathcal{F}} = \frac{h^e_{|\mathcal{F}}}{nf_2} \tag{13}$$

$$p^f_{0|\mathcal{F}} = \frac{\left(1 - h^f_{|\mathcal{F}}\right)}{nf_3}, \quad p^f_{1|\mathcal{F}} = \frac{h^f_{|\mathcal{F}}}{nf_4}, \tag{14}$$

where nf_p, $p = 1, 2, 3, 4$ are normalizing factors that guarantee $h_{00} + h_{01} = 1 - h^e$, $h_{10} + h_{11} = h^e$, $h_{00} + h_{10} = 1 - h^f$, $h_{01} + h_{11} = h^f$.

Let the odds ratio $\eta \in R^+$ be

$$\eta = \frac{h_{00|\mathcal{F}} \cdot h_{11|\mathcal{F}}}{h_{01|\mathcal{F}} \cdot h_{10|\mathcal{F}}} = \frac{\gamma_{00|\mathcal{F}} \cdot \gamma_{11|\mathcal{F}}}{\gamma_{01|\mathcal{F}} \cdot \gamma_{10|\mathcal{F}}}. \tag{15}$$

The odds ratio is a measure of association for binary random variables. For ease of interpretation, it can be rewritten as $\eta = \frac{(\gamma_{11|\mathcal{F}}/\gamma_{10|\mathcal{F}})}{(\gamma_{01|\mathcal{F}}/\gamma_{00|\mathcal{F}})}$, where the numerator gives the "odds" of country f event occurring versus not occurring given that country e event occurs, while the denominator gives the "odds" of country f event occurring given that country e event does not occur. Thus the odds ratio indicates how much the odds of country f changing its target rate increase when country e changes its target. Independence is $\eta = 1$.

For a pair of binary random variables with uniform marginals, the following property holds:

$$\gamma_{01|\mathcal{F}} = \frac{1}{2} - \gamma_{00|\mathcal{F}}, \quad \gamma_{10|\mathcal{F}} = \frac{1}{2} - \gamma_{00|\mathcal{F}}, \quad \gamma_{11|\mathcal{F}} = \gamma_{00|\mathcal{F}}. \tag{16}$$

Therefore, $\gamma_{00|\mathcal{F}}$ can be obtained as the solution[7] to the following quadratic equation:

$$\gamma^2_{00|\mathcal{F}}\ (\eta - 1) - \eta\ \gamma_{00|\mathcal{F}} + \frac{\eta}{4} = 0. \tag{17}$$

Hence $h_{ij|\mathcal{F}}\ i, j = 0, 1$ can be computed.

This copula-type representation allows me to construct the joint hazard rates to be used in the likelihood.

Some assumptions must be made. First, in my setup, both marginals depend on t. According to equation (15), the odds ratio should also depend on t. I instead assume that $\eta_t = \eta,\ \forall t$. Second, I define the conditioning set $\mathcal{F}$ as the information available as of time $t - 1$. The conditioning set must be the same for both marginal distributions.[8]

Therefore, the joint probability of events e and f occurring (given $\mathcal{F}_{t-1}$) is

$$\begin{aligned} g\big(x^e_t x^f_t | \mathcal{F}_{t-1}; \theta_1\big) = {} & (h_{00,t|t-1})^{1(x^e_t=0, x^f_t=0)} (h_{10,t|t-1})^{1(x^e_t=1, x^f_t=0)} \\ & \times (h_{01,t|t-1})^{1(x^e_t=0, x^f_t=1)} (h_{11,t|t-1})^{1(x^e_t=1, x^f_t=1)}, \end{aligned} \tag{18}$$

where $1(\cdot, \cdot)$ is an indicator function. Equation (18) yields the following likelihood function:

$$\begin{aligned} \mathcal{L}_1(\theta_1) = \sum_{t=1}^{T} & 1\big(x^e_t = 0, x^f_t = 0\big)\ \log(h_{00,t|t-1}) \\ & + 1\big(x^e_t = 1, x^f_t = 0\big)\ \log(h_{10,t|t-1}) \\ & + 1\big(x^e_t = 0, x^f_t = 1\big)\ \log(h_{01,t|t-1}) \\ & + 1\big(x^e_t = 1, x^f_t = 1\big)\ \log(h_{11,t|t-1}). \end{aligned} \tag{19}$$

[7] Only one of the two roots belongs to $[0, 1/2)$ and is therefore admissible. The root $\gamma^+_{00,t} \notin [0, 1/2)$.

[8] Thus in principle the marginal hazard rate for process e, h^e, depends on all the conditioning variables (even the one from process f). I will impose the restriction that process f variables have no effect on the duration. The same applies to h^f.

2.2 Conditional Bivariate Ordered (CBO) Probit Model

I use a special bivariate ordered probit model to analyze the interest rate magnitude changes (y_t^e and y_t^f). My framework is a special case of the standard bivariate ordered probit model, because I am interested in the distribution of y_t^e and y_t^f conditioned on the information set $\mathcal{W}_{t-1}$ and conditioned on the timing decision (x_t^e, x_t^f).

Assume there are two latent variables, one for each country, representing the optimal (but unobserved) target change

$$\widetilde{y}_t^e = w_{t-1}^{e\prime}\pi^e + \varepsilon_t^e \tag{20}$$

$$\widetilde{y}_t^f = w_{t-1}^{f\prime}\pi^f + \varepsilon_t^f, \tag{21}$$

where π^e and π^f are parameter vectors, w_{t-1}^e and $w_{t-1}^f \in \mathcal{W}_{t-1}$ are vectors of variables observed as of time $t-1$, and $(\varepsilon_t^e, \varepsilon_t^f)|w_{t-1}, \sim \mathcal{N}(0, \Sigma)$ with $\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$.

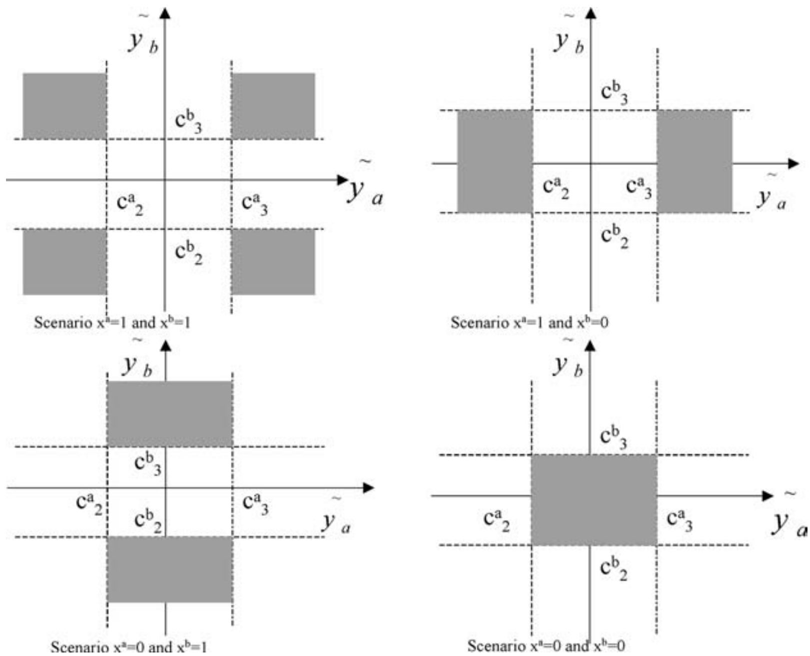
If the observable target change y_t^i could assume the discrete values $s^i \in \{-50, -25, 0, 25, 50\}$ measured in basis points (bps), $i = e, f$, then it would be related to the unobservable optimal target change, so that[9]

$$y_t^i = \begin{cases} s_1 \leq -50 & \text{if } \widetilde{y}_t^i \in \left(-\infty = c_0^i, c_1^i\right] \\ s_2 = -25 & \text{if } \widetilde{y}_t^i \in \left(c_1^i, c_2^i\right] \\ s_3 = 0 & \text{if } \widetilde{y}_t^i \in \left(c_2^i, c_3^i\right] \\ s_4 = 25 & \text{if } \widetilde{y}_t^i \in \left(c_3^i, c_4^i\right] \\ s_5 = 50 & \text{if } \widetilde{y}_t^i \in \left(c_4^i, c_5^i = \infty\right). \end{cases} \tag{22}$$

I observe x_t^e and x_t^f and am interested in the conditional distribution of y_t^e and y_t^f given x_t^e and x_t^f. The questions I want to address are as follows: What is the joint probability of y_t^e and y_t^f taking values $s_m, s_n \in \{-50, -25, 25, 50\}$, respectively, given $x_t^e = x_t^f = 1$ and given $\mathcal{W}_{t-1}$? What is the probability of y_t^e

[9]In our framework, I also observe changes of −75 bps. Because these changes are not enough to identify a separate cut point, in the estimated model, I cluster the –75 bps changes together with the –50 bps changes.

Figure 2. Rescaling Necessary to Condition on (x_t^a, x_t^b)



Notes: The shaded areas of the top-left panel show the feasible regions for (y_t^a, y_t^b) when $(x_t^a = 1, x_t^b = 1)$; the top-right panel shows the feasible region for y^a when $(x_t^a = 1, x_t^b = 0)$, in which case $y^b = 0$ with probability 1; the bottom-left panel shows the feasible region for y^b when $(x_t^a = 0, x_t^b = 1)$, in which case $y^a = 0$ with probability 1; and the bottom-right panel shows the feasible region when $(x_t^a = 0, x_t^b = 0)$, in which case $y^a = y^b = 0$ with probability 1.

being equal to $s_m \in \{-50, -25, 25, 50\}$ when $y_t^f = 0$ (no change for country f occurs)? What is the probability of y_t^f being equal to $s_n \in \{-50, -25, 25, 50\}$ when $y_t^e = 0$?

Thus, starting from the bivariate normal distribution that characterizes $\binom{\widetilde{y}_t^e}{\widetilde{y}_t^f}|\mathcal{W}_{t-1}$, I want to retrieve $f(\widetilde{y}_t^e, \widetilde{y}_t^f | w_{t-1}, x_t^e, x_t^f)$. Conditioning on x_t^e and x_t^f, the bivariate normal density that characterizes the distributions of $(\varepsilon_t^e, \varepsilon_t^f)$ and $(\widetilde{y}_t^e, \widetilde{y}_t^f)$ is rewritten so as to redistribute the probability mass. Figure 2 contains a visual illustration of the necessary rescaling.

The log-likelihood relative to the magnitude decision can be written as

$$\mathcal{L}_2(\theta_2) = \sum_{t=1}^{T} \mathit{1}\left(x_t^e = 1, x_t^f = 0\right) \log P_{10} + \mathit{1}\left(x_t^e = 0, x_t^f = 1\right) \log P_{01}$$
$$+ \mathit{1}\left(x_t^e = 1, x_t^f = 1\right) \log P_{11}, \tag{23}$$

where P_{10} denotes the probability of y_t^e being equal to $s_m \in \{-50, -25, 25, 50\}$ when $y_t^f = 0$ (no change for country f occurs), once I have rescaled to condition on the timing decision ($x_t^e = 1, x_t^f = 0$). A similar interpretation is given to P_{01} and P_{11}.[10] A detailed derivation of these probabilities is presented in the appendix.

3. Data

The raw data that I use to analyze Fed and ECB decisions are the dates and size of changes in the FFTR and the MRO rate. Table 1 displays the FFTR level, dates on which it was changed, and the size of the change. Table 2 displays similar data for the Eurosystem. Dummies for FOMC and Governing Council meetings have also been included. Due to the youth of the EMU, my sample spans the period January 1, 1999 to the last week of 2009 for a total of 575 weeks.

As is clear from both table 1 and table 2, the Fed has changed rates more frequently than the ECB. The average duration for the United States is about 80 days as opposed to 117 in the EMU. Figure 3 shows that there have been a total of forty-six changes in the United States and thirty-one in the EMU. In the United States, three were –75 bps changes, thirteen were –50 bps changes, seven were –25 bps changes, twenty-two were +25 bps changes, and one was a +50 bps change. In the EMU, one was a –75 bps change, nine were –50 bps changes, five were –25 bps changes, fourteen were +25 bps changes, and two were +50 bps changes. In the CBO probit estimation, I combined the –75 bps changes with –50 bps changes due to identificability issues of the cut points with so few observations.

[10]Note that, given $x_t^i = 0,\ i = e, f$, then $y^i = 0$ with probability 1. Thus $\log P_{00} = 0$.

Table 1. Calendar of Federal Funds Target Rate Changes

	FFTR	FFTR Change	Weekday	Duration in Days
17-Nov-98	4.75		Tue	
30-Jun-99	5	0.25	Wed	225
24-Aug-99	5.25	0.25	Tue	55
16-Nov-99	5.5	0.25	Tue	84
2-Feb-00	5.75	0.25	Wed	78
21-Mar-00	6	0.25	Tue	48
16-May-00	6.5	0.25	Tue	56
3-Jan-01	6	0.5	Wed	232
31-Jan-01	5.5	−0.5	Wed	28
20-Mar-01	5	−0.5	Tue	48
18-Apr-01	4.5	−0.5	Wed	29
15-May-01	4	−0.5	Tue	27
27-Jun-01	3.75	−0.25	Wed	43
21-Aug-01	3.5	−0.25	Tue	55
17-Sep-01	3	−0.5	Mon	27
2-Oct-01	2.5	−0.5	Tue	15
6-Nov-01	2	−0.5	Tue	35
11-Dec-01	1.75	−0.25	Tue	35
6-Nov-02	1.25	−0.5	Wed	330
25-Jun-03	1	−0.25	Wed	231
30-Jun-04	1.25	0.25	Wed	371
10-Aug-04	1.5	0.25	Tue	41
21-Sep-04	1.75	0.25	Wed	42
10-Nov-04	2	0.25	Wed	50
14-Dec-04	2.25	0.25	Wed	34
2-Feb-05	2.5	0.25	Wed	50
22-Mar-05	2.5	0.25	Wed	48
3-May-05	3	0.25	Tue	42
30-Jun-05	3.25	0.25	Tue	58
9-Aug-05	3.5	0.25	Thu	40
20-Sep-05	3.75	0.25	Tue	42
1-Nov-05	4	0.25	Tue	42
13-Dec-05	4.25	0.25	Tue	42
31-Jan-06	4.5	0.25	Tue	49
28-Mar-06	4.75	0.25	Tue	56
10-May-06	5	0.25	Wed	43
29-Jun-06	5.25	0.25	Thu	50

(continued)

Table 1. (Continued)

	FFTR	FFTR Change	Weekday	Duration in Days
18-Sep-07	4.75	−0.5	Tue	446
31-Oct-07	4.5	−0.25	Wed	43
11-Dec-07	4.25	−0.25	Tue	41
22-Jan-08	3.5	−0.75	Tue	42
30-Jan-08	3	−0.5	Wed	8
18-Mar-08	2.25	−0.75	Tue	48
30-Apr-08	2	−0.25	Wed	43
8-Oct-08	1.5	−0.5	Wed	161
29-Oct-08	1	−0.5	Wed	21
16-Dec-08	0.25	−0.75	Tue	48

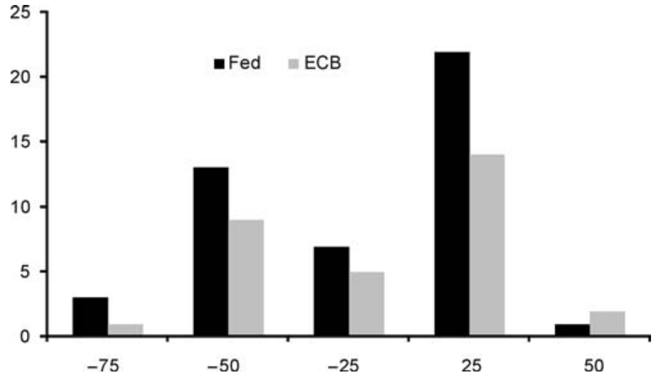
Table 2 deserves a few comments. Main refinancing operations which settled before June 28, 2000 were conducted on the basis of fixed-rate tenders, in which the ECB would specify the interest rate in advance and participating counterparties would bid the amount of money (volume) they were willing to transact at that rate. A side effect of the system was chronic overbidding by financial institutions. On June 8, 2000, the ECB announced that, starting with the operation to be settled on June 28, 2000, the main refinancing operations would be conducted as variable-rate tenders, in which counterparties would specify both the amount and the interest rate at which they want to transact. Starting from the operation to be settled on October 15, but announced on October 8, the ECB introduced a fixed-rate tender procedure with full allotment for all its refinancing operations.

Together with these key policy rates, I create dummy variables to control for the FOMC schedule in the United States and the Governing Council schedule in the EMU. These dummies are important since the majority of interest rate changes happen on these scheduled meeting dates. The Fed has made five intermeeting changes (January, April, and September 2001; January and October 2008), while the ECB has changed rates on a non-meeting day only twice, in the immediate aftermath of September 11, 2001 and in October 2008 at the height of the financial crisis.

Table 2. Calendar of MRO Rate Changes

	MRO		MRO Change	Weekday	Duration in Days
	Fix. Rate	Var. Rate			
1-Jan-99	3	—		Fri	
4-Jan-99	3	—		Mon	
21-Jan-99	3	—		Thu	
8-Apr-99	2.5	—	−0.5	Thu	77
4-Nov-99	3	—	0.5	Thu	210
3-Feb-00	3.25	—	0.25	Thu	91
16-Mar-00	3.5	—	0.25	Thu	42
27-Apr-00	3.75	—	0.25	Thu	42
8-Jun-00	4.25	—	0.5	Thu	42
28-Jun-00	—	4.25		Wed	20
31-Aug-00	—	4.5	0.25	Thu	64
5-Oct-00	—	4.75	0.25	Thu	35
10-May-01	—	4.5	−0.25	Thu	217
30-Aug-01	—	4.25	−0.25	Thu	112
17-Sep-01	—	3.75	−0.5	Mon	18
8-Nov-01	—	3.25	−0.5	Thu	52
5-Dec-02	—	2.75	−0.5	Thu	392
6-Mar-03	—	2.5	−0.25	Thu	91
5-Jun-03	—	2	−0.5	Thu	91
1-Dec-05	—	2.25	0.25	Thu	910
2-Mar-06	—	2.5	0.25	Thu	91
8-Jun-06	—	2.75	0.25	Thu	98
3-Aug-06	—	3	0.25	Thu	56
5-Oct-06	—	3.25	0.25	Thu	63
7-Dec-06	—	3.5	0.25	Thu	63
8-Mar-07	—	3.75	0.25	Thu	91
6-Jun-07	—	4	0.25	Wed	90
3-Jul-08	—	4.25	0.25	Thu	393
8-Oct-08	3.75	—	−0.50	Wed	97
6-Nov-08	3.25	—	−0.50	Thu	29
4-Dec-08	2.5	—	−0.75	Thu	28
15-Jan-09	2	—	−0.50	Thu	42
5-Mar-09	1.5	—	−0.50	Thu	49
2-Apr-09	1.25	—	−0.25	Thu	28
7-May-09	1	—	−0.25	Thu	35

Figure 3. Basis-Point-Change Distribution in the United States and in the EMU



Moreover, I construct a weekly real-time data set.[11] U.S. variables include the CPI and GDP deflator as inflation measures; GDP growth, industrial production (IP), and the unemployment rate as output measures; and the euro/dollar exchange rate. EMU variables include the euro-zone CPI[12] and GDP deflator as inflation measures; GDP growth, industrial production, and the unemployment rate as output measures; and the euro/dollar exchange rate. See table 3.[13] I take weekly average exchange rate data. Notice that some of these variables are released at a frequency which is lower than weekly, and therefore the latest number can potentially be quite old and stale. This might explain why variables that are updated more frequently—such as the CPI, IP, and the unemployment rate—will be preferred to GDP and GDP deflator in the estimation results. Evans (2005), among others, has focused on deriving daily or weekly

[11]The original data is available daily. That is, on a given day I observe whatever data is released, and for variables for which there is no release on that day, I consider the latest available number. To make this a weekly data set, I am forced to cut off the information on Fridays prior to the meetings. For the ECB, Governing Council meetings are only on Thursdays, so I disregard all the information that arrives thereafter. FOMC meetings are normally held on Tuesdays; thus considering information until the Friday of the previous week is not very restrictive.

[12]Euro-zone inflation is measured by the Harmonized Index of Consumer Prices (HICP).

[13]Inflation and output variables, together with their released dates, are taken from Bloomberg.

Table 3. Explanatory Variables Included in the Data Set

U.S. Variables		**EMU Variables**	
Meeting Dummies			
FOMC		Governing Council	
Inflation Measures			
CPI Excl. FE Index GDP Deflator	YOY% YOY%	MUCPI GDP Deflator	YOY% YOY%
Output Measures			
GDP Growth Industrial Production Unemployment Rate	YOY% MOM%	GDP Growth Industrial Production Unemployment Rate	YOY% MOM%
Exchange Rate			
Eurodollar Rate	Weekly Average	Eurodollar Rate	Weekly Average
Decision Dummies			
U.S. Decision		EMU Decision	

estimates of GDP and other macroeconomic variables. Including those "sophisticated" variables might be a way to overcome this problem, but it goes beyond the scope of this paper.

The aim of collecting real-time data is to consider all the available information that the ECB and the Fed have at the beginning of week t. I am interested in knowing all estimates, provisional, or final data released up until the end (Friday) of week $t-1$. In order to construct the euro-zone GDP and CPI series, I make use of actual released data as well as flash estimates. Euro-zone CPI[14] data for month t are released in the second half of month $t+1$. The same release schedule also applies to the United States. CPI flash estimates represent a considerable enhancement in the available information because they are released within five to ten days from the end of month t. Thus

[14]To compute the HICP flash estimates, Eurostat uses early price information for the reference month from member states for which data are available as well as early information about energy prices. The estimation procedure for the HICP flash estimate combines historical information with partial information on price developments in the most recent months to give a total index for the euro zone. No detailed breakdown is available.

I include flash estimates as soon as they become available and substitute those with final data when they are released. Whereas in the United States GDP data for the quarter ending in month t become available as early as the end of month $t+1$, in the euro area they used to become available at the beginning of month $t+3$. Flash estimates improve the available information because flash GDP estimates are now released as early as the middle of month $t+2$. Flash estimates for the CPI started being released in November 2001 for the October 2001 CPI. Flash estimates for GDP only began in May 2003, for 2003:Q1 GDP.[15] Unemployment data relative to month t are released in the first week of month $t+1$ in the United States and in the first week of month $t+2$ in the EMU.

I also construct two *decision* dummy variables that will be used to assess interdependence in timing decisions. The U.S. dummy variable takes the value one from the last EMU interest rate change until the first FOMC meeting. The EMU dummy variable takes the value one from the last U.S. interest rate change until the second subsequent Governing Council meeting. The asymmetry comes from the fact that Governing Council meetings are more frequent than FOMC meetings (especially in the first part of the sample, when the Governing Council was meeting every two weeks; the FOMC meets only eight times a year), and I want to allow sufficient time for both central banks to react to policy changes.

4. Estimation Strategy and Empirical Results

4.1 Bayesian Implementation[16]

I conduct the estimation in a Bayesian framework. A Bayesian model is characterized by the probability distribution of the data, $p(Y^T|\theta)$, and by the prior distribution $p(\theta)$. I look at the probability of θ given the realized Y^T:

$$p(\theta|Y^T) = \frac{p(Y^T|\theta)p(\theta)}{\int p(Y^T|\theta)p(\theta)d\theta} \tag{24}$$

[15] Thus I do not use flash estimates but only provisional and final estimates before those dates.

[16] I thank Frank Schorfheide for providing Gauss code for the Bayesian estimation, which can be found at www.ssc.upenn.edu/~schorf/programs/gauss-bayesdsge.zip.

so that the parameters θ are treated as random. Equation (24) simply shows how to recover the posterior distribution of θ by applying Bayes's Theorem. The prior belief, which is chosen by the researcher based on economic considerations, can be thought of as an augmentation of the data set which is particularly useful when there are many parameters to be estimated from a short data sample. Bayesian analysis is therefore particularly well suited to my framework because it allows me to augment the data set by including pre-sample information, about the United States and Germany, into the prior.[17] ACD- and ACH-type models generally require fairly large data sets.[18] Because of the youth of the ECB and the type of events I analyze, the data set is small. Bayesian estimation helps in this respect, although it does not completely eliminate the problem.[19]

I use the posterior odds test to select between models. Let M_0 be the baseline model with prior probability $\pi_{0,0}$. The posterior odds of M_0 versus M_1 are

$$\frac{\pi_{0,T}}{\pi_{1,T}} = \left(\frac{\pi_{0,0}}{\pi_{1,0}}\right)\left(\frac{p(Y^T|M_0)}{p(Y^T|M_1)}\right), \tag{25}$$

where $\left(\frac{p(Y^T|M_0)}{p(Y^T|M_1)}\right)$ is the Bayes factor containing the sample evidence and $p(Y^T|M_i)$ is the data density, which I approximate with the modified harmonic mean estimation.[20]

[17]I use data on the German Bundesbank because it is the central bank in Europe that most closely resembles the ECB. Faust, Rogers, and Wright (2001) study the monetary policy of the ECB and compare it with a simple empirical representation of the monetary policy of the Bundesbank before 1999.

[18]Omitted simulation results show that the BACH model, as well as all ACD- and ACH-type models, needs a long data sample to identify the expected duration parameters (α and β) and the constants.

[19]This data problem could have been avoided by analyzing the Fed and the Bank of Japan, or the Fed and the Bank of Canada. However, I believe that investigating the Fed and the ECB is more interesting.

[20]Interpretation of the posterior odds is as follows: $log(\pi_{0,T}/\pi_{1,T}) > 2$ is decisive evidence against H_1; $1 < log(\pi_{0,T}/\pi_{1,T}) < 2$ is strong evidence against H_1; $1/2 < log(\pi_{0,T}/\pi_{1,T}) < 1$ is substantial evidence against H_1; and $0 < log(\pi_{0,T}/\pi_{1,T}) < 1/2$ is not worth more than a bare mentioning.

Table 4. Information on the BACH Parameter Priors

Parameter	BACH Model—Prior Distribution				
	Range	Density	Mean	St. Dev.	90% Interval
α^{US}	[0, 1)	Beta	0.14	0.08	[0.02, 0.25]
β^{US}	[0, 1)	Beta	0.56	0.14	[0.33, 0.79]
w^{US}	$R+$	Gamma	7.00	2.00	[3.71, 10.11]
$FOMC$	$R+$	Gamma	8.00	1.50	[5.48, 10.38]
CPI^{US}	$R+$	Gamma	1.50	0.50	[0.69, 2.27]
U^{US}	$R+$	Gamma	1.20	0.70	[0.16, 2.20]
d^{US}	$R+$	Gamma	2.00	1.00	[0.49, 3.48]
α^{EMU}	[0, 1)	Beta	0.14	0.08	[0.02, 0.25]
β^{EMU}	[0, 1)	Beta	0.56	0.14	[0.34, 0.80]
w^{EMU}	$R+$	Gamma	10.00	2.00	[6.71, 13.21]
GC	$R+$	Gamma	10.00	1.50	[7.53, 12.43]
CPI^{EMU}	$R+$	Gamma	1.50	0.40	[0.84, 2.11]
U^{EMU}	$R+$	Gamma	1.20	0.40	[0.55, 1.82]
d^{EMU}	$R+$	Gamma	2.00	1.00	[0.47, 3.46]
η	$R+$	Gamma	5.00	2.98	[0.63, 9.25]

4.1.1 Choice of Priors

I choose the priors for the parameters according to a number of considerations. I assume parameters are a priori independent of each other. Parameter restrictions are implemented by appropriately truncating the distribution or by redefining the parameters to be estimated.

Table 4 describes the distributional form, means, and 90 percent confidence intervals of the BACH model priors. According to the Taylor-rule literature, policy rates depend on their lagged values, on some measures of inflation and output deviations, and on exchange rate depreciation. Thus I assume that the probability of a rate change depends on the absolute deviation of inflation and output from a norm, and on the absolute exchange rate depreciation. I take absolute deviations because I want the probability of an interest rate change to increase with large deviations, regardless of their signs.

Let R_t, π_t, y_t, and er_t be the interest rate, inflation rate, a measure of output growth, and nominal exchange rate depreciation, respectively. Also, let π^* and y^* be the optimal level of inflation and output growth, and $0 < \rho^R < 1$ be the smoothing term. Then we can write the Taylor rule as[21]

$$\begin{aligned}\Delta R_t = (\rho^R - 1)R_{t-1} + (1 - \rho^R)[b_1(\pi_t - \pi^*) \\ + b_2(y_t - y^*) + b_3 er_t] + \varepsilon_t.\end{aligned} \tag{26}$$

I should also include the lagged interest rate among the covariates. However, the duration in the model generates the dynamics that a lagged interest rate would normally generate. I will, however, verify that this is in fact the case. FOMC and Governing Council meeting dummies have also been included as covariates, following Hamilton and Jordà (2002).

I choose priors for α, β, and the constant w in equation (8), so that the marginal hazard rate, when all the covariates are at their average value,[22] matches the probability of an interest rate change over the ten-year pre-sample period January 1, 1989 to December 31, 1998. I approximate this probability by dividing the number of changes by the number of periods. Since I do not have any pre-sample data for the ECB, I use information about the German Lombard rate. I choose α, β, and w such that

$$h^{US} = \frac{1}{\alpha^{US}\ \overline{u}^{US} + \beta^{US}\ \overline{\psi}^{US} + w^{US}} = \frac{39}{552} = 0.07 \tag{27}$$

$$h^{EMU} = \frac{1}{\alpha^{G}\ \overline{u}^{G} + \beta^{G}\ \overline{\psi}^{G} + w^{G}} = \frac{19}{552} = 0.035, \tag{28}$$

where $\overline{u}$ and $\overline{\psi}$ represent the average values for duration and expected duration over the pre-sample period 1989–98. With this approach, of course, I cannot identify the three parameters involved in each equation. Thus I decide to fix α and β to values close to those that have been estimated in the literature (see Hamilton and Jordà 2002 for the United States) and I vary w to match the probability.

[21] See Lubik and Schorfheide (2007).

[22] I actually assume that π^* and y^* are the average inflation and output growth rates over the sample period.

I choose priors for the covariates z using the following relationships:

$$h^{US} = \frac{1}{\{\alpha^{US}\overline{u}^{US} + \beta^{US}\overline{\psi}^{US} + w^{US} - \delta^{US}|z^{US}|\}} \tag{29}$$

$$h^{EMU} = \frac{1}{\{\alpha^{G}\overline{u}^{G} + \beta^{G}\overline{\psi}^{G} + w^{G} - \delta^{EMU}|z^{EMU}|\}}. \tag{30}$$

In particular, assuming all the other variables are at their average value, the prior for U.S. inflation implies a 0.25 increase in the probability (from 0.07 to 0.32) when the inflation rate increases or decreases by 100 basis points and it is an FOMC day. Similarly, the prior on output/unemployment implies that, on FOMC weeks, a 100-basis-point change in targeted output growth/unemployment rate increases the probability by 0.22. The asymmetry in treating inflation and output is justified by the fact that inflation always has a greater coefficient in the Taylor-rule literature. Very similar priors are given to EMU inflation and output. The meeting dummy coefficients (d^i, $i = US$, EMU) are not treated equally in the two countries, due to the greater number of EMU meetings. Notice that, since I expect all the coefficients to be positive,[23] I parameterized them as gamma distributions.

I compute the mean of the odds ratio prior (η) by using the pre-sample proportions for scenarios $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$, over the period January 1, 1989 to December 31, 1998. Again, I use data about the Bundesbank to construct priors about the ECB. The pre-sample odds ratio is about 5.

I also choose priors for the conditional bivariate ordered probit model based on pre-sample information. The means of the cut points are those that I would expect if I were to estimate an ordered probit with no covariates, based on the data 1989–98 for the United States and Germany. The first cut point $c_1 \in R$, hence the normal distribution. The other coefficients are appropriately redefined so as to guarantee that the cut points are ordered. Priors for U.S. inflation and output coefficients are centered at values that have been commonly estimated for Taylor-type rules (see equation (26)). Table 5 describes the distributional form, means, and 90 percent

[23]That is, I expect $-\delta < 0$.

Table 5. Information on the CBO Probit Parameter Priors

CBO Probit Model—Prior Distribution					
Parameter	**Range**	**Density**	**Mean**	**St.Dev.**	**90% Interval**
c_1^{US}	R	Normal	−2.15	2.00	[−7.00, 2.85]
$c_2^{US} - c_1^{US}$	R^+	Gamma	0.70	1.98	[0.00, 1.99]
$c_3^{US} - c_2^{US}$	R^+	Gamma	3.11	1.99	[0.30, 5.90]
$c_4^{US} - c_3^{US}$	R^+	Gamma	0.67	2.00	[0.00, 1.83]
CPI^{US}	R^+	Gamma	1.54	0.50	[0.74, 2.32]
U^{US}	R^+	Gamma	0.25	0.40	[0.00, 0.71]
Δer_t^{US}	R^+	Gamma	0.25	0.20	[0.00, 0.51]
c_1^{EMU}	R	Normal	−2.15	2.00	[−7.17, 2.76]
$c_2^{EMU} - c_1^{EMU}$	R^+	Gamma	0.92	2.01	[0.00, 2.78]
$c_3^{EMU} - c_2^{EMU}$	R^+	Gamma	4.28	2.00	[1.13, 7.23]
$c_4^{EMU} - c_3^{EMU}$	R^+	Gamma	0.81	1.97	[0.00, 2.46]
CPI^{EMU}	R^+	Gamma	1.54	0.50	[0.74, 2.33]
U^{EMU}	R^+	Gamma	0.25	0.20	[0.00, 0.52]
Δer_t^{UEMU}	R^+	Gamma	0.25	0.20	[0.00, 0.52]
ρ	$[-1, 1]$	Normal	0.00	0.40	[−0.66, 0.65]

confidence intervals of the priors of the CBO probit model. To better understand the meaning of these numbers, I consider two scenarios for the United States: a 100-basis-point increase in target inflation, and a 100-basis-point increase in target inflation with an additional 1-percentage-point decrease in the targeted unemployment rate. Conditioning on the Fed having decided to change its target rate, i.e., $x_t^f = 1$, the priors for the United States imply that a 100-basis-point increase in the target level of inflation will raise the probability of a 25-basis-point increase in the FFTR by approximately 0.20 and the probability of a 50-basis-point increase in the FFTR by approximately 0.40. A 100-basis-point increase in the target level of inflation and a 1-percentage-point decrease in targeted unemployment will raise the probability of a 25- and 50-basis-point increase in the FFTR by approximately 0.15 and 0.45, respectively, compared with the baseline scenario. A qualitatively similar analysis holds for the euro area, though in this case the priors imply a

Table 6. BACH Model Posterior Means and Intervals for (i) the Basic Specification and (ii) the Specification with the Odds Ratio Fixed at Its Independence Value (Odds Ratio = 1)

	BACH Parameter Estimation Results			
	Basic Specification		No Synchronization	
Parameter	Mean	90% Interval	Mean	90% Interval
α^{US}	0.05	[0.00, 0.09]	0.05	[0.00, 0.10]
β^{US}	0.44	[0.23, 0.66]	0.44	[0.22, 0.66]
w^{US}	15.05	[12.54, 17.55]	15.10	[12.59, 17.72]
$FOMC$	11.00	[8.80, 13.18]	10.82	[8.66, 13.07]
CPI^{US}	0.26	[0.14, 0.37]	0.28	[0.15, 0.40]
U^{US}	0.04	[0.01, 0.07]	0.04	[0.00, 0.08]
d^{US}	—	—	—	—
α^{EMU}	0.11	[0.00, 0.20]	0.12	[0.02, 0.10]
β^{EMU}	0.56	[0.35, 0.77]	0.57	[0.35, 0.78]
w^{EMU}	19.85	[16.17, 23.50]	19.94	[16.21, 23.66]
GC	9.24	[7.16, 11.27]	9.22	[7.15, 11.25]
CPI^{EMU}	0.28	[0.17, 0.38]	0.29	[0.18, 0.40]
U^{EMU}	0.35	[0.18, 0.53]	0.36	[0.18, 0.53]
d^{EMU}	—	—	—	—
η	4.27	[1.21, 7.14]	1	—

stronger reaction to inflation than unemployment, consistent with the sole mandate of price stability for the ECB.

4.2 BACH Estimates

I estimate a number of different specifications in order to assess which variables are in fact relevant for U.S. and EMU timing decisions. Table 3 shows the covariates I have considered. The basic specification I have selected includes meeting dummies, and inflation and unemployment absolute deviations.[24] Table 6 reports 90 percent posterior probabilities intervals and posterior means as point estimates. The constant parameters for both the United States and the

[24]Thus unemployment and the CPI dominate GDP and GDP deflator measures. Intuitively, unemployment and the CPI are monthly statistics and therefore more promptly incorporate new information.

EMU turn out to have a higher value compared with the prior means, possibly meaning a lower average probability over the sample. The increased value of the constant terms goes together with a smaller value for the coefficients of the other covariates, with the exception of the meeting dummies. The FOMC meeting dummy has a bigger coefficient than the Governing Council meeting dummy (GC): given that FOMC meetings are less frequent than Governing Council meetings, I expect a bigger increase in the probability of a rate change when the FOMC meets.[25] Inflation deviation seems to play a bigger role than unemployment in determining the timing of a rate change in the United States, while inflation and unemployment deviations have a similar weight in the ECB timing decisions. The estimated U.S. inflation parameter implies a 0.15 increase in the probability of an FFTR change when the inflation rate increases or decreases by 100 basis points and it is an FOMC day. Similarly, the estimated unemployment rate coefficient implies that, on FOMC weeks, a 100-basis-point change in the unemployment rate increases the probability of a rate change by 0.13. Consistent with the fact that the ECB is more resilient in changing its target rate, the estimate of the euro-area inflation parameter only implies a 0.04 increase in the probability of a target rate change when the inflation rate increases or decreases by 100 basis points and it is a meeting week for the ECB Governing Council. The odds ratio parameter η has a posterior mean of 4.27, suggesting interdependence between the two central banks.

An interesting by-product of the BACH model is that it generates persistence in the interest rate without including past interest rates. The basic specification with meeting dummies, inflation, and output has been tested against a specification that also includes lagged interest rates, and the former has been selected. I have also tested for a specification that includes exchange rate data in the covariates. Once again, the specification with meeting dummies, inflation, and output has been favored. Finally, unemployment is favored over industrial production and GDP growth as a measure of output, and the CPI is preferred to the GDP deflator as a measure of inflation (results omitted).

[25] The covariates enter with a negative sign in the denominator of the hazard rate; hence, a bigger coefficient on a covariate means a decrease in the denominator and an increase in the probability of a change.

Hu and Phillips (2004) apply a discrete choice approach to model the FFTR during the period 1994–2001, drawing information from the announced target rate as well as from explanatory variables that may be included in a Taylor-type rule. In their model, Hu and Phillips use a triple-choice specification to explain either a decrease, an increase, or no change in the FFTR. They find that four main economic variables contribute to explaining FFTR movements: M2, unemployment claims, consumer confidence, and new orders. While these variables are different from the one that I found significant in my model, their results also suggest that economic conditions trigger the timing of interest rate changes contrary to Hamilton and Jordà (2002), who found that only the absolute value of the spread between the effective federal funds rate and the six-month Treasury-bill rate is the main determinant of the timing, together with the FOMC dummy.

4.3 Conditional Bivariate Ordered Probit Estimates

The basic specification of the CBO probit model that I estimate includes inflation, unemployment, and exchange rates. Table 7 reports 90 percent posterior probabilities intervals and posterior means as point estimates. Inflation and unemployment results exhibit interesting features. While inflation continues to play the foremost role in explaining the size of changes in the U.S. and EMU policy rates, unemployment and exchange rate dynamics share a secondary role. Inflation and unemployment posterior means are, respectively, 0.53 and 0.09 for the United States in the basic model, whereas they are 0.47 and 0.11 for the EMU. The correlation coefficient has a posterior mean of 0.36, with a 90 percent interval equal to [0.06, 0.67].

Conditioning on the Fed having decided to change its target rate, i.e., $x_t^f = 1$, the parameter estimates for the United States imply that a 100-basis-point increase in the target level of inflation will increase the probability of a 25-basis-point rise in the FFTR by approximately 0.18. A 100-basis-point increase in the target level of inflation and a 1-percentage-point decrease in targeted unemployment will increase the probability of a 25- and 50-basis-point rise in the FFTR by approximately 0.21 and 0.05, respectively, compared with the baseline scenario. Results for the EMU suggest that,

Table 7. CBO Probit Model Posterior Means and Intervals for (i) the Basic Specification and (ii) the Specification with the Correlation Coefficient Fixed to Zero

CBO Probit Parameter Estimation Results				
	Basic Model		$\rho = 0$	
Parameter	**Mean**	**90% Interval**	**Mean**	**90% Interval**
c_1^{US}	1.35	[0.35, 2.78]	0.69	[−0.30, 1.69]
$c_2^{US} - c_1^{US}$	0.37	[0.16, 0.56]	0.29	[0.12, 0.46]
$c_3^{US} - c_2^{US}$	0.07	[0.01, 0.12]	0.82	[0.07, 1.56]
$c_4^{US} - c_3^{US}$	1.80	[1.17, 2.50]	1.75	[1.06, 2.42]
CPI^{US}	0.53	[0.25, 0.78]	0.52	[0.25, 0.78]
U^{US}	0.11	[0.00, 0.28]	0.04	[0.00, 0.12]
Δer_t^{US}	0.09	[0.00, 0.18]	0.09	[0.00, 0.18]
c_1^{EMU}	0.65	[0.08, 1.22]	1.96	[0.30, 3.80]
$c_2^{EMU} - c_1^{EMU}$	0.25	[0.07, 0.42]	0.27	[0.08, 0.46]
$c_3^{EMU} - c_2^{EMU}$	2.04	[0.84, 3.20]	1.90	[0.67, 3.12]
$c_4^{EMU} - c_3^{EMU}$	0.97	[0.50, 1.42]	1.14	[0.60, 1.65]
CPI^{EMU}	0.47	[0.28, 0.66]	0.51	[0.29, 0.72]
U^{EMU}	0.11	[0.03, 0.20]	0.25	[0.07, 0.43]
Δer_t^{UEMU}	0.13	[0.00, 0.26]	0.17	[0.01, 0.32]
ρ	0.36	[0.06, 0.67]	0	—

conditioning on the ECB having decided to change its target rate (i.e., $x_t^e = 1$), parameter estimates imply that a 100-basis-point increase in the target level of inflation will increase the probability of a 25-basis-point rise in the MRO rate by approximately 0.22. A 100-basis-point increase in the target level of inflation and a 1-percentage-point decrease in targeted unemployment will increase the probability of a 25- and 50-basis-point rise in the MRO rate by approximately 0.28 and 0.05, respectively, compared with the base scenario.

Magnitude results are therefore showing that, for both central banks, inflation has been crucial and unemployment has had a minor role during the period analyzed. This is expected in view of the fact that the ECB's primary objective is to maintain price stability; a policy of targeting output growth would probably be more problematic, given the intrinsic differences in the economies of the EMU countries. For the Fed, this result may hinge on the fact that most of the period analyzed in this paper is characterized by relatively low unemployment. Because of the non-linearities in my model, it is difficult to compare my results with the existing Taylor-rule literature, such as Evans (1998), Orphanides (2001), Clarida, Galí, and Gertler (2002), and Molodtsova, Nikolsko-Rzhevskyy, and Papell (2008), among others. However, my model seems to validate the idea that the inflation parameter tends to be bigger than the output/unemployment parameter. Moreover, in line with the real-time literature which suggests that real-time policy recommendations differ widely from those obtained with revised published data, my results for the United States differ from Hamilton and Jordà (2002), who employ a revised data set. In their paper, most of the candidate explanatory variables turn out to be insignificant, and they find that the spread between the effective federal funds rate and the six-month Treasury-bill rate is the main determinant of the size of the FFTR change as well as of the timing of the change. The importance of using real-time data is also confirmed by the fact that variables that are updated more frequently—such as CPI, IP, and the unemployment rate—are preferred, according to my estimation results, to variables that are updated less frequently, like GDP and GDP deflator. GDP information tends to be old and stale due to long reporting lags. Monetary policy committees might be more concerned about the current state of the economy and, as such, care more about the information contained in macroeconomic announcements that are more frequently and timely released.

4.4 Interdependence

The interdependence test of U.S. and EMU timing decisions is twofold: on the one hand, I am interested in assessing "contemporaneous" interdependence, after controlling for each country's macroeconomic conditions, which I refer to as *synchronization*; on the other hand, I investigate the possibility of *follower behaviors*, after controlling for each country's macroeconomic conditions. Assessing *synchronization* involves testing whether the odds ratio is different from 1 (1 meaning independence). The odds ratio indicates how

Table 8. BACH Model Posterior Means and Intervals for the Specifications with (i) the U.S. Dummy Variable in the EMU Decision Variables (EMU Follower), (ii) the EMU Dummy Variable in the U.S. Decision Variables (U.S. Follower), and (iii) Both Dummy Variables

	BACH Parameter Estimation Results					
	EMU Follower		**U.S. Follower**		**Both Dummies**	
Parameter	**Mean**	**90% Interval**	**Mean**	**90% Interval**	**Mean**	**90% Interval**
α^{US}	0.05	[0.00, 0.09]	0.05	[0.00, 0.10]	0.05	[0.00, 0.09]
β^{US}	0.44	[0.21, 0.66]	0.44	[0.22, 0.66]	0.44	[0.21, 0.66]
w^{US}	15.12	[12.64, 17.63]	15.31	[12.75, 17.86]	15.35	[12.64, 17.63]
$FOMC$	11.03	[8.83, 13.20]	10.68	[8.45, 12.87]	10.68	[8.83, 13.20]
CPI^{US}	0.26	[0.14, 0.38]	0.25	[0.13, 0.37]	0.25	[0.14, 0.38]
U^{US}	0.04	[0.01, 0.08]	0.04	[0.00, 0.08]	0.04	[0.01, 0.08]
d^{US}	—	—	0.78	[0.20, 1.35]	0.78	[0.20, 1.34]
α^{EMU}	0.12	[0.02, 0.21]	0.11	[0.06, 0.20]	0.12	[0.02, 0.21]
β^{EMU}	0.57	[0.35, 0.78]	0.56	[0.36, 0.77]	0.57	[0.36, 0.78]
w^{EMU}	19.95	[16.24, 23.74]	19.91	[16.18, 23.55]	19.99	[16.27, 23.74]
GC	9.21	[7.18, 11.24]	9.26	[7.21, 11.27]	9.23	[7.14, 11.24]
CPI^{EMU}	0.29	[0.18, 0.40]	0.29	[0.17, 0.39]	0.29	[0.18, 0.40]
U^{EMU}	0.35	[0.18, 0.52]	0.36	[0.18, 0.53]	0.35	[0.18, 0.52]
d^{EMU}	1.82	[0.50, 3.09]	—	—	1.81	[0.49, 3.08]
η	4.38	[1.23, 7.34]	3.84	[1.12, 6.26]	4.20	[1.15, 7.02]

much the odds of one country changing its target rate increase when the other country changes its target. Columns 4 and 5 in table 6 display the estimation results for the independence setup. Setting the odds ratio to 1 does not significantly affect the other coefficients: both means and 90 percent probability intervals are very similar to the basic specification. However, as table 9 shows, the posterior odds of the model M_1 with the *odds ratio* = 1 versus the alternative basic model M_0 seems to support model M_0. Thus the BACH model tends to favor a setup with synchronization between the two central banks, after controlling for each country's macroeconomic conditions.

Follower behaviors are studied by including the two *decision* dummies to account for the effect of the other country's decisions (see section 3 for a more detailed explanation of the dummy variables). Results are shown in table 8. Table 9 suggests that the

Table 9. BACH Model Posterior Odds for the Synchronization and the Leader/Follower Models

	Log Marginal Data Densities	Posterior Odds
Basic Model	−286.61	
No Synchronization ($\eta = 1$)	−287.36	2.12 (M_0: Basic Model)
U.S. Leader/EMU Follower[a]	−286.80	1.21 (M_0: Basic Model)
EMU Leader/U.S. Follower[b]	−290.19	35.88 (M_0: Basic Model)
Both U.S. and EMU Dummies	−290.12	33.45 (M_0: Basic Model)

[a]Only dummy^{EMU}
[b]Only dummy^{US}

Table 10. CBO Probit Model Posterior Odds for the Correlation in Magnitude Hypothesis

	Log Marginal Data Densities	Posterior Odds
Basic Model	−130.05	
$\rho = 0$	−125.49	95.58 (M_0: $\rho = 0$)

posterior odds ratio supports the hypothesis of no follower behaviors when both dummies are included. I obtain similar results by testing for the Fed's follower behavior: there is strong evidence against the Fed following what the ECB does. On the other hand, though there is some evidence against the reverse scenario of the ECB following the Fed's timing decisions, this evidence is slim.

I analyze interdependence in the CBO probit framework by testing whether the correlation coefficient between the latent variables in equations (20) and (21) is different from zero. Table 7 presents the estimation results for this scenario. The posterior odds ratio in table 10 shows evidence in favor of the model specification in which the correlation is set to zero. The correlation coefficient measures the correlation between the shocks in the unobservable variable equations—the omitted factors.[26] The results seem to suggest that although there is *synchronization* in the timing of interest rate changes, each central bank then sets the target level based exclusively on its own macroeconomic conditions.

The BACH model with the odds ratio set to 1 and the CBO probit model with $\rho = 0$ can be thought of as the Hamilton and Jordà (2002) univariate model estimated for both the Fed and the ECB. Please note that results for the United States cannot be compared because the sample is different in term of length and included variables. Moreover, Hamilton and Jordà (2002) do not use a real-time data set.

5. Posterior Predictive Checks

To check the goodness of fit of the model described in the paper, I run some posterior predictive checks on the BACH and CBO probit models that were selected above.[27] The general idea is that if the model fits well enough, then the replicated data will look somehow similar to the observed data. I run predictive checks for the BACH model separately from the CBO probit model.

To generate data based on the BACH model, I use the following algorithm:

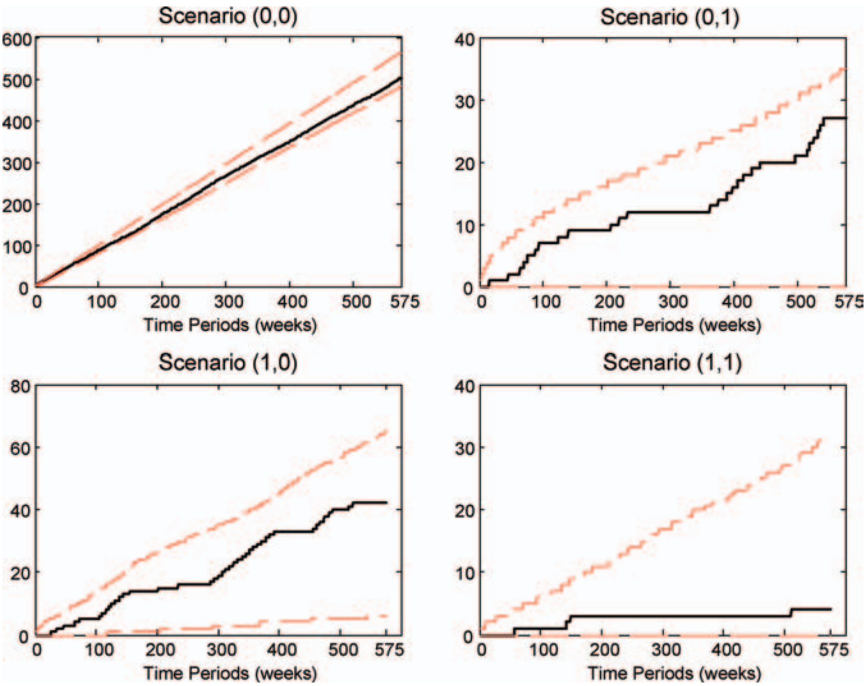
- Sample θ from the posterior distribution $p(\theta|x^{e,\ obs}, x^{f,\ obs}, z)$, where $x^{e,\ obs}, x^{f,\ obs}$ are defined as in equation (1) and z are the exogenous data (for $nrep = 100{,}000$).
- Using each one of the θ^i realizations above, sample $x^{i,rep}$ from the model $p(x|\theta, z)$, where each sample $x^{i,rep}$ has length $T = 575$ (length of my data set).
- Consider $(x^{e,rep}, x^{f,rep}) = \{(0,0), (0,1), (1,0), (1,1)\}$ and compare it with $(x^{e,\ obs}, x^{f,\ obs})$.

Figure 4 shows the results of the posterior predictive checks on the BACH model. The four panels each represent one of the four

[26]By relating these shocks to the VAR literature, it turns out that, given the assumption that interest rates only depend on past values of output and inflation, the disturbances in equations (20) and (21) are purely monetary shocks.

[27]See Bauwens, Lubrano, and Richard (1999).

Figure 4. Results of BACH Predictive Checks



Notes: The panels show the cumulative count of the (0,0), (0,1), (1,0), and (1,1) scenarios over the 575 weeks under consideration. The thick black lines represent the cumulative count of the observed data. The broken lines show the range of paths for the replicated data.

possible outcomes $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$. The thick black line represents the cumulative count of the observed data. The upper-left panel shows the cumulative count of the $(0, 0)$ scenarios over the 575 weeks under consideration. Similarly, the thick black lines in the other panels show the cumulative counts of scenarios $(0, 1)$, $(1, 0)$, and $(1, 1)$. The value at the end of week 575 corresponds to the total amount of $(0, 0)$, $(0, 1)$, $(1, 0)$, and $(1, 1)$ scenarios that we observed from 1999 to 2009. The broken lines show the range of paths for the replicated data. As clearly shown by the figure, the observed path lies within the paths of the replicated data, endorsing the plausibility of the BACH model. Of note, because scenario $(1, 1)$ is so little observed in reality, it is also perhaps the more difficult to simulate.

The algorithm for the predictive checking of the CBO probit model is similar to the one described above for the BACH model. After sampling from the parameters' posterior distribution, I sample $y^{i,rep}$ from the likelihood distribution $p(y^{obs}|\theta, z)$, where $y^{i,rep} = \{-50, -25, 25, 50\}$. For each repetition, I count the number of times $y^i = -50$, –25, 25, and 50. Results are shown in figure 5, where both the histogram and some statistics are shown. Although accuracy is not perfect, the mean and the median are not far off from the true realizations, endorsing the plausibility of the CBO probit model.

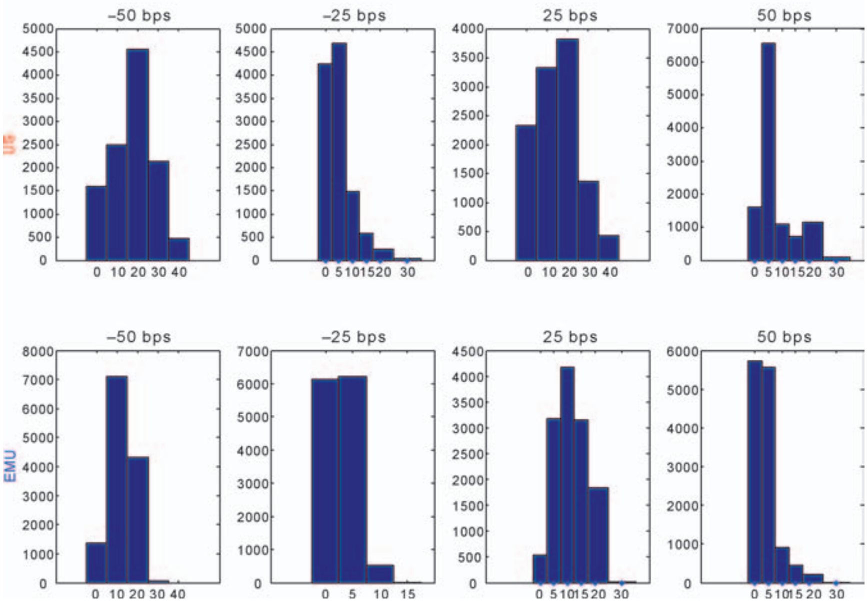
6. Conclusions

In this paper I have derived and estimated with Bayesian techniques a bivariate model to account for interdependence between Fed and ECB decisions. I have operationalized interest rate timing decisions with a bivariate autoregressive conditional hazard (BACH) model and magnitude decisions with a conditional bivariate ordered (CBO) probit model. The timing model yields evidence supporting the hypothesis that (i) institutional factors (scheduled FOMC and Governing Council meetings) and inflation rates are relevant variables for both central banks, (ii) output plays a minor role in the U.S. timing decisions, and (iii) there exists some evidence of synchronization but no follower behavior. The magnitude model illustrates that (i) inflation rates are the most important variables in determining interest rate levels, (ii) output and exchange rates play a secondary role in magnitude decisions, and (iii) the posterior odds ratio favors a model with zero correlation in magnitude changes. I also find that, based on posterior predictive checks, my model has a good fit and is able to generate data that are similar to the observed data.

My findings are necessarily based on a relatively small sample; however, there seems to be evidence suggesting that timing and magnitude changes are in fact quite interesting issues. The paper provides evidence in favor of interaction in U.S. and EMU interest rate timing and magnitude decisions, after controlling for traditional variables that have commonly been used in the literature. The paper offers a new methodology to analyze interdependence; however, it does not provide a complete answer to the underlying problem about what is in fact the source of the interdependence and

Figure 5. Results of CBO Probit Model Predictive Checks

		–50bps	–25bps	25bps	50bps
Fed	Observed	16	7	22	1
	Mean Realization	18	5	16	4
	Median Realization	19	4	15	4
ECB	Observed	10	5	14	2
	Mean Realization	13	3	11	4
	Median Realization	12	3	12	3



Notes: The top part of the figure shows the count of the times the Fed and ECB changed their respective target rate by −50, −25, 25, and 50 basis points. It also shows the mean and median of those counts for the simulated data. The bottom part of the figure shows the histogram of the simulated data for each one of the four outcomes (−50, −25, 25, and 50 basis points).

whether interdependence is optimal. Identifying where the interdependence comes from and analyzing whether results are robust to the inclusion of a larger set of explanatory variables remain important topics for further research.

Appendix

CBO Probit Model

The log-likelihood relative to the magnitude decision is

$$\mathcal{L}_2(\theta_2) = \sum_{t=1}^{T} \mathit{1}\left[x_t^e = 1, x_t^f = 0\right] \log P_{10} + \mathit{1}\left[x_t^e = 0, x_t^f = 1\right] \log P_{01} + \mathit{1}\left[x_t^e = 1, x_t^f = 1\right] \log P_{11} \tag{31}$$

with

$$\begin{aligned} P_{10} &= \Pr\left(y_t^e = s_m,\ y_t^f = 0 \mid w_{t-1}, x_t^e = 1, x_t^f = 0\right) \\ &= \Pr\left(c_{m-1}^e < \widetilde{y}_t^e \le c_m^e,\ c_2^f < \widetilde{y}_t^f \le c_3^f \mid w_{t-1}, x_t^e = 1, x_t^f = 0\right) \\ m &= 1, 2, 4, 5, \end{aligned} \tag{32}$$

where the probability is computed using the density

$$f\left(\widetilde{y}_t^e, \widetilde{y}_t^f | w_{t-1},\ x_t^e = 1,\ x_t^f = 0\right) = \frac{f\left(\widetilde{y}_t^e, \widetilde{y}_t^f | w_{t-1}\right)}{\Pr\left\{\begin{array}{c}\left[\left(\widetilde{y}_t^e \le c_2^e\right) \vee \left(\widetilde{y}_t^e > c_3^e\right)\right] \\ \wedge\left(c_2^f < \widetilde{y}_t^f \le c_3^f\right)\end{array}\right\}}; \tag{33}$$

with

$$\begin{aligned} P_{01} &= \Pr\left(y_t^e = 0,\ y_t^f = s_n \mid w_{t-1}, x_t^e = 0, x_t^f = 1\right) \\ &= \Pr\left(c_2^e < \widetilde{y}_t^e \le c_3^e,\ c_{n-1}^f < \widetilde{y}_t^f \le c_n^f \mid w_{t-1}, x_t^e = 0, x_t^f = 1\right) \\ n &= 1, 2, 4, 5, \end{aligned} \tag{34}$$

where the probability is computed using the density

$$f\left(\widetilde{y}_t^e, \widetilde{y}_t^f | w_{t-1},\ x_t^e = 0,\ x_t^f = 1\right) = \frac{f\left(\widetilde{y}_t^e, \widetilde{y}_t^f | w_{t-1}\right)}{\Pr\left\{\begin{array}{c}\left(c_2^e < \widetilde{y}_t^e \le c_3^e\right) \\ \wedge\left[\left(\widetilde{y}_t^f \le c_2^f\right) \vee \left(\widetilde{y}_t^f > c_3^f\right)\right]\end{array}\right\}}; \tag{35}$$

and with

$$\begin{aligned} P_{11} &= \Pr\left(y_t^e = s_m,\ y_t^f = s_n \mid w_{t-1}, x_t^e = x_t^f = 1\right) \\ &= \Pr(c_{m-1}^e < \widetilde{y}_t^e \le c_m^e,\ c_{n-1}^f < \widetilde{y}_t^f \le c_n^f \mid w_{t-1}, x_t^e = x_t^f = 1) \\ m, n &= 1, 2, 4, 5, \end{aligned} \tag{36}$$

where the probability is computed using the density

$$f\left(\widetilde{y}_t^e, \widetilde{y}_t^f | w_{t-1},\ x_t^e = x_t^f = 1\right) = \frac{f\left(\widetilde{y}_t^e, \widetilde{y}_t^f | w_{t-1}\right)}{\Pr\left\{\begin{array}{c} \left[\left(\widetilde{y}_t^e \le c_2^e\right) \vee \left(\widetilde{y}_t^e > c_3^e\right)\right] \\ \wedge\left[\left(\widetilde{y}_t^f \le c_2^f\right) \vee \left(\widetilde{y}_t^f > c_3^f\right)\right] \end{array}\right\}}. \tag{37}$$

Bayesian Implementation

Following Schorfheide (2000), I compute the mode $\widetilde{\theta}$ of the posterior density $p(\theta|Y^T)$ through a numerical optimization routine and then evaluate the inverse Hessian $\widetilde{\Sigma}$. I use a random-walk Metropolis algorithm to generate n_{sim} draws $\widetilde{\theta}^s$ from the posterior $p(\theta|Y^T)$.[28] At each iteration s, I draw a candidate parameter vector ϑ from a jumping distribution $J_s(\vartheta|\theta^{(s-1)})$ and I accept the jump from $\theta^{(s-1)}$ so that $\theta^{(s)} = \vartheta$ with probability $min(r, 1)$, where r is defined as

$$r = \frac{p(Y^T|\vartheta)p(\vartheta)}{p(Y^T|\theta^{(s-1)})p(\theta^{(s-1)})}, \tag{38}$$

and reject otherwise. The Markov chain sequence $\{\theta^{(s)}\}_{s=1}^{n_{sim}}$ converges to the posterior distribution as $n_{sim} \to \infty$. I use a Gaussian jumping distribution $J_s \sim \mathcal{N}(\theta^{(s-1)}, c^2\widetilde{\Sigma})$ with $c = 0.3$.

References

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[28] I use $n_{sim} = 150{,}000$ for the BACH model and $n_{sim} = 100{,}000$ for the CBO probit model.

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