Endogenous Exposure to Systemic Liquidity Risk*

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Traditionally, aggregate liquidity shocks are modeled as exogenous events. This paper analyzes the adequate policy response to endogenous exposure to systemic liquidity risk. We analyze the feedback between lender-of-last-resort policy and incentives of private banks, determining the aggregate amount of liquidity available. We show that imposing minimum liquidity standards for banks ex ante is a crucial requirement for sensible lender-of-last-resort policy. In addition, we analyze the impact of equity requirements and narrow banking, in the sense that banks are required to hold sufficient liquid funds so as to pay out in all contingencies. We show that both policies are strictly inferior to imposing minimum liquidity standards ex ante combined with lender-of-last-resort policy.

JEL Codes: E5, G21, G28.

"The events earlier this month leading up to the acquisition of Bear Stearns by JP Morgan Chase highlight the importance of liquidity management in meeting obligations during stressful market conditions. . . . The fate of Bear Stearns was the result of a lack of confidence, not a lack of capital. . . . At all

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times until its agreement to be acquired by JP Morgan Chase during the weekend, the firm had a capital cushion well above what is required to meet supervisory standards calculated using the Basel II standard."

(SEC Chairman Christopher Cox, "Letter to Basel Committee in Support of New Guidance on Liquidity Management," March 20, 2008)

"Bear Stearns never ran short of capital. It just could not meet its obligations. At least that is the view from Washington, where regulators never stepped in to force the investment bank to reduce its high leverage even after it became clear Bear was struggling last summer. Instead, the regulators issued repeated reassurances that all was well. Does it sound a little like a doctor emerging from a funeral to proclaim that he did an excellent job of treating the late patient?"

(Floyd Norris, "The Regulatory Failure Behind the Bear Stearns Debacle," *New York Times*, April 4, 2008)

1. Introduction

Before the financial crisis, financial markets seemed to have been awash with excessive liquidity. But suddenly, in August 2007, liquidity dried up nearly completely as a response to doubts about the quality of subprime mortgage-backed securities. Despite massive central bank interventions, the liquidity freeze did not melt away, but rather spread slowly to other markets such as those for auction-rate bonds. On March 16, 2008, the investment bank Bear Stearns, which (according to the Securities and Exchange Commission chairman) was adequately capitalized even a week before, had to be rescued via a Fed-led takeover by JP Morgan Chase.

Following the turmoil on financial markets, there has been a strong debate about the adequate policy response. Some have warned that central bank actions may encourage dangerous moral hazard behavior of market participants in the future. Others instead criticized central banks for responding far too cautiously. The most prominent voice has been Willem Buiter, who—jointly with Ann Sibert—right from the beginning of the crisis in August 2007

strongly pushed the idea that in times of crises, central banks should act as market maker of last resort (see Buiter and Sibert 2007). As an adaptation of the Bagehot principle to modern times with globally integrated financial systems, central banks should actively purchase and sell illiquid private-sector securities and so play a key role in assessing and pricing credit risk. In his *Financial Times* blog "Maverecon," Willem Buiter stated the intellectual arguments behind such a policy very clearly on December 13, 2007:

> Liquidity is a public good. It can be managed privately (by hoarding inherently liquid assets), but it would be socially inefficient for private banks and other financial institutions to hold liquid assets on their balance sheets in amounts sufficient to tide them over when markets become disorderly. They are meant to intermediate short maturity liabilities into long maturity assets and (normally) liquid liabilities into illiquid assets. Since central banks can create unquestioned liquidity at the drop of a hat, in any amount and at zero cost, they should be the liquidity providers of last resort, both as lender of last resort and as market maker of last resort. There is no moral hazards as long as central banks provide the liquidity against properly priced collateral, which is in addition subject to the usual "liquidity haircuts" on this fair valuation. The private provision of the public good of emergency liquidity is wasteful. It's as simple as that.

Buiter's statement represents the prevailing mainstream view that there is no moral hazard risk as long as the Bagehot principle is followed as best practice in liquidity management.

According to Goodfriend and King (1988), a lender-of-last-resort policy should target liquidity provision to the market, but not to specific banks. Central banks should "lend freely at a high rate against good collateral." This way, public liquidity support is supposed to be targeted toward solvent yet illiquid institutions, since insolvent financial institutions should be unable to provide adequate collateral to secure lending. This paper wants to challenge the view that a policy following the Bagehot principle does not create moral hazard. The key argument is that this view neglects the endogeneity of aggregate liquidity risk. Starting with Allen and Gale (1998) and Holmström and Tirole (1998), there have been quite a few models

recently analyzing private and public provision of liquidity. But in most of these models, exposure to aggregate systemic risk is assumed to be exogenous.

In Holmström and Tirole (1998), for instance, liquidity shortages arise when financial institutions and industrial companies scramble for and cannot find the cash required to meet their most urgent needs or undertake their most valuable projects. They show that credit lines from financial intermediaries are sufficient for implementing the socially optimal (second-best) allocation, as long as there is no aggregate uncertainty. In the case of aggregate uncertainty, however, the private sector cannot satisfy its own liquidity needs, so the existence of liquidity shortages vindicates the injection of liquidity by the government. In their model, the government can provide (outside) liquidity by committing future tax income to back up the reimbursements.

In the model of Holmström and Tirole (1998), the lender of last resort indeed provides a free lunch: Public provision of liquidity in the presence of aggregate shocks is a pure public good, with no moral hazard involved. The reason is that the probability for being hit by an aggregate shock is not affected by the amount of investment in liquid assets carried out by the private sector. The same holds in Allen and Gale (1998), even though they analyze a quite different mechanism for public provision of liquidity: the adjustment of the price level in an economy with nominal contracts. We adopt Allen and Gale's mechanism, but we endogenize the exposure of financial intermediaries to aggregate (systemic) liquidity risk.

The basic idea of our model is fairly straightforward: Financial intermediaries choose the share invested in projects which might turn out to be illiquid. We model (real) illiquidity in the following way: Some fraction of those projects turns out to be realized late. The aggregate share of late projects is endogenous, since it depends on the incentives of financial intermediaries to invest in those illiquid projects. When intermediaries would invest only in liquid assets, they would never be hit by shocks affecting illiquid projects. The larger the share invested in those assets, however, the higher the exposure to aggregate liquidity risk. This endogeneity allows us to capture the feedback from liquidity provision to risk-taking incentives of financial intermediaries. We show that the share invested in illiquid projects rises endogenously with central bank liquidity provision: The

anticipation of unconditional central bank liquidity provision encourages excessive risk taking (moral hazard). It turns out that in the absence of liquidity requirements, there will be overinvestment in risky activities, creating excessive exposure to systemic risk.

In contrast to what the Bagehot principle suggests, unconditional provision of liquidity to the market (lending of central banks against good collateral) is exactly the wrong policy: It distorts incentives of banks to provide sufficient private liquidity, thus reducing investors' payoff. In our model, we concentrate on pure illiquidity risk: There will never be insolvency unless triggered by illiquidity (by a bank run). Illiquid projects promise a higher, yet possibly delayed, return. Relying on sufficient liquidity provided by the market (or by the central bank), financial intermediaries are inclined to invest more heavily in high-yielding, but illiquid, long-term projects. A central bank's liquidity provision, helping to prevent bank runs with inefficient early liquidation, encourages banks to invest more in illiquid assets. At first sight, this seems to work fine, even if systemic risk increases: After all, public insurance against aggregate risks should allow agents to undertake more profitable activities with higher social return. As long as public insurance is a free lunch, there is nothing wrong with providing such a public good.

The problem, however, is that due to limited liability some banks will be encouraged to free-ride on liquidity provision. This competition will force the other banks to reduce their efforts for liquidity provision, too. Chuck Prince, at that time chief executive of Citigroup, stated the dilemma posed in fairly poetic terms on July 10, 2007 in an infamous interview with *Financial Times*[1]:

> When the music stops, in terms of liquidity, things will be complicated. But as long as the music is playing, you've got to get up and dance. We're still dancing.

[1]The key problem is best captured by the following remark about Citigroup in the *New York Times* report "Treasury Dept. Plan Would Give Fed Wide New Power" on March 29, 2008: "Mr. Frank said he realized the need for tighter regulation of Wall Street firms after a meeting with Charles O. Prince III, then chairman of Citigroup. When Mr. Frank asked why Citigroup had kept billions of dollars in 'structured investment vehicles' off the firm's balance sheet, he recalled, Mr. Prince responded that Citigroup, as a bank holding company, would have been at a disadvantage because investment firms can operate with higher debt and lower capital reserves."

The dancing banks simply enjoy liquidity provided in good states of the world and just disappear (go bankrupt) in bad states. The incentive of financial intermediaries to free-ride on liquidity in good states results in excessively low liquidity in bad states. Even worse, as long as they do not suffer runs, "dancing" banks can always offer more attractive collateral in bad states—so they are able to outbid prudent banks in a liquidity crisis. For that reason, the Bagehot principle, rather than providing correct incentives, is the wrong medicine in modern times with a shadow banking system relying on liquidity being provided by other institutions.

This paper extends a model developed in Cao and Illing (2008). In that paper we did not allow for banks holding equity, so we could not analyze the impact of equity requirements. As we will show, imposing equity requirements can be inferior even to the outcome of a mixed-strategy equilibrium with free-riding (dancing) banks. In contrast, imposing binding liquidity requirements ex ante combined with lender-of-last-resort policy ex post is able to implement the second-best outcome. In our model, it yields a strictly superior outcome compared with imposing equity requirements. We also prove that "narrow banking" (banks being required to hold sufficient equity so as to be able to pay out demand deposits in all states of the world) is inferior to ex ante liquidity regulation.

Allen and Gale (2007, p. 213f) notice that the nature of market failure leading to systemic liquidity risk is not yet well understood. They argue that "a careful analysis of the costs and benefits of crises is necessary to understand when intervention is necessary." In this paper, we try to fill this gap, providing a cost/benefit analysis of different forms of banking regulation to better understand what type of intervention is required. We explicitly compare the impact both of liquidity and of equity requirements. To the best of our knowledge, this is the first paper providing such an analysis.

The paper is organized as follows. In section 2, we present the structure of the model with real deposit contracts and characterize the central planner's constrained efficient solution and the market equilibrium. In section 3 we introduce a central bank and show that in an economy with nominal deposit contracts, lender-of-last-resort policy eliminates bank runs but is subject to the time-inconsistency problem. We show that ex ante liquidity regulation, combined with lender-of-last-resort policy, can implement the constrained efficient

solution. The effectiveness of imposing equity requirements is analyzed in section 4. Section 5 concludes.

2. The Structure of the Model

2.1 The Agents, Time Preferences, and Technology

In this economy, there are three types of agents: investors, banks (run by bank managers), and entrepreneurs. All agents are risk neutral. The economy extends over three periods, $t = 0, 1, 2$, and the details of timing will be explained later. We assume the following:

(i) There is a continuum of investors, each initially (at $t = 0$) endowed with one unit of resources. The resource can be either stored (with a gross return equal to 1) or invested in the form of bank deposits.

(ii) There is a finite number N of active banks engaged in Bertrand competition, competing for investors' deposits. Using these deposits, the banks as financial intermediaries can fund projects of entrepreneurs.

(iii) There is a continuum of entrepreneurs. There are two types of them (denoted by i, $i = 1, 2$), characterized by their project returns R_i:

- Projects of type 1 (safe projects) are realized early at period $t = 1$ with a safe return $R_1 > 1$.
- Projects of type 2 (risky projects) give a higher return $R_2 > R_1 > 1$. With probability p, these projects will also be realized at $t = 1$, but they may be delayed (with probability $1 - p$) until $t = 2$. Therefore, in the aggregate, the share p of type 2 projects will be realized early. The aggregate share p, however, is not known at $t = 0$. It will only be revealed between periods 0 and 1 at some intermediate period; call it $t = \frac{1}{2}$. In the following, we are interested in the case of aggregate shocks. We model them in the simplest way: The aggregate share of type 2 projects realized early, p, can take on just two values—either p_H or p_L with $p_H > p_L$. The "good" state with a high share of early type 2 projects p_H, i.e., the state with plenty of liquidity, will

be realized with probability π. In the following, we assume that $1 < p_s R_2 < R_1$ ($s \in \{H, L\}$) to focus on the relevant case (to be explained later).

Investors are impatient: They want to consume early (at $t = 1$). In contrast, both entrepreneurs and bank managers are indifferent between consuming early ($t = 1$) or late ($t = 2$).

Focusing on the case of liquidity constraints being binding, we assume that resources of investors are scarce in the sense that there are more projects of each type available than the aggregate endowment of investors. Thus, in a first-best market economy (in the absence of commitment problems, as explained in the next paragraph), total surplus would go to the investors. They would simply put all their funds in early projects and capture the full return. We take this frictionless market outcome as our reference point and seek to minimize the distance in terms of the investors' welfare between this reference point and the equilibrium outcome under various policies. Hold-up problems prevent realization of the frictionless market outcome, creating a demand for liquidity. Since there is a market demand for liquidity only if investors' funds are the limiting factor, we choose the investors' payoff as the policymaker's objective and concentrate on deviations from this market outcome. With investors' payoff as the relevant criterion, we analyze those equilibria coming closest to implementing the frictionless market outcome.

Due to hold-up problems as modeled in Hart and Moore (1994), or Holmström and Tirole (1997), entrepreneurs can only commit to pay a fraction $\gamma < 1$ of their return with $\gamma R_i > 1$. Banks as financial intermediaries can pool investment; they have superior collection skills (a higher γ, which justifies their role as intermediaries). In the following, we also assume that $p_s \leq \gamma$ ($s \in \{H, L\}$) to concentrate on the relevant case that investors care about investment in liquid projects (see section 2.4). Following Diamond and Rajan (2001), banks offer deposit contracts with a fixed payment d_0 payable at any time after $t = 0$ as a credible commitment device not to abuse their collection skills. The threat of a bank run disciplines bank managers to fully pay out all available resources pledged in the form of bank deposits. Deposit contracts, however, introduce a fragile structure into the economy: Whenever investors have doubts about their bank's liquidity (the ability to pay investors the promised amount

d_0 at $t = 1$), they run on the bank at the intermediate date, forcing the bank to liquidate all its projects (even those funding entrepreneurs with safe projects) at high costs: Early liquidation of projects gives only the inferior return $c < 1$. In the following, we do not consider pure sunspot bank runs of the Diamond and Dybvig type. Instead, we concentrate on the runs happening if liquid funds are not sufficient to pay out investors.

2.2 Timing and Events

At date $t = 0$, banks competing for funds offer deposit contracts with payment d_0 which maximize the expected return of investors. Banks compete by choosing the share α of deposits invested in type 1 projects, taking their competitors' choice as given. Investors have rational expectations about each bank's default probability; they are able to monitor all banks' investment. Remember that, at this stage, the share p of type 2 projects that will be realized early is not yet known.

At date $t = \frac{1}{2}$, the value of p is revealed, as is the expected return of the banks at $t = 1$. A bank will experience a run if it cannot meet the investors' demand. If this happens, all the assets—even the safe projects—have to be liquidated.

Those banks which do not suffer a run trade with early entrepreneurs in a perfectly competitive market for liquidity at $t = 1$, clearing at interest rate r. Note that because of the hold-up problem, entrepreneurs retain a rent—their share $(1 - \gamma)R_i$. Since early entrepreneurs are indifferent between consuming at $t = 1$ or $t = 2$, they are willing to provide liquidity (using their rent to deposit at banks at $t = 1$ at the market rate r). Banks use the liquidity provided to pay out investors. In this way, impatient investors can profit indirectly from the investment in high-yielding long-term projects. So banking allows the transformation between liquid claims and illiquid projects.

At date $t = 2$, the banks collect the return from the late projects and pay back the early entrepreneurs at the predetermined interest rate r.

Note that the aggregate liquidity available at date $t = 1$ depends on the total share of funds, α, invested in liquid type 1 projects at

Figure 1. Timing and Payoff Structure, When Banks Are Liquid

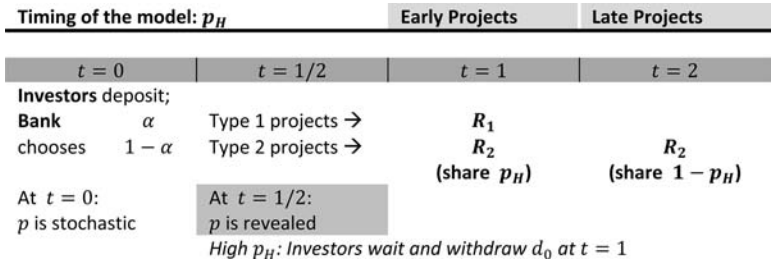
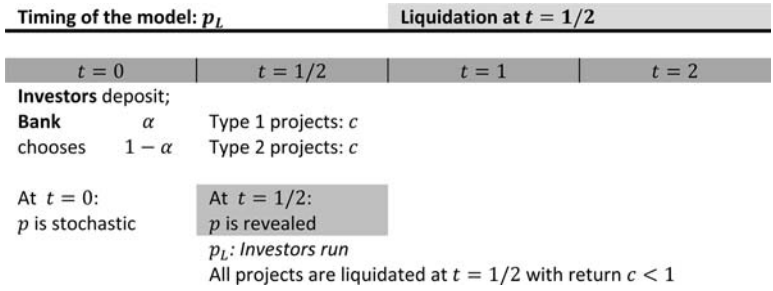


Figure 2. Timing and Payoff Structure, When Banks Are Illiquid



date $t = 0$. As long as the banks are liquid, the payoff structure is described as in figure 1. But if α is so low that the banks cannot honor deposits when p_L occurs, investors will run at $t = \frac{1}{2}$. The payoff in that case is captured in figure 2.

2.3 The Central Planner's Constrained Efficient Solution

We first analyze the problem of a central planner maximizing the investors' payoff. This provides the reference point for the market equilibrium with banks as financial intermediaries characterized in the next section. Investors being impatient, the central planner would choose the share invested in illiquid projects so as to maximize the resources available to investors at period 1. Since $p_s R_2 < R_1$,

in the absence of hold-up problems, the planner would invest only in liquid type 1 projects, this way maximizing resources available at period 1. But due to the hold-up problem caused by entrepreneurs, the central planner can implement only a constrained efficient solution. If the central planner had unlimited taxation authority, he or she could eliminate the hold-up problem completely by taxing the entrepreneurs' rent and redistributing the resources to the investors. Again, all resources would be invested only in liquid type 1 projects, and the entrepreneurs' rents would be transferred to the investors in period 1.

Obviously, allowing for non-distortionary taxation biases the comparison between the market and the planner's solution, giving the planner an unfair advantage. Effectively, redistribution via lump-sum taxation would make both hold-up and liquidity constraints non-binding, assuming the relevant issues away. To make the planner's constrained optimization problem interesting, we assume that non-distortionary taxation is not feasible in period 1. In order to impose sensible restrictions, we take private endowments as a binding constraint and assume that the entrepreneur has to receive an equivalent compensation when he or she is asked to give up resources in period 1. Being indifferent between consuming at $t = 1$ and $t = 2$, the entrepreneur needs to be compensated by an appropriate transfer in period 2. In order not to distort the comparison in favor of banks, we furthermore assume that the planner has the same collection skills (the same γ) as financial intermediaries.

Given these constraints, the constrained efficient solution is characterized in the following proposition.

PROPOSITION 1. *The optimal solution for the central planner's problem is as follows:*

(i) If there is no aggregate risk, i.e., when p_s is known at $t = 0$, the planner invests the share $\alpha_s = \frac{\gamma - p_s}{\gamma - p_s + (1-\gamma)\frac{R_1}{R_2}}$ $(s \in \{H, L\})$ in liquid projects and the investors' return is maximized at $\gamma E[R_s] = \gamma[\alpha_s R_1 + (1 - \alpha_s)R_2]$.

(ii) In the presence of aggregate risk, the central planner implements the following state-contingent strategy, depending on

the probability π for p_H being realized: The planner invests the share α_H in liquid projects as long as $\overline{\pi}'_2 = \frac{\gamma E[R_L]-\kappa}{\gamma E[R_H]-\kappa} \leq \pi \leq 1$ with $\kappa = \alpha_H R_1 + (1-\alpha_H)p_L R_2$, and the share $\alpha_L > \alpha_H$ for $0 \leq \pi < \overline{\pi}'_2$.

Proof. See appendix 1.

The first part of the proposition says that if p is known, the planner simply chooses α so as to maximize the investors' return. The second part says that if p is unknown, the planner faces a trade-off: the investors' return is maximized under p_H if the planner chooses α_H, but will be low if p_L is realized; the investors' return is maximized under p_L if the planner chooses α_L, but will be low if p_H is realized. So the optimal solution depends on the likelihood of p_H, that is, on π. When π is high enough, the planner will choose α_H; otherwise, he or she will pick α_L.

Obviously, hold-up and liquidity constraints are bound to have a distributional impact: If resources were taken away from investors in the initial period and redirected toward type 2 entrepreneurs, the commitment problem would no longer be relevant, nor would the need for liquidity provision. Even though such a reallocation would result in higher aggregate resources (all funds being invested in high-return projects), it would yield inferior payoff to investors. Since $p_s R_2 < R_1$, investing less than α_s in liquid projects reduces resources available in period 1 and so makes investors worse off.

In contrast to our modeling strategy, Holmström and Tirole (1998) assume that the lender of last resort has unlimited power to tax real resources and so is always able to redistribute resources ex post. This assumption, however, effectively makes liquidity constraints non-binding: The central planner can always redistribute resources ex post in such a way as to make them irrelevant. The planner could simply redirect resources to the constrained agents (and potentially compensate the unconstrained). Interestingly, in our model, giving the planner taxation power in period 2 cannot help to improve upon the investors' allocation: The investors being impatient, any redistribution from illiquid projects realized late at $t = 2$ is simply not feasible.

2.4 The Market Equilibrium

Let us now characterize the market equilibrium with banks as financial intermediaries. First, let us again start with the simplest case with no aggregate uncertainty; i.e., the share p of type 2 projects realized early is known at $t = 0$. The market equilibrium of the model is characterized by bank i's strategic profile $(\alpha_i, d_{0i}), \forall i \in \{1, \ldots, N\}$ such that

- Bank i's profit is maximized by

$$\alpha_i = \arg \max_{\alpha_i \in [0,1]} \gamma \left\{ \alpha_i R_1 + (1 - \alpha_i) \left[pR_2 + \frac{(1-p)R_2}{r} \right] \right\}. \tag{1}$$

  Bank i chooses the share of liquid projects α_i so as to maximize expected discounted returns;
- Bank i makes zero profit from offering deposit contract d_{0i},

$$d_{0i} = \max_{\alpha_i \in [0,1]} \gamma \left\{ \alpha_i R_1 + (1 - \alpha_i) \left[pR_2 + \frac{(1-p)R_2}{r} \right] \right\}. \tag{2}$$

  Investors deposit their funds at those banks offering the highest return. Thus, with Bertrand competition in the deposit market, the deposit rate d_{0i} offered to investors in equilibrium will be equal to expected returns, maximizing resources available at period 1;
- The market interest rate is determined in the following way:

  (i) In equilibrium, all resources available at $t = 1$ will be paid out to investors, so $d_{0i} = \alpha_i R_1 + (1 - \alpha_i) pR_2$. Banks receive funds $\gamma[\alpha_i R_1 + (1 - \alpha_i) pR_2]$ from those projects realized early. In addition, early entrepreneurs are willing to provide liquidity at $t = 1$ (depositing their rent at the market rate $r \geq 1$) to solvent banks that are able to meet their liabilities to the investors, that is, to banks with $d_{0i} \leq \gamma \left\{ \alpha_i R_1 + (1 - \alpha_i) \left[pR_2 + \frac{(1-p)R_2}{r} \right] \right\}$. So the liquidity supplied by early entrepreneurs is $(1 - \gamma)[\alpha_i R_1 + (1 - \alpha_i) pR_2]$ as long as bank i is expected to stay solvent—that is, as long as it is able to pay out early entrepreneurs at

the market rate r at $t = 2$ from its late project's return $\gamma(1 - \alpha_i)(1 - p)R_2$.

Furthermore, as the market clearing condition, aggregate liquidity supply and demand at $t = 1$ have to be equal, given that banks stay solvent at the interest rate $r \geq 1$: $\sum_{i=1}^{N} r(1 - \gamma)[\alpha_i R_1 + (1 - \alpha_i)pR_2] = \sum_{i=1}^{N} \gamma(1 - \alpha_i)(1 - p)R_2$;

(ii) Finally, when there is excess liquidity supply at $t = 1$, i.e., when total intermediate output exceeds the payoff promised to the investors, $r = 1$.

If there is no aggregate uncertainty, the market equilibrium with $r = 1$ is equivalent to the solution of the social planner's problem: Banks will invest such that—on aggregate—they are able to fulfill investors' claims in period 1, so there will be no run.

Proposition 2. *If there is no aggregate uncertainty, the allocation in the market equilibrium with $r = 1$ is identical to the solution of the social planner's problem, characterized by the following:*

- *All banks set* $\alpha = \frac{\gamma - p}{\gamma - p + (1 - \gamma)\frac{R_1}{R_2}}$.
- *The market interest rate* $r = 1$.

Proof. See appendix 1.

The proposition says that in the absence of aggregate uncertainty, the banks will choose α (the share invested in liquid projects) so as to maximize the depositors' return and to stay solvent at $t = 1$, given that entrepreneurs are willing to provide liquidity at that time. This coincides with the solution of the social planner's problem. Since $R_1 > pR_2$ and $\gamma > p$, α will be strictly positive in equilibrium. For given p, there is a unique α maximizing resources available for investors at $t = 1$. A bank investing less than this value of α would not be able to pay out the amounts promised to investors at $t = 1$ and thus would experience a run at $t = \frac{1}{2}$. A bank investing more than α would be outbid by competitors offering a higher d_{0i}. Note that α is decreasing in p: The larger the share p of type 2 projects realized early, the less need for investment in liquid type 1 projects. For $p > \gamma$, liquid projects are dominated by the risky ones,

so there would be no demand for liquid projects at $t = 0$. Similarly, there would be no demand for liquid projects at $t = 0$ either when $R_1 < pR_2$. Since liquidity is not an issue for these cases, they are ruled out by assumption.

It becomes tricky to find the market equilibrium when there is aggregate uncertainty. Let us briefly sketch the market equilibrium in the following proposition.

PROPOSITION 3. *When there is aggregate uncertainty,*

- *(i) there is a symmetric pure-strategy equilibrium such that all banks set* $\alpha = \alpha_H$ *for all* $\overline{\pi}_2 < \pi \leq 1$, *with* $\overline{\pi}_2 = \frac{\gamma E[R_L]-c}{\gamma E[R_H]-c}$ *and* $E[R_s] = \alpha_s R_1 + (1-\alpha_s)R_2$ $(s \in \{H, L\})$;
- *(ii) there is a symmetric pure-strategy equilibrium such that all banks set* $\alpha = \alpha_L$ *for all* $0 \leq \pi < \overline{\pi}_1$, *with* $\overline{\pi}_1 = \frac{\gamma E[R_L]-c}{\gamma R_2 - c}$; *and*
- *(iii) there exists no symmetric pure-strategy equilibrium for all* $\overline{\pi}_1 \leq \pi \leq \overline{\pi}_2$. *However, there exists a unique equilibrium in mixed strategies such that*
  - *(a) at* $t = 0$, *with probability* θ *a bank chooses to be a free-riding bank that sets* $\alpha = 0$ *and with probability* $1 - \theta$ *a bank chooses to be a prudent bank that sets* $0 < \alpha_s^* < \alpha_L$; *and*
  - *(b) in the mixed-strategy equilibrium, investors are worse off than if all banks would coordinate on the prudent (non-equilibrium) strategy* α_L.

Proof. See appendix 1.

The intuition behind proposition 3 is as follows: With uncertainty about p, a bank seems to have just two options available—it may either invest so much in safe type 1 projects (α_L) that it will be able to pay out the investors all the time (that is, even if the bad state occurs), or it may invest just enough, α_H, so as to pay out investors only in the good state and experience a run in the bad state. If π is very high (close to 1), a bank should choose α_H—to reap the high yields in the good state, since the cost of the bank run in the bad state is rather low. Alternatively, if π is very low (close to 0), it always pays to be prepared for the worst case, so the bank should choose $\alpha_L > \alpha_H$ in safe projects. Since α_s $(s \in \{H, L\})$ is

the share invested in safe projects with return R_1, the total payoff by choosing α_s is $E[R_s] = \alpha_s R_1 + (1-\alpha_s)R_2$, with $E[R_H] > E[R_L]$.

With a high share α_L of safe projects, the banks will be able to pay out investors in all states. There will never be a bank run. So, independent of π, the expected payoff for investors is $\gamma E[R_L]$. In contrast, with strategy α_H there will be a bank run in the bad state, giving just the bankruptcy payoff c with probability $1-\pi$. So the return to strategy α_H is $\pi\gamma E[R_H] + (1-\pi)c$, which is increasing in π. Investors get a higher payoff under α_H, if $\pi\gamma E[R_H] + (1-\pi)c > \gamma E[R_L]$ or

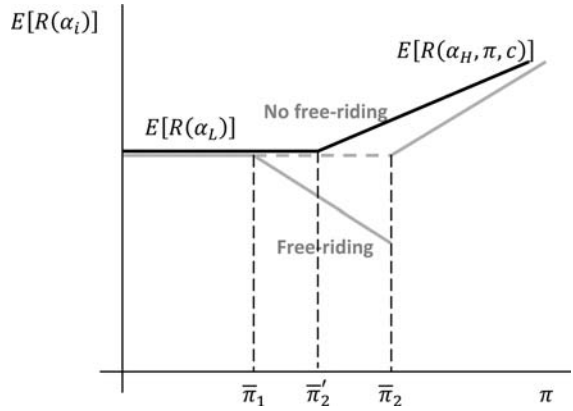
$$\pi > \overline{\pi}_2 = \frac{\gamma E[R_L] - c}{\gamma E[R_H] - c}.$$

For $\pi < \overline{\pi}_2$, the investors' payoff is higher with strategy α_L. But if all banks would choose strategy α_L, there will be excess liquidity at $t = 1$ if the good state occurs (with a large share of type 2 projects realized early). A bank anticipating this event has a strong incentive to invest all funds in type 2 projects, reaping the benefit of excess liquidity in the good state. As long as the music is playing, such a deviating bank gets up and dances. In the good state, such a free-riding bank can credibly rely on entrepreneurs' excess liquidity at $t = 1$, promising to pay back at $t = 2$ out of highly profitable projects. After all, at that stage, this bank, free-riding on liquidity, can offer a capital cushion with expected returns well above what prudent banks are able to promise. Of course, if the bad state happens, there is no excess liquidity. Liquidity dries up. The free-riding banks would just bid up the interest rates, urgently trying to get funds. Rational investors, anticipating that these banks will not succeed, will have already triggered a bank run on these banks at $t = \frac{1}{2}$.

As long as the free-riding banks are not supported in the bad state, they are driven out of the market, providing just the return c. Nevertheless, these banks can offer the return $\pi\gamma R_2 + (1-\pi)c$ as expected payoff for investors. Thus, a free-riding bank will be able to offer a higher expected return than a prudent bank, provided the probability π for the good state is not too low. The condition is

$$\pi > \overline{\pi}_1 = \frac{\gamma E[R_L] - c}{\gamma R_2 - c}.$$

Figure 3. Investors' Expected Return under Free-Riding Compared with the Case of No Free-Riding and the Central Planner's Solution



Since $R_2 > E[R_H]$, it pays to free-ride within the range $\overline{\pi}_1 \leq \pi < \overline{\pi}_2$.

Obviously, there cannot be an equilibrium in pure strategies within that range. As long as the music is playing, all banks would like to "get up and dance." But then, there would be no prudent bank left providing the liquidity needed to be able to free-ride. In the resulting mixed-strategy equilibrium, a proportion of banks behave prudently, investing some amount $\alpha_s^* < \alpha_L$ in liquid assets, whereas the rest free-ride on liquidity in the good state, choosing $\alpha = 0$. Prudent banks reduce α in order to cut down the opportunity cost of investing in safe projects. Interest rates and α_s^* adjust so that investors are indifferent between the two types of banks. At $t = 0$, both prudent and free-riding banks offer the same expected return to investors. The proportion of free-riding banks is determined by aggregate market clearing conditions in both states. Free-riding banks experience a run for sure in the bad state, but the high return in the good state R_2 compensates investors for that risk.

As shown in proposition 3, free-riding drives down the return for investors. They are definitely worse off than they would be if all banks coordinated on the prudent strategy α_L—similar to the inefficient mixed-strategy equilibrium in Allen and Gale (2004). The solid gray lines in figure 3 illustrate the investors' expected return in the market equilibrium; as a result of free-riding behavior, the effective return on deposits for investors deteriorates in the range

$\overline{\pi}_1 \leq \pi < \overline{\pi}_2$, compared with the outcome if all banks would coordinate (off equilibrium) on α_L as the dashed gray line shows.

Compared with the central planner's solution (the solid black line in figure 3), the investor's payoff is lower in the market equilibrium with banks as financial intermediaries for two different reasons: First, free-riding banks reduce the investor's payoff in the mixed-strategy equilibrium in the intermediate case. Second, for high values of π ($\overline{\pi}_2 < \pi \leq 1$), a representative bank, choosing α_H, accepts the risk of a bank run if the bad state occurs (with a low share p_L of illiquid projects realized early). If that state occurs, a bank run is triggered with inefficient liquidation, resulting in an inferior payoff $c < 1$. In the following sections, we will carefully analyze how different mechanism designs may help to raise the investor's payoff, bringing the market outcome closer to the constrained efficient solution as stated in section 2.3. In the next section, we show that in an economy with nominal deposit contracts, lender-of-last-resort policy is able to tackle the problem of bank runs, but at the same time aggravates banks' incentives for free riding.

3. Lender-of-Last-Resort Policy

3.1 Nominal Contracts and the Lender of Last Resort

A lender of last resort, usually the central bank, cannot create real liquidity in period 1. But a central bank can add nominal liquidity at the stroke of a pen. Following Allen and Gale (1998) and Diamond and Rajan (2006), assume from now on that deposit contracts are arranged in nominal terms. The bail-out mechanism of the central bank is similar to that in Allen, Carletti, and Gale (2010). Here is the timing of the model:

(i) At $t = 0$, the banks provide nominal deposit contracts to investors, promising a fixed nominal payment d_0 at $t = 1$. The central bank announces a minimum level $\underline{\alpha}$ of investment on safe projects, and only those banks that meet the requirement will be eligible for liquidity support in the time of a crisis.

(ii) At $t = \frac{1}{2}$, the banks decide whether to borrow liquidity from the central bank. If yes, the central bank will provide liquidity for the banks, provided they fulfill the requirement $\underline{\alpha}$.

(iii) At $t = 1$, the liquidity injection with the banks' illiquid assets as collateral is carried out so that the banks are able to honor their nominal contracts, which reduces the real value of deposits just to the amount of real resources available at that date.
(iv) At $t = 2$, the banks repay the central bank using the returns from the late projects, with gross nominal interest rate r^M agreed at $t = 1$.

As a new element in this extended model, we allow the central bank to impose minimal liquidity holdings in addition to lender-of-last-resort policy as a way to implement the allocation maximizing the investors' payoff.

3.2 Liquidity Regulation and Lender-of-Last-Resort Policy

With nominal contracts, the central bank's optimal policy as lender of last resort can be summarized in the following proposition.

PROPOSITION 4. *With nominal contracts, the central bank can act as lender of last resort. The central bank's optimal policy that maximizes the investors' return is as follows:*

(i) Set $\underline{\alpha} = \alpha_H$ *for all* $\pi \in [\overline{\pi}'_2, 1]$, *where* $\overline{\pi}'_2 = \frac{\gamma E[R_L]-\kappa}{\gamma E[R_H]-\kappa} < \overline{\pi}_2$ *and* $\kappa = \alpha_H R_1 + (1 - \alpha_H) p_L R_2$.
(ii) Set $\underline{\alpha} = \alpha_L$ *for all* $\pi \in [0, \overline{\pi}'_2)$.
(iii) Set $r^M = 1$.

What's more, under such a policy bank runs are eliminated for the eligible banks; i.e., the eligible banks will not experience runs when p_L *is revealed.*

Proof. See appendix 1.

The investors' return is maximized when the banks get liquidity injection at the lowest cost, the central bank setting $r^M = 1$. With liquidity injection, bank runs are prevented when the bad state (with low payoffs at $t = 1$) occurs. Essentially, nominal deposits allow the central bank to implement state-contingent payoffs, and such a policy replicates the optimal allocation in the central planner's

problem. This argument seems to confirm the view that the lender of last resort can indeed provide a free lunch, delivering a public good at no cost. It turns out, however, that the anticipation of these actions has an adverse impact on the amount of aggregate liquidity provided by the private sector, affecting endogenously the exposure to systemic risk.

Proposition 5. *Assume that a market equilibrium exists, i.e.,* $\pi p_H R_2 + (1 - \pi) p_L R_2 \geq 1$. *If the central bank is willing to provide liquidity to the entire market in times of crisis, all banks have an incentive to free-ride, choosing* $\alpha = 0$, *and investors are made worse off.*

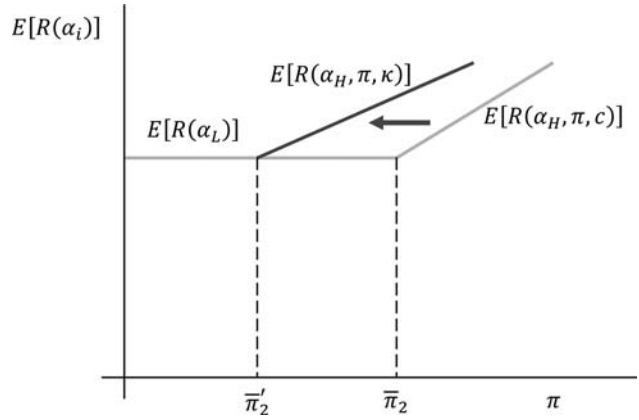
Proof. See appendix 1.

The reason for this surprising result is the following: If the central bank targets liquidity provision to the market instead of to specific banks, the optimal policy as stated by proposition 4 is not enforceable. Since we concentrate on the case of pure illiquidity risk, in our model all projects will certainly be realized at $t = 2$. So there is no doubt about solvency of the projects, unless insolvency is triggered by illiquidity. If the central bank follows the Bagehot principle and creates artificial liquidity at the drop of a hat—against allegedly good collateral—all private incentives to care about ex ante liquidity provision will be destroyed, exacerbating the moral hazard problem: The free-riding banks, investing all their funds in the projects with higher returns, can always get liquidity support and thus are able to offer more attractive terms to investors at $t = 0$. This drives out all the prudent banks and leaves the investors worse off.[2]

So what policy options should be taken? One might argue that a central bank should provide liquidity support only to prudent banks (conditional on banks having invested sufficiently in liquid assets). But such a commitment is simply not credible: As emphasized by Rochet (2004) and Cao and Illing (2008), there is a serious problem of dynamic consistency.

[2]In reality, there is no clear-cut distinction between insolvency and illiquidity. We leave it to future research to allow for the risk of insolvency. But we doubt that our basic argument will be affected.

Figure 4. Investors' Real Expected Return $E[R(\alpha_H, \pi, \kappa)]$ for the Case of Ex Ante Liquidity Regulation Combined with Ex Post Lender-of-Last-Resort Policy for High π, in Comparison with the Market Equilibrium, $E[R(\alpha_H, \pi, c)]$



Rather than relying on an implausible commitment mechanism, the obvious solution is a mix of two instruments: ex ante liquidity regulation combined with ex post lender-of-last-resort policy. The second-best outcome from the investors' point of view needs to be implemented by the following policy: In a first step, a banking regulator has to impose ex ante liquidity requirements. Requesting minimum investment in liquid type 1 assets of at least α_L for $\pi < \overline{\pi}'_2$ and α_H for $\pi \geq \overline{\pi}'_2$ would give investors the highest expected payoff, as characterized in figure 4. When banks are not allowed to operate with insufficiently low liquidity holdings, there are no incentives for free-riding. For high values $\pi \geq \overline{\pi}'_2$, the central bank acts as lender of last resort in the bad state, eliminating costly bank runs. This raises the expected payoff for investors, even though it increases the range of parameter values with systemic risk.

In a quite different setting, using a framework with asymmetric information, Farhi and Tirole (2009) derive related results. They show that monetary policy (with the real interest rate as policy variable) faces a commitment problem. They also derive a role for a minimum liquidity ratio. Our setup shows that the key challenge for regulators and the central bank is to cope with incentives for financial intermediaries to free-ride on liquidity provision. Furthermore,

it allows us to compare liquidity regulation with alternative mechanism designs. One might expect that imposing equity requirements is sufficient to provide a cushion against liquidity shocks. As a further alternative, one might impose narrow banking in the sense that banks are required to hold sufficient liquid funds so as to pay out in all contingencies. As shown in the next section, both these options turn out to be strictly worse than imposing minimum liquidity standards ex ante combined with lender-of-last-resort policy. They can even be inferior to the outcome of a mixed-strategy equilibrium with free-riding banks.

4. The Role of Equity and Narrow Banking

Let us now introduce equity requirements in the model; i.e., banks are required to hold some equity as a share of their assets. Instead of pure fixed deposit contracts, the banks now issue a mixture of deposit contracts and equity for attracting funds from the investors (Diamond and Rajan 2000, 2005, 2006). Equity can reduce the fragility by providing a cushion against negative shocks. This, however, comes at a cost, since it allows the bank manager to capture a rent. So the regulator needs to strike a balance between benefit and cost.

Being a renegotiable claim, in contrast to deposits, equity is subject to the hold-up problem; i.e., equity holders will only get a share ζ ($\zeta \in [0,1]$) of the surplus, the bank manager extracting the remaining part $1-\zeta$ as rent from his or her superior collection skills. Without changing the nature of the problem, in the following we simply assume that $\zeta = \frac{1}{2}$.

With $\zeta = \frac{1}{2}$, the bank manager and equity holders share the surplus over deposits equally. So the equity value of a bank not suffering from a run is $\frac{\gamma E[R_s]-d_0}{2}$ in state s with expected return $\gamma E[R_s]$ and deposit claims d_0. Assume that some equity requirement k is imposed—that is, the share of equity to bank assets is k, with

$$k = \frac{\frac{\gamma E[R_s]-d_0}{2}}{\frac{\gamma E[R_s]-d_0}{2} + d_0}.$$

Solving for d_0 gives the return to depositors as

$$d_0 = \frac{1-k}{1+k}\gamma E[R_s],$$

with equity holders receiving $\frac{k}{1+k}\gamma E[R_s]$. Thus investors providing funds both in the form of deposits and equity to the banks will receive the payoff $\frac{1}{1+k}\gamma E[R_s] < \gamma E[R_s]$ at $t = 1$.

In the absence of aggregate risk, introducing equity requirements is a pure cost, reducing the investors' payoff. Somewhat counterintuitively, without aggregate risk, equity requirements even reduce the share α invested in safe projects. The reason is that with equity financing bank managers get the rent $\frac{\gamma E[R]-d_0}{2}$, extracting part of the surplus over deposits from equity holders. Since the return at $t = 2$ is higher than at $t = 1$, bank managers are willing to consume late, so the amount of resources needed at $t = 1$ is lower in the presence of equity. Consequently, the share α will be reduced. Obviously, banks holding no equity provide more attractive conditions for investors, so equity could not survive. This seemingly counterintuitive result simply demonstrates that there is no role (or rather only a payoff-reducing role) for costly equity in the absence of aggregate risk.

The benefit of equity comes in when there is aggregate risk: Equity helps to absorb aggregate shocks and avoid the costly bank runs. In the simple two-state setup, equity holdings need to be just sufficient to cushion the bad state. With sufficient equity, the bank can choose $\alpha = \alpha_H$, profiting from the high return in the good state and still staying solvent in the bad state. In that case, it just needs to be able to pay out the fixed claims of investors, wiping out all equity.

With equity k and investment $\alpha = \alpha_H$, the total amount that can be pledged to investors providing funds both as depositors and equity holders is $\frac{1}{1+k}\gamma E[R_H]$ in the good state, with claims of depositors being $d_0 = \alpha_H R_1 + (1-\alpha_H)p_L R_2$ and return on equity $\frac{k}{1+k}\gamma E[R_H]$. In the bad state, a marginally solvent bank is able to pay out $d_0 = \alpha_H R_1 + (1-\alpha_H)p_L R_2$ to depositors. So the minimum k^* to prevent bank runs is determined by the condition

$$\frac{1-k^*}{1+k^*}\gamma E[R_H] = \alpha_H R_1 + (1-\alpha_H)p_L R_2,$$

and we solve to get

$$k^* = \frac{\gamma E[R_H] - d_0}{\gamma E[R_H] + d_0}. \tag{3}$$

Obviously, k^* is decreasing in p_L: The higher p_L, the lower the cushion k^* which is needed to stay solvent in the bad state.

Condition (3) determines the minimum equity requirement k^* a regulator needs to impose in order to eliminate the risk of costly bank runs. Setting k lower ($k < k^*$) would not help to prevent bank runs; setting k too high ($k > k^*$) would just raise the cost of holding equity without additional benefit. Thus from now on we can concentrate on the level k^* without loss of generality. In the following, we compare the investors' payoff in an economy subject to equity requirements with the payoff in the absence of any regulation (as derived in proposition 3) and then with the case of liquidity requirements, combined with the central bank acting as lender of last resort. Finally, we conclude with an analysis of narrow banking.

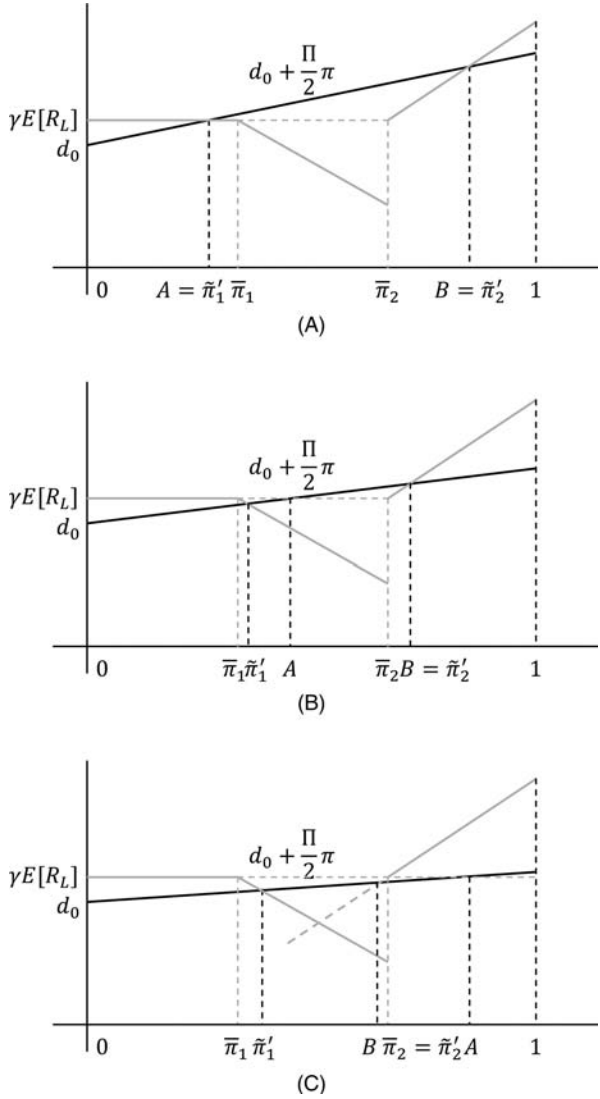
4.1 Equity Requirements versus Market Equilibrium

We first ask whether equity requirements can improve the investors' allocation in this economy, relative to the payoff they get in the market equilibrium we characterized in proposition 3. As shown in section 2, the investors' payoff depends on the probability π of the good state. If π is low enough, banks choose the safe strategy α_L. For high π, they pick α_H, with payoff increasing in π. In an intermediate range, free-riding banks drive down investors' return relative to what they could earn from investment in the safe strategy α_L. The overall payoff as a function of π is shown by the gray lines in figure 3. Let us call this function $\Pi(\pi)$. It seems natural to expect that equity requirements are superior at least for the intermediate range. As we will show, this intuition does not hold.

In figure 5A, the solid black lines show the investors' expected return $\Pi_e(\pi) = d_0 + \frac{\Pi}{2}\pi$ for the case of equity requirements. With equity requirements, the investors' expected return is uniformly increasing in π. The equity requirement k^* is chosen such that deposits will always be paid out fully, even in the bad state. Thus, the fixed deposit payment d_0 is independent of π. In contrast, the return on equity $\frac{\Pi}{2}$ is paid out only in the good state. The more likely the good state (the higher π), the higher is the expected return on equity. Its value is determined by

$$\frac{\Pi}{2} = \frac{\gamma E[R_H] - d_0}{2} = \frac{\gamma E[R_H] - \frac{1-k}{1+k}\gamma E[R_H]}{2} = \frac{k}{1+k}\gamma E[R_H].$$

Figure 5. Investors' Expected Return in the Market Equilibrium (Solid Gray Lines)/under Equity Requirements (Solid Black Lines)



Note: In each of the three cases, the range where the equity regulation regime dominates the market equilibrium is denoted by the interval $(\tilde{\pi}_1', \tilde{\pi}_2')$.

Under what conditions will a banking system with equity requirements outperform the investors' return in the market equilibrium? Intuition suggests that relative performance depends on parameter values. As lemma 1 proves, equity requirements can never dominate the market outcome uniformly. It is straightforward to compare the investors' payoff under equity requirements with the market equilibrium with free-riding for the extreme values $\pi = 0$ and $\pi = 1$.

Lemma 1. *The investors' expected return under the equity requirement is lower than the market equilibrium outcome when* $\pi = 0$ *or* $\pi = 1$.

Proof. See appendix 1.

The intuition of lemma 1 is straightforward: Since there is no uncertainty when $\pi = 0$ or $\pi = 1$, it is inferior to hold costly equities as explained above.

Figure 5A suggests, however, that equity requirements might uniformly improve the investor's expected return for the range of parameter values resulting in the mixed-strategy equilibrium with free-riding banks. Unfortunately, proposition 6 shows that this need not be the case. The equity regulation regime may even be sometimes dominated by the market equilibrium with free-riding.

Proposition 6. *Imposing the equity requirement* k^* *may make investors better off than the mixed-strategy equilibrium with free-riding banks for some range of parameter values. But the costs of imposing equity requirements may be so high that the equity regulation regime may be dominated even by the market equilibrium with free-riding. There are three possible cases:*

(i) The equity regulation regime dominates the market equilibrium in the case of free-riding—that is, for $\overline{\pi}_1 \leq \pi \leq \overline{\pi}_2$.

(ii) In the range $\overline{\pi}_1 \leq \pi \leq \overline{\pi}_2$, *the equity regulation regime dominates for high values of* π, *whereas the market equilibrium with free-riding dominates for low values. In addition, the equity regulation regime dominates the market equilibrium for the low values of* π *in the range* $\overline{\pi}_2 \leq \pi \leq 1$.

(iii) In the range $\overline{\pi}_1 \leq \pi \leq \overline{\pi}_2$, *the equity regulation regime dominates for high values of* π, *whereas the market equilibrium*

with free-riding dominates for low values. In addition, the equity regulation regime is uniformly dominated by the market equilibrium in the range $\overline{\pi}_2 \leq \pi \leq 1$.

Proof. See appendix 1.

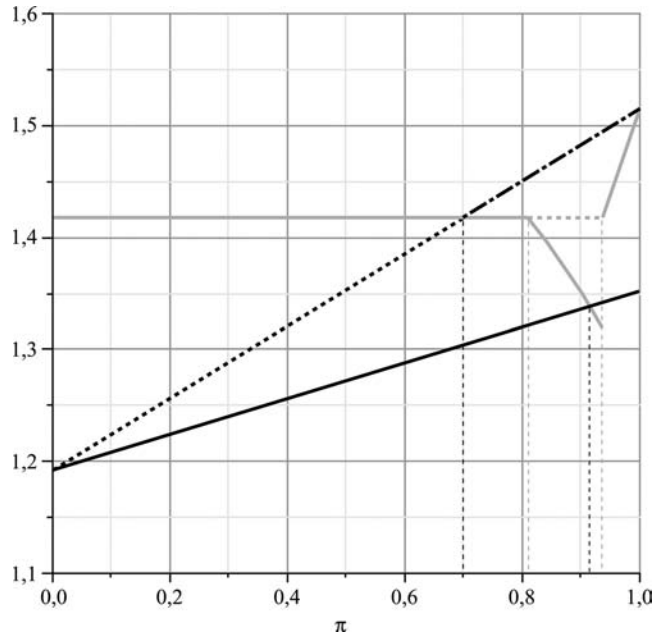
The three possible cases are characterized in figures 5A, B, and C, respectively. The quantitative conditions which separate these cases can be found in appendix 2. Numerical examples illustrating these cases are presented in appendix 3.

Proposition 6 says that the effectiveness of imposing equity requirements is dubious. Equity requirements may give investors a higher payoff than the mixed-strategy equilibrium with free-riding banks for all parameter values with mixed-strategy equilibrium $\overline{\pi}_1 \leq \pi \leq \overline{\pi}_2$. This case is captured as case (A) in figure 5A (and as proposition 6 (i)). Since free-riding partly destroys the value of assets held by prudent banks (forcing them to hold a riskier portfolio), it might seem that imposing equity requirements will always dominate the market equilibrium outcome with mixed strategies. But according to proposition 6, it is quite likely that equity requirements result in inferior payoffs for some range of parameter values (for example, when c is not very low and p_H is close to γ, i.e., the bank run cost is not very high), as shown in case (B) in figure 5B (when $A \in (\overline{\pi}_1, \overline{\pi}_2)$, as in proposition 6 (ii), equity requirements result in inferior payoffs to mixed-strategy equilibrium for $(\overline{\pi}_1, \tilde{\pi}_1')$ but superior to market equilibrium for some high values of $(\overline{\pi}_2, \tilde{\pi}_2')$). It might be that imposing equity requirements makes investors even worse off, as in figure 5C, representing case (C) (when $A > \overline{\pi}_2$, as in proposition 6 (iii), equity requirements result in inferior payoffs to mixed-strategy equilibrium for $(\overline{\pi}_1, \tilde{\pi}_1')$ and inferior payoffs for all $\pi \in [\overline{\pi}_2, 1]$).

The intuition behind this result is that holding equity can be quite costly[3]; if so, it may be superior to accept the fact that systemic risk is a price to be paid for higher returns on average.

[3]The cost of holding equity comes from the fact that equity holders can only get the share ζ from the surplus in the good state. In this paper, we set $\zeta = 0.5$ for a simpler exposition. The investors' expected return increases when ζ gets higher and, ceteris paribus, outperforms the return in the market equilibrium for a wider range of π. In the limit, when $\zeta = 1$, issuing equity creates no hold-up problems and so does not incur costs. In that case, financing via equity would be able to implement the planner's solution.

Figure 6. Real Expected Return with Credible Liquidity Injections (Black Chain Line, for the Case of Figure 12), Compared with Investors' Expected Return in the Market Equilibrium (Solid Gray Lines)/under Equity Requirements (Solid Black Lines)

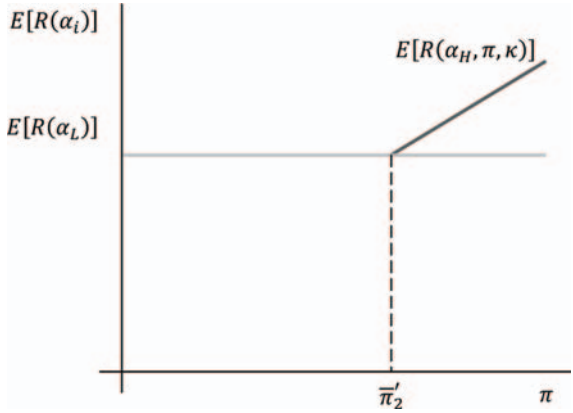


4.2 Equity Requirements versus Conditional Lender-of-Last-Resort Policy

As just shown, there is no clear ranking between market equilibrium without regulation and a regime with equity requirements. In contrast, the mix of ex ante liquidity requirements with ex post lender-of-last-resort policy is always dominating equity requirements. See figure 6. The reason is as follows: Consider that the banks are required to hold $\underline{\alpha} = \alpha_H$ when π is high. Then when p_H is revealed, the investors' real return is $\gamma E[R_H]$; and when p_L is revealed, the investors' real return is $\alpha_H R_1 + (1 - \alpha_H)p_L R_2$. Therefore the investors' overall expected return turns out to be

$$\Pi_m = \gamma E[R_H]\pi + (1 - \pi)[\alpha_H R_1 + (1 - \alpha_H)p_L R_2],$$

Figure 7. Real Expected Return with Narrow Banking Compared with Ex Ante Liquidity Regulation



which is linear in π, as the chain line of figure 6 shows. Note that when $\pi = 1$, $\Pi_m = \gamma E[R_H] > d_0 + \frac{\Pi}{2}$; and when $\pi = 0$, $\Pi_m = \alpha_H R_1 + (1 - \alpha_H) p_L R_2 = d_0$. Therefore, Π_m line is above $d_0 + \frac{\Pi}{2}\pi$, $\forall \pi \in (0, 1]$; i.e., the mix of liquidity requirements with lender-of-last-resort policy is always dominating equity requirements when aggregate uncertainty exists.

4.3 Liquidity Requirements versus Narrow Banking

In times of crises, frequently there are calls to go back to narrow banking in order to avoid the risk of runs. Under narrow banking, institutions with deposits would be required to hold as assets only the most liquid instruments so as to be always able to meet any deposit withdrawal by selling their assets. Obviously, narrow banking can be extremely costly. In our model, banks would be required to hold sufficient liquid funds to pay out in all contingencies: $\alpha \geq \alpha_L$. As figure 7 illustrates, under narrow banking an investor's payoff (the solid gray line) can be much lower for high π compared with ex ante liquidity regulation combined with ex post lender-of-last-resort policy (the solid black line). Just as with equity requirements, narrow banking (imposing the requirement that banks hold sufficient

equity so as to be able to pay out demand deposits in all states of the world) can be quite inferior: If the bad state is a rare probability event, it simply makes no sense to dispense with all the efficiency gains from investing in high-yielding illiquid assets despite its impact on systemic risk.

5. Conclusion

Traditionally, exposure to aggregate liquidity shocks has been modeled as an exogenous event. In this paper, we derive the aggregate share of liquid projects endogenously. It depends on the incentives of financial intermediaries to invest in risky, illiquid projects. This endogeneity allows us to capture the feedback between financial market regulation and incentives of private banks, determining the aggregate amount of liquidity available. As a consequence of limited liability, banks are encouraged to free-ride on liquidity provision. Relying on sufficient liquidity provided by the market, they are inclined to invest excessively in illiquid long-term projects.

Liquidity provision by central banks can help to prevent bank runs with inefficient early liquidation. However, the anticipation of unconditional liquidity provision results in overinvestment in risky activities (moral hazard), creating excessive exposure to systemic risk. We show that it is crucial for efficient lender-of-last-resort policy to impose ex ante minimum liquidity standards for banks. In addition, we analyze the impact of equity requirements. We show that it is even likely to be inferior to the outcome of a mixed-strategy equilibrium with free-riding banks. For similar reasons, imposing narrow banking (requiring banks to hold sufficient liquid funds to pay out in all contingencies) turns out to be strictly inferior relative to the combination of liquidity requirements with lender-of-last-resort policy.

In modern economies, a significant part of intermediation is provided by the shadow banking sector. These institutions (like hedge funds and investment banks) are not financed via deposits, but they are highly leveraged. Incentives to dance (to free-ride on liquidity provision) seem to be even stronger for the shadow banking industry. So imposing liquidity requirements only for the banking

sector will not be sufficient to cope with free-riding. In future work, we plan to analyze incentives for leveraged institutions within our framework.

Appendix 1. Proofs

Proof of Proposition 1

In the absence of aggregate risk, given p_s ($s \in \{H, L\}$), the social planner maximizes the investors' return by setting α_s such that

$$\alpha_s = \arg \max_{\alpha_s \in [0,1]} \gamma \left\{ \alpha_s R_1 + (1 - \alpha_s) \left[p_s R_2 + \frac{(1 - p_s) R_2}{r} \right] \right\}, \quad \text{with } r_s \geq 1. \tag{4}$$

Solve to get $\alpha_s = \frac{\gamma - p_s}{\gamma - p_s + (1-\gamma)\frac{R_1}{R_2}}$, with $r_s = 1$.

In the presence of aggregate risk, to find the social planner's optimal α which may depend on π, one just has to find the α that maximizes the investors' return for each $\pi \in [0, 1]$.

That the gross interest rate offered to the entrepreneurs at $t = 1$ is no less than 1 implies that for any given α, the investors' expected payoff is

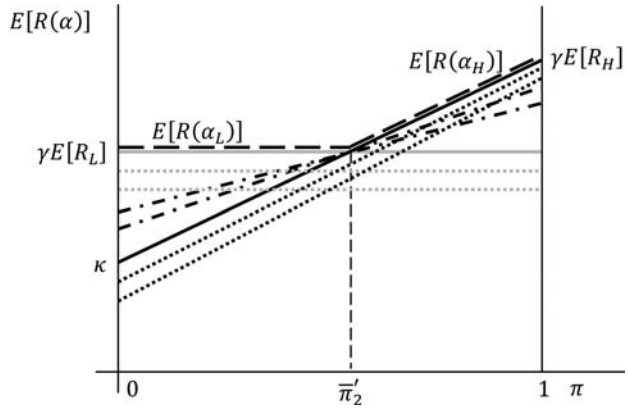
$$\begin{aligned} E[R(\alpha)] = {} & \pi \min\{\alpha R_1 + (1 - \alpha) p_H R_2, \gamma[\alpha R_1 + (1 - \alpha) R_2]\} \\ & + (1 - \pi) \min\{\alpha R_1 + (1 - \alpha) p_L R_2, \gamma[\alpha R_1 + (1 - \alpha) R_2]\}, \end{aligned}$$

which is linear in π. Then it is easy to depict $E[R(\alpha)]$ as a function of π, when $\alpha = \alpha_H$ or α_L, as figure 8 shows. These two lines intersect at $\overline{\pi}_2' = \frac{\gamma E[R_L] - \kappa}{\gamma E[R_H] - \kappa}$. Note that $E[R(\alpha_H)] = \gamma E[R_H] > \gamma E[R_L]$ when $\pi = 1$, and $E[R(\alpha_H)] = \kappa < \gamma E[R_L]$ when $\pi = 0$.

For any $\alpha \in (\alpha_L, 1]$, $E[R(\alpha)] = \gamma[\alpha R_1 + (1 - \alpha) R_2] < \gamma E[R_L]$, as shown by the dotted gray lines in figure 8. For any $\alpha \in [0, \alpha_H)$, $E[R(\alpha)] = \pi[\alpha R_1 + (1-\alpha) p_H R_2] + (1-\pi)[\alpha R_1 + (1-\alpha) p_L R_2]$. Note that $E[R(\alpha)] < \kappa$ when $\pi = 0$ and $E[R(\alpha)] < \gamma E[R_H]$ when $\pi = 1$, as shown by the dotted black lines in figure 8.

For any $\alpha \in (\alpha_H, \alpha_L)$, $E[R(\alpha)] = \pi\gamma[\alpha R_1 + (1 - \alpha) R_2] + (1 - \pi)[\alpha R_1 + (1 - \alpha) p_L R_2]$. Denote $\alpha R_1 + (1 - \alpha) R_2$ by $E[R_\alpha]$, and

Figure 8. The Investors' Expected Return for any $\alpha \in [0, 1]$



Notes: The solid gray line denotes $E[R(\alpha_L)]$, the black line denotes $E[R(\alpha_H)]$, the dotted gray lines denote those $E[R(\alpha)]$ with $\alpha \in (\alpha_L, 1)]$, the dotted black lines denote those $E[R(\alpha)]$ with $\alpha \in (0, \alpha_H]$, and the chain lines denote those $E[R(\alpha)]$ with $\alpha \in (\alpha_H, \alpha_L]$.

$\alpha R_1 + (1-\alpha)p_L R_2$ by κ'. Note that $\kappa < E[R(\alpha)] < \gamma E[R_L]$ when $\pi = 0$ and $\gamma E[R_L] < E[R(\alpha)] < \gamma E[R_H]$ when $\pi = 1$. Such $E[R(\alpha)]$ are depicted as the chain lines in figure 8.

Suppose that the intersection between $E[R(\alpha)]$ and $E[R(\alpha_L)]$ is $\overline{\pi}''_2 = \frac{\gamma E[R_L]-\kappa'}{\gamma E[R_\alpha]-\kappa'}$. To determine the value of $\overline{\pi}''_2$, note that $\overline{\pi}''_2 \gtrless \overline{\pi}'_2$ only if $\frac{\gamma E[R_L]-\kappa'}{\gamma E[R_\alpha]-\kappa'} \gtrless \frac{\gamma E[R_L]-\kappa}{\gamma E[R_H]-\kappa}$. This is equivalent to

$$\gamma E[R_L](\gamma E[R_H] - \gamma E[R_\alpha]) + (\gamma E[R_\alpha] - \gamma E[R_L])\kappa + (\gamma E[R_L] - \gamma E[R_H])\kappa' \gtrless 0. \tag{5}$$

Using the fact that $\gamma E[R_s] = \alpha_s R_1 + (1-\alpha_s)p_s R_2$ $(s \in \{H, L\})$, the left-hand side of the inequality (5) can be written as

$$\gamma(R_1 - p_L R_2)\{E[R_H](\alpha_L - \alpha) - E[R_\alpha](\alpha_L - \alpha_H) + E[R_L](\alpha - \alpha_H)\}.$$

Further, since $\alpha \in (\alpha_H, \alpha_L)$, we can replace α with the linear combination of α_H and α_L, $\alpha = \omega\alpha_H + (1-\omega)\alpha_L$ with $\omega \in (0, 1)$. It is easily seen that

$$\gamma(R_1 - p_L R_2)\{E[R_H](\alpha_L - \alpha) - E[R_\alpha](\alpha_L - \alpha_H) + E[R_L](\alpha - \alpha_H)\} = 0,$$

which implies that $\overline{\pi}_2'' = \overline{\pi}_2'$.

Combining all the cases, figure 8 shows the investors' expected return for any $\alpha \in [0, 1]$. The social planner's optimal solution is given by the frontier of the investors' expected return (as shown by the dashed black lines in figure 8), which is a state-contingent strategy depending on the probability π: The planner invests the share α_H in liquid projects as long as $\overline{\pi}_2' \leq \pi \leq 1$, and the share α_L in liquid projects as long as $0 \leq \pi < \overline{\pi}_2'$.

Proof of Proposition 2

To show that the optimal allocation of the central planner's problem is supported by the market equilibrium, one has to show that (i) the allocation is feasible in the market economy, and (ii) it is not profitable to unilaterally deviate from such allocation.

In the planner's economy, the central planner picks up the optimal α_s as equation (4) suggests, and transfers the maximized return to the investors. This coincides with equations (1) and (2), implying that claim (i) holds.

To show that claim (ii) holds, suppose an arbitrary bank i deviates from such allocation by choosing $\alpha_i \neq \alpha_s$ for a given $s \in \{H, L\}$:

(i) If $\alpha_i < \alpha_s$, by the market clearing condition, the liquidity market interest rate r' at $t = 1$ is now determined by

$$r'\{(1-\gamma)[\alpha_i R_1 + (1-\alpha_i)p_s R_2] + (N-1)(1-\gamma)[\alpha_s R_1 + (1-\alpha_s)p_s R_2]\} = \gamma(1-\alpha_i)(1-p_s)R_2 + (N-1)\gamma(1-\alpha_s)(1-p_s)R_2.$$

Comparing with the condition in the central planner's problem in which $r = 1$,

$$r(1-\gamma)[\alpha_s R_1 + (1-\alpha_s)p_s R_2] = \gamma(1-\alpha_s)(1-p_s)R_2,$$

one can see that $r' > 1$. For the non-deviators, the depositors' return becomes

$$\gamma \left\{ \alpha_s R_1 + (1 - \alpha_s) \left[p_s R_2 + \frac{(1 - p_s) R_2}{r'} \right] \right\} < d_0.$$

Knowing that the non-deviators will not be able to meet the contracted d_0 at $t = 1$, the depositors will only deposit at bank i at $t = 0$. If so, the deposit return that bank i can offer is at maximum

$$d_{0i} = \alpha_i R_1 + (1 - \alpha_i) p_s R_2 < d_0,$$

implying that the deviator gets worse off.

(ii) If $\alpha_i > \alpha_s$, the aggregate liquidity supply at $t = 1$ exceeds the the aggregate liquidity demand because

$$\begin{aligned}
&(1-\gamma)[\alpha_i R_1 + (1-\alpha_i) p_s R_2] + (N-1)(1-\gamma)[\alpha_s R_1 + (1-\alpha_s) p_s R_2] \\
&\quad > N(1-\gamma)[\alpha_s R_1 + (1-\alpha_s) p_s R_2] \\
&\quad = N\gamma(1-\alpha_s)(1-p_s) R_2.
\end{aligned}$$

Therefore, the liquidity market interest rate remains at $r = 1$ and the non-deviators are able to meet d_0. However, the deposit return that bank i can offer is

$$d_{0i} = \gamma[\alpha_i R_1 + (1 - \alpha_i) R_2] < \gamma[\alpha_s R_1 + (1 - \alpha_s) R_2] = d_0,$$

implying that the deviator will not get any deposit at $t = 0$ and is hence worse off.

Therefore, the planner's optimal allocation is indeed supported by the market equilibrium.

Proof of Proposition 3

The mixed-strategy equilibrium, proposition 3(i)–3(iii)(c), is characterized as proposition 2 of Cao and Illing (2008). By choosing to hold a share of safe assets—call it α_s^*—a prudent bank should have equal return at both states, $d_0^s = d_0^s(p_H) = d_0^s(p_L)$, i.e.,

$$\gamma\left[\alpha_s^* R_1 + \left(1-\alpha_s^*\right)p_H R_2 + \frac{\left(1-\alpha_s^*\right)(1-p_H)R_2}{r_H}\right]$$
$$= \gamma\left[\alpha_s^* R_1 + \left(1-\alpha_s^*\right)p_L R_2 + \frac{\left(1-\alpha_s^*\right)(1-p_L)R_2}{r_L}\right].$$

With some simple algebra, this is equivalent to

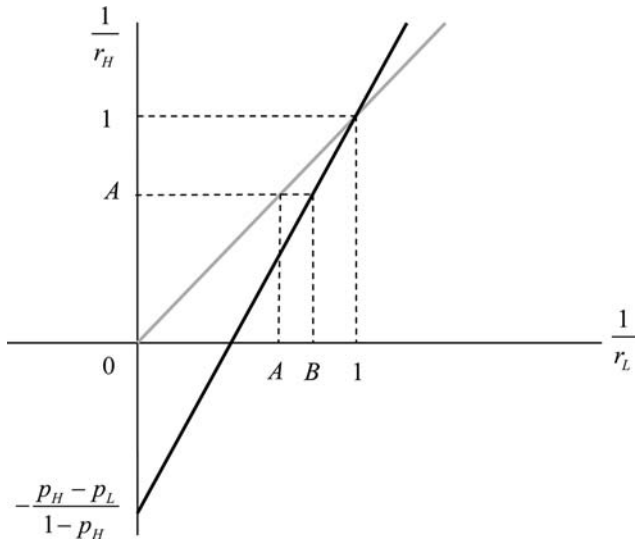
$$\frac{1}{r_H} = \frac{1-p_L}{1-p_H}\frac{1}{r_L} - \frac{p_H - p_L}{1-p_H}.$$

Plot $\frac{1}{r_H}$ as a function of $\frac{1}{r_L}$, as the solid black line in figure 9 shows.

The slope $\frac{1-p_L}{1-p_H} > 1$ and intercept $-\frac{p_H-p_L}{1-p_H} < 0$, and the line goes through $(1,1)$. But $r_H = r_L = 1$ cannot be the equilibrium outcome here, because α_L is the dominant strategy in this case and subject to deviation. So whenever $r_H > 1$ (suppose $\frac{1}{r_H} = A$ in the graph), there must be $r_H > r_L > 1$ (because $\frac{1}{r_H} < \frac{1}{r_L} = B < 1$).

At p_L, given that $r_L > 1$, the prudent bank's return is equal to $d_0^s = \kappa(\alpha_s^*(p_L, r_L)) < \kappa(\alpha_L)$, since the latter maximizes the bank's

Figure 9. Higher Interest Rates in the Mixed-Strategy Equilibrium



expected return with $r^* = 1$ by lemma 2 of Cao and Illing (2008). Therefore in the mixed-strategy equilibrium, investors are worse off than if all banks would coordinate on the prudent strategy α_L.

Proof of Proposition 4

The central bank's optimal policy is to restore the constrained efficiency, as stated in proposition 1. Therefore, the optimal liquidity requirement, which is captured by $\underline{\alpha}$, should be exactly the same as α_H (α_L) for high (low) π. So is it with r^M.

In addition, any bank that observes $\underline{\alpha}$ will get bailed out whenever necessary. This only happens when a bank follows $\underline{\alpha} = \alpha_H$ but p_L is revealed. In this case, the investors will get $\kappa = \alpha_H R_1 + (1 - \alpha_H) p_L R_2$ real return plus $d_0 - \kappa$ fiat money if they do not run on the bank at $t = \frac{1}{2}$. In contrast, the investors will only get the liquidated value $c < 1 < \kappa$ as the real return if they run on the bank. Of course, they will wait instead of run.

Proof of Proposition 5

Suppose that a representative bank chooses to be prudent with $\alpha_i = \underline{\alpha}$, and promises a nominal deposit contract $d_0^i = \gamma[\underline{\alpha} R_1 + (1 - \underline{\alpha}) R_2]$ in order to maximize its investors' return. Then when the bad state with high liquidity needs is realized, the central bank has to inject enough liquidity into the market to keep the interest rate at $r = 1$ in order to ensure bank i's survival. However, given $r = 1$, a naughty bank j can always profit from setting $\alpha_j = 0$, promising the nominal return $d_0^j = \gamma R_2 > d_0^i$ to its investors. Thus, surely the banks prefer to play naughty.

For those parameter values such that $\pi p_H R_2 + (1 - \pi) p_L R_2 < 1$, there exists no equilibrium with liquidity injection. The reason is the following:

(i) Any symmetric strategic profile cannot be in equilibrium, because
   (a) if there is no trade under such strategic profile—i.e., α is so small that the real return is less than 1—one bank can deviate by setting $\alpha = 1$ and trading with investors;
   (b) if there is trade under such strategic profile—i.e., $\alpha > 0$ for all the banks—then one bank can deviate by setting

$\alpha = 0$ and getting higher nominal return than the other banks.

(ii) Any asymmetric strategic profile, or profile of mixed strategies, cannot be in equilibrium, because
   (a) if there is no trade under such strategic profile, then the argument of (i)(a) applies here;
   (b) if there is trade under such strategic profile, then one bank can deviate by choosing a pure strategy, $\alpha = 0$, and get better off—there is no reason to mix with the other dominated strategies.

Proof of Lemma 1

When $\pi = 0$,

$$d_0 + \frac{\Pi}{2} \cdot 0 = \alpha_H R_1 + (1 - \alpha_H) p_L R_2 < \alpha_L R_1 + (1 - \alpha_L) p_L R_2 = \gamma E[R_L].$$

When $\pi = 1$,

$$d_0 + \frac{\Pi}{2} = \frac{\alpha_H R_1 + (1 - \alpha_H) p_L R_2 + \alpha_H R_1 + (1 - \alpha_H) p_H R_2}{2} < \alpha_H R_1 + (1 - \alpha_H) p_H R_2 = \gamma E[R_H].$$

Proof of Proposition 6

As lemma 1 shows, the investors' expected return with equity requirements $d_0 + \frac{\Pi}{2}\pi$ is a linear increasing function of π, starting from $d_0 < \gamma E[R_L]$ when $\pi = 0$ and ending with $d_0 + \frac{\Pi}{2} < \gamma E[R_H]$ when $\pi = 1$. Whether imposing equity requirements improves investors' expected return (such "improved" region is denoted by the interval $(\tilde{\pi}_1', \tilde{\pi}_2')$ in figure 5) depends on the intersection between $d_0 + \frac{\Pi}{2}\pi$ and $\gamma E[R_L]$, denoted by A as in figure 5. Generically, there are three cases concerning the relative positions of $\Pi(\pi)$ and $\Pi_e(\pi)$:

(i) As figure 5A shows, when $A \in (0, \overline{\pi}_1]$, $\Pi_e(\pi)$ is higher than $\Pi(\pi)$ for $\pi \in [\overline{\pi}_1, \overline{\pi}_2]$.

(ii) As figure 5B shows, when $A \in (\overline{\pi}_1, \overline{\pi}_2)$, $\Pi_e(\pi)$ is only higher than $\Pi(\pi)$ for part of $\pi \in [\overline{\pi}_1, \overline{\pi}_2]$. In addition, $\Pi_e(\pi)$ is higher for part of $\pi \in (\overline{\pi}_2, 1]$.

(iii) As figure 5C shows, when $A \geq \overline{\pi}_2$, $\Pi_e(\pi)$ is only higher than $\Pi(\pi)$ for part of $\pi \in [\overline{\pi}_1, \overline{\pi}_2]$. In addition, $\Pi_e(\pi)$ is no higher for all $\pi \in (\overline{\pi}_2, 1]$.

Appendix 2. Effectiveness Conditions for Equity Requirements

To economize the notations, define the investors' expected return function in the market equilibrium as follows:

Definition 1. *Define a representative investor's expected return function without equity requirements as* $\Pi(\pi)$*, such that*

$$\Pi(\pi, \cdot) = \begin{cases} \gamma E[R_L], & \text{if} \quad \pi \in [0, \overline{\pi}_1]; \\ \alpha_s^* R_1 + \big(1 - \alpha_s^*\big) p_L R_2, & \text{if} \quad \pi \in (\overline{\pi}_1, \overline{\pi}_2); \\ \gamma E[R_H]\pi + (1 - \pi)c, & \text{if} \quad \pi \in [\overline{\pi}_2, 1] \end{cases}$$

and his or her expected return function under equity requirements as $\Pi_e(\pi)$*, as well as the set* S *in which the investor's payoff is improved under equity requirements, such that*

$$S := \{\hat{\pi} | \Pi_e(\hat{\pi}) \geq \Pi(\hat{\pi})\}.$$

In the following, we are interested in the cases, captured in the set S (denoted by the area $(\tilde{\pi}_1', \tilde{\pi}_2')$ in figure 5), where the banking system with equity requirements outperforms that in the market equilibrium.

Denote the intersection of $\Pi_e(\pi) = d_0 + \frac{\Pi}{2}\pi$ and $\gamma E[R_L]$ by A, which is equal to (see the proof below for detail)

$$A = \frac{2(R_1 - p_L R_2)}{(1 - \gamma)R_1 + (\gamma - p_L)R_2},$$

as well as the intersection of $\Pi_e(\pi) = d_0 + \frac{\Pi}{2}\pi$ and $\gamma E[R_H]\pi + (1 - \pi)c$ by B, which is equal to (see the proof below for detail)

$$B = \frac{2[(1-\gamma)(cR_1 - p_L R_1 R_2) + (\gamma - p_H)(cR_2 - R_1 R_2)]}{2(1-\gamma)cR_1 + 2(\gamma - p_H)cR_2 + [\gamma(p_H - 1) - (\gamma - p_H) - (1-\gamma)p_L]R_1 R_2}.$$

Then proposition 7 characterizes the improvement in investors' payoff achieved by introducing equity requirements.

PROPOSITION 7. *Given the equity requirement k imposed by the regulator,*

(i) When $A \in (0, \overline{\pi}_1]$, i.e.,

$$(2\gamma R_2 - \gamma E[R_H] - d_0)(\gamma E[R_L] - d_0) + (2\gamma E[R_L] - \gamma E[R_H] - d_0)(d_0 - c) \leq 0,$$

then $S = [A, B] \supseteq [\overline{\pi}_1, \overline{\pi}_2]$;

(ii) When $A \in (\overline{\pi}_1, \overline{\pi}_2)$, i.e.,

$$(2\gamma R_2 - \gamma E[R_H] - d_0)(\gamma E[R_L] - d_0) + (2\gamma E[R_L] - \gamma E[R_H] - d_0)(d_0 - c) > 0,$$

and

$$\gamma(E[R_H] - E[R_L])(d_0 - c) \geq (\gamma E[R_H] - c)(\gamma E[R_L] - d_0),$$

then $S = [\tilde{\pi}, B]$ in which $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2)$ and $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$;

(iii) When $A \geq \overline{\pi}_2$, i.e.,

$$2(\gamma E[R_L] - d_0)(\gamma E[R_H] - c) \geq (\gamma E[R_H] - d_0)(\gamma E[R_L] - c),$$

then $S \subseteq [\tilde{\pi}, B]$ in which $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2)$ and $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$.

Proof. The intersection A takes the value of π, such that

$$\gamma E[R_L] = d_0 + \frac{\Pi}{2}\pi.$$

Solve to get

$$A = \frac{2(\gamma E[R_L] - d_0)}{\gamma E[R_H] - d_0} = \frac{2(R_1 - p_L R_2)}{(1-\gamma)R_1 + (\gamma - p_L)R_2}.$$

The intersection B takes the value of π, such that

$$\gamma E[R_H]\pi + (1-\pi)c = d_0 + \frac{\Pi}{2}\pi.$$

Solve to get

$$B = \frac{d_0 - c}{\frac{\gamma E[R_H]+d_0}{2} - c}$$
$$= \frac{2[(1-\gamma)(cR_1 - p_L R_1 R_2) + (\gamma - p_H)(cR_2 - R_1 R_2)]}{2(1-\gamma)cR_1 + 2(\gamma - p_H)cR_2 + [\gamma(p_H - 1) - (\gamma - p_H) - (1-\gamma)p_L]R_1 R_2}.$$

Then the set S can be determined in each case:

(i) As figure 5A shows, when $A \in (0, \overline{\pi}_1]$,

$$\frac{2(\gamma E[R_L] - d_0)}{\gamma E[R_H] - d_0} \leq \overline{\pi}_1 = \frac{\gamma E[R_L] - c}{\gamma R_2 - c}.$$

Rearrange to get

$$\begin{aligned}(2\gamma R_2 - \gamma E[R_H] - d_0)(\gamma E[R_L] - d_0) \\ + (2\gamma E[R_L] - \gamma E[R_H] - d_0)(d_0 - c) \quad \leq 0.\end{aligned}$$

Since $\Pi_e(\pi)$ is strictly increasing in π, then

$$\begin{aligned}\Pi_e(\pi)|_{\pi=B} > \Pi_e(\pi)|_{\pi=A} \geq \gamma E[R_L]|_{\pi=\overline{\pi}_1} \\ = [\gamma E[R_H]\pi + (1-\pi)c]|_{\pi=\overline{\pi}_2} \\ \geq \Pi(\pi)|_{\pi\in[\overline{\pi}_1, \overline{\pi}_2]},\end{aligned}$$

which implies $S = [A, B] \supseteq [\overline{\pi}_1, \overline{\pi}_2]$;

(ii) As figure 5B shows, when $A \in (\overline{\pi}_1, \overline{\pi}_2]$,

$$\overline{\pi}_1 = \frac{\gamma E[R_L] - c}{\gamma R_2 - c} < \frac{2(\gamma E[R_L] - d_0)}{\gamma E[R_H] - d_0}.$$

Rearrange to get

$$\begin{aligned}(2\gamma R_2 - \gamma E[R_H] - d_0)(\gamma E[R_L] - d_0) \\ + (2\gamma E[R_L] - \gamma E[R_H] - d_0)(d_0 - c) \quad > 0.\end{aligned}$$

What's more, in this case $B \in [\overline{\pi}_2, 1]$, and this is equivalent to

$$\frac{\gamma E[R_L] - c}{\gamma E[R_H] - c} = \overline{\pi}_2 < \frac{d_0 - c}{\frac{\gamma E[R_H] + d_0}{2} - c}.$$

Rearrange to get

$$\gamma(E[R_H] - E[R_L])(d_0 - c) \geq (\gamma E[R_H] - c)(\gamma E[R_L] - d_0).$$

Similarly,

$$\begin{aligned}\Pi_e(\pi)|_{\pi \leq A} &\leq \gamma E[R_L]|_{\pi=\overline{\pi}_1} = [\gamma E[R_H]\pi + (1-\pi)c]|_{\pi=\overline{\pi}_2} \\ &\leq \Pi(\pi)|_{\pi \in [\overline{\pi}_2, B]} \\ &\leq \Pi_e(\pi)|_{\pi \geq B},\end{aligned}$$

which implies $S = [\tilde{\pi}, B]$ in which $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2]$ and $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$;

(iii) As figure 5C shows, when $A \geq \overline{\pi}_2$,

$$\overline{\pi}_2 = \frac{\gamma E[R_L] - c}{\gamma E[R_H] - c} \leq \frac{2(\gamma E[R_L] - d_0)}{\gamma E[R_H] - d_0}.$$

Rearrange to get

$$2(\gamma E[R_L] - d_0)(\gamma E[R_H] - c) \geq (\gamma E[R_H] - d_0)(\gamma E[R_L] - c).$$

Similarly,

$$\begin{aligned}&\Pi_e(\pi)|_{\pi \leq B} < \Pi_e(\pi)|_{\pi \geq A} \leq \gamma E[R_L]|_{\pi=\overline{\pi}_1} \\ &= [\gamma E[R_H]\pi + (1-\pi)c]|_{\pi=\overline{\pi}_2},\end{aligned}$$

which implies $S \subseteq [\tilde{\pi}, B]$ in which $\tilde{\pi} \in (\overline{\pi}_1, \overline{\pi}_2]$ and $S \bigcap [\overline{\pi}_1, \overline{\pi}_2] = [\tilde{\pi}, \overline{\pi}_2]$.

Appendix 3. Numerical Examples

The following figures present numerical illustrations representing the three different cases.

Figure 10. Investors' Expected Return in the Market Equilibrium (Solid Gray Lines)/under Equity Requirements (Solid Black Lines), with $p_H = 0.3$, $p_L = 0.25$, $\gamma = 0.6$, $R_1 = 1.8$, $R_2 = 5.5$, and $c = 0.9$

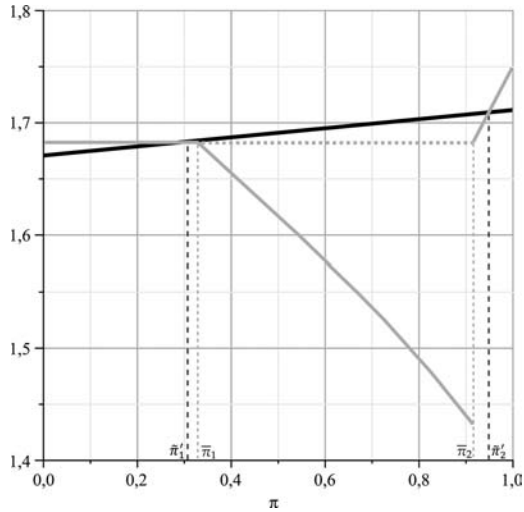


Figure 11. Investors' Expected Return in the Market Equilibrium (Solid Gray Lines)/under Equity Requirements (Solid Black Lines), with $p_H = 0.4$, $p_L = 0.3$, $\gamma = 0.6$, $R_1 = 2$, $R_2 = 4$, and $c = 0.8$

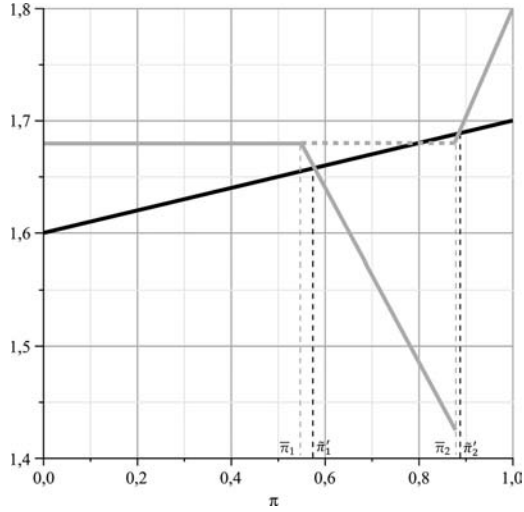
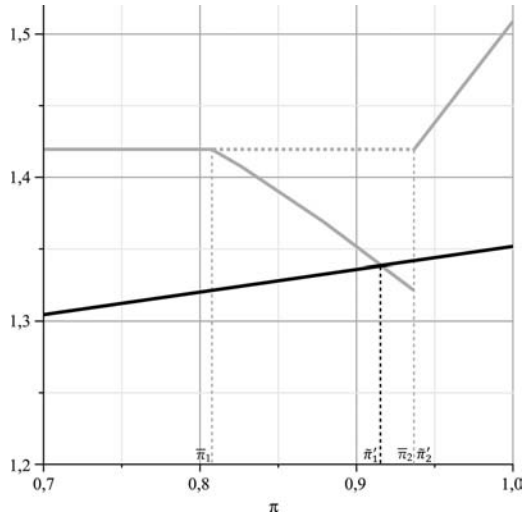


Figure 12. Investors' Expected Return in the Market Equilibrium (Solid Gray Lines)/under Equity Requirements (Solid Black Lines), with $p_H = 0.5$, $p_L = 0.25$, $\gamma = 0.7$, $R_1 = 1.8$, $R_2 = 2.5$, and $c = 0$



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