Volume 5, Number 4 December 2009

IJCB
INTERNATIONAL JOURNAL OF CENTRAL BANKING
Volume 5, Number 4
December 2009

IJCB

INTERNATIONAL JOURNAL OF CENTRAL BANKING

Special Issue: Provision and Pricing of Liquidity Insurance

IJCB International Journal of Central Banking

Volume 5, Number 4 December 2009

Special Issue: Provision and Pricing of Liquidity Insurance

The contents of this journal, together with additional materials provided by article authors, are available without charge at ***www.ijcb.org****.*

The *International Journal of Central Banking* is published quarterly (ISSN: 1815-4654). Online access to the publication is available free of charge at **www.ijcb.org**. Individual print subscriptions are available at an annual rate of $100 (USD).

Print subscription orders may be placed online at www.ijcb.org, by phone (+49 69 1344 7623), via fax (+49 69 1344 8553), or by e-mail (editor@ijcb.org).

Renewals, claims, address changes, and requests for permission to reprint material from this journal should be addressed to:

International Journal of Central Banking
DG Research
European Central Bank
Postfach 16 03 19
D-60066 Frankfurt
Germany

Phone: +49 69 1344 7623
Fax: +49 69 1344 6575
E-mail: editor@ijcb.org

ISSN: 1815-4654

Provision and Pricing of Liquidity Insurance

Introduction to a Special Issue of the International Journal of Central Banking

Douglas Gale, Rafael Repullo, Til Schuermann, and Frank Smets

The evaporation of liquidity has been at the center of the financial crisis that erupted in August 2007 and intensified following the collapse of a number of financial institutions in October 2008. Banks and other financial institutions with funding needs faced increasingly serious problems in drawing funds from the interbank money market, triggering central banks to intervene as lenders of last resort. At the same time, market liquidity dried up in a number of markets such as the one for asset-backed securities, making it more difficult for financial institutions to off-load the more complex assets on their balance sheet. Again, in a number of cases central banks stepped in to become market makers of last resort. The drying up of both funding and market liquidity and their interaction played an important role in the propagation of the crisis.

This volume of the *International Journal of Central Banking* (IJCB) deals with some of the key aspects of the provision and pricing of liquidity insurance. It contains the papers, discussions, and commentaries that were presented at the inaugural Financial Stability Conference organized by the IJCB and held at the Federal Reserve Bank of New York on June 11, 2009. The purpose of the IJCB conferences is to stimulate research on issues of topical importance for central banking and provide a forum for interaction between central bank researchers, academics, and policymakers. In addition to the contributions published in this volume, the conference also contained a policy panel, in which Nigel Jenkinson (Bank of England), Martin Helwig (Max Planck Institute), and Eli Remolona (Bank for International Settlements) debated the need for adjusting the global regulatory framework in order to ensure stronger liquidity buffers. In addition, Ben Bernanke (Chairman of the Federal Reserve Board) gave the dinner speech, discussing the response of the Federal Reserve to the financial crisis through the lens of the central bank's balance sheet and addressing how the growth of the Federal Reserve's balance sheet could be reversed in case the need for an exit from the credit-easing policies was needed.

The first paper, "Interbank Lending, Credit-Risk Premia, and Collateral" by Florian Heider and Marie Hoerova (European Central Bank), provides a model that illustrates how tensions in the unsecured interbank market and the market secured by risky collateral affect repo rates in the market for safe collateral. The scarcity of safe collateral exacerbates the volatility of repo rates. The model generates empirical predictions that are in line with developments during the 2007–09 financial crisis. This paper is discussed by Ernst-Ludwig von Thadden (University of Mannheim). The second paper, "Liquidity, Moral Hazard, and Interbank Market Collapse" by Enisse Kharroubi and Edouard Vidon (Banque de France), discusses the incentives of banks to provide liquidity in a model where banks face moral hazard when confronted with liquidity shocks and those shocks are private information. The paper shows how the collapse of the interbank market for liquidity can occur in equilibrium and that the likelihood of such an equilibrium is higher when the individual probability of the liquidity shock is lower. The paper is discussed by Tano Santos (Columbia University).

The third paper, "Credit, Asset Prices, and Financial Stress" by Miroslav Misina and Greg Tkacz (Bank of Canada), asks whether some combination of measures of credit and asset prices can help predict periods of financial stress in Canada. They find that at the two-year horizon, credit and real-estate prices emerge as important predictors of financial stress, confirming some of the general findings in the early-warning literature. The paper is discussed by Stijn Claessens (International Monetary Fund). In their commentary, "Prices and Quantities in the Monetary Policy Transmission Mechanism," Tobias Adrian (Federal Reserve Bank of New York) and Hyun Song Shin (Princeton University) argue in favor of a revaluation of credit quantities and factors affecting the supply of credit in the conduct of monetary policy. They emphasize and illustrate fluctuations in leverage and the associated changes in haircuts in collateralized credit markets and argue that central banks' interest rate and liquidity management policies should take into account leverage in the pursuit of price and financial stability.

The fourth paper, "Strategic Trading in Multiple Assets and the Effects on Market Volatility" by Chenghuan Sean Chu, Andreas Lehnert, and Wayne Passmore (Federal Reserve Board), studies equilibrium price dynamics in an environment where some distressed firms are forced to sell assets and others specialize in strategic trading around such fire sales. The model is used to conduct a welfare analysis of extending government guarantees to less liquid securities and allows one to consider the optimal exit strategy for a government that accumulates assets on its balance sheet. The paper is discussed by Bruce Carlin (UCLA). In his commentary, "When Everyone Runs for the Exit," Lasse Pedersen (New York University) analyzes the dangers of rushing for the exit in financial markets. He analyzes why people crowd into trades, why they run, what determines the risk, whether to trade when the dust settles, and how much to pay for assets in light of this risk.

Interbank Lending, Credit-Risk Premia, and Collateral*

Florian Heider and Marie Hoerova
European Central Bank

We study the functioning of secured and unsecured interbank markets in the presence of credit risk. The model generates empirical predictions that are in line with developments during the 2007–09 financial crisis. Interest rates decouple across secured and unsecured markets following an adverse shock to credit risk. The scarcity of underlying collateral may amplify the volatility of interest rates in secured markets. We use the model to discuss various policy responses to the crisis.

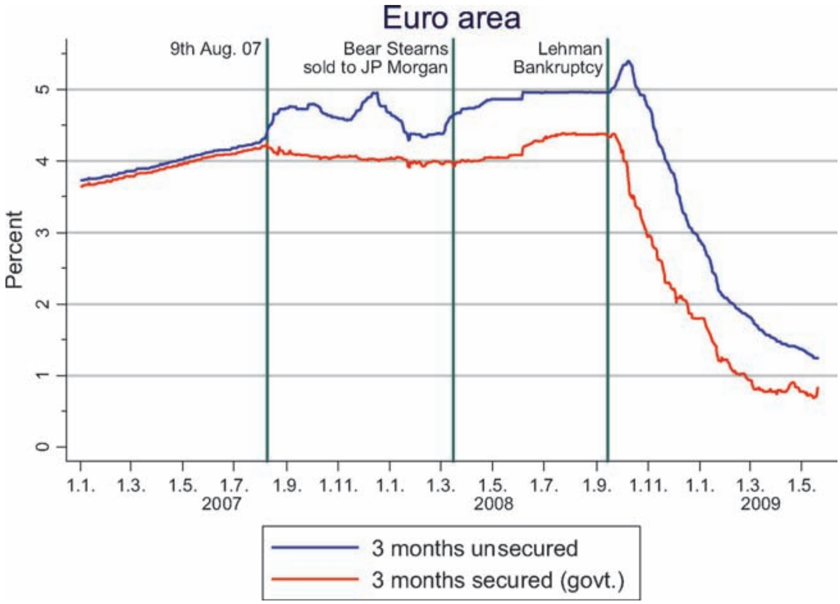
JEL Codes: G01, G21, E58.

1. Introduction

Interbank markets play a key role in the financial system. They are vital for banks' liquidity management. Secured, or repo, markets have been a fast-growing segment of money markets: they have doubled in size since 2002 with gross amounts outstanding of about $10 trillion in the United States and comparable amounts in the euro area just prior to the start of the crisis in August 2007. Since repo transactions are backed by collateral securities similar to those used in the central bank's refinancing operations, repo markets are a key part of the transmission of monetary policy. At the same time, the interest rate in the unsecured three-month interbank market acts

*We thank Douglas Gale, Rafael Repullo, Elu von Thadden, and seminar participants at the European Central Bank and the Federal Reserve Bank of New York (conference on "Pricing and Provision of Liquidity Insurance") for helpful comments. Dimitrios Rakitzis and Francesca Fabbri provided excellent research assistance. The views expressed do not necessarily reflect those of the European Central Bank or the Eurosystem. The authors are at the European Central Bank, Financial Research Division, Kaiserstrasse 29, D-60311 Frankfurt; E-mails: florian.heider@ecb.int and marie.hoerova@ecb.int.

Figure 1. Decoupling of Secured and Unsecured Interbank Rates in the Euro Area

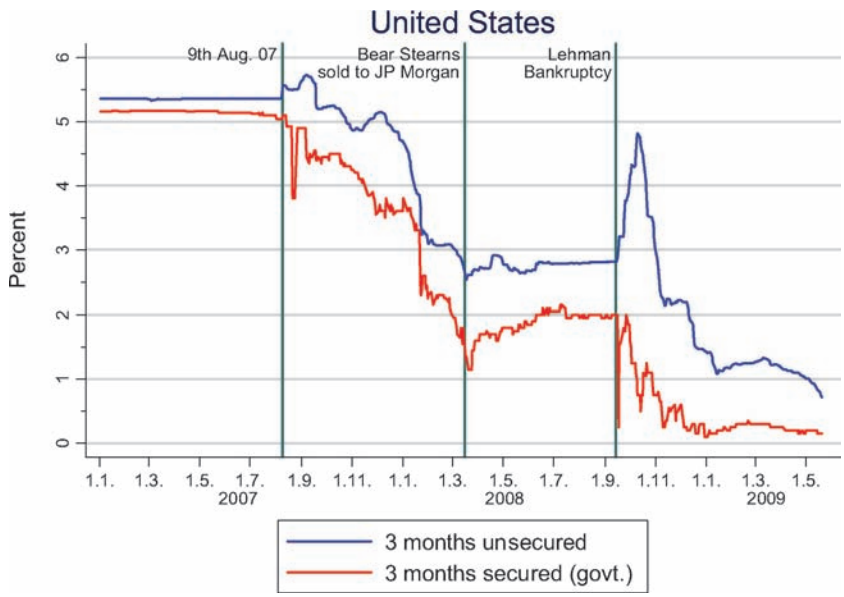


as a benchmark for pricing fixed-income securities throughout the economy.

In normal times, interbank markets function smoothly. Rates are broadly stable across secured and unsecured segments, as well as across different collateral classes. Since August 2007, however, the functioning of interbank markets has become severely impaired around the world. The tensions in the interbank markets have become a key feature of the 2007–09 crisis (see, for example, Allen and Carletti 2008, and Brunnermeier 2009).

One striking manifestation of the tensions in the interbank markets has been the decoupling of interest rates between secured and unsecured markets. Figure 1 shows the unsecured and secured (by government securities) three-month interbank rates for the euro area since January 2007. Prior to the outbreak of the crisis in August 2007, the rates were closely tied together. In August 2007, they moved in opposite directions, with the unsecured rate increasing and the secured rate decreasing. They decoupled again following the Lehman bankruptcy and, to a lesser extent, just prior to the sale of Bear Stearns.

Figure 2. Decoupling of Secured and Unsecured Interbank Rates in the United States



A second, related feature of the tensions in the interbank markets has been the difference in the severity of the disruptions in the United States and in the euro area. Figure 2 shows rates in secured and unsecured interbank markets in the United States. As in the euro area, there is a decoupling of the rates at the start of the financial crisis and a further divergence after the sale of Bear Stearns and the bankruptcy of Lehman. However, the decoupling and the volatility of the rates is more pronounced than in the euro area.

Why have secured and unsecured interbank interest rates decoupled? Why has the U.S. repo market experienced significantly more disruptions than the euro-area market? What underlying friction can explain these developments? And what policy responses are possible to tackle the tensions in interbank markets?

To examine these questions, we present a model of interbank markets with both secured and unsecured lending in the presence of credit risk. Credit risk and the accompanying possibility of default, stemming from the complexity of securitization, was at the heart of the financial crisis (see Gorton 2008, 2009, and Taylor 2009). We model the interbank market in the spirit of Bhattacharya and Gale (1987), who in turn build on Diamond and Dybvig (1983). Banks face liquidity demand of varying intensity. Some may need to realize cash quickly due to demands of customers who draw on committed lines of credit or on their demandable deposits. Since idiosyncratic liquidity shocks are noncontractible, this creates a scope for an interbank market where banks with excess liquidity trade with banks in need of liquidity.

Banks can invest in liquid assets (cash), illiquid assets (loans), and in bonds. In their portfolio choice, they face a trade-off between liquidity and return. Illiquid investments are profitable but risky.[1] Banks can obtain funding liquidity in the unsecured interbank market by issuing claims on the illiquid investment, which has limited market liquidity.[2] Due to the risk of illiquid investments, banks may become insolvent and thus unable to repay their interbank loan. This makes unsecured interbank lending risky. To compensate lenders, borrowers have to pay a premium for funds obtained in the unsecured interbank market.

To model the secured interbank market, we introduce bonds that provide a positive net return in the long run. Unlike the illiquid asset, they can also be traded for liquidity in the short term. We consider the case of safe bonds, e.g., government bonds. Since unsecured borrowing is costly due to credit risk, banks in need of liquidity will sell bonds to reduce their borrowing needs. We assume that government bonds are in fixed supply and that they are scarce enough not to crowd out the unsecured market. The risk of banks' illiquid assets will affect the price of safe government bonds since banks with a liquidity surplus must be willing to both buy the bonds offered and lend in the unsecured interbank market. In equilibrium there must not be an arbitrage opportunity between secured and unsecured lending. We use the link between secured and unsecured markets to derive a number of empirical predictions.

[1]Illiquidity as a key factor contributing to the fragility of modern financial systems is emphasized by Diamond and Rajan (2008) and Brunnermeier (2009), for example.

[2]See also Brunnermeier and Pedersen (2009), who distinguish between market liquidity and funding liquidity.

This paper is part of a growing literature analyzing the ability of interbank markets to smooth out liquidity shocks. We use the framework developed by Freixas and Holthausen (2005), who examine the scope for the integration of unsecured interbank markets when cross-country information is noisy. They show that introducing secured interbank markets reduces interest rates and improves conditions when unsecured markets are not integrated. Their introduction may, however, hinder the process of integration.

Several recent papers examine various frictions in interbank markets that can justify a policy intervention. The role of asymmetric information about credit risk is emphasized in Heider, Hoerova, and Holthausen (2009). The model generates several possible regimes in the interbank market, including one in which trading breaks down. The regimes are akin to the developments prior to and during the 2007–09 financial crisis. Imperfect competition is examined in Acharya, Gromb, and Yorulmazer (2008). If liquidity-rich banks use their market power to extract surplus from liquidity-poor banks, a central bank can provide an outside option for the latter. Freixas, Martin, and Skeie (2008) show that when multiple, Pareto-ranked equilibria exist in the interbank market, a central bank can act as a coordination device for market participants and ensure that a more efficient equilibrium is reached. Freixas and Jorge (2008) analyze the effects of interbank market imperfections for the transmission of monetary policy. Bruche and Suarez (2009) explore implications of deposit insurance and spatial separation for the ability of money markets to smooth out regional differences in savings rates. Acharya, Shin, and Yorulmazer (2009) study the effects of financial crises and their resolution on banks' choice of liquid asset holdings. In Allen, Carletti, and Gale (2009), secured interbank markets can be characterized by excessive price volatility when there is a lack of opportunities for hedging aggregate and idiosyncratic liquidity shocks. By using open-market operations, a central bank can reduce price volatility and improve welfare.[3]

[3]Aggregate shortages are also examined in Diamond and Rajan (2005) where bank failures can be contagious due to a shrinking of the pool of available liquidity. Freixas, Parigi, and Rochet (2000) analyze systemic risk and contagion in a financial network and its ability to withstand the insolvency of one bank. In Allen and

The remainder of the paper is organized as follows. In section 2, we describe the setup of the model. In section 3, we solve the benchmark case when banks can only trade in the unsecured interbank market. In section 4, we allow banks to invest in safe bonds. In section 5, we present empirical implications and relate them to the developments during the 2007–09 financial crisis. In section 6, we discuss policy responses to mitigate the tensions in interbank markets, and in section 7 we offer concluding remarks. All proofs are in the appendix.

2. The Model

The model is based on Freixas and Holthausen (2005). There are three dates—$t = 0$, 1, and 2—and a single homogeneous good that can be used for consumption and investment. There is no discounting between dates.

There is a [0,1] continuum of identical, risk-neutral, profit-maximizing banks. We assume that the banking industry is perfectly competitive. Banks manage the funds on behalf of risk-neutral households with future liquidity needs.[4] To meet the liquidity needs of households, banks offer them claims worth c_1 and c_2 that can be withdrawn at $t = 1$ and $t = 2$, e.g., demand deposits or lines of credit. We assume that $c_1 > 0$. Households do require some payout in response to their liquidity need at $t = 1$.[5] The aggregate demand for liquidity is certain: a fraction λ of households withdraws their

Gale (2000), the financial connections leading to contagion arise endogenously as a means of insurance against liquidity shocks.

[4]We do not address the question of why households use banks to manage their funds, nor why banks offer demandable debt in return. Moreover, we abstract from any risk-sharing concerns and side-step the question whether interbank markets are an optimal arrangement. There is a large literature dealing with these important normative issues, starting with Diamond and Dybvig (1983), Bhattacharya and Gale (1987), and Jacklin (1987). For recent examples, see Diamond and Rajan (2001), Allen and Gale (2004), or Farhi, Golosov, and Tsyvinski (2008).

[5]In principle, risk-neutral households are indifferent between consuming at $t = 1$ and $t = 2$. In order to have an active interbank market, we assume that some households will have a strictly positive need for early consumption, which must be satisfied by banks. For example, some households may have to pay a tax at $t = 1$.

claims at $t = 1$. The remaining fraction $1 - \lambda$ withdraws at $t = 2$. At the individual bank level, however, the demand for liquidity is uncertain. A fraction π_h of banks face a high liquidity demand $\lambda_h > \lambda$ at $t = 1$, and the remaining fraction $\pi_l = 1 - \pi_h$ of banks face a low liquidity demand $\lambda_l < \lambda$. Hence, we have $\lambda = \pi_h \lambda_h + \pi_l \lambda_l$. Let the subscript $k = l, h$ denote whether a bank faces a low or a high need for liquidity at $t = 1$. Since aggregate liquidity needs are known, a bank with a high liquidity shock at $t = 1$ will have a low liquidity shock at $t = 2$: $1 - \lambda_h < 1 - \lambda_l$. We assume that banks' idiosyncratic liquidity shocks are not contractible. A bank's demandable liabilities cannot be contingent on whether it faces a high or a low liquidity shock at $t = 1$ and $t = 2$. This is the key friction that will give rise to an interbank market.

At $t = 0$, banks invest the funds of households either into a long-term illiquid asset (*loans*), a short-term liquid asset (*cash*), or into government bonds. We assume that each bank has one unit of the good under management at $t = 0$. Each unit invested in the liquid asset offers a return equal to one unit of the good after one period (costless storage). Each unit invested in the illiquid asset yields an uncertain payoff at $t = 2$. The investment into the illiquid asset can either succeed with probability p or fail with probability $1 - p$. If it succeeds, the bank is solvent and receives a return on the illiquid investment worth R units of the good at $t = 2$. If the investment fails, we assume that the bank is insolvent and is taken over by a deposit insurance fund. The fund assumes all the liabilities of an insolvent bank.[6] The investment into the illiquid asset does not produce any return at $t = 1$. Moreover, the illiquid asset is nontradable.

Government bonds yield a certain return equal to Y at $t = 2$. We assume that $pR > Y > 1$ so that bonds do not dominate the illiquid asset. Like the illiquid long-term investment, government bonds do not offer a return at $t = 1$. Unlike the illiquid asset, however, government bonds can be traded at $t = 1$ at a price P_1. Since we employ the term "liquidity" as the ability to produce cash flow at $t = 1$, the liquidity of government bonds is therefore *endogenous.* Government

[6]Thus, banks are protected by limited liability. Note that the deposit insurance fund only intervenes if the bank is insolvent, i.e., if the illiquid investment has failed.

bonds are in fixed supply. Let B denote the supply of government bonds to the banking sector at $t = 0$.[7]

Banks face a trade-off between liquidity and return when making their portfolio decision at $t = 0$. The short-term liquid asset allows banks to satisfy households' need for liquidity at $t = 1$. The illiquid asset is more profitable in the long run. Government bonds lie in between and are in fixed supply. Let α denote the fraction of bank assets at $t = 0$ invested in the illiquid asset, β denote the fraction invested in government bonds, and $1 - \alpha - \beta$ denote the remaining fraction invested in the liquid asset.

Since banks face different liquidity demands at $t = 1$, interbank markets can develop. Banks with low levels of withdrawals can provide liquidity to banks with high levels of withdrawals. We consider both secured and unsecured interbank markets. For ease of exposition, we model the secured market (repo agreements) as the trading of government bonds and treat $\frac{1}{P_1}$ as the repo rate.[8] The unsecured market consists of borrowing and lending amounts L_l and L_h, respectively, at an interest rate r. Given that banks can be insolvent when their illiquid investment fails, lenders in the unsecured interbank market will be exposed to credit risk. The deposit insurance fund does not cover interbank loans. However, borrowers always have to repay their interbank loan if they are solvent. Should a borrowers' counterparty be insolvent, the repayment goes to the deposit insurance fund. We denote the probability that an unsecured interbank loan is repaid by $\hat{p}$.

We assume that the interbank markets for unsecured loans and for government bonds are anonymous and competitive. Banks are price takers and are completely diversified across unsecured interbank loans. That is, a lender's expected return per unit lent in the unsecured interbank market is $p\hat{p}(1+r)$. With probability p a lender is solvent, in which case he collects the interest repayment $1 + r$ on a proportion $\hat{p}$ of the interbank loans made. The per-unit expected cost to a borrower is $p(1 + r)$.

[7]As we will show, if bonds were in unlimited supply, banks would prefer to satisfy their liquidity needs at $t = 1$ solely by trading bonds to avoid the risk premium of unsecured borrowing.

[8]In interbank repo markets, government bonds serve as collateral. This arrangement is different from an outright sale of bonds in that the original owner of the bond still collects the interest payment Y.

Figure 3. Assets and Financial Claims

	$t = 0$	$t = 1$	$t = 2$	time
Cash	-1	1 -1	1	
Loans	-1	0	p: R $1-p$: 0	
Govt. Bonds	$-\frac{1}{P_0}$	P_1 $-\frac{1}{P_1}$	Y	
Risky Interbank Debt		-1	$\hat{p}$: $1+r$ $1-\hat{p}$: 0	
		1	$-(1+r)$	

Figure 3 summarizes the payoffs of assets and financial claims. Note that the payoff shown for risky interbank debt is conditional on banks being solvent at $t = 2$.

The sequence of events is summarized in figure 4. At $t = 0$, banks invest households' funds in illiquid loans, government bonds, and cash. Government bonds are in fixed supply to the banking sector and their price at $t = 0$, P_0 must be such that (i) the market

Figure 4. The Timing of Events

$t = 0$	$t = 1$	$t = 2$
Banks offer deposit contracts (c_1, c_2).	Idiosyncratic liquidity shocks are realized.	The return of the illiquid asset and the government bond realize.
Banks invest into a risky illiquid asset, a safe liquid asset, and government bonds.	Banks borrow and lend in secured and/or unsecured interbank markets. Additionally, they can reinvest into the liquid asset.	Interbank loans are repaid.
		The remaining fraction of households withdraws and consumes c_2.
	A fraction of households withdraws and consumes c_1.	

for government bonds at $t = 0$ clears and (ii) it is consistent with banks' optimal holding of government bonds. At $t = 1$, after receiving an idiosyncratic liquidity shock, banks manage their liquidity by borrowing or lending in the unsecured interbank market, buying or selling government bonds and possibly reinvesting into the liquid asset in order to maximize bank profits at $t = 2$, taking their portfolio allocation $(\alpha, \beta, 1-\alpha-\beta)$ and the payout to households (c_1, c_2) as given. Both the interbank market for unsecured loans and for government bonds must clear. Prices are set by a Walrasian auctioneer so that (i) decentralized trading is consistent with banks' portfolios of bonds, illiquid loans, and cash, and (ii) there is no arbitrage opportunity between government bonds and unsecured interbank loans. At $t = 2$, returns on the illiquid asset and bonds are realized, interbank loans are repaid, and solvent banks pay out all their cash flow to households.

3. Benchmark: No Government Bonds

In this section we solve the model without government bonds (i.e., $\beta = 0$). The analysis clarifies how the model works and provides a benchmark. The main text gives the outline of the arguments. The details of the proofs are in the appendix. We proceed backwards by first considering banks' liquidity management at $t = 1$.

3.1 Liquidity Management

Having received liquidity shocks, $k = l, h$, banks manage their liquidity at $t = 1$ while taking their assets $(\alpha, 1 - \alpha)$ and liabilities (c_1, c_2) as given.

A bank that faces a low level of withdrawals at $t = 1$, $k = l$, has spare liquidity. The bank can thus choose to lend an amount L_l at a rate r in the interbank market. The bank can also reinvest a fraction γ_l^1 of funds left over in the liquid asset. At $t = 1$, a type l bank maximizes $t = 2$ profits

$$\max_{\gamma_l^1, L_l} p\left[R\alpha + \gamma_l^1(1-\alpha) + \hat{p}(1+r)L_l - (1-\lambda_l)c_2\right] \tag{1}$$

subject to

$$\lambda_l c_1 + L_l + \gamma_l^1(1-\alpha) \leq (1-\alpha)$$

and feasibility constraints: $0 \leq \gamma_l^1 \leq 1$ and $L_l \geq 0$.

Conditional on being solvent (with probability p), the profits at $t=2$ of a bank with a surplus of liquidity at $t=1$ are the sum of the proceeds from the illiquid investment, $R\alpha$, from the reinvestment into the liquid asset, $\gamma_l^1(1-\alpha)$, and the repayments of risky interbank loans, $\hat{p}(1+r)L_l$, minus the payout to households withdrawing at $t=2$, $(1-\lambda_l)c_2$. The budget constraint requires that the outflow of liquidity at $t=1$ (deposit withdrawals, $\lambda_l c_1$; reinvestment into the liquid asset, $\gamma_l^1(1-\alpha)$; and interbank lending, L_l) is matched by the inflow (return on the liquid asset, $1-\alpha$).

A bank that has received a high liquidity shock, $k=h$, will be a borrower in the interbank market, solving

$$\max_{\gamma_h^1, L_h} p\big[R\alpha + \gamma_h^1(1-\alpha) - (1+r)L_h - (1-\lambda_h)c_2\big] \qquad (2)$$

subject to

$$\lambda_h c_1 + \gamma_h^1(1-\alpha) \leq (1-\alpha) + L_h$$

and feasibility constraints: $0 \leq \gamma_h^1 \leq 1$ and $L_h \geq 0$.

There are two differences between the optimization problems of a lender and a borrower. First, a borrower expects having to repay $(1+r)L_h$ with probability p, while a lender expects a repayment $\hat{p}(1+r)L_l$ with probability p. A lender is exposed to credit risk. Second, interbank loans are an outflow for a lender and an inflow for a borrower.

Given that banks must provide some liquidity to households, $c_1 > 0$, the interbank market will be active as banks trade to smooth out the idiosyncratic liquidity shocks, $L_l > 0$ and $L_h > 0$.

The marginal value of (inside) liquidity at $t=1$, $1-\alpha$, is given by the Lagrange multiplier, denoted by μ^k, on the budget constraints of the optimization problems (1) and (2).

LEMMA 1 (Marginal Value of Liquidity). *The marginal value of liquidity is* $\mu^l = p\hat{p}(1+r)$ *for a lender and* $\mu^h = p(1+r)$ *for a borrower.*

A lender values liquidity at $t = 1$ since he can lend it out at an expected return of $p\hat{p}(1 + r)$. A borrower values liquidity since it saves the cost of borrowing in the interbank market, $p(1 + r)$. The marginal value of liquidity is lower for a lender because of credit risk.

The following result describes a bank's decision to reinvest into the liquid asset.

LEMMA 2 (Reinvestment into the Liquid Asset). *A borrower does not reinvest in the liquid asset at $t = 1$: $\gamma_h^1 = 0$. A lender does not reinvest in the liquid asset if and only if $\hat{p}(1 + r) \geq 1$.*

It cannot be optimal for a bank with a shortage of liquidity to borrow in the interbank market at rate $1 + r$ and to reinvest the obtained liquidity in the liquid asset since it would yield a negative net return. The same is not true for a lender since his rate of return on lending in the interbank market is only $\hat{p}(1+r)$ due to credit risk. If a lender reinvests his liquidity instead of lending it out, then the interbank market cannot be active. Thus, we have to check whether $\hat{p}(1+r) \geq 1$ once we have obtained the interest rate in the interbank market.

Market clearing in the interbank market, $\pi_l L_l = \pi_h L_h$, yields the following:

LEMMA 3 (Interbank Market Clearing). *The amount of the liquid asset held by banks exactly balances the aggregate payout at $t = 1$:*

$$\lambda c_1 = 1 - \alpha.$$

The interbank market fully smoothes out the idiosyncratic liquidity shocks, λ_k.

3.2 Pricing Liquidity

The price of unsecured interbank loans, $1 + r$, which banks take as given when making their portfolio choice, must be consistent with an interior portfolio allocation, $0 < \alpha < 1$. The profitability of the illiquid asset implies that a bank would never want to invest everything into the liquid asset and thus $\alpha > 0$. The need for a positive payout to households at $t = 1$, $c_1 > 0$, implies that banks will not be able to invest everything into the illiquid asset, $\alpha < 1$.

An interior portfolio allocation α solves

$$\max_{0<\alpha<1} \quad \pi_l p\big[R\alpha + \hat{p}(1+r)L_l - (1-\lambda_l)c_2\big] + \pi_h p\big[R\alpha - (1+r)L_h - (1-\lambda_h)c_2\big] \tag{3}$$

subject to

$$L_l = (1-\alpha) - \lambda_l c_1 \tag{4}$$

$$L_h = \lambda_h c_1 - (1-\alpha), \tag{5}$$

where we have used that $\gamma_k^1 = 0$ (lemma 2).

The first-order condition requires that

$$\pi_h p(1+r) + \pi_l p\hat{p}(1+r) = \pi_h pR + \pi_l pR$$

or, equivalently,

$$(\pi_h + \pi_l\hat{p})(1+r) = R. \tag{6}$$

The interbank interest rate r—i.e., the price of liquidity traded in the unsecured interbank market—is effectively given by a no-arbitrage condition. The right-hand side is the expected return from investing an additional unit into the illiquid asset, R. The left-hand side is the expected return from investing an additional unit into the liquid asset. With probability π_h, a bank will have a shortage of liquidity at $t = 1$, and one more unit of the liquid asset saves on borrowing in the interbank market at an expected cost of $(1+r)$ (conditional on being solvent). With probability π_l, a bank will have excess liquidity, and one more unit of the liquid asset can be lent out at an expected return $\hat{p}(1+r)$ (again conditional on being solvent). Note that banks' own probability of being solvent at $t = 2$, p, cancels out in (6) since it affects the expected return on the liquid and the illiquid investment symmetrically.

What is the level of credit risk? Since lenders hold a fully diversified portfolio of unsecured interbank loans, the proportion of loans that will not be repaid is given by the proportion of borrowers whose illiquid investment failed and who are thus insolvent at $t = 2$,

$$1 - \hat{p} = 1 - p. \tag{7}$$

We therefore have the following result:

PROPOSITION 1 (Pricing). *The price of liquidity at $t = 1$ is given by*

$$1 + r = \frac{R}{\delta}, \tag{8}$$

where

$$\frac{1}{\delta} \equiv \frac{1}{\pi_h + \pi_l p} > 1 \tag{9}$$

is the premium of lending in the interbank market due to banks' risky assets.

Given the price of liquidity (8), a bank with a surplus of liquidity will always want to lend it out rather than reinvest it. That is, the condition in lemma 2 is always satisfied: $p\frac{R}{\delta} > 1$ since $pR > 1$ and $\delta < 1$.

Liquidity becomes more costly when (i) asset risk increases (lower p) and (ii) a bank is more likely to become a lender (higher π_l) and thus is more likely to be subject to credit risk.

3.3 Portfolio Allocation

A bank's portfolio allocation α must be consistent with the promised payout to households, as well as market clearing and competition. We assume that banks pay out everything to households at $t = 2$. For a solvent bank that has lent in the unsecured interbank market, this means that

$$R\alpha + \hat{p}(1 + r)[(1 - \alpha) - \lambda_l c_1] - (1 - \lambda_l)c_2 = 0,$$

while for a solvent bank that has borrowed, it must be that

$$R\alpha - (1 + r)[\lambda_h c_1 - (1 - \alpha)] - (1 - \lambda_h)c_2 = 0.$$

Both types of banks must break even at $t = 2$ when solvent.[9] Note that a bank's payout to households at $t = 2$ cannot be contingent on whether it has lent or borrowed at $t = 1$. Using (i) market clearing at $t = 1$ (lemma 3), which links the proportion of the investment into the liquid asset $1 - \alpha$ to the payout c_1, (ii) the price of liquidity at $t = 1$ (equation (8)), and (iii) the link between credit and asset risk (equation (7)), we arrive at the following result:

PROPOSITION 2 (Portfolio Allocation). *Banks' portfolio allocation across the liquid and the illiquid asset satisfies*

$$\frac{\alpha}{1-\alpha} = \frac{1}{\delta} \frac{(1-\lambda_l)\pi_l + (1-\lambda_h)\pi_h p}{\lambda_l \pi_l + \lambda_h \pi_h}. \tag{10}$$

A bank chooses to hold a more liquid portfolio if it expects a higher level of withdrawals at $t = 1$ (λ_k increases). With respect to the probability of becoming a lender, π_l, and asset risk, p, there are two effects at play: the risk premium $\frac{1}{\delta}$ and the ratio between withdrawals at $t = 1$ versus $t = 2$ (the second fraction on the right-hand side of (10)). With respect to the probability of becoming a lender, both effects go in the same direction: higher π_l increases the risk premium and the relative proportion of $t = 2$ withdrawals.[10] Consequently, a higher probability of having a liquidity surplus at $t = 1$ leads to a less liquid portfolio at $t = 0$.

With respect to the risk of banks' illiquid assets, p, the two effects work in opposite directions. More asset risk increases the risk premium in the unsecured market but lowers the ratio of $t = 2$ versus $t = 1$ withdrawals. Higher asset risk means more credit risk for lenders and, consequently, fewer profits and a lower payout at $t = 2$. At the same time, lenders have more withdrawals than borrowers at $t = 2$, yet banks' withdrawable claims cannot be made contingent

[9]We also assume that the deposit insurance fund only intervenes if banks' illiquid investment fails (see footnote 6). If the investment succeeds, banks are not allowed to default on their deposits at $t = 2$ for regulatory reasons. The assumption that deposit insurance only intervenes when the illiquid investment fails is for simplicity only. The assumption is responsible for the clean link between asset risk and credit risk in the interbank market, $\hat{p} = p$.

[10]The derivative with respect to π_l of the second fraction on the right-hand side of (10) is positive if and only if $\lambda_h(1-\lambda_l) > p\lambda_l(1-\lambda_h)$. This always holds since $\lambda_h > \lambda_l$.

on banks' idiosyncratic liquidity shocks. To counter this imbalance at $t = 2$, a bank holds more liquid assets when asset risk is higher. This allows it to lend more and thus to increase revenue at $t = 2$ in case it received a low liquidity shock at $t = 1$. Similarly, it decreases its revenue at $t = 2$ in case it received a high liquidity shock and ends up being a borrower. The derivative of the right-hand side of equation (10) with respect to p is negative if and only if

$$(1 - \lambda_h)\pi_h^2 < (1 - \lambda_l)\pi_l^2. \tag{11}$$

A sufficient condition for more credit risk leading to less liquid investments is that banks are (weakly) more likely to have a liquidity surplus than a shortage, $\pi_l \geq \pi_h$ or $\pi_l \geq \frac{1}{2}$.

3.4 A Benchmark—No Risk

It is useful to consider the benchmark case when there is no asset risk and hence no credit risk. Substituting $p = 1$ into (10) yields the following result:

Corollary 1 (No Risk). *Without risk, $p = 1$, the interest rate in the unsecured interbank market $1 + r$ is equal to R, and the fraction invested in the illiquid asset is equal to the expected amount of withdrawals at $t = 2$: $\alpha^* = 1 - \lambda$.*

Without asset risk there is no credit risk for lenders in the unsecured interbank market. The amount invested in the liquid asset exactly covers the expected amount of withdrawals at $t = 1$. The interbank market smoothes out the problem of uneven demand for liquidity across banks at no cost. The fraction invested in the illiquid investment exactly covers the expected amount of withdrawals at $t = 2$. Without credit risk, lenders no longer lose revenue at $t = 2$.

4. Access to Government Bonds

In this section we allow banks to invest a fraction β of their portfolio into government bonds at $t = 0$ and to trade these bonds at $t = 1$. To solve the model we follow the same steps as in the previous section. The main text gives the outline of the arguments. The detailed proofs are in the appendix.

4.1 Liquidity Management

In order to manage their liquidity needs at $t = 1$, banks choose a fraction of government bond holdings to sell, β_k^S; a fraction of liquid asset holdings to be reinvested in the liquid asset, γ_k^1; a fraction of liquid asset holdings to be used to acquire more government bonds, γ_k^2; and how much to borrow/lend in the interbank market, L_k.

A bank that faces a low level of withdrawals at $t = 1$, type l, solves the following problem:

$$\max_{\beta_l^S,\gamma_l^1,\gamma_l^2,L_l} p\left[R\alpha + \left(\gamma_l^1 + \gamma_l^2\frac{Y}{P_1}\right)\left((1-\alpha-\beta) + \beta_l^S\frac{\beta}{P_0}P_1\right)\right.$$
$$\left. + (1-\beta_l^S)\frac{\beta}{P_0}Y + \hat{p}(1+r)L_l - (1-\lambda_l)c_2\right] \quad (12)$$

subject to

$$\lambda_l c_1 + L_l + \left(\gamma_l^1 + \gamma_l^2\right)\left((1-\alpha-\beta) + \beta_l^S\frac{\beta}{P_0}P_1\right)$$
$$\leq (1-\alpha-\beta) + \beta_l^S\frac{\beta}{P_0}P_1 \quad (13)$$

and feasibility constraints: $0 \leq \beta_l^S \leq 1$, $0 \leq \gamma_l^1$, $0 \leq \gamma_l^2$, $\gamma_l^1 + \gamma_l^2 \leq 1$, and $L_l \geq 0$.

A bank that has received a high liquidity shock, type h, will be a borrower in the interbank market, solving

$$\max_{\beta_h^S,\gamma_h^1,\gamma_h^2,L_h} p\left[R\alpha + \left(\gamma_h^1 + \gamma_h^2\frac{Y}{P_1}\right)\left((1-\alpha-\beta) + \beta_h^S\frac{\beta}{P_0}P_1\right)\right.$$
$$\left. + \left(1-\beta_h^S\right)\frac{\beta}{P_0}Y - (1+r)L_h - (1-\lambda_h)c_2\right] \quad (14)$$

subject to

$$\lambda_h c_1 + \left(\gamma_h^1 + \gamma_h^2\right)\left((1-\alpha-\beta) + \beta_h^S\frac{\beta}{P_0}P_1\right)$$
$$\leq (1-\alpha-\beta) + \beta_h^S\frac{\beta}{P_0}P_1 + L_h \quad (15)$$

and feasibility constraints: $0 \leq \beta_h^S \leq 1$, $0 \leq \gamma_h^1$, $0 \leq \gamma_h^2$, $\gamma_h^1 + \gamma_h^2 \leq 1$, and $L_h \geq 0$.

Access to bonds changes the liquidity management of banks as follows. Banks hold $\frac{\beta}{P_0}$ units of bonds. They can sell a fraction β_k^S of their bond holdings at price P_1. Hence, the amount of funds available at $t = 1$ is the sum of liquid asset holdings, $1 - \alpha - \beta$, and the proceeds from selling bonds, $\beta_k^S \frac{\beta}{P_0} P_1$. Banks can also acquire new bonds using γ_k^2 fraction of their liquid asset holdings.

At $t = 2$, banks earn return Y per unit of bond holdings. The return is earned on bonds bought at $t = 0$ that were not sold at $t = 1$, $(1 - \beta_k^S)\frac{\beta}{P_0}$ units, and on additional bonds bought at $t = 1$, $\frac{\gamma_k^2}{P_1}\left((1 - \alpha - \beta) + \beta_k^S \cdot \frac{\beta}{P_0} P_1\right)$ units.

Market clearing in the bond market requires that

$$\left(\pi_l \beta_l^S + \pi_h \beta_h^S\right) \frac{\beta}{P_0} P_1 = \pi_l \gamma_l^2 \left((1 - \alpha - \beta) + \beta_l^S \frac{\beta}{P_0} P_1\right) + \pi_h \gamma_h^2 \left((1 - \alpha - \beta) + \beta_h^S \frac{\beta}{P_0} P_1\right). \quad (16)$$

The left-hand side of (16) is the value of bonds sold by banks at $t = 1$, while the right-hand side is the amount available to buy them. The demand for bonds at $t = 1$ will depend on how much banks decide to hold in liquid assets at $t = 0$, $1 - \alpha - \beta$.

As before, banks need to satisfy households' demand for liquidity at $t = 1$. Access to safe government bonds will, however, reduce the amount that banks in need of liquidity must borrow unsecured. Acquiring liquidity through the sale of bonds is cheaper since the provider of liquidity (the buyer of the bond) does not need to be compensated for credit risk. To focus on the more interesting case in which the trading of bonds and unsecured interbank lending coexist, we assume that there are not enough bonds to fully cover banks' liquidity shortage at $t = 1$.

The introduction of bonds does not change the marginal value of liquidity. It is still given by lemma 1.

A bank with a shortage of liquidity at $t = 1$ will neither sell its bonds to reinvest in the liquid asset nor will it hold on to them. It will sell them in order to reduce the amount it needs to borrow in the unsecured interbank market.

LEMMA 4 (Liquidity Management of a Bank with a Shortage). *A bank with a liquidity shortage will not reinvest, neither in bonds nor in the liquid asset,* $\gamma_h^1 = 0, \gamma_h^2 = 0$*, and it will sell all its bonds:* $\beta_h^S = 1$.

Since bonds are scarce and the unsecured market is active, banks with a surplus of liquidity must still find it attractive to lend unsecured. The return on bonds must not be larger than the return on unsecured lending. Since lenders need to be compensated for credit risk in unsecured lending, banks with a shortage of liquidity will sell all their bonds first and then borrow the remaining amount.

Given that banks with a liquidity shortage sell bonds and borrow in the unsecured market, banks with a liquidity surplus must buy bonds and lend unsecured.

LEMMA 5 (Liquidity Management of a Bank with a Surplus). *A bank with a liquidity surplus will buy additional bonds:* $\gamma_l^1 = 0$, $\gamma_l^2 > 0$, *and* $\beta_l^S = 0$.

Using the results in lemmas 4 and 5, we can simplify (16), the market-clearing condition in the bond market:

$$\pi_h \frac{\beta}{P_0} P_1 = \pi_l \gamma_l^2 (1 - \alpha - \beta). \tag{17}$$

Market clearing in the bond market and the unsecured interbank market yields the following:

LEMMA 6 (Interbank Market Clearing). *The amount of the liquid asset held by banks exactly balances the aggregate payout to households at* $t = 1$*:*

$$\lambda c_1 = 1 - \alpha - \beta.$$

As before, trading at $t = 1$ fully smoothes out the idiosyncratic liquidity shocks.

4.2 Pricing Liquidity

Since both the bond market and the unsecured loan market are open, there must not be an arbitrage opportunity between the two markets at $t = 1$:

$$\frac{Y}{P_1} = \hat{p}(1+r) \geq 1. \tag{18}$$

Buying bonds and unsecured lending must offer the same return. Moreover, the return must be weakly larger than one since otherwise banks would prefer to reinvest into the liquid asset.

As before, banks are price takers in the market for unsecured interbank loans. The price at which banks engage in decentralized trading must be consistent with an interior portfolio allocation, $0 < \alpha < 1$ and $0 < \beta < 1$. Investing everything into bonds is inconsistent with satisfying households' need for liquidity at $t = 1$. All banks would have to sell bonds and no bank would be able to buy bonds. Hence, $\beta < 1$. Since bonds are not subject to credit risk, banks with high liquidity shocks prefer to sell them rather than borrow unsecured. Since there is a positive probability of a high liquidity shock ex ante, banks want to hold some bonds, $\beta > 0$. Banks will also have to hold the liquid asset to satisfy the demand for liquidity at $t = 1$, $\alpha < 1$. Finally, the profitability of the illiquid asset implies $\alpha > 0$.

An interior portfolio allocation solves

$$\max_{0<\alpha<1, 0<\beta<1} \quad \pi_l p[R\alpha + \left(\frac{\pi_h}{\pi_l} + 1\right)\frac{Y}{P_0}\beta + \hat{p}(1+r)L_l - (1-\lambda_l)c_2] + \pi_h p\big[R\alpha - (1+r)L_h - (1-\lambda_h)c_2\big] \tag{19}$$

subject to

$$L_l = (1-\alpha-\beta) - \frac{\pi_h}{\pi_l}\frac{P_1}{P_0}\beta - \lambda_l c_1$$

and

$$L_h = \lambda_h c_1 - (1-\alpha-\beta) - \frac{P_1}{P_0}\beta,$$

where we have used the results in lemmas 4 and 5 on banks' liquidity management and market clearing in the bond market (17) in order to substitute for γ_l^2.

The first-order condition for an interior allocation of the illiquid asset, α, is

$$p\big[R - \pi_l\hat{p}(1+r) - \pi_h(1+r)\big] = 0. \tag{20}$$

As in the case without access to bonds, the proportion of interbank loans that are not repaid is given by the probability of being insolvent since lenders hold a fully diversified portfolio of loans. Using (7) to simplify (20) yields

$$1 + r = \frac{R}{\delta}. \tag{21}$$

Condition (21) is identical to condition (8). The cost of unsecured borrowing is not affected by the access to bonds.

Due to the no-arbitrage condition between bonds and unsecured loans (equation (18)), condition (21) ties down the price of bonds at $t = 1$:

$$\frac{Y}{P_1} = \frac{pR}{\delta}. \tag{22}$$

The condition immediately implies that $\frac{Y}{P_1} > 1$ since $pR > 1$ and $\delta < 1$. That is, bonds trade at a discount at $t = 1$. Lenders must be compensated for providing liquidity. If bonds did not trade at a discount, then holding the liquid asset to lend it out unsecured is not very attractive. The no-arbitrage condition would imply that $p(1 + r) = 1$ and hence $1 + r = \frac{1}{p} < \frac{R}{\delta}$, which is inconsistent with $\alpha < 1$. Banks would invest everything into the illiquid asset.

The requirement on an interior portfolio allocation for bonds ties down the price of bonds at $t = 0$.

LEMMA 7. *The price of bonds at $t = 0$ is equal to the price at $t = 1$:*

$$P_0 = P_1. \tag{23}$$

Given that bonds and liquid asset holdings must coexist at $t = 0$ in order to manage liquidity shocks at $t = 1$, it is intuitive that the first-period yield on bonds equals the return on the liquid asset.

The following proposition summarizes the pricing of bonds and unsecured interbank loans:

PROPOSITION 3 (Pricing). *The interest rate in the unsecured market is $1 + r = \frac{R}{\delta}$. The yield to maturity of the bond at $t = 0$ and at $t = 1$ is identical. It is given by $\frac{Y}{P_0} = \frac{Y}{P_1} = \frac{pR}{\delta} > 1$.*

4.3 Portfolio Allocation

As in the case without access to bonds, banks' portfolio allocation (α, β) must lead to a payout to households that is consistent with (i) market clearing at $t = 1$ and (ii) a full payout at $t = 2$ such that banks make zero profits if they are solvent and the payout is not contingent on $k = l, h$. The following proposition and its derivation are analogous to proposition 2.

PROPOSITION 4 (Portfolio Allocation). *Banks' portfolio allocation across the liquid and illiquid asset satisfies*

$$\frac{\alpha}{1-\alpha-\beta} = \frac{1}{\delta}\frac{(1-\lambda_l)\pi_l + (1-\lambda_h)\pi_h p}{\lambda_l\pi_l + \lambda_h\pi_h} - \frac{BY}{pR}\frac{1}{1-\alpha-\beta}\frac{(1-\lambda_l)-(1-\lambda_h)p}{\lambda_h-\lambda_l}. \tag{24}$$

The fraction invested in bonds is given by market clearing at $t = 0$: $\beta = BP_0 = \delta\frac{BY}{pR}$.

The left-hand side of (24) is as in (10), except that we have to subtract the bond holdings β to obtain the investment in the liquid asset. The first term on the right-hand side is the same as in the case without bonds. The second term captures the effect of having access to government bonds. Note that if $\beta = 0$ and $B = 0$, then equation (24) reduces to equation (10). Banks hold a more liquid portfolio ceteris paribus when they have access to bonds. Bonds provide banks with a safe return at $t = 2$ so that banks need to invest less into the illiquid investment in order to satisfy withdrawals at $t = 2$. At the same time, trading bonds is a valuable alternative to the unsecured interbank market.

The size of the banking sector relative to the amount of bonds available matters. The effect of bonds on the investment in liquid and illiquid assets is stronger when the ratio of the value of bonds to the expected value of banks' productive assets, $\frac{BY}{pR}$, is larger. Finally, banks' bond holdings are proportional to the relative size of bonds to banks' productive assets, $\frac{BY}{pR}$, and the constant of proportionality is given by the risk discount factor in the unsecured market, δ.

The effect of bonds on ex ante liquidity holdings is stronger when there is more credit risk (lower p), there are more withdrawals at banks with a liquidity surplus (higher λ_l), and there are fewer withdrawals at banks with a liquidity shortage (lower λ_h).

Suppose that banks are equally likely to have a liquidity shortage or a liquidity surplus, $\pi_l = \pi_h = \frac{1}{2}$. Then more credit risk increases the first term on the right-hand side (see condition (11)). It also increases the second term on the right-hand side, making the overall impact of more credit risk on banks' portfolio choice ambiguous.

5. Empirical Implications

Looking at figures 1 and 2, it seems that repo markets secured by government bonds in the United States and in the euro area followed different dynamics between August 2007 and May 2009. Below, we discuss empirical predictions of the model that may help explain the developments.

5.1 Decoupling of Secured and Unsecured Rates

Government bonds become relatively more valuable if the credit-risk problem becomes more severe, i.e., as p decreases. To see this, note that the following no-arbitrage conditions hold ex ante (see proposition 3):

$$\frac{R}{\delta} = 1 + r \text{ and } \frac{pR}{\delta} = \frac{Y}{P_1}.$$

Higher credit risk increases the credit-risk premium $\frac{1}{\delta}$, leading to a higher interest on unsecured lending $1+r$. A higher interest on unsecured lending decreases the price of bonds ceteris paribus; see equation (18). But there is a second, countervailing effect since the yield on *safe* bonds must be equal to the expected return on *risky* interbank loans. Overall, the second effect dominates since the credit-risk premium $\frac{1}{\delta}$ does not increase one for one with changes in p. Ex ante, a bank is uncertain whether it will be a lender, and thus exposed to credit risk, or not. In sum, following a shock to credit risk, unsecured rates and rates secured by government bonds move in opposite directions.

Changes in the perception of credit risk can explain why secured and unsecured rates decoupled with the onset of the financial crisis in August 2007, around the sale of Bear Stearns and following the Lehman bankruptcy. In the summer of 2007, the discovery of subprime mortgages in the portfolios of banks and bank-sponsored conduits led to a marketwide reassessment of credit risk. The sale of Bear Stearns caused a temporary turbulence, while the failure of Lehman led to a dramatic revision of expected default probabilities.

5.2 Spillovers and Credit Risk Levels

The potential for spillover effects from the unsecured to the secured market increases as the level of credit risk increases. To see this, note that

$$R = \frac{Y}{P_1}\left[\pi_l + \frac{1}{p}\pi_h\right]$$

must hold ex ante (see proposition 3), which implies that

$$-\frac{\frac{\partial P_1}{\partial p}}{\frac{P_1}{p}} = \frac{\pi_h}{\delta} < 1$$

for all $p > 0$.

It follows that the elasticity of the price of government bonds to credit risk is the lowest for $p = 1$ (no credit risk) since the elasticity is decreasing in p. This is consistent with the fact that the decoupling between the secured and unsecured rates was most pronounced in the aftermath of the Lehman failure, when the perceived level of credit risk in the banking sector was very high.

5.3 Relative Scarcity of Collateral

How does the scarcity of the underlying collateral affect the dynamics of repo rates when credit risk increases? Our model implies that the sensitivity of the price of government bonds to credit risk is lower in a country with a larger supply of government bonds. In other words, we have that, ex post,[11]

---

[11]Ex ante, the supply of bonds B has no impact on the price of bonds (see proposition 3).

$$\frac{\partial^2 P_1}{\partial B \partial p} > 0.$$

To see this, note that equation (24) and the fact that $\frac{pR}{\delta} = \frac{Y}{P_1}$ imply that

$$\frac{R\alpha}{1-\alpha-\beta} = \frac{1}{p}\frac{Y}{P_1}\frac{(1-\lambda_l)+(1-\lambda_h)\frac{\pi_h}{\pi_l}p}{\lambda_l+\lambda_h\frac{\pi_h}{\pi_l}} - \frac{1}{p}\frac{BY}{1-\alpha-\beta}\frac{(1-\lambda_l)-(1-\lambda_h)p}{\lambda_h-\lambda_l}. \quad (25)$$

Applying the implicit function theorem to the equation above, we have that

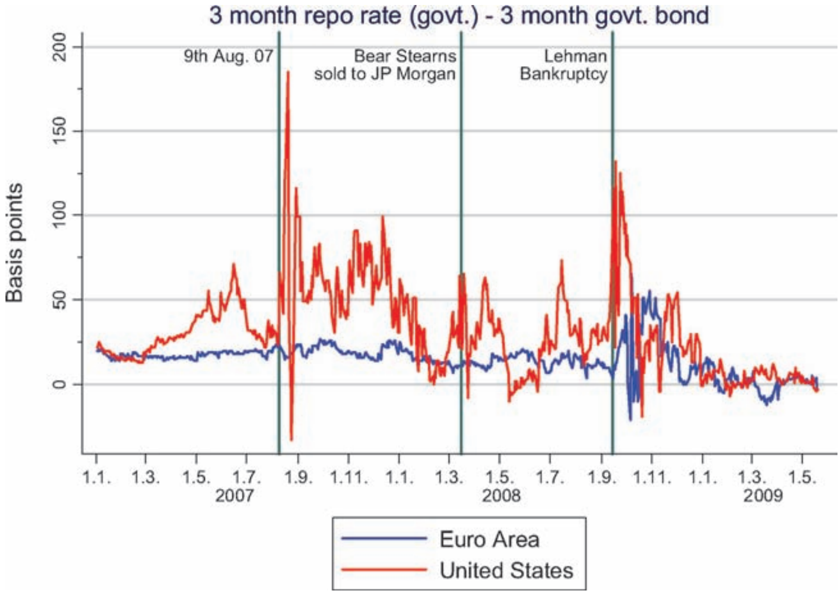
$$\frac{dP_1}{dB} = -P_1^2\frac{1}{1-\alpha-\beta}\frac{(1-\lambda_l)-(1-\lambda_h)p}{\lambda_h-\lambda_l} \times \frac{\pi_l\lambda_l+\pi_h\lambda_h}{\pi_l(1-\lambda_l)+\pi_h(1-\lambda_h)p} < 0. \quad (26)$$

Hence, if there is an unexpected shock to the amount of government bonds available in the banking sector—i.e., B decreases (say, due to a high demand for these securities outside the banking sector at the time when banks need to cope with liquidity shocks)—then the price of bonds P_1 must increase. Taking the derivative of (26) with respect to p, we get the desired result, since $\frac{\partial P_1}{\partial p} < 0$ and

$$\frac{\partial}{\partial p}\left(\frac{(1-\lambda_l)-(1-\lambda_h)p}{\pi_l(1-\lambda_l)+\pi_h(1-\lambda_h)p}\right) = \frac{-(1-\lambda_h)(1-\lambda_l)}{(\pi_l(1-\lambda_l)+\pi_h(1-\lambda_h)p)^2} < 0.$$

With the onset of the crisis in August 2007, repo rates in the United States became much more volatile than in the euro area. We document this in figure 5, which plots the spread between the three-month repo rate secured by government bonds and the yield on the three-month government bond. Hördahl and King (2008) argue that the higher volatility in the United States can be partly explained by the increased "safe haven" demand for U.S. Treasury securities, which made Treasuries relatively scarce. The Federal Reserve responded by introducing measures that increased the supply of high-quality collateral for private repo markets. We discuss policy responses further in section 6.

Figure 5. Tensions in Treasury-Backed Repo Markets, Euro Area and United States



5.4 Aggregate Liquidity Shocks

If there is an unanticipated shock to the relative proportion of banks with high and low liquidity demands, $\frac{\pi_h}{\pi_l}$, then liquidity becomes more scarce and even the price of government bonds declines. To show that this is the case, we apply the implicit function theorem to (25). It is straightforward to show that the sign of the derivative of P_1 with respect to $\frac{\pi_h}{\pi_l}$ is determined by the sign of the following expression:

$$\frac{\partial}{\partial \frac{\pi_h}{\pi_l}} \left(\frac{(1-\lambda_l) + (1-\lambda_h)\frac{\pi_h}{\pi_l} p}{\lambda_l + \lambda_h \frac{\pi_h}{\pi_l}} \right) = \frac{p\lambda_l(1-\lambda_h) - \lambda_h(1-\lambda_l)}{\left(\lambda_l + \lambda_h \frac{\pi_h}{\pi_l}\right)^2}.$$

As long as

$$p < \frac{\lambda_h(1-\lambda_l)}{\lambda_l(1-\lambda_h)},$$

this derivative is negative. Note that for $\lambda_h > \lambda_l$, which is what we assume, $\frac{\lambda_h(1-\lambda_l)}{\lambda_l(1-\lambda_h)} > 1$ and thus the inequality above always holds. It follows that P_1 declines after an unexpected increase in the aggregate demand for liquidity, $\frac{\pi_h}{\pi_l}$.

Following the same steps, it follows that the unsecured rate $1+r$ must increase if the aggregate demand for liquidity increases. Consequently, both secured and unsecured rates would move in the same direction after an unexpected aggregate liquidity shock; i.e., the decoupling of rates cannot be explained by such a shock.

6. Policy Implications

Unsecured markets are particularly vulnerable to changes in the perceived creditworthiness of counterparties. In repo transactions, such concerns are mitigated to some extent by the presence of collateral. Yet, our model illustrates how tensions in the unsecured market spill over to the market secured by collateral of the highest quality. Moreover, the volatility of repo rates can be exacerbated by structural characteristics such as the scarcity of securities that are used as collateral.

Central banks are particularly concerned with the well functioning of interbank markets because it is an important element in the transmission of monetary policy, and because persisting tensions may affect the financing conditions faced by nonfinancial corporations and households. In many countries, central banks have reacted to the observed tensions by introducing measures to support the interbank market, trying to avoid marketwide liquidity problems turning into solvency problems for individual institutions. The goal of this section is to examine a number of policy responses implemented since August 2007 that aimed at reducing tensions in interbank markets.

Specifically, we examine how the range of collateral accepted by a central bank affects the liquidity conditions of banks and how central banks can help alleviate tensions associated with the scarcity of high-quality collateral. In line with the predictions of the model, we present evidence that these measures can be effective in reducing tensions in secured markets. At the same time, they are not designed to resolve the underlying problems in the unsecured segment and the associated spillovers, if those are driven by credit-risk concerns.

6.1 Collateral Accepted by the Central Bank

Central banks provide liquidity to the banking sector against eligible collateral. The range of acceptable collateral varies across countries. Since the onset of the crisis, however, central banks have generally lowered the minimum credit rating and increased the quantity of lending they provide. For example, the Federal Reserve expanded its collateral list for repo operations in March, May, and September 2008, in response to market tensions. Moreover, it established the Term Auction Facility (TAF) in December 2007. The TAF provides term credit through periodic auctions to a broader range of counterparties and against a broader range of collateral than open-market operations. The Federal Reserve stressed that "this facility could help ensure that liquidity provisions can be disseminated efficiently even when the unsecured interbank markets are under stress."[12] The European Central Bank (ECB) headed into the crisis with the broadest list of eligible collateral among its peers, including nonmarketable securities and commercial loans. As a result, the ECB made no changes until mid-October 2008, when it expanded the eligible collateral significantly and lowered the minimum credit rating from A– to BBB– as the crisis intensified.

What are the implications of a wider range of collateral accepted in a central bank's operations according to our model? First, allowing securities other than Treasuries can reduce the volatility of the repo rates backed by Treasuries, as it reduces the pressure on acquiring Treasury securities and the limits imposed by their fixed supply. Moreover, we showed that if there is an unexpected aggregate liquidity shock, funding pressures can appear in all interbank market segments. By providing liquidity, a central bank can counter the effects of aggregate shocks and ensure that financial institutions do not sell their assets, including Treasuries, at distressed prices.

How effective were changes to the collateral framework of central banks during the crisis? McAndrews, Sarkar, and Wang (2008) provide evidence that the introduction of the TAF was associated with

[12]See Board of Governors of the Federal Reserve System (2007).

downward shifts of the LIBOR by reducing the liquidity risk premium. Christensen, Lopez, and Rudebusch (2009) analyze the role of the TAF in reducing the spreads between term LIBOR rates and the yield on Treasuries of corresponding maturity. They construct a counterfactual path and conclude that in the absence of the TAF, the LIBOR would have been higher. On the other hand, Taylor and Williams (2009) argue that the TAF had no significant impact on interest rate spreads, as it did not address the fundamental problem of credit risk on banks' balance sheets.

6.2 Upgrading Collateral

If concerns about the creditworthiness of counterparties make it expensive to borrow in the unsecured market, financial institutions try to obtain more funds in the secured market. However, we show that if the underlying collateral is scarce, repo market rates will be volatile. Measures aimed at increasing the supply of collateral can thus improve the allocation of liquidity in interbank markets.

For example, the Federal Reserve introduced the Term Securities Lending Facility (TSLF) in March 2008. It lends Treasury securities to dealers, taking less liquid securities, including agency debt securities and mortgage-backed securities (MBS), as collateral. The Treasury securities are allocated to dealers via auctions.[13] The primary dealers then use those Treasury securities to obtain financing in private repo markets. The TSLF thus increases the ability of dealers to obtain financing and decreases their need to sell assets into illiquid markets. The direct benefits that can be expected from the TSLF are, first, an increase in the supply of Treasury collateral in the private repo market and, second, a reduction of the supply of less liquid collateral.

[13]The TSLF is divided into two schedules: schedule 1 TSLF operations (i.e., auctions for Treasury and agency securities) are separated from schedule 2 TSLF operations (i.e., schedule 1 plus other investment-grade collateral). Schedule 2 collateral originally included schedule 1 collateral plus AAA/Aaa-rated non-agency residential MBS, commercial MBS, and agency collateralized mortgage obligations (CMOs). Schedule 2 collateral was expanded to include AAA/Aaa-rated asset-backed securities starting with the May 8, 2008 auction and all investment-grade debt securities starting with the September 17, 2008 auction.

The TSLF is closely related to the Primary Dealer Credit Facility (PDCF), which is also available to primary dealers. A key difference is that the PDCF is a standing facility whereas the TSLF is an auction facility. As a standing facility, the PDCF offers the advantage of availability on a continuous basis. It also accepts a broader class of securities as collateral. Whereas the TAF (discussed in the previous section) is only available to depository institutions, the TSLF is available to primary dealers. Both programs address the tensions in interbank markets via different market participants.

Fleming, Hrung, and Keane (2009) provide evidence of the impact of the introduction of the TSLF on repo spreads between Treasury collateral and lower-quality collateral. They document that the introduction of the TSLF was associated with an increase in repo rates relative to the federal funds rate. This is consistent with the predictions of our model that reducing the scarcity of high-quality collateral should result in higher Treasury repo rates. Moreover, the introduction of the TSLF narrowed financing spreads during spring 2008, particularly after the first auction. Much of the narrowing seems to have come from an increase in Treasury rates rather than a decrease of the rates for non-Treasury collateral.

7. Conclusion

Despite the presence of collateral, the disruptions in the unsecured interbank market during the 2007–09 financial crisis have also affected secured markets. This paper presents a model of secured and unsecured interbank lending in the presence of credit risk. Credit-risk premia in the unsecured market will affect the price of riskless bonds when they are used to manage banks' liquidity shocks.

Going forward, our analysis points to a number of issues for further research. First, the size of the banking sector relative to the amount of collateral matters. We saw that the presence of bonds reduces the amount banks have to borrow in unsecured markets. The positive effect of bonds is stronger when the ratio of banks' balance sheets to the value of bonds is larger. Hence, the interplay between the relative size of banking, financial markets, and the economy deserves further attention.

Second, our analysis abstracted from risk-sharing concerns. Banks were simply maximizing the total amount of demandable liabilities. Still, we obtain a credit-risk premium in unsecured interbank markets. Introducing risk aversion of banks' customers is beyond the scope of this paper and constitutes a fruitful avenue for further research. With respect to the spillover of credit risk across interbank markets, we anticipate that risk aversion can add an additional premium that would exacerbate the tensions that we identified.

Third, we assumed that the various shocks in our model are uncorrelated. The financial crisis has made it painfully clear that, in reality, the risk embedded in banks' illiquid assets, their liquidity needs, and shocks to collateral values are interlinked. The challenge will therefore be to model and analyze the joint distribution of the risks in banks' balance sheets, especially "at the tail." Banks' risk management practices have to take into account the forces affecting different collateral classes and the market's response in times of stress when liquidity and high-quality collateral are scarce.

Appendix

Proof of Lemma 1

The interbank market is active so that the constraints $L_k \geq 0$ are slack. Let μ^k be the Lagrange multiplier on the budget constraint. The first-order condition for a lender with respect to L_l is

$$p\hat{p}(1+r) - \mu^l = 0, \tag{27}$$

while the first-order condition for a borrower with respect to L_h is

$$-p(1+r) + \mu^h = 0. \tag{28}$$

Proof of Lemma 2

Let μ_1^k be the Lagrange multipliers on $0 \leq \gamma_k^1$. The constraint $\gamma_k^1 \leq 1$ cannot be binding since otherwise all available funds at $t = 1$ are reinvested and nothing can be paid or lent out. The first-order condition for a type k bank with respect to γ_k^1 is

$$(1-\alpha)(p - \mu^k) + \mu_1^k = 0. \tag{29}$$

Substituting $\mu^h = p(1+r)$ (lemma 1) yields

$$(1-\alpha)(-pr) = -\mu_1^h < 0, \tag{30}$$

since the left-hand side is negative. It cannot be zero since $\alpha = 1$ cannot be optimal. A type h bank would have to finance its entire need for liquidity by borrowing in the interbank market at a rate $r > 0$, whereas it could just store some liquidity without cost using the short-term asset. Since $-\mu_1^h < 0$, we have $\gamma_h^1 = 0$.

Consider now the case of a lender. Substituting $\mu^l = p\hat{p}(1+r)$ (lemma 1) into (29) yields

$$(1-\alpha)p(1-\hat{p}(1+r)) = -\mu_1^l.$$

Again, $\alpha = 1$ cannot be optimal. A type l bank cannot invest everything into the illiquid asset and still lend in the interbank market. Hence, $\gamma_l^1 = 0$ if and only if $\hat{p}(1+r) \geq 1$ (we assume that a type l bank does not reinvest into the liquid asset when the condition holds as an equality).

Proof of Lemma 3

Using the binding budget constraints from the optimization problems (1) and (2) (lemma 1) to substitute for L_l and L_h in the market-clearing condition $\pi_l L_l = \pi_h L_h$ and using $\gamma_k^1 = 0$ (lemma 2) gives the result.

Proof of Lemma 4

The first-order condition of a borrower with respect to reinvesting into the liquid asset at $t = 1$, γ_h^1, is

$$\left((1-\alpha-\beta) + \beta_h^S \frac{\beta}{P_0} P_1\right)(p - \mu^h) + \mu_1^h = 0,$$

where μ^h is the marginal value of liquidity for a borrower (given by lemma 1) and μ_1^h is the multiplier on the feasibility constraint $\gamma_h^1 \geq 0$. Note that $(1-\alpha-\beta) + \beta_k^S \frac{\beta}{P_0} P_1 > 0$ since we are considering interior portfolio allocations, $1-\alpha-\beta > 0$. Since $\mu^h = p(1+r) > p$, we have that $\mu_1^h > 0$ and thus $\gamma_h^1 = 0$. As in the case without bonds, a borrower does not reinvest into the liquid asset.

From the first-order condition of a lender with respect to bond purchases at $t = 1$, γ_l^2, we have that

$$\left((1-\alpha-\beta) + \beta_l^S \frac{\beta}{P_0} P_1\right)(p\frac{Y}{P_1} - \mu^l) + \mu_2^l = 0, \tag{31}$$

where μ^l is the marginal value of liquidity for a lender (given by lemma 1) and μ_2^l is the multiplier on the feasibility constraint $\gamma_l^2 \geq 0$. Note that the feasibility constraint $\gamma_l^1 + \gamma_l^2 \leq 1$ must be automatically satisfied since otherwise all available funds at $t = 1$ are reinvested and nothing can be paid or lent out. Since $\mu^l = p\hat{p}(1+r)$, the first-order condition holds if

$$\frac{Y}{P_1} \leq \hat{p}(1+r). \tag{32}$$

The yield on the bond at $t = 1$ must be less than or equal to the expected return of unsecured interbank lending (given that the unsecured interbank market is open).

The first-order condition of a borrower with respect to bond purchases at $t = 1$, γ_h^2, is

$$\left((1-\alpha-\beta) + \beta_h^S \frac{\beta}{P_0} P_1\right)(p\frac{Y}{P_1} - \mu^h) + \mu_2^h = 0,$$

where μ_2^h is the multiplier on the feasibility constraint $\gamma_h^2 \geq 0$. Due to condition (32), we have that $\mu_2^h > 0$ and hence $\gamma_h^2 = 0$. A borrower does not reinvest using bonds either.

The first-order condition of a borrower with respect to bond sales at $t = 1$, β_h^S, is

$$\frac{\beta}{P_0}\left[p\big(P_1\gamma_h^1 + Y(\gamma_h^2 - 1)\big) - \mu^h P_1\big(\gamma_h^1 + \gamma_h^2 - 1\big)\right] + \mu_3^h - \mu_4^h = 0,$$

where μ_3^h and μ_4^h are the Lagrange multipliers on $0 \leq \beta_h^S \leq 1$. Using $\gamma_h^1 = 0$, $\gamma_h^2 = 0$, and $\mu^h = p(1+r)$, we have

$$p\frac{\beta}{P_0}\left[-Y + (1+r)P_1\right] + \mu_3^h - \mu_4^h = 0.$$

Due to condition (32), the term in squared brackets is positive. For the condition to hold, it must be that $\mu_4^h > 0$, and hence $\beta_h^S = 1$. The borrower sells all his bonds at $t = 1$.

Proof of Lemma 5

Market clearing for bonds at $t = 1$ requires that

$$(\pi_l \beta^S + \pi_h)\frac{\beta}{P_0}P_1 = \pi_l \gamma_l^2 \left((1-\alpha-\beta) + \beta_l^S \frac{\beta}{P_0}P_1\right),$$

where we have used $\beta_h^S = 1$ and $\gamma_h^2 = 0$. Market clearing therefore requires that $\gamma_l^2 > 0$. Since borrowers sell bonds, lenders must buy them.

Given that $\gamma_l^2 > 0$, and hence $\mu_2^l = 0$, the lender's first-order condition with respect to bond purchases (31) requires that

$$\frac{Y}{P_1} = \hat{p}(1+r). \tag{33}$$

The yield on safe bonds must be equal to the expected return on risky interbank loans, as both markets are open.

The first-order condition of a lender with respect to bond sales at $t = 1$, β_l^S, is

$$\frac{\beta}{P_0}\left[p\big(P_1\gamma_l^1 + Y(\gamma_l^2 - 1)\big) - \mu^l P_1\big(\gamma_l^1 + \gamma_l^2 - 1\big)\right] + \mu_3^l - \mu_4^l = 0,$$

where μ_3^l and μ_4^l are the Lagrange multipliers on $0 \leq \beta_l^S \leq 1$. Using (33), the condition becomes

$$p\frac{\beta}{P_0}\big((P_1 - Y)\gamma_l^1\big) + \mu_3^l - \mu_4^l = 0. \tag{34}$$

It cannot be that $P_1 > Y$ since lenders would not want to buy any bonds at $t = 1$. When $P_1 < Y$, then $\mu_3^l > 0$ and hence $\beta_l^S = 0$. If $P_1 = Y$, then we can let $\beta_l^S = 0$ without loss of generality. To see this, plug $P_1 = Y$ and $\hat{p}(1+r) = 1$ (see condition (33)) into the lender's problem at $t = 1$ (equations (12) and (13)):

$$p\left[R\alpha + (1-\alpha-\beta) + \frac{\beta}{P_0}Y - \lambda_l c_1 - (1-\lambda_l)c_2\right], \tag{35}$$

where we substituted the budget constraint into the objective function using L_l. The objective function is independent of β_l^S.

The first-order condition of a lender with respect to reinvesting into the liquid asset at $t = 1$, γ_l^1, is

$$(1 - \alpha - \beta)p\left(1 - \frac{Y}{P_1}\right) + \mu_1^l = 0,$$

where we used $\beta_l^S = 0$, the lender's marginal value of liquidity $\mu^l = p\hat{p}(1 + r)$, and (33). We have ruled out that $P_1 > Y$. When $P_1 = Y$, the lender's problem is independent of γ_l^1 (see (35)) and we can set $\gamma_l^1 = 0$ without loss of generality. When $P_1 < Y$, then $\mu_1^l > 0$ and hence $\gamma_l^1 = 0$.

Proof of Lemma 6

As in the proof of lemma 3, the extra element is the presence of γ_l^2, the amount of bonds bought by banks with a liquidity surplus. But we can use the condition on market clearing in the bond market (17) to solve for γ_l^2.

Proof of Lemma 7

The first-order condition with respect to the allocation into the government bond, β, requires that

$$p\left[\frac{Y}{P_0} + \hat{p}(1 + r)\left(-\pi_l - \pi_h\frac{P_1}{P_0}\right) - (1 + r)\pi_h\left(1 - \frac{P_1}{P_0}\right)\right] = 0.$$

Solving the first-order condition for P_0 and using (7) yields

$$P_0 = \frac{1}{p\pi_l + \pi_h}\left(\frac{Y}{1 + r} + \pi_h(1 - p)P_1\right).$$

Using (18) to substitute for $\frac{Y}{1+r}$ results in $P_0 = P_1\frac{(1-\pi_h)p+\pi_h}{p\pi_l+\pi_h}$, which gives the desired result since $1 - \pi_h = \pi_l$.

Proof of Proposition 4

We require that

$$R\alpha + \gamma_l^2 \frac{Y}{P_1}(1-\alpha-\beta) + p(1+r)\left[\left(1-\gamma_l^2\right)(1-\alpha-\beta) - \lambda_l c_1\right] + \frac{\beta}{P_0} Y - (1-\lambda_l)c_2 = 0$$

and that

$$R\alpha - (1+r)\left[\lambda_h c_1 - (1-\alpha-\beta) - \frac{\beta}{P_0}P_1\right] - (1-\lambda_h)c_2 = 0,$$

where we have used the results from lemmas 1, 4, and 5, and equation (7). The amount of bonds purchased, γ_l^2, is given by market clearing in the bond market (equation (16) or, after simplification, (17)).

Using the result for c_1 from lemma 6, using one condition above to solve for c_2 and substituting back into the other condition yields

$$\frac{R}{1+r}\frac{\alpha}{1-\alpha-\beta} = \frac{(1-\lambda_l)\pi_l + (1-\lambda_h)\pi_h p}{\lambda_l \pi_l + \lambda_h \pi_h} - \frac{BP_1}{1-\alpha-\beta}\frac{(1-\lambda_l)-(1-\lambda_h)p}{\lambda_h - \lambda_l}. \tag{36}$$

Combining (36) with the result in proposition 3 gives the desired result.

References

Acharya, V. V., D. Gromb, and T. Yorulmazer. 2008. "Imperfect Competition in the Interbank Market for Liquidity as a Rationale for Central Banking." Working Paper, London Business School.

Acharya, V., H. Shin, and T. Yorulmazer. 2009. "Crisis Resolution and Bank Liquidity." Working Paper, New York University Stern School of Business.

Allen, F. and E. Carletti. 2008. "The Role of Liquidity in Financial Crises." Paper presented at the Federal Reserve Bank of Kansas City Symposium on Maintaining Stability in a Changing Financial System, Jackson Hole, Wyoming, August 21–23.

Allen, F., E. Carletti, and D. Gale. 2009. "Interbank Market Liquidity and Central Bank Intervention." *Journal of Monetary Economics* 56 (5): 639–52.

Allen, F., and D. Gale. 2000. "Financial Contagion." *Journal of Political Economy* 108 (1): 1–33.

———. 2004. "Financial Intermediaries and Markets." *Econometrica* 72 (4): 1023–61.

Bhattacharya, S., and D. Gale. 1987. "Preference Shocks, Liquidity, and Central Bank Policy." In *New Approaches to Monetary Economics*, ed. W. A. Barnett and K. J. Singleton, 69–88. Cambridge: Cambridge University Press.

Board of Governors of the Federal Reserve System. 2007. "Federal Reserve and Other Central Banks Announce Measures Designed to Address Elevated Pressures in Short-Term Funding Markets." Press Release, December 12.

Bruche, M., and J. Suarez. 2009. "The Macroeconomics of Money Market Freezes." CEPR Discussion Paper No. 7304.

Brunnermeier, M. K. 2009. "Deciphering the Liquidity and Credit Crunch 2007–2008." *Journal of Economic Perspectives* 23 (1): 77–100.

Brunnermeier, M. K., and L. H. Pedersen. 2009. "Market Liquidity and Funding Liquidity." *Review of Financial Studies* 22 (6): 2201–38.

Christensen, J., J. Lopez, and G. Rudebusch. 2009. "Do Central Bank Liquidity Operations Affect Interbank Lending Rates?" Mimeo, Federal Reserve Bank of San Francisco.

Diamond, D. W., and P. H. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91 (3): 401–19.

Diamond, D. W., and R. G. Rajan. 2001. "Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking." *Journal of Political Economy* 109 (2): 287–327.

———. 2005. "Liquidity Shortages and Banking Crises." *Journal of Finance* 60 (2): 615–47.

———. 2008. "The Credit Crisis: Conjectures about Causes and Remedies." Working Paper, University of Chicago Booth School of Business.

Farhi, E., M. Golosov, and A. Tsyvinski. 2008. "A Theory of Liquidity and Regulation of Financial Intermediation." *Review of Economic Studies* 76 (3): 973–92.

Fleming, M. J., W. B. Hrung, and F. M. Keane. 2009. "The Term Securities Lending Facility: Origin, Design, and Effects." *Current Issues in Economics and Finance* (Federal Reserve Bank of New York) 15 (2).

Freixas, X., and C. Holthausen. 2005. "Interbank Market Integration under Asymmetric Information." *Review of Financial Studies* 18 (2): 459–90.

Freixas, X., and J. Jorge. 2008. "The Role of Interbank Markets in Monetary Policy: A Model with Rationing." *Journal of Money, Credit, and Banking* 40 (6): 1151–76.

Freixas, X., A. Martin, and D. R. Skeie. 2008. "Bank Liquidity, Interbank Markets, and Monetary Policy." Working Paper.

Freixas, X., B. M. Parigi, and J. C. Rochet. 2000. "Systemic Risk, Interbank Relations, and Liquidity Provision by the Central Bank." *Journal of Money, Credit, and Banking* 32 (3): 611–38.

Gorton, G. 2008. "The Panic of 2007." Paper presented at the Federal Reserve Bank of Kansas City Symposium on Maintaining Stability in a Changing Financial System, Jackson Hole, Wyoming, August 21–23.

———. 2009. "Information, Liquidity, and the (Ongoing) Panic of 2007." *American Economic Review: Papers and Proceedings* 99 (2): 567–72.

Heider, F., M. Hoerova, and C. Holthausen. 2009. "Liquidity Hoarding and Interbank Market Spreads: The Role of Counterparty Risk." ECB Working Paper.

Hördahl, P., and M. R. King. 2008. "Developments in Repo Markets During the Financial Turmoil." *BIS Quarterly Review* (December).

Jacklin, C. 1987. "Demand Deposits, Trading Restrictions, and Risk Sharing." In *Contractual Arrangements for Intertemporal Trade*, ed. E. C. Prescott and N. Wallace, 26–47. Minneapolis: University of Minnesota Press.

McAndrews, J., A. Sarkar, and Z. Wang. 2008. "The Effect of the Term Auction Facility on the London Inter-Bank Offered Rate." Federal Reserve Bank of New York Staff Report No. 335.

Taylor, J. B. 2009. "The Financial Crisis and the Policy Responses: An Empirical Analysis of What Went Wrong." NBER Working Paper No. 14631.

Taylor, J. B., and J. C. Williams. 2009. "A Black Swan in the Money Market." *American Economic Journal: Macroeconomics* 1 (1): 58–83.

Discussion of "Interbank Lending, Credit-Risk Premia, and Collateral"*

Ernst-Ludwig von Thadden
University of Mannheim

The paper by Heider and Hoerova (2009) is ambitious. It studies the interaction between secured interbank lending, unsecured interbank lending, and banks' portfolio choices. It is motivated by a puzzling empirical observation, namely the "decoupling of secured and unsecured lending rates" in the Great Financial Crisis of 2007–09.

The observation made by Heider and Hoerova is that on August 9, 2007, not only did secured three-month interbank rates start to exhibit historically unprecedented discounts to unsecured rates, but the time-series behavior of both rates began to diverge as never before. Furthermore, the discount of secured interbank rates fluctuated more strongly in the United States than in the euro area. While the evidence consists of only two plots of time series, it is obvious without any further econometric analysis that the phenomenon is there and is significant.

The simple observation that unsecured lending became more expensive than secured lending in a period of financial turbulence is not surprising. Any model of lending will produce this result. What is puzzling is the decoupling of the time-series behavior. In order to address this puzzle, Heider and Hoerova propose a model of interbank borrowing that imbeds both types of interbank markets into a model of bank portfolio management under investment risk. This model exhibits a key feature of what seems to have occurred after August 2007: the dramatic change in the risk of bank portfolios has affected secured and unsecured interbank rates differently. While this result is derived in an essentially static model, it is plausible and contributes greatly to our understanding of the crisis.

*Author contact: Ernst-Ludwig von Thadden, University of Mannheim, Department of Economics, L7, D-68131 Mannheim. E-mail: vthadden@uni-mannheim.de.

The model is ambitious because it integrates three assets into a model of liquidity provision by the banking sector—namely cash, bank loans, and government bonds—and links banks by an unsecured interbank market. The model remains tractable because the authors use a clever shortcut to model the repo market: instead of modeling collateralized borrowing literally, they consider the outright sale of safe bonds as a means to generate liquidity, thus ignoring the repurchase leg of the transaction. This means that there are essentially two markets to be considered: the market for unsecured lending and the market for safe bonds, which are linked through a no-arbitrage condition.

Much of the mechanics of the analysis and some of its shortcomings can be understood by dropping the bond market and studying the case of only two assets. This is what I will do in most of the comments that follow. In these comments I will highlight the following conceptual issues:

- balance-sheet liquidity
- equilibrium in the interbank market
- provision of economy-wide liquidity

All of these issues are simplified in one way or another in the paper by Heider and Hoerova and thus provide fertile ground for further research. But while the paper may be too brief or oversimplify on some of these issues, it stands out by blazing a trail through a highly complex set of questions and by providing a benchmark model that is very useful and that I expect to become widely used.

The model features competitive banks in the tradition of Diamond and Dybvig (1983) that manage funds on behalf of households with uncertain liquidity needs. In exchange for collecting the funds of the household sector at date $t = 0$, banks promise households a repayment on demand of either c_1 in $t = 1$ or of c_2 in $t = 2$. In the spirit of Bhattacharya and Gale (1987), aggregate liquidity demand by households in the economy is certain, but liquidity demand at each bank is either λ_l or λ_h, with $\lambda_l < \lambda_h$ and $\lambda = \pi_l\lambda_l + \pi_h\lambda_h$, $\pi_l + \pi_h = 1$. At date $t = 0$, banks are identical, have no equity capital, and can invest their funds into a liquid asset with zero net return ("cash"), or an illiquid risky asset, or in a safe long-term bond that is in limited supply. The illiquid asset repays nothing at date 1 but has a risky return of R with probability p at date 2 (and pays nothing with the complementary probability $1 - p$).

The initial balance sheet of a representative bank is therefore

α	Illiquid Risky Asset		
β	Safe Bond	1	Deposits
$1 - \alpha - \beta$	Cash		

As noted above, I will here focus mostly on the case $\beta = 0$.

At date 1, each bank holds liquid reserves of $1 - \alpha$ and faces liquidity demand of either $\lambda_l c_1$ or $\lambda_h c_1$. In the latter case, the bank borrows inelastically on the (unsecured) interbank market the amount

$$D_h = 1 - \alpha - \lambda_h c_1. \tag{1}$$

In the former case, the bank has excess liquidity and can hold it in terms of cash or lend it in the interbank market at the rate r. Cash holdings C_l and interbank loans L_l are related by the date 1 budget constraint

$$C_l + L_l + \lambda_l c_1 = 1 - \alpha. \tag{2}$$

The bank's date 2 profits now are a random variable with four possible realizations: its illiquid risky asset can pay off or not, and the counterparty in the interbank market can repay or not. Heider and Hoerova assume that banks' portfolio risks are identically and independently distributed. Since a bank that borrows in the interbank market only has illiquid risky assets, the counterparty failure risk $1 - \hat{p}$ for the l-type bank is simply the common failure risk of the illiquid risky asset, $1 - p$. Assuming that

$$(1 - \lambda_l)c_2 \leq \alpha R \text{ and } \lambda_l c_1 \leq 1 - \alpha \tag{3}$$

(as Heider and Hoerova implicitly do), the bank's expected date 2 profit is

$$\begin{aligned} \Pi = p[\alpha R + C_l + \hat{p}(1 + r)L_l - (1 - \lambda_l)c_2] \\ + (1 - p)\hat{p} \max(0, C_l + (1 + r)L_l - (1 - \lambda_l)c_2). \end{aligned} \tag{4}$$

The term in square brackets in (4) is the bank's return if its illiquid asset pays off, and the maximum term is its return if its illiquid asset does not pay off but the counterparty is solvent. Let

$$S_l = 1 - \alpha - \lambda_l c_1 - (1 - \lambda_l)c_2 \tag{5}$$

denote the difference of the bank's liquid assets and its expected fixed payout obligations. S_l is a measure of the bank's balance-sheet liquidity.

Then, using (2) to eliminate C_l and rearranging (4), we have

$$\Pi = \begin{cases} p\alpha R + pS_l + p(\hat{p}(1+r) - 1)L_l & \text{if } rL_l + S_l \leq 0 \\ p\alpha R + (p + (1-p)\hat{p})S_l + (\hat{p}r - p(1-\hat{p}))L_l & \text{if } rL_l + S_l \geq 0 \end{cases} \tag{6}$$

At date 1, the bank maximizes Π over $0 \leq L_l \leq 1 - \alpha - \lambda_l c_1$. In their analysis, Heider and Hoerova implicitly assume that $rL_l + S_l \leq 0$. Under this assumption, their analysis is correct, but it is not clear that this assumption is justified. The bank's balance-sheet liquidity is an important part of the bank's date 0 optimization problem, and it may well be optimal to set it at levels that invalidate Heider and Hoerova's assumption.

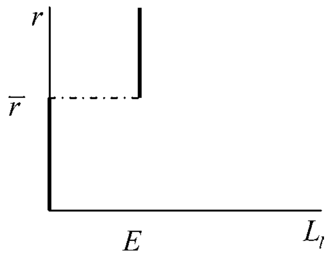
Turning to the supply of loanable funds in the interbank market, (6) shows that this supply is bang-bang: for small interest rates r it is 0, and for sufficiently large rates it is equal to the total amount of excess cash (per lending bank):

$$E = 1 - \alpha - \lambda_l c_1. \tag{7}$$

In between, there is an interest $\overline{r}$ at which the banks with high liquidity (l-type) are indifferent as to how much they want to lend. Hence, the supply function of loanable funds by the individual bank looks like that shown in figure 1.

Using (6), it can be shown that the critical interest rate satisfies $\overline{r} < \frac{1}{\hat{p}} - 1$. Equilibrium in the interbank market requires that the total supply of loanable funds, $\pi_l L_l$, be equal to the total demand for loans, $\pi_h D_h$. There can be two types of equilibria—either $L_l = E$ with an arbitrary equilibrium interest rate $r \geq \hat{r}$, or $L_l < E$ and $r = \hat{r}$. Figure 2 provides examples of both types of equilibria.

Figure 1. Individual Loan Supply

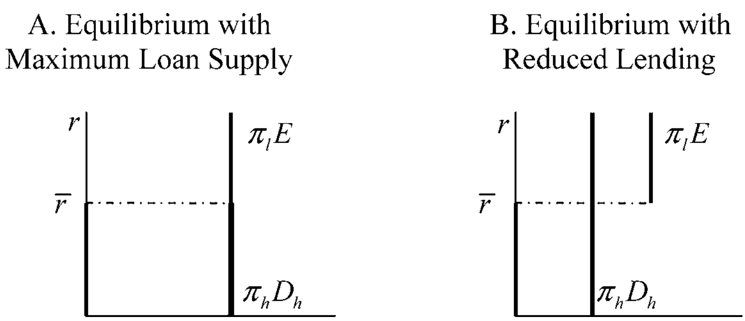


In the former equilibrium (figure 2A), (1) and (7) imply that

$$1 - \alpha = \lambda c_1, \tag{8}$$

which is the situation considered in lemma 3 (and correspondingly, in lemma 6) of the paper. But what about equilibria of the type depicted in figure 2B? There are parameter constellations under which such equilibria can exist as well, and it would be interesting to know more about them. In such equilibria, banks hoard liquidity at date 0. Therefore, banks with liquidity needs at date 1 only need to borrow little, and banks with excess liquidity only provide what is needed, keeping the rest on their balance sheet. Such behavior corresponds to more "prudent" banking and therefore may not be a good description of what happened in the run-up to the Great Financial Crisis. But they exist, and it may be interesting to investigate also theoretically why they may not be selected.

Figure 2. Equilibrium in the Interbank Market



A final question that is left open by the analysis in the paper is the question of what determines the consumption allocations (c_1, c_2). In the paper, this is taken as exogenous, and the equilibrium allocations are computed as a function of (c_1, c_2). This is certainly a useful first step but should be carried further in subsequent work. One way to endogenize bank liabilities is, of course, the classical motive of liquidity insurance by Diamond and Dybvig (1983). It may well be that this way to close the model can simply be added to the existing analysis, but care needs to be taken with respect to the many implicit assumptions in the model such as (3) above. But if this is done properly, it may well be that endogenizing bank liabilities even solves the multiplicity problem noted earlier. There are a number of questions that this paper leaves open, but the research agenda opened by Heider and Hoerova is very promising.

References

Diamond, D. W., and P. H. Dybvig. 1983 "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91 (3): 401–19.

Bhattacharya, S., and D. Gale. 1987. "Preference Shocks, Liquidity, and Central Bank Policy." In *New Approaches to Monetary Economics*, ed. W. A. Barnett and K. J. Singleton. Cambridge: Cambridge University Press.

Heider, F., and M. Hoerova. 2009. "Interbank Lending, Credit-Risk Premia, and Collateral." *International Journal of Central Banking* 5 (4): 5–43.

Liquidity, Moral Hazard, and Interbank Market Collapse*

Enisse Kharroubi[a] and Edouard Vidon[a,b]
[a]Banque de France
[b]International Monetary Fund

This paper proposes a framework to analyze the functioning of the interbank liquidity market and the occurrence of liquidity crises. The model relies on three key assumptions: (i) ex ante investment in liquid assets is not verifiable—it cannot be contracted upon, (ii) banks face moral hazard when confronted with liquidity shocks—unobservable effort can help overcome the shock, and (iii) liquidity shocks are private information—they cannot be diversified away. Under these assumptions, the aggregate volume of capital invested in liquid assets is shown to exert a positive externality on individual decisions to hoard liquid assets. Due to this property, the collapse of the interbank market for liquidity is an equilibrium. Moreover, such an equilibrium is more likely when the individual probability of the liquidity shock is lower. Banks may therefore provision too few liquid assets compared with the social optimum.

JEL Codes: D53, D82, D86.

*We thank Nuno Cassola, Edouard Challe, Denis Gromb, Arvind Krishnamurthy, Henri Pagès, Jean-Charles Rochet, Rafael Repullo, Tano Santos, Jean Tirole, and two anonymous referees for helpful comments and suggestions. All remaining errors are ours. A previous version of the paper was circulated under the title "Increasing Returns in Inter-Bank Liquidity Market." The views expressed herein are those of the authors and should not be attributed to the Banque de France, the Eurosystem, or the International Monetary Fund, its executive board, or its management. Corresponding author: Enisse Kharroubi. Address: Banque de France. 49-1374. 1, rue de la Vrillière. 75049 Paris cedex 01. E-mail: enisse.kharroubi@banque-france.fr. Tel: + 33 1 42 92 47 39. Fax: + 33 1 42 92 49 50.

1. Introduction

The financial market turmoil that has been under way since the summer of 2007 hit the core of the global financial system, the interbank market for liquidity. While this paper does not endeavor to account for all the features of the recent crisis, be it hard evidence or casual stories about the motivations of market players, it argues that a proper modeling of the collapse in the market for liquidity involves a close look at incentives to provision/hoard liquidity and moral hazard mechanisms in the interbank market. In addition, it makes sense to do so in a framework where banks can actually fail and default on their borrowing. Both of these assumptions are strongly vindicated by salient features of the recent crisis. Many observers have argued that securitization may have provided the wrong incentives regarding the monitoring of underlying asset quality, in a clear-cut case of moral hazard. In addition, recent developments have shown that bank failure scenarios are only too realistic.

We investigate the possible role of insufficient ex ante liquidity provision in paving the way to an interbank market collapse. We thus highlight the benefits of situations where banks set aside large amounts of liquid assets in order to better deal with shocks affecting their illiquid investments. By liquidity provisions, we mean specifically holdings of assets that can be used to safely transfer wealth over a short period of time. This may be seen as a form of "balance-sheet liquidity." In practice, such liquid holdings could be remunerated reserves held at the central bank, or short-term Treasury securities.[1] Indeed, the secular decline in the share of liquid assets on banks' balance sheets is a striking stylized fact that has been underscored by Goodhart (2008) as a troubling feature of risk management. A situation where market and funding liquidity appeared to be high may thus have hidden vulnerabilities stemming from limited holdings of liquid assets.

Against such a background, this paper shows that across equilibria, the risk-adjusted return on liquid assets can be increasing with

[1]We do not model a risk-free asset market as such; however, we will simply assume that a technology providing a risk-free rate of return is available as an alternative to illiquid investments on the one hand, and to interbank lending on the other hand.

the aggregate volume of such assets in the economy. When a bank faces a liquidity shock, it needs to reinvest in its impaired assets. Moreover, success in reinvestment depends on the effort the bank undertakes. When it has provisioned a large volume of liquidity ex ante, reinvestment is mostly financed through internal funds. Hence, the distressed bank pays particular attention to improving the probability that reinvestment succeeds. Consequently, the moral hazard problem is mitigated and the distressed bank benefits from a large capacity to borrow liquidity on the interbank market. This tends to raise the demand for liquidity and hence the price of liquidity, which in turn raises incentives to provision liquid assets ex ante. As a result, both the risk-adjusted return on liquidity provisioning and the total volume of liquidity in the economy are large.

By contrast, with low ex ante liquidity provision, the argument is reversed: the moral hazard problem is amplified through the aforementioned channel—reinvestment is mostly financed through external funds. Intact lending banks then impose a tight constraint on the volume of liquidity distressed banks can borrow on the interbank market so as to restore their incentives to deliver effort. This, however, reduces the demand for liquidity and drives down the price of liquidity, which in turn depresses banks' incentives to provision liquidity ex ante. Consequently, the risk-adjusted return on liquidity provisioning and the total volume of liquidity in the economy are low. The two polar cases of high and low liquidity provisions can therefore both be equilibrium outcomes.

Turning to comparative statics, the credit-rationing equilibrium happens to be more likely when the liquidity shock is less likely. We call this property *the curse of good times*. When the probability of facing the liquidity shock is low, banks reduce their liquidity holdings because they are less likely to need these liquid assets for reinvestment. This tightens the moral-hazard-induced liquidity constraint, reducing the demand for liquid assets and thereby the return on liquid assets on the interbank market, which in turn reduces incentives to provision liquidity ex ante. Conversely, the equilibrium with large liquidity provision and high risk-adjusted return is more likely when the liquidity shock is more likely, a property we call *the virtue of bad times*. When the probability of facing the liquidity shock is high, banks raise their liquidity holdings because they are more likely to need these provisions for reinvestment. This relaxes the moral-hazard-induced liquidity constraint, raising the demand for liquidity and thereby the price of liquidity on the interbank market, which in turn raises incentives to provision liquidity ex ante. Hence, when the probability of the liquidity shock is intermediate, multiple equilibria emerge: large (resp. low) aggregate investment in liquid assets tends to raise (resp. reduce) the return on liquid assets and thereby raise (resp. reduce) individual incentives to invest in liquid assets.

Finally, the paper investigates how policy can prevent or dampen a collapse of the market for liquidity. The main result is that policies aimed at tackling the collapse of the interbank market ex post—i.e., after the collapse has happened—are unlikely to reach their goal. In particular, liquidity injections as well as interest rate cuts cannot help distressed banks overcome their liquidity shocks. By contrast, ex ante policies, especially those that modify the relative return of liquid assets compared with illiquid assets, can be successful in preventing a collapse of the interbank market. In other words, monetary policy, by setting short-term interest rates which provide incentives to invest in liquid assets, can be helpful in reducing the occurrence of liquidity crises. Regulatory policies requiring liquidity provision can also be useful.

The model in this paper builds on the standard literature on moral hazard and liquidity crisis. The demand for liquidity is modeled in a basic, standard fashion, similar to that of Holmström and Tirole (1998). Agents (in our case, banks) with long-term assets face stochastic liquidity shocks which trigger a reinvestment need and a moral hazard problem: success in reinvestment depends on unobservable effort by banks.[2] We, however, depart from this seminal paper in an important way, by assuming that idiosyncratic liquidity shocks cannot be diversified away: this opens the door to an interbank market where liquidity can be reallocated interim. Because of this feature, our framework is closely related to the

[2]The main alternative modeling of liquidity is based on the Diamond and Dybvig (1983) approach—enriched by Diamond and Rajan (2001)—in which banks with illiquid assets supply liquidity to consumers through liquid deposits (funding liquidity). While this approach can account for bank runs that have taken place during the current financial crisis, the Holmström and Tirole (1998) approach, focused on market liquidity, seems more relevant given the particular initial circumstances of the crisis.

model of liquidity demand developed by Caballero and Krishnamurthy in a series of papers (in particular, Caballero and Krishnamurthy 2004) dealing with access to international financing. Our model shares their features that (i) idiosyncratic shocks cannot be written into insurance contracts, generating the need for domestic financial transactions, and (ii) borrowers cannot transfer the full surplus generated by reinvestment resources. Likewise, we therefore have situations where private decisions are biased against hoarding liquidity.

Our paper is connected to the literature on interbank markets, as a mechanism for managing, and potentially eliminating, risks stemming from idiosyncratic liquidity shocks. Bhattacharya and Gale (1987) in particular studied the case where neither banks' investments in the illiquid technology nor liquidity shocks are observable. In their framework, banks have an incentive to underprovision liquidity ex ante and free-ride the common pool of liquidity. Rochet and Tirole (1996) adapted the Holmström-Tirole framework to the interbank market in order to study systemic risk and "too-big-to-fail" policy. The existence of interbank market imperfections has been established empirically by Kashyap and Stein (2000), which showed the role of liquidity positions, the so-called "liquidity effect." Building on such evidence, Freixas and Jorge (2008) analyzed the functioning of the interbank market in order to show the consequences of its imperfections for monetary policy. In particular, they established the relevance of heterogeneity in banks' liquid asset holdings for policy transmission.

Our work is also related to recent work on liquidity crises. A recent strand of literature has explored the propagation of crises through banks' balance sheets, while treating the level of liquidity held by banks as endogenous. This approach builds on Allen and Gale's (1998) analysis of distressed liquidation of risky assets, to explore the mechanism whereby anticipation of fire-sale pricing of such assets determines banks' ex ante portfolio allocation. Allen and Gale (2004) as well as Acharya, Shin, and Yorulmazer (2007, 2009) have concentrated on this interaction between equilibrium liquidity and endogenously determined fire sales. In particular, Acharya, Shin, and Yorulmazer (2007) showed that banks' holdings of liquidity may be too low or too high compared with the social optimum, depending on the pledgeability of their assets and the possibility to take advantage of fire sales. Interestingly, in their model, liquidity holdings are decreasing in the health of the economy, a result similar to our *curse of good times* property.[3]

In related work, Acharya, Gromb, and Yorulmazer (2008) studied the consequences of imperfect competition in the interbank market for liquidity. In a model where there are frictions in the money and asset markets, if banks that provide liquidity have market power, they may strategically underprovide liquidity and thus precipitate fire sales.

Finally, Caballero and Krishnamurthy (2008) provided a model of crises that features liquidity hoarding and provides a motivation for lender-of-last-resort intervention. Their approach is primarily based on Knightian uncertainty that leads each agent to hedge against the worst-case scenario.

A common feature of this literature is that bank holdings of liquidity are not necessarily optimal. The public provision of liquidity, such as liquidity injections, can therefore often improve on the allocation of liquidity resulting from the decentralized outcome. Our work shares these features. It also rejoins the result of Acharya, Shin, and Yorulmazer (2007, 2009) by which banks or outside arbitrageurs hold too little liquidity in good times.

Our paper, however, departs from this literature in two key aspects. First, the motivation for banks' ex ante provisioning of liquidity is not to have the possibility to purchase low-priced distressed assets, but rather to have the resources to reinvest in its own distressed projects, or to lend on the interbank market. Second, these papers do not feature interbank liquidity crises in the sense of a breakdown in the money market, simply because they typically do not consider interbank lending. While in the "fire-sales" literature the source of inefficiency is liquidation to outsiders, the focus of this paper is the interbank market collapse. Namely, we provide conditions under which the market for liquidity itself (as opposed to the distressed asset market) may cease to function. In addition, we show that the equilibrium where liquidity-affected banks face credit

[3]Acharya, Shin, and Yorulmazer (2009) also feature the result that arbitrage capital is lower in good times, leading to bigger fire-sale discounts.

rationing remains when allowing for liquidation of risky assets once the liquidity shock hits.[4]

In sum, this paper's contribution consists in combining standard features of the moral hazard literature in order to account for a collapse in interbank lending. To the best of our knowledge, the feedback loop between aggregate investment in liquid assets and the return to liquid assets as well as implications in terms of insufficient aggregate liquidity provision and multiple equilibria have not been studied previously.

The paper is organized as follows. The following section lays down the main assumptions of the model. The first-best allocation is derived in section 3. The problem of intact and distressed banks in a second-best environment is analyzed in section 4. Section 5 details the decentralized equilibrium, characterizing the full-reinvestment and credit-rationing equilibria. Section 5 also discusses the nature of the externality at the source of the multiple equilibria property. Section 6 looks at its robustness by relaxing some of the model's assumptions. Section 7 derives some policy implications. Section 8 concludes.

2. Timing and Technology Assumptions

We consider an economy with a unit mass continuum of banks. Banks are risk neutral and maximize expected profits. The economy lasts for three dates: 0, 1, and 2. At date 0, each bank has a unit capital endowment and two investment possibilities. The first is to invest in a liquid asset: a unit of capital invested in the liquid technology at date $t \in \{0; 1\}$ yields one unit of capital at date $t + 1$. The volume of capital that a bank invests at date 0 in the liquid technology is denoted l. Alternatively, each bank can invest in an illiquid project. The volume of capital a bank can invest in an illiquid project at date 0 is hence equal to $1 - l$. The volume of capital invested in each technology is observable but not

[4]This result holds assuming that liquidity-affected banks can borrow on the interbank market against the product of liquidation. If this is not possible—if liquidation takes time, for instance—then interbank market total collapse is still an equilibrium even if liquidity-affected banks can liquidate their risky assets.

verifiable. Contingent contracts on ex ante liquidity provisioning are thus precluded.[5]

Illiquid projects are invested in at date 0. At date 1, they may face a liquidity shock. With probability $1-q$, the liquidity shock is avoided and the bank that has financed the project is said to be "intact." The illiquid project yields R units of capital at date 2 per unit of date 0 investment. With probability q, the liquidity shock occurs and the bank that has financed the project is said to be "distressed." Following Holmström and Tirole (1998), a liquidity shock at date 1 triggers (i) a reinvestment need and (ii) a shirking possibility: a distressed bank which reinvests ck units of capital ($0 < c \leq 1$ and $k \leq 1-l$) and delivers an effort e at date 1 reaps $R(e)k$ units of capital at date 2 with a probability e. With probability $1-e$, it gets nothing. Importantly, effort e is private information and hence a source of moral hazard. Similarly, the liquidity shock is private information and hence cannot be diversified away across banks.[6]

To simplify, and without any implications for further analysis, effort e can be either high, $e = e_h$, or low, $e = e_l$ ($e_h > e_l$), with $R(e_h) = R$ and $R(e_l) = \mu R$ with $\mu > 1$. High effort e_h is efficient and low effort is dominated: $e_l \mu R < 1 < e_h R$.[7] Finally we add the following parameter restrictions: (i) parameter c is normalized to 1, (ii) the illiquid project is more profitable on average than the liquid technology $(1-q)R > 1$, and (iii) moral hazard—scaled by the μ parameter—is sufficiently large, i.e., $\frac{e_h - e_l}{e_h - \mu e_l} > R$.[8]

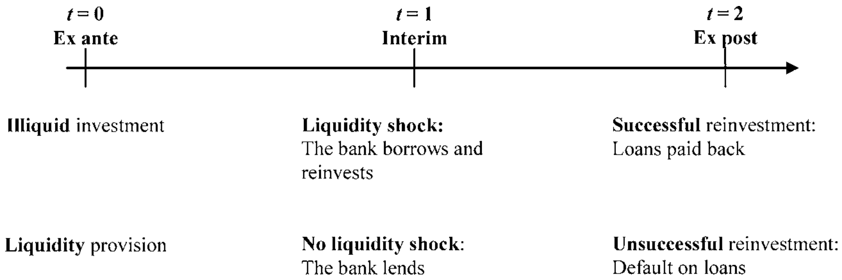
[5]The assumption that ex ante liquidity provisioning is neither observable nor verifiable is a sufficient condition (although not necessary), under which the results of the model hold. In reality, the volume of liquid assets a bank holds at a given point in time may be observable. However, the funding source for these assets—capital or short-term deposits, for instance—is much more difficult to assess for an outside agent in real time. Hence, even observability can be an issue in practice.

[6]The alternative arrangement under which banks would sign ex ante insurance contracts against liquidity shock is not possible here. If banks receive a payment when they declare to be distressed, then "intact" banks would always report untruthfully their situation as "distressed" since (i) liquidity shocks are unobservable and (ii) banks can invest the payment from the insurance contract in the liquid technology from date 1 to date 2 and finally consume the output at date 2.

[7]The parameter μ incorporates private benefits stemming from delivering low effort e_l.

[8]This last parameter restriction ensures that the moral hazard problem does not disappear when the interest rate on the interbank market is sufficiently low.

Figure 1. Timing of the Model



Timing, shown in figure 1, is as follows. At date 0, banks decide on capital allocation between liquid and illiquid assets. At date 1, a fraction q of banks face the liquidity shock. The interbank market then opens, and intact banks can lend to distressed banks. Distressed banks reinvest their own liquidity plus borrowed funds in their illiquid project and deliver some effort. Banks, both intact and distressed, can also invest in the risk-free liquid technology at date 1 if they prefer to do so. Finally, at date 2, distressed banks learn if reinvestment has been successful. If so, they pay back their liabilities.

3. The First-Best Allocation

To derive the first-best allocation, we remove two assumptions regarding market imperfections. First, date 0 allocation between liquid and illiquid assets is now verifiable. Second, both the liquidity shock at date 1 and the effort e delivered by distressed banks are now public information.

Let $(l; k; e)$ be a generic contract where l is date 0 investment in the liquid technology, k is date 1 reinvestment in a project that faces a liquidity shock, and e is effort undertaken in case of reinvestment. The first-best allocation solves

$$\max_{l;k;e} \ (1-q)(1-l)R + qkeR(e) + (l - qk) \quad \text{s.t. } qk \leq \min\{l; q(1-l)\}. \tag{1}$$

Each unit of capital endowment is divided between l units of capital invested in the liquid asset and $1 - l$ units of capital invested in the illiquid asset. The illiquid asset is intact with probability $1 - q$. In this case, it returns $(1-l)R$ at date 2. With probability q, the illiquid asset is distressed. If k units of capital are reinvested in each distressed project, total date 1 reinvestment is equal to qk. Since there are l units of capital available at date 1 for reinvestment, and given that reinvestment k in each distressed project cannot be larger than $(1-l)$, total reinvestment qk cannot be larger than l and $q(1-l)$. Moreover, each distressed project in which k is reinvested yields an expected return $keR(e)$. Finally, when total capital available at date 1 is larger than aggregate reinvestment, $l > qk$, the remaining available capital $l - qk$ is invested in the liquid technology with a unit marginal return. We can then derive the following result.

PROPOSITION 1. *The first-best capital allocation is such that each bank invests l^* units of capital in the liquid technology at date 0 with*

$$l^* = \frac{q}{1+q}\mathbf{1}[e_h > 1-q].$$

Proof. Optimality requires that $e = e_h$ since $e_h R(e_h) > e_l R(e_l)$ and $qk = \min\{l; q(1-l)\}$ since $e_h R > 1$. The problem therefore simplifies as

$$\max_l \ (1-q)(1-l)R + \min\{l; q(1-l)\}e_h R + (l - \min\{l; q(1-l)\}).$$

This problem is piecewise linear in l. So one extreme value of l must be optimal. When $l \leq q(1-l)$, the optimal capital allocation writes as

$$l^* = \frac{q}{1+q}\mathbf{1}[e_h \geq 1-q],$$

where $\mathbf{1}[x]$ is equal to 1 if x is true and zero otherwise. On the contrary, when $l \geq q(1-l)$, then given that $(1-q)R > 1$ and $e_h R > 1$, optimal capital allocation writes as $l^* = \frac{q}{1+q}$.

The first-best optimal ex ante liquidity provision is $l^* = \frac{q}{1+q}$ when $e_h \geq 1-q$ and $l^* = 0$ when $e_h < 1-q$. Typically, when the probability q of the liquidity shock is sufficiently low—i.e., $q < 1-e_h$—then it is not worth provisioning liquidity, because there will be very few illiquid projects hit by the liquidity shock. Put differently, the expected return to illiquid investments without any ex ante liquidity provision $(1-q)R$ is very large. The social planner then prefers to maximize illiquid investments. In what follows, we will assume that the parameter restriction $e_h > 1-q$ always holds so that first-best ex ante liquidity provision is always $l^* = \frac{q}{1+q}$.

4. Intact and Distressed Banks

We now turn to the resolution of the model described in section 2, which can be done by backward induction. We first solve the problem of intact and distressed banks at date 1. Then we solve the date 0 problem of optimal ex ante liquidity provision.

4.1 Distressed Banks' Optimal Demand for Liquidity

Consider bank i which, at date 0, invested l_i units of capital in the liquid technology and $1-l_i$ in an illiquid project. If bank i is distressed at date 1, it can either reinvest in its illiquid project or give up this project and lend its liquid assets on the interbank market. In case a distressed bank reinvests in its illiquid project, d_i denotes the volume of capital it borrows at date 1 and e_i the effort it undertakes. Its date 2 expected profit then writes as

$$\pi_b = e_i[(l_i + d_i)R(e_i) - rd_i]. \tag{2}$$

At date 1, a distressed bank uses the proceeds of its date 0 liquid investments l_i and borrows d_i to reinvest in the illiquid project initiated at date 0. Hence, reinvestment is equal to $l_i + d_i$. Conditional on success, date 2 output net of nonpecuniary cost of delivering effort is $(l_i + d_i)R(e_i)$, the face value of liabilities is rd_i, and e_i is the probability of successful reinvestment. Note that the interest rate r is independent of bank i decisions and in particular of its effort e_i, because effort is unobservable. The problem at date 1 of a distressed bank which reinvests in its illiquid project consists in choosing the effort level e_i and the volume of borrowing d_i which solve the problem

$$\max_{d_i;e_i} \pi_b = e_i[(l_i + d_i)R(e_i) - rd_i] \quad \text{s.t. } l_i + d_i \leq 1 - l_i. \tag{3}$$

The constraint that total reinvestment $(l_i + d_i)$ cannot be larger than the reinvestment need $(1 - l_i)$ imposes a limit on the volume d_i that can be borrowed on the interbank market. We can then derive the following proposition.

PROPOSITION 2. *Denoting* $\psi = \frac{e_h - e_l \mu}{e_h - e_l}$, *if the interest rate on the interbank market verifies* $r \leq R$, *a distressed bank's demand for liquidity* d_i *is such that* $l_i + d_i = 1 - l_i$. *It delivers effort* e_i *such that*

$$e_i = \begin{cases} e_h \text{ if } (r - \psi R)d_i \leq \psi R l_i \\ e_l \text{ if } (r - \psi R)d_i > \psi R l_i. \end{cases} \tag{4}$$

Proof. If bank i is distressed and reinvests in its illiquid project, then optimal borrowing d_i^* writes as

$$d_i^* = (1 - l_i - l_i)\mathbf{1}[R(e_i^*) \geq r]. \tag{5}$$

Consequently, as long as $r < R$, $d_i^* = (1 - l_i - l_i)$ and optimal effort e_i^* is given by

$$e_i^* = \begin{cases} e_h \text{ if } rd_i^* \leq \psi R\big(l_i + d_i^*\big) \\ e_l \text{ if } rd_i^* > \psi R\big(l_i + d_i^*\big). \end{cases} \tag{6}$$

A distressed bank is more likely to deliver high effort e_h when reinvestment is proportionally more financed through internal funds—i.e., when ex ante liquidity provisioning l_i is larger and/or borrowing d_i is lower.

Having determined optimal borrowing and effort conditional on reinvestment, we can now examine whether distressed banks prefer to reinvest in their illiquid assets or to give up their illiquid project and lend their liquid holdings on the interbank market. The following lemma derives this choice.

LEMMA 1. *If the interest rate on the interbank liquidity market verifies* $r \leq R$, *then distressed banks always prefer to reinvest in their illiquid project rather than lend their liquid assets on the interbank market.*

Proof. Denoting d_i^* the volume of capital a distressed bank borrows, when the interest rate on the interbank market verifies $r \leq R$, its expected profits from reinvestment π_b then write as

$$\pi_b = e_i \left[R(e_i)\big(l_i + d_i^*\big) - rd_i^* \right],$$

with e_i being the distressed bank's optimal effort. Expected profits π_b' from lending liquid assets on the interbank market are simply $\pi_b' = e_i r l_i$ because the repayment probability of distressed banks is e_i. Given the assumption $R(e_i) \geq r$, d_i^* is always positive and profits from reinvestment π_b are always larger than profits from lending liquid assets on the interbank market.

4.2 Intact Banks' Optimal Supply of Liquidity

We now turn to the case where bank j is intact at date 1. Recall that at date 0 it invested l_j units of capital in the liquid technology and $1 - l_j$ in an illiquid project. It hence reaps $(1 - l_j)R$ at date 2. Moreover, it can lend its liquid assets to distressed banks at date 1. When the interest rate on the interbank market is r, and distressed banks deliver effort e, intact bank j enjoys date 2 expected profits:

$$\pi_g(l_j) = (1 - l_j)R + l_j \max\{er; 1\}. \tag{7}$$

An intact bank can always invest its liquid assets l_j at date 1 in the liquid technology. Hence, intact banks supply their liquid holdings on the interbank market if and only if $er \geq 1$. A distressed bank delivers high effort e_h if and only if its ex ante liquidity provision l_i and its interbank market borrowing d_i verify

$$(l_i + d_i)\psi R \geq rd_i. \tag{8}$$

Given that it borrows at most $(1 - l_i - l_i)$ on the interbank market, there can be two different situations:

(i) If (8) holds for $d_i = (1 - l_i - l_i)$, then the distressed bank always delivers high effort e_h. Intact banks then supply their liquid holdings on the interbank market as long as the interest rate r verifies $e_h r \geq 1$.

(ii) If (8) does not hold for $d_i = (1 - l_i - l_i)$, then the distressed bank delivers low effort e_l and intact banks' participation constraint $er \geq 1$ cannot be met.

When a distressed bank delivers low effort e_l, the interest rate r it is charged cannot be larger than μR—otherwise, the distressed bank would not borrow—and by assumption we have $e_l \mu R < 1$. To make sure that the distressed bank delivers high effort e_h, intact lending banks impose a liquidity constraint. The volume of liquidity the distressed bank can then borrow verifies the incentive constraint:

$$e_h((l_i + d_i)R - d_i r) \geq e_l((l_i + d_i)\mu R - d_i r).$$

Denoting $[x]^+ = \max(x; 0)$, this condition simplifies as a borrowing constraint:

$$d_i \leq \overline{d}(l_i) \equiv \frac{\psi R}{[r - \psi R]^+} l_i. \tag{9}$$

In this case, a distressed bank's total borrowing from the interbank market is a positive function of its ex ante liquidity provision.[9]

5. The Decentralized Equilibrium

In the previous section, we derived the optimal date 1 decision rules for intact and distressed banks in terms of lending, borrowing, and effort. Based on these results, we now turn to the optimal date 0 liquidity provision policy in order to characterize the different equilibria of the economy.

Definition 1. *An equilibrium is an ex ante liquidity provision policy l and an interest rate r on the interbank market such that banks' date 0 expected profits are maximized:*

$$\max_l (1-q)[(1-l)R + l \max\{e_h r; 1\}] + q e_h[(l+d)R - rd]$$
$$s.t.\ d = \mathbf{1}(R \geq r) \min\left\{\tfrac{\psi R}{[r-\psi R]^+} l; 1 - l - l\right\}$$

[9] Recall that ex ante liquidity provisions are observable, so that the size of illiquid projects as well as reinvestment needs, assuming a shock has occurred, are also observable. However, the implementation of a borrowing constraint by intact banks on distressed banks requires the additional (implicit yet standard) assumption that total interbank borrowing is observable by lenders. Without such an assumption, no borrowing constraint can ever be enforced.

and the interest rate r balances the supply and the demand of liquidity at date 1; i.e., $L_s = L_d$, with

$$L_s = (1-q)l \text{ and } L_d = q \min\left\{\frac{\psi R}{[r-\psi R]^+} l; 1-l-l\right\}.$$

Aggregate liquidity supply L_s is the sum of intact banks' available liquid assets $(1-q)l$. Aggregate demand of liquidity L_d is the minimum of distressed banks' liquidity constraint and the maximal amount of liquidity these banks need to borrow. The following two subsections are devoted to laying down the conditions under which each of these two situations can be an equilibrium.

5.1 The Full-Reinvestment Equilibrium

5.1.1 Optimal Ex Ante Liquidity Provision with Full Reinvestment

Let us focus first on the case where distressed banks are able to reinvest fully in their illiquid project. Assuming the interest rate on the interbank market verifies $R > r$, the problem of bank i at date 0 then writes as

$$\begin{aligned} \max_{l_i} E\pi_i = (1-q)[(1-l_i)R + l_i e_h r] + q e_h[(l_i+d_i)R - r d_i] \\ \text{s.t. } d_i = 1 - l_i - l_i \text{ and } d_i \leq \overline{d}(l_i). \end{aligned} \tag{10}$$

PROPOSITION 3. *Denoting $r_1 = \frac{1-q+qe_h}{1+q}\frac{R}{e_h}$, optimal individual ex ante liquidity provision for a bank that reinvests fully in its illiquid project when distressed is given by*

$$l_i^* = \begin{cases} \dfrac{\frac{r-\psi R}{r}}{1+\frac{r-\psi R}{r}} & \text{if } r \leq r_1 \\ 1 & \text{if } r \geq r_1. \end{cases} \tag{11}$$

Proof. Expected profits are decreasing in ex ante liquidity provision for $r \leq r_1$, since

$$\frac{\partial E\pi_i}{\partial l_i} = (1+q)e_h\left[r - \frac{1-q+qe_h}{1+q}\frac{R}{e_h}\right] \leq 0.$$

Banks then choose to provision as little liquidity as they can. Optimal ex ante liquidity provision then verifies $l_i + \overline{d}(l_i) = 1 - l_i$. On the contrary, expected profits are increasing in ex ante liquidity provision for $r \geq r_1$. Banks then choose to provision as much liquidity as they can; i.e., $l_i^* = 1$. In between—i.e., for $r = r_1$—they are indifferent to ex ante liquidity provisioning.

5.1.2 Equilibrium Interbank Interest Rate with Full Reinvestment

The equilibrium with distressed banks achieving full reinvestment exists if and only if two conditions are met: First, ex ante liquidity provision l_i^* maximizes expected profits; i.e., there should be no profitable deviation ex ante for banks. Second, the aggregate supply of liquidity must balance the aggregate demand for liquidity:

$$(1-q)\int_{[0;1]} l_i^* di = q \int_{[0;1]} (1 - l_i^* - l_i^*) di. \tag{12}$$

Moreover, the cost of liquidity in the interbank market r must be such that distressed banks are willing to borrow and intact banks are willing to lend their liquid assets on the interbank market:

$$1 \leq e_h r \leq e_h R. \tag{13}$$

Let us denote $r_2 = \frac{\psi R}{1-q}$ and $r^* = \min\{r_1; r_2\}$. We can then derive the following proposition.

PROPOSITION 4. *The first-best allocation—where banks provision liquidity $l_i = l^*$ and fully reinvest in their project when distressed—is an equilibrium if and only if*

$$(1-q)R \leq e_h r^* \leq e_h R. \tag{14}$$

Proof. See the appendix.

Conditions (14) are more likely to be verified when the individual probability q of the liquidity shock is high. In other words, the equilibrium with full reinvestment is more likely to hold in deteriorated environments. More precisely, when the equilibrium interest rate is $r^* = r_1$, the individual rationality constraint for intact banks, $e_h r_1 \geq (1-q)R$, is always verified. Similarly, the individual rationality constraint for distressed banks, $r^* \leq R$, always holds since by assumption $e_h \geq 1-q$. Alternatively, when the equilibrium interest rate is $r^* = r_2$, the individual rationality constraint for distressed banks, $r^* \leq R$, is necessarily verified since $r^* = r_2$ implies $r_2 \leq r_1$ and we always have $r_1 \leq R$. Finally, the individual rationality constraint for intact banks, $e_h r_2 \geq (1-q)R$, is more likely to be verified when the probability q to face the liquidity shock is relatively large, since r_2 increases with the probability q.

When the probability q to face the liquidity shock is high, there are on the one hand more distressed banks, but on the other hand, banks raise their liquidity holdings because they are more likely to need these ex ante provisions for reinvestment. At the aggregate level, the former effect dominates and the demand of liquidity from distressed banks on the interbank market is large. This drives up the interbank market interest rate, which provides incentives for banks to provision liquidity ex ante. The full-reinvestment equilibrium is therefore more likely when the liquidity shock is more likely, a property we refer to as *the virtue of bad times*. Note finally that the equilibrium where distressed banks achieve full reinvestment is efficient in the sense that it replicates the first-best capital allocation between liquid and illiquid assets.

5.2 The Credit-Rationing Equilibrium

In the equilibrium described in the previous subsection, distressed banks are able to carry out full reinvestment thanks to their relatively large ex ante liquidity provision. This subsection examines what happens when the volume of liquidity that banks provision ex ante is not sufficiently large to ensure both full reinvestment and high effort.

5.2.1 Optimal Ex Ante Liquidity Provision under Credit Rationing

When the constraint $d_i \leq \overline{d}(l_i)$ on the volume of liquidity that can be borrowed from the interbank market is binding, each distressed bank borrows $\overline{d}(l_i)$ from intact banks. Assuming the cost of borrowing liquidity is lower than the return on reinvestment—i.e., $r < R$—the problem of an individual bank i at date 0 therefore consists in choosing the volume of ex ante liquidity provision l_i which solves

$$\begin{aligned} \max_{l_i} E\pi_i &= (1-q)[(1-l_i)R + e_h r l_i] + q e_h[(l_i+d_i)R - rd_i] \\ &\text{s.t. } d_i = \overline{d}(l_i) \text{ and } d_i \leq 1 - l_i - l_i. \end{aligned} \tag{15}$$

Proposition 5. *Optimal individual ex ante liquidity provision for a bank whose liquidity constraint binds is given by*

$$l_i^* = \begin{cases} 0 \text{ if } \dfrac{\partial E\pi_i}{\partial l_i} \leq 0 \\ \dfrac{\frac{r-\psi R}{r}}{1+\frac{r-\psi R}{r}} \text{ if } \dfrac{\partial E\pi_i}{\partial l_i} \geq 0. \end{cases} \tag{16}$$

Proof. When expected profits are decreasing in ex ante liquidity provision, then banks choose to provision as little liquidity as they can; i.e., $l_i^* = 0$. On the contrary, when expected profits are increasing in ex ante liquidity provision, then banks choose to provision as much liquidity as they can. This level of ex ante liquidity provisioning solves $l_i + \overline{d}(l_i) = 1 - l_i$.

The function $\frac{\partial E\pi_i}{\partial l_i}$ is potentially nonmonotonic in the interest rate on the interbank market. On the one hand, a high interbank market interest rate r raises the return to liquidity for intact banks. On the other hand, however, it raises the cost of borrowing liquidity for distressed banks, and it reduces the volume of liquidity they can borrow on the interbank market. Banks therefore choose low ex ante liquidity provisioning when the interest rate on the interbank market is either very low or very high.

5.2.2 Equilibrium Collapse of the Interbank Market

Given optimal date 0 ex ante liquidity provisioning (16), the aggregate demand of liquidity L_d at date 1 is

$$L_d = q\frac{\psi R}{r-\psi R}\int_{[0;1]} l_i^* di$$

and the aggregate supply of liquidity L_s at date 1 is

$$L_s = (1-q)\int_{[0;1]} l_i^* di.$$

We define a collapse of the interbank market as a situation where banks do not provision liquidity ex ante, and intact banks do not lend to distressed banks. We can then derive the following proposition.

PROPOSITION 6. *The collapse of the interbank market is the unique equilibrium of the credit-rationing regime. It exists if and only if*

$$1 + q\frac{e_h R - 1}{1 - \psi e_h R} < (1-q)R. \tag{17}$$

In this equilibrium, the interest rate verifies $e_h r = 1$.

Proof. See the appendix.

Condition (17)—under which the interbank market collapse equilibrium exists—is more likely to be satisfied when the probability q to face the liquidity shock is relatively low. When the liquidity shock is less likely, banks provision less liquidity ex ante and invest more in illiquid assets. Distressed banks are then more likely to deliver low effort when they reinvest in their illiquid project, as reinvested funds will be mostly borrowed. Intact lending banks then impose credit rationing to ensure that distressed banks deliver high effort. However, credit rationing reduces the demand for liquidity and thereby depresses the return on ex ante liquidity provision for intact banks. This in turn reduces ex ante incentives to provision liquidity, especially when the probability to remain intact is large. The credit-rationing equilibrium is therefore more likely when the liquidity shock is less likely, a property we refer to as *the curse of good times:* an environment with good fundamentals is conducive to credit rationing and interbank market collapse.

5.3 Multiple Equilibria and the General Equilibrium Externality

5.3.1 Multiple Equilibria

When ex ante liquidity provisioning is low, then both liquidity supply and liquidity demand are relatively low. Supply is low because intact banks have relatively few provisions. Demand is also low because the liquidity constraint stemming from moral hazard introduces a positive relationship between aggregate liquidity provisioning and the aggregate demand for liquidity. Hence, with little provisioning, the demand for liquidity is also low. In this case it turns out that the equilibrium interest rate on interbank liquidity is relatively low. This has two opposite consequences: On the one hand, this reduces the return to ex ante liquidity provisioning for intact (lending) banks. On the other hand, it raises the return to ex ante liquidity provisioning for distressed (borrowing) banks because (i) borrowing liquidity is not expensive and (ii) the volume of liquidity that can be borrowed on the interbank market increases with ex ante liquidity provisioning. When the probability q of facing the liquidity shock is relatively low, then the former effect (for intact banks) dominates the latter (for distressed banks), which gives rise to a negative feedback loop: a low expected return on ex ante liquidity provisioning reduces bank incentives to provision liquidity, and low ex ante liquidity provisioning generates a low demand for liquidity, which depresses the expected return on such provisioning. An equilibrium of low ex ante provisioning and low expected return on provisions therefore emerges. As a matter of fact, the necessary and sufficient condition (17) under which the interbank-market-collapse equilibrium exists can be simplified as an upper bound on the probability q of liquidity shocks:

$$q < \overline{q} \equiv \frac{R-1}{R+\frac{e_h R-1}{1-\psi e_h R}}.$$

Conversely, when ex ante liquidity provisioning is large, then both liquidity supply and liquidity demand are relatively high. Supply is high because intact banks hold a large volume of liquid assets. Demand is also high because with large ex ante liquidity provisioning, the liquidity constraint is not binding and distressed banks can therefore achieve full reinvestment. When the probability q of facing the liquidity shock is high, the interest rate at date 1 is relatively high because a larger number of banks are distressed, which raises the relative demand for liquidity. The expected return on ex ante liquidity provisioning is then high. This gives rise to a positive feedback loop: a large expected return on ex ante liquidity provisioning raises bank incentives to provision liquidity, while large ex ante liquidity provisions translate into a large expected return on liquid assets. As a result, an equilibrium with high ex ante liquidity provisioning and high expected return on provisions emerges. As a matter of fact, the necessary and sufficient condition (14) under which the full-reinvestment equilibrium appears can be simplified as a lower bound on the probability q of liquidity shocks:

$$q \geq \underline{q} \equiv 1 - \sqrt{e_h \min(e_h; \psi)}.$$

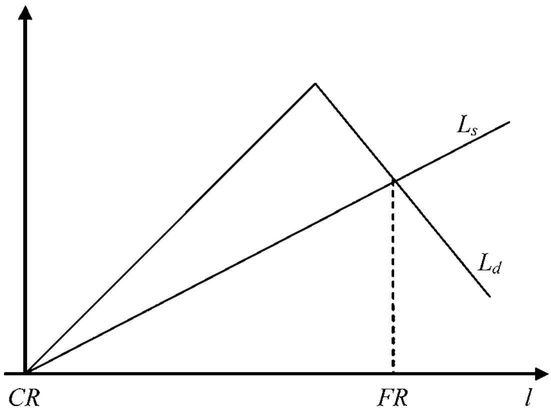
The economy is therefore subject to multiple equilibria when the probability q to face the liquidity shock verifies $\underline{q} \leq q < \overline{q}$. In this region, there is a probability p that agents coordinate on the interbank-market-collapse equilibrium and a probability $1 - p$ that agents coordinate on the full-reinvestment equilibrium. Outside this region the equilibrium is unique. When the probability q is sufficiently high, the full-reinvestment equilibrium occurs with probability one while when the probability q is sufficiently low, the interbank-market-collapse equilibrium occurs with probability one.[10]

5.3.2 Aggregate Supply of and Aggregate Demand for Liquidity

The multiple equilibria property can be examined in a diagram (figure 2) representing aggregate liquidity supply L_s and aggregate

[10]The interbank-market-collapse equilibrium could be eliminated if banks could sign contracts contingent on the volume of date 0 liquidity provisioning. For instance, banks could agree at date 0 to make the cost of borrowing liquidity at date 1 contingent on individual ex ante liquidity provisioning. If the interest rate r charged to distressed bank i writes as $r(l_i) = r^* + (R - r^*)\mathbf{1}[l_i < l^*]$, then bank i's ex ante liquidity provision l_i would always verify $l_i \geq l^*$ and the credit-rationing equilibrium would be ruled out. The assumption that ex ante liquidity provisioning is not verifiable is therefore required to obtain the credit-rationing equilibrium.

Figure 2. Aggregate Supply of and Aggregate Demand for Liquidity



demand L_d as a function of the aggregate ex ante liquidity provision l. Due to the existence of moral hazard, the aggregate demand for liquidity L_d is decreasing in the volume of aggregate ex ante liquidity provisioning l if and only if l is sufficiently large. When provisioning is low, the moral hazard problem binds and the demand for liquidity increases with aggregate ex ante liquidity provisioning.

Liquidity supply L_s is increasing in the volume of aggregate ex ante liquidity provisioning l. As a consequence, there are two equilibria. The credit-rationing equilibrium is situated at point CR, where banks provision no liquidity. The moral-hazard-induced liquidity constraint then binds for distressed banks which cannot borrow liquidity, and intact banks have no liquidity to offer at date 1. If intact banks had liquidity—e.g., assuming intact illiquid projects did generate some output at date 1—they would be compelled to store it in the liquid technology. The full-reinvestment equilibrium is situated at point FR. In this case, the date 1 market for liquidity clears and banks' capital allocation between liquid and illiquid assets is identical to the first-best allocation.

As can be noted from the above discussion, the risk-adjusted return to ex ante liquidity provisioning and the aggregate volume of ex ante liquidity provisioning are higher under the full-reinvestment equilibrium. Hence, across equilibria, the expected return on liquid assets increases with the volume of liquid assets that banks provision ex ante.

6. Extensions

In this section we investigate the robustness of our main result, i.e., the existence of multiple equilibria including the possibility of a collapse in the market for liquidity. To do so, we consider the consequences of relaxing two assumptions made so far.

6.1 Aggregate Shocks

While this model shows that the fragility of the market for liquidity does not necessarily stem from the presence of aggregate shocks, it can easily be extended to allow for such shocks. Suppose, for instance, that the individual probability q to face a liquidity shock can take different values, F denoting the cumulative distribution function for q. Then when $q < \underline{q}$, which happens with probability $F(\underline{q})$, the interbank market collapse is the unique equilibrium and therefore happens with probability one. When $\underline{q} < q < \overline{q}$, which happens with probability $F(\overline{q}) - F(\underline{q})$, there are multiple equilibria and the interbank-market-collapse equilibrium happens with probability p. Finally, when $q > \overline{q}$, which occurs with probability $1 - F(\overline{q})$, the interbank market never collapses. Hence, the unconditional probability θ of an interbank market collapse is given by

$$\theta = F(\underline{q}) + p[F(\overline{q}) - F(\underline{q})].$$

Proposition 7. *An increase in the return to illiquid investment R reduces the unconditional probability θ of a market collapse if and only if*

$$R > \sqrt{\frac{1}{\psi e_h}}.$$

Proof. Deriving the expression for θ with regard to R yields

$$\frac{\partial \theta}{\partial R} = \frac{e_h(1-\psi)}{1-\psi e_h R} \frac{R - \frac{R-1}{1-\psi e_h R}}{\left(R + \frac{e_h R - 1}{1-\psi e_h R}\right)^2} pf \left(\frac{R-1}{R + \frac{e_h R - 1}{1-\psi e_h R}} \right),$$

where $f(.)$ is the distribution function for q. This expression is positive if and only if

$$R > \frac{R-1}{1-\psi e_h R},$$

which simplifies as $e_h \psi R^2 > 1$.

An increase in the return to illiquid investment R has two opposite effects. On the one hand, it raises the return to illiquid investments and hence raises banks' incentives to invest in illiquid assets. On the other hand, it raises the return to liquid investment in the credit-rationing regime because a larger return R raises the borrowing capacity on the interbank market and thereby raises incentives to invest in the liquid technology. When the return to illiquid investment is low, the former effect dominates the latter: an increase in R then raises incentives to invest in illiquid assets. As a consequence, the probability of liquidity shocks $\overline{q}$ below which the market-collapse equilibrium is possible tends to increase. On the contrary, when the return to illiquid investment is large, an increase in R reduces incentives to invest in illiquid assets. As a consequence, the probability $\overline{q}$ below which the market-collapse equilibrium exists tends to decrease.

6.2 Interim Liquidation of Illiquid Assets

We have assumed so far the liquidation value of distressed illiquid projects to be zero. Let us assume instead that distressed banks can liquidate (part of) their illiquid projects with a strictly positive liquidation value. Specifically, a distressed bank can liquidate a fraction α of its illiquid project ($0 < \alpha < 1$). It then gets ρ units of capital for each unit of capital liquidated ($0 < \rho < 1$).

Denoting l_i the amount of capital bank i has invested in the liquid technology at date 0, and v_i the part of the illiquid project liquidated at date 1, the date 1 problem of bank i when distressed now writes as

$$\begin{gathered} \max_{d_i; v_i} \pi_b = e_h[(l_i + \rho v_i + d_i)R - r d_i] \\ \text{s.t. } l_i + \rho v_i + d_i \leq 1 - l_i - v_i \text{ and } v_i \leq \alpha(1 - l_i) \\ d_i \leq \frac{\psi R}{r - \psi R}(l_i + \rho v_i). \end{gathered} \tag{18}$$

The distressed bank reinvests $(l_i + \rho v_i + d_i)$, with d_i being what the distressed bank borrows on the interbank market. Hence, its profit conditional on reinvestment being successful is $(l_i+\rho v_i+d_i)R-rd_i$, while e_h is both the effort the distressed bank undertakes and the probability that reinvestment is successful. Finally, the distressed bank i faces the following three constraints. First, reinvestment $(l_i + \rho v_i + d_i)$ cannot be larger than the illiquid project's size $(1-l_i-v_i)$. Second, the distressed bank cannot liquidate more than a fraction α of its illiquid project. Third, the distressed bank faces an incentive constraint stemming from the moral hazard problem: what a distressed bank can borrow on the interbank market is at most a fraction $\frac{\psi R}{r-\psi R}$ of the distressed bank's own available capital $(l_i + \rho v_i)$ at date 1.[11]

Expected profits of an intact bank are modified as follows: if bank i has invested ex ante l_i units of capital in the liquid technology and $1-l_i$ units of capital in the illiquid technology, then it reaps $(1 - l_i)\beta + l_i$ at date 1, with β being the interim marginal return to an intact illiquid project. Intact bank expected profits therefore write as

$$\pi_g = (1 - l_i)R + [(1 - l_i)\beta + l_i] \max\{e_h r; 1\}. \qquad (19)$$

PROPOSITION 8. *When distressed banks can liquidate interim a fraction α of their illiquid assets with a marginal return ρ, and intact banks enjoy an interim marginal return β on their illiquid assets, then a credit-rationing equilibrium where*

(i) banks make no liquidity provision at date 0,

(ii) distressed banks are unable to achieve full reinvestment, and

[11]This incentive constraint is based on the implicit assumption that ex ante liquidity provision l_i and interim liquidation v_i are both observable when the interbank market opens. In reality, interim liquidation v_i is likely to be more difficult to observe than ex ante liquidity provision when the interbank market opens because liquidating assets takes time. Put differently, there are very few assets that banks can liquidate over night. Moreover, interim liquidation could well happen at the same time or even *after* distressed banks borrow on the interbank market. In this case—where liquidation would happen after borrowing on the interbank market takes place—then allowing for interim liquidation does not change any of the properties of the model.

(iii) intact banks store part of their liquid assets in the liquid technology at date 1, exists if and only if the parameters α, β, and ρ verify

$$1 + q\frac{e_h R - 1}{1 - e_h \psi R} \leq (1-q)(R+\beta) + q\alpha\rho\frac{e_h R[1-\psi]}{1 - e_h \psi R}$$

$$q\frac{e_h \psi R}{1 - e_h \psi R}\rho\alpha < (1-q)\beta$$

$$\rho\frac{\alpha}{1-\alpha} \leq 1 - e_h \psi R.$$

Proof. See the appendix.

Here the possibility for banks to borrow in the interbank market based on liquid assets l and liquidated distressed projects v prevents a total collapse of the interbank market. However, when the share α of illiquid assets that distressed banks can liquidate is sufficiently low, there is still a credit-rationing equilibrium in which some liquidity is traded on the interbank market as opposed to the previous credit-rationing equilibrium where a total collapse of the interbank market takes place. However, distressed banks still face credit rationing and are still unable to achieve full reinvestment.

7. Policy Implications

In this section we investigate whether and how policy can avoid a collapse of the interbank market. To do so, we focus on two types of public interventions. First we look at ex post interventions, i.e., policies that take place after the interbank market has collapsed. Then we focus on ex ante interventions, i.e., interventions aiming at preventing the collapse of the interbank market.

7.1 Ex Post Interventions

There are basically two types of interventions that can take place after the interbank market has collapsed: liquidity injections and changes in interest rates which modify the return on the liquid technology. Typically, a central bank can lend liquidity to distressed banks when the interbank market does not function. It can also influence the cost of liquidity by modifying short-term interest rates. In our case, both these policies are unlikely to be successful in helping distressed banks to achieve reinvestment. Given that banks do not make any ex ante liquidity provision in the equilibrium where the interbank market collapses, any loan from the central bank or from any intact bank violates the incentive constraint stemming from moral hazard. This implies that liquidity injections from the central bank toward distressed banks—assuming the central bank can distinguish between intact and distressed banks—would end up financing negative net-present-value projects, as distressed banks would deliver low effort given that reinvestment is fully financed with external funds. In other words, unless the central bank has access to a monitoring technology that market participants do not have access to, liquidity injections are doomed to fail.

Similarly, cutting interest rates to dampen the effects of a market collapse is unlikely to work. In theory, a reduction in interest rates relaxes the moral hazard problem and raises distressed banks' incentives to deliver high effort. As a consequence, the incentive-compatible level of interbank borrowing is larger with a lower interest rate. However, this effect depends on banks' ex ante liquidity provisions. Given that banks make no ex ante liquidity provision in the equilibrium with a market collapse, the reduction in interest rates does not modify distressed banks' borrowing capacity, which remains at zero. The positive impact of an interest rate cut on distressed banks' borrowing capacity depends positively on banks' ex ante liquidity provision. Hence, interest rate cuts are most effective when banks have made relatively large ex ante liquidity provisions. In a nutshell, interest rate cuts are most effective when not needed.

7.2 Ex Ante Interventions

A regulator can affect the banks' date 0 allocation of capital by imposing a liquidity ratio, requiring that banks invest at least some fraction of their portfolio in liquid assets. Imposing this type of regulation eliminates the equilibrium characterized by a collapse in the interbank market. However, one of the important assumptions of the model is that liquidity is not contractible; i.e., it is not possible to write contracts contingent on the share of assets invested in the liquid technology. Yet imposing a liquidity ratio is equivalent to writing such a contingent contract between the regulator and banks, stating that the bank would be shut down if the share of liquid assets was lower than a given threshold. Imposing such a regulation in this type of model therefore ends up giving discretion to the regulator, which can be costly for reasons outside the scope of this paper (e.g., in terms of capture of the regulator by the regulated agents).

A central bank can, however, affect the return to liquid assets through its policy rates. In particular, the central bank can raise the return to liquid assets between date 0 and date 1 to raise banks' incentives to invest in liquid assets and thereby prevent the collapse of the interbank market at date 1. Assume that the central bank can (at no cost) modify the return r_0 on liquid assets between date 0 and date 1. We can then derive the following result.

PROPOSITION 9. *The central bank can always prevent the collapse of the interbank market by imposing an interest rate* r_0 *such that*

$$r_0 > \frac{R}{1 + \frac{q}{1-q}\frac{e_h(1-\psi)R}{1-e_h\psi R}}.$$

Proof. Let us consider the credit-rationing regime where distressed banks' borrowing constraint binds. Denoting r_0 the return to liquid assets between date 0 and date 1, bank i date 0 expected profits write as

$$\pi_i = (1-q)R(1-l_i) + e_h r\left[1 - q + q\frac{(1-\psi)R}{r-\psi R}\right] r_0 l_i$$

and bank i optimal liquidity provision l_i^* writes as follows:

$$l_i^* = \begin{cases} 0 \text{ if } (1-q)R \geq e_h r\left[1 - q + q\dfrac{(1-\psi)R}{r-\psi R}\right] r_0 \\ \dfrac{\frac{r-\psi R}{r}}{r_0 + \frac{r-\psi R}{r}} \text{ if } (1-q)R \leq e_h r\left[1 - q + q\dfrac{(1-\psi)R}{r-\psi R}\right] r_0. \end{cases}$$

Hence, any return r_0 verifying

$$r_0 > \frac{(1-q)R}{e_h r\left[1 - q + q\frac{(1-\psi)R}{r-\psi R}\right]}$$

will preclude the collapse of the interbank market since then banks will make ex ante liquidity provision $l_i^* = \frac{\frac{r-\psi R}{r}}{r_0+\frac{r-\psi R}{r}}$ and thus distressed banks will be able to carry out full reinvestment.

Given that the right-hand side of the above inequality is decreasing in the interest rate r, the above inequality always holds if it holds for the lowest possible interest rate, i.e., when $e_h r = 1$. In this case, the inequality simplifies as

$$r_0 > \frac{(1-q)R}{1-q+q\frac{(1-\psi)e_h R}{1-e_h\psi R}}.$$

The bottom line is therefore that a sufficiently high interest rate ex ante, by raising banks' incentives to invest in liquid assets, can help avoid the collapse of the interbank market.

8. Conclusion

The model we analyzed in this paper provides a framework for analyzing the occurrence of liquidity crises and discussing policy responses to situations of interbank market collapse. To the extent that such a collapse may be explained by the ingredients we focus on (in particular, moral hazard and nonverifiability of ex ante liquidity provisions), this model provides some insights on the scope for ex ante policies to prevent this outcome. In addition, this framework presumably lends itself well to the analysis of the role of international liquidity and its impact on domestic liquidity provision in an open-economy setting. These are possible research avenues for future work.

Appendix

Proof of Proposition 4: The Full-Reinvestment Equilibrium

When distressed banks achieve full reinvestment, the equilibrium interest rate cannot verify $r > r_1$, since banks would then invest their capital in liquid assets and the interbank market would be in excess supply at date 1. The equilibrium interest rate therefore always verifies $r \leq r_1$. When $r < r_1$, then each bank makes ex ante liquidity provisions $l(r) \equiv \frac{\frac{r-\psi R}{r}}{1+\frac{r-\psi R}{r}}$. The equilibrium interest rate is $r = r_2$, which yields an equilibrium ex ante liquidity provision $l = l(r_2) = l^*$. When $r = r_1$, then the equilibrium volume of liquidity each bank provisions ex ante is $l = l^*$. When banks achieve full reinvestment, they always provision the first-best volume of liquidity, and the equilibrium interest rate on the interbank market is $r^* = \min\{r_1; r_2\}$. To determine whether this case is an equilibrium, let us examine if there are profitable deviations. A bank can deviate by provisioning a lower level of liquidity. Assuming the interest rate on the interbank market verifies $r \leq R$, then the profit of a deviating bank is

$$\pi_d = (1-q)(1-l_i)R + e_h r \left(1 - q + q\frac{[1-\psi]R}{r - \psi R}\right) l_i.$$

Denoting $\frac{\partial E\pi}{\partial l_i} = e_h r(1 - q + q\frac{(1-\psi)R}{r-\psi R}) - (1-q)R$, the optimal ex ante liquidity provision policy of the deviating bank l_d is given by

$$l_d = \begin{cases} 0 \text{ if } \dfrac{\partial E\pi}{\partial l_i} \leq 0 \\ l(r) \text{ if } \dfrac{\partial E\pi}{\partial l_i} \geq 0 \end{cases},$$

where r is the equilibrium interest rate when banks achieve full reinvestment; $r = r^*$. If the interest rate r^* is such that $\frac{\partial E\pi}{\partial l_i} \geq 0$, then the deviating bank provisions $l_d = l(r^*)$. In this case, deviation is not strictly profitable, since we have $\pi_d = \pi_h$. On the contrary, if the interest rate on the interbank market r^* is such that $\frac{\partial E\pi}{\partial l_i} \leq 0$, then the deviating bank chooses to make no ex ante liquidity provision $l_d = 0$. Deviation is then profitable if and only if

$$(1-q)R > e_h r^*.$$

When $r^* = r_2$, this inequality simplifies as $e_h < 1 - q$. By assumption, this inequality never holds, since we have $e_h \geq 1-q$. When the interest rate is $r^* = r_1$, deviation is profitable if and only if

$$1 - q < \frac{1 - q + e_h\phi}{1-q}.$$

However, since by assumption we have $e_h \geq 1 - q$, this condition cannot be satisfied. As a consequence, there are no profitable deviations, and the situation where banks achieve full reinvestment is an equilibrium.

Proof of Proposition 6: The Credit-Rationing Equilibrium

This proof is divided into two parts. The first part establishes that (17) is a necessary and sufficient condition for the existence of a market-collapse equilibrium. The second part shows that the market-collapse equilibrium is the unique equilibrium in the credit-rationing regime.

To establish that (17) is a necessary and sufficient condition for the existence of a market-collapse equilibrium, we proceed in two steps.

First Step. Assume that the liquidity constraint $d_i \leq \overline{d}(l_i)$ binds. Then distressed banks borrow $d_i = \overline{d}(l_i)$ from the interbank market and the first-order condition to the problem of an individual bank implies that zero ex ante liquidity provision is optimal if and only if $\frac{\partial E\pi}{\partial l_i} < 0$; i.e.,

$$e_h r \left[1 - q + q\frac{(1-\psi)R}{r - \psi R}\right] < (1-q)R.$$

When optimal ex ante liquidity provision l_i^* is zero, the demand for liquidity is $L_d = 0$ and the supply of liquidity is $L_s = 0$. Hence, any interest rate r verifying $r > \psi R$ and

$$e_h r \left[1 - q + q\frac{(1-\psi)R}{r - \psi R}\right] < (1-q)R$$

is an equilibrium interest rate of the interbank market. In particular, $r = e_h^{-1}$ is such an equilibrium interest rate if and only if $e_h \psi R < 1$—which by assumption always holds—and

$$1 + q\frac{e_h R - 1}{1 - e_h \psi R} < (1-q)R.$$

When this last condition is verified, the situation where banks do not provision liquidity ex ante is possibly an equilibrium and the liquidity constraint $d_i \leq \overline{d}(l_i)$ is indeed binding.

Second Step. Let us now show that the liquidity constraint $d_i \leq \overline{d}(l_i)$ is always binding when (17) holds and $e_h r = 1$. To do so, consider a bank that decides to provision ex ante a volume of liquidity such that the liquidity constraint $d_i \leq \overline{d}(l_i)$ does not bind. Given that the interest rate on the interbank market verifies $e_h r = 1$, the bank's expected profits π_d writes as

$$\pi_d = (1-q)[(1-l_i)R + l_i] + q[(1-l_i)(e_h R - 1) + l_i].$$

Moreover, the liquidity constraint does not bind if and only if the bank's ex ante liquidity provision l_i verifies

$$l_i \geq l(e_h^{-1}) = \frac{1 - e_h \psi R}{1 + 1 - e_h \psi R}.$$

The bank can then achieve full reinvestment. Expected profits π_d are strictly decreasing in ex ante liquidity provisioning l_i because

$$\frac{\partial \pi_d}{\partial l_i} = -[(1-q)R - 1 + q(e_h R - 1)]$$

and, by assumption, we have $(1-q)R > 1$ and $e_h R > 1$. As a consequence, the optimal ex ante liquidity provision l_d of a bank seeking to maximize π_d is $l_d = l(e_h^{-1})$. Its optimal expected profits π_d can hence be written as

$$\pi_d(l_d) = (1-q)[(1-l_d)R + l_d] + q e_h \frac{(1-\psi)R}{1 - e_h \psi R} l_d.$$

However, when a bank does not provision liquidity, expected profits are $(1-q)R$. The zero ex ante liquidity provision policy is therefore optimal if and only if $(1-q)R > \pi_d(l_d)$. This inequality simplifies as (17), which by assumption is supposed to hold. As a consequence, when (17) holds, it is never optimal for a bank to provision ex ante a volume of liquidity such that the liquidity constraint $d_i \leq \overline{d}(l_i)$ does not bind. The situation where banks do not provision liquidity is hence an equilibrium if and only if (17) holds, in which case the interest rate on the interbank market verifies $e_h r = 1$.

We now turn to establishing the unicity of the market-collapse equilibrium in the credit-rationing regime. Suppose banks make strictly positive ex ante liquidity provision while being credit constrained. There may be two types of such equilibria.

First, we examine whether the case where banks' optimal ex ante liquidity provision is $l(r)$ and

$$\left[1+q\frac{R-r}{r-\psi R}\right]e_h r \geq (1-q)\,R \tag{20}$$

can indeed be an equilibrium of the economy. When banks' ex ante optimal liquidity provision is $l(r)$, the equilibrium interbank market interest rate is necessarily $r = r_2$. Otherwise, the interbank market would not be balanced. Banks' expected profits then write as $e_h r_1$. Let us now show that the strategy which consists in provisioning a larger volume of liquidity is more profitable. When $r_2 < r_1$, then a bank that wants to achieve full reinvestment chooses to provision the same volume of liquid assets $l(r)$, and expected profits are identical. On the contrary, if $r_2 > r_1$, then a bank that wants to achieve full reinvestment chooses to invest all its capital in liquid assets, $l_d = 1$, and its expected profit is $e_h r_2$, which by assumption is larger than $e_h r_1$. As a consequence, the situation where banks provision a volume of liquidity $l(r)$ and (20) holds cannot be an equilibrium.

Second, we examine the case where banks are indifferent to provisioning ex ante any volume of liquid assets in $[0; l(r)]$ and

$$\left[1+q\frac{R-r}{r-\psi R}\right]e_h r = (1-q)R. \tag{21}$$

In this case, banks' expected profits write as $(1-q)R$. If the interest rate r which solves (21) is such that $r < r_1$, then a bank that wants to achieve full reinvestment would choose to provision a volume of liquidity $l(r)$. In this case, expected profits are identical. On the contrary, if the interest rate r which solves (21) is such that $r > r_1$, then a bank that wants to achieve full reinvestment would invest all its capital in liquid assets $l = 1$, and its expected profit would be $e_h r$. This situation is an equilibrium if and only if the interest rate r which solves (21) verifies the condition

$$e_h r < (1-q)R. \tag{22}$$

Given that we consider the case where $r > r_1$, a necessary condition for this situation to be an equilibrium is that (22) must hold for $r = r_1$. This necessary condition simplifies as

$$e_h < 1-q,$$

which by assumption does not hold. Consequently, the situation where the interbank market interest rate verifies (21) and banks are indifferent to provisioning any amount l_i of liquid asset such that $0 \leq l_i \leq l(r)$ cannot be an equilibrium. The equilibrium with zero ex ante liquidity provision and interbank market collapse is therefore the only equilibrium, when it exists, in the credit-rationing regime.

Proof of Proposition 8: Credit-Rationing Equilibrium with Interim Liquidation

Given its program (18), a distressed bank's optimal choices are as follows: optimal liquidation is such that $v = \alpha(1-l)$ and optimal borrowing d on the interbank market writes as

$$d = \min\left\{1 - 2l - \alpha(1+\rho)(1-l); \frac{\psi R}{r - \psi R}(l + \rho\alpha(1-l))\right\},$$

with l being the bank's optimal ex ante liquidity provision. Assuming the distressed bank's liquidity constraint is binding and assuming the interest rate r on the interbank market satisfies participation constraints—i.e., $1 < e_h r < e_h R$—the equilibrium at date 1 on the interbank market writes as

$$(1-q)[(1-l)\beta + l] = q\frac{\psi R}{r - \psi R}(l + \rho\alpha(1-l)).$$

Having determined banks' date 1 decisions, we can turn to banks' date 0 problem, which writes as

$$\max_l (1-q)[(1-l)R + [(1-l)\beta + l]\max\{e_h r; 1\}] + q e_h r\frac{R[1-\psi]}{r - \psi R}(l + \rho\alpha(1-l)).$$

The solution is given by

$$l = \begin{cases} \lambda(r) \text{ if } \left[(1-q)(1-\beta) + q\frac{R[1-\psi]}{r-\psi R}(1-\rho\alpha)\right] e_h r \geq (1-q)R \\ 0 \text{ if } \left[(1-q)(1-\beta) + q\frac{R[1-\psi]}{r-\psi R}(1-\rho\alpha)\right] e_h r \leq (1-q)R, \end{cases}$$

where $\lambda(r)$ is such that

$$1 - 2\lambda(r) - \alpha(1+\rho)(1-\lambda(r)) = \frac{\psi R}{r - \psi R}(\lambda(r) + \rho\alpha(1-\lambda(r))).$$

When banks choose $l = 0$, the equilibrium interest rate on the interbank market would write as $r = \psi R(1 + \frac{\rho\alpha}{\beta(1-q)})$. However, for α and/or ρ sufficiently low compared with β, i.e.,

$$q\frac{e_h\psi R}{1 - e_h\psi R}\rho\alpha < (1-q)\beta,$$

intact banks' participation constraint is violated. As a consequence, the "equilibrium" interest rate is such that $e_h r = 1$, distressed banks face rationing in their demand for liquid assets, and there is an excess supply on the interbank market.

Hence, banks make no liquidity provision ex ante and intact banks store part of their liquid assets in the liquid technology at date 1 instead of lending on the interbank market if and only if

$$q\frac{e_h\psi R}{1 - e_h\psi R}\rho\alpha < (1-q)\beta$$

$$1 + q\frac{e_h R - 1}{1 - e_h\psi R} \leq (1-q)(R+\beta) + q\alpha\rho\frac{e_h R[1-\psi]}{1 - e_h\psi R}.$$

Finally, distressed banks are unable to achieve full reinvestment if and only if

$$\rho\alpha + \frac{\psi R}{r - \psi R}\rho\alpha < 1 - \alpha,$$

where the interest rate r verifies $e_h r = 1$. This inequality can be simplified as

$$\rho\frac{\alpha}{1-\alpha} \leq 1 - e_h\psi R.$$

References

Acharya, V., D. Gromb, and T. Yorulmazer. 2008. "Imperfect Competition in the Interbank Market for Liquidity as a Rationale for Central Banking." IFA Working Paper No. 473.

Acharya, V., H. S. Shin, and T. Yorulmazer. 2007. "Fire-Sales, Foreign Entry and Bank Liquidity." CEPR Discussion Paper No. 6309.

———. 2009. "A Theory of Slow-Moving Capital and Contagion." CEPR Discussion Paper No. 7147.

Allen, F., and D. Gale. 1998. "Optimal Financial Crises." *Journal of Finance* 53 (4): 1245–84.

———. 2004. "Financial Fragility, Liquidity and Asset Prices." *Journal of the European Economic Association* 2 (6): 1015–48.

Bhattacharya, S., and D. Gale. 1987. "Preference Shocks, Liquidity and Central Bank Policy." In *New Approaches to Monetary Economics*, ed. W. A. Barnett and K. J. Singleton, 69–88. Cambridge University Press.

Caballero, R. J., and A. Krishnamurthy. 2004. "Smoothing Sudden Stops." *Journal of Economic Theory* 119 (1): 104–27.

———. 2008. "Collective Risk Management in a Flight to Quality Episode." *Journal of Finance* 63 (5): 2195–2230.

Diamond, D. W., and P. H. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91 (3): 401–19.

Diamond, D. W., and R. G. Rajan. 2001. "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking." *Journal of Political Economy* 109 (2): 287–327.

Freixas, X., and J. Jorge. 2008. "The Role of Interbank Markets in Monetary Policy: A Model with Rationing." Economics Working Paper No. 1027, Department of Economics and Business, Universitat Pompeu Fabra.

Goodhart, C. 2008. "Liquidity Risk Management." *Financial Stability Review* (Banque de France) 11 (February): 39–44.

Holmström, B., and J. Tirole. 1998. "Private and Public Supply of Liquidity." *Journal of Political Economy* 106 (1): 1–40.

Kashyap, A. K., and J. C. Stein. 2000. "What Do a Million Observations on Banks Say about the Transmission of Monetary Policy?" *American Economic Review* 90 (3): 407–28.

Rochet, J.-C., and J. Tirole. 1996. "Interbank Lending and Systemic Risk." *Journal of Money, Credit, and Banking* 28 (4): 733–62.

Discussion of "Liquidity, Moral Hazard, and Interbank Market Collapse"

Tano Santos
Columbia University

Financial intermediaries, such as banks, perform many roles: they screen risks, evaluate and fund worthy entrepreneurs, pool risks, monitor borrowers, refinance projects, and—when confronted with the default in their loans—they perform a valuable role in loss discovery and assessing whether the bankrupt concern is economically viable or not. All these activities, which constitute the actions of a bank in what concerns the asset side of its balance sheet, need to be funded, and much depends on how the bank structures its liabilities to obtain these funds, whether with time deposits, short- or long-term debt, equity, or other more novel forms of short-term financing such as commercial paper or repo transactions. Independently of the particular liability structure adopted by financial intermediaries, it typically involves some form of maturity transformation. For instance, bank deposits, which are available on demand, are transformed into longer-maturity loans to finance projects. Maturity mismatch is almost inherent in the intermediation business.

A liquidity shock for banks occurs whenever at some interim stage additional funds are needed to bridge the maturity mismatch between assets and liabilities. Given the nature of the bank's balance sheet, there are two potential sources of liquidity problems. First, it may be that projects that were funded at some past date need to be refinanced in some contingency and that failure to provide additional funds may result in some loss or even the wasteful liquidation of those projects. Alternatively, it may be that liabilities are coming due before the proceeds of the investments are realized and that funds need to be produced to meet those liabilities; a bank run is the classic example of the latter. Of course, both things can happen simultaneously: projects on the asset side of the bank's balance sheet may need additional financing while liabilities are coming due, and in fact there are obvious theoretical reasons to expect this simultaneous occurrence. In summary, the liquidity shock can happen on the asset side of the bank's balance sheet, on the liability side, or on both.

Roughly, liquidity is of concern to banks (and financial economists) because the inability of financial institutions to contract ex ante on their potential funding needs may induce inefficiencies, either ex ante or ex post. For example, in order to meet a liquidity shock, a bank may be forced to liquidate some projects when it would be ex post efficient to carry them to maturity. That is, liquidity arises as a problem because it can only be addressed in a spot market transaction at the aforementioned interim stage and it may be that there are limits to what banks can achieve via these spot transactions.

These questions have been at the center of the literature on financial intermediation for a long time but more specifically since the seminal paper of Diamond and Dybvig (1983). The paper by Enisse Kharroubi and Edouard Vidon fits nicely in this tradition. It offers, though, some novel insights and implications that should be of interest to specialists in the field as well as to policymakers struggling with the largest financial crisis since the Great Depression.

Given all this, one can immediately ask, how can banks address potential liquidity shortfalls? Here there are essentially two possibilities. First, banks can meet their own liquidity needs via the reserves they carry, which is a form of self-insurance, and/or second, they can appeal to the reserves carried by *other* banks, which—not being subject to the liquidity shock—are willing to supply these reserves to banks in distress. A natural question is then, what determines the reliance on one form of liquidity provision versus the other?

Consider first the benefits of self-insurance via internal reserves. A bank can always protect itself against a shock on the liability side of its balance sheet by simply carrying every dollar raised via deposits as cash or a close substitute such as Treasuries. This is an extreme version of a narrow bank. If a liquidity shock occurs, the bank is well protected by its own large reserves and it does not need to appeal to potentially expensive outside sources of funding. An additional benefit of carrying reserves is that if the bank does not suffer a liquidity shock, it can deploy these reserves opportunistically and provide funding to other banks in distress at attractive Sharpe ratios. In doing so, of course, it compromises the narrow-bank business model. The cost of carrying large reserves is that every dollar invested in cash or a close substitute is a dollar not invested in an attractive, but potentially risky, project: the return on carrying cash is typically very low, and depositors may prefer alternative institutions to keep their funds.

The benefit for the bank of lowering its own reserves and increasing its reliance on outside funding to confront liquidity shocks is now obvious: it frees funds to invest in attractive projects. If confronted with a liquidity shock, the bank can appeal to the liquidity carried by others in different ways. A first and obvious one is the sale of the assets carried by the bank in distress. There are several problems associated with this outright asset sale, though. For instance, it may be that there is a classic adverse-selection issue confronting the potential buyers of the assets in question: some assets are good and others less so, and buyers cannot readily distinguish one from the other. Sellers may be able to signal the quality of the assets sold, an issue to which I shall return, and this may imply some form of inefficiency that the seller of good assets must bear. If signaling is not feasible or pooling is the only equilibrium, the price incorporates an adverse-selection premium; effectively, the seller of the good asset gets a price below the one he would fetch if there was full information. Alternatively, it may be that there is some form of specific capital between the bank and the asset or project that is lost when the sale occurs; for example, the bank has some intangible information that—not being codifiable—cannot be conveyed to the acquiring party, which, being aware of this friction, pays less for the project than it would if it had a more complete picture of the asset.

Alternatively, the bank may raise equity at the interim stage to meet those liquidity needs, but here there are also problems. In particular, the amount of equity that can be raised is limited by the need to preserve incentives inside the bank. If the bank raises "too much," it can only come at the expense of lowering the stake of insiders, who remain in control of exercising the costly actions that guarantee the successful completion of the projects funded by the bank. Thus there is a third benefit of carrying reserves and it is that insiders can refinance projects with internal funds, which in turn encourages the provision of external funds, as now outsiders can remain confident that incentives for high-effort provision are preserved. Another way of conveying this powerful insight, which is originally due to Holmström and Tirole (1998), is that internal funds relax the amount of outside funds that can be raised to reinvest in distressed projects, as when the former are reinvested, inside equity is preserved and with it the incentives to supply costly effort. To express this in a simple expression, if ℓ is the amount of inside reserves carried by the bank and d is the amount of external funds raised, then it is has to be that

$$d \leq A^{*}\ell, \tag{1}$$

where A^{*} is some constant to be determined in equilibrium and which determines the rate at which one dollar of inside liquidity can translate into outside liquidity.

Kharroubi and Vidon's model combines several of these ingredients to understand the workings of the interbank market. Specifically, they consider a model where banks choose the amount of internal funds they carry, trading off the opportunity costs of the foregone investment opportunities against the benefit of opportunistic liquidity provision in the absence of idiosyncratic liquidity shocks and the role that internal funds have in relaxing the constraint (1). Roughly, the model combines the analysis of the interbank market as in Bhattacharya and Gale (1987) with the frictions considered in Holmström and Tirole (1998) when it comes to the provision of outside liquidity, a strategy that has been followed by other researchers, such as Acharya, Shin, and Yorulmazer (2007), to address related, albeit different, questions.

The main result is that there are potentially two equilibria in the model. In the first one, which exists when the probability of a liquidity shock is relatively high, there are many banks in distress, and banks anticipating distress provision generously. The first effect dominates, and as a result the interest rate for ex post funding is also large, which further reinforces the incentives of banks to carry substantial internal funds, because if they are not subject to the liquidity shock they will be able to redeploy their funds at attractive rates; this they do in the knowledge that the internal funds of distressed banks which are reinvested preserve incentives for effort provision. Moreover, when the probability of an idiosyncratic liquidity shock is high, this equilibrium implements the constrained efficient level of ex ante liquidity provision.

A second equilibrium occurs precisely when the probability of an idiosyncratic liquidity shock is low. In this case few banks are in distress and, in addition, banks don't carry internal funds. This limits the amount of funds that healthy banks are willing to supply to distressed banks, for given the limited amount of internal funds reinvested, the incentives for effort provision are also low, which further reduces the incentives to carry liquidity and so on. The resulting equilibrium features no ex ante provision of liquidity and thus the breakdown of the interbank market. In this case good times (periods where idiosyncratic liquidity shocks are less likely) lead to breakdowns in the interbank market. Both equilibria can coexist when the probability of an idiosyncratic shock is in some intermediate range.

The model offers some novel insight in investigating carefully how the strategic complementarity between inside and outside liquidity may result in an equilibrium that supports a constrained efficient level of provisioning, which is precisely when the probability of idiosyncratic liquidity shocks is high—that is, when crises are most likely. Several policy implications follow from the analysis in the model.

First notice that in the paper the interbank market breakdown is simply a chronicle of a crisis foretold; it is precisely the low levels of internal funds which leads to a breakdown of the interbank market because healthy banks—were they to invest in distressed banks—could not elicit the high effort provision from them. It is thus obvious what the repair should be to restore efficiency to the interbank market: force the banks to carry a minimum level of reserves. This will guarantee that the banks in distress have enough to reinvest and thus preserve incentives for effort provision. But this obvious implication of the model is not robust to some sensible modifications of the framework. Consider the case where there are participants other than banks who can provide financing in case of liquidity shocks. Assume further that banks are good at finding and funding projects and thus, socially, it is wasteful to have banks carry "excessive" reserves. In this scenario (constrained) efficiency may call for banks to carry low internal funds and instead have them sell projects, even at distressed prices, and fully rely on external funds to meet liquidity shocks. Imposing a minimum reserve requirement can only come at the expense of lowering the returns of carrying liquidity for financial intermediaries other than banks. In fact, it may be optimal in this case to impose maximum reserve requirements on banks in order to force them to sell projects when affected by the liquidity shocks, depress prices, and increase the returns of outside liquidity. This result has been shown recently in a paper by Bolton, Santos, and Scheinkman (2009), which offers an alternative view of the trade-offs involved between inside and outside liquidity.

Another comment concerns the particular friction that makes the supply of outside liquidity problematic in this setup, which is the moral hazard problem that follows the interim stage. It is important to note that—as Holmström and Tirole (2008) themselves emphasize—the insight is very general. Indeed, consider the case where the amount of a project's proceeds that can be pledged to outsiders $\underline{v}$ is less than what the project is worth to the party undertaking it, the insider, $\overline{v} > \underline{v}$; if the amount needed to implement the project i is somewhere between these two quantities, it trivially follows that the insider needs to have sufficient internal funds, $\ell \geq i - \underline{v}$. It follows that if the insider has insufficient internal funds, positive net-present-value projects will not be implemented. As mentioned before, there are several reasons why internal and external values may differ. Holmström and Tirole (2008) divide these reasons into two categories—exogenous and endogenous. An example of the first is when the project yields some private benefits to the party undertaking it—benefits that cannot be transferred to outsiders. An example of the second is the moral hazard problem emphasized by the authors and that was also the original modeling route chosen by Holmström and Tirole. One suspects that naturally the different frictions that open a wedge between internal and external values may lead in turn to different policy implications.

This is, in my opinion, of importance because if one thinks about the critical moral hazard in the financial crisis that started in the summer of 2007, it seems that, notwithstanding the potential moral hazard problems going forward in the banking sector, the critical one was related to the origination of bad risks that occurred in the real estate market. Roughly, it was the fact that intermediaries expected to distribute risks easily that led, ex ante, to the origination of assets of dubious credit quality. It was not that the amount of liquidity was limited because of the need to preserve incentives going forward but rather that the considerable amounts of liquidity available to absorb risks led to a moral hazard problem ex ante. An interesting extension of the model would be to consider the incentives of financial intermediaries to originate risks of different credit qualities depending on their ability to refinance ex post.

In addition, it is worth emphasizing that there is a multiplicity of intermediaries, not just banks, willing to supply liquidity to banks in distress. Policy recommendations that focus exclusively on banks run the risk of ignoring the incentives that other parties may have to supply liquidity to banks in distress. To put it differently, financial markets have changed considerably over the last twenty-five years, and one of the more striking changes is precisely the fact that financing comes now from all quarters—insurance companies, investment banks, hedge funds, and even sovereign funds. It is important that banking models consider the fact that banks interact with other intermediaries and that the risks faced by banks are inextricably linked to the overall financial system, as the current crisis so painfully illustrates.

Finally, it would be useful to incorporate in our models some of the institutions that are in place in the interbank market to understand better the incentive problems faced by banks. Indeed, it is common to consider the policy implications of models without properly considering the effect that the expectation of the policy intervention may have on the incentives of the different parties to take particular actions—for example, portfolio decisions regarding the amount of reserves carried. Given that there has been much discussion on whether the actions of, for example, the Federal Reserve have led to excessive risk taking by large banks, it seems that adding to our banking models the possibility of a potential policy intervention and investigating the effects that such intervention may have on the private provision of liquidity would be a useful step in this interesting research agenda.

References

Acharya, V., H. Shin, and T. Yorulmazer. 2007. "Fire Sales, Foreign Entry and Bank Liquidity." Manuscript, New York University.

Bhattacharya, S., and D. Gale. 1987. "Preference Shocks, Liquidity and Central Bank Policy." In *New Approaches to Monetary Economics,* ed. W. A. Barnett and K. J. Singleton. Cambridge: Cambridge University Press.

Bolton, P., T. Santos, and J. Scheinkman. 2009. "Outside and Inside Liquidity." Manuscript, Columbia Business School.

Diamond, D. W., and P. H. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91 (3): 401–19.

Holmström, B., and J. Tirole. 1998. "Private and Public Supply of Liquidity." *Journal of Political Economy* 106 (1): 1–40.

———. 2008. "Inside and Outside Liquidity." Manuscript, Massachussetts Institute of Technology.

Credit, Asset Prices, and Financial Stress*

Miroslav Misina and Greg Tkacz
Bank of Canada

Historical narratives typically associate financial crises with credit expansions and asset price misalignments. The question is whether some combination of measures of credit and asset prices can be used to predict these events. Borio and Lowe (2002) answer this question in the affirmative for a sample of thirty-four countries, but the question is surprisingly difficult to answer for individual developed countries that have faced very few, if any, financial crises in the past. To circumvent this problem, we focus on financial stress and ask whether credit and asset price movements can help predict it. To measure financial stress, we use the financial stress index (FSI) developed by Illing and Liu (2006). Other innovations include the estimation and forecasting using both linear and endogenous threshold models, and a wide range of asset prices (stock and housing prices, for example). The exercise is mainly performed for Canada, but in our robustness checks we also consider data for Japan and the United States. Our sample also includes the financial crisis of 2007–08.

JEL Codes: G10, E5.

1. Introduction

Despite the apparent uniqueness of each financial cycle—from the conditions that lead to boom times, to triggers that result in reversals—historical narratives (e.g., Kindleberger and Aliber 2005)

*We would like to thank the participants and discussants at the Bank of Canada, the "Forecasting Financial Markets" conference (Aix-en-Provence), the "Measuring and Forecasting Financial Stability" workshop (Dresden), the "Provision and Pricing of Liquidity Insurance" conference (New York), Andrew Levin, and an anonymous referee for valuable comments and suggestions, as well as Stephen Doxey and Thomas Carter for excellent research assistance. Author contact: Misina: mmisina@bankofcanada.ca; Tel: 613-782-8271; Tkacz: gtkacz@bankofcanada.ca; Tel: 613-782-8591.

suggest that most cycles display common features: boom times are typically associated with periods of credit expansion and persistent increases in asset prices, often followed by rapid reversals.

These commonalities, confirmed by recent empirical work (e.g., Borio and Lowe 2002, Kaminsky and Reinhart 1999), suggest that developments in the credit and asset markets of individual countries may provide an early-warning indicator of vulnerability in the financial system that would be useful in assessing the current situation and in discussions of possible policy actions. In light of this, it is somewhat surprising that the empirical work in this area is scarce (Borio and Lowe 2002, 11). Whatever reasons there may be at a general level, the problem in doing this type of analysis for developed countries is compounded by the scarcity of events that would qualify as financial crises in those countries.[1] Absence of financial crises does not, however, mean that financial systems of developed countries have not, or cannot, come under stress, but it does raise the issue of the best way to proceed.

In this work we propose a methodology that can be used to assess the role of credit and asset prices as early-warning indicators of vulnerability in the financial system of countries that have experienced very few or no financial crises over the sample period of interest. A typical example is Canada, which will be the basis of our empirical work in this paper: in Bordo et al. (2001) dating, Canada has not experienced any “twin crises” (banking and currency crises) since the beginning of their sample in 1883, and has experienced only four currency crises since 1945.[2] These features of the sample preclude a meaningful country-level analysis based on binary indicators of crises. Instead, we suggest that in such circumstances one focuses on incidences of financial stress. In our work we use the financial stress index (FSI), a continuous measure of financial stress developed by Illing and Liu (2006). The measure was originally developed for Canada, but the underlying approach can be applied to any country. In our examination of the role of credit and

[1]Bordo et al. (2001, 55) define financial crises as episodes of financial market volatility marked by significant problems of illiquidity and insolvency among financial market participants and/or by official intervention in order to contain such consequences.

[2]For details and dating of crises, see the appendix to Bordo et al. (2001).

asset price in episodes of financial stress we consider both linear and nonlinear models, since the latter may be more suitable in capturing any behavioral asymmetries of financial market participants.

It is important to emphasize that the objective of this type of work is not to forecast idiosyncratic events that cause reversals (an impossible task using any econometric model), but rather to assess whether, historically, there has been a relationship between the various measures of movements in credit and asset prices at time t and the FSI k periods ahead. The working hypothesis is that movements in credit and asset prices are indicators of the health of the system and its ability to withstand various types of shocks. Since the impact of a shock depends not only on the state of the system but also on the magnitude of the shock, one would expect that, everything else being the same, excessive growth of credit and persistent increases in asset prices reduce the ability of the system to withstand the shocks.

To preview the main results, we find that within a linear framework, domestic credit growth is the best predictor of the FSI at all horizons, resulting in marginally lower prediction errors relative to our base-case model, although we do not observe the combination of credit and asset prices observed by Borio and Lowe (2002). Our results suggest that asset prices tend to be better predictors of stress when we allow for nonlinearities, suggesting that extreme asset price movements have disproportionate impact on financial stress. Finally, at the two-year horizon, business credit and real estate prices emerge as important predictors of financial stress, confirming the general findings of Borio and Lowe.

The presentation is organized as follows. In section 2 we review the related literature and describe the nature of the problem addressed in this paper. Section 3 discusses in detail the data used. In section 4, we describe the model and present our results. Section 5 contains the results of our robustness checks, including the application of our approach to the United States and Japan. The last section concludes.

2. Related Literature

Broadly speaking, the present work forms part of the literature attempting to arrive at a set of early-warning indicators. The general problem in this literature has been to identify a subset of macroeconomic and other relevant variables that would help predict the probability of a financial crisis.[3]

Borio and Lowe (2002) investigate the usefulness of asset prices as indicators of financial crises. The authors establish some stylized facts regarding the behavior of asset prices over the last thirty years and conclude that there is a relationship between asset price movements, credit cycles, and developments in the real economy. Given this, they asked whether a useful indicator of financial crises can be constructed. The exercise performed is to assess whether credit, asset prices, and investments—either separately or in some combination—can predict financial crises.[4] The methodology used is that of Kaminsky and Reinhart (1999), and it is based on threshold values of each series. The dating of crises is taken from Bordo et al. (2001). The key finding is that some combination of asset prices and credit gap can help predict crises.

Hanschel and Monnin (2005) focus on the banking sector and propose an index that can be used to measure stress in the Swiss banking sector. The paper then investigates whether the values of the index can be predicted by a set of macro variables. In assessing the latter, the authors follow Borio and Lowe (2002), focusing on the imbalances rather than levels of variables.

This paper is related to Hanschel and Monnin's work since it focuses on a single country and investigates the predictive ability of a set of variables for financial stress rather than as indicators of

[3]There have been a variety of papers that have explored a range of indicators for different types of crises. Recent work has tended to cluster around specific financial crises. For example, following the Mexican Crisis in 1994, papers such as Sachs, Tornell, and Velasco (1996) and Frankel and Rose (1996) explored whether a variety of variables—such as bank credit growth, currency reserves, capital inflows, and level of the exchange rate—affect the likelihood of a crisis. Following the Asian crisis in 1997, there were more efforts, such as Goldstein and Hawkins (1998) and Rodrik and Velasco (1999). Berg and Pattillo (1998) test whether crisis prediction measures constructed after the Mexican crisis would have predicted the Asian crisis. Sorge (2004) provides an excellent survey of stress-testing literature and its relationship to macroeconomic forecasting and early-warning-signals literature.

[4]The idea that credit expansions may lead to imbalances and eventual crises is certainly not new. Hayek (1932) is an early example in the twentieth century, but the views go further back. Schumpeter (1954) contains a historical survey of the key ideas by period, and their proponents.

crises. In our work, however, the indicator of financial stress used is the one developed by Illing and Liu (2006). This indicator is broader based than in Hanschel and Monnin, since it tries to capture stress in the financial system rather than only focusing on the banking sector.

In exploring the forecasting ability of credit and asset prices for financial stress, we look at both linear and nonlinear (threshold) specifications. In the latter, we follow Borio and Lowe (2002) but, rather than specifying the threshold exogenously, in our work the thresholds are determined endogenously.

3. Data

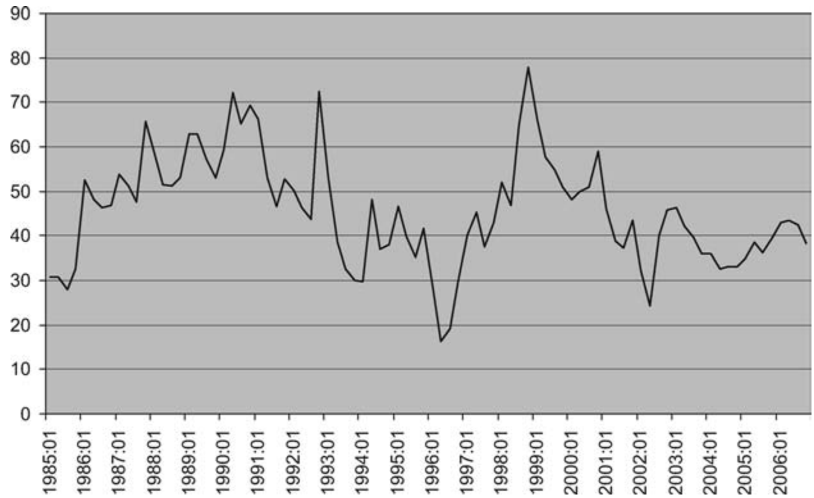
3.1 A Measure of Financial Stress

Financial stress can be characterized as a situation in which large parts of the financial sector face the prospects of large financial losses. These situations are usually accompanied by an increased degree of perceived risk (a widening of the distribution of probable losses) and uncertainty (decreased confidence in the shape of that distribution).

To capture these features of financial stress, Illing and Liu (2006) constructed a weighted average of various indicators of expected loss, risk, and uncertainty in the financial sector. The resulting financial stress index (FSI) is a continuous, broad-based measure that includes the following indicators from equity, bond, and foreign exchange markets, as well as indicators of banking-sector performance:

- the spread between the yields on bonds issued by Canadian financial institutions and the yields on government bonds of comparable duration
- the spread between yields on Canadian nonfinancial corporate bonds and government bonds
- the inverted term spread (i.e., the ninety-day Treasury bill rate minus the ten-year government yield)
- the beta derived from the total return index for Canadian financial institutions
- Canadian trade-weighted dollar GARCH volatility
- Canadian stock market (TSX) GARCH volatility

Figure 1. Financial Stress Index



- the difference between Canadian and U.S. government short-term borrowing rates
- the average bid-ask spread on Canadian Treasury bills
- the spread between Canadian commercial paper rates and Treasury bill rates of comparable duration

In constructing the FSI, Illing and Liu considered several weighting options and settled on weights that reflect relative shares of credit for particular sectors in the economy. The resulting index, shown in figure 1, was most effective in correctly signaling events that are widely associated with high financial stress (e.g., the stock market crash in October 1987, the peso crisis in 1994, the long-term capital management crisis in 1998, etc.). This is not surprising, given that Canada is a small open economy whose markets are well integrated internationally. As such, it is not insulated from international financial developments. Turmoil in international financial markets will be reflected in increased stress in Canadian markets. This does not mean that financial stress is not or cannot be domestically generated, but it may indicate that the level of "internal" stress is secondary to the level of "external" stress that spills over into Canadian financial markets. To assess the importance of external factors in predicting financial stress in Canada, we include a set of international explanatory variables described below.

3.2 Explanatory Variables

Because Canada is a small open economy, its financial stress will necessarily be impacted by international events—the 1994 peso and 1997 Asian crises being well-known recent examples. For this reason, our data set incorporates, in addition to a broad set of domestic variables, several foreign variables. However, international developments will also be felt in many domestic variables. For example, Canadian stock prices move in response to expected future earnings of Canadian firms, which in turn are largely dependent on international factors such as the economic health of Canada's trading partners or on world commodity prices. In addition, real estate prices follow similar patterns across major international cities (e.g., see Shiller 2005, 19). As a result, many of our variables will necessarily move in response to the ultimate source of the stress, be it domestic or international factors.

The explanatory variables are divided into four major categories:[5]

(i) Credit measures: the growth rate of total household credit (HouseCR), total business credit (BusCR), and total credit/GDP (CR/Y)

(ii) Asset prices (growth rates): stock prices (TSX), commercial real estate indices (real (ComREI) and nominal (real Com REI)), residential real estate indices ("New house price" and existing (RoyalLePage)), average price to personal disposable income ratio (AvgP/PDI), and Canadian dollar price of gold (GoldC$)

(iii) Macroeconomic variables: Investment/GDP (I/Y), GDP growth rate, money (M1++ and M2++), and inflation (Total CPI and Core CPI)

[5]Unless otherwise indicated, the source of the data is the Bank of Canada.

(iv) Foreign variables: crude oil, asset price indices (United States, Australia, Japan),[6] world gold price, U.S. bank credit,[7] U.S. federal funds rate, and world GDP

The data is quarterly and spans the period 1984–2006. The forecasting exercise is performed over the period 1996–2006. The last observation is 2006:Q4. The explanatory variables are converted into growth rates, so all variables are stationary.[8] We consider both quarterly and annual growth rates, since it is possible that longer-run cumulative growth rates in the explanatory variables may contain more information about financial stress than quarterly growth rates. In our output we use $d = 1$ to denote quarterly growth rates and $d = 4$ for annual (year-over-year) growth rates.

4. Models and Results

4.1 Linear Models and Forecast Evaluation

In order to evaluate the marginal contributions of the various explanatory variables, we compare all our models with a simple linear benchmark, whereby the current FSI is simply a function of the k-quarter lagged FSI:

$$FSI_t = \alpha + \beta FSI_{t-k} + \varepsilon_{1,t}. \tag{1}$$

At this time, the explanatory variables will be added to (1) in isolation and in pairs; given the multitude of horizons and variables under consideration, this alone results in several thousand models to be assessed. The augmented models are thus

$$FSI_t = \alpha + \beta_1 FSI_{t-k} + \gamma X_{t-k} + \varepsilon_{2,t}, \tag{2}$$

[6]These indices are the same ones used by Borio and Lowe (2002). In general, they are the aggregates of stock prices, bond prices, and real estate prices, but the components vary by country depending on data availability. The reader is referred to their paper for details.

[7]Source: Federal Reserve Board, H.8 Release: Assets and Liabilities of Commercial Banks in the United States. Available at http://www.federalreserve.gov/releases/h8/.

[8]Commercial real estate investment is not transformed, since it was found to be stationary.

where X is a vector containing one or two explanatory variables. Since we are primarily interested in forecast performance, we summarize the forecast performance according to the ratio of the root mean squared error (RMSE) of model (2) relative to that of (1):

$$rmr = \frac{\sqrt{\sum_{t=1996Q1}^{2006Q4} (F\hat{S}I_{2,t} - FSI_t)^2 \Big/ 44}}{\sqrt{\sum_{t=1996Q1}^{2006Q4} (F\hat{S}I_{1,t} - FSI_t)^2 \Big/ 44}}, \quad (3)$$

where $F\hat{S}I_{1,t}$, $F\hat{S}I_{2,t}$ are forecasts of the FSI originating from models (1) and (2), respectively. When the *rmr* is above 1.0, this indicates that the additional explanatory variables worsen the forecast performance relative to the base-case model; when it is below 1.0, the forecast performance is improved.[9,10]

To determine whether the ratio of mean-squared errors is statistically less than 1.0, we employ a test proposed by McCracken (2004) that can test for equality of the mean-squared errors of nested models. Let D_{t+k} denote the difference between the squared forecast errors at $t + k$ of the base-case model (i.e., the model which includes only the lagged FSI) and the alternative model (i.e., the model augmented with one or more explanatory variables):

$$D_{t+k} = \hat{\varepsilon}^2_{1,t+k} - \hat{\varepsilon}^2_{2,t+k}. \quad (4)$$

With n forecast periods, the statistic for testing the equality of mean-squared errors between the base-case and alternative model is computed as

[9]In comparisons of models that contain the same dependent variable, the above is equivalent to a comparison of the adjusted R-squared of these models. The adjustment factor for the R-squared imposes a penalty on the inclusion of additional explanatory variables. The adjusted R-squared of the resulting model will be lowered if the additional variable's contribution does not exceed the penalty factor. Consequently, the ratio of the adjusted R-squared statistics could be greater than 1.

[10]This model selection criterion gives the same weight to the errors on the upside and the downside. To the extent that the users might give more weight to increases than decreases, a selection criterion that penalizes downside errors more than the upside ones would be more appropriate. We thank the referee for bringing this point to our attention.

$$MSE - F = n \sum \frac{n^{-1} \sum_{t=R-k}^{T} \left(\hat{\varepsilon}_{1,t+k}^{2} - \hat{\varepsilon}_{2,t+k}^{2} \right)}{n^{-1} \sum_{t=R-k}^{T} \hat{\varepsilon}_{2,t+k}^{2}}, \qquad (5)$$

where R represents the first out-of-sample forecast period (1996:Q1). Intuitively, note that the numerator represents the difference in mean-squared errors (MSEs) between the base-case and alternative model, and the denominator represents the MSE of the alternative. If both models produce equally accurate forecasts, then the numerator and test statistic are zero; if the base-case model has a lower MSE, then the statistic will be negative, and it will be positive if the alternative model has a lower MSE. The distribution is nonstandard due to the fact that the models are nested, and so we use the critical values computed by McCracken (2004). Results presented by McCracken show that this test has good size and power for sample sizes as small as fifty. Our own application has a sample size of forty-four (1996:Q1 to 2006:Q4), so this test should be appropriate for our purposes. Instances where the alternative model is found to have a statistically lower MSE than the base-case model are highlighted in our figures.

The details of the forecasting exercise are as follows:

- We initially estimate (1) and (2) with data from 1984 to 1996:Q1$-k$, where $k = 1$, 2, 4, 8, or 12.
- Using the estimated parameters, we produce a forecasted FSI for 1996:Q1.
- We reestimate the parameters with data from 1984 to 1996:Q2$-k$.
- We use the newly estimated parameters to obtain a forecast of the FSI for 1996:Q2.
- We continue in this fashion until forecasts have been generated for 2006:Q4, for a total of forty-four forecast periods.

Note that the above attempts to replicate actual real-time forecasts, whereby the forecaster uses data available up to time t to produce a forecast at $t + k$ (or, equivalently, data up to $t - k$ to produce forecasts at time t). The issue of data revisions does not apply in the case of most of our financial variables, as these observations are not revised. However, it is known that GDP and monetary aggregates are subject to revision, so some caution should be used in interpreting some of these forecast results, as the data that we use in these particular cases do not produce true real-time forecasts.

4.2 Threshold Specification

Equation (2) supposes that financial stress is a linear function of asset price movements and other variables. However, if one believes that unusually large movements in asset prices, credit, monetary expansion, etc., may lead to greater financial uncertainty if, for example, herding mentality replaces rational financial decisions, then the relationship between some of our explanatory variables and the FSI may be nonlinear.[11] We can approximate such relationships by allowing for threshold effects between the explanatory variables and the FSI, such that the parameters of the models are allowed to differ when the explanatory variables lie above or below their threshold values. A similar strategy was employed by Borio and Lowe (2002), but the thresholds used in that study were explicitly specified by the authors. We employ a more general approach, whereby we estimate the threshold values; these endogenous thresholds therefore maximize the probability of locating a threshold effect in the data.

The threshold models take the form

$$FSI_t = \alpha^1 + \beta^1 FSI_{t-k} + \gamma^1 X_{t-k} + \delta^1 z_{t-k} + \xi_t \quad \text{for } z_{k,t-k} \leq \tau \tag{6}$$

$$FSI_t = \alpha^2 + \beta^2 FSI_{t-k} + \gamma^2 X_{t-k} + \delta^2 z_{t-k} + \xi_t \quad \text{for } z_{k,t-k} > \tau, \tag{7}$$

where z is some variable extracted from the vector X, and τ represents the level of z that triggers a regime change. We allow for a threshold effect for each of our twenty-four explanatory variables. Superscripts denote the values taken in regimes 1 and 2, respectively.

To estimate the parameters of the threshold model (6)–(7), we follow Hansen (2000) who derives an approximation of the asymptotic distribution of the least-squares estimator of the threshold parameter $\hat{\tau}$. To understand how the parameters are estimated, we

[11] Regardless of the underlying mechanism, Misina and Tessier (2008) show that nonlinearities play the key role in capturing extreme events associated with stress, as well as in generating plausible responses to shocks.

introduce an indicator function w and can rewrite equations (6) and (7) as a single equation:

$$FSI_t = \alpha^2 + \beta^2 FSI_{t-k} + \gamma^2 X_{t-k} + \delta^2 z_{t-k} + Aw + BwFSI_{t-k} + CwX_{t-k} + Dwz_{t-k} + \xi_t, \qquad (8)$$

where

$$w = \begin{cases} 1 & z_{t-k} \leq \tau \\ 0 & z_{t-k} > \tau \end{cases},$$

$$\alpha^2 + A = \alpha^1, \beta^2 + B = \beta^1, \gamma^2 + C = \gamma^1, \text{ and } \delta^2 + D = \delta^1.$$

By assuming that $\hat{\tau}$ is bounded by the largest and smallest values of the threshold variables, we can estimate the parameters in (8) by least squares conditional on a given value of $\hat{\tau}$. By iterating through the possible values of τ in the range of available threshold values, we select the $\hat{\tau}$ that minimizes the sum of squared residuals in (8).

The forecast exercise using the threshold models proceeds in exactly the same manner as for the linear models described above, so the parameters and threshold values are reestimated each period. The *rmr* is computed as the ratio of the RMSE from (8) relative to the RMSE of a modified version of the simple base-case model (1) which allows for threshold effects in the lagged value of the FSI.

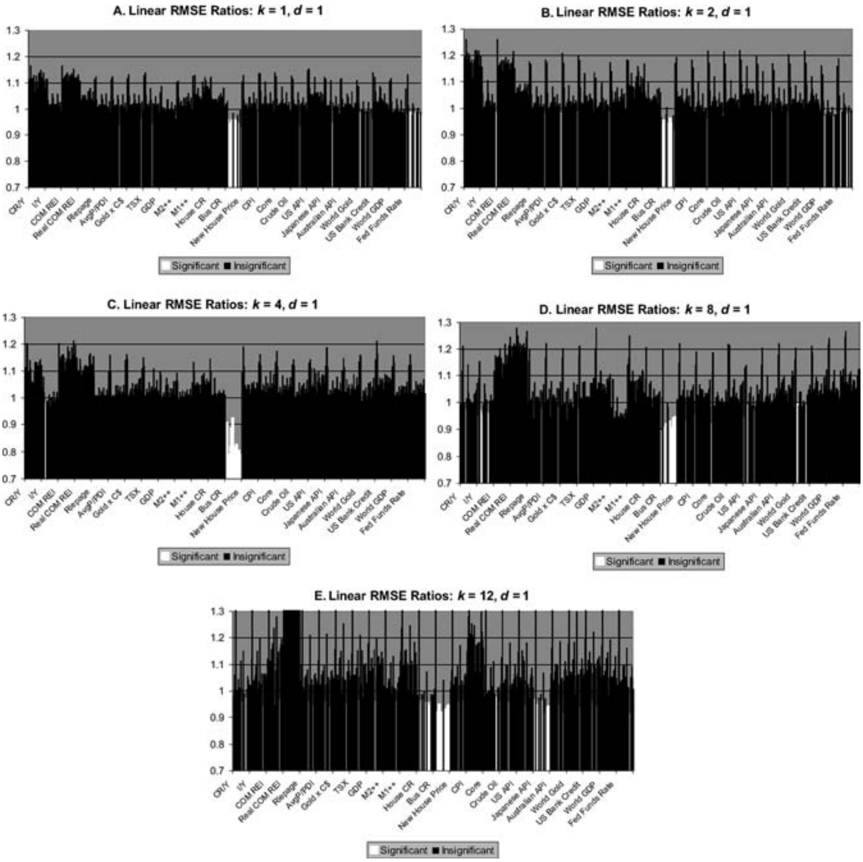
4.3 Results

Given all the combinations of variables, horizons, and specifications, we consider 11,520 models relative to the base-case model (1).[12] To summarize these results in the least cumbersome manner, we present the ratio of root mean squared errors for each horizon (k) and differencing operator (d) and model specification (linear or threshold) in twenty different graphs. This provides a simple visual approach to judge the usefulness of various variables. Since the results for $d = 1$ and $d = 4$ are very similar, we place the latter in an appendix.

The forecast performance of the linear models is summarized in figure 2. To interpret these figures, consider panel A. The horizontal axis contains labels for all the explanatory variables considered

[12] This is based on 24×24 variable combinations, five horizons, two differencing operators, and two model specifications: $576 \times 5 \times 2 \times 2 = 11{,}520$.

Figure 2. Linear Models, Forecast Performance, $d = 1$



(twenty-four variables). When a variable is listed along the horizontal axis, this indicates that it is included as the first regressor in the next twenty-four models. After each label there are twenty-four bars, corresponding to the *rmr*'s associated with models using different combinations of the labeled explanatory variable with other variables. For example, the first variable on the horizontal axis is the credit-to-GDP ratio, CR/Y. The first bar is the *rmr* for a model that includes only the CR/Y ratio as an additional explanatory variable, so that the estimated model is

$$FSI_t = \alpha + \beta_1 FSI_{t-k} + \gamma_1 (CR/Y)_{t-k} + \varepsilon_t.$$

The second bar is the *rmr* for a model including the CR/Y as well as the investment-to-GDP (I/Y) variable:

$$FSI_t = \alpha + \beta_1 FSI_{t-k} + \gamma_1 (CR/Y)_{t-k} + \gamma_2 (I/Y)_{t-k} + \varepsilon_t.$$

The third bar is the *rmr* for the model

$$FSI_t = \alpha + \beta_1 FSI_{t-k} + \gamma_1 (CR/Y)_{t-k} + \gamma_2 (ComREI)_{t-k} + \varepsilon_t,$$

etc.

The results associated with different models are assessed against the benchmark value of $rmr = 1$, which indicates that the inclusion of additional explanatory variables did not impact the forecasting performance of the base-case model. As stated earlier, $rmr > 1$ indicates that the inclusion of the variable has resulted in deterioration of the forecasting performance of the model relative to the benchmark. Finally, $rmr < 1$ indicates improved performance of the new model relative to the benchmark. Models for which the *rmr* is statistically lower than 1.0 according to the McCracken (2004) test are denoted in white.

Returning to figure 2, it is clear that the only variable that consistently helps forecast the FSI is domestic business credit, although the federal funds rate is significant at shorter horizons (up to two quarters ahead). For both these variables we find that, regardless of which variable they are paired with, they often produce mean-squared errors that are statistically lower than 1.0.

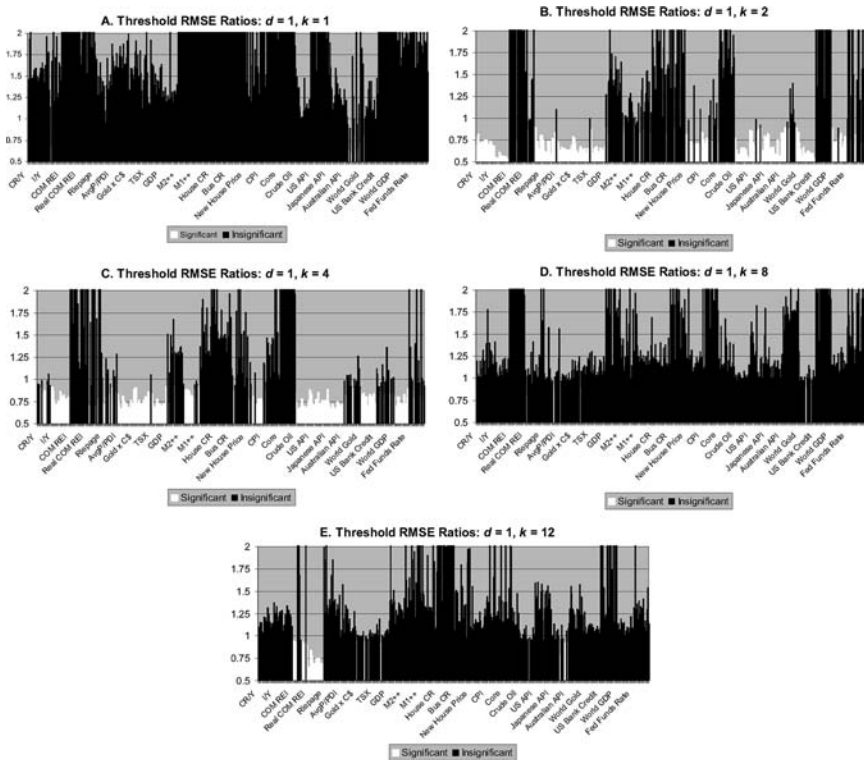
To understand the effect of business credit on the FSI, we can analyze the estimated parameters of the best forecasting models at each horizon, which are presented in table 1. We note several interesting results. First, the explanatory power of the lagged FSI decreases as the forecast horizon k increases, as evidenced by the adjusted R^2 which steadily decreases from 0.58 for $k = 1$ to 0.00 for $k = 12$. Second, the federal funds rate is retained in the best forecasting models at the shorter horizons ($k = 1$, 2), while domestic credit is retained at all horizons. Third, the parameters on the credit variables are all positive and statistically significant. This signals that a 1 percent quarterly increase in credit will cause the FSI to increase by between one and two points in the following quarters, which signals higher stress. If business credit is expanding, this could indicate that financial institutions are adding more risk to

Table 1. In-Sample Regression Results, Linear Models, Base-Case and Best Forecasting Models, $d = 1$ Sample: 1984:Q1 to 2006:Q4

Variable	$k = 1$		$k = 2$		$k = 4$		$k = 8$		$k = 12$	
Constant	7.66 (2.27)	11.25 (3.56)	14.31 (3.77)	21.21 (5.47)	19.40 (5.62)	28.46 (7.04)	30.43 (6.32)	40.01 (7.80)	45.80 (8.27)	52.19 (10.48)
FSI_{t-k}	0.75 (12.29)	0.76 (11.42)	0.54 (7.23)	0.54 (6.53)	0.30 (4.31)	0.39 (4.43)	0.04 (0.39)	0.15 (1.49)	−0.19 (−2.09)	−0.11 (−1.07)
Business $Credit_{t-k}$	0.68 (2.54)	—	1.13 (3.32)	—	2.13 (7.47)	—	1.34 (4.08)	—	1.59 (4.64)	—
World GDP_{t-k}	—	—	−0.01 (−1.89)	—	—	—	—	—	—	—
Japan API_{t-k}	—	—	—	—	—	—	—	—	0.36 (2.76)	—
Core CPI_{t-k}	—	—	—	—	—	—	2.29 (3.11)	—	—	—
Fed $Funds_{t-k}$	2.07 (1.99)	—	1.58 (5.47)	—	−2.32 (−1.11)	—	—	—	—	—
$\bar{R}^2$	0.62	0.58	0.41	0.29	0.44	0.14	0.29	0.01	0.35	0.00
RMSE Ratio	0.92		0.89		0.78		0.87		0.82	

Note: The base-case model includes only the lagged FSI. *RMSE Ratio* is the ratio of root mean squared error of the model containing exogenous regressors relative to the base-case model. t-statistics corrected for serial correlation and heteroskedasticity are in parentheses.

Figure 3. Threshold Models, Forecast Performance, $d = 1$



their balance sheets, and so results in a rise in the FSI. Conversely, when business credit falls, the opposite occurs. At shorter horizons, the federal funds rate is positively correlated with financial stress in Canada. That result is reversed at the one-year horizon, although the parameter is not statistically significant.

The results for threshold models are presented in figure 3, and the estimated coefficients for the best specifications are presented in table 2. The interpretation of these figures is similar to the linear model, except that in each model a threshold effect is allowed in the variable labeled in the figure. For example, the first variable on the horizontal axis in figure 3 is CR/Y, and the first bar is the *rmr* for the model that includes only the CR/Y ratio, with threshold effect, as an additional explanatory variable. The second bar is the *rmr* for

Table 2. In-Sample Regression Results, Threshold Models, Best Forecasting Models, $d = 1$ Sample: 1984:Q1 to 2006:Q4

Variable	**$k = 1$**		**$k = 2$**		**$k = 4$**		**$k = 8$**		**$k = 12$**	
Threshold Variable	Australia API		Japan API		AvgP/PDI		RLePage		Real Com REI	
$\hat{\tau}$	−3.05		−10.49		−5.79		12.93		88.22	
Regime	$\leq \hat{\tau}$	$> \hat{\tau}$	$\leq \hat{\tau}$	$> \hat{\tau}$	$\leq \hat{\tau}$	$> \hat{\tau}$	$\leq \hat{\tau}$	$> \hat{\tau}$	$\leq \hat{\tau}$	$> \hat{\tau}$
Constant	42.87	9.44	−4.22	19.25	79.88	21.29	39.65	27.37	190.02	10.72
	(2.32)	(2.97)	(−0.35)	(4.15)	(14.18)	(6.33)	(8.30)	(2.31)	(10.85)	(0.45)
FSI_{t-k}	0.47	0.73	0.22	0.56	0.06	0.24	−0.01	0.49	0.03	−0.28
	(1.95)	(10.56)	(0.91)	(6.16)	(0.79)	(3.34)	(−0.06)	(2.93)	(0.43)	(−2.39)
$CORE_{t-k}$	−1.41	0.32	—	—	—	—	—	—	—	—
	(−1.00)	(0.59)								
Australia API_{t-k}	0.98	0.20	—	—	—	—	—	—	—	—
	(1.84)	(1.83)								
Japan API_{t-k}	—	—	−2.65	0.09	—	—	—	—	—	—
			(−3.45)	(0.62)						
GDP_{t-k}	—	—	0.69	0.52	—	—	—	—	—	—
			(0.44)	(1.34)						
Business $Credit_{t-k}$	—	—	—	—	−0.22	2.03	1.09	1.63	—	—
					(−0.82)	(6.45)	(3.32)	(2.07)		
$AvgP/PDI_{t-k}$	—	—	—	—	1.79	0.36	—	—	—	—
					(13.99)	(3.07)				
$RLePage_{t-k}$	—	—	—	—	—	—	−0.50	−0.33	—	—
							(−2.25)	(−1.27)		
Real Com REI_{t-k}	—	—	—	—	—	—	—	—	−1.95	0.46
									(−8.31)	(2.05)
New House $Prices_{t-k}$	—	—	—	—	—	—	—	—	−1.18	0.86
									(−2.31)	(5.02)
Obs.	11.00	80.00	13.00	77.00	7.00	81.00	70.00	14.00	43.00	37.00
$\bar{R}^2$	0.41	0.60	0.51	0.35	0.95	0.53	0.17	0.53	0.71	0.46
RMSE Ratio	0.54		0.48		0.54		0.69		0.55	

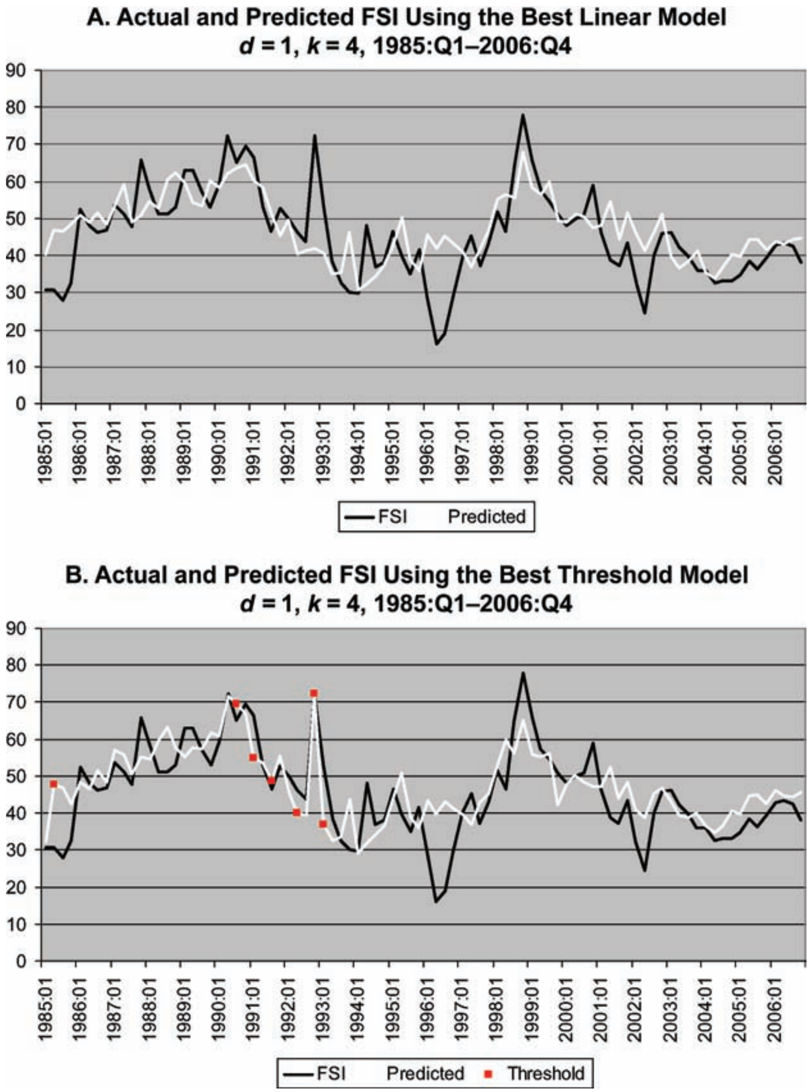
Note: The base-case model includes only the lagged FSI. *RMSE Ratio* is the ratio of root mean squared error of the model containing exogenous regressors relative to the base-case model. t-statistics corrected for serial correlation and heteroskedasticity are in parentheses.

the model that allows for the threshold effect in CR/Y and includes I/Y as an additional explanatory variable, etc.

The key features of the results at given horizon k, are as follows:

- $k = 1, 2$: No single variable appears to universally perform well at forecasting the FSI at very short horizons, as evidenced by the large number of black bars in one-quarter-ahead forecasting models. The situation improves noticeably two quarters ahead, but in both cases, the forecast performance varies according to the specific combinations of variables that are retained, as well as the choice of the threshold variable. The best forecasting equations at these horizons retain core inflation and GDP, with international asset price indices (Australia, Japan) identified as threshold variables (table 2). The *rmr* at these horizons is quite low, indicating a significant improvement in forecast performance relative to the base-case model.
- $k = 4, 8$: At these horizons, both business credit and asset prices emerge as significant predictors of financial stress (table 2). In both cases, a variable related to housing prices appears as a significant threshold variable. The *rmr* in both specifications remains quite low, indicating improvements in the forecast performance. For $k = 8$, the equation shows that when house prices rise by more than 13 percent during a quarter, the impact of additional business credit on the FSI rises from 1.1 to 1.6, so business credit expansion during a housing boom can add additional financial stress two years later.
- $k = 12$: At the longer horizons, we observe that few variables retain any forecasting power, the notable exception being the commercial real estate variable. At this horizon, commercial real estate investment is the regime-change trigger, while new house prices is retained as a significant regressor. Regime effects are quite pronounced for this equation, as the signs on both parameters actually change depending on which regime we are in. Furthermore, the number of observations in each regime are almost equal. When commercial real estate investment is low, increases in this variable and in new house prices lower stress; when commercial real estate investment is high, increases in these variables increase stress.

Figure 4. Actual and Fitted FSI



Finally, to provide a sense of how these models track the actual data, we plot in figure 4 the actual and fitted values of the best linear and threshold forecasting models for $k = 4$, a horizon which could be of interest to policymakers. In both cases, business credit is retained as a regressor and we see that, in general, both models perform reasonably well in tracking the trend and turning points of the FSI. The improvement of the threshold model relative to the linear model centers on the seven observations early in the sample, where the threshold model succeeds in picking up a few extreme movements of the FSI. One would therefore conclude that at this horizon business credit offers some hope in forecasting the FSI, regardless of the specification used.

5. Robustness Checks

To verify whether credit and asset prices are useful predictors of financial stress more generally, we first assess whether some of the more promising variables are good predictors of stress in Japan and the United States; second, we consider how these variables moved prior to the 2007–08 financial crisis.

5.1 The Crisis of 2007–08

In August 2007, the FSI increased sharply, pointing to considerable stress in the Canadian financial system. Indeed, the FSI in the recent episode reached its historical high, indicating that this is the most stressful episode since 1985. To assess the predictive power of the best forecasting model identified in the previous section, we have extended the sample to 2008:Q2 and performed a forecasting exercise along the lines described in the previous section. The results are shown in figure 5. The results show that whereas the best forecasting model does generate an increase in the FSI, the magnitude of that increase underestimates the increase in the FSI by a large margin. This is not surprising, given that the increase in stress captured by the FSI was largely triggered by exogenous events (collapse of the U.S. subprime market), but an analysis of the behavior of the explanatory variables can provide additional insights.

A look at the two key explanatory variables retained in the best threshold specification (figure 6) reveals that while both variables peaked in 2007:Q2, neither was anywhere near their historical highs. This may be an important contributing factor to the relatively good

Figure 5. 2007–08 Crisis

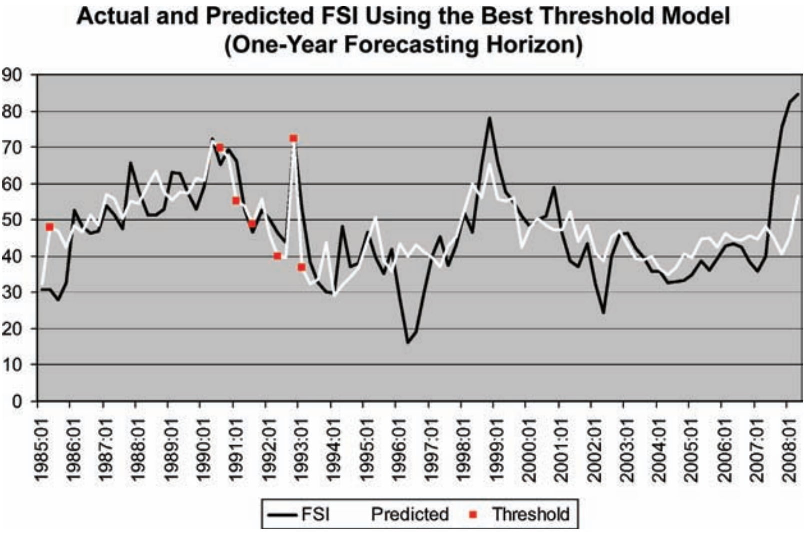
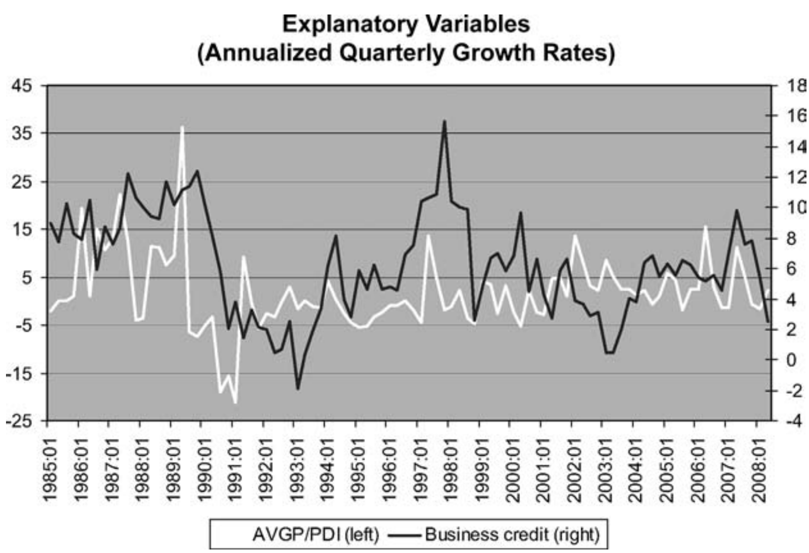


Figure 6. Behavior of the Explanatory Variables



health of the Canadian system and its resilience to date. Of course, the impact of a shock on the system is a function of the magnitude of the shock as well, and the peak in the FSI in spite of the good health of the system indicates that this is a large shock, by historical standards.

5.2 Results for Japan and the United States

We begin by constructing financial stress indexes for Japan and the United States that are as comparable as possible to the one we use for Canada.[13] We choose these two countries since their financial systems are quite different and have experienced different shocks over the last several years, so the degree of predictability of financial stress by credit and asset prices for these two countries can be informative with regard to their robustness as indicators. We notice in figures 7 and 8 that the movements in the U.S. FSI are generally similar to that of Canada's, with notable peaks in 2001 and 2008, while the Japan FSI experienced more independent movements.

To assess the usefulness of credit and asset prices in predicting the FSIs in these countries, we focus solely on the case of $d = 1$ and $k = 8$ (the eight-quarter forecast horizon). In this context, we found for Canada that the lowest forecast errors could be obtained using housing prices and business credit, with the former serving as the threshold variable. The Japan and U.S. variables that most closely matched the Canadian definitions of these variables are residential land prices[14] (from the Japanese Real Estate Institute) and total private-sector credit (from the International Monetary Fund) for Japan, and housing prices (Case-Shiller composite ten-city index) and total business credit (from the Federal Reserve bank of St. Louis's FRED database) for the United States. The threshold variable is land/housing prices. The out-of-sample forecasted values for the period 1998 to 2008 are presented in figures 7 and 8, allowing

[13]Details about the FSIs for Japan and the United States are available from the authors upon request.

[14]Land prices are the key driver of housing prices in Japan, so this is why we use this variable to capture real estate prices in Japan.

Figure 7. Forecasting the FSI for Japan

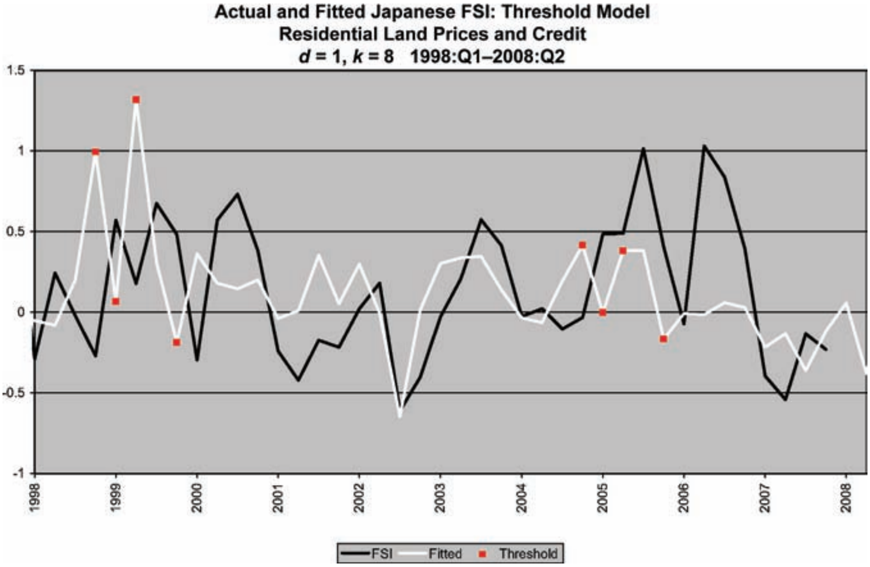
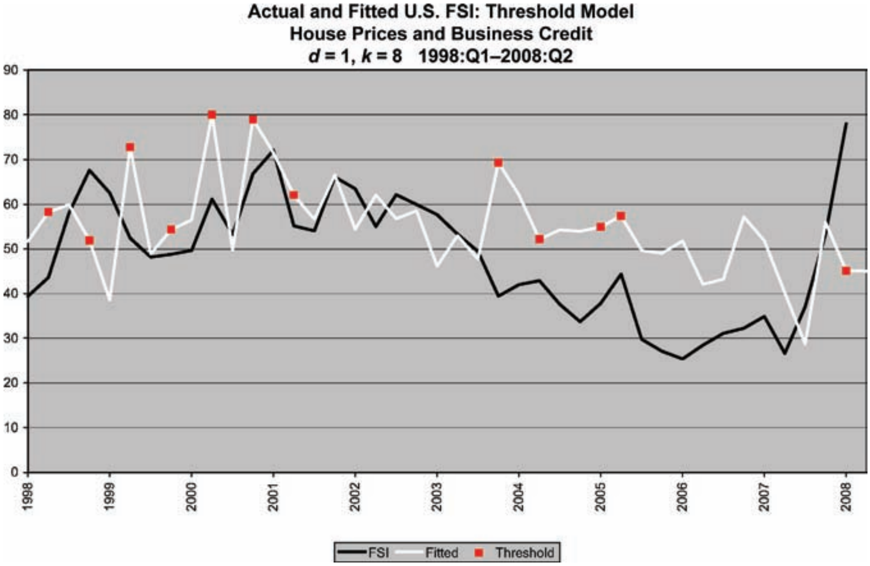


Figure 8. Forecasting the FSI for the United States



us to assess whether such variables could have predicted the recent spike in the U.S. FSI.

For Japan, although its FSI is somewhat more volatile than that of the United States and Canada, land prices and credit growth predicted some peaks and troughs relatively well; for example, the cycle from early 2002 to mid-2004 was captured quite accurately eight quarters in advance. Other episodes, such as the financial stress spike in 2006, were not captured by these variables. This suggests that financial stress was being driven by other factors in this period, so forecasters should consider additional predictors of financial stress for Japan.[15]

In the United States (figure 8) we observe that the forecasted values generally captured the level and volatility of actual FSI until about 2003, but from 2004 to 2007 it overpredicted stress, and it underpredicted stress in 2008. Most recently, housing and credit growth predicted a sharp increase in stress in late 2007, which materialized, but the subsequent stress actually overshot the predicted values. Given that housing prices are known to have increased dramatically until 2007 and then suffered a serious correction, the extent of the bubble characteristics of the U.S. housing market was only partially captured by the simple threshold relationship that we consider.

In short, credit growth and housing prices appear to predict the direction of the FSI relatively well eight quarters in advance. The challenge appears to obtain better predictions of the level of stress. To this end, although the threshold model that we use can capture some of the "bubble" features of asset prices, in future work one may wish to consider models with richer nonlinear dynamics. In the most recent episode, housing prices experienced a long and sustained buildup and then suffered a dramatic crash. Since our model overpredicted stress during the buildup phase, forecasters may wish to focus their attention on building models that more adequately capture the dynamics between asset prices and financial stress in such periods.

[15] Our last observation for the Japan FSI is 2007:Q3, so it predates the most recent global financial crisis.

6. Conclusion

The literature on financial stress typically equates financial stress with the occurrence of financial crises, and attempts to forecast the latter using different sets of macroeconomic variables. This procedure runs into difficulties when applied to countries where financial crises are rare or non-existent events, and this is evident especially when the analysis is constrained by data availability to the last twenty-five to thirty years. The absence of financial crises, however, does not imply that a country has not been subjected to financial stress in the past, or that accumulated financial imbalances could not result in financial crises in the future.

To deal with the problem of measurement of financial stress in the absence of financial crises, Illing and Liu (2006) constructed a financial stress index for Canada. The question we asked in this paper is whether a set of explanatory variables commonly considered in the macroprudential literature could help forecast financial stress. To do this we have considered both linear and threshold models and assessed their performance by comparing them with the benchmark model in which the future value of the FSI is predicted using only its lagged value.

We find that, in line with the macroprudential literature, *some* combination of credit and asset price variables are important predictors of financial stress, although the results depend on the type of model used (linear or threshold) and the forecast horizon. As a general rule, we find that these indicators offer greater value added at forecasting the FSI than the benchmark model as the forecast horizon increases. A specific indicator worthy of being highlighted is business credit, which emerges as the prominent leading indicator in both linear and nonlinear models at the one- and two-year horizons. A combination of this variable with a threshold in a housing-sector asset price leads to significant improvements in performance over the same horizon. At shorter horizons, the federal funds rate emerges as a predictor of financial stress in linear models. In general, however, international variables seem to play a smaller role than one would expect they would in a small open economy.

At the one-year horizon, which could be of interest to forward-looking policymakers, in practical terms there is little to distinguish the linear and threshold specifications, as both models track the FSI relatively well at this horizon. What matters most is the monitoring of business credit, which emerges as an important leading indicator among all variables considered in our study.

The empirical results reported here are country specific, and a more in-depth comparative study of determinants of financial stress in countries with few or no financial crises would be instructive. The methodology proposed in our work is well suited for that task.

Appendix

Figure 9. Linear Models, Forecast Performance, $d = 4$

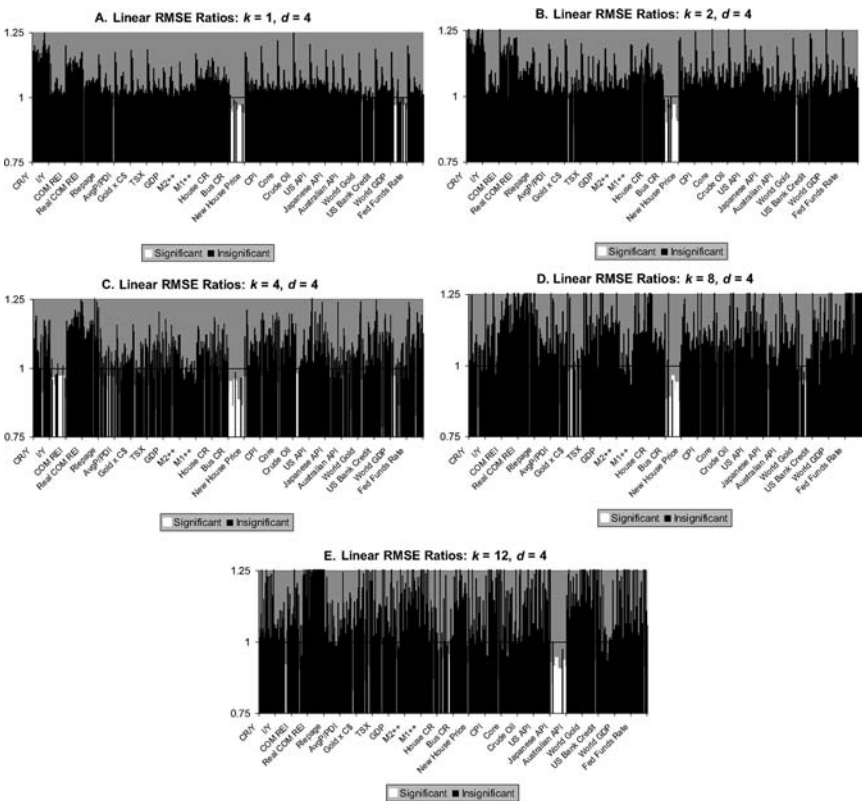
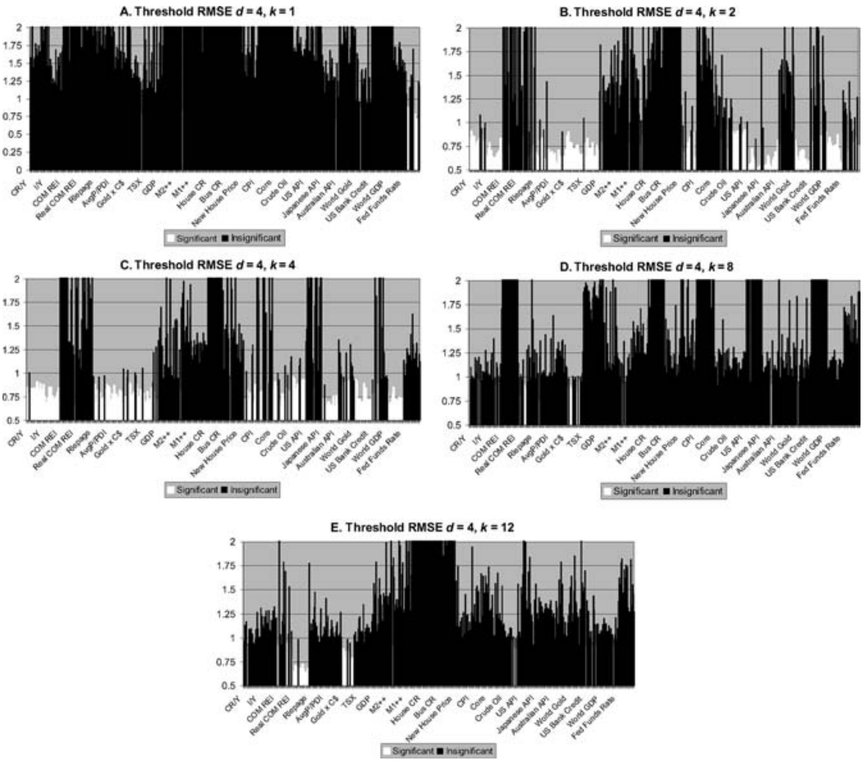


Figure 10. Threshold Models, Forecast Performance, $d = 4$



References

Berg, A., and C. A. Pattillo. 1998. "Are Currency Crises Predictable? A Test." IMF Working Paper No. 98/154.

Bordo, M., B. Eichengreen, D. Klingebiel, and M. S. Martinez-Peria. 2001. "Is the Crisis Problem Growing More Severe?" *Economic Policy* 16 (32): 51–82.

Borio, C., and P. Lowe. 2002. "Asset Prices, Financial and Monetary Stability: Exploring the Nexus." BIS Working Paper No. 114 (July).

Frankel, J. A., and A. K. Rose. 1996. "Currency Crashes in Emerging Markets: An Empirical Treatment." *Journal of International Economics* 41 (3–4): 351–66.

Goldstein, M., and J. Hawkins. 1998. "The Origin of the Asian Financial Turmoil." Reserve Bank of Australia Research Discussion Paper No. 980 (May).

Hanschel, E., and P. Monnin. 2005. "Measuring and Forecasting Stress in the Banking Sector: Evidence from Switzerland." In *BIS Papers No. 22: Investigating the Relationship Between the Financial and Real Economy*, 431–49. Bank for International Settlements.

Hansen, B. E. 2000. "Sample Splitting and Threshold Estimation." *Econometrica* 68 (3): 575–603.

Hayek, F. A. 1932. "Monetary Theory and the Trade Cycle." In *Prices & Production and Other Works* (2008), ed. J. T. Salerno. Ludwig von Mises Institute.

Illing, M., and Y. Liu. 2006. "Measuring Financial Stress in a Developed Country: An Application to Canada." *Journal of Financial Stability* 2 (3): 243–65.

Kaminsky, G. L., and C. M. Reinhart. 1999. "The Twin Crises: The Causes of Banking and Balance-of-Payments Problems." *American Economic Review* 89 (3): 473–500.

Kindleberger, C. P., and R. Aliber. 2005. *Manias, Panics, and Crashes: A History of Financial Crises.* 5th ed. John Wiley & Sons.

McCracken, M. W. 2004. "Asymptotics for Out-of-Sample Tests of Granger Causality." Working Paper, University of Missouri.

Misina, M., and D. Tessier. 2008. "Non-linearities, Model Uncertainty, and Macro Stress Testing." Bank of Canada Working Paper No. 2008–30.

Rodrik, D., and A. Velasco. 1999. "Short-Term Capital Flows." NBER Working Paper No. 7364.

Sachs, J. D., A. Tornell, and A. Velasco. 1996. "Financial Crises in Emerging Markets: The Lessons from 1995." In *Brookings Papers on Economic Activity, 1:1996*, ed. W. C. Brainard and G. L. Perry, 147–215. Washington, DC: Brookings Institution.

Schumpeter, J. A. 1954. *History of Economic Analysis.* Oxford University Press.

Shiller, R. J. 2005. *Irrational Exuberance.* 2nd ed. New York: Doubleday.

Sorge, M. 2004. "Stress-Testing Financial Systems: An Overview of Current Methodologies." BIS Working Paper No. 165 (December).

Discussion of "Credit, Asset Prices, and Financial Stress"*

Stijn Claessens
International Monetary Fund

Let me start with praise for the paper and also introduce the structure of my comments. The paper by Misina and Tkacz is part of recent, broader literature investigating how asset prices and other financial variables can help document and predict financial stress and financial distress. Clearly, since we have faced the largest financial crisis in advanced countries since the Great Depression, nobody needs to be reminded how important this type of research is. It would have been extremely useful for policymakers to have an objective tool on the basis of which they could have anticipated at least some of the turmoil over the past year. This would also have been helpful for private market participants, since that could have provided a check on the excessive risk taking that went on in many financial markets.

In this context, the paper is a very useful addition to the literature on financial stress indexes (FSIs) and their counterpart—the financial condition indexes (FCIs)—which try to capture respectively the stress and buoyancy of financial markets. It is, by the way, remarkable how the Bank of Canada has been at the forefront of the development of financial stress indexes, first with the paper by Illing and Liu (2006) and now with this paper. Given that Canada seems to be one of the few countries that have largely escaped the global financial crisis, it is tempting to attribute this to development and application of FSIs in policymaking. It will be hard to provide evidence whether this is the case, but I think it would be a great example of the Lucas critique working to the benefit of financial stability.

*I would like to thank Rocco Huang and Lev Ratnovski for discussions. Author contact: sclaessens@imf.org.

My comments will start with a review of what FSIs are. It is useful to go through the development of FSIs and their analytical bases since it will help set out the major questions this literature in general is trying to answer. It will also explain how this paper specifically fits in with these questions. I will then review the paper and suggest some improvements in terms of data, offer some suggestions on the methodology, and hint at some possible extensions.

But let me first review the main result. Basically, the authors find that credit and asset prices help predict movements in an FSI specifically developed for Canada. Since others, notably Illing and Liu (2006), have shown that movements in this FSI are related to specific, financially stressful events in Canada, they can thus conclude that credit and asset prices help predict financial stress. They also show that this same relationship holds for other countries, confirming results by others that FSIs help predict financial stress. This in turn suggests that authorities need to consider these indicators in policymaking. Presumably, this would be in a variety of areas: monetary policy; the design and conduct of regulation and supervision; fiscal policy, both tax policy and overall fiscal management; and possibly some other policy areas as well. Here, by the way, the paper does not go into as much detail as I would have liked, but at least we now have a basis for policymaking.

The main results of the paper, I would say, are new and yet reassuring, but also disappointing. They are reassuring in that credit and asset-price variables help improve with predicting FSIs. This confirms the results of others that have shown, in individual country and cross-country work, that credit growth and asset price increases can predict financial stress situations (starting with Borio and Lowe 2002). It also relates to work that has shown that elevated FSIs, in combination with prior credit and asset price growth, have higher economic costs in the aftermath of financial crises (International Monetary Fund 2008). As noted, the paper is also reassuring in that results apply, besides to Canada, to the United States and Japan.

At the same time, the results are somewhat disappointing. For one, adding credit and asset variables still leaves large errors in the ability to predict financial stress. In particular, the predicted FSI does poorly in anticipating the 2007-08 crisis. The work—and this applies not just to this paper but to the broader literature as well—is also less satisfying in that we still know little about the channels for financial stress. In essence, we find out that "a bust is preceded by a bubble in credit and asset prices," but this is something we had known for some time (Borio and Lowe 2002 document this clearly, but Keynes, Kindleberger 1996, and many others had alerted us to it before). At this point, we need more specificity to avoid these large financial crises. Policymakers and market participants need to know better how these events come about, as that will help them develop specific policy and investment implications.

It is for this reason worthwhile to go back to the objectives of this literature. The story pursued in these papers is well known: financial stress can lead to financial crises which have adverse impacts on the real economy. Of course, there are many feedback loops here and one needs to consider endogeneity and simultaneity, so cause and effects are likely mixed up in this literature. Nevertheless, the objective is to construct FSIs that help predict financial distress, or at least explain distress in the sense of finding that an elevated FSI is correlated with financial stress ex post. This was done for Canada first and around the same time by some private-sector participants, and is now a little cottage industry. The objective of the paper is to build on this literature and to try to predict the Canadian FSI using lagged FSI, financial, and other variables, including macroeconomic variables. This way, one can anticipate financial stress periods better.

But this raises the question, why not relate a bunch of variables directly to financial stress or distress? Choosing between the two alternative approaches—predicting the FSI and thereby financial stress or directly trying to predict financial stress—involves some trade-offs. There are some reasons why one would not want to relate a bunch of variables directly to financial distress or the buildup of vulnerabilities. First, there is the timeliness of available data: financial data, especially prices, are much more readily available than data on the real economy. Constructing an FSI thus leads to a much timelier indicator than building a model that predicts distress on the basis of a bunch of real and financial variables. Second, and related, there are often too few episodes of financial distress. Canada, for example, did not experience a major financial crisis in recent decades, not even in the current period. Also, when using the approach of directly trying to predict distress or the buildup of vulnerabilities, one cannot analyze crises that are prevented. Yet it would be useful to know about these prevented crises as well, which financial market indicators may have signaled.

One neither may want to relate explanatory variables directly to economic outcomes. That way, one learns little about the channels through which the crisis came about: was it indeed the financial sector or was it something else? Furthermore, financial stability is an objective in itself: from theory, but also from empirical evidence, we know the importance of a well-functioning financial sector for overall development. On balance, there are thus very good reasons to use the approach pursued here of predicting the FSI using its own and other variables.

This then raises the question of how best to create FSIs/FCIs. Most of the development has been done on the basis of some related theory and a mixture of priors and ad hoc reasoning. But it also has involved tests of the ability to explain or predict financial stress or crises. Here, the variables or weights may be chosen on the basis of the fewest missed stress periods. And some approaches, as noted, have involved the ability to explain or predict real economic outcomes. Note the differences here between *explain* and *predict*; in-sample explaining involves, of course, a lower statistical threshold. The various FSIs are derived using a mixture of these objectives and corresponding methodologies—and are thus difficult to compare, but most fall in the explaining category.

The FSI construction raises many methodological choices: there are numerous variables to choose from (one can use fixed, optimally derived, or time-varying weights for each of the variables included in the index), there are different econometric methods to use, etc. In the particular case here, the choice was based on the ability to classify (not predict) correctly "expert-identified" periods of financial stress in Canada. The variables were chosen on the basis of a combined lowest type I and II errors. The nine variables used are mostly asset prices from Canadian capital markets. The index uses fixed weights, based on the "credit" shares of the financial markets' subsegment that the price variable represents in overall financial intermediation in Canada.

In terms of data, my comment would be that while many indicators have already been included, one could consider some additions or changes. For one, the Canadian system is very bank oriented, yet most indicators used in the FSI are capital market oriented. The logical suggestion would be to focus more on banking and less on capital market conditions in both constructing the FSI and in selecting the explanatory variables that can help predict the FSI. In the FSI, for example, one could add all sorts of bank interest rates—short- and long-term debt, CD/interbank, subdebt, or preferred shares—whichever are more economically and financially important for Canada. In the set of explanatory variables used to predict the FSI, one could add bank data, such as capital ratios, liquidity positions, funding gap, credit growth, interest rate margins, etc. Timelines may be a constraint for some of these variables, yet there are nevertheless some that can be used and that fit closer the heavily banking-dominated system in Canada.

My second comment on the data is to acknowledge that Canada is closely integrated with the global financial market and economy. Canadian corporations finance themselves abroad; at least they were able to do so before the crisis. Canadian markets are also very interlinked and correlated with other markets, especially with the U.S. capital and financial markets. And Canada is dependent on global economic conditions through its trade. The implication would be that financial stress globally, and in the United States especially, is important for Canada's financial sector and real economy, maybe more so than domestic stress is. This in turn means one should expand the list of U.S. variables on the right-hand side of the regressions. For example, one can try to predict the Canadian FSI from a U.S. FSI and then try to explain the residuals, using other Canadian and U.S. data.

Other suggestions related to the data are to use the subindexes of the existing FSI. This breakdown would give a measure of the relative contribution of the banking-system data versus capital market data: this would be worth studying since it would help in getting at the channels by which stress comes about. Also, others creating FSIs have included data on volatility (such as the U.S. S&P 500 volatility index, VIX) and high-yield, TED, and other spread measures. These could be easily added here.

There are also some surprising results that could be explored more. For example, the Japanese and Australian equity indexes are found to be significant in predicting the Canadian FSI. A direct link is not obvious, however, from Australia or Japan to financial stress in Canada. It rather suggests that these equity indexes are proxies for similarity in commodity exports (as in the case of Australia), the general state of global demand (perhaps the Japan index), or something else. It would thus be useful to try to find other indexes that can show these links more directly. There is also the issue of ex post data revisions, which is now ignored. The paper could adjust for this by going back through the announcements of the data at the time or by adjusting for the proper lag structure. And finally, the data stop before the 2007–08 financial crisis. Of course, it is tempting to ask the authors to extend the analysis fully to cover the current financial crisis.

In terms of empirical methodology, the paper tests many models, and it seems robust in its findings. It includes an endogenous threshold, which is a nice innovation. My main concern is that these approaches, as is common to the literature, suffer from data mining. This is especially a problem for these studies since there are few financial crises or stress periods. The small sample size (fewer than fifty observations) in general is a statistical problem—unfortunately, not easily solvable—and the paper acknowledges this openly.

There are also other choices to be made. For criteria, one can choose (just) to minimize the mean-squared errors (MSEs). But one could also weigh type I versus II errors to account for the fact that a missed financial crisis is "worse" than having "overpredicted" financial crises. In this context, sign tests could be useful since negative errors may be less severe than positive errors. Also, one can try to base the forecasting quality on cumulative abnormal errors; that is, when does the model predict the FSI to be out of line for more than just that single period? Lastly, one can try to weigh the accuracy of the forecast by the impact of the financial stress or crisis predicted.

More generally for this literature, not necessarily for this paper, I would suggest some alternative tests. For example, one can use a signal approach instead of regressions (as this paper does). It is somewhat a matter of preferences, but I am struck by how much use is made of the signal approach ("raising a number of red flags") in international financial institutions and presumably elsewhere. This suggests that for policymakers, the signal approach is easier to communicate, which in turn matters for policy impact. Then there are a variety of other econometric techniques that can be used. For example, I think quantile regressions has a lot to offer since it might allow one to better capture the tails that are of most concern.

Lastly, and most importantly, the literature can benefit from more guidance through formal modeling. This is a new field, so robust models are just coming on line, but there are some models and empirical work already that have specific suggestions for what matters in terms of financial conditions. For example, the recent models on financial intermediation using frictions (Adrian and Shin 2008; Brunnermeier and Pedersen 2009) suggest that leverage and liquidity ought to be used in forecasting models. Insights from the asset pricing literature suggest that one ought to use (principal components from) interest rate yield-curve data, since those help predict macroeconomic and financial events (for example, Ang, Piazzesi, and Wei 2006). Perhaps futures for some asset, commodity, and other prices ought to be used as well, as they can have predictive power. These more formal approaches may help design better machinery and models that help to identify not just stress, but also risks.

References

Adrian, T., and H. S. Shin. 2008. "Liquidity and Financial Cycles." BIS Working Paper No. 256 (July).

Ang, A., M. Piazzesi, and M. Wei. 2006. "What Does the Yield Curve Tell Us about GDP Growth?" *Journal of Econometrics* 131 (1–2): 359–403.

Borio, C., and P. Lowe. 2002. "Asset Prices, Financial and Monetary Stability: Exploring the Nexus." BIS Working Paper No. 114 (July).

Brunnermeier, M. K., and L. H. Pedersen. 2009. "Market Liquidity and Funding Liquidity." *Review of Financial Studies* 22 (6): 2201–38.

Illing, M., and Y. Liu. 2006. "Measuring Financial Stress in a Developed Country: An Application to Canada." *Journal of Financial Stability* 2 (3): 243–65.

International Monetary Fund. 2008. "Financial Stress and Economic Downturns." In *World Economic Outlook*, 129–58 (chapter 4). Washington, DC: International Monetary Fund.

Kindleberger, C. P. 1996. *Manias, Panics and Crashes.* 3rd ed. Cambridge: Cambridge University Press.

Prices and Quantities in the Monetary Policy Transmission Mechanism*

Tobias Adrian[a] and Hyun Song Shin[b]
[a]Federal Reserve Bank of New York
[b]Princeton University

Central banks have a variety of tools for implementing monetary policy, but the tool that has received the most attention in the literature has been the overnight interest rate. The financial crisis that erupted in the summer of 2007 has refocused attention on other channels of monetary policy, notably the transmission of policy through the supply of credit and overall conditions in the capital markets. In 2008, the Federal Reserve put into place various lender-of-last-resort programs under section 13(3) of the Federal Reserve Act in order to cushion the strains on financial intermediaries' balance sheets and thereby target the unusually wide spreads in a variety of credit markets. While classic monetary policy targets a price (for example, the federal funds rate), the liquidity facilities affect balance-sheet quantities. The financial crisis forcefully demonstrated that the collapse of the financial sector's balance-sheet capacity can have powerful adverse effects on the real economy. We reexamine the distinctions between prices and quantities in monetary policy transmission.

JEL Codes: E44, E52, E58, G18, G28.

1. Introduction

Central banks have a variety of tools for implementing monetary policy, but the tool that has received most attention in the mainstream

*Adrian: Federal Reserve Bank of New York (e-mail: tobias.adrian@ny.frb.org). Shin: Princeton University (e-mail: hsshin@princeton.edu). The authors thank Douglas Gale and Lasse Pedersen for helpful comments. This paper was prepared for the inaugural International Journal of Central Banking (IJCB) conference "Provision and Pricing of Liquidity Insurance," held at the Federal Reserve Bank of New York on June 11, 2009. The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

literature has been the overnight interest rate. In the case of the United States, the Federal Reserve targets the "federal funds rate," an overnight interbank interest rate among U.S. banks with reserves at the Federal Reserve. However, mainstream models of monetary economics have virtually no direct role for the overnight interest rate to affect the economy. The conventional view is summarized forcefully by Alan Blinder (1998), who states:

> Central banks generally control only the overnight interest rate, an interest rate that is relevant to virtually no economically interesting transactions. Monetary policy has important macroeconomic effects only to the extent that it moves financial market prices that really matter—like long-term interest rates, stock market values, and exchange rates.

Instead, the mainstream approach in monetary economics has been to emphasize the importance of managing market expectations. By charting a path for future short rates and communicating this path clearly to the market, the central bank can influence long rates and thereby influence mortgage rates, corporate lending rates, and other prices that affect consumption and investment. This "expectations channel" of monetary policy had become the dominant theme in many central banks, especially among those that practice inflation targeting.[1]

However, the financial crisis that erupted in the summer of 2007 has refocused attention on other channels of monetary policy, notably the channel that works through the supply of credit and overall capital market conditions. Borio and Zhou (2008) have coined the term "risk-taking channel" of monetary policy to describe the broad set of effects that work through expansive behavior of financial intermediaries and the feedback effects that result from the increased supply of credit.

One taxonomy that sheds light on the debate is to make a distinction along two dimensions:

(i) prices versus quantities

(ii) demand for credit versus the supply of credit

[1] The expectations channel is explained in more detail in Blinder (1998), Bernanke (2004), Svensson (2004), and Woodford (2005).

The crisis management efforts since the eruption of the financial crisis illustrate well the first distinction between prices versus quantities. Central bank liquidity facilities that attempt to counter the shrinking of intermediary balance sheets have become a key plank of policy, especially after short-term interest rates were pushed close to their zero lower bound.

The Federal Reserve has put into place various lender-of-last-resort programs under section 13(3) of the Federal Reserve Act in order to cushion the strains on balance sheets and thereby target the unusually wide spreads in a variety of credit markets. Liquidity facilities have been aimed at the repo market (TSLF and PDCF), the commercial paper (CP) market (CPFF and AMLF), the foreign exchange (FX), and asset-backed securities (ABS) markets (TALF). In addition, the Federal Reserve made outright purchases of Treasury and agency securities, and liquidity backstops to money markets were increased via the Term Auction Facility (TAF) and foreign exchange Swap lines (FX Swaps). The common element in these liquidity facilities is to alleviate the strains associated with the shrinking balance sheets of intermediaries. The narrowing of spreads is a by-product of such actions. While classic monetary policy targets a price (e.g., the federal funds rate), the liquidity facilities affect balance-sheet quantities.

The balance-sheet expansion of the Federal Reserve as a result of the 13(3) liquidity facilities has refocused the monetary policy debate on the role of quantities in the monetary policy transmission mechanism. The financial crisis forcefully demonstrated that the collapse of balance-sheet capacity of the financial sector can have powerful adverse effects on the real economy.

It may be argued that the crisis management efforts of a central bank are driven by special considerations that are not operative under "normal" conditions. Against this, the counterargument is that the crisis did not erupt out of the blue, but instead culminated from a long accumulation of vulnerabilities that were left unchecked. The relevant question, then, is whether a focus on the risk-taking channel of monetary policy may have led to a better outcome.

In this respect, a reexamination of the conventional tenets of monetary policy is in order. The distinctions between prices and quantities and between the supply of credit and the demand for credit take on added weight. In order to understand the distinctions better, it is useful to review the effect of monetary policy on the demand for and supply of credit.

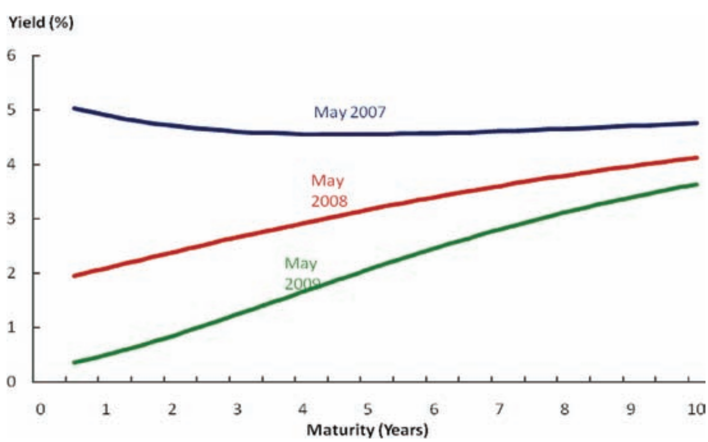
2. Demand for Credit

Credit is demanded by nonfinancial corporations, households, and the government sector (both central government and local government). Nonfinancial firms need to finance new investment in the physical capital stock, while households primarily finance the acquisition of housing and consumption. For consumption purposes, durable goods consumption such as auto purchases and consumer loans tends to be particularly important for the credit markets. The federal, state, and local government debt is also an important determinant of the demand for credit. Most of the public debt is financed via Treasury or municipal bond issuance.

For the corporate and household sector, the demand for credit is influenced by the level of interest rates, as well as credit spreads. In addition, households' and firms' net worth determines the extent of their borrowing capacity.

The standard visualization of the cost of demand for credit is the yield curve; it plots the yields at different maturities. In figure 1,

Figure 1. The Term Structure of Interest Rates



Source: Board of Governors of the Federal Reserve System.

we plot yield curves for three dates: May 2007, May 2008, and May 2009. We can see that the yield curve was more or less flat in the spring of 2007, before the onset of the crisis. By May 2008, immediately following the Bear Stearns crisis, the federal funds target was cut from 5.25 percent to 2 percent, leading to a sharp steepening of the yield curve. By May 2009, the federal funds target was set to a 0–0.25 percent corridor, giving rise to a further steepening of the curve. The federal funds target cuts were designed to lower the cost of borrowing, thus stimulating the demand for credit, and hence overall economic activity.

If the demand for credit is all that matters, then the position and slope of the yield curve (as well as the net worth of the borrower) are paramount. The expectations channel of monetary policy and the importance of the management of expectations are then well motivated.

3. Supply of Credit

Credit is supplied by financial intermediaries such as traditional commercial banks, as well as institutions of the "shadow banking system" such as security broker-dealers and ABS issuers. If credit supply is the main determinant of the quantity of credit in the economy, the key is to understand the motivation of financial intermediaries and how capital market conditions determine their behavior. For monetary policy, the question is how the central bank affects capital market conditions that ultimately affect the supply of credit.

The starting point in understanding credit supply is the delegation of capital allocation decisions to financial intermediaries. Savers—including households and nonfinancial corporations—delegate capital allocation decisions to intermediaries. This delegation raises agency problems, which are (at least potentially) solved by constraints on leverage, risk management, and credit ratings, as well as targets for measures such as ROE or ROA targets. One particularly simple way to summarize such constraints is to look at haircuts—the amount of overcollateralization that is required for borrowing against risky asset collateral.

Monetary policy and lender-of-last-resort policies affect overall capital market conditions through the balance sheets of financial intermediaries. The variation of the federal funds target primarily moves around the slope of the yield curve, making the lend-long/borrow-short carry more or less profitable. Central bank liquidity facilities work through the equilibrium trade-off between credit spreads and haircuts. An increase of central bank lending against a particular asset class will tend to lower haircuts and spreads. As the financial crisis can be viewed as a shortage of financial intermediary balance-sheet capacity, lender-of-last-resort operations tend to offset the decline of that capacity. The Federal Reserve's balance-sheet expansion can thus be viewed as an emergency replacement of lost private-sector balance-sheet capacity by the public sector.

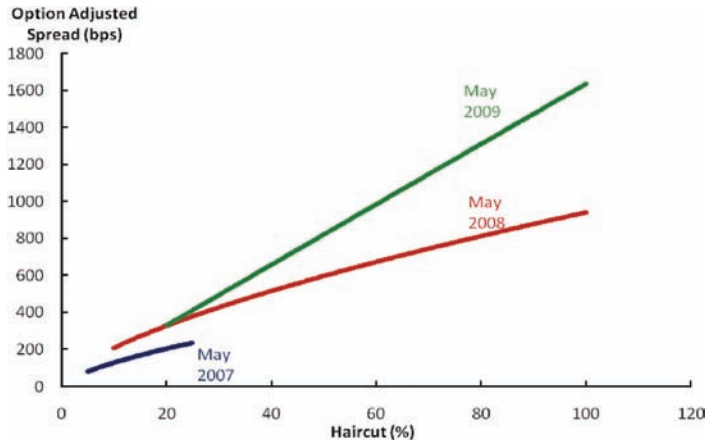
4. Refocusing Monetary Policy

Prior to 1980, the monetary policy literature primarily focused on the role of monetary aggregates for the supply of credit. However, with the emergence of the market-based financial system, the ratio of high-powered money to total credit (the money multiplier) became highly unstable. As a consequence, monetary aggregates faded from both the policy debate and the monetary policy literature.

However, there is a sense in which the focus on balance-sheet quantities is appropriate. The mechanism that has amplified fluctuations in capital market conditions is the fluctuations in leverage and the associated changes in haircuts in collateralized credit markets. As the uncertainty of the future of mortgage-backed securities (MBS) increased in 2007, haircuts on MBS and ABS increased, forcing institutions to either unwind or move assets from off-balance-sheet vehicles onto bank balance sheets. This shifted funding of long-term assets from collateralized asset-backed commercial paper (ABCP) and repo markets into the uncollateralized money markets, with the effect of massively increasing money market spreads, such as the LIBOR-Treasury spread.

Financial intermediaries tend to hold long-term assets, financed by short-term collateralized liabilities. In order to obtain funds, intermediaries lend out assets they already own and receive cash, which in turn can be invested in additional assets. The constraint on how much of such collateralized lending can be done is imposed by the level of haircuts. Haircuts can be thought of as the percentage downpayment an intermediary has to make in order to finance an asset. When a haircut is 20 percent, the intermediary can take out

Figure 2. The Haircut Curve



Note: Credit spreads are option adjusted. Haircuts are for DTC intraday borrowing.
Source: DTC, Bloomberg.

a maximum leverage of $1/20\% = 5$. When haircuts increase from 20 percent to 50 percent, the intermediary has to unwind to $1/50\% = 2$ times leverage.

In this way, haircuts determine the amount of leverage that investors can obtain in repo markets. The haircut is the overcollateralization of a specific type of collateralized borrowing agreement such as the repurchase agreement (or repo) or other forms of collateralized borrowing such as agreements of the DTC.

One way to visualize the effect is through shifts in the haircut curve, plotted in figure 2. The curve plots option-adjusted credit spreads for corporate and ABS of different ratings against the percent haircut. The credit spread is a proxy for the expected return for being long the particular security and short the Treasury security of matching duration.

Recall that a 5 percent haircut means that an investor has to overcollateralize a particular repo transaction by 5 percent, thus allowing a leverage of $1/5\% = 20/1$. If haircuts for a particular security increase—say from 5 percent to 10 percent—the investor's maximum leverage is reduced from $20/1$ to $1/10\% = 10/1$.

The haircut curves of figure 2 represent equilibrium relationships between credit spreads and haircuts. Before the financial crisis, in May 2007, the maximum haircut for the most illiquid security of the

Table 1. Spreads and Haircuts for Fixed-Income Securities

		Haircuts			Spreads		
		May 07	**May 08**	**May 09**	**May 07**	**May 08**	**May 09**
Corporate Debt	A	5	10	20	86	235	364
Corporate Debt	Baa	5	20	30	115	278	508
ABS	Aaa	10	25	35	73	327	350
Corporate Debt	Ba	25	30	40	177	433	833
Corporate Debt	B	25	40	50	239	618	996
Corporate Debt	Caa	25	100	100	396	932	1,573

Note: Credit spreads are option adjusted. Haircuts are for DTC intraday borrowing.
Source: DTC, Bloomberg.

sample—Caa-rated corporate debt securities—was 25 percent (see table 1). In May 2008, this haircut had increased to 100 percent, implying that Caa corporate had become totally illiquid: these securities could no longer be used as collateral in repo transactions. At the same time, spreads of the Caa corporate had increased from 396 basis points to 932 basis points. Part of this increase in the spread likely reflected the deterioration of the underlying credit quality. However, part of the increase in spread simply reflected the inability of Caa securities to be used by levered financial institutions. For securities with a 100 percent haircut, the acquisition of the security has to be fully funded, as it has become impossible to borrow against the security. Caa corporate continued to have a 100 percent haircut in May 2009, while the credit spread increased to 1,573 basis points. Higher-rated securities, such as the Aaa-rated ABS that is used in figure 2 and in table 1, tell a similar story. Haircuts increased from 10 percent to 25 percent between May 2007 and May 2008, and then to 35 percent in May 2009. Over that time horizon, spreads increased from 73 to 327, and then to 350. Such a relationship between haircuts and credit spreads is a prediction of a general equilibrium asset pricing theory presented by Gârleanu and Pedersen (2009).

The haircut curve has three important dimensions: level, slope, and length. As the crisis unwound, the curve shifted up (i.e., credit spreads increased for any given haircut), the curve became steeper (i.e., each additional unit of haircut demanded a higher compensation in terms of credit spread), and the curve became longer, shifting

Figure 3. Repos and Financial CP as a Fraction of M2 (weekly)

Source: Board of Governors of the Federal Reserve System and Federal Reserve Bank of New York.

to the right (the haircuts on the most liquid and least liquid securities both increased). Such shifts in level, slope, and length can be compared with the traditional level, slope, and curvature shifts of the Treasury yield curve (see figure 1). The major advantage of the plotting of the haircut curve is that it clearly shows the impact of the crisis: haircut increases are both causes and consequences of a financial crisis. In contrast, the term structure of interest rates does not unambiguously show a crisis. For example, the three yield curves of figure 1 happen to coincide with the financial crisis, but such shapes are not necessarily related to the crisis.

One consequence of the fluctuation in haircuts is the associated fluctuations in balance-sheet quantities. Figure 3 plots the sum of financial-sector commercial paper (CP) and primary dealer repos as a fraction of M2. Financial CP is primarily used to finance the activities of the "market-based banking system" (sometimes called the "shadow banking system"). The market-based banking system comprises institutions that act like banks but that do not possess the "carrots and sticks" of banks. In particular, they are institutions that hold securitized credit on the asset side of their balance sheets (for example, pools of credit card receivables, pools of car loans, or pools of mortgages) and that finance those assets by issuing commercial paper. However, such institutions are not banks; i.e., they do not usually have access to the discount window or FDIC insurance (the "carrots") and are often subject to less regulation than traditional commercial banks (the "stick"). Such market-based banking institutions could be structured investment vehicles (SIVs), or finance companies. The other important set of market-based financial intermediaries are security broker-dealers. Broker-dealers often use repo transactions to finance their balance sheet.

Figure 3 shows that the short-term liabilities of the market-based banking system had reached a similar order of magnitude as the main short-term liabilities of the traditional banking system in August 2007 (money). The period of August 8, 2007–September 10, 2008 saw a slow decline in the share of market-based short-term liabilities relative to M2. This decline rapidly accelerated after the bankruptcy of Lehman Brothers in September 2008. By April 2009, the share of the market-based system liabilities had returned to a level below 50 percent, comparable to the mid-1990s.

5. Central Bank Policy Rules

Baseline macroeconomic models usually comprise three key state variables: the short-term interest rate (the federal funds target), real activity (GDP growth), and inflation (core CPI inflation). In addition, such models implicitly define a pricing kernel, allowing the calculation of the whole term structure of interest rates from the dynamics of the state variables. However, as we have argued above, such an approach is missing a key state variable: the leverage of the financial sector. Central bank policy rules can thus be expressed as functions of the three state variables:

$$\text{Federal funds target} = F(\text{GDP, inflation, leverage})$$
$$\text{Quantitative policy} = Q(\text{GDP, inflation, leverage})$$

The second rule superficially resembles monetary aggregate targeting. However, it is more directly related to broader aggregates, such as repos and commercial paper, which finance the shadow banking system. The liquidity facilities mentioned in the introduction (TALF, CPFF, etc.) can be directly viewed as such quantity policies.

Empirical asset pricing evidence strongly points toward the importance of the leverage factor in determining the cost of capital. Adrian and Shin (2007), Adrian, Etula, and Shin (2009), and Adrian, Moench, and Shin (2009) demonstrate for a wide variety of asset classes that leverage proxies can forecast excess returns, controlling for macro factors. These findings are significant, as they link balance-sheet quantities to asset prices, and hence provide the link that makes liquidity provision policies affect real economic activity. In addition, these results have the potential to solve long-standing asset pricing puzzles described in the macroeconomic literature.

6. Monetary Policy Lessons

Financial intermediaries lie at the heart of both monetary policy transmission and liquidity policies. The interaction of financial intermediaries' balance-sheet management with changes in asset prices and measured risks represents an important component in the transmission mechanism of monetary policy.

Financial intermediary balance-sheet management matters for the real economy, as well as for the soundness of the financial system. The lesson for the conduct of monetary policy is that the interaction of leverage constraints of financial intermediaries, short-term interest rates, and financial asset quantities are important to consider in conjunction. Our discussion suggests that tracking measures of financial market liquidity derived from the balance sheets of intermediaries has some information value in the conduct of monetary policy.

References

Adrian, T., E. Etula, and H. S. Shin. 2009. "Risk Appetite and Exchange Rates." Federal Reserve Bank of New York Staff Report No. 361. Available at http://newyorkfed.org/research/staff_reports/sr361.html.

Adrian, T., E. Moench, and H. S. Shin. 2009. "Financial Intermediation, Asset Prices and Macroeconomic Dynamics." Working Paper, Federal Reserve Bank of New York.

Adrian, T., and H. S. Shin. 2007. "Liquidity and Leverage." Forthcoming in *Journal of Financial Intermediation*.

Bernanke, B. 2004. "The Logic of Monetary Policy." Remarks before the National Economists Club, Washington, DC, December 2, 2004. Available at http://www.federalreserve.gov/boarddocs/speeches/2004/20041202/default.htm.

Blinder, A. S. 1998. *Central Banking in Theory and Practice*. Cambridge, MA: MIT Press.

Borio, C., and H. Zhou. 2008. "Capital Regulation, Risk-Taking and Monetary Policy: A Missing Link in the Transmission Mechanism?" BIS Working Paper No. 268. Available at http://www.bis.org/pub1/work268.htm.

Gârleanu, N., and L. H. Pedersen. 2009. "Margin-Based Asset Pricing and Deviations from the Law of One Price." Unpublished Working Paper, New York University and University of California, Berkeley.

Svensson, L. 2004. "Challenges for Monetary Policy." Comments presented at the meeting of the Bellagio Group of the G-10, National Bank of Belgium, Brussels, January 26–27. Available at http://www.princeton.edu/∼svensson.

Woodford, M. 2005. "Central-Bank Communication and Policy Effectiveness." Paper presented at the Federal Reserve Bank of Kansas City's Symposium on The Greenspan Era: Lessons for the Future, Jackson Hole, Wyoming, August 25–27. Available at http://www.kc.frb.org/publicat/sympos/2005/sym05prg.htm.

Strategic Trading in Multiple Assets and the Effects on Market Volatility*

Chenghuan Sean Chu, Andreas Lehnert, and Wayne Passmore
Federal Reserve Board

We study government policies designed to increase liquidity by extending government guarantees to fundamentally illiquid assets. We characterize the effects of such policies on equilibrium price dynamics, trading strategies, and welfare. We build on the strategic trading framework of Brunnermeier and Pedersen (2005) and Carlin, Lobo, and Viswanathan (2007) by adding multiple assets and by modeling all agents' utility functions. The assets in our model differ in their fundamental liquidity, i.e., the price reaction of the nonstrategic (or "retail") traders to amounts sold by the strategic traders. Nonstrategic traders are willing to accept greater amounts of the more liquid asset with less short-term price reaction. These additions allow us to consider the welfare implications of, for example, shifting some assets from the illiquid category to the liquid category, a proxy for government guarantees on a risky asset. As in other models of this type, the strategic players "predate" on each other when one becomes distressed and is forced to liquidate its holdings. As others have shown, the more liquid the asset, the cheaper it is to predate on a distressed firm. Our model features an additional channel between liquidity and predation: because of the cross-elasiticies of demand across assets, firms can create liquidity in one asset by shorting a complementary asset. We find that when firms begin with larger endowments in highly liquid assets, forced liquidation of those assets tends to result in higher price volatility than would otherwise be the case. For plausible parameter ranges, such a policy also results in lower welfare for the nonstrategic traders. This finding suggests that market interventions designed to alleviate

*The views expressed are those of the authors and do not necessarily reflect the official positions of the Federal Reserve System. Correspondence: sean.chu@frb.gov; andreas.lehnert@frb.gov; wayne.passmore@frb.gov.

illiquidity in particular asset markets may instead unintentionally exacerbate price volatility.

JEL Codes: G0, G01, G12.

1. Introduction

Policymakers have considered several different policies to stabilize financial markets and institutions since the onset of the financial crisis in August 2007. A feature of the crisis has been a sharply decreased willingness of investors to accept illiquid assets, citing mark-to-market risk, difficulty in valuation, or simple preference for assets that can quickly be converted to cash. One policy response has been to harden the implicit government guarantee on certain assets important for basic intermediation, such as obligations of Fannie Mae and Freddie Mac.

The financial crisis has also been marked by forced sales of assets by distressed firms. Other firms, however, have specialized in strategic trading around such fire sales. Prices in related markets, such as private-label mortgage-backed securities backed by prime mortgages, have fallen when firms liquidated a different asset, such as MBS backed by subprime mortgages. Presumably, strategic traders have also been taking advantage of these correlated price movements.

We study equilibrium price dynamics in such an environment and conduct a welfare analysis of extending government guarantees to less liquid securities. Our results also allow us to consider the optimal exit strategy for a government that accumulates assets on its balance sheet, either directly through purchases or indirectly by guaranteeing assets.

Specifically, we investigate the role of predatory trading and substitution effects across assets when a subset of the firms are forced to liquidate a portion of their portfolios. Following Brunnermeier and Pedersen (2005) and Carlin, Lobo, and Viswanathan (2007), we define "predation" in this context as trading that exploits "distressed" investors' need to alter their positions. Substitution effects arise whenever there is an incentive to actively trade assets other than ones that actually must be liquidated, and we propose one mechanism by which these incentives may arise.

An example of a scenario in which both predation and substitution across assets have been important determinants of trading behavior is the financial market turmoil of late 2008, during which banks attempted to liquidate holdings of mortgage-backed securities (MBS), resecuritizations of MBS, and other related assets, many of which lacked an informative market price due to the general cessation of trading. Illiquidity in markets for assets not directly related to mortgages provides evidence of substitution.

Our study relates to two main strands of theoretical research on market microstructure, as well as the current policy debate about the effect of government interventions designed to make certain assets more liquid. First, we extend the literature on the role of strategic trading when a subset of firms become distressed and are forced to liquidate their assets (Brunnermeier and Pedersen 2005; Carlin, Lobo, and Viswanathan 2007). Brunnermeier and Pedersen introduce the notion that profit-maximizing behavior by the nondistressed firms can cause the price of the asset being liquidated to overshoot its "fundamental" value (i.e., its long-term value after netting out the market impact of trading). Carlin, Lobo, and Viswanathan extend the model by setting it in the context of a repeated game and by allowing for trading to have market impact, such that prices depend upon not only the stock but also the flow of assets being traded in each instant. A key feature of both models is that nondistressed traders initially race to sell the asset at the same time that the distressed traders are trying to liquidate, and then buy back the asset at a discount toward the end of the trading window. The dynamics implied by this "race-and-fade" behavior leads to higher price volatility than what we would observe if the nondistressed firms were passive. As well, predation generates inefficiencies in the stage game and implies the existence of Pareto improvements that can be sustained through supergame strategies. Our key contribution to the existing literature comes from the introduction of multiple goods. While the race-and-fade phenomenon is preserved under heterogeneous goods, heterogeneity also leads to novel properties that are absent when there is just a single, homogeneous asset.

To understand the implications of having multiple assets, first note that when there is just a single asset, profit opportunities come from two sources. To begin with, by selling high and buying low (for expositional convenience, we refer to this behavior as the "predatory" component of the nondistressed firm's optimal response), nondistressed firms can exploit the declining fundamental value of the asset driven by the forced liquidation. Here, we define the fundamental value of the asset, at a moment, to be its price after netting out the liquidity premium or discount caused by the trading in that instant. Second, by causing the speed of trading to vary over time, predation also generates a competing incentive for nondistressed firms to make profits by supplying liquidity to distressed firms—i.e., purchasing the asset when the distressed firms are selling it at a faster rate, and selling when the distressed firms are selling at a slower rate. The first effect is a pure transfer across firms and does not affect the total surplus summed over all of the strategic players. However, because traders do not internalize the effect of their own trading on the liquidity costs of others, the latter effect implies that equilibrium outcomes are socially inefficient relative to the cooperative outcome, which would entail setting a net trading speed that is constant over time.

When there are multiple assets, linkages in the fundamental values of different assets imply that forced liquidation in any one of the assets will also result in declines in the fundamental values of the remaining assets. Therefore, there are opportunities for nondistressed firms to obtain predatory profits through trading in any of the assets. Similar to the one-asset case, there is also a competing incentive to supply liquidity. However, because of cross-asset linkages in the short-term market impact of trading, the act of purchasing (selling) any of the assets to a greater or lesser degree also enhances liquidity for sellers (buyers) of all the other assets. In the one-asset case, the competing incentives to predate and to provide liquidity must necessarily involve the same asset, and the latter incentive tends to dampen the volatility-inducing effects of predation. By contrast, when there are multiple assets, nondistressed firms can predate in some assets (usually the ones in which distressed firms hold a large initial endowment) while generating liquidity cross-subsidies through trading in the other assets, which lowers the overall liquidity cost of predation.

An important implication is that overall market volatility will actually tend to be higher when the distressed firms hold a large initial endowment in relatively liquid assets. Because the short-term, own-price market impact of trading is—by definition—high for the illiquid assets, firms are less able to use liquidity cross-subsidies to buffer price volatility induced by predation in the liquid good. The opposite is true when distressed firms must dispose of large quantities of the relatively illiquid assets. Even though nondistressed firms also engage in predation in this latter case, the cost of providing the liquidity cross-subsidy—in terms of the short-term impact of trading in the liquid assets on their own price—is relatively low.

In other words, cross-elasticities in the fundamental values create an incentive to predate in assets other than the chief ones being liquidated, while cross-elasticities in the short-term market impact of trading provide opportunities to reduce the liquidity cost of predation. When the bulk of the assets being liquidated are relatively liquid, the overall impact of the cross-subsidy is relatively weak, implying that predation results in high price volatility.

Our work is also related to the incomplete-information approach to modeling strategic trading, in which market participants have private information about the fundamental value of an asset. Both Brunnermeier and Pedersen as well as Carlin, Lobo, and Viswanathan treat as exogenous the parameters determining the market-clearing price. By contrast, in a second group of papers (Kyle 1985; Holden and Subrahmanyam 1992; and Back, Cao, and Willard 2000), the market impact of trading arises endogenously because firms have private information about the fundamental value of the asset. Similar to the first group of authors, we do not attach a structural interpretation to the parameters of the demand system. However, we can think of them as being plausible reduced-form effects implied by a multi-asset extension of the incomplete-information literature. Namely, if there is correlation across either the asset values themselves or the agents' private signals about those values, then trading activity in any one asset is informative about the other assets. In equilibrium, such correlations would give rise to the cross-elasticities in our model.

Finally, our paper relates to the ongoing debate over the effect of policies intended to increase the liquidity of particular assets. For example, our analysis suggests that the extension of government guarantees to particular classes of illiquid assets—which reduces the price impact of trading by effectively decreasing the degree of asymmetric information about the assets' payoffs—may actually increase price volatility by making it "cheaper" (in terms of trading costs) to predate.

In effect, the cross-elasticities among assets work against orderly trading if firms' initial endowments contain a high proportion of liquid assets. The analysis thus suggests that government interventions may be more effective at stemming market disruptions if they focus on relieving the distress of financially troubled firms, as opposed to extending government guarantees to illiquid assets.

The remainder of this paper proceeds as follows. The remainder of this section discusses a motivating example. In the next section, we set up a model of strategic interaction among N firms trading in M assets. In section 3, we characterize the equilibrium outcome as well as the cooperative solution that maximizes total surplus for the strategic players. Note that we do not explicitly model the preferences or behavior of the retail investors. As an alternative, in section 4 we use a nonstructural approach to capture the effect of strategic trading on the welfare of the retail investors. Using a hazard specification, we investigate the mapping from model primitives to the probability of financial distress among retail traders. Section 5 discusses potential policy implications, and section 6 concludes.

1.1 An Example

To fix ideas, consider the following example: two firms, A and B, own and trade in two assets: private-label residential mortgage-backed securities (RMBS) and Treasury bonds. Firms A and B act strategically, trading in the short run in response to perceived deviations from fundamental values. A large number of nonstrategic or retail investors follow a "buy-and-hold" strategy in RMBS and bonds as well. For concreteness, assume that A and B each initially holds $200 billion in RMBS and zero bonds, and that the retail investors hold the remaining amounts of each asset, where outstanding quantities total $1,000 and $5,000 billion, respectively.

Firm A encounters unmodeled financial difficulties and must liquidate its entire portfolio of RMBS to satisfy its creditors within a relatively short amount of time—say, a week. Firm B knows this. As a convenient normalization, we will require that firm A and B hold the same amount of RMBS and bonds at the end of the week as they did at the beginning (before A became distressed), with the exception that A's holdings of RMBS run down to zero. Thus at the end of the period, A will hold $0 of RMBS and $0 in bonds, B will hold \$200 billion in RMBS and \$0 in bonds, and the retail investors will hold \$800 billion in RMBS and \$5,000 billion in bonds. During the week of trading, the strategic traders may go long or short in bonds. (The trajectories of prices and quantities for this example are shown in figure 5.)

The retail investors thus, on net, increase their holdings of RMBS and will have to be compensated by higher yields (lower prices) on RMBS. They may also have to be compensated by higher yields on bonds, depending on the degree to which RMBS and bonds are seen as similar assets by retail investors. These changes in yields are the long-run or fundamental change. However, as trading proceeds within the week, the strategic firms A and B may quickly buy and sell large volumes of each asset in response to actions by the other. The retail investors will require extra compensation the faster A or B trade because they have only limited capacity to evaluate each asset as it is sold to them. They require less extra compensation for the same volume of bond sales because bonds are largely homogeneous. They require extra compensation for RMBS sales because each security differs.

Importantly, they will demand more compensation on bonds even if sales of bonds do not increase but sales of RMBS do. This cross-elasticity can be motivated in a variety of ways: retail investors have only limited capacity to evaluate any kind of asset within a short time frame or they are reluctant to rebalance their portfolios on short notice. So-called "buy-and-hold investors" or "real money accounts" might view their holdings of financial assets as one part of a broader portfolio that might contain real assets and other risk exposures correlated with financial asset returns, limiting their willingness and ability to quickly switch positions. Strategic traders, by contrast, are driven primarily by the returns available on the financial assets in the model.

Thus, there will be spillover effects from the standard strategic "racing" and "fading" effects. With multiple assets, there are further effects at work. Distressed firms face an incentive to "create liquidity" for themselves by trading in opposite directions in each asset. In turn, nondistressed firms will use these cross-asset price effects to further predate on the distressed firm. For relatively small cross-elasticities, this liquidity creation motive is relatively muted and the optimal trading strategies are much the same as those in Brunnermeier and Pedersen (2005) or Carlin, Lobo, and Viswanathan (2007). For extremely large cross-elasticities—for example, cross-elasticities greater than unity—the optimal trading strategies look very different as firms predate on one another by trading in the nondistressed asset. We do not focus on such cases in this paper, in part because they are often inconsistent with standard conditions required for uniqueness. However, one could imagine assets whose prices are correlated in this manner; for example, changes in the value of a junior tranche of a structured asset would be larger than changes to the senior tranche of the same asset. Thus, one could imagine modeling the equity tranches of asset-backed securities (ABS) as the illiquid asset and the investment-grade tranches as the liquid asset. In such a case, large cross-elasticities would be perfectly plausible.

The main predatory effect is that firm B sells RMBS at the same time that firm A is forced to. Firm B then buys the RMBS back at distressed prices, booking a profit.

Note that neither firm starts with bonds or faces any requirement to trade in them. However, the fundamental value of bonds will decrease because the retail investors will demand greater compensation to hold a permanently higher amount of RMBS. This is due to the long-run cross-elasticity between the two assets, which can be different from the short-run cross-asset effects. Thus, there is a standard motive for bond trading.

In addition, firm A can provide itself some liquidity by buying bonds from the retail investors while selling them RMBS. In effect, firm A mitigates the price decrease in RMBS by temporarily buying bonds from the retail investors. Later, when trading volumes subside, firm A unwinds its position in bonds. However, knowing that firm A will engage in such behavior, firm B predates against firm A.

We model retail traders' utility as a function of their probability of financial distress; this, in turn, is driven by the fluctuations in the prices of the assets they hold. Thus sudden asset price drops, even if followed by recoveries, will harm retail investors. For example, retail investors could be thought of as mutual fund managers who are fired if a fund's net asset value falls too much.

The government could attempt to protect retail investors and generally lower volatility by providing guarantees on the illiquid asset. In our example, this would correspond to transforming some portion of RMBS outstanding into government bonds. For example, the government might guarantee all RMBS issued by a particular firm. Imagine that the government guarantees 10 percent of outstanding RMBS. Now, firm A and firm B each begins the period holding \$180 billion in RMBS and \$20 billion in bonds. The retail investors hold \$540 billion in RMBS and \$5,060 billion in bonds at the start of the period.

Effectively, then, such guarantees change the endowments of all parties prior to the start of trading. We study the effects on asset price volatility and retail traders' financial distress of such a change in endowments. We find that increasing holdings of the liquid asset increases price volatility. For plausible parameter values, this increases the probability of financial distress among retail investors, decreasing their welfare. Volatility increases because of the standard effect—predation is cheaper when assets are more liquid—but also because of the spillover effects. Increased predation in one asset leads to increased trading in the other asset. (We show the probability of financial distress among retail investors in figures 7 and 8.)

2. Model

We consider an economy in which there are M assets traded in continuous time and N strategic players. In addition, we assume that there is a continuum of nonstrategic "retail" investors who comprise the residual demand for the assets. The incomplete-information approach to modeling strategic trading actually derives the residual demand based on the information available in the market about the fundamental value of the assets. However, for simplicity we treat the residual demand function as exogenously given. Each strategic firm maximizes returns given the actions of its competitors. On the other hand, the retail investors are "unaware" of the temporary liquidity needs in the market. Instead, the residual demand is determined mechanistically as a function of the total stocks of assets held by the strategic traders as well as the current volume of trading activity.

We let the vector $x_i(t) \in R^m$ denote player i's position in each of the M assets at time t, and correspondingly let the stacked vector $x(t) \equiv [x_1(t) \dots x_n(t)]$ denote all of the players' positions.

Each strategic firm i begins with initial endowments $x_i(0) = x_{0i}$. We assume that for exogenous reasons, the traders must reach positions $\{x_{1i}\}_{i=1...n}$ by the end of the trading period, normalized to $t = 1$. For example, setting x_{1i} to zero implies that firm i must liquidate all of its assets by the end of trading.

The trading speeds at time t are given by the vector $y(t) \equiv dx(t)/dt$. Demand for assets is imperfectly elastic and is described by the following system of inverse demand equations:

$$p(t) = u + \Gamma X(t) + \Lambda Y(t), \tag{1}$$

where $X(t) \equiv \sum_{i=1}^{n} x_i(t)$ captures the strategic traders' total positions and $Y(t) \equiv \sum_{i=1}^{n} y_i(t)$ captures the net instantaneous speed of trading. The elasticity matrices, Λ and Γ, are positive-definite by assumption. Here, we treat the demand functions as primitives. However, as discussed in the introduction, a similar relationship arises endogenously under the assumption that traders have differing degrees of private information and there is a risk-neutral market maker who sets prices in order to make zero profits (e.g., Kyle 1985). Alternatively, without explicitly specifying the information structure or the price-setting process of our model, we can also interpret the dependence of prices on $X(t)$ through the matrix of elasticities Γ as reflecting heterogeneity in nonstrategic traders' valuation of the assets. The dependence of prices on $Y(t)$ through the matrix of elasticities Λ captures the reduced-form effect of the instantaneous speed of trading: the more quickly traders are buying or selling the assets, the higher the "liquidity" costs.

If the elasticity matrices Γ and Λ are diagonal, the markets for each of the M assets are completely segmented. In this case, the greater the j'th diagonal term of Λ is, the greater is the market impact of trading in asset j. Crucially, our model allows for nonzero off-diagonal terms in Γ and Λ, reflecting cross-elasticities in the permanent and short-term effects of trading. The importance of these off-diagonal terms is explored in the following section.

3. Equilibrium Analysis

3.1 Competitive Equilibrium

Similar to Brunnermeier and Pedersen (2005) and to Carlin, Lobo, and Viswanathan (2007), we focus on open-loop equilibria for the sake of tractability. That is, we assume that strategies are deterministic, are chosen at $t = 0$, and are not conditioned on subsequent actions. Under this framework, each trader i solves the following optimal control problem (where we use a superscripted T to indicate the transpose of a vector or matrix):

$$\begin{gathered}\max_{x_i(t),y_i(t)} -\int_{t=0}^{1} p(t)^T y_i(t)\; dt \\ s.t. \\ y_i(t) = dx_i(t)/dt, \\ x_i(0) = x_{0i}, \quad x_i(1) = x_{1i}.\end{gathered} \tag{2}$$

In the above expression, $p(t)^T y_i(t)$ is firm i's instantaneous revenue at time t; $y_i(t) = dx_i(t)/dt$ are the equations of motion governing i's position $x_i(t)$; and $x_i(0) = x_{0i}$ and $x_i(1) = x_{1i}$ are the boundary conditions. The latter boundary condition captures the notion that some firms become distressed. In general, we assume that for nondistressed firms, $x_{1i} = x_{0i}$, while for distressed firms, $x_{1i} < x_{0i}$ (in the vector sense). In practice, our later examples focus on the specific case in which $x_{1i} = 0$, but the general intuition holds even without assuming this particular endpoint.

Substituting in the price equations (1), the Hamiltonian for trader i can be expressed as $H_i = [-u - \Gamma X(t) - \Lambda Y(t)]^T y_i(t) + z_i(t)^T y_i(t)$, where $z_i(t) \in R^m$ are the shadow costs of the equations of motion. Following Carlin, Lobo, and Viswanathan, we limit the set of admissible strategies to differentiable trading profiles satisfying $\int_0^T (y_i(t))^2 < \infty$. The necessary conditions defining a competitive equilibrium are as follows:

$$\begin{aligned} -\frac{\partial H_i(t)}{\partial x_i(t)} &= \Gamma^T y_i(t) = \frac{dz_i(t)}{dt} \\ -\frac{\partial H_i(t)}{\partial y_i(t)} &= u + \Gamma X(t) + \Lambda Y(t) + \Lambda^T y_i(t) - z_i(t) = 0, \end{aligned} \qquad i = 1 \dots n.$$

Differentiating the second equation with respect to t and substituting in the first equation yields

$$\Gamma Y(t)dt + \Lambda dY(t) + \Lambda^T dy_i(t) - \Gamma^T y_i(t)dt = 0, \quad i = 1 \dots n.$$

We can reexpress the above system in matrix form by defining $\Upsilon(t) = [y_1(t) \ldots y_i(t) \ldots y_n(t)]$ (i.e., $y(t) = vec(\Upsilon(t))$):

$$\Gamma Y(t)dt\mathbf{1}^T + \Lambda dY(t)\mathbf{1}^T + \Lambda^T d\Upsilon(t) - \Gamma^T \Upsilon(t)dt =$$
$$\Gamma\Upsilon(t)dt\mathbf{1}\mathbf{1}^T + \Lambda d\Upsilon(t)\mathbf{1}\mathbf{1}^T + \Lambda^T d\Upsilon(t) - \Gamma^T \Upsilon(t)dt = 0,$$

where 1 denotes an $N \times 1$ vector of ones. The matrices Γ and Λ are symmetric by assumption.[1] Thus, the above expression immediately implies that the optimal trajectory when $N = 1$ is for the monopoly firm to liquidate each asset at a uniform speed ($dy(t) = 0$). For $N > 1$, we have

$$\begin{aligned}
&\Lambda d\Upsilon(t)[I + \mathbf{1}\mathbf{1}^T]\Gamma\Upsilon(t)dt[I - \mathbf{1}\mathbf{1}^T] \\
&\Longleftrightarrow \\
&d\Upsilon(t) = \Lambda^{-1}\Gamma\Upsilon(t)dtA \\
&\Longleftrightarrow \\
dy(t) &= vec(\Lambda^{-1}\Gamma\Upsilon(t)dtA) \\
&= (A^T \otimes \Lambda^{-1}\Gamma)vec(\Upsilon(t)dt) \\
&= (A^T \otimes \Lambda^{-1}\Gamma)y(t)dt,
\end{aligned} \tag{3}$$

where $A \equiv [I - 11^T][I + 11^T]^{-1}$.[2]

The general solution to the system of differential equations given by (3) is $y(t) = exp(tA^T \otimes \Lambda^{-1}\Gamma) \cdot c_0$. Integrating $y(t)$ over t yields the following:

Proposition 1. *There is a unique open-loop equilibrium to the trading game, in which the strategic traders' asset positions have the following trajectory:*

$$x(t) = \int y(t)dt = (A^T \otimes \Lambda^{-1}\Gamma)^{-1}exp(tA^T \otimes \Lambda^{-1}\Gamma) \cdot c_0 + c_1. \tag{4}$$

[1]If $\Lambda = 0$ and the demand system is linear, symmetry of Γ is implied by the symmetry of the Slutsky matrix.

[2]The final expression comes from applying the *vec* operator to both sides of the equation and using the fact that for any three matrices A, B, and C, $vec(ABC) = c^T \otimes A\ vec(B)$.

Here, "$exp(\cdot)$" is the matrix exponential operator and the constants c_0 and c_1 are implied by the boundary conditions.[3] Uniqueness follows from the fact that each firm's objective function is globally concave in the control function[4] and from the linearity of the equation of motion.[5] Thus, there is a unique trading profile that satisfies the necessary conditions.

3.2 Features of Equilibrium Outcome

Figure 1A plots the firms' asset positions $x_i(t)$ as a function of time for the case of two firms ($N = 2$) and two assets ($M = 2$), and specific values of the elasticities Λ and Γ. By assumption, asset 2 is more "liquid" than asset 1, in the sense of having a lower market impact of trading (i.e., $\lambda_{22} < \lambda_{11}$, where λ_{ij} indicates element i, j of Λ). The graphs show what happens when firm 1 becomes distressed at time 0 and must liquidate all of its assets.

Figure 1B plots the corresponding asset prices as a function of time. The example is instructive because it shows that even though the fundamental values of both assets are expected to fall, only trading in asset 2 (the more liquid asset) exhibits the "race-and-fade" effect, which can be seen in the convexity of the nondistressed firm's trajectory in that asset. By contrast, the nondistressed firm's trajectory in asset 1 is concave: toward the beginning of the trading window, the firm actually purchases asset 1, providing liquidity for the disposition of the distressed firm's portfolio.

Several competing effects are at play. To begin with, ignoring the cross-elasticities (for the time being), note that the nondistressed

---

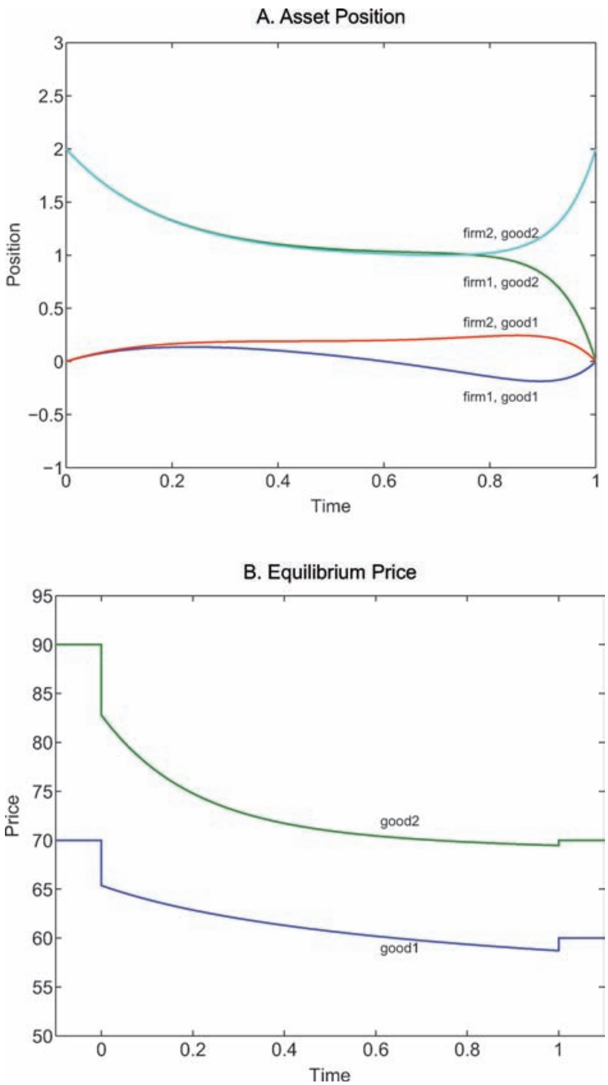
[3]To compute the exponent of a matrix, first express the matrix in terms of its orthogonal factorization PVP^{-1}, where P is a matrix of eigenvectors and V is a diagonal matrix of eigenvalues $v_1 \ldots v_n$. $B^m = PV^mP^{-1}$ for any power m, and because the exponential operator is defined as a power series of B, we have $exp(B) = Pexp(V^m)P^{-1}$, where $exp(V^m)$ can be computed element by element. The constants are determined by the boundary conditions $x(0) = x_0$ and $x(1) = x_1$. Defining $B = A^T \otimes \Lambda^{-1}\Gamma$, we have

$$\begin{bmatrix} B^{-1} & I \\ B^{-1}exp(B) & I \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}.$$

[4]This assertion relies on the assumption that Λ is positive-definite.

[5]See Kirk (1970).

Figure 1. Trajectories for Baseline Case with Two Assets and Two Firms



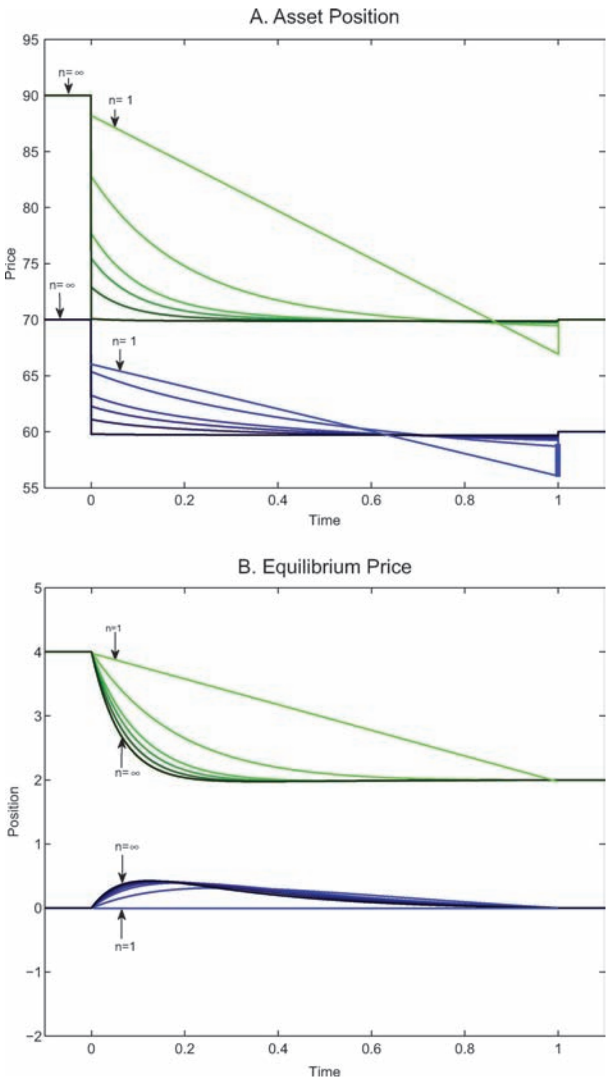
Note: Figures were generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 1.25 \\ 1.25 & 1 \end{bmatrix}$, both strategic players begin with endowments $(0, 2)$, and player 1 must liquidate all assets by $t = 1$.

firm's trading activity in the distressed asset is driven by two competing incentives. Just as in Carlin, Lobo, and Viswanathan's single-asset case, there is an incentive to predate (by selling rapidly when the fundamental price is high and then "fading" back when the fundamental price is low). Of course, the distressed firm best responds by racing the nondistressed firm to sell early, resulting in faster net selling toward the beginning of trading and slower net selling toward the end of trading, relative to the rate that would be realized if firms traded at a uniform speed over time. However, because the speed of trading affects the liquidity discount or premium, the incentive to predate is partially mitigated by a desire to smooth the speed of trading over time. In effect, the predatory incentive to race and fade simultaneously generates a second, countervailing incentive to provide liquidity by buying the asset being predated on at a discount during the "race" phase of the game—when it is being sold more rapidly than the average over time—and selling the asset at a premium during the "fade" phase, when it is being sold more slowly than the average over time. Moreover, because the individual players internalize less of the liquidity impact of predation when there are more firms, the trading speed deviates more and more from a uniform rate as the number of firms $N \to \infty$ (figure 2A). As shown in figure 2B, in the limit as the number of competing firms, N, goes to ∞, prices immediately jump down to the final level at the very start of trading and then remain constant thereafter.[6]

In addition to the previous effects, which our model shares in common with the single-asset case, having multiple assets introduces two novel features. First, cross-elasticities in the permanent values of assets (off-diagonal terms of Γ) imply that *forced liquidation of any one asset puts downward pressure on the fundamental values of all remaining assets, creating incentives to trade strategically even in assets that do not have to be liquidated.* Trading activity in assets that do not need to be liquidated reflects the fact that the goods are imperfect substitutes. We can isolate the effect of cross-elasticities in Γ by examining the dynamics when only Λ is diagonal (figures 3A

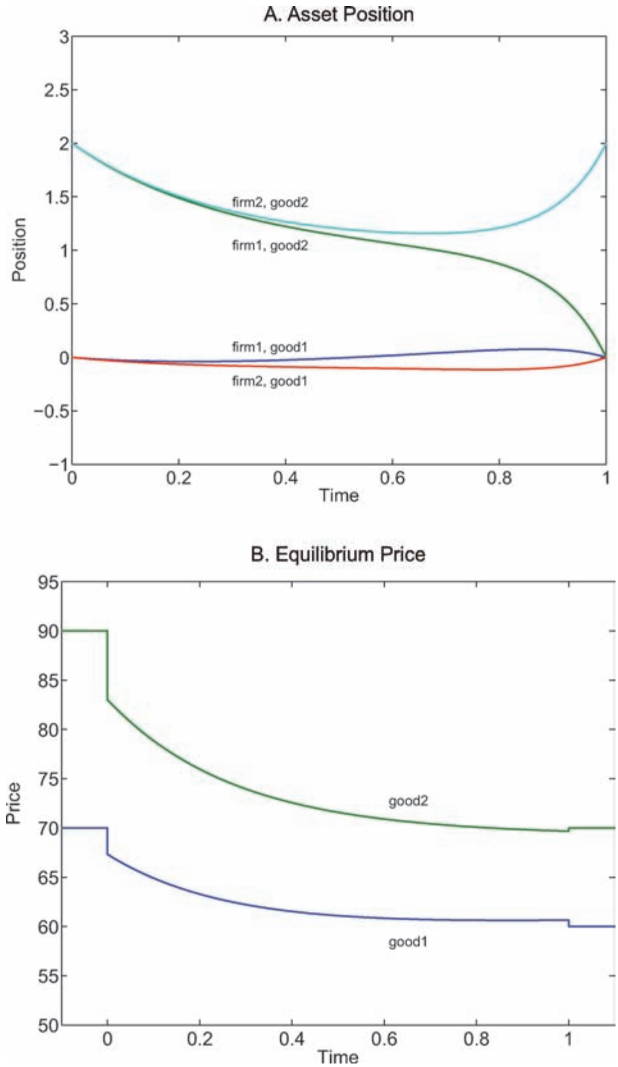
[6]Intuitively, because under perfectly competitive markets each individual trader is a price taker, fluctuations in prices during the trading window would imply the presence of arbitrage opportunities. Thus, prices must be constant throughout the trading window when $N = \infty$.

Figure 2. Trajectories with 50 Percent of Firms Distressed, Varying Number of Firms N



Note: Figures were generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 1.25 \\ 1.25 & 1 \end{bmatrix}$, all firms begin with endowments $(0, 2/N)$, and half of all firms must liquidate all assets by $t = 1$.

Figure 3. Trajectories for Zero Cross-Elasticities in Short-Term Market Impact



Note: Figures were generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$, both strategic players begin with endowments $(0, 2)$, and player 1 must liquidate all assets by $t = 1$.

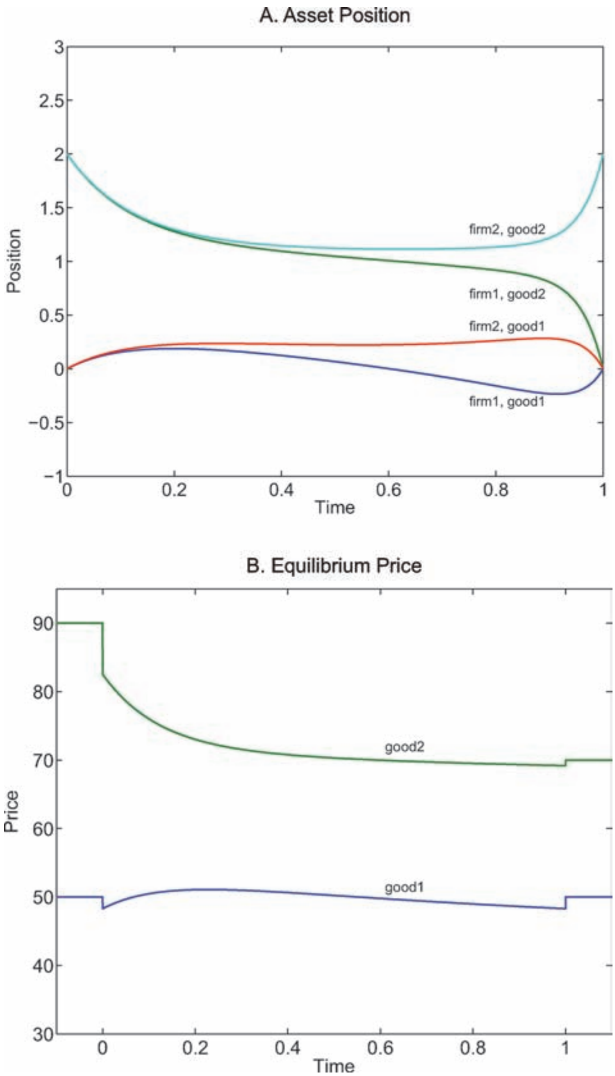
and 3B). In this case, we observe the race-and-fade phenomenon in both assets, with accelerated early sales for both the asset held in the distressed firm's initial portfolio and the other asset. Note that relative to the case shown in figures 1A and 1B, the extent of racing and fading in the liquid asset (asset 2) is weaker: in the former case, the cross-subsidization effect essentially reduces the liquidity cost of trading, encouraging more predation.

Clearly, when declines are expected in the fundamental values of all assets, race-and-fade behavior must be manifested to some extent for at least a subset of the assets. At the same time, the cross-elasticities in the short-term impact of trading (Λ) introduce a second effect: *firms have the opportunity to provide liquidity "cross-subsidies" to themselves.* Except when Λ is diagonal, firms can lower their overall trading costs at each moment by trading in opposite directions in the various assets. In particular, purchasing activity in one asset increases the price premium on all other assets. Therefore, racing and fading in a particular asset (due to an expected decline in its fundamental value) simultaneously generates an incentive to make purchases in other assets during the "race" phase and then to sell back those other assets during the "fade" phase. Figures 4A and 4B isolate the self-subsidization effect for the two-asset case by setting Γ to be a diagonal matrix, such that only Λ has off-diagonal terms. Similar to figures 1A and 1B, in this example, the nondistressed firm tends to trade in opposite directions in each of the assets at every moment.

In the examples discussed so far, the nondistressed firm predates in the liquid asset and trades in the opposite direction in the illiquid asset. However, this need not be the case in general, and it is possible to generate examples for which the reverse is true. Because of the cross-elasticities in Λ and Γ, both the incentive to predate and the incentive to provide liquidity exist for all assets. In particular, the trading profile will be concave for certain assets when the incentive to provide liquidity through those assets actually outweighs the incentive to predate on those assets. In other words, whether the trajectory of a given asset position exhibits race-and-fade behavior or, instead, the opposite depends upon the net liquidity costs of trading in that asset, relative to the expected decline in its fundamental value.

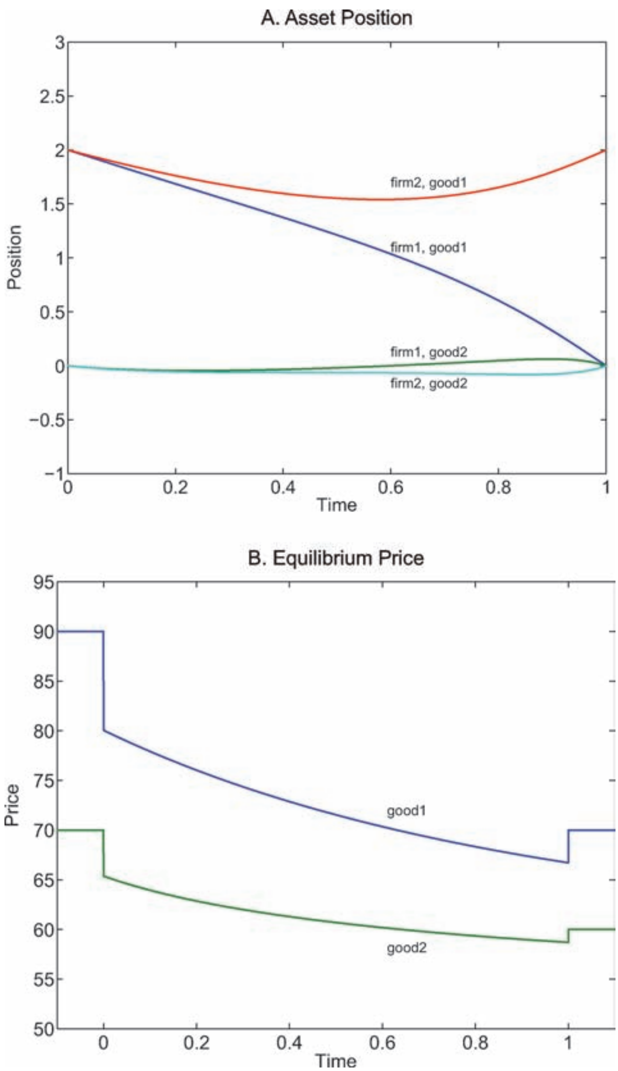
For example, if firms hold a large initial endowment in the illiquid asset (figures 5A and 5B), the nondistressed firm's trading profiles

Figure 4. Trajectories for Zero Cross-Elasticities in Fundamental Values



Note: Figures were generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 1.25 \\ 1.25 & 1 \end{bmatrix}$, both strategic players begin with endowments $(0, 2)$, and player 1 must liquidate all assets by $t = 1$.

Figure 5. Trajectories with More Illiquid Initial Endowment



Note: Figures were generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 1.25 \\ 1.25 & 1 \end{bmatrix}$, both firms begin with endowments $(2, 0)$, and firm 1 must liquidate all assets by $t = 1$.

in both assets are convex, with early selling and later buying-back of the assets. Furthermore, relative to the case in which the strategic firms begin with a liquid endowment (figures 1A and 1B), overall trading takes place at a much more constant speed and prices fall more gradually. Intuitively, when the distressed firm must dispose of a more illiquid endowment, there is a weaker incentive to predate because the liquidity costs associated with racing and fading in the illiquid asset are high relative to the expected decline in its fundamental value. Also, for this particular case, the nondistressed firm's incentive to race and fade in the liquid good (asset 2) outweighs the incentive to provide liquidity, though the overall trading volume is light.

3.3 Monopoly Case and Cooperative Outcome

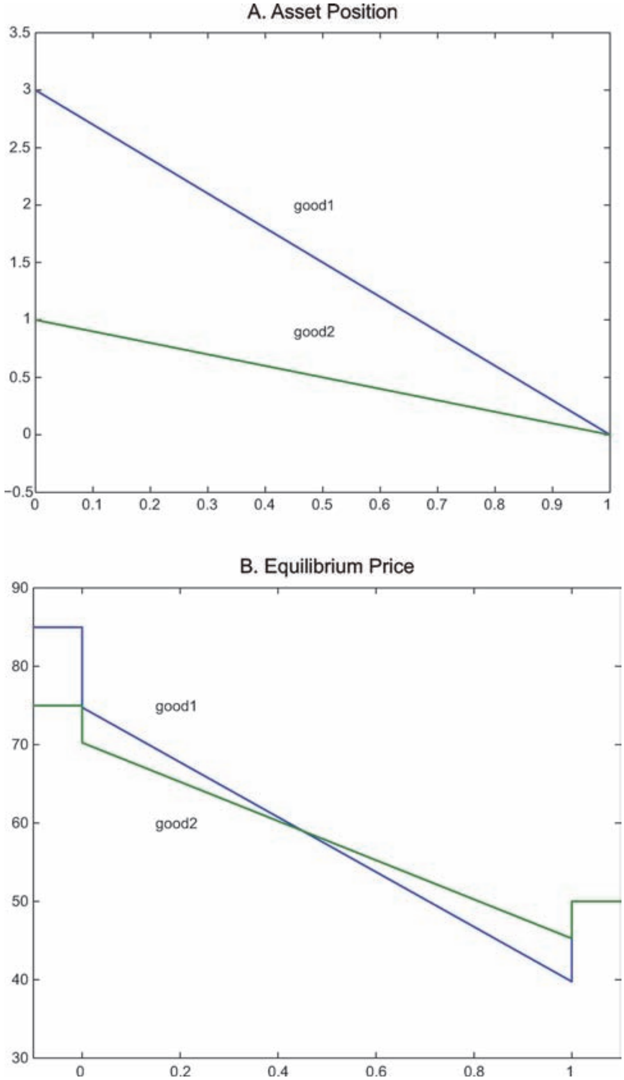
In the monopoly case or when firms behave cooperatively, agents entirely internalize the liquidity impact of trading. Note that in the monopoly case, the total trading costs associated with liquidating the portfolio are minimized by selling each asset at a uniform rate (as formally proved in section 3.1). The minimized trading costs amount to $\frac{1}{T}(x(1) - x(0))^T \Lambda (x(1) - x(0))$. There is effectively no cross-subsidization across assets: because there are no additional firms whose strategic responses must be taken into account, there is no variation over time in the cost of liquidating given quantities of the assets. Thus the optimal trading speed in each asset is constant (figures 6A and 6B), and there are no benefits to trading in assets other than the ones that actually must be liquidated.

Similarly, if there are multiple firms, the first-best cooperative outcome is attained by setting net trading speed to be constant over time. The trading profile that achieves the cooperative optimum is not unique and may be any trading profile such that net trading occurs at a uniform speed, so long as the boundary conditions are met. In particular, all trading could be assigned to a single firm, and then at the last moment firms "clear accounts" by trading infinitely fast to attain the required endpoint positions.

4. Effect on Nonstrategic Investors

A key issue of policy relevance is the effect of strategic trading on the nonstrategic, "retail" investors. The preferences and behavior of

Figure 6. Trajectories for Monopoly Firm



Note: Figures were generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 1.25 \\ 1.25 & 1 \end{bmatrix}$, and the single firm begins with endowment $(3, 1)$ and must liquidate all assets by $t = 1$.

the retail traders are not explicitly modeled in the setup. However, common intuition suggests that investors are harmed when asset prices decline suddenly, relative to the more gradual declines that would be observed if assets were liquidated at a constant rate.

For reasons already discussed in subsection 3.2, when the distressed firm begins with a relatively more illiquid endowment, prices may actually fall more gradually than when it is initially endowed with relatively liquid assets. In order to construct a summary measure of the distress that sudden drops in price are likely to create among retail investors, we consider the following scenario. Suppose there is a "representative" retail investor who begins with a vector of endowments, x_{0r}. At time t, the investor's net worth, $w_r(t)$, is given by

$$w_r(t) = p(t)' x_r(t) - \int_{\tau=0}^{t} p(\tau) y_r(\tau)\, d\tau, \tag{5}$$

where $x_r(t)$ is the retail investor's portfolio at time t and $-\int_{\tau=0}^{t} p(\tau) y_r(t)\, d\tau$ is the cash position generated by her trading activity between time 0 and t. (Note that summing over the retail investors and the strategic players yields zero net trading.)

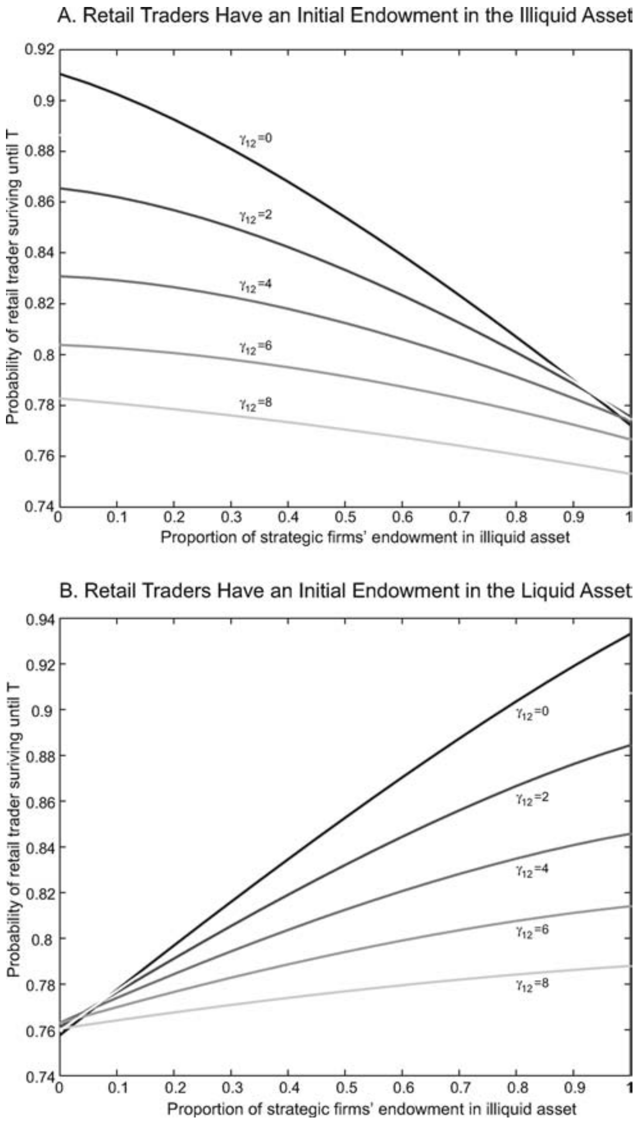
At each point in time, the investor's hazard of becoming "distressed," $h(t)$, depends upon her net worth as a proportion of initial wealth. While it is inessential how we actually interpret the event of becoming distressed, as a conceptual matter, we can think of distress as the prospect of facing a margin call.[7] Specifically, we assume that the probability of the retail investor becoming distressed before the end of the trading period (at time T) is given by

$$\begin{gathered} 1 - \int_0^T e^{-h(t)\cdot t} dt \\ h(t) = -log[min(1, w_r(t)/w_r(0))]. \end{gathered} \tag{6}$$

The functional form of $h(t)$ implies that the retail investor only faces a nonzero hazard of distress if her net worth falls below the initial level. Figures 7A and 7B graph the retail investor's probability

[7] We abstract from any potential price impact of the retail investors having to liquidate.

Figure 7. Probability of Retail-Trader Distress vs. Proportion of Strategic Traders' Initial Allocation in the Illiquid Asset



Note: Figures were generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & \gamma_{12} \\ \gamma_{12} & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & 1.25 \\ 1.25 & 1 \end{bmatrix}$, each strategic player has a total endowment of two units (whose allocation between the two assets is captured by the x-axis), and player 1 must liquidate all assets by $t = 1$.

of survival as a function of the strategic traders' initial asset allocation for the two-good case. Each curve in the graph corresponds to a particular value of γ_{12}, the cross-price elasticity in the permanent values of the assets. Figure 7A assumes that the retail traders have an initial endowment in the illiquid asset while figure 7B assumes that the retail traders have an initial endowment in the liquid asset. Similarly, each curve in figures 8A and 8B corresponds to a particular value of λ_{12}, the cross-price elasticity in the short-term market impact of trading.[8] There are four main points to take from these graphs:

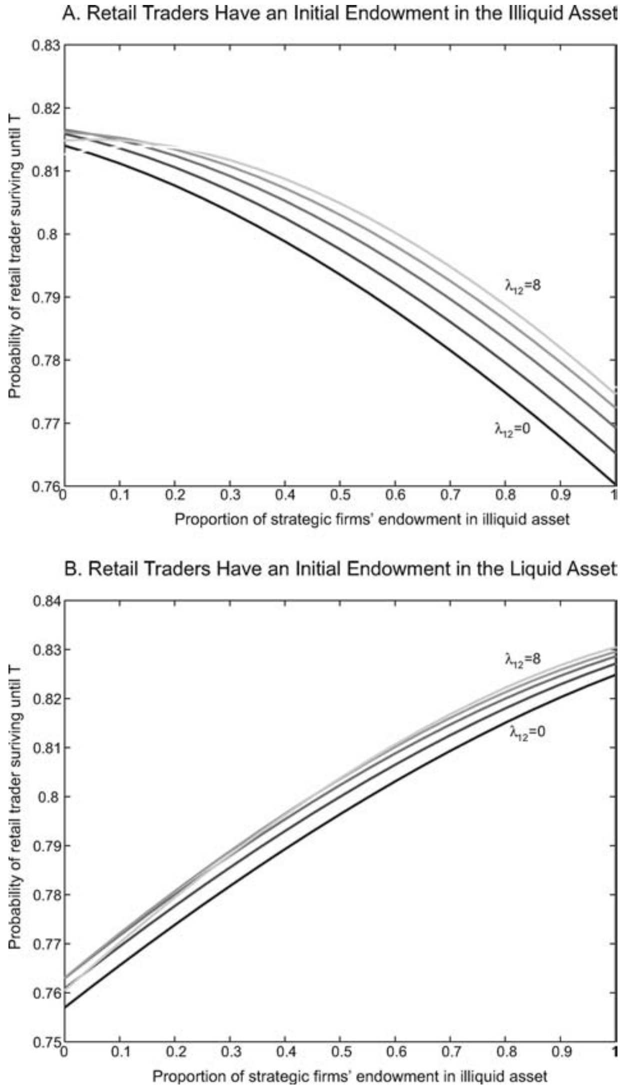
(i) Shifts along each of the curves indicate that as we increase the initial proportion of the strategic players' endowment in whichever asset is held by the retail traders, the retail traders are less likely to survive. Because trading by the strategic traders depresses the prices of the assets being liquidated, when the two types of traders hold the same types of assets, the retail traders are more likely to become distressed.

(ii) As shown by the flatness of the curves corresponding to high values of γ_{12}, as the cross-price elasticity of the permanent effect increases, survival of the retail investors becomes less sensitive to the initial allocation of the strategic traders. Essentially, higher γ_{12} implies that the two assets are closer substitutes, meaning that retail investors are less affected by changes in the composition of the portfolio that must be liquidated.

(iii) Shifts across the curves corresponding to values of λ_{12} indicate that as the short-term cross-elasticity increases, the retail investor's probability of survival generally increases over most ranges of the initial endowments for the strategic traders. To understand why, note that a higher cross-elasticity strengthens the cross-subsidization effect, making it cheaper (in terms of overall price impact) for the nondistressed firms to predate. The reduction in price impact has not only a direct impact but also indirect effects. (The situation is loosely analogous to the effect of a decrease in the price of a good in standard

[8] In the graphs, values of γ_{12} and λ_{12} are limited to ranges such that Γ and Λ are positive-definite.

Figure 8. Probability of Retail-Trader Distress vs. Proportion of Strategic Traders' Initial Allocation in the Illiquid Asset



Note: Figures were generated under the following assumptions: $\Gamma = \begin{bmatrix} 10 & 5 \\ 5 & 10 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 3 & \lambda_{12} \\ \lambda_{12} & 1 \end{bmatrix}$, each strategic player has a total endowment of two units (whose allocation between the two assets is captured by the x-axis), and player 1 must liquidate all assets by $t = 1$.

consumer theory: the overall expenditure share may either increase or decrease, depending on the relative strength of the income and substitution effects). In particular, higher values of λ_{12} encourage the nondistressed firms to engage in more racing and fading with respect to the asset being liquidated by the distressed firm, while their trajectory in the other asset becomes more concave. The overall change in price volatility is theoretically ambiguous. However, when the strategic firms' initial endowment is not too heavily skewed toward the liquid good, the direct price effect dominates, and higher values of λ_{12} tend to reduce price volatility and increase the probability of survival for the retail investor.

(iv) On the other hand, if the strategic traders' initial allocation in the liquid good is sufficiently close to 1.0, the indirect effect of λ_{12} on the extent of predation in the more liquid asset predominates, which may actually decrease the retail investor's probability of survival. To see why, note that the trading trajectories for the illiquid good are relatively insensitive with respect to λ_{12}. On the other hand, high values of λ_{12} make it less costly for the nondistressed firm to predate in the liquid asset, with the overall effect of increasing price volatility.

To summarize, increasing the share of the liquid good is likely to benefit the retail traders only under certain conditions: the retail traders must not also hold large endowments of the liquid good, and the assets must not be close substitutes (as indicated by the cross-elasticities of the permanent effects). Moreover, increasing the share of the liquid good is less likely to benefit the retail traders when there are strong cross-elasticities in the market impact of trading various assets.

5. Policy Implications

Our model suggests that a constant trading speed is a feature of the first-best outcome (defined as the trading profile that maximizes the strategic players' total surplus). This feature corresponds to the idea of an "orderly resolution" of distressed or insolvent firms. The concept is recognized by bankruptcy laws, which give judges broad latitude to wind down firms in a manner that tries to conserve value for liability holders. However, at times the government has taken actions to intervene in the liquidation of large and complex distressed firms with many counterparties, based on the concern that standard corporate bankruptcy law and practice are insufficient to avoid market disruptions.

Most recently, the government has contemplated or engaged in four different types of actions with regard to banks and other large financial institutions: (i) providing liquidity facilities for illiquid assets (financing collateralized by the illiquid assets), (ii) injecting equity into firms so that they do not need to liquidate assets in the first place, (iii) buying assets from distressed firms, and (iv) extending government guarantees to certain classes of asset.

In the context of our model, the first three of these policies are similar in the sense that each reduces the overall quantity of illiquid assets that must be sold by the distressed firms. As the amount of assets that must be liquidated by distressed firms declines (say, from more widespread capital injections), trading in the markets becomes more orderly in all assets. Similarly, as the holdings of the illiquid assets by distressed firms decline (either because the government buys the asset or is willing to finance the asset for a time longer than the trading period), trading becomes more orderly across asset markets.

In contrast, when the government increases the relative proportion of liquid assets by guaranteeing some of the illiquid assets, asset price volatility and predation increase. There are two channels for this increase. First, increasing the proportion of liquid assets directly increases predatory behavior because the strategic players find it cheaper to trade against one another in the liquid asset. Second, this increased predation leads to increased trading in the illiquid asset because of the cross-elasticities between the two. Thus, our analysis suggests that government actions to relieve disorderly trading conditions should focus on relieving the distress of financially troubled firms rather than extending government guarantees to illiquid assets.

6. Conclusion

The current financial crisis has been marked by a strong preference by investors for liquid assets amid heightened price volatility. One policy response has been to increase liquidity by extending government guarantees. We study the effects of such a policy on equilibrium prices and welfare in an endogenous liquidity model containing both strategic and nonstrategic traders. We augment the standard model by adding multiple assets and by modeling the utility of the nonstrategic traders.

We find that extending guarantees, modeled as shifting assets from the illiquid to the liquid category, increases predatory trading when one of the strategic traders becomes distressed. In part this is because of the decreased cost of predation when assets are more liquid, but in part it reflects the linkages between prices in the two markets. For broad and plausible parameter ranges, the increase in volatility from such a policy leads to a decrease in the welfare of the nonstrategic traders.

This model can also be used to consider the policy choices facing a government wishing to unwind its position in the liquid asset. After the financial crisis abates, government officials may want to reduce public holdings or guarantees on assets. Policymakers would then be in a position analogous to that of a distressed firm in our analysis. However, they presumably would also want to minimize price volatility, and associated welfare costs, further exposing themselves to predation.

Finally, we can conjecture about what the financial system would look like following a widespread increase in government guarantees. Such a post-crisis system would feature a smaller amount of private assets, and these assets would trade in thinner, less liquid markets with associated increased price volatility. Restarting issuance of risky securities such as consumer ABS in such a market would seem particularly difficult.

References

Back, K., C. H. Cao, and G. A. Willard. 2000. “Imperfect Competition among Informed Traders.” *Journal of Finance* 55 (5): 2117–55.

Brunnermeier, M. K., and L. H. Pedersen. 2005. “Predatory Trading.” *Journal of Finance* 60 (4): 1825–63.

Carlin, B. I., M. S. Lobo, and S. Viswanathan. 2007. "Episodic Liquidity Crises: Cooperative and Predatory Trading." *Journal of Finance* 62 (5): 2235–74.

Holden, C. W., and A. Subrahmanyam. 1992. "Long-Lived Private Information and Imperfect Competition." *Journal of Finance* 47 (1): 247–70.

Kirk, D. E. 1970. *Optimal Control Theory*. Englewood Cliffs, NJ: Prentice-Hall.

Kyle, A. S. 1985. "Continuous Auctions and Insider Trading." *Econometrica* 53 (6): 1315–35.

Forced Liquidation of an Investment Portfolio*

Bruce Ian Carlin
Anderson Graduate School of Management,
University of California at Los Angeles

Following the crisis that surrounded the downfall of long-term capital management (LTCM) in 1998, Myron Scholes raised an important and timeless problem: how should an investment manager liquidate a portfolio when competitors in the market trade to profit from one's misfortune? Specifically, he stated:

> In an unfolding crisis, most market participants respond by liquidating their most liquid investments first to reduce exposures and reduce leverage. . . . However, after the liquidation, the remaining portfolio is most likely unhedged and more illiquid. Without new inflows of liquidity, the portfolio becomes even more costly to unwind and manage.
>
> (Scholes 2000)

This problem, which I call the Scholes liquidation problem, is ubiquitous during unstable financial periods. Indeed, in the recent financial crisis, banks incurred large losses during the forced contraction of their balance sheets, as access to short-term financing through repo markets dried up (e.g., Adrian and Shin 2008; Brunnermeier 2009). A systemic deleveraging process propagated through the banking sector, in which careful liquidation became tantamount to preserving wealth and surviving the storm.

Previous work by Carlin, Lobo, and Viswanathan (2007) analyzes a simplified version of this problem: in an economy in which one asset is traded, how should a distressed participant sell off his holdings

*Author contact: Anderson Graduate School of Management, University of California at Los Angeles, 110 Westwood Plaza, Suite C519, Los Angeles, CA 90095-1481; E-mail: bruce.carlin@anderson.ucla.edu; Tel: 310-825-7246.

when there are predators in the market who can trade against him? In that paper, the asset is traded at the price

$$P_t = U_t + \gamma X_t + \lambda Y_t.$$

The first term is the expected value of future dividends, which is modeled as a martingale stochastic diffusion process with zero drift. The second and third terms partition the price impact of trading into permanent and temporary components. As described in Carlin, Lobo, and Viswanathan (2007), the permanent component of liquidity refers to a change in the price that is independent of the rate at which the asset is traded. This impact is likely to be high when the amount of asymmetric information associated with the asset is high or ownership of the asset in the market is concentrated. The temporary component of liquidity measures the instantaneous, reversible price pressure that results from trading. This component is likely to be high when the asset is thinly traded or there is a paucity of counterparties in the market.

In equilibrium, predators practice a "race-and-fade" strategy in which they first race the distressed trader to the market, but then reverse their trades to secure their profits. The amount of predation in the market, and hence the price volatility that ensues, is a function of both γ and λ. When the $\frac{\gamma}{\lambda}$ ratio is high, there is more price volatility. This occurs because there are greater incentives to predate when someone in the market needs liquidity. Indeed, when γ is high, there is a greater benefit to racing others to the market since predators would like to capture the profits associated with the permanent component of liquidity. Additionally, when λ is low, the cost of predation is also low because less surplus is given up during trade to long-term traders who form the competitive fringe in the market.

The analysis in Carlin, Lobo, and Viswanathan (2007) has important policy implications, especially if it is socially optimal to avoid price volatility. Specifically, their results shed light on how liquidity injection into the market impacts price volatility. From the discussion presented already, it should be clear that any policy change that decreases γ or increases λ should decrease both predation and price volatility. For example, when the government guarantees a residential mortgage-backed security, this decreases the requirement for investors to consider the monitoring incentives present during the origination of the loans. Such a decrease in asymmetric information should lower the permanent component of liquidity, and therefore decrease price volatility. In contrast, injecting liquidity in the form of lowering λ may increase price volatility. For example, establishing an entity whose mission is to purchase securities in the market will decrease the temporary impact of trading. Such a move may have the unintended consequence of increasing predatory activity and price volatility.

These implications are also central in the model posed by Chu, Lehnert, and Passmore (2009). Their work explores a multidimensional extension of Carlin, Lobo, and Viswanathan (2007) in which the prices of the assets in the market are given by the vector P_t, where

$$P_t = U_t + \Gamma X_t + \Lambda Y_t.$$

The three components of prices have the same interpretation, with one main difference: there are cross-price elasticities associated with trading each of the assets. That is, Γ and Λ are matrices in which the off-diagonal entries represent the permanent and temporary effects that trading in one asset has on the prices of other assets.

The work by Chu, Lehnert, and Passmore (2009) explores the effects that these cross-price elasticities have on predation, liquidity provision, and the policies outlined above. For example, they explore in detail the case in which there are two traders and two assets: one illiquid and the other liquid. They compare cases in which one trader is distressed and is exogenously required to liquidate either the illiquid asset or the liquid asset. In particular, they characterize the effects of such liquidation on trading in the alternate asset. They show that when an illiquid asset is sold in the market, the distressed trader will buy more of the liquid asset to improve the liquidity of their portfolio. Of course, the nondistressed trader will do the same to take advantage of price movements in both assets. The cross-price elasticities between the two assets determine the equilibrium rate of trade that is optimal for the distressed trader, given that they face adversity in the market from the nondistressed trader.

This advance does get us one step closer to understanding the solution to the Scholes liquidation problem, but does not allow us to address it directly. Specifically, the problems posed by Carlin, Lobo, and Viswanathan (2007) and Chu, Lehnert, and Passmore (2009) exogenously assume that there is a specific asset that needs to be liquidated. The model by Chu, Lehnert, and Passmore (2009) does explore the effect that this liquidation has on other assets in the portfolio. However, we still do not know how a trader chooses which assets to sell during a liquidation. That is, the equilibrium solution should endogenously describe which assets should be traded, given their effects on other assets.

One parsimonious way to approach this might be to model a distressed trader who chooses his trading strategies to generate a particular quantity of cash, given the cross-effects noted above. In such a setting, multiple assets might be bought or sold, but the goal would be to preserve value in the remaining portfolio, given that other traders are present who wish to again capitalize on the misfortune of the distressed trader. This is a more realistic, albeit more complicated, problem. Hopefully, though, with such an approach, the Scholes liquidation problem might be tackled. Characterization of this profound problem would have important academic and practical implications. Indeed, the derivation of this solution is ongoing work.

References

Adrian, T., and H. S. Shin. 2008. "Financial Intermediary Leverage and Value-at-Risk." Federal Reserve Bank of New York Staff Report No. 338.

Brunnermeier, M. K. 2009. "Deciphering the Liquidity and Credit Crunch 2007–2008." *Journal of Economic Perspectives* 23 (1): 77–100.

Carlin, B. I., M. S. Lobo, and S. Viswanathan. 2007. "Episodic Liquidity Crises: Cooperative and Predatory Trading." *Journal of Finance* 62 (5): 2235–74.

Chu, C. S., A. Lehnert, and W. Passmore. 2009. "Strategic Trading in Multiple Assets and the Effects on Market Volatility." *International Journal of Central Banking* 5 (4): 143–172.

Scholes, M. S. 2000. "Crisis and Risk Management." *American Economic Review* 90 (2): 17–21.

When Everyone Runs for the Exit*

Lasse Heje Pedersen
New York University, CEPR, and NBER

The dangers of shouting "fire" in a crowded theater are well understood, but the dangers of rushing to the exit in the financial markets are more complex. Yet, the two events share several features, and I analyze why people crowd into theaters and trades, why they run, what determines the risk, whether to return to the theater or trade when the dust settles, and how much to pay for assets (or tickets) in light of this risk. These theoretical considerations shed light on the recent global liquidity crisis and, in particular, the quant event of 2007.

JEL Codes: G11, G12.

1. Introduction

People choose to crowd into a theater or a trade because they share a common goal: in one case, they all want to see the best play in town; in the other, they all pursue the highest risk-adjusted return. They run for the exit because staying is associated with real risk, namely being caught in the theater fire or being forced to liquidate at the most distressed prices. Many people running introduces a second, and endogenous, risk: theater guests risk being trampled by running feet, and traders risk being trampled by falling prices, margin

*I am grateful for helpful comments from Yakov Amihud, Jacob Bock Axelsen, David Backus, Markus Brunnermeier, Jennifer Carpenter, Douglas Gale (the editor), Arvind Krishnamurthy, Bill Silber, and Jeff Wurgler, as well as from participants at the Inaugural Financial Stability Conference on Provision and Pricing of Liquidity Insurance hosted by the International Journal of Central Banking and the Federal Reserve Bank of New York. This paper has benefitted from the insights gained at AQR Capital Managements, including those from Michele Aghassi, Cliff Asness, Ronen Israel, Toby Moskowitz, Lars Nielsen, and Humbert Suarez. The author is at New York University, CEPR, and NBER, 44 West Fourth Street, NY 10012-1126; E-mail: lpederse@stern.nyu.edu; Web site: http://www.stern.nyu.edu/∼lpederse/.

calls, and vanishing capital—a negative externality that increases the aggregate risk.

The risk of running for the exit depends on how crowded the theater or trade is, and the quality of risk management. The liquidity risk can be reduced by restricting reliance on funding that cannot be depended on during crises, by limiting how large and levered positions one takes, or, even better, if the leveraged players limit how large an aggregate position they take relative to their capital.

Finally, investors return to these markets as liquidity crises create opportunities. Indeed, the expected return on liquidity provision rises during crisis. Just like fear of a theater fire would reduce ticket prices, liquidity risk reduces asset prices.

A panic run to the exit is so dangerous that Justice Oliver Wendell Holmes Jr.'s opinion in the U.S. Supreme Court's decision in *Schenck v. U.S.* (1919) states: "The most stringent protection of free speech would not protect a man falsely shouting fire in a theater and causing a panic." Panic runs have been studied extensively by physicists,[1] who document and model how people try to move faster and start pushing, passing of bottlenecks becomes uncoordinated, exits are clogged, alternative exits are used inefficiently, and the risk depends on the design of the exit routes. As evidence of the danger of such runs, jammed crowds can cause pressures up to 4,500 newtons per meter, which can bend steel barriers or tear down brick walls, and escape is further slowed by fallen or injured people turning into "obstacles."

A panic run in the financial markets is also serious, and, indeed, the global crisis that started in 2007 provides ample evidence of the importance of liquidity risk. Subprime credit losses put highly levered financial institutions into a tailspin, their sources of funding dried up, and each institution's liquidations and risk reductions added stress to the other institutions as the crisis spilled over to other credit markets, money markets, currency markets, convertible bonds, stocks, and over-the-counter derivatives. Central banks' balance sheets increased significantly as they tried to address the funding problems using various lending programs and unconventional monetary policy tools.

[1] See Helbing, Farkas, and Vicsek (2000), Helbing (2001), Helbing et al. (2005), and references therein.

I focus on the "quant" event of August 2007 as it illustrates well the nature of liquidity crises. While this event was almost invisible to the public, it can be seen very clearly through the lens of a diversified long-short strategy. Quantitative traders running for the exit had a significant impact on some of the most liquid markets in the world, and I show how prices dropped and rebounded in early August 2007. Using high-frequency data, I document an amazing short-term predictability and volatility driven by the run to the exit (figure 1, panel A). In hindsight, the quant crisis was an early warning signal of how the levered system would face trouble as the liquidity spirals caused havoc in the global markets.

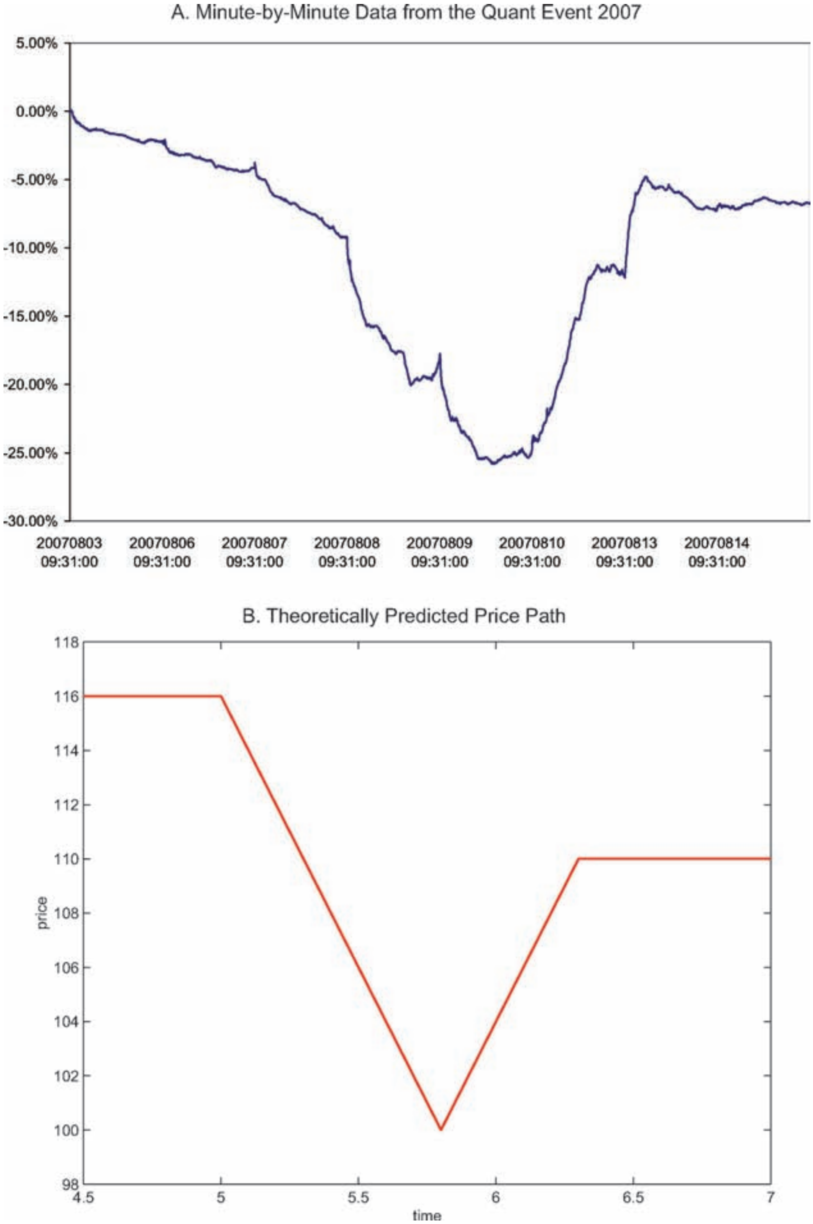
The recent liquidity crisis is the last in a string of earlier ones throughout history, such as the crash of 1987, the crisis following the Russian default in 1998, the convertible bond episode in 2005, and currency crashes when carry trades unwind. To reduce the risk of the next one, it is important to understand the mechanisms that drive these crises. Therefore, after presenting the evidence from the global liquidity crisis and the quant event in section 2, I consider some models of liquidity risk in section 3.[2]

I analyze the underlying causes of forced selling, the reasons why other investors may sell even if they are not forced to do so, and the resulting price path (figure 1, panel B). I show how "liquidity spirals" amplify and spread the initial shock when selling leads to more selling, higher margin requirements, tighter risk management, and withdrawal of capital, consistent with the evidence from the crisis that I present.

Finally, I discuss the implications for asset pricing and monetary policy. I explain how securities with larger and more varying transaction costs must offer higher expected returns as compensation for the larger market liquidity risk. Further, securities with higher margin requirements must offer higher expected returns as compensation for their larger use of capital and funding liquidity risk. For instance, securities backed by loans and other credit instruments

[2]My understanding of the crisis is largely based on my own research, and this is reflected in this note. Amihud, Mendelson, and Pedersen (2006) review the broad literature on liquidity and asset pricing, and Gorton (2008), Brunnermeier (2009), and Krishnamurthy (2009) review the crisis and amplification mechanisms.

Figure 1. Everyone Runs for the Exit



Note: Panel A shows the cumulative return to a long-short market-neutral value and momentum strategy for U.S. large-cap stocks, scaled to 6 percent annualized volatility during August 3–14, 2007. Panel B illustrates the Brunnermeier and Pedersen (2005) model's predicted price path when everyone runs for the exit.

have higher yields (or lower prices) if their market liquidity risk and margin requirements are higher. Hence, monetary policy that affects margin requirements (or "haircuts"), funding, and market liquidity can thus affect asset prices and credit availability.

2. Running for the Exit in the Real World

I first study the recent global financial crisis and how it spilled over across asset classes, with a special focus on the quant event of August 2007.

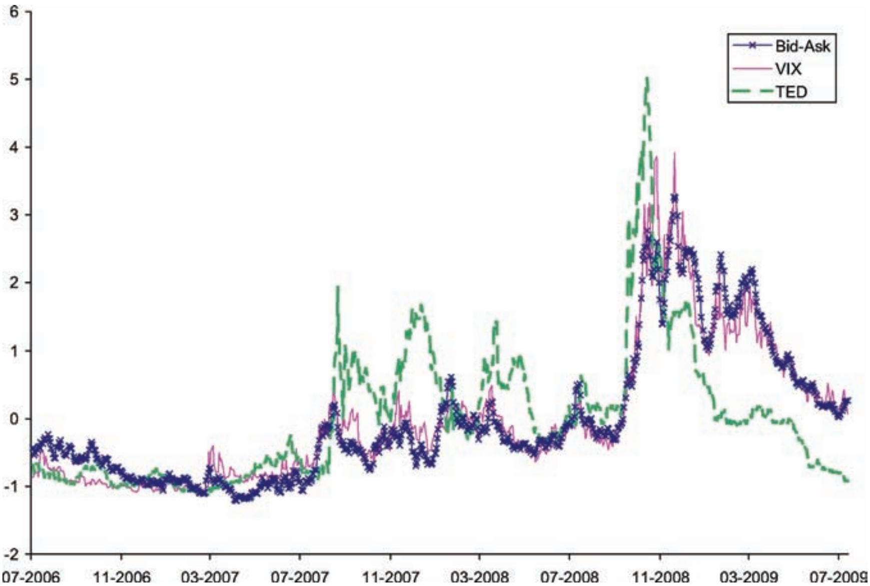
2.1 The Global Liquidity Crisis That Started in 2007

In the years preceding the crisis, the global financial markets were flush with liquidity due to low interest rates, high savings rates in Asia, economic growth, and low volatility. As a response to low borrowing costs and low apparent risk, financial institutions became highly levered (a positive liquidity spiral). This made them vulnerable. When house prices started to decline and it started to become clear in 2007 that subprime borrowers would default in large numbers, an adverse liquidity spiral was kicked off. Many banks experienced significant mark-to-market losses, and two hedge funds at Bear Stearns blew up due to subprime-related collateralized debt obligations (CDOs) in June 2007.

Market liquidity dried up in one market after another as volatility picked up, funding became tight, and risk premia rose, as seen in figure 2. The figure shows the evolution of market liquidity as measured by bid-ask spread, in percent of mid quote, averaged across large-cap U.S. stocks.[3] The figures also shows the TED spread and the VIX index. The TED spread is the difference between the interest rate on three months' uncollateralized interbank LIBOR loans and the interest rates on Treasury bills. A high TED spread indicates reluctance to provide interbank loans, that is, risks and funding problems in the financial sector. VIX is the volatility of the S&P 500 equity index as implied by the option markets, and may also be related to funding liquidity as many financial institutions

[3] This is computed using tick data, using the best bid and ask quotes at 3:00 p.m. each day.

Figure 2. Bid-Ask Spreads during the Global Liquidity Crisis



Note: The chart shows average bid-ask spread for large-cap U.S. stocks, the equity volatility index VIX, and the interest rate spread between LIBOR and Treasury bills (TED) from July 2006 to July 2009. Each of the series has been scaled to have a zero mean and a unit standard deviation.

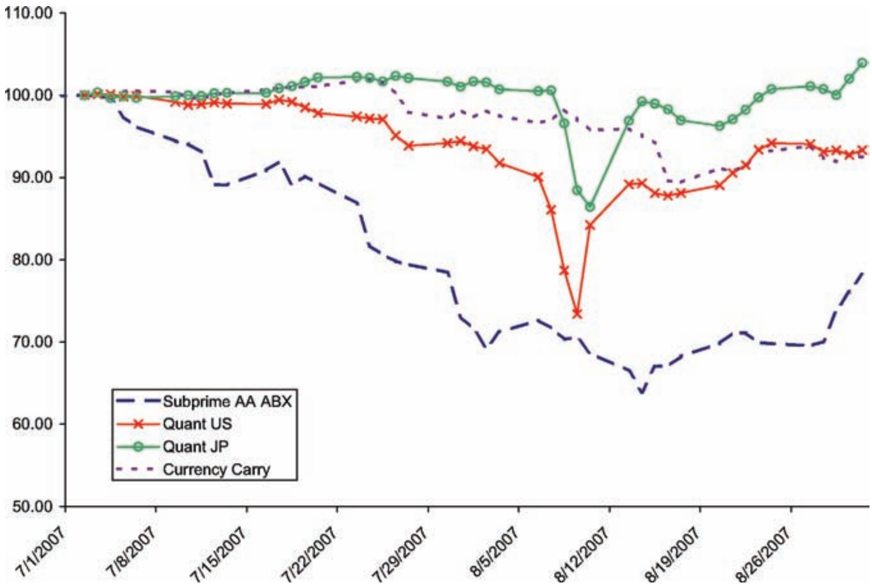
are exposed to the VIX directly or indirectly. We see that there is a close co-movement between bid-ask spreads and VIX throughout the crisis, and also a visible connection to the TED spreads, indicating a link between market liquidity, funding liquidity, and volatility as explained by the theory in section 3.[4]

2.2 Spirals and Spillover

The shock to the subprime credit market spread quickly spilled over to other markets, as seen in figure 3. It spread to credit markets more broadly, to money markets, to quant equity in the United States, and later to quant equity in Japan and beyond. Next, the

[4]See Nagel (2009) for further evidence.

Figure 3. Spillover in the Beginning of the Crisis (July–August 2007)



Note: The figure shows how the crisis started with a decline in the price of subprime credit. In July 2007, quantitative long-short stock-selection strategies based on value and momentum in the United States (Quant US) began to experience losses, and subsequently this spilled over to similar strategies in Japan (Quant JP). The currency carry trade experienced an unwinding in the end of August. The price series and cumulative returns have been normalized to be 100 in the beginning of July.

liquidity shock started to affect the currency carry trade, commercial mortgage-backed securities, convertible bonds, event arbitrage, and fixed-income markets. Investors ran for the exit in one market after another, and the rush to the exits reached its peak after Lehman failed in September 2008. Market liquidity deteriorated in most markets and vanished almost completely in many over-the-counter markets. For instance, dealers in emerging-market interest rate swaps largely stopped quoting bid and ask prices. The extreme market liquidity risk was complemented by extreme funding liquidity risk as haircuts and margin requirements went up and certain securities became unacceptable as collateral for many counterparties. As the funding situation for banks and other financial institutions deteriorated, central banks globally had to expand their balance sheets substantially to deal with the ramifications.

In many different markets, it turned out that levered liquidity-providing traders had some common features in their portfolios. Despite their different investment philosophies and analysis, one manager's long position was another manager's long more often than it was a short. Is this herding in the sense that traders are buying something because they have heard that others are buying it? It need not be. Consider the theoretical benchmark of each trader doing his independent analysis on common data: in a standard Markowitz/CAPM world, all investors are holding exactly the same portfolio, namely the "tangency" portfolio with the highest risk-adjusted return. While real-world traders are far more diverse than this theoretical benchmark since they use different methods to estimate risk and expected return, it is natural to expect that at least the most sophisticated traders in a specific market have some overlap in their portfolios since they are striving toward the same goal.

2.3 What Is Quant?

Before showing more detailed evidence from the quant event, it is useful to briefly explain what quant is. Most traders—e.g., proprietary traders and hedge funds—engage in "discretionary trading," meaning that the decision to buy or sell is at the trader's discretion given his overall assessment based on experience, various kinds of information, intuition, etc.

This traditional form of trading can be seen in contrast to "quantitative trading," or "quant" for short.[5] Quants define the trading rules explicitly and build systems that implement them systematically. They try to develop a small edge on each of many small diversified trades using sophisticated processing of ideas that cannot be easily processed using nonquantitative methods. To do this, they use tools and insights from economics, finance, statistics, math,

[5]Quantitative traders are close cousins to, but perform different roles than, the "sell-side quants" described in Emanuel Derman's interesting autobiography *My Life as a Quant.* Sell-side quants provide analytical tools that are helpful for hedging, risk management, discretionary traders, clients, and other purposes.

computer science, and engineering, combined with lots of data (public and proprietary) to identify relationships that market participants may not have incorporated in the price immediately. They build computer systems that generate trading signals based on these relationships, perform portfolio optimization in light of trading costs, and trade using automated execution schemes that route hundreds of orders every few seconds. In other words, trading is done by feeding data into computers that run various programs with human oversight.

Some quants focus on high-frequency trading where they exit a trade minutes or days after it was entered. Others focus on lower-frequency trades, and still others do some of both. For instance, "value" strategies seek to buy cheap securities (and short overvalued ones), and, since such securities often stay cheap for months, this is a low-frequency (that is, low-turnover) strategy. "Momentum" strategies buy securities that recently performed relatively well, while shorting underperforming securities, based on the idea that such recent performance has tended to continue more often than it has reversed. (Value and momentum in many asset classes is discussed in Asness, Moskowitz, and Pedersen 2008). Another strategy is to provide liquidity to securities with temporary order imbalances that are associated with short-term price reversals, and this is inherently a high-frequency strategy.

While discretionary trading has the advantages of a tailored analysis of each trade and the use of soft information such as private conversations, its labor-intensive method implies that only a limited number of securities can be analyzed in depth, and the discretion exposes the trader to psychological biases. Quantitative trading has the advantage of discipline, an ability to apply a trading idea to a wide universe of securities with the benefits of diversification, efficient portfolio construction, and an ability to "back test" the strategy, meaning that one can check how well one would have done by following such a strategy in the past. Of course, past success does not guarantee future success, but at least it rules out using rules that never worked and, to a degree, psychological biases. The quant method's disadvantage is its reliance on hard data and the computer program's limited ability to incorporate real-time human judgment.

2.4 The Quant Event of August 2007

Quants trade in many markets and, in particular, take significant long and short positions in stocks. By mid-2007, quant-managed stock portfolios had about $300–$400 billion long and short positions in equities by some estimates. In August, a significant liquidity event occurred in which some quants were forced to unwind and others also reduced positions. The buying and selling pressure was immense. It consisted of hundreds of billions of dollars as aggregate positions were reduced approximately by half according to some prime broker estimates. While the effects were clear to quants, they were at first largely hidden to outsiders since the trades were spread over thousands of stocks, with some stock prices being pushed up and others pushed down. To “see” the event, one must look through the lens of a typical quant’s diversified long-short portfolio at a high frequency.

While the precise origin of the event is hard to determine with certainty, the following is a likely sequence of events (see also Khandani and Lo 2007, 2009). In June 2007, many banks and some hedge funds experienced significant losses due to credit exposure or to the ripple effects of the credit turmoil. In July, some started to reduce risk and raise cash by selling liquid instruments such as their stock positions, hurting the returns of common stock-selection strategies. Some banks even closed down some of their trading desks, including quant proprietary trading operations. Simultaneously, some hedge funds were experiencing redemptions. For instance, some funds of funds (hedge funds investing in other hedge funds) hit loss triggers and were forced to redeem from the hedge funds they were invested in, including quants.

The quant value strategy in particular experienced losses in July. Money was pulled out of stocks that were potential leveraged buyout (LBO) candidates because of the reduced access to leverage. These were stocks that LBO firms considered cheap based on strong value and cash flow characteristics, and, since quants typically consider similar characteristics, this hurt value strategies. Value strategies were also hurt because the cheap stocks on the long side had more leverage and therefore more sensitivity to widening credit spreads.

On Monday, August 6, 2007, a major delevering of quant strategies began. Figure 1, panel A shows the cumulative return to an industry-neutral long-short portfolio based on value and momentum signals. We see that the portfolio incurs substantial losses from Monday, August 6 through Thursday, August 9, as quants were unwinding, and then recovers much of its losses on Friday and Monday as the unwinding ended and some traders may have reentered their positions.

The smoothness of the graph is noteworthy. It is not an artifact of drawing the graph by connecting a few dots—the graph uses minute-by-minute data. The smoothness is due to a remarkable short-term predictability arising from the selling pressure and subsequent snap back. For instance, on Tuesday, August 7, the strategy was down 90 percent of the ten-minute intervals, and it was up 75 percent of the ten-minute intervals on that Friday. This predictability provides strong evidence of a liquidity event, as it is statistically significantly different from the behavior of a random walk.

Another striking feature of the graph is the sheer magnitude of the drop and rebound. The strategy has been scaled to have an annualized volatility of about 6 percent using a well-known commercial risk model. The strategy loses about 25 percent in four days, about four annual standard deviations and more than thirty standard deviations based on the four-day volatility of $(4/260)^{1/2} * 6\% = 0.74\%$. The thirty standard deviations must be interpreted correctly. This number does not mean that this was a thousand-year flood and can never happen again. It means that the event was a liquidity event, not based on stock fundamentals, and that this risk model does not capture liquidity risk and the endogenous amplification by the liquidity spirals. Stock price fluctuations are driven primarily by economic news about fundamentals most of the time, but during a liquidity crisis, price pressure can have a large effect. Hence, the distribution of stock returns can be seen as a mixture of two distributions: shocks driven by fundamentals mixed with shocks driven by liquidity effects. Since fundamentals are usually the main driver, conventional risk models are calibrated to capture fundamental shocks, and liquidity tail events are not well captured by such models. Hence, thirty standard deviations means that the event is statistically significantly different from a fundamental shock and, hence, must have been driven by a liquidity event.

The quant event started with the U.S. value strategy and spilled over to global markets—e.g., Japan, as seen in figure 3—and to certain other types of quant factors, though not all. For instance, even though momentum is normally negatively correlated to value, these strategies became positively correlated as they both experienced significant losses during the unwind. Also, certain high-frequency strategies that rely on price reversals were affected due to the unusual amount of price continuation.

It is curious to notice the resemblance between the actual data from 2007 in panel A and the price path in panel B predicted by the Brunnermeier and Pedersen (2005) model that we discuss next: both graphs go down smoothly, go back up smoothly, and, finally, level off below where they started.

3. Theoretical Background

To understand the mechanisms that drive liquidity crises, I first consider a stylized model of running for the exit. I then show how endogenous systemic liquidity risk arises as agents run for fear of being trampled, and, lastly, I discuss the asset pricing implications.

3.1 Running for the Exit

There are two "arbitrageurs," and we seek to model the notion that they might run for the exit using a simplified version of the model of Brunnermeier and Pedersen (2005). They trade with each other and with a group of "long-term traders," who in the aggregate give rise to a demand curve with a slope of 1. In order words, if the arbitrageurs buy one share, the equilibrium price goes up by 1, and if they sell one share, the price drops by 1.[6]

The agents trade at times 0, 1, and 2, and the asset pays off its dividend at the final time 3. The arbitrageurs can hold at most ten shares due to limited capital and margin constraints.

At time 0, the arbitrageurs each buy eight shares since their information indicates that the asset is undervalued at its price per share of, say, \$116. For simplicity, we take the size of this initial purchase as given, but Brunnermeier and Pedersen (2005) show how it can be derived as an optimal trade-off between potential liquidation

[6] Evidence on downward-sloping demand curves is provided by Shleifer (1986), Wurgler and Zhuravskaya (2002), and others.

costs (discussed below) and the benefits of buying an undervalued asset early. The arbitrageurs have similar positions because they are looking for the same thing, namely securities that offer high returns.

At time 1, one of the arbitrageurs might lose money in another trade (say, subprime debt), which forces him to sell his entire position of eight shares in the asset under consideration, or, alternatively, neither arbitrageur suffers such a shock. In case neither suffers a shock, both arbitrageurs buy two additional shares to be fully invested up to their limit, which pushes the price to $116 + 2 + 2 =$ \$120, where it stays until the final dividend is paid at time 3 (a dividend which the arbitrageurs expect to be above \$120).

Let us consider the more interesting case where one arbitrageur is distressed and is forced to sell. Suppose first that the other arbitrageur doesn't trade anything at this time (e.g., because he does not know about the other arbitrageur's distress). In this case, the distressed selling pushes the price down to $116 - 8 =$ \$108 for an average execution price of \$112.

What if the other arbitrageur knew that this selling pressure was coming? Then he would be able to predict that the price would drop and, as a result, that he would incur a mark-to-market loss. Therefore, he would optimally also sell at time 1 and buy back his position at time 2. The price path associated with this selling followed by buying is illustrated in figure 1, panel B. Since both traders are selling at time 1, the price drops from \$116 to $116 - 8 - 8 =$ \$100, with an average execution price of \$108. When the nondistressed trader is buying back his eight shares at time 2, he pushes the price back up to $100 + 8 =$ \$108, with an average execution price of \$104, and he then buys two additional shares, pushing the price to \$110. Importantly, given that he sold for \$108 on average and bought back at \$104, his wealth is \$4 higher than if he had not traded.

The drop and rebound in prices resembles that of the quant event (panel A) and other liquidity events, and it means that the distressed trader's losses are worsened. When he liquidates alone, he receives an average execution price of \$112, but with both traders selling, the average execution price is \$108 since the price drops more sharply (and he does not enjoy the rebound that happens after he is out of the market).

If the other trader manages to sell before the distressed trader (front running), then the distressed trader would realize an even lower liquidation value. Hence, the distressed trader rushes to the exit as fast as he can. In fact, both arbitrageurs run for the exit, and this exacerbates the distressed trader's losses.

In this simple example, the nondistressed trader sold and bought back to enhance his overall profit in the long run. However, the real world is more complex than that. If someone else is pushing down the price of the assets you hold, you could soon become distressed yourself. Bernardo and Welch (2004) and Brunnermeier and Pedersen (2005) consider this endogenous distress and show that there can be multiple equilibria: "panics" can occur when people sell because they fear others will sell, leading to more failures than in equilibria where traders stay "calm." In a panic equilibrium, the traders thus "step on" each other as they run for the exit. This leads to systemic risk: a fear of forced selling leads to selling, and selling leads to forced selling, as discussed further in the next subsection.

We have seen that liquidity evaporates exactly when it is most needed in this setting, but, with more than one nondistressed trader, will competitive forces ensure that the price is at its efficient level? The answer is: not generally, since running for the exit occurs even with multiple traders in this setting.

Could this be because there are only a few time periods? In other words, since the nondistressed traders ultimately want to hold this position, why don't they compete in being the first to buy back the position in a way that makes the price stabilize without overshooting? Brunnermeier and Pedersen (2005) address this by allowing many trading opportunities in a continuous-time model and show that this competition does in fact occur if the nondistressed traders have enough excess capital to absorb the shares that the distressed traders need to sell.[7] However, if the nondistressed traders don't have enough capital to absorb all these shares, then the price must ultimately fall, and this price drop gives an incentive to sell early (at a high price) and buy back later (at a low price), leading to price overshooting.

---

[7]Figure 1, panel B is in fact from a continuous-time example in that paper (the numbers in the discrete-time example above are chosen so the examples match).

Hence, running for the exit happens only when the selling pressure is large relative to the available capital on the sideline. Said differently, it happens when the trade is crowded and a significant part of the crowd is near or over the edge, with few "outsiders" ready to step in (e.g., because it takes time to build the expertise and infrastructure to trade these assets).

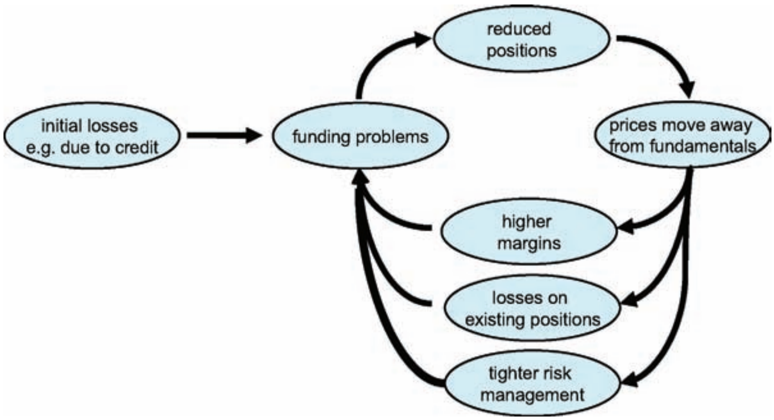
In related work, Carlin, Lobo, and Viswanathan (2007) consider a model in which traders cooperate most of the time due to repeated interaction, but episodic liquidity crises occur. Chu, Lehnert, and Passmore (2009) extend the framework to multiple assets and consider cross-asset effects and possible interventions. Duffie, Gârleanu, and Pedersen (2007) consider a funding shock among a group of investors in an over-the-counter search market and show how the speed of the drop-and-rebound price signature depends on the market liquidity. In a more illiquid market (with more time-consuming search), the drop-and-rebound occurs over a longer time period and with a deeper drop in prices. Lagos, Rocheteau, and Weill (2007) model dealers' liquidity provision following a crash.

3.2 Running for Fear of Being Trampled: Endogenous Liquidity Risk

It is interesting to dig a level deeper into the main drivers of forced selling and the mechanisms that make the run spiral into a panic liquidation. As illustrated in figure 4, suppose an initial shock (a shout of "fire") leads to losses of traders—e.g., subprime losses due to a drop in house prices, as in the most recent liquidity crisis. Traders then reduce their positions, which pushes prices away from fundamentals, and liquidity spirals make the effect disproportionally larger than the initial shock. This endogenous liquidity risk has several key elements, discussed below.

First, as prices move away from fundamentals, the market becomes illiquid, volatility picks up, and these effects make it riskier for others to finance the traders' positions. Therefore, margin requirements (or haircuts) increase, and, in extreme cases, counterparties refuse to lend against certain securities as collateral. For instance, after Lehman's failure in September 2008, it became difficult to borrow against certain illiquid fixed-income securities. High margins and inability to finance positions naturally worsens levered

Figure 4. Liquidity Spirals



Note: The chart shows how an initial shock to financial institutions' funding is amplified by increasing margins (margin spirals), losses on existing positions (loss spiral), and tightened risk management (risk management spiral).

traders' funding problems, leading to further selling, and so on as the margin spiral swirls (Brunnermeier and Pedersen 2009).

The second effect is that prices start moving against the liquidity-providing traders' positions, leading to losses, inducing further unwinding, and the losses spiral. This is worsened if poor performance leads to further reductions in capital—for instance, if a hedge fund has redemptions, or a bank (or multistrategy hedge fund) moves capital away from one trading desk to use it elsewhere (Shleifer and Vishny 1997, Xiong 2001, Gromb and Vayanos 2002, and Vayanos 2004).

Thirdly, many traders' risk management tightens at these times since volatility increases—especially the volatility measured over a possible liquidation period which lengthens due to illiquidity—and one trader's prudent risk management can be another trader's vanishing market liquidity and funding. As risk management tightens, traders sell to reduce risk and banks cut back the funding they provide, leading to further funding problems, and a risk management spiral arises (Gârleanu and Pedersen 2007). Portfolio insurance is an extreme example of this, and stop-loss orders is another example. Further, when banks face losses, depositors may withdraw capital to limit their risk, other creditors may not roll over debt, counterparties shy away, and this can lead to a bank run (Diamond and Dybvig 1983, Holmström and Tirole 1997, Allen and Gale 2007, and Acharya, Gale, and Yorulmazer 2009).

Figure 2 shows how market liquidity, funding liquidity, and volatility spiraled in the liquidity crisis that started in 2007. Mitchell, Pedersen, and Pulvino (2007) document how these liquidity spirals played out in the convertible bond market in 1998 and 2005, and in the merger market in 1987. Similar—and, in fact, larger—liquidity spirals have caused havoc in the convertible bond and fixed-income markets during the recent crisis. Brunnermeier and Pedersen (2009) discuss how the interplay between market liquidity and funding liquidity can help explain commonality in liquidity across securities and markets, flight to quality, the fact that liquidity is poor in down markets, and other empirical phenomena documented by Chordia, Roll, and Subrahmanyam (2000), Hasbrouck and Seppi (2001), Huberman and Halka (2001), Coughenour and Saad (2004), Chordia, Sarkar, and Subrahmanyam (2005), Hameed, Kang, and Viswanathan (2005), and Hendershott, Moulton, and Seasholes (2006). Adrian and Shin (2009) provide evidence that broker-dealers have procyclical leverage, consistent with the margin spiral. Brunnermeier, Nagel, and Pedersen (2008) document that speculators use the carry trade in currency markets, and their unwinding during funding crises leads to currency crash risk. The crash risk discourages speculators from taking large enough positions to enforce the uncovered interest rate parity (UIP), and thus funding liquidity risk can help explain the forward premium puzzle.

3.3 Implications for Asset Pricing

Given the costs incurred in liquidity crises, investors must manage their liquidity risk and be compensated for taking on liquidity risk. Hence, securities with more liquidity risk must offer a higher return to compensate investors for incurring the risk. Acharya and Pedersen (2005) consider such a model in which transaction costs change unpredictably over time.[8] Since investors care about return net of costs, the CAPM holds in net returns. A security's required return

[8]Amihud and Mendelson (1986) show how required returns depend on average market liquidity.

depends, therefore, on its net-return beta, that is, the covariance $cov_t(r^i_{t+1}-c^i_{t+1}, r^M_{t+1}-c^M_{t+1})$ between the return r^i net of trading costs c^i with the market return r^M net of market trading costs c^M. This covariance can be separated into four terms, namely the standard market beta coming from $cov_t(r^i_{t+1}, r^M_{t+1})$ as well as three liquidity risks: (i) $+cov_t(c^i_{t+1}, c^M_{t+1})$, the compensation for the commonality of liquidity as investors require higher returns for securities with higher trading costs during liquidity crises when trading costs are high in general; (ii) $-cov_t(r^i_{t+1}, c^M_{t+1})$ with a negative sign, meaning that investors require higher return for a security that has a low return during liquidity crises when c^M is high; and (iii) $-cov_t(c^i_{t+1}, r^M_{t+1})$, implying compensation for high illiquidity in a down market.

This market-liquidity-adjusted CAPM implies that, when the market becomes illiquid and liquidity risk goes up, the required return rises ($\frac{\partial}{\partial C^i_t}E_t(r^i_{t+1} - r^f) > 0$) and, therefore, contemporaneous returns are low ($cov_t(c^i_{t+1}, r^i_{t+1}) < 0$). For instance, due to the liquidity risk that arose when the banking system faced trouble in 2008, the required return rose, which contributed to the downfall in prices. Amihud (2002), Pastor and Stambaugh (2003), and Acharya and Pedersen (2005) find empirical evidence for the pricing of liquidity risk.

Gârleanu and Pedersen (2009) consider the asset pricing effects of funding liquidity risk—as opposed to market liquidity risk discussed above—by showing how a security's margin requirement can increase its required return.[9] The paper considers a relatively minimal extension of the Lucas tree model, having two groups of agents with different risk aversion facing margin requirements. The more risk-tolerant agents (which can be interpreted as the financial institutions) use leverage. Hence, after negative fundamental shocks, they incur large losses and ultimately hit their margin constraint.

The paper shows that a security's required return is the sum of its beta times the risk premium (as in the standard CAPM or consumption CAPM), and its margin requirement times the cost of capital—a compensation for funding liquidity risk. The paper finds large asset pricing effects by explicitly solving the model and calibrating it using

[9] He and Krishnamurthy (2008) consider a model where intermediaries are constrained in raising equity instead of debt.

realistic parameters, and the model can help explain the deviations from the law of one price during the recent and previous crises. For instance, corporate bonds have traded at higher yield spreads than corresponding credit default swaps (CDS), giving rise to an apparent arbitrage called the CDS-bond basis. This can be explained by the fact that the margin requirement on the bond is higher than that of the CDS. Hence, investors require a higher yield on a high-margin bond than on a low-margin CDS when capital is scarce. Further, the basis varies in the time series with the tightness of credit and in the cross-section with the margin differential, consistent with the model. Another stark failure of the law of one price is the failure of covered interest rate parity (CIP), which was driven by a dollar funding need by global financial institutions combined with a limited ability to arbitrage the deviation due to binding margin requirements.

The Federal Reserve and other central banks have tried to improve the financing environment by providing lending programs that offer collateralized loans with lower margins than otherwise available. Since lower margins lead to lower required returns, this leads to higher prices of debt securities, which ultimately results in improved credit conditions for businesses and households. Ashcraft, Gârleanu, and Pedersen (2009) use a survey conducted by the Federal Reserve to see how market participants change their bids in response to lower margins/haircuts. The evidence suggests that the effect of margins is very large.

Margins, and financing conditions more broadly, are important both for asset prices and for financial firms' ability to operate, and these affect the real economy through consumers' and firms' access to credit and ability to issue securities. Hence, lending programs, broadening the acceptable collateral, and setting margins/haircuts are important monetary policy tools during liquidity crises.

4. Conclusion

The severe consequences for the global economy brought about by the recent liquidity crisis highlight the importance of liquidity risk. Liquidity shocks are sudden, spill over across markets where levered traders have positions, and affect mostly risky and illiquid securities with large increases in margins. Liquidity events can happen even in the most liquid markets in the world, as was clearly illustrated by the sharp drop and rebound in the values of quant positions in U.S. large-cap stocks during August 2007.

Investors need to manage both their funding liquidity risk—including their cash management, the financing terms (margins/haircuts), and the risk of changes in financing or equity redemptions—and their market liquidity risk, including the trading costs, possible hikes in trading costs, the time it takes to unwind positions in an orderly fashion, and the risk of predatory trading (Brunnermeier and Pedersen 2005, 2009).

Further, investors need to be compensated for taking liquidity risk. Their pricing models should capture market liquidity risk (Acharya and Pedersen 2005) and funding liquidity risk (Gârleanu and Pedersen 2009).

While predicting liquidity crises in advance is very challenging, it is useful to understand whether price drops that already occurred were due to liquidity or fundamentals. This is because liquidity events present both risks and opportunities—liquidity-induced price drops tend to revert, and investors with dry powder can try to capture this rebound.

During a liquidity crisis, central banks can use unconventional monetary tools that improve the financing environment—e.g., by offering collateralized loans at lower (but still prudent) haircuts/margins—and, in good times, central banks need to reduce banks' incentive to take on systemic risk (Acharya et al. 2009, Ashcraft, Gârleanu, and Pedersen 2009, Cúrdia and Woodford 2009, Gertler and Karadi 2009, and Reis 2009).

References

Acharya, V. V., D. M. Gale, and T. Yorulmazer. 2009. "Rollover Risk and Market Freezes." Working Paper, New York University.

Acharya, V. V., and L. H. Pedersen. 2005. "Asset Pricing with Liquidity Risk." *Journal of Financial Economics* 77 (2): 375–410.

Acharya, V. V., L. H. Pedersen, T. Philippon, and M. Richardson. 2009. "Regulating Systemic Risk." In *Restoring Financial Stability: How To Repair a Failed System*, ed. V. V. Acharya and M. Richardson, 283–304. Wiley.

Adrian, T., and H. Shin. 2009. "Liquidity and Leverage." Forthcoming in *Journal of Financial Intermediation.*

Allen, F., and D. Gale. 2007. *Understanding Financial Crises.* Oxford University Press (USA).

Amihud, Y. 2002. “Illiquidity and Stock Returns: Cross-Section and Time-Series Effects.” *Journal of Financial Markets* 5 (1): 31–56.

Amihud, Y., and H. Mendelson. 1986. “Asset Pricing and the Bid-Ask Spread.” *Journal of Financial Economics* 17 (2): 223–49.

Amihud, Y., H. Mendelson, and L. H. Pedersen. 2006. “Liquidity and Asset Prices.” *Foundations and Trends in Finance* 1 (4).

Ashcraft, A., N. Gârleanu, and L. H. Pedersen. 2009. “Haircuts or Interest-Rate Cuts: New Evidence on Monetary Policy.” Working Paper, Federal Reserve Bank of New York, University of California, Berkeley, and New York University.

Asness, C. S., T. J. Moskowitz, and L. H. Pedersen. 2008. “Value and Momentum Everywhere.” Working Paper, AQR Capital Management, University of Chicago Booth School of Business, and New York University.

Bernardo, A. E., and I. Welch. 2004. “Liquidity and Financial Market Runs.” *Quarterly Journal of Economics* 119 (1): 135–58.

Brunnermeier, M. K. 2009. “Deciphering the Liquidity and Credit Crunch 2007–2008.” *Journal of Economic Perspectives* 23 (1): 77–100.

Brunnermeier, M. K., S. Nagel, and L. H. Pedersen. 2008. “Carry Trades and Currency Crashes.” *NBER Macroeconomics Annual* 23: 313–48.

Brunnermeier, M. K., and L. H. Pedersen. 2005. “Predatory Trading.” *Journal of Finance* 60 (4): 1825–63.

———. 2009. “Market Liquidity and Funding Liquidity.” *Review of Financial Studies* 22 (6): 2201–38.

Carlin, B. I., M. S. Lobo, and S. Viswanathan. 2007. “Episodic Liquidity Crises: Cooperative and Predatory Trading.” *Journal of Finance* 62 (5): 2235–74.

Chordia, T., R. Roll, and A. Subrahmanyam. 2000. “Commonality in Liquidity.” *Journal of Financial Economics* 56 (1): 3–28.

Chordia, T., A. Sarkar, and A. Subrahmanyam. 2005. “An Empirical Analysis of Stock and Bond Market Liquidity.” *Review of Financial Studies* 18 (1): 85–129.

Chu, C. S., A. Lehnert, and W. Passmore. 2009. “Strategic Trading in Multiple Assets and the Effects on Market Volatility.” *International Journal of Central Banking* 5 (4): 143–172.

Coughenour, J. F., and M. M. Saad. 2004. "Common Market Makers and Commonality in Liquidity." *Journal of Financial Economics* 73 (1): 37–69.

Cúrdia, V., and M. Woodford. 2009. "Credit Frictions and Optimal Monetary Policy." Working Paper, Federal Reserve Bank of New York.

Diamond, D. W., and P. H. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91 (3): 401–19.

Duffie, D., N. Gârleanu, and L. H. Pedersen. 2007. "Valuation in Over-the-Counter Markets." *Review of Financial Studies* 20 (6): 1865–1900.

Gârleanu, N., and L. H. Pedersen. 2007. "Liquidity and Risk Management." *American Economic Review* 97 (2): 193–97.

———. 2009. "Margin-Based Asset Pricing and Deviations from the Law of One Price." Working Paper, University of California, Berkeley, and New York University.

Gertler, M., and P. Karadi. 2009. "A Model of Unconventional Monetary Policy." Working Paper, New York University.

Gorton, G. B. 2008. "The Panic of 2007." Yale School of Management.

Gromb, D., and D. Vayanos. 2002. "Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs." *Journal of Financial Economics* 66 (2–3): 361–407.

Hameed, A., W. Kang, and S. Viswanathan. 2005. "Asymmetric Comovement in Liquidity." Mimeo, Duke University.

Hasbrouck, J., and D. J. Seppi. 2001. "Common Factors in Prices, Order Flows, and Liquidity." *Journal of Financial Economics* 59 (3): 383–411.

He, Z., and A. Krishnamurthy. 2008. "Intermediated Asset Prices." Working Paper, Northwestern University.

Helbing, D. 2001. "Traffic and Related Self-Driven Many-Particle Systems." *Reviews of Modern Physics* 73 (4): 1067–1141.

Helbing, D., L. Buzna, A. Johansson, and T. Werner. 2005. "Self-Organized Pedestrian Crowd Dynamics: Experiments, Simulations, and Design Solutions." *Transportation Science* 39 (1): 1–24.

Helbing, D., I. Farkas, and T. Vicsek. 2000. "Simulating Dynamical Features of Escape Panic." *Nature* 407 (6803): 487–90.

Hendershott, T., P C. Moulton, and M. S. Seasholes. 2006. "Capital Constraints and Stock Market Liquidity." Working Paper, University of California, Berkeley.

Holmström, B., and J. Tirole. 1997. "Financial Intermediation, Loanable Funds, and the Real Sector." *Quarterly Journal of Economics* 112 (3): 663–91.

Huberman, G., and D. Halka. 2001. "Systematic Liquidity." *Journal of Financial Research* 24: 161–78.

Khandani, A. E., and A. W. Lo. 2007. "What Happened to the Quants in August 2007?" *Journal of Investment Management* 5 (4): 29–78.

———. 2009. "What Happened to the Quants in August 2007? Evidence from Factors and Transactions Data." Working Paper, Massachusetts Institute of Technology.

Krishnamurthy, A. 2009. "Amplification Mechanisms in Liquidity Crises." NBER Working Paper No. 15040.

Lagos, R., G. Rocheteau, and P.-O. Weill. 2007. "Crashes and Recoveries in Illiquid Markets." Working Paper, New York University.

Mitchell, M., L. H. Pedersen, and T. Pulvino. 2007. "Slow Moving Capital." *American Economic Review* 97 (2): 215–20.

Nagel, S. 2009. "Evaporating Liquidity." Working Paper, Stanford University.

Pastor, L., and R. F. Stambaugh. 2003. "Liquidity Risk and Expected Stock Returns." *Journal of Political Economy* 111 (3): 642–85.

Reis, R. 2009. "Where Should Liquidity Be Injected During a Financial Crisis?" Working Paper, Columbia University.

Shleifer, A. 1986. "Do Demand Curves for Stocks Slope Down?" *Journal of Finance* 41 (3): 579–90.

Shleifer, A., and R. W. Vishny. 1997. "The Limits of Arbitrage." *Journal of Finance* 52 (1): 35–55.

Vayanos, D. 2004. "Flight to Quality, Flight to Liquidity, and the Pricing of Risk." NBER Working Paper No. 10327.

Wurgler, J., and E. V. Zhuravskaya. 2002. "Does Arbitrage Flatten Demand Curves for Stocks?" *Journal of Business* 75 (4): 583–608.

Xiong, W. 2001. "Convergence Trading with Wealth Effects: An Amplification Mechanism in Financial Markets." *Journal of Financial Economics* 62 (2): 247–92.