



INTERNATIONAL JOURNAL OF CENTRAL BANKING

Special Issue: Stress Testing—The Link between Macro and Micro

Introduction

Iman van Lelyveld

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Antonella Foglia

Towards a Framework for Quantifying Systemic Stability
Piergiorgio Alessandri, Prasanna Gai, Sujit Kapadia, Nada Mora, and Claus Puh

Interdependencies between Expected Default Frequency and the Macro Economy

Per Åsberg Sommar and Hovick Shahnazarian

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Eivind Bernhardsen and Bjørne Dyre Syversten

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Introduction <i>Iman van Lelyveld</i>	1
Stress Testing Credit Risk: A Survey of Authorities' Approaches <i>Antonella Foglia</i>	9
Towards a Framework for Quantifying Systemic Stability <i>Piergiorgio Alessandri, Prasanna Gai, Sujit Kapadia, Nada Mora, and Claus Pühr</i>	47
Interdependencies between Expected Default Frequency and the Macro Economy <i>Per Åsberg Sommar and Hovick Shahnazarian</i>	83
Stress Testing the Enterprise Sector's Bank Debt: A Micro Approach <i>Eivind Bernhardsen and Bjørne Dyre Syversten</i>	111
Crash Testing German Banks <i>Klaus Duellmann and Martin Erdelmeier</i>	139
Macroeconomic Default Modeling and Stress Testing <i>Dietske Simons and Ferdinand Rolwes</i>	177
How to Find Plausible, Severe, and Useful Stress Scenarios <i>Thomas Breuer, Martin Jandačka, Klaus Rheinberger, and Martin Summer</i>	205

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Stress Testing—The Link between Macro and Micro

Introduction to a Special Issue of the International Journal of Central Banking

Iman van Lelyveld

The events since autumn 2007 have highlighted a need to test the assumptions of industry risk measurement and pricing models and anticipate the potential impact of changes in financial market conditions that are significantly outside the bounds of historical norms. Stress testing is a generic term that is frequently used to describe a variety of techniques that are applied to assess the importance of assumptions that underlie economic models and forecasts.

Stress tests can be simple or complex, and they can be performed at many different levels of aggregation, including at the portfolio level, at the institution level, or at an aggregate level for a nation's entire financial system. Stress tests may, for example, be used to assess the economic impact of a single economic shock like a drop in equity prices or a spike in default rates. Alternatively, stress tests may involve the use of multiple econometric models to arrive at an assessment of the systemwide implications of a complex macroeconomic scenario. Various types of stress tests are regularly employed by individual banks, central banks, and financial supervisory authorities to assess the potential impacts of events that include slowdowns in economic activity, breakdowns in market mechanisms, violations of modeling assumptions, or changes in historical correlation and investor behavior patterns.

Stress testing features prominently in the context of Basel II, where stress tests are (i) a means for adjusting average probabilities of default (PDs) for stress conditions (Pillar 1); (ii) a procedure for assessing the robustness of IRB risk parameters such as PD or loss given default (LGD) (Pillar 1); (iii) a requirement to assess the impact of economic recession on regulatory capital requirements (Pillar 1 “cyclicality stress test”); and (iv) a means to assess the global impact on a bank's risk profile and capital adequacy of adverse events or changes in market conditions including counterparties' ability to honor financial contracts (Pillar 2).

While the need for sound stress-testing practices is a cornerstone of modern risk management and a requirement in both Pillar 1 and

2 of Basel II, stress-testing practices within banks remain underdeveloped. The 2005 report by the Committee on the Global Financial System (CGFS 2005) found that stress-testing practices in the institutions it surveyed included only a limited number of risks and focused on traded portfolios. Subsequent events have shown that banks have indeed relied too much on models that were based on normal states of the world.

In response to recent events, many supervisory groups have reemphasized the need for enhanced stress-testing practices, and many are in the process of undertaking studies that will lead to enhanced supervisory stress-testing guidance.¹ In contrast to these newly initiated supervisory efforts, most of the papers in this special issue were initiated much earlier by the Research Task Force Stress Testing subgroup (RTF ST group, mandated by the Basel Committee on Banking Supervision). This group was formed more than three years ago with a remit to study stress-testing issues related to infrequently traded and illiquid portfolios and best practice in modeling the link between macroeconomic risk drivers and the microeconomic models that are typically used to estimate credit losses in financial stability stress-testing models. Note that the boundary between what constitutes a traded or a nontraded portfolio is sometimes an arbitrary one, and recent events have shown that this boundary will shift in times of turmoil. Nevertheless, some exposures are more difficult to trade (and evaluate) than others.

For the longer term, formulating macroprudential policy might be another area where the papers in this volume might inform the debate. A consensus on what macroprudential policy entails precisely is yet to emerge, but understanding the link between macro risk drivers and micro credit-risk measures is surely important if we, for instance, want to curb excessive risk buildup in financial systems.

The RTF ST group has delivered on its mandate, and its findings and recommendations have been taken on board by the other, more policy-oriented groups of the Basel Committee. In addition, the group members produced research papers and organized a conference

¹Supervisory groups that have studied stress tests include the FSF Working Group on Market and Institutional Resilience, the Senior Supervisors Group, and various Basel Committee subgroups.

at which academics presented their insights.² The present special issue provides a selective overview of what the state of play is in modeling the link between macro risk drivers and micro risk measures, especially under stressed conditions.

This special issue has four types of papers. First, we start with a thorough overview of how central banks and supervisory agencies have approached this issue, written by Foglia. It is thus an overview of where we stand. Next, we have a paper (Allesandri et al.) that proposes a framework for modeling these issues. The third type consists of those papers that, in a sense, would fit in the framework just sketched, although they are well-rounded modeling exercises in their own right. The papers written by Åsberg and Shahnazarian, Bernhardsen and Syversten, Duellmann and Erdelmeier, and Simons and Rolwes fit in this category. Finally, we have a more methodological paper (Breuer et al.) on choosing scenarios. Let me now provide a little more detail on each of these papers.

The first paper, written by Antonella Foglia, documents how many central banks and authorities have built their own stress-testing tools in order to assess the stability of financial institutions.³ These tools differ from those of financial institutions in that they are more focused on the stability of the financial system as a whole. Considerable research has been carried out in this area with regard to the macro-micro link of credit risk, which might also be helpful for advancing credit-risk stress testing of infrequently traded portfolios. Therefore, a review of the analytical expertise developed at central banks and supervisory authorities can offer useful suggestions to supervisors to build up a common analytical background that may be useful in their review process of the various stress-testing requirements of the Basel II framework. This review might also be a helpful input in designing currently hotly debated macroprudential tools and policies.

In the surveyed approaches, system-focused stress testing can be seen as a multistage process with four steps. The first step is

² Available RTF ST group papers not included in this issue are Castrén, Déés, and Zaher (2008), Castrén, Fitzpatrick, and Sydow (2009), Gutiérrez Girault (2008), and Huang, Zhou, and Zhu (2009).

³ The paper is based on a survey on the quantitative methods used at selected central banks and supervisory agencies and has benefited greatly from group members' comments.

the design of the macroeconomic stress scenario. These are simulated using (i) structural macroeconomic models used by the central bank for macroeconomic forecasts and monetary policy analysis, (ii) (reduced-form) VAR models, and (iii) pure statistical methods that model the multivariate distribution of macro-financial variables using nonlinear dependency structures. In the second step of the process, macro variables are mapped to microeconomic indicators of banks' credit risk, usually by means of a "satellite" or auxiliary model. Unlike the macroeconomic model, the credit-risk satellite model is often estimated on individual banks' and even individual borrower data. These auxiliary models provide the link between macroeconomic stress scenario conditions and loan performance at the sector or bank level. One can distinguish between two modeling approaches: one based on data on *loan performance*, such as nonperforming loans (NPLs) and loan loss provisions (LLPs), and a second approach based on *micro-level data* related to the default risk of the household and/or the corporate sector. In both cases the dependent variable is regressed against various macroeconomic variables, such as the nominal interest rate, the inflation rate, the percentage change of real GDP, and the percentage change in the terms of trade. The macroeconomic model is then used in the third step to project the time path of macro variables under stress conditions. These estimates are fed into the auxiliary model of credit risk in order to determine the stressed credit quality indicators. The final step of the stress-testing process is usually an assessment of whether banks can withstand the assumed shock.

An important extension to the typical stress-test process focuses specifically on the impact measure. Instead of producing point estimates under any assumed stress scenarios, one important objective of the latest work is to derive a *credit loss distribution* for the banking system as a whole, extending to a systemwide analysis the framework adopted at a micro level by many financial institutions in their risk-management systems.

Finally, depending on the availability of detailed data, it is important to calculate the impact at the bank-by-bank level and not only for an aggregated portfolio of the entire system, in order to be able to understand the distribution throughout the system, which can be useful for understanding the potential for contagion and confidence effects on stability.

The paper by Allesandri et al. moves us from the current state of play to a proposed framework for assessing financial stability. In this model, the authors model the effects of macro shocks to UK banks on a highly detailed level. They incorporate asset-side feedbacks and network effects, and discern market, credit, and interest rate risk. The incorporation of network effects allows for contagious defaults as well.

Next we turn to four papers that each take a different approach to capture macro-micro linkage and answer the question of whether macroeconomic fluctuations are important for understanding credit risk. Researchers take two general approaches to answer this question. Some researchers analyze the dependence of credit-risk measures on observables like fluctuations in GDP. Others start from the idea that the business cycle is inherently unobservable.⁴ With regard to the first approach, researchers either model the (expected) default of individual firms⁵ or aggregate default numbers, sometimes at the sector level.⁶ The most important lessons from these papers seem to be as follows:

- Adding macroeconomic information improves our understanding of credit risk. The study by Åsberg and Shahnazarian shows that forecasting default frequencies can be improved by including macroeconomic factors. Bernhardsen and Syversten show a similar result.
- There is a need to be mindful of hidden risks. Duellmann and Erdelmeier show that although direct exposures to one sector (the automobile sector) might be limited, risks might still be significant because this particular sector affects many other sectors in case of stress. Simons and Rolwes show that our understanding of aggregate defaults improves if shocks common to all sectors are accounted for. This is an alternative way to model the sometimes unobserved correlation between risks across sectors.

⁴Papers following this approach which were presented at the conference are Bruche and González-Aguado (2009), Koopman, Lucas, and Schwaab (2008), and Jiménez and Mencía (2009).

⁵Åsberg and Shahnazarian (2009), Bernhardsen and Syversten (2009), Duellmann and Erdelmeier (2009).

⁶Simons and Rolwes (2009).

Last but not least, we turn to one of the key challenges in implementing stress tests: the scenarios chosen should be sufficiently bad to yield stressful results but not so outlandish that decision makers decide that such circumstances will not happen, at least not in their lifetime. This dynamic results in scenarios that are generally relatively mild. Breuer et al. propose a method to identify those scenarios that are extreme but not too extreme. As it is a flexible approach and straightforward to implement, it might be useful for practitioners and supervisors and central banks alike.

Taken together, the papers in this special issue provide a good overview of where the modeling of linkage between macro factors and micro risk drivers stands at this moment. It's clear that these models do not answer all the questions that the current crisis has brought to light. The almost universal collapse of the interbank money market and other wholesale markets and the importance of feedback effects are important aspects that are not well captured in these models, to mention just two issues. Nevertheless, the lessons drawn are relevant, especially once the financial system reverts to functioning normally. Credit risk at the portfolio level cannot be well captured if (exceptional) movements in macro factors are not incorporated in their modeling.

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Stress Testing Credit Risk: A Survey of Authorities' Approaches*

Antonella Foglia

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This paper reviews the quantitative methods developed at selected authorities for stress testing credit risk, focusing in particular on the methods used to link macroeconomic drivers of stress with bank-specific measures of credit risk (macro stress test). Authorities with a mandate for financial stability are particularly interested in quantifying the macro-to-micro linkages and have developed specific modeling expertise in this field. Stress testing credit risk is also an essential element of the Basel II framework, and some stress-testing requirements of Basel II are formulated by making explicit reference to the economic cycle. The paper highlights recent developments in macro stress testing and details a number of methodological challenges that may be useful for supervisors in their review process of banks' models as required by Basel II. It also contributes to the ongoing macroprudential research efforts to integrate macroeconomic oversight and prudential supervision, for early detection of key vulnerabilities and assessment of macro-financial linkages.

JEL Codes: E32, E37, G21.

1. Introduction

This paper reviews the quantitative methods developed at selected central banks and supervisory authorities to assess the

*The views expressed in this paper are those of the author and do not necessarily reflect those of Banca d'Italia. Work on this paper was initiated by the Basel Committee Research Task Force Group on Stress Testing. The author would like to thank the group members for their input, comments, and suggestions and especially A. Garcia Pascual, P. Kupiec, and I. van Lelyveld. Author contact: Banca d'Italia, Servizio Normativa e Politiche di vigilanza, Via Milano 53, Roma. Tel: +39647924553. E-mail: antonella.foglia@bancaditalia.it.

vulnerabilities of financial systems to credit risk, focusing in particular on methods used to link macroeconomic drivers of stress with bank-specific measures of credit risk. It is based on a number of recent papers and internal documentation provided by supervisors and central banks. The models included in this survey are listed in table 1 in the appendix.

Financial sector stress tests provide information on a system's potential losses under exceptional but plausible shocks, helping policymakers assess the significance of the system's vulnerabilities. The value added by system stress tests derives from a consultative process that combines a forward-looking macroeconomic perspective, a focus on the financial system as a whole, and a uniform approach to the assessment of risk exposures across institutions.¹ System stress tests can complement those of individual institutions and provide a cross-check for other types of analysis.

For many authorities the practice of stress testing was introduced as part of the Financial Sector Assessment Programs (FSAPs) conducted by the International Monetary Fund and the World Bank. The FSAP stress tests stimulated widespread research interest in developing new techniques, and many additional studies are under way. The survey includes methodologies that were used during the FSAPs and other studies developed afterward at the individual agencies.

The focus on credit risk and on the "macro-to-micro" models reflects a number of concerns: (i) stress testing credit risk is an essential element of the Basel II framework (Basel Committee on Banking Supervision 2005), and some stress-testing requirements of Basel II—such as the IRB-cyclicality stress tests (par. 435–37) and the forward-looking stress testing in the internal capital adequacy and assessment process (ICAAP) (par. 726)—are formulated by making explicit reference to the economic cycle (e.g., mild recession scenarios) and the macroeconomic background of a stress event; (ii) in this area, sound industry practices have not yet been established, and the translation of a stress event defined in terms of macroeconomic variables into movements in bank micro variables often represents a challenge for individual banks; and (iii) both in the FSAP context

¹See International Monetary Fund and the World Bank (2003).

and more generally for financial stability analysis, it is also one of the modeling areas most in need of further development.²

Because of their mandate for financial stability, central banks and supervisors are particularly interested in quantifying the macro-to-micro linkages and have developed specific modeling expertise. Such expertise can be a useful starting point to develop a common analytical background because, in this field, supervisors and banks often face the same methodological challenges. Sections 2–6 review the current stress-testing practices across various supervision and financial stability authorities, comparing features and outlining the latest developments. Section 7 discusses a number of technical issues that may be relevant for supervisors in reviewing the Basel II stress-testing requirements. As methods to better incorporate macro/systemwide conditions as drivers of default risk and macro stress testing in general are among the main tools of supervisors with a “macroprudential” orientation, the discussion can also contribute to the ongoing analytical efforts to integrate macroeconomic oversight and prudential supervision, as recently advocated by the Financial Stability Forum.³ Section 8 concludes and sets out a research agenda.

2. The Stress-Testing Process

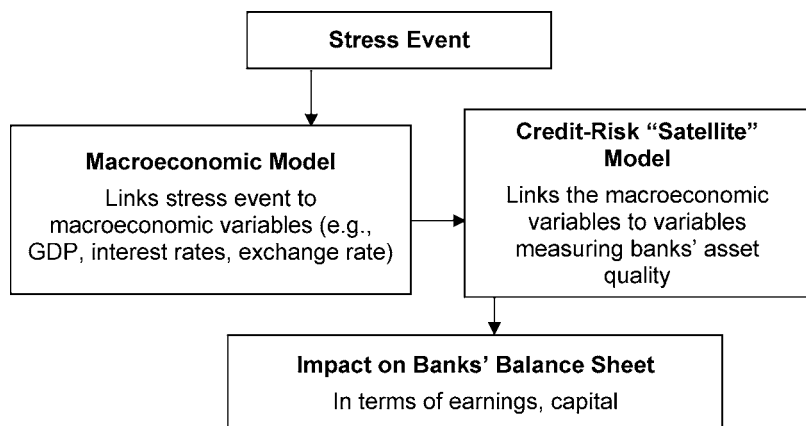
In all the approaches surveyed, macro stress testing can be seen as a multistage process, as shown in figure 1.

The first step is to put together a coherent stress-test scenario, typically using a macroeconomic model. The scenario or the model may include endogenous policy responses. Given that such models do not generally include financial sector variables, the stress-testing framework usually includes “satellite” models (i) to map macroeconomic variables to some “key” financial variables, such as asset prices (typically, housing prices) and credit growth and (ii) to map macroeconomic and financial variables into financial sector measures of asset quality and potential credit losses. Total bank losses are calculated by aggregating credit and market losses, in some cases including additional allowances for the impact on net interest

²See the discussion in Swinburne (2007).

³See Financial Stability Forum (2008).

Figure 1. Credit Risk—A Typical Macro Stress-Testing Process



Notes: This figure is adapted from Čihák (2007). For an overview of a typical stress-testing process, see also Jones, Hilbers, and Slack (2004).

income and on funding costs. Losses are then compared with the buffers of profits and capital.

This approach has valuable strengths, but it also suffers from some important limitations. Generally, current models are weak in the treatment of key financial system interactions. For example, they only rarely model the impact of funding and market liquidity stresses or the correlation between credit, market, and liquidity risks. Feedback effects are often absent or modeled in rudimentary fashion. Existing methods are generally unable to endogenously account for cross-border transmission channels for risk, including cross-border contagion between financial institutions. They often ignore potential nonlinearities and structural breaks in estimated relationships. In addition, some approaches focus on a projected conditional mean stress scenario outcome and fail to consider the distribution of the losses that will be borne by individual financial institutions in a real-world stress situation.⁴

⁴The strengths and limitations of traditional stress tests were examined extensively at the ECB conference on "Simulating Financial Instability," Frankfurt, July 12–13, 2007.

The market turmoil that began in mid-2007 in the U.S. “sub-prime” mortgage market has highlighted the crucial importance of the links between credit risk, funding liquidity risk, market risk, and counterparty credit risk, as well as other limitations of current stress tests. Addressing these limitations is therefore an important priority for both banks and financial stability authorities, as is shown by the recent initiatives undertaken by various international bodies.⁵ Some of the enhancements designed to overcome specific problems of traditional stress-testing techniques are addressed in the latest research projects initiated by the various agencies and are also reviewed in the following sections.

Table 1 summarizes the common stress-testing framework of the various agencies and classifies the models used in the different stages of the process according to their methodologies and assumptions.

3. The Design of the Macroeconomic Stress Scenario

In macro-scenario stress testing, the financial sector effects of multiple shocks to macroeconomic and financial variables are estimated using different models. The stress scenario’s effects on macroeconomic conditions are typically measured using (i) a structural econometric model, (ii) vector autoregressive methods, and (iii) pure statistical approaches.

Many stress-testing approaches use an existing structural macroeconomic model (e.g., one used by the central bank for forecasts and policy analysis) to project the levels of key macroeconomic indicators under the stress conditions assumed. A set of initial shocks are taken as exogenous inputs, and their interactions with the other macroeconomic variables are projected over the scenario horizon. The simulations will produce a range of economic and financial variables as outputs, such as GDP, interest rates, the exchange rate, and other variables.

The use of structural models imposes consistency across predicted values in the stress scenario. Moreover, they may allow for endogenous policy reactions to the initial shock. The feasibility of

⁵As for the work of the Basel Committee related on stress testing liquidity risk, see Basel Committee on Banking Supervision (2008).

the approach for stress-scenario analysis varies with modeling expertise and the type of macro model. Some considerations involved in using a macro structural model are discussed in Jones, Hilbers, and Slack (2004), such as the choice of the baseline assumptions, the policy responses, the time horizon, and which variables are assumed fixed and which are shocked. Another frequently mentioned concern is the inability of linear models to capture relationships between macroeconomic variables that may become nonlinear at times of stress, as well as the difficulty in determining the likelihood of a specific macro scenario.

In the context of the FSAP exercise, all the authorities used the domestic macroeconomic models developed for monetary policy purposes. However, since a domestic model does not provide all the information that is needed when shocks arise from international linkages, in some cases the models were extended to incorporate international effects.

If a well-developed macroeconomic model is not available, or it is not considered feasible to generate consistent relevant shocks, a second possibility is vector autoregression (VAR) or vector error-correction models (VECMs). In these models, a set of macroeconomic variables are jointly affected by the initial shock, and the vector process is used to project the stress scenario's combined impact on this set of variables. VAR models have appeal because they are a flexible and relatively simple way of producing a set of mutually consistent shocks, although they do not include the economic structure that is incorporated in the macro modeling approach.⁶ As detailed in table 1, these models are used in the studies developed at the central banks of the United Kingdom (BoE), Japan (BoJ), Spain (BoS), and the Netherlands (DNB), and at the European Central Bank (ECB).

In its *Financial System Report* (Bank of Japan 2007), the BoJ estimates a VAR model comprising five macroeconomic variables (GDP, inflation rate, bank loans outstanding, effective exchange rate, and the overnight call rate). Van den End, Hoeberichts, and Tabbæ (2006) and Jiménez and Mencía (2007) use a VAR structure

⁶As noted by Åsberg and Shahnazarian (2008), an important operational advantage of VAR models based on a few variables is that they do not require particularly substantial resources and their results are easy to interpret.

to model the response to a shock of the two macroeconomic factors included in an auxiliary credit-risk model (see subsection 4.1).⁷ The model used by Castrén, Déés, and Zaher (2008) is a global vector autoregressive (GVAR) model based on country- or region-specific VECMs, where domestic and foreign variables interact simultaneously; the endogenous variables included in the country-specific models are real output, the rate of inflation, real equity prices, and short- and long-term interest rates.⁸ The BoE prototype model uses a two-country version of the GVAR approach, modeling the UK and U.S. economies only, with the same macroeconomic variables as in the ECB paper.⁹

For stress-testing purposes, the Norges Bank developed an “ad hoc” small macro model that is used to design extreme stress scenarios and follow the transmission of initial macro shocks through a set of micro-data-based models for the corporate, household, and bank sector. The macro model focuses on risks that originate and develop outside the financial system (the corporate and household sector, and asset prices). In particular, household credit is a function of housing prices, interest rate, and income; housing prices are determined by housing credit, interest rates, unemployment, income, housing stock, and expectations. The model includes feedback effects from housing prices and credit to GDP.¹⁰

⁷Van den End, Hoeberichts, and Tabbæ (2006) also use a well-developed macro model (Norkmon/NIGEM) to simulate macro stress scenarios. The projected path of macro variables (GDP, interest rate) is subsequently used as input in an auxiliary VAR model (see section 4).

⁸The GVAR is a set of twenty-six VAR(2,1) models specific to twenty-five countries and one specific to the euro area. Each model includes a set of domestic/regional macroeconomic variables (usually five or six) and a vector of foreign variables specific to the respective country/region. In addition to the usual macro variables, the specification also includes the stock market return.

⁹In the BoE approach, the GVAR is a set of two VAR(2,1) models, one for the United Kingdom and one for the United States; the United Kingdom is treated as a small open economy and the United States represents the rest of the world. The BoE approach (discussed in Haldane, Hall, and Pezzini 2007) models income, market, and credit risk jointly, as well as some key feedbacks across banks, such as network and market liquidity externalities; a prototype version of the model is described in Alessandri et al. (2007).

¹⁰See Andersen et al. (2008). Much emphasis is put on linking the different macro-micro models together as a system. The corporate sector model is designed to analyze the default probability of all Norwegian limited companies (see subsection 4.2). For a discussion of the feedback effects, see also section 4.

In contrast to structural macroeconometric and VAR/VECM models, the Oesterreichische Nationalbank (OeNB), in its Systemic Risk Monitor (SRM), has developed a pure statistical approach to scenario design.¹¹ Macroeconomic and financial variables are modeled through a multivariate t-copula. The copula approach has two important advantages. First, the marginal distributions can be different from the multivariate distribution that characterizes the joint behavior of the variables. Second, the co-dependence between the macro-financial variables displays tail dependence (i.e., “correlation” increases under stress scenarios). However, as a “purely” statistical approach, it is not as well suited for policy analysis. Drehmann (2008) underlies how important is the suitability for storytelling for a proper communication of policy evaluations and how using general equilibrium structural macroeconomic models may be appropriate in highlighting the key macroeconomic transmission channels from shocks to impact on credit risk. By contrast, risk managers at financial institutions are less interested in the unwinding of the transmission mechanism and are more focused on the model forecast performance, which can guide their day-to-day decision-making process (see the discussion in section 7).

4. The Credit-Risk Models

Both the structural econometric and the VAR approaches require a method to map macroeconomic variables into indicators that can be used to estimate the implications of the stress scenario for banks’ balance sheets. Macroeconomic models, in fact, typically do not include a measure of credit risk, so the second stage of a stress-testing process usually involves estimating satellite or auxiliary models that link a measure of credit risk to the macroeconomic model variables, thus mapping external shocks onto banks’ asset quality shocks.

In these credit-quality regression models, loan performance measures are typically related to measures of macroeconomic conditions. Blaschke et al. (2001) report an example in which the nonperforming loan (NPL) ratio is regressed against the nominal interest rate, the

¹¹SRM is a model developed by the Austrian central bank for systemic financial stability analysis and stress testing; the framework includes credit risk, market risk, and interbank contagion risk. See Boss et al. (2006).

inflation rate, the change in real GDP, and the change in the terms of trade. The coefficients of the regression provide an estimate of the sensitivity of loan performance to those macroeconomic factors.

The assumption is that loan quality is sensitive to the economic cycle. The estimation strategy normally requires the selection of an initial set of macroeconomic and financial variables that, according to theory and empirical evidence, affect credit risk. Variables such as economic growth, unemployment, interest rates, equity prices, and corporate bond spreads contribute to default risk. In particular, interest rates are a crucial variable, as they represent the direct cost of borrowing. Among alternative specifications, the preferred one is selected on the basis of the consistency of macroeconomic variables with economic theory (that is, the variable's sign has to be "right"; otherwise, it is dropped) and on the specifications' goodness of fit.¹²

A satellite model that treats the macroeconomic variables as exogenous ignores—by construction—the feedback effects from a situation of distress in the banking system to the macro economy, which is one of the many limitations of traditional stress testing. Castrén, Dées, and Zaher (2008) argue that there are several possible reasons for this approach, such as a lack of sufficiently long time-series data, more modeling flexibility, easiness of implementation, and interpretation.¹³

Unlike the macroeconomic model, the credit-risk satellite model can be estimated on data for individual banks and even individual borrowers. Various modeling techniques have been applied so far, mostly depending on data availability. Čihák (2007) divides approaches into two classes: one is based on data on loan

¹²A detailed description of such an estimation strategy is provided in Segoviano (2006).

¹³The analysis of feedback effects is a core concern for financial stability work, as the recent intensification of the financial crisis has aggravated the downside risks to growth. The typical econometric framework that allows for feedback effects between the financial sector and the real economy is the VAR methodology, in which a vector of endogenous variables includes both a measure of credit quality (or another proxy of financial distress) and aggregate economic variables associated with the state of the business cycle (see the discussion in Chan-Lau 2006). Two studies reviewed in this section actually apply this methodology: Marcucci and Quagliariello (2008), who explicitly address this issue, and Åsberg and Shahnazarian (2008), who discuss only the sensitivity of credit risk to shocks to the macroeconomic variables.

performance, such as NPLs, loan loss provisions (LLPs), and historical default rates; the other is based on micro-level data related to the default risk of the household and/or the corporate sector. The same classification is used in this section, highlighting the distinct features of the different approaches. The capstone of many credit-risk satellite models is the estimation of the credit-portfolio loss distribution, which summarizes its overall risk profile and permits a thorough assessment of the impact of a shock (see section 6).

4.1 Models Based on Loan Performance

In this approach the key dependent variables are the NPL ratio, the LLP ratio, and historical default frequencies. As shown in table 1, these models include various macroeconomic factors, ranging in a number from two to five depending on the country. In some cases variables more directly related to the creditworthiness of firms are added, such as measures of indebtedness; in other cases, market-based indicators of credit risk, such as equity prices and corporate bond spreads, are also used.¹⁴

As regards the level of aggregation, and depending on the availability of data, models based on loan performance data can be run at the aggregate level, at the industry level, or even at the level of the individual banks.

Alessandri et al. (2007) and Marcucci and Quagliariello (2008) model credit quality using observed default frequencies at the household/corporate level of aggregation. Aggregate data allow Marcucci and Quagliariello to use a VAR approach to estimate the satellite credit-risk model, whereas previously VAR models had been used in the first stage of the stress-testing process.¹⁵ Their model for the corporate sector includes the default rate and four macroeconomic variables (output gap, inflation, short-term interest rate, and real exchange rate). In the identification scheme, the default rate is assumed to be contemporaneously exogenous to the output gap

¹⁴Introducing market variables such as interest rates, foreign exchange rates, equity, and real estate price indices into credit-risk models is a way of explicitly integrating the analysis of market and credit risks.

¹⁵Åsberg and Shahnazarian (2008) also use a similar approach, estimating a VECM model (see subsection 4.2).

and all the other macroeconomic variables. The impulse-response functions indicate significant impact of the various macroeconomic variables (except inflation) on the default rate.

The credit-risk models of Lehmann and Manz (2006), van den End, Hoeberichts, and Tabbae (2006), and the German Bundesbank (Deutsche Bundesbank 2006) use the LLP ratio to measure credit quality at the individual bank level, with static or dynamic panel data estimation. The panel estimation of individual banks' LLPs controls for individual bank characteristics that affect credit risk and captures the banks' different sensitivities to macroeconomic developments.

Jiménez and Mencía (2007), Fiori, Foglia, and Iannotti (2008), and the OeNB's SRM all model historical default rates grouped by industry.¹⁶ The sectoral breakdown allows the use of different macroeconomic variables to explain default frequencies in different industry sectors and the inclusion of sector-specific explanatory variables to improve the goodness of fit. For example, in the OeNB's SRM model, the number of statistically and economically most reasonable explanatory macroeconomic variables ranges from two to four depending on the sector, with some variables common to all the sectors.¹⁷

In such models, macroeconomic variables that are found to be significant for many sectors represent the systematic risk component; *intersectoral* default correlation is due to the common dependence on the systematic component. The idiosyncratic risk component is measured by potential sector-specific variables and/or by the residuals of the sectoral equations. When systematic risk is taken into account, default events should be independent, and the cross-equation residuals should be uncorrelated (conditional independence). If that is not the case, macroeconomic factors do not fully explain the default correlations across sectors; an important implication is that a portfolio's credit risk can be significantly underestimated (see discussion in section 7).

¹⁶In addition to ten industry equations, Jiménez and Mencía (2007) model one mortgage sector and a sector of consumer loans.

¹⁷GDP or industrial production, the unemployment rate, investment in equipment, and the price of oil are significant in more than one sector; see Boss (2002) for an explanation of the model selection procedure.

Jiménez and Mencía (2007) and Fiori, Foglia, and Iannotti (2008) argue that micro-contagion effects between sectors create an additional channel of default correlation. Using a different estimation strategy, both papers allow sectoral default frequencies to depend on macroeconomic conditions as well as on latent factors that can capture contagion effects. Accordingly, they are able to distinguish “cyclical” sectors (those highly sensitive to systematic risk) from those more dependent on idiosyncratic risk. Both studies find significant micro-contagion effects and similarly identify agriculture, manufacturing, construction, and trade as “cyclical” sectors, and mining and quarrying and utilities as “idiosyncratic.”¹⁸

The use of loan performance data to measure credit quality raises some important questions. Loan performance is a lagged or “retrospective” indicator of asset quality, in that it reflects past defaults. Loan loss provisioning rules may vary across jurisdictions, and legal protocols may determine whether or not institutions actually write off nonperforming loans or keep them on their financial statements with appropriate provisioning. Variations in loan loss provisions, in addition, may be only partly driven by changes in credit risk; other bank-specific factors, such as income-smoothing policies, might also come into play.

Another frequent problem in interpreting macroeconomic models of credit risk concerns the use of linear statistical models: the linear approximation may be reasonable when shocks are small, but when they are large, nonlinearities are likely to be important. In fact, almost all the studies reviewed here, following Wilson (1997), have used nonlinear specifications, such as the logit and probit transformation, to model the default rate. As van den End, Hoeberichts, and Tabbæ (2006) argue, nonlinear transformations of the default rate extend the domain of the dependent variable to negative values and take into account the possible nonlinear relationships between macroeconomic variables and the default rate that are likely in stress situations.

¹⁸Latent factors are orthogonal to the observable macroeconomic conditions. Jiménez and Mencía (2007) use a Kalman filter to deal with the unobserved factors; Fiori, Foglia, and Iannotti (2008) use factor analysis to identify the latent factors that account for the contagion component.

To address nonlinearities, the specification of the credit-risk model used in the macro stress-testing exercise of the Bank of Canada includes nonlinear terms. The model analyzes the relationship between a logit transformation of Canadian sectoral default rates and two macroeconomic variables (GDP and interest rate), adding higher-order terms as explanatory variables.¹⁹ It shows a better performance with respect to the same model without higher-order terms—in particular, in stressful periods, when the default rate reaches its historical peak; without nonlinearities, even the extreme shocks would have had a very limited impact on default rates.

4.2 Models Based on Data for Individual Borrowers

In this approach the credit-risk satellite model is estimated on individual borrower data. In this case, the model specification may also include macro-financial data as explanatory variables. When no macroeconomic variables are included, an additional satellite model may be used to link the macro-financial variables to borrower-specific data.

Using a database of yearly accounting data for all limited liability companies in Norway, Eklund, Larsen, and Bernhardsen (2001) relate the probability of default to borrower characteristics such as firm age, size, industry, and accounting variables measuring corporate earnings, liquidity, and financial strength. In this model, the projected figures for the main macroeconomic variables are used to estimate the future income statement and balance sheet of each company and on this basis to calculate individual probabilities of default (PDs). Data are then aggregated to estimate the banking sector's total loan loss.

Individual measures of credit quality can be exploited to estimate a direct relationship with macroeconomic variables. Åsberg and Shahnazarian (2008) and Castrén, Fitzpatrick, and Sydow (2008) use Moody's KMV expected default frequencies (EDFs) to model the average credit quality of listed companies. The EDF is a forward-looking, market-based measure of credit risk that gauges a firm's

¹⁹The macro stress-testing exercise of the Bank of Canada is described in Coletti et al. (2008). The treatment of nonlinearities is discussed in Misina and Tessier (2008).

probability of defaulting within a year, based on the volatility of its share price.

Åsberg and Shahnazarian (2008) analyze the median EDF of all Swedish nonfinancial listed companies and estimate a vector error-correction model (VECM) for this aggregate EDF and three macroeconomic variables (industrial production index, consumer price index, and short-term interest rate). Assuming a long-term correlation between variables, a VECM can discern shared trends between series as well as short-term fluctuations. The results indicate that the macroeconomic variable with the strongest (positive) impact on EDF is the interest rate, and that a fall in manufacturing output and an increase in inflation lead to a higher EDF.²⁰

The model by Castrén, Fitzpatrick, and Sydow (2008) also measures credit risk by the median EDF of euro-area companies, but at the sector level (eight economic sectors). The model relates the credit quality of European companies to five macroeconomic variables, including real equity prices, measured for the whole euro area; the parameters are statistically significant and with the expected sign for real equity prices and, in four of the eight sectors, for GDP.²¹

In contrast to the use of market-based measures of credit risk, the French Banking Commission (FBC) and the BoJ use internal data sets of individual nonfinancial company ratings, whose evolution over time is summarized by transition matrices.²² Both models estimate the sensitivity of a nonlinear transformation of the probability that borrowers will migrate to a different rating class with respect to

²⁰ Åsberg and Shahnazarian (2008) observe that higher inflation implies higher factor prices, which lead to increased costs and tend to impair credit quality. Moreover, high inflation is usually considered a signal of macroeconomic mismanagement and a source of uncertainty. Thus the relation between the default rate and the rate of inflation should be positive. However, higher inflation also implies higher product prices, which can boost earnings, and a lower debt burden in real terms, thereby improving creditworthiness.

²¹ The fact that the interest rate is not significant may seem to be a counterintuitive result in view of its importance as a driver of corporate credit quality. The authors explain by reference to the characteristics of the dependent variable: the main drivers of EDFs are the value of asset/equity (market capitalization) and the default point (which is a function of liabilities), so it is not surprising that the econometric analysis confirms the role of equity prices and not of interest rates.

²² See Commission Bancaire (2007) and Bank of Japan (2007).

a limited number of macroeconomic variables.²³ In the FBC model, the macroeconomic variables are GDP and short-term and long-term interest rates. In the BoJ model, a system of five equations (one for each rating class) is estimated by seemingly unrelated regression to account for possible correlations between error terms. Explanatory variables are GDP growth rate and a leverage ratio as proxies of profit and liability conditions. GDP is significant in all but the lowest rating class; the results for the debt ratio are more mixed.

In sum, the survey shows a wide array of approaches to credit-risk modeling in terms of measures of credit quality, level of aggregation, and estimation methodology. Methods that use current financial market data to predict bankruptcies (as contrasted with modeling LLPs or NPLs) within a given time horizon may be able to detect problems in the loan portfolio earlier than those based on loan classification data. Such methods, however, are restricted to listed companies and so may not be readily applicable in some countries. A common feature is that the macroeconomic variables used as explanatory variables are not numerous. As for the level of aggregation, models based on individual data can in principle lead to more accurate results; if these data are not available, there can still be benefits associated with parsimonious models using more aggregate data, as noted by Åsberg and Shahnazarian (2008).

5. Stress-Test Implementation

In the third stage of a typical stress-testing process, the macroeconomic models (structural, vector autoregressive, or purely statistical) are used to project the values of the macroeconomic variables under stress conditions and are applied in an auxiliary model of credit risk to estimate credit quality under stress.

As noted, all the authorities reviewed used a macroeconometric structural model for the FSAP exercises. In the Italian FSAP, one macroeconomic scenario involved a shock to oil and share prices. The effects on domestic macroeconomic variables were simulated using the Bank of Italy quarterly macroeconometric model to generate deviations from a baseline projection over several time horizons.

²³In the BoJ model, the banks' borrower classification data available at the central bank were supplemented with credit scores provided by a Japanese rating agency.

The macroeconomic projections for output gap and short-term interest rate were used in the credit-risk model to calculate an aftershock PD; the result was an estimated increase of 83 percent.

In such an approach, however, the structural macroeconomic model generates point estimates associated with a single future path, the conditional mean path under the stress scenario, with no probabilistic interpretation.

The VAR/VECM framework can generate stress scenarios that do allow for probabilistic interpretations. Shock sizes are specified in terms of the unconditional standard deviation of the innovation in an autoregressive series, and under a normality assumption they can be given a probabilistic interpretation. Thus scenarios do not follow from the economic reasoning behind a structural macro model but are based only on a probabilistic method. Tail outcomes of such simulations present extreme scenarios.

Pesaran et al. (2006) were the first to present a VAR model to generate a probabilistic scenario for credit-risk analysis. Impulse-response functions are used to examine how an isolated shock to one macroeconomic variable affects all the others. Impulse-response functions assume that the other variables are displaced according to their historical covariances with the variable being shocked, so that the correlations across shocks are accounted for in an appropriate manner. The authors examine the impact on a hypothetical corporate loan portfolio and its exposure to a range of macroeconomic shocks. For example, they find that a -2.33 -standard-deviation drop in real U.S. equity prices causes an expected loss of 80 basis points over four quarters. This approach is particularly valuable in addressing specific risk-management questions and, in particular, producing a rank order of the possible shock scenarios.

Examples of scenarios generated by this probabilistic method are given in the stress exercises conducted at the BoJ and in Jiménez and Mencía (2007) and Castrén, Fitzpatrick, and Sydow (2008). In the BoJ model, the stress test assesses the impact of a negative GDP shock of a size that has a 1 percent probability. Jiménez and Mencía (2007) apply a three-standard-deviation shock to the GDP and interest rate variables; similarly, Castrén, Fitzpatrick, and Sydow (2008) use a five-standard-deviation shock for one macroeconomic variable of the GVAR model.

The OeNB's SRM multivariate t-copula approach is used to draw risk-factor changes randomly according to their estimated

multivariate distribution. During the scenario simulation, one or more of the factor changes are set to a fixed value according to the given shock; changes for all other (nonstressed) risk factors are drawn from the conditional distribution given the stress scenario. For example, the SRM model documentation evaluates the impact of a drop in GDP or of a rise in interest rates; the t-copula approach ensures consistency with the overall dependency structure between risk factors.²⁴

Van den End, Hoeberichts, and Tabbæ (2006) propose an alternative method that accounts for simultaneous changes in the macroeconomic variables and their interactions as typically present in the macro scenarios derived from structural macro models. To simulate the hypothetical stress scenario, the projected values of the macroeconomic factors are used to reestimate a VAR model including GDP and interest rate. Reestimating a VAR that includes stressed values for the macroeconomic factors can take into account changes in the correlations and overcome the objection that stress-testing models posit constant statistical relationships, which might not be the case in stress situations.

A similar procedure is applied in the paper by Åsberg and Shah-nazarian (2008): they use the impulse responses of the Riksbank's macroeconometric model to a given shock (e.g., a supply shock) to estimate stressed values for the three macroeconomic variables of a VEC model that also include EDFs (see section 4.2). The VEC model is then used to forecast the stressed EDFs conditional on the stressed values of the macroeconomic variables.

6. Impact Measures

The final step in the stress-testing process is evaluating the impact on the banks' loan portfolio and judging whether banks can withstand the shock assumed. This means comparing the loss with an appropriate benchmark. Issues that arise concern the choice of the variable to measure the banking systems' ability to face shocks, the estimation of a loan portfolio's loss distribution, and the assessment of the impact for the systemwide portfolio as well as at the level of individual banks.

²⁴The model can also simulate the effect of one single-factor shock (unconditional simulation). See Boss et al. (2006).

Depending on the credit-risk model used, the results of the simulation can be expressed in terms of either provisions or projected default rates. In the latter case, given a (usually ad hoc) figure for the recovery rate, one can estimate banks' expected losses, which determines the volume of provisions to be set aside. As observed in Čihák (2007), in a normal situation ("baseline scenario"), banks would typically be profitable. When carrying out stress tests, it is important to evaluate impacts against such a baseline, as banks would exhaust profits before undergoing reductions in their balance-sheet or regulatory capital position. Expressing shocks only in terms of capital may result in overestimating the actual impacts if banks remain profitable in the baseline scenario. However, to accommodate the view that it is prudent to disregard profits, one can measure losses directly against capital or capitalization (capital or equity to assets, or capital to risk-weighted assets). The effects on capital adequacy ratios are obviously particularly important for agencies with supervisory responsibilities.

An important extension to the typical stress-test process focuses specifically on the impact measure. Instead of producing accounting measures of distress as point estimates under the assumed stress scenarios, more recent work has sought to derive a profit and loss distribution for the loan portfolio of the banking system as a whole, extending to systemwide scale the risk-management framework adopted at a micro level by many financial institutions in their risk-management systems.

The loss distribution shows the probability of loan losses of various sizes—from the possibility of no losses occurring to the loss of the entire loan portfolio. The expected loss—the mean of the distribution—is normally covered by earnings; banks need to hold a capital buffer to cover losses above those expected (unexpected loss or value-at-risk).²⁵ The estimation of a loss distribution for the banking system's loan portfolio makes it possible to calculate the size of the aggregate capital buffer given a tolerance level (the economic capital).

²⁵The shape of the loss distribution of a given portfolio is to a large extent determined by the presence of name concentration and/or correlations between the different exposures/sectors. The shape is typically skewed and has a relatively fat right tail, indicating that, although losses less than or around the expected value are most frequent, more extreme outcomes may also occur.

In the context of a loan loss distribution, the stress exercise can be couched in terms of deterministic shifts in the parameters, such as the PD and the loss given default (LGD), as, for instance, the sensitivity analyses reported in Sveriges Riksbank (2006). Alternatively, macro stress scenarios like those discussed in section 3 can be used to simulate adverse macroeconomic conditions that—using the satellite models described in section 4—generate a stressed aggregate PD or a set of stressed sectoral PDs. Via this link, the stress test has a clear economic interpretation.

The idea of measuring the impact of credit shocks in terms of an overall systemwide credit loss distribution—as opposed to banks' expected losses—was first discussed in Sorge and Virolainen (2006) and applied in the OeNB's SRM stress-test model.²⁶ Research projects along these lines are planned or under way at many authorities, with a view to improving the existing framework.

In a first approach, used in van den End, Hoeberichts, and Tabbae (2006), Alessandri et al. (2007), and Jiménez and Mencía (2007), the portfolio loss distribution is estimated using Monte Carlo simulation techniques, taking random draws of the innovations in the macroeconomic factors (GDP, interest rates, etc.).²⁷ The estimation can be performed at the aggregate level for the banking system or for individual banks.

In a second approach, used by the Sveriges Riksbank (2006), by Castrén, Fitzpatrick, and Sydow (2008), and in the OeNB's SRM, the simulation of random innovations in the macroeconomic factors is supplemented with a readily available portfolio model, such as Credit Risk Plus. The use of a full-blown portfolio model can combine predictions on default frequencies with more granular information on the credit quality of individual borrowers.

The OeNB's SRM calculates a loss distribution using a modified version of Credit Risk Plus. Sectoral default frequencies from the model are combined with individual borrowers' default probabilities from the central credit register by adapting the latter according to the difference with the model-predicted default frequencies.

²⁶ A similar analysis is conducted also in Pesaran et al. (2006).

²⁷ In the paper by Jiménez and Mencía (2007), the simulation also includes random draws of the innovations in the latent factors; the BoE's model combines various sources of risk (see footnote 9) and the corresponding output is a distribution of total banks' assets rather than pure credit losses.

If, for example, the model-predicted default frequency doubles due to changes in macroeconomic variables, this will result in a doubling of default probabilities of individual borrowers, which is then used to calculate the overall credit loss distribution using Credit Risk Plus.

A similar but simpler procedure is used by Castrén, Fitzpatrick, and Sydow (2008) and by the Riksbank. Instead of using individual default probabilities, both studies make assumptions about the creditworthiness of borrowers and classify loan portfolios into three quality classes; aggregate Moody's KMV EDFs of the lower and higher credit quality portions of the portfolio are adjusted accordingly.

Moving from a baseline to a stress scenario is likely to produce a shift in the conditional loss distribution and in the corresponding value-at-risk measure; in order to assess whether the banking system can withstand the assumed shock, the stressed value-at-risk (economic capital) should then be compared with a measure of actual capital held for credit risk by the banking system.

As is noted by Bonti et al. (2006), stress tests performed within a portfolio credit-risk model enable one to assess the outcomes of a stress scenario consistently with the quantitative framework used in a normal, nonstressed situation, because the stress scenario is translated into movements of "internal" risk drivers (the macroeconomic risk factors). The risk measures of the model (expected loss, value-at-risk) can be studied relative to the baseline simulation derived from the unconditional (nonstressed) risk-factor distribution. Using the same quantitative framework for normal and stressed situations implies that the relationships between nonstressed risk factors remain intact and the experience gained in the day-to-day use of the model can be used to interpret the results from stress testing.²⁸

Finally, depending on the availability of micro data, it is important that central banks and supervision authorities calculate the impact at the individual bank level and not only for an aggregate systemwide portfolio. In fact, seeing the distribution throughout the system is essential to assessing the threat of contagion and the possible impact of confidence effects on stability.

²⁸Consistency is one of the desirable properties of stress testing mentioned also in a Basel Committee study on credit-risk concentration (Basel Committee on Banking Supervision 2006).

7. Discussion and Evaluations

This section discusses the main findings of the survey, highlighting a number of methodological issues that may be relevant to supervisors in reviewing stress-testing requirements under Basel II. From a financial stability perspective, the discussion contributes to the ongoing macroprudential research efforts to integrate macroeconomic oversight and prudential supervision, by facilitating early detection of key vulnerabilities and the assessment of macro-financial linkages.

7.1 *Characteristics of the Credit-Risk Models*

One application of the macro credit-risk models is the calculation of IRB capital requirements in stress scenarios: the impact of a macro stress scenario on regulatory capital can be evaluated by recalculating the Basel II formula with the stressed PDs from the credit-risk model. The models surveyed here differ significantly in such areas as the measure of credit quality chosen, the level of aggregation, and the estimation methodology.

- Borrower credit quality is modeled either on the basis of loan performance data, requiring time-series data on different proxies for default rates (such as NPLs or LLPs), or on the basis of market-based indicators (such as Moody's KMV EDFs). The use of different variables raises several issues that must be considered in interpreting the results. For example, loan performance is a "retrospective" indicator of asset quality: loan loss provisioning rules or policies may affect the financial statement data that are used. Market-based indicators, on the other hand, are fully reliable only for listed firms. Moreover, the magnitude and statistical significance of the relevant macroeconomic variables' estimated coefficients may differ with the indicator of credit quality.
- The studies reviewed here use different levels of aggregation for the dependent variable. Whenever possible, disaggregated data are essential to capture the differing response of sectors/banks/portfolios to stress scenarios. One major shortcoming of econometric models based on aggregate data is that the conditional means may conceal significant variation at the portfolio or bank level. More specifically, this

procedure fails to detect uncertainty about (variations in) the actual defaults at the level of the single sector, bank, or individual obligor. Thus, the loss distribution obtained (see below) is more concentrated than the underlying overall loss distribution and so misses information about the extreme tails.

- The survey shows the importance of the model development stage, i.e., the statistical model-building technique. In general, a parsimonious selection of uncorrelated (or weakly correlated), statistically significant, and intuitively understandable variables makes the model more attractive for stress testing. In particular, economic plausibility is a key requirement in all the models: the economic meaning of the macroeconomic factors used must be clear, with no counterintuitive relationships with the dependent variable.
- The most important aspect in assessing the model specification process is overall performance in sample and out of sample. A common feature of macroeconometric-based models of credit risk is that macroeconomic variables alone tend to explain a fairly small part of the variation of the dependent variable, especially when only one or two macroeconomic variables are considered (omitted variables). The goodness of fit is considerably improved by the inclusion of latent variables (unobserved common factors), possibly accounting for micro-contagion effects. Failing to include such variables can result in significant underestimation of tail risk.
- Other important aspects to emerge from our examination of the model specification process include (i) the model's ability to handle low-quality data (missing values, outliers, structural breaks), (ii) the time period used for calibration, which should span at least one full business cycle to ensure capturing the cyclical effects on default probabilities, and (iii) an evaluation of parameter stability and model robustness.

7.2 *The Formulation of Stress Scenarios and Stress-Test Methods*

The models used to simulate macroeconomic scenarios range from more structural methods, which are better suited for policy analysis, to pure statistical methods that model the multivariate distribution

of macro-financial variables using nonlinear dependence structures (e.g., based on multivariate copulas). An intermediate option consists of reduced-form models such as VAR or VECM, which retain some of the desirable policy-analysis features of a structural model combined with some of the flexibility of a more statistical approach. However, macro models are generally local approximations of equilibrium relationships. They are not necessarily suitable for assessing the effect of large shocks, which are very likely to produce nonlinearities and regime shifts. This results in uncertainty over the size (and sometimes the sign) of the response. Whichever model is chosen, it is essential that the stressed macroeconomic variables be internally consistent.

So far, financial institutions have had trouble selecting “big picture” macroeconomic scenarios and have preferred to calibrate shocks directly in terms of micro variables. For individual firms, therefore, models such as VAR or VECM based on just a few variables can be feasible for designing internally consistent macroeconomic scenarios in a simple and transparent way and conducting macro stress tests without requiring particularly substantial resources.

7.3 Impact Measures and the Estimation of a Loss Distribution

While early stress-testing exercises concentrated mostly on expected losses, most of the recent methodologies estimate the entire portfolio loss distribution.

The loss distribution provides a measure of the credit VaR (economic capital or capital at risk) as well as other measures of “tail risk” under stress. In particular, the shape of the right-hand tail of the portfolio loss distribution is to a large extent dependent on key risk factors such as portfolio concentration and on correlations between risk components (PD, LGD, and exposure-at-default, EAD), which are not captured by other risk metrics such as expected losses. Stressed loss distributions can be used to examine stress scenarios in a consistent setting, and in particular to evaluate the future capital needs of banks to comply with their economic capital constraints under stress conditions, as required in the more general Pillar II stress test.

The survey found basically two approaches to estimating credit loss distributions in the context of a macroeconomic multifactor credit-risk model. A first approach applies only Monte Carlo simulations of innovations in macroeconomic factors to obtain stressed aggregate or sectoral PDs. This implies (i) treating every loan in the estimation bucket (aggregate or sector) as equally risky regardless of the credit quality of individual borrowers and (ii) assuming that realized losses are always equal to expected losses or else that banks hold an infinitely granular portfolio. This would result in an underestimation of risk.²⁹ In a second approach, the simulation of random innovations of the macroeconomic factors is supplemented with a full-blown portfolio credit-risk model, which generates loss distributions with greater variance and fatter tails.

In estimating the baseline and stressed loss distributions, much attention has been given to modeling default rates; there has not been much progress in modeling LGDs and EADs, and in most cases, ad hoc values for LGDs are assumed (e.g., LGDs and EADs are typically kept constant). However, in stressed scenarios, PDs often increase as the financial strength of households and firms deteriorates, LGDs increase as recovery rates fall with asset prices, and EADs increase as credit lines are drawn on in worsening financial conditions. Ignoring these correlations among PDs, LGDs, and EADs can result in a considerable underestimation of tail losses. It is accordingly important to model the joint behavior of these three variables in stress scenarios, as their correlations tend to increase in stress conditions.

8. Concluding Remarks

This paper reviews the quantitative methods developed at selected central banks and supervision authorities for stress testing credit risk. The focus is on macro stress testing—i.e., the linkage of the macroeconomic drivers of stress with bank-specific measures of credit risk—with a view to helping supervisors in reviewing stress tests for compliance with Basel II and contributing to the ongoing

²⁹For the first remark, see Boss et al. (2006); for the second, see Alessandri et al. (2007).

research efforts to integrate macroeconomic oversight and prudential supervision.

As a result of the IMF's Financial Sector Assessment Programs, central banks have acquired specific modeling expertise in this sector. The review shows the modeling and organizational complexity of macro stress testing, which involves a number of stages. The first step is to design a coherent macroeconomic stress scenario that is consistent with the application of a macroeconometric model. The second step, since these models generally do not include the financial sector, is to apply "satellite" models to measure credit risk, mapping the macroeconomic variables onto some measures of banks' asset quality. The third step is the assessment of losses under stress scenarios, evaluating them in connection with variables that gauge the banking system's ability to withstand shocks.

The paper outlines and compares features of the approaches adopted at the various authorities and traces the latest developments. In particular, (i) in devising scenarios, central bank researchers increasingly adopt models that are more flexible and easier to use, such as VARs and other strictly statistical rather than structural models; (ii) the "satellite" models for credit risk display a great variety of statistical methods, dependent variables, and levels of aggregation, while the explanatory variables are more uniform and not numerous; the most recent models, when data are available, incline toward a sectoral aggregation, which permits distinguishing between cyclical and acyclical sectors; and (iii) unlike the early macro stress testing, which assessed the impact of stress scenarios on expected losses, current research projects tend to envisage an assessment of the entire portfolio loss distribution and unexpected losses under stress conditions.

Finally, the paper analyzes and discusses a series of methodological aspects with a view to improving macro stress-testing models. A number of research programs are working to overcome some of these limitations. In particular, the current objectives are to extend time horizons and to build in banks' management actions to adjust balance sheets in response to the stress scenarios (for example, by changing lending and borrowing policy). In this way it would be possible to take account of the potential transmission, and amplification, of a shock within the financial system to the real economy.

Appendix

Table 1. Macro Stress Testing of Credit Risk—Methodologies of Selected Authorities

Agency	Credit-Risk Model		Data and Estimation	Macroeconometric Model	Stress Methodology	Impact Measure	Reference
	Dependent Variable	Independent Variables					
Bank of Canada	Logit transformation of sectoral default rates (six nonfinancial corporate sectors, household sector)	• GDP growth rate • Unemployment rate • Medium-term business loans rate • Credit/GDP	Nonlinear regressions (higher-order polynomials) Sample: 1988:Q1–2005:Q4	Bank of Canada's Global Economic Model (GEM), which is a version of the IMF's GEM	Paths of the macro variables under stress, coming from the macro model, were used to obtain the paths of sectoral default rates under stress. These were applied to loan portfolios of individual banks to obtain loss distributions. Expected and unexpected losses were used to assess the impact on banks' capital and their CAR.	Portfolio loss distribution (expected and unexpected losses), mapped into impact on CAR	Coletti et al. (2008)
	Logit transformation of aggregate default rates	• GDP growth • Short-term interest rate • Equity return	Linear OLS regressions on quarterly data (various samples)	Macroeconomic scenarios are generated by a two-country GVAR (UK, US) model, which includes six country variables and a foreign variable (see ECB box).	Conditional/unconditional GVAR simulations, historical recessions, parameter breaks	Stressed asset distribution	Alessandri et al. (2007)

(continued)

Table 1. (Continued)

Agency	Credit-Risk Model		Data and Estimation	Macroeconometric Model	Stress Methodology	Impact Measure	Reference
	Dependent Variable	Independent Variables					
Bank of Italy	• Corporate default rate • Output gap • Inflation rate • Three-month interest rate • Real exchange rate		1990:Q1–2005:Q2; quarterly data; VAR(1) estimation	The BOI's quarterly macroeconomic model. For shocks affecting the euro area and/or the world economy, satellite models used for the Eurosystem projections or IMF models were also applied.	The outputs of the macro model (stressed output gap, stressed leverage, and interest rate) are the input of the credit-risk VAR model.	Stressed default rates and expected losses	Laviola, Marcucci, and Quagliariello (2006); Marcucci and Quagliariello (2008)
	Logit transformation of sectoral default rates (eight corporate sectors)	GDP growth, equity index, competitiveness index, interest rate, two contagion latent factors depending on the sector	1990:Q1–2006:Q3; quarterly data; SUR estimation	The BOI's quarterly macroeconomic model (under way)	The outputs of the macro model (stressed macroeconomic variables) are the input of the sectoral credit-risk model (under way)	Stressed credit loss distributions (under way)	Fiori, Foglia, and Iannotti (2008)

(continued)

Table 1. (Continued)

Agency	Credit-Risk Model		Data and Estimation	Macroeconometric Model	Stress Methodology	Impact Measure	Reference
	Dependent Variable	Independent Variables					
Bank of Japan	Probit transformation of the probability of a rating transition	• GDP growth • Ratio of interest-bearing liability to cash flow	Data on bank borrowers SUR regression for a system of five equations (one for each rating category) 1985–2005	A VAR model comprising five variables: • GDP • CPI • Bank loan outstanding • Effective exchange rate • Call rate	VAR forecasts to (i) a negative GDP shock, of which probability is 1 percent; (ii) a negative GDP shock equivalent to the financial crisis since 1997	Maximum loss to capital derived from a Monte Carlo simulation	Bank of Japan (2007)
	Probit transformation of the default rate	• Quarterly change in real GDP growth • Variation of three-month real IR • Term spread • Six sectoral variables • Two latent factors	Ten sectoral equations for corporates; two equations for households; 1984:Q4–2006:Q4	VAR(1) estimation for the macroeconomic variables and for the latent factors	An artificial shock (three standard deviations) to the GDP and interest rate variables is introduced in the vector of innovations.	Stressed credit loss distribution	Jiménez and Mencía (2007)

(continued)

Table 1. (Continued)

Agency	Credit-Risk Model		Data and Estimation	Macroeconometric Model	Stress Methodology	Impact Measure	Reference
	Dependent Variable	Independent Variables					
De Nederlandsche Bank (DNB)	Logit transformation of the default rate	• Real GDP growth • Term spread	A system of two simultaneous equations; annual data 1990–2004; panel estimation	(i) The domestic macroeconomic model developed at the DNB plus NIGEM world model. They are used to generate projections of the macroeconomic variables given the initial shock to the exogenous variables; (ii) A VAR(2) model for the macroeconomic variables included in the credit-risk equations.	First type of stress: The deviations of the macro variable from the baseline scenarios—obtained as output of the macroeconomic model—are input in the credit-risk model. Second type of stress: The stressed (future) values of the macro variables as projected by the macroeconomic model are used to estimate an AR(2) or a VAR(2) model for the macroeconomic variables of the credit-risk equations.	First type of stress: stressed PDs, expected losses Second type of stress: A stressed credit loss distribution is simulated by taking random draws of the innovations in the macro variables used in the “stressed” VAR(2) model.	van den End, Hoerberichts, and Tabbae (2006)
	Logit transformation of the LLP ratio	• Real GDP growth • Long-term interest rate • Logit transformation of the default rate					

(continued)

Table 1. (Continued)

Agency	Credit-Risk Model		Data and Estimation	Macroeconometric Model	Stress Methodology	Impact Measure	Reference
	Dependent Variable	Independent Variables					
Deutsche Bundesbank	Logit transformation of the LLP ratio	• Lagged dependent variable • Credit growth • Real GDP growth • Variation short-term IR	A system of two simultaneous equations; panel data from 1993; dynamic panel estimation	The macroeconomic model developed at the Bundesbank used to generate projections of the macroeconomic variables	Given the initial shock to the exogenous variables, the stressed values of the macroeconomic variables are used to project an aftershock value of the variables that are input of the credit-risk model.	Loan loss provisions	Deutsche Bundesbank (2006)
	Credit growth	• Lagged credit growth • Real GDP growth • Variation short-term IR					

(continued)

Table 1. (Continued)

Agency	Credit-Risk Model		Data and Estimation	Macroeconometric Model	Stress Methodology	Impact Measure	Reference
	Dependent Variable	Independent Variables					
ECB	EDF of euro-area corporates	• Euro-area real GDP • CPI inflation • Real equity prices • Real euro/US\$ exchange rate • Short-term interest rate	Regression model of the median EDF (one aggregate/eight sector specific); quarterly data, 1992-2005	Macroeconomic scenarios are generated by a global VAR (GVAR) model which includes seven variables (six country/region variables and a vector of foreign variables specific to each country/region) and thirty-three countries, where eight of the eleven countries that originally formed the euro area are grouped together and the remaining twenty-five countries are modeled individually by a VECM.	The impulse responses from the GVAR model to five standard deviation shocks to one of the macro variables of the GVAR model	Stressed credit loss distribution	Castrén, Déas, and Zaher (2008); Castrén, Fitzpatrick, and Sydow (2008)
	Logit transformation of the probability of a rating transition	• GDP • Short-term interest rate • Long-term interest rate	Logit/probit estimation based on observed transition matrix and macroeconomic variables	The Mascotte macroeconometric model developed by the Banque de France for macroeconomic forecasts.	The outputs of the macro model (stressed GDP, short-term and long-term interest rates) are the input of the credit-risk model.	Stressed solvency ratio as a result of stressed risk-weighted assets (via credit-risk model) and a stressed capital (via an intermediation income model, not described here)	Commission Bancaire (2007)

(continued)

Table 1. (Continued)

Agency	Credit-Risk Model		Data and Estimation	Macroeconometric Model	Stress Methodology	Impact Measure	Reference
	Dependent Variable	Independent Variables					
Norges Bank	Loan loss ratio RWD=PD*DEBT Probability of bankruptcy (PD)	• Lagged risk-weighted debt (RWD) • House prices (first difference)	A logit model that predicts individual bankruptcy probabilities estimated using the entire population of enterprises in Norges Bank's accounts database for the period 1990–2002.	The scenarios are developed using a small macro model (SMM) that is designed for stress-testing purposes. Scenarios are compared with the official baseline scenario of Norges Bank.	The change in the macro variables from the macroeconomic model are translated into changes in accounting variables and a stressed PD is obtained. A bank model takes the output from the macro model and the distribution of PDs across industries from the enterprise model as input.	Expected losses, capital adequacy ratio, and results of the five largest Norwegian banks	Eklund, Larsen, and Bernhardt-sen (2001); Hagen et al. (2005); Andersen et al. (2008)
		Age, size, and financial ratios measuring corporate earnings, liquidity, and financial strength					

(continued)

Table 1. (Continued)

Agency	Credit-Risk Model		Data and Estimation	Macroeconometric Model	Stress Methodology	Impact Measure	Reference
	Dependent Variable	Independent Variables					
Oesterreichische Nationalbank (OeNB)	First difference of the logit transformation of industry default rates	Depending on the industry: • Real GDP • Industrial production • Unemployment rate • Investment in equipment • Oil (Brent) in euro • Real short-term IR • Real five-year IR All variables (except unemployment rate) were taken as logarithmic changes of the moving average over four quarters.	ML estimation of the first difference of the logit transformation of observed industry default rates; independent estimation for seven sectors (total eleven sectors); quarterly data; 1969–2007.	(i) Within SRM: Modeling of the joint distribution of macroeconomic and market risk factors through a t-grouped copula approach with four groups (macroeconomic variables, interest rates, fx-rates, and equity price indices) (ii) For FSAP 2007: Domestic model developed at the OeNB plus NIGEM world model to project macroeconomic variables given an initial shock	(i) Within SRM: Risk factors (macroeconomic variables and market risk factors) are increased by percentage or percentage points or set to the stressed value. (ii) For FSAP: Projected outputs of the macroeconomic metric model are used as input for the credit-risk model.	• Stressed capital adequacy ratio (CAR) and expected losses; • With the Systemic Risk Monitor (SRM) wherein the credit-risk model is integrated, a loss distribution is estimated using a modified version of Credit Risk Plus.	Previous version in Boss et al. (2006); current version is planned to be published.

(continued)

Table 1. (Continued)

Agency	Credit-Risk Model		Data and Estimation	Macroeconometric Model	Stress Methodology	Impact Measure	Reference
	Dependent Variable	Independent Variables					
Sveriges Riksbank	EDF of Swedish listed companies • Domestic industrial product index • Domestic consumer price index • Nominal domestic three-month interest rate		Monthly data 1997–2006 VECM estimation	The DSGE model used for policy simulation generates forecasts and stress scenarios for the three macro variables included in the VEC model.	The VEC model is used to forecast a stressed EDF by conditioning the model on ad hoc stressful scenario based on the DSGE model.	• Conditioned or stressed EDFs • The conditioned or stressed EDFs are also used as inputs for the simulation of a credit loss distribution.	Sveriges Riksbank (2006); Åsberg and Shahnazar-ian (2008)
	Logit transformation of the LLP ratio	• GDP growth • Unemployment rate • Level of three-month IR • Corporate bond spread • Bank control variables	1987–2004; static and dynamic panel estimation		Macroeconomic variables are replaced by the values assumed in the stress scenarios. Given an initial shock to one of those variables, the change in the remaining variables is determined through historical correlations.	Loan loss provisions	Lehmann and Manz (2006)
Swiss National Bank							

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Towards a Framework for Quantifying Systemic Stability*

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This paper describes a prototype quantitative framework for gauging systemic risk which explicitly characterizes banks' balance sheets and allows for macro credit risk, interest income risk, market risk, network interactions, and asset-side feedback effects. In presenting our results, we focus on projections for systemwide banking assets in the United Kingdom, considering both unconditional distributions and stress scenarios. We show how a combination of extreme credit and trading losses can

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precipitate fundamental defaults and trigger contagious default associated with network effects and fire sales of distressed assets. Despite the joint normality of all risk factors, the model generates a bimodal asset distribution.

JEL Codes: G01, G21, G32.

1. Introduction

Whilst central banks have long used models to guide monetary policy decisions, models of financial stability and systemic risk are much less well developed. The lack of an adequate analytical framework has meant that the financial stability risk-assessment work of central banks is often couched in more qualitative terms. As a result, the analyses presented in some financial stability reports all too often resemble a laundry list of things that could go wrong and, more recently, have gone wrong.

This paper describes work in progress at the Bank of England to develop a quantitative framework to help guide and sharpen macroprudential analysis. As with macroeconomic models in the monetary policy context, a quantitative approach provides a means of filtering news and assessing interrelationships between variables. It can provide insights to policymakers on the probability and potential impact of major threats to the financial system and can help them communicate their views clearly. In times of crisis, it can also be used as a guide to assess the vulnerability and systemic importance of individual institutions. For example, an approach of this sort could be used to shed light on whether the failure of an institution like Bear Stearns would pose a greater threat to systemic stability than that of Lehman Brothers.

The Risk Assessment Model for Systemic Institutions (RAMSI for short) focuses on the health of the UK banking system, with particular emphasis on risks over and above those priced and managed by financial institutions themselves. In the current version of RAMSI, the externalities that generate such “systemic risk” stem from the connectivity of bank balance sheets via interbank exposures and the interaction between balance sheets and asset prices. Default cascades can arise as a result of the direct interlinkages of claims and obligations in the financial network and may be reinforced

by asset-price spirals, particularly when the market for key system assets is illiquid.

The analytical foundation of our approach stems from recent theoretical work on modeling systemic financial crises. Allen and Gale (2000) explore the spread of contagion in a banking network and Cifuentes, Ferrucci, and Shin (2005) examine how default across the network can be amplified by asset-price effects. Gai and Kapadia (2008) examine the nonlinearities implied by both these externalities and suggest that modern banking systems may be robust-yet-fragile in nature. The greater connectivity of financial networks enhances risk sharing and lowers the probability of a crisis. But direct and indirect balance-sheet interdependencies mean that the impact of crises, when they occur, can be much greater than in less-connected systems. This result is reinforced by Gai et al. (2008), who suggest that financial innovation and greater macroeconomic stability may have intensified the robust-yet-fragile nature of modern banking systems.¹

In what follows, we show how models of the macro economy and banks' balance sheets can be brought together and integrated with models of the interbank network and asset-side feedbacks to generate illustrative forecast distributions for institution-specific and systemwide losses and banking assets over arbitrary horizons. Although we impose jointly normal shocks, we show that the network and asset-price feedback effects induce nonlinearities. These make our distributions bimodal in character, with a main peak associated with a healthy banking sector and a considerably smaller second peak in the extreme tail associated with outbreaks of contagious default.

At root, bankruptcy costs, which erode the assets of defaulting banks in our model, are the key source of this bimodality because they create a large, discrete loss at the point of default. But network effects and adverse asset-price feedbacks have a critical role: following the default of one bank, other banks may be tipped into default due to counterparty credit losses and mark-to-market write-downs on some of their assets. If default occurs, and especially if contagion breaks out, the cumulative bankruptcy costs therefore yield a systemwide outcome that is discretely and considerably worse than if the initial default is just avoided. Our model thus generates bimodal

¹See Rajan (2005), Tucker (2005), and Gieve (2006) for policymaker perspectives on this issue.

distributions in a context where banks do not optimize and there is no strategic interaction—this contrasts with existing literature (e.g., Morris and Shin 1999), which focuses on how bimodal distributions may arise in illiquid markets due to strategic interaction amongst traders with heterogeneous information sets.

Comparing Lehman Brothers with banks that have just avoided default serves to highlight how this bimodality may manifest itself in reality. Upon the failure of Lehman Brothers in September 2008, its creditors are likely to have incurred a range of direct expenses such as legal, accounting, and trustee fees.² Over time, financial institutions may also suffer losses from the cheap (fire-sale) liquidation of assets, which may be severe when the wider banking industry is also in distress (Shleifer and Vishny 1992), and can represent a real cost if the cheap sale is due, for example, to a disruption to established bank-borrower relationships. On their own, these losses represent a discrete, deadweight loss to the system which would not have been incurred if Lehman Brothers had survived. But it was the contagion from the failure of Lehman Brothers that caused a major amplification in systemwide distress and arguably moved the entire banking system from a precarious, but possibly sustainable, state to a full-blown crisis.³

A key value of RAMSI is its potential as a policy tool for risk-assessment and stress-testing exercises. For example, it can yield point estimates for losses and future assets if paths for all macroeconomic variables are specified ex ante. Alternatively, it can be used to generate conditional distributions by perturbing parameters and/or a set of macroeconomic and other variables in any period. In this paper, we illustrate the latter type of stress test by examining the conditional systemwide asset distributions that are obtained from an illustrative scenario which combines adverse market sentiment with distress in the U.S. household and global corporate sectors.

²Bris, Welch, and Zhu (2006) find that these costs are considerable for a wide sample of corporate bankruptcies in the United States, sometimes amounting to up to 20 percent of assets. Meanwhile, for the banking sector, James (1991) estimates direct bankruptcy costs to be equivalent to approximately 10 percent of the failed bank's assets.

³For a more detailed discussion of the financial crisis and the events following the failure of Lehman Brothers, see the Bank of England *Financial Stability Report* (2008).

Central banks and regulators are increasingly seeking to use formal models to support their financial stability work, and various modeling approaches have emerged in recent years. Some authors (e.g., Goodhart, Sunirand, and Tsomocos 2006) have attempted to model systemic risk in a general equilibrium framework, but these models are highly stylized and extremely difficult to operationalize. A much more common alternative is to rely on a modular approach. In this case, a macroeconomic model is combined with models that describe how the risk profiles—and notably the default probabilities—of key financial institutions respond to changes in macroeconomic conditions.

The link between macroeconomic factors and default probabilities can be formalized in two ways (see Sorge and Virolainen 2006). One possibility is to rely on an option-pricing framework and extract risk from observed security prices. This “asset-pricing” approach can be applied to individual banks (Elsinger, Lehar, and Summer 2006a; Segoviano and Padilla 2006; Frisell et al. 2007) or indeed entire sectors of the economy (Gray, Merton, and Bodie 2007). These models typically use publicly available, high-frequency data, can track changes in market perceptions of banks’ risk profiles in a timely fashion, and capture the market’s view on the risks in the system. However, they rely on strong assumptions on the completeness and efficiency of the markets in which the securities are traded. And, as market prices may embed the possibility of official support, asset-pricing models may be unable to identify the extent to which intervention may help to mitigate systemic risks (for a formalization of this argument, see Birchler and Facchinetti 2007).

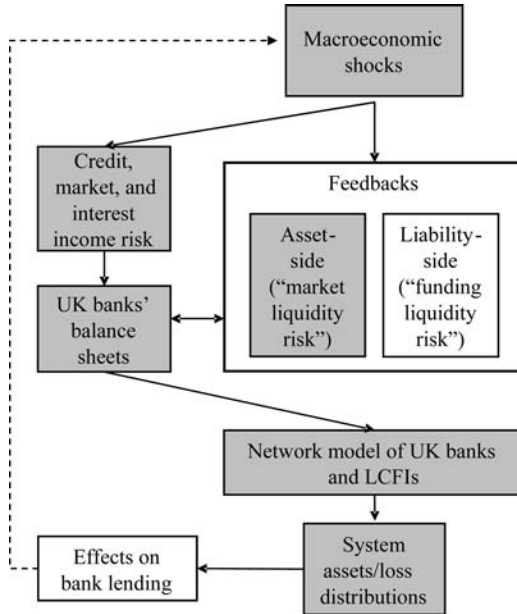
A second option, pursued in this paper, is to use balance-sheet data and separately estimate the performance of the bank’s exposures. Although they are not microfounded and typically rely on behavioral “rules of thumb,” these “balance-sheet” models offer a flexible and operational means of capturing a wide range of risks and transmission channels. And, compared with the asset-pricing approach, the transparency of the transmission channels allows for a more articulated analysis and interpretation of the outputs of stress-testing exercises. Because of this “story-telling” ability, many central banks use this type of framework as an input to their financial stability analysis (Borio and Drehmann 2009; Foglia 2009). But existing models tend to focus solely on credit risk.

In terms of contagion, some central banks model counterparty credit risk through an interbank network (see Upper 2007 for a comprehensive survey). Most of these models focus on the exogenous failure of particular institutions, though there is an increasing tendency, especially within the asset-pricing approach, to integrate aggregate shocks into the framework (see, e.g., Elsinger, Lehar, and Summer 2006a or Frisell et al. 2007).

The framework developed by the Oesterreichische Nationalbank (2006) for the Austrian banking system (see also Elsinger, Lehar, and Summer 2006b) is most closely related to our work in that it takes a balance-sheet approach and integrates a network model with models of credit and market risk to evaluate the probability of bank default. Our paper builds on their approach, developing a model of net interest income and capturing the feedback effects associated with the asset side of bank balance sheets. Recent events suggest that these feedback effects are crucial to systemic risk in modern banking systems. Further, whilst the Austrian model is limited to a one-quarter forecasting horizon, restricting its ability to assess the full impact of slow-burn risks to the financial system, our model is dynamic and allows for arbitrary horizons. The duration of the ongoing crisis has emphasized the importance of being able to make projections over several years.

Our paper is intended to provide an illustrative framework for how systemic stability in a modern banking system might begin to be quantified, and it is very much work in progress. As such, the calibration and estimation adopted in the prototype version of the model described in this paper is deliberately broad-brush in nature to emphasize the qualitative results of the model. More importantly, several key channels are excluded in this initial version. These include funding liquidity risk, off-balance-sheet risks, and feedbacks from the banking sector to the macro economy, all of which have been central to the ongoing financial turmoil and represent the focus of our current model development efforts. Therefore, the numerical results should not be construed to be an accurate measure of systemic risk in the UK banking system. And though some of our results may shed qualitative light on recent events, the model keys off end-2005 data and should not be viewed as quantifying any aspect of the ongoing financial crisis.

Figure 1. Suite of Models



The structure of the paper is as follows. Section 2 describes the current components of RAMSI and explains how they fit together. Section 3 discusses the systemwide distributions obtained from the stochastic simulation of risk factors and illustrates the possibility of contagious default associated with network effects and asset fire sales. Section 4 presents the outcome of our illustrative stress scenario, and a final section concludes with suggestions for future research.

2. The Modeling Framework

Figure 1 illustrates the structure of RAMSI and the mapping from shocks to systemic risk.⁴ The transmission dynamics hinge crucially on two factors—the nature and scale of shocks and the structural

⁴We use the word “shocks” to refer to unexpected changes in macroeconomic variables. No econometric identification strategy is pursued in this paper, so our shocks are generic random innovations to the macroeconomic data-generating process (see section 3).

characteristics of the financial system, such as the heterogeneity of balance sheets, the connectivity of the interbank network, and so on. In such an environment, balance-sheet interdependencies and asset-price feedbacks make for complex, nonlinear behavior. RAMSI can produce asset distributions for the banking system by linking together the shaded modules presented in figure 1—the unshaded modules are discussed briefly in the conclusion but are left for future work. In what follows, we discuss the overall modeling strategy in RAMSI before briefly discussing each of its components.

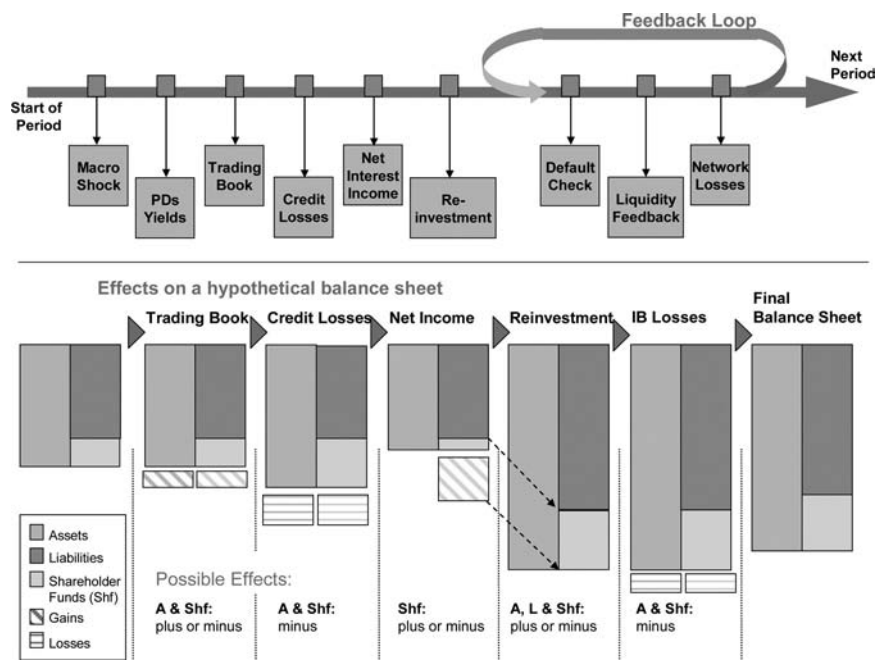
2.1 Overview and Sequencing

The model can be run over an arbitrary forecasting horizon as it takes account of the reinvestment of profits between quarters. In this paper, we adopt a time frame of twelve quarters. A three-year horizon seems appropriate given the duration of business and credit cycles—in particular, it takes time for some adverse shocks to be reflected in credit losses (Bunn, Cunningham, and Drehmann 2005; De Nederlandsche Bank 2006). And central banks often use a three-year horizon when stress testing their financial systems (see, e.g., Hagen et al. 2005, Bank of England 2007, and Sveriges Riksbank 2007).

The sequence of events is illustrated in figure 2. Macroeconomic risk factors for each quarter are generated using a simple two-country macroeconomic model estimated for the United Kingdom and the United States. These risk factors are used to derive both a yield curve and probabilities of default on UK banks' household and non-financial corporate credit exposures. For each combination of risk factors, we model three first-round effects on each of ten major UK banks.⁵ First, we model gains/losses on net trading and other financial assets held by the banks. Second, we account for credit losses. And third, we capture the effects on net interest income. For now,

⁵Membership of the major UK banks group is based on the provision of customer services in the United Kingdom, regardless of country of ownership. The following financial groups, in alphabetical order, were members at the end of 2007 and are included in the model: Alliance & Leicester, Banco Santander, Barclays, Bradford & Bingley, Halifax Bank of Scotland, HSBC, Lloyds TSB, Nationwide, Northern Rock, and Royal Bank of Scotland.

Figure 2. Model Timeline



we do not model the evolution of other income and costs, though these can be incorporated into the framework.

After computing the first-round impact on each bank, we update the balance sheets of profitable banks using a rule of thumb for reinvestment behavior. Specifically, we suppose that banks try to maintain their initial leverage and tier 1 capital ratios and invest in assets in proportion to their shares on their initial balance sheet. For banks that incur losses, however, we apply a “threshold rule” based on the Basel I regulatory minimum for tier 1 capital to determine whether any of them default. If no bank fails, the simulation for the quarter ends and we immediately proceed to model the effects of the following quarter’s risk factors on the banking system.

When a bank fails, it incurs a bankruptcy cost. A fraction of its assets are lost, reducing the amount available to its creditors. The bank then defaults on obligations that it cannot fulfill in the inter-bank network, imposing counterparty credit losses on other banks in the system. The trading and other financial assets of failed banks are

sold in a secondary asset market, creating asset-side feedbacks which cause other banks to suffer temporary mark-to-market losses on these asset classes. We account for both counterparty credit losses and mark-to-market losses on net trading and other financial assets before reapplying the threshold default rule to banks which initially survived.⁶ If any of these banks now default, we iterate around the network and asset-side feedback mechanism again. If not, we proceed to the next quarter after rebalancing all balance sheets to account for counterparty credit losses. We assume, however, that mark-to-market losses are not carried forward. In other words, the price of the fire-sale assets recovers to its (fundamental) pre-feedback level.

Throughout the paper, we assume that there are no regulatory or other policy interventions, aside from any short-term interest rate response embedded in and endogenous to the macroeconomic model. This is partly because modeling policy reaction to extreme events is inherently difficult, especially given that there is no single, standard response to financial crises. But we also feel that it is particularly interesting from a practical perspective to assess how the financial system would fare *without* any policy response, as it allows judgments to be drawn on the potential benefits and costs of intervening.

2.2 Macroeconomic Model

The macroeconomic risk factors are simulated using a two-country version of the global VAR (GVAR) model of Pesaran, Schuermann, and Weiner (2004). We treat the United Kingdom as a small open economy and take the United States to represent the rest of the world. The model is estimated quarterly over 1979:Q1–2005:Q4 and has the following reduced form:

$$x_t^{uk} = a_0^{uk} + a_1^{uk}t + \Phi_1^{uk}x_{t-1}^{uk} + \Phi_2^{uk}x_{t-2}^{uk} + \Lambda_0x_t^{us} + \Lambda_1x_{t-1}^{us} + \epsilon_t^{uk}, \quad (1)$$

$$x_t^{us} = a_0^{us} + a_1^{us}t + \Phi_1^{us}x_{t-1}^{us} + \Phi_2^{us}x_{t-2}^{us} + \epsilon_t^{us}. \quad (2)$$

⁶These network and asset-side feedback effects are applied to the balance sheets of other banks *after* they have been updated to account for the reinvestment of any surplus (see figure 2). As such, network and asset-price externalities influence banks' balance sheets at the end of each quarter.

Variables and data are the same as in Déés et al. (2007). For the United Kingdom, these are real output (GDP), CPI inflation (CPI), real equity prices (EQP), an overnight nominal interest rate (SR), a twenty-year nominal interest rate (LR), and the sterling-dollar real exchange rate (EX). For the United States, the real exchange rate is replaced by the oil price (OIL). The GDP and CPI series are seasonally adjusted. Output, equity prices, and the exchange rate are modeled in logarithms. A zero bound is imposed on nominal interest rates. For simplicity, we approximate the yield curve by linearly interpolating the short- and long-term interest rates implied by the GVAR. This is the source of all risk-free rates used in the model.

2.3 First-Round Impact on Banks

2.3.1 Asset and Liability Classes on the Balance Sheet

We split balance sheets into fifteen asset and eight liability classes. Assets are divided into domestic and foreign exposures; for simplicity, we assume that exchange rate risk is fully hedged. Where possible, we also combine information from published accounts with confidential information provided by the ten UK banks in our sample to break down the total figures into five repricing buckets: zero to three months, three to six months, six to twelve months, one to five years, and greater than five years. Non-interest-bearing items are grouped separately. We then model the balance-sheet gains and losses and cash flows on each of these asset and liability classes. Table 1 summarizes this information.

Throughout this analysis, we simply take balance sheets as given—we do not model off-balance-sheet items such as commitments, or attempt to adjust for credit-risk transfer, securitization, hedging of interest rate risk, or other similar activities which are likely to mean that balance sheets will not fully reflect the risks to which banks are exposed. Clearly, a proper treatment of these issues is an important area for future work.

2.3.2 Trading-Book Gains and Losses

In the absence of formal top-down models of gains and losses on the trading book, we suppose that trading assets increase in value when the equity market is buoyant and interest rates are falling. The

Table 1. Asset and Liability Classes on the Balance Sheet and Associated Modeling

Assets ^a			
Asset Class	Modeling of Losses/Gains	Modeling of Cash Flow	Risk Weight
Trading and other financial assets ^b	Using trading-book equation	Risk-free rate + 15 bps ^c	50%
UK interbank unsecured ^d	Via network	Risk-free rate + 15 bps	35%
UK household secured (mortgage)	UK Mortgage PD*UK Mortgage LGD (17%)*Exposure	Coupon from net interest income model + 50 bps	35%
UK household unsecured	UK Unsecured PD*UK Unsecured LGD (100%)*Exposure	Coupon from net interest income model + 50 bps	75%
UK government	Assumed not to default	Risk-free rate	0%
UK PNFC	UK Corporate PD*UK Corporate LGD (84%)*Exposure	Coupon from net interest income model + 50 bps	100%
UK OFC	15 bps spread implied PD*UK OFC LGD (13%)*Exposure ^e	Risk-free rate + 15 bps	40%
UK other assets ^f	Assumed not to default	Risk-free rate	0%

^aAll footnotes referring to UK asset classes also apply to foreign asset classes.

^bIncludes trading portfolio assets (minus certificates of deposit), available-for-sale financial investments, and financial assets designated at fair value.

^cAt the short end, this is roughly in line with LIBOR prior to the recent financial crisis.

^dUnsecured interbank loans + derivatives + certificates of deposit.

^e15 bps spread implied PD is equivalent to the PD implied by the average difference between the LIBOR in pre-crisis times and the short risk-free rate under the assumption that the spread entirely reflects credit risk.

^fIncludes reverse repos.

(continued)

Table 1. (Continued)

Assets			
Asset Class	Modeling of Losses/Gains	Modeling of Cash Flow	Risk Weight
Foreign interbank unsecured	Assumed not to default ^g US Mortgage PD*UK Mortgage LGD (17%)*Exposure US Unsecured PD*UK Unsecured LGD (100%)*Exposure Assumed not to default US Corporate PD*UK Corporate LGD (84%)*Exposure 15 bps spread implied PD*UK OFC LGD (13%)*Exposure Assumed not to default	Risk-free rate + 15 bps	35%
Foreign household secured (mortgage)		Coupon from net interest income model + 50 bps	35%
Foreign household unsecured		Coupon from net interest income model + 50 bps	75%
Foreign government		Risk-free rate	2%
Foreign PNFC		Coupon from net interest income model + 50 bps	100%
Foreign OFC		Risk-free rate + 15 bps	40%
Foreign other assets		Risk-free rate	0%
Liabilities			
Liability Class	Modeling of Losses/Gains	Modeling of Cash Flow	
Trading liabilities	Using trading-book equation N/A N/A N/A N/A N/A N/A N/A	Risk-free rate + 15 bps	
Unsecured interbank ^h		Risk-free rate + 15 bps	
Household		Risk-free rate minus variable negative spread ⁱ	
Government		Risk-free rate	
PNFC ^j		Risk-free rate minus variable negative spread ^j	
OFC		Risk-free rate + 15 bps	
Subordinated liabilities		Risk-free rate + 15 bps	
Other liabilities ^k		Risk-free rate + 15 bps	
^g The only exception is if a foreign bank defaults in the network as a result of the failure of a UK bank. In this case, UK banks can suffer losses on foreign interbank assets.			
^h Unsecured interbank deposits + derivatives.			
ⁱ The negative spread on household deposits is 200 bps in the 0–3 months repricing bucket, 150 bps in the 3–6 month bucket, 100 bps in the 6–9 month bucket, 50 bps in the 9–12 month bucket, and 0 bps at longer maturities.			
^j The negative spread on household deposits is 100 bps in the 0–3 months repricing bucket, 75 bps in the 3–6 month bucket, 50 bps in the 6–9 month bucket, 25 bps in the 9–12 month bucket, and 0 bps at longer maturities.			
^k Includes debt securities and repos.			

reverse is assumed to hold for trading-book liabilities. Changes in the value of the trading book evolve according to a linear relationship for net trading assets, NTA .⁷ Specifically,

$$\frac{NTA_t}{NTA_{t-1}} = 1 + \gamma_1(\Delta EQP_t - \Delta EQP^*) + \gamma_2 \Delta SR_t + \gamma_3 \Delta LR_t, \quad (3)$$

where ΔEQP^* is the historical mean return on equity over 1979:Q1–2005:Q4 and the right-hand-side macroeconomic variables are simple averages of UK and U.S. realizations. The use of net trading assets means that we are implicitly assuming that banks' liability positions are the exact reverse of their asset positions. In what follows, we take $\gamma_1 = 0.208$ and $\gamma_2 = \gamma_3 = -1.25$. Ceteris paribus, a 12 percent fall in equity prices relative to the trend growth rate or a 2-percentage-point rise in either short- or long-term interest rates causes a 2.5 percent decline in the value of net trading assets.

It is important to stress that the modeling and calibration of the trading book is intended to be illustrative. Indeed, the parameters in this prototype model have been deliberately chosen to ensure that fundamental defaults occur in some scenarios, thus allowing us to explore the feedback channels built into the framework. A more plausible model would break down banks' trading assets and liabilities into more granular classes (e.g., equities, bonds, etc.) and attempt to model each of these categories individually. In addition, it would allow for asymmetries and nonlinearities in trading gains and losses.

2.3.3 Credit Losses

Credit losses on household and corporate exposures are computed by multiplying an appropriate default probability (PD) for the asset class by a constant loss given default (LGD) to obtain a write-off rate.⁸ With the exception of the PD for other financial companies, which is backed out from an assumed LIBOR spread (see table 1),

⁷Net trading assets are taken to be trading and other financial assets, as defined in table 1, net of trading liabilities. Note that this exaggerates banks' true exposure to market movements, as it does not account for derivatives and other hedging activity.

⁸Most other asset classes are assumed not to default. See table 1 for full details.

PDs are estimated as linear functions of the macroeconomic outputs of the GVAR as follows:

$$L_t = \alpha + \beta_1 \Delta GDP_{t-1} + \beta_2 \Delta EQP_{t-1} + \beta_3 SRR_{t-1} + \varepsilon_t, \quad (4)$$

where SRR_t is the short-term real interest rate and L_t is the quarterly log-odds transform of the default ratio for each of the following sectors in the United Kingdom and the United States: household secured (mortgages), household unsecured, and private nonfinancial corporate (PNFC). In specifying the equations, we impose $\beta_{1,2} < 0$ and $\beta_3 > 0$, deliberately ignoring any correlations that are at odds with our theoretical priors. The resulting models have R-squared coefficients between 5 percent and 30 percent. This simple specification may underestimate the volatility and persistence of actual PDs, but it allows us to capture some of the cyclicity of credit risk, and we consider it adequate given the focus of the paper.

We assume LGDs of 100 percent for unsecured household loans, 17 percent for mortgages, 84 percent for nonfinancial corporate loans, and 13 percent for financial corporate loans. The LGDs are assumed to be the same for domestic and foreign exposures. We choose these LGDs to allow us to match the average write-off rates observed over the available data sample (1993–2005 for corporates and 1997–2005 for households). The implied LGDs appear to be fairly high, but this is likely to reflect underrecording of defaults.⁹

At any given point in time, credit losses are calculated by simply multiplying (state-contingent) write-off rates by the relevant exposures (we assume that U.S. write-off rates apply to all foreign exposures). This calculation is underpinned by a fairly strong assumption—namely, that banks hold infinitely granular portfolios. It implies that realized credit losses are linear functions of the underlying PDs. In reality, portfolio concentration (or “lumpiness”) is an

⁹Because of informal debt restructurings, recorded data on bankruptcies tend to underestimate the true scale of default, especially in relation to household unsecured debt. As a result, for some asset classes, even an LGD of 100 percent implies a write-off rate that is well below historical figures. In those cases, we scale up the measured PDs so that we can match observed write-off rates using LGDs which fall between 0 and 100 percent.

important driver of credit risk, and it is responsible for various well-documented stylized facts, including significant skewness in realized credit losses. Though we could introduce this ingredient in our framework (for instance, by using a Bernoulli mixture model), we sacrifice some realism to keep our analysis simple and maintain a transparent, direct link between macroeconomic risk factors and realized credit losses.

2.3.4 *Net Interest Income*

An important novelty of our framework is the modeling of endogenous interest income. Banks price their loans on the basis of the prevailing yield curve and the perceived riskiness of their debtors: an increase in actual or expected credit risk translates into a higher cost of borrowing. However, banks' ability to reset coupons is constrained by the repricing structure of their balance sheets. Since assets and liabilities typically do not have matched repricing frequencies, these constraints generate significant income risk. And possible shifts in the yield curve intensify this risk.

In this paper, we use the risk-neutral asset pricing model of Drehmann, Sorensen, and Stringa (2008) to capture both sources of income risk in a consistent fashion. Consider a risky asset, A , with a repricing maturity equal to T , implying that the asset pays a fixed coupon C over the next T periods. The economic value of the asset today is the risk-adjusted discounted value of future coupon payments and the principal:

$$EV(A_0) = \sum_{t=1}^T D_t C A_0 + D_T A_0, \quad (5)$$

where the discount factors are given by

$$D_t = \prod_{l=1}^t (1 + R_{l-1,l})^{-1}, \quad (6)$$

$$R_{l-1,l} = \frac{r_{l-1,l} + PD_{l-1,l} * LGD}{1 - PD_{l-1,l} * LGD}, \quad (7)$$

and $r_{l-1,l}$ and $PD_{l-1,l}$ represent, respectively, the forward risk-free interest rate and the expected default probability between time $l-1$

and l .¹⁰ We can use equation (5) to calculate a “fair” time-zero coupon that guarantees that $EV(A_0) = A_0$:

$$C_0 = (1 - D_T) / \sum_{t=1}^T D_t. \quad (8)$$

Whenever the bank can update C (i.e., at time $T, 2T, \dots$), it will do so using (8), so that expected interest income covers expected losses and book and economic value coincide. Between 0 and T , though, interest rates and PDs may change, whereas the coupon is fixed: any change in discount factors that is unexpected as of time zero will thus prevent the zero-profit condition from holding. For each bank, we use balance-sheet information to determine what fraction of assets and liabilities can be repriced at any point in time. The model implies that the pricing structure of the balance sheet, and particularly the mismatch between assets and liabilities, influences a bank’s vulnerability to interest rate and PD shocks.

Since we do not have a domestic-foreign split of liabilities, we use the UK yield curve to determine the appropriate risk-free rate for both domestic and foreign assets. Though clearly unrealistic, this avoids the severe distortion to net interest income that would arise if some assets were repriced on a different basis to corresponding liabilities. But it implies that foreign macroeconomic risk factors only affect PDs and hence the credit-risk component of the coupon.

The model-implied coupons are calibrated to better accord with actual observed spreads, as these may also partly reflect compensation for fixed costs associated with arranging loans and additional profits derived by banks. In particular, for household and nonfinancial sector corporate assets, the model-implied coupon is increased by 50 basis points. For other parts of the balance sheet, including all of the liability side, we simply calibrate spreads to accord with reality. For example, we impose negative spreads on some retail and corporate deposits (if the negative spread implies a negative interest rate, the interest rate paid is assumed to be zero). Table 1 provides a detailed summary of the cash flows paid on assets and liabilities.

¹⁰The risk-free yield curve is known at the time of pricing; we assume that banks take future PDs to be equal to the most recent observation.

2.4 *Reinvestment Behavior*

As noted above, banks in RAMSI rely on a rule of thumb to update their balance sheets when they make profits.¹¹ Specifically, when shareholder funds grow, banks follow three distinct rules in increasing their liability base and investing extra resources:

- (i) “Leverage” Target: When shareholder funds increase, a bank raises extra resources (liabilities) up to the point where the initial shareholder funds to liabilities ratio is restored.
- (ii) “Tier 1 Ratio” Target: In investing the available cash (i.e., net profits plus, potentially, the increase in liabilities), banks aim to maintain or restore the initial ratio of shareholder funds to risk-weighted assets.
- (iii) “Portfolio Composition” Rule: Subject to (ii), banks invest in assets in proportion to their shares on the bank’s initial balance sheet (e.g., mortgage banks will, *ceteris paribus*, invest in mortgage assets rather than trading assets).

Note that banks may be unable to fully meet their targets at the start of each period. In particular, if they have suffered a series of losses and make another loss or only a small profit, they will be unable to restore their tier 1 capital ratio to its initial level. Therefore, accumulated losses may weaken banks’ capital positions for several quarters.

The reinvestment assumptions are motivated by the presumption that the initial balance sheets represent desirable equilibrium outcomes which banks seek to preserve in the face of changes in size. Some empirical justification for leverage targeting is provided by Adrian and Shin (2008, figure 2.4), who find signs in the data which suggest that commercial banks target a fixed leverage ratio. Meanwhile, historical evidence from UK banks can be used to support capital-ratio targeting. In particular, over the 1997–2004 period, the mean ratio of capital to risk-weighted assets for the major UK banks has been relatively stable, and institution-specific standard

¹¹We rule out equity buybacks in profitable states. If banks make losses, we assume that their shareholder funds are eroded but they are unable to disinvest or raise capital. Rather, they simply raise new liabilities. For simplicity, our analysis also currently ignores the balance-sheet effects of taxes and dividends.

deviations of this ratio have been low.¹² Support for the rule that banks grow their balance sheets in proportion to the initial composition of their portfolio is more difficult to defend over a long horizon. However, drastic changes in portfolio are typically associated with a change in the bank's "business model." Within a given business model, the assumed portfolio composition rule seems reasonable, especially over the three-year horizon considered in this paper.

2.5 Default Thresholds and Bankruptcy

2.5.1 Default Thresholds

Banks default if the ratio of shareholder funds to risk-weighted assets falls below 4 percent—the Basel regulatory minimum for tier 1 capital. Risk-weighted assets are computed by applying the risk weights listed in table 1. These are calibrated on the basis of the Basel II standardized approach.¹³

The Basel threshold ratio is an extreme (solvency-based) definition of the failure of a financial institution. In practice, a funding liquidity crisis is likely to result before such a ratio is breached. In future work, we intend to develop a broader set of indicators and ratios that can be used to refine the point at which crises are precipitated.

2.5.2 Bankruptcy Costs

When a bank defaults, we follow James (1991) and suppose that it incurs costs equivalent to 10 percent of its remaining assets. This is also in line with the mean figure reported in Bris, Welch, and Zhu (2006). These bankruptcy costs are designed to capture the direct legal, accounting, and redundancy costs which are incurred upon default. They may also be viewed as capturing the erosion in the real value of a bank's assets that may occur upon default due to

¹²Specifically, the average standard deviation is under one-tenth of the average capital ratio. Note also that the existence of fixed trigger ratios for capital below which regulators might take disciplinary action also points toward a desire amongst banks to maintain capital at a relatively constant buffer above the minimum requirement.

¹³In future work, we intend to compute endogenous risk weights on the basis of the Basel II internal ratings-based approach.

disruptions to established bank-borrower relationships or the loss of human capital. They imply that even if banks fail with positive shareholder funds, they will be unable to fulfill all of their obligations upon default.

2.6 *Second-Round Impact on Banks—Feedback Effects*

2.6.1 *Price Effects of Asset Sales*

When a bank fails, financial markets may have a limited capacity to absorb assets sold onto the market and, as a result, asset prices may be depressed. Following Schnabel and Shin (2004) and Cifuentes, Ferrucci, and Shin (2005), we suppose the following relation between forced sales and the asset price, q :

$$q = e^{-\theta x}, \quad (9)$$

where $x > 0$ is the fraction of system assets sold onto the market and the fundamental price of the asset is set at $q = 1$. The value of θ is based on calibrations of a search-based model of asset prices developed by Duffie, Gârleanu, and Pedersen (2007). In the baseline calibration, we assume that UK banks hold 10 percent of system trading assets and set $\theta = 0.81$. Our choice implies that the asset price falls by 8 percent when one-tenth of system assets have been sold in a fire sale.

We integrate equation (9) into RAMSI by assuming that trading assets can be treated as a single generic asset class on banks' balance sheets, whose price depends on the volume of assets being sold onto the market. Specifically, when a bank defaults, all of its net trading assets are sold onto the market in the same quarter.¹⁴ This reduces the generic asset price, and other banks suffer mark-to-market losses as a result. Subsequent problems in other banks could depress the price still further, setting off the feedback loop described below.

2.6.2 *Network Model*

When a bank defaults, counterparty credit losses incurred by other banks are determined using a network model. A matrix of interbank

¹⁴This could be interpreted as deriving from attempts by the bank to save itself before formally defaulting.

exposures for the ten major banks in our model, along with some smaller UK institutions and a selection of large, complex financial institutions (LCFIs), is built using reported large exposure data, where available. Though it is only the major UK banks that can default for fundamental reasons, the additional banks in the network may transmit contagion. Furthermore, since we can force the idiosyncratic default of any institution, their inclusion allows us to obtain a partial assessment of the likely implications of the failure of a given institution on the rest of the system—something which has become increasingly important as the current crisis has progressed.

Since we have information on total interbank asset and liability positions, we use maximum entropy techniques to fill in any missing gaps in the network, ensuring that none of the estimated entries exceed the reporting threshold for large exposures.¹⁵ If any interbank assets or liabilities are unallocated following this procedure, we assume that they are associated with interbank business with a residual sector. Once constructed, the estimated exposure matrix remains static over the forecasting horizon.

To clear the network following the default of one or more institutions, we use the Eisenberg and Noe (2001) algorithm. The approach assumes that interbank claims are junior to nonbank claims. The LGD incurred by interbank creditors on their exposures is determined endogenously based on the shortfall in assets relative to liabilities of the defaulting bank (recall that bankruptcy costs imply that a bank's assets will generally be insufficient to fulfill all of its obligations upon default). Counterparty credit losses can lead to the failure of another bank, in which case it too incurs bankruptcy costs and defaults on part of its interbank obligations. The clearing algorithm solves for the unique outcome of this iterative process, determining all contagious defaults and returning final, total counterparty credit losses for each institution.

2.6.3 Feedback Loop

After accounting for counterparty credit losses and mark-to-market losses on net trading and other financial assets, we check for further

¹⁵The techniques adopted are similar to those discussed by Wells (2004), Elsinger, Lehar, and Summer (2006b), and Oesterreichische Nationalbank (2006). See Upper (2007) for an overview of this literature.

defaults by reapplying the default rule to banks which initially survived. In the event of a further default, we iterate around the network and asset-side feedback mechanism again. If not, we proceed to the next quarter after rebalancing all balance sheets to account for counterparty credit losses. As noted earlier, we assume that asset prices recover to pre-feedback levels, so mark-to-market losses are not carried forward. Clearly, recent events suggest that it might be more appropriate to assume a gradual adjustment process. This would impose higher systemic costs on the banking system and is something we intend to explore in future work.

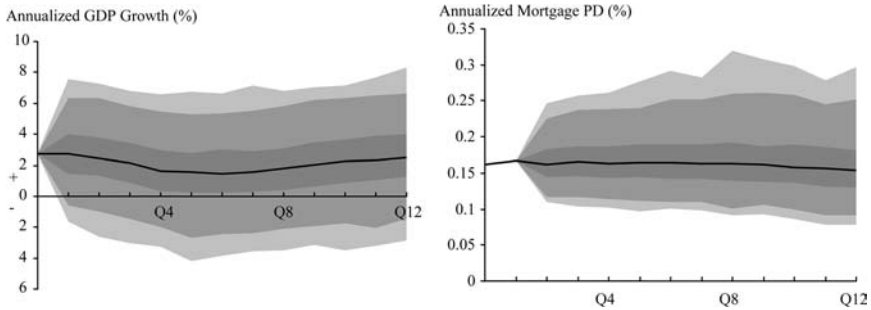
3. Simulations

We now consider the output of two sets of stochastic simulations based on alternative parameterizations of the trading book. In both cases, we use data up to 2005:Q4 (so that all balance-sheet information is on the basis of end-2005 data) and run 1,000 simulations on a three-year forecast horizon stretching to the end of 2008. Both configurations use the same seed for the random-number generator—this implies that the underlying risk factors in each case are identical. The GVAR is currently the only source of exogenous randomness in the stochastic simulations; each simulation is thus driven by a sequence of macroeconomic shocks $[(\varepsilon_t^{uk})' (\varepsilon_t^{us})']'$ drawn from a multivariate normal distribution.¹⁶

Throughout this section, we discuss results for the UK banking system in aggregate. But, since individual banks' balance sheets are at the core of RAMSI, the model produces a rich set of information and may be used both to obtain baseline projections for specific institutions and to analyze their performance under stress.

¹⁶In other words, we draw 1,000 realizations of the macroeconomic risk factors in the first quarter. In subsequent periods, we draw a single set of macroeconomic risk factors for each of the 1,000 draws. The number of simulations is admittedly rather small; furthermore, by simulating a single (multivariate) innovation in each quarter and scenario, we are effectively sampling from the underlying random tree of the model. Both limitations can be bypassed at the cost of higher computational complexity. We believe, however, that the current setup provides a good description of the properties of the model and yields several interesting qualitative results.

Figure 3. UK Risk Factors (Median, 50%, 95%, and 99% Confidence Bands)



Such information can be used to assess the vulnerability of particular institutions to different risks and may thus, in time, feed into the internal institution-specific risk-assessment work undertaken by regulators and central banks.

3.1 Risk Factors

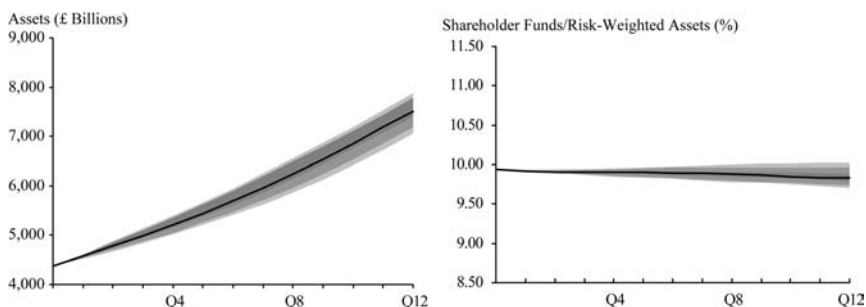
Figure 3 presents fan charts for UK GDP growth and mortgage PDs over the forecasting horizon. These are representative of the paths of macroeconomic risk factors and PDs generated by the model based on end-2005 data.¹⁷ It is evident that there are several recessionary scenarios in the simulations. Though not shown, equity prices are highly volatile, as might be expected, and the slope of the yield curve varies considerably. Meanwhile, the variability of PDs reflects macroeconomic outcomes as implied by equation (4).

3.2 The Baseline Asset Distribution

In the first simulation exercise, we assume that no gains or losses are made on net trading assets (i.e., we set $\gamma_1 = \gamma_2 = \gamma_3 = 0$ in equation (3)). Given our somewhat arbitrary modeling of the trading book, this parameterization represents a natural benchmark. Figure 4 shows how the systemwide distributions of some of the key banking sector variables evolve over the forecasting horizon in this case.

¹⁷These distributions are intended to be illustrative and do not represent the views of Monetary Policy Committee members as reported in the Bank of England's *Inflation Report*.

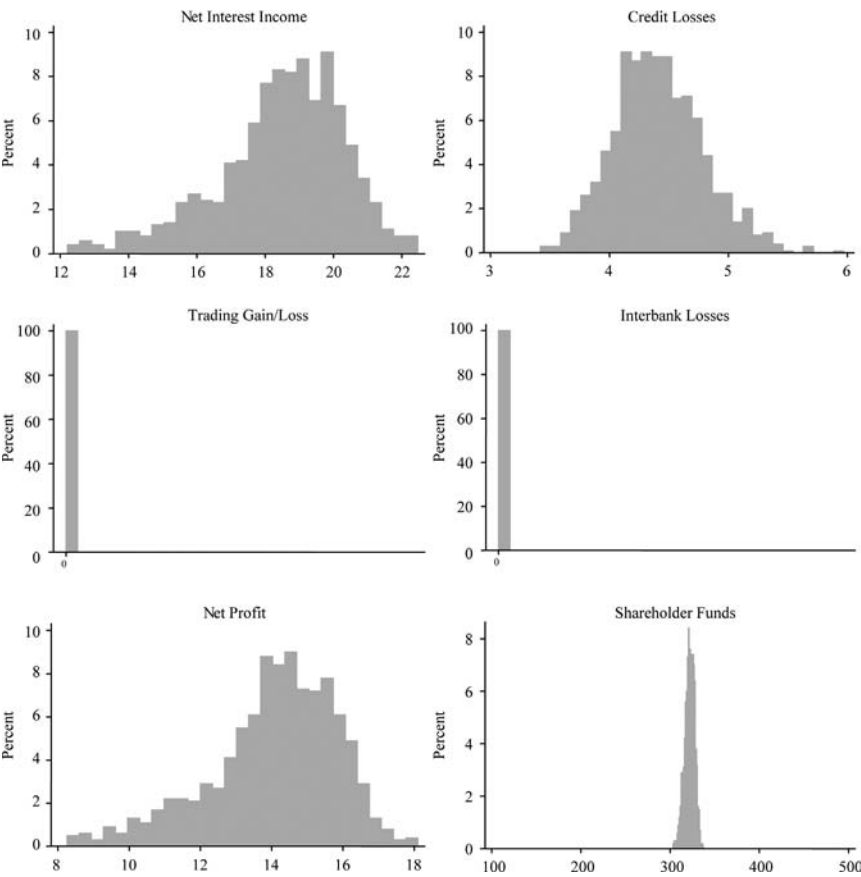
**Figure 4. Banking Sector Dynamics, No Trading Book
(Median, 50%, 95%, and 99% Confidence Bands)**



As is clear, total banking system assets rise consistently over time. This reflects our pricing assumptions and the supplementary spreads added onto certain asset classes which imply that net interest income exceeds credit losses in expected (mean) terms by construction. Moreover, even in the worst-case outcomes in this calibration, net interest income still exceeds credit losses (see figure 5, discussed below). Partly because banks cannot make the types of trading loss experienced over the past couple of years in this simulation exercise, they are always profitable and balance sheets always expand. But, consistent with the reinvestment-rule targeting, the ratio of shareholder funds to risk-weighted assets barely changes, as seen in the right-hand panel of figure 4.

Figure 5 illustrates some of the output in a slightly different way, using the distribution of a few key variables in the final quarter (in principle, these distributions can be generated for any quarter). As can be seen, there is variation in both credit losses and net interest income, but the variance of both these distributions is relatively low in the current calibration. Interestingly, however, the net interest income distribution has a fat negative tail and hence so does the profit distribution. This reflects the zero lower bound on nominal interest rates. In scenarios for which the risk-free rate falls close to zero, banks are constrained in their ability to pay negative spreads on household and corporate deposits. As a result, their net interest income margins are squeezed. Since there is no corresponding upside effect, the net interest income distribution exhibits negative skew. By contrast, credit losses are normally distributed. A skewed, fat-tailed

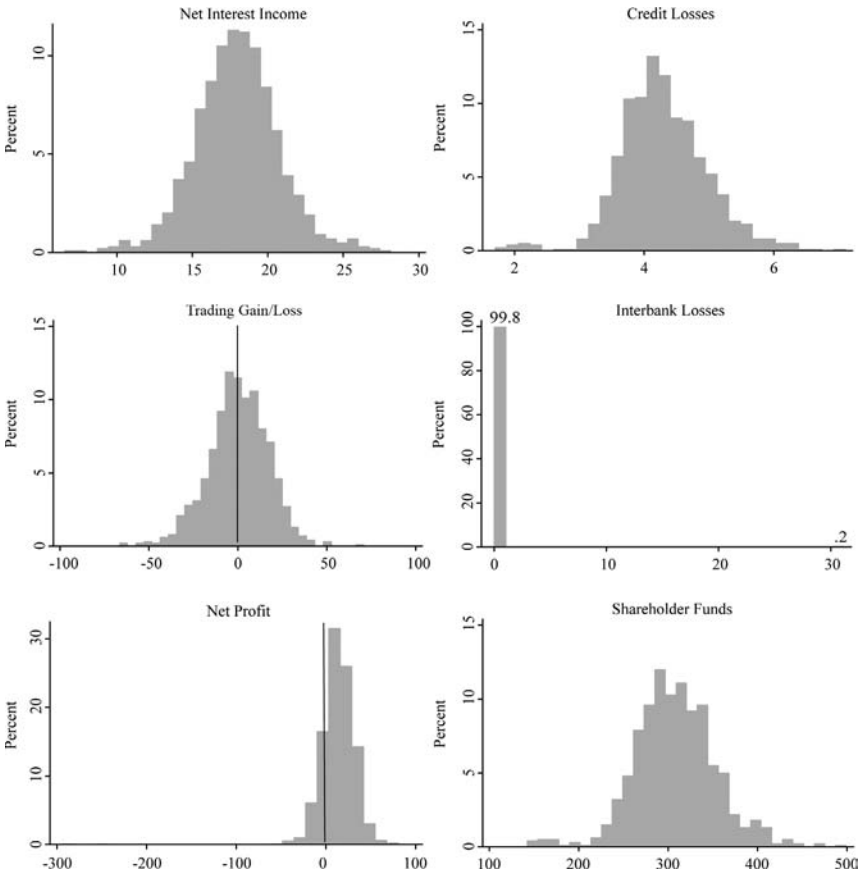
Figure 5. Final-Quarter Banking Sector Distributions (No Trading Book)—Unit: £ Billions



Note: Net profit is defined as net interest income – credit losses + trading gain/loss – bankruptcy costs.

credit loss distribution could be generated by modeling lumpy rather than granular exposures, introducing nonlinearities and correlated defaults in the PD equations, or modeling dependencies between PDs and LGDs (see, e.g., Chava, Stefanescu, and Turnbull 2006, and Das et al. 2007). Moreover, incorporating a richer set of macroeconomic risk factors, including house prices, can generate greater variation in the distribution of credit losses.

Figure 6. Final-Quarter Banking Sector Distributions (With Trading Book)—Unit: £ Billions

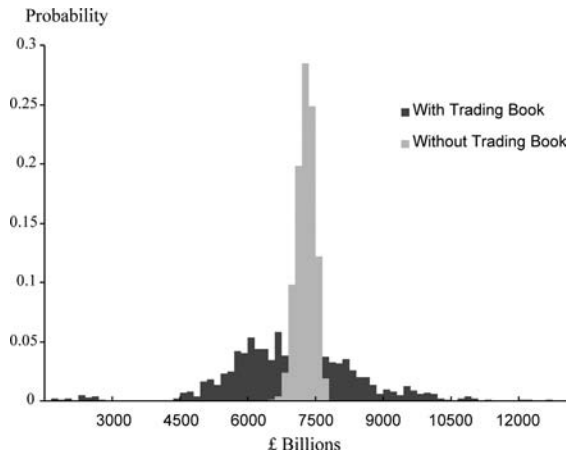


Note: Net profit is defined as net interest income – credit losses + trading gain/loss – bankruptcy costs.

3.3 Introducing Trading-Book Volatility

The second simulation exercise uses the trading-book calibration specified in section 2.3.2. As discussed, this calibration was partly chosen to ensure that fundamental defaults occur in some scenarios. Unsurprisingly, it leads to large gains and losses on net trading assets. Consequently, as is made clear by the bottom left-hand panel of figure 6, the system is no longer profitable in all scenarios. And,

**Figure 7. Final System Assets Distributions
(With and Without Trading Book)**

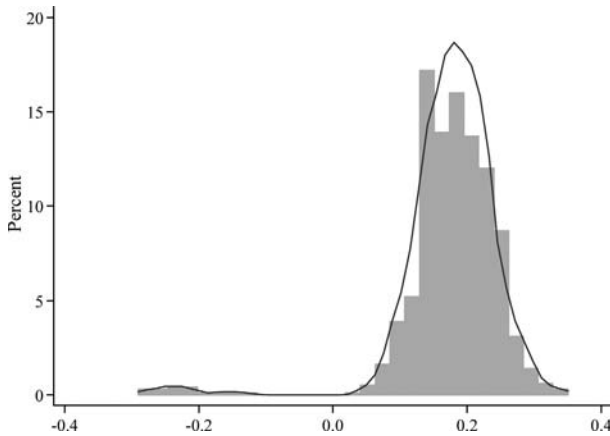


as shown in figure 7, the distribution for final banking system assets is much more volatile. Further, in some of the scenarios, the trading losses are large enough to generate defaults; there are two cases of this in the last quarter of the simulation, giving rise to the nonzero interbank losses displayed in the middle right-hand panel of figure 6.

3.4 Fundamental and Contagious Default

From figure 7, it is also clear that the final asset distribution is bimodal, with a main peak associated with a healthy banking sector and a considerably smaller second peak in the left-hand tail. This is despite the Gaussian nature of the underlying shocks. At root, bankruptcy costs are the key source of this bimodality because they create a large, discrete loss at the point of default. But network effects and adverse asset-price feedbacks have a critical role: following the default of one bank, other banks may be tipped into default due to counterparty credit losses and mark-to-market write-downs on some of their assets. If default occurs, and especially if contagion breaks out, the cumulative bankruptcy costs thus yield a systemwide outcome that is discretely and considerably worse than if the initial default is just avoided. Therefore, beyond a certain threshold, “extreme” negative outcomes become relatively more likely than “moderate” negative outcomes. This result captures a

**Figure 8. Aggregate Return on Assets (%)
(Quarterly Average)**



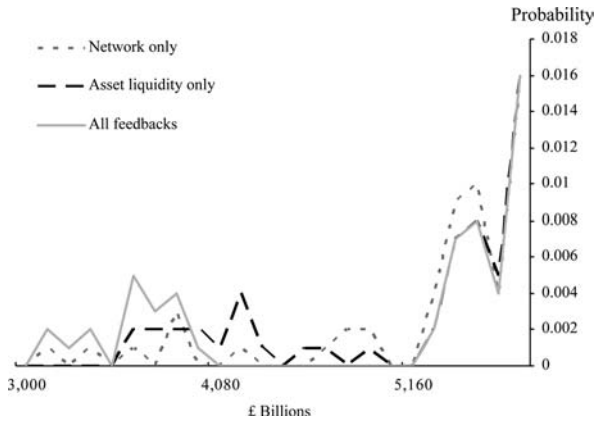
Notes: Return on assets is defined as system net profits relative to beginning-of-period assets. The line shows an estimate of the density based on normal kernels and an optimal (i.e., mean square error-minimizing) bandwidth.

phenomenon that is commonly perceived as a key feature of financial risk.

As a stock variable, total assets also reflect the step-wise nature of the default process (a bank's assets adjust smoothly in normal times, but they are no longer counted as part of the system once they have defaulted). To better illustrate that bankruptcy costs and feedbacks are material to the bimodality, their effect can be isolated by examining aggregate profits. In particular, the bimodality is clearly evident in figure 8, which shows the distribution of the aggregate return on assets averaged over the twelve quarters.

Figure 9 analyzes the bimodality in more detail by considering how our results differ if either network effects or asset-side feedbacks are excluded. The figure illustrates how these changes affect the tail of the total asset distribution. It is clear that network and asset-price feedbacks independently contribute to shaping the tail of the distribution and that, when combined, they generate a significant amplification effect. There are eighteen scenarios in which at least one fundamental default is observed. Independently of whether one or two fundamental defaults occur, a total of at least three banks

Figure 9. Tail of the Final System Assets Distribution (Various Feedback Specifications)



end up defaulting if both feedback effects are taken into account. But, even in isolation, both the network and the asset-side feedback mechanism can generate contagion. However, two contagious defaults are observed less than half the time when only one of the feedback mechanisms is active. This makes clear that the interaction of network effects and asset-side feedbacks is of prime importance when modeling contagion (see also Cifuentes, Ferrucci, and Shin 2005).

4. A Stress Scenario

To highlight the flexibility of our framework and to illustrate some of the channels captured in more detail, we now consider a specific application of the model. This takes the form of a stress test based on an illustrative scenario which combines adverse market sentiment with distress in the U.S. household and global corporate sectors. Whilst we could specify the path of all macroeconomic variables associated with this stress scenario to obtain a point estimate of its impact, we instead implement it by only imposing our priors on a subset of risk factors and parameters. This allows us to retain some of the randomness in the macroeconomic model and generate distributions conditional on the set of adverse events associated with the stress scenario.

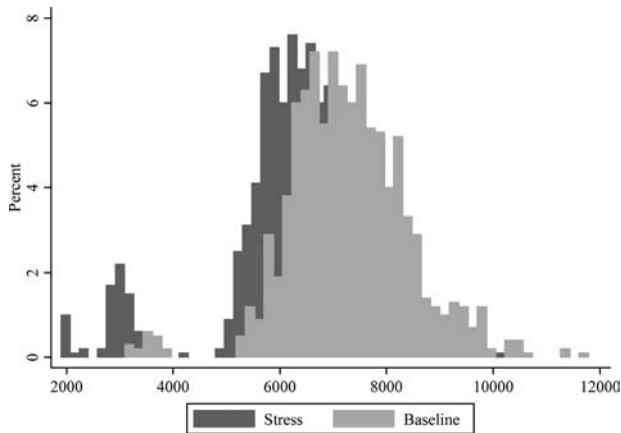
Our stress-test has four elements: real equity prices fall for one quarter; the market interbank spread goes up; PDs rise unexpectedly; and finally, markets become less liquid (i.e., asset prices become more sensitive to fire sales than in the baseline calibration). Specifically, we assume that real equity prices in both the United Kingdom and United States fall by an average of 10 percent in the first quarter, which we apply by shifting the mean of the distribution of errors of the equity equations in the GVAR. The spread of the interbank borrowing rate over the risk-free rate permanently increases from 15 basis points to 100 basis points, a level which has been reached or exceeded for much of the ongoing crisis. We also impose permanent increases in the U.S. secured (mortgage) default rate, and both the UK and U.S. corporate default rates, with the household default rate assumed to pick up from the first quarter of the simulation but the corporate default rates only increasing in the fourth quarter. These shifts represent a large but plausible shock, roughly matching the unexplained increases in PDs during the early 1990s recession in the United Kingdom.¹⁸ Finally, we change θ in equation (9) so that fire sales cause the trading asset price to fall by 12 percent when 10 percent of system assets are sold, instead of 8 percent as in the baseline case. It is important to emphasize that this stress test is entirely meant to be illustrative and its calibration is somewhat arbitrary.

Figure 10 depicts the final asset distribution of the UK banking sector under stress. Relative to the baseline, it is clear that banks are adversely affected—the entire distribution shifts left. The greater mass in the tail in the stress scenario reflects a higher incidence of default. We also observe that the stronger liquidity feedback effect can amplify default contagion further than in the baseline; indeed, there are scenarios in which six defaults are generated.

However, there are several factors which mitigate the overall impact of the stress test. First, the fall in equity returns is not very persistent—the shock is imposed for one quarter only, so the impact

¹⁸Specifically, equation (4) is reestimated from 1980 to 2006 in five-year rolling regressions, holding β_1 , β_2 , and β_3 equal to their estimated values over the baseline sample period. The maximum intercept corresponds to the period from about 1991 through 1996. This translates to, for example, a shock of approximately 50 percent to the UK corporate PD.

Figure 10. Final System Assets under Stress-Test Scenario—Three-Year Horizon (Unit: £ Billions)



on trading assets is relatively short lived. The equity market fall causes significant losses to materialize early on before equity prices recover. Second, the higher PDs in the corporate and household sectors are gradually priced in as coupons on longer-maturity assets are repriced. Credit risk is initially underpriced, but the rise in net interest income eventually offsets higher credit losses. Finally, the rise in interbank spreads washes out to a certain extent—net borrowers in the interbank market are worse off, but lenders profit. There can, therefore, be important distributional effects within the major UK banks which are not apparent in a distribution of total system assets.

5. Conclusion

This paper developed a quantitative framework in which to gauge financial stability and assess risks to the UK banking system. The unified modeling approach sheds light on risks over and above those priced and managed by financial institutions themselves. In particular, absent intervention by the authorities, defaulting financial institutions may directly trigger default cascades as exposed counterparties take a hit to their capital and these losses are amplified by mark-to-market losses on assets due to fire sales by failed banks. Nonlinearities arising from the combination of bankruptcy costs and

network and asset-price feedback effects yield bimodal simulated asset and profit distributions. This bimodality arises despite the joint normality of all risk factors and the linear modeling of credit risk and trading-book gains and losses.

Since this paper was designed to present a broad, illustrative *framework* for modeling systemic risk, there are clearly a number of areas in which the model could be developed. For example, a more sophisticated macroeconomic model including a wider range of risk factors, such as unemployment and house prices, may help to improve the fit of the probability of default equations. Additional variables would also assist with endogenizing LGDs in a meaningful way. Both of these extensions would probably increase the variation in credit losses. There is also clear scope for improving the modeling of the trading book. And the changing nature of financial intermediation highlights the importance of modeling non-interest income and attempting to capture the effects of securitization, credit-risk transfer, and off-balance-sheet items.

A more substantial area for further work is to develop our models of feedback effects on both the asset and liability sides of banks' balance sheets. RAMSI currently captures one such feedback: the impact of post-default fire sales of assets on asset prices. But the ongoing crisis has highlighted the importance of modeling the causes and implications of funding liquidity stress. Therefore, we are currently developing a set of indicators of funding stress which may be used as a guide to suggest when different funding markets may close to particular institutions. Further, by analyzing the cash-flow constraints of banks experiencing funding stress, we intend to model how banks' defensive actions, including liquidity hoarding and pre-default fire sales, may affect the rest of the financial system.

A longer-term challenge is to incorporate feedbacks from the banking sector to the real economy. In principle, this could be done by adding a reduced-form equation to the model linking realized banking sector profits and losses to the future path of the macro economy. Such an equation, though, would inevitably be difficult to formulate and estimate. Furthermore, the underlying mechanism may be partially captured by the estimated macroeconomic model, making it hard to identify the precise feedback from the banking sector. Despite these complications, the issue is highly relevant from a policy perspective and merits further investigation.

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Interdependencies between Expected Default Frequency and the Macro Economy*

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We use a vector error-correction model to study interdependencies between the aggregate expected default frequency (EDF) and the macroeconomic development. The model is used to forecast the median EDF. Evaluations of the model show that it yields low forecast errors and that the interest rate has the strongest impact on expected default frequency. Forecasts indicate that a lower short-term interest rate reduces the EDF and, in turn, risk premiums. This reduces the marginal cost for corporate investments and household consumption and stimulates growth through these two components of aggregate demand. At the same time, it imposes a downward pressure on the product prices of firms and thereby on inflation.

JEL Codes: C32, C52, C53, G21, G33.

1. Introduction

The turmoil that started in 2007 has led to a large-scale global crisis of confidence. The origin of this crisis can be found in reckless lending practices. It began when American financial institutions lent money to borrowers with weak credit histories. The high-risk subprime loans were sold on to a large extent by bundling them together with low-risk loans in what are known as structured credit products.

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Investors found it difficult to conduct their own risk assessments and largely relied on the classifications of credit-rating agencies. When the losses on subprime loans became unexpectedly large, genuine uncertainty arose about the value of the structured products and where losses associated with them would occur. Banks and other financial institutions became cautious about lending money to one another. They began to hoard liquidity to meet their own obligations and to avoid losses on loans to counterparties about whose status they felt uncertain in the wake of the general crisis. The cost for bank borrowing on the interbank market rose, causing other market interest rates to climb. As a result, the banks' borrowing costs have risen, access to loans has decreased, and companies and households are paying more for loans. In summary, financial institutions lending to high-credit-risk borrowers led to the confidence crisis in the financial system, which led to high risk premiums and high lending rates, which in this case have intensified the downswing in the global economy. An economic downswing will in turn increase the credit risk for all borrowers and especially for corporations. To alleviate the real economic effects of the financial crisis and to stress-test credit risk in banks, it is therefore necessary to have a good understanding of the interdependencies between credit risk and the macro economy.

Operations of banks are typically dominated by the granting of credit; therefore, credit risk is the largest individual risk in the banking system by far. In recent years, central banks and commercial banks have begun to use models that make it possible to more coherently probe the development of the banks' credit risks on the basis of different assumptions and events. A proper assessment of credit risk within banks requires some kind of a credit portfolio model. The idea behind a credit portfolio model is that the resilience of the banks is reflected in the size of the capital buffer they hold in relation to the credit risk measured in their loan portfolios. A portfolio model makes it possible to calculate the probability that loan losses of various sizes may arise in existing portfolios. Information regarding the composition of the portfolio, the probability of default, and recoveries is needed in order to calculate the risk in the loan portfolio. Two measures are often used to quantify the credit losses the banks may incur. One is a measure of the expected loss that indicates how much a bank can expect to lose in its current credit portfolio. This is calculated by multiplying the likelihood of default by exposure at

default (exposure*LGD, where LGD is loss given default). The other is a measure of the size of the losses that can occur in addition to the expected losses (unexpected losses) and for which the bank must have capital cover (required risk capital). In this way, it is possible to study how changes in the credit quality of the banks' borrowers influence the credit risk in the banks' loan portfolios. The banks compensate themselves for both the expected and the unexpected loss through a risk premium on the price of loans in their regular operations. If there is an increase in the expected loss in the portfolio, this may mean that the banks' costs increase as a result of increased reserve funds. The banks hold a buffer to cover possible loan losses above those expected; let us call this the risk capital requirement. Loan loss distribution makes it possible for banks to calculate the size of this need, given a certain tolerance level. The unexpected loan loss—and thereby the need for risk capital—also affects the prices banks set for their loans, since holding capital entails a cost for the banks in the form of a return-on-investment requirement from the shareholders, and the banks must compensate themselves for this. The amount of capital the bank requires to cover unexpected losses depends on the loan loss distribution. The greater the probability of extreme outcomes—that is to say, outcomes that lie in the tail of the loss distribution—the greater is the need for risk capital.

The above discussion indicates that one of the most important variables for assessing credit risk in banking is the likelihood of default, which reflects the borrowers' credit quality. It is quite likely that macroeconomic variables play an important role in determining the direction of the future development of borrowers' credit quality.¹ Linking credit quality to the development of macroeconomic variables makes it possible to undertake endogenous forecasts for credit risk as well as scenario analyses where the credit risk for the banks can be appraised on the basis of the paths of different macroeconomic development curves. We present a model that creates a link between the assessment of credit risks and macroeconomic appraisals.

Several papers address the empirical relationship between fundamentals and default probabilities among companies. Chan-Lau (2006) offers a survey of this literature where macroeconomic-based

¹See, e.g., Jacobson, Lindé, and Roszbach (2005).

models constitute one class of such models. These models study how default probabilities are affected by the state of the economy, and they can be divided into models that allow for feedback between default probabilities and explanatory economic variables and models that do not. Virolainen (2004) is an example of the latter category, while Alves (2005), Jacobson, Lindé, and Roszbach (2005), Pesaran et al. (2006), and Castrén, Déés, and Zehar (2007) are time-series models that do take feedback effects into account.

Jacobson, Lindé, and Roszbach present an empirical model that consists of a system made up of three blocks. The first is a vector autoregressive (VAR) model for the macroeconomic variables they consider. They include domestic output, inflation, the nominal interest rate, and the real exchange rate as endogenous variables in the VAR model. The foreign macro variables as well as the aggregated default frequency of incorporated firms enter the model exogenously. In the second block, they have a logit model for the default risk at the firm level, where the macroeconomic variables as well as various balance-sheet variables enter as regressors. The third block in their empirical model estimates the dependence of the balance-sheet variables included in the logit model on the macroeconomic variables. Alves (2005), Pesaran et al. (2006), and Castrén, Déés, and Zehar (2007) are models that allow for influence from explanatory economic variables on default probabilities, but not the other way around. They use VAR models for forecasting the development of the macroeconomic variables. These forecasts are then used in a satellite model for credit risk. Unlike Pesaran et al. (2006) and Castrén, Déés, and Zehar (2007), Alves (2005) takes into account that the likelihood of defaults and the macroeconomic variables display common trends. This is also the case in our model. Moreover, we also take feedback effects into account, which means that we allow for influence from macroeconomic variables on default probabilities, and the other way around.

In this paper, we depart from the literature in one important respect. We analyze the likelihood of defaults in the corporate sector using a forward-looking measure of the likelihood of defaults. One example of a structural credit-risk model of this kind is Credit Monitor (Moody's KMV), which offers a theoretically attractive model for calculating the empirical expected default frequency

(*EDF*) for individual companies.² Forward-looking measurement of the capacity of listed companies to make payments can be calculated using the market value of their assets in relation to the book value of their debts. The market value of equity is a function of the current value of all future cash flows the company can be expected to generate. General economic developments play an important role for the development of company cash flows. Therefore, it is reasonable to assume that *EDF* and the macroeconomic variables display common trends. We study the long-term relationship between expected default frequencies and macroeconomic development using a vector error-correction model (VECM), i.e., a VAR model that includes an error-correction term. The choice of a VECM can be justified by its ability to discern shared trends between series as well as allowing for feedback between default probabilities and explanatory macroeconomic variables. Estimates of the coefficients may be improved if the existence of shared trends in series is taken into account. Including shared trends becomes even more important when the model is estimated on high-frequency data, which is the case in this paper. A principal feature of cointegrated variables is that their time paths are influenced to the extent that any of these deviate from their long-run relationship. Moreover, the short-run dynamics must be influenced by the deviation from the long-run relationship.

This paper has three objectives. One is to explore whether the development over time of the aggregate EDF for listed companies provided by Credit Monitor can be explained by macroeconomic development.³ Another objective is to make forecasts for future default frequencies as well as the macroeconomic variables. By this, we mean that the forecasts are made taking into account the feedback effects from the EDF on the macroeconomic variables and vice versa. The third objective is to conduct a stress test of the aggregate EDF, given different alternative macroeconomic development.

In section 1, we give an intuitive discussion of the impact of various macroeconomic factors on aggregate EDF. Section 2 presents the

²This model is based on Merton's approach for the evaluation of credit risk as refined by Vasicek and Kealhofer, which is why it is known as Kealhofer Merton Vasicek (KMV).

³Aggregate EDF is represented by the monthly median of EDF for individual Swedish nonfinancial companies in Moody's KMV Credit Monitor.

database used for the empirical analysis, followed by an empirical time-series model for aggregate EDF in section 3. Section 4 contains an evaluation of this model. In section 5, forecasts are given for future default frequency in the corporate sector. In section 6, a stress test is conducted of the expected default frequency of companies. The discussion that follows sums up and concludes the paper.

2. Expected Default Frequency and Macroeconomics

Moody's KMV Credit Monitor calculates *EDF* as a function of the distance to default. The premise for Moody's KMV model is that a company is bankrupt when the market value of its assets (MV_A) is lower than its default barrier, i.e., the company's debts (D): $MV_A < D$ or $MV_E = MV_A - D < 0$, where MV_E is the market value of the company's equity. The distance to default (DD) is measured in the number of standard deviations, which makes it possible to compare default frequencies between different companies, irrespective of their size:

$$DD = \frac{MV_A - D}{\sigma_A} = \frac{MV_E}{\sigma_A}, \quad (1)$$

where σ_A is the volatility (standard deviation) of the market value of the company's assets. The market value of the company's equity is a function of the current value of all future cash flows the company can be expected to generate. General economic developments play an important role for the development of company cash flows. Therefore, it is reasonable to assume that the EDF and the macroeconomic variables display common trends. The existence of common trends is estimated and tested using a vector error-correction model. We have decided to present the macroeconomic conditions using three different variables: the domestic industrial production index (*INDY*), the domestic consumer price index (*CPI*), and the nominal domestic three-month rate for Treasury bills (*R3M*). On this basis, we estimate the following relationship:

$$\log(EDF) = \beta_0 + \beta_1 R3M + \beta_2 \log(INDY) + \beta_3 \log(CPI) + u. \quad (2)$$

The selection of the macroeconomic variables included in the empirical assessment in this paper is based on Jacobson, Lindé,

and Roszbach (2005). They model the macro economy using a set of macroeconomic variables, including aggregate bankruptcy frequency, in a quarterly VAR model. Based on work by Lindé (2002), they choose to include the following endogenous variables: the gap in domestic production, domestic inflation, the Riksbank's repo rate, and actual exchange rates. The exogenous variables included in their paper are the gap in foreign production, foreign inflation, and foreign three-month interest rates. In addition to these macroeconomic variables, Jacobson, Lindé, and Roszbach (2005) also include a measurement of the aggregate proportion of defaults as another exogenous variable in the VAR model. This consists of the number of actual defaults in relation to the total number of existing companies.

The model in this paper is deliberately based on just a few variables in order to keep the analysis simple and transparent. This means that operationalizing the model in the ongoing analysis does not require any large amount of resources and that the results are not too difficult to interpret. Further, the aim of the model is to provide a platform for scenario analysis to study the effects of major macroeconomic shocks—an analysis that is by nature relatively rough. Moreover, the model in this paper will be used to make both conditional and unconditional forecasts on the EDF. Unconditional forecasts mean that we let the model forecast the development of both the EDF and the macroeconomic variables. By this, we mean that the forecasts are made taking into account the feedback effects from the EDF on the macroeconomic variables and vice versa. Conditional forecasts are made by conditioning EDF forecasts on external forecasts on macro variables. The external forecasts for these macro variables are made taking the foreign macroeconomic developments into account. This is one of the reasons why we do not include foreign macroeconomic variables as exogenous variables in the model. Finally, we have opted for including industrial production in the estimates instead of the non-observable production gap since the estimates in this paper are based on monthly data. The estimates indicate that, on the whole, there is a one-to-one relationship between changes in the GDP and changes in industrial production.

It is difficult to know a priori what effect each of the macroeconomic variables may have on EDF. From the model specification in appendix 1 (equation (10)), the impact of the macroeconomic

variables on EDF is not unambiguously decided, and thus it is ultimately an empirical issue. Nevertheless, it can be argued that the different macro variables should affect the EDF in certain ways. A negative correlation is expected between manufacturing output and EDF because increased output implies higher economic activity and higher corporate earnings. A higher interest rate increases the interest expenditure on corporate loans, which tends to raise EDF. The link between inflation and EDF is mainly twofold, through factor prices and the prices companies charge for their goods and services. Higher factor prices lead to increased production costs and tend to impair credit quality. Higher product prices can boost earnings and thereby improve creditworthiness. The relative strength of these two effects of inflation is determined by the structure of the markets for factors of production and the company's output.

It is also difficult to know a priori what effect the EDF may have on each of the macroeconomic variables. As we mentioned in the introduction to this paper, the banks compensate themselves for the expected loss by including a risk premium in their lending interest rates. Increased lending interest rates have a negative impact on economic growth through reduced household consumption and corporate investments. Household consumption decreases as higher lending rates make consumption more expensive. Corporate investments decrease as higher lending rates increase the marginal cost of capital, which makes investments more expensive. At the same time, increased marginal cost of capital imposes an upward pressure on product prices of firms in a monopolistic competition market, thereby causing an upward pressure on inflation. This is because the product prices of firms in a monopolistic competition market are set as a markup to their marginal cost, which also includes the marginal cost of capital.

The link between short-term interest rate and EDF is mainly twofold because the short-term interest rate is a policy variable. First, the interest rate might be increased by the monetary authorities because of the fact that higher EDF leads to higher inflation. The reason for this is that the rule for the monetary policy is to use the interest rate to restore inflation to its target level. Second, as we mentioned earlier, higher interest rates imply higher EDF. Therefore, by decreasing the interest rate, the monetary authorities can decrease the EDF and the risk premiums banks charge through

their lending interest rates. In this case, it implies higher growth and lower inflation. The relative strength of these two effects of interest rate is determined by the structure of the lending market.

3. Data

The estimations are based on monthly data of expected default frequency covering the period from November 1997 through September 2008. Data on the empirical expected default frequency for nonfinancial listed companies (*EDF*) are from the Credit Monitor (Moody's KMV). The index for industrial production (*INDY*) has been taken from Reuters EcoWin.⁴ The consumer price index (*CPI*) and interest rates on three-month Treasury bills (*R3M*) come from Statistics Sweden. Data for *INDY* are available for the period from January 1990 through August 2008. Data for *CPI* and *R3M* are available for the period from January 1970 through September 2008.

Testing to find out whether series are stationary is carried out using unit-root tests. These tests indicate that all variables used in the estimates of the VECM in this paper appear to be nonstationary.⁵ However, the test for the occurrence of cointegration in the next section does not presume that all series are $I(1)$ variables. Even if a specific series is not stationary, combinations of such series may have a cointegrating connection.

4. VECM for the Aggregate Expected Default Frequency

In the analysis of the time series, it is possible to show that even though all series prove to be nonstationary, a linear combination of them may nevertheless be stationary, i.e., integrated at the order of

⁴Reuters EcoWin is a provider of economic and financial market data. For more information about EcoWin, the reader is referred to <http://www.ecowin.com/>.

⁵*EDF*, *INDY*, and *R3M* have unit roots and are $I(1)$ variables. However, even if we realize that *EDF* may not be a genuine $I(1)$ variable, it seems to behave as an $I(1)$ variable during the sample period studied in this paper. Moreover, *CPI* seems to be an $I(2)$ variable, which indicates that CPI inflation is a nonstationary variable. That CPI inflation and *R3M* have unit roots is due to a shift in levels around 1993. It is more than possible that CPI inflation and *R3M* are not genuinely $I(1)$. Nor is it necessary for the analysis in this paper for CPI inflation and *R3M* to be $I(1)$ variables, even though this is what the test indicates.

Table 1. Johansen's Test for Cointegrating Relationships

Null Hypothesis	λ_{trace}	5% Critical Value	P-value
$r = 0$	57.4765	47.8561	0.0048
$r \leq 1$	23.9003	29.7970	0.2047
Notes: Since the cointegrating vector is not identified, we impose different identifying restrictions on the cointegrating vector. The likelihood-ratio test indicates that the variables are cointegrated notwithstanding which variable we use for the normalization. This indicates that a subset of the variables cannot be cointegrated.			

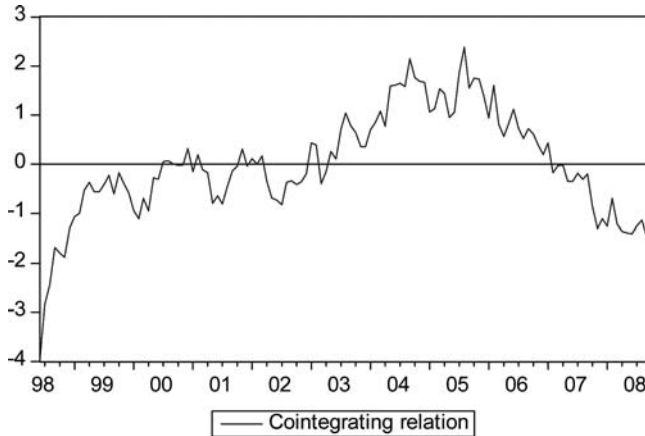
zero. If this is the case for the data series on which our model is based, we can conclude that *EDF*, *INDY*, *CPI*, and *R3M* are cointegrated. This means that the linear combination cancels out the stochastic trends in these series.

Using Johansen's (1988) method is one way of testing whether a data series is cointegrated. Table 1 presents the test statistics for this method (λ_{trace}). The test indicates that we can reject the hypothesis that there exists no cointegrated relationship at the 5 percent significance level; i.e., there is at least one cointegrating vector.⁶ The test also indicates that there is only one cointegrating relationship out of the possible four. Figure 1 contains a graphic representation of the cointegrating relationship. The relationship has been normalized on the basis of *EDF*, as our primary interest is in the effects of the macroeconomic variables on the EDF. The test provides support for a long-term relationship between *EDF*, *INDY*, *CPI*, and *R3M*.⁷

After having tested for cointegrating relationships, we also need to decide on the appropriate lag structure for the model. The choice of lag structure is basically an empirical question, and the lag structure chosen in the specified empirical model is based on three different criteria. First, a residual test is made using a serial correlation Lagrange-multiplier (LM) test. This test is an alternative to the Q-statistics for testing serial correlation. The LM test is used to test for higher-order ARMA errors. Second, when the possibility that the

⁶However, it should be noted that we cannot reject the null hypothesis that the number of cointegrating vectors is less than or equal to one ($r \leq 1$).

⁷A maximum characteristic root test also indicates that there is only one cointegrating relationship.

Figure 1. Cointegrated Relationship

errors exhibit autocorrelation has been excluded for a given lag structure, we also investigate whether the estimated coefficients in the cointegrating relationship are stable. This is done by investigating whether the estimated coefficients in the cointegrating relationship change when new observations are added to the database. Finally, we investigate the out-of-sample forecasting accuracy for different models with different lag structures. We choose the model that gives the lowest forecast error in terms of root mean squared error (RMSE).⁸ After having determined the lag structure, we specify the VECM:

$$\begin{aligned} \Delta x_t = & \delta_0 + \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \Gamma_3 \Delta x_{t-5} \\ & + \Gamma_4 \Delta x_{t-6} + \alpha \beta' x_{t-1} + \varepsilon_t, \end{aligned} \quad (3)$$

where $x_t = [\log(EDF_t), R3M_t, \log(CPI_t), \log(INDY_t)]$, $\delta_0 = \Gamma_0 - \alpha\beta_0$, and $\varepsilon_t \sim N(0, \Omega)$.⁹ Two important parameters estimated in this model are β and α . The cointegrating vector, summarized by matrix β , describes the long-run relationships between the endogenous variables. The loading (or adjustment) coefficients forming matrix α

⁸The outcomes of these tests and evaluations can be provided by the authors upon request.

⁹A specification of the model where the implied volatility of the stock market was included has been estimated to capture some measure of market risk. This caused some problems with multicollinearity in the model, and therefore we chose the specification where the volatility measure is excluded.

Table 2. Estimated Beta Parameters for Long-Term Relationship

	β	α
$\log(EDF)$	1	-0.064 (-5.04)
$R3M$	1.08 (-4.21)	0.009 (0.83)
$\log(CPI)$	30.00 (-2.90)	-1.11E-06 (0.00)
$\log(INDY)$	-22.03 (3.11)	-0.002 (-2.02)
Note: T-values are presented in parentheses.		

describe the dynamic adjustment of the endogenous variables to deviations from long-run equilibrium by $\beta'x$.¹⁰

Table 2 summarizes the maximum likelihood estimate (ML estimate) for the beta parameters in the long-run relationships in the estimated model. All the coefficients are significant and have the expected signs. This means that in the long term, industrial production, *INDY*, has a negative effect on *EDF*, while *CPI* and *R3M* have a positive effect on *EDF*.

The test that $\beta = 0$ and $\alpha = 0$ entails restrictions on cointegrating vectors or the adjustment parameters. The likelihood-ratio tests indicate that we can reject each and every one of the following hypotheses: $\beta_2 = 0$, $\beta_3 = 0$, $\beta_4 = 0$, $\alpha_1 = 0$, and $\alpha_4 = 0$. However, we cannot reject that $\alpha_2 = 0$, $\alpha_3 = 0$, which is also indicated by the t-values. We were also able to reject the hypothesis that $\beta = (0, 1, x, x)$, where x denotes a free parameter. The hypothesis excludes *EDF* from the cointegrating vector, imposes an identifying restriction on *R3M*, and estimates *INDY* and *CPI*. This indicates that the macroeconomic variables cannot be integrated with each other.

Adjustments of the variables to the long-run level after shocks have occurred take place via adjustment coefficients or the “error-correction terms,” α . Table 2 offers a summary of the estimated adjustment coefficients. The error-correction terms for *EDF* and

¹⁰When the *EDF* deviates from its estimated long-term level, ($\alpha = 0.1$) thus indicates that 10 percent of the deviation will be corrected in the subsequent period. How long it will take before the system returns to long-term equilibrium can be calculated using α .

INDY (α_1 and α_4) are negative, indicating convergence toward the long-run equilibrium. The fact that α_2 and α_3 are not significant does not constitute a problem, as Johansen's test reveals that the model converges toward long-run equilibrium. To save space, the rest of the results of the estimation are presented in appendix 2.¹¹ The results for various specification tests of the estimated model do not reveal any problems.¹²

5. Evaluation of the VECM

In this section, we evaluate the VECM by analyzing its within-sample properties as well as its out-of-sample properties.

5.1 *Within-Sample Forecasts*

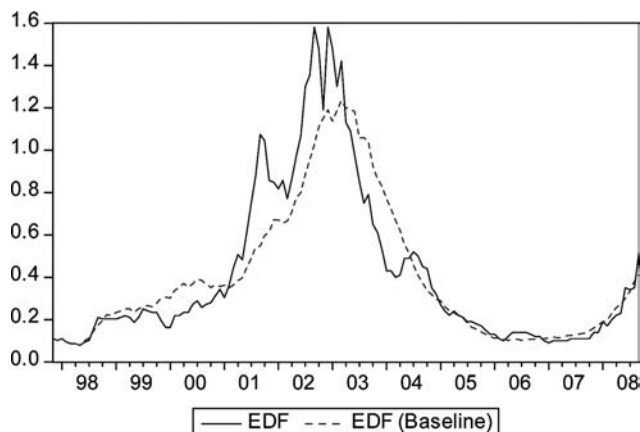
A comparison between within-sample forecasts for the VECM with the actual outcomes for the sample period shows that the model replicates the actual distribution of the EDF relatively well; see figure 2. However, the important question is, how good will the out-of-sample forecasts of the EDF be?

5.2 *Out-of-Sample Forecasts*

The VECM is evaluated in two ways. One way is by comparing the RMSE for the VECM with the RMSE for the forecasts made using a naive model (such as a random-walk model or an AR(1) model). A second evaluation method is through the comparison of the RMSE for the forecasted EDF for one month, three months, six months, one year, and two years ahead with the standard deviation for EDF.

¹¹Due to lack of space, we have opted not to present t-values for the short-term coefficients. Naturally, these can be supplied by the authors upon request.

¹²The autocorrelation LM test indicates that there is no autocorrelation in the residuals. White's heteroskedasticity test indicates that there are no heteroskedasticity problems in the residuals. The normality test reports the multivariate extensions of the Jarque-Bera residual normality test. For the multivariate test, a factorization of the residuals that are orthogonal to each other must be chosen. We have used the factorization method suggested by Urzúa (1997). Testing for whether the residuals are normally distributed confirms that this is the case.

Figure 2. Within-Sample Forecasts for EDF Over Time

5.2.1 RMSE for the VECM vs. RMSE for Naive Models

The procedures used are as follows. The VECM for EDF is estimated on data from June 1998 through December 2002. The estimated VECM is used to make four different forecasts for the period from January 2003 through September 2008. First, we make endogenous forecasts, which means that the models generate trajectories for all variables included in the model. Unconditional forecasts mean that forecasts are made by taking into account the feedback effects from the EDF on the macroeconomic variables and vice versa. Second, we also make semi-endogenous forecasts for *EDF*, *CPI*, and *INDY* by conditioning the forecasts for these variables on the outcome for the short-term interest rate. The reason for making semi-endogenous forecasts is the fact that the interest rate is a policy variable. Many central banks use the interest rate to control the inflation rate. This policy instrument also has an impact on expected default frequencies in an economy. However, given the path for the interest rate, the macroeconomic variables as well as the EDF can have important feedback effects on each other. By making semi-endogenous forecasts, we are able to take these feedback effects into account, given the path for the interest rate. Third, conditional forecasts are made for the period from January 2003 through September 2008. This is made possible by conditioning the EDF forecasts on the outcomes for the macroeconomic variables for the period from January

2003 through September 2008. The reason for making conditional forecasts is to examine how accurate the forecasts for the EDF are when uncertainty regarding the development for the macroeconomic variables is removed from the model. Fourth, the estimated VECM is used to make conditional forecasts for the period from January 2003 through September 2008—in this case, conditioned on the Riksbank's official forecasts for the macroeconomic variables presented in its Monetary Policy Reports 2003:1–2008:2.

EDF is also estimated using a simpler AR(1) model: $EDF_t = \alpha EDF_{t-1} + \varepsilon_t$. The estimated AR(1) model for EDF is used to make alternative EDF forecasts. In addition, a random-walk model is used to make EDF forecasts. The premise for this naive forecast is that the best forecast for future EDF is provided by the most recent information about the outcomes for the same variable.

Then, we proceed step by step by increasing the sample one month at a time and making new estimates of the model and forecasts using the various models as described above. Finally, the RMSE is calculated for the four forecasts that have been produced using the VECM, the AR(1) model, and the random-walk model. The RMSE for the different models is calculated as follows:

$$RMSE = \sqrt{\frac{1}{T} \sum_{t=s}^{2006:3} (EDF_t^{forecast} - EDF_t^{actual})^2}, \quad (4)$$

where $EDF_t^{forecast} - EDF_t^{actual}$ is the forecast error. The forecast errors are squared so that both overestimates and underestimates will have the same weight. Then, the average of the squared forecast errors is calculated. The square root of this figure then provides the RMSE. For endogenous and exogenous forecasts, the number of periods selected is $T = 69$ and the starting period for the forecast is $s = 2003 : 1$ for those with one-month forecast horizons.¹³

Table 3 presents a summary of the RMSE results for the four EDF forecasts produced using the VECM (endogenous forecasts, semi-endogenous forecasts, forecasts conditioned on the outcomes

¹³However, $T = 67$ and $s = 2003 : 3$ for forecasts with three-month forecast horizons; $T = 64$ and $s = 2003 : 6$ for forecasts with six-month forecast horizons; $T = 58$ and $s = 2003 : 12$ for forecasts with one-year forecast horizons; and $T = 46$ and $s = 2004 : 12$ for forecasts with two-year forecast horizons.

Table 3. RMSE for EDF

Period	Conditioned Forecasts Using Riksbank's Macro Forecasts	Endogenous Forecasts	Semi- Endogenous Forecasts	Conditioned Forecasts Using the Outcomes for Macro Variables	AR(1)	Random Walk
One Month	0.21	0.08	0.08	0.08	0.07	0.07
Three Months	0.37	0.16	0.12	0.12	0.15	0.13
Six Months	0.58	0.29	0.19	0.23	0.27	0.23
One Year	0.86	0.60	0.35	0.25	0.46	0.34
Two Years	2.20	1.21	0.90	0.78	0.79	0.49

for the macroeconomic variables, and forecasts conditioned on the Riksbank's forecasts of the macroeconomic variables in its monetary policy reports), the AR(1) model, and the random-walk model.

The VEC forecasts conditioned on the Riksbank's macro forecasts in its Monetary Policy Reports have consistently higher RMSE figures than the two forecasts based on the AR(1) model and a random-walk model.¹⁴ However, the endogenous forecasts have much better forecasting accuracy than forecasts conditioned on the Riksbank's forecasts of the macroeconomic variables in its monetary policy reports. Moreover, the endogenous forecasts are as good as forecasts made by AR(1) and random-walk models up to six months. Semi-endogenous forecasts outperform both the forecasts conditioned on the Riksbank's forecasts of the macroeconomic variables in the Monetary Policy Reports and the endogenous forecasts. Moreover, semi-endogenous forecasts are as good as, or even better than, the forecasts made by AR(1) and random-walk models up to one year. Finally, semi-endogenous forecasts are as good as forecasts conditioned on the actual outcomes for macro variables.

What is interesting in this context is that the forecasting accuracy of VEC forecasts conditioned on the actual outcomes for macroeconomic variables is better than the accuracy of other forecasts made by the VECM. The RMSE declines for the entire period when uncertainty about macroeconomic developments is eliminated by conditioning the forecasts on the outcomes for macroeconomic variables. This indicates that the estimated equation for the EDF has good forecasting properties. Moreover, the RMSE falls below the RMSE for forecasts made using the AR(1) model up to two years. Finally, the RMSE falls below the RMSE for forecasts made using the random-walk model up to one year.¹⁵

¹⁴The model in this paper is estimated using monthly data. To make conditioned forecasts with this model, we need monthly forecasts for the macroeconomic variables. However, the Riksbank's macro forecasts are usually presented quarterly (this is especially the case for the GDP and the interest rate forecasts). This means that we are obliged to transform these quarterly forecasts into monthly forecasts using a rather simple method. In this way, we introduce a measurement error when making the forecasts that may magnify the RMSE.

¹⁵Yet another way of evaluating the VECM is to conduct a sign test to study the extent to which the model's forecasts develop in the same direction as EDF outcomes. The results of a test of this kind can lie between 0 and 100 percent. A

5.2.2 RMSE vs. the Standard Deviation of EDF

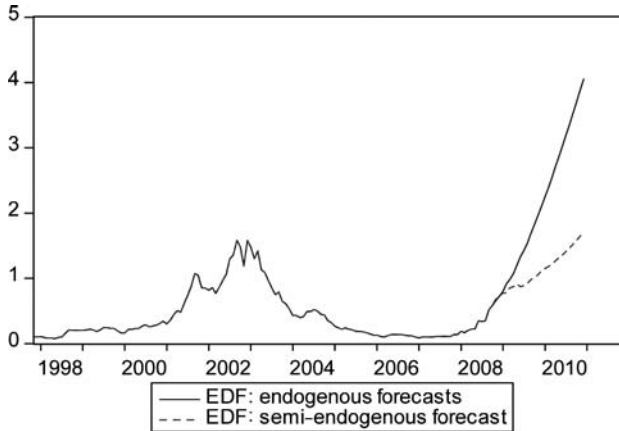
Another way of evaluating the RMSE for the different forecasts (and the different periods of time) is to relate the RMSE to the standard deviation of the EDF, which in this case is 0.38. This comparison indicates that conditional forecasts using the Riksbank's macro forecast are reliable between three and six months ahead, and endogenous forecasts as well as forecasts made by the AR(1) model are reliable between six months and one year ahead. However, the semi-endogenous forecasts, conditioned on forecasts using the outcome for the macro variables as well as forecasts made by the random-walk model, are reliable between one and two years ahead.

6. Endogenous and Semi-Endogenous Forecasts

The estimated VECM is used to forecast EDF in two different ways. First, we make unconditional forecasts, which means that the forecasts are made by taking into account the feedback effects from the EDF on the macroeconomic variables and vice versa. Second, we also make semi-endogenous forecasts for *EDF*, *CPI*, and *INDY* by conditioning the forecasts for these variables on the interest rate forecasts made by the Riksbank that is presented in its Monetary Policy Report 2008:3. The reason for making semi-endogenous forecasts is the fact that the interest rate is a policy variable and exogenous in its nature.

The EDF forecasts using these two different methods are summarized in figure 3. The endogenous forecast indicates that the aggregate EDF will rise from 0.52 percent in September 2008 to about

zero result means that the forecast model never indicates whether EDF will rise or fall during the forecast period, and 100 percent indicates perfect predictions. A sign test indicates that the model specifications that exclude macro variables (AR(1) and random-walk models) have much lower prediction capacity compared with VECMs that include macro variables. The predictive capacity of the VECM is particularly good if the EDF forecasts are conditioned on macroeconomic forecasts with a high degree of precision. The outcome of this test can be provided by the authors by request.

Figure 3. Conditioned Forecasts of Aggregate EDF

4.05 percent in December 2010.¹⁶ However, according to the semi-endogenous forecast, where we have taken the Riksbank's monetary policy into account, the EDF will rise from 0.52 percent in September 2008 to about 1.71 percent in December 2010. Both forecasts indicate that there will be a turn in the credit cycle in terms of higher default frequencies during the forecast period, a turn that actually began already in May 2008. Both the endogenous and the semi-endogenous forecasts for the EDF reveal that the interest rate has a very strong and important impact on the expected default frequency.

Since the semi-endogenous forecasts use an exogenous path for future interest rate levels, as explained above, while in the endogenous forecasts the interest path is determined endogenously, the paths for future interest levels will differ between the two models. The endogenous forecast model generates a path for future interest rates that is higher than the path for future interest levels in the semi-endogenous model. This is because in the endogenous forecasts, the VECM tries to stabilize the future paths for all included variables, *R3M*, *CPI*, *INDY*, and the EDF and allows for feedback

¹⁶The endogenous forecast made by the VECM is much like the forecasts made by the VECM conditioned on the Riksbank's Monetary Policy Update of September 2008. We present these forecasts in the next section.

effects between all of them. In the semi-endogenous case, the VECM will try to stabilize future paths for all included variables except for the interest level that is exogenous. Moreover, it will not allow for feedback effects on the short-term interest level from the other included variables. Therefore, the semi-endogenous forecasts for the EDF are much lower than the endogenous forecasts for the EDF.

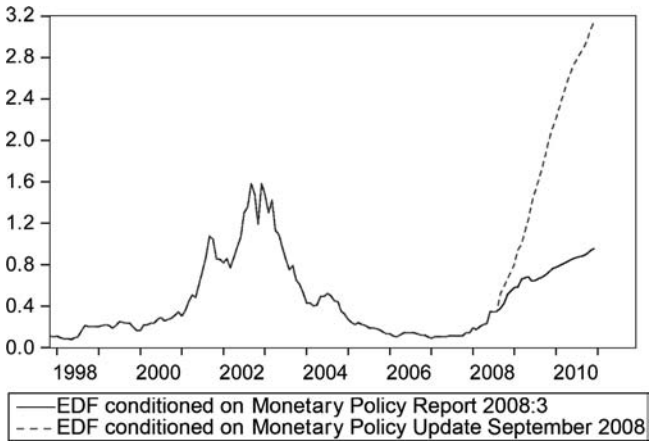
As we mentioned earlier, banks compensate themselves for expected credit losses by including a risk premium in their lending interest rates. The EDF can be seen as a measure of this risk premium. A higher (lower) EDF therefore implies a higher (lower) risk premium and ultimately a higher (lower) lending interest rate. By this reasoning, banks' lending interest rates should be lower in the case of semi-endogenous forecasts than in the case of endogenous forecasts. Reduced lending interest rates have a positive impact on economic growth through increased consumption and increased corporate investments. At the same time, decreased lending interest rates reduce the marginal cost of capital and impose a downward pressure on the product prices of firms in a monopolistic competition market. Therefore, inflation is expected to be lower in the semi-endogenous model compared with the endogenous model. This is exactly what the forecasts for the CPI reveal.

7. Stress Testing Using Conditional Forecasts

The estimated model can be used to make conditional EDF forecasts based on different developments for the macroeconomic variables. The VECM's conditional EDF forecasts are obtained by allowing the macroeconomic forecasts made by the Riksbank and presented in its Monetary Policy Report to determine the development of the macroeconomic variables in the model. We use the macroeconomic forecasts presented in the Riksbank's Monetary Policy Update of September 2008 and the Riksbank's Monetary Policy Report 2008:3.

In the Monetary Policy Update of September 2008, the Riksbank feared that inflation measured in terms of *CPI* would rise by 3.9 percent in 2008, 3.2 percent in 2009, and 2.0 percent in 2010. This was believed to be the case because of different supply shocks heating the economy. The Riksbank assumed that a short-term interest rate of 4.57 percent in 2007, 4.55 percent in 2009, and 4.34 percent in 2010 would help to restore inflation to its target. GDP in Sweden

Figure 4. Development of Aggregate EDF Given Two Different Macroeconomic Developments



was at the same time estimated to increase by 1.4 percent in 2008, by 0.8 percent in 2009, and by 26 percent in 2010.

In its Monetary Policy Report 2008:3, the Riksbank argued that the macroeconomic development will be less favorable than anticipated because the ongoing worldwide financial crises will be followed by high risk premiums required by the investors. Because of this crisis, the Riksbank revised its GDP and CPI forecasts downward. GDP in Sweden was estimated to increase by 1.2 percent in 2008, by 0.1 percent in 2009, and by 2.5 percent in 2010. Inflation measured in terms of *CPI* was expected to rise by 3.7 percent in 2008, 2.1 percent in 2009, and 1.6 percent in 2010. These forecasts were made assuming a short-term interest rate of 4.34 percent in 2007, 3.29 percent in 2009, and 3.47 percent in 2010.

The EDF forecasts for the same period are summarized in figure 4. The forecast conditioned on the Riksbank's Monetary Policy Update of September 2008 indicates that the aggregate EDF will rise from 0.52 percent in September 2008 to about 3.14 percent in December 2010. However, because of the Riksbank's new monetary policy and the new forecasts of GDP and CPI, the EDF will rise from 0.52 percent in September 2008 to about 0.95 percent in December 2010.

These conditional forecasts reveal that the macroeconomic development is very important for the development of future corporate defaults measured by the EDF. The fact is that the short-term interest rate has the strongest impact on expected default frequency among the included macroeconomic variables. Both the endogenous and the semi-endogenous forecasts indicate that there will be a turn in the credit cycle in terms of higher default frequencies during the forecast period, which actually began already in May 2008. However, the stress test shows that a supply shock in a situation where the credit cycle is expected to turn upward (in terms of higher default frequencies) during the forecast period and which is countered by the Riksbank with increased interest rates leads to a threefold increase in aggregate expected default frequency at the end of the forecast period.

Yet another way of interpreting the forecasting results is that the Riksbank will succeed in lowering the high risk premiums required by market players because of the ongoing financial crises. This is evident from the forecast conditioned on the macroeconomic development presented in the Riksbank Monetary Policy Report 2008:3. The EDF forecast is much lower in this case compared with the forecast conditioned on the Riksbank's Monetary Policy Update of September 2008. By lowering the interest rate, the Riksbank could reduce future EDFs and thereby also help reduce the risk premiums required by market players that went up because of the financial crisis.

8. Conclusions

In this paper, we estimate a time-series model for predicting future credit quality in the corporate sector. The model is based on aggregated data and a few variables only. This means that it is relatively straightforward and can be used in the ongoing analysis. The variable that represents credit quality is expected default frequency (EDF), a market-based indicator of the probability of a company not being able to meet its commitments within a specified period. The model uses the median EDF for the corporate sector, which is an aggregated measure of credit quality with all company-specific risks eliminated, so that it is solely affected by risk factors that all companies have in common. The model estimates the relationships between

the EDF and three macroeconomic variables: inflation, industrial production, and the short-term interest rate. The estimates are then used to predict credit quality in the future.

A vector error-correction model (VECM) is used to catch long-run relationships between the variables studied as well as short-run fluctuations around these relationships. The effects of different factors on credit quality are ultimately an empirical matter. Estimations using monthly data for the period November 1997 through September 2008 show that increased industrial production is accompanied by a lower expected default frequency. Rising inflation leads to the opposite scenario: a higher expected default frequency and thereby poorer corporate credit quality. However, the short-term interest rate has the strongest impact on corporate credit quality among the three macroeconomic variables. A higher interest rate leads to a higher expected default frequency.

The model's performance is evaluated using two tests. One compares the root mean squared error (RMSE) for predicted credit quality with the standard deviation of recorded credit quality. Another test compares the RMSE for predicted credit quality in the VECM with credit quality predicted with a naive model (i.e., based on an AR(1) model or a random-walk model). Tests indicate that the model's ability to predict corporate credit quality is satisfactory. This means that the model's predictions of future credit quality can be expected to co-vary with actual credit quality to a high degree. Less uncertainty in the macro forecast naturally leads to greater precision in the model's predictions.

The predictions of credit quality are made in two different ways. First, we make unconditional forecasts, which means that the predictions of credit quality are made endogenously, taking into account the feedback effects from the EDF on the macroeconomic variables and vice versa. Second, we also make semi-endogenous forecasts for *EDF*, *CPI*, and *INDY* by conditioning the forecasts for these variables on the interest rate forecasts made by the Riksbank and presented in its Monetary Policy Report 2008:3. The reason for making semi-endogenous forecasts is the fact that the interest rate is a policy variable and is exogenous by nature. Traditionally, a reduced interest rate is expected to increase inflation while increasing the growth rate in the economy. However, when expected defaults are

included in the model, a reduced interest rate gives both lower inflation and higher growth. The reason for this is that a lower interest rate lowers EDF, a measure of the risk premium. Lower risk premiums contribute to lower lending interest rates that firms and consumers meet at the market and lower marginal cost of capital. This stimulates higher growth because corporate investments and households' consumption increase. At the same time, a decreased marginal cost of capital imposes a downward pressure on the product prices of firms in a monopolistic competition market. Therefore, inflation is expected to be lower in the semi-endogenous model compared with the endogenous model. This is exactly what the forecasts for the CPI reveal.

The VECM's EDF forecasts conditioned on the Riksbank's official view of macroeconomic developments in its Monetary Policy Report 2008:3 and its Monetary Policy Update of September 2008 show that there will be a gradual increase in aggregate default frequencies during the forecast period. This development, in turn, indicates a turn in the credit cycle. At the same time, the stress test shows that a supply shock in a situation where the credit cycle is expected to turn during the forecast period, and which is countered by the Riksbank with a rise in interest rates, leads to a threefold increase in aggregate expected default frequency at the end of the forecast period. Yet another way of interpreting the conditioned forecasts is that the Riksbank will succeed in lowering the high risk premiums currently required by market players because of the ongoing financial crises. This is because the EDF forecast is much lower when it is conditioned on the Riksbank macro forecasts presented in its Monetary Policy Report 2008:3 compared with the forecast conditioned on the Riksbank's Monetary Policy Update of September 2008.

The model can be used as one of a number of instruments for forward assessments of banks' credit risks. The EDF predictions can be used as inputs to calculate the economic capital individual banks should hold to cover unexpected credit losses, which is a clear indicator of the credit risk in each bank's loan portfolio. The model can also be used for the analysis of scenarios to test alternative assumptions about macroeconomic developments. Finally, the model can also be used to analyze the interdependencies between the expected default frequency and the macro economy.

Appendix 1. Expected Default Frequency and Macroeconomics

Moody's KMV Credit Monitor calculates *EDF* as a function of distance to default (*DD*): $EDF = f(DD)$.¹⁷ The premise for Moody's KMV model is that a company becomes bankrupt when $MV_A < D$ or $MV_E = MV_A - D < 0$.¹⁸ The distance to default (*DD*) is defined as follows:

$$DD = \frac{MV_A - D}{\sigma_A} = \frac{MV_E}{\sigma_A}. \quad (5)$$

As neither MV_A nor σ_A can be directly observed, an additional assumption must be made to be able to calculate *DD*. Merton (1974) drew attention to the fact that the cost of guaranteeing the value of a company's loans corresponds to the value of a call option on the market value of the company's assets (MV_A) with a redemption price (D) at time T . In parity with this argument, the yield on a company's equity, MV_E , corresponds to the yield of a call option on the company's assets: $MV_E = \max[0, MV_A - D]$. The lenders either receive the market value of the company's assets (if the market value of these assets is less than the company's debts) or full repayment of the loan when it becomes due for settlement: $MV_D = \min[MV_A, D] = \min[MV_A - D, 0] + D$. This yield is equivalent to holding a bond with the nominal value of D and issuing a call option on the company's assets with D as the redemption price. The market value of a company's assets and their volatility can be calculated with the help of Black and Scholes (1973).

The approach used in this paper is to estimate the empirical relationship between *EDF* and the macroeconomic conditions. We know that *EDF* is a function of *DD*; i.e., $EDF = f(DD)$. The elasticity between *EDF* and *DD* is defined as $\varepsilon = \frac{dEDF}{dDD} \frac{DD}{EDF}$. Thereby, we assume *EDF* to be a nonlinear function of *DD*: $EDF = DD^\varepsilon e^u$,¹⁹ where u denotes the residual. Taking the logarithms on both sides

¹⁷For a short description of the method used by Moody's KMV Credit Monitor to derive *EDF*, the interested reader is referred to Rehm and Rudolf (2001).

¹⁸The default barrier is assumed to be deterministic and consists of the nominal value of the debt, i.e., short-term debt + 0.5*long-term debt.

¹⁹This function is known as the exponential regression model.

of this equation, we obtain

$$\log(EDF) = \varepsilon \log(DD) + u. \quad (6)$$

Further, taking the logarithms on both sides of equation (5), we obtain

$$\log(DD) = \log(MV_E) - \log(\sigma_A). \quad (7)$$

The market value of the company's equity is a function of the current value of all future cash flows the company can be expected to generate. General economic developments play an important role for the development of company cash flows. Therefore, it can be assumed that $MV_E = f_1(X)$, where $X = [INDY, CPI, R3M]$. This means, in turn, that the volatility of the market value of the assets should be a function of macroeconomic development: $\sigma_A = f_2(X)$. It is difficult to know a priori what effect the macroeconomic variables can be considered to have on MV_E and σ_A . The elasticity between MV_E and X is defined as $E^1 = \frac{dMV_E}{dX} \frac{X}{MV_E}$. Further, the elasticity between σ_A and X is defined as $E^2 = \frac{d\sigma_A}{dX} \frac{X}{\sigma_A}$. Thereby, we assume that MV_E and σ_A have the following functional forms: $MV_E = E_0^1 e^{E_1^1 R3M} INDY^{E_2^1} CPI^{E_3^1}$ and $\sigma_A = E_0^2 e^{E_1^2 R3M} INDY^{E_2^2} CPI^{E_3^2}$. Taking the logarithms on both sides of these equations, we obtain²⁰

$$\log(MV_E) = \log(E_0^1) + E_1^1 R3M + E_2^1 \log(INDY) + E_3^1 \log(CPI) \quad (8)$$

$$\log(\sigma_A) = \log(E_0^2) + E_1^2 R3M + E_2^2 \log(INDY) + E_3^2 \log(CPI). \quad (9)$$

Inserting (8) and (9) into (7), we can rewrite (6) as follows:

$$\log(EDF) = \beta_0 + \beta_1 R3M + \beta_2 \log(INDY) + \beta_3 \log(CPI) + u, \quad (10)$$

where $\beta_0 = \varepsilon [\log(E_0^1) - \log(E_0^2)]$, $\beta_1 = \varepsilon(E_1^1 - E_1^2)$, $\beta_2 = \varepsilon(E_2^1 - E_2^2)$, and $\beta_3 = \varepsilon(E_3^1 - E_3^2)$ are the coefficients we want to estimate. As is

²⁰Using differential calculus, it can be shown that $E_1^1 = d(\log MV_E)/dR3M = [d(MV_E)/dR3M]/MV_E$, which is the relative change in the regressand divided by the absolute change in the regressor. If we multiply the relative change in MV_E by 100, E_1^1 will then give the percentage change, or the growth rate, in MV_E for an absolute change in $R3M$. In the literature, this is known as the semi-elasticity of MV_E with respect to $R3M$.

evident, it is difficult to know a priori what effect the macroeconomic variables can be considered to have on EDF.

Appendix 2. Results of Estimations

Cointegrating Eq:	CointEq1			
LOG(EDF(−1))	1.0000			
R3M(−1)	−1.0841			
LOG(CPI(−1))	−30.0009			
LOG(INDY(−1))	22.0265			
C	70.9596			
Error Correction:	D(LOG(EDF))	D(R3M)	D(LOG(CPI))	D(LOG(INDY))
CointEq1	−0.06375	0.009408	−1.11E-06	−0.00245
D(LOG(EDF(−1)))	0.053327	0.06509	0.00198	−0.00887
D(LOG(EDF(−2)))	−0.01575	−0.131	−0.00099	−0.01792
D(LOG(EDF(−5)))	0.011637	−0.06766	0.001602	−0.01509
D(LOG(EDF(−6)))	−0.13644	0.030463	−0.00029	−0.00794
D(R3M(−1))	0.164029	0.642813	0.003366	−0.00044
D(R3M(−2))	−0.16209	−0.13218	9.71E-05	0.004215
D(R3M(−5))	0.12757	0.149394	−0.00092	−0.00122
D(R3M(−6))	−0.03116	−0.08816	−8.61E-05	0.001807
D(LOG(CPIM(−1)))	−0.45858	−0.39609	0.077999	0.18397
D(LOG(CPIM(−2)))	1.030785	4.772407	−0.13908	−0.27444
D(LOG(CPIM(−5)))	0.372754	2.651221	0.066178	0.182219
D(LOG(CPIM(−6)))	1.808998	−0.82136	0.502338	−0.17895
D(LOG(INDY(−1)))	0.129055	0.201066	−0.01871	−0.36274
D(LOG(INDY(−2)))	0.722394	−0.64399	−0.05448	−0.25407
D(LOG(INDY(−5)))	0.505888	−0.51408	−0.01363	−0.1226
D(LOG(INDY(−6)))	−1.22083	0.287994	0.02263	−0.07238
C	0.01243	−0.00533	0.000888	0.004113
R^2	0.3064	0.4013	0.3829	0.2819
Adj. R^2	0.1952	0.3052	0.2839	0.1667

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Stress Testing the Enterprise Sector's Bank Debt: A Micro Approach*

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This paper describes Norges Bank's micro stress-testing framework for assessing the Norwegian banking sector's losses on loans to the nonfinancial enterprise sector. Using projected macro variables and a stock-flow approach, annual financial statements of every firm in Norway are projected five years ahead. The loan loss potential is then assessed using a credit-scoring model. We present a backtest of projections, taking the history of macro variables as given. Our results are fairly good using a relatively simple setup, and we conclude that stock-flow projections of financial statements can be useful for stress testing banks' loan portfolios.

JEL Codes: G21, G32, G33, M49.

1. Introduction

The credit quality of banks' loans to the enterprise sector is of great importance when measuring banks' credit risk. During the Norwegian banking crisis of 1988–93, a large part of banks' losses were on loans to the enterprise sector. Even today we expect that an economic downturn will lead to larger losses on loans to the enterprise sector than on loans to the household sector.

Norges Bank's SEBRA database contains annual financial statements for every limited liability company in Norway. The financial

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statements provide valuable information about the risk characteristics of individual firms. Even in favorable macroeconomic periods, there will be some firms in a vulnerable financial situation. Financial statements for such firms provide useful information about the banking sector's loss potential on loans to the enterprise sector.

Norges Bank uses several models in its stress testing of financial stability; see Andersen et al. (2008). Our paper deals with Norges Bank's micro approach for stress testing the banking sector's losses on loans to the nonfinancial enterprise sector.¹ This approach consists of four steps:

- (i) Norges Bank's macroeconomic models are used to predict the development of key macro variables in a given stress scenario. The stress scenario will normally cover a period of five years. The macroeconomic models used in connection with stress testing are described in Andersen et al. (2008). The macro variables we use in our micro approach are GDP growth, the borrowing rate for the nonfinancial enterprise sector, the growth in households' employment income, the inflation rate, the real exchange rate, and the growth in house prices.
- (ii) Annual financial statements for every firm are projected, primarily based on empirical models which depend on the aforementioned macro variables. Based on the projected financial statements, Norges Bank's SEBRA model is used to predict a yearly probability of default (PD) for each firm.
- (iii) The expected potential loss on the banking sector's loans to the enterprise sector is calculated by multiplying each firm's bank debt by its PD and summing over all firms. This expected potential loss is transformed to an estimated loss for the banking sector. This transformation involves the use of an empirical model for loss given default (LGD).
- (iv) The final step is the measurement of the impact of the estimated loan loss on the banking sector. The impact will depend on the banking sector's initial capital adequacy and its lending growth, profit before loan losses, and dividend payouts in the

¹Norges Bank has also modeled the banking sector's stock of nonperforming and doubtful loans to the enterprise sector by a macro approach; see Berge and Boye (2007).

stress years. Projections of these quantities are carried out in a separate bank model; see Andersen et al. (2008).

Feedback effects to the macroeconomic models from the loan losses projected in step (iii) are not considered, which constitutes a clear limitation to the setup. In particular, we would expect that the magnitude of loan losses typically prevailing in a stress scenario would limit the availability of credit to both households and firms. A weakening in credit conditions may in turn affect GDP growth and further depress the financial strength of the enterprise sector. In this paper we do not take this issue further. Our focus is on steps (ii) and (iii). We analyze how to best translate the output from the macroeconomic models into projections of financial items that are used by the SEBRA model in its predictions of PDs. To project financial items, we utilize the stock-flow structure of financial statements and a set of simple assumptions for liquidity management and dividend policy. The accuracy of the projections is evaluated through backtesting of the predicted PD for the enterprise sector. We do this by taking the historical development of the macro variables in step (i) as given and projecting the financial statements five years ahead, using financial information only from the initial year together with the macro variables.

Our approach gives ample opportunities for tailoring a specific stress scenario. Each independent financial item represents a handle we might pull to create a tailored stress scenario. For example, to measure the impact of a wage shock, we can sharply increase payroll expenses. In such a scenario, labor-intensive enterprises will be hit the hardest. In other scenarios the shock may come through a sharp increase in interest expenses or through write-downs of fixed assets and investments, and so forth.

The paper proceeds as follows. Section 2 describes the SEBRA model and the associated database, which is the basis for our stress testing of the banking sector's losses on loans to the enterprise sector. In section 3 we describe how the (debt-weighted) PD for the enterprise sector and a measure of LGD are calculated. Section 4 gives an overall description of how financial statements are projected at the firm level and presents the empirical models and auxiliary assumptions used in the projections. The results of the backtests are reported in section 5. Section 6 presents an empirical model

for LGD and shows an application of our stress-testing framework using scenarios from Norges Bank's Financial Stability Report 1/08. Section 7 concludes.

2. The SEBRA Model and the Associated Database

An important part of Norges Bank's stress-testing framework for banks' losses on loans to the enterprise sector is a bankruptcy prediction model, called the SEBRA model. The associated database contains annual financial statements for every limited liability company in Norway in the period 1989–2006.² Some additional information, such as industry code, year of formation, and location, is also stored in the database. In the SEBRA model we utilize data from the firms' unconsolidated financial statements.³ We receive financial statements for most firms about nine months after the end of the financial year. For each financial year, the database contains about thirty items from the profit-and-loss account and about fifty items from the balance sheet of each firm. The financial statements provide information about the total bank debt of each firm. We do not have information about which bank(s) the firm has borrowed from.⁴

The SEBRA model predicts bankruptcy probabilities as logistic functions of variables that are based on key financial items from the financial statement. The model belongs to the class of "generalized additive models" (GAMs).⁵ All variables in the SEBRA model utilize data from the same financial year. Thus, based on the financial statement of a firm for one year, we are able to predict the bankruptcy probability for the next year. Two versions of the SEBRA model have been developed: a basic version and an extended version. These versions are described in Bernhardsen and Larsen (2007),

²The database also contains annual financial statements for a smaller set of firms back to 1981.

³We also have access to consolidated financial statements for a limited number of firms for the period 1992–2006, but we do not use this information.

⁴Kredittilsynet (The Financial Supervisory Authority of Norway), which also uses the SEBRA model, does have access to such information. Kredittilsynet regularly uses the SEBRA model in its on-site inspections of banks.

⁵The general functional expression for the SEBRA model is given in appendix 1.

while an earlier version of the model is described in Eklund, Larsen, and Bernhardsen (2001).

The last versions of the SEBRA model have been estimated on financial statements from the period 1990–2002 and bankruptcies in the period 1991–2005.⁶ The three core variables in the basic version of the SEBRA model are as follows:⁷

$$x_1 = \frac{\text{Earnings Before Depreciation and Amortization (EBDA)}}{\text{Total Debt}} \quad (1)$$

$$x_2 = \frac{\text{Book Value of Equity}}{\text{Book Value of Total Assets}} \quad (2)$$

$$x_3 = \frac{\text{Cash and Deposits} - \text{Short-Term Debt}}{\text{Operating Revenue}} \quad (3)$$

The numerator in x_1 is an estimate of cash earnings after taxes. Since x_1 measures cash earnings as a share of total debt, this variable is a measure of the debt-servicing capacity of the firm. The inverse of x_1 shows how many years it will take to repay the debt given the current year's cash earnings. x_2 is the equity ratio, which is a measure of financial strength. The variable x_3 is a measure of liquidity. In addition to the three core variables, there are two sets of indicator variables in the basic version of the SEBRA model. The first set measures whether or not the book value of equity is less than paid-in equity, i.e., whether the firm has an accumulated net loss. The second set measures the age of the firm. This set consists of a dummy variable for each age in the interval from one year to eight years. The inclusion of age dummies means that the model can be interpreted within the class of hazard-rate models.

The basic version of the SEBRA model was developed with stress testing in mind. An important criterion in the development process was that the model should depend on few variables, but include the most critical ones. Bernhardsen and Larsen (2007) document that the deterioration of predictive power compared with the extended

⁶The reason for the different time periods is that the estimation is based on the event bankruptcy, which occurs with a time lag compared with the release of the last financial statement.

⁷The balance-sheet items included in equations (1)–(3) are measured at year end.

version of the SEBRA model is relatively small. Another consideration when developing the basic version of the SEBRA model was that it should be possible to make good projections of the explanatory variables included in the model.

By using the misclassification approach to binary dependent models outlined in Hausman, Abrevaya, and Scott-Morton (1998), we take the observed event of bankruptcy as a noisy proxy for the unobserved event of default.⁸ Under the assumption that the conditional probability of bankruptcy given default does not depend on the explanatory variables, the basic version of the SEBRA model is able to predict PDs.⁹ The advantage of predicting PDs instead of bankruptcy probabilities is that it makes our stress-testing framework more compatible with the Basel II framework, where PD and LGD are central credit-risk parameters.

3. PD for the Enterprise Sector and Implied LGD

This section describes how we aggregate *probability of defaults* (PDs) for individual firms to arrive at the (debt-weighted) PD for the enterprise sector. This PD is used to calculate an implied measure of *loss given default* (LGD).

We denote the probability of default for firm i in year t by $PD_{i,t}$. Moreover, let $D_{i,t-1}$ denote the bank debt of firm i at the end of year $t - 1$, i.e., at the start of year t . The *expected potential loss* (EPL) for the banking sector regarding this firm in year t is then given by

$$EPL_{i,t} = PD_{i,t} \cdot D_{i,t-1}. \quad (4)$$

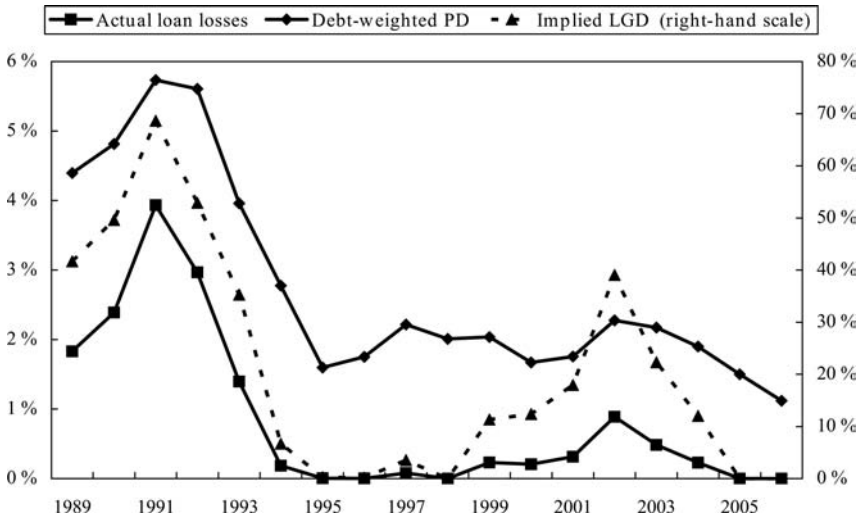
By aggregating over all firms, we obtain the expected potential loss for the banking sector in year t . This aggregate, denoted EPL_t , can be interpreted as an estimate of the total loss for the banking sector before realization of collateral. By dividing EPL_t by the aggregate bank debt, we find the (debt-weighted) PD for the enterprise sector in year t :

$$PD_t = \frac{EPL_t}{D_{t-1}}. \quad (5)$$

⁸Each bank has information about defaults among its customers. Norges Bank does not have access to such information.

⁹For details, see appendix 2.

Figure 1. Implied LGD for Banks' Loans to the Enterprise Sector in the Period 1989–2006



3.1 Calculation of Implied LGD

Normally, the banking sector's actual loan losses (ALL)—i.e., the losses after realization of collateral—will be lower than EPL. Based on the historical series of ALL and our prediction of EPL based on historical data, we get an implied estimate of LGD in year t :

$$\text{LGD}_t = \frac{\text{ALL}_t}{\text{EPL}_t}. \quad (6)$$

LGD varies strongly over time and its peaks are associated with economic downturns. This is due to the impairment of the value of collateral in downturns. Figure 1 illustrates the implied LGD for the period 1989–2006 calculated as the actual loan losses as a percentage of total loans divided by the predicted (debt-weighted) PD.¹⁰

The main objective with stress testing is to predict the banking sector's future loan losses in various scenarios. By using predicted financial items as input to the SEBRA model, we obtain annual estimates of EPL for future years. We then need a model for LGD

¹⁰To avoid negative values for the implied LGD, we have set $\text{ALL}_t = 0$ in years with net reversals of earlier loan losses, i.e., in years when $\text{ALL}_t < 0$.

to convert these EPLs to annual estimates of the loan losses. We have modeled the implied LGD, i.e., the dotted line in figure 1. The empirical model is presented in section 6.

4. Projections of Financial Statements

The starting point for our stress testing of banks' losses on loans to the enterprise sector is the development of macro variables in a given stress scenario. This information must be translated to the micro level, which in our case means the financial statement of each firm. We use aggregate growth rates from empirical models to project the main items in the financial statement of every firm. In addition, we use simple "rule-of-thumb" assumptions regarding the liquidity management and dividend policy of each firm.

A financial statement consists of a profit-and-loss account (table 1) and a balance sheet (table 2). There are some differences between projections of items in the profit-and-loss account (flow variables) and projections of items in the balance sheet (stock variables). Most of our projections are of flow variables, but we also project important stock variables like debt and paid-in equity. The development of some stock variables, like total assets and cash & deposits, is partly determined by the projected flow variables.

The distribution of net profit between dividend payment and retained earnings affects both cash & deposits and the equity ratio. We assume that firms follow a pecking-order rule regarding the distribution of net profit. The starting point of the rule is the projected cash earnings of the firm. We assume that this is the amount that is available for strengthening the stock of cash & deposits and for payment of dividends. This means that negative cash earnings automatically reduce the stock of cash & deposits. On the other hand, higher operating revenue will normally necessitate a higher stock of cash & deposits and other current assets. For firms with positive cash earnings, we assume that cash & deposits are targeted to grow by the same rate as operating revenue. If the cash earnings are not sufficient to cover this increase, we assume that cash & deposits only grow by the available amount. If there is still something left of the cash earnings after the allocation to cash & deposits, and the equity ratio is above 10 percent, we assume that 35 percent of the remaining cash earnings are paid as dividends. This is a rough estimate

Table 1. Main Items in the Profit-and-Loss Account

Profit-and-Loss Account	
Operating Revenue	(E)
– Cost of Goods Sold	(E)
– Payroll Expenses	(E)
– Depreciation	(H)
– Write-Downs	(E)
– Other Operating Expenses	(H)
<u>= Operating Profit</u>	
+ Interest Income	(H)
– Interest Expenses	(E)
+ Net Other Financial Items*	(H)
<u>= Profit Before Taxes</u>	
– Income Tax	(H)
<u>= Net Profit</u>	
– Dividend	(H)
<u>= The Year’s Retained Earnings</u>	
* = Includes write-downs on investments.	
(E) = Projected based on empirical model.	
(H) = Projected based on heuristic.	

based on results of a study we have conducted of dividend payments in Norwegian firms in the period 1989–2005.¹¹

The equity ratio is influenced by the current year’s retained earnings and debt growth. Write-downs have a pronounced impact on net profit, and thereby on retained earnings. Write-downs for

¹¹The study shows that the dividend level in the enterprise sector varies largely during the period. One reason is frequently changing tax regimes for dividends. Over the period 1992–2005, about 40 percent of firms with a positive net profit paid dividends. Among these firms, about 70 percent of their net profit was paid out as dividend. If the total dividend is divided among all firms with a positive net profit, this gives a payout ratio of about 28 percent. Our assumption is related to “the remaining cash earnings,” which is a somewhat different measure than net profit.

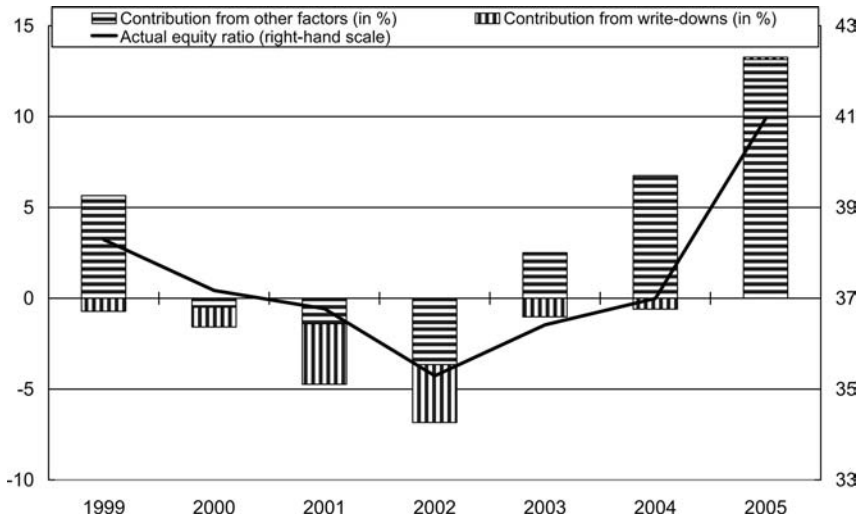
Table 2. Main Items in the Balance Sheet

Balance Sheet		
<i>Assets</i>		
Long-Term Assets		
Intangible Assets		
Fixed Assets		
Long-Term Investments		
+ Current Assets		
Cash & Deposits	(H)	
Other Current Assets		
= Total Assets		
<i>Equity and Liabilities</i>		
Equity		
Paid-In Equity	(H)	
Retained Earnings	(H)	
+ Debt	(E)	
Long-Term Debt		
Short-Term Debt		
= Sum Equity and Liabilities		
(E) = Projected based on empirical model.		
(H) = Projected based on heuristic.		

impairment are often associated with economic downturns. The normal cycle is a low level of write-downs in economic upturns, and increasing write-downs as the economy moves deeper into a downturn. Figure 2 shows the relative effects of write-downs and other factors, like net profit (before write-downs) and debt growth, on the equity ratio in the mild economic downturn in Norway in 2001–03. Our decomposition shows that about two-thirds of the decrease in the equity ratio in 2000 and 2001, and about half the decrease in 2002, can be attributed to write-downs. It is therefore important to project write-downs in order to get good projections of the equity ratio of each firm.

To sum up our procedure for projecting financial statements, first the flow variables needed to calculate next year’s cash earnings

Figure 2. Effect of Write-Downs on the Equity Ratio



are projected. Then a heuristic assumption for the firm's use of cash earnings to strengthen liquidity holdings and pay dividends is applied. Next, the remaining flow variables are projected to calculate retained earnings. By adding the estimate for injections of paid-in equity, the total equity is determined. Finally, the stock variable debt is projected, thereby determining total assets. We then have the means for calculating the variables used in the SEBRA model. The procedure is repeated up to five years ahead. Subsection 4.1 describes in more detail how each financial item is projected.

4.1 Projections of Financial Items

We expect close relationships between the macro variables at hand and many of the financial items we want to project, at least at the aggregate level. Payroll expenses are expected to be closely related to households' employment income. Interest expenses should be closely related to credit growth and to changes in interest rates, and so forth. Such presumptions give a good starting point for finding useful forecasting models.

Because our panel database is not complete, level information on aggregated financial items is not entirely consistent over time. Also, since our goal is to project financial items for a stationary

population of firms, we do not want to capture effects from entries and exits of firms. We therefore choose to construct series of growth rates for aggregated financial items, including only firms that are represented in the data in any two consecutive years.¹² The estimation sample consists of yearly series covering the period 1985–2006. In our development of forecasting models, we choose to fit autoregressive distributed lag models (ADLs) in the constructed growth rates. By this formulation we may force long-run restrictions on growth rates but still remain open for flexibility in the short run. For example, we would like projected revenue to grow in line with GDP over time. We generally find that the growth rates align relatively rapidly, so predictions are not drastically different from that of a static model in growth rates. However, since the fit of the ADLs are generally better than with a static model, and since we appreciate the separation between long-run interpretations and short-term flexibility, we decide to go with the ADLs.¹³ The resulting empirical models are presented in table 3. Variable names in lowercase letters and italics denote growth rates measured in percent, and Δ denotes the absolute change from the previous year. The dependent variables (DEP) are operating revenue (REV), payroll expenses (PAY), total debt (DEBT), interest expenses (IEX), and costs of goods sold (CGS). Explanatory variables include (real) GDP, the consumer price index (CPI), the real export weighted exchange rate (RX), household's employment income (INC), and the average borrowing rate for the enterprise sector (BOR). All financial items are measured in nominal terms.

Operating revenue is found to depend positively on both nominal GDP and the real exchange rate, the latter representing the competitive power of the export sector.¹⁴ In steady state, i.e., when $\Delta rev = \Delta gdp = \Delta RX = 0$, the model gives that the growth rate of operating revenue will be exactly 5 percent when nominal GDP

¹²The constructed series will then imitate growth rates from a stationary population even though the sample composition changes over time.

¹³Another alternative would be to calculate synthetic levels of the financial items using the constructed growth rates and then fit an error-correction model in the levels. We left this alternative unexplored.

¹⁴A higher value of the real exchange rate variable represents a real depreciation, thus the positive sign.

Table 3. Regression Results

	dep_{t-1}	Δgdp_t	gdp_{t-1}	Δcpi_t	cpi_{t-1}	ΔRX	Δinc_t	inc_{t-1}	BOR_{t-1}	bor_t	$debt_t$	Δrev_t	constant
Δrev_t	-0.79 (-4.02)	2.06 (5.71)	1.64 (4.56)	2.06 (*)	1.64 (*)	0.62 (2.82)	1.45 (4.1)	1.65 (6.82)				0.38 (3.22)	-4.25 (-2.45)
Δpay_t	-0.85 (-4.35)												-3.57 (-1.61)
$\Delta debt_t$	-0.84 (-4.56)	2.11 (2.21)	1.60 (3.06)	2.49 (1.97)	3.10 (4.60)				-0.73 (-2.57)				0 (***)
$ie x_t$										1 (**)	1 (**)	1 (**)	
Δcgs_t													

(*) Coefficient restriction $gdp = cpi$ imposed.
(**) Definition relation (not rejected).
(***) Constant = 0 imposed (not rejected).

growth is 5 percent, which is in line with commonly applied figures for the sum of long-term GDP and inflation.

Payroll expenses are found to depend on households' employment income and operating revenue. In steady state, i.e., when $\Delta pay = \Delta inc = \Delta rev = 0$, estimated coefficients give that the growth rate of payroll expenses will be 4.5 percent, which is somewhat low but still fairly reasonable.

Total debt is found to depend on GDP, CPI, and the borrowing rate for the enterprise sector. In steady state, i.e., when $\Delta debt = \Delta gdp = \Delta cpi = 0$, debt can grow in line with nominal GDP at 5 percent only if the borrowing rate is set to 10.3 percent. This is about 3 percentage points above what we initially expected, and may be due to the unusually high interest rates in the first part of the sample or to the fact that debt growth was somewhat higher than GDP growth over the sample period. However, our assessment was that the potential misalignment of growth rates was not of such magnitude that we needed to impose restrictions on coefficients in the estimation.

For interest expenses, which for each firm may be approximated by multiplying the average borrowing rate over the financial year by the firm's average interest-bearing debt, simply setting $iex = bor + debt$ did fit nearly as well as any empirical model we estimated. On similar grounds, we ended up setting the growth rate of costs of goods sold equal to growth in operating revenue. Figures 6–9 in appendix 3 show actual and fitted growth rates for operating revenue, payroll expenses, debt, and interest expenses during the sample period, and the projected paths in the baseline scenario and stress scenario from Norges Bank's Financial Stability Report 1/08.

We also need empirical models for write-downs of assets. We find that write-downs at the aggregate level are best modeled as a percentage of aggregate book value of assets, instead of as growth rates. Since there are great differences in the magnitude of write-downs on different types of assets, we found it necessary to model fixed assets, long-term investments, and short-term investments differently. The financial reporting of write-downs became more detailed in 1999, so we only have the period 1999–2006 available for estimation. This gives us an absolute minimum degrees of freedom, and we therefore turned to the method of static regressions with some a priori determined coefficient restrictions applied. Write-downs on

Table 4. Regression Results for Write-Downs

Models for Write-Downs	<i>eqi</i>	<i>cpp</i>	constant	$\sqrt{MSE}$
Fixed Assets	−0.016 (−7.43)	−0.04 (−4.19)	1.27 (*)	0.14
Long-Term Investments	−0.04 (−2.78)	−0.04 (−1.26)	2.6 (*)	0.95
Short-Term Investments	−0.13 (−10.2)		−0.54 (*)	1.02
(*) Constrained.				

all three categories of assets were modeled as a function of the growth in a broad equity index (EQI) and the growth in commercial property prices (CPP). The coefficients were constrained such that the aggregate write-downs equaled their averages over the period 2004–06¹⁵ when the growth rates for EQI and CPP were set to their long-run averages of 5 percent. The resulting regression results are shown in table 4.

In order to project write-downs, we need projections of both equity prices and commercial property prices. We have elected to project commercial property prices by assuming that they will grow in line with the predicted house prices. To obtain a forecast of the growth rate of the broad equity index, we use a simple dividend-discounting model; see appendix 4. Figure 10 in appendix 3 shows actual and fitted write-down percentages for long-term investments.

The other financial items to be projected are depreciation, paid-in equity, interest income, other operating costs, and income tax. Below we explain our assumptions regarding these items. Based on the historical ratios for the period 1989–2005, we project yearly depreciation as 8.5 percent of the stock of tangible fixed assets at year end. We find that changes in paid-in equity are difficult to model. We have therefore set the annual growth rate of paid-in equity to 9 percent, which is equal to the median for the period 2000–04. Interest income is shifted with the changes in the enterprise

¹⁵We view this period as the most normal subperiod in the estimation sample 1999–2006. For short-term investments we consider valuation adjustments which can be either positive or negative.

sector's borrowing rate, which seems conceptually fine but unfortunately does not fit too well with data. Other operating costs vary little from year to year, so we elected to let this item grow with the consumer price index. In order to construct the liquidity variable that enters the SEBRA model, we have to project short-term debt specifically. Similarly, to calculate our measure of (debt-weighted) PD for the enterprise sector, we need projections of the bank debt in each firm. We have elected to apply the growth rate of total debt to all types of debt. Finally, income taxes are calculated as the projected profit before taxes, multiplied by the income tax rate of 28 percent.

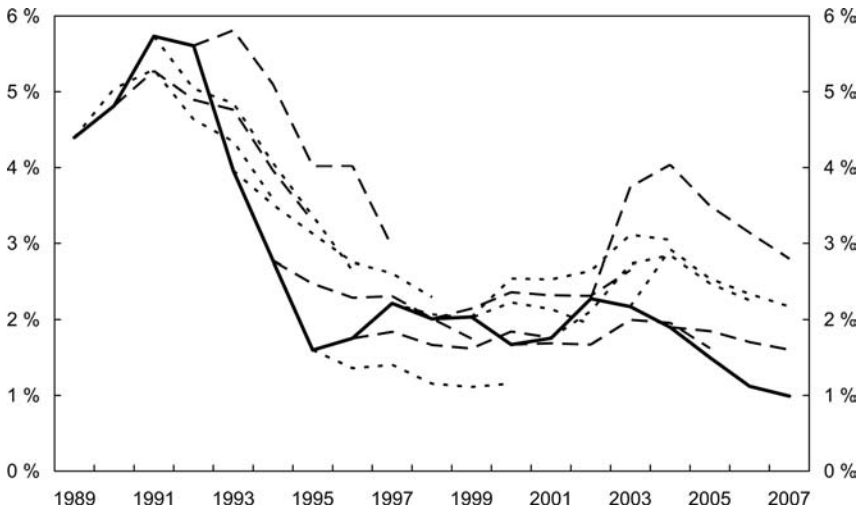
5. Results of Backtests

To evaluate the fit of our projections of (debt-weighted) PDs for the enterprise sector, we carry out backtests. Financial statements for each year from 1988 to 2003 serve as the starting points for the backtests. The projections in each backtest cover a five-year period; e.g., if we start from the observed financial statement of 1991, the backtest covers the projections of financial statements for the period 1992–96 and the predictions of PDs for the period 1993–97.¹⁶ Since our task is not to evaluate the uncertainty in forecasting core macro-economic variables, but rather to evaluate if a portfolio of financial statements can be projected conditional on the macro variables, we use the actual development of the macro variables in the backtest period. The actual values of the macro variables for the first year of the backtesting period are used to make forecasts of the growth rates of financial items that year, using the empirical models and heuristics presented in section 4. This forecasting process is repeated for each year in the backtesting period.

The backtest period includes two economic downturns: the banking-crisis years 1988–93 and the mild downturn in 2001–03. We have elected not to use financial statements from the period 1982–87, due to incompleteness of the SEBRA database before 1988. This means that we are only partly able to backtest the period prior to

¹⁶Keep in mind that the predicted PD for year t is based on the financial statements for year $t - 1$.

Figure 3. Backtests of Projected PDs for the Enterprise Sector



the banking-crisis years, when risk built up. Some of the empirical models and heuristics, like the models for write-downs and the heuristic for the growth of paid-in equity, are based on the period 1999–2005. The backtests that do not cover this period are therefore partly out of sample. The results of the backtests are presented in figure 3. The solid line shows PDs for the enterprise sector based on the actual financial statements. The dotted and dashed lines show projections of PDs from the different starting points.

With the exception of the projected paths starting in 1992 and 2002, both the direction and the development over time of the projected paths seem satisfactory for stress-testing purposes. All paths starting in the early banking-crisis years seem to drag the PDs down in a reasonable manner, while paths starting in the late 1990s seem to pull the PDs moderately upward and down again as the economy recovers. The two projected paths that do not fare that well—i.e., the paths starting in 1992 and 2002—are both related to a weak initial year and a temporary fall in operating revenue in the enterprise sector.

The fit of the projected paths is somewhat poorer than what we would expect from an empirical model estimated on the historical series of PDs. However, such a comparison is not completely fair

since we have not designed our stress-testing framework to fit this measure specifically. It is important to bear in mind the simplicity of our setup. The fit can probably be improved markedly if the setup is modified in light of the experiences from the backtests. As long as one sticks to theoretically sound modifications of the setup, it should be possible to make improvements without significantly increasing the risk of overfitting.

6. Application of the Stress-Testing Framework

In order to apply our framework to assess banks' loan losses, we need to project the measure of LGD defined in section 3. We have modeled LGD as a function of GDP growth and the change in the real growth rate of commercial property prices. The rationale is that GDP growth says something about the general economic condition, while commercial property usually is the most important collateral for banks' loans to firms. Changes in the growth rate of commercial property prices may therefore capture surprises to the value of banks' collateral. The estimated model is

$$\Delta \text{LGD}_t = \frac{28}{(4.98)} - \frac{0.43}{(2.05)} \cdot \Delta \text{cpp}_t + 0.31 \cdot \text{Dummy}_{1991} - \frac{0.62}{(4.97)} \cdot \left[\text{LGD}_{t-1} + \frac{9.88}{(2.08)} \cdot \text{gdp}_{t-1} \right], \quad (7)$$

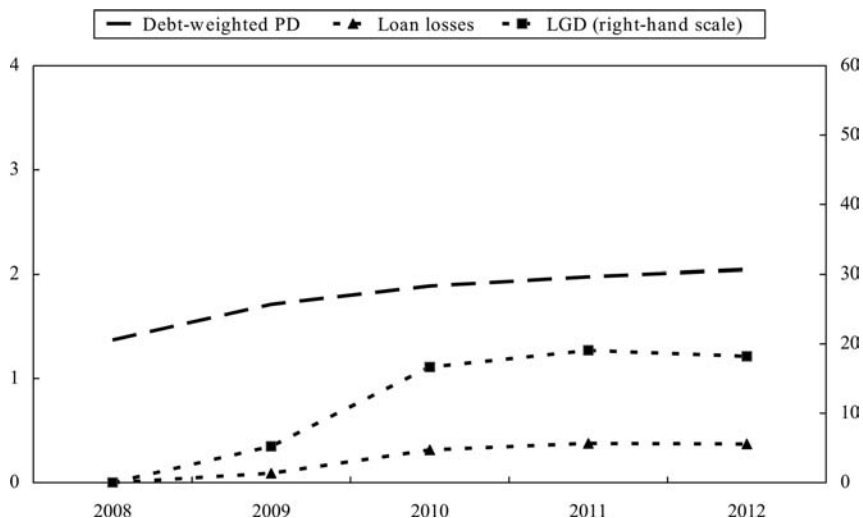
where *gdp* denotes the growth rate of the real gross domestic product in mainland Norway and *cpp* denotes the real growth rate of commercial property prices. We use a dummy for the year 1991, which was the peak of the banking crisis in Norway. In the long run, commercial property prices grow at a steady rate, so LGD only depends on *gdp*. Assuming a GDP growth of 2.5 percent gives a long-run LGD at 20.5 percent, which seems plausible.

We have applied the micro stress-testing framework to forecast banks' loan losses for the period 2008–12. We take the baseline scenario and stress scenario outlined in Norges Bank's Financial Stability Report 1/08 as the basis for projections. In the stress scenario, a weakening of households' confidence in their own financial situation and in the general outlook for the Norwegian economy lead

Table 5. Forecasted Paths for Macro Variables in Two Scenarios

Year	Real GDP		Wage Income		Real FX Rate		Borrowing Rate		Property Prices	
	Baseline	Stress	Baseline	Stress	Baseline	Stress	Baseline	Stress	Baseline	Stress
2007	5.99	5.99	10.10	10.10	92.98	92.98	6.44	6.44	11.34	11.34
2008	3.54	2.59	8.21	8.00	89.30	85.79	7.07	8.02	-0.53	-9.19
2009	2.02	-1.49	5.50	3.33	90.39	81.43	6.66	9.38	5.18	-19.75
2010	2.35	-0.22	5.16	3.52	91.61	83.58	6.21	8.49	4.82	-8.87
2011	2.65	3.58	4.87	6.16	92.03	89.83	6.28	6.64	4.80	13.09

Figure 4. Projection of Loan Losses in the Baseline Scenario (in percent)

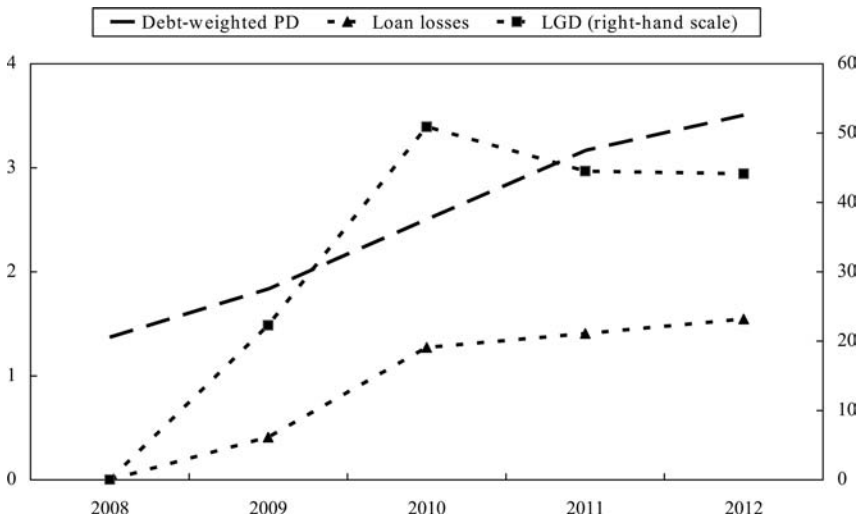


to a sharp fall in house prices. At the same time, interest rates are increased substantially in response to prospects for higher inflation. In table 5 the baseline and stress scenarios are outlined in terms of the macro variables we employ, while figures 6–10 in appendix 3 show the corresponding projections for firms' operating revenues, payroll expenses, debt growth, interest expenses, and write-downs of long-term investments.

In the baseline scenario, PD normalizes somewhat from a historically low level; see figure 4. LGD increases from its level of zero in 2008 and converges to the estimated long-run average of 20 percent. The combined effect is that the projected loan losses increase only slightly, to approach levels close to the average losses over the last two decades.

In the stress scenario (figure 5), both PD and LGD increase sharply, and by 2012 the projected loan losses exceed the level experienced in 2002 by almost 70 percent. However, loan losses are still well below the level experienced at the peak of the Norwegian banking crisis in 1991. This holds even if we replace our projected value of LGD with the implied LGD derived for the banking-crisis period.

Figure 5. Projection of Loan Losses in the Stress Scenario (in percent)



7. Concluding Remarks

In this paper we presented Norges Bank’s micro stress-testing framework for assessing banks’ losses on loans to the enterprise sector. We presented empirical models and heuristics for projecting financial statements at the micro level. We find that average growth rates for financial items like operating revenue, various operating expenses, and debt are well forecasted conditional on macroeconomic variables, even on a five-year horizon. Back tests of (debt-weighted) PDs for the enterprise sector indicate that our framework performs reasonably well. Our conclusion is that stock-flow projections of financial statements can be a powerful tool for stress-test analyses. The approach has several advantages that make it a useful supplement to empirical models fitted directly to historical series of loan losses or default rates. First, series of historical loan losses typically have little variation except from crisis periods, which often are distant in time. Thus, the problem of overfitting can be severe, and stress testing will always involve large comparative shifts in models fitted mostly to small data variations. Stock-flow projections of financial statements are more structural in nature and may relieve us from some of

these obstacles. Moreover, our approach puts emphasis on the initial financial situation of each firm rather than on initial losses or average default rates. Default rates can change rapidly, particularly in a situation where many firms are close to defaulting and the macroeconomic condition starts to worsen. Our approach lets the initial situation in the enterprise sector be of great significance for the outcome of the stress tests. The approach is also applicable for stress testing individual banks' loan portfolios, and can easily be broken down to the industry level. The approach seems particularly useful for smaller banks with little or incomplete data on their historical losses.

There are several possibilities for improvement that could be investigated. First, the macroeconomic models, the bank model, and our model for the enterprise sector should be integrated in order to assess feedback effects. Norges Bank's macroeconomic model for stress testing does in fact include credit as a driving factor of GDP growth. Hence, if the effect of loan losses on bank lending can be reliably assessed in Norges Bank's bank model, it may be possible to iterate feedback effects using the micro and macro models interchangeably. In such an integrated exercise, we would probably see a deeper impact on GDP, higher loan losses, increasing risk premiums on loans, and lower credit growth in stress scenarios.

Appendix 1. Functional Expression for the SEBRA Model

Let $p(B)$ denote the bankruptcy probability. The general functional expression for the SEBRA model is

$$p(B) = F[\alpha + \beta_1 \cdot T_1(x_1) + \beta_2 \cdot T_2(x_2) + \cdots + \beta_n \cdot T_n(x_n)] \\ = \frac{1}{1 + \exp(-[\alpha + \beta_1 \cdot T_1(x_1) + \beta_2 \cdot T_2(x_2) + \cdots + \beta_n \cdot T_n(x_n)])} \quad (8)$$

where

$$T_j(x_j) = \frac{1}{1 + \exp\left(-\left[\frac{x_j - m_j}{s_j}\right]\right)}. \quad (9)$$

$T_j(x_j)$ is a cumulative logistic function, and the parameters m_j and s_j are estimated for each variable x_j using (9).

The SEBRA model has flexible compensation rates. How much the variable x_j has to increase when the variable x_k decreases for the bankruptcy probability to remain constant will therefore depend on both x_j and x_k . This is not true for the standard logit model.

Appendix 2. Use of the Misclassification Approach to Predict PDs

Even though we only have data for bankruptcies, the misclassification approach makes it possible, given certain assumptions, to transform bankruptcy probabilities to probabilities of default (PDs). Our method for predicting PDs may be described as a combination of two parts: generalization of the logit model for bankruptcies and a linear scaling of probabilities.

Let X denote the vector of variables in the basic version of the SEBRA model, and β a vector of coefficients associated with these variables. Assume that the true model relates to the probability of default and that bankruptcy occurs with a fixed probability given default. Then the probability of default can be expressed as

$$PD = F(\beta \cdot X), \quad (10)$$

where $F(\cdot)$ is the cumulative logistic density function.

Let B , D , and D^C denote the events of bankruptcy, default, and nondefault, respectively. Moreover, let $p(\cdot)$ denote an arbitrary probability. Then the bankruptcy probability can be written as

$$p(B) = p(B|D) \cdot PD + p(B|D^C) \cdot p(D^C). \quad (11)$$

By definition, $p(D^C) = 1 - PD$. By utilizing this relationship and (10), the bankruptcy probability can be written as

$$p(B) = r + (1 - q - r) \cdot F(\beta \cdot X), \quad (12)$$

where

$$r = p(B|D^C) \text{ and } q = 1 - p(B|D). \quad (13)$$

r and q are misclassification probabilities. r is the probability of registering bankruptcy when there has not been a default, while q is the probability of not registering bankruptcy when there has been

a default. Since it seems unlikely that an enterprise that has not defaulted should be registered as bankrupt, one would expect very few misclassifications of the first type; i.e., one would expect r to be zero. Historically, only a portion of the defaults eventually leads to bankruptcy. One would therefore expect $q \in \langle 0, 1 \rangle$.

The model in (12), with r and $(1 - q - r)$ taken as constants, gives a generalization of our original logit model for bankruptcy probabilities. Let us now estimate the following model for bankruptcy probabilities, where $\hat{g}$, $\hat{h}$, and $\hat{\beta}$ are estimated simultaneously by the method of maximum likelihood:

$$\hat{p}(B) = \hat{g} + \hat{h} \cdot F(\hat{\beta} \cdot X). \quad (14)$$

Then $\hat{g}$ can be interpreted as an estimate for r and $(1 - \hat{g} - \hat{h})$ as an estimate for q . If the estimation results in $\hat{g} = 0$ and $\hat{h} = 1$, this indicates that there are no misclassifications. The estimated equation based on data from the SEBRA database for the period 1990–96 is

$$\hat{p}(B) = \underset{(-0.21)}{0.00} + \underset{(-17.10)}{0.49} \cdot F(\hat{\beta} \cdot X). \quad (15)$$

The t -values in (15) mean that the hypothesis $\hat{g} = 0$ is not rejected, while the hypothesis $\hat{h} = 1$ is rejected. The rejection of $\hat{g} > 0$ backs up the hypothesis that there is a zero probability of registering bankruptcy in the case that there has not been a default. The rejection of $\hat{h} = 1$ rejects the original logit specification. Thus, we have rejected the logit model in favor of the misclassification model. We interpret the scaled individual bankruptcy probability $F(\hat{\beta} \cdot X) = \frac{\hat{p}(B) - \hat{g}}{\hat{h}}$ as an estimate of the probability of default. However, this interpretation requires that the true model relates to the probability of default and that bankruptcy occurs conditional on default. This assumption, which seems reasonable, cannot be truly validated. Moreover, it is required that the misclassification probabilities r and q can be estimated as constants, which is only true if they are independent of X . If one suspects that this latter requirement does not hold, one could turn to the semiparametric approach suggested in Lewbel (2000). For a further description of the misclassification approach regarding binary choice models, see Hausman, Abrevaya, and Scott-Morton (1998).

Appendix 3. Figures of Fit of Empirical Models

Figure 6. Actual and Fitted Growth Rates for Operating Revenue (in percent)

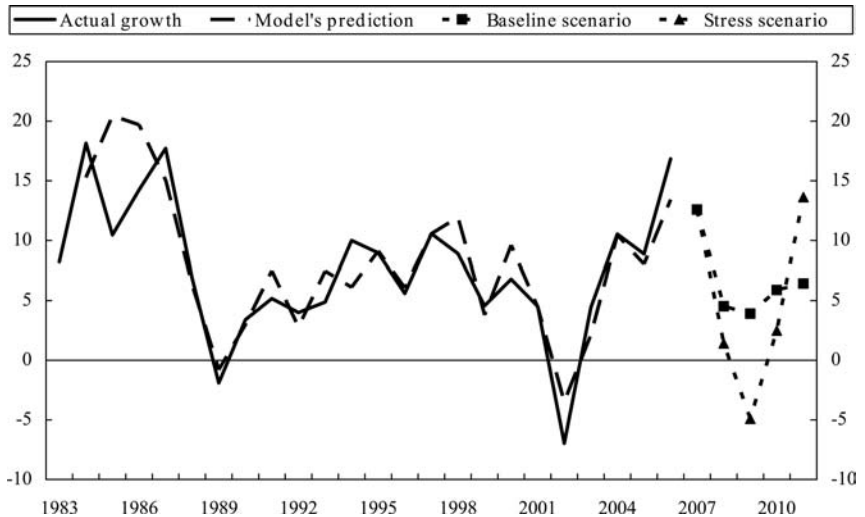


Figure 7. Actual and Fitted Growth Rates for Payroll Expenses (in percent)

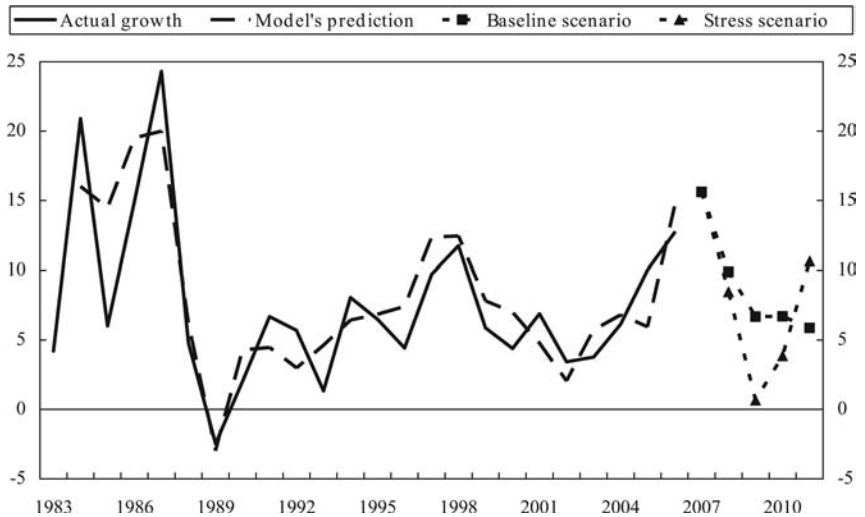


**Figure 8. Actual and Fitted Growth Rates for Debt
(in percent)**

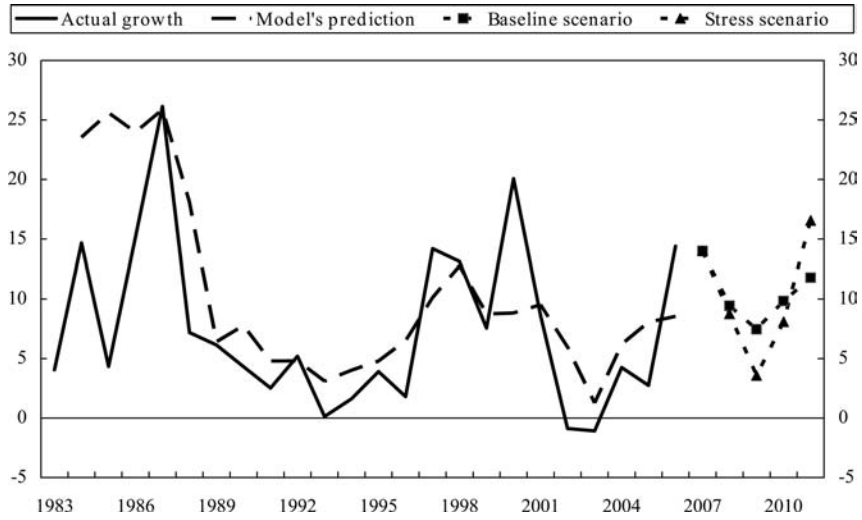


**Figure 9. Actual and Fitted Growth Rates for Interest
Expenses (in percent)**

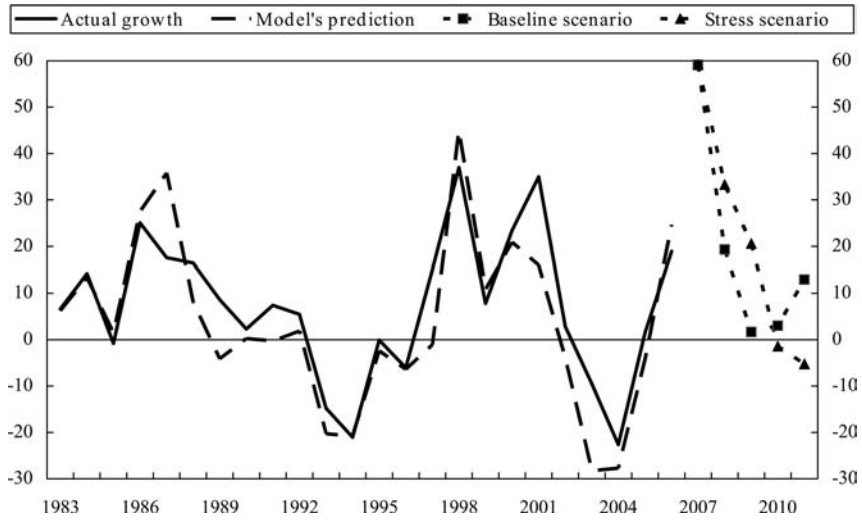
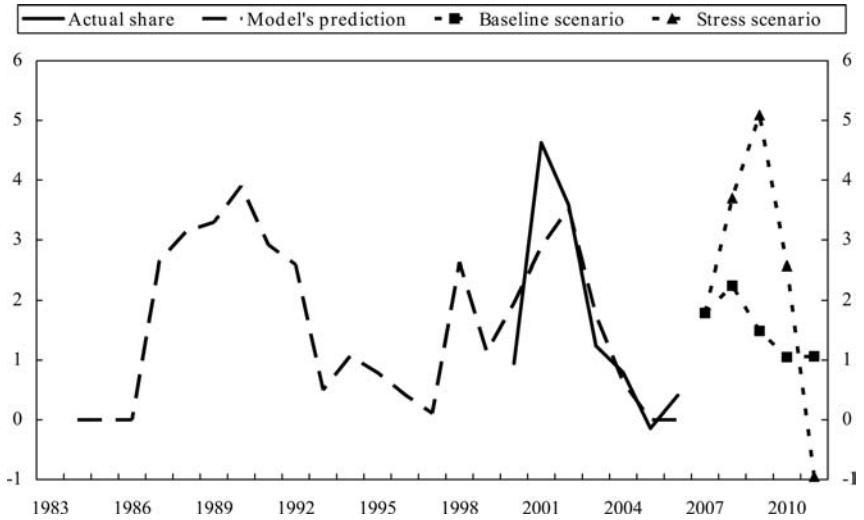


Figure 10. Actual and Fitted Write-Down Percentages for Long-Term Investments

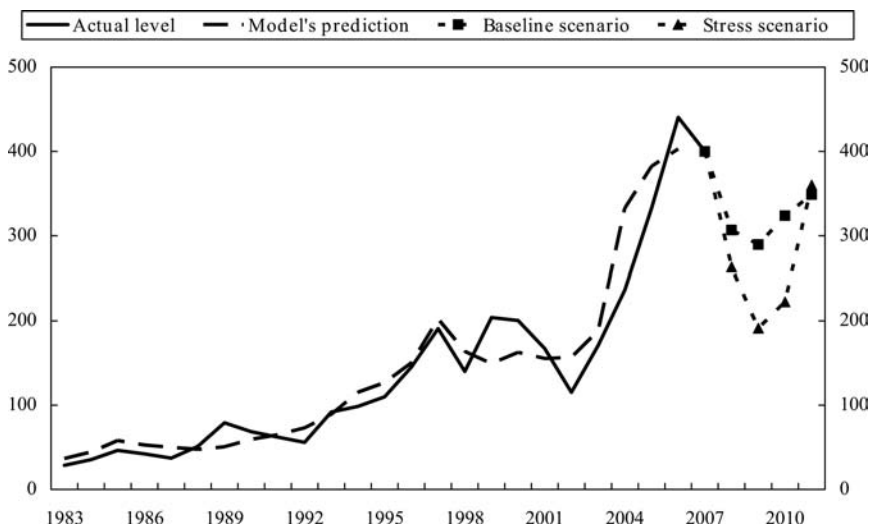


Appendix 4. Use of a GDP Discounting Model to Project Equity Prices

We assume that the yearly dividend payment from the equity market constitutes a constant share λ of GDP. Moreover, we assume that each period, investors forecast GDP to grow by a constant yearly rate g , which they update by placing a weight of 0.2 on the current year’s GDP growth and a weight of 0.8 on the long-term average of 5 percent (nominal). Furthermore, they calculate the discounting rate i the same way, using the weight 0.2 for the current year’s borrowing rate for the enterprise sector and the weight 0.8 for the borrowing rate’s long-term average of 6.5 percent (nominal). Given assumptions which ensure that the infinite geometric series of discounted dividend payments converges, the formula for the level of the equity index (EQI) at the end of year t can be written as

$$EQI_t = \lambda \cdot NGDP_t \cdot \frac{1 + i_t}{i_t - g_t}, \tag{16}$$

where NGDP is nominal GDP. We use the parameter λ to link the right-hand side of (16) to the current level of the equity index, i.e., to the level at the start of the projections. Given equation (16), shifts

Figure 11. Actual and Fitted Levels for the Equity Market

in NGDP and i will shift the level of the equity market. Figure 11 shows actual and fitted levels of the equity index.

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Crash Testing German Banks*

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In this paper we stress-test credit portfolios of twenty-eight German banks based on a Merton-type multifactor credit-risk model. The stress scenario is an economic downturn in the automobile sector. Although the percentage of loans in the automobile sector is relatively low for all banks in the sample, the expected loss conditional on the stress event increases substantially by 70–80 percent for the total portfolio. This result confirms the need to account for hidden sectoral concentration risk because the increase in expected loss is driven mainly by correlation effects with related industry sectors. Therefore, credit-risk dependencies between sectors have to be adequately captured even if the trigger event is confined to a single sector. Finally, we calculate the impact on banks' own-funds ratios, which decrease on average from 12 percent to 11.4 percent due to the stress event, which indicates that banks overall remain well capitalized. These main results are robust against various robustness checks, namely those concerning the granularity of the credit portfolio, the level of intersector asset correlations, and a cross-sectional variation of intrasector asset correlations.

JEL Codes: G21, G33, C13, C15.

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1. Introduction

Credit risk in loan portfolios, which is closely linked to changes in the economic environment, is widely perceived as the most relevant risk faced by banks. In an increasingly volatile financial environment, stress tests have recently become more important as an instrument to gauge the impact of specific adverse developments in the economy. It is, therefore, no surprise that regulators in the Basel II framework emphasize their use—in particular, in connection with credit concentrations.¹ Credit concentrations become important in extreme events (“tail risk”), and portfolio models which capture sectoral as well as name concentrations are an obvious tool to assess this type of risk.

In this paper, we stress-test credit portfolios of large German banks based on a one-period default-mode version of a standard Merton-type portfolio model in the spirit of Gupton, Finger, and Bhatia (1997) and Finger (1999). The stress scenario refers to a single sector, the automobile sector. It is based on a downturn prediction of 10 percent for the German automobile production index, suggested by historical data. The stress scenario reflects an exceptional but plausible event because it summarizes a continuum of stress events which, together, occur under baseline conditions with a probability of 33 percent and include the downturn forecast as the expected value under stress conditions. Rather than focusing on a particular stress forecast, however, this paper focuses on the main drivers of the stress impact on banks’ credit portfolios—including, for instance, the role of borrower-dependent compared with pooled probabilities of default or the influence of sectoral and name concentrations.

The stress-test methodology is based on recent work by Bonti et al. (2006). Our approach differs from their work and other work on stress tests of credit risk by the following five characteristics:

- (i) As the automobile sector is regarded as a key sector of the German economy, a downturn in this sector is expected to have severe repercussions in other business sectors. Therefore, intersector dependencies need to be accounted for, which is achieved by using a multifactor portfolio risk model.

¹See Basel Committee on Banking Supervision (2005).

- (ii) A common drawback of traditional stress tests is that they concentrate on a single-event scenario, which occurs only with a marginal probability. The sensitivity to deviations from this single event are rarely taken into account. In our setup, we consider instead a stress scenario comprising a range of stress events such that the probability of the stress scenario is quite significant.
- (iii) Our approach can also be used to identify hidden sectoral credit concentrations, as it allows us to identify risk concentrations under stress conditions across highly correlated sectors. Previous studies have found that sectoral concentration and, to a lesser extent, also name concentration have a material impact on the portfolio risk.²
- (iv) The use of the German credit register allows us to apply our stress-test methodology consistently to a sample of twenty-eight banks, taking into account their credit portfolios to the extent that loans are included in the credit register. Name concentration is automatically accounted for by using credit information aggregated to risk-oriented “borrower units,” which is more appropriate for risk assessment than the facility level or the legal entity level.
- (v) Traditionally, the focus of stress tests is on the expected loss (EL) conditional on the stress event and its increase relative to baseline conditions. We also consider the impact on economic capital (EC), defined as the difference between a 99.9 percent value-at-risk (VaR) and the (unconditional or baseline) EL.³ As a robustness check, we also calculate the *expected shortfall* (ES) or tail conditional expectation. Since these risk measures refer only to the potential loss of the credit portfolios, they do not convey immediate information about the impact of the stress scenario on a bank’s solvency. For this purpose, we calculate and compare in addition banks’ own-funds ratios before and after stress.

²See, for example, Heitfield, Burton, and Chomsisengphet (2006) or Duellmann and Masschelein (2007).

³Since we assume that under baseline conditions we have no further information on future realizations of the risk factors, the expected values under baseline conditions are always unconditional.

A key challenge in any stress-test design is how an adverse change in macroeconomic variables is incorporated into the model. In our case, this is achieved by judiciously truncating the distribution of the risk factor that belongs to the automobile sector. The threshold where the distribution of the risk factor is truncated is set so that the event that the risk factor falls below the threshold has the same probability as the event that the automobile production index falls below a corresponding threshold. This corresponding threshold of the automobile production index is in turn defined so that the expected value of the index, conditional on being below the threshold, equals the forecast of a downturn in the automobile sector. In this way the stress forecast is linked to the threshold of the unobservable risk factor without having to specify a functional relationship between this risk factor and the production index. This stress-test methodology is plausible in the sense that the stress scenario should be believable and have a certain probability of actually occurring. It is also consistent with the existing quantitative framework since we employ the same model which is also used under baseline conditions and we make use of all information contained in the parameter estimates of this model.

The need to take into account the reaction of other risk factors if one or more risk factors are stressed in order to avoid a material underestimation of the stress impact has been recognized in Kupiec (1998). Our stress-test design and the underlying credit-risk model draw heavily on the work by Bonti et al. (2006) but differ in important ways. Firstly, since we have access to the German central credit register, we can apply it to a cross-section of twenty-eight different banks. Secondly, we extend our analysis by additionally considering the impact on banks' capitalization, in this case measured by the own-funds ratio. Thirdly, since we do not have access to borrower-specific default probabilities, we have to revert to sector-dependent average default probabilities, which we consider to be one of the most severe limitations of our analysis.⁴ A related methodology was also applied by Elsinger, Lehar, and Summer (2006), with a stronger focus on financial stability aspects.

⁴This restriction will be lifted in future work when the German credit register is extended to include PD estimates of all banks adopting the internal ratings-based approach of Basel II.

Our results can be useful from the perspective of a risk manager, a central bank, or a supervisor. From a risk-management perspective, our results provide an empirical implementation of the stress-testing methodology proposed by Bonti et al. (2006). Although the number of twenty-eight banks in the sample is relatively low compared with the total number of 2,301 German banks in 2006, their aggregated total assets amount to almost 60 percent of the total assets of the German banking system.⁵ Therefore, from a financial stability perspective, our results can give valuable information as to the resiliency of a major part of the German banking system against an external shock to the automobile sector. Finally, the performance of individual banks, particularly the change of their own-funds ratios, may be useful information for supervisory purposes.

Our main results are the following:

- EL increases under stress conditions by 70–80 percent for all banks in the sample. As a consequence, the own-funds ratio decreases on average from 12 percent to 11.4 percent. Therefore, the German banks in the overall sample could sustain losses from a stress event in the automobile sector, at least up to the extent captured by our stress test.
- EC increases under stress by 8–20 percent and somewhat more sharply if ES is applied as a risk measure (12 percent to 22 percent). In both cases, it is still significantly lower than the increase in EL, always measured relative to the value under baseline conditions. Expressed in percentage points, referring to the nominal loan exposure, the average increase in EL across banks (0.34 percentage points) is, however, lower than the average increase in EC (0.54 percentage points).
- The significant impact on EC and the even stronger impact on EL are mainly driven by the effect of intersector correlations. If only the isolated impact on the automobile sector is considered, EL of the total portfolio increases by less than 2.5 percent. This low number is explained by the relatively small portfolio share of the automobile sector. Therefore, the results underline the need to account carefully for intersector

⁵Furthermore, the total credit exposure of the twenty-eight banks amounts to 75 percent of the total credit exposure of German banks to nonfinancial firms, measured in terms of banks' credit volume captured by the credit register.

dependencies even if a stress scenario in a single sector is analyzed.

- The level of EC is substantially higher (on average about 16 percent) for portfolios of real banks compared with highly fine-grained or *infinitely granular* portfolios with otherwise the same risk characteristics. The relative increase in EC due to the stress scenario, however, is similar in both cases. This indicative finding suggests that the computationally more tractable case of an infinitely granular portfolio can provide a reasonable proxy of the stress impact on the VaR. Further robustness checks, however, are needed if PDs are heterogeneous at the borrower level.
- Our results are robust against replacing a constant intrasector asset correlation by sector-dependent correlation estimates. More specifically, the average increase in EL, EC, and ES under stress is approximately only 3 percent lower than in the case of a constant intrasector asset correlation.
- A robustness check with larger intersector correlations shows a materially higher relative increase in EL of up to 16.4 percentage points, whereas the relative increase in EC is slightly lower. Therefore, good estimates of the asset correlations are a key prerequisite for meaningful stress-test results.

The paper is structured as follows. Section 2 describes the data on banks' credit portfolios and the correlation estimates. The design of the stress scenario and the portfolio credit-risk model are presented in section 3. The impact of the stress scenario on banks' portfolios is measured and discussed in section 4. Section 5 contains a sensitivity analysis with respect to the granularity of the exposures in the portfolio, the use of constant versus sector-dependent intrasector asset correlations, and the level of intersector correlations. Section 6 summarizes and concludes.

2. Data and Descriptive Analysis

In order to base our stress-test results on realistic input parameters, we employ information on credit portfolios of German banks that was extracted from the credit register maintained by the Deutsche Bundesbank. The reference date is September 2006. The credit

register contains bank loans exceeding €1.5 million; i.e., smaller loans are not considered. Credit information is available only at the borrower level, not at the facility level. As a particularity, the credit register aggregates borrowers to *borrower units*, which are treated as single credit entities because of business ties or legal linkages.⁶ Companies not belonging to a borrower unit are treated as single entities. Loans granted within borrower units are omitted in this exercise. Credit-risk mitigation techniques in the form of guarantees and plain-vanilla credit default swaps are taken into account in the exposure amount.

The sample of twenty-eight banks comprises all German banks that have at least 1,000 borrowers/borrower units included in the credit register. This limit was imposed in order to ensure that the loan information in the credit register is sufficiently representative of the bank's actual credit portfolio.

The analysis requires every borrower and borrower unit to be assigned to one industrial sector. For single firms, the sector can be assigned directly according to their field of business. In the case of borrower units, this information is not available in the database. The industrial sector covering the largest percentage of the borrower unit's total loan amount is used instead. This assignment is reasonable since, on average for all borrower units, the share of the largest industrial sector amounts to 89 percent.

Since the credit register does not contain information on the credit quality of single borrowers, we have to revert to sector-dependent average probabilities of default (PDs) which are deduced from historical insolvency rates, available from the German Federal Statistical Office.⁷ In order to calculate PDs, the ratio of average default events in 2005 and 2006 to the number of existing companies is used.

The definition of sectors follows the Industry Classification Benchmark (ICB), which is convenient for the estimation of inter-sector correlations. The ICB classification was originally developed by the Financial Times Stock Exchange and Dow Jones to create a

⁶A borrower unit comprises, for example, companies that are formally independent but that are considerably influenced or controlled by one of these companies.

⁷See table 4 in the appendix.

standard for trading and investment decisions. It distinguishes four hierarchical sector levels that comprise ten sectors at the top level and 104 subsectors at the base level. For this study, we use the second aggregation level, which comprises eighteen sectors. For the analyses, the ICB classification has two main advantages: Firstly, stock indices are readily available which can be directly mapped to the ICB classification. Secondly, the industrial sectors used in the credit register of the Bundesbank can also be easily matched to the ICB classification.

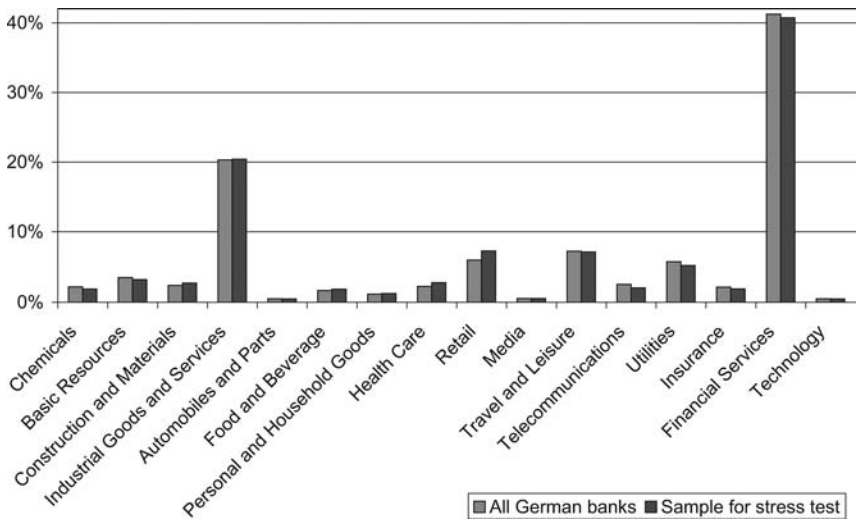
The banking sector is excluded from the study owing to its specific characteristics—for example, the monitoring by banking supervisors and the particularities of the interbank market, which constitutes a major section of interbank exposures. Furthermore, since no German company is listed in the *oil and gas* sector, the analyses are limited to sixteen sectors instead of eighteen.

The intersector correlations are estimated from weekly log-returns of stock indices over a time frame of two years just before the reference date. In order to differentiate between industry sectors, Dow Jones Eurostoxx sub-indices are used, which can be matched to the sixteen ICB sectors. The correlation matrix was estimated from index returns during 2005 and 2006 and is shown in table 5 in the appendix.

Figure 1 shows the distribution of the aggregated credit claims among the sectors, both for the twenty-eight selected banks and for all domestic banks. The twenty-eight chosen banks cover approximately 75 percent of the volume of claims granted to nonfinancial companies included in the credit register. This explains why their credit distributions among the different sectors are quite similar to those of all domestic banks. The distribution indicates high concentrations in two sectors, *financial services* (approximately 40 percent) and *industrial goods and services* (approximately 20 percent). Since banks in their function as borrowers are excluded from our analyses and since insurance firms are assigned to a separate sector, a considerable percentage of loans to the *financial services* sector is given to other financials—in particular, to capital investment companies.

The share of the *automobiles and parts* sector appears relatively small. Yet it has to be considered that, owing to the sector correlation matrix, the stress event also affects other industries with

Figure 1. Sectoral Distribution of Credit Exposures



Note: This figure shows the relative loan share of individual sectors relative to the total credit volume, both for banks in the sample and all German banks.

economic ties to this sector. In order to draw conclusions on the contribution of a specific sector to the entire portfolio risk, both the credit exposure *and* the correlations with other sectors have to be considered.

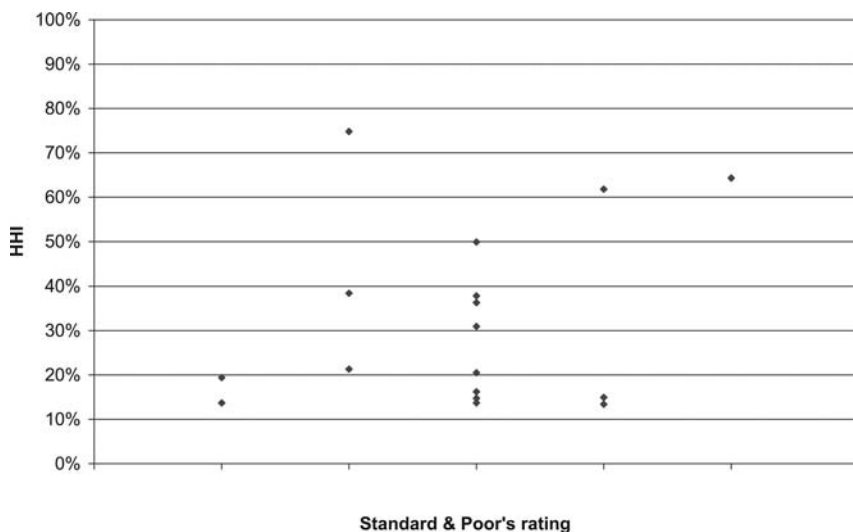
Table 1 provides some more detailed balance-sheet information on the sample of banks. Four banks are large private banks; eight are “other” private banks (i.e., excluding large private banks); thirteen

Table 1. Average Balance-Sheet Ratios

Banking Sector	Number of Banks	Average Balance-Sheet Total (€ million)	Average Market Capitalization (€ million)
Large Private Banks	4	764,603	30,883
Other Private Banks	8	108,639	4,304
Savings Banks	13	217,650	—
Cooperative Banks	3	154,864	—

Notes: This table shows selected balance-sheet ratios of the sample of twenty-eight banks. The balance-sheet ratios are averaged for different banking sectors.

Figure 2. Rating of Standard & Poor's and Herfindahl-Hirschman Index



Note: This figure shows the rating of Standard & Poor's against the Herfindahl-Hirschman Index at the sector level for all rated banks in the sample.

belong to the savings bank sector, which also comprises the *Landesbanken*; and the remaining three are cooperative banks.

In order to measure if sectoral concentrations have an impact on a bank's credit quality, we employ the Herfindahl-Hirschman Index (*HHI*). If such a relation existed, it would underline the importance of sectoral concentrations for the credit quality. If not, no conclusion is possible because it may simply be due to limitations of heuristic risk measures or because the bank accounts for sectoral risks otherwise—for example, through holding an additional capital buffer. Figure 2 suggests that any relation between rating grades and sector concentrations measured by the *HHI* is weak at best. The same holds for a modified *HHI**, which also captures the borrowers' credit risk:

$$HHI = \sum_i w_i^2 \text{ and } HHI^* = \sum_i \frac{PD_i}{\overline{PD}} w_i^2, \quad (1)$$

where PD_i denotes the default probability in the i -th sector, $\overline{PD}$ the average default probability of all sectors and all banks in the

sample, and w_i the portfolio share of the i -th sector defined as the ratio of the loan amount in the i -th sector to the total loan amount of the portfolio. Comparisons between, firstly, the capital ratio and both HHI indices and, secondly, PDs and HHI indices do not suggest any strong interrelation either. Although the exact reason for the weak relation between the concentration indices and the banks' risk ratings (or other risk indicators) is left for further research, a model-based assessment of sectoral concentration risks, which is carried out in this paper, offers at least a viable and theoretically superior alternative.

3. Stress Scenarios and Methodology

3.1 Credit-Risk Model

In order to capture all aspects of credit risk, including default dependencies, a CreditMetrics-type portfolio model is applied, which is frequently used in the banking business for credit-risk modeling. Our implementation of this model type considers a one-period time horizon and differentiates between two states of a default-trigger variable—default and nondefault—at the end of a one-year risk horizon.⁸ An obligor defaults if the default trigger—corresponding to the asset value in the classic Merton model—falls below an exogenously determined default barrier.

The portfolio losses due to credit defaults are described by the following loss function L_N :

$$L_N = \sum_{i=1}^N w_i \cdot LGD_i \cdot 1_{\{Y_i \leq c_i\}}. \quad (2)$$

L_N denotes the total loss of the bank portfolio which is composed of credit claims to N borrowers or borrower units. The relative share of a single loan in the entire portfolio is indicated by w_i , whereas the corresponding probability of default and the expected loss severity are described by PD_i and LGD_i . Since we do not have information

⁸A generalization of the model framework toward a mark-to-market valuation which considers migration risk in addition to default risk would be possible, but is not implemented in the current approach owing to data constraints.

on the ratings or PDs of individual borrowers, the PDs are estimated from historical default rates on a sector basis. Table 4 in the appendix shows the PDs sector by sector, which were calculated as average default rates over two years. The LGDs of all borrowers are set to 45 percent, which is the value set by supervisors for senior unsecured corporate exposures in the foundation internal ratings-based approach of Basel II. The indicator function $1_{\{\dots\}}$ denotes a binary random variable which takes the value of one if a loan defaults, and zero otherwise. A default event occurs if the default trigger Y_i falls below the default threshold c_i . Since Y_i has a standard normal distribution by construction (see below), the default threshold $c_i = \Phi^{-1}(PD_i)$ can be directly derived from the probability of default, where $\Phi(\cdot)^{-1}$ denotes the inverse of the cumulative normal distribution function.

The default trigger Y_i economically represents the change in the unobservable and appropriately normalized asset value of the company up to the end of the risk horizon. It has two components:

$$Y_i = r \cdot X_{s(i)} + \sqrt{1 - r^2} \cdot \epsilon_i. \quad (3)$$

The first risk component is the sector-dependent systematic risk factor (sector factor) $X_{s(\cdot)}$ and the second component is the borrower-dependent (or idiosyncratic) risk factor ϵ_i . Both components are mutually and pairwise independent and have a joint standard normal distribution. As initially assumed, each loan is uniquely assigned to one out of S business sectors. Let $s : \{1, \dots, N\} \rightarrow \{1, \dots, S\}$ denote a mapping of the borrower to a sector. The sector factors $X_{s(\cdot)}$ are normally distributed. The estimate of their correlation matrix Ω is given by table 5 in the appendix. For simulating the loss distribution of the portfolio, it is convenient to express $X_{s(i)}$ as a linear combination of independent standard normal systematic risk factors Z_k :

$$X_{s(i)} = \sum_{k=1}^S \alpha_{s(i),k} Z_k. \quad (4)$$

The linear coefficients $\alpha_{s(i),k}$ are obtained from a Cholesky decomposition of the correlation matrix Ω .

The coefficient r determines the relative weight of the systematic and nonsystematic risk factor; i.e., the closer its value is to one, the

higher the systematic risk. Since the asset correlation of any pair of borrowers i and j is given by

$$\rho_{i,j} \equiv \text{cor}(Y_i, Y_j) = r^2 \omega_{s(i),s(j)}, \quad (5)$$

the parameter r can be determined if the asset correlation and the correlation between the two sector factors are known. The intra-sector correlation equals r^2 and is the same for all sectors.⁹ For practical purposes, we take the average asset correlation $\bar{\rho}$ of small and medium-sized German companies,¹⁰ an empirical value of 0.09, and the mean value $\bar{\omega} = 0.648$ of the correlation matrix given by table 5 in the appendix. With these values, r is calculated by $\sqrt{\bar{\rho}/\bar{\omega}}$ and equals 0.373.

In order to calculate the risk measures, the loss distribution is determined by Monte Carlo simulations. In every simulation run, $S + N$ independent and standardized normally distributed random numbers are generated. The sector factors can be calculated as linear combinations of the first S random numbers, whereas the idiosyncratic risk factors are determined by the remaining N realizations of the random numbers. The portfolio loss can subsequently be calculated by means of equations (2) and (3). EL, EC, and ES are used as risk measures for the credit portfolio before and after stress. Both EC and ES refer to the 99.9 percent quantile of the loss distribution. Following common industry practice, both risk measures are defined after subtraction of (unconditional) EL.

3.2 Design of the Stress Scenario

The key idea of our stress-testing methodology is based on Bonti et al. (2006). The stress scenario is defined by constraints on the systematic risk factors of those sectors that we want to stress. Constraining the sample space of these factors offers several advantages. The probability of stress conditions before stress is automatically known, which gives an indication about the severity of the scenario. Furthermore, the original model parameters are kept with the consequence that all the information used for their estimation is still

⁹The assumption of constant intrasector correlations across sectors is relaxed in section 5.

¹⁰See Hahnenstein (2004).

harnessed. Contrary to stressing correlations directly, the problem of keeping the stressed correlation matrix positive semidefinite is avoided.

In the baseline scenario—i.e., before the stress event occurs—a standard normal distribution is assumed for all sector factors. In the stress scenario, only realizations of the sector factor that are below a scenario-specific threshold are considered. Technically speaking, the marginal distribution associated with the sector factor is restricted to a lower half space limited by the upper threshold of the scenario.

In principle, this scenario threshold can be derived from a macroeconomic model. Our stress test follows a pragmatic approach in which the expectation value of an observable macroeconomic variable closely related to the risk factor of the stressed industry sector is used as input. In order to determine the threshold value of the unobservable risk factor, we also need the distribution function of the macroeconomic variable. This distribution function can be approximated by the empirical distribution of the production index. Accordingly, the truncated distribution of the risk factor considered in the stress test reflects realistic stress conditions observed in the past.

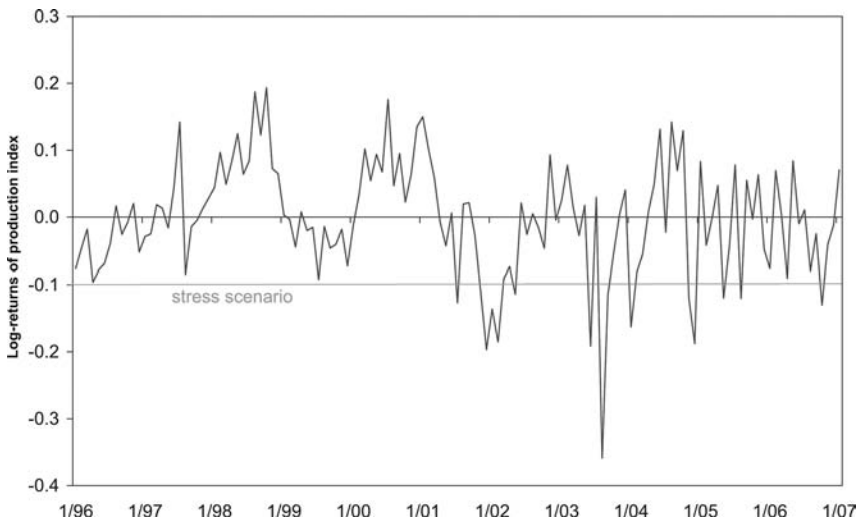
As we are dealing with a special/predetermined sector, we need to consider the available information on sector-specific expected developments and trends. We thus take into account forecasts that, owing to stricter environmental regulations, the demand for cars could increasingly shift toward less expensive, more fuel-efficient models over the coming years. The German automobile industry, which is traditionally mostly present in the segment of powerful vehicles in the upper price range, would be particularly affected by such developments, which could trigger a drop in German automobile production. Yet how strongly it will be affected depends on its ability to adapt to these emerging trends.

A sudden decline in automobile production, however, can also have other explanations. Market disturbances such as the subprime crisis starting in summer 2007 could also negatively affect the automobile industry. A declining demand for vehicles due to stricter credit conditions could cause the situation of an already fragile U.S. car market to deteriorate. Since automobile exports have made an increasingly important contribution to the economic success of

German automobile producers in recent years, this could also have material repercussions for the German automobile industry.

In light of these economic considerations, we assess the impact of a stress scenario in the automobile sector—more specifically, of a sudden decline in automobile production—on the credit portfolios of our sample of twenty-eight banks. Our trigger event refers to an expected decline in automobile production by 10 percent, which is motivated by historical data. The detrended log-returns of the underlying automobile production index between 1996 and 2007 are illustrated in figure 3. The values can be used as an empirical frequency distribution of the yearly index variations. The horizontal line at the ordinate value of -0.1 indicates a 10 percent decline of the index value, subsequently assumed as a reference point for the stress scenario. Since various more pronounced drops in the index value occurred during the observation period (e.g., in fall 2003), a decrease of 10 percent is regarded as an exceptional, but not an extreme, scenario.

Figure 3. Log-Returns of the Production Index of the Automobile Sector



Note: This figure shows the yearly log-returns of the production index of the automobile sector from January 1996 to January 2007.

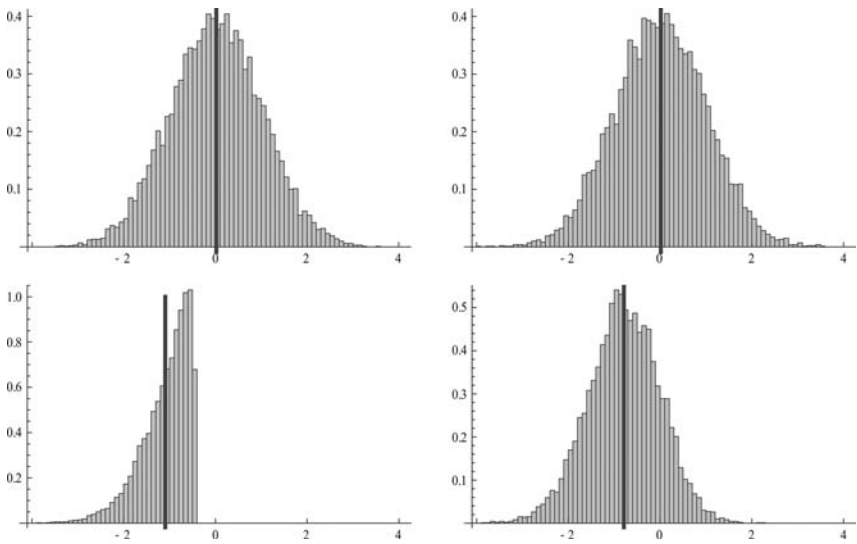
In order to consider the stress scenario within the portfolio model, the expected decrease in the index value induced by the stress event needs to be mapped to the systematic and unobservable risk factor of the automobile sector. For this purpose, the empirical distribution of the historical yearly log-returns of the index is restricted by an upper threshold in such a way that the log-returns of the truncated distribution average the expected index decline of 10 percent under stress conditions. Given a conditional expected index return of -10 percent, the upper threshold of the log-returns implies a probability of 33 percent that returns are below the threshold. The probability of 33 percent is transferred to the risk model, i.e., to the unobservable systematic risk factor of the *automobiles and parts* sector. Because of a standard normal distribution of the sector factor (before stress), the scenario threshold amounts to -0.44 .

In principle, it is well possible that the production index and the sector factor are not highly correlated—for example, in the case of a nonlinear dependence. Our approach takes this into account by distinguishing between both of them and linking them such that the probabilities of the scenarios are the same under stress conditions.

The impact of the stress event is also captured for the remaining sectors, which is a crucial advantage of employing the underlying multifactor risk model. Since the sector factors are correlated with one another, the stress event is transferred to other sectors and affects the distributions of the remaining sector factors. Figure 4 illustrates the distribution of the risk factors before (upper part) and after (lower part) the application of the stress scenario, both for the *automobiles and parts* (left side) and *industrial goods and services* (right side) sectors. The mean values of the distributions are marked as vertical lines.

In the left-hand part of figure 4, the impact of the stress scenario and the restriction of the risk factor in the 33 percent quantile can be clearly identified. Owing to correlation effects, the stress event also affects the remaining sectors, which is illustrated in the right-hand part for the *industrial goods and services* sector. As a consequence, the conditional distribution of this sector factor and its mean are likewise shifted toward the negative domain. Due to the above average correlation with the *automobiles and parts* sector (see table 5 in the appendix), this shift is also above average.

Figure 4. Frequency Distribution of the Systematic Risk Factors



Note: This figure shows simulated frequency distributions of the systematic risk factors before stress (upper part) and after stress (lower part) of the *automobiles and parts* sector (left) and the *industrial goods and services* sector (right).

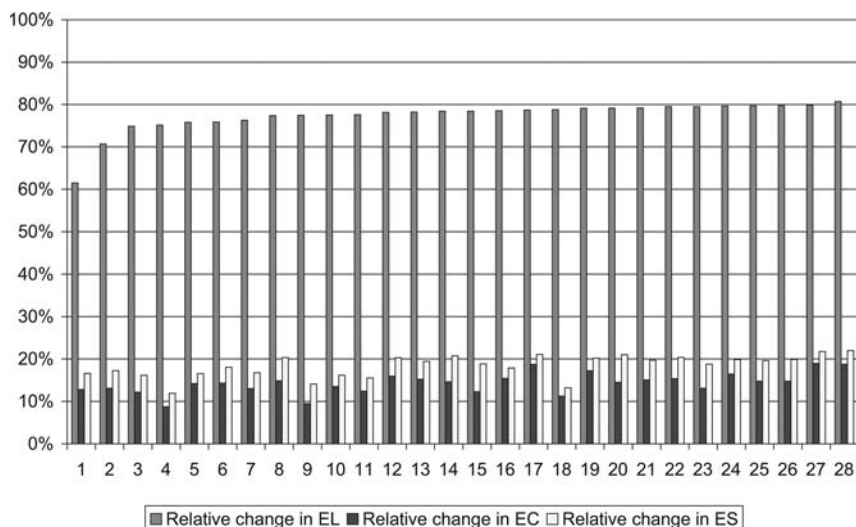
4. Results for the Stress Scenario

In this section we present the results of the stress test—first, in terms of EL of the credit portfolios of the banks in the sample and, second, in terms of the impact on the banks’ own-funds ratios.

The results for EL, EC, and ES are based only on loans to non-financial companies and are shown in figure 5. The changes in EL, EC, and ES due to the stress event are sorted in ascending order according to the relative increase in EL. Based on the chosen stress scenario, the results indicate a considerable and relatively similar increase in EL in a range generally between 70 percent and 80 percent.¹¹ A higher relative increase in EL than in EC is only observable

¹¹Compared with the other institutions, the increase in EL of one particular bank amounting only to approximately 60 percent is considerably lower. The reason for this is the business model of this bank, which has the consequence that loans are granted to sectors with relatively low correlations with the automobile sector.

Figure 5. Impact of Stress Scenario on Expected Loss, Economic Capital, and Expected Shortfall



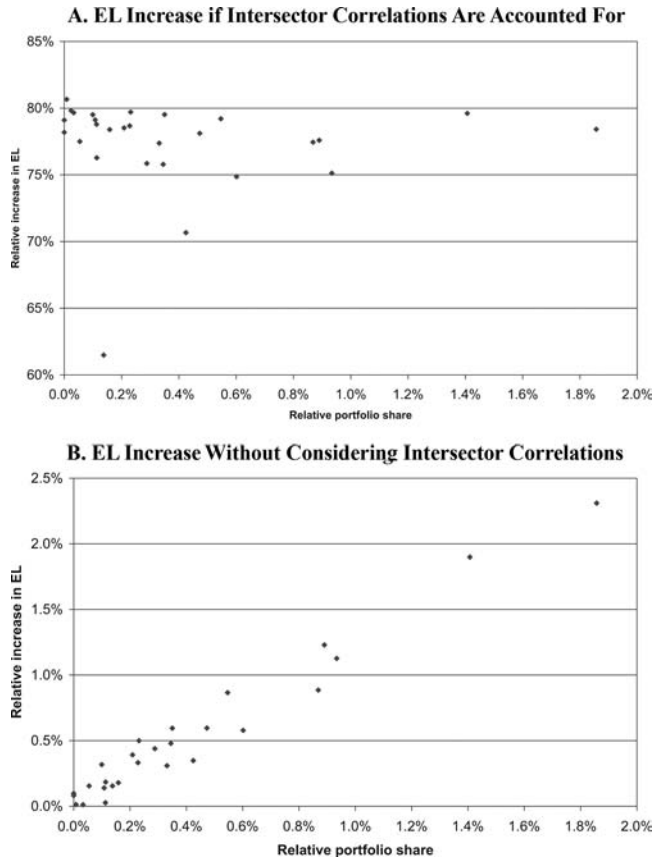
Notes: This figure shows the relative change of expected loss (EL), economic capital (EC), and expected shortfall (ES) in the stress scenario for all twenty-eight banks in the sample. The figures are shown as percentages.

if measured relative to the baseline values of EL and EC. Expressed in percentage points (referring to the nominal loan exposure), the average increase in EL across banks (0.34 percentage points) is, however, lower than the average increase in EC (0.54 percentage points).

Note that, for all banks, the share of loans granted to the *automobiles and parts* sector is below 2 percent and thus is only a very minor share of the entire credit portfolio. Therefore, compared with the overall bank portfolio, the overall effect of the stress event on loans to the automobile sector is rather limited. This is confirmed by panel A of figure 6, which plots the relative increase in EL against the relative portfolio share of the automobile sector for the twenty-eight banks in the sample. As expected, only a weakly positive relation between the portfolio share of the automobile sector and the EL increase is observable.

In order to explain the relatively large increase in EL across all banks, it is important to consider the correlations between the sectors. Owing to these correlations, the stress is transferred from the

Figure 6. Portfolio Share of Automobile Sector and Expected Loss



Notes: This figure shows the relative portfolio share of the *automobiles and parts* sector per bank compared with the relative increase in the expected loss of the total portfolio conditional on the stress scenario. Intersector correlations are accounted for in the first diagram but not in the second diagram.

automobile sector to other sectors that can have a considerably bigger share of the credit portfolio. The *industrial goods and services* sector, for example, with a relatively high correlation with the automobile sector, has a portfolio share between 3.4 percent and 33.5 percent among all chosen banks. Since the declining credit quality of the automobile sector affects this sector due to a high correlation,

the overall increase in EL is more pronounced than if the automobile sector were considered in isolation. Thus, the increase in EL cannot be attributed primarily to loans granted to the automobile sector, but rather to the impact of the stress event on the remaining sectors due to correlation effects. Note that the impact of correlations should not be interpreted in the sense of economic causality such that the model would explain a stress impact in those sectors in reaction to a stress event in the automobile sector. What is captured instead is an impact in a statistical sense such that “bad” draws of the automobile risk factor are more often accompanied by “bad” rather than “good” draws of the other risk factors. The correlation with other sector factors can explain in this sense the strong increase in EL among all banks despite the relatively low percentage of loans in the automobile sector.

In order to quantify explicitly the importance of intersector correlations for the loss distribution under stress, we measure in an auxiliary calculation the difference in the EL increase between two cases: first, the case in which only the impact on the automobile sector is included and, second, the case in which the impact on other sectors driven by the intersector correlations is also considered. In detail, for the first case we estimate losses under baseline conditions for all sectors except the automobile sector. For loans in the automobile sector, we consider instead losses conditional on stress conditions. For the second case, which also captures the stress propagation through intersector correlations, we use the previous results.

The relative increase in EL in the first case is depicted in panel B of figure 6, depending on the portfolio share of the automobile sector. The level of the EL increase, which is below 2.5 percent for all banks, is low compared with the increase of 70–80 percent if the intersector correlation effects are also considered. It is, however, well explained by the very minor exposure share of the automobile sector, which is below 2 percent of their total portfolio exposure for all banks in the sample. Furthermore, the scattergram reveals a positive, broadly linear relation between the increase in EL and the portfolio share of the automobile sector. Such a relation is not observable in panel A of figure 6, in which the overall change in EL is mainly driven by correlations with other sectors rather than by the exposure size in the automobile sector.

Another striking observation in figure 5 is that there are relatively small differences in the EL increase between banks when disregarding one outlier bank with a lower increase of around 60 percent. This is all the more surprising given that the portfolio distribution among sectors varies from bank to bank such that different correlations take effect. One possible explanation is the similar portfolio share of certain sectors—in particular, *industrial goods and services* and *financial services*, which have not only a relatively high correlation with the automobile sector but also a large share of the banks' credit portfolios (see also figure 1). Since both sectors cover 60 percent of the entire credit portfolio on average, the increase in EL is mainly driven by their correlation with the automobile sector. Since the portfolio shares of both sectors are relatively similar across banks, the EL also rises in a similar range.

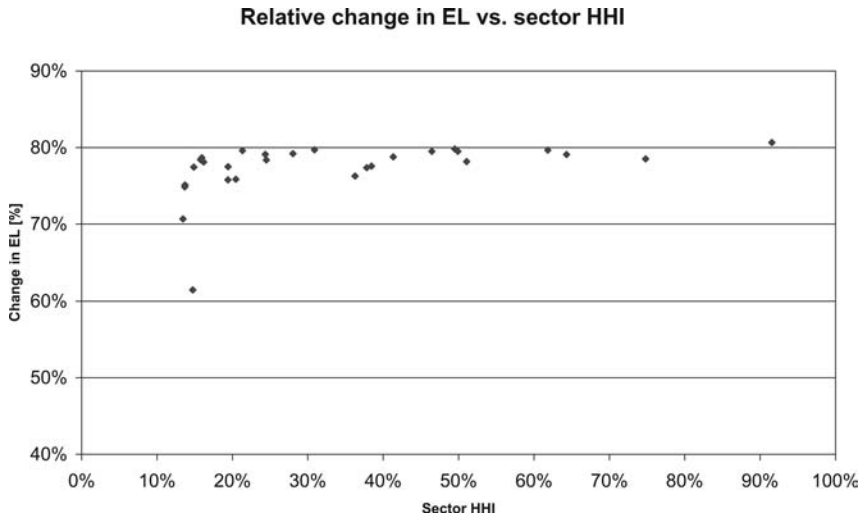
According to figure 5, the increase in EC measured relative to its value under baseline conditions is lower than the overall increase in EL (8.7–18.8 percent for EC compared with 70–80 percent for EL). An increase in EL is a first-order effect, as it immediately affects net income and can trigger a bank failure if capital is exhausted and a bank becomes overindebted.¹² An increase in EC, instead, means that a larger capital buffer is needed at the end of one year for potential future losses. This is still important, as it could affect the capital ratio, which is an important indicator of a bank's risk-absorbing capacity. Compared with an increase in actual losses, however, it is only a second-order effect, as it concerns the solvency under a high percentile, which is, in turn, conditional on the stress scenario. Therefore, EL is considered as the primary concern of bank's risk management and serves as the key risk measure in the subsequent impact analysis on regulatory own-funds ratios.

The results for ES in figure 5 differ from those for EC in that the measured relative increase in risk is slightly higher. This is to be expected since—given the same confidence level—ES refers to a point further in the tail of the loss distribution than EC.

Figure 7 plots the percentage change in EL of all banks in the sample against the *HHI*, calculated on a sector basis. The diagram suggests a slightly positive relation between both measures, yet it

¹²According to the German insolvency code, overindebtedness automatically causes insolvency.

Figure 7. EL Impact of Stress Scenario Against Sector-Based HHI



Notes: This figure shows the impact of the stress scenario on the expected loss (EL) against the sectoral concentration of the twenty-eight banks in the sample. Sectoral concentration is measured by the Herfindahl-Hirschman Index (HHI) calculated from the portfolios' sectoral exposures according to the ICB sector classification.

also points out the limits of relatively simple yardsticks for concentration risk such as the *HHI* or *HHI**. Hence, only model-based analyses are able to provide robust results on the impact of the stress event.

From a risk-management perspective, it is not only important how the level of risk changes under stress conditions. It is also important to consider the impact on the banks' solvency. Below, the regulatory own-funds ratios (OFR) of the chosen banks are used in order to approximate the impact of the stress event on banks' minimum required capital. The regulatory requirements for own funds after stress are approximated as follows:

OFR^{stress}

$$= \frac{\text{regulatory own funds} - \Delta EL_{\%}^{\text{stress}} \cdot \text{credit exposure}^{\text{corporates}}}{\text{risk-weighted assets incl. market risk}}. \quad (6)$$

$\Delta EL_{\%}^{stress}$ denotes the change in EL due to the stress event. It is measured in percentage points, i.e., relative to the nominal credit amount. Although a proxy of the credit exposure to corporate borrowers could be extracted from the credit register, we use a different data source from banks' regular reports for the following reason. The credit register only contains loans above the reporting limit of €1.5 million, with the consequence that a comparison across banks could be biased.¹³ Therefore, the stress effects would be underestimated for banks focusing on clients with single credit exposures below this reporting limit. As a consequence, *credit exposure^{corporates}* is taken from regular reports by banks (*Bilanzstatistik*) encompassing the total credit exposure to nonbanks. Although these aggregate numbers do not account for off-balance-sheet exposures, as do exposure numbers in the credit register, we believe that the greater coverage justifies their use. The impact of off-balance-sheet exposures is still reflected in $\Delta EL_{\%}^{stress}$, which is based on credit-register data. In other words, our approach intends to combine more risk-relevant information from the credit register with data from a different source which better reflect banks' total credit exposures. It is conservative because $\Delta EL_{\%}^{stress}$ is based on the granularity level of the credit register. This granularity level is lower than for the total credit portfolio because the loans below the reporting limit will increase the portfolio's granularity.

Conditional on the stress scenario, the mean own-funds ratio decreases by 0.62 percentage points from 12.04 percent to 11.42 percent, which indicates that banks in the sample overall remain well capitalized.

5. Sensitivity Analysis

5.1 Impact of Name Concentration

The stress-test results for EL, EC, and ES presented in the previous section can be considered conservative in the sense that the granularity or name concentration of the portfolio is overestimated because the credit register does not contain credit exposures below the reporting limit. Therefore, diversification benefits from smaller

¹³See section 2 for further details.

exposures in the portfolio are not captured. Although data constraints prevent us from measuring this effect directly, it is possible to estimate an upper bound of potential diversification effects by assuming that the portfolio is infinitely fine-grained in every business sector. Under this assumption, applying the law of large numbers conditionally on the factors shows that the limiting loss is given by the expected loss conditional on the (orthogonal) systematic risk factors $Z_1, \dots, Z_S$,¹⁴

$$L^\infty \equiv \mathbb{E}[L|Z_1, \dots, Z_S] = \sum_{k=1}^S \bar{w}_k LGD \Phi\left(\frac{\Phi^{-1}(p_k) - r \sum_{j=1}^S \alpha_{k,j} Z_j}{\sqrt{1 - r^2}}\right), \quad (7)$$

with sectoral exposure weights $\bar{w}_k = \sum_{\{i: s(i)=k\}} w_i$. The simplified “asymptotic” model represented by the loss distribution from (7) is computationally much more tractable. Although it still requires Monte Carlo simulation, random numbers now only need to be generated for the systematic risk factors but no longer for the idiosyncratic risk components.

Regarding terminology, we refer below to the original bank portfolios as “finite” portfolios, and the portfolios with the same risk characteristics except infinite granularity in every business sector are referred to as “infinitely granular” portfolios. Table 2 compares summary statistics of EL, EC, and ES, both for the finite portfolio analyzed in the previous section and for the infinitely granular portfolio under a baseline and a stress scenario. All statistics refer to the sample of twenty-eight banks. The statistics for the finite portfolios summarize the results depicted in figure 5.

We first discuss the results for the risk measure EL. The EL statistics under baseline conditions are necessarily the same for both portfolios because, in the case of homogenous and independent PDs and LGDs, the expected value does not depend on the exposure distribution inside a business sector. Under stress, the mentioned EL statistics likewise increase by almost the same amount in the case of both finite and infinite granularities. This result suggests that the asymptotic approximation of the loss distribution as given by (7) properly reproduces the EL impact of the stress scenario in

¹⁴See Gordy (2003).

Table 2. Summary Statistics of Risk for Real and Infinitely Granular Portfolios

Portfolio Granularity Scenario	Finite		Infinite	
	Baseline	Stress	Baseline	Stress
Expected Loss				
Maximum	0.54	0.92	0.54	0.92
75% Quantile	0.45	0.80	0.45	0.80
Mean	0.44	0.77	0.44	0.77
25% Quantile	0.40	0.73	0.40	0.72
Minimum	0.38	0.68	0.38	0.68
Economic Capital				
Maximum	5.98	6.65	3.64	4.28
75% Quantile	3.97	4.48	3.38	3.87
Mean	3.84	4.38	3.22	3.68
25% Quantile	3.43	3.96	3.07	3.44
Minimum	3.05	3.48	2.72	3.07
Expected Shortfall				
Maximum	7.41	8.39	4.94	5.91
75% Quantile	5.20	6.16	4.58	5.49
Mean	5.07	5.99	4.39	5.24
25% Quantile	4.65	5.52	4.20	4.98
Minimum	4.14	4.82	3.73	4.45
Notes: This table shows summary statistics of expected loss, economic capital, and expected shortfall for a sample of twenty-eight banks. We differentiate, firstly, between banks' real portfolios and infinitely granular portfolios with otherwise the same risk characteristics and, secondly, between a normal and a stress scenario. All results are given as percentages.				

the finite portfolios. This result is plausible for the following reason. Name concentration becomes important in the extreme adverse tail of the loss distribution. In our stress test, we consider, however, a half space of the stressed systematic factor such that many factor realizations of this and other sectors are predominantly still relatively close to the center of the distribution.

Contrary to the risk measure EL, for which we find quite similar results for the infinitely granular portfolio and the finite portfolio, the level of EC is significantly lower in the infinitely granular case,

for both the baseline and the stress scenarios. The difference is 16 percent for the mean and 10–40 percent, depending on the statistic. This increase in EC due to name concentration is moderately stronger than observed in previous studies by Heitfield, Burton, and Chomsisengphet (2006) and Duellmann and Masschelein (2007). Comparing EC under normal and stress conditions, we find that the increase in EC is similar in both cases, amounting to a range of 9–19 percent, depending on the statistic.

Summarizing these results, portfolio granularity has a significant impact on the *level* of EC, but it does not seem to affect in the same way its *relative* increase from the stress event. This finding confirms that results based on using an infinitely granular portfolio as a proxy can substantially underestimate the *level* of required EC. They seem to provide, however, a good proxy for the *relative stress impact* on EC.

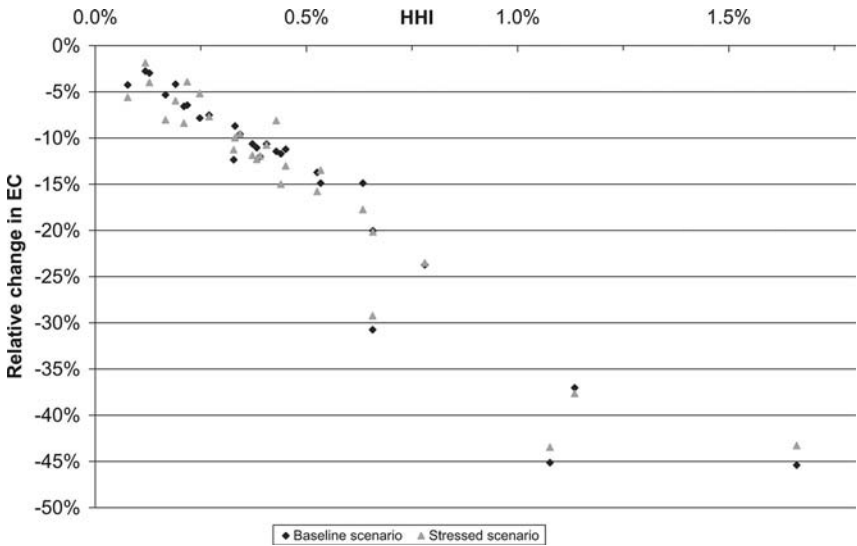
Figure 8 illustrates the impact of portfolio granularity measured by the decrease in EC if the bank's portfolio is replaced by a portfolio of infinite granularity but otherwise the same risk characteristics. The results are shown for the baseline and the stressed scenario. In both cases, the relation between EC and HHI appears to be broadly linear.¹⁵ Furthermore, the magnitude of the impact on EC is almost identical in both cases.

A diagram similar to figure 8 in which EC is replaced as risk measure by EL does not show a similar dependence on HHI. This is to be expected, as exposure concentrations become more important in the tail of the loss distribution. The EL conditional on the 33 percent quantile of the automobile risk factor, however, is still too close to the center of the distribution to show a similar relation between EC and HHI.

Finally turning toward the risk measure ES, the numbers in table 2 show a similar, albeit somewhat stronger increase under stress conditions than was observed for EC. A stronger increase is plausible, as the ES refers to a point higher in the tail of the loss distribution than the EC.

¹⁵In the case of a single-factor credit-risk model and an otherwise homogenous portfolio, a “granularity adjustment” to the EC figure calculated for an infinitely granular portfolio is linear in the HHI. (See Gordy and Lütkebohmert 2007 for an example of such a granularity adjustment.)

Figure 8. Impact of Portfolio Granularity on Economic Capital



Notes: This figure shows the HHI calculated on exposure level against the percentage change in EC for portfolios of twenty-eight banks if the portfolio is replaced by a portfolio with infinite granularity in every business sector but otherwise the same risk characteristics. Results are further differentiated between baseline and stress conditions.

In summary, we find that the level of EC—contrary to EL—is rather different in the portfolios with finite and infinite granularities. The relative increase in EC due to the stress event, however, is similar for the finite and the infinitely granular portfolio. If EC is replaced by ES, the results are similar except that the increase under stress conditions is more pronounced.

5.2 Sector-Dependent vs. Constant Intrasector Asset Correlations

From an economic perspective, it is plausible to assume that the average level of intrasector asset correlations between firms differs between sectors. It will, for example, be higher in more cyclical industry sectors. This would suggest replacing the constant factor loading r of the systematic risk factor in (3) with a sector-dependent value.

The previous assumption of intrasector asset correlations being constant across sectors is not only common practice. It is also motivated by empirical obstacles in estimating differences in asset correlations of borrowers belonging to the same sector. The arguably most natural way to estimate asset correlations inside a sector is to use stock returns of listed companies and determine the R^2 in an index model using the portfolio of companies in the respective sector as the index portfolio. The work by Hahnenstein (2004) demonstrates how the composition of the index portfolios can substantially bias the results because the R^2 is then driven by the firm's weight in the index portfolio rather than by the "true" asset correlations. This sample bias becomes even more important if the percentage of listed firms in a specific sector is relatively small and the listed companies are less representative of the whole sector.¹⁶

Even if the *level* of R^2 could be biased, the cross-sectoral differences between the average R^2 values, each computed for all firms in the same sector, may still be indicative of *relative* differences in asset correlations across sectors. Therefore, we calibrate the sector-dependent R^2 values such that their average value over all sectors is the same as the (constant) asset correlation level used in section 4. In other words, we use R^2 only for information on relative differences between the intrasector asset correlations of different sectors. The correlation level is—averaged over sectors—still the same as before. For this purpose, $\hat{R}_j$, the square root of the R^2 value for sector j , is scaled by the ratio of the original factor weight $\bar{r}$ in section 4 and the average of the square root of the $\hat{R}_j$ values over all sectors. The sector-dependent factor weights r_j are then defined as follows:

$$r_j = \frac{\bar{r}}{\frac{1}{S} \sum_{j=1}^S \hat{R}_j} \hat{R}_j. \quad (8)$$

Table 3 summarizes both the original and the normalized factor weights of each sector. Depending on the differing factor weights and the exposure distribution among sectors of a respective bank, the resulting impact of the stress scenario can be either more or less severe compared with the use of a unique factor weight.

¹⁶For this reason, we also refrain from using an empirically estimated function that relates asset correlations, for example, to firm size, as is common procedure in the original CreditMetrics model.

Table 3. Sector-Dependent Factor Weights before and after Normalization

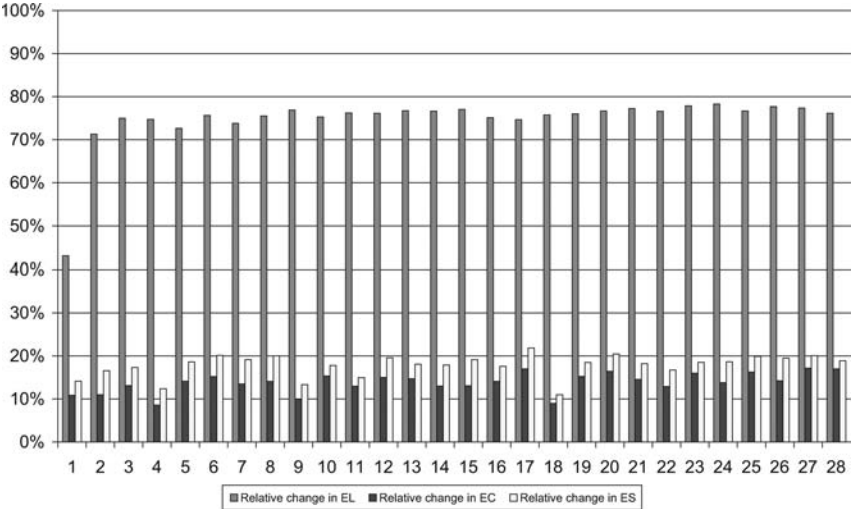
Sector	Factor Weight before Normalization	Factor Weight after Normalization
Chemicals	0.68	0.43
Basic Resources	0.63	0.40
Construction and Materials	0.66	0.42
Industrial Goods and Services	0.59	0.38
Automobiles and Parts	0.66	0.42
Food and Beverage	0.46	0.29
Personal and Household Goods	0.63	0.40
Health Care	0.28	0.18
Retail	0.54	0.34
Media	0.52	0.33
Travel and Leisure	0.59	0.38
Telecommunications	0.59	0.38
Utilities	0.55	0.35
Insurance	0.65	0.42
Financial Services	0.55	0.35
Technology	0.57	0.37

Notes: This table shows both the original and the normalized factor weights calculated for each industry sector. The values are derived from the asset correlations of all companies within a certain sector (R^2 -statistic). The normalized values are adapted to the unique factor weight used in the main analysis (see section 3.1).

Figure 9 illustrates the impact on EL, EC, and ES for the twenty-eight banks. For better comparability, all banks are sorted according to the results of the main analysis (see figure 5). Compared with the main results, the EL, EC, and ES increase in a similar magnitude for almost all banks. However, depending on the individual factor weight of each sector and varying exposure distributions among sectors for a respective bank, the mentioned statistics deviate slightly from the main results. Hence, the EL does not exhibit the same monotonic increase as in figure 5.

As a general tendency, the increase in EL, EC, and ES is marginally lower (approximately 3 percent on average) than in the results based on a uniform factor weight. One possible explanation is that the *financial services* sector, which covers an average portfolio share

Figure 9. Impact of Stress Scenario on Expected Loss, Economic Capital, and Expected Shortfall in Case of Sector-Dependent Factor Weights



Notes: This figure shows the relative change of expected loss (EL), economic capital (EC), and expected shortfall (ES) in the stress scenario for all twenty-eight banks in the sample. The figures are shown as percentages. In contrast to figure 5, the results are based on sector-dependent factor weights calibrated to the uniform weight used in the main analysis.

of around 40 percent and which is highly correlated with the automobile sector, is now assigned a slightly smaller factor weight. As a consequence, the impact from cross-sector correlations is slightly dampened.

In the case of the first bank in figure 9, the EL increase is notably lower than in the results based on a uniform factor weight (around 43 percent compared with 61 percent before). This difference is mainly driven by considerable exposure concentrations within a specific sector which is assigned a below-average factor weight.

5.3 Sensitivity to Higher Intersector Correlations

The results presented in section 4 are based on correlation estimates from stock index returns observed between 2005 and 2006. This time span was selected because it comprises the last two years of our

sample of bank portfolios. It is commonly known that asset correlations are difficult to estimate. Because we use equity returns as the basis of our correlation estimation, one could argue that the co-movement in stock prices is also driven by factors unrelated to credit risk and also that asset correlations appear to be unstable over time.¹⁷ In order to measure the robustness of our results against errors in the correlation estimates, we carry out a straightforward “correlation stress test.” For this purpose, we replace the intersector correlation matrix with a correlation matrix estimated for the time period from 1997 to 1998.¹⁸ This period exhibits the highest correlation estimates for the automobile sector over two-year periods between 1995 and 2006.

With this new correlation matrix, we repeat the stress test on the portfolios of the twenty-eight banks (see figure 10). The relative increase in EL is again calculated relative to the unconditional EL, which is the same as before. As expected, the relative increase in EL, which ranges from 78–93 percent across banks, is stronger than in the case of the original correlation matrix (see figure 5). The additional increase does not exceed 16.4 percentage points.

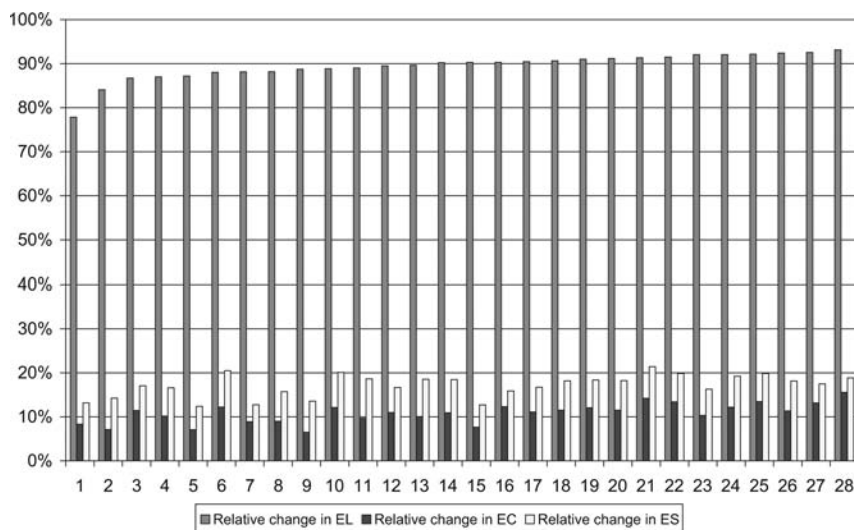
In the case of higher intersector correlations, the relative EC increase is far weaker than the increase in EL and even less than the increase in EC measured in the original stress test in figure 5. Because of the “correlation stress,” the loss distribution is shifted to the right-hand side. This shift, however, seems mostly to affect the losses closer to the center of the distribution rather than in the tail such that EL is more affected than EC.

We finally analyzed the effect of the “stressed” correlations on banks’ regulatory own-funds ratios. The increased correlations have only a secondary impact on this ratio in all four categories of banks. These results suggest that our stress-test results are robust against “stressed” correlations insofar as the impact on the banks’ solvency is concerned.

¹⁷See, for example, Bollerslev, Engle, and Wooldridge (1988), Ang and Chen (2002), or Duellmann, Scheicher, and Schmieder (2007).

¹⁸Since the coefficient r of the systematic risk factor depends on the average of the correlation matrix Ω (see section 3), this coefficient becomes 0.343 for this robustness check.

Figure 10. Impact of Stress Scenario on Expected Loss and Economic Capital in a High-Correlation Scenario



Notes: This figure shows the relative change of expected loss (EL), economic capital (EC), and expected shortfall (ES) in the stress scenario for all twenty-eight banks in the sample. The figures are shown as percentages. In contrast to figure 5, the results are based on sector correlations observed from 1997 to 1998, a period in which the highest correlations are measured.

6. Summary and Outlook

In this paper we stress-test credit portfolios of twenty-eight large German banks based on a Merton-type multifactor default-mode credit-risk model. Rather than focusing on a particular stress forecast, however, the paper focuses on the main drivers of the stress impact on banks' credit portfolios. The ad hoc stress scenario assumes a downturn in the automobile sector. Following Bonti et al. (2006), it is captured by truncating the distribution of the risk factor assigned to this sector. In this way, a wide range of stress events is considered instead of only a single "point scenario." Therefore, the typical assumption that the stress is no more severe than its forecast can be avoided.

Our results reveal a strong increase of EL in the corporate credit portfolio across banks which ranges between 70 percent and 80 percent, measured relative to the EL under baseline conditions.

From a bankwide perspective, however, the impact appears to be less serious. The own-funds ratio decreases on average from 12 percent to 11.4 percent. Therefore, the German banks in the sample overall could sustain losses from our stress scenario. Furthermore, this discrepancy in numbers between the single-portfolio perspective and the bankwide perspective suggests that it is important to look beyond actual portfolio losses in order to assess the stress impact on a bank. In addition to EL, we also determine the impact on EC and ES, which increase under stress by 8–20 percent and 12–22 percent, respectively, again measured relative to baseline conditions. In both cases, this increase is significantly less than for EL. Expressed in percentage points, referring to the nominal loan exposure, the average increase in EL across banks (0.34 percentage points) is, however, lower than the average increase in EC (0.54 percentage points).

The impact on EL, EC, and ES is mainly driven by intersector correlations propagating the stress impact into other sectors. If only the impact on the automobile sector is considered, EL of the total portfolio, for example, increases by less than 2.5 percent. These findings argue in favor of accounting carefully for intersector dependencies, even for stress scenarios that are related only to a single sector.

The level of EC is, on average, about 16 percent and, therefore, substantially higher for portfolios of real banks than for highly fine-grained or *infinitely granular* portfolios with otherwise the same risk characteristics. Since the relative increase in EC and ES under stress conditions is similar in both cases, the computationally more tractable case of an infinitely granular portfolio can provide a reasonable proxy of the *relative* stress impact, at least if PDs are homogenous in every sector, as assumed in our study.

Our results are robust against replacing a constant intrasector asset correlation with sector-dependent correlation estimates. A further robustness check with higher intersector correlations shows a relative increase in EL of up to 16.4 percentage points, which is material. The relative increase in EC and ES, however, is slightly lower than in our benchmark case.

Further research is warranted in particular on the following two issues, which will both be addressed in a future paper. The current downturn forecast for the automobile industry sector has been obtained from the historical frequency distribution of production index returns for that sector. Using forecasts based on a dynamic

macro model of the economy would not only improve the forecasting power but also allow to stress several sectors simultaneously in a consistent way.

In order to consider also name concentration in the credit portfolio, the sector-dependent default probabilities need to be replaced by borrower-dependent PDs conditional on data availability. Recent research confirms that borrower-dependent PDs can have a material impact on portfolio risk.¹⁹ Furthermore, any comparison between different banks can be distorted if the borrower selection and monitoring abilities of the individual institutions are not taken into account.

Appendix

Table 4. Insolvency Rates of Sixteen Business Sectors in 2005 and 2006

Sector	2005	2006	Average
Chemicals	1.4%	0.9%	1.1%
Basic Resources	1.1%	0.8%	1.0%
Construction and Materials	2.4%	1.8%	2.1%
Industrial Goods and Services	1.3%	1.1%	1.2%
Automobiles and Parts	1.4%	0.8%	1.1%
Food and Beverage	0.9%	0.7%	0.8%
Personal and Household Goods	1.0%	0.8%	0.9%
Health Care	1.3%	1.2%	1.2%
Retail	0.9%	0.8%	0.9%
Media	1.5%	1.2%	1.3%
Travel and Leisure	1.1%	1.0%	1.0%
Telecommunications	3.3%	3.0%	3.2%
Utilities	0.1%	0.1%	0.1%
Insurance	0.0%	0.8%	0.4%
Financial Services	0.9%	0.7%	0.8%
Technology	1.0%	0.8%	0.9%
Notes: This table shows historical insolvency rates from the German Federal Statistical Office for sixteen sectors according to the ICB sector classification. The insolvency rates are calculated separately for 2005 and 2006 and averaged in the last column.			

¹⁹See, for example, Hanson, Pesaran, and Schuermann (2005) and Duellmann and Masschelein (2007).

Table 5. Correlation Matrix of the Sector Indices

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1 Chemicals	1	0.64	0.77	0.80	0.73	0.74	0.82	0.58	0.67	0.70	0.56	0.43	0.70	0.77	0.75	0.59
2 Basic Resources	0.64	1	0.70	0.75	0.54	0.59	0.61	0.35	0.52	0.54	0.45	0.38	0.57	0.65	0.71	0.48
3 Construction & Materials	0.77	0.70	1	0.90	0.70	0.74	0.79	0.46	0.64	0.71	0.70	0.45	0.74	0.78	0.82	0.62
4 Indus. Goods and Services	0.80	0.75	0.90	1	0.73	0.74	0.84	0.49	0.64	0.73	0.71	0.48	0.73	0.83	0.80	0.70
5 Automobiles & Parts	0.73	0.54	0.70	0.73	1	0.65	0.72	0.46	0.62	0.65	0.63	0.50	0.54	0.74	0.66	0.61
6 Food & Beverage	0.74	0.59	0.74	0.74	0.65	1	0.82	0.50	0.68	0.66	0.62	0.43	0.69	0.73	0.73	0.57
7 Pers. & Household Goods	0.82	0.61	0.79	0.84	0.72	0.82	1	0.55	0.68	0.72	0.72	0.50	0.70	0.83	0.72	0.68
8 Health Care	0.58	0.35	0.46	0.49	0.46	0.50	0.55	1	0.53	0.45	0.34	0.29	0.40	0.45	0.43	0.40
9 Retail	0.67	0.52	0.64	0.64	0.62	0.68	0.68	0.53	1	0.54	0.52	0.40	0.60	0.67	0.62	0.47
10 Media	0.70	0.54	0.71	0.73	0.65	0.66	0.72	0.45	0.54	1	0.62	0.66	0.58	0.67	0.65	0.68
11 Travel and Leisure	0.56	0.45	0.70	0.71	0.63	0.62	0.72	0.34	0.52	0.62	1	0.59	0.47	0.69	0.57	0.59
12 Telecommunications	0.43	0.38	0.45	0.48	0.50	0.43	0.50	0.29	0.40	0.66	0.59	1	0.39	0.51	0.45	0.51
13 Utilities	0.70	0.57	0.74	0.73	0.54	0.69	0.70	0.40	0.60	0.58	0.47	0.39	1	0.65	0.70	0.49
14 Insurance	0.77	0.65	0.78	0.83	0.74	0.73	0.83	0.45	0.67	0.67	0.69	0.51	0.65	1	0.73	0.71
15 Financial Services	0.75	0.71	0.82	0.80	0.66	0.73	0.72	0.43	0.62	0.65	0.57	0.45	0.70	0.73	1	0.57
16 Technology	0.59	0.48	0.62	0.70	0.61	0.57	0.68	0.40	0.47	0.68	0.59	0.51	0.49	0.71	0.57	1

Notes: This table shows intersector correlations of sixteen sector indices following the ICB sector classification. The correlations were estimated from weekly stock index returns in 2005 and 2006.

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Macroeconomic Default Modeling and Stress Testing*

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This paper applies a macroeconomic-based model for estimating probabilities of default. The first part of the paper focuses on the relation between macroeconomic variables and the default behavior of Dutch firms. A convincing relationship with GDP growth and oil price and, to a lesser extent, the interest and exchange rate exists. The second part of the paper assesses the default behavior based on a stress scenario of two consecutive quarters of zero GDP growth as required by the Basel II framework. It can be concluded that a stress-test scenario covering two quarters of zero GDP growth does not influence the default rate significantly and thus does not seem to be very severe.

JEL Codes: C12, C13, C15, E32, E44, E47, G21, G28.

1. Introduction

Estimating probabilities of default is the first step in assessing the credit exposure and potential losses faced by financial institutions. Probabilities of default are also the basic inputs when evaluating systemic risk and stress testing financial systems. Therefore, predictors of credit risk are of natural interest to practitioners in the financial industry, as well as to regulators—especially under the new capital adequacy framework (Basel II), which encourages the active involvement of banks in measuring the likelihood of defaults.

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Financial literature has brought forth a variety of models that attempt to measure the probability of default, e.g., macroeconomic-variable-based models. Macroeconomic-based models are motivated by the observation that default rates in the financial, corporate, and household sectors increase during recessions. This observation has led to the implementation of econometric models that attempt to explain default indicators, such as probabilities of default or default rates, using macroeconomic variables.

In this paper we consider the default rate in relation to macroeconomic variables. Specifically, we will explore the relationship between the default rate and the macro economy by developing a logit model with macroeconomic parameters. This fairly simplistic model has three advantages: First, the model is relatively easy to understand. Second, it presents robust results. Third, the model takes the correlation of default rates amongst sectors into account. We call this special feature the “correlation” factor. Relatively little work has been done in previous research on estimating such a “correlation” factor, although it has often been confirmed in the literature that default rates are highly correlated amongst sectors.

A direct consequence of this model and its advantages is that the direction of causality is only one way—from macro variables to default—and that we do not allow for feedback from financial factors to the macro variables. We chose not to incorporate the banking sector, in order to develop a relatively simple, easy-to-implement model and focus solely on the relation between macroeconomic variables and the default rate.

By means of the logit model and selected variables, first we will assess which macroeconomic variables are related to the default behavior of firms and, second, examine the default behavior in 2007 on the basis of an unfavorable macroeconomic scenario of two quarters of zero GDP growth in the third and fourth quarters of 2006. We will compare it with a base scenario and the situation where we took the 2.5 percent worst cases of the base scenario.

The remainder of this paper is organized as follows. Section 2 discusses related studies. Section 3 describes the construction of the data set. Section 4 formulates the estimation model. Section

5 discusses the estimation results. Section 6 studies the stress-test scenarios. Section 7 concludes.

2. The Concept of Macroeconomic-Based Models

Estimating probabilities of default is a challenging subject. The types of models used to assess credit risk can be broadly classified either as market-based models or as fundamental-based models. Market-based models build on Merton's option-pricing theory and rely on security prices. Chan-Lau (2006) distinguishes four approaches within fundamental-based modeling to model probabilities of default: macroeconomic-based, accounting-based, rating-based, and hybrid models.

Macroeconomic models, as used in this research, explain changes in the default rate on the basis of macroeconomic conditions. These macroeconomic variables are cyclical indicators—for instance, GDP growth or interest rates—and financial market indicators—for instance, stock market prices and stock market volatility. Accounting-based models, on the other hand, generate probabilities of default for individual firms using accounting information. Furthermore, rating-based models can be used to infer probabilities of default when external ratings information is available. Hybrid models generate probabilities of default using a combination of economic variables, financial ratios, and ratings data.

Chan-Lau (2006) lists three advantages of macroeconomic models. First, an advantage is that this type of model is very suitable for designing stress scenarios. Second, because long data series are available for most countries, it is also possible to conduct cross-country comparative studies. Third, the default rate used to estimate the model is observed historically, so that one can avoid making assumptions.

On the other hand, a disadvantage of macroeconomic models is that the time span of the data needs to be longer than one business cycle; otherwise, the model would not capture the impact of the business cycle on probabilities of default. Furthermore, this type of model is subject to Lucas critique since the parameters or functional forms are unlikely to remain stable; i.e., it is virtually impossible to capture the complex interaction between the state

of the economy and the default risk. Finally, aggregate economic data are usually reported with substantial lags. This makes it difficult to estimate or forecast macroeconomic models with up-to-date information.

Macroeconomic models can be classified into exogenous and endogenous models, depending on whether the model allows feedback between financial distress and the explanatory economic variables. The first category of macroeconomic-based models assumes that the economic variables are exogenous and not affected by financial distress. The general approach to modeling this category is described by the following equation:

$$pd_t = g(x_1, x_2, \dots, x_n) + \varepsilon, \quad (1)$$

where pd is the probability of default over a given period t . A general aggregate model sets pd_t equal to a function g , $X = (x_1, x_2, \dots, x_n)$, a function of a set of economic variables and a random variable ε .

A problem of the exogenous approach is that the relationship between macroeconomic variables and the default rate is assumed to be the same during periods of economic downturn and expansion. This seems intuitively to be implausible.

The second category of macroeconomic-based models assumes that the economic variables are endogenous and differ in times of financial distress. The typical econometric framework used in these models is the vector autoregressive (VAR) methodology. See, for example, Hoggarth, Logan, and Zicchino (2005). We can write the VAR in a more general form as

$$Z_{t+1} = \alpha_t + \sum_{j=1}^p \beta_j z_{t+1-j} + \varepsilon_{t+1}, \quad (2)$$

where α_t is a constant vector, β_j are (lagged) coefficients matrices, ε_{t+1} is a vector of residuals/shocks, and z is the vector of endogenous variables that includes both probabilities of default and the aggregate economic variables associated with the state of the business cycle. In principle, inference in VAR models is sensitive to the

choice of lags. If a large number of lags is included, degrees of freedom are lost. If the lag length is too short, important lag dependencies may be omitted.

We end this section with a short overview of which macroeconomic variables are related to the default rate. Appendix 1 lists an extensive set of papers on macroeconomic default modeling, with a short description of each. It seems that the literature can be divided into that on quoted firms and that on unquoted firms. The papers mainly confirm the significance of GDP growth in relation to the default rate. Relations with stock market variables have been identified several times, but only for the literature on quoted firms. Furthermore, some papers show that the influence of the interest rate and the exchange rate is significant in certain sectors. Remarkably, none of the papers on our list examines the oil price as an explanatory variable.

3. The Model

3.1 Aggregate Default Modeling

Consider a general aggregate model that can be estimated by maximum likelihood. Let pd_t be the fraction (proportion) of firms that default in period t . We set pd_t equal to a function $g(\cdot)$ of the relevant explanatory variables $\mathbf{z}_t$, a parameter vector $\boldsymbol{\theta}$, and a disturbance v_t . In addition, controlling the distribution of v_t controls the distribution of pd_t .

$$pd_t = g(\boldsymbol{\theta}, \mathbf{z}_t, v_t) \quad (3)$$

More specifically, $pd_{t,i}$ is the fraction of firms in sector i , $\forall i \in \{0, 1, \dots, s\}$ that default in period t , with s the total number of sectors in the economy. The economy as a whole is denoted by $i = 0$. Furthermore, $\mathbf{z}_t$ is a vector of variables including intercept, relevant for the default rate at time t . $v_{t,i}$ is a disturbance and β_i is a vector of parameters.

$$pd_{t,i} = \frac{\exp(\mathbf{z}_t' \boldsymbol{\beta}_i + v_{t,i})}{1 + \exp(\mathbf{z}_t' \boldsymbol{\beta}_i + v_{t,i})} \quad (4)$$

Taking the logit¹ of both sides, we find

$$\tilde{p}d_{t,i} : \text{logit}(pd_{t,i}) = \mathbf{z}'_t \boldsymbol{\beta}_i + v_{t,i}. \quad (5)$$

From this general model, we obtain two separate models, an economy model ($i = 0$) and a sector model $\forall i \in \{1, 2, \dots, s\}$. For the economy default rate ($i = 0$), we assume the disturbances $v_{t,0}$ are independent and identically distributed (i.i.d.). Letting $\sigma_{\psi,0}^2 = \text{var}(v_{t,0})$, the economy model is

$$\begin{aligned} \tilde{p}d_{t,0} &= \mathbf{z}'_t \boldsymbol{\beta}_0 + v_{t,0} \\ \text{where } v_{t,0} &\overset{iid}{\sim} (0, \sigma_{\psi,0}^2). \end{aligned} \quad (6)$$

For the sector default rates, $\forall i \in \{1, 2, \dots, s\}$, the disturbances are divided into a latent systematic (ξ_t) and an idiosyncratic ($\psi_{t,i}$) part. The systematic part captures the correlation between the sector default rates. Estimation and inference of the parameters are based on maximizing the Gaussian quasi log-likelihood. Letting $\sigma_{\xi,i}$ and $\sigma_{\psi,i}$ be non-negative parameters, $\forall i \in \{1, \dots, s\}$. The sector model is

$$\tilde{p}d_{t,i} = \mathbf{z}'_t \boldsymbol{\beta}_i + v_{t,i} \quad (7)$$

$$\text{where } v_{t,i} = \sigma_{\xi,i} \xi_t + \sigma_{\psi,i} \psi_{t,i}$$

$$\xi_t \overset{iid}{\sim} (0, 1), \psi_{t,i} \overset{iid}{\sim} (0, 1).$$

An advantage of taking the correlation of the sector default rates into account is that combining such a factor with macroeconomic indicators provides a natural test of the specification of the macro relationship. If the macroeconomic indicators are indeed informative, then the fluctuations explained by the factor will be relatively small.

¹The logit transformation is given by $\text{logit}(x) = \ln\left(\frac{x}{1-x}\right)$. Since $\frac{\exp(\text{logit}(x))}{1+\exp(\text{logit}(x))} = \frac{x/(1-x)}{1+x/(1-x)} = x$, the equation $x = \frac{\exp(y)}{1+\exp(y)}$ is solved for y by $y = \text{logit}(x)$.

3.2 Dynamic Effects of Shocks in the Aggregate Default Model

This section assesses the effect of shocks in $\mathbf{z}_t$ over the long and short term. Therefore, we differentiate (4) with respect to $\mathbf{z}_t$ to find

$$D_{\mathbf{z}_t} p d_{t,i} = \frac{p d_{t,i} \beta_i}{\left(1 + \exp\left(\tilde{p} d_{t,i}\right)\right)^2}. \quad (8)$$

Given that $\tilde{p} d_{t,i}$ is generally low, the denominator can be ignored. Accordingly, for small $\Delta \mathbf{z}_t$, the elements of β_i are approximate semi-elasticities:

$$\Delta \% p d_{t,i} \approx \beta'_i \Delta \mathbf{z}_t. \quad (9)$$

The next step is to capture persistence in the default rate. Therefore, we include the lagged default rate as an explanatory variable. Letting $\mathbf{z}_t^*$ denote explanatory variables other than the lagged default rate and the intercept, and β_i^* the corresponding parameter vector, we may write equation (5) as

$$\tilde{p} d_{t,i} = \beta_{i,0} + \beta_{i,1} \tilde{p} d_{t-1,i} + \beta_i^{*'} \mathbf{z}_{t-1}^* + v_{t,i} \quad (10)$$

or, equivalently,

$$\tilde{p} d_{t,i} = \frac{\beta_{i,0}}{1 - \beta_{i,1}} + \beta_i^{*'} \sum_{j=0,1}^{\infty \beta_j} \mathbf{z}_{t-1-j}^* + \sum_{j=0,1}^{\infty \beta_j} v_{t-j,i}. \quad (11)$$

Specifically, the inclusion of the lagged default rate makes the current default rate depend on all lags of the explanatory variables with coefficients declining at rate $\beta_{i,1}$. In other words, a default depends not only on the previous period but also on the entire history, with more recent developments being more important.

Furthermore, to estimate the short- and long-term effects of a small shock $\Delta \mathbf{z}^*$ that occurs in period t_0 and persists indefinitely through time, equations (9) and (11) are combined. Applying equation (9) to (11) and considering $\sum_{j=0,1}^{\infty \beta_j} \mathbf{z}_{t-1-j}^*$ as the explanatory vector, the estimated effects are as follows:

Short-term effect:

$$\Delta \%pd_{t_0+1,i} \approx \beta_i^{*'} \Delta \left(\sum_{j=0i,1}^{\infty \beta_j} \mathbf{z}_{t_0-j}^* \right) = \beta_i^{*'} \Delta \mathbf{z}^* \quad (12)$$

Long-term effect:

$$\lim_{t \rightarrow \infty} \Delta \%pd_{t+1,i} \approx \lim_{t \rightarrow \infty} \beta_i^{*'} \Delta \left(\sum_{j=0i,1}^{\infty \beta_j} \mathbf{z}_{t-j}^* \right) = \frac{\beta_i^{*'}}{1 - \beta_{i,1}} \Delta \mathbf{z}^* \quad (13)$$

It must be noted that, at time $t > t_0$, the effect on the default rate is

$$\Delta \%pd_{t,i} \approx \beta_i^{*'} \Delta \left(\sum_{j=0i,1}^{\infty \beta_j} \mathbf{z}_{t-1-j}^* \right) = (1 - \beta_{i,1}^{t-t_0}) \frac{\beta_i^{*'}}{1 - \beta_{i,1}} \Delta \mathbf{z}^*. \quad (14)$$

Equation (14) shows that at time t , a fraction $(1 - \beta_{i,1}^{t-t_0})$ of the long-term effect is more or less realized. For this reason, $\beta_{i,1}$ controls the speed at which the default rate responds to shocks. For shocks that do not persist indefinitely, the long-term effect can be interpreted as an upper bound to the maximum effect that will be attained.

4. Data and Definitions

4.1 Definitions

In this section we give a short description of both the default rate and the macroeconomic variables. The default rate is defined as the ratio of the number of firms in default to the total number of firms in quarter t . If $pd_{t,0}$ is the fraction of all firms that default in quarter t and $pd_{t,i}$ is the fraction of firms in sector i that default in quarter t , $pd_{t,0}$ and $pd_{t,i}$ are the economy and the sector i default rates, respectively.

$$\begin{aligned} pd_{t,0} &= \frac{\text{Number of defaults in all sectors in quarter } t}{\text{Average number of firms in all sectors in quarter } t} \\ pd_{t,i} &= \frac{\text{Number of defaults in sector } i \text{ in quarter } t}{\text{Average number of firms in sector } i \text{ in quarter } t} \end{aligned} \quad (15)$$

With regard to the macroeconomic variables, we chose macroeconomic variables that give rise to particular concerns about movements in unfavorable directions, so that the variables are useful for stress testing or scenario analysis. It must be emphasized that these variables need not be those that add the most explanatory or forecasting power. The following variables were selected:

- **Gross Domestic Product:** GDP equals aggregate demand of an economy. Aggregate demand relates to sales of firms. Lower GDP growth means lower growth in sales of firms. The lower GDP growth is, the harder it is for firms to generate income through sales. Lower income thus increases the possibility that firms cannot meet their obligations and will default.
- **Interest Rate:** Firms often finance part of their activities by debt. The funding costs of firms are therefore positively related to interest rates. If interest rates are high, firms have higher costs and are more likely to default.
- **Exchange Rate:** The exchange rate is the price of the domestic currency expressed in terms of a foreign currency. Firms in sectors that have a great deal of international business are expected to be affected by exchange rates. However, the sign of the relationship varies. Business conditions of importing firms are positively affected by the exchange rate because imports become cheaper if the exchange rate is high. Business conditions of exporting firms are negatively affected by the exchange rate because exports become more expensive if the exchange rate is high.
- **Stock Market Return and Volatility:** Merton's theory predicts that the probability of default is negatively related to the stock market return and positively related to volatility. The stock market return and, thus, also volatility are variables that are popular in scenario analysis.
- **Oil Price:** Oil prices—or, more broadly, energy prices—affect the price of most of the products used by firms. Therefore, the cost of firms and thus their probability of defaulting are positively related to the oil price.

4.2 *Interesting Properties of the Default Rate in the Netherlands*

In this subsection we analyze the properties of the default rate in the Netherlands. We observe that the default rate in the Netherlands is persistent, negatively related to the business cycle, and correlated between sectors. No seasonal effects are observed.

The data set consists of defaults in companies in the Netherlands per quarter in the period from the first quarter (Q1) of 1983 and the second quarter (Q2) of 2006 (ninety-four quarters). The average economy default rate is about 0.23 percent per quarter, or just below 1 percent per year. The total number of firms in the economy varies between 408,665 (1983:Q1) and 652,367 (2006:Q2). Most firms in the Netherlands are rather small.

First of all, we examined if the economy default rate is persistent by estimating an autoregressive model of order 1 (AR(1)) with ordinary least squares (OLS), which results in

$$pd_{t,0} = 0.00 + .85pd_{t-1,0} + v_t. \quad (16)$$

The AR(1) model captures most of the serial correlation. The high coefficient of the first lag confirms that the default rate is persistent.

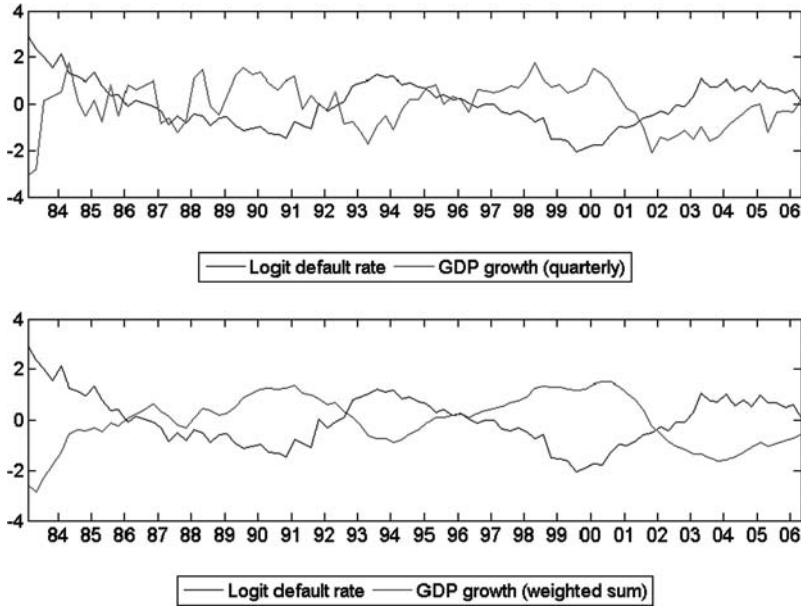
Second, the upper panel of figure 1 plots the economy default rate and $\Delta\%GDP_{t-1}$ against time. The relation is not obvious because GDP growth fluctuates a lot, while the default rate is persistent. Intuitively, however, one would expect a negative relation of the default rate with the business cycle. Accordingly, upon adding $\Delta\%GDP_{t-1}$ to (16) and carrying out a new estimate, we obtain

$$pd_{t,0} = .00 + .82pd_{t-1,0} - .00\Delta\%GDP_{t-1} + v_t \quad (17)$$

or, equivalently,

$$pd_{t,0} = \frac{.00}{1 - .82} - .00 \sum_{j=0}^{\infty} .82^j \Delta\%GDP_{t-1-j} + \sum_{j=0}^{\infty} .82^j v_{t-j}. \quad (18)$$

The intercept and the coefficient of GDP growth are rounded to 0.00. Nevertheless, they deviate significantly from zero. We can conclude

Figure 1. Economy Default Rate and GDP Growth

Notes: The upper panel plots $pd_{t,0}$ and $\Delta\%GDP_{t-1-j}$ against time. The lower panel plots $pd_{t,0}$ and $\sum_{j=0}^{19} 0.82^j \Delta\%GDP_{t-1-j}$ against time. All series are standardized to have a zero mean and unit variance.

that persistency actually implies that the default rate is related to a weighted sum of lags in GDP growth. This relation is shown in the lower panel of figure 1, which plots the economy default rate and $\sum_{j=0}^{19} 0.82^j \Delta\%GDP_{t-1-j}$ against time.

Third, with regard to the separate sectors, we found that all correlations of the default rates between sectors are significant at the 1 percent level. Thus, a highly significant correlation exists between the sectors.

Finally, we checked for seasonal effects of the default rate. The following simple model is estimated by OLS. In this model, the function 1_A is the indicator function for the event A .

$$pd_{t,i} = \sum_{j=1}^4 \delta_j 1_{t \in \text{quarter } j}(t) + v_t \quad (19)$$

We test $\delta_1 = \delta_2 = \delta_3 = \delta_4$. No indication of seasonal effects is found.

5. Estimation Results

5.1 Results for the Netherlands

First, we substitute the selected macroeconomic variables for $\mathbf{z}_t^*$ in $\tilde{pd}_{t,i} = \beta_{i,0} + \beta_{i,1}\tilde{pd}_{t-1,i} + \beta_i^{*'}\mathbf{z}_{t-1}^* + v_{t,i}$ (see equation (10)) and identify which macroeconomic variables are significant. We find that the GDP growth and the oil price are related to the default rate in the Netherlands as well as, to a lesser extent, the interest and the exchange rate. The stock market return and volatility have no relation with the default rate.

Gross Domestic Product. The estimated coefficients of GDP growth have a negative sign, as expected. The “industry and mining,” “transport, storage, and communications,” “financial services,” and “rental and corporate services” sectors are significant at the 1 percent level, which means they have the strongest link with the default rate. For the overall economy, the hypothesis of no relation with GDP growth is firmly rejected.

Interest Rate. Only the “construction” sector has a significant relation between the level of the short-term interest rate and the default rate. A reason why the results in this sector are different from those in other sectors is that construction firms are substantially affected by interest rates through another channel than the cost of debt; households find it easier to finance construction work on their homes when interest rates are low. This should affect, in particular, small construction firms. Indeed, about 86 percent of all construction firms have fewer than ten employees. The strong rejection of the hypothesis of equal coefficients supports the view that the “construction” sector is an exception. Demand for construction work is negatively related to interest rates, and it is interesting that the level of (and not the change in) the interest rate is significant. Apparently, people or firms react to the level of (and hardly to changes in) the interest rate.

We also tested the long-term instead of the short-term interest rate in $\mathbf{z}_t^*$. This yielded somewhat stronger but qualitatively

similar results. Due to the correlation between the short-term and the long-term rate, it is not sensible to include both rates. Various term spreads can be included, however. They tested insignificant with a one-quarter lag. Given that the term structure forecasts GDP growth, the term structure does have forecasting power when used with a lag of several years.

Exchange Rate. The most significant sector is “transport, storage, and communications,” as could be expected, particularly in the Netherlands. For the overall economy, the relation is moderately strong. No relation with the change in the exchange rate was found. Apparently, the level of the exchange rate is decisive in trading. This is remarkable since one would expect firms to become used to the level and only react to changes.

Stock Market Return and Volatility. The signs of the coefficients are both negative and positive, and in the case of both the return and volatility, the coefficients do not deviate significantly from zero. We may thus conclude that the default rate is unrelated to the stock market return and volatility. A reason for this is that the Dutch data set consists of mostly unlisted firms.

Oil Price. Most coefficients of the level of the oil price are significant. All coefficients have the expected sign. Again, the level of the oil price is apparently more important than the change. Furthermore, remarkably, the oil price is the only significant variable for which there is no statistical reason to doubt the hypothesis of equal coefficients. This suggests that the dependence of sectors on the oil price is equal.

Second, we estimate equation (10) without the stock market return and volatility, both of which were found to be insignificant. This leaves GDP growth, the short-term interest rate (level), the exchange rate (level), and the oil price (level) in $\mathbf{z}_t^*$. In general, the behavior of the variables is the same as noted above. Table 8 in appendix 2 lists the results.

Of this model, we measure short- and long-run effects of shocks in macroeconomic variables and their 95 percent confidence intervals. By applying the coefficients of the first lag in equation (14), we estimate that a year after a persistent shock, about 60–95 percent of the long-run effect is realized; see table 1.

Table 1. Short- and Long-Run Effects of Macroeconomic Shocks

	Lower Bound	Mean Effect	Upper Bound
GDP Growth: +.01 (Economy)	−2.38%	−1.40%	−.43%
Short Run	−14.91%	−9.01%	−3.11%
Long Run			
Short Rate: +.01 (Construction)	0.32%	1.52%	2.73%
Short Run	0.96%	6.60%	12.24%
Long Run			
Exchange Rate: +1% (Transport, Storage, and Communications)			
Short Run	0.75%	1.48%	2.22%
Long Run	1.39%	2.95%	4.51%
Oil Price: +10% (Transport, Storage, and Communications)			
Short Run	0.52%	1.79%	3.05%
Long Run	−6.46%	3.55%	13.57%
Notes: Short- and long-run effects (percentage changes) on the default rate of the economy or a certain sector are computed using (12) based on estimation results from table 8 in appendix 2. Upper and lower bounds of a 95 percent confidence interval are reported as well.			

Although GDP growth shocks will be examined more closely in the next section, we can already note that the effects of GDP growth shocks are somewhat low. A persistent 3 percent decrease in GDP growth raises the long-run default rate by only 25 percent.

The explanatory power of the model can be assessed by comparing the variances of the macroeconomic variables, the latent systematic (ξ_t) disturbances, and the idiosyncratic ($\psi_{t,i}$) disturbances. The first lag is excluded from the decomposition because (i) it explains most of the variance and (ii) it is not independent from the macroeconomic variables.

Recall that $v_{t,i} = \sigma_{\xi,i}\xi_t + \sigma_{\psi,i}\psi_{t,i}$. We can rewrite (10) as

$$\tilde{p}d_{t,i} - \beta_{i,0} - \beta_{i,1}\tilde{p}d_{t-1,i} = \beta_i^{*'}\mathbf{z}_{t-1}^* + \sigma_{\xi,i}\xi_t + \sigma_{\psi,i}\psi_{t,i}.$$

This leads to the following variance decomposition, given that the systematic and idiosyncratic disturbances are independent:

$$\text{var}(\tilde{pd}_{t,i} - \beta_{i,1}\tilde{pd}_{t-1,i}) = \beta_i^{*'} \text{var}(\mathbf{z}_{t-1}^*) \beta_i^* + \sigma_{\xi,i}^2 + \sigma_{\psi,i}^2$$

or, equivalently,

$$\begin{aligned} \frac{\beta_i^{*'} \text{var}(\mathbf{z}_{t-1}^*) \beta_i^*}{\text{var}(\tilde{pd}_{t,i} - \beta_{i,1}\tilde{pd}_{t-1,i})} + \frac{\sigma_{\xi,i}^2}{\text{var}(\tilde{pd}_{t,i} - \beta_{i,1}\tilde{pd}_{t-1,i})} \\ + \frac{\sigma_{\psi,i}^2}{\text{var}(\tilde{pd}_{t,i} - \beta_{i,1}\tilde{pd}_{t-1,i})} = 1. \end{aligned} \quad (20)$$

The three fractions are referred to as the macroeconomic, the latent systematic, and the idiosyncratic parts, and are presented in table 2. More variance is explained by the latent systematic part than by the macroeconomic part. Although the macroeconomic variables were not selected because of their explanatory power, this result does illustrate the difficulty of finding relevant systematic variables.

5.2 Sensitivity Analysis

An advantage of the model used in this paper is that it is easy to implement for other countries. We received data from Austria. The default frequency data are from the Austrian business information provider and debt collector Kreditschutzverband (KSV). The KSV database provides us with time series of insolvencies and the total number of firms in most NACE branches at a quarterly frequency. All other data are obtained from the Austrian Statistics Bureau and the IMF International Financial Statistics. We ran the model from 1991 onward.

The results are not the same as the results for the Dutch data series. It seems that the model is country specific. No significant relationship was found between the macroeconomic variables and the default rate. In other words, the model did not detect a relation between the macro economy and the default rate. This does not mean that the model of this research is incorrect. A reason might be the relatively short period we ran the model with. It might also be that another model is appropriate for the Austrian case.

Table 2. Variance Decomposition

	Macroeconomic Part	Latent Systematic Part	Idiosyncratic Part
Industry and Mining	22%	20%	58%
Construction	13%	35%	52%
Trade and Repair, Consumer Products	12%	51%	38%
Catering	10%	29%	60%
Transport, Storage, and Communications	33%	23%	44%
Financial Services	25%	27%	48%
Rental and Corporate Services	27%	42%	31%
Other	8%	13%	79%
Average	19%	30%	51%
Note: This table shows the variance decomposition (20) based on the model estimated in table 8 in appendix 2.			

6. Scenario Analysis of Zero GDP Growth

In this section, results from the scenario analysis are presented. We will examine the default behavior in 2007, given an unfavorable macroeconomic scenario of zero GDP growth in the third and fourth quarters of 2006. We will compare it with the average of a base scenario and the average of all 2.5 percent worst cases of the base scenario.

A framework for stress testing the credit exposure to macroeconomic shocks was developed on the basis of Virolainen (2004). In this framework, stress tests are conducted by comparing the average result of a stressed scenario, where an artificial adverse macroeconomic development is introduced, with that of the average of a base scenario, where no adverse shock takes place. Estimated averages of the default rates for each sector corresponding to the stressed and base scenarios are obtained from simulating a large number of future default rates by applying a Monte Carlo method. This is

partly governed by the simulated future path of the macroeconomic variables.

For the scenario analysis, we use the macroeconomic model (10) with only GDP included in $\mathbf{z}_t$. This allows us to examine a GDP growth scenario without the need to make assumptions on the other macroeconomic variables. In formula,

$$\begin{aligned} \tilde{p}d_{t,i} &= \beta_{i,0} + \beta_{i,1}\tilde{p}d_{t-1,i} + \beta_{i,2}\Delta\%GDP_{t-1} + v_{t,i} \\ v_{t,i} &\stackrel{iid}{\sim} (0, \sigma_{\xi,i}^2 + \sigma_{\psi,i}^2). \end{aligned} \quad (21)$$

Table 3 shows the estimation results. The coefficients of GDP growth are somewhat closer to zero than in the model where more macroeconomic variables were included. This is consistent with macroeconomic theory, which states that an increase in GDP lowers the default rate but also leads to higher interest rates and an appreciating exchange rate, which in turn causes the default rate to rise.

A model to forecast the behavior of GDP growth is also required. It seems that an AR(1) model fits GDP growth quite well. Let γ be a parameter vector and σ_ν a non-negative parameter.

$$\begin{aligned} \Delta\%GDP_t &= \gamma_0 + \gamma_1\Delta\%GDP_{t-1} + \nu_t \\ \nu_t &\stackrel{iid}{\sim} (0, \sigma_\nu^2) \end{aligned} \quad (22)$$

Estimation with OLS using data over the period 1978:Q1 to 2006:Q2 (114 observations) leads to $\Delta\%GDP_t = .01 + .61\Delta\%GDP_{t-1} + .02$. All parameters are significantly different from zero at the 1 percent level.

To apply the Monte Carlo simulation, we need to draw realizations of the disturbances $v_{t,i}$ in (21) and ν_t in (22). We assume the disturbances $v_{t,i}$ and ν_t follow, after standardizing, a standardized t-distribution with df degrees of freedom. The probability density function (pdf) of a standardized t-distribution evaluated at a real number x is given by

$$\frac{\Gamma\left(\frac{df+1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)\sqrt{(df-2)\pi}}\left(1 + \frac{x^2}{df-2}\right)^{-\frac{df+1}{2}}.$$

Table 3. Estimated Parameters Model (10) with GDP Growth Only

	Intercept	First Lag	GDP Growth	Systematic s.d.	Idiosyncratic s.d.
Industry and Mining	-1.49***	0.73***	-2.43***	0.05***	0.10***
Construction	-0.86***	0.85***	-0.83	0.07***	0.10***
Trade and Repair, Consumer Products	-0.92***	0.85***	-1.23**	0.07***	0.07***
Catering	-0.54***	0.91***	-1.16	0.09***	0.13***
Transport, Storage, and Communications	-1.69***	0.70***	-2.93***	0.08***	0.13***
Financial Services	-0.50***	0.89***	-2.67***	0.09***	0.09***
Rental and Corporate Services	-1.44***	0.76***	-2.64***	0.09***	0.06***
Other	-2.32***	0.68***	-1.14	0.05***	0.16***
Economy	-0.84***	0.86***	-1.46***	NA	0.08***
Pooled	-0.75***	0.86***	-1.47***	0.07***	0.11***
(P-Value Equal Coefficients)	0.0000	0.0000	0.2619	0.0315	0.0000

Notes: Model (10) is estimated with only GDP included in z_t . All estimated parameters are reported. Pooled results are obtained by estimating the sector model under the restriction $\beta_{1,j} = \dots = \beta_{8,j}$ or $\sigma_{j,1} = \dots = \sigma_{j,8}$ for certain j while allowing the other parameters to differ per sector. These restrictions are tested and the p-values are reported. The 1, 5, and 10 percent levels are denoted ***, **, and *, respectively.

Table 4. Sample Kurtosis and Degrees of Freedom of Disturbances in Equations (21) and (22)

	Sample Kurtosis	Degrees of Freedom
Sector Model	4.31	8.60
Economy Model	4.12	9.38
GDP Model	5.49	6.41
Notes: The sector and economy models refer to $v_{t,i} \forall i \{1, \dots, 8\}$ and $v_{t,0}$ in (21), respectively; the GDP model refers to v_t in equation (22). The sample kurtoses and the degrees of freedom of the fitted t-distributions are shown.		

The standardized t-distribution with df degrees of freedom allows us to adjust the kurtosis, which is important for worst-case scenarios. Moreover, the degrees of freedom are set equal to match the sample kurtoses. Table 4 shows the sample kurtoses of the disturbances and the degrees of freedom of the fitted t-distributions. The kurtoses are pooled for the sector model because they differ between the sectors.

$$\frac{3df - 6}{df - 4}$$

To analyze the scenarios, we generate 200,000 paths of the logit default rate using equations (21) and (22), given certain starting values for $\tilde{pd}_{t,i}$ and $\Delta\%GDP_t$. Disturbances $v_{t,i}$ and ν_t are generated by multiplying draws from the t-distributions by their respective standard deviations $\sqrt{\sigma_{\xi,i}^2 + \sigma_{\psi,i}^2}$ and σ_{ν} . Finally, we invert equation (5) to find the default rate:

$$pd_{t,i} = \frac{\exp(\tilde{pd}_{t,i})}{1 + \exp(\tilde{pd}_{t,i})}.$$

Based on the average of the simulations, the average default rate of the year 2007 is defined as

$$\bar{pd}_{2007,i} = \frac{1}{4} \sum_{t=1}^4 pd_{2007.t,i}.$$

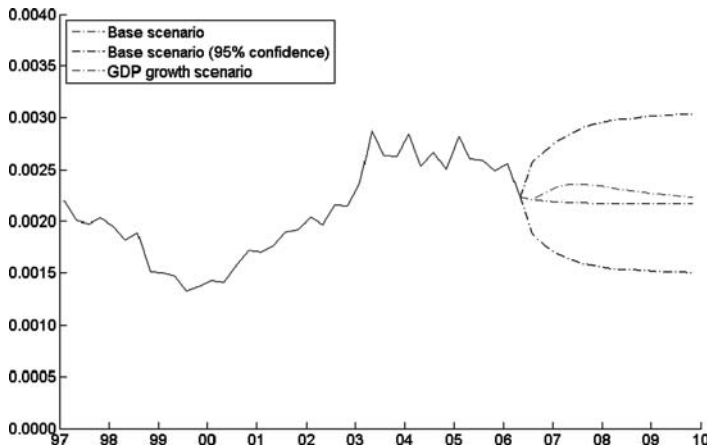
Table 5. Base Scenario

	Default Rate
Industry and Mining	0.31%
Construction	0.26%
Trade and Repair, Consumer Products	0.22%
Catering	0.31%
Transport, Storage, and Communications	0.29%
Financial Services	0.83%
Rental and Corporate Services	0.20%
Other	0.06%
Economy	0.22%
Note: This table shows the average generated $\bar{pd}_{2007,i}$ given the base scenario.	

Base Scenario. The expected average 2007 default rate is computed without making assumptions on what will happen after 2006:Q2. We do so by setting the starting values for $pd_{t,i}$ and $\Delta\%GDP_t$ equal to the known values from 2006:Q2, generating the logit default rates and computing $\bar{pd}_{2007,i}$. The results in table 5 serve as a benchmark for the stress scenario. The average economy default rate is plotted against time in figure 2. It remains approximately constant because both the default rate and GDP growth were already close to their long-run averages in 2006:Q2.

The 2.5 Percent Worst Cases. Table 6 compares the 2.5 percent worst cases of $\bar{pd}_{2007,i}$ (the 0.975th percentile of the base scenario) with the overall average of the base scenario. The percentage difference and its 95 percent confidence interval are reported. The 2.5 percent worst cases of the base scenario result in a 31–62 percent rise in the default rate (depending on the sector). The “industry and mining” and “trade and repair, consumer products” sectors have relatively the smallest difference in default rate. The “catering” and “financial services” sectors have a relatively large difference in the default rate.

Zero GDP Growth. The zero GDP growth scenario assumes GDP growth to be zero in 2006:Q3 and 2006:Q4. After 2006:Q4, GDP growth develops in accordance with equation (22). Logit

Figure 2. Forecasting the Economy Default Rate

Notes: The figure shows (i) the average and (ii) the .025th and .975th percentiles of the generated economy default rates given the base scenario, as well as (iii) the average generated economy default rate given the zero GDP growth scenario plotted against time.

default rates evolve according to equation (21), with the $\tilde{pd}_{t,i}$ from 2006:Q2 as the starting value. We generate the logit default rates and compute $\bar{pd}_{2007,i}$. Table 7 compares the average generated $\bar{pd}_{2007,i}$ of the zero GDP growth scenario to the base scenario. The percentage difference and its 95 percent confidence interval are reported. The confidence interval captures uncertainty in the percentage difference of the expected effect caused by uncertainty in the estimated parameters of models (21) and (22). The effect of the GDP growth scenario in 2007 is a 4–15 percent rise in the default rate, depending on the sector. The effects are surprisingly small even if we look at the upper bounds. Note that, in accordance with the estimation results from table 3, the “industry and mining,” “transport, storage, and communications,” “financial services,” and “rental and corporate services” sectors are affected most by the zero GDP growth scenario.

The question arises as to whether this small effect is realistic in comparison with historical results. During the period 1983–91, there were about three brief, sharp drops in GDP growth (figure

Table 6. The 2.5 Percent Worst-Case Scenario

	Lower Bound	% Difference Default Rate	Upper Bound
Industry and Mining	25%	33%	41%
Construction	31%	39%	48%
Trade and Repair, Consumer Products	23%	31%	39%
Catering	54%	62%	70%
Transport, Storage, and Communications	35%	43%	51%
Financial Services	43%	51%	59%
Rental and Corporate Services	27%	35%	43%
Other	32%	40%	48%
Economy	20%	28%	36%

Notes: This table shows the percentage difference between the .975th percentile of all generated $pd_{2007,i}$ and the average generated $\bar{pd}_{2007,i}$ given the base scenario. The percentage difference is an estimate because of uncertainty in the estimated parameters of models (21) and (22). Upper and lower bounds of a 95 percent confidence interval for the percentage difference are reported.

1). In these cases, the default rate did not react visibly. However, during the more lengthy GDP growth slowdowns of 1991–93 and 2000–03, the default rate approximately doubled. It appears that the default rate only reacts substantially to long-lasting GDP growth developments.

The average of the 2.5 percent worst-case scenarios is a lot worse than the average of the zero GDP growth scenarios. For most sectors, the 2.5 percent worst-case scenario is three to four times as bad as the zero GDP growth scenario. For the “construction,” “catering,” and “other” sectors, which are relatively insensitive to GDP growth, the worst-case scenario is more than nine times as bad. Figure 2 plots the average economy default rates. The figure clearly shows that the 2.5 percent worst-case scenario is a lot worse than the zero GDP growth scenario.

Table 7. Zero GDP Growth Scenario

	Lower Bound	% Difference Default Rate	Upper Bound
Industry and Mining	5%	10%	16%
Construction	-1%	4%	10%
Trade and Repair, Consumer Products	1%	6%	12%
Catering	1%	7%	12%
Transport, Storage, and Communications	7%	12%	18%
Financial	10%	15%	21%
Rental and Corporate Services	6%	12%	18%
Other	-1%	4%	10%
Economy	2%	8%	13%
Notes: This table shows the percentage difference between the average generated $\bar{p}d_{2007,i}$ given the zero GDP growth scenario and the base scenario. The percentage difference is an estimate because of uncertainty in the estimated parameters of models (21) and (22). Upper and lower bounds of a 95 percent confidence interval for the percentage difference are reported.			

7. Discussion and Conclusion

The focus of this paper is to (i) assess which macroeconomic variables are related to the default behavior of Dutch firms and (ii) assess the default behavior given two quarters of zero GDP growth in the third and fourth quarters of 2006. We will discuss these two aspects in turn.

To assess which macroeconomic variables are related to the default behavior of Dutch firms, we studied GDP growth, interest rates, exchange rates, stock market returns and volatility, and oil prices. A convincing negative relation with the default rate and GDP growth was found. The relation with the oil price is also significant in several sectors. Furthermore, there is some indication of a positive relation with the (short-term) interest rate for the “construction” sector and with the (logarithm of the real) exchange rate for the

“transport, storage, and communications” sector, the “financial services” sector, and the “rental and corporate services” sector. No relation with stock market return and volatility was found. Remarkably, for the interest rate, exchange rate, and oil price, it is not the change but the level of the variables that is significant to the default rate.

For the overall economy, the relations with the default rate and the macroeconomic variables are stable through time. The macroeconomic relations with the sector default rates are mostly unstable, except for the oil price. A reason for the instability is that results amongst sectors can differ according to the growth opportunities of the sector of economic activity to which firms belong, the sector’s degree of internationalization, and its dependence on other sectors.

The first lag of the logit default rate has a highly significant coefficient. This implies that the effect of persistent macroeconomic shocks gradually increases over time. Without taking account of the lagged default rate, the macroeconomic variables explain, on average, about a fifth of the variance of the default rate. A latent factor affecting all sectors explains about 30 percent, and the rest is explained by sector-specific disturbances. Other literature mainly confirms the results on GDP growth and, to a limited extent, interest and exchange rates. Furthermore, the stock market is often found to be related, but always for firms listed on a stock exchange.

To assess what the behavior would be for another country, we ran the model with Austrian data. We can conclude that each country has its own dynamics. For the Dutch data, the results showed a relationship between the macro economy and the default rate. For Austrian data, the model showed no significant relationship between the macroeconomic variables and the default rate.

The effect in Dutch default behavior given two quarters of zero GDP growth in the third and fourth quarters of 2006 is that in 2007, there is a 4–15 percent rise of the default rate, depending on the sector. Historic recessions of similarly short duration are in accordance with these small numbers: the default rate does not visibly react to short recessions. However, historic recessions lead to higher long-run effects of more persistent recessions. It can be concluded that a short recession of two quarters does not influence the default rate significantly.

Concluding, a stress-test scenario of two quarters of zero GDP growth, as required by Basel II, might underestimate the true risk. We would advise to do the stress test with a more severe scenario to gain a better estimate of the true risk.

Appendix 1. Literature

- Couderc and Renault (2005) estimate the default rate of firms listed in the S&P index over the period 1981–2003 by means of a continuous time model. They also investigate lags of variables. They show that past economic conditions are of prime importance in explaining probability changes: current shocks and long-term trends jointly determine default probabilities. Significant variables are the stock market return and volatility, the term and credit spread, and GDP growth.
- Carling et al. (2002) estimate a duration model to explain the survival time to default for borrowers in the business loan portfolio of a major Swedish bank over the period 1994–2000. The model takes both firm-specific characteristics and the prevailing macroeconomic conditions into account. The output gap, the yield curve, and consumers' expectations of future economic development have significant explanatory power for the default risk of firms.
- Koopman and Lucas (2005) estimate the default rate of U.S. firms over the period 1933–97 for a general class of periodic unobserved-components time-series models with stochastic trend, seasonal, and cycle components. They take into account the correlation between stochastic cycle effects. GDP growth is found to be significant.
- Fiori, Foglia, and Ianotti (2006) find that the explanatory power of macro factors for defaults is relatively limited, but that a residual cross-section correlation of default rates suggests the presence of contagion effects from the impact of sector-specific risk on the default rates of other sectors.
- Jakubik (2006) estimates the default rate of Finnish firms over the period 1988–2003 by means of a linear vector autoregressive model. Jakubik found GDP growth to be significant, interest rates to be somewhat insignificant, and the exchange rate to be significant for the trading sector.

- Hamerle, Liebig, and Scheule (2004) estimate the default rate of German firms over the period 1991–2000 by means of a discrete time model, also including firm-specific variables. They show that systematic variables make a latent factor insignificant. They find that the inclusion of variables that are correlated with the business cycle improves the forecasts of default probabilities. Asset and default correlations depend on the factors used to model default probabilities. They conclude that correlations and default probabilities should always be estimated simultaneously. GDP growth is found to be significant.
- Lucas et al. (2006) study the relation between the credit cycle and macroeconomic fundamentals using rating transition and default data of U.S. corporations from Standard and Poor's over the period 1980–2005. They conclude that many of the variables thought to explain the credit cycle turn out to be insignificant. The main exceptions are GDP growth and, to some extent, stock market returns and stock market return volatilities. Their economic significance appears low, however.
- Kavvathas (2001) assesses the potential of conditioning on economy-wide state variables in improving the forecasting of the Credit Rating Transition Probability Matrix over the period 1981–98. He finds that an increase in nominal short-term, long-term, and real interest rates, a lower equity return, and a higher equity return volatility are associated with higher relative downgrade intensities.
- Vlieghe (2001), using UK data over the period 1975–99, suggests that the substantial rise in the number of defaults during the recession in the early 1990s mainly reflected deteriorating company finances, including a marked buildup of indebtedness. In the subsequent recovery, however, rising GDP relative to trend and other macroeconomic factors seem to have had greater explanatory power than changes in company finances in accounting for the fall in the rate of corporate liquidations to its currently low level.
- Virolainen (2004), using Finnish data over the period 1986–2003, finds a significant relationship between corporate sector default rates and macroeconomic factors, including GDP, interest rates, and corporate indebtedness.

Appendix 2. Results

Table 8. Estimated Coefficients of the Parsimonious Specification

	Intercept	First Lag	GDP Growth	Short Rate	ln(ER)	ln(Oil Price)	Systematic s.d.	Idiosyncratic s.d.
Industry and Mining	−3.41***	.67***	−2.75***	.97*	.28	.06	.10***	.06***
Construction	−1.77	.77***	−1.24	1.52***	.05	.03	.09***	.07***
Trade and Repair, Cons. Products	−3.39***	.78***	−1.25**	.71	.40*	.07	.06***	.07***
Catering	−4.84**	.84***	−.50	.30	.71*	.16**	.13***	.09***
Transport, Storage, and Comm.	−10.27***	.50***	−3.22***	−.58	1.48***	.18***	.12***	.08***
Financial Services	−4.79**	.83***	−2.89***	−.18	.78**	.12**	.10***	.07***
Rental and Corp. Services	−4.71***	.69***	−3.07***	1.18**	.55**	.08*	.07***	.08***
Other	−2.82	.59***	−1.15	.33	−.10	.06	.15***	.06***
Economy	−2.61**	.84***	−1.40***	.71	.32*	.05*	.08***	.00
Pooled	−3.23***	.79***	−1.77***	.78*	.32*	.07**	.10***	.07***
P-Value Equal Coefficients	.0003	.0000	0.421	.0011	.0001	.1305	.0000	.1153

Notes: Model (10) is estimated with the macroeconomic variables selected in section 5 included in z_t . All estimated parameters are reported. Pooled results are obtained by estimating the sector model under the restriction $\beta_{1,j} = \dots = \beta_{8,j}$ or $\sigma_{j,1} = \dots = \sigma_{j,8}$ for certain j while allowing the other parameters to differ per sector. These restrictions are tested and the p-values are reported. The 1, 5, and 10 percent levels are denoted ***, **, and *, respectively.

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How to Find Plausible, Severe, and Useful Stress Scenarios*

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We give a precise operational definition to three requirements the Basel Committee on Banking Supervision specifies for stress tests: plausibility and severity of stress scenarios as well as suggestiveness of risk-reducing actions. The basic idea of our approach is to define a suitable region of plausibility in terms of the risk-factor distribution and search systematically for the scenario with the worst portfolio loss over this region. One key innovation compared with the existing literature is the solution of two open problems. We suggest a measure of plausibility that is not prone to the problem of dimensional dependence of maximum loss and we derive a way to consistently deal with situations where some but not all risk factors are stressed. We show that setting the nonstressed risk factors to their conditional expected value given the value of the stressed risk factors maximizes plausibility among the various approaches used in the literature.

JEL Codes: G28, G32, G20, C15.

1. Introduction

The current regulatory framework of the Basel Committee on Banking Supervision (2005) requires banks to perform stress tests which meet three requirements: *plausibility* of stress scenarios, *severity* of

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stress scenarios, and *suggestiveness of risk-reducing action*.¹ How do we find stress scenarios which are at the same time plausible, severe, and suggestive for the design of risk-reducing action? Our paper gives a systematic answer to this question. We suggest a method that can be implemented for a wide class of stress-testing problems usually encountered in practice. We illustrate the method and the issues in the context of an example: stress tests for a portfolio of adjustable-rate loans in home and foreign currency.

The quality of a stress test crucially depends on the definition of stress scenarios. Defining stress scenarios is a thought experiment. It is a counterfactual exercise where a risk manager tries to imagine what adverse or even catastrophic events might strike his or her portfolio. Such a thought experiment is prone to two major pitfalls: consideration of implausible scenarios and neglect of plausible scenarios. Thinking about scenarios requires one to imagine situations that have perhaps not yet occurred but might occur in the future. Bias toward historical experience can lead to the risk of ignoring plausible but harmful scenarios which did not yet happen in history. This creates a dangerous blind spot. If the imagination of a stress tester puts excessive weight on very unplausible scenarios, management faces an embarrassing decision: should one react to alarming results of highly implausible stress scenarios? Our method allows a precise trade-off between plausibility and severity. In this way we can ensure that in a model of portfolio risk, no harmful but plausible scenarios are missed. Furthermore, our stress-test method suggests ways to reduce risk if desired.

We analyze the problem of finding extreme but plausible scenarios in a classical quantitative risk-management framework. A portfolio of financial instruments, say a portfolio of loans, is given. The value of each loan at some given horizon in the future is described by the realization of certain risk factors. In the case of a loan portfolio,

¹The respective references in Basel Committee on Banking Supervision (2005) are as follows: “Quantitative criteria should identify plausible stress scenarios to which banks could be exposed” (par. 718 (LXXIX)); “A bank should also develop its own stress tests which it identifies as most adverse based on the characteristics of its portfolio” (par. 718 (LXXXIII)); and “Qualitative criteria should emphasize that two major goals of stress testing are to evaluate the capacity of the bank’s capital to absorb potential large losses and to identify steps the bank can take to reduce its risk and conserve capital” (par. 718 (LXXIX)).

for example, these risk factors will comprise the macroeconomic environment (because of its impact on the payment ability and thus on the solvency of borrowers) and market factors like interest rates (or exchange rates in the case of foreign currency loans) but also idiosyncratic factors that influence a borrower's solvency. The uncertainty about the realization of risk factors is described by a risk-factor distribution that is estimated from historical data. Plausibility is captured by specifying how far we go into the tails of the distribution in our search for stress scenarios. The severity of scenarios is maximized by systematically searching for the worst case, the maximum portfolio loss, in a risk-factor region of given plausibility.

This general idea of looking at extreme scenarios has been formulated in the literature before. It is informally discussed by Čihák (2004, 2007). More formally, the idea is discussed in Studer (1997, 1999) and in Breuer and Krenn (1999). This literature leaves, however, two open issues that seem technical at first sight but are of great practical relevance: the problem of partial scenarios and the problem of dimensional dependence of maximum loss.

The partial-scenario problem comes from the situation that a portfolio may depend on many risk factors, but the modelers are interested in stressing not all but only a few factors at a time. For example, in a loan portfolio we are often interested in stress scenarios for particular variables: a certain move in the exchange rate or a particular drop in GDP. How do we deal with the other risk factors consistently? Do we leave them at their last observed value, at some average value, or should we condition on the stressed macro factor and, if so, how? We show that the way to deal with the partial-scenario problem that maximizes plausibility is to set the nonstressed systematic risk factors to their conditional expectation for the given value of the stressed factors. We show furthermore that this has the same plausibility as the computationally more intensive full-loss simulation from the conditional stress distribution as in Bonti et al. (2005).

If we look for maximum loss in a risk-factor region of given plausibility, we want the maximum loss not to depend on the inclusion of irrelevant risk factors or risk factors that are highly correlated with factors already included in the analysis. The plausibility measures that were used in the previous literature (see Studer 1997, 1999) suggested to define plausibility regions as regions with a

given probability mass. This definition of plausibility has an undesirable property, known as the problem of dimensional dependence of maximum loss. To get an intuitive understanding of the problem, consider an example from Breuer (2008). We have a bond portfolio with risk factors consisting of two yield curves in ten foreign currencies. One risk manager chooses to model the yield curve with seven maturity buckets and another risk manager uses fifteen buckets. In this case the first risk manager uses 150 risk factors in the analysis and the second manager uses 310. As plausibility region, both of them choose an ellipsoid of mass 95 percent. Breuer (2008) shows that the second risk manager will calculate a maximum loss that is 1.4 times higher than the maximum loss calculated by the first risk manager. This is problematic because both of them look at the same portfolio and use the same plausibility level. We suggest an approach to define plausibility that does not have this problem.

The paper is organized as follows: In section 2 we define a quantitative measure of plausibility and explain why it is not subject to the dimensional dependence problem. We discuss how to deal with the problem of partial scenarios and explain the technique of worst-case analysis. We also discuss how measures for risk-reducing actions can be deduced from the stress test. In section 3 we analyze an example of a portfolio of foreign currency loans that illustrates the practical applicability as well as the potential improvement compared with a standard stress-testing procedure. Finally, section 4 concludes.

2. Finding Scenarios That Are Plausible, Severe, and Suggestive of Counteraction

We consider the problem of stress testing a loan portfolio. The value of each position in the portfolio depends on n systematic risk factors $\mathbf{r} = (r_1, \dots, r_n)$ and on m idiosyncratic risk factors $\epsilon_1, \dots, \epsilon_m$. In our approach, we have to restrict the distribution of the systematic risk factors $\mathbf{r}$ to a class called the elliptical distributions. For the definition and some basic facts about elliptical distributions, we refer to the standard work of Fang, Kotz, and Ng (1987). For our purpose it is enough to note that the standard distributions used in classical risk-management problems are in fact from this class. We

denote the covariance matrix and expectations of the distribution of $\mathbf{r}$ by Cov and $\boldsymbol{\mu}$. The distribution of the idiosyncratic risk factors may be arbitrary.

2.1 Plausible Scenarios

In a stress test of a loan portfolio, we imagine extreme realizations of one or more of the systematic risk factors. How would we quantify the plausibility of this thought experiment?

An intuitive approach could be to compare the extreme realization of a risk factor with its average. Intuitively, the further we are away from this average value, the less plausible the stress scenario becomes. The distance should be measured in standard deviations. For multivariate moves, the plausibility should depend additionally on the correlations. A multivariate move which is in agreement with the correlations is more plausible than a move against the correlations.

A statistical concept that formalizes these ideas is the so-called Mahalanobis distance given by

$$\text{Maha}(\mathbf{r}) := \sqrt{(\mathbf{r} - \boldsymbol{\mu})^T \cdot \text{Cov}^{-1} \cdot (\mathbf{r} - \boldsymbol{\mu})}.$$

The Mahalanobis distance is simply the distance of the test point $\mathbf{r}$ from the center of mass $\boldsymbol{\mu}$ divided by the width of the ellipsoid in the direction of the test point. Intuitively, $\text{Maha}(\mathbf{r})$ can be interpreted as the number of standard deviations of the multivariate move from $\boldsymbol{\mu}$ to $\mathbf{r}$. Maha takes into account the correlation structure and the standard deviations of the risk factors.

In contrast to the previous literature, we define plausibility directly in terms of $\text{Maha}(\mathbf{r})$: a high value of Maha implies a low plausibility of the scenario $\mathbf{r}$. Earlier work defined plausibility in terms of the *probability mass* of the ellipsoid of all scenarios of equal or lower Maha; see Studer (1997, 1999) or Breuer and Krenn (1999). This approach creates the problem of dimensional dependence. If one defined plausibility in terms of the ellipsoid containing some fixed probability mass, then the maximum loss would depend on the number of systematic risk factors, which is to some degree arbitrary. In our approach this problem does not occur.

2.2 Partial Scenarios

Typically, portfolios are modeled with hundreds or thousands of risk factors. Stress scenarios involving the full plethora of risk factors are hardly tractable numerically and overwhelmingly complex to interpret. A feasible answer to this problem is to design partial stress scenarios, which involve only a handful of risk factors. How should the other risk factors be treated?

Kupiec (1998) discussed four different ways to deal with the risk factors not fixed by some partial scenario:

- (i) The other systematic risk factors remain at their *last observed* value.
- (ii) The other macro risk factors take their *unconditional expectation* value.
- (iii) The other systematic risk factors take their *conditional expected value* given the values of the fixed risk factors. Denote by $\mathbf{r}_C$ the resulting vector of values of the systematic risk factors.
- (iv) The other systematic factors are not fixed but are distributed according to the conditional distribution given the values of the fixed risk factors. Denote by $\mathbf{r}_D$ the vector of values of the fixed systematic risk factors.

Our first result suggests a choice between these alternatives based on our concept of plausibility. The result says that the specification of partial scenarios as in method (iii) or (iv) both maximize plausibility. In the literature on stress testing of loan portfolios, Bonti et al. (2005) have suggested to use method (iv). This is indeed an approach that maximizes plausibility. From our result, we learn that we can achieve an equivalent plausibility by using the computationally more efficient approach (iii).

We state this result more formally in the following.

PROPOSITION 1. *Assume the distribution of systematic risk factors is elliptical, with density strictly decreasing as a function of Maha. Then:*

1. $\text{Maha}(\mathbf{r}_C) = \text{Maha}(\mathbf{r}_D)$.
2. *This is the maximal plausibility which can be achieved among all macro scenarios which agree on the fixed risk factors.*

A proof of this proposition is in the working paper Breuer et al. (2009b). This proposition is of high practical relevance. It is the basis of partial-scenario analysis. It implies that two choices of macro stress distributions are preferable, namely (iii) or (iv). Assigning to the nonfixed risk factors other values than the conditional expected values given the fixed risk factors leads to less plausible macro stress scenarios.

2.3 *Severe Scenarios*

An important disadvantage of stress testing with hand-picked scenarios is the danger of ignoring harmful but plausible scenarios. This may create an illusion of safety. A way to overcome this disadvantage is to search systematically for those macro scenarios in some plausible admissibility domain which are most harmful to the portfolio. By searching systematically over admissible domains of plausible macro scenarios, one can be sure not to ignore any harmful but plausible scenarios. This is our approach to construct a stress test: find the relevant scenarios which are most harmful yet above some minimal plausibility threshold. This problem can be formulated as an optimization problem which can be solved numerically by using an algorithm of Pistovčák and Breuer (2004).

The admissibility domain is determined by our concept of plausibility. It contains all scenarios with $\text{Maha}(\mathbf{r})$ below a threshold k :

$$\text{Ell}_k := \{\mathbf{r} : \text{Maha}(\mathbf{r}) \leq k\}.$$

Geometrically, this domain is an ellipsoid whose shape is determined by the covariance matrix of the systematic risk factors.

Partial scenarios do not specify a unique portfolio value but just a distribution, namely the distribution of portfolio values conditional on the values of the risk factors fixed by the scenario. In order to measure the severity of scenarios, one needs to quantify the severity of the corresponding conditional portfolio value distribution. In this paper we use the expectation value, although other risk measures could be used as well. Thus we call a partial scenario severe if it has a low conditional expected profit (CEP). To sum up, our stress-testing method amounts to solving the following optimization problem:

$$\min_{\mathbf{r} \in \text{Ell}_k} \text{CEP}(\mathbf{r}).$$

The difference between the lowest CEP in the admissibility domain and the CEP in the expected scenario is the maximum expected loss in the admissibility domain. This concept of maximum loss overcomes the problem of dimensional dependence that we mentioned in the introduction. Maximum expected loss over the admissibility domain Ell_k is not affected by excluding or including macro risk factors that are irrelevant to the portfolio value (see Breuer 2008).

What is the advantage of this worst-case search over standard stress testing? First, it achieves a controlled trade-off between plausibility and severity of scenarios. If we want to get more severe scenarios, we choose a higher k and get less plausible worst-case scenarios. If we want to get more plausible scenarios, we choose a lower k and get less severe worst-case scenarios. Second, it overcomes the historic bias by considering all scenarios that are plausible enough. In this way we can be sure not to miss scenarios that are plausible but did not yet happen in history. Thirdly, worst-case scenarios reflect portfolio-specific dangers. What is a worst-case scenario for one portfolio might be a harmless scenario for another portfolio. This is not taken into account by standard stress testing. Portfolio-specific dangers suggest possible counteraction to reduce risk if desired.

2.4 Scenarios Suggesting Risk-Reducing Action

Risk-reducing action is suggested by identifying the key risk factors that contribute most to the expected loss in the worst-case scenario. We define key risk factors as the risk factors with the highest maximum loss contribution (MLC). The loss contribution (LC) of risk factor i to the loss in some scenario $\mathbf{r}$ is

$$LC(i, \mathbf{r}) := \frac{CEP(\boldsymbol{\mu}) - CEP(\mu_1, \dots, \mu_{i-1}, r_i, \mu_{i+1}, \dots, \mu_n)}{CEP(\boldsymbol{\mu}) - CEP(\mathbf{r})}, \quad (1)$$

if $CEP(\mathbf{r}) \neq CEP(\boldsymbol{\mu})$. $LC(i, \mathbf{r})$ is the loss if risk factor i takes the value it has in scenario $\mathbf{r}$, and the other risk factors take their expected values $\boldsymbol{\mu}$, as a percentage of the loss in scenario $\mathbf{r}$. In particular, one can consider the worst-case scenario, $\mathbf{r} = \mathbf{r}^{WC}$. In this case the loss contribution of some risk factor i can be called the maximum loss contribution:

$$MLC(i) := LC(i, \mathbf{r}^{WC}). \quad (2)$$

$MLC(i)$ is the loss if risk factor i takes its worst-case value and the other risk factors take their expected values, as a percentage of maximum loss.

The maximum loss contributions of the macro risk factors in general do not add up to 100 percent. Sometimes the sum is larger; sometimes it is smaller. If this sum is equal to one, the loss in the scenario is exactly equal to the sum of losses from individual risk-factor moves. This happens if and only if the risk factors do not interact.

PROPOSITION 2. *Assume CEP as a function of the macro risk factors has continuous second-order derivatives. The loss contributions of the risk factors add up to 100 percent for all scenarios $\mathbf{r}$,*

$$\sum_{i=1}^n LC(i, \mathbf{r}) = 1,$$

if and only if CEP is of the form

$$CEP(r_1, \dots, r_n) = \sum_{i=1}^n g_i(r_i). \quad (3)$$

This is the case if and only if all cross-derivatives of CEP,

$$\frac{\partial^2 CEP(\mathbf{r})}{\partial r_i \partial r_j} = 0,$$

vanish identically for $i \neq j$.

For the proof, we refer to the working paper of Breuer et al. (2009b). This characterization has a substantial practical relevance. The sum of loss contributions measures the role of interaction of systematic risk for the portfolio value. If the sum is larger than one, the interaction between risk factors is positive. The total loss in the scenario is smaller than the sum of losses from individual risk-factor moves.

Most dangerous is the case of negative interaction between risk factors. If the sum is smaller than one, the total loss in the scenario is larger than the sum of losses from individual risk-factor moves. The harm of the scenario cannot be fully explained by individual risk-factor moves. The simultaneous move of some risk factors causes

harm on top of the single risk-factor moves. In this case it will be necessary to consider maximum loss contributions not of single risk-factor moves but of pairs or even of larger groups of risk factors.

A consequence of this insight outside of the stress-testing problem is that it reveals a weakness in current regulatory thinking. Analyzing portfolio risk along the categories market and credit risk, and determining risk capital based on the aggregation of these separately calculated risk numbers, may in fact underestimate the true portfolio risk because it ignores the risks stemming from simultaneous moves in market and credit-risk factors. For a detailed discussion of this problem see Breuer et al. (2009a).

Possible risk-reducing action can be designed with knowledge of the key risk factors. One strategy could be to buy hedges that pay off exactly when the key risk factors take their worst-case value. Another, more comprehensive but also more expensive strategy is to buy hedges that neutralize the harm done not just by the worst-case moves of the key risk factors but by all moves of the key risk factors. For the example of the foreign currency loan portfolios discussed in the next section, this strategy is demonstrated in Breuer et al. (2008).

3. Application: Stress Testing a Portfolio of Foreign Currency Loans

We now illustrate the concepts and their quantitative significance in an example: a stress test for a portfolio of adjustable-rate loans in home (EUR) or foreign (CHF) currency. In the current downturn, the additional risk of foreign currency plays a major role in some CEE economies. Our sample portfolio consists of loans to 100 borrowers in the rating class B+, corresponding to a default probability of $p_i = 2$ percent, or in rating class BBB+, corresponding to a default probability of $p_i = 0.1$ percent. At time 0, in order to receive the home currency amount $l = \text{€}10,000$, the customer of a foreign currency loan takes a loan of $le(0)$ units in a foreign currency, where $e(0)$ is the home currency value of the foreign currency at time 0. The bank borrows $le(0)$ units of the foreign currency on the interbank market. After one period, at time 1, which we take to be one year, all the loans expire and the bank repays the foreign currency at the interbank market with an interest rate r_f (e.g., LIBOR),

and it receives from the customer a home currency amount that is exchanged at the rate $e(1)$ to the foreign currency amount covering repayment of the principal plus interest rolled over from four quarters, plus a spread s . So the borrower's payment obligation to the bank at time 1 in home currency is

$$o_f = l(1 + r_f)E + s_f l E \quad (4)$$

$$o_h = l(1 + r_h) + s_h l \quad (5)$$

for the foreign and home currency loan $E := e(0)/e(1)$ is the exchange rate change between times 0 and 1. The first term on the right-hand side is the part of the payment that the bank uses to repay its own loan on the interbank market. The second term is profits remaining with the bank. For all loans in the portfolio, we assume they expire at time 1. The model can be extended to a multiperiod setting allowing for loans not maturing at the same time and requiring payments at intermediate times.

In order to evaluate idiosyncratic and systematic risk of a portfolio of such loans, we use a one-period structural model specifying default frequencies and losses given default endogenously. For details of the model, we refer to Breuer et al. (2008). The basic structure of the model is given by the payment obligation distribution derived from the payment obligation function (4) and a log-normal payment ability distribution, which involves log-normally distributed idiosyncratic changes and an additional dependence of the mean one future GDP changes. (Pesaran, Schuermann, and Treutler 2005 use a model of this type for the returns of firm value.) Each customer defaults in the event that their payment ability at the expiry of the loan is smaller than their payment obligation. In the case of default, the borrower pays what he or she is able to pay. The difference to the payment obligation first is lost profit and then loss for the bank.

The spread s_h (resp. s_f) and the variance of the idiosyncratic payment ability changes are determined jointly in a calibration procedure. The first calibration condition ensures that the model default probability coincides with the default probability determined in some external rating procedure. The second calibration condition ensures that expected profit from each loan reaches some target level of €160, which amounts to a return of 20 percent on a regulatory capital of 8 percent. Both calibration conditions depend on the spread s and the variance of the idiosyncratic payment ability changes.

The systematic risk factors entering the portfolio valuation are GDP, the home interest rate r_h and the foreign interest rate r_f , and the exchange rate change E . The probability law driving these risk factors is modeled by a time-series model that takes account of economic interaction between countries and regions. Estimating the parameters of this model, we can simulate scenarios for the systematic risk factors. For details of this model, known in the literature as the global VAR model, see Pesaran, Schuermann, and Weiner (2001), Pesaran, Smith, and Smith (2005), Garrett, Pesaran, and Shin (2006), and Déés et al. (2007).

The profit distribution was calculated in a Monte Carlo simulation by generating 100,000 scenario paths of four steps each. The resulting distribution of risk factors after the last quarter, which is not normal, was used to estimate the covariance matrix of one-year macro risk-factor changes. In each macro scenario, defaults of the customers were determined by 100 draws from the idiosyncratic changes in the payment ability distribution. From these we evaluated the profit distribution at the one-year time horizon.

3.1 Hand-Picked Versus Systematic Stress Tests

Let us compare the severity of the hand-picked scenario “GDP shrinks by 3 percent,” which is a 5.42σ event, with the worst-case scenario of the same plausibility. Conditional expected profits (CEP) for the standard scenario “GDP -3 percent and other risk factors at their conditional expected value” and of worst-case scenarios of the same plausibility are shown in table 1.

We observe that for all portfolios the conditional expected profits are considerably lower in the worst-case scenarios than in the hand-picked GDP scenario. This is evidence of the danger that lies in relying solely on hand-picked scenarios. Expected profits in this rather extreme hand-picked GDP scenario are only moderately lower, namely by amounts between €129 and €1,751 on a loan portfolio worth €1 million giving an unconditional expected profit of €16,000. These moderate profit reductions in such an extreme scenario might provide a feeling of safety. But this is an illusion. There are other scenarios out there that are equally plausible but much more harmful. There are scenarios that reduce expected profit by amounts between €374 (resp. €2,709) for the home currency loans,

Table 1. Comparison of the Severity of the Hand-Picked Scenario with the Worst-Case Scenario of the Same Plausibility

Scenario	Maha	CEP
Foreign B+		
Expected	0	16,001
GDP −3%	5.42	15,950
Worst Case	5.42	−98,101
Foreign BBB+		
Expected	0	15,999
GDP −3%	5.42	15,870
Worst Case	5.42	−95,591
Home B+		
Expected	0	16,000
GDP −3%	5.42	14,249
Worst Case	5.42	13,291
Home BBB+		
Expected	0	16,000
GDP −3%	5.42	15,811
Worst Case	5.42	15,626

and by €114,101 (resp. €111,591) for the foreign currency loan portfolios. These huge losses of roughly 11 percent are higher than the total regulatory capital of 8 percent for the loan portfolio.

3.2 Key Risk Factors and Risk-Reducing Actions

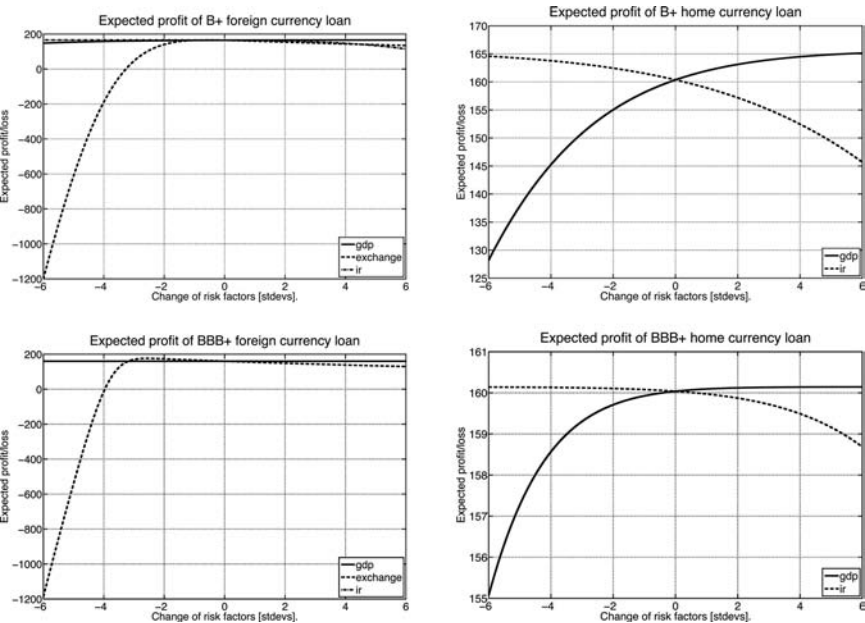
What is a worst-case scenario for one portfolio might be a harmless scenario for another portfolio. This is not taken into account by standard stress testing. Stress testing is relevant only if the choice of scenario takes into account the portfolio. In a systematic way, this is done by worst-case search.

Key risk factors are ones with highest maximum loss contributions (MLCs). The worst-case scenarios, together with the MLC for each risk factor, are given in table 2 for different sizes of the admissibility domain. For each scenario, the risk factors with the highest MLCs are printed in bold face. These results identify which risk factor is key for which portfolio.

- For the foreign currency loan portfolio, the exchange rate is clearly the key risk factor. This becomes apparent from table 2. In the worst-case scenario, the FX rate alone contributes between 65.3 percent and 100 percent of the losses in the worst-case scenarios; the other risk factors contribute less than 1 percent. This indicates that the FX rate is the key risk factor of the foreign currency loan portfolio.
- For the home portfolio, GDP is the key risk factor. The moves in GDP alone contribute between 46.8 percent and 70.0 percent of the losses in the worst-case scenarios. The MLC of the home interest rate is comparatively small. The negative interaction between GDP and interest rate moves explains about one-third of the worst-case loss—more for larger k , less for smaller k .
- There is another interesting effect. The dependence of expected profits of foreign currency loans on the CHF/€ rate is not only nonlinear but also not monotone. For the BBB+ FX loan portfolio (bottom left plots in figure 1), focusing on changes smaller than 4σ it becomes evident that a small increase in the exchange rate has a positive influence on the portfolio value, but large increases have a very strong negative influence. Correspondingly, in table 2, if we restrict ourselves to small moves (Maha smaller than 4σ), the worst-case scenario is in the direction of increasing exchange rates, but if we allow larger moves, the worst-case scenario is in the direction of decreasing exchange rates. This effect also shows up in the worst macro scenarios of table 2. The reason for this nonmonotonicity is that a small decrease in the FX rate increases the EUR value of spread payments received. For larger moves of the FX rate, this positive effect is outweighed by the increases in defaults due to the increased payment obligations of customers. For the bad-quality B+ portfolios, the positive effect of a small FX rate decrease persists only up to a maximal Maha radius of $k = 2$.

The diagnosis that the FX rate is the key risk factor for the foreign currency loans and GDP is the key risk factor for the home currency loans is confirmed by the right- and left-hand plots in figure 1, which show the expected profits in dependence of single

Figure 1. Key Risk Factors of Foreign and Home Currency Loans



Notes: This figure shows expected profit or loss of a single foreign (left) and home currency (right) loan as a function of changes of the macro risk factors with other macro risk factors fixed at their expected values. At the top are B+ loans. At the bottom are BBB+ loans. The left-hand plots show that for the foreign portfolio the exchange rate is the key risk factor. We also observe the negative effect of small foreign currency depreciations, which is particularly pronounced for the BBB+ portfolio. The right-hand plots show that for the home portfolio GDP is the key risk factor. Note the different scales of the two plots.

macro risk-factor moves, keeping the other macro risk factors fixed at their expected values. Note the different scales of the two plots. Expected losses of the FX loan are considerably larger than for the home currency loan. This plot also shows that expected profits of both loan types depend nonlinearly on the relevant risk factors. The profiles of expected profits in figure 1 resemble those of short options. A home currency loan behaves like a short put on GDP together with a short call on the home interest rate. From the point of view of the bank, a foreign currency loan behaves largely like a short call on the FX rate.

One could ask why the effort to search for worst-case scenarios is necessary to identify key risk factors. Wouldn't it be easier to read the key risk factors from the plots in figure 1? This would be true if losses from moves in different risk factors *added up*. But for certain kinds of portfolios, the worst case is a *simultaneous* move of several risk factors—and the loss in this worst case might be considerably worse than adding up the losses resulting from moves in single risk factors. This is the message of proposition 2. The effects of simultaneous moves are not reflected in figure 1, but they do show up in the worst-case scenario.

As an example, consider a B+ home currency loan and assume we are restricting ourselves to moves with Maha smaller than $k = 6$. From table 2 we see that the MLC of the two risk factors sum up to 62.0 percent + 9.9 percent = 71.9 percent, which is considerably lower than 100 percent. This indicates that the loss of a joint move is considerably larger than the sum of losses of individual risk-factor moves. This is not reflected in figure 1, which only displays the effects of single risk-factor moves.

The same occurs for foreign currency loans. They show a dangerous interaction of market and credit risk. At $k = 4$ the exchange rate has an MLC of 65.3 percent, the interest rate has an MLC of 0.4 percent, and GDP has an MLC of 0.1 percent, which is a total of 65.8 percent instead of 100 percent. Single risk-factor moves leave about 35 percent of the maximum loss unexplained. The reason is that adverse exchange rate moves drive up payment obligations. This increases default probabilities and losses given default.

The identification of key risk factors suggests risk-reducing counteractions. Knowing that the exchange rate is the key risk factor for FX loans, one can plot the behavior of CEP in dependence of exchange rate moves, as in the left-hand plots of figure 1. Breuer et al. (2008) show how FX derivatives can be used to construct hedges reducing the exchange rate risk of foreign currency loans. It turns out that FX options can be used to virtually eliminate the dependence of expected loss on exchange rates—at some fixed level of interest rates and other macroeconomic factors. But the hedge is not perfect: Firstly, it cannot fully remove dependence of expected losses on exchange rates at other levels of interest rates, and secondly, it can bring to zero only the expectation but not the variance of losses caused by adverse exchange rate moves.

4. Conclusion

The central message of our paper is that the three principles of the Basel Committee on Banking Supervision (2005) required for stress tests—*plausibility*, *severity* of stress scenarios, and *suggestiveness of risk-reducing action*—can be systematically implemented within a standard quantitative risk-management framework. In order to do so, we need a measure of plausibility that can be formulated using the probability distribution of the risk factors but that does not suffer from the dimensional dependence of maximum loss. We show that this concept of plausibility can be formulated by working with regions of a given Mahalanobis radius rather than working with regions of given probability mass. We need to replace the common practice of hand-picked scenarios with a systematic worst-case search over the given region of plausibility. Finally, we have to identify the key risk factors and their contributions to maximum loss. The key contribution to maximum loss may only be revealed if we take into account simultaneous moves in risk factors.

Our approach has three major advantages compared with standard stress tests. First, it ensures that no harmful scenarios are missed and therefore prevents a false sense of safety. Second, it does not analyze scenarios that are too implausible and would therefore jeopardize the credibility of stress analysis. Third, it allows for a portfolio-specific identification of key risk factors. We hope that the compatibility of our concepts with the standard quantitative risk-management framework used by practitioners makes the insights of this paper useful in practical stress-testing problems.

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