Interdependencies between Expected Default Frequency and the Macro Economy*

Per Åsberg Sommar and Hovick Shahnazarian
Financial Stability Department, Sveriges Riksbank

We use a vector error-correction model to study interdependencies between the aggregate expected default frequency (EDF) and the macroeconomic development. The model is used to forecast the median EDF. Evaluations of the model show that it yields low forecast errors and that the interest rate has the strongest impact on expected default frequency. Forecasts indicate that a lower short-term interest rate reduces the EDF and, in turn, risk premiums. This reduces the marginal cost for corporate investments and household consumption and stimulates growth through these two components of aggregate demand. At the same time, it imposes a downward pressure on the product prices of firms and thereby on inflation.

JEL Codes: C32, C52, C53, G21, G33.

1. Introduction

The turmoil that started in 2007 has led to a large-scale global crisis of confidence. The origin of this crisis can be found in reckless lending practices. It began when American financial institutions lent money to borrowers with weak credit histories. The high-risk subprime loans were sold on to a large extent by bundling them together with low-risk loans in what are known as structured credit products.

*We would like to thank Iman van Lelyveld, Tor Jacobson, Michael Andersson, Mattias Persson, Lars Frisell, and Martin W. Johansson for useful comments and suggestions. This paper has benefited considerably from comments by members of BIS Research Task Force subgroups on Stress Testing and Interaction of Market and Credit Risk. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Executive Board of Sveriges Riksbank. Authors contact: Financial Stability Department, Sveriges Riksbank, SE-103 37, Stockholm, Sweden. E-mail: per.asberg.sommar@riksbank.se and hovick.shahnazarian@riksbank.se.

Investors found it difficult to conduct their own risk assessments and largely relied on the classifications of credit-rating agencies. When the losses on subprime loans became unexpectedly large, genuine uncertainty arose about the value of the structured products and where losses associated with them would occur. Banks and other financial institutions became cautious about lending money to one another. They began to hoard liquidity to meet their own obligations and to avoid losses on loans to counterparties about whose status they felt uncertain in the wake of the general crisis. The cost for bank borrowing on the interbank market rose, causing other market interest rates to climb. As a result, the banks' borrowing costs have risen, access to loans has decreased, and companies and households are paying more for loans. In summary, financial institutions lending to high-credit-risk borrowers led to the confidence crisis in the financial system, which led to high risk premiums and high lending rates, which in this case have intensified the downswing in the global economy. An economic downswing will in turn increase the credit risk for all borrowers and especially for corporations. To alleviate the real economic effects of the financial crisis and to stress-test credit risk in banks, it is therefore necessary to have a good understanding of the interdependencies between credit risk and the macro economy.

Operations of banks are typically dominated by the granting of credit; therefore, credit risk is the largest individual risk in the banking system by far. In recent years, central banks and commercial banks have begun to use models that make it possible to more coherently probe the development of the banks' credit risks on the basis of different assumptions and events. A proper assessment of credit risk within banks requires some kind of a credit portfolio model. The idea behind a credit portfolio model is that the resilience of the banks is reflected in the size of the capital buffer they hold in relation to the credit risk measured in their loan portfolios. A portfolio model makes it possible to calculate the probability that loan losses of various sizes may arise in existing portfolios. Information regarding the composition of the portfolio, the probability of default, and recoveries is needed in order to calculate the risk in the loan portfolio. Two measures are often used to quantify the credit losses the banks may incur. One is a measure of the expected loss that indicates how much a bank can expect to lose in its current credit portfolio. This is calculated by multiplying the likelihood of default by exposure at default (exposure*LGD, where LGD is loss given default). The other is a measure of the size of the losses that can occur in addition to the expected losses (unexpected losses) and for which the bank must have capital cover (required risk capital). In this way, it is possible to study how changes in the credit quality of the banks' borrowers influence the credit risk in the banks' loan portfolios. The banks compensate themselves for both the expected and the unexpected loss through a risk premium on the price of loans in their regular operations. If there is an increase in the expected loss in the portfolio, this may mean that the banks' costs increase as a result of increased reserve funds. The banks hold a buffer to cover possible loan losses above those expected; let us call this the risk capital requirement. Loan loss distribution makes it possible for banks to calculate the size of this need, given a certain tolerance level. The unexpected loan loss—and thereby the need for risk capital—also affects the prices banks set for their loans, since holding capital entails a cost for the banks in the form of a return-on-investment requirement from the shareholders, and the banks must compensate themselves for this. The amount of capital the bank requires to cover unexpected losses depends on the loan loss distribution. The greater the probability of extreme outcomes—that is to say, outcomes that lie in the tail of the loss distribution—the greater is the need for risk capital.

The above discussion indicates that one of the most important variables for assessing credit risk in banking is the likelihood of default, which reflects the borrowers' credit quality. It is quite likely that macroeconomic variables play an important role in determining the direction of the future development of borrowers' credit quality.[1] Linking credit quality to the development of macroeconomic variables makes it possible to undertake endogenous forecasts for credit risk as well as scenario analyses where the credit risk for the banks can be appraised on the basis of the paths of different macroeconomic development curves. We present a model that creates a link between the assessment of credit risks and macroeconomic appraisals.

Several papers address the empirical relationship between fundamentals and default probabilities among companies. Chan-Lau (2006) offers a survey of this literature where macroeconomic-based

[1] See, e.g., Jacobson, Lindé, and Roszbach (2005).

models constitute one class of such models. These models study how default probabilities are affected by the state of the economy, and they can be divided into models that allow for feedback between default probabilities and explanatory economic variables and models that do not. Virolainen (2004) is an example of the latter category, while Alves (2005), Jacobson, Lindé, and Roszbach (2005), Pesaran et al. (2006), and Castrén, Dées, and Zehar (2007) are time-series models that do take feedback effects into account.

Jacobson, Lindé, and Roszbach present an empirical model that consists of a system made up of three blocks. The first is a vector autoregressive (VAR) model for the macroeconomic variables they consider. They include domestic output, inflation, the nominal interest rate, and the real exchange rate as endogenous variables in the VAR model. The foreign macro variables as well as the aggregated default frequency of incorporated firms enter the model exogenously. In the second block, they have a logit model for the default risk at the firm level, where the macroeconomic variables as well as various balance-sheet variables enter as regressors. The third block in their empirical model estimates the dependence of the balance-sheet variables included in the logit model on the macroeconomic variables. Alves (2005), Pesaran et al. (2006), and Castrén, Dées, and Zehar (2007) are models that allow for influence from explanatory economic variables on default probabilities, but not the other way around. They use VAR models for forecasting the development of the macroeconomic variables. These forecasts are then used in a satellite model for credit risk. Unlike Pesaran et al. (2006) and Castrén, Dées, and Zehar (2007), Alves (2005) takes into account that the likelihood of defaults and the macroeconomic variables display common trends. This is also the case in our model. Moreover, we also take feedback effects into account, which means that we allow for influence from macroeconomic variables on default probabilities, and the other way around.

In this paper, we depart from the literature in one important respect. We analyze the likelihood of defaults in the corporate sector using a forward-looking measure of the likelihood of defaults. One example of a structural credit-risk model of this kind is Credit Monitor (Moody's KMV), which offers a theoretically attractive model for calculating the empirical expected default frequency

(*EDF*) for individual companies.[2] Forward-looking measurement of the capacity of listed companies to make payments can be calculated using the market value of their assets in relation to the book value of their debts. The market value of equity is a function of the current value of all future cash flows the company can be expected to generate. General economic developments play an important role for the development of company cash flows. Therefore, it is reasonable to assume that *EDF* and the macroeconomic variables display common trends. We study the long-term relationship between expected default frequencies and macroeconomic development using a vector error-correction model (VECM), i.e., a VAR model that includes an error-correction term. The choice of a VECM can be justified by its ability to discern shared trends between series as well as allowing for feedback between default probabilities and explanatory macroeconomic variables. Estimates of the coefficients may be improved if the existence of shared trends in series is taken into account. Including shared trends becomes even more important when the model is estimated on high-frequency data, which is the case in this paper. A principal feature of cointegrated variables is that their time paths are influenced to the extent that any of these deviate from their long-run relationship. Moreover, the short-run dynamics must be influenced by the deviation from the long-run relationship.

This paper has three objectives. One is to explore whether the development over time of the aggregate EDF for listed companies provided by Credit Monitor can be explained by macroeconomic development.[3] Another objective is to make forecasts for future default frequencies as well as the macroeconomic variables. By this, we mean that the forecasts are made taking into account the feedback effects from the EDF on the macroeconomic variables and vice versa. The third objective is to conduct a stress test of the aggregate EDF, given different alternative macroeconomic development.

In section 1, we give an intuitive discussion of the impact of various macroeconomic factors on aggregate EDF. Section 2 presents the

[2]This model is based on Merton's approach for the evaluation of credit risk as refined by Vasicek and Kealhofer, which is why it is known as Kealhofer Merton Vasicek (KMV).

[3]Aggregate EDF is represented by the monthly median of EDF for individual Swedish nonfinancial companies in Moody's KMV Credit Monitor.

database used for the empirical analysis, followed by an empirical time-series model for aggregate EDF in section 3. Section 4 contains an evaluation of this model. In section 5, forecasts are given for future default frequency in the corporate sector. In section 6, a stress test is conducted of the expected default frequency of companies. The discussion that follows sums up and concludes the paper.

2. Expected Default Frequency and Macroeconomics

Moody's KMV Credit Monitor calculates *EDF* as a function of the distance to default. The premise for Moody's KMV model is that a company is bankrupt when the market value of its assets (MV_A) is lower than its default barrier, i.e., the company's debts (D): $MV_A < D$ or $MV_E = MV_A - D < 0$, where MV_E is the market value of the company's equity. The distance to default (DD) is measured in the number of standard deviations, which makes it possible to compare default frequencies between different companies, irrespective of their size:

$$DD = \frac{MV_A - D}{\sigma_A} = \frac{MV_E}{\sigma_A}, \tag{1}$$

where σ_A is the volatility (standard deviation) of the market value of the company's assets. The market value of the company's equity is a function of the current value of all future cash flows the company can be expected to generate. General economic developments play an important role for the development of company cash flows. Therefore, it is reasonable to assume that the EDF and the macroeconomic variables display common trends. The existence of common trends is estimated and tested using a vector error-correction model. We have decided to present the macroeconomic conditions using three different variables: the domestic industrial production index ($INDY$), the domestic consumer price index (CPI), and the nominal domestic three-month rate for Treasury bills ($R3M$). On this basis, we estimate the following relationship:

$$\log(EDF) = \beta_0 + \beta_1 R3M + \beta_2 \log(INDY) + \beta_3 \log(CPI) + u. \tag{2}$$

The selection of the macroeconomic variables included in the empirical assessment in this paper is based on Jacobson, Lindé, and Roszbach (2005). They model the macro economy using a set of macroeconomic variables, including aggregate bankruptcy frequency, in a quarterly VAR model. Based on work by Lindé (2002), they choose to include the following endogenous variables: the gap in domestic production, domestic inflation, the Riksbank's repo rate, and actual exchange rates. The exogenous variables included in their paper are the gap in foreign production, foreign inflation, and foreign three-month interest rates. In addition to these macroeconomic variables, Jacobson, Lindé, and Roszbach (2005) also include a measurement of the aggregate proportion of defaults as another exogenous variable in the VAR model. This consists of the number of actual defaults in relation to the total number of existing companies.

The model in this paper is deliberately based on just a few variables in order to keep the analysis simple and transparent. This means that operationalizing the model in the ongoing analysis does not require any large amount of resources and that the results are not too difficult to interpret. Further, the aim of the model is to provide a platform for scenario analysis to study the effects of major macroeconomic shocks—an analysis that is by nature relatively rough. Moreover, the model in this paper will be used to make both conditional and unconditional forecasts on the EDF. Unconditional forecasts mean that we let the model forecast the development of both the EDF and the macroeconomic variables. By this, we mean that the forecasts are made taking into account the feedback effects from the EDF on the macroeconomic variables and vice versa. Conditional forecasts are made by conditioning EDF forecasts on external forecasts on macro variables. The external forecasts for these macro variables are made taking the foreign macroeconomic developments into account. This is one of the reasons why we do not include foreign macroeconomic variables as exogenous variables in the model. Finally, we have opted for including industrial production in the estimates instead of the non-observable production gap since the estimates in this paper are based on monthly data. The estimates indicate that, on the whole, there is a one-to-one relationship between changes in the GDP and changes in industrial production.

It is difficult to know a priori what effect each of the macroeconomic variables may have on EDF. From the model specification in appendix 1 (equation (10)), the impact of the macroeconomic variables on EDF is not unambiguously decided, and thus it is ultimately an empirical issue. Nevertheless, it can be argued that the different macro variables should affect the EDF in certain ways. A negative correlation is expected between manufacturing output and EDF because increased output implies higher economic activity and higher corporate earnings. A higher interest rate increases the interest expenditure on corporate loans, which tends to raise EDF. The link between inflation and EDF is mainly twofold, through factor prices and the prices companies charge for their goods and services. Higher factor prices lead to increased production costs and tend to impair credit quality. Higher product prices can boost earnings and thereby improve creditworthiness. The relative strength of these two effects of inflation is determined by the structure of the markets for factors of production and the company's output.

It is also difficult to know a priori what effect the EDF may have on each of the macroeconomic variables. As we mentioned in the introduction to this paper, the banks compensate themselves for the expected loss by including a risk premium in their lending interest rates. Increased lending interest rates have a negative impact on economic growth through reduced household consumption and corporate investments. Household consumption decreases as higher lending rates make consumption more expensive. Corporate investments decrease as higher lending rates increase the marginal cost of capital, which makes investments more expensive. At the same time, increased marginal cost of capital imposes an upward pressure on product prices of firms in a monopolistic competition market, thereby causing an upward pressure on inflation. This is because the product prices of firms in a monopolistic competition market are set as a markup to their marginal cost, which also includes the marginal cost of capital.

The link between short-term interest rate and EDF is mainly twofold because the short-term interest rate is a policy variable. First, the interest rate might be increased by the monetary authorities because of the fact that higher EDF leads to higher inflation. The reason for this is that the rule for the monetary policy is to use the interest rate to restore inflation to its target level. Second, as we mentioned earlier, higher interest rates imply higher EDF. Therefore, by decreasing the interest rate, the monetary authorities can decrease the EDF and the risk premiums banks charge through their lending interest rates. In this case, it implies higher growth and lower inflation. The relative strength of these two effects of interest rate is determined by the structure of the lending market.

3. Data

The estimations are based on monthly data of expected default frequency covering the period from November 1997 through September 2008. Data on the empirical expected default frequency for nonfinancial listed companies (*EDF*) are from the Credit Monitor (Moody's KMV). The index for industrial production (*INDY*) has been taken from Reuters EcoWin.[4] The consumer price index (*CPI*) and interest rates on three-month Treasury bills (*R3M*) come from Statistics Sweden. Data for *INDY* are available for the period from January 1990 through August 2008. Data for *CPI* and *R3M* are available for the period from January 1970 through September 2008.

Testing to find out whether series are stationary is carried out using unit-root tests. These tests indicate that all variables used in the estimates of the VECM in this paper appear to be nonstationary.[5] However, the test for the occurrence of cointegration in the next section does not presume that all series are I(1) variables. Even if a specific series is not stationary, combinations of such series may have a cointegrating connection.

4. VECM for the Aggregate Expected Default Frequency

In the analysis of the time series, it is possible to show that even though all series prove to be nonstationary, a linear combination of them may nevertheless be stationary, i.e., integrated at the order of

[4]Reuters EcoWin is a provider of economic and financial market data. For more information about EcoWin, the reader is referred to http://www.ecowin.com/.

[5]*EDF*, *INDY*, and *R3M* have unit roots and are I(1) variables. However, even if we realize that *EDF* may not be a genuine I(1) variable, it seems to behave as an I(1) variable during the sample period studied in this paper. Moreover, *CPI* seems to be an I(2) variable, which indicates that CPI inflation is a nonstationary variable. That CPI inflation and *R3M* have unit roots is due to a shift in levels around 1993. It is more than possible that CPI inflation and *R3M* are not genuinely I(1). Nor is it necessary for the analysis in this paper for CPI inflation and *R3M* to be I(1) variables, even though this is what the test indicates.

Table 1. Johansen's Test for Cointegrating Relationships

Null Hypothesis	λ_{trace}	5% Critical Value	P-value
$r = 0$	57.4765	47.8561	0.0048
$r \leq 1$	23.9003	29.7970	0.2047

Notes: Since the cointegrating vector is not identified, we impose different identifying restrictions on the cointegrating vector. The likelihood-ratio test indicates that the variables are cointegrated notwithstanding which variable we use for the normalization. This indicates that a subset of the variables cannot be cointegrated.

zero. If this is the case for the data series on which our model is based, we can conclude that *EDF*, *INDY*, *CPI*, and *R3M* are cointegrated. This means that the linear combination cancels out the stochastic trends in these series.

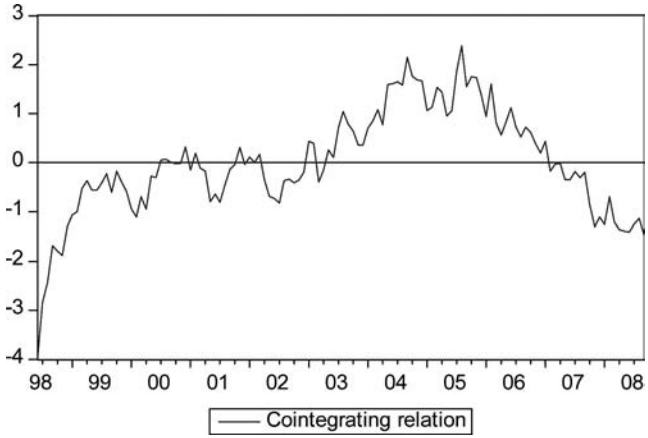
Using Johansen's (1988) method is one way of testing whether a data series is cointegrated. Table 1 presents the test statistics for this method (λ_{trace}). The test indicates that we can reject the hypothesis that there exists no cointegrated relationship at the 5 percent significance level; i.e., there is at least one cointegrating vector.[6] The test also indicates that there is only one cointegrating relationship out of the possible four. Figure 1 contains a graphic representation of the cointegrating relationship. The relationship has been normalized on the basis of *EDF*, as our primary interest is in the effects of the macroeconomic variables on the EDF. The test provides support for a long-term relationship between *EDF*, *INDY*, *CPI*, and *R3M*.[7]

After having tested for cointegrating relationships, we also need to decide on the appropriate lag structure for the model. The choice of lag structure is basically an empirical question, and the lag structure chosen in the specified empirical model is based on three different criteria. First, a residual test is made using a serial correlation Lagrange-multiplier (LM) test. This test is an alternative to the Q-statistics for testing serial correlation. The LM test is used to test for higher-order ARMA errors. Second, when the possibility that the

[6]However, it should be noted that we cannot reject the null hypothesis that the number of cointegrating vectors is less than or equal to one ($r \leq 1$).

[7]A maximum characteristic root test also indicates that there is only one cointegrating relationship.

Figure 1. Cointegrated Relationship



errors exhibit autocorrelation has been excluded for a given lag structure, we also investigate whether the estimated coefficients in the cointegrating relationship are stable. This is done by investigating whether the estimated coefficients in the cointegrating relationship change when new observations are added to the database. Finally, we investigate the out-of-sample forecasting accuracy for different models with different lag structures. We choose the model that gives the lowest forecast error in terms of root mean squared error (RMSE).[8] After having determined the lag structure, we specify the VECM:

$$\begin{aligned}\Delta x_t = \delta_0 + \Gamma_1 \Delta x_{t-1} + \Gamma_2 \Delta x_{t-2} + \Gamma_3 \Delta x_{t-5} \\ + \Gamma_4 \Delta x_{t-6} + \alpha\beta' x_{t-1} + \varepsilon_t,\end{aligned} \tag{3}$$

where $x_t = [\log(EDF_t), R3M_t, \log(CPI_t), \log(INDY_t)]$, $\delta_0 = \Gamma_0 - \alpha\beta_0$, and $\varepsilon_t \sim N(0, \Omega)$.[9] Two important parameters estimated in this model are β and α. The cointegrating vector, summarized by matrix β, describes the long-run relationships between the endogenous variables. The loading (or adjustment) coefficients forming matrix α

[8]The outcomes of these tests and evaluations can be provided by the authors upon request.

[9]A specification of the model where the implied volatility of the stock market was included has been estimated to capture some measure of market risk. This caused some problems with multicolinearity in the model, and therefore we chose the specification where the volatility measure is excluded.

Table 2. Estimated Beta Parameters for Long-Term Relationship

	$\boldsymbol{\beta}$	$\boldsymbol{\alpha}$
$\log(EDF)$	1	−0.064 (−5.04)
$R3M$	1.08 (−4.21)	0.009 (0.83)
$\log(CPI)$	30.00 (−2.90)	−1.11E-06 (0.00)
$\log(INDY)$	−22.03 (3.11)	−0.002 (−2.02)

Note: T-values are presented in parentheses.

describe the dynamic adjustment of the endogenous variables to deviations from long-run equilibrium by $\beta' x$.[10]

Table 2 summarizes the maximum likelihood estimate (ML estimate) for the beta parameters in the long-run relationships in the estimated model. All the coefficients are significant and have the expected signs. This means that in the long term, industrial production, *INDY*, has a negative effect on *EDF*, while *CPI* and *R3M* have a positive effect on *EDF*.

The test that $\beta = 0$ and $\alpha = 0$ entails restrictions on cointegrating vectors or the adjustment parameters. The likelihood-ratio tests indicate that we can reject each and every one of the following hypotheses: $\beta_2 = 0$, $\beta_3 = 0$, $\beta_4 = 0$, $\alpha_1 = 0$, and $\alpha_4 = 0$. However, we cannot reject that $\alpha_2 = 0$, $\alpha_3 = 0$, which is also indicated by the t-values. We were also able to reject the hypothesis that $\beta = (0, 1, x, x)$, where x denotes a free parameter. The hypothesis excludes *EDF* from the cointegrating vector, imposes an identifying restriction on $R3M$, and estimates *INDY* and *CPI*. This indicates that the macroeconomic variables cannot be integrated with each other.

Adjustments of the variables to the long-run level after shocks have occurred take place via adjustment coefficients or the "error-correction terms," α. Table 2 offers a summary of the estimated adjustment coefficients. The error-correction terms for *EDF* and

[10]When the EDF deviates from its estimated long-term level, ($\alpha = 0.1$) thus indicates that 10 percent of the deviation will be corrected in the subsequent period. How long it will take before the system returns to long-term equilibrium can be calculated using α.

INDY (α_1 and α_4) are negative, indicating convergence toward the long-run equilibrium. The fact that α_2 and α_3 are not significant does not constitute a problem, as Johansen's test reveals that the model converges toward long-run equilibrium. To save space, the rest of the results of the estimation are presented in appendix 2.[11] The results for various specification tests of the estimated model do not reveal any problems.[12]

5. Evaluation of the VECM

In this section, we evaluate the VECM by analyzing its within-sample properties as well as its out-of-sample properties.

5.1 Within-Sample Forecasts

A comparison between within-sample forecasts for the VECM with the actual outcomes for the sample period shows that the model replicates the actual distribution of the EDF relatively well; see figure 2. However, the important question is, how good will the out-of-sample forecasts of the EDF be?

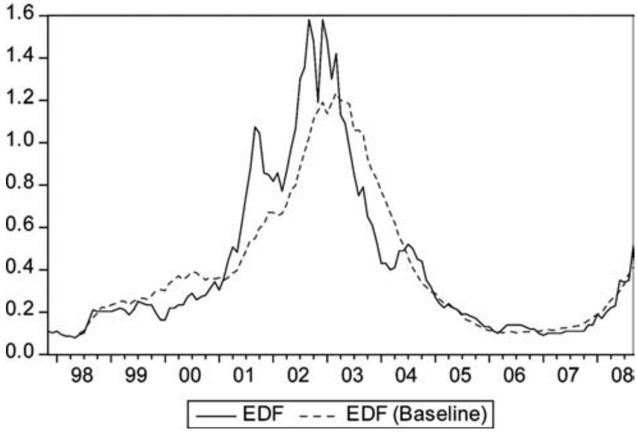
5.2 Out-of-Sample Forecasts

The VECM is evaluated in two ways. One way is by comparing the RMSE for the VECM with the RMSE for the forecasts made using a naive model (such as a random-walk model or an AR(1) model). A second evaluation method is through the comparison of the RMSE for the forecasted EDF for one month, three months, six months, one year, and two years ahead with the standard deviation for EDF.

[11] Due to lack of space, we have opted not to present t-values for the short-term coefficients. Naturally, these can be supplied by the authors upon request.

[12] The autocorrelation LM test indicates that there is no autocorrelation in the residuals. White's heteroskedasticity test indicates that there are no heteroskedasticity problems in the residuals. The normality test reports the multivariate extensions of the Jarque-Bera residual normality test. For the multivariate test, a factorization of the residuals that are orthogonal to each other must be chosen. We have used the factorization method suggested by Urzúa (1997). Testing for whether the residuals are normally distributed confirms that this is the case.

Figure 2. Within-Sample Forecasts for EDF Over Time



5.2.1 RMSE for the VECM vs. RMSE for Naive Models

The procedures used are as follows. The VECM for EDF is estimated on data from June 1998 through December 2002. The estimated VECM is used to make four different forecasts for the period from January 2003 through September 2008. First, we make endogenous forecasts, which means that the models generate trajectories for all variables included in the model. Unconditional forecasts mean that forecasts are made by taking into account the feedback effects from the EDF on the macroeconomic variables and vice versa. Second, we also make semi-endogenous forecasts for *EDF*, *CPI*, and *INDY* by conditioning the forecasts for these variables on the outcome for the short-term interest rate. The reason for making semi-endogenous forecasts is the fact that the interest rate is a policy variable. Many central banks use the interest rate to control the inflation rate. This policy instrument also has an impact on expected default frequencies in an economy. However, given the path for the interest rate, the macroeconomic variables as well as the EDF can have important feedback effects on each other. By making semi-endogenous forecasts, we are able to take these feedback effects into account, given the path for the interest rate. Third, conditional forecasts are made for the period from January 2003 through September 2008. This is made possible by conditioning the EDF forecasts on the outcomes for the macroeconomic variables for the period from January

2003 through September 2008. The reason for making conditional forecasts is to examine how accurate the forecasts for the EDF are when uncertainty regarding the development for the macroeconomic variables is removed from the model. Fourth, the estimated VECM is used to make conditional forecasts for the period from January 2003 through September 2008—in this case, conditioned on the Riksbank's official forecasts for the macroeconomic variables presented in its Monetary Policy Reports 2003:1–2008:2.

EDF is also estimated using a simpler AR(1) model: $EDF_t = \alpha EDF_{t-1} + \varepsilon_t$. The estimated AR(1) model for EDF is used to make alternative EDF forecasts. In addition, a random-walk model is used to make EDF forecasts. The premise for this naive forecast is that the best forecast for future EDF is provided by the most recent information about the outcomes for the same variable.

Then, we proceed step by step by increasing the sample one month at a time and making new estimates of the model and forecasts using the various models as described above. Finally, the RMSE is calculated for the four forecasts that have been produced using the VECM, the AR(1) model, and the random-walk model. The RMSE for the different models is calculated as follows:

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=s}^{2006:3}(EDF_t^{forecast} - EDF_t^{actual})^2}, \qquad (4)$$

where $EDF_t^{forecast} - EDF_t^{actual}$ is the forecast error. The forecast errors are squared so that both overestimates and underestimates will have the same weight. Then, the average of the squared forecast errors is calculated. The square root of this figure then provides the RMSE. For endogenous and exogenous forecasts, the number of periods selected is $T = 69$ and the starting period for the forecast is $s = 2003:1$ for those with one-month forecast horizons.[13]

Table 3 presents a summary of the RMSE results for the four EDF forecasts produced using the VECM (endogenous forecasts, semi-endogenous forecasts, forecasts conditioned on the outcomes

[13]However, $T = 67$ and $s = 2003:3$ for forecasts with three-month forecast horizons; $T = 64$ and $s = 2003:6$ for forecasts with six-month forecast horizons; $T = 58$ and $s = 2003:12$ for forecasts with one-year forecast horizons; and $T = 46$ and $s = 2004:12$ for forecasts with two-year forecast horizons.

Table 3. RMSE for EDF

Period	**Conditioned Forecasts Using Riksbank's Macro Forecasts**	**Endogenous Forecasts**	**Semi-Endogenous Forecasts**	**Conditioned Forecasts Using the Outcomes for Macro Variables**	**AR(1)**	**Random Walk**
One Month	0.21	0.08	0.08	0.08	0.07	0.07
Three Months	0.37	0.16	0.12	0.12	0.15	0.13
Six Months	0.58	0.29	0.19	0.23	0.27	0.23
One Year	0.86	0.60	0.35	0.25	0.46	0.34
Two Years	2.20	1.21	0.90	0.78	0.79	0.49

for the macroeconomic variables, and forecasts conditioned on the Riksbank's forecasts of the macroeconomic variables in its monetary policy reports), the AR(1) model, and the random-walk model.

The VEC forecasts conditioned on the Riksbank's macro forecasts in its Monetary Policy Reports have consistently higher RMSE figures than the two forecasts based on the AR(1) model and a random-walk model.[14] However, the endogenous forecasts have much better forecasting accuracy than forecasts conditioned on the Riksbank's forecasts of the macroeconomic variables in its monetary policy reports. Moreover, the endogenous forecasts are as good as forecasts made by AR(1) and random-walk models up to six months. Semi-endogenous forecasts outperform both the forecasts conditioned on the Riksbank's forecasts of the macroeconomic variables in the Monetary Policy Reports and the endogenous forecasts. Moreover, semi-endogenous forecasts are as good as, or even better than, the forecasts made by AR(1) and random-walk models up to one year. Finally, semi-endogenous forecasts are as good as forecasts conditioned on the actual outcomes for macro variables.

What is interesting in this context is that the forecasting accuracy of VEC forecasts conditioned on the actual outcomes for macroeconomic variables is better than the accuracy of other forecasts made by the VECM. The RMSE declines for the entire period when uncertainty about macroeconomic developments is eliminated by conditioning the forecasts on the outcomes for macroeconomic variables. This indicates that the estimated equation for the EDF has good forecasting properties. Moreover, the RMSE falls below the RMSE for forecasts made using the AR(1) model up to two years. Finally, the RMSE falls below the RMSE for forecasts made using the random-walk model up to one year.[15]

[14]The model in this paper is estimated using monthly data. To make conditioned forecasts with this model, we need monthly forecasts for the macroeconomic variables. However, the Riksbank's macro forecasts are usually presented quarterly (this is especially the case for the GDP and the interest rate forecasts). This means that we are obliged to transform these quarterly forecasts into monthly forecasts using a rather simple method. In this way, we introduce a measurement error when making the forecasts that may magnify the RMSE.

[15]Yet another way of evaluating the VECM is to conduct a sign test to study the extent to which the model's forecasts develop in the same direction as EDF outcomes. The results of a test of this kind can lie between 0 and 100 percent. A

5.2.2 RMSE vs. the Standard Deviation of EDF

Another way of evaluating the RMSE for the different forecasts (and the different periods of time) is to relate the RMSE to the standard deviation of the EDF, which in this case is 0.38. This comparison indicates that conditional forecasts using the Riksbank's macro forecast are reliable between three and six months ahead, and endogenous forecasts as well as forecasts made by the AR(1) model are reliable between six months and one year ahead. However, the semi-endogenous forecasts, conditioned on forecasts using the outcome for the macro variables as well as forecasts made by the random-walk model, are reliable between one and two years ahead.

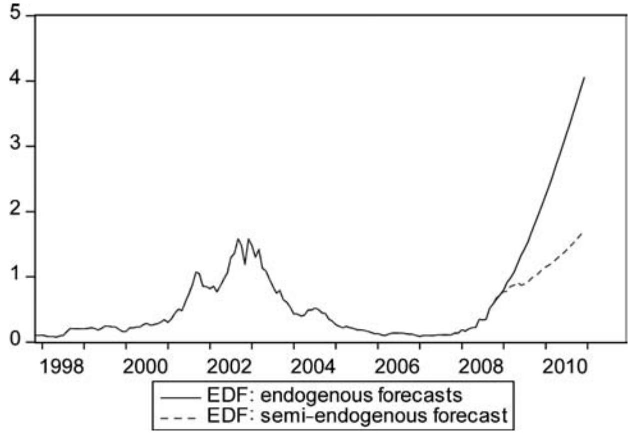
6. Endogenous and Semi-Endogenous Forecasts

The estimated VECM is used to forecast EDF in two different ways. First, we make unconditional forecasts, which means that the forecasts are made by taking into account the feedback effects from the EDF on the macroeconomic variables and vice versa. Second, we also make semi-endogenous forecasts for *EDF*, *CPI*, and *INDY* by conditioning the forecasts for these variables on the interest rate forecasts made by the Riksbank that is presented in its Monetary Policy Report 2008:3. The reason for making semi-endogenous forecasts is the fact that the interest rate is a policy variable and exogenous in its nature.

The EDF forecasts using these two different methods are summarized in figure 3. The endogenous forecast indicates that the aggregate EDF will rise from 0.52 percent in September 2008 to about

zero result means that the forecast model never indicates whether EDF will rise or fall during the forecast period, and 100 percent indicates perfect predictions. A sign test indicates that the model specifications that exclude macro variables (AR(1) and random-walk models) have much lower prediction capacity compared with VECMs that include macro variables. The predictive capacity of the VECM is particularly good if the EDF forecasts are conditioned on macroeconomic forecasts with a high degree of precision. The outcome of this test can be provided by the authors by request.

Figure 3. Conditioned Forecasts of Aggregate EDF



4.05 percent in December 2010.[16] However, according to the semi-endogenous forecast, where we have taken the Riksbank's monetary policy into account, the EDF will rise from 0.52 percent in September 2008 to about 1.71 percent in December 2010. Both forecasts indicate that there will be a turn in the credit cycle in terms of higher default frequencies during the forecast period, a turn that actually began already in May 2008. Both the endogenous and the semi-endogenous forecasts for the EDF reveal that the interest rate has a very strong and important impact on the expected default frequency.

Since the semi-endogenous forecasts use an exogenous path for future interest rate levels, as explained above, while in the endogenous forecasts the interest path is determined endogenously, the paths for future interest levels will differ between the two models. The endogenous forecast model generates a path for future interest rates that is higher than the path for future interest levels in the semi-endogenous model. This is because in the endogenous forecasts, the VECM tries to stabilize the future paths for all included variables, *R3M*, *CPI*, *INDY*, and the EDF and allows for feedback

[16]The endogenous forecast made by the VECM is much like the forecasts made by the VECM conditioned on the Riksbank's Monetary Policy Update of September 2008. We present these forecasts in the next section.

effects between all of them. In the semi-endogenous case, the VECM will try to stabilize future paths for all included variables except for the interest level that is exogenous. Moreover, it will not allow for feedback effects on the short-term interest level from the other included variables. Therefore, the semi-endogenous forecasts for the EDF are much lower than the endogenous forecasts for the EDF.

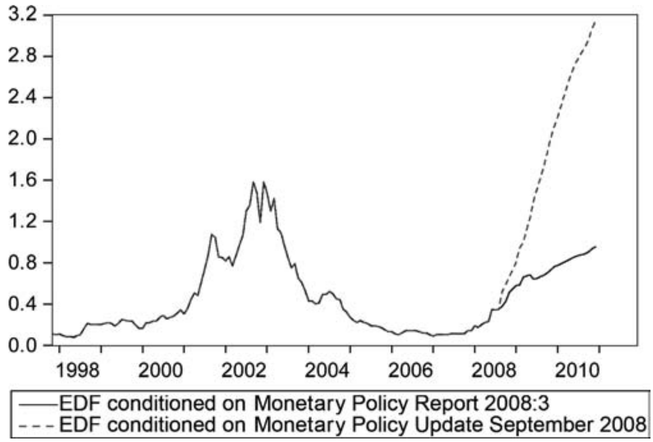
As we mentioned earlier, banks compensate themselves for expected credit losses by including a risk premium in their lending interest rates. The EDF can be seen as a measure of this risk premium. A higher (lower) EDF therefore implies a higher (lower) risk premium and ultimately a higher (lower) lending interest rate. By this reasoning, banks' lending interest rates should be lower in the case of semi-endogenous forecasts than in the case of endogenous forecasts. Reduced lending interest rates have a positive impact on economic growth through increased consumption and increased corporate investments. At the same time, decreased lending interest rates reduce the marginal cost of capital and impose a downward pressure on the product prices of firms in a monopolistic competition market. Therefore, inflation is expected to be lower in the semi-endogenous model compared with the endogenous model. This is exactly what the forecasts for the CPI reveal.

7. Stress Testing Using Conditional Forecasts

The estimated model can be used to make conditional EDF forecasts based on different developments for the macroeconomic variables. The VECM's conditional EDF forecasts are obtained by allowing the macroeconomic forecasts made by the Riksbank and presented in its Monetary Policy Report to determine the development of the macroeconomic variables in the model. We use the macroeconomic forecasts presented in the Riksbank's Monetary Policy Update of September 2008 and the Riksbank's Monetary Policy Report 2008:3.

In the Monetary Policy Update of September 2008, the Riksbank feared that inflation measured in terms of *CPI* would rise by 3.9 percent in 2008, 3.2 percent in 2009, and 2.0 percent in 2010. This was believed to be the case because of different supply shocks heating the economy. The Riksbank assumed that a short-term interest rate of 4.57 percent in 2007, 4.55 percent in 2009, and 4.34 percent in 2010 would help to restore inflation to its target. GDP in Sweden

Figure 4. Development of Aggregate EDF Given Two Different Macroeconomic Developments



was at the same time estimated to increase by 1.4 percent in 2008, by 0.8 percent in 2009, and by 26 percent in 2010.

In its Monetary Policy Report 2008:3, the Riksbank argued that the macroeconomic development will be less favorable than anticipated because the ongoing worldwide financial crises will be followed by high risk premiums required by the investors. Because of this crisis, the Riksbank revised its GDP and CPI forecasts downward. GDP in Sweden was estimated to increase by 1.2 percent in 2008, by 0.1 percent in 2009, and by 2.5 percent in 2010. Inflation measured in terms of *CPI* was expected to rise by 3.7 percent in 2008, 2.1 percent in 2009, and 1.6 percent in 2010. These forecasts were made assuming a short-term interest rate of 4.34 percent in 2007, 3.29 percent in 2009, and 3.47 percent in 2010.

The EDF forecasts for the same period are summarized in figure 4. The forecast conditioned on the Riksbank's Monetary Policy Update of September 2008 indicates that the aggregate EDF will rise from 0.52 percent in September 2008 to about 3.14 percent in December 2010. However, because of the Riksbank's new monetary policy and the new forecasts of GDP and CPI, the EDF will rise from 0.52 percent in September 2008 to about 0.95 percent in December 2010.

These conditional forecasts reveal that the macroeconomic development is very important for the development of future corporate defaults measured by the EDF. The fact is that the short-term interest rate has the strongest impact on expected default frequency among the included macroeconomic variables. Both the endogenous and the semi-endogenous forecasts indicate that there will be a turn in the credit cycle in terms of higher default frequencies during the forecast period, which actually began already in May 2008. However, the stress test shows that a supply shock in a situation where the credit cycle is expected to turn upward (in terms of higher default frequencies) during the forecast period and which is countered by the Riksbank with increased interest rates leads to a threefold increase in aggregate expected default frequency at the end of the forecast period.

Yet another way of interpreting the forecasting results is that the Riksbank will succeed in lowering the high risk premiums required by market players because of the ongoing financial crises. This is evident from the forecast conditioned on the macroeconomic development presented in the Riksbank Monetary Policy Report 2008:3. The EDF forecast is much lower in this case compared with the forecast conditioned on the Riksbank's Monetary Policy Update of September 2008. By lowering the interest rate, the Riksbank could reduce future EDFs and thereby also help reduce the risk premiums required by market players that went up because of the financial crisis.

8. Conclusions

In this paper, we estimate a time-series model for predicting future credit quality in the corporate sector. The model is based on aggregated data and a few variables only. This means that it is relatively straightforward and can be used in the ongoing analysis. The variable that represents credit quality is expected default frequency (EDF), a market-based indicator of the probability of a company not being able to meet its commitments within a specified period. The model uses the median EDF for the corporate sector, which is an aggregated measure of credit quality with all company-specific risks eliminated, so that it is solely affected by risk factors that all companies have in common. The model estimates the relationships between the EDF and three macroeconomic variables: inflation, industrial production, and the short-term interest rate. The estimates are then used to predict credit quality in the future.

A vector error-correction model (VECM) is used to catch long-run relationships between the variables studied as well as short-run fluctuations around these relationships. The effects of different factors on credit quality are ultimately an empirical matter. Estimations using monthly data for the period November 1997 through September 2008 show that increased industrial production is accompanied by a lower expected default frequency. Rising inflation leads to the opposite scenario: a higher expected default frequency and thereby poorer corporate credit quality. However, the short-term interest rate has the strongest impact on corporate credit quality among the three macroeconomic variables. A higher interest rate leads to a higher expected default frequency.

The model's performance is evaluated using two tests. One compares the root mean squared error (RMSE) for predicted credit quality with the standard deviation of recorded credit quality. Another test compares the RMSE for predicted credit quality in the VECM with credit quality predicted with a naive model (i.e., based on an AR(1) model or a random-walk model). Tests indicate that the model's ability to predict corporate credit quality is satisfactory. This means that the model's predictions of future credit quality can be expected to co-vary with actual credit quality to a high degree. Less uncertainty in the macro forecast naturally leads to greater precision in the model's predictions.

The predictions of credit quality are made in two different ways. First, we make unconditional forecasts, which means that the predictions of credit quality are made endogenously, taking into account the feedback effects from the EDF on the macroeconomic variables and vice versa. Second, we also make semi-endogenous forecasts for *EDF*, *CPI*, and *INDY* by conditioning the forecasts for these variables on the interest rate forecasts made by the Riksbank and presented in its Monetary Policy Report 2008:3. The reason for making semi-endogenous forecasts is the fact that the interest rate is a policy variable and is exogenous by nature. Traditionally, a reduced interest rate is expected to increase inflation while increasing the growth rate in the economy. However, when expected defaults are included in the model, a reduced interest rate gives both lower inflation and higher growth. The reason for this is that a lower interest rate lowers EDF, a measure of the risk premium. Lower risk premiums contribute to lower lending interest rates that firms and consumers meet at the market and lower marginal cost of capital. This stimulates higher growth because corporate investments and households' consumption increase. At the same time, a decreased marginal cost of capital imposes a downward pressure on the product prices of firms in a monopolistic competition market. Therefore, inflation is expected to be lower in the semi-endogenous model compared with the endogenous model. This is exactly what the forecasts for the CPI reveal.

The VECM's EDF forecasts conditioned on the Riksbank's official view of macroeconomic developments in its Monetary Policy Report 2008:3 and its Monetary Policy Update of September 2008 show that there will be a gradual increase in aggregate default frequencies during the forecast period. This development, in turn, indicates a turn in the credit cycle. At the same time, the stress test shows that a supply shock in a situation where the credit cycle is expected to turn during the forecast period, and which is countered by the Riksbank with a rise in interest rates, leads to a threefold increase in aggregate expected default frequency at the end of the forecast period. Yet another way of interpreting the conditioned forecasts is that the Riksbank will succeed in lowering the high risk premiums currently required by market players because of the ongoing financial crises. This is because the EDF forecast is much lower when it is conditioned on the Riksbank macro forecasts presented in its Monetary Policy Report 2008:3 compared with the forecast conditioned on the Riksbank's Monetary Policy Update of September 2008.

The model can be used as one of a number of instruments for forward assessments of banks' credit risks. The EDF predictions can be used as inputs to calculate the economic capital individual banks should hold to cover unexpected credit losses, which is a clear indicator of the credit risk in each bank's loan portfolio. The model can also be used for the analysis of scenarios to test alternative assumptions about macroeconomic developments. Finally, the model can also be used to analyze the interdependencies between the expected default frequency and the macro economy.

Appendix 1. Expected Default Frequency and Macroeconomics

Moody's KMV Credit Monitor calculates *EDF* as a function of distance to default (*DD*): $EDF = f(DD)$.[17] The premise for Moody's KMV model is that a company becomes bankrupt when $MV_A < D$ or $MV_E = MV_A - D < 0$.[18] The distance to default (*DD*) is defined as follows:

$$DD = \frac{MV_A - D}{\sigma_A} = \frac{MV_E}{\sigma_A}. \tag{5}$$

As neither MV_A nor σ_A can be directly observed, an additional assumption must be made to be able to calculate *DD*. Merton (1974) drew attention to the fact that the cost of guaranteeing the value of a company's loans corresponds to the value of a call option on the market value of the company's assets (MV_A) with a redemption price (D) at time T. In parity with this argument, the yield on a company's equity, MV_E, corresponds to the yield of a call option on the company's assets: $MV_E = \max[0, MV_A - D]$. The lenders either receive the market value of the company's assets (if the market value of these assets is less than the company's debts) or full repayment of the loan when it becomes due for settlement: $MV_D = \min[MV_A, D] = \min[MV_A - D, 0] + D$. This yield is equivalent to holding a bond with the nominal value of D and issuing a call option on the company's assets with D as the redemption price. The market value of a company's assets and their volatility can be calculated with the help of Black and Scholes (1973).

The approach used in this paper is to estimate the empirical relationship between *EDF* and the macroeconomic conditions. We know that *EDF* is a function of *DD*; i.e., $EDF = f(DD)$. The elasticity between *EDF* and *DD* is defined as $\varepsilon = \frac{dEDF}{dDD}\frac{DD}{EDF}$. Thereby, we assume *EDF* to be a nonlinear function of *DD*: $EDF = DD^{\varepsilon}e^{u}$,[19] where u denotes the residual. Taking the logarithms on both sides

[17] For a short description of the method used by Moody's KMV Credit Monitor to derive *EDF*, the interested reader is referred to Rehm and Rudolf (2001).

[18] The default barrier is assumed to be deterministic and consists of the nominal value of the debt, i.e., short-term debt + 0.5*long-term debt.

[19] This function is known as the exponential regression model.

of this equation, we obtain

$$\log(EDF) = \varepsilon \log(DD) + u. \tag{6}$$

Further, taking the logarithms on both sides of equation (5), we obtain

$$\log(DD) = \log(MV_E) - \log(\sigma_A). \tag{7}$$

The market value of the company's equity is a function of the current value of all future cash flows the company can be expected to generate. General economic developments play an important role for the development of company cash flows. Therefore, it can be assumed that $MV_E = f_1(X)$, where $X = [INDY, CPI, R3M]$. This means, in turn, that the volatility of the market value of the assets should be a function of macroeconomic development: $\sigma_A = f_2(X)$. It is difficult to know a priori what effect the macroeconomic variables can be considered to have on MV_E and σ_A. The elasticity between MV_E and X is defined as $\mathrm{E}^1 = \frac{dMV_E}{dX}\frac{X}{MV_E}$. Further, the elasticity between σ_A and X is defined as $\mathrm{E}^2 = \frac{d\sigma_A}{dX}\frac{X}{\sigma_A}$. Thereby, we assume that MV_E and σ_A have the following functional forms: $MV_E = \mathrm{E}_0^1 e^{\mathrm{E}_1^1 R3M} INDY^{\mathrm{E}_2^1} CPI^{\mathrm{E}_3^1}$ and $\sigma_A = \mathrm{E}_0^2 e^{\mathrm{E}_1^2 R3M} INDY^{\mathrm{E}_2^2} CPI^{\mathrm{E}_3^2}$. Taking the logarithms on both sides of these equations, we obtain[20]

$$\log(MV_E) = \log\left(\mathrm{E}_0^1\right) + \mathrm{E}_1^1 R3M + \mathrm{E}_2^1 \log(INDY) + \mathrm{E}_3^1 \log(CPI) \tag{8}$$

$$\log(\sigma_A) = \log\left(\mathrm{E}_0^2\right) + \mathrm{E}_1^2 R3M + \mathrm{E}_2^2 \log(INDY) + \mathrm{E}_3^2 \log(CPI). \tag{9}$$

Inserting (8) and (9) into (7), we can rewrite (6) as follows:

$$\log(EDF) = \beta_0 + \beta_1 R3M + \beta_2 \log(INDY) + \beta_3 \log(CPI) + u, \tag{10}$$

where $\beta_0 = \varepsilon\lfloor\log(\mathrm{E}_0^1) - \log(\mathrm{E}_0^2)\rfloor$, $\beta_1 = \varepsilon(\mathrm{E}_1^1 - \mathrm{E}_1^2)$, $\beta_2 = \varepsilon(\mathrm{E}_2^1 - \mathrm{E}_2^2)$, and $\beta_3 = \varepsilon(\mathrm{E}_3^1 - \mathrm{E}_3^2)$ are the coefficients we want to estimate. As is

[20] Using differential calculus, it can be shown that $\mathrm{E}_1^1 = d(\log MV_E)/dR3M = [d(MV_E)/dR3M]/MV_E$, which is the relative change in the regressand divided by the absolute change in the regressor. If we multiply the relative change in MV_E by 100, E_1^1 will then give the percentage change, or the growth rate, in MV_E for an absolute change in $R3M$. In the literature, this is known as the semi-elasticity of MV_E with respect to $R3M$.

evident, it is difficult to know a priori what effect the macroeconomic variables can be considered to have on EDF.

Appendix 2. Results of Estimations

Cointegrating Eq:	**CointEq1**			
LOG(EDF(−1))	1.0000			
R3M(−1)	−1.0841			
LOG(CPI(−1))	−30.0009			
LOG(INDY(−1))	22.0265			
C	70.9596			
Error Correction:	**D(LOG(EDF))**	**D(R3M)**	**D(LOG(CPI))**	**D(LOG(INDY))**
CointEq1	−0.06375	0.009408	−1.11E-06	−0.00245
D(LOG(EDF(−1)))	0.053327	0.06509	0.00198	−0.00887
D(LOG(EDF(−2)))	−0.01575	−0.131	−0.00099	−0.01792
D(LOG(EDF(−5)))	0.011637	−0.06766	0.001602	−0.01509
D(LOG(EDF(−6)))	−0.13644	0.030463	−0.00029	−0.00794
D(R3M(−1))	0.164029	0.642813	0.003366	−0.00044
D(R3M(−2))	−0.16209	−0.13218	9.71E-05	0.004215
D(R3M(−5))	0.12757	0.149394	−0.00092	−0.00122
D(R3M(−6))	−0.03116	−0.08816	−8.61E-05	0.001807
D(LOG(CPIM(−1)))	−0.45858	−0.39609	0.077999	0.18397
D(LOG(CPIM(−2)))	1.030785	4.772407	−0.13908	−0.27444
D(LOG(CPIM(−5)))	0.372754	2.651221	0.066178	0.182219
D(LOG(CPIM(−6)))	1.808998	−0.82136	0.502338	−0.17895
D(LOG(INDY(−1)))	0.129055	0.201066	−0.01871	−0.36274
D(LOG(INDY(−2)))	0.722394	−0.64399	−0.05448	−0.25407
D(LOG(INDY(−5)))	0.505888	−0.51408	−0.01363	−0.1226
D(LOG(INDY(−6)))	−1.22083	0.287994	0.02263	−0.07238
C	0.01243	−0.00533	0.000888	0.004113
R^2	0.3064	0.4013	0.3829	0.2819
Adj. R^2	0.1952	0.3052	0.2839	0.1667

References

Alves, I. 2005. "Sectoral Fragility: Factors and Dynamics." *BIS Papers* 22 (April): 450–80.

Black, F., and M. Scholes. 1973. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81 (3): 637–54.

Castrén, O., S. Dées, and F. Zaher. 2007. "How Do Global Macro-Financial Shocks Affect Sector Expected Default Frequencies in the Euro Area?" Forthcoming in *Journal of Financial Stability.*

Chan-Lau, J. A. 2006. "Fundamentals-Based Estimation of Default Probabilities: A Survey." IMF Working Paper No. 149.

Jacobson, T., J. Lindé, and K. Roszbach. 2005. "Exploring Interactions Between Real Activity and the Financial Stance." *Journal of Financial Stability* 1 (3): 308–41.

Johansen, S. 1988. "Statistical Analysis of Cointegration Vectors." *Journal of Economic Dynamics and Control* 12 (2–3): 231–54.

Lindé, J. 2002. "Monetary Policy Shocks and Business Cycle Fluctuations in a Small Open Economy: Sweden 1986–2002." Manuscript, Sveriges Riksbank.

Merton, R. C. 1974. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *Journal of Finance* 29 (2): 449–70.

Pesaran, M. H., T. Schuermann, B.-J. Treutler, and S. Weiner. 2006. "Macroeconomic Dynamics and Credit Risk: A Global Perspective." *Journal of Money, Credit, and Banking* 38 (5): 1211–61.

Rehm, F., and M. Rudolf. 2001. "KMV Credit Risk Modeling." In *Risk Management — Challenge and Opportunity*, ed. M. Frenckel, U. Hommel, and M. Rudolf, 141–54. Berlin: Springer.

Urzúa, C. M. 1997. "Omnibus Tests for Multivariate Normality Based on a Class of Maximum Entropy Distributions." *Advances in Econometrics* 12: 341–58.

Virolainen, K. 2004. "Macro Stress Testing with a Macroeconomic Credit Risk Model for Finland." Discussion Paper No. 18, Bank of Finland.