How to Find Plausible, Severe, and Useful Stress Scenarios*

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We give a precise operational definition to three requirements the Basel Committee on Banking Supervision specifies for stress tests: plausibility and severity of stress scenarios as well as suggestiveness of risk-reducing actions. The basic idea of our approach is to define a suitable region of plausibility in terms of the risk-factor distribution and search systematically for the scenario with the worst portfolio loss over this region. One key innovation compared with the existing literature is the solution of two open problems. We suggest a measure of plausibility that is not prone to the problem of dimensional dependence of maximum loss and we derive a way to consistently deal with situations where some but not all risk factors are stressed. We show that setting the nonstressed risk factors to their conditional expected value given the value of the stressed risk factors maximizes plausibility among the various approaches used in the literature.

JEL Codes: G28, G32, G20, C15.

1. Introduction

The current regulatory framework of the Basel Committee on Banking Supervision (2005) requires banks to perform stress tests which meet three requirements: *plausibility* of stress scenarios, *severity* of

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stress scenarios, and *suggestiveness of risk-reducing action.*[1] How do we find stress scenarios which are at the same time plausible, severe, and suggestive for the design of risk-reducing action? Our paper gives a systematic answer to this question. We suggest a method that can be implemented for a wide class of stress-testing problems usually encountered in practice. We illustrate the method and the issues in the context of an example: stress tests for a portfolio of adjustable-rate loans in home and foreign currency.

The quality of a stress test crucially depends on the definition of stress scenarios. Defining stress scenarios is a thought experiment. It is a counterfactual exercise where a risk manager tries to imagine what adverse or even catastrophic events might strike his or her portfolio. Such a thought experiment is prone to two major pitfalls: consideration of implausible scenarios and neglect of plausible scenarios. Thinking about scenarios requires one to imagine situations that have perhaps not yet occurred but might occur in the future. Bias toward historical experience can lead to the risk of ignoring plausible but harmful scenarios which did not yet happen in history. This creates a dangerous blind spot. If the imagination of a stress tester puts excessive weight on very unplausible scenarios, management faces an embarrassing decision: should one react to alarming results of highly implausible stress scenarios? Our method allows a precise trade-off between plausibility and severity. In this way we can ensure that in a model of portfolio risk, no harmful but plausible scenarios are missed. Furthermore, our stress-test method suggests ways to reduce risk if desired.

We analyze the problem of finding extreme but plausible scenarios in a classical quantitative risk-management framework. A portfolio of financial instruments, say a portfolio of loans, is given. The value of each loan at some given horizon in the future is described by the realization of certain risk factors. In the case of a loan portfolio,

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[1] The respective references in Basel Committee on Banking Supervision (2005) are as follows: "Quantitative criteria should identify plausible stress scenarios to which banks could be exposed" (par. 718 (LXXIX)); "A bank should also develop its own stress tests which it identifies as most adverse based on the characteristics of its portfolio" (par. 718 (LXXXIII)); and "Qualitative criteria should emphasize that two major goals of stress testing are to evaluate the capacity of the bank's capital to absorb potential large losses and to identify steps the bank can take to reduce its risk and conserve capital" (par. 718 (LXXIX)).

for example, these risk factors will comprise the macroeconomic environment (because of its impact on the payment ability and thus on the solvency of borrowers) and market factors like interest rates (or exchange rates in the case of foreign currency loans) but also idiosyncratic factors that influence a borrower's solvency. The uncertainty about the realization of risk factors is described by a risk-factor distribution that is estimated from historical data. Plausibility is captured by specifying how far we go into the tails of the distribution in our search for stress scenarios. The severity of scenarios is maximized by systematically searching for the worst case, the maximum portfolio loss, in a risk-factor region of given plausibility.

This general idea of looking at extreme scenarios has been formulated in the literature before. It is informally discussed by Čihák (2004, 2007). More formally, the idea is discussed in Studer (1997, 1999) and in Breuer and Krenn (1999). This literature leaves, however, two open issues that seem technical at first sight but are of great practical relevance: the problem of partial scenarios and the problem of dimensional dependence of maximum loss.

The partial-scenario problem comes from the situation that a portfolio may depend on many risk factors, but the modelers are interested in stressing not all but only a few factors at a time. For example, in a loan portfolio we are often interested in stress scenarios for particular variables: a certain move in the exchange rate or a particular drop in GDP. How do we deal with the other risk factors consistently? Do we leave them at their last observed value, at some average value, or should we condition on the stressed macro factor and, if so, how? We show that the way to deal with the partial-scenario problem that maximizes plausibility is to set the nonstressed systematic risk factors to their conditional expectation for the given value of the stressed factors. We show furthermore that this has the same plausibility as the computationally more intensive full-loss simulation from the conditional stress distribution as in Bonti et al. (2005).

If we look for maximum loss in a risk-factor region of given plausibility, we want the maximum loss not to depend on the inclusion of irrelevant risk factors or risk factors that are highly correlated with factors already included in the analysis. The plausibility measures that were used in the previous literature (see Studer 1997, 1999) suggested to define plausibility regions as regions with a given probability mass. This definition of plausibility has an undesirable property, known as the problem of dimensional dependence of maximum loss. To get an intuitive understanding of the problem, consider an example from Breuer (2008). We have a bond portfolio with risk factors consisting of two yield curves in ten foreign currencies. One risk manager chooses to model the yield curve with seven maturity buckets and another risk manager uses fifteen buckets. In this case the first risk manager uses 150 risk factors in the analysis and the second manager uses 310. As plausibility region, both of them choose an ellipsoid of mass 95 percent. Breuer (2008) shows that the second risk manager will calculate a maximum loss that is 1.4 times higher than the maximum loss calculated by the first risk manager. This is problematic because both of them look at the same portfolio and use the same plausibility level. We suggest an approach to define plausibility that does not have this problem.

The paper is organized as follows: In section 2 we define a quantitative measure of plausibility and explain why it is not subject to the dimensional dependence problem. We discuss how to deal with the problem of partial scenarios and explain the technique of worst-case analysis. We also discuss how measures for risk-reducing actions can be deduced from the stress test. In section 3 we analyze an example of a portfolio of foreign currency loans that illustrates the practical applicability as well as the potential improvement compared with a standard stress-testing procedure. Finally, section 4 concludes.

2. Finding Scenarios That Are Plausible, Severe, and Suggestive of Counteraction

We consider the problem of stress testing a loan portfolio. The value of each position in the portfolio depends on n systematic risk factors $\boldsymbol{r} = (r_1, \ldots, r_n)$ and on m idiosyncratic risk factors $\epsilon_1, \ldots, \epsilon_m$. In our approach, we have to restrict the distribution of the systematic risk factors $\boldsymbol{r}$ to a class called the elliptical distributions. For the definition and some basic facts about elliptical distributions, we refer to the standard work of Fang, Kotz, and Ng (1987). For our purpose it is enough to note that the standard distributions used in classical risk-management problems are in fact from this class. We denote the covariance matrix and expectations of the distribution of $\boldsymbol{r}$ by Cov and $\boldsymbol{\mu}$. The distribution of the idiosyncratic risk factors may be arbitrary.

2.1 Plausible Scenarios

In a stress test of a loan portfolio, we imagine extreme realizations of one or more of the systematic risk factors. How would we quantify the plausibility of this thought experiment?

An intuitive approach could be to compare the extreme realization of a risk factor with its average. Intuitively, the further we are away from this average value, the less plausible the stress scenario becomes. The distance should be measured in standard deviations. For multivariate moves, the plausibility should depend additionally on the correlations. A multivariate move which is in agreement with the correlations is more plausible than a move against the correlations.

A statistical concept that formalizes these ideas is the so-called Mahalanobis distance given by

$$\mathrm{Maha}(\boldsymbol{r}) := \sqrt{(\boldsymbol{r} - \boldsymbol{\mu})^T \cdot \mathrm{Cov}^{-1} \cdot (\boldsymbol{r} - \boldsymbol{\mu})}.$$

The Mahalanobis distance is simply the distance of the test point $\boldsymbol{r}$ from the center of mass $\boldsymbol{\mu}$ divided by the width of the ellipsoid in the direction of the test point. Intuitively, Maha($\boldsymbol{r}$) can be interpreted as the number of standard deviations of the multivariate move from $\boldsymbol{\mu}$ to $\boldsymbol{r}$. Maha takes into account the correlation structure and the standard deviations of the risk factors.

In contrast to the previous literature, we define plausibility directly in terms of Maha($\boldsymbol{r}$): a high value of Maha implies a low plausibility of the scenario $\boldsymbol{r}$. Earlier work defined plausibility in terms of the *probability mass* of the ellipsoid of all scenarios of equal or lower Maha; see Studer (1997, 1999) or Breuer and Krenn (1999). This approach creates the problem of dimensional dependence. If one defined plausibility in terms of the ellipsoid containing some fixed probability mass, then the maximum loss would depend on the number of systematic risk factors, which is to some degree arbitrary. In our approach this problem does not occur.

2.2 Partial Scenarios

Typically, portfolios are modeled with hundreds or thousands of risk factors. Stress scenarios involving the full plethora of risk factors are hardly tractable numerically and overwhelmingly complex to interpret. A feasible answer to this problem is to design partial stress scenarios, which involve only a handful of risk factors. How should the other risk factors be treated?

Kupiec (1998) discussed four different ways to deal with the risk factors not fixed by some partial scenario:

- (i) The other systematic risk factors remain at their *last observed* value.
- (ii) The other macro risk factors take their *unconditional expectation* value.
- (iii) The other systematic risk factors take their *conditional expected value* given the values of the fixed risk factors. Denote by $\boldsymbol{r}_C$ the resulting vector of values of the systematic risk factors.
- (iv) The other systematic factors are not fixed but are distributed according to the conditional distribution given the values of the fixed risk factors. Denote by $\boldsymbol{r}_D$ the vector of values of the fixed systematic risk factors.

Our first result suggests a choice between these alternatives based on our concept of plausibility. The result says that the specification of partial scenarios as in method (iii) or (iv) both maximize plausibility. In the literature on stress testing of loan portfolios, Bonti et al. (2005) have suggested to use method (iv). This is indeed an approach that maximizes plausibility. From our result, we learn that we can achieve an equivalent plausibility by using the computationally more efficient approach (iii).

We state this result more formally in the following.

PROPOSITION 1. *Assume the distribution of systematic risk factors is elliptical, with density strictly decreasing as a function of* Maha. *Then:*

1. $\mathrm{Maha}(\boldsymbol{r}_C) = \mathrm{Maha}(\boldsymbol{r}_D)$.
2. *This is the* maximal *plausibility which can be achieved among* all *macro scenarios which agree on the fixed risk factors.*

A proof of this proposition is in the working paper Breuer et al. (2009b). This proposition is of high practical relevance. It is the basis of partial-scenario analysis. It implies that two choices of macro stress distributions are preferable, namely (iii) or (iv). Assigning to the nonfixed risk factors other values than the conditional expected values given the fixed risk factors leads to less plausible macro stress scenarios.

2.3 Severe Scenarios

An important disadvantage of stress testing with hand-picked scenarios is the danger of ignoring harmful but plausible scenarios. This may create an illusion of safety. A way to overcome this disadvantage is to search systematically for those macro scenarios in some plausible admissibility domain which are most harmful to the portfolio. By searching systematically over admissible domains of plausible macro scenarios, one can be sure not to ignore any harmful but plausible scenarios. This is our approach to construct a stress test: find the relevant scenarios which are most harmful yet above some minimal plausibility threshold. This problem can be formulated as an optimization problem which can be solved numerically by using an algorithm of Pistovčák and Breuer (2004).

The admissibility domain is determined by our concept of plausibility. It contains all scenarios with $\mathrm{Maha}(\boldsymbol{r})$ below a threshold k:

$$\mathrm{Ell}_k := \{\boldsymbol{r} : \mathrm{Maha}(\boldsymbol{r}) \leq k\}.$$

Geometrically, this domain is an ellipsoid whose shape is determined by the covariance matrix of the systematic risk factors.

Partial scenarios do not specify a unique portfolio value but just a distribution, namely the distribution of portfolio values conditional on the values of the risk factors fixed by the scenario. In order to measure the severity of scenarios, one needs to quantify the severity of the corresponding conditional portfolio value distribution. In this paper we use the expectation value, although other risk measures could be used as well. Thus we call a partial scenario severe if it has a low conditional expected profit (CEP). To sum up, our stress-testing method amounts to solving the following optimization problem:

$$\min_{\boldsymbol{r} \in \mathrm{Ell}_k} \mathrm{CEP}(\boldsymbol{r}).$$

The difference between the lowest CEP in the admissibility domain and the CEP in the expected scenario is the maximum expected loss in the admissibility domain. This concept of maximum loss overcomes the problem of dimensional dependence that we mentioned in the introduction. Maximum expected loss over the admissibility domain Ell_k is not affected by excluding or including macro risk factors that are irrelevant to the portfolio value (see Breuer 2008).

What is the advantage of this worst-case search over standard stress testing? First, it achieves a controlled trade-off between plausibility and severity of scenarios. If we want to get more severe scenarios, we choose a higher k and get less plausible worst-case scenarios. If we want to get more plausible scenarios, we choose a lower k and get less severe worst-case scenarios. Second, it overcomes the historic bias by considering all scenarios that are plausible enough. In this way we can be sure not to miss scenarios that are plausible but did not yet happen in history. Thirdly, worst-case scenarios reflect portfolio-specific dangers. What is a worst-case scenario for one portfolio might be a harmless scenario for another portfolio. This is not taken into account by standard stress testing. Portfolio-specific dangers suggest possible counteraction to reduce risk if desired.

2.4 Scenarios Suggesting Risk-Reducing Action

Risk-reducing action is suggested by identifying the key risk factors that contribute most to the expected loss in the worst-case scenario. We define key risk factors as the risk factors with the highest maximum loss contribution (MLC). The loss contribution (LC) of risk factor i to the loss in some scenario $\boldsymbol{r}$ is

$$LC(i, \boldsymbol{r}) := \frac{CEP(\boldsymbol{\mu}) - CEP(\mu_1, \ldots, \mu_{i-1}, r_i, \mu_{i+1}, \ldots \mu_n)}{CEP(\boldsymbol{\mu}) - CEP(\boldsymbol{r})}, \qquad (1)$$

if $CEP(\boldsymbol{r}) \neq CEP(\boldsymbol{\mu})$. $LC(i, \boldsymbol{r})$ is the loss if risk factor i takes the value it has in scenario $\boldsymbol{r}$, and the other risk factors take their expected values $\boldsymbol{\mu}$, as a percentage of the loss in scenario $\boldsymbol{r}$. In particular, one can consider the worst-case scenario, $\boldsymbol{r} = \boldsymbol{r}^{WC}$. In this case the loss contribution of some risk factor i can be called the maximum loss contribution:

$$MLC(i) := LC(i, \boldsymbol{r}^{WC}). \qquad (2)$$

$MLC(i)$ is the loss if risk factor i takes its worst-case value and the other risk factors take their expected values, as a percentage of maximum loss.

The maximum loss contributions of the macro risk factors in general do not add up to 100 percent. Sometimes the sum is larger; sometimes it is smaller. If this sum is equal to one, the loss in the scenario is exactly equal to the sum of losses from individual risk-factor moves. This happens if and only if the risk factors do not interact.

Proposition 2. *Assume CEP as a function of the macro risk factors has continuous second-order derivatives. The loss contributions of the risk factors add up to 100 percent for all scenarios* $\boldsymbol{r}$*,*

$$\sum_{i=1}^{n} LC(i, \boldsymbol{r}) = 1,$$

if and only if CEP is of the form

$$CEP(r_1, \ldots, r_n) = \sum_{i=1}^{n} g_i(r_i). \tag{3}$$

This is the case if and only if all cross-derivatives of CEP,

$$\frac{\partial^2 CEP(\boldsymbol{r})}{\partial r_i \partial r_j} = 0,$$

vanish identically for $i \neq j$*.*

For the proof, we refer to the working paper of Breuer et al. (2009b). This characterization has a substantial practical relevance. The sum of loss contributions measures the role of interaction of systematic risk for the portfolio value. If the sum is larger than one, the interaction between risk factors is positive. The total loss in the scenario is smaller than the sum of losses from individual risk-factor moves.

Most dangerous is the case of negative interaction between risk factors. If the sum is smaller than one, the total loss in the scenario is larger than the sum of losses from individual risk-factor moves. The harm of the scenario cannot be fully explained by individual risk-factor moves. The simultaneous move of some risk factors causes harm on top of the single risk-factor moves. In this case it will be necessary to consider maximum loss contributions not of single risk-factor moves but of pairs or even of larger groups of risk factors.

A consequence of this insight outside of the stress-testing problem is that it reveals a weakness in current regulatory thinking. Analyzing portfolio risk along the categories market and credit risk, and determining risk capital based on the aggregation of these separately calculated risk numbers, may in fact underestimate the true portfolio risk because it ignores the risks stemming from simultaneous moves in market and credit-risk factors. For a detailed discussion of this problem see Breuer et al. (2009a).

Possible risk-reducing action can be designed with knowledge of the key risk factors. One strategy could be to buy hedges that pay off exactly when the key risk factors take their worst-case value. Another, more comprehensive but also more expensive strategy is to buy hedges that neutralize the harm done not just by the worst-case moves of the key risk factors but by all moves of the key risk factors. For the example of the foreign currency loan portfolios discussed in the next section, this strategy is demonstrated in Breuer et al. (2008).

3. Application: Stress Testing a Portfolio of Foreign Currency Loans

We now illustrate the concepts and their quantitative significance in an example: a stress test for a portfolio of adjustable-rate loans in home (EUR) or foreign (CHF) currency. In the current downturn, the additional risk of foreign currency plays a major role in some CEE economies. Our sample portfolio consists of loans to 100 borrowers in the rating class B+, corresponding to a default probability of $p_i = 2$ percent, or in rating class BBB+, corresponding to a default probability of $p_i = 0.1$ percent. At time 0, in order to receive the home currency amount $l =$ €10,000, the customer of a foreign currency loan takes a loan of $le(0)$ units in a foreign currency, where $e(0)$ is the home currency value of the foreign currency at time 0. The bank borrows $le(0)$ units of the foreign currency on the interbank market. After one period, at time 1, which we take to be one year, all the loans expire and the bank repays the foreign currency at the interbank market with an interest rate r_f (e.g., LIBOR), and it receives from the customer a home currency amount that is exchanged at the rate $e(1)$ to the foreign currency amount covering repayment of the principal plus interest rolled over from four quarters, plus a spread s. So the borrower's payment obligation to the bank at time 1 in home currency is

$$o_f = l(1 + r_f)E + s_f\ l\,E \tag{4}$$

$$o_h = l(1 + r_h) + s_h\ l \tag{5}$$

for the foreign and home currency loan $E := e(0)/e(1)$ is the exchange rate change between times 0 and 1. The first term on the right-hand side is the part of the payment that the bank uses to repay its own loan on the interbank market. The second term is profits remaining with the bank. For all loans in the portfolio, we assume they expire at time 1. The model can be extended to a multiperiod setting allowing for loans not maturing at the same time and requiring payments at intermediate times.

In order to evaluate idiosyncratic and systematic risk of a portfolio of such loans, we use a one-period structural model specifying default frequencies and losses given default endogenously. For details of the model, we refer to Breuer et al. (2008). The basic structure of the model is given by the payment obligation distribution derived from the payment obligation function (4) and a log-normal payment ability distribution, which involves log-normally distributed idiosyncratic changes and an additional dependence of the mean one future GDP changes. (Pesaran, Schuermann, and Treutler 2005 use a model of this type for the returns of firm value.) Each customer defaults in the event that their payment ability at the expiry of the loan is smaller than their payment obligation. In the case of default, the borrower pays what he or she is able to pay. The difference to the payment obligation first is lost profit and then loss for the bank.

The spread s_h (resp. s_f) and the variance of the idiosyncratic payment ability changes are determined jointly in a calibration procedure. The first calibration condition ensures that the model default probability coincides with the default probability determined in some external rating procedure. The second calibration condition ensures that expected profit from each loan reaches some target level of €160, which amounts to a return of 20 percent on a regulatory capital of 8 percent. Both calibration conditions depend on the spread s and the variance of the idiosyncratic payment ability changes.

The systematic risk factors entering the portfolio valuation are GDP, the home interest rate r_h and the foreign interest rate r_f, and the exchange rate change E. The probability law driving these risk factors is modeled by a time-series model that takes account of economic interaction between countries and regions. Estimating the parameters of this model, we can simulate scenarios for the systematic risk factors. For details of this model, known in the literature as the global VAR model, see Pesaran, Schuermann, and Weiner (2001), Pesaran, Smith, and Smith (2005), Garrett, Pesaran, and Shin (2006), and Dées et al. (2007).

The profit distribution was calculated in a Monte Carlo simulation by generating 100,000 scenario paths of four steps each. The resulting distribution of risk factors after the last quarter, which is not normal, was used to estimate the covariance matrix of one-year macro risk-factor changes. In each macro scenario, defaults of the customers were determined by 100 draws from the idiosyncratic changes in the payment ability distribution. From these we evaluated the profit distribution at the one-year time horizon.

3.1 Hand-Picked Versus Systematic Stress Tests

Let us compare the severity of the hand-picked scenario "GDP shrinks by 3 percent," which is a 5.42σ event, with the worst-case scenario of the same plausibility. Conditional expected profits (CEP) for the standard scenario "GDP -3 percent and other risk factors at their conditional expected value" and of worst-case scenarios of the same plausibility are shown in table 1.

We observe that for all portfolios the conditional expected profits are considerably lower in the worst-case scenarios than in the hand-picked GDP scenario. This is evidence of the danger that lies in relying solely on hand-picked scenarios. Expected profits in this rather extreme hand-picked GDP scenario are only moderately lower, namely by amounts between €129 and €1,751 on a loan portfolio worth €1 million giving an unconditional expected profit of €16,000. These moderate profit reductions in such an extreme scenario might provide a feeling of safety. But this is an illusion. There are other scenarios out there that are equally plausible but much more harmful. There are scenarios that reduce expected profit by amounts between €374 (resp. €2,709) for the home currency loans,

Table 1. Comparison of the Severity of the Hand-Picked Scenario with the Worst-Case Scenario of the Same Plausibility

Scenario	Maha	CEP
Foreign B+		
Expected	0	16,001
GDP −3%	5.42	15,950
Worst Case	5.42	−98,101
Foreign BBB+		
Expected	0	15,999
GDP −3%	5.42	15,870
Worst Case	5.42	−95,591
Home B+		
Expected	0	16,000
GDP −3%	5.42	14,249
Worst Case	5.42	13,291
Home BBB+		
Expected	0	16,000
GDP −3%	5.42	15,811
Worst Case	5.42	15,626

and by €114,101 (resp. €111,591) for the foreign currency loan portfolios. These huge losses of roughly 11 percent are higher than the total regulatory capital of 8 percent for the loan portfolio.

3.2 Key Risk Factors and Risk-Reducing Actions

What is a worst-case scenario for one portfolio might be a harmless scenario for another portfolio. This is not taken into account by standard stress testing. Stress testing is relevant only if the choice of scenario takes into account the portfolio. In a systematic way, this is done by worst-case search.

Key risk factors are ones with highest maximum loss contributions (MLCs). The worst-case scenarios, together with the MLC for each risk factor, are given in table 2 for different sizes of the admissibility domain. For each scenario, the risk factors with the highest MLCs are printed in bold face. These results identify which risk factor is key for which portfolio.

Table 2. Systematic Macro Stress Tests of the Home and Foreign Currency Loan Portfolios

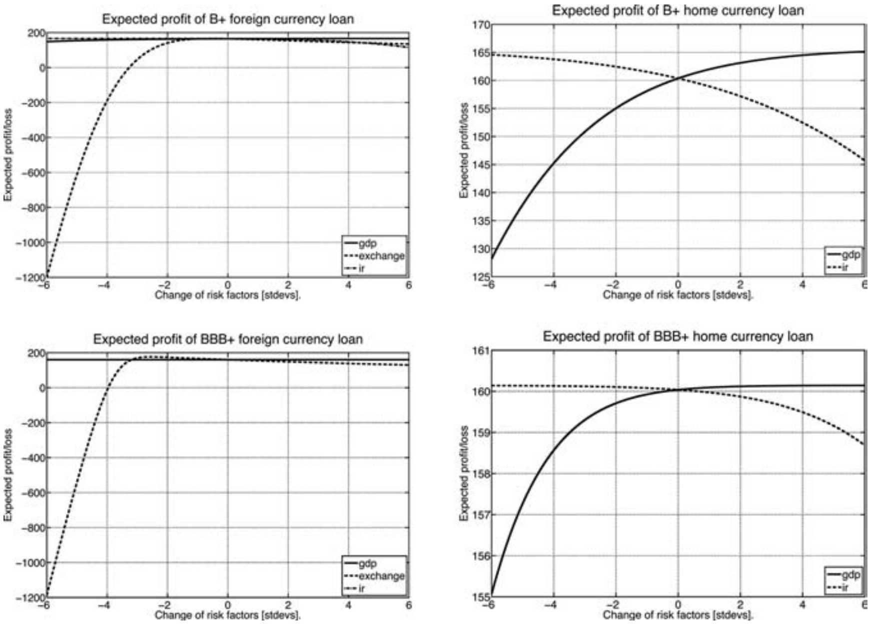
	Worst Macro Scenario												
Maximum Maha	**GDP**			**Home IR**			**Foreign IR**			**CHF/€**			**CEP**
	Abs.	**St. Dev.**	**MLC**	**Abs.**	**St. Dev.**	**MLC**	**Abs.**	**St. Dev.**	**MLC**	**Abs.**	**St. Dev.**	**MLC**	
Foreign B+													
2	231.64	−0.14	0.5%				0.022	0.04	0.4%	1.646	2.00	**100.0%**	15,400
4	230.98	−0.10	0.1%				0.039	1.17	0.4%	1.306	−3.78	**65.3%**	−26,084
6	230.86	−0.03	0.0%				0.046	1.59	0.2%	1.191	−5.74	**77.0%**	−136,000
Foreign BBB+													
2	231.65	−0.14	0.0%				0.022	0.03	0.0%	1.646	2.00	**100.0%**	14,855
4	231.47	−0.27	0.0%				0.022	0.07	0.0%	1.765	4.00	**100.0%**	13,859
6	230.83	−0.04	0.0%				0.045	1.58	0.0%	1.191	−5.74	**75.8%**	−135,203
Home B+													
2	228.31	−1.30	**70.0%**	0.040	1.07	18.5%							15,482
4	224.70	−2.64	**65.7%**	0.045	2.10	13.5%							14,458
6	221.04	−4.01	**62.0%**	0.049	3.10	9.9%							12,676
Home BBB+													
2	228.28	−1.31	**63.9%**	0.040	1.06	12.9%							15,969
4	224.65	−2.67	**54.2%**	0.045	2.07	6.4%							15,848
6	220.99	−4.03	**46.8%**	0.049	3.07	3.3%							15,476

Notes: We search for the macro scenarios with the worst expectation value of the profit distribution, under the condition that macro scenarios lie in elliptical admissibility domain of maximal Mahalonobis radius k. Macro scenarios are specified by the macro risk factors GDP, exchange rate, and interest rates. We give the absolute values of these risk factors in the worst-case scenario, as well as their change in standard deviations and their maximum loss contributions (MLC). For the key risk factors, MLC is printed in bold face.

- For the foreign currency loan portfolio, the exchange rate is clearly the key risk factor. This becomes apparent from table 2. In the worst-case scenario, the FX rate alone contributes between 65.3 percent and 100 percent of the losses in the worst-case scenarios; the other risk factors contribute less then 1 percent. This indicates that the FX rate is the key risk factor of the foreign currency loan portfolio.
- For the home portfolio, GDP is the key risk factor. The moves in GDP alone contribute between 46.8 percent and 70.0 percent of the losses in the worst-case scenarios. The MLC of the home interest rate is comparatively small. The negative interaction between GDP and interest rate moves explains about one-third of the worst-case loss—more for larger k, less for smaller k.
- There is another interesting effect. The dependence of expected profits of foreign currency loans on the CHF/€ rate is not only nonlinear but also not monotone. For the BBB+ FX loan portfolio (bottom left plots in figure 1), focusing on changes smaller than 4σ it becomes evident that a small increase in the exchange rate has a positive influence on the portfolio value, but large increases have a very strong negative influence. Correspondingly, in table 2, if we restrict ourselves to small moves (Maha smaller than 4σ), the worst-case scenario is in the direction of increasing exchange rates, but if we allow larger moves, the worst-case scenario is in the direction of decreasing exchange rates. This effect also shows up in the worst macro scenarios of table 2. The reason for this nonmonotonicity is that a small decrease in the FX rate increases the EUR value of spread payments received. For larger moves of the FX rate, this positive effect is outweighed by the increases in defaults due to the increased payment obligations of customers. For the bad-quality B+ portfolios, the positive effect of a small FX rate decrease persists only up to a maximal Maha radius of $k = 2$.

The diagnosis that the FX rate is the key risk factor for the foreign currency loans and GDP is the key risk factor for the home currency loans is confirmed by the right- and left-hand plots in figure 1, which show the expected profits in dependence of single

Figure 1. Key Risk Factors of Foreign and Home Currency Loans



Notes: This figure shows expected profit or loss of a single foreign (left) and home currency (right) loan as a function of changes of the macro risk factors with other macro risk factors fixed at their expected values. At the top are B+ loans. At the bottom are BBB+ loans. The left-hand plots show that for the foreign portfolio the exchange rate is the key risk factor. We also observe the negative effect of small foreign currency depreciations, which is particularly pronounced for the BBB+ portfolio. The right-hand plots show that for the home portfolio GDP is the key risk factor. Note the different scales of the two plots.

macro risk-factor moves, keeping the other macro risk factors fixed at their expected values. Note the different scales of the two plots. Expected losses of the FX loan are considerably larger than for the home currency loan. This plot also shows that expected profits of both loan types depend nonlinearly on the relevant risk factors. The profiles of expected profits in figure 1 resemble those of short options. A home currency loan behaves like a short put on GDP together with a short call on the home interest rate. From the point of view of the bank, a foreign currency loan behaves largely like a short call on the FX rate.

One could ask why the effort to search for worst-case scenarios is necessary to identify key risk factors. Wouldn't it be easier to read the key risk factors from the plots in figure 1? This would be true if losses from moves in different risk factors *added up*. But for certain kinds of portfolios, the worst case is a *simultaneous* move of several risk factors—and the loss in this worst case might be considerably worse than adding up the losses resulting from moves in single risk factors. This is the message of proposition 2. The effects of simultaneous moves are not reflected in figure 1, but they do show up in the worst-case scenario.

As an example, consider a B+ home currency loan and assume we are restricting ourselves to moves with Maha smaller than $k = 6$. From table 2 we see that the MLC of the two risk factors sum up to 62.0 percent + 9.9 percent = 71.9 percent, which is considerably lower than 100 percent. This indicates that the loss of a joint move is considerably larger than the sum of losses of individual risk-factor moves. This is not reflected in figure 1, which only displays the effects of single risk-factor moves.

The same occurs for foreign currency loans. They show a dangerous interaction of market and credit risk. At $k = 4$ the exchange rate has an MLC of 65.3 percent, the interest rate has an MLC of 0.4 percent, and GDP has an MLC of 0.1 percent, which is a total of 65.8 percent instead of 100 percent. Single risk-factor moves leave about 35 percent of the maximum loss unexplained. The reason is that adverse exchange rate moves drive up payment obligations. This increases default probabilities and losses given default.

The identification of key risk factors suggests risk-reducing counteractions. Knowing that the exchange rate is the key risk factor for FX loans, one can plot the behavior of CEP in dependence of exchange rate moves, as in the left-hand plots of figure 1. Breuer et al. (2008) show how FX derivatives can be used to construct hedges reducing the exchange rate risk of foreign currency loans. It turns out that FX options can be used to virtually eliminate the dependence of expected loss on exchange rates—at some fixed level of interest rates and other macroeconomic factors. But the hedge is not perfect: Firstly, it cannot fully remove dependence of expected losses on exchange rates at other levels of interest rates, and secondly, it can bring to zero only the expectation but not the variance of losses caused by adverse exchange rate moves.

4. Conclusion

The central message of our paper is that the three principles of the Basel Committee on Banking Supervision (2005) required for stress tests—*plausibility, severity* of stress scenarios, and *suggestiveness of risk-reducing action*—can be systematically implemented within a standard quantitative risk-management framework. In order to do so, we need a measure of plausibility that can be formulated using the probability distribution of the risk factors but that does not suffer from the dimensional dependence of maximum loss. We show that this concept of plausibility can be formulated by working with regions of a given Mahalanobis radius rather than working with regions of given probability mass. We need to replace the common practice of hand-picked scenarios with a systematic worst-case search over the given region of plausibility. Finally, we have to identify the key risk factors and their contributions to maximum loss. The key contribution to maximum loss may only be revealed if we take into account simultaneous moves in risk factors.

Our approach has three major advantages compared with standard stress tests. First, it ensures that no harmful scenarios are missed and therefore prevents a false sense of safety. Second, it does not analyze scenarios that are too implausible and would therefore jeopardize the credibility of stress analysis. Third, it allows for a portfolio-specific identification of key risk factors. We hope that the compatibility of our concepts with the standard quantitative risk-management framework used by practitioners makes the insights of this paper useful in practical stress-testing problems.

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