Volume 3, Number 3 September 2007

IJCB
INTERNATIONAL JOURNAL OF CENTRAL BANKING

International Journal of Central Banking

Volume 3, Number 3 September 2007

The contents of this journal, together with additional materials provided by article authors, are available without charge at ***www.ijcb.org***.

The *International Journal of Central Banking* is published quarterly (ISSN: 1815-4654). Online access to the publication is available free of charge at **www.ijcb.org**. Individual print subscriptions are available at an annual rate of $100 (USD).

Print subscription orders may be placed online at www.ijcb.org, by phone (202-452-3245), via fax (202-728-5886), or by e-mail (editor@ijcb.org).

Renewals, claims, address changes, and requests for permission to reprint material from this journal should be addressed to:

International Journal of Central Banking
Publications Fulfillment, MS-127
Federal Reserve Board
Washington, DC 20551

Phone: 202-452-3245
Fax: 202-728-5886
E-mail: editor@ijcb.org

ISSN: 1815-4654

Interest Rate Setting by the ECB, 1999–2006: Words and Deeds*

Stefan Gerlach
Institute for Monetary and Financial Stability
Johann Wolfgang Goethe University, Frankfurt am Main

We estimate empirical reaction functions for the European Central Bank (ECB) with ordered-probit techniques, using the ECB's *Monthly Bulletin* to guide the choice of variables. The results show that policy reacts to the state of the real economy, M3 growth, and exchange rate changes but not to inflation. We develop quantitative indicators of the Governing Council's assessment of economic conditions to understand its interest rate decisions and argue that the ECB has not reacted to inflation shocks because they were seen as temporary. By contrast, policy responses to economic activity are strong because it impacts on the outlook for inflation.

JEL Codes: E43, E52, E58.

1. Introduction

A number of authors have studied the interest-rate-setting behavior of the Governing Council of the European Central Bank (ECB) by estimating empirical reaction functions.[1] However, it is unclear

*I am grateful to participants at the 2005 Konstanz Seminar and the Second HKIMR Summer Workshop (in particular, my discussants, Klaus Adam and Corrinne Ho); to participants at seminars at the Austrian National Bank, the Bank for International Settlements, the Bundesbank, the European Central Bank, De Nederlandsche Bank, and the University of Frankfurt; and to Katrin Assenmacher, Michael Chui, Hans Genberg, Petra Gerlach, Edi Hochreiter, Paul Mizen, John Taylor, and Cees Ullersma for comments. E-mail: stefan.gerlach@wiwi.uni-frankfurt.de.

[1]The literature estimating reaction functions has grown too large to survey here. See Berger, de Haan, and Sturm (2006) and Carstensen (2006) for recent contributions. The working paper version of this paper (Gerlach 2004) contains a review of the early literature on estimating empirical reaction functions on euro-area data.

whether studies that focus solely on the ECB's *deeds*—its policy actions—can be fully informative about the way the Governing Council sets interest rates. Estimates of reaction functions in which policy-controlled interest rates are regressed on macroeconomic variables disregard the fact that policymakers' assessment of these variables may vary over time. For instance, the extent to which central banks react to movements in inflation is likely to depend on whether they expect the movements to be temporary or permanent. To understand the ECB's policy decisions, it is therefore helpful to consider how the Governing Council interprets incoming data by considering its public statements regarding macroeconomic developments—that is, by also studying the *words* of the ECB.

This paper seeks to do so. In particular, it extends the literature on empirical reaction functions for the euro area by using information from the statements made in the ECB's *Monthly Bulletin* to develop indicators capturing the Governing Council's assessment of inflation pressures, developments in real economic activity, and M3 growth. The paper studies how these indicators evolve over time, what factors explain them, and how they are related to decisions to change the repo rate, the ECB's main monetary policy instrument.

The indicators are constructed by reading the editorials in the ECB's *Monthly Bulletin.* Doing so also clarifies what variables the Governing Council does or does not respond to in conducting policy. For instance, empirical reaction functions for the euro area typically use a measure of the output gap constructed using monthly industrial production data to explore how the ECB responds to changes in real activity. However, the editorials never refer to output gaps and suggest instead that the Governing Council attaches great weight to business and consumer confidence and survey measures of expected output growth. For this reason we use measures of economic sentiment, constructed by the European Commission, and of expected real GDP growth, constructed from data reported in *The Economist.* Interestingly, these variables are much more significant in the regressions than output gaps that are traditionally used to capture the state of the economy.

The rest of the paper is organized as follows. Section 2 provides a brief review of the related literature that analyzes the ECB's statements. Section 3 looks at the ECB's deeds by estimating reaction functions using ordered-probit techniques. Interestingly, we find that while the ECB has not responded to (past) headline or core inflation, it has reacted to the state of the real economy, the rate of growth of M3, and the rate of change of the nominal effective exchange rate of the euro. We also find that a change in the interest rate in the past month *reduces* the likelihood of a change this month. Interest rate changes thus seem to be made in order to "clear the air"—that is, to reduce the need for further changes in the immediate future. There is thus little evidence of interest rate smoothing.

Section 4 turns to the ECB's words. We construct indicators using the editorials in the ECB's *Monthly Bulletin* in order to capture how the Governing Council judges economic developments and the risks to price stability. Moreover, we study how the indicator variables are correlated with economic conditions. We find that the indicator variable for inflation is not correlated with (past) inflation but is correlated with real economic activity, M3 growth, and changes in the nominal effective exchange rate of the euro. This latter finding suggests that the reason inflation is insignificant in the estimated reaction functions is that the Governing Council has interpreted movements in inflation as being temporary and due to price-level shocks.

In section 5 we study how the probabilities of the different policy choices evolve over the sample period. Since M3 growth was significant in the empirical reaction functions, we also investigate how money growth has an impact on the probability of interest rate changes. The results show that while money growth is not an important factor explaining repo-rate changes under normal economic conditions, it plays an important role in situations in which real economic activity is strong.

Finally, section 6 concludes.

2. Related Literature

This paper argues that in seeking to understand the interest-rate-setting behavior of the ECB, it is useful to consider the information about policymakers' assessment of economic conditions that is contained in the ECB's official communications. While the paper is part of the literature on empirical reaction functions for the euro area, in the interest of space, below we focus on papers studying the information contained in the introductory statements made by the president of the ECB at the monthly press conferences following the meetings of the Governing Council. Some authors analyze the reaction of financial markets to this information. For instance, Rosa and Verga (2005) use a glossary to convert the statements into an ordered scale and find that forward interest rates respond to the introductory statements, even when controlling for changes in repo rates. Musard-Gies (2006) also codes the information in the statements and studies how the term structure of interest rates reacts to it.[2]

Another set of papers uses the information in the press statements to understand the ECB's interest rate setting. Rosa and Verga (2007) extend their earlier analysis and show that the statements contain information useful for forecasting future changes in monetary policy in the euro area, and that this information is not contained in macroeconomic aggregates or market interest rates. Berger, de Haan, and Sturm (2006) also quantify the information in the introductory statements. They distinguish between statements concerning price stability, the real economy, and monetary factors, and study how they account for the Governing Council's interest rate decisions. One finding of importance for the current paper is that monetary factors do not appear to play an important role in the setting of monetary policy. Heinemann and Ullrich (2005) also quantify the information in the introductory statements and find that the resulting variable is significant in an empirical reaction function for the euro area.

While related to the literature reviewed above, this paper uses the information in the ECB's statements to study how the Governing Council's assessment of economic conditions varies with objective measures of those conditions. This is an important question that is likely to shed light on the ECB's thinking about the economy. For instance, in most years since the introduction of the euro, euro-area inflation has exceeded 2 percent, which is the upper limit of the ECB's definition of price stability, and many observers have noted that the ECB appears to react strongly to economic activity

[2]In a related literature, Ehrmann and Fratzscher (2005a, 2005b, 2005c) study the communication of central bank committee members through speeches, testimony, etc., and analyze its impact on interest rates and the predictability of monetary policy.

but not to inflation.[3] While this may be interpreted as the ECB's having been willing to risk overshooting its inflation objective in order to stabilize economic activity, the analysis here suggests that the ECB has viewed movements in inflation as reflecting price-level shocks that have temporary effects on inflation and has therefore not reacted to them. By contrast, it has reacted strongly to economic activity because it sees it as an important determinant of the outlook for inflation.

3. Deeds: What the ECB Does

We start by studying the ECB's interest rate decisions—its deeds—by estimating empirical reaction functions. This section discusses the model estimated, the choice of variables, and the econometric findings.

3.1 The Model

Since the Governing Council leaves the repo rate unchanged in most months and changes it by a discrete amount when it judges it necessary, it is inappropriate to fit the model using OLS. Therefore, below we estimate ordered-probit models using data for the period February 1999 through June 2006.[4] As a first step, we consider the pattern of interest rate changes. Table 1 shows that there was no change in the repo rate in seventy-one of the eighty-nine months in the sample (or 80 percent) and that it was raised ten times and cut eight times. On eleven occasions the change was ±0.25 percent and on seven occasions it was ±0.50 percent. Since the size of policy changes varies over time, below we distinguish between "small" and "large" changes in interest rates. Interestingly, the table also shows that while increases tended to be small, cuts tended to be large.

Next we derive the equation estimated below. Let i_t denote the repo rate and i_t^T the Governing Council's "target" for the repo rate. These may differ because the ECB and most other central banks

[3] For instance, see the discussion in Carstensen (2006, footnote 14).

[4] See Ruud (2000) and Greene (2003) for a discussion of ordered probits. See Galí et al. (2004) and Carstensen (2006) for applications to the ECB. Kim, Mizen, and Thanaset (2005) estimate ordered-logit models for the Bank of England.

Table 1. Changes in Repo Rate: February 1999–June 2006 (Eighty-Nine Observations)

	Small Change (±25 Basis Points)	Large Change (±50 Basis Points)	Subtotal
Increase	8	2	10
Decrease	3	5	8
Subtotal	11	7	Total: 18

set interest rates at discrete levels, typically 0.25 percent apart, and because of interest rate smoothing. Let π_t, y_t, μ_t, and ε_t denote (some measure of) inflation, real economic activity, money growth, and the rate of appreciation of the nominal effective exchange rate. Consider next the following expression for the target level for the interest rate:

$$i_t^T = \alpha_y y_t + \alpha_\pi \pi_t + \alpha_\mu \mu_t + \alpha_\varepsilon \varepsilon_t, \tag{1}$$

where the constant is omitted; α_y, α_π, and α_μ are positive; and α_ε is negative.[5] Next, we allow for gradual adjustment of the actual interest rate as in Judd and Rudebusch (1998):

$$i_t - i_{t-1} = \beta_0 \big(i_t^T - i_{t-1}\big) + \beta_1 \Delta i_{t-1} + e_t, \tag{2}$$

where the constant is omitted and e_t is a residual. Equation (2) implies that changes in interest rates should be distributed continuously. However, because the ECB sets interest rates in steps, only discrete changes are observed. Using equations (1) and (2), and incorporating the fact that the ECB sets interest rates in steps, we have

$$i_t^* - i_{t-1} = \tilde{\alpha}_y y_t + \tilde{\alpha}_\pi \pi_t + \tilde{\alpha}_\mu \mu_t + \tilde{\alpha}_\varepsilon \varepsilon_t - \beta_0 i_{t-1} + \beta_1 \Delta i_{t-1} + e_t, \tag{3}$$

[5]Svensson (1997) presents a simple model in which the target interest rate depends on the state of the economy, as measured by the output gap, and the deviation of inflation from the central bank's target or objective.

where $\tilde{\alpha}_i \equiv \alpha_i\beta_0$ and the asterisk, *, indicates that the interest rate should be thought of as an unobserved, or latent, variable.[6] What is observed is the actual change in the interest rate, which depends on where the latent variable is relative to a set of threshold values, γ_i:

$$\begin{aligned}
\Delta i_t &= -0.50\% && \text{if } i_t^* - i_{t-1} \leq \gamma_1 \\
\Delta i_t &= -0.25\% && \text{if } \gamma_1 < i_t^* - i_{t-1} \leq \gamma_2 \\
\Delta i_t &= 0 && \text{if } \gamma_2 < i_t^* - i_{t-1} \leq \gamma_3 \\
\Delta i_t &= +0.25\% && \text{if } \gamma_3 < i_t^* - i_{t-1} \leq \gamma_4 \\
\Delta i_t &= +0.50\% && \text{if } \gamma_4 < i_t^* - i_{t-1}.
\end{aligned} \tag{4}$$

Equations (3) and (4) constitute an ordered-response model that says that the Governing Council will adopt one of the policy options depending on the level of inflation, economic activity, money growth, the rate of appreciation, and the lagged level (and the lagged change) of the repo rate.

Below we estimate the model, reporting the parameter estimates, the value of the likelihood function, and the McFadden pseudo-R^2.[7] In addition, we show p-values from tests of the hypothesis of no first-order serial correlation in the residuals, constructed as suggested by Gourieroux, Monfort, and Trognon (1985, 326).

3.2 Data

Next we describe our choice of data, which, unless otherwise noted, was taken from the ECB's web site. As noted above, the lagged level of the repo rate and the change in the repo rate are used as regressors in the equations we estimate. While the *Monthly Bulletin* suggests that money and credit growth both are important in the Governing Council's thinking about policy, the emphasis put on M3 growth in the ECB's public statements suggests that it is the single most important indicator of monetary developments.

[6]This formulation differs from the dynamic-probit models estimated by Eichengreen, Watson, and Grossman (1985) and Davutyan and Parke (1995), who assume that $\Delta i*$ depends on observables.

[7]Greene (2003, 683) discusses the McFadden pseudo-R^2.

We therefore concentrate on this variable in the econometric analysis. Since the editorials suggest that the Governing Council's deliberation focuses on the three-month moving average of the annual rate of M3 growth, this definition is used in the empirical analysis below.

The choice of the inflation variable is less clear cut. It seems natural to use headline inflation computed using the Harmonized Index of Consumer Prices (HICP) in the euro area. However, inflation rates across the world have been subject to large energy-price shocks in recent years, which central banks can presumably disregard since they should arguably be seen as price-level shocks that have a temporary effect on inflation. It is therefore of interest to consider a measure of core inflation in the regressions. While the ECB never uses the term *core inflation*, in discussing inflation pressures it frequently refers to a measure of the HICP excluding fresh-food and energy prices. We consequently use this variable as a measure of core inflation. Finally, since monetary policy is forward looking, another natural possibility would be to use a measure of expected inflation. We therefore construct a measure of expected inflation over the coming twelve months, using data from the polls of forecasters tabulated in *The Economist*.[8]

Following Heinemann and Ullrich (2005), we also explore whether the Governing Council has reacted to the exchange rate by including in the reaction function the percentage change over twelve months in the nominal effective exchange rate of the euro against a basket of forty-three currencies. It should be noted that this variable is defined such that an increase indicates an appreciation of the euro.

The issue of selecting a measure of real economic activity is more complicated and is discussed in the next section.

[8] *The Economist* surveys forecasts of inflation and real output growth for this year and the next made by a number of financial institutions, and publishes the means of these forecasts on a monthly basis. Following Begg et al. (1998) and Alesina et al. (2001), we compute measures of expected inflation and real output growth for the coming twelve months as a weighted average of the two forecasts, with the weights depending on the month in which the forecasts are made. To illustrate, the expected rate of inflation in February is computed as 10/12 of the expected rate of inflation for this year and 2/12 of the expected rate of inflation for next year.

3.3 Measuring Real Economic Activity

Following the seminal paper by Taylor (1993), the empirical literature on monetary-policy reaction functions focuses on the role of the output gap as the measure of real economic activity best able to explain interest rate decisions taken by central banks. However, the national accounts are released with considerable delay and are subject to one or more revisions. Comments in the editorials on the behavior of real GDP therefore typically refer to developments that occurred some time ago. For instance, the March 2004 editorial states, "According to Eurostat's first estimate, in the fourth quarter of 2003 real GDP in the euro area grew by 0.3% quarter on quarter, following growth of 0.4% in the third quarter. These data confirm that a gradual recovery in economic activity in the euro area took place in the second half of 2003. More recent indicators, including those from business and consumer surveys, point to a moderate economic growth also in early 2004."

Since output gaps consequently can only be constructed with long time lags and are highly uncertain, they are never discussed in the editorials and do not appear to play much of a role in the ECB's interest rate setting (although, of course, they may be highly significant in empirical reaction functions).[9,10] By contrast, and as indicated by the quote above, the editorials frequently comment on survey measures of economic conditions, which are typically available with very short lags and are never updated. If subjective measures of economic activity such as these are strongly correlated with estimates of the output gap, it would be sensible for the ECB to rely on them in thinking about the state of the economy and consequently appropriate for applied econometricians to focus on them in modeling interest rate setting in the euro area.

In the econometric analysis below we consider an economic sentiment indicator, which is developed by the European Commission,

---

[9] Orphanides (2001) shows that estimates of empirical reaction functions for the Federal Reserve that rely on output gaps are highly sensitive to the choice of ex post or real-time data.

[10] As noted earlier, many authors have estimated reaction functions for the ECB using output gaps computed from industrial production data, which are available at a monthly frequency. This approach has the additional problem that industrial production is only a small part of euro-area GDP.

as a subjective indicator of real economic activity.[11] We also construct a measure of expected real GDP growth in the coming twelve months using the information contained in the poll of forecasters reported on a monthly basis in *The Economist*. Since these forecasts are subjective, we think of them as akin to the sentiment indicator.

To explore the information content of these subjective measures of real activity, we compute their cross-correlations with a monthly measure of the output gap using the industrial production index and a quarterly measure of the gap using real GDP, in both cases starting in 1999.[12] Interestingly, in the case of the monthly data, the highest cross-correlations are obtained when sentiment ($\rho = 0.60$) and expected real growth ($\rho = 0.59$) lead by two months the output gap computed using the industrial production data. Redoing these calculations using the quarterly real GDP data, we find that sentiment leads the output gap by two quarters ($\rho = 0.80$) and that expected output growth leads the output gap by one quarter ($\rho = 0.80$). Thus, both subjective indicators of economic activity are strongly correlated with, and lead, data on the state of the real economy. Since the indicators of sentiment and expected real growth are available with much shorter time lags than industrial production and real GDP data, it makes good sense for the Governing Council and applied econometricians alike to rely on subjective measures of economic activity.

3.4 Estimates

Before turning to the estimates, it is important to note that the lags by which the data are available to the ECB need to be taken into account. The Governing Council generally discusses policy at its first

[11]The economic sentiment index pertains to the euro area and is based on a large survey of firms and consumers. For more information about the index, see http://ec.europa.eu/economy_finance/indicators/business_consumer_surveys/userguide_en.pdf.

[12]Since the output gap is measured in percentage points, we define the sentiment as the percentage deviation from its mean in the sample period. The quarterly data on sentiment are obtained by using the data point for the first month of the quarter.

meeting in the month. Since most of the data we use stem from the *Monthly Bulletin*, which has a cutoff date for the data before the policy meeting, it is straightforward to establish what data are available at the time of the interest rate decision. Thus, among the measures of real economic activity, the output gap computed using industrial production is available with a three-month lag, whereas sentiment and expected real GDP growth are available for the previous month. Headline inflation and expected inflation are also available with a one-month lag, but core inflation is only available with a two-month lag. Money growth is available with a two-month lag, and the ECB's preferred measure of M3 growth using a three-month centered moving average is available with a three-month lag. In estimating the reaction functions below, we thus lag the variables appropriately. To avoid simultaneity, we lag the exchange rate change by one month.

The estimates of the model in equations (3) and (4) are presented in columns 1–9 of table 2 (the estimates in column 10 are discussed in section 5). Before drawing conclusions from the estimates, we briefly consider those in the first column. These show that the parameter on sentiment (our proxy for real economic activity) is positive and significant. Thus, stronger sentiment has led the ECB to raise interest rates. The parameter on headline inflation, by contrast, is insignificant, suggesting no reaction to (past) inflation. Interestingly, the parameter on M3 growth is positive and significant, and the parameter on the change in the exchange rate is negative and highly significant. Thus, faster money growth and a depreciation of the euro in effective terms have been associated with a monetary tightening. Finally, the lagged level of the interest rate and the change in the interest rate are significant.

Rather than commenting on the regressions individually, in the interest of brevity we summarize the most interesting aspects of the results in the table. First, the two subjective indicators of economic activity—economic sentiment and expected real growth—are both highly significant, while the output gap is not. Moreover, the pseudo-R^2 is much lower when the output gap is used. This suggests that the common practice of estimating reaction functions for the ECB employing a measure of the output gap computed using industrial production data is problematic. Note also that the t-values on expected real growth are systematically higher than

Table 2. Ordered-Probit Estimates of Reaction Function: February 1999–June 2006

Model	1	2	3	4	5	6	7	8	9	10
Sentiment	20.24***	23.52***	17.08***							
	(3.15)	(3.14)	(2.66)							
Expected Growth				2.84***	2.46***	2.20***				2.28***
				(3.38)	(3.22)	(2.77)				(3.61)
Output Gap							19.88	2.65	31.33	
							(0.82)	(0.12)	(1.33)	
Headline Inflation	0.17			0.52			−0.71			
	(0.34)			(0.95)			(1.44)			
Core Inflation		0.78			0.05			−0.87		
		(0.98)			(0.06)			(1.34)		
Expected Inflation			−0.78			−0.60			−2.14***	
			(0.90)			(0.67)			(2.59)	
M3 Growth	0.77**	0.72**	0.86***	0.80**	0.85**	0.90***	0.46*	0.48*	0.58**	0.61**
	(2.53)	(2.51)	(2.75)	(2.48)	(2.52)	(2.81)	(1.91)	(1.80)	(2.21)	(2.31)
Exchange Rate	−0.19***	−0.18***	−0.22***	−0.21***	−0.22***	−0.24***	−0.27***	−0.25***	−0.32***	−0.16***
	(2.93)	(3.12)	(3.02)	(3.34)	(3.60)	(3.43)	(4.73)	(4.17)	(4.57)	(3.06)
Lagged Change in Repo Rate	−3.90***	−3.91***	−4.15***	−3.96**	−4.06**	−4.24***	−2.87**	−2.85*	−3.66**	−3.23**
	(2.62)	(2.68)	(2.72)	(2.41)	(2.48)	(2.60)	(1.97)	(1.89)	(2.31)	(2.31)
Lagged Level of Repo Rate	−0.70*	−0.77*	−0.46	−1.30**	−0.99**	−0.79*	−0.51	−0.49	−0.42	−1.04***
	(1.69)	(1.93)	(1.20)	(2.28)	(2.40)	(1.74)	(1.44)	(1.37)	(1.22)	(2.83)
Pseudo-R^2	0.44	0.45	0.45	0.46	0.46	0.46	0.36	0.36	0.40	0.38
AR(1), p-val.	0.82	0.72	0.91	0.50	0.56	0.50	0.81	0.77	0.71	0.55

Notes: Absolute value of t-statistics in parentheses. *, **, and *** denote significance at the 10 percent, 5 percent, and 1 percent level, respectively. "AR(1), p-val." shows the p-value for a test of the hypothesis of no first-order serial correlation of the residuals (see Gourieroux, Monfort, and Trognon 1985).

those on sentiment, as is the pseudo-R^2 when expected real growth is used.

Second, irrespective of how it is measured, the inflation rate is insignificant, except in the case of expected inflation when the output gap is used, in which case the parameter is negative. While this suggests that the ECB has not reacted to past inflation, it is premature to assess this finding before having reviewed the Governing Council's interpretation of economic conditions.

Third, the parameter on M3 growth is positive and significant in all cases. This suggests that the Governing Council has reacted to money growth. One reason money is significant may be that the models include several rarely used variables (such as lagged changes in interest rates and the exchange rate) that are highly significant. Furthermore, the measures of the state of real economic activity also have higher explanatory power than the output gap. These models arguably fit better than more-standard specifications, which would tend to raise the significance of individual parameters.

Fourth, the change in the nominal effective exchange rate is highly significant in all cases. The negative sign indicates that the Governing Council is likely to reduce interest rates when the currency is appreciating, presumably because this is expected to reduce inflation pressures.

Fifth, the parameter on the lagged change in the interest rate is significant and negative. This result implies that, holding economic conditions constant, if the Governing Council decided to raise interest rates last month, it is less likely to do so again this month. In turn, this suggests that policymakers wait for some time before changing interest rates, and when they do change rates, they do so sufficiently so that they do not expect to have to change them again soon. The Governing Council seems to change rates to "clear the air" rather than to smooth interest rates.

Sixth, and finally, the coefficient on the lagged level of the interest rate is negative but only significant in the cases in which expected GDP growth is used together with headline or core inflation—that is, in the cases of the two best-fitting equations.

The results discussed above raise three sets of questions. First, why does the Governing Council react to real economic activity but not to inflation? In particular, is this because it is more concerned by the state of the real economy than inflation pressures? Furthermore, why does it react to money growth but not inflation? Second, how well do these models predict the Governing Council's interest rate decisions? Third, how does money growth affect the probability of interest rate changes? Next we turn to these questions.

4. Words: What the ECB Says

As already noted, central banks' responses to macroeconomic news depend critically on how policymakers interpret the incoming data. To understand the ECB's interest rate setting, it is therefore desirable to consider also the Governing Council's judgments about the outlook for inflation and economic activity and its assessment of monetary developments. To do so, we construct indicator variables of the Governing Council's view of the outlook of the economy by reading the editorials of the ECB's *Monthly Bulletin* in the period between January 1999 and July 2006.

The reason for focusing on the editorials, rather than the full report, is as follows. The *Monthly Bulletin* contains an exhaustive analysis of macroeconomic conditions in the euro area. While there is little doubt that the members of the Governing Council are in general agreement with that analysis, it is arguably best interpreted as expressing the views of the ECB's senior staff. By contrast, the editorials contain a short explanation for why interest rates were or were not changed in the previous month and frequently include a summary statement of the Governing Council's view of the economy. For instance, the June 1999 editorial states that "the Governing Council did not consider that recent monetary developments were indicative of future price pressures," and the January 2000 editorial notes that "recent data confirm the Governing Council's previous assessment regarding the outlook." The editorials must thus receive considerable scrutiny by the members of the Governing Council.

4.1 Construction of the Indicator Variables

The discussion of the risks to price stability in the editorials is structured in three parts. First, there is a discussion of real activity, presumably because the Governing Council views this as an important determinant of future inflation. Second, recent inflation trends are reviewed. Finally, monetary developments in the euro area are commented on. We therefore construct indicator variables that are intended to capture the Governing Council's views of the "risks to price stability" arising from recent developments in economic activity, realized inflation, and M3 growth. Since the ECB has emphasized the importance of M3 growth for its policy decisions and this variable is highly significant in the econometric analysis, it is particularly interesting to explore whether the Governing Council's assessment of inflation risks depends on money growth.

The indicator variables can take five values: −2, −1, 0, 1, and 2.[13] The value of 0 should be interpreted as the editorial's suggesting that the Governing Council believes that *given the current level of the repo rate*, a change in the level of interest rates is not warranted. As an illustration, consider the editorial in the first *Monthly Bulletin*, in January 1999, which states that "on balance, the evidence suggests that there are no indications of significant upward or downward pressures on price development." Since it more generally suggests that the Governing Council viewed inflation as stable at the then-current rate, the assessment of price pressures is coded as 0.

The value −1 indicates that the editorial suggests that the current level of the repo rate is too high. For instance, the April 1999 *Monthly Bulletin* notes that "many projections for inflation rates in the euro area have been revised downward recently." Moreover, the editorial states that "downward pressure on inflation stems from the current economic situation." Since this and the overall reading of the editorial suggest that the Governing Council had become more concerned that inflation might fall too low, the inflation indicator is coded as −1.

The value −2 is used when the Governing Council appears increasingly persuaded that the behavior of the variable in question warrants a cut in interest rates. Consider, for instance, the Governing Council's assessment of real economic activity in early 1999. In January 1999 the editorial discusses "expectations of a slowdown in the growth of economic activity in the short term" (coded as −1), and in February it notes that "while there are indications of a slowdown in real GDP growth, the extent and duration of such

[13]It should be emphasized that the coding was done by reading the full editorials. To illustrate how this is done, appendix 1 contains quotes from the editorials for (in the interest of brevity) 1999. Appendix 2 contains the indicators.

a weakening of economic activity remain a matter of uncertainty" (also coded as −1). By contrast, by the time of the March issue, it was clearer that real economic activity was slowing and that it was doing so more rapidly than had been anticipated earlier. This is indicated by the phrasing "recent information on indicators of economic activity . . . provided evidence of a sizeable slowdown in the fourth quarter of 1998" and "the deterioration of confidence has continued into 1999." We code this as −2. The values +1 and +2 are used in cases in which in the Governing Council appears to be somewhat or strongly concerned that developments in inflation, real economic activity, or M3 growth warrant a tightening of policy.

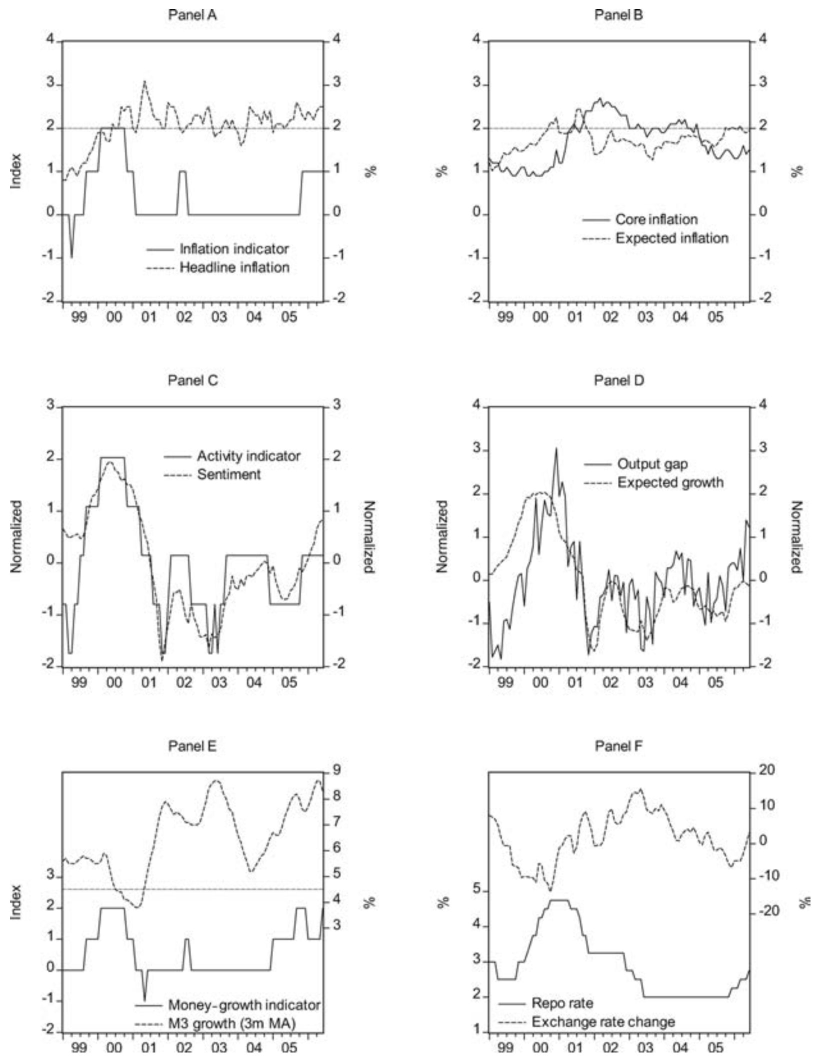
We emphasize that the indicator variables are intended to capture the Governing Council's assessment of whether economic conditions suggest that a change in policy is warranted, which does not necessarily map into the actual behavior of macroeconomic aggregates in this short sample. Indeed, the rationale for using the indicators is that macroeconomic data are not fully informative about the Governing Council's view of the economy.

4.2 Inflation

We start by considering the Governing Council's assessment of inflation. Panel A of figure 1 contains plots of the inflation indicator together with headline inflation, and panel B of the same figure contains plots of core inflation and expected inflation. The 2 percent upper limit of the ECB's definition of price stability is also indicated in these figures.

The editorials suggest that the concerns the Governing Council expressed about declining inflation in the spring of 1999 before the interest rate cut in April soon gave way to worries that inflation risks had increased. This coincided with rising headline and expected future inflation. In late 2000 and in early 2001, the Governing Council viewed inflation risks as having become more balanced, despite the fact that headline inflation was generally above 2 percent. However, that judgment looked appropriate as headline and expected future inflation declined during the later part of 2001. With both rising toward the end of the year and in early 2002, the editorials indicate that the Governing Council became concerned in the middle of 2002 as expected inflation started to rise

Figure 1. The Data



toward the 2 percent level. But with inflation staying just above (and expected inflation just below) 2 percent, the Governing Council soon judged the risks as more balanced and maintained that judgment until late 2005, when it took the view that inflationary pressures had risen.

Exploring more formally the correlations between the inflation indicator and the different measures of inflation, we note the correlations are generally low. The highest correlation is that between the inflation indicator and expected inflation ($\rho = 0.25$), followed by the correlation with current inflation ($\rho = 0.02$). Interestingly, the correlation between the inflation indicator and core inflation is larger in absolute value but negative ($\rho = -0.54$).[14] This suggests that core inflation does not play an important role in the Governing Council's thinking about the economy.

The above analysis of the Governing Council's assessments suggests that *realized* inflation and the ECB's *outlook* for price stability have been quite different. However, since the ECB also reacts to other variables, we postpone a discussion of what to infer from this for the moment.

4.3 Real Economic Activity

While the overriding objective of the ECB is to ensure price stability, the editorials contain frequent statements about developments in real economic activity, presumably because it has an impact on the rate of inflation with a lag. Panel C of figure 1 shows the indicator variable together with the sentiment variable, and panel D shows the output gap and expected real GDP growth.[15] The figure displays a striking correlation between the indicator and sentiment or expected GDP growth (the correlation is 0.79 in the first case and 0.82 in the second case), and a somewhat lower correlation, 0.67, between the indicator and the output gap. The correlation between sentiment and expected output growth is even higher at 0.92, which further supports the view that sentiment captures expected future growth in the economy.

Again we emphasize that actual real GDP growth and the output gap are not included in the econometric analysis, since the editorials suggest that these variables do not play much of a role

[14]These correlations generally rise when future values of the inflation measures are considered, peaking at 0.45 when expected inflation is led by ten months, 0.23 when actual inflation is led by nine months, and 0.43 when core inflation is led by twenty-two months.

[15]To permit easy comparison, the data have been normalized by subtracting the mean and dividing by the standard error.

in the Governing Council's assessment of inflation risks because of reporting lags and data revisions.

4.4 Money Growth

Since the ECB has repeatedly stated that it attaches a prominent role to money in conducting monetary policy, next we turn to its interpretation of M3 growth. Panel E of figure 1 contains a plot of the indicator variable for money together with a three-month average of M3 growth over twelve months. For clarity, the 4.5 percent "reference value" for money growth that the ECB has announced is also indicated. The figure suggests that the Governing Council viewed money growth as indicating risks to price stability between mid-1999 and late 2000. Except during a brief period in 2002, the Governing Council did not view money growth as indicating risks to price stability again until early 2005, despite the fact that money growth had exceeded the reference value since early 2001. As is clear from the editorials, the reason for this was that the rapid increase in money growth between 2001 and 2003 was interpreted as largely reflecting increases in the demand for money that did not generate inflation risks.

4.5 Exchange Rate and Repo Rate

Finally, panel F shows that the euro depreciated in effective terms between 1999 and late 2000, a period during which the repo rate was rising, and that it appreciated between late 2000 and late 2004 as the ECB's repo rate was cut repeatedly and then held constant. The euro subsequently started to depreciate again but then appreciated as monetary policy was tightened from late 2005 onward.

4.6 The Determinants of the Indicators

The indicators are intended to summarize the Governing Council's views of the outlook for inflation and real economic activity and its interpretation of the information on money growth. As is clear from the figures discussed above, the different indicators—in particular, those for inflation and money growth—evolve in similar ways over time. This suggests that they may in fact be driven by the same factors. To explore this issue in an informal way, we regress the indicator variables on inflation, expected real growth (which was more significant than sentiment or the output gap in table 2), M3 growth, and the rate of appreciation of the effective exchange rate. Since these regressions are subject to serial correlation and heteroskedasticity, we assume first-order autoregressive errors and compute standard errors using the White approach. Overall, the regressions should be thought of as a way to capture the correlations between the indicators and the macroeconomic variables and should not be given any structural interpretation.

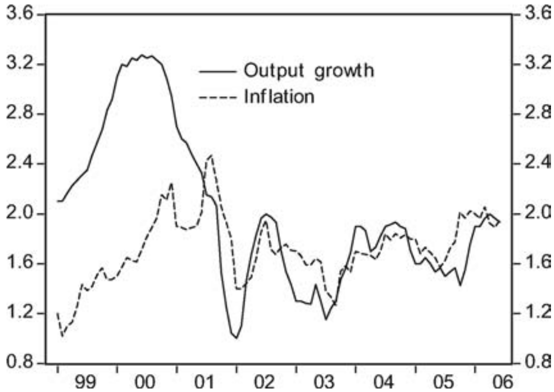
The results in table 3 show that expected real growth is correlated with both the inflation indicator and the output indicator. Thus, the Governing Council may react to the state of real activity because it sees stronger growth as suggesting that inflation risks have risen. This interpretation is supported by figure 2, which demonstrates that there is a strikingly close relationship between

Table 3. OLS Regressions of Indicators on Macroeconomic Variables: January 1999–June 2006

	Dependent Variable		
Regressors	**Inflation Indicator**	**Output Indicator**	**Money-Growth Indicator**
Inflation	−0.00	−0.82	−0.19
	(0.03)	(0.40)	(0.77)
Expected Growth	0.98***	0.88**	0.32
	(5.19)	(2.62)	(1.07)
M3 Growth	0.25***	−0.08	0.01
	(3.72)	(0.81)	(0.04)
Exchange Rate Change	−0.02*	−0.04*	0.01
	(1.95)	(1.91)	(0.60)
ρ	0.56***	0.72***	0.90***
	(4.66)	(8.06)	(15.17)
R^2	0.83	0.85	0.79

Note: Regressions include an unreported constant and allow for first-order autoregressive errors (ρ). t-values are in parentheses. Standard errors are computed using the White correction. *, **, and *** denote significance at the 10 percent, 5 percent, and 1 percent level, respectively.

Figure 2. Expected Output Growth and Expected Inflation



expected real growth and expected inflation since the middle of 2001.[16]

The results in table 3 also show that money growth is correlated with the inflation indicator. Faster money growth is thus associated with greater concerns being expressed by the Governing Council about the outlook for inflation. Finally, exchange rate changes are negatively correlated with the inflation and real-growth indicators. Thus, an appreciation of the exchange rate (a positive change in the exchange rate) leads the Governing Council to be less concerned about the outlook for inflation and, perhaps, more concerned about a slowing of the economy. Interestingly, none of the macroeconomic variables are significant in the regression for the indicator variable for money growth.

4.7 Indicators and Economic Conditions

At this stage it is useful to summarize what we can learn from figure 1 and the empirical analysis of the determination of the indicator variables in table 3. Several conclusions appear warranted.

First, there is no close link between headline or core inflation and the Governing Council's outlook for inflation. As suggested earlier, this may be because shocks to inflation largely reflect price-level

[16]The correlation coefficient over the sample June 2001–June 2006 is 0.63.

disturbances that have little implication for future inflation and therefore do not have an impact on the Governing Council's assessment of the risk to price stability. That interpretation is compatible with the finding that headline and core inflation are insignificant in the estimated reaction functions discussed above. More surprising is the finding that expected inflation is insignificant in the reaction function and, as suggested by panel A in figure 1, does not appear correlated with the indicator variable for inflation. We return to this issue in the next paragraph.

Second, there are strong correlations between data on, and the Governing Council's assessment of, real economic activity. Furthermore, real economic activity is also an important determinant of the Governing Council's assessment of the outlook for price stability. This suggests that the reason expected real growth is so strongly significant in the estimated reaction functions is that it is seen as containing information about future inflation pressures.

Third, the relationship between money growth and interest rates appears complex. Since the Governing Council has repeatedly stated that it attaches importance to monetary developments as an indicator of "risks to price stability," one would have expected that high money growth would have been associated with high or rising interest rates. Panel E of figure 1 suggests that the opposite is the case: periods of above-average interest rates are associated with money growth below average and vice versa. However, money growth is significant in the estimated reaction functions and, furthermore, is correlated with the indicator variable capturing the Governing Council's assessment of the risks to price stability. One way of reconciling these findings is to note that the figure captures the bivariate relationship between money growth and the outlook for price stability. By contrast, multivariate reaction functions control for economic activity, past interest rates, and the rate of depreciation of the exchange rate and are therefore more informative about the role of money in the Governing Council's conduct of monetary policy.

5. Assessing the Model

This section considers what can be learned about the interest rate setting of the Governing Council from the econometric model. To that end, we reestimate the model without including actual or expected inflation since these variables were insignificant in the econometric analysis. The results are provided in column 10 in table 2. All variables are significant at the 5 percent level and have the expected signs. Thus, increases in expected real growth and money growth raise, and faster exchange rate appreciation reduces, the probability of an interest rate increase, given the level of interest rates last month. Furthermore, and as already noted, interest rate changes are of the "clearing the air" variety in that, holding economic fundamentals constant, the Governing Council is less likely to change interest rates this month if it did so last month.

5.1 *Estimated Probabilities of Policy Changes*

Table 4 presents information regarding the model's ability to account for interest rate changes in the sample. There are eighty-nine observations, of which seventy-one involve no change of the interest rate. Since a model with zero explanatory power would predict these correctly, it is more informative to ask how well the model predicts the eighteen interest rate changes that did occur. Interestingly, it correctly predicts four of the five 0.50 percent cuts in interest rates but none of the three 0.25 percent cuts.[17] Moreover, it predicts four of the eight 0.25 percent increases and one of the two 0.50 percent increases in rates. Overall, the model thus predicts nine of the eighteen policy changes. We also estimated a version of the model that does not distinguish between small and large changes in the repo

Table 4. Actual and Predicted Interest Rate Changes (Using the Model in Column 10 of Table 2)

	Actual	Predicted	Error
Large Cut	5	4	1
Small Cut	3	0	3
No Change	71	80	−9
Small Increase	8	4	4
Large Increase	2	1	1

[17] By "predict" an outcome, we mean that the fitted probability is highest for that outcome.

rate. That simpler model correctly predicts five of the eight cuts in interest rates and eight of the ten increases. It therefore appears that one reason why the model has difficulties in predicting interest rate changes is that it is asked to distinguish between small and large changes. A second reason is no doubt the fact that we use monthly data. Since the explanatory macroeconomic variables evolve slowly over time, the probability of a policy change is likely to be high for an extended period of time. It is difficult to predict exactly when in that period the policy change occurs—in particular, since it may partially depend on factors outside the model.[18]

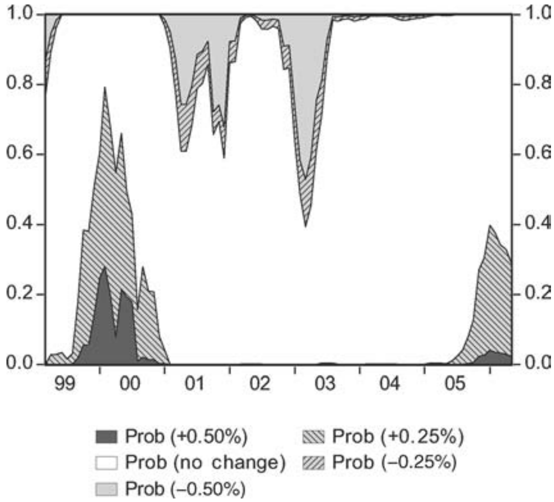
Figure 3 shows the evolution over time of the fitted probabilities of the different outcomes (since the probabilities are somewhat jagged, we plot three-month centered moving averages of the probabilities; the sample period is therefore March 1999–May 2006). The figure indicates that the tightening in monetary policy in 1999–2000 is associated with increases in the predicted probabilities of interest rate increases, and the cuts between 2001 and late 2003 occur in a period when the estimated probabilities of a relaxation of monetary policy are high. The process of monetary policy tightening that started in late 2005 also coincides with an increased probability of increases in interest rates. However, the fitted probabilities are quite low at this time.

To assess whether the estimated probabilities are plausible, we explore how well they are able to account for movements in the short end of the term structure of money-market rates, which were not used in the estimation of the ordered-probit models. More precisely, we regress the spread between three-month and one-month money-market rates ($SLOPE$) on a constant, its own lagged value and the difference between the probability of a 0.25 percentage point increase in interest rates and the probability of a 0.25 percentage point cut in interest rates ($DPROB$). Since money-market rates moved a lot in the final months of 1999, the sample is January 2000 through June 2006.[19] The results are as follows (with t-statistics in parentheses):

[18]For instance, central banks typically avoid changing interest rates when this may be misinterpreted as a response to outside pressure or as evidence that they "follow" another central bank.

[19]In the run-up to the new millennium, widespread concerns about computer malfunctioning on January 1, 2000 ("Y2K"), led to sharp increases in the demand for liquidity, which caused money-market rates to rise significantly.

Figure 3. Smoothed Probabilities of a Change in Monetary Policy



Note: Three-month centered moving averages. Regressors are assumed to be at their means.

$$SLOPE_t = \underset{(0.01)}{0.01} + \underset{(5.39)}{0.48}\ SLOPE_{t-1} + \underset{(5.03)}{0.21}\ DPROB_t + Error_t$$

with $R^2 = 0.73$ and $DW = 2.07$. The fact that *DPROB* is highly significant suggests that the model is useful for predicting the future course of monetary policy.

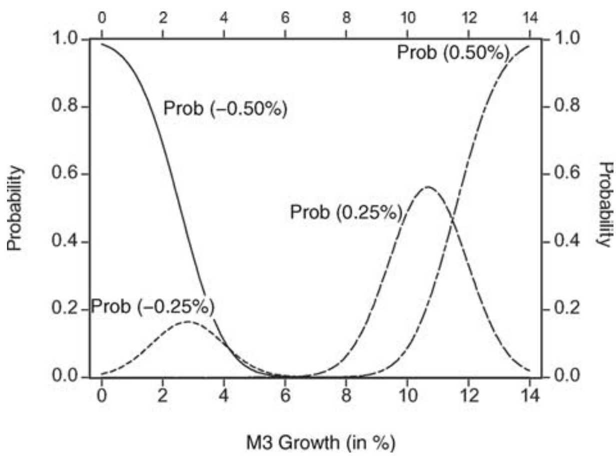
5.2 Money Growth

To explore whether and, if so, how the Governing Council has reacted to money growth, we calculate the probabilities of the five possible policy outcomes as a function of the growth rate of M3. Before considering the results, it is important to recall that the fitted probabilities depend on *all* variables and not only on money growth. To construct the plots, values for expected output growth, the lagged repo rate, and the change in the exchange rate must therefore be assumed. Since the results below serve as benchmarks for the subsequent analysis, it is natural to assume that all variables are at their unconditional sample means and that there was no change in the repo rate last month.[20]

The results are shown in figure 4, which shows that the probability of a policy change is minimized at the average money-growth rate in the sample, which was 6.4 percent. Faster money-growth rates raise relatively quickly the probability of a 0.25 percent increase in the repo rate. As increasingly higher money-growth rates are considered, the probability of a 0.50 percent increase in the repo rate rises rapidly and the probability of a 0.25 percent increase starts to decline. Of course, similar relationships hold for the probability of interest rate cuts if money-growth rates below 6.4 percent are considered.

For comparison purposes, figure 5 contains a plot of the estimated probabilities of an increase in interest rates of 0.25 percent or 0.50 percent shown in figure 4, but it is drawn in such a way as to show the sum of the probabilities. To understand the figure, assume

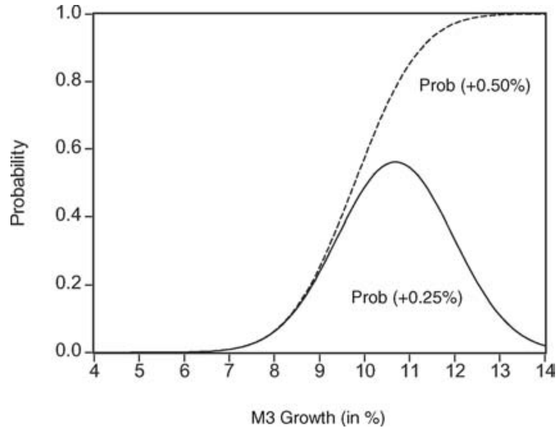
Figure 4. Probability of Policy Choices as a Function of Money Growth



Note: Regressors are assumed to be at their means.

[20]The sample means of the regressors are as follows: repo rate, 2.9 percent; expected output growth, 2.9 percent; and change of the exchange rate, 1.5 percent. For comparison, the average rate of inflation is 2.1 percent.

Figure 5. Probability of an Increase in Interest Rates



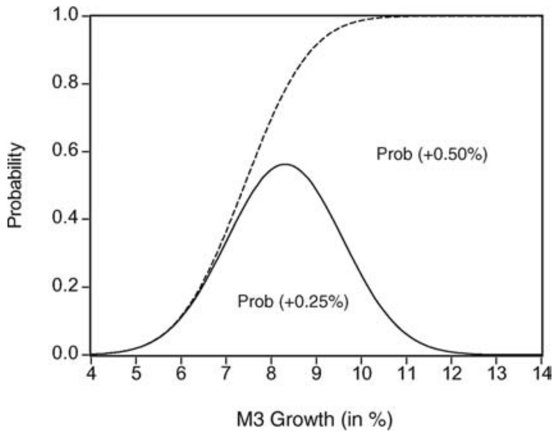
Note: Regressors are assumed to be their means.

that the money-growth rate is 8.7 percent—that is, the highest rate observed in the sample. If so, the estimated overall probability of an increase in interest rates is 18 percent. Decomposing that probability further, the estimated probability of a 0.25 percent increase is 17 percent and the probability of a 0.50 percent increase is 1 percent.

The figure suggests that variations in money-growth rates within the range observed in the sample—that is, annual rates of between 3.8 and 8.7 percent—have essentially no impact on the probability of interest rate changes. However, these estimated probabilities are computed under the assumption that the other variables are at their sample means. Of course, if the business cycle is at neutral, interest rates are at their mean, and the exchange rate is stable, a policy change in response solely to money growth being unusually high or low would be unlikely. Figure 6 shows the impact of money growth on interest rate decisions in an environment in which a policy change is more likely.

The probabilities are in this case constructed under the assumption that expected output growth is one standard deviation above its mean, which would be observed quite commonly in the sample. Furthermore, we assume that the lagged repo rate and the change in the exchange rate are at their means, conditional on expected output growth being one standard deviation above its mean, and

Figure 6. Probability of an Increase in Interest Rates



Note: Expected real output growth is assumed to be one standard deviation above its mean.

that the interest rate was not changed the previous month. Assuming a money-growth rate of 8.7 percent, in this case the estimated probability of a 0.25 percent increase in interest rates is 54 percent and the probability of a 0.50 percent increase is 33 percent, for an overall probability of a tightening of monetary policy of 87 percent.

The conclusion we draw from this analysis is that under "normal" economic conditions, when a change in monetary policy in any case is unlikely, money growth has little impact on the probability of a policy change. When economic conditions are weaker or stronger, however, the role of money growth in interest rate setting is much greater. Furthermore, this analysis suggests an explanation for why it is sometimes claimed that the ECB has disregarded money growth in setting interest rates. In the sample there is a strong, negative correlation ($\rho = -0.71$) between money growth and expected output growth. Thus, when money growth has been high, expected growth has tended to be low, reducing the overall probability of an interest rate increase.[21]

[21] Since the correlation between the repo rate and expected output growth is 0.61, it may be that variations in M3 growth largely reflect changes in the stance

6. Conclusions

The main conclusions of the analysis of the ECB's words and deeds are as follows. First, subjective measures of economic growth play an important role in the ECB's policy decisions. They are frequently referred to in the Governing Council's discussion of the economy in the editorials of the *Monthly Bulletin* and are statistically highly significant in the estimated reaction functions. The use of such subjective measures of economic conditions is sensible since they are strongly correlated with future output gaps. Furthermore, because of publication lags, the Governing Council uses real GDP data largely to assess its past judgment of economic activity. Output gaps appear to play no role in its analysis of current economic conditions.

Second, interest rate changes are more closely tied to economic activity than to inflation. The reason for this appears to be that while economic activity has an impact on the Governing Council's assessment of the outlook for inflation, shocks to actual inflation have been seen as largely temporary, reflecting price-level shocks, and thus as having little implication for future inflation. This has been the case even in situations in which inflation has exceeded the 2 percent level that constitutes the upper limit of the ECB's definition of price stability.

Third, the Governing Council reacts to M3 growth. The extent to which it does so, however, depends also on the other arguments in its reaction function. In "normal" times, the probability of a policy change is not particularly sensitive to variations in money growth. In times in which expected growth is high and the Governing Council perceives a greater risk of inflation, money growth has had a much larger impact on the probability of interest rate changes.

Fourth, and finally, by studying the ECB's statements about its assessment and outlook of economic conditions, we can obtain a better understanding about its conduct of monetary policy than is possible by solely estimating empirical reaction functions.

---

of monetary policy. Of course, it would be natural for the ECB not to react to such movements.

Appendix 1. Summary of the Editorials in the ECB's *Monthly Bulletin* (January–December 1999)

This appendix illustrates the coding of the indicators. The coding of indicator variables has been made on the basis of the full editorials. The quotes provided below are intended to give a brief rationale for the coding.

***Monthly Bulletin*, Dates of Meetings, and Interest Rate Decisions**	**Outlook for Prices**	**Outlook for Real Activity**	**Monetary Analysis**
January 1999 (Jan. 7, 3 percent)	"Financial indicators support the view that market participants expect the current climate of price stability to continue." "no indications of significant upward or downward pressure on price developments" "The outlook for price developments . . . can be regarded as being broadly balanced."	"negative impact on industrial confidence" "less optimistic view of future growth" "expectations of a slowdown in the growth of economic activity in the short term"	"M3 showed a stable trend in 1998."

February 1999 (Jan. 21 and Feb. 4, no change)	"Recent price developments are consistent with future price stability."	"Industrial output growth slowed down, capacity utilization in manufacturing decreased, and the decline in unemployment appears to have stalled." "Overall, while there are indications of a slowdown in real GDP growth, the extent and duration of such a weakening of economic activity remain a matter of uncertainty."	"Monetary data . . . bear witness of the continuation of favorable prospects for price stability." "The 12-month growth rate of M3 . . . was 4.7%, . . . very close to the reference value."
March 1999 (March 4, no change)	"Recent price developments in the euro area do not appear to signal a threat to future price stability."	"Recent information on indicators of economic activity . . . provided evidence of a sizable slowdown in the fourth quarter of 1998."	"Since monthly data for monetary aggregates can be rather volatile, the Governing Council decided to focus . . . not on outturns for a single month but instead on the three-month moving average of the 12-month growth rate."

(continued)

***Monthly Bulletin*, Dates of Meetings, and Interest Rate Decisions**	**Outlook for Prices**	**Outlook for Real Activity**	**Monetary Analysis**
March 1999 (continued)	"The pattern of upward and downward risks to price stability has remained broadly unchanged."	"The deterioration in industrial confidence has continued into 1999." "On the downside, there is a slowdown in the . . . economy."	"The acceleration of M3 was largely attributable to low levels of short-term and long-term interest rates and the environment of price stability . . . as well as to technical factors." "In view of the uncertainty relating to special factors . . ., the Governing Council did not consider the acceleration of M3 in January 1999 as a signal of upcoming inflationary pressures."

April 1999 (April 8, cut 50 basis points, to 2.5 percent)	“this stable rate of price increases” “Many projections for inflation rates . . . have been revised downward recently.”	“Recent data on economic activity . . . confirmed a weakening toward the end of last year.” “The euro area may need longer than previously expected to recover from the slowdown.” “downward revisions in the growth forecasts” “reinforced expectations of somewhat lower inflationary pressure arising from economic activity”	“Money growth should not be seen as signaling upcoming inflationary pressures.” “M3 growth . . . may have been affected by special factors.” “Monetary growth cannot, at the moment, be considered to be a risk to . . . price stability.”

(continued)

***Monthly Bulletin*, Dates of Meetings, and Interest Rate Decisions**	**Outlook for Prices**	**Outlook for Real Activity**	**Monetary Analysis**
May 1999 (April 22 and May 6, no change)	"The outlook for price stability . . . is favorable." "Current monetary developments and other available indicators do not point to risks for price stability."	"Data . . . do not yet indicate . . . a rebound in economic growth." "The slowdown . . . recorded in the last quarter of 1998 continued into early 1999." "Confidence indicators point toward some first signs of improvement. . . . Preliminary April figures indicate a slight recovery."	"Considering the special circumstances . . . and the fact that the three-month moving average of M3 growth still remained close enough to the reference value . . . , the Governing Council confirmed the judgment . . . that current monetary trends should not be seen as a warning signal with regard to future inflationary pressures."

June 1999 (April 22 and May 6, no change)	“Consumer price developments in the euro area have been affected by higher oil prices. . . . However, this . . . is likely to constitute only a temporary influence on price developments.” “Recent surveys and forecasts of inflation in the euro area covering the next one to two years indicate that consumer price increases are expected to remain below 2%.”	“Available national data do not yet provide clear evidence of an improvement in the economic situation. . . . This notwithstanding, most forecasts point to a strengthening of activity.”	“The latest three-month moving average of M3 growth . . . decreased.” “Against this background, and taking into account the specific circumstances . . . , the Governing Council did not consider that recent monetary developments were indicative of future inflationary pressures.”

(continued)

***Monthly Bulletin*, Dates of Meetings, and Interest Rate Decisions**	**Outlook for Prices**	**Outlook for Real Activity**	**Monetary Analysis**
July 1999 (June 17 and July 1, no change)	"In the short and in the medium term price developments should continue to be compatible with the Eurosystem's definition of price stability." "The outlook for the maintenance of price stability in the euro area remains favorable." "Downward risks to future price stability have receded." "Some moderate upward pressure on HICP increases still appears to be the most likely outcome."	"New data point to a stabilization of overall output growth in early 1999 and to an economic recovery in the second part of 1999." "The risks to this economic outlook seem to have become more balanced, as it appears that the likelihood of further downward pressures on economic activity is now less than in previous months." "There have been positive indications regarding economic activity . . . in recent business and consumer surveys conducted in several euro-area countries."	"The three-month moving average of the annual growth rates of M3 . . . increased." "Consequently, M3 growth remained above the reference value." "While this situation is not seen as signaling inflationary pressures at the present juncture, a reassessment may be appropriate once economic growth in the euro area starts to accelerate."

August 1999 (July 15 and July 29, no change)	"The outlook for . . . price stability remains favorable. However, careful monitoring of the evolution of monetary and credit aggregates, and of the indicators which are now pointing more firmly toward an acceleration of economic activity in the euro area, will be necessary in the months to come."	"The outlook for the external environment continued to support the view of an acceleration of growth in the euro area." "Recent evidence has confirmed that output growth . . . should recover in the second part of 1999." "Overall, the outlook for economic activity . . . is more favorable now than it was a month ago."	"M3 growth remained above the reference value." "The annual increase in M3 fell." "The high rate of growth of credit . . . and the development of M3 are not seen as signaling inflationary pressures at the present juncture."

(continued)

***Monthly Bulletin*, Dates of Meetings, and Interest Rate Decisions**	**Outlook for Prices**	**Outlook for Real Activity**	**Monetary Analysis**
September 1999 (Aug. 26 and Sept. 9, no change)	"Consumer price increases have been picking up moderately." "Further upward pressures on consumer prices can be expected." "While the prospects for continued price stability are good, it is necessary to remain vigilant with regard to upside risks."	"a number of signs that economic growth . . . has started to recover" "The most recent areawide industrial production data support the picture of an ongoing cyclical improvement. Forecasts . . . point to a strengthening of overall activity during the course of the year." "The downside risks pertaining to these projections have tended to recede."	"From a forward-looking perspective, upward risks to price stability merit close attention as monetary growth has been moving upward from the reference value." "However, as Monetary Union is still in a very early phase and figures for broad money growth have been subject to a number of revisions in recent months, the short-term monetary developments need to be interpreted with caution and the data need to be analyzed carefully."

October 1999 (Sept. 23 and Oct. 7, no change)	“The balance of risks to price stability remains on an upward trend.” “Consumer prices data . . . show a further rise in the annual rate of change of the headline HICP.” “This was mainly due to the increase in oil prices. . . . Seen in isolation, the increase in energy prices should only have a temporary effect upon consumer price increases, but it is essential that this effect should not trigger wage claims which prove incompatible with price stability.”	“the more favorable expectations for real GDP growth” “There has been an upturn in growth this year.” “Data available for the first half of 1999 are consistent with the view that there has been an upturn in growth.” “The outlook for a continuing improvement in economic activity therefore remains favorable.”	“The rate of growth of . . . M3 has gradually been moving away from the reference value.” “Both the rising trend in M3 and high credit growth call for great vigilance on the part of monetary policy at a time of accelerating economic activity.”

(continued)

***Monthly Bulletin*, Dates of Meetings, and Interest Rate Decisions**	**Outlook for Prices**	**Outlook for Real Activity**	**Monetary Analysis**
November 1999 (Nov. 4, raised 50 basis points, to 3 percent)	"Inflation rates are expected to increase, mainly as a result of the increase in oil prices earlier this year working its way through to consumer prices." "The annual rate of change in consumer prices . . . remained unchanged at 1.2% in September. . . . But there are still expectations of some overall upward movement in the HICP rate in the short term, mainly connected with energy prices."	"Developments over the past few months indicate expectations of increasing economic growth." "Information available on economic activity continues to support the view that the prospects for the euro-area economy have continued to improve in recent months."	"Both pillars concurred in indicating that the balance of risks to future price stability had gradually been moving toward the upside." "The continued upward deviation of broad monetary growth from the reference value over recent months indicates there is ample liquidity in the euro area."

November 1999 (continued)	"To summarize, the downside risks to price stability which motivated the cut in ECB interest rates in April 1999 are no longer present."	"Real GDP growth increased in the second quarter of 1999, while data on industrial production indicate that the recovery in economic activity progressed further in the third quarter of 1999." "A further strengthening of economic activity can be expected in the near future."	"important to prevent the generous liquidity situation from translating into upward pressures on prices over the medium term" "M3 growth has been on a rising trend." "The deviation from the reference value, which has to be monitored and interpreted with caution, has been growing steadily in 1999." "Overall, the sustained and growing deviation of M3 growth from the reference value implied the existence of a very generous liquidity situation . . . which could generate upward risks for price stability in the medium term."

(continued)

***Monthly Bulletin*, Dates of Meetings, and Interest Rate Decisions**	**Outlook for Prices**	**Outlook for Real Activity**	**Monetary Analysis**
December 1999 (Nov. 18 and Dec. 2, no change)	"Consumer price developments . . . reflect the impact of higher oil prices." "Overall, the outlook for price developments will very much depend on a number of factors, and in particular on wage developments." "On balance, risks to price stability are on the upside."	"confirmation of a gradual recovery in domestic activity" "Available forecasts indicate a pickup in real GDP growth over the next one to two years." "The general picture is one of an ongoing economic expansion." "Real GDP growth is now widely projected to increase." "cyclical upturn . . . clearly established"	"The annual rate of growth of M3 has been rising since the beginning of 1999." "These . . . developments appear to have been determined mainly by the low level of interest rates and the pickup in economic activity in the euro area."

Appendix 2. Indicator Variables (January 1999–June 2006)

	1999–2000				**2001–2002**				**2003–2004**				**2005–2006**		
	Inf.	**Act.**	**Mon.**		**Inf.**	**Act.**	**Mon.**		**Inf.**	**Act.**	**Mon.**		**Inf.**	**Act.**	**Mon.**
Jan	0	−1	0	Jan	1	1	1	Jan	0	−1	0	Jan	0	−1	1
Feb	0	−1	0	Feb	0	1	0	Feb	0	−1	0	Feb	0	−1	1
Mar	0	−2	0	Mar	0	1	0	Mar	0	−2	0	Mar	0	−1	1
Apr	−1	−2	0	Apr	0	0	0	Apr	0	−2	0	Apr	0	−1	1
May	0	−1	0	May	0	0	−1	May	0	−1	0	May	0	−1	1
Jun	0	−1	0	Jun	0	0	0	Jun	0	−2	0	Jun	0	−1	1
Jul	0	0	0	Jul	0	0	0	Jul	0	−1	0	Jul	0	−1	1
Aug	0	0	0	Aug	0	−1	0	Aug	0	−1	0	Aug	0	−1	1
Sep	1	1	1	Sep	0	−1	0	Sep	0	0	0	Sep	0	−1	2
Oct	1	1	1	Oct	0	−1	0	Oct	0	0	0	Oct	0	−1	2
Nov	1	1	1	Nov	0	−2	0	Nov	0	0	0	Nov	1	0	2
Dec	1	1	1	Dec	0	−2	0	Dec	0	0	0	Dec	1	0	2
Jan	1	1	1	Jan	0	−1	0	Jan	0	0	0	Jan	1	0	1
Feb	2	2	2	Feb	0	0	0	Feb	0	0	0	Feb	1	0	1
Mar	2	2	2	Mar	0	0	0	Mar	0	0	0	Mar	1	0	1
Apr	2	2	2	Apr	0	0	0	Apr	0	0	0	Apr	1	0	1
May	2	2	2	May	1	0	0	May	0	0	0	May	1	0	1
Jun	2	2	2	Jun	1	0	0	Jun	0	0	0	Jun	1	0	2
Jul	2	2	2	Jul	1	0	1	Jul	0	0	0				
Aug	2	2	2	Aug	0	0	1	Aug	0	0	0				
Sep	2	2	2	Sep	0	−1	0	Sep	0	0	0				
Oct	2	2	2	Oct	0	−1	0	Oct	0	0	0				
Nov	1	1	1	Nov	0	−1	0	Nov	0	0	0				
Dec	1	1	1	Dec	0	−1	0	Dec	0	−1	0				

Note: Inf. = Inflation, Act. = Real Economic Activity, and Mon. = Money Growth.

References

Alesina, A., O. Blanchard, J. Galí, F. Giavazzi, and H. Uhlig. 2001. *Defining a Macroeconomic Framework for the Euro Area: Monitoring the European Central Bank 3.* London: Centre for Economic Policy Research.

Begg, D., P. De Grauwe, F. Giavazzi, H. Uhlig, and C. Wyplosz. 1998. *The ECB: Safe at Any Speed? Monitoring the European Central Bank 1.* London: Centre for Economic Policy Research.

Berger, H., J. de Haan, and J.-E. Sturm. 2006. "Does Money Matter in the ECB Strategy? New Evidence Based on ECB Communication." KOF Working Paper No. 125.

Carstensen, K. 2006. "Estimating the ECB Policy Reaction Function." *German Economic Review* 7: 1–34.

Davutyan, N., and W. R. Parke. 1995. "The Operations of the Bank of England, 1880–1908: A Dynamic Probit Approach." *Journal of Money, Credit, and Banking* 27 (4): 1099–1112.

Ehrmann, M., and M. Fratzscher. 2005a. "Communication and Decision-Making by Central Bank Committees: Different Strategies, Same Effectiveness?" ECB Working Paper No. 488.

———. 2005b. "How Should Central Banks Communicate?" ECB Working Paper No. 557.

———. 2005c. "The Timing of Central Bank Communication." ECB Working Paper No. 565.

Eichengreen, B., M. W. Watson, and R. S. Grossman. 1985. "Bank Rate Policy under the Interwar Gold Standard: A Dynamic Probit Model." *Economic Journal* 95 (379): 725–45.

Galí, J., S. Gerlach, J. Rotemberg, H. Uhlig, and M. Woodford. 2004. *The Monetary Policy Strategy of the ECB Reconsidered: Monitoring the European Central Bank 5.* London: Centre for Economic Policy Research.

Gerlach, S. 2004. "Interest Rate Setting by the ECB: Words and Deeds." CEPR Discussion Paper No. 4775 (December).

Gourieroux, C., A. Monfort, and A. Trognon. 1985. "A General Approach to Serial Correlation." *Econometric Theory* 1 (3): 315–40.

Greene, W. H. 2003. *Econometric Analysis*, 5th ed. Englewood Cliffs, NJ: Prentice-Hall.

Heinemann, F., and K. Ullrich. 2005. "Does It Pay to Watch Central Bankers' Lips? The Information Content of ECB Wording." ZEW Discussion Paper No. 05-70.

Judd, J. P., and G. D. Rudebusch. 1998. "Taylor's Rule and the Fed: 1970–1997." *Economic Review* (Federal Reserve Bank of San Francisco) 3: 3–16.

Kim, T.-H., P. Mizen, and A. Thanaset. 2005. "Predicting Directional Changes in the UK Interest Rate: The Usefulness of Information from the Taylor Rule versus a Wider Alternative." Unpublished Manuscript.

Musard-Gies, M. 2006. "Do European Central Bank's Statements Steer Interest Rates in the Euro Zone?" *Manchester School* 74 (S1): 116–39.

Orphanides, A. 2001. "Monetary Policy Rules Based on Real-Time Data." *American Economic Review* 91 (4): 964–85.

Rosa, C., and G. Verga. 2005. "Is ECB Communication Effective?" CEP Discussion Paper No. 0682.

———. 2007. "On the Consistency and Effectiveness of Central Bank Communication: Evidence from the ECB." *European Journal of Political Economy* 23 (1): 146–75.

Ruud, P. A. 2000. *An Introduction to Classical Econometric Theory.* New York: Oxford University Press.

Svensson, L. E. O. 1997. "Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets." *European Economic Review* 41 (6): 1111–46.

Taylor, J. 1993. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* 39 (December): 195–214.

Imperfect Knowledge, Adaptive Learning, and the Bias Against Activist Monetary Policies*

Alberto Locarno
Research Department, Banca d'Italia and London School of Economics

The paper studies the implications for the effectiveness of discretionary monetary policymaking of departing from the assumption of rational expectations. Society, whose welfare function is quadratic, can appoint a central banker whose preferences are either quadratic or lexicographic, to achieve the best mix of inflation and output stability. The focus on lexicographic preferences is justified on the grounds that they imply a strict ordering of policy objectives, which is typical of the mandate of several central banks. Both the private sector and the monetary policymaker have incomplete knowledge of the working of the economy and rely upon adaptive learning to form expectations and decide policy moves. The model economy is assumed to be subject to recurrent unobserved shifts, and the monetary authority, who has private information on the shocks hitting the economy, cannot credibly commit. The main finding of the paper is that when agents rely on an adaptive learning technology, a bias against activist policies arises. The paper also shows that when society has quadratic utility, a strategy based on a strict ordering of objectives is close to optimal for a wide range of values of the inflation aversion parameter.

JEL Codes: E52, E31, D84.

1. Introduction

Effective policymaking requires that the monetary authority commit to a systematic approach to policy. As long as price setting depends

*This paper benefited from conversation with Margaret Bray, Giuseppe Ferrero, Alessandro Secchi, and Marco Vega.

on expectations of the future, a credible central bank may be able to face an improved short-run trade-off between inflation and output and, accordingly, it may reduce inflation at lower costs. However, for policymakers to succeed in steering expectations and reaping the benefits of commitment, agents must be able to fully anticipate the future impact of monetary decisions on the economy, which is feasible only if the economic environment is stationary and expectations are rational. When, on the contrary, the structure of the economy is subject to recurrent shifts and knowledge is imperfect, agents must rely on alternative methods for anticipating future events, and this may dramatically alter the policy trade-offs.

A recent stream of literature, epitomized by Orphanides and Williams (2002), has shown that imperfect knowledge makes stabilization policies more difficult. This happens when central banks put too much weight on output stabilization, because overly activist policies are prone to generate episodes in which the public's expectations of inflation become uncoupled from the policy objective. An additional complication arises from the fact that if knowledge is incomplete, committing to a systematic policy becomes problematic (because the private sector cannot easily verify whether the central bank is delivering on its promises) and the gains from commitment are severely reduced (because expectations, being backward rather than forward looking, cannot be manipulated to increase policy effectiveness).

This paper focuses on how the relaxation of the rational expectations hypothesis changes the way monetary policy is set. It applies a principal-agent approach to the time-inconsistency problem: in order to improve the discretionary equilibrium, society (the principal) assigns a loss function to the central bank (the agent) that may differ from society's preferences. The assumption underlying this approach is that it is possible to commit the monetary authority to a particular loss function, whereas the minimization of the loss function occurs under discretion. Society has standard quadratic preferences on output and inflation, and the central banker is endowed with either a quadratic or a lexicographic preference ordering. The literature on monetary policy almost always relies on the assumption of quadratic preferences, but such a loss function does not reflect the task assigned to the monetary authority of most developed countries and does not account for the role of factors that are vital in actual policymaking.[1] Lexicographic preferences overcome a few of these shortcomings: they capture a hierarchical ordering of alternatives, which is typical of the mandate of virtually every inflation-targeting central bank,[2] and allow the policymaker to focus on different policy objectives under different circumstances.

The main findings of the paper are the following. First, it is confirmed that when agents do not possess complete knowledge about the structure of the economy and rely on an adaptive learning technology, a bias toward conservatism arises, suggesting that society is better off by appointing a policymaker whose degree of inflation aversion is higher than its own; even if there is no intrinsic dynamics in the economy, agents' and policymakers' attempts to learn adaptively introduce inertia in the system, which makes it costly for the central bank not to respond promptly and forcefully to shocks. Second, what matters for society's welfare is that the degree of inflation aversion of the monetary authority is high enough to prevent expectations from fluctuating too much: the specific form the loss function of the central bank takes is of second-order importance. Third, the bias against stabilization policies and toward conservatism, and the relative efficiency of alternative monetary strategies, do not depend on whether the memory of the learning process is finite or infinite.

The paper is related to the literature in several ways. It parallels, under more general conditions, the work of Terlizzese (1999) and Driffil and Rotondi (2003) in deriving the properties of a monetary strategy that has price stability as its primary objective and does not allow inflation to rise above an upper limit; it models two-sided learning in the vein of Evans and Honkapohja (2002); and it uses the study of Orphanides and Williams (2002) as a

[1]The following are just a few of these factors: (i) uncertainty (see al-Nowaihi and Stracca 2002); (ii) imperfect observability of the state variables (see Svensson and Woodford 2000); and (iii) path dependence and differential valuation of deviations from the inflation and output targets (see Orphanides and Wilcox 1996).

[2]Buiter (2006) presents a list of the central banks whose mandates reflect a lexicographic preference ordering, with price stability ranking first and all other desiderata coming after.

benchmark for the quantitative analysis. The most closely related of these contributions is the work of Orphanides and Williams: model simulations are designed so as to replicate their experiments, and the objective of this paper is to a large extent the same as theirs—namely, to understand how the economy responds to alternative monetary strategies when agents have bounded rationality and imperfect knowledge.

The original contribution of this work is to extend the findings by Orphanides and Williams (2002). It tries to do that in three different ways. First, it assumes that not only private agents but also the policymaker have imperfect knowledge; under this framework, policy effectiveness ends up depending on both inflation and output variability so that the bias toward conservatism, if confirmed, cannot be attributed to the limited role of output volatility in the reference model. Second, it tests whether society can increase welfare by appointing a policymaker whose preference ordering is lexicographic. Third, it tests the claim about the negative impact of imperfect knowledge on economic stabilization under a set of alternative learning mechanisms.

The paper is organized as follows. Section 2 outlines the model used in the paper and contrasts the implications of assuming quadratic or, alternatively, lexicographic preferences. Section 3 introduces econometric learning and studies how different policies affect the speed at which learning algorithms converge to the rational expectations equilibrium. Section 4 presents some evidence, obtained by means of simulation, on the distortions on monetary policymaking caused by assuming that agents have bounded rationality; the focus is on whether adaptive learning induces a bias toward conservatism and whether appointing a central banker whose preferences are lexicographic is welfare improving. Section 5 concludes.

2. The Model

The model presented in this section has two basic components: (i) the unobservability of the supply shock and (ii) an unknown and time-varying output-inflation trade-off. The model is first solved under rational expectations and then under adaptive learning.

2.1 The Structure of the Economy and the Delegation Problem

The economy is characterized by an expectations-augmented Phillips-curve relationship, linking inflation surprises $\pi - \pi^e$ to the output gap y:

$$y = \alpha(\pi - \pi^e) + \varepsilon. \quad (1)$$

Inflation is the policy instrument and is controlled without error by the monetary authority; the natural level of output is normalized at 0. Output also responds to a zero-mean supply shock ε, unobservable to the central bank and the private sector and uniformly distributed on the closed interval $[-\mu, \mu]$. A signal z, conveying noisy information on ε, is observed by the policymaker after expectations have been determined; it is assumed that $z = \varepsilon + \xi$, with ξ following a uniform distribution with the same support as ε, i.e., $\xi \sim U$ $[-\mu, \mu]$.[3]

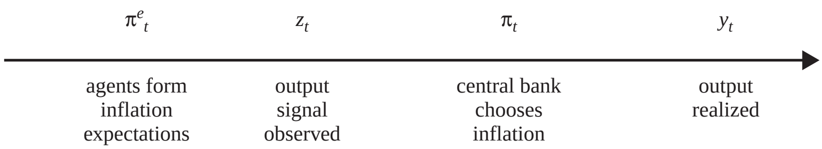
The final component of the model is the assumption that α, the output-inflation trade-off, is a random variable. Since α is time varying, the effects of monetary policy on output depend on the value of the trade-off. It is assumed that $\alpha = \bar{\alpha} + \widetilde{\alpha} \sim IID(\bar{\alpha}, \sigma_{\alpha}^2)$ and that it is independent of all the other shocks in the economy.[4] Notice that the model is entirely static, so that no issue of strategic interaction between the monetary authority and the private sector arises.

The timing of the model is shown in figure 1. The signal z materializes before the central bank chooses the inflation rate but after private agents have set their inflation expectations for the period. The information advantage of the central bank creates a role for

[3]The assumption that both ε and ξ follow a uniform distribution ensures that in the rational expectations equilibrium a closed-form solution exists. The additional hypothesis that both shocks share the same support helps to keep the distribution of z simple.

[4]The stochastic variable α can be interpreted as an index of monetary policy effectiveness. It can be either discrete or continuous. What is relevant is the i.i.d. assumption, which avoids introducing dynamic elements and strategic interactions into the optimization problem of the central bank. Ellison and Valla (2001) show that strategic interactions create a connection between the activism of the central bank and the volatility of inflation expectations: the latter reacts to the former because an activist policy produces more information, helping private agents learn. The value of experimentation in policymaking is also studied in Wieland (2003).

Figure 1. Timing of the Model



stabilization policies and takes into account the fact that policy decisions can be made more frequently than most wage and price decisions.

Society has quadratic utility and dislikes both inflation and output variability. Its welfare function is

$$W^S = -E[(y-k)^2 + \beta^S \pi^2], \tag{2}$$

where k is the target level of output; the expectation operator is due to the unobservability of the output-inflation trade-off α and output shock ε. The assumption that $k > 0$ is usually justified on the grounds that the presence of distortions in the labor and goods markets leads to an inefficiently low level of output in equilibrium; alternatively, $k > 0$ is interpreted as arising from political pressures on the central bank, as in Mishkin and Westelius (2006). In the principal-agent approach, society, whose preferences are quadratic, can appoint a central banker whose loss function may differ from its own. It is assumed that the choice is restricted to policymakers with either quadratic or lexicographic preferences; in the first case, the inflation aversion parameter β can be different from β^S.

2.2 Central Bank Loss Function

Though in general a lexicographic-preference ordering cannot be represented by a function, in the simplified case in which the monetary authority has been given only two objectives, such an ordering can be described by a loss function involving only the secondary objective, subject to a constraint involving the primary target. It is therefore assumed that the central bank aims at stabilizing output around a nonzero level, provided that inflation is kept below a known upper bound.[5] Though there is some dispute about the correctness of formulations depicting central bankers as affected by an inflation bias, the assumption is retained because otherwise the only rational expectation for inflation would be the zero target itself and the inflation constraint would never be binding.

In formal terms, the problem solved by the central bank is[6]

$$\begin{array}{ll} \min_\pi & \frac{1}{2}E(y-k)^2 \\ s.t. & \begin{cases} \pi \leq \bar{\pi} \\ y = \alpha(\pi - \pi^e) + \varepsilon. \end{cases} \end{array} \tag{3}$$

Notice that k cannot exceed μ, the upper bound of the output shock: in that which follows, it will be assumed that k is not too high and, in particular, that $k = \frac{\mu}{6}$. Under the standard hypothesis of time separability of preferences, the problem is static and involves no trade-off between current and future utility, so that the optimal policy does not have to rely on the strategic interactions described by Bertocchi and Spagat (1993), Ellison and Valla (2001), and Wieland (2003).

To highlight the implications of endowing the monetary authority with lexicographic preferences, the policy problem is also analyzed under the standard assumption that the loss function is quadratic. In this case, the problem solved by the central bank can be formulated as

$$\begin{array}{ll} \min_\pi & \frac{1}{2}E[(y-k)^2 + \beta\pi^2] \\ s.t. & y = \alpha(\pi - \pi^e) + \varepsilon, \end{array} \tag{4}$$

[5]The existence of a lower bound on inflation is neglected in this paper. In a model where the output shock is observable and the trade-off between output and inflation is time invariant, Terlizzese (1999) shows that the main features of the monetary policy problem are largely unaffected by the inclusion of a lower bound on inflation. Intuitively, what explains this result is the asymmetric nature of the inflation bias that is assumed to characterize the monetary authority's preferences: if the central bank aims at pushing output above the natural level, it will tend to inflate, so that while the upper bound will often be binding, the lower one will not.

[6]Buiter (2006) provides an alternative, more restrictive representation of a lexicographic-preference ordering.

where β measures the weight attached to the inflation objective relative to output stabilization.

Regardless of the specific form of the loss function, be it (3) or (4), the model features an inflation bias, arising from the policymaker's incentive to create surprise inflation in order to keep output above the natural level. Many economists, however, think that such a feature makes the model irrelevant for monetary policy analysis.[7] Jensen (2003) argues that the Barro-Gordon-type models can be rescued by noting that their main implications—notably, the inflation bias result—can be maintained without having to resort to the presumption that the central bank engenders inflation surprises to fool the public.

2.3 Signal Extraction and the Rational Expectations Equilibrium

Given the structure of the problem, the issues of estimating the unobserved output shock and setting the optimal inflation rate can be kept separated and solved sequentially. Before deciding the optimal policy, the central bank has to solve a signal extraction problem. The first step is therefore to derive the probability distribution of $z = \varepsilon + \xi$ and the conditional mean $E(\varepsilon|z)$. In lemma 1 the density function of the signal z is derived, while in proposition 1 the first moment of the distribution of the output shock ε conditional on z is calculated.

LEMMA 1. *If $z = \varepsilon + \xi$ and ε and ξ are independent uniform random variables, with support on the interval $[-\mu, \mu]$, then the density function of z is equal to $f(z) = \frac{1}{2\mu} + \frac{1}{4\mu^2}[\min(z, 0) - \max(0, z)]$.*

Proof. See Locarno (2006).

PROPOSITION 1. *If ε and ξ are uniform random variables, defined on the same close interval $[-\mu, \mu]$, and $z = \varepsilon + \xi$, then the optimal estimate of ε conditional on z is $E(\varepsilon|z) = \frac{z}{2}$.*

Proof. See Locarno (2006).

[7] See, for instance, the quotations from Blinder, Vickers, and Issing listed in Jensen (2003).

Given the assumption regulating the flow of information, the central bank sets the inflation rate on the basis of the observed signal and the private-sector inflation expectations. Under lexicographic preferences, it will choose the inflation rate that solves the first-order condition $E[\alpha(\alpha(\pi - \pi^e) + \varepsilon - k) \mid z] = 0$—provided the inflation constraint is satisfied—and will choose $\pi = \bar{\pi}$ otherwise, i.e.,

$$\pi = \begin{cases} \pi^e - \dfrac{\bar{\alpha}}{\bar{\alpha}^2 + \sigma_\alpha^2}\left(\dfrac{z}{2} - k\right) = \pi^e - \dfrac{z}{\phi} + \dfrac{2k}{\phi} & \text{if } z \geq 2k + \phi(\pi^e - \bar{\pi}), \\ \bar{\pi} & \text{otherwise,} \end{cases} \quad (5)$$

where $\phi^{-1} \equiv \frac{\bar{\alpha}}{\bar{\alpha}^2+\sigma_\alpha^2}\frac{1}{2}$. The optimal policy depends, in a nonlinear way, on the value of z: when output shocks are strongly negative and the primary objective is at risk, the central bank acts as an inflation nutter; when the signal indicates more-favorable disturbances, it displays more activism, favoring output stabilization. Notice that the optimal policy depends on the parameters of the distribution of the output-inflation trade-off α. Two cases are considered, one corresponding to the rational expectations equilibrium (REE), and the other assuming bounded rationality and least-squares learning. In the first case, it is assumed that α is not observed but $\bar{\alpha}$ and σ_α^2 are known by both the central bank and the private sector;[8] in the second case, $\bar{\alpha}$ and σ_α^2 are unknown and must be estimated.

A few points illustrating the main properties of optimal policy are worth stressing. First, the optimal policy is not altered by the unobservability of the output shock, except that the policymaker responds to an efficient estimate of the state variable rather than to its actual value. It is a well-known fact that a linear model with a quadratic loss function and a partially observable state of the economy is characterized by certainty equivalence; since the assumed preference ordering is not quadratic, the result applies only when inflation is within the admissible range. Second, uncertainty about the multiplier of the policy instrument makes it optimal to react less than completely to the output shock. There is nothing to be gained

[8]This assumption could be justified if ε, though unobserved at the time when expectations and the inflation rate were set, was observed with a one-period lag.

by adopting more-activist policies in order to learn from experimentation, since the model is static and the loss in current welfare incurred in overreacting will not be compensated by future gains. The reduction in policy activism caused by parameter uncertainty, originally shown by Brainard (1967), reflects the direct impact of the monetary authority's instrument on the variability of the target variable.

For the equilibrium to be fully characterized, the solution for expected inflation must be provided. Under rational expectations, agents understand the incentives driving the actions of the central bank and have expectations that coincide on average with realizations. Accordingly, $\pi^e = \int \pi dF(z)$, where $F(z)$ is the distribution function of the signal z. Proposition 2 gives the full characterization of the rational expectations equilibrium under the simplifying assumption that $k = \frac{\mu}{6}$.[9]

PROPOSITION 2. *If the central bank has lexicographic preferences and output is determined as in (1), there is a unique REE, where* $\pi = \min\left[\pi^e - \frac{\bar{\alpha}}{\bar{\alpha}^2+\sigma_\alpha^2}\left(\frac{z}{2} - k\right), \bar{\pi}\right]$ *and* $\pi^e = \bar{\pi} - \frac{2k}{\phi} = \bar{\pi} - \frac{\mu}{3\phi}$.

Proof. See Locarno (2006).

Equilibrium is noncooperative Nash: the central bank and the private sector try to maximize their respective objective functions, taking as given the other player's actions. The assumption of rational expectations implicitly defines the loss function of the private sector as $E(\pi - \pi^e)^2$: given the public's understanding of the central bank's decision problem, its choice of π^e is the one minimizing disutility.

From the expression for π^e, it is apparent that the existence of an upper bound on inflation contributes to stabilizing expected inflation: for any value of k, the lower $\bar{\pi}$ is, the lower expected inflation is. Another feature of the policy is that the larger the support of the output shock, the closer to zero π^e. The reason for this result is straightforward: positive (and higher than k) output shocks trigger a

[9]Setting $k = \frac{\mu}{6}$ amounts to assuming that $2k + \phi(\pi^e - \bar{\pi}) = 0$ and implies that the central bank chooses its best static response when $z > 0$, while it has no discretion for negative values of the signal.

reaction from the central bank, which creates negative inflation surprises to stabilize output; large negative disturbances, on the other hand, cannot be neutralized, because too-high inflation rates are not admissible. Widening the support of ε has an asymmetric effect on the actions of the monetary authority: it increases the cases in which the central bank finds it optimal to deflate, but has no influence on its incentives to inflate.

Two features of the optimal monetary policy are worth stressing. First, for values of the signal in the nonempty interval $[-2\mu, 0)$, inflation is constant and equal to $\bar{\pi}$, which is higher than π^e: the central bank keeps price dynamics above inflation expectations and, in so doing, it sustains output, though it cannot cushion it against shocks. Second, even in the face of favorable output shocks, the policymaker is unable to fully stabilize output at the desired level. Two factors contribute to attenuate the policymaker's response: uncertainty about the output-inflation trade-off and unobservability of ε. The former reduces the response of the central bank by a factor of $\frac{2\alpha}{\phi}$, while the latter leaves part of the output shock, namely, $\varepsilon - \frac{z}{2}$, unchecked. Since $\frac{z}{2}$ is an unbiased estimate of ε, unobservability of the output shock increases volatility but does not affect the degree of activism of the policy response; on the contrary, unobservability of the output-inflation trade-off has a bearing on the policy strategy, since it favors more cautious policies. It turns out that output volatility is therefore smaller than $\frac{\mu^2}{3}$, the variance of the output shock, implying some degree of stabilization on the side of monetary policy.

To assess the distinguishing traits of the policy pursued by a central bank with lexicographic preferences, it is useful to contrast it with the optimal policy arising under the standard assumption of quadratic loss function. If (4) describes the central bank's problem, the policy instrument is set according to the rule

$$\pi = \frac{\bar{\alpha}^2 + \sigma_\alpha^2}{\bar{\alpha}^2 + \sigma_\alpha^2 + \beta}\pi^e + \frac{\bar{\alpha}}{\bar{\alpha}^2 + \sigma_\alpha^2 + \beta}\left(k - \frac{z}{2}\right) = \rho\pi^e + \frac{\rho}{\phi}(2k - z)$$
$$= \pi^e - \frac{\rho}{\phi}z, \tag{6}$$

where $\rho \equiv \frac{\bar{\alpha}^2 + \sigma_\alpha^2}{\bar{\alpha}^2 + \sigma_\alpha^2 + \beta}$, with $0 < \rho \leq 1$.

A few differences are apparent when comparing the two optimal policies. When output shocks are not too negative, rule (5) ensures more output stabilization, as the increase in the inflation rate that must be engendered to counteract the supply disturbance does not have a negative impact on welfare and hence does not bring on a trade-off between the output and the inflation objectives. The reverse is true when ε is large and negative, because in that case output stabilization is sacrificed to the primary objective of price stability. More activist policies are possible at the cost of larger inflation variability: in general, there exists a value $\bar{\beta}$ such that, for $\beta \in (\bar{\beta}, \infty)$, output variability under lexicographic preferences (henceforth, strategy 1) is lower than under quadratic utility (henceforth, strategy 2), and there exists a value $\underline{\beta}$ such that, for $\beta \in [0, \underline{\beta})$, $E(\pi - \pi^e)^2$ is smaller under strategy 1.[10] Since $\underline{\beta} < \bar{\beta}$, strategy 1 apparently cannot outperform strategy 2 in terms of both objectives, but in reality this is not necessarily the case, since what is important for social welfare is not $E(\pi - \pi^e)^2$ but $E\pi^2$. Under strategy 1 the central bank can use an additional instrument—the upper bound on inflation $\bar{\pi}$—to try to achieve both lower output volatility and lower mean-square inflation: the reason that strategy 1 is more appealing compared with Rogoff's (1985) solution is that the reduction of the inflation bias does not come at the cost of the output-stabilization objective;[11] the reason it is less valuable is that it tends to stabilize output too much when $\pi < \bar{\pi}$.

3. Adaptive Learning and Monetary Policy Regimes

In the real world, where shifts in policies and in the economic structure are by no means rare events, people often face the problem of understanding whether and how the environment has changed and which is the least costly way to adapt decision rules to suit the new framework. In such a context, a strict application of the rational expectations hypothesis (REH) would not be a convincing

[10] See Locarno (2006), proposition 4, for a formal proof.

[11] This is easily seen by noting that the choice of $\bar{\pi}$ has no effect on the set of values of z corresponding to inactive policies. This implies that it is always optimal for the policymaker to set $\bar{\pi}$ so that π^e is equal to 0.

theoretical solution. Alternatives have long been suggested. Herbert Simon (1957), for instance, supported some kind of bounded rationality and proposed to create a theory with behavioral foundations where agents learn in the same way as econometricians do. An increasingly important stream of literature, which builds on the pioneering work of Bray (1982) and Marcet and Sargent (1989), recently revived in particular by Evans and Honkapohja (2001), has introduced a specific form of bounded rationality, called adaptive learning, where agents adjust their forecast rule as new data becomes available over time. This approach provides an asymptotic justification for the REH and makes nonlearnable solutions in models with multiple equilibria irrelevant.

The central idea behind adaptive learning is that at each period t, private agents have a perceived law of motion (PLM) that they use to make forecasts. The PLM relates the variables of interest, whose future values are to be anticipated, to a set of state variables; the projection parameters are estimated using least squares. Forecasts generated in this way are used in decisions for period t, which yields the temporary equilibrium, also called the actual law of motion (ALM). The temporary equilibrium provides a new data point, and agents are then assumed to reestimate the projection parameters with data through period t and to use the updated forecast functions for period $t+1$ decisions. The learning dynamics continue with the same steps in subsequent periods.

The updating of the projection coefficients may be represented in terms of a system of recursive equations having, under reasonable assumptions for the PLM, the rational expectations equilibrium as a fixed point. The recursive equations describe the mapping between the PLM and the ALM. Convergence of the adaptive learning process may be studied by means of the associated ordinary differential equation (ODE):[12] stability holds whenever the real parts of the eigenvalues of the Jacobian of the ODE are negative, i.e., whenever the system is E-stable.

[12]For large t, the stochastic recursive algorithm is well approximated by an ordinary differential equation, provided a few regularity conditions are met. These regularity conditions involve the stochastic process driving the state variables, the deterministic gain sequence, and the function governing the revision in the projection coefficients.

The connection between E-stability and the convergence of least-squares learning is a great advantage, since E-stability conditions are often easier to work out. However, focusing on asymptotic approximations puts aside any consideration of the learning speed. Benveniste, Metiver, and Priouret (1990) show that root-t convergence of the learning process holds when the real part of the largest eigenvalue of the Jacobian of the ODE is less than $-\frac{1}{2}$.[13] When this condition on the eigenvalues is not met, no analytic results on the asymptotic distribution are known, since the importance of initial conditions fails to die out quickly enough. Marcet and Sargent (1992) suggest a numerical procedure to obtain an estimate of the rate of convergence. The starting point is the assumption that there is a δ for which

$$t^{\delta}(\theta_t - \theta) \xrightarrow{D} F, \tag{7}$$

where θ_t is the vector of parameters of the PLM, θ is its asymptotic limit, and F is some nondegenerate, well-defined, mean-zero distribution. Marcet and Sargent show that, for large t, a good approximation of the rate of convergence δ is given by the expression

$$\delta = \frac{1}{\log l} \log \sqrt{\frac{E(\theta_t - \theta)^2}{E(\theta_{tl} - \theta)^2}}. \tag{8}$$

Given t and l, the expectations can be approximated by simulating a large number of independent realizations of length t and $l \times t$, and calculating the mean square across realizations.

In this section, adaptive learning is introduced to analyze the implications of imperfect knowledge on policy outcomes. The question to be answered is how the interaction between learning and central bank preferences affects aggregate welfare. For the sake of clarity, the case where only the private sector learns is considered first; then the model is expanded to incorporate central bank learning.

[13] Root-t convergence means convergence at a rate of the same order as the root square of the sample size.

3.1 Private-Sector Learning

Suppose that private agents have nonrational expectations, which they try to correct through adaptive learning. Assume also that the policymaker does not explicitly take agents' learning into account and continues to set policy according to either (5) or (6). The evolution of output and inflation is therefore described by the system

$$y = \alpha(\pi - \hat{E}^P\pi) + \varepsilon$$
$$\pi = \begin{cases} \min\left[\hat{E}^{CB}\pi - \frac{z}{\phi} + \frac{2k}{\phi}, \bar{\pi}\right] \\ \rho\hat{E}^{CB}\pi - \frac{\rho}{\phi}z + \frac{\rho}{\phi}2k, \end{cases} \tag{9}$$

where the inflation rate depends on the monetary authority's preferences. $\hat{E}^P\pi$ represents the current estimate of the inflation rate of the private sector, while $\hat{E}^{CB}\pi$ is the value of inflation expectations used in the central bank's control rule. It is assumed that private agents run regressions to set $\hat{E}^P\pi$, while the monetary authority, which observes $\hat{E}^P\pi$ before moving, has rational expectations and therefore sets $\hat{E}^{CB}\pi = \hat{E}^P\pi$.

At each period t, private agents have a PLM for inflation, which they use to make forecasts. This PLM takes the form $\hat{E}^P\pi_t = a_{Pt}$,[14] where

$$a_{Pt} = a_{Pt-1} + \frac{1}{t}(\pi_{t-1} - a_{Pt-1}). \tag{10}$$

The estimate a_{Pt} is updated over time using least squares; $\frac{1}{t}$ represents the gain parameter, which is a decreasing function of the sample size. Equation (10), in line with the literature, is in recursive form, uses data up to period $t-1$, and requires a starting value at time $t = 0$. The PLM has the same form as the RE solution for expected inflation: private agents estimate the parameter of the reduced form and set $\hat{E}^P\pi_t = a_{P_t}$.

[14] A time index is used only when strictly necessary—for instance, when tracking the evolution over time of least-squares learning.

Consider first the case in which the central bank acts as if it had a lexicographic ordering of preferences. The ALM turns out to be

$$\pi_t = \begin{cases} a_{Pt} - \dfrac{z_{t-1}}{\phi} + \dfrac{2k}{\phi} & \text{if } z_{t-1} \geq 0, \\ \bar{\pi} & \text{otherwise.} \end{cases} \tag{11}$$

The mapping between the PLM and the ALM generates the stochastic recursive algorithm

$$a_{Pt} = \begin{cases} a_{Pt-1} + \dfrac{1}{t}\left(-\dfrac{z_{t-1}}{\phi} + \dfrac{2k}{\phi}\right) & \text{if } z_{t-1} \geq 0, \\ a_{Pt-1} + \dfrac{1}{t}\left(\bar{\pi} - a_{Pt-1}\right) & \text{otherwise,} \end{cases} \tag{12}$$

which is approximated by the following ODE,

$$\frac{d}{d\tau} a_P = h(a_P) = \lim_{t \to \infty} E(\pi_{t-1} - a_P), \tag{13}$$

where

$$\begin{aligned} \lim_{t \to \infty} E(\pi_{t-1} - a_P) &= (\bar{\pi} - a_P) \int_{-2\mu}^{0} \left(\frac{1}{2\mu} + \frac{z}{4\mu^2}\right) dz \\ &\quad + \int_{0}^{2\mu} \left(-\frac{z}{\phi} + \frac{2k}{\phi}\right)\left(\frac{1}{2\mu} - \frac{z}{4\mu^2}\right) dz \\ &= \frac{1}{2}(\bar{\pi} - a_P) + \frac{k - \mu/3}{\phi} \\ &= \frac{1}{2}(\bar{\pi} - a_P) - \frac{\mu}{6\phi}. \end{aligned}$$

Notice that the fixed point of the ODE, namely, $a_P = h^{-1}(0) = \bar{\pi} - \frac{2k}{\phi} = \bar{\pi} - \frac{\mu}{3\phi}$, coincides with the unique REE for expected inflation. The theorems on the convergence of stochastic recursive algorithms can be applied so that convergence is governed by the stability of the associated ODE.[15] Since $\frac{d}{da_P} h(a_P) = -\frac{1}{2} < 0$, the ODE is

[15] Chapter 6 of Evans and Honkapohja (2001) studies the conditions under which convergence of stochastic recursive algorithms is ensured. In the case of

(globally) stable and hence adaptive learning asymptotically converges to the REE. Notice that the size of the eigenvalue of $h(a_P)$ depends on the share of the support of z corresponding to an active policy: in general, root-t convergence does not hold.

Consider now the situation in which the central bank has quadratic preferences. The optimal policy for the monetary authority is to set $\pi_t = \rho a_{Pt} - \frac{\rho}{\phi} z_t + \frac{\rho}{\phi} 2k$. Compared with the RE case, the central bank does not completely offset inflation expectations and the parameter k enters explicitly the control rule: both features disappear asymptotically, provided $a_P \rightarrow \pi^e = \frac{\bar{\alpha}}{\beta} k$.

If agents use recursive least squares, then expectations evolve according to the equations

$$\begin{aligned} a_{Pt} &= a_{Pt-1} + \frac{1}{t}(\pi_{t-1} - a_{Pt-1}) \\ &= a_{Pt-1} + \frac{1}{t}\left[(\rho - 1)a_{Pt-1} - \frac{\rho}{\phi} z_{t-1} + \frac{\rho}{\phi} 2k\right] \end{aligned} \tag{14}$$

and

$$h(a_P) = \lim_{t\rightarrow\infty} E\left[(\rho - 1)a_{Pt} - \frac{\rho}{\phi} z_{t-1} + \frac{\rho}{\phi} 2k\right] = (\rho - 1)a_P + \frac{\rho}{\phi} 2k. \tag{15}$$

Also in this case, the fixed point of the ODE, namely, $a_P = h^{-1}(0) = \frac{\rho}{(1-\rho)\phi} 2k = \frac{\bar{\alpha}}{\beta} k$, coincides with the unique REE and the system is (globally) stable. In fact, $\frac{d}{da_P} h(a_P) = \rho - 1 < 0$, since ρ is positive and smaller than 1. Whether root-t convergence holds depends on the size of β: the higher the weight the central bank attaches to the inflation objective, the faster agents learn. The explanation of this result is quite intuitive: the attempt to offset output shocks requires generating inflation surprises—i.e., moving the inflation rate away from expectations—so that every period agents will have to revise their estimate with values of π that may substantially differ from the unconditional mean. A similar result applies

---

interest, they amount to show that the process π, which is a linear function of z, is bounded and stationary and that the function driving the updating of the projection parameter, namely, $\pi_{t-1} - a_{Pt-1}$, is bounded and is twice countinuously differentiable (with respect to both π_{t-1} and a_{Pt-1}), with bounded second derivatives. Whether stability holds locally or globally depends on whether the regularity conditions hold on an open set around the equilibrium or for all admissible values of a_P. These regularity conditions, in the present case, are clearly met.

to the case of lexicographic preferences: by reducing the support of the signal corresponding to an active policy (i.e., to a policy that aims at avoiding excessive output fluctuations), expectations adjust more quickly to the long-term equilibrium. Notice that for reasonable parameterization of the model, the value of β must be large for $\frac{d}{da_P}h(a_P)$ to be less than $-\frac{1}{2}$, meaning that convergence holds only in the case of a highly inflation-averse central bank.

The previous result is interestingly similar to the one in Orphanides and Williams (2002), who use a dynamic model based on aggregate supply and demand equations. They find that, with imperfect knowledge, the ability of private agents to forecast inflation depends on the monetary policy in place, with forecast errors on average smaller when the central bank responds more aggressively to inflationary pressures. Significantly improved economic performances can be achieved by placing greater emphasis on controlling inflation: indeed, more-aggressive policies reduce the persistence of inflation and facilitate the formation of expectations, which in turn enhances economic stability and mitigates the influence of imperfect knowledge on the economy. The conclusion of the paper turns out to be quite similar to Rogoff's (1985) solution to the central bank's credibility problem under discretion: to improve welfare, the responsibility of the conduct of monetary policy must be delegated to a policymaker who is more inflation averse than society.

3.2 Private-Sector and Central Bank Learning

Consider now the case in which $\bar{\alpha}$ and σ_α^2 (or, alternatively, ϕ) are not known to the policymaker. The central bank needs to estimate them since both parameters affect the policy rule through the degree of responsiveness to the signal z. As usual, it is assumed that they are gauged by means of least squares and that the estimate is updated every time new realizations of y and π are available. This form of bounded rationality corresponds to the case in which ε is never observed, so that $\bar{\alpha}$ and σ_α^2 cannot be directly estimated on the basis of past realizations of the output shock.

To account for central bank learning, the previous model must be augmented with a new set of recursive equations, which are the same irrespective of the monetary authority's preferences, as learning involves parameters rather than variables so that the values to be estimated are not related to agents' behavior.

The system of recursive least-squares equations is now the following:

$$\begin{aligned} a_{Pt} &= a_{Pt-1} + \frac{1}{t}(\pi_{t-1} - a_{Pt-1}) \\ \hat{\bar{\alpha}}_t &= \hat{\bar{\alpha}}_{t-1} + \frac{1}{t} S_{\pi,t}^{-1}(\pi_{t-1} - a_{Pt-1}) \\ &\quad \times \left[\left(y_{t-1} - \frac{z_{t-1}}{2} \right) - \hat{\bar{\alpha}}_{t-1}(\pi_{t-1} - a_{Pt-1}) \right] \\ S_{y,t} &= S_{y,t-1} + \frac{1}{t} \left[\left(y_{t-1} - \frac{z_{t-1}}{2} \right)^2 - S_{y,t-1} \right] \\ S_{\pi,t} &= S_{\pi,t-1} + \frac{1}{t} [(\pi_{t-1} - a_{Pt-1})^2 - S_{\pi,t-1}] \end{aligned} \tag{16}$$

or, more compactly,

$$\theta_t = \theta_{t-1} + \frac{1}{t} Q(t, \theta_{t-1}, X_t),$$

where $\theta_t = (a_{Pt}, \hat{\bar{\alpha}}_t, R_{y,t}, R_{\pi,t})'$, $R_{y,t-1} = S_{y,t}, R_{\pi,t-1} = S_{\pi,t}$, and $X_t = (1, \alpha_t, z_t, \varepsilon_t)'$. The first equation is the same as in the previous section and captures private-sector learning, while the others refer to the central bank's inference problem: $\hat{\bar{\alpha}}_t$ is an estimate of the mean value of the output-inflation trade-off; $R_{y,t}$ measures the sample variance of $y - \frac{z}{2}$, the policy-driven component of the output gap;[16] and $R_{\pi,t}$ is the second moment of the inflation surprise. As shown below, the central bank calculates the statistics $R_{y,t}$ and $R_{\pi,t}$ as an intermediate step in estimating the optimal response coefficient to the signal z in the policy rule.

While the recursion for $R_{\pi,t}$ is obvious, as it is simply the estimate of the variance of the inflation surprise, the other two equations require some explanations. To understand the recursion for $\hat{\bar{\alpha}}_t$, notice that the output equation can be rearranged as

$$y - \frac{z}{2} = \bar{\alpha}(\pi - a_P) + \left[\varepsilon - \frac{z}{2} + \widetilde{\alpha}(\pi - a_P) \right]. \tag{17}$$

[16]Since $E(\varepsilon|z) = \frac{z}{2}$, the difference $y - \frac{z}{2}$ represents (an unbiased estimate of) the share of the output gap that depends on the inflation surprise only.

The central bank observes the signal z and can efficiently estimate $\bar{\alpha}$ by regressing $y - \frac{z}{2}$ on the inflation surprise $(\pi - a_P)$. Using $y - \frac{z}{2}$ as the regressand allows for the elimination of the simultaneity bias: the inflation surprise, being a linear function of the signal z only, is uncorrelated with $\varepsilon - \frac{z}{2}$ (the residual of the regression of ε on z) and with $\widetilde{\alpha}$, by assumption orthogonal to all other shocks in the model.

The justification for the recursion for $R_{y,t}$ is somewhat more complicated. A biased estimator of $E(\alpha^2)$ can be obtained from the sample average of the squared (policy-driven component of the) output gap, scaled by the second moment of the inflation surprise,

$$\frac{E\left(y - \frac{z}{2}\right)^2}{E(\pi - a_P)^2} = \frac{E(\alpha^2)E(\pi - a_P)^2 + E\left(\varepsilon - \frac{z}{2}\right)^2}{E(\pi - a_P)^2}$$
$$= \bar{\alpha}^2 + \sigma_\alpha^2 + \frac{2\frac{\mu^2}{3}}{E(\pi - a_P)^2}.$$

The bias can be easily calculated, since it depends on $E(\pi - a_P)^2$ and on known parameters. The sample estimate of $\bar{\alpha}^2 + \sigma_\alpha^2$ is therefore obtained by using the expression $\psi_t \equiv \frac{R_{y,t} - 2\frac{\mu^2}{3}}{R_{\pi,t}}$.[17]

Whether the stochastic recursive algorithm converges depends on the associated ODE, i.e., on the Jacobian of the matrix $h(\theta) = \lim_{t\to\infty} EQ(t, \theta, X_t)$. In the case of lexicographic preferences, it can be shown that the ODE is

$$\begin{bmatrix} \frac{d}{d\tau} a_P \\ \frac{d}{d\tau}\hat{\bar{\alpha}} \\ \frac{d}{d\tau} R_y \\ \frac{d}{d\tau} R_\pi \end{bmatrix} = h(\theta) = \begin{bmatrix} \frac{1}{2}(\bar{\pi} - a_P) - \frac{\hat{\bar{\alpha}}}{2\frac{R_y - 2\mu^2/3}{R_\pi}}\frac{\mu}{6} \\ R_\pi^{-1} E(\pi - a_P)^2(\bar{\alpha} - \hat{\bar{\alpha}}) \\ E\left(y - \frac{z}{2}\right)^2 - R_y \\ E(\pi - a_P)^2 - R_\pi \end{bmatrix}, \quad (18)$$

[17] One alternative could have been to regress $(y - \frac{z}{2})^2$ on $(\pi - a_P)^2$. The drawback of this approach is that the bias is a more convoluted function of the model parameters than it is in the case considered in the paper.

while for the standard quadratic case it is equal to

$$\begin{bmatrix} \frac{d}{d\tau}a_P \\ \frac{d}{d\tau}\hat{\hat{\alpha}} \\ \frac{d}{d\tau}R_y \\ \frac{d}{d\tau}R_\pi \end{bmatrix} = h(\theta) = \begin{bmatrix} -\frac{\beta}{\frac{R_y - 2\mu^2/3}{R_\pi} + \beta} a_P + \frac{\hat{\hat{\alpha}}}{\frac{R_y - 2\mu^2/3}{R_\pi} + \beta} \frac{\mu}{6} \\ R_\pi^{-1} E(\pi - a_P)^2 (\bar{\alpha} - \hat{\hat{\alpha}}) \\ E\left(y - \frac{z}{2}\right)^2 - R_y \\ E(\pi - a_P)^2 - R_\pi \end{bmatrix}. \tag{19}$$

It is apparent that while the specific form of the loss function does not affect the inference problem of the central bank, it has a bearing on private-sector learning.

Both systems are recursive. $R_\pi \to E(\pi - a_P)^2$ from any starting point, which implies that $R_\pi^{-1}E(\pi - a_P)^2 \to I$, provided R_π is invertible along the path. The same happens for R_y. Hence, the stability of the differential equation for $\hat{\hat{\alpha}}$ may be assessed regardless of the remaining part of the system. Conditional on $\hat{\hat{\alpha}}$, R_y, and R_π approaching the true parameter values, convergence to the REE of private-sector expectations is determined on the basis of the eigenvalues of the ODE for a_P. It is noticeable that the probability limit of the latter does not depend on the information set of the central bank and is the same whether or not the monetary authority knows the full structure of the economy. Conditions for learnability of the REE under both lexicographic and quadratic preferences are stated in the next proposition.

PROPOSITION 3. *Assume that the economy is endowed with agents that rely on adaptive learning to form expectations; moreover, assume that the central bank has only incomplete information about the structure of the economy and uses recursive least squares (RLS) to estimate the unknown parameters. Then, the asymptotic behavior of the system is described by (18) and (19) and, regardless of whether the policymaker has quadratic or lexicographic preferences, the discretionary REE is unique and E-stable: the estimates* $(\hat{\hat{\alpha}}_t, \psi_t)$ *converge locally to* $(\bar{\alpha}, \bar{\alpha}^2 + \sigma_\alpha^2)$ *and the expectations of private agents tend in the limit to the RE values.*

Proof. See the appendix.

As in the case when only the private sector learns, the effect of preferences on the speed of convergence is not clear. For small values of β, a central bank setting policy so as to minimize a quadratic loss function seems to be less effective in driving the economy toward the REE, while the opposite is true when β is high. However, central banks' imperfect knowledge introduces an additional layer of interaction between monetary policy and economic outcomes, and model dynamics cannot be properly analyzed by focusing only on asymptotic distributions. In particular, when the learning process is disturbed by several sources of shocks, the ODE becomes an acceptable approximation to the stochastic recursive algorithm only for large values of t, and the asymptotic distribution is not of much help in understanding the properties of the system. The problem is even more serious in models where there are multiple equilibria, since in such cases, in early time periods, when estimates are based on very few degrees of freedom, large shocks can displace θ_t outside the domain of attraction of the ODE, and the system can therefore converge to any of the equilibrium points.[18] It follows then that when the agents' information set is severely constrained, both the asymptotic and the finite sample behavior of the system are relevant. Theoretical results are therefore no longer sufficient and it becomes necessary to rely on simulation experiments and numerical results.

4. Imperfect Knowledge and Policy Effectiveness

Model simulations are used to illustrate how learning affects the dynamic properties of inflation, inflation expectations, and output. First the performance of the forecasting rules is assessed; then the issue of the relative speed of convergence is considered; finally, the output-inflation variability trade-off under alternative monetary regimes is assessed. To account for the finding by Orphanides and

[18]When there is a unique equilibrium and the ODE is stable, it can be shown that $\theta_t \to \theta^*$ with probability 1 from any starting point. When there are multiple equilibria, however, such a strong result does not apply, unless one artificially constrains θ_t to an appropriate neighborhood of the locally stable equilibrium θ^*. In the earlier literature, local convergence was obtained by making an additional assumption about the algorithm, known as the *projection facility.* As a reference, see Evans and Honkapohja (2001, section 6.4).

Table 1. Baseline Calibrated Parameters

Parameter	Value
$\bar{\alpha}$	1.75
σ_α	0.5
μ	0.0175
k	0.0029

Williams (2002)—that policies that put too much weight on output stabilization can generate episodes in which the public's expectations of inflation become uncoupled from the policy objective—additional simulations are run mimicking the impact of a string of negative supply shocks on the economy. Finally, as a further check on how much the results depend on the chosen learning mechanism, the assumption of infinite memory is dropped and the case of perpetual learning is considered.[19]

Each experiment is based on 500 replications, and all simulations cover an interval of 2,000 periods. Subsamples of 500 observations are also considered in order to estimate the convergence speed. Initial conditions for the lagged variables in the RLS algorithm are randomly drawn from the distribution corresponding to the REE. Results reported in the tables are calculated excluding the first 150 periods so as to minimize the impact of initial observations, which could be too far away from the equilibrium solution. The model is calibrated according to the estimates in Ellison and Valla (2001); the selected parameter values are reported in table 1. Concerning β, the relative weight in the loss function of the inflation objective, three values are considered—namely, $\beta = \{.176, 1, 5.667\}$. Under lexicographic preferences, it is assumed that $\bar{\pi}$ is chosen so as to drive inflation expectations to 0.

Tables 2 and 3 report the simulation results: the first table describes the "plain" RLS learning rule (unconstrained estimation, or UE), while the second shows results for the case of constrained

[19]Perpetual learning is sometimes used as a synonym of constant-gain learning. A constant-gain algorithm is preferable when the agents believe that the economic environment is subject to frequent structural changes. In such cases, observations from the distant past are no longer informative and can instead become a source of distortion.

Table 2. Least-Squares Learning and the Volatility of Output and Inflation (Unconstrained Estimator—Decreasing-Gain Sequence)

The table reports the estimated value (in 500 replications) of expected inflation, the coefficient of the optimal policy rule, the standard deviation of output and inflation, the performance index, and the rate at which estimates of α_p, $\hat{\hat{\alpha}}$, and ψ converge to the REE. The performance index is the ratio between the social loss function under the delegated policymaker and that which would have been obtained by appointing a central banker with the same preferences as society's. Agents are assumed to have infinite memory, implying a decreasing-gain sequence. Recursive least-squares estimates are unconstrained (UE case). In columns 1 and 3, the RE values for lexicographic and, respectively, quadratic preferences are shown; in columns 2 and 4, the same statistics are presented for the case when agents learn.

	Lexicographic Preferences		Quadratic Preferences	
	RE	**T = 2,000**	**RE**	**T = 2,000**
			$\beta = 0.176$	
Mean α_p	0.0000	−0.0005	0.0289	0.0283
Policy Rule Coefficient	0.2642	0.1807	0.2508	0.2308
Output Variability	0.0084	0.0088	0.0074	0.0076
Inflation Variability	0.0022	0.0023	0.0291	0.0298
Performance Index: $\beta^s = 0.176$	0.3494	0.3628	1.0000	1.0000
$\beta^s = 1.0$	0.8203	0.8453	9.8428	9.7605
$\beta^s = 5.667$	1.0925	1.1462	54.5380	55.0000
Convergence Speed	α_p: $\delta = 0.3351$		α_p: $\delta = 0.3570$	
	α: $\delta = 0.4951$		α: $\delta = 0.2435$	
	ψ: $\delta = 0.4609$		ψ: $\delta = 0.4884$	
			$\beta = 1.0$	
Mean α_p	0.0000	−0.0005	0.0051	0.0048
Policy Rule Coefficient	0.2642	0.1807	0.2029	0.1844
Output Variability	0.0084	0.0088	0.0076	0.0078
Inflation Variability	0.0022	0.0023	0.0059	0.0059
Performance Index: $\beta^s = 0.176$	0.3494	0.3628	0.3105	0.3168
$\beta^s = 1.0$	0.8203	0.8453	1.0000	1.0000
$\beta^s = 5.667$	1.0925	1.1462	2.8307	2.8287
Convergence Speed	α_p: $\delta = 0.3351$		α_p: $\delta = 0.2185$	
	α: $\delta = 0.4951$		α: $\delta = 0.2728$	
	ψ: $\delta = 0.4609$		ψ: $\delta = 0.4892$	
			$\beta = 5.667$	
Mean α_p	0.0000	−0.0005	0.0009	0.0008
Policy Rule Coefficient	0.2642	0.1807	0.0975	0.0775
Output Variability	0.0084	0.0088	0.0086	0.0088
Inflation Variability	0.0022	0.0023	0.0017	0.0016
Performance Index: $\beta^s = 0.176$	0.3494	0.3628	0.3627	0.3680
$\beta^s = 1.0$	0.8203	0.8453	0.8319	0.8340
$\beta^s = 5.667$	1.0925	1.1462	1.0000	1.0000
Convergence Speed	α_p: $\delta = 0.3351$		α_p: $\delta = 0.4080$	
	α: $\delta = 0.4951$		α: $\delta = 0.3642$	
	ψ: $\delta = 0.4609$		ψ: $\delta = 1.0000$	

Table 3. Least-Squares Learning and the Volatility of Output and Inflation (Constrained Estimator—Decreasing-Gain Sequence)

The table reports the estimated value (in 500 replications) of expected inflation, the coefficient of the optimal policy rule, the standard deviation of output and inflation, the performance index, and the rate at which estimates of α_p, $\hat{\hat{\alpha}}$, and ψ converge to the REE. The performance index is the ratio between the social loss function under the delegated policymaker and that which would have been obtained by appointing a central banker with the same preferences as society's. Agents are assumed to have infinite memory, implying a decreasing-gain sequence. Recursive least-squares estimates are constrained to belong to a subset of the parameter space (CE case). In columns 1 and 3, the RE values for lexicographic and, respectively, quadratic preferences are shown; in columns 2 and 4, the same statistics are presented for the case when agents learn.

	Lexicographic Preferences		**Quadratic Preferences**	
	RE	**T = 2,000**	**RE**	**T = 2,000**
			$\beta = 0.176$	
Mean α_p	0.0000	−0.0003	0.0289	0.0267
Policy Rule Coefficient	0.2642	0.2365	0.2508	0.2411
Output Variability	0.0084	0.0087	0.0074	0.0075
Inflation Variability	0.0022	0.0025	0.0291	0.0271
Performance Index: $\beta^s = 0.176$	0.3494	0.4153	1.0000	1.0000
$\beta^s = 1.0$	0.8203	0.9092	9.8428	8.7535
$\beta^s = 5.667$	1.0925	0.7491	54.5380	28.4679
Convergence Speed	α_p: $\delta = 0.4193$		α_p: $\delta = 0.2972$	
	α: $\delta = 0.4670$		α: $\delta = 0.4557$	
	ψ: $\delta = 0.4765$		ψ: $\delta = 0.4892$	
			$\beta = 1.0$	
Mean α_p	0.0000	−0.0003	0.0051	0.0047
Policy Rule Coefficient	0.2642	0.2365	0.2029	0.1913
Output Variability	0.0084	0.0087	0.0076	0.0077
Inflation Variability	0.0022	0.0025	0.0059	0.0056
Performance Index: $\beta^s = 0.176$	0.3494	0.4153	0.3105	0.3475
$\beta^s = 1.0$	0.8203	0.9092	1.0000	1.0000
$\beta^s = 5.667$	1.0925	0.7491	2.8307	1.5986
Convergence Speed	α_p: $\delta = 0.4193$		α_p: $\delta = 0.1616$	
	α: $\delta = 0.4670$		α: $\delta = 0.4525$	
	ψ: $\delta = 0.4765$		ψ: $\delta = 0.4918$	
			$\beta = 5.667$	
Mean α_p	0.0000	−0.0003	0.0009	0.0014
Policy Rule Coefficient	0.2642	0.2365	0.0975	0.0805
Output Variability	0.0084	0.0087	0.0086	0.0105
Inflation Variability	0.0022	0.0025	0.0017	0.0026
Performance Index: $\beta^s = 0.176$	0.3494	0.4153	0.3627	0.6033
$\beta^s = 1.0$	0.8203	0.9092	0.8319	1.2990
$\beta^s = 5.667$	1.0925	0.7491	1.0000	1.0000
Convergence Speed	α_p: $\delta = 0.4193$		α_p: $\delta = 0.3331$	
	α: $\delta = 0.4670$		α: $\delta = 0.4544$	
	ψ: $\delta = 0.4765$		ψ: $\delta = 0.4870$	

estimation (CE). The latter amounts to impose some minimal restriction on the admissible regions of the estimates of $\bar{\alpha}$ and $\bar{\alpha}^2 + \sigma_\alpha^2$: in the first case, it is assumed that only positive values of $\hat{\bar{\alpha}}_t$ are admissible, since surprises cannot have a negative impact on output; in the second, it is assumed that only values of ψ_t greater than $\hat{\bar{\alpha}}_t^2$ are sensible, since variances cannot be negative. The constraints act as substitutes for a *projection facility*:[20] though they cannot guarantee almost-sure convergence of the learning algorithm, they can in principle contribute to reducing the number of nonconvergent replications.

Expected inflation, the estimate of the policy rule coefficient, the standard deviation of output and inflation, and the performance index are reported in the tables, as is the convergence speed for a_P, $\hat{\bar{\alpha}}_t$, and ψ_t. The performance index is defined as

$$\frac{Ey_{LEX}^2 + \beta^S E\pi_{LEX}^2}{Ey_{QUA(\beta^S)}^2 + \beta^S E\pi_{QUA(\beta^S)}^2} \text{ or } \frac{Ey_{QUA(\beta)}^2 + \beta^S E\pi_{QUA(\beta)}^2}{Ey_{QUA(\beta^S)}^2 + \beta^S E\pi_{QUA(\beta^S)}^2},$$

depending on which strategy is evaluated. Ey_{LEX}^2 is the second moment of output under strategy 1, and the other terms have a similar meaning; β is in general different from β^S, though the case $\beta = \beta^S$ is also considered. The index is equal to the ratio between the social loss function under the delegated policymaker and the loss that would be obtained by appointing a central banker with the same preferences as society's: when $PI < 1$, it is efficient to delegate to a policymaker with a utility function that is different from society's. For ease of comparison, the values of the performance index are also shown for the REE.

Regardless of the central bank type, the estimates of expected inflation are precise but biased downward: the higher the values of

[20]Convergence of the learning process to the REE holds almost surely when there is a unique solution and the ODE is globally stable; in general, convergence with probability 1 is not guaranteed, since the ODE is not a reliable approximation of the stochastic recursive algorithm for small values of t. Almost-sure convergence holds only when the algorithm is supplemented with a *projection facility*, i.e., when θ_t is artificially constrained to remain in an appropriate neighborhood of θ (see section 6.4 and corollary 6.8 in Evans and Honkapohja 2001). The hypothesis of a projection facility, however, is often criticized because it cannot be easily justified on economic grounds and it is clearly inappropriate for decentralized markets.

β, the higher the accuracy of the estimate of a_p. Imposing constraints on the support of $\bar{\alpha}$ and $\bar{\alpha}^2 + \sigma_\alpha^2$ reduces the size of the bias in the case of lexicographic preferences but increases it if the policymaker has quadratic utility. One of the most striking findings of the simulation experiment concerns the minor effect on output and inflation variability of assuming imperfect central bank knowledge: the increase in output and inflation volatility, compared with the REE case, is surprisingly small—in most cases limited to a few percentage points. This is remarkable, since the estimation problem faced by the monetary authority is quite convoluted, requiring dealing with nonlinearities and computing higher-order moments. In the vast majority of cases, the increase in volatility of output and inflation remains well below 10 percent regardless of the preferences of the central bank, suggesting that the cost for the policymaker of having partial knowledge of the working of the economy is not disproportionately large. It is worth remembering, however, that the model lacks intrinsic dynamics, which explains why deviations from the REE tend to be short lived.

The analysis of the performance index provides a few interesting insights to the relative efficiency of the two strategies. The first finding is that, with decreasing gain, learning strategy 1 performs well regardless of the exact value of β^S: in the UE case, it is at least as good as strategy 2 unless the degree of inflation aversion is extremely high; in the CE case, it is uniformly better. The second finding is that the shorter the memory of the learning process, the poorer the performance of strategy 1, possibly because of the nonlinearity of the policy rule: when the policymaker has no discretion, the inflation surprises are not informative about the output-inflation trade-off and the recursive estimates become less accurate—in particular, when the size of the sample is small. The third finding is that the relative ranking of the two strategies is the same regardless of the way expectations are formed, which suggests that the learning process converges quite quickly to the REE. A closer inspection of the simulation evidence shows that for low values of β, a central bank endowed with lexicographic preferences is more effective in keeping inflation expectations under control, and it is also successful in stabilizing output fluctuations, even when bounds are not imposed on the RLS algorithm. The situation is reversed when β is high. A downward bias is evident in the estimate of the parameter measuring the response to the signal z, but it mostly disappears in the CE case; the imprecision in guessing the value of $\frac{1}{\phi}$ is responsible for some undesired fluctuations in output, while the excessive volatility of inflation is not attributable to the surprise component but rather to movements in private-sector inflation expectations, which under adaptive learning are not constant as in the REE. Since the bounds imposed on the RLS algorithm mimic the working of a projection facility, the rejection rate in the CE case turns out to be substantially lower.[21] Indeed, because of the complexity of the filtering problem facing the monetary authority, in a large number of replications, shocks displace the recursive algorithm outside of the domain of attraction of the ODE, and the estimate of the optimal response coefficient in the policy rule remains far off the true value. If the estimate of ϕ is very large at time t, the monetary authority has no incentive to respond aggressively to the signal z, and changes in y mostly reflect output shocks ε: the data become uninformative about the output-inflation trade-off and the estimate of ϕ gets larger and larger. Expectations become self-fulfilling and the economy gets stuck indefinitely on a suboptimal path, characterized by too passive a monetary policy, as if the policymaker's degree of inflation aversion were enormously higher than society's.

Except for high βs, strategy 2 underperforms strategy 1. It is, however, more effective in enhancing agents' learning process, as witnessed by the more precise estimate of the coefficient of the policy rule, which guarantees that the equilibrium under imperfect

[21] The rejection rate is calculated on the basis of the RLS estimate of the second moment of the output-inflation trade-off. Replications are considered diverging if the estimate of $\bar{\alpha}^2 + \sigma_\alpha^2$ is at least three times as large as the true value. The initial 150 observations are not used in the calculation. In the case of lexicographic preferences, absent constraints, the rejection rate turns out to be quite high (some 20 percent); it falls by a factor of 4 if the RLS algorithm is augmented with lower bounds. In the case of quadratic preferences, the number of diverging replications is on average much smaller and so is the gain obtained by imposing constraints on the learning process; for high βs, however, the rejection rate gets larger and becomes similar to the one observed under lexicographic preferences. The estimated number of diverging replications decreases sizably if less-restrictive criteria are used. Notice that divergence pertains to central bank learning and is defined in terms of the estimates of the policy parameters $\frac{1}{\phi}$ and $\frac{\rho}{\phi}$, which become very close to 0: neither the output gap nor the inflation rate are actually deviating boundlessly from equilibrium.

knowledge matches very closely the REE. There is no clear evidence that too-low or too-high inflation aversion can have a negative impact on the accuracy of the estimate of $\frac{\rho}{\phi}$. Concerning the speed of convergence, the two rules are more or less equivalent, though under CE, strategy 1 seems to be preferable; δ is very close to one half, so that convergence to a Gaussian distribution of both sequences $\{\theta_t\}^{LEX}$ and $\{\theta_t\}^{QUA}$ cannot be ruled out. As expected, strategy 2 reduces output variability more than strategy 1 when β is low, while the opposite result holds when inflation aversion is very high. The simulation results confirm that when learning substitutes rational expectations, a bias against activist policies arises. Hawkish policies are welfare enhancing because they offset the negative impact of two distortions: (i) the policymaker's desire to push output above the natural level and (ii) the uncoupling of expectations from policy objectives. While the latter distortion is present in the paper by Orphanides and Williams (2002), the former is not.

Tables 4 and 5 present evidence for the case of perpetual learning; the statistic for the speed of convergence is of course not shown since, under constant-gain learning, θ_t may at most converge to a probability distribution but not to a nonstochastic point. No meaningful differences are apparent compared with the previous case. Given the structure of the model, there is no benefit in discarding observations, so it is no surprise that in most cases RLS estimates are less accurate and policies are less successful in stabilizing both output and inflation; the deterioration in policy effectiveness seems to be relatively stronger for strategy 1.

The previous evidence suggests that a benevolent government may be better off appointing a central banker whose preferences are lexicographic if the degree of inflation aversion is not known with certainty or if it changes over time, since strategy 1 can get very close to maximize welfare for a large set of values of β^S. Strategy 1 can be implemented by giving the central bank a mandate that specifies an upper (and possibly a lower) bound on inflation and does not require that the government finds the perfect policymaker with the right preferences.[22]

[22]This feature also belongs to inflation-zone targeting; see Mishkin and Westelius (2006). It is easily seen that when the cost of overshooting (undershooting) the upper (lower) bound gets extremely large ($C \longrightarrow \infty$ in

Table 4. Least-Squares Learning and the Volatility of Output and Inflation (Unconstrained Estimator—Constant-Gain Sequence)

The table reports the estimated value (in 500 replications) of expected inflation, the coefficient of the optimal policy rule, the standard deviation of output and inflation, and the performance index. The performance index is the ratio between the social loss function under the delegated policymaker and that which would have been obtained by appointing a central banker with the same preferences as society's. Agents are assumed to use a finite number of observations in computing RLS estimates, implying a constant-gain sequence. Recursive least-squares estimates are unconstrained (UE case). In columns 1 and 3, the RE values for lexicographic and, respectively, quadratic preferences are shown; in columns 2 and 4, the same statistics are presented for the case when agents learn.

	Lexicographic Preferences		**Quadratic Preferences**	
	RE	**T = 2,000**	**RE**	**T = 2,000**
			$\beta = 0.176$	
Mean α_p	0.0000	−0.0005	0.0289	0.0288
Policy Rule Coefficient	0.2642	0.2084	0.2508	0.2001
Output Variability	0.0084	0.0090	0.0074	0.0076
Inflation Variability	0.0022	0.0026	0.0291	0.0291
Performance Index: $\beta^s = 0.176$	0.3494	0.3974	1.0000	1.0000
$\beta^s = 1.0$	0.8203	0.9326	9.8428	1.2276
$\beta^s = 5.667$	1.0925	1.2008	54.5380	48.8786
			$\beta = 1.0$	
Mean α_p	0.0000	−0.0005	0.0051	0.0051
Policy Rule Coefficient	0.2642	0.2084	0.2029	0.4747
Output Variability	0.0084	0.0090	0.0076	0.0077
Inflation Variability	0.0022	0.0026	0.0059	0.0059
Performance Index: $\beta^s = 0.176$	0.3494	0.3974	0.3105	0.3163
$\beta^s = 1.0$	0.8203	0.9326	1.0000	1.0000
$\beta^s = 5.667$	1.0925	1.2008	2.8307	2.5821
			$\beta = 5.667$	
Mean α_p	0.0000	−0.0005	0.0009	0.0009
Policy Rule Coefficient	0.2642	0.2084	0.0975	0.0503
Output Variability	0.0084	0.0090	0.0086	0.0090
Inflation Variability	0.0022	0.0026	0.0017	0.0018
Performance Index: $\beta^s = 0.176$	0.3494	0.3974	0.3627	0.3944
$\beta^s = 1.0$	0.8203	0.9326	0.8319	0.8952
$\beta^s = 5.667$	1.0925	1.2008	1.0000	1.0000

Mishkin's and Westelius's notation) and the policymaker does not care much about inflation variability inside the range ($\omega_\pi \longrightarrow 0$), strategy 1 and inflation-zone targeting become more and more alike.

Table 5. Least-Squares Learning and the Volatility of Output and Inflation (Constrained Estimator—Constant-Gain Sequence)

The table reports the estimated value (in 500 replications) of expected inflation, the coefficient of the optimal policy rule, the standard deviation of output and inflation, and the performance index. The performance index is the ratio between the social loss function under the delegated policymaker and that which would have been obtained by appointing a central banker with the same preferences as society's. Agents are assumed to use a finite number of observations in computing RLS estimates, implying a constant-gain sequence. Recursive least-squares estimates are constrained to belong to a subset of the parameter space (CE case). In columns 1 and 3, the RE values for lexicographic and, respectively, quadratic preferences are shown; in columns 2 and 4, the same statistics are presented for the case when agents learn.

	Lexicographic Preferences		**Quadratic Preferences**	
	RE	**T = 2,000**	**RE**	**T = 2,000**
			$\beta = 0.176$	
Mean α_p	0.0000	−0.0003	0.0289	0.0288
Policy Rule Coefficient	0.2642	0.1917	0.2508	0.2070
Output Variability	0.0084	0.0101	0.0074	0.0075
Inflation Variability	0.0022	0.0023	0.0291	0.0290
Performance Index: $\beta^s = 0.176$	0.3494	0.5040	1.0000	1.0000
$\beta^s = 1.0$	0.8203	1.1546	9.8428	9.6551
$\beta^s = 5.667$	1.0925	1.4084	54.5380	51.4546
			$\beta = 1.0$	
Mean α_p	0.0000	−0.0003	0.0051	0.0051
Policy Rule Coefficient	0.2642	0.1917	0.2029	0.1603
Output Variability	0.0084	0.0101	0.0076	0.0077
Inflation Variability	0.0022	0.0023	0.0059	0.0058
Performance Index: $\beta^s = 0.176$	0.3494	0.5040	0.3105	0.3192
$\beta^s = 1.0$	0.8203	1.1546	1.0000	1.0000
$\beta^s = 5.667$	1.0925	1.4084	2.8307	2.6668
			$\beta = 5.667$	
Mean α_p	0.0000	−0.0003	0.0009	0.0009
Policy Rule Coefficient	0.2642	0.1917	0.0975	0.0588
Output Variability	0.0084	0.0101	0.0086	0.0089
Inflation Variability	0.0022	0.0023	0.0017	0.0016
Performance Index: $\beta^s = 0.176$	0.3494	0.5040	0.3627	0.3900
$\beta^s = 1.0$	0.8203	1.1546	0.8319	0.8799
$\beta^s = 5.667$	1.0925	1.4084	1.0000	1.0000

An additional set of simulations have been run to analyze the dynamic response of output and inflation to a sequence of unanticipated shocks. The experiment is designed supposing that the economy is perturbed by a string of negative output shocks, declining gradually in magnitude and vanishing after twelve periods. With rational expectations, the impact of the shocks is short lived and causes only a temporary fall in output and a rise in inflation, while under imperfect knowledge the response of the economy is prolonged and amplified by agents' learning. The objective of the experiment is to test whether the evidence reported by Orphanides and Williams (2002)—namely, that activist policies end up causing the perceived process for inflation to become uncoupled from the policymaker's objectives—is a general one and extends also to the theoretical framework adopted in this paper.

Tables 6 and 7 report the outcome of the experiment. Regardless of the central bank preferences, activist policies do not seem to pay off: the lower β is, the more volatile inflation and output are, especially the latter. Simulation evidence supports Rogoff's (1985) claim that it is welfare improving to appoint a central banker who places greater relative importance on the inflation objective than society does. No policy seems to be able to offset effectively the impact on activity of a sequence of negative output shocks: it still pays to be hawkish, but the attempts to reduce inflation volatility translate into output fluctuations that are much wider than under rational expectations. What worsens the performance of monetary policy is the uncoupling between expectations and policy targets: since expectations depend upon past values of inflation, they cannot be easily anchored unless the policymaker behaves as an inflation nutter. According to the performance index, strategy 1 is highly successful in promoting social welfare: in the UE case, except for very high values of β, it is more effective than strategy 2; in the CE case, it is uniformly better. A central banker endowed with lexicographic preferences ensures both robustness and effectiveness of monetary policy.

5. Conclusions

This paper focuses on the implications for the effectiveness of monetary policymaking of departing from the benchmark of rational expectations and applies a principal-agent approach to deal with the time-inconsistency problem that arises when the central bank cannot commit. It is assumed that society can delegate monetary policy to a central banker endowed with either quadratic or lexicographic preferences. Special attention is paid to the latter, which seems to describe more accurately the objectives of inflation-targeting central

Table 6. Dynamic Response to Contractionary Shocks (Decreasing-Gain Sequence)

The table reports a few statistics measuring how the equilibrium outcome under learning differs from the perfect knowledge—i.e., rational expectations—benchmark. Results are presented for both the "plain" RLS algorithm (UE) and the constrained version (CE); agents are assumed to have infinite memory, implying a decreasing-gain sequence. The first two columns refer to lexicographic preferences, while the next two refer to quadratic (dis)utility. To describe the dynamic response of output and inflation and to compare the outcomes under adaptive learning and rational expectations, three measures are computed: (1) the ratio of the volatility of the target variables under adaptive learning and under rational expectations, (2) the trough, and (3) the peak of the responses of output and inflation. In the last three lines of each section of the table, the value of the performance index is presented. All statistics are computed on the first 50 observations.

	Lexicographic Preferences		Quadratic Preferences	
	UE	**CE**	**UE**	**CE**
			$\beta = 0.176$	
$\sigma_y^{AL}/\sigma_y^{RE}$	6.6936	6.0532	1.9096	2.1572
Min y	−0.0177	−0.0149	−0.0132	−0.0196
Max y	0.0025	0.0025	0.0067	0.0067
$\sigma_\pi^{AL}/\sigma_\pi^{RE}$	2.8320	1.0090	1.0370	0.9868
Min π	−0.0028	−0.0012	0.0286	0.0261
Max π	0.0013	0.0013	0.0330	0.0330
Performance Index: $\beta^s = 0.176$	0.1634	0.1409	1.0000	1.0000
$\beta^s = 1.0$	0.5891	0.4687	17.9880	16.7520
$\beta^s = 5.667$	1.4124	0.8177	190.9600	155.5900
			$\beta = 1.0$	
$\sigma_y^{AL}/\sigma_y^{RE}$	6.6936	6.0532	2.6290	2.6582
Min y	−0.0177	−0.0149	−0.0138	−0.0144
Max y	0.0025	0.0025	0.0054	0.0054
$\sigma_\pi^{AL}/\sigma_\pi^{RE}$	2.8320	1.0090	1.0060	0.9812
Min π	−0.0028	−0.0012	0.0048	0.0048
Max π	0.0013	0.0013	0.0084	0.0084
Performance Index: $\beta^s = 0.176$	0.1634	0.1409	0.1460	0.1564
$\beta^s = 1.0$	0.5891	0.4687	1.0000	1.0000
$\beta^s = 5.667$	1.4124	0.8177	7.2675	6.2611
			$\beta = 5.667$	
$\sigma_y^{AL}/\sigma_y^{RE}$	6.6936	6.0532	5.7385	5.5524
Min y	−0.0177	−0.0149	−0.0162	−0.0149
Max y	0.0025	0.0025	0.0028	0.0028
$\sigma_\pi^{AL}/\sigma_\pi^{RE}$	2.8320	1.0090	0.6207	0.9806
Min π	−0.0028	−0.0012	−0.0003	0.0008
Max π	0.0013	0.0013	0.0025	0.0025
Performance Index: $\beta^s = 0.176$	0.1634	0.1409	0.1361	0.1365
$\beta^s = 1.0$	0.5891	0.4687	0.4769	0.4726
$\beta^s = 5.667$	1.4124	0.8177	1.0000	1.0000

Table 7. Dynamic Response to Contractionary Shocks (Constant-Gain Sequence)

The table reports a few statistics measuring how the equilibrium outcome under learning differs from the perfect knowledge—i.e., rational expectations—benchmark. Results are presented for both the "plain" RLS algorithm (UE) and the constrained version (CE); agents are assumed to use a finite number of observations in computing RLS estimates, implying a constant-gain sequence. The first two columns refer to lexicographic preferences, while the next two refer to quadratic (dis)utility. To describe the dynamic response of output and inflation and to compare the outcomes under adaptive learning and rational expectations, three measures are computed: (1) the ratio of the volatility of the target variables under adaptive learning and under rational expectations, (2) the trough, and (3) the peak of the responses of output and inflation. In the last three lines of each section of the table, the value of the performance index is presented. All statistics are computed on the first 50 observations.

	Lexicographic Preferences		**Quadratic Preferences**	
	UE	**CE**	**UE**	**CE**
			$\beta = 0.176$	
$\sigma_y^{AL}/\sigma_y^{RE}$	7.0633	5.5241	2.0835	1.6911
Min y	−0.0170	−0.0138	−0.0156	−0.0111
Max y	0.0025	0.0025	0.0067	0.0067
$\sigma_\pi^{AL}/\sigma_\pi^{RE}$	1.3607	0.7174	0.9863	0.9938
Min π	−0.0017	−0.0002	0.0283	0.0286
Max π	0.0025	0.0013	0.0330	0.0330
Performance Index: $\beta^s = 0.176$	0.1938	0.1217	1.0000	1.0000
$\beta^s = 1.0$	0.6838	0.4498	17.8460	19.4540
$\beta^s = 5.667$	1.2303	0.7748	168.8800	182.7800
			$\beta = 1.0$	
$\sigma_y^{AL}/\sigma_y^{RE}$	7.0633	5.5241	2.6372	2.3021
Min y	−0.0170	−0.0138	−0.0148	−0.0122
Max y	0.0025	0.0025	0.0054	0.0054
$\sigma_\pi^{AL}/\sigma_\pi^{RE}$	1.3607	0.7174	0.9321	0.9540
Min π	−0.0017	−0.0002	0.0046	0.0048
Max π	0.0025	0.0013	0.0084	0.0084
Performance Index: $\beta^s = 0.176$	0.1938	0.1217	0.1528	0.1281
$\beta^s = 1.0$	0.6838	0.4498	1.0000	1.0000
$\beta^s = 5.667$	1.2303	0.7748	6.2019	6.7111
			$\beta = 5.667$	
$\sigma_y^{AL}/\sigma_y^{RE}$	7.0633	5.5241	5.5729	5.3428
Min y	−0.0170	−0.0138	−0.0149	−0.0143
Max y	0.0025	0.0025	0.0028	0.0028
$\sigma_\pi^{AL}/\sigma_\pi^{RE}$	1.3607	0.7174	0.7987	0.8049
Min π	−0.0017	−0.0002	0.0005	0.0007
Max π	0.0025	0.0013	0.0025	0.0025
Performance Index: $\beta^s = 0.176$	0.1938	0.1217	0.1383	0.1308
$\beta^s = 1.0$	0.6838	0.4498	0.4987	0.4982
$\beta^s = 5.667$	1.2303	0.7748	1.0000	1.0000

banks. The main focus of the paper is on validating the claim that policies that are designed to be efficient under rational expectations can perform very poorly when knowledge is incomplete and agents learn adaptively.

The evidence shown in the paper confirms that, when agents do not possess complete knowledge of the structure of the economy and rely instead on an adaptive learning technology, a bias toward conservatism arises, suggesting that society is better off by appointing a policymaker whose degree of inflation aversion is higher than its own. The rationale for this finding is that agents' and policymakers' attempts to learn adaptively introduce inertia into the system and induce prolonged deviations of output and inflation from target, thereby raising the costs for the central bank of not responding promptly and forcefully to shocks. The paper also shows that the strategy that implements a lexicographic-preference ordering performs, on average, very well: it comes close to maximizing social welfare for a wide range of values of β^S and outperforms the strategies implementing quadratic preferences unless society is extremely inflation averse.

The findings of the paper closely resemble those of Orphanides and Williams (2002), which is surprising given the differences in the theoretical framework. First, the model adopted in this paper has no intrinsic dynamics, and the only source of persistence comes from the assumption that agents learn adaptively: the uncoupling between actual and perceived inflation is much harder to achieve with such a simple dynamic structure, though presumably, the lack of dynamics in the economy is compensated by the inertia induced by the attempts of the central bank to estimate the mean and variance of the output-inflation trade-off. Second, though only inflation expectations have a direct impact on the equilibrium outcome, output gap uncertainty affects the central bank's estimates of the moments of α and hence the policy setting: it is by no means obvious that a strategy that penalizes output variability might be conducive to higher welfare. The justification for the existence of a bias in favor of hawkish policies is to be found in the role of central bank learning: too-activist policies reduce the information content of the output gap and make estimates of the coefficients of the policy rule too volatile and unreliable.

Appendix

Proof of Proposition 3. Regardless of the preferences of the monetary authority, the recursive system representing the learning process is of the form $\theta_t = \theta_{t-1} + \frac{1}{t}Q(t, \theta_{t-1}, X_t)$, where $\theta_t = (a_{Pt}, \hat{\hat{\alpha}}_t, R_{y,t}, R_{\pi,t})'$ and $X_t = (1, \alpha_t, z_t, \varepsilon_t)$. To show the asymptotic stability of the REE under learning, one has to proceed as follows: first, it must be verified that there exists a nontrivial open domain containing the equilibrium point where the learning algorithm satisfies a few regularity conditions concerning the updating function $Q(t, \theta_{t-1}, X_t)$ and the stochastic process driving the state variables X_t; second, the local (or global) stability of the ODE associated with the stochastic recursive system must be established.

Consider first the case of lexicographic preferences. The system (16) has a unique equilibrium point θ^*, at which $a_P = \bar{\pi} - \frac{\mu/3}{\phi}$, $\hat{\hat{\alpha}} = \bar{\alpha}$, $R_\pi = \frac{2}{3\phi^2}(\frac{\mu^2}{3})$, and $R_y = (\bar{\alpha}^2 + \sigma_\alpha^2)[\frac{2}{3\phi^2}(\frac{\mu^2}{3})] + 2\frac{\mu^2}{3}$. It can easily be seen that θ^* is the REE. The stochastic process X_t is white noise, with finite absolute moments, so that regularity conditions (B.1) and (B.2) in Evans and Honkapohja (2001) are satisfied.[23] In addition, the gain sequence approaches zero asymptotically and is not summable. Finally, provided that R_π and R_y are nonzero along the learning path, $Q(t, \theta_{t-1}, X_t)$ satisfies a Lipschitz condition on a compact set containing the equilibrium point θ^*,[24] which ensures that regularity conditions (A.1)–(A.3) in Evans and Honkapohja (2001) are also met. Convergence of the learning process to the REE hinges therefore on the stability of the associated ODE (18). Notice that the system is recursive and the asymptotic behavior of the subsystem describing central bank learning can be assessed independently of the expectations formation mechanism of the private agents. Indeed, provided R_π and R_y are invertible along the convergence path, $R_y \to E(y - \frac{z}{2})^2$ and $R_\pi \to E(\pi - a_P)^2$ from any starting point; since $R_\pi^{-1}E(\pi - a_P)^2 \to I$, it is easily seen that $\hat{\hat{\alpha}}_t \to \bar{\alpha}$, since

[23] Chapter 6 in Evans and Honkapohja (2001) lists the regularity conditions required for the analysis of the asymptotic behavior of the stochastic recursive algorithm. Local stability is treated in section 6.2, while global convergence is analyzed in section 6.7.

[24] $Q(\theta_{t-1}, X_t)$ satisfies a Lipschitz condition if it is bounded and twice continuously differentiable, with bounded second derivatives.

the eigenvalue of the Jacobian of the corresponding differential equation has a negative real part. Conditional on $\hat{\bar{\alpha}}_t \to \bar{\alpha}$, convergence of private-sector inflation forecasts follows, since the associated ODE is stable.

A more formal proof of the convergence of the learning process to the REE requires proving that the Jacobian of the ODE, evaluated at the REE θ^*, has eigenvalues whose real parts are negative. In order to show that this is indeed the case, first notice that $R_\pi = E(\pi - a_P)^2$ at θ^*, which implies that R_π does not appear in the first three equations of the ODE evaluated at θ^*. A similar result holds for R_y. In the last two equations, the derivatives of R_π and R_y (and, accordingly, of $E(\pi - a_P)^2$ and $E(y - \frac{z}{2})^2$), though different from zero, cancel out, so that the Jacobian has the following upper-triangular, block-recursive structure:

$$Dh(\theta^*) = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{\bar{\alpha}^2 + \sigma_\alpha^2}\frac{\mu}{12} & \frac{3}{2\mu}\frac{1}{\bar{\alpha}} & -\frac{3}{2\mu}\frac{\bar{\alpha}^2 + \sigma_\alpha^2}{\bar{\alpha}} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

It is easily checked that its eigenvalues are $(-\frac{1}{2}, -1, -1, -1)$. They are all negative, and the system is therefore E-stable.

Consider now the case of quadratic preferences. The unique equilibrium point θ^* of the system (16) is now $a_P = \frac{\bar{\alpha}}{\beta}k$, $\hat{\bar{\alpha}} = \bar{\alpha}$, $R_\pi = E(\pi - a_P)^2 = 2(\frac{\rho}{\phi})^2(\frac{\mu^2}{3})$, and $R_y = E(y - \frac{z}{2})^2 = (\bar{\alpha}^2 + \sigma_\alpha^2)[2(\frac{\rho}{\phi})^2(\frac{\mu^2}{3})] + 2\frac{\mu^2}{3}$. The stochastic process X_t is the same as in the previous case, so that regularity conditions (B.1) and (B.2) in Evans and Honkapohja (2001) are satisfied. The same holds for the assumptions (A.1)–(A.3) on the gain sequence and the updating function $Q(t, \theta_{t-1}, X_t)$. The stability of the associated ODE (19) can be proved in the same way as for the system (18): provided that R_π and R_y are invertible along the convergence path, $R_\pi \to E(\pi - a_P)^2$ from any starting point; central bank estimates converge to the true parameter values $\bar{\alpha}$, since the eigenvalue of the Jacobian of the corresponding differential equation has a negative real part, and $a_P \to \frac{\bar{\alpha}}{\beta}k$, since the associated ODE is stable.

As in the previous case, the structure of the Jacobian justifies the sequential solution of the system. At θ^*, the derivative matrix of the ODE (19) is equal to

$$Dh(\theta^*) = \begin{bmatrix} -\dfrac{\beta}{\bar{\alpha}^2 + \sigma_\alpha^2 + \beta} & \dfrac{1}{\bar{\alpha}^2 + \sigma_\alpha^2 + \beta}\dfrac{\mu}{6} & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

The lower block for R_π and R_y can be solved first; then, triangularity of the upper block ensures that convergence for $\hat{\bar{\alpha}}_t$ does not depend on the asymptotic behavior of a_{Pt}. The eigenvalues of the Jacobian are $(-\frac{\beta}{\bar{\alpha}^2+\sigma_\alpha^2+\beta}, -1, -1, -1)$, and the system is therefore E-stable.

References

Benveniste, A., M. Metivier, and P. Priouret. 1990. *Adaptive Algorithms and Stochastic Approximations.* Berlin and Heidelberg: Springer-Verlag.

Bertocchi, G., and M. Spagat. 1993. "Learning, Experimentation and Monetary Policy." *Journal of Monetary Economics* 32 (1): 169–83.

Brainard, W. 1967. "Uncertainty and the Effectiveness of Policy." *American Economic Review* 57 (2): 411–25.

Bray, M. M. 1982. "Learning, Estimation and Stability of Rational Expectations." *Journal of Economic Theory* 26 (2): 318–39.

Buiter, W. H. 2006. "How Robust Is the New Conventional Wisdom? The Surprising Fragility of the Theoretical Foundations of Inflation Targeting and Central Bank Independence." CEPR Discussion Paper No. 5772.

Driffill, J., and Z. Rotondi. 2003. "Monetary Policy and Lexicographic Preference Ordering." CEPR Discussion Paper No. 4247.

Ellison, M., and N. Valla. 2001. "Learning, Uncertainty and Central Bank Activism in an Economy with Strategic Interactions." *Journal of Monetary Economics* 48 (1): 153–71.

Evans, G. W., and S. Honkapohja. 2001. *Learning and Expectations in Macroeconomics.* Princeton, NJ: Princeton University Press.

———. 2002. "Expectations and the Stability Problem for Optimal Monetary Policy." Mimeo.

Jensen, H. 2003. "Explaining an Inflation Bias without Using the Word 'Surprise.'" Mimeo.

Locarno, A. 2006. "Imperfect Knowledge, Adaptive Learning and the Bias Against Monetary Policy." Temi di Discussione No. 590, Banca d'Italia.

Marcet, A., and T. J. Sargent. 1989. "Convergence of Least-Squares Learning Mechanisms in Self-Referential Linear Stochastic Models." *Journal of Economic Theory* 48 (2): 337–68.

———. 1992. "Speed of Convergence of Recursive Least Squares Learning with ARMA Perceptions." UPF Economics Working Paper No. 15.

Mishkin, F. S., and N. J. Westelius. 2006. "Inflation Band Targeting and Optimal Inflation Contracts." NBER Working Paper No. 12384.

al-Nowaihi, A., and L. Stracca. 2002. "Non-standard Central Bank Loss Functions, Skewed Risks and Certainty Equivalence." ECB Working Paper No. 129.

Orphanides, A., and D. W. Wilcox. 1996. "The Opportunistic Approach to Disinflation." FEDS Discussion Paper No. 24, Board of Governors of the Federal Reserve System.

Orphanides, A., and J. C. Williams. 2002. "Imperfect Knowledge, Inflation Expectations and Monetary Policy." Working Paper No. 27, Board of Governors of the Federal Reserve System.

Rogoff, K. 1985. "The Optimal Commitment to an Intermediate Monetary Target." *Quarterly Journal of Economics* 100 (4): 1169–89.

Simon, H. A. 1957. *Models of Man: Social and Rational.* New York: Wiley.

Svensson, L. E. O., and M. Woodford. 2000. "Indicator Variables for Optimal Policy." NBER Working Paper No. 7953.

Terlizzese, D. 1999. "A Note on Lexicographic Ordering and Monetary Policy." Mimeo, Bank of Italy.

Wieland, V. 2003. "Monetary Policy and Uncertainty about the Natural Unemployment Rate." CEPR Discussion Paper No. 3811.

Economic and Regulatory Capital in Banking: What Is the Difference?*

Abel Elizalde[a] and Rafael Repullo[b]
[a]CEMFI and UPNA
[b]CEMFI and CEPR

We analyze the determinants of regulatory capital (the minimum required by regulation), economic capital (that chosen by shareholders without regulation), and actual capital (that chosen with regulation) in a dynamic model of a bank with a loan-portfolio return described by the single-risk-factor model of Basel II. We show that variables that only affect economic capital, such as the intermediation margin and the cost of capital, can account for large deviations from regulatory capital. Actual capital is closer to regulatory capital, but the threat of closing undercapitalized banks generates significant capital buffers. Market discipline, proxied by the coverage of deposit insurance, increases economic and actual capital, although the effects are small.

JEL Codes: G21, G28.

1. Introduction

Economic capital and *regulatory capital* are two terms frequently used in the analysis of the new framework for bank capital regulation proposed by the Basel Committee on Banking Supervision (2004). Known as Basel II, this framework is in the process of being implemented worldwide. According to the chairman of the Basel Committee, the primary objective of the new regulation is to set

*We thank Jaime Caruana, Douglas Gale, Charles Goodhart, Esa Jokivuolle, Nobu Kiyotaki, Rosa Lastra, Julio Segura, Andrew Winton, and especially Javier Suarez for helpful comments. This paper was written when the first author was a PhD student at CEMFI and Universidad Pública de Navarra. The views expressed in the paper are those of the authors. Address: CEMFI, Casado del Alisal 5, 28014 Madrid, Spain. E-mail: abel_elizalde@hotmail.com, repullo@cemfi.es.

"... more risk-sensitive minimum capital requirements, so that regulatory capital is both adequate and closer to economic capital" (Caruana 2005, 9).

To compare economic and regulatory capital, we must first clarify the meaning of each term. In principle, regulatory capital should be derived from the maximization of a social welfare function that takes into account the costs (e.g., an increase in the cost of credit) and the benefits (e.g., a reduction in the probability of bank failure) of capital regulation.[1] However, in this paper we define *regulatory capital* as the minimum capital required by the regulator, which we identify with the capital charges in the internal-ratings-based (IRB) approach of Basel II. Economic capital is usually defined as the capital level that is required to cover the bank's losses with a certain probability or confidence level, which is related to a desired rating.[2] However, it is our view that such desired solvency standard should not be taken as a primitive, but should be derived from an underlying objective function such as the maximization of the value of the bank. For this reason, *economic capital* may be defined as the capital level that bank shareholders would choose in absence of capital regulation.[3] This is, in fact, the definition we will use hereafter.

The purpose of this paper is to analyze the differences between economic and regulatory capital in the context of a dynamic model of a bank with a loan-portfolio return described by the single-risk-factor model that underlies the IRB capital requirements of Basel II. Economic capital is the level of capital chosen by shareholders at the beginning of each period in order to maximize the value of the bank, taking into account the possibility that the bank will be closed if the losses during the period exceed the initial level of capital. This closure rule may be justified by assuming that a bank run takes place before the shareholders can raise new equity to cover the losses. Thus economic capital trades off the costs of funding the

[1]See Repullo and Suarez (2004) for a discussion of Basel II from this perspective.

[2]See, e.g., Jones and Mingo (1998) or Carey (2001).

[3]As noted by Allen (2006, 45), "The two concepts reflect the needs of different primary stakeholders. For economic capital, the primary stakeholders are the bank's shareholders, and the objective is the maximization of [their] wealth. For regulatory capital, the primary stakeholders are the bank's [depositors], and the objective is to minimize the possibility of loss."

bank with costly equity against the benefits of reducing the probability of losing the bank's franchise value, which appears as a key endogenous variable in the bank's maximization problem.

We show that economic and regulatory capital do not depend on the same variables: the former (but not the latter) depends on the intermediation margin and the cost of bank capital, while the latter (but not the former) depends on the confidence level set by the regulator. Moreover, economic and regulatory capital do not respond in the same manner to changes in the common variables that affect them, such as the loans' probability of default and loss given default.

Due to the difficulty of obtaining analytical results for economic capital, we use a numerical procedure to compute it. The results show that Basel II regulatory capital only approaches economic capital for a limited range of parameter values. Our analysis also shows that the relative position of economic and regulatory capital is mainly determined by the cost of bank capital: economic capital is higher (lower) than regulatory capital when the cost of capital is low (high).

Another key variable in the shareholders' economic capital decision is the intermediation margin, which has two opposite effects. On the one hand, a higher margin increases the bank's franchise value and, consequently, shareholders' incentives to contribute capital. On the other hand, a higher margin increases bank revenues and therefore reduces the role of capital as a buffer to absorb future losses, acting as a substitute for economic capital. We show that the net effect of the intermediation margin on economic capital is positive in very competitive loan markets and negative otherwise. Finally, the numerical results show that increases in the loans' probability of default and loss given default increase regulatory capital, while they only increase economic capital for a range of plausible values of these variables.

The paper also addresses the determinants of *actual capital*, which is defined as the capital chosen by bank shareholders taking into account regulatory constraints. In particular, two regulations are considered. First, at the beginning of each period, banks must have a capital level no lower than regulatory capital. Second, following U.S. banking regulation and in particular the prompt corrective action (PCA) provisions of the Federal Deposit Insurance Corporation Improvement Act (FDICIA), banks whose capital level at the end of a period falls below a minimum (positive) threshold are considered critically undercapitalized and are closed. We show that, for a wide range of parameter values, the threat of closing undercapitalized banks induces them to choose a capital level above regulatory capital. Therefore, in situations in which the cost of capital is such that economic capital is below regulatory capital, PCA provides an explanation for why banks typically hold a buffer of capital above the minimum required by regulation.[4]

The model proposed in the paper allows us to analyze the effect of market discipline, proxied by the coverage of deposit insurance, on economic and actual capital. We consider two alternative scenarios: one in which depositors are fully insured and where the deposit interest rate is equal to the risk-free rate, and another one in which depositors are uninsured. In this second scenario, depositors require an interest rate such that the expected return of their investment is equal to the risk-free rate. The results suggest that measures aimed at increasing market discipline have a positive effect on economic capital, though the magnitude of this effect is generally small, except in very competitive markets for high-risk loans. The impact of market discipline on actual capital is even lower and almost negligible.

Two limitations of our analysis are the assumption that the risk of the bank's loan portfolio is exogenous and the use of the single-risk-factor model to derive the probability distribution of loan default rates. The inclusion of the bank's level of risk as an endogenous variable, together with capital, in the shareholders' maximization problem, as well as the analysis of more-complex models of bank risk are left for future research.

The academic literature on this topic is small, and in no case economic and regulatory capital are compared. The literature, both empirical and theoretical, deals with the impact of different regulations on capital (as we do) and risk-taking decisions (as we do not do).

[4]An alternative explanation would be the banks' incentives to maintain high credit ratings to support their derivatives counterparty business; see Jackson, Perraudin, and Saporta (2002). This could be modeled as an extra revenue that comes from this business as long as the probability of failure is sufficiently low (or the rating is sufficiently high). We plan to explore this issue in future work.

From an empirical perspective, it is not possible to analyze economic capital in the sense defined above, given that some form of capital regulation has been in place for many years. In terms of actual capital, the predictions of our model coincide with several stylized facts supported by empirical studies of the drivers of banks' capital. Flannery and Rangan (2004) analyze the relationship between regulatory and actual bank capital between 1986 and 2000 for a sample of U.S. banks. They conclude that the increase in regulatory capital during the first part of the 1990s could explain the increase in the capital levels of the banking industry during those years, but that the additional increase in capital in the second part of the 1990s is mainly driven by market discipline. These results support two predictions of our model: actual capital is an increasing function of regulatory capital and of the level of market discipline. However, our results suggest that mandatory restrictions and penalties to undercapitalized banks could have played a more important role in boosting bank capital levels than market discipline. This is in line with the evidence provided by Aggarwal and Jacques (2001) showing that both adequately capitalized and undercapitalized banks increased their capital ratios in response to the PCA provisions of FDICIA during both the announcement period, 1992, and the years after the standards went into effect, 1993–96.

From a theoretical perspective, the literature provides a wide range of assumptions and modeling frameworks, which complicates the comparison with ours (and between them). There are a few papers that share the focus on dynamic models with endogenous franchise values. The most interesting paper—which is the foundation of our analysis of economic capital—is Suarez (1994), who constructs a dynamic model of bank behavior in which shareholders choose not only the capital level but also the asset risk.

Calem and Rob (1999) present a dynamic model similar to ours where the bank's franchise value is endogenous. The model is calibrated with empirical data from the banking industry for 1984–93, focusing on the impact of risk-based versus flat-rate capital requirements on banks' risk taking, which is shown to be ambiguous across banks depending on their capital levels. Repullo (2004) analyzes capital and risk-taking decisions in a dynamic model of imperfect competition with endogenous franchise values. He shows that capital requirements reduce the banks' incentives to take risk, and that risk-based requirements are more-efficient regulatory tools. Estrella (2004) presents a dynamic model in which banks choose their capital subject to risk-based capital regulation and adjustment costs in both raising capital and paying dividends. He focuses on capital levels over the cycle, concluding that risk-based capital regulation, if binding, is likely to be procyclical.

This paper is organized as follows. Section 2 presents the model and characterizes the determinants of economic, regulatory, and actual capital. Section 3 derives the numerical results, and section 4 concludes. Appendix 1 discusses the comparative statics of economic capital, and appendix 2 contains a proof of the negative relationship between bank capital and the interest rate on uninsured deposits.

2. The Model

Consider a bank that, at the beginning of each period $t = 0, 1, 2, \ldots$ in which it is open, has an asset size that is normalized to 1.[5] The bank is funded with deposits, $1 - k_t$, that promise an interest rate c, and capital, k_t, that requires an expected return δ. We assume that the deposit rate c is smaller than the cost of capital δ. The bank is owned by risk-neutral shareholders who have limited liability and, in the absence of minimum capital regulation, choose the capital level k_t in the interval $[0, 1]$. When $k_t = 0$, the bank is fully funded with deposits, and when $k_t = 1$, the bank is fully funded with equity capital.

In each period t in which the bank is open, its funds are invested in a portfolio of loans paying an exogenously fixed interest rate r. The return of this investment is stochastic: a random fraction $x_t \in [0, 1]$ of these loans default, in which case the bank loses the interest r as well as a fraction $\lambda \in [0, 1]$ of the principal. Therefore, the bank gets $1 + r$ from the fraction $1 - x_t$ of the loans that do not default, and it recovers $1 - \lambda$ from the fraction x_t of defaulted loans, so the value of its portfolio at the end of period t is given by

$$a_t = (1 - x_t)(1 + r) + x_t(1 - \lambda). \tag{1}$$

[5]This normalization is related to the size of the bank's loan customer base, which is assumed to be fixed over time. Introducing a (small) growth rate of the customer base would be straightforward and would not change the qualitative results.

Since the bank has to pay depositors an amount $(1-k_t)(1+c)$, assuming zero intermediation costs, its capital at the end of period t is

$$k_t' = a_t - (1-k_t)(1+c) = k_t + r - (1-k_t)c - (\lambda + r)x_t. \quad (2)$$

We assume that the probability distribution of the default rate x_t is the one derived from the single-risk-factor model of Vasicek (2002) that is used for the computation of the capital charges in the IRB approach of Basel II.[6] Its cumulative distribution function is given by

$$F(x_t) = N\left(\frac{\sqrt{1-\rho}\, N^{-1}(x_t) - N^{-1}(p)}{\sqrt{\rho}}\right), \quad (3)$$

where $N(\cdot)$ denotes the distribution function of a standard normal random variable, $p \in [0,1]$ is the loans' (unconditional) probability of default, and $\rho \in [0,1]$ is their exposure to the systematic risk factor: when $\rho = 0$, defaults are statistically independent, so $x_t = p$ with probability 1, and when $\rho = 1$, defaults are perfectly correlated, so $x_t = 0$ with probability $1-p$, and $x_t = 1$ with probability p. We also assume that the default rate x_t is independent over time.

The distribution function $F(x_t)$ in (3) is increasing, with $F(0) = 0$ and $F(1) = 1$. Moreover, it can be shown that

$$E(x_t) = \int_0^1 x_t \, dF(x_t) = p$$

and

$$Var(x_t) = \int_0^1 (x_t - p)^2 \, dF(x_t) = N_2(N^{-1}(p), N^{-1}(p); \rho) - p^2,$$

where $N_2(\cdot,\cdot;\rho)$ denotes the distribution function of a zero-mean bivariate normal random variable with standard deviation equal to 1 and correlation coefficient ρ; see Vasicek (2002). Therefore, the

[6] As shown by Gordy (2003), this model has the property that the contribution of a given asset to value-at-risk (and hence the corresponding IRB capital charge) is portfolio invariant; i.e., it depends on the asset's own characteristics and not on those of the portfolio in which it is included.

expected value of the default rate is the probability of default p, while its variance is increasing in the correlation parameter ρ, with $Var(x_t) = 0$ for $\rho = 0$, and $Var(x_t) = p(1-p)$ for $\rho = 1$.

2.1 Economic Capital

To derive the level of capital chosen by the bank shareholders in the absence of minimum capital regulation, we solve a dynamic programming problem in which the state variable $I_t \in \{0,1\}$ indicates whether the bank is closed ($I_t = 0$) or open ($I_t = 1$) at the beginning of period t. Let $V(I_t)$ denote the value function of this problem. Clearly, the value $V(0)$ of a bank that is closed is 0, while $V(1)$ is the franchise value of a bank that is open, which henceforth will simply be denoted by V.

Following Suarez (1994), a closure rule may be described by a function $h\colon \mathbb{R} \to \{0,1\}$ that specifies the values of end-of-period capital k'_t for which a bank that is open at the beginning of period t is closed at the end of this period, i.e.,

$$I_{t+1} = I_t h\big(k'_t\big). \tag{4}$$

Notice that $I_t = 0$ implies $I_{t+1} = 0$, so a bank that is closed cannot be reopened.

Two particular closure rules will be considered. The first rule assumes that the shareholders can freely recapitalize the bank when its capital at the end of period t is negative, so the bank is only closed when its shareholders do not exercise the recapitalization option. This will happen whenever the funds that they have to inject to pay back depositors are greater than the franchise value of the bank, i.e., whenever $k'_t + V < 0$. The second closure rule assumes that the bank is closed at the end of period t whenever $k'_t < 0$, i.e., whenever the losses during the period exceed the initial capital k_t. The rationale for this rule is that when the liabilities take the form of demandable deposits, a shock that depletes all the bank's capital triggers a run before the shareholders are able to raise fresh equity to cover the shortfall.

Formally, the first closure rule is described by

$$h_1\big(k'_t\big) = \begin{cases} 0 & \text{if } k'_t + V < 0, \\ 1 & \text{otherwise,} \end{cases} \tag{5}$$

while the second is described by

$$h_2(k_t') = \begin{cases} 0 & \text{if } k_t' < 0, \\ 1 & \text{otherwise.} \end{cases} \tag{6}$$

The Bellman equation that characterizes the solution to the bank's maximization problem for closure rule (5) is

$$V = \max_{k_t \in [0,1]} \left[-k_t + \frac{1}{1+\delta} E\big(\max \{ k_t' + V, 0 \} \big) \right]. \tag{7}$$

According to this expression, the franchise value V of a bank that is open results from maximizing with respect to k_t an objective function that has two terms: the first one, with a negative sign, is the capital contribution of the shareholders at the beginning of period t; the second one is the discounted expected payoff at the end of period t, which comprises the value k_t' of its end-of-period capital plus the value V of remaining open in period $t+1$, whenever their sum $k_t'+V$ is non-negative, and 0 otherwise. Notice that the discount rate used in this second term is the return required by bank shareholders, or cost of capital δ.

Therefore, assuming that the bank is open at the beginning of period t, there are two possible scenarios at the end of period t: (i) if $k_t' + V < 0$, the bank fails and the shareholders get a final payoff of 0, and (ii) if $k_t' + V \geq 0$, the bank remains open in period $t+1$ and the shareholders receive a dividend payment (or make a capital contribution, depending on the sign) of $k_t' - k_{t+1}$, i.e., the difference between the capital at the end of period t and the capital that they would like to keep in the bank for period $t+1$.

Using the definition of k_t', (2), we have $k_t' + V \geq 0$ if and only if the default rate x_t is below the critical value

$$x(k_t, V) = \frac{k_t + r - (1-k_t)c + V}{\lambda + r}, \tag{8}$$

so we can rewrite Bellman equation (7) as

$$V = \max_{k_t \in [0,1]} \left[-k_t + \frac{1}{1+\delta} \int_0^{x(k_t,V)} \times \left[k_t + r - (1-k_t)c - (\lambda+r)x_t + V \right] dF(x_t) \right]. \tag{9}$$

Differentiating the bank's objective function with respect to k_t, and using the assumption that the deposit rate c is smaller than the cost of capital δ, we get

$$-1 + \frac{1}{1+\delta} \int_0^{x(k_t,V)} (1+c)\, dF(x_t) \leq -1 + \frac{1+c}{1+\delta} = \frac{c-\delta}{1+\delta} < 0.$$

Hence we conclude that for the first closure rule, (5), shareholders always choose the corner solution $k_t = 0$, i.e., zero economic capital. The intuition for this result is clear: bank shareholders do not have an incentive to contribute costly capital ex ante when they are able to provide it ex post via the recapitalization option.

However, as argued above, raising equity capital after a large negative shock may not be feasible, especially when the liabilities take the form of demandable deposits that may be subject to runs. For this reason, in what follows we focus exclusively on the second closure rule, (6), according to which the bank is closed at the end of period t whenever $k'_t < 0$.

The Bellman equation that characterizes the solution to the bank's maximization problem for closure rule (6) is

$$V = \max_{k_t \in [0,1]} \left[-k_t + \frac{1}{1+\delta} \left[E\big(\max \left\{ k'_t, 0 \right\} \big) + \Pr(k'_t \geq 0) V \right] \right]. \quad (10)$$

According to this expression, the franchise value V of a bank that is open results from maximizing with respect to k_t an objective function that has three terms: the first one, with a negative sign, is the capital contribution of the shareholders at the beginning of period t; the second one is the discounted expected payoff at the end of period t; and the third one is the discounted expected value of remaining open in period $t+1$. As before, the discount rate used in the last two terms is the return required by bank shareholders, or cost of capital δ.

Therefore, assuming that the bank is open at the beginning of period t, there are two possible scenarios at the end of period t: (i) if $k'_t < 0$, the bank fails and the shareholders get a final payoff of 0, and (ii) if $k'_t \geq 0$, the bank remains open in period $t+1$ and the shareholders receive a dividend payment (or make a capital contribution, depending on the sign) of $k'_t - k_{t+1}$, i.e., the difference between the capital at the end of period t and the capital that they would like to keep in the bank for period $t+1$.

Using the definition of k_t', (2), we have $k_t' \geq 0$ if and only if the default rate x_t is below the critical value

$$x(k_t) = \frac{k_t + r - (1-k_t)c}{\lambda + r}, \tag{11}$$

so we can rewrite Bellman equation (10) as

$$V = \max_{k_t \in [0,1]} \left[-k_t + \frac{1}{1+\delta} \int_0^{x(k_t)} \times \left[k_t + r - (1-k_t)c - (\lambda + r)x_t + V \right] dF(x_t) \right]. \tag{12}$$

Notice that for

$$k_t \geq k_{\max} = \frac{\lambda + c}{1 + c}, \tag{13}$$

we have $x(k_t) \geq 1$, so the probability of bank failure is 0. In this case the derivative with respect to k_t of the bank's objective function equals $(c-\delta)/(1+\delta)$, which is negative by the assumption that the deposit rate c is smaller than the cost of capital δ. Hence economic capital will never be above $k_{\max}$. This result is easy to explain: bank shareholders might be willing to contribute capital, instead of funding the bank with cheaper deposits, as long as capital provides a buffer that reduces the probability of failure and consequently increases the probability of receiving a stream of future dividends. However, if $k_t \geq k_{\max}$, capital covers the bank's losses at the end of period t even when 100 percent of the loans in its portfolio default, which means that any additional capital will only increase the bank's funding costs without reducing its probability of failure (which is 0).

The solution of Bellman equation (12) gives the level of economic capital k^* that bank shareholders would like to hold in the absence of minimum capital regulation, as well as the bank's franchise value V^*. In addition, this equation allows us to identify the determinants of economic capital k^*, which are the loans' probability of default p, loss given default λ, and exposure to systematic risk ρ; the loan rate r; the deposit rate c; and the cost of bank capital δ.

Appendix 1 shows that economic capital can be at the corner $k^* = 0$ and that if there is an interior solution, comparative static results are, in general, ambiguous, except for the cost of capital δ, for which we obtain

$$\frac{\partial k^*}{\partial \delta} < 0.$$

Thus the higher the bank's equity funding costs, the lower the capital provided by its shareholders.

2.2 Regulatory Capital

As noted above, in this paper we follow the IRB approach of Basel II, according to which regulatory capital must cover losses due to loan defaults with a given probability (or confidence level) $\alpha = 99.9$ percent. Specifically, let $\hat{x}$ denote the α-quantile of the distribution of the default rate x_t, i.e., the critical value such that

$$\Pr(x_t \leq \hat{x}) = F(\hat{x}) = \alpha.$$

Hence we have $\hat{x} = F^{-1}(\alpha)$, so making use of (3), we get the capital requirement

$$\hat{k} = \lambda\hat{x} = \lambda N\left(\frac{N^{-1}(p) + \sqrt{\rho}\, N^{-1}(\alpha)}{\sqrt{1-\rho}}\right). \tag{14}$$

This is the formula that appears in Basel Committee on Banking Supervision (2004, paragraph 272), except for the fact that we are assuming a one-year maturity (which implies a maturity adjustment factor equal to 1) and that the correlation parameter ρ is, in Basel II, a decreasing function of the probability of default p. It should also be noted that in the IRB approach, expected losses, λp, are to be covered with general loan-loss provisions, while the remaining charge, $\lambda(\hat{x} - p)$, should be covered with capital. However, from the perspective of our analysis, provisions are just another form of equity capital, and thus the distinction between the expected and unexpected components of loan losses is immaterial.

From IRB formula (14), we can immediately identify the determinants of regulatory capital $\hat{k}$, which are the loans' probability of default p, loss given default λ, and exposure to systematic risk ρ, as well as the confidence level α set by the regulator.

To analyze the effects on regulatory capital $\hat{k}$ of changes in its determinants, we differentiate function (14), which gives

$$\frac{\partial \hat{k}}{\partial p} > 0, \quad \frac{\partial \hat{k}}{\partial \lambda} > 0, \quad \text{and} \quad \frac{\partial \hat{k}}{\partial \alpha} > 0.$$

Moreover, we also get

$$\frac{\partial \hat{k}}{\partial \rho} > 0 \quad \text{if and only if} \quad N^{-1}(\alpha) + \sqrt{\rho}\, N^{-1}(p) > 0,$$

which for $\alpha = 99.9$ percent and $\rho \leq 0.24$ (the maximum value in Basel II for corporate, sovereign, and bank exposures) holds for all $p \geq 0.03$ percent (the minimum value in Basel II). Therefore, we conclude that regulatory capital $\hat{k}$ is an increasing function of its four determinants.[7]

It is important to highlight the different determinants of economic and regulatory capital. Both economic and regulatory capital depend on the loans' probability of default p, loss given default λ, and exposure to systematic risk ρ. However, while an increase in any of these variables increases regulatory capital, its effect on economic capital is, in general, ambiguous. Moreover, economic capital depends on the loan rate r, the deposit rate c, and the cost of bank capital δ, whereas regulatory capital depends on the confidence level α set by the regulator.

2.3 Actual Capital

We next derive the level of capital chosen by the bank shareholders when their choice is restricted by two regulatory constraints. First, we assume that there is a supervisor that audits the bank at the beginning of each period and requires it to hold at least the regulatory capital $\hat{k}$ in order to operate. Second, in line with U.S.

[7]In contrast, regulatory capital in the 1988 Accord (Basel I) was largely independent of risk. Basel I required a minimum capital equal to 8 percent of the bank's risk-weighted assets. Two basic criteria were used to compute these weights: the institutional nature of the borrower and the collateral provided. In particular, the weights were 0 percent for sovereign risks with OECD countries, 20 percent for interbank risks, 50 percent for mortgages, and 100 percent for all other risks. See Basel Committee on Banking Supervision (1988).

regulation, and in particular the PCA provisions of FDICIA, we assume that banks whose capital at the end of a period falls below a certain critical level $\hat{k}_{\min}$ are closed by the supervisor.[8]

In this setup the Bellman equation that characterizes the solution to the shareholders' maximization problem is

$$V = \max\left\{ \max_{k_t \in [\hat{k},1]} \left[-k_t + \frac{1}{1+\delta} \left[E\big(\max\left\{ k_t', 0 \right\} \big) + \Pr\big(k_t' \geq \hat{k}_{\min} \big) V \right] \right], 0 \right\}. \tag{15}$$

There are two differences between this equation and the one for economic capital, (10). First, the bank is not allowed to operate with an initial capital k_t below the minimum required by regulation $\hat{k}$, so the choice of k_t is restricted to the interval $[\hat{k}, 1]$. But with this constraint the shareholders may find it optimal not to operate the bank, in which case $V = \max\{\cdot, 0\} = 0$. Second, equation (15) takes into account that the bank is closed by the supervisor when its end-of-period capital k_t' falls below the critical level $\hat{k}_{\min}$, so the discounted value of remaining open in period $t+1$ is multiplied by $\Pr(k_t' \geq \hat{k}_{\min})$.

The solution of Bellman equation (15) gives the actual level of capital k^a that bank shareholders would like to hold given the assumed regulation, as well as the corresponding franchise value V^a. This equation also identifies the determinants of actual capital k^a, which are the same six variables that determine economic capital plus the minimum capital requirement $\hat{k}$ and the critical level $\hat{k}_{\min}$.

As in the case of economic capital, actual capital can be at the corner $k^a = \hat{k}$, in which case none of the other variables matter for actual capital. And if there is an interior solution, comparative static

[8] According to FDICIA, banks whose tangible equity ratio falls below 2 percent are considered critically undercapitalized and are required to be placed in receivership or conservatorship within ninety days of becoming critically undercapitalized; see Comptroller of the Currency (1993). Tangible equity ratio is defined as tier 1 capital plus cumulative preferred stock and related surplus, less intangibles except qualifying purchased mortgage servicing rights (PMSRs), divided by total assets, less intangibles except qualifying PMSRs.

results are, in general, ambiguous, except for the cost of capital δ and the minimum capital requirement $\hat{k}$, for which we obtain

$$\frac{\partial k^a}{\partial \delta} < 0 \quad \text{and} \quad \frac{\partial k^a}{\partial \hat{k}} = 0.$$

Thus, when the shareholders choose an interior solution, an increase in the bank's equity funding costs reduces the level of actual capital, while an increase in the minimum capital requirement does not have any effect on their choice.

An important difference between economic and actual capital is that in choosing the former, bank shareholders have the option of providing no capital, which implies that the bank always has a positive franchise value, while in choosing the latter, they have to provide at least the minimum capital required by regulation, which in some cases may lead them to prefer not to operate the bank ($V^a = 0$). Whenever shareholders choose to operate the bank ($V^a > 0$), actual capital will, by construction, be greater than or equal to regulatory capital. In contrast, economic capital may be below regulatory capital. Obviously, the bank's franchise value will be, in general, higher for economic capital than for actual capital ($V^* > V^a$), because the constraints imposed by the regulator reduce the value of the bank.

It should also be noted that for $\hat{k}_{\min} = 0$, the bank's objective function in Bellman equation (15) coincides with the objective function in Bellman equation (10) that characterizes economic capital, except for the fact that in the former the bank's choice of capital is restricted to the interval $[\hat{k}, 1]$. But this implies that $k^a = \max\{k^*, \hat{k}\}$, except when $k^* < \hat{k}$ and the shareholders find it optimal not to operate the bank. In words, when the critical end-of-period capital $\hat{k}_{\min}$ below which the bank is closed by the supervisor is 0, and the shareholders choose to operate the bank, actual capital will be equal to the maximum of economic and regulatory capital.

Since comparative static results are, in general, ambiguous, in the following section we resort to numerical solutions to discuss the relationship between regulatory, economic, and actual capital.

3. Results

This section compares the values of regulatory capital $\hat{k}$, economic capital k^*, and actual capital k^a obtained by, respectively, computing

IRB formula (14), and solving Bellman equations (10) and (15), for plausible values of the parameters of the model.[9] The implicit assumption is that the bank invests all its portfolio in a single class of loans, with the same probability of default p and loss given default λ.

For the benchmark case, we assume a probability of default p of 2 percent and a loss given default λ of 45 percent (the value specified in the IRB foundation approach of Basel II for senior claims on corporates, sovereigns, and banks not secured by recognized collateral). For computing regulatory capital, we use the confidence level $\alpha = 99.9$ percent also set in Basel II.

The exposure-to-systematic-risk parameter ρ will be assumed to be a decreasing function of the probability of default p, according to the functional form specified in Basel II for corporate, sovereign, and bank exposures, which is

$$\rho(p) = 0.24 - 0.12 \, \frac{1 - e^{-50p}}{1 - e^{-50}}.$$

Thus the maximum value of the exposure to systematic risk is $\rho(0) = 0.24$, the minimum value is $\rho(1) = 0.12$, and for the benchmark probability of default we have $\rho(0.02) = 0.16$. The effect of this assumption is to flatten (relative to the case with a constant ρ) the function that relates regulatory capital $\hat{k}$ to the probability of default p. However, our conclusions do not vary qualitatively when ρ is constant.

With regard to the loan rate r, instead of taking it as exogenous, we assume that it is determined by equating the expected return of a loan, $(1 - p)r - p\lambda$, to a margin μ over the risk-free rate, which is normalized to 0, i.e.,

$$(1 - p)r - p\lambda = \mu. \tag{16}$$

The margin μ is intended to capture the market power of the bank in the market for loans; i.e., it is taken to be exogenous.[10] Rearranging

[9]Bellman equations (10) and (15) are solved by an iterative procedure. For example, in the case of (10), if we denote by $G(k, V)$ the bank's objective function, given an initial franchise value V_0, we compute $V_1 = \max_k G(k, V_0)$ and iterate the process until convergence to a value V^*. Economic capital is then given by $k^* = \arg\max_k G(k, V^*)$.

[10]Endogenizing μ would require an equilibrium model of imperfect competition in the loan market, which is beyond the scope of this paper.

the loan pricing equation, (16), we obtain

$$r = \frac{\mu + p\lambda}{1 - p},$$

so the loan rate r is an increasing function of the probability of default p, the loss given default λ, and the intermediation margin μ. In the benchmark case we take a value of μ of 1 percent.

For the deposit rate c, we assume that the return required by depositors is equal to the risk-free rate, which has been normalized to 0, and we consider two alternative scenarios. In the first scenario, depositors are fully insured (at a 0 premium) by a deposit insurance agency, so the deposit rate c is equal to the risk-free rate, i.e., $c = 0$. In the second scenario, depositors are completely uninsured, so under the assumption of risk neutrality, the deposit rate c has to verify the participation constraint

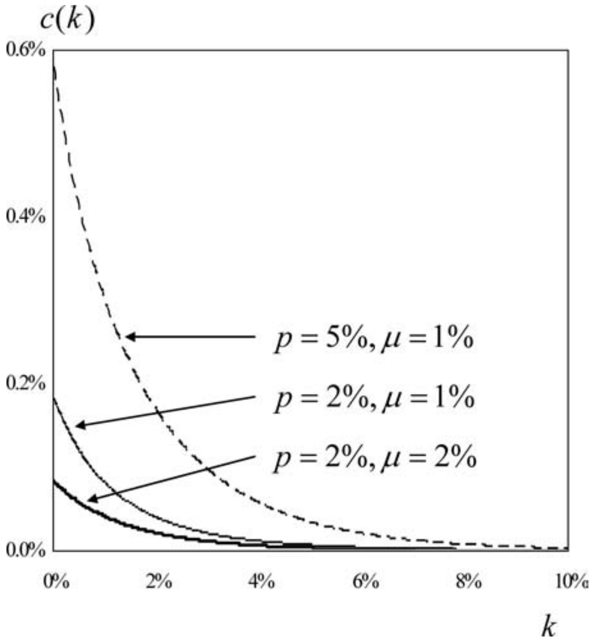
$$E[\min\{a, (1 - k)(1 + c)\}] = 1 - k. \tag{17}$$

To understand this equation, notice that when the value of the bank's end-of-period assets is greater than or equal to the depositors' principal and interest, i.e., when $k' = a - (1 - k)(1 + c) \geq 0$, depositors receive $(1 - k)(1 + c)$,[11] whereas when $k' < 0$, depositors receive the liquidation value of the bank, which (ignoring bankruptcy costs) is equal to a. Thus the left-hand side of equation (17) is the expected value of the depositors' claim at the end of each period, while the right-hand side is the gross return that they require on their investment. Appendix 2 shows that this equation has a unique solution $c(k) \geq 0$ for all k and that $c'(k) < 0$, except for $k \geq \lambda$, in which case $c'(k) = c(k) = 0$.

Figure 1 represents the cost of uninsured deposits c as a function of the capital level k—i.e., the function $c(k)$—for the benchmark case parameters, $p = 2$ percent and $\mu = 1$ percent, as well as the effects of an increase in the probability of default p and in the intermediation margin μ. The negative effect of k on the uninsured deposit rate c is significant for small values of k, for which the probability of bank failure is relatively high. An increase in the intermediation margin

[11] In the model of actual capital, the bank is closed for $0 \leq k' < \hat{k}_{\min}$, but in this case uninsured depositors also receive $(1 - k)(1 + c)$.

Figure 1. Effect of Bank Captial on the Uninsured Deposits' Interest Rate



μ from 1 percent to 2 percent reduces this probability and consequently the deposit rate c, whereas an increase in the probability of default p from 2 percent to 5 percent has the opposite effect.

The last parameter that has to be specified is the expected return δ required by bank shareholders, or cost of bank capital, which in the benchmark case is set equal to 6 percent.[12] Since we have normalized the risk-free rate to 0, this value should be interpreted as a spread over the risk-free rate.

Table 1 summarizes the parameter values in the benchmark case, as well as the range of values for which regulatory, economic, and

[12]Maccario, Sironi, and Zazzara (2002) estimate the cost of tier 1 capital for G-10 countries' major banks over the period 1993–2001, obtaining yearly averages between 6 and 10 percent. McCauley and Zimmer (1991) estimate banks' cost of equity for six countries during the period 1984–90, obtaining average estimates of around 10 percent for Canadian, UK, and U.S. banks; 6 percent for German and Swiss banks; and 3 percent for Japanese banks.

Table 1. Parameter Values Used in the Numerical Exercise

Parameter	Benchmark Case	Range of Values
Probability of Default p	2%	0–20%
Intermediation Margin μ	1%	0–5%
Cost of Bank Capital δ	6%	0–10%
Loss Given Default λ	45%	0–100%

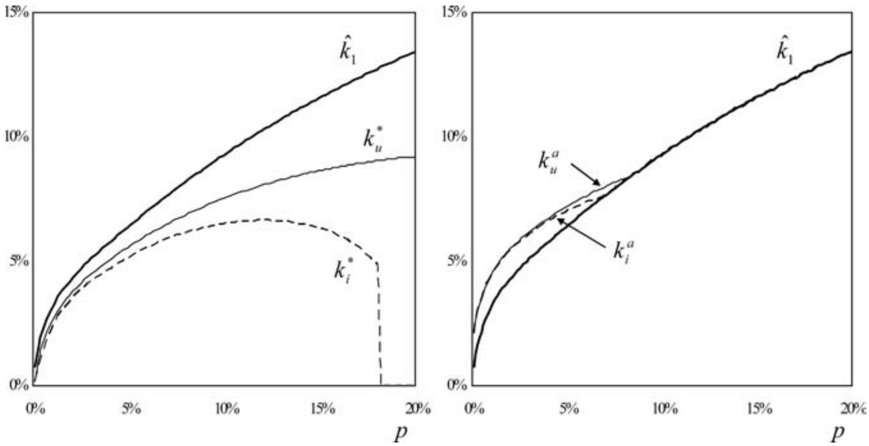
actual capital will be computed, keeping the rest of the parameters at their benchmark levels.

Our model only considers deposits and equity capital as sources of bank funding, but one should bear in mind that, in reality, there are many instruments in between. For regulatory purposes, Basel II distinguishes between tier 1 and tier 2 capital.[13] Tier 1 comprises equity capital and reserves from retained earnings, while tier 2 represents "supplementary capital" such as undisclosed reserves, revaluation reserves, general loan-loss reserves, hybrid (debt/equity) capital instruments, and subordinated debt. Ignoring for simplicity the special treatment of loan-loss provisions, Basel II involves two constraints: (i) tier 1 plus tier 2 capital should be greater than the minimum capital requirement, and (ii) tier 1 capital should be greater than 50 percent of the minimum requirement.

In what follows, we restrict attention to the tier 1 minimum capital requirement $\hat{k}_1 = \hat{k}/2$, where $\hat{k}$ is computed from IRB formula (14). In the case where deposits are uninsured, this requires no justification, since these deposits could be identified with subordinated debt, so tier 1 plus tier 2 capital would equal 100 percent of the bank's assets. In the case where deposits are insured, we would be effectively ignoring the tier 2 capital requirement. But since, as we will see below, the effect on the bank's capital choice of going

[13]The definition of eligible regulatory capital has not changed from Basel I; see Basel Committee on Banking Supervision (1988, paragraph 14; 2004, paragraphs 40 and 41).

Figure 2. Effect of the Loans' Probability of Default on Regulatory, Economic, and Actual Capital



from no insurance to full insurance is generally small, the effect of ignoring a small tier 2 requirement is negligible.

Finally, to compute actual capital, we follow FDICIA and set the threshold for critically undercapitalized banks at $\hat{k}_{\min} = 2$ percent.

3.1 Effect of the Loans' Probability of Default

The left panel of figure 2 plots regulatory (tier 1) capital $\hat{k}_1$ and economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the loans' probability of default p, and the right panel plots $\hat{k}_1$ and actual capital with insured and uninsured deposits, k_i^a and k_u^a, as functions of p.

As discussed in section 2, an increase in the probability of default p increases regulatory capital but has an ambiguous effect on economic capital. In particular, the left panel of figure 2 shows that economic capital with insured deposits k_i^* is increasing in the probability of default for values of p below 12 percent, is decreasing for values of p between 12 and 18 percent, and jumps to the corner solution $k_i^* = 0$ for higher values of p. Economic capital with uninsured deposits k_u^* is also first increasing and then decreasing in the probability of default p, although the change in slope takes place for much higher levels of p.

Economic capital with insured deposits k_i^* is always below economic capital with uninsured deposits k_u^*, because in the latter case shareholders have an additional incentive to provide capital in order to reduce the cost of uninsured deposits. This effect is more important when the loans' probability of default p is high because of the higher impact of the capital level k on the uninsured deposits' interest rate c noted above. Hence we conclude that the market discipline introduced by uninsured depositors leads to higher bank capital.

With respect to actual capital, the right panel of figure 2 shows that actual capital with insured and uninsured deposits, k_i^a and k_u^a, is strictly greater than regulatory capital $\hat{k}_1$ for default probabilities p below 7.7 and 8.5 percent, respectively. It can be shown that this buffer is increasing in the critical end-of-period capital $\hat{k}_{\min}$ below which the bank is closed by the supervisor, with $k_i^a = k_u^a = \hat{k}_1$ for $\hat{k}_{\min} = 0$. This result indicates that PCA provisions are an effective instrument to induce banks to hold capital levels above the minimum required by regulation.

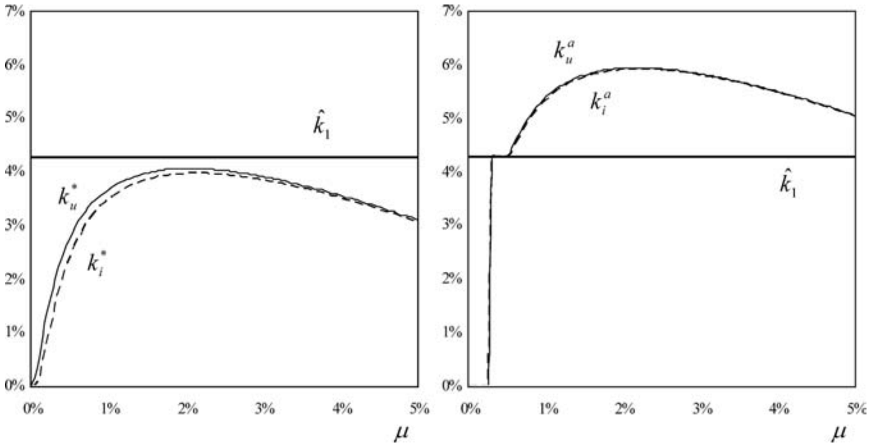
The right panel of figure 2 also shows that, for those cases where an interior solution exists, actual capital with insured deposits k_i^a is always below actual capital with uninsured deposits k_u^a. For higher default probabilities, those capital levels are equal to the minimum requirement $\hat{k}_1$, except when p is greater than 32 percent, in which case shareholders do not operate the bank when deposits are uninsured. Finally, the gap between actual capital with uninsured and insured deposits, $k_u^a - k_i^a$, is smaller than the corresponding gap for economic capital, $k_u^* - k_i^*$, because since actual capital is greater than economic capital, shareholders have fewer incentives to provide capital in order to reduce the cost of uninsured deposits. As we shall see below, this is a general result.

3.2 Effect of the Intermediation Margin

The left panel of figure 3 plots regulatory (tier 1) capital $\hat{k}_1$ and economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the intermediation margin μ, and the right panel plots $\hat{k}_1$ and actual capital with insured and uninsured deposits, k_i^a and k_u^a, as functions of μ.

The intermediation margin μ has two opposite effects on economic capital. On the one hand, a higher margin increases the bank's

Figure 3. Effect of the Intermediation Margin on Regulatory, Economic, and Actual Capital



franchise value V and therefore shareholders' incentives to provide capital in order to preserve it. On the other hand, by assumption (16), a higher margin increases the loan rate r, which increases (in the sense of first-order stochastic dominance) the bank's portfolio return, and consequently reduces the need to hold capital in order to protect V.

The left panel of figure 3 shows that, for values of the intermediation margin μ below 2.1 percent, increases in the margin increase both k_i^* and k_u^*, bringing them closer to regulatory capital (which does not vary with μ), but the relationship becomes negative for higher values of the margin μ. Thus, for sufficiently competitive banking markets, the positive effect of the intermediation margin on economic capital (via an increase in the bank's franchise value) outweighs its negative effect (via the substitution between economic capital and the margin), while for oligopolistic markets, the negative effect dominates.

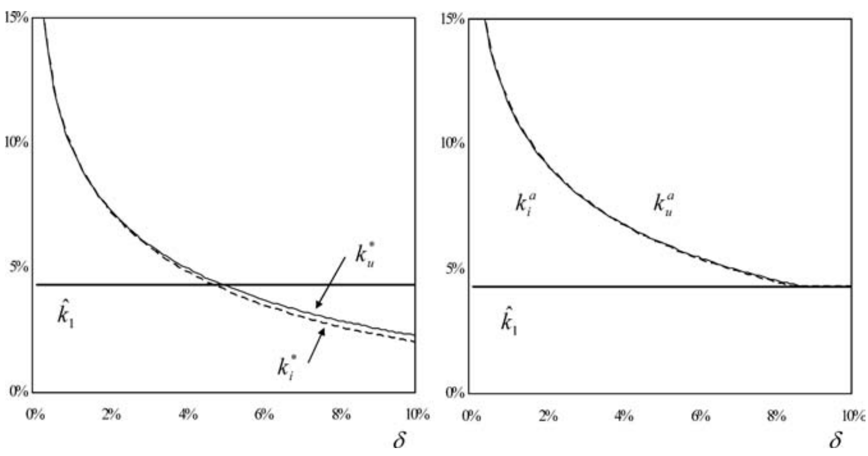
With respect to actual capital, the right panel of figure 3 shows that when the intermediation margin is below 0.25 percent, the shareholders prefer not to operate the bank rather than provide the minimum capital $\hat{k}_1$. Beyond this point, and for those values of the margin for which the restriction $k^a \geq \hat{k}_1$ is not binding, actual capital has a shape similar to that of economic capital. Again, whenever the bank operates, actual capital is higher than economic capital, which explains why the effect of the market discipline introduced by uninsured depositors on actual capital is almost negligible.

3.3 Effect of the Cost of Bank Capital

In all cases analyzed so far, we have found economic capital to be below regulatory capital. This is mainly due to our benchmark parameter value for the cost of bank capital δ. The left panel of figure 4 plots regulatory (tier 1) capital $\hat{k}_1$ and economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the cost of capital δ, and the right panel plots $\hat{k}_1$ and actual capital with insured and uninsured deposits, k_i^a and k_u^a, as functions of δ.

As shown in appendix 1, economic capital is a decreasing function of the cost of capital ($\partial k^*/\partial\delta < 0$). Moreover, for values of the cost of capital δ below approximately 5 percent, both levels of economic capital, with and without insured deposits, are above regulatory capital. The reason is obvious: the lower the cost of capital δ, the higher the incentives of bank shareholders to contribute

Figure 4. Effect of the Cost of Bank Capital on Regulatory, Economic, and Actual Capital



capital. In fact, for values of δ sufficiently close to 0, shareholders choose capital levels that effectively guarantee the bank's survival regardless of the fraction of the loans in its portfolio that default.

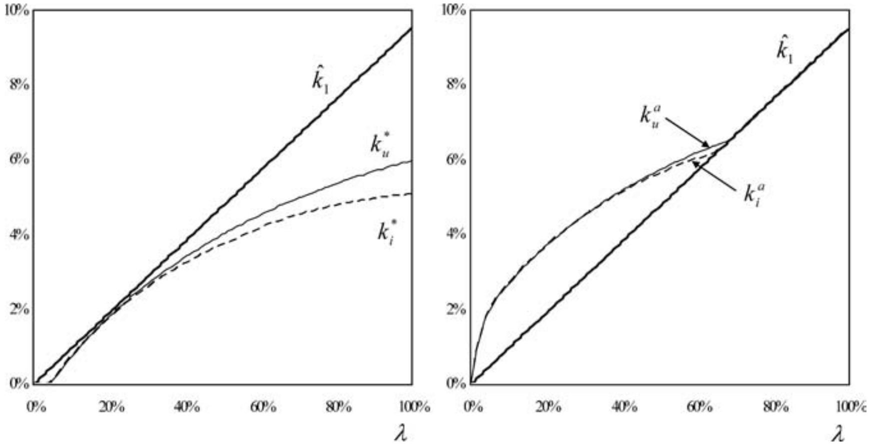
The relative position of actual capital with respect to economic and regulatory capital follows the same pattern as in figures 2 and 3. Actual capital is higher than regulatory capital for values of δ below 8.4 and 8.8 percent, respectively, for the insured and uninsured deposits cases. From those levels onward, they are equal to $\hat{k}_1$. The shareholders do not operate the bank for unreasonably high values of δ (above 23.5 percent).

3.4 Effect of the Loans' Loss Given Default

The left panel of figure 5 plots regulatory (tier 1) capital $\hat{k}_1$ and economic capital with insured and uninsured deposits, k_i^* and k_u^*, as functions of the bank loans' loss given default λ, and the right panel plots $\hat{k}_1$ and actual capital with insured and uninsured deposits, k_i^a and k_u^a, as functions of λ.

According to IRB formula (14), regulatory capital $\hat{k}_1$ is a linear function of the loss given default λ. While the effect of λ on economic

Figure 5. Effect of the Loans' Loss Given Default on Regulatory, Economic, and Actual Capital



capital is positive in figure 5, as noted in section 2, this is not true in general (for example, if the probability of default p equals 7 percent, k_i^* starts to decrease for values of λ greater than 57 percent). Finally, actual capital is strictly above regulatory capital for most values of λ.

To sum up, we have found that both regulatory and economic capital depend positively on the loans' probability of default and loss given default for reasonable values of these variables. However, variables that only affect economic capital, such as the intermediation margin and the cost of capital, may significantly move it away from regulatory capital. Actual capital, which by definition is higher than regulatory capital, always lies above economic capital, and it is increasing in the critical capital level below which the supervisor closes the bank. We have also found that market discipline, proxied by the coverage of deposit insurance, has a positive impact on economic capital, but the effect is, in general, small and very sensitive to the values of the rest of the determinants of economic capital. Since actual capital is higher than economic capital, it is affected even less by market discipline.

4. Conclusion

Defining economic capital as the capital that shareholders would choose in the absence of regulation, this paper analyzes the determinants of economic and regulatory capital for a bank whose loan default rates are derived from the single-risk-factor model that underlies the capital charges in the IRB approach of Basel II. Our results show that there does not exist a direct relationship between both capital levels.

First, economic and regulatory capital do not depend on the same variables: regulatory (but not economic) capital depends on the confidence level set by the regulator, while economic (but not regulatory) capital depends on the intermediation margin and the cost of bank capital. These last two variables play a key role in determining the differences between economic and regulatory capital. Economic capital is above regulatory capital for low values of the cost of capital, and when this cost increases, the former quickly falls below the latter. The effect of the intermediation margin on economic capital is nonmonotonic, which is explained by the existence of two opposite effects: on the one hand, a higher margin increases the bank's franchise value and hence shareholders' incentives to contribute capital in order to preserve it, but on the other hand, a higher margin provides a source of income that reduces the need to hold capital as a buffer against losses. The first (positive) effect outweighs the second (negative) in sufficiently competitive credit markets. Therefore, changes in the market power of banks—due, e.g., to entry of new banks or mergers and acquisitions—may have very different effects on economic capital, depending on the initial level of competition.

Second, variables that affect both economic and regulatory capital, such as the loans' probability of default and loss given default, have a positive impact on both capital levels for reasonable values of these variables. But when they reach certain critical values, their effect on economic capital becomes negative, increasing the gap between economic and regulatory capital.

However, it is important to note that, in reality, banks choose their capital structure considering the regulations in place; i.e., they choose actual capital rather than economic capital. We define actual capital as the equity capital chosen by the bank shareholders when their choice is restricted by two regulations: (i) an initial capital greater than or equal to the minimum required by regulation and (ii) a closure rule for critically undercapitalized banks. The first regulation alone makes actual capital equal to the maximum of economic and regulatory capital, which according to our results coincides almost always with the latter (except for small values of the cost of capital). Therefore, whenever actual capital is higher than regulatory capital, this is likely to be explained by the second regulation. Our results indicate that the threat of closing critically undercapitalized banks (banks with tier 1 capital below 2 percent) significantly increases actual bank capital for reasonable ranges of parameter values. This regulation was introduced in the United States by the Federal Deposit Insurance Corporation Improvement Act, and it is not explicitly contemplated in Basel II. However, under pillar 2 (supervisory review process) of Basel II, national supervisors have discretion to introduce prompt corrective action provisions. According to our results, PCA provisions would be an effective instrument to induce banks to hold capital buffers.

Finally, the comparison of economic capital with insured and uninsured deposits reveals that, even though the latter is never below the former, their differences are, in general, small and very sensitive to the values of the rest of the determinants of economic capital. In the case of actual capital, those differences are almost negligible. Therefore, we conclude that the effects on the banks' capital structure of policies aimed at increasing market discipline, such as those contemplated in pillar 3 of Basel II, may be very limited.

Appendix 1. Comparative Statics of Economic Capital

This appendix discusses the effects on economic capital k^* of changes in its determinants—namely, the loans' probability of default p, loss given default λ, and exposure to systematic risk ρ; the loan rate r; the deposit rate c; and the cost of bank capital δ. It is shown that when there is an interior solution ($k^* > 0$), we can only determine the effect of the cost of capital.

To this end we first note that, integrating by parts and taking into account the restriction $k \leq k_{\max}$, we can rewrite Bellman equation (12) as

$$V = \max_{k \in [0, k_{\max}]} G(k, V), \tag{18}$$

where

$$G(k, V) = -k + \frac{1}{1+\delta}\left[(\lambda + r)\int_0^{x(k)} F(x)dx + F(x(k))V\right]. \tag{19}$$

The derivatives of the function $G(k, V)$ with respect to k are given by

$$\frac{\partial G}{\partial k} = -1 + \frac{1+c}{1+\delta}\left[F(x(k)) + \frac{f(x(k))V}{\lambda + r}\right], \tag{20}$$

$$\frac{\partial^2 G}{\partial k^2} = \frac{(1+c)^2}{(1+\delta)(\lambda + r)}\left[f(x(k)) + \frac{f'(x(k))V}{\lambda + r}\right], \tag{21}$$

where $f(x) = F'(x)$ is the density function of the default rate and $f'(x)$ is its derivative. While the first term of (21) is non-negative (since $f(\cdot)$ is a density), the second term can be either positive or negative. Thus $G(k, V)$ is not, in general, a convex or a concave function of k, which implies that we may have both corner and interior solutions. However, since $F(x(k_{\max})) = F(1) = 1$ and $f(x(k_{\max})) = f(1) = 0$, our assumption $\delta > c$ implies that the derivative of $G(k, V)$ with respect to k evaluated at $k_{\max}$ is always negative, so a corner solution at $k_{\max}$ can be ruled out. Therefore, the only possible corner solution is $k^* = 0$.

If an interior solution exists, it would be characterized by the first-order condition $\partial G/\partial k = 0$ and the second-order condition $\partial^2 G/\partial k^2 < 0$. Differentiating the first-order condition gives

$$\frac{\partial k^*}{\partial z} = -\left(\frac{\partial^2 G}{\partial k^2}\right)^{-1}\left(\frac{\partial^2 G}{\partial k \partial z} + \frac{\partial^2 G}{\partial k \partial V}\frac{\partial V}{\partial z}\right),$$

where z is any of the six variables that determine economic capital k^*. Since

$$\frac{\partial^2 G}{\partial k \partial V} = \frac{(1+c)f(x(k))}{(1+\delta)(\lambda+r)} > 0,$$

and by the second-order condition we have $\partial^2 G/\partial k^2 < 0$, we need to find the signs of $\partial^2 G/\partial k \partial z$ and $\partial V/\partial \delta$. For $z = \delta$ it is easy to check that (20) implies

$$\frac{\partial^2 G}{\partial k \partial \delta} = -\frac{1+c}{(1+\delta)^2}\left[F(x(k)) + \frac{f(x(k))V}{\lambda+r}\right] = -\frac{1}{1+\delta} < 0,$$

and by the definition of the franchise value V, (18), and the envelope theorem, we have

$$\frac{\partial V}{\partial \delta} = -\frac{1}{(1+\delta)^2}\left[1 - \frac{F(x(k))}{1+\delta}\right]^{-1} \times \left[(\lambda+r)\int_0^{x(k)} F(x)\, dx + F(x(k))V\right] < 0,$$

which implies $\partial k^*/\partial \delta < 0$. However, for $z = p$, λ, and ρ, the sign of $\partial^2 G/\partial k \partial z$ is ambiguous; for $z = r$ we have $\partial^2 G/\partial k \partial r < 0$ and $\partial V/\partial r > 0$; and for $z = c$ we have $\partial^2 G/\partial k \partial c > 0$ and $\partial V/\partial c < 0$, so we would need additional assumptions to get comparative statics results for these five variables.

Appendix 2. Uninsured Deposits' Interest Rate

The uninsured deposits' interest rate c is obtained by solving the participation constraint, (17), that equates the expected value of the depositors' claim at the end of each period, $E[\min\{a, (1-k)(1+c)\}]$, to the gross return that the depositors require on their investment, $1-k$. This appendix shows that the equation,

$$U(c,k) = E[\min\{a,(1-k)(1+c)\}] - (1-k) = 0, \tag{22}$$

has a unique solution $c(k) \geq 0$ for all k, and that $c'(k) < 0$, except for $k \geq \lambda$, in which case $c'(k) = c(k) = 0$.

For $k \geq \lambda$ it is immediate to check that $U(c,k) \geq U(0,k) = 0$, with strict inequality for $c > 0$, so $c = 0$ is the unique solution.

For $k < \lambda$, given that $0 \leq x(k) < 1$ for all $0 < c \leq (k+r)/(1-k)$, substituting the definition of a, (1), into (22), integrating by parts, and making use of the definition of $x(k)$, (11), gives

$$U(c,k) = k - \lambda + (\lambda + r)\int_{x(k)}^{1} F(x)\ dx. \tag{23}$$

To prove that (22) has a unique solution $c(k) > 0$, it suffices to show that $U(0,k) < 0 < \max_c U(c,k)$ and that $\partial U/\partial c > 0$. First, since $F(x(k)) < 1$, using (23) and the definition of $x(k)$, (11), implies

$$U(0,k) < k - \lambda + (\lambda + r)(1 - x(k)) = 0.$$

Second, using the fact $\int_0^1 F(x)\ dx = 1-p$, together with the fact that by equation (16) we have $(1-p)r - p\lambda = \mu > 0$, (23) implies

$$\begin{aligned}\max_c U(c,k) &= k - \lambda + (\lambda + r)\int_0^1 F(x)\ dx = k + (1-p)r - p\lambda \\ &= k + \mu > 0.\end{aligned}$$

And third, differentiating (23) with respect to c gives

$$\frac{\partial U}{\partial c} = (1-k)F(p(k)) > 0. \tag{24}$$

Finally, totally differentiating $U(c,k) = 0$ and using (24), we have $c'(k) < 0$ if

$$\frac{\partial U}{\partial k} = 1 - (1+c)F(x(k)) > 0.$$

But for $c = c(k)$, we have

$$U(c,k) = (1-k)(1+c)F(x(k)) + \int_{x(k)}^{1} a \; dF(x) - (1-k) = 0,$$

which implies

$$1 - (1+c)F(x(k)) = \frac{1}{1-k} \int_{x(k)}^{1} a \; dF(x) > 0.$$

References

Aggarwal, R., and K. T. Jacques. 2001. "The Impact of FDICIA and Prompt Corrective Action on Bank Capital and Risk: Estimates Using a Simultaneous Equations Model." *Journal of Banking and Finance* 25 (6): 1139–60.

Allen, B. 2006. "Internal Affairs." *Risk* 19 (June): 45–49.

Basel Committee on Banking Supervision. 1988. *International Convergence of Capital Measurement and Capital Standards*. Bank for International Settlements.

———. 2004. *International Convergence of Capital Measurement and Capital Standards: A Revised Framework*. Bank for International Settlements.

Calem, P., and R. Rob. 1999. "The Impact of Capital-Based Regulation on Bank Risk-Taking." *Journal of Financial Intermediation* 8 (4): 317–52.

Carey, M. 2001. "Dimensions of Credit Risk and Their Relationship to Economic Capital Requirements." In *Prudential Supervision: What Works and What Doesn't*, ed. F. S. Mishkin. Chicago: University of Chicago Press and NBER.

Caruana, J. 2005. "Basel II: Back to the Future." 7th Hong Kong Monetary Authority Distinguished Lecture. Available at www.bde.es/prensa/intervenpub/gobernador/040205e.pdf.

Comptroller of the Currency. 1993. "Banking Circular 268." Available at www.occ.treas.gov/ftp/bc/bc-268.doc.

Estrella, A. 2004. "The Cyclical Behavior of Optimal Bank Capital." *Journal of Banking and Finance* 28 (6): 1469–98.

Flannery, M., and K. Rangan. 2004. "What Caused the Bank Capital Build-up of the 1990s?" Working Paper No. 2004-03, FDIC Center for Financial Research.

Gordy, M. 2003. "A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules." *Journal of Financial Intermediation* 12 (3): 199–232.

Jackson, P., W. Perraudin, and V. Saporta. 2002. "Regulatory and 'Economic' Solvency Standards for Internationally Active Banks." *Journal of Banking and Finance* 26 (5): 953–76.

Jones, D., and J. Mingo. 1998. "Industry Practices in Credit Risk Modeling and Internal Capital Allocations: Implications for a Models-Based Regulatory Capital Standard." *Economic Policy Review* (Federal Reserve Bank of New York) 4 (3): 53–60.

Maccario, A., A. Sironi, and C. Zazzara. 2002. "Is Banks' Cost of Equity Capital Different Across Countries? Evidence from the G10 Countries' Major Banks." Working Paper No. 02-77, SDA Bocconi Research Division.

McCauley, R. N., and S. A. Zimmer. 1991. "Bank Cost of Capital and International Competition." *Quarterly Review* (Federal Reserve Bank of New York) 15 (3–4): 33–59.

Repullo, R. 2004. "Capital Requirements, Market Power, and Risk-Taking in Banking." *Journal of Financial Intermediation* 13 (2): 156–82.

Repullo, R., and J. Suarez. 2004. "Loan Pricing under Basel Capital Requirements." *Journal of Financial Intermediation* 13 (4): 496–521.

Suarez, J. 1994. "Closure Rules, Market Power and Risk-Taking in a Dynamic Model of Bank Behaviour." Discussion Paper No. 196, LSE Financial Markets Group.

Vasicek, O. 2002. "Loan Portfolio Value." *Risk* 15 (December): 160–62.

Stock Liquidity Requirements and the Insurance Aspect of the Lender of Last Resort*

Spyros Pagratis
Bank of England

This paper considers a model of information-based bank runs where a central bank sets its lender of last resort (LOLR) policy in order to maximize welfare. To mitigate the risks associated with overinvestment by the banking sector, the central bank sets prudential liquidity requirements for the banking sector in the form of a ratio of liquid assets to deposits. Liquidity requirements then provide a buffer against early deposit withdrawals, but they also allow the central bank to manufacture a distribution of costs to LOLR funding with an expected value equal to 0. It is shown that liquidity requirements, along with an appropriate LOLR policy, become welfare improving if the banking sector is characterized by high-profit opportunities, low leverage, and a relatively volatile deposit base. Otherwise, forgone productive investment due to liquidity restrictions may result in a disproportional cost to the banking sector relative to the insurance value of LOLR.

JEL Codes: E58, G28.

1. Introduction

The lender of last resort (LOLR) function of a central bank insures the banking system against a run on liquidity, although no premia

*This paper is based on my PhD thesis at the London School of Economics. I am grateful to Hyun Song Shin for very valuable advice. I wish to thank Viral Acharya, Nicola Anderson, Hassan Naqvi, Erlend Nier, Jean-Charles Rochet, and Tanju Yorulmazer for stimulating discussions, and the two anonymous referees for thoughtful comments and suggestions. I would also like to thank participants at the Econometric Society World Congress 2005 and seminar participants at the London School of Economics for helpful comments. This paper represents the views and analysis of the author and should not be taken to represent those of the Bank of England. Any errors also remain my responsibility. Author contact: Bank of England, Threadneedle Street, London EC2R 8AH, United Kingdom. E-mail: spyros.pagratis@bankofengland.co.uk.

are explicitly charged against it. Instead, to forestall LOLR exposures from running at unsustainable levels, both in terms of public money at stake and political capital at risk, the official sector can influence the likelihood and extent of LOLR intervention by means of regulatory restrictions on banks. Liquidity regulation, however, may entail welfare losses due to forgone productive investments, implying a trade-off between the welfare benefits from LOLR insurance and the welfare losses from regulatory restrictions. This paper examines this trade-off in a setting in which the official sector aims to maximize welfare conditional on the expected cost of LOLR intervention being equal to 0.

The interaction between LOLR policy and liquidity requirements is discussed in the context of a three-period model with three active risk-neutral agents—namely, a bank, a continuum of depositors, and a central bank that represents the official sector. The bank is the only agent in the economy that has access to a production loan technology and finances its investment through equity and wholesale deposits. In addition, the bank voluntarily holds an amount of riskless assets that pay no return so as to target an optimal probability of solvency. The loan investment is illiquid, and the bank needs to wait for two periods before returns are realized. The probability that the loan *succeeds*, paying a nonzero return, depends on the realization of a risk factor, which we refer to as a *systemic shock*.

When depositors have full information about the systemic shock, liquidity problems do not arise and there is no role for LOLR policy. Relaxing the assumption of full information, the systemic shock is observed with noise by depositors who may then decide either to leave their money with the bank or to run, causing a liquidity shock to the bank. Strategic interaction among depositors, which may give rise to this liquidity shock, is modeled using a global-games methodology as in Morris and Shin (2002) and Rochet and Vives (2004). If the bank has a sufficient cushion of liquid assets in place, it may be able to sustain the liquidity shock. Otherwise, the central bank may have to intervene and extend LOLR provision.

The LOLR policy is defined here as a commitment by the central bank to extend emergency funding to the banking sector. But such a commitment is not unconditional; if a bank runs into liquidity difficulties due to a solvency problem that is severe enough to result in a major hit on the central bank's balance sheet in case of

LOLR intervention, then the central bank may refrain from providing emergency liquidity funding. Otherwise, the LOLR policy would be expected to involve unsustainable losses. To keep things simple, we assume that LOLR intervention takes the form of a capital injection, which may result in a loss to the central bank if bank illiquidity is due to a (moderate) solvency problem; otherwise, the central bank earns a positive return from its LOLR activity.

The central bank is assumed to be able to influence the expected cost of extending LOLR provision by requiring the banking sector to maintain a prescribed ratio of liquid (riskless) assets to deposits. This regulation restricts credit extension by the banking sector, limiting its exposure to risks and mitigating the impact on the central bank's balance sheet should those risks crystallize and LOLR provision has to be extended. The central bank chooses the required ratio with the objective of maximizing welfare conditional on there being zero expected cost of LOLR provision.

While LOLR insurance is welfare enhancing, the associated liquidity regulation may be costly for banks. Absent liquidity requirements, a profit-maximizing bank may have the incentive to free-ride on LOLR insurance and hold a lower stock of liquidity than would be consistent with a zero expected cost of LOLR provision. This is because funding is scarce in this model, and liquid assets are non–interest bearing. The scarcity of funds is modeled along the lines of Dewatripont and Tirole (1994), where an ex post moral hazard problem from the bank's side limits the amount of deposits the bank can raise.

The welfare benefit of LOLR insurance is modeled along the lines of Holmström and Tirole (1998). They examine the impact of liquidity shocks prior to realization of a production process and describe how an irrevocable line of credit—liquidity insurance—can improve welfare. However, Holmström and Tirole (henceforth H&T) assume an exogenously given liquidity shock, which in our model is derived endogenously on the basis of an information-induced bank run. That allows us to consider how the official safety net (LOLR policy complemented with liquidity regulation) influences the magnitude of the liquidity shock and vice versa. Another departure from the H&T framework is that credit lines in their model are extended free of charge, while here liquidity insurance is conditional on liquidity regulation.

By examining the trade-off between the welfare benefit of LOLR insurance and the cost of liquidity regulation, we follow the spirit of Buser, Chen, and Kane (1981), who consider banking regulations by the Federal Deposit Insurance Corporation (FDIC) as a condition for banks receiving deposit insurance and interpret the deadweight cost of regulatory rules as implicit insurance premia. But Buser, Chen, and Kane simply sketch a model of interaction between regulation and the official safety net, stopping short of analyzing the optimal FDIC response. Thus, they offer no insights into welfare implications, which is a key objective of our analysis.

We find that, in line with Posner (1971), who introduced the idea of taxation by regulation, whatever raises the desirability of LOLR activity also makes liquidity regulation more or less desirable ex ante. All things equal, we find that LOLR activity is more desirable the more capital a bank holds. That is because capital provides a cushion to the bank against losses, also offering the central bank some leeway to extend the LOLR insurance without violating its budget constraint. Another way to consider the welfare-enhancing role of LOLR, along with liquidity regulation, is in terms of the bank's leverage ratio. It turns out that LOLR insurance is more valuable when the ratio of bank deposits to bank equity is low. That is because, under low leverage, the bank has a weak incentive to undertake positive-net-present-value (positive-NPV) investment. By increasing the marginal expected return on investment, LOLR insurance increases the bank's marginal propensity to invest, which means that liquidity regulation is more likely to be welfare improving when the bank's leverage ratio is low.

Given the bank's capital ratio, LOLR insurance tends to become more valuable the riskier the liquidity shock that may hit the bank due to a run on deposits. In fact, the liquidity shock becomes riskier, and the LOLR insurance more valuable, the higher the extent of asymmetric information among deposits and the more volatile the bank's deposit base. For example, if the bank relies extensively on interbank funding, then the value of LOLR insurance could be higher than if the bank was raising funds through core deposits, which are generally perceived as more stable over time. In addition, a higher potential return per unit of loan investment increases the value of LOLR insurance and, as a result, the desirability of quid pro quo liquidity restrictions.

The rest of the paper is organized as follows. Section 2 describes the three-period model. Section 3 presents the benchmark case in which depositors have full information and there is no LOLR intervention or regulation. Section 4 introduces asymmetric information and characterizes the liquidity shock to the bank, and section 5 solves for the regulatory contract. Section 6 considers welfare implications of prudential liquidity regulation and LOLR insurance, and section 7 concludes. Proofs are included in the appendix.

2. Basic Environment

Consider an economy with three risk-neutral classes of agents: a bank, bank depositors, and a central bank that acts both as liquidity regulator to the bank and as an LOLR. The bank operates for three periods: $t = 0, 1, 2$. At $t = 0$, it holds capital A and raises a volume of uninsured wholesale deposits D. After deposits have been raised, the bank makes a risky loan investment I with constant returns to scale, and any unused funds l are placed in liquid assets that pay no interest.

The loan investment is risky because its payoff at $t = 2$ is binary, paying a gross return $R > 1$ per unit of investment if it succeeds and 0 otherwise. The probability of investment success is characterized by a random variable $\widetilde{\phi}$ with a uniform prior $U(1 - \bar{\phi}, \bar{\phi})$ that is interpreted as a systemic shock affecting the quality of the bank's assets. The bank also acts as a delegated monitor of its loan investment, undertaking an unobservable decision to manage it prudently or to engage in excess risk taking. Excess risk taking implies that the bank's residual claimholders receive a private benefit B per unit of investment if the bank does not fail, but the probability of loan success is scaled down by $\beta > 1$ to $\frac{\widetilde{\phi}}{\beta}$. Loan investment by the bank is assumed to have positive net present value, which implies the following parameter restriction.[1]

[1]Let ϕ_c be the minimum value of ϕ above which the bank is not liquidated and operates for two periods, and let $F(\phi)$ be the cumulative distribution of ϕ. Then, the expected return per unit of investment is $E(\widetilde{R}) = [1 - F(\phi_c)]\frac{\bar{\phi}+\phi_c}{2}R$. If $R \leq \frac{2(2\bar{\phi}-1)}{\bar{\phi}^2}$, then $E(\widetilde{R}) = 1 - \frac{\phi_c}{\bar{\phi}} < 1$, implying a negative-net-present-value investment.

Assumption 1. $R > \frac{2(2\bar{\phi}-1)}{\bar{\phi}^2}$.

Also, the risky investment is assumed to have a higher net present value when the bank does not engage in excess risk taking, which implies that $R - \frac{B}{\beta-1} > 0$.[2] In addition, we assume that due to moral hazard considerations à la Dewatripont and Tirole (1994), the bank is unable to raise an unlimited amount of deposits and may also need capital to finance its chosen level of investment. That is equivalent to assuming that $R - \frac{B}{\beta-1} < 1$.[3] Together, these inequalities lead to the following parameter restriction.

Assumption 2. $0 < R - \frac{B}{\beta-1} < 1$.

The bank makes its optimal choice over the amount of loan investment given the actions of two other classes of agents in the economy—namely, depositors and the central bank. A continuum of deposit managers—*depositors* for short—of total measure D are assumed to make no profits and to be able to monitor costlessly the realization of ϕ and manage a unit of wholesale deposits à la Rochet and Vives (2004). Depositors may decide to withdraw prematurely at $t = 1$ on the basis of private signals about ϕ and given their state-contingent payoffs. If a high proportion of depositors receive bad signals, then early withdrawals may occur on a large scale and the bank may face a *liquidity shortfall*. That is a situation where the bank does not have a sufficient amount of liquid assets to meet the withdrawal of deposits. But whether a signal is interpreted as bad or good will eventually depend not only on the realization of ϕ but also on what official measures are in place, as well as on depositors' beliefs about other depositors' actions.

[2] In the absence of excess risk taking and for a given continuation threshold ϕ_c, the expected return per unit of loan investment is given by $E(\widetilde{R}|\text{no excess risk taking}) = [1 - F(\phi_c)]\frac{\bar{\phi}+\phi_c}{2}R$. If the bank engages in excess risk taking, then the expected return per unit of investment becomes $E(\widetilde{R}|\text{excess risk taking}) = [1 - F(\phi_c)]\frac{\bar{\phi}+\phi_c}{2\beta}(R + B)$. It is easy then to show that $E(\widetilde{R}|\text{no excess risk taking}) > E(\widetilde{R}|\text{excess risk taking})$ if and only if $R - \frac{B}{\beta-1} > 0$.

[3] This inequality follows from $D < I$ and the incentive compatibility condition (3b) for bank deposits that we introduce in section 3.1.

Under the possibility of a bank run, the central bank sets its LOLR policy in a way that satisfies a twofold objective: first, to maximize social welfare, which in this model is equivalent to the expected surplus from bank lending, and second, to maintain a zero expected cost of LOLR intervention, i.e., to avoid losses from occurring on a systematic basis. In addition, the central bank is able to control the probability and the extent of a bank run by regulating the amount of cash that the bank holds in relation to deposits. LOLR policy is characterized by a level ϕ^{**} of the systemic shock ϕ such that the bank is bailed out if and only if $\phi \geq \phi^{**}$. In that sense, ambiguity about LOLR intervention does not emanate from strategic behavior by the central bank (i.e., it is not *constructive*); rather, it stems from uncertainty about the systemic shock.

LOLR intervention is assumed to occur via a capital injection,[4] whereby deposit withdrawals are met in full and the central bank acquires control of the bank, repaying all remaining deposits in the final period. Should the proceeds from realized loan returns be sufficient to cover all outstanding deposits, the central bank may make a profit; otherwise, it incurs a loss that could be financed through distortionary taxation.

The following sections examine the optimal behavior of the bank given the actions of depositors and the central bank. In section 3, depositors have full information about the quality of loans and so do not withdraw deposits prematurely as long as the bank is solvent. Therefore, the central bank does not operate an LOLR policy and, consequently, places no restrictions in the actions of the bank. In sections 4 to 6, we relax the assumption of full information and we introduce central bank intervention.

3. Full-Information Benchmark

We envisage a situation in which depositors have full information about the realization of ϕ and, as a result, the bank faces no

[4]In reality, central banks may require preferential terms under LOLR intervention, such as enhanced seniority and extra collateral. Such terms may enhance central banks' willingness to bail out ailing institutions but may also negate the catalyzing impact of the LOLR safety net on private creditors' incentives to roll over, crowding out private credit.

liquidity shortfall as long as it is solvent. Moreover, the bank faces no regulatory restrictions on the amount of liquid assets it can hold, and there is no LOLR insurance. We first solve for a critical level ϕ_0 of the systemic shock above which the bank is solvent and, as a result, it is not liquidated at $t = 1$. Then we solve for the bank's optimal investment and we characterize the bank's voluntary holdings of liquid assets.

Suppose that I_0 is the bank's chosen level of investment, l_0 is the amount of voluntary liquid holdings, and D_0 is the amount of deposits that the bank would be able to raise at $t = 0$. The solvency threshold ϕ_0 satisfies the breakeven condition

$$\phi_0 R I_0 = D_0 - l_0, \tag{1}$$

where the left-hand side of (1) is the expected return on investment, conditional on $\phi = \phi_0$, and the right-hand side is the amount of deposits that are not covered by liquid assets. Given that $I_0 + l_0 = A + D_0$, from equation (1) we find that

$$\phi_0 = \frac{I_0 - A}{R I_0}. \tag{2}$$

3.1 Voluntary Holdings of Liquid Assets

Let $m(I_0)$ be the marginal net expected return per unit of investment at a total level of investment I_0. From equation (2) it follows that $m(I_0)$ is decreasing in I_0 since the riskier the investment that the bank undertakes, the more vulnerable the bank becomes to a systemic shock and, as a result, the higher its probability of default. Let also $U(I_0) = m(I_0)I_0$ be the expected surplus of the bank's investment. Given that depositors and the central bank make no profits, the surplus of the bank's investment is equal to the bank's net expected utility from loan investment. Then, the optimal amount of investment I_0 solves the following:

$$\max_{I_0} U(I_0) \tag{3a}$$

subject to

$$\int_{\phi_0}^{\bar{\phi}} \phi (R I_0 - D_0) dF(\phi) \geq \int_{\phi_0}^{\bar{\phi}} \frac{\phi}{\beta} (R I_0 - D_0 + B I_0) dF(\phi) \tag{3b}$$

$$I_0 + l_0 = A + D_0. \tag{3c}$$

Inequality (3b) is the bank's incentive compatibility condition, (3c) is the bank's budget constraint, and ϕ_0 is the solvency threshold given by (2). For a given amount of deposits D_0, the optimal choice of I_0 is such that the bank has no incentive to take excessive risks for the private benefit B. Thus, the incentive compatibility condition (3b) can be written as follows:

$$D_0 \leq \left(R - \frac{B}{\beta - 1}\right) I_0. \tag{4}$$

By assumption 1, the bank's investment has positive net present value, and the cost of borrowing from depositors (normalized to 0) is less than the expected net payoff per unit of investment. Consequently, given risk neutrality, the bank always has the incentive to take on more deposits in order to increase its investment, which implies that the incentive compatibility condition (3b) binds

$$D_0 = \left(R - \frac{B}{\beta - 1}\right) I_0. \tag{5}$$

We may now show that the proportion of deposits that the bank keeps in liquid assets is given by the following result.

Proposition 1. *Under full information and no central bank intervention, the liquidity ratio* $\frac{l_0}{D_0}$ *that the bank opts to maintain is given by*

$$\frac{l_0}{D_0} = 1 - \frac{1}{\left(R - \frac{B}{\beta-1}\right)} + \frac{R\sqrt{\frac{1}{R^2} - C_1^2}}{\left(R - \frac{B}{\beta-1}\right)}, \tag{6}$$

where $C_1 \equiv \sqrt{\bar{\phi}^2 - \frac{2(2\bar{\phi}-1)}{R}}$.

Proof. See the appendix.

Proposition 1 offers a consistent benchmark for comparison with the regulatory case under asymmetric information that we examine next. Moreover, under full information, any voluntary liquidity holdings can be considered *spare liquidity* in the presence of bank

Table 1. Depositors' Payoff Structure

	No Liquidity Shortfall	Liquidity Shortfall and LOLR	Liquidation
Withdraw	$\pi(1-k)$	$\pi(1-k)$	$\pi(1-k)$
Roll Over	π	$\pi(1-k)$	0

capital A, deposits D_0, and an optimal amount of loan investment I_0. Thus, a stock of liquidity under the full-information benchmark plays solely a residual role, rather than serving a deeper economic purpose, relating to the fact that for a given level of capital there are diminishing expected returns on investment since this increases the probability of default. In the following section, we relax the assumption of perfect observability of ϕ by depositors. We then characterize the liquidity shock that hits the bank at $t = 1$ in terms of depositors' equilibrium strategy following the realization of their private signals.

4. Liquidity Shock under Asymmetric Information

Let us assume that at $t = 1$ depositors observe private signals $s_i = \widetilde{\phi} + \widetilde{\varepsilon}_i$, where $\{\widetilde{\varepsilon}_i\}$ are i.i.d. innovations with uniform priors $U(-\varepsilon, +\varepsilon)$. To keep matters simple, we assume that depositors' payoffs are given exogenously in a way that captures one's incentive to run or stay with the bank if others act in the same manner. In particular, depositors receive a bonus π if they roll over and the bank faces no liquidity shortfall. But they receive a reduced bonus in the case of premature withdrawal of deposits for whatever reason, or if they fail to foresee a liquidity shortfall and shift their deposits elsewhere. Finally, in the case of bank liquidation, depositors who did not run lose their total bonus. See table 1.

Having observed signals $\{s_i\}$, depositors are assumed to follow a trigger strategy s^* that is defined as follows.

DEFINITION 1. *A trigger strategy s^* is a rule of action that maps the realization of a depositor's signal s_i to one of the following actions: to withdraw if signal s_i is less than s^*, or to roll over if s_i is greater than or equal to s^*.*

Given the above structure of payoffs, it is easy to verify that a depositor's incentive to withdraw or to roll over increases with the proportion of other depositors undertaking the same action. That is, payoffs satisfy global strategic complementarities and, as shown by Morris and Shin (2002), if a trigger strategy s^* exists and is unique, then it is the only dominant solvable equilibrium strategy.[5] In the remainder of this section, we aim to describe the liquidity shock to a bank, defined as the proportion of depositors that withdraw given ϕ. A uniquely determined s^* allows us to characterize this shock not only in terms of the realized value of the systemic shock ϕ but also in terms of the general economic environment, such as the extent of prudential liquidity maintained by the bank, the central bank's LOLR policy ϕ^{**}, and the degree of noise ε in depositors' signals. Assuming that all model parameters, the LOLR policy ϕ^{**}, and the prior distribution of ϕ are common knowledge, and that the realized sample distribution of depositors is the common distribution of their signals $\{s_i\}$, we prove the following result.

LEMMA 1. *The critical level of systemic shock ϕ^* below which the bank faces a liquidity shortfall is given by*

$$\phi^* = s^* + \varepsilon\left(1 - 2\frac{l}{D}\right). \tag{7}$$

Proof. See the appendix.

We observe that ϕ^* decreases in the ratio of liquid assets to deposits and increases in the strategy s^* that is followed by depositors. ϕ^* also increases with the extent of asymmetric information ε among depositors, implying that the higher the dispersion of beliefs about ϕ, the more likely it is for the bank to face a liquidity shortfall. With ϕ^* in hand, we may now solve for the depositors' equilibrium trigger strategy.

[5] In other words, s^* is the only strategy that survives the iterated deletion of strictly dominated strategies. That typically requires non-empty upper and lower dominance regions—namely, the existence of a level of fundamentals above (below) which *all* depositors accept (refuse) to roll over.

LEMMA 2. *Depositors' equilibrium trigger strategy* s^* *is given by*

$$s^* = \phi^{**} + \varepsilon \left(1 - 2\frac{k}{1-k}\frac{l}{D}\right), \tag{8}$$

where ϕ^{**} *characterizes the central bank's LOLR policy and* $\frac{l}{D}$ *is the ratio of liquid assets to deposits that is maintained by the bank.*

Proof. See the appendix.

Equilibrium strategy s^* essentially captures depositors' reaction to news about $\widetilde{\phi}$. The lower the s^*, the less sensitive depositors are to bad news and vice versa. Equation (8) implies that the higher the extent of asymmetric information about $\widetilde{\phi}$, the less willing depositors are to roll over. Moreover, depositors' willingness to roll over increases with the bank's liquidity ratio $\frac{l}{D}$ and the extent of the central bank's readiness to extend the LOLR provision, as captured by ϕ^{**}.

There are also levels ϕ_U and ϕ_L of the systemic shock such that all depositors roll over if $\widetilde{\phi} \geq \phi_U$, or withdraw if $\widetilde{\phi} \leq \phi_L$. The range $[\phi_U, \bar{\phi}]$ is referred to as the *upper dominance region*, corresponding to realizations of fundamentals that are high enough to prevent any information-induced deposit withdrawals. Similarly, the range $[1 - \bar{\phi}, \phi_L]$ is the *lower dominance region*, where fundamentals are very weak and there is a massive outflow of deposits from the bank. It is easy to show that ϕ_U and ϕ_L are given by

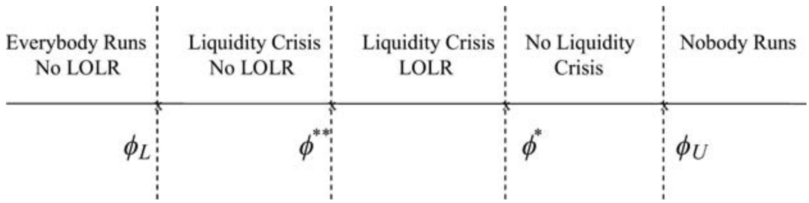
$$\phi_U = \phi^{**} + 2\varepsilon \left(1 - \frac{k}{1-k}\frac{l}{D}\right) \tag{9}$$

$$\phi_L = \phi^{**} - 2\varepsilon \frac{k}{1-k}\frac{l}{D}. \tag{10}$$

Conditional on the realization ϕ of the systemic shock, depositors' signals are drawn independently from a uniform distribution with support $[\phi - \varepsilon, \phi + \varepsilon]$. Then, for small values of $\varepsilon > 0$,[6] the standard global-game argument applies where depositors roll over their funds

[6]For a uniform prior in the real line, size limits for ε are dictated by the bounded support of $\widetilde{\phi}$ in the interval $[1 - \bar{\phi}, \bar{\phi}]$. So, we assume that $2\varepsilon < \min\{\phi_L - (1 - \bar{\phi}), \bar{\phi} - \phi_U\}$, where ϕ_U and ϕ_L are given by equations (9) and (10). We can easily show that such a condition holds if $2\varepsilon < \frac{1}{R} min\{R - \frac{B}{\beta - 1}, \bar{\phi}R - 1\}$.

Figure 1. Event Line Depending on Realization of $\widetilde{\phi}$



if and only if $s_i \geq s^*$, and the event line of the model looks as shown in figure 1.

For values of ϕ within the interval $[\phi_L, \phi_U]$, lemmas 1 and 2 imply that the proportion ρ of depositors who withdraw—the liquidity shock—is given by

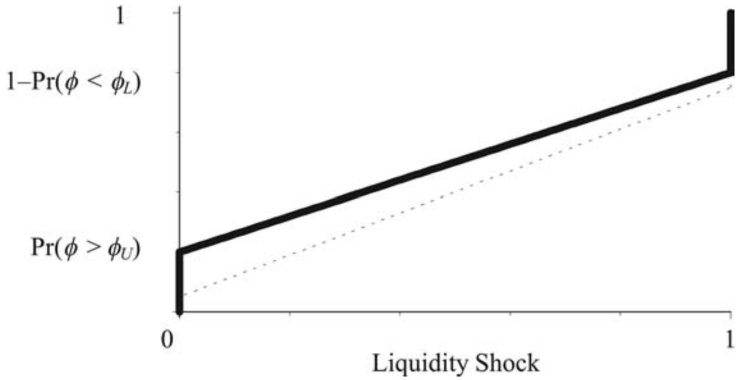
$$\rho(\widetilde{\phi}) = \frac{\phi^{**} + 2\varepsilon\left(1 - \frac{k}{1-k}\frac{l}{D}\right) - \widetilde{\phi}}{2\varepsilon}. \tag{11}$$

We observe that the liquidity shock $\widetilde{\rho}$ increases in ϕ^{**}; i.e., the more partial the LOLR insurance, the higher the proportion of depositors who run. Moreover, $\widetilde{\rho}$ decreases in the ratio of liquid assets to deposits, given that the higher the stock of liquidity, the more confident depositors are that the bank will not face a liquidity shortfall.

Finally, the uncertainty about the size of the liquidity shock $\widetilde{\rho}$ increases with the degree of asymmetric information among depositors, meaning that $\widetilde{\rho}$ becomes *riskier* the higher the ε. To see this, consider the probability distribution of $\widetilde{\rho}$, as implied by equations (9), (10), and (11). That distribution is uniform but with discrete jumps at 0 and 1 of size $\Pr(\phi > \phi_U)$ and $\Pr(\phi < \phi_L)$, respectively, as shown in figure 2 (bold line). In section 5.2 we prove that ϕ^{**} increases with ε, which implies that ϕ_U and ϕ_L also increase with ε. That said, an increase in ε leads to a decrease in $\Pr(\phi > \phi_U)$ and $\Pr(\phi < \phi_L)$ and, as a result, a downward shift in the distribution of $\widetilde{\rho}$ (dotted line in figure 2), meaning that $\widetilde{\rho}$ becomes riskier in a first-order stochastic-dominance sense.

In terms of welfare implications, recognizing that $\widetilde{\rho}$ becomes riskier with increasing information asymmetry is important because the riskiness of $\widetilde{\rho}$ will have an impact on the trade-off between the benefits of LOLR insurance and the costs of liquidity regulation. That is discussed in more detail in section 6.

Figure 2. Probability Distribution of Liquidity Shock ($\widetilde{\rho}$)



5. Regulatory Contract

For the purposes of our analysis, the regulatory contract provides for an optimal LOLR policy and a ratio of liquid assets to deposits that needs to be maintained by the bank. As far as the LOLR policy is concerned, we need to solve for an intervention threshold ϕ^{**} that maximizes the expected surplus from bank lending, conditional on a zero expected cost of LOLR intervention. Such a ϕ^{**} corresponds to an optimal amount of loan investment by the bank, which is then implemented by a requirement for the bank to maintain a certain ratio of liquid assets to deposits.

The central bank's optimization problem at $t = 0$ can be written as follows:

$$\max_{\phi^{**}} U(\phi^{**}) \tag{12a}$$

subject to

$$\int_{\phi^*}^{\bar{\phi}} \phi(RI - D)dF(\phi) \geq \int_{\phi^*}^{\bar{\phi}} \frac{\phi}{\beta}(RI - D + BI)dF(\phi) \tag{12b}$$

$$\int_{\phi^{**}}^{\phi^*} [\phi RI - (1 - \rho(\phi))D]dF(\phi) \geq \int_{\phi^{**}}^{\phi^*} \left(\rho(\phi) - \frac{l}{D}\right) DdF(\phi) \tag{12c}$$

$$I + l = A + D. \tag{12d}$$

Inequality (12b) is the bank's incentive compatibility condition, (12c) is the central bank's breakeven condition, and (12d) is the budget constraint of the bank. Also, $\rho(\phi)$ is the proportion of depositors who run—the liquidity shock; l is the amount of liquid assets; and A is the amount of capital that the bank holds. In order to determine the central bank's optimal LOLR policy, we first need to establish what is the optimal amount of loan investment $I(\phi^{**})$ for a given LOLR policy ϕ^{**}.

5.1 Optimal Loan Investment

As with the no-regulation benchmark, the amount of deposits D is set before the bank's choice of its optimal investment and is such that the bank has no incentive to engage in excess risk taking for a private benefit B. Thus, the incentive compatibility condition (12b) binds and simplifies as

$$D = \left(R - \frac{B}{\beta - 1}\right) I. \tag{13}$$

We note that the ratio of loans to deposits is fixed and equal to $(R - \frac{B}{\beta-1})^{-1}$ regardless of the choice of investment amount I. Given also positive returns on investment and the fact that the central bank makes no profits, the breakeven condition (12c) of the central bank also binds and can be written as

$$\frac{\phi^* + \phi^{**}}{2} RI = D - l. \tag{14}$$

We are now able to prove the following result.

LEMMA 3. *For a given LOLR policy ϕ^{**}, the amount of investment $I(\phi^{**})$ that satisfies simultaneously the bank's incentive compatibility condition (13), the central bank's breakeven condition (14), and the bank's budget constraint (12d) is given by*

$$I(\phi^{**}) = \frac{A\left(\frac{1}{R} - \frac{a\varepsilon}{R - \frac{B}{\beta-1}}\right)}{(C_2 - \phi^{**})}, \tag{15}$$

where $C_2 \equiv \frac{1}{R} - \varepsilon\frac{a + (1-a)\left(R - \frac{B}{\beta-1}\right)}{\left(R - \frac{B}{\beta-1}\right)}$ *and* $a \equiv \frac{1}{1-k}$.

Proof. See the appendix.

Having calculated the bank's optimal loan investment for a given LOLR policy ϕ^{**}, we may now evaluate the optimal LOLR policy ϕ^{**} that maximizes the expected surplus from the bank's loan investment.

5.2 LOLR Policy

Lemma 3 provides the optimal investment $I(\phi^{**})$ by the bank for a given LOLR policy ϕ^{**}. With $I(\phi^{**})$ in hand, the optimal LOLR policy maximizes the expected surplus from the bank's investment and is given by the following result.

PROPOSITION 2. *The central bank's optimal LOLR policy is to bail out the bank if and only if the level of systemic shock $\widetilde{\phi}$ is such that $\widetilde{\phi} \geq \phi^{**}$, where*

$$\phi^{**} = C_2 - \sqrt{C_2^2 - C_1^2}, \tag{16}$$

where $C_1 \equiv \sqrt{\bar{\phi}^2 - \frac{2(2\bar{\phi}-1)}{R}}$ *and* $C_2 \equiv \frac{1}{R} - \varepsilon \frac{a+(1-a)(R-\frac{B}{\beta-1})}{(R-\frac{B}{\beta-1})}$, *with* $a \equiv \frac{1}{1-k}$.

Proof. See the appendix.

An optimal LOLR policy ϕ^{**} could also be considered in terms of the induced probability of a liquidity shortfall at the optimum. In fact, lemmas 1 and 2 imply a one-to-one mapping from the central bank's optimal intervention threshold ϕ^{**} to the critical level of systemic shock ϕ^* below which a liquidity shortfall occurs. Consequently, by choosing an LOLR policy ϕ^{**}, the central bank implicitly induces a certain probability of liquidity shortfall in equilibrium. While this model is stylized, what this optimal policy captures is a balancing act whereby the central bank has to trade off welfare benefits of the official safety net against the intervention cost to the central bank. That is consistent with the principle of *proportionality*, which is widely used in the political debate about the extent and intensity of actions by the official sector.[7]

[7]Under the proportionality principle, the content and form of actions by the official sector shall not exceed what is necessary for the official sector to achieve its

5.3 Prudential Liquidity Ratio

Having established what is the optimal LOLR response to a liquidity shortfall by the bank, the corresponding optimal loan investment $I(\phi^{**})$ can be implemented by means of prudential liquidity regulation, i.e., by requiring the bank at $t = 0$ to maintain a ratio of liquid assets to deposits that is given by the following result.

PROPOSITION 3. *Under the optimal regulatory contract, the ratio of liquid assets to deposits that implements the optimal loan investment by the bank is given by the following expression:*

$$\frac{l}{D} = 1 - \frac{1}{\left(R - \frac{B}{\beta-1}\right)} + \frac{R\sqrt{C_2^2 - C_1^2}}{\left(R - \frac{B}{\beta-1} - aR\varepsilon\right)}, \tag{17}$$

where $C_1 \equiv \sqrt{\bar{\phi}^2 - \frac{2(2\bar{\phi}-1)}{R}}$ *and* $C_2 \equiv \frac{1}{R} - \varepsilon\frac{a+(1-a)(R-\frac{B}{\beta-1})}{(R-\frac{B}{\beta-1})}$, *with* $a \equiv \frac{1}{1-k}$. *Such a ratio is higher than what the bank would voluntarily maintain with no official-sector involvement.*

Proof. See the appendix.

Consequently, in the presence of bank funding constraints, prudential liquidity requirements imply an opportunity cost of funds, which is basically the price to be paid for quid pro quo LOLR insurance. We may also notice that the liquidity requirements increase with the extent of asymmetric information among depositors, which is not surprising given that the riskiness of the liquidity shock, as discussed in section 4, also increases. The question then becomes, Under what circumstances is it worth bearing such a cost? Or, to put it differently, Under what conditions is combining prudential liquidity regulation with LOLR insurance, if at all, welfare improving of the laissez-faire regime? The following section reflects on those questions by identifying a set of conditions under which prudential liquidity is warranted from a welfare perspective.

policy objective. In our case, such an official-sector objective is the maximization of the surplus from financial intermediation, conditional on avoiding systematic losses under the LOLR facility.

6. Welfare Analysis

From section 5, the expected surplus U of the bank's investment under the optimal regulatory contract is given by

$$U = \frac{A}{2(2\bar{\phi}-1)} \frac{\left(C_1^2 - \phi^{**2}\right)}{\left(C_2 - \phi^{**}\right)} \left(1 - \frac{aR}{R - \frac{B}{\beta-1}} \varepsilon\right). \tag{18}$$

By substituting the optimal LOLR policy ϕ^{**} from proposition 2 into equation (18), we get

$$U = \frac{A}{2\bar{\phi}-1} \left(C_2 - \sqrt{C_2^2 - C_1^2}\right) \left(1 - \frac{aR}{R - \frac{B}{\beta-1}} \varepsilon\right). \tag{19}$$

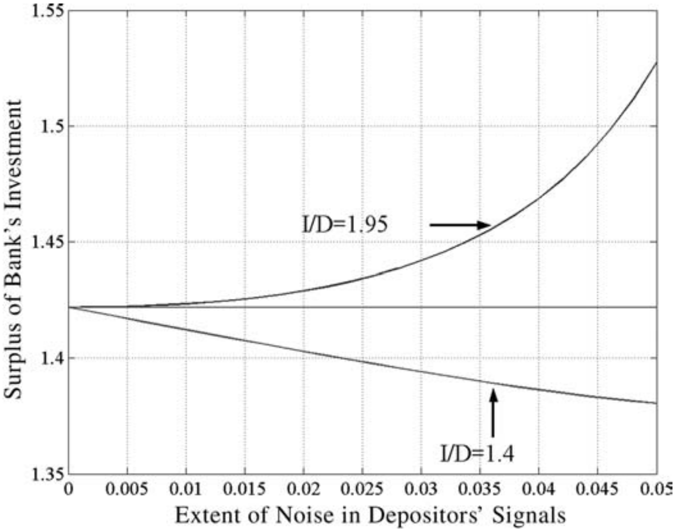
Similarly, from section 3, the expected surplus U_0 of the bank's investment, absent any official-sector involvement, is given by

$$U_0 = \frac{A}{2\bar{\phi}-1} \left[\frac{1}{R} - \sqrt{\frac{1}{R^2} - C_1^2}\right]. \tag{20}$$

From equations (19) and (20) it follows that, as asymmetric information among depositors dissipates, the expected surplus of the bank's investment under the optimal regulatory contract tends toward its level under the no-regulation benchmark. However, for $\varepsilon > 0$, liquidity regulation may or may not lead to a welfare improvement of the no-regulation case. As we show next, that depends on the extent of the bank's funding constraints as measured by the ratio of loan investments to deposits. In figure 3, for example, we plot the expected surplus of loan investments against different values of ε and for a ratio of loans to deposits equal to 1.4 and 1.95, while the horizontal line corresponds to the expected surplus under the no-regulation benchmark.

As figure 3 shows, prudential liquidity regulation may become socially desirable the more debt constrained the banking sector is, i.e., the higher the ratio of loans to deposits. Otherwise, a requirement for the bank to hoard liquidity may be too costly, even after taking into account the social value of the LOLR safety net. Next,

Figure 3. Welfare Implications of Liquidity Regulation



we derive a threshold for the ratio of loans to deposits above which liquidity requirements, quid pro quo for LOLR insurance, lead to a welfare improvement of the no-regulation case.

PROPOSITION 4. *For small values of* ε, *liquidity regulation improves welfare, relative to the no-regulation benchmark, if and only if the bank's loans-to-deposits ratio is such that*

$$\frac{I}{D} > \frac{a-1}{a\left(1-\sqrt{1-R^2C_1^2}\right)}, \tag{21}$$

where $C_1 \equiv \sqrt{\bar{\phi}^2 - \frac{2(2\bar{\phi}-1)}{R}}$ *and* $a \equiv \frac{1}{1-k}$.

Proof. See the appendix.

Proposition 4 results from a trade-off between the benefit of LOLR insurance and the cost of hoarding liquid assets for regulatory purposes. Whatever raises the desirability of LOLR activity, it also makes liquidity regulation more or less desirable ex ante. As in Rothschild and Stiglitz (1970, 1971), liquidity insurance becomes more valuable the riskier the liquidity shock from premature withdrawal of deposits. The higher the extent of asymmetric information ε among depositors, the riskier the liquidity shock and, as a result, the more valuable the LOLR insurance. Also, the value of LOLR insurance increases with the prima facie incentive of depositors to withdraw, which is captured by parameter k in depositors' payoffs. The lower the value of k, the less stable the deposit base and the easier it is for (21) to be satisfied. By the same token, the higher the potential return R on investment, the easier it is for (21) to hold and the higher the benefit of LOLR insurance relative to the cost of liquidity regulation.

We also stated in section 5.1 that the ratio of loans to deposits is fixed and depends on parameters B and β, which determine the bank's moral hazard problem. The stronger the moral hazard incentive of the bank—i.e., the higher the B and the lower the β—the lower the amount of deposits that the bank can raise relative to assets, which makes it easier for (21) to hold and for liquidity regulation to be welfare improving. To put it differently, when moral hazard incentives are strong, the ratio of external to internal funding of the bank is low. Proposition 4 can then be interpreted in terms of the bank's *leverage ratio*—namely, the ratio of deposits to bank's equity. By substituting equations (12d) and (17) into (21), it turns out that prudential liquidity restrictions, accompanied by an optimal LOLR policy, are welfare improving if the leverage ratio is sufficiently low.

The intuition that underlies this point is subtle: Given that the marginal expected return on investment decreases with the total amount of loan investment since this increases the probability of default, low leverage implies weak incentives to undertake positive-NPV investments. But LOLR insurance increases the marginal expected return on investment and, as a result, the bank's marginal propensity to invest in positive-NPV projects. That makes liquidity regulation more likely to be welfare improving exactly when the leverage ratio of the banking sector is low. Lower bank leverage also implies a higher level of bank capital relative to investment. But the higher the capitalization of the banking sector, the higher the cushion against losses, which also offers some leeway to the central bank to extend the LOLR insurance without violating its budget constraint. That said, the lower the leverage ratio—or the higher the capital cushion—the lower the liquidity requirements and, as a result, the more likely it is for central bank intervention to be warranted.

7. Conclusion

We presented a model of information-based bank runs where prudential liquidity requirements of the banking sector allow the central bank to run a balanced budget under its LOLR facility. In the presence of bank funding constraints and diverse beliefs among depositors about the quality of bank loans, it was shown that optimal liquidity regulation implies a ratio of liquid assets to deposits that is higher than voluntary liquidity holdings, absent any official-sector involvement. Such a higher liquidity ratio is to enable the banking sector to meet liquidity shocks from its own resources and up to a certain level of confidence. In other words, we viewed stock liquidity requirements as serving as a first line of defense against deposit withdrawals, allowing the central bank to maintain a zero expected cost of LOLR intervention, while counteracting any excess risk taking by the bank due to the existence of LOLR insurance. It was shown that prudential liquidity is warranted, from a welfare perspective, if the banking sector is characterized by high-profit opportunities, low leverage, and a relatively volatile deposit base (e.g., interbank funding, as opposed to funding through core deposits). Otherwise, stock liquidity becomes too costly, even after accounting for the social value of LOLR insurance.

Appendix

Proof of Proposition 1. Equation (2) implies that the net expected return $m(I_0)$ per unit of investment is

$$m(I_0) = \frac{R}{2(2\bar{\phi} - 1)} \left[C_1^2 - \left(\frac{I_0 - A}{RI_0} \right)^2 \right], \tag{22}$$

where $C_1 \equiv \sqrt{\bar{\phi}^2 - \frac{2(2\bar{\phi}-1)}{R}}$. Then, the bank's expected utility U_0 from investment I_0 is given by

$$U_0 = m(I_0)I_0. \tag{23}$$

Given (22), (23) can be written as

$$U_0 = \frac{R}{2(2\bar{\phi} - 1)} \left[C_1^2 - \left(\frac{I_0 - A}{RI_0} \right)^2 \right] I_0, \tag{24}$$

where the amount of investment I_0 that maximizes U_0 in (24) solves

$$I_0 = \frac{A}{R\sqrt{\frac{1}{R^2} - C_1^2}}. \tag{25}$$

Given that $I_0 + l_0 = A + D_0$, (3b) and (25) imply that the amount of liquid assets l_0 that the bank would hold voluntarily is given by

$$l_0 = \left[1 - \frac{1}{\left(R - \frac{B}{\beta - 1} \right)} + \frac{R\sqrt{\frac{1}{R^2} - C_1^2}}{\left(R - \frac{B}{\beta - 1} \right)} \right] \left(R - \frac{B}{\beta - 1} \right) I_0. \tag{26}$$

Finally, from (3b) and (26), the ratio of liquid assets to deposits under no official-sector involvement is

$$\frac{l_0}{D_0} = 1 - \frac{1}{R - \frac{B}{\beta - 1}} + \frac{R\sqrt{\frac{1}{R^2} - C_1^2}}{\left(R - \frac{B}{\beta - 1} \right)}. \tag{27}$$

Proof of Lemma 1. Conditional on systemic shock ϕ, depositors' signals $\{s_i\}$ are i.i.d. uniform with support on $[\phi - \varepsilon, \phi + \varepsilon]$. Given ϕ and depositors' equilibrium strategies s^*, the proportion of withdrawn deposits is equal to the probability that a signal s_i is lower than s^*:

$$\Pr(s_i \leq s^* | \phi) = \frac{s^* - \phi + \varepsilon}{2\varepsilon}. \tag{28}$$

From (28), the critical level of shock ϕ^* solves

$$\phi^* = s^* + \varepsilon \left(1 - 2\frac{l}{D} \right). \tag{29}$$

Proof of Lemma 2. Conditional on signal $s_i = s^*$, let P_{00} be the probabilities of no liquidity shortfall and P_{10} be the probability of liquidity shortfall and LOLR intervention. P_{00} and P_{10} are given by

$$P_{00} = \frac{s^* + \varepsilon - \phi^*}{2\varepsilon}. \tag{30}$$

By substituting ϕ^* from lemma 1 into equation (30), we get

$$P_{00} = \frac{l}{D}, \tag{31}$$

where l is the amount of liquid assets held by the bank and D is the total amount of deposits. Similarly, given ϕ^* and ϕ^{**}, P_{10} is given by

$$P_{10} = \frac{\phi^* - \phi^{**}}{2\varepsilon}. \tag{32}$$

With P_{00} and P_{10} in hand, depositors' equilibrium in trigger strategies s^* is such that a depositor who observes a signal $s_i = s^*$ is indifferent between withdrawing and rolling over; i.e., s^* solves

$$\pi P_{00} + \pi(1-k)P_{10} = \pi(1-k). \tag{33}$$

The left-hand side of (33) is the expected payoff from rolling over, conditional on s^*, while the right-hand side is the (certain) payoff from withdrawing. By substituting P_{00} and P_{10} from (31) and (32) into (33), we get

$$\frac{l}{D}\pi + \frac{\phi^* - \phi^{**}}{2\varepsilon}\pi(1-k) = \pi(1-k). \tag{34}$$

Then, by substituting ϕ^* from lemma 1 into equation (34), we derive s^*:

$$s^* = \phi^{**} + \varepsilon\left(1 - 2\frac{k}{1-k}\frac{l}{D}\right). \tag{35}$$

Proof of Lemma 3. By substituting (12d) into (14), we get

$$\frac{\phi^* + \phi^{**}}{2}RI = I - A. \tag{36}$$

From lemmas 1 and 2, the term $\frac{\phi^*+\phi^{**}}{2}$ on the left-hand side of (36) can be written as

$$\frac{\phi^* + \phi^{**}}{2} = \phi^{**} + \varepsilon\left(1 - \frac{1}{1-k}\frac{l}{D}\right). \tag{37}$$

By substituting (37) into (36) and setting $a \equiv \frac{1}{1-k}$, the central bank's breakeven condition becomes

$$\left[\phi^{**} + \varepsilon\left(1 - a\frac{l}{D}\right)\right] RI = I - A. \quad (38)$$

From the incentive compatibility condition (13) and budget constraint (12d), (38) becomes

$$(C_2 - \phi^{**})I = A\left[\frac{1}{R} - \frac{a\varepsilon}{R - \frac{B}{\beta-1}}\right], \quad (39)$$

where $C_2 \equiv \frac{1}{R} - \varepsilon\frac{a+(1-a)(R-\frac{B}{\beta-1})}{(R-\frac{B}{\beta-1})}$ and $a \equiv \frac{1}{1-k}$. Then, equation (39) implies that for a given ϕ^{**}, the amount of investment $I(\phi^{**})$ that satisfies simultaneously constraints (12b), (12c), and (12d) solves

$$I(\phi^{**}) = \frac{A\left(\frac{1}{R} - \frac{a\varepsilon}{R-\frac{B}{\beta-1}}\right)}{(C_2 - \phi^{**})}. \quad (40)$$

Proof of Proposition 2. For a given LOLR policy ϕ^{**}, the bank's net expected return per unit of investment is given by

$$m(\phi^{**}) = [1 - F(\phi^{**})]\frac{\bar{\phi} + \phi^{**}}{2}R - 1, \quad (41)$$

which can be restated as

$$m(\phi^{**}) = \frac{R}{2(2\bar{\phi} - 1)}\left(C_1^2 - \phi^{**2}\right), \quad (42)$$

where parameter $C_1 \equiv \sqrt{\bar{\phi}^2 - \frac{2(2\bar{\phi}-1)}{R}}$. Given (42), the central bank's optimization problem (12a) is equivalent to the following unconstrained problem:

$$\max_{\phi^{**}} U(\phi^{**}), \quad (43)$$

where $U(\phi^{**}) = m(\phi^{**})I(\phi^{**})$, $I(\phi^{**})$ is given by lemma 3, and $m(\phi^{**})$ is the net expected return per unit of investment. Given (42), (43) can be expressed as

$$\max_{\phi^{**}} \frac{A}{2(2\bar{\phi}-1)} \frac{\left(C_1^2 - \phi^{**2}\right)}{(C_2 - \phi^{**})} \left(1 - \frac{a\varepsilon R}{R - \frac{B}{\beta-1}}\right), \tag{44}$$

where $C_1 \equiv \sqrt{\bar{\phi}^2 - \frac{2(2\bar{\phi}-1)}{R}}$, $C_2 \equiv \frac{1}{R} - \varepsilon \frac{a+(1-a)(R-\frac{B}{\beta-1})}{(R-\frac{B}{\beta-1})}$, and $a \equiv \frac{1}{1-k}$. However, maximizing the expression in (44) is equivalent to maximizing the following expression of ϕ^{**}:

$$f(\phi^{**}) = C_2 + \phi^{**} - \frac{C_1^2 - C_2^2}{\phi^{**} - C_2}. \tag{45}$$

The first derivative of $f(\cdot)$ with respect to ϕ^{**} is

$$\frac{\partial f(\phi^{**})}{\partial \phi^{**}} = 1 + \frac{C_1^2 - C_2^2}{(C_2 - \phi^{**})^2}, \tag{46}$$

while the second derivative is given by

$$\frac{\partial^2 f(\phi^{**})}{\partial \phi^{**2}} = 2\frac{C_1^2 - C_2^2}{(C_2 - \phi^{**})^3}. \tag{47}$$

We consider two cases: (i) $C_2 \leq C_1$ and (ii) $C_2 > C_1$. However, we can easily show that $C_2 \leq C_1$ holds if and only if the bank's loans-to-deposits ratio is such that $\frac{I}{D} \geq \frac{1-(C_1-\varepsilon)R}{2R\varepsilon}$, which for small values of ε implies a high value of $\frac{I}{D}$. Nevertheless, given that the loans-to-deposits ratio under both the regulation and the no-regulation case is equal to $(R - \frac{B}{\beta-1})^{-1}$, the case where $C_2 \leq C_1$ can be easily ruled out by the fact that the ratio of liquid assets to deposits, as given by proposition 1, is non-negative. Thus, the only relevant case to consider is $C_2 > C_1$, under which (47) becomes negative and the optimal LOLR policy ϕ^{**} is given by

$$\phi^{**} = C_2 - \sqrt{C_2^2 - C_1^2}. \tag{48}$$

Proof of Proposition 3. Given $I + l = A + D$, the liquidity ratio $\frac{l}{D}$ can be expressed as

$$\frac{l}{D} = \frac{D + A - I}{D}. \tag{49}$$

By substituting (13) and (40) into (49), the ratio of liquid assets to deposits is given by

$$\frac{l}{D} = 1 - \frac{1}{R - \frac{B}{\beta - 1}} + \frac{R(C_2 - \phi^{**})}{\left(R - \frac{B}{\beta-1} - aR\varepsilon\right)}, \tag{50}$$

where $C_2 \equiv \frac{1}{R} - \varepsilon \frac{a + (1-a)(R - \frac{B}{\beta - 1})}{(R - \frac{B}{\beta-1})}$ and $a \equiv \frac{1}{1-k}$. Then, by substituting ϕ^{**} from proposition 2 into equation (50), the optimal liquidity ratio is given by

$$\frac{l}{D} = 1 - \frac{1}{\left(R - \frac{B}{\beta-1}\right)} + \frac{R\sqrt{C_2^2 - C_1^2}}{\left(R - \frac{B}{\beta-1} - aR\varepsilon\right)}. \tag{51}$$

Moreover, from proposition 1 and equation (51), the ratio of liquid assets to deposits under the optimal regulatory contract is higher than under no official-sector involvement if and only if

$$\frac{\sqrt{\left[\frac{1}{R} - \varepsilon \frac{a + (1-a)\left(R - \frac{B}{\beta-1}\right)}{\left(R - \frac{B}{\beta-1}\right)}\right]^2 - C_1^2}}{\left(R - \frac{B}{\beta-1} - aR\varepsilon\right)} > \frac{\sqrt{\frac{1}{R^2} - C_1^2}}{\left(R - \frac{B}{\beta-1}\right)}. \tag{52}$$

By recasting (52) we derive the following inequality:

$$\left(\varepsilon - \frac{K}{aR}\right)^2 < \frac{1}{(1-L)}\left(\frac{a + (1-a)K}{a}\right)^2 \times \left(\varepsilon - \frac{K}{R[a + (1-a)K]}\right)^2 - \frac{K^2 L}{a^2 R^2 (1-L)}, \tag{53}$$

where $K \equiv R - \frac{B}{\beta-1}$, $L \equiv (RC_1)^2$, and $C_1 \equiv \sqrt{\bar{\phi}^2 - \frac{2(2\bar{\phi}-1)}{R}}$. With respect to ε, the geometric loci defined by the left-hand side and right-hand side of (53) are parabolas. The parabola defined by the left-hand side of (53) has its vertex at the point $[\frac{K}{aR}, 0]$, and its focal parameter is $p_{LHS} = \frac{1}{2}$. Similarly, the parabola defined by the right-hand side of (53) has its vertex at the point $[\frac{K}{R[a+(1-a)K]}, -\frac{K^2 L}{a^2 R^2 (1-L)}]$, and its focal parameter is $p_{RHS} = \frac{(1-L)}{2}\left[\frac{a}{a+(1-a)K}\right]^2$. From analytical geometry we know that if the focal parameter p of a parabola is positive, then the parabola faces upward. That is definitely the case for the left-hand side of (53)—i.e., $\frac{1}{2} > 0$—while for the right-hand side of (53), that is true if and only if $1 - L > 0$, or $C_1 < \frac{1}{R}$. But, given $C_2 > C_1$ and $C_2 < \frac{1}{R}$, it also follows that $C_1 < \frac{1}{R}$. Consequently, the parabola defined by the right-hand side of (53) is also facing upward. Finally, we observe that both parabolas intersect at $[0, (\frac{K}{aR})^2]$ and their vertices lie to the right of their intersection point. Thus, for relatively small values of ε, a sufficient condition for (53) to hold is $0 > -\frac{K^2 L}{a^2 R^2 (1-L)}$, or $1 - LR > 0$. But this has already been shown to be true, implying that the optimal regulatory contract stipulates a higher liquidity ratio than what the bank would voluntarily maintain under no official-sector involvement.

Proof of Proposition 4. Given that $U|_{\varepsilon=0} = U_0$, a necessary and sufficient condition for $U(\varepsilon) > U_0$ for small values of ε is that $\frac{\vartheta U(\varepsilon)}{\vartheta \varepsilon}|_{\varepsilon=0} > 0$. With a bit of algebra, we get

$$\left.\frac{\partial U}{\partial \varepsilon}\right|_{\varepsilon=0} = \frac{A}{(2\bar{\phi}-1)} \frac{(1-\sqrt{1-L})}{K\sqrt{1-L}} \left[a(1-\sqrt{1-L}) + (1-a)K\right], \quad (54)$$

where $K \equiv R - \frac{B}{\beta - 1}$, $L \equiv R^2 C_1^2$, $C_1 \equiv \sqrt{\bar{\phi}^2 - \frac{2(2\bar{\phi}-1)}{R}}$, and $a \equiv \frac{1}{1-k}$. Then, a necessary and sufficient condition for $\frac{\vartheta U(\varepsilon)}{\vartheta \varepsilon}|_{\varepsilon=0} > 0$ is

$$\frac{I}{D} > \frac{a-1}{a\left(1-\sqrt{1-R^2 C_1^2}\right)}. \quad (55)$$

References

Buser, S. A., A. H. Chen, and E. J. Kane. 1981. "Federal Deposit Insurance, Regulatory Policy, and Optimal Bank Capital." *Journal of Finance* 36 (1): 51–60.

Dewatripont, M., and J. Tirole. 1994. *The Prudential Regulation of Banks*. Cambridge, MA: MIT Press.

Holmström, B., and J. Tirole. 1998. "Private and Public Supply of Liquidity." *Journal of Political Economy* 106 (1): 1–40.

Morris, S., and H. S. Shin. 2002. "Global Games: Theory and Applications." In *Advances in Economics and Econometrics*, Vol. 1, ed. M. Dewatripont, L. Hansen, and S. Turnovsky. Cambridge University Press.

Posner, R. A. 1971. "Taxation by Regulation." *Bell Journal of Economics and Management Science* 2 (1): 22–50.

Rochet, J.-C., and X. Vives. 2004. "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?" *Journal of the European Economic Association* 2 (6): 1116–47.

Rothschild, M., and J. E. Stiglitz. 1970. "Increasing Risk I: A Definition." *Journal of Economic Theory* 2 (3): 225–43.

———. 1971. "Increasing Risk II: Its Economic Consequences." *Journal of Economic Theory* 3 (1): 66–84.

Technology Diffusion within Central Banking: The Case of Real-Time Gross Settlement*

Morten L. Bech and Bart Hobijn
Federal Reserve Bank of New York

We examine the diffusion of the real-time gross settlement (RTGS) technology across the world's 174 central banks. RTGS reduces settlement risk and facilitates financial innovation in, for example, the settlement of foreign exchange trades. In 1985 only three central banks had implemented RTGS systems; by year-end 2006 that number had increased to ninety-three. We find that the RTGS diffusion process is consistent with a standard S-shaped curve. Real GDP per capita, the relative price of capital, and trade patterns explain a significant part of the cross-country variation in RTGS adoption. These determinants are remarkably similar to those that seem to drive cross-country adoption patterns of other technologies.

JEL Codes: C72, E58.

1. Introduction

Modern growth theory suggests that financial-sector development affects long-run economic growth. One of the most fundamental functions of the financial sector is to provide efficient mechanisms to make and receive payments. By reducing transaction costs, the payment system facilitates trade and allows greater specialization by agents in the economy. Over the past few decades, there has been a rapid increase in technological innovation in the financial sector.

*We thank Allison Fleming and Ariel Stern for research assistance. We are grateful for comments from Peter Allsopp and an anonymous referee. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System.
Corresponding author: Morten Bech, Research and Statistics Group, Federal Reserve Bank of New York, 33 Liberty Street, Third Floor, New York, NY 10045; Tel: (212) 720-6935; Fax: (212) 720-8363; E-mail: morten.bech@ny.frb.org.

Nowhere is such innovation more apparent than in the electronification of means of payment.

In most countries, the payment system is provided by commercial banks in a symbiotic partnership with the central bank. Central banks tend to provide the medium to settle small payments (i.e., cash) and to support an interbank system that settles large-value and time-critical payments. Historically, interbank payments have been settled via netting systems. In a netting system, the settlement of payments is deferred for some period of time, usually until the end of the business day. At the end of the deferment period, all payments are tallied up, and money is exchanged on either a bilateral or multilateral net basis among the participants. Commonly, this process is referred to as deferred net settlement (DNS).

The volume of interbank payments has increased dramatically over the last thirty years, mainly due to the aforementioned rapid financial innovation and the integration and globalization of financial markets. As the volume and value of transactions increased, central banks became worried about settlement risks inherent in netting systems. In particular, central banks were concerned about the potential for contagion (or even a systemic event) due to the unwinding of the net positions that would result if a participant failed to make good on its obligations when due. Consequently, over the last couple of decades, many countries have chosen to modify the settlement procedure employed by their interbank payment system with a view to reducing settlement risks and the potential for systemwide implications. Most central banks have opted for the implementation of a real-time gross settlement (RTGS) system. An RTGS system reduces settlement risk, as payments are settled individually and irrevocably on a gross basis in real time. This ensures immediate finality.

By 1985 three central banks had implemented an RTGS system. A decade later, that number had increased to sixteen, but RTGS was still utilized predominately by industrialized countries. In recent years, however, both transitional and developing countries have begun investing heavily in improving their financial systems, and now RTGS is a common choice for the interbank payment system. At the end of 2006, the use of RTGS systems had diffused to 93 of the world's 174 central banks.

In this paper, we describe the diffusion of RTGS systems within the central banking community and seek to identify the determinants of adoption by using a new data set on the implementation of RTGS systems. A small, but growing, literature analyzes how and why commercial banks adopt new technologies (e.g., Hannan and McDowell 1984, Gowrisankaran and Stavins 2004, and Akhavien, Frame, and White 2005). However, to our knowledge, no studies have focused on central bank adoption of new technologies, including RTGS systems.[1] This is surprising, given the key role the central bank plays in the financial sector. Moreover, the adoption decision by a central bank is potentially interesting in its own right, as it might be different from the profit considerations driving technology adoption in the private sector.

We analyze our data in two different ways. First, we consider the pattern of RTGS adoption over time and space. We estimate an S-shaped adoption curve that is a generalized version of those estimated by Griliches (1957) and find that, in all likelihood, it will take at least another fifteen years before RTGS is fully adopted. Second, we consider the main determinants of RTGS adoption using a discrete-time duration model, based on a logistic hazard rate. We find that the probability that a country adopts RTGS in a given year increases significantly with the level of real GDP per capita. Moreover, countries with a lower relative price of capital and countries whose major trading partners adopted RTGS are also more likely to adopt. This suggests that beyond market forces reflected by real GDP and capital costs, spillovers seem to play a significant role in the adoption of this financial innovation. These spillovers seem to disseminate mainly through trade relationships.

The structure of the rest of this paper is as follows. In the next section, we describe the role of the interbank payment systems as well as the features of netting and real-time gross settlement using the U.S. large-value payment system as an example. In section 3 we introduce our data on the adoption of RTGS, map out the

[1]Rogers (1995) defines diffusion as a process by which some type of innovation is communicated through certain channels over time and space among the members of a social system. An innovation is any kind of idea, object, or practice that members of the social system perceive as new.

diffusion of RTGS across the world, and study the S-shaped nature of the world RTGS diffusion curve. Based on the theoretical and empirical literature on technology adoption, we review the potential determinants of RTGS adoption in section 4, and we briefly discuss our econometric methodology in section 5. In section 6 we present our empirical results. We conclude with section 7. A more detailed description of our data can be found in the appendix.

2. Interbank Payment Systems and Real-Time Gross Settlement

At the apex of the financial system are a number of critical financial markets that provide the means for agents to allocate capital and manage their risk exposures. Instrumental to the smooth functioning of these markets is a set of financial infrastructures that facilitate clearing and settlement.[2] Many of these infrastructures use the interbank payment system to achieve final settlement. In addition, most central banks use the interbank payment system to implement monetary policy, and the system also serves as the platform for the interbank money market. An efficient and resilient interbank payment system reduces transaction costs for agents in the economy and is a precondition for the successful implementation of monetary policy and ensuring financial stability.

In the United States, there are two principal systems that settle interbank payments: the Federal Reserve's Fedwire Funds Transfer System® (Fedwire) and the Clearing House Inter-Bank Payments System (CHIPS), a private-sector enterprise. Commercial banks use Fedwire and CHIPS to handle large-value or time-critical payments, such as payments for the purchase, sale, and financing of securities transactions; foreign exchange transactions; the settlement of interbank purchases and sales of federal funds; the disbursement or repayment of loans; and the settlement of real

[2]Clearing is the process of transmitting and confirming payment orders or security transfers arising from market trades, as well as establishing, possibly by way of netting, final positions for settlement. Settlement is the act that discharges the obligations between two or more parties arising from the market trades. Settlement is final when it is irrevocable and unconditional. Ultimate settlement is sometimes used to denote final settlement in central bank money.

estate transactions and other large-value purchases on behalf of customers.[3]

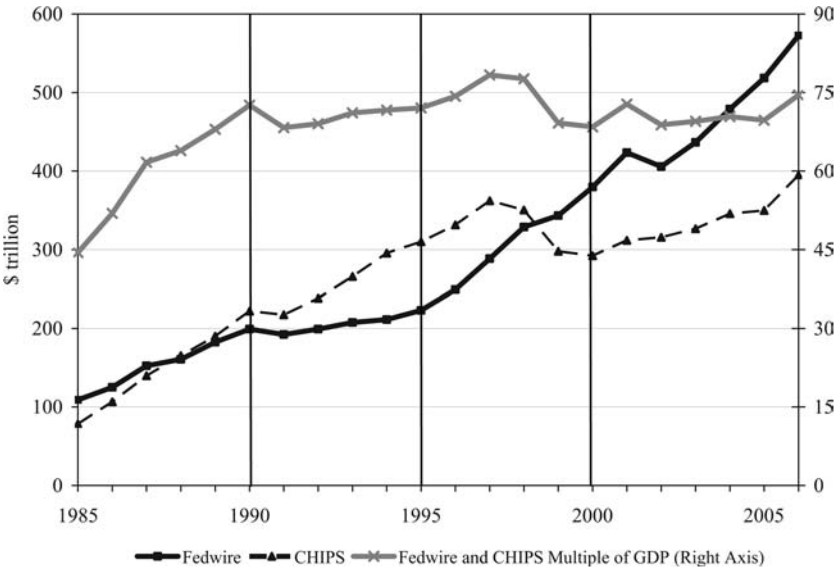
Through Fedwire, participants initiate funds transfers that are immediate, final, and irrevocable when processed. Like other systems with these key features, Fedwire is an RTGS system. In fact, Fedwire is the world's oldest RTGS system. Its origins go back to 1918, when the Federal Reserve inaugurated a network of wire communications among the individual Reserve Banks. The new system of wire-initiated book entries allowed funds to be transferred on behalf of the member banks and significantly reduced the need for physical shipment of gold and currency. In the early 1970s, the Fedwire system migrated to a fully computerized platform, and settlement in "real time" was achieved. During the first year of Fedwire operation, the Federal Reserve Bank of New York processed around 100 wires per day; ten years later, the Bank was processing about 600 wires per day. Today, more than 7,500 participants across the twelve Reserve Banks originate an average of over 530,000 transfers per day. The value of transfers originated has seen tremendous growth. The annual turnover increased from around $100 trillion in 1985 to over $570 trillion in 2006, equivalent to an annual growth rate of over 8 percent (see figure 1).

The New York Clearing House Association organized CHIPS in 1970 for eight of its members to settle the dollar side of foreign exchange transactions and other large-value payments. Participation in CHIPS expanded gradually in the 1970s and 1980s to include other commercial banks, branches of foreign banks, and other financial institutions. CHIPS began as a deferred net settlement system. Until 1981, final settlement occurred on the morning of the next business day through the transfer of balances across the books of the Federal Reserve Bank of New York. A sharp increase in settlement volumes raised concerns that next-day settlement unduly exposed participants to various overnight and overweekend risks. In response, CHIPS began providing same-day settlement through Fedwire in August 1981.

Netting substantially reduces the amount of money needed to settle a given set of obligations. As a rough rule of thumb, $100 in

[3]Several payment and securities settlement systems (including CHIPS) use Fedwire to square final positions over the course of the business day.

Figure 1. Value of Payments Settled on Fedwire and CHIPS



gross payments can be settled by an end-of-day net transfer of $1. A major drawback of (unprotected) netting systems, however, is the risk that a party will default on its payment obligations. If even one participant fails to meet its net obligation when due, all processed payment orders involving that participant may be unwound and, as a result, other participants may fail to make full on their own net obligations.[4] Such an event could conceivably set off a cascade of settlement failures that could disrupt the smooth functioning of the financial system.[5] According to Humphrey's (1986) simulation study of two randomly selected business days in January 1983, the failure of a major CHIPS participant, given the rules at the time, could have caused dozens of large banks to fail, which could have

[4]It would be particularly troublesome if funds were credited to customers' accounts by banks expecting final settlement at a later time. This could translate into a credit exposure for those banks that have a limited ability to reclaim such funds.

[5]See Selgin (2004) for a critique of this line of reasoning.

triggered a systemwide crisis.[6] Accordingly, in response to the wishes of both participants and regulators, CHIPS moved in 2001 to a new settlement system that provides intraday finality.[7]

The current version of CHIPS is referred to as a hybrid system, as it combines features of DNS and RTGS.[8] Currently, CHIPS has 45 participants (down from 142 in 1985) representing nineteen different countries. In 1985, CHIPS settled $78 trillion, which increased to $362 trillion in 1997. However, settlements over CHIPS dipped in the late 1990s, and it was not until 2006 (with $395 trillion settled) that the previous peak was surpassed. As illustrated in figure 1, settlements over CHIPS were larger than those over Fedwire from 1988 to 1998. Seen over the entire period, the annual growth rate is comparable to that of Fedwire at almost 8 percent. Measured relative to the economic output of the United States, Fedwire and CHIPS settled payments equivalent to seventy-five times the gross domestic product in 2006. In other words, payments equivalent to the U.S. GDP run through the two systems on average every three-and-a-half business days.

3. Diffusion of Real-Time Gross Settlement

The U.S. experience with interbank payment systems is different from that of most countries in the sense that there have been two systems operating for the last thirty-five years and that RTGS was implemented before DNS. In the rest of the world, there is typically only one large-value interbank payment system per country, and prior to the 1980s, that system settled payments using deferred net settlement. As interbank payment systems around the world saw growth in the value of payments similar to that experienced by the United States, settlement risk issues emerged on the forefront of the payments policy agenda of the world's central banks. Because many aspects of settlement risk are eliminated in an RTGS system, many

---

[6]Similar simulation studies of interbank netting schemes in other countries have not found risks of the same magnitude. See, e.g., Angelini, Maresca, and Russo (1996) and Bech, Natorp, and Madsen (2002).

[7]For a description of the current CHIPS system, see www.chips.org.

[8]For a discussion of hybrid systems and new developments in large-value payment systems, see McAndrews and Trundle (2001) and Bank for International Settlements (2005).

central banks began to consider this option.[9] However, the reduction in risk comes at the cost of a greatly increased need for intraday liquidity to smooth the nonsynchronized payment flows.[10]

In the 1980s a number of Western European countries began implementing RTGS systems. By 1988, RTGS systems operated in five of the six major currencies (sterling being the other). During the early 1990s, RTGS adoption continued at a rate of roughly one country per year. In 1992, the Treaty of Maastricht created the foundation for the Economic and Monetary Union (EMU). A year later, the central banks within the European Union (EU) agreed that each member state should have an RTGS system. Furthermore, in 1995 it was decided to interlink the national RTGS systems through the Trans-European Automated Real-time Gross settlement Express Transfer (TARGET) system in order to facilitate the European Central Bank's (ECB) single monetary policy and to promote sound and efficient payment mechanisms in euros.[11] These decisions led to a flurry of new systems and upgrades to existing ones. TARGET went live on January 4, 1999, and even EU countries that did not join the EMU at the outset were allowed to participate in the system.

As the ECB made RTGS a prerequisite for membership to the EMU, the prospective members in the rest of Europe began to implement RTGS as well. Furthermore, as hostilities ended in the Balkans in the late 1990s, governments began to rebuild their respective economies. They considered the establishment of sound and efficient financial systems a priority. RTGS systems were implemented with support from the EU, the International Monetary Fund (IMF), and

[9] RTGS can also help reduce settlement risk by facilitating payment versus payment or delivery versus payment in the settlement of foreign exchange and securities transactions, respectively. For a discussion of settlement risk in gross and net settlement systems, see Kahn, McAndrews, and Roberds (2003).

[10] Initially, central banks provided intraday credit for free to commercial banks. But this policy is no longer considered a viable option by central banks, as it exposes them (and ultimately the taxpayers), as guarantor of the finality of payments, to credit risk.

[11] The ECB also operates its own RTGS system for its account holders—the ECB payment mechanism (EPM). EPM is connected to TARGET and is used for settlement of euro positions arising from the EBA Clearing Company (through the EURO 1 system) and CLS, the continuous linked settlement system for foreign exchange transactions.

the World Bank. With ongoing projects in Russia and Cyprus, the diffusion of RTGS in Europe is nearly complete.

In the late 1980s, the central banks of the Group of Ten (G-10)[12] countries published the Angell and Lamfalussy reports, which study different aspects of interbank netting schemes. In 1990, the permanent Committee on Payment and Settlement Systems (CPSS) was created by the G-10 governors. The CPSS has focused on disseminating information on payment system design and has been instrumental in defining the norms and best practices for the central bank community in the area of payments. For example, in 1997, the CPSS published a report on real-time gross settlement that laid out general features as well as the specifics of the systems in operation in the G-10 countries. In 2002, the CPSS published a set of core principles for systemically important payment systems. These CPSS recommendations are now part of the toolkit of the Financial Sector Assessment Program (FSAP), jointly established by the IMF and the World Bank. The FSAP reviews seek to identify strengths and vulnerabilities in order to reduce the potential for financial crisis.

Outside Europe, the rate of adoption of RTGS since the mid-1990s has been equally impressive. Australia and New Zealand implemented RTGS in 1998 and remain the only countries in Oceania that have gone live with RTGS. In Asia, the rate of RTGS implementation has been fairly steady; on average, about one country has adopted RTGS per year. Hong Kong is interesting in that RTGS is used to settle payments not only in the local currency but also in U.S. dollars and euros. Six countries in the Middle East have implemented RTGS. In Africa, the South African Reserve Bank (SARB) spearheaded adoption in 1998. Through the South African Development Community (SADC),[13] the SARB has participated in developing and strengthening the financial infrastructure in the rest of southern Africa. As of 2006, eleven African central banks have implemented RTGS, many with the support of the World Bank.

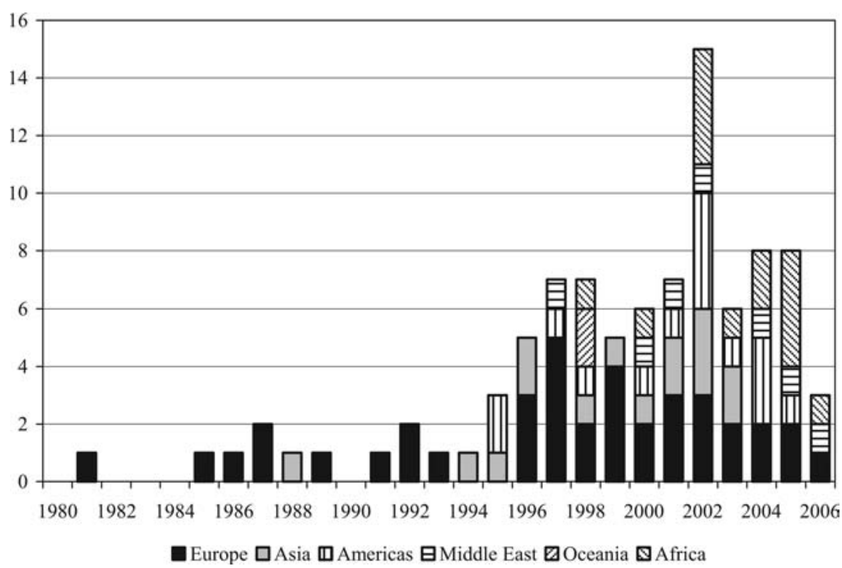
[12]The G-10 countries are Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, the United Kingdom, and the United States. In 1997 the Hong Kong Monetary Authority and the Monetary Authority of Singapore joined the Committee on Payment and Settlement Systems.

[13]The member states of the SADC are Angola, Botswana, the Democratic Republic of the Congo, Lesotho, Malawi, Mauritius, Mozambique, Namibia, Seychelles, South Africa, Swaziland, Tanzania, Zambia, and Zimbabwe.

In the Western Hemisphere, Canada is an interesting case. It is the only G-10 country that has decided not to implement an RTGS system. Instead, Canada opted for a hybrid system that primarily employs multilateral netting by novation. The Canadian Large Value Transfer System is considered equivalent to RTGS in terms of finality, as the Bank of Canada provides an explicit guarantee of settlement in case of participant failure. In South America, Uruguay was the first country to adopt RTGS, in 1995. In 2006, seven of thirteen South American countries had adopted RTGS. RTGS implementation in Central America and the Caribbean has only started recently, but the Inter-American Development Bank is assisting the efforts to implement RTGS systems in the region.[14]

As illustrated in figure 2, the world RTGS adoption rate was about one central bank per year in the latter part of the 1980s and in the beginning of the 1990s. In the mid-1990s, however, the adoption

Figure 2. Annual Number of Central Banks That Adopt RTGS



[14]A central-bank-specific account of the diffusion of RTGS is available in Bech (forthcoming).

rate started to accelerate, with three central banks in 1995 and five central banks in 1996. The annual adoption rate has not dipped below three new central banks since. It peaked in 2002, when a total of fifteen central banks implemented new RTGS systems. By the end of 2006, 93 out of the 174 central banks in the world had adopted an RTGS system for their interbank payments, up from only a total of 3 central banks in 1985 (see figure 3).

3.1 Diffusion Curve and Adopter Categories

An important regularity in empirical studies of technology diffusion is that the rate of adoption follows a predictable intertemporal pattern (e.g., Griliches 1957). At first, the rate of adoption is slow. If the technology is ultimately a success, however, the rate of adoption starts to accelerate. Rapid adoption continues until a substantial share of the agents have adopted the technology. At this point, the rate of adoption levels off and eventually falls. This pattern implies that the share of adopters follows a sigmoidal, or S-shaped, curve as a function of time, as illustrated in figure 4.

An extensive literature spanning many disciplines has used logistic curves to model and predict growth and adoption data. The choice of a particular functional form is often somewhat arbitrary. For our application, we use a generalized logistic model proposed by Richards (1959) to fit the share of central banks that have adopted RTGS at time t, denoted by a_t. Specifically, we have

$$a_t = \frac{1}{(1 + \kappa \exp(-\gamma(t - t_{\max}))^{1/\kappa})} + \varepsilon_t, \qquad (1)$$

where γ is the rate at which growth initially accelerates, $t_{\max}$ is the time at which the rate of adoption is highest (i.e., the inflection point of the curve), and κ is a shape parameter that determines whether the point of inflection is closer to the lower or the upper asymptote. If κ is equal to 1, then the curve is symmetric around the median.

In accordance with the stylized fact from studies on the adoption of other technologies, we find that the diffusion of RTGS thus far is consistent with an S-shaped pattern over time. The curve of equation (1) was fitted using nonlinear least squares. The estimated

Figure 3. Adoption of RTGS in 1985, 1995, and 2006

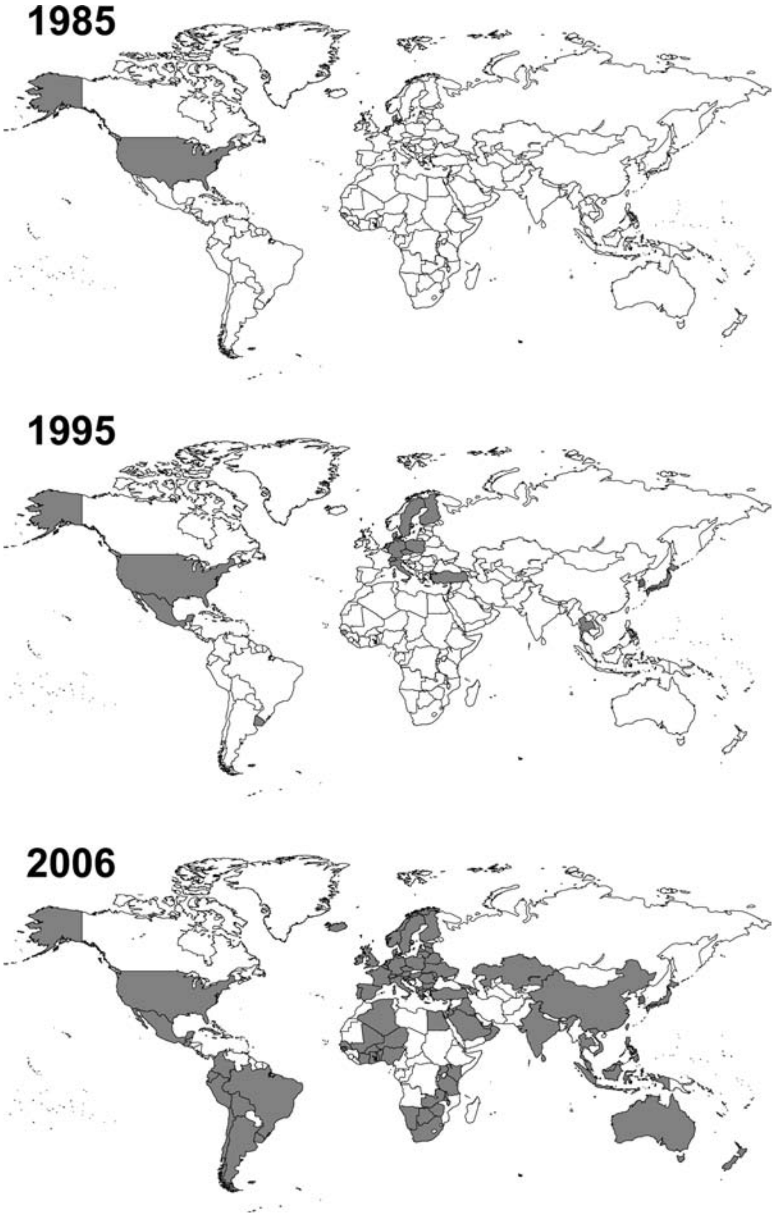
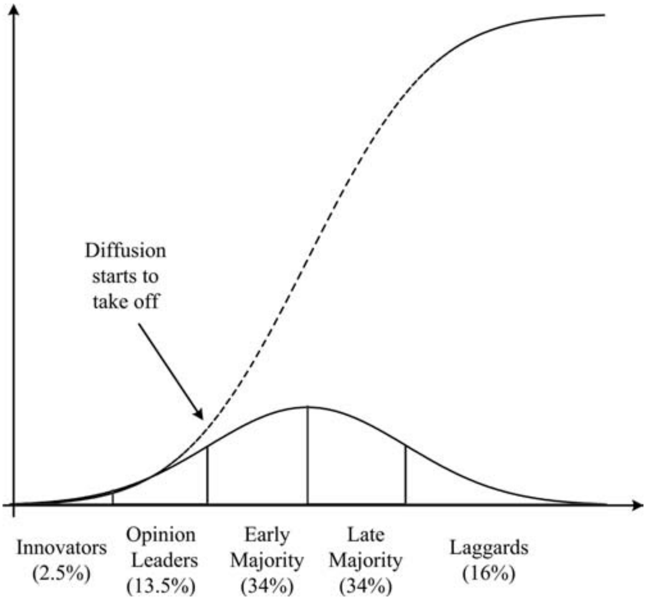


Figure 4. Stylized Sigmoidal Adoption Curve and Adopters Categories



parameters and standard errors are given in table 1. The actual and fitted share of central banks that have adopted RTGS are shown in figure 5. The overall fit of the curve is good, but the time of maximum growth is estimated to be 2004, not 2002, as found in sample. The shape parameter is found to be significantly different from 1, implying a degree of asymmetry.

The estimated generalized logistic curve suggests that the diffusion of RTGS will continue at a rapid pace until around 2010, at which time it will start to slow down. According to our estimates, the last 1 percent of central banks will not adopt until 2020. Predictions about the tails of the adoption curve do, however, depend a lot on the particular functional form chosen and thus have to be considered, at best, as only indicative of the pace of adoption.

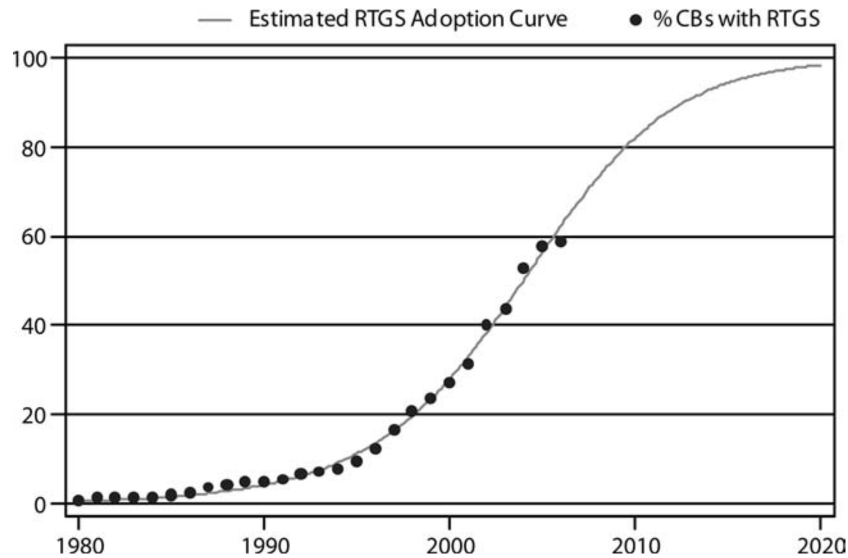
Rogers (1995) classifies agents into five categories that reflect their "innovativeness," or predisposition to adopt a new technology, based on how early or late they adopt relative to the median adopter, as illustrated in figure 5. The rate of adoption is assumed to follow a bell curve over time. The first 2.5 percent of adopters (i.e., those for which the time of adoption occurs more than two standard

Table 1. Estimated Coefficients for Logistic Adoption Curve

Coefficient	$t_{\mathbf{max}}$	γ	$\boldsymbol{\kappa}$
Value	2004**	0.27**	1.27**
Standard Error	.019	0.02	0.016

Note: ** denotes significance at the 5 percent level (two-sided).

Figure 5. Actual and Fitted Adoption Rates of RTGS (Generalized Logistic)



deviations earlier than the median agent) are labeled *innovators.* The following 13.5 percent (i.e., those for which the time of adoption is between one and two standard deviations earlier than the median adoption time) are labeled *opinion leaders.* The adoption time of the last opinion leaders corresponds to the first inflection point of the bell curve (i.e., where the rate of adoption takes off). The *early majority* is the next 34 percent of adopters up to the median, while the *late majority* is the 34 percent above the median. These two majority groups reflect those agents that adopt within one standard deviation of the median agent. The remaining 16 percent of adopters are called *laggards.*

Rogers's (1995) classification implies that central banks that adopted RTGS prior to 1987 are considered innovators. Central banks that adopted RTGS between 1986 and 1998 are opinion leaders. Central banks that adopted RTGS from 1998 through 2004 belong to the early majority, and the most recent adopters are in the late majority. Interestingly, but not surprisingly, we find that all the central bank members of the CPSS belong to either the group of innovators or opinion leaders (see table 2). Moreover, many central banks belonging to the early and late majorities explicitly cite the recommendations put forward by the CPSS as a reason for implementing RTGS.

4. Determinants of RTGS Diffusion

In economic theories (e.g., Jensen 1982 and Jovanovic and Lach 1989) of technology diffusion, agents—usually firms—adopt a technology up to the point where the adoption cost equals the expected present discounted value of future private profits. Empirical analyses thus seek to identify the determinants of the adoption costs and the future profits. For example, Hannan and McDowell (1984) base their empirical investigation of automated teller machine (ATM) adoption on the presumption that "an innovation will appear more attractive to a potential adopter, the greater the positive differential between profits obtainable with and without the innovation."

However, as argued by, e.g., Blix, Daltung, and Heikensten (2003), central banking has certain features that make it quite different from the operations of private firms. Central banks tend not to operate for the single goal of profit maximization. They are instead charged with pursuing multiple public policy goals such as monetary and financial stability as well as the smooth operation of the payment system. Moreover, central banks often face soft budget constraints that may lead to less pressure to achieve cost efficiency.

Lester, Millard, and Willison (2006) provide a theoretical model of RTGS adoption. They argue that deferred net settlement is less costly due to the higher information and communication technology (ICT) costs associated with an RTGS environment. If these costs fall, then the settlement risk reduction achieved in a real-time gross settlement system may be enough to justify a switch from deferred net settlement. As in any theoretical model of payment systems,

Table 2. Central Banks That Have Adopted RTGS and Their Respective Adoption Years

Central Bank	Year
Innovators	
Federal Reserve System	1918
Danish National Bank	1981
Dutch Central Bank	1985
The Riksbank (Sweden)	1986
Opinion Leaders	
Bundesbank (Germany)	1987
Swiss National Bank	1987
Bank of Japan	1988
Bank of Italy	1989
Bank of Finland	1991
Central Bank of the Republic of Turkey	1992
State Bank of Czechoslovakia	1992
National Bank of Poland	1993
Bank of Korea (South Korea)	1994
Central Bank of Uruguay	1995
Bank of Mexico	1995
Bank of Thailand	1995
Bank of England	1996
National Bank of Kazakstan	1996
Hong Kong Monetary Authority	1996
National Bank of Belgium	1996
Bank of Portugal	1996
Bank of Spain	1997
Central Bank of Argentina	1997
National Bank of Austria	1997
Central Bank of Ireland	1997
Bank of Norway	1997
Bank of France	1997
Saudi Arabian Monetary Agency	1997
Early Majority	
Bank of Slovenia	1998
Bank of the Republic of Belarus	1998

(*continued*)

Table 2 (continued). Central Banks That Have Adopted RTGS and Their Respective Adoption Years

Central Bank	Year
Early Majority (continued)	
Central Bank of Colombia	1998
Monetary Authority of Singapore	1998
Reserve Bank of Australia	1998
Reserve Bank of New Zealand	1998
South African Reserve Bank	1998
Croatian National Bank	1999
Central Bank of Luxembourg	1999
Bank of Greece	1999
Central Bank of Malaysia	1999
National Bank of Hungary	1999
Central Bank of Peru	2000
Central Bank of Iceland	2000
Qatar Central Bank	2000
Bank of Indonesia	2000
Bank of Latvia	2000
Bank of Mauritius	2000
Central Bank of the United Arab Emirates	2001
State Bank of Vietnam	2001
National Bank of the Republic of Macedonia	2001
National Bank of Azerbaijan	2001
Central Bank of Bosnia and Herzegovina	2001
Central Bank of Netherlands Antilles	2001
National Bank of Georgia	2001
Central Bank of Costa Rica	2001
Central Bank of Cuba	2001
Central Bank of Brazil	2002
Central Bank of Malta	2002
Central Bank of the Philippines	2002
Bank of Estonia	2002
Bank of Namibia	2002
Reserve Bank of Malawi	2002
Reserve Bank of Zimbabwe	2002
Central Bank of Barbados	2002
Bank of Ghana	2002

(*continued*)

Table 2 (continued). Central Banks That Have Adopted RTGS and Their Respective Adoption Years

Central Bank	Year
Early Majority (continued)	
The Peoples Bank of China	2002
Central Bank of Jordan	2002
Central Bank of China (Taiwan)	2002
Central Bank of Bulgaria	2002
Bank of Botswana	2003
Reserve Bank of India	2003
National Bank of the Slovak Republic	2003
National Bank of Serbia	2003
Central Bank of Ecuador	2003
Central Bank of Sri Lanka	2003
Central Bank of Kuwait	2004
Central Bank of Chile	2004
Bank of Tanzania	2004
Central Bank of Bolivia	2004
Bank of Albania	2004
Bank of Lithuania	2004
Central Bank of West African States	2004
Bank of Zambia	2004
Central Bank of Trinidad and Tobago	2004
Late Majority	
National Bank of Romania	2005
Bank of Guatemala	2005
Bank of Uganda	2005
National Bank of Ukraine	2005
Central Bank of Montenegro	2005
Central Bank of Kenya	2005
Central Bank of Oman	2005
Central Bank of Nigeria	2005
National Bank of Moldova	2006
Bank of Algeria	2006
Central Bank of Iraq	2006

a switch from one system to another generally requires a facilitator that coordinates the choices of all market participants. This is where the public policy role of the central bank comes in.

Hence, our study of RTGS adoption is importantly different from other empirical studies of technology adoption (e.g., Griliches 1957, Mansfield 1961, Gort and Klepper 1982, Jovanovic and Lach 1989, and Skinner and Staiger 2005). These studies focus on the adoption of technologies by individuals or firms that have a profit incentive. Such a market incentive is only indirectly present in a central bank's decisions to adopt RTGS systems. The set of factors taken into account by central banks when assessing the benefits of adopting a new technology is potentially much bigger than that of private-sector firms.

Technology is an input in the production function of clearing and settlement services, just as labor and capital are. Keller (2004) highlights two key aspects of technology that are applicable in our context as well. First, technology is a nonrival in the sense that the marginal costs of an additional agent using the technology are negligible and that the adoption of RTGS by one central bank does not in any way hinder another central bank from adopting. Second, the returns to technological investments are partly private and partly public.

The basic premise of our analysis is that a central bank that has not yet adopted RTGS will adopt at such time τ_i when the perceived (present discounted) value of benefits equals or exceeds the (present discounted) cost of adoption. As argued above, we do not necessarily interpret the costs and benefits here in the neoclassical sense of a profit function, but instead interpret them in the broader context of both private and social costs and benefits.

We regard an RTGS system to be an irreversible settlement process innovation with a considerable ICT investment embedded in it. The adoption of an RTGS is virtually impossible to reverse due to the central role of the interbank payment system and its many interconnections and interdependencies with other financial infrastructures. As other clearing and settlement mechanisms come to rely on instant finality in real time, changing their modus operandi becomes increasingly expensive or even infeasible. Moreover, the implementation of a specific settlement process imposes certain requirements in terms of ICT. For example, DNS requires batch-based computing and communication capabilities with participants, while RTGS requires real-time computing and communication.

Economic theory and evidence on technology adoption provide us with guidance on the set of potentially significant determinants of perceived value and costs. In the rest of this section, we discuss the potential determinants of innovativeness and the method by which we approximate them in our empirical analysis. We limit ourselves to a brief description of the data here. The underlying details, definitions, and sources are provided in the appendix. The cost and benefit factors we identify for our empirical analysis can be classified into two groups. The first group comprises direct indicators of costs and benefits. The second consists of the effects the actions of other central banks have on a particular central bank's cost and benefit perception.

Investment Price Index. Our main indicator of direct adoption costs is the price of investment goods from the Penn World Table (Heston, Summers, and Aten 2002). We use this as a proxy for the price of the ICT equipment that is necessary to execute settlements in an RTGS system. Hence, we expect countries in which ICT is cheaper, as reflected in a lower investment price, to adopt RTGS more quickly. We are aware that our investment price measure is not ICT specific. For this reason, we also use personal computers per capita as an additional proxy, where our main assumption is that more personal computers reflect lower ICT prices and implementation costs.

Education. Brynjolfsson and Hitt (2000) provide evidence that the costs of physical capital and software are only a small part of the total costs involved in implementing ICT-related process innovations in the U.S. private sector. A large part of the cost of implementation has to do with the acquisition of the knowledge about how to use and benefit from the new process, which mainly involves skilled labor inputs. That is, adoption costs are lower if employees' pre-RTGS skills and knowledge help them learn the best way to use RTGS systems more quickly, as pointed out in Nelson and Phelps (1966). This is often referred to as the new technology being "appropriate" for the existing endowments (see, e.g., Basu and Weil 1998).

Since we use a measure of education of the overall workforce, our measure also includes the skill level of the (potential) users of the RTGS system. Because of capital-skill complementaries, an RTGS system increases the productivity of educated users more than that of other users. As a consequence, we expect the benefits of RTGS to be higher in countries with a more educated workforce.

Independent of the cost versus benefit interpretation of education for RTGS adoption, we expect education to have an unambiguously positive effect on the probability of a central bank adopting an RTGS system.

Financial Market Development. On the direct benefits side, central banks themselves are not assumed to pursue a profit objective per se. They are, however, presumed to contribute to the efficiency, stability, and competitiveness of domestic financial markets. The main benefit of adopting an RTGS system is that it improves the efficiency of domestic financial markets, primarily by significantly reducing settlement risks.

Unfortunately, cross-country data on the amount of settlements on interbank payment systems is not available. This means we will have to use indirect proxy variables for the level of financial development and its effects on RTGS adoption. We use three proxy variables, shown by Levine, Loayza, and Beck (2000) to be correlated with GDP growth. These variables are liquid liabilities, which measure the financial depth of a country; the ratio of commercial to central bank assets, which measures the degree to which the private part of the banking sector allocates savings; and private credit issued by financial intermediaries to the private sector. The latter is the preferred measure of financial development of Levine, Loayza, and Beck (2000).

Real GDP per Capita and Population. Real GDP per capita seems to matter in almost all studies of cross-country technology adoption patterns (e.g., Caselli and Coleman 2001 and Comín and Hobijn 2004). Because the fixed setup costs of an RTGS system are, in large part, independent of the size of the market, while the benefits of an RTGS system, as in the reduction of the settlement risk, are increasing with the size of the market, we would expect to observe the adoption of RTGS to be subject to scale effects. These effects are similar to those in the endogenous growth literature (Jones 1995) in which a fixed innovation cost would imply that the endogenous growth rate would be increasing with market size. To the extent such scale effects exist, we expect the likelihood of adoption of RTGS to be higher in countries with a bigger population and a larger economy, as measured by GDP per capita.

So far, we have focused on a central bank's adoption costs and benefits of implementing an RTGS system that does not depend on the actions of its counterparts.[15] There are, however, several reasons to believe that the RTGS adoption decisions of central banks are likely to be interdependent.

First, the decision to adopt is influenced by competitive pressures from global financial markets. To the extent that an RTGS system provides benefits that enhance the competitiveness of the domestic financial markets or financial assets denominated in the currency of the central bank, such pressures may speed up or force the adoption decision. For example, a prerequisite for a currency to be part of the Continuous Linked Settlement (CLS) foreign exchange settlement system is the ability to make real-time transfers in central bank money.[16]

Second, just as is the case for personal computers (e.g., Goolsbee and Klenow 2002), the adoption of RTGS is subject to network externalities. That is, given international financial integration, it is more beneficial for a central bank to adopt an RTGS system when this allows access to a broad network of other countries' RTGS systems. This consideration seems to have been an important driving force behind the EU's requirement that countries have an RTGS system in place before joining the monetary union.

Third, given the many cooperative structures set up among central banks (like the Bank for International Settlements and the European Central Bank), as well as the many other interlinkages between central banks (such as their historical linkages), it is very likely that the knowledge acquired through experiences with RTGS systems spills over to other central banks and helps them make their

[15]It is difficult to measure central bank efficiency (see Blix, Daltung, and Heikensten 2003 for a discussion). For example, the functions carried out by central banks vary significantly across countries. In fact, as noted by Brione (2006), it is hard to get an idea of the actual operating expenses of central banks, even across the OECD, let alone the rest of the world.

[16]CLS Bank International is currently the leading settlement institution for foreign exchange transactions. It is an industry-owned bank, chartered in the United States as an Edge corporation and supervised by the Federal Reserve System. CLS was founded in response to concerns raised by the G-10 central banks about settlement risk in foreign exchange transactions. It began operations in September 2002 and now settles transactions in fifteen currencies for a sizable share of the foreign exchange market.

respective adoption decisions. Spillovers of this type have been extensively documented as they relate to R&D and trade flows (see, e.g., Coe and Helpman 1995), and they are likely to be an important factor in the diffusion of payments systems as well.

Fourth, central banks do not necessarily have to install and develop the ICT component involved with the RTGS adoption themselves. Several private companies have entered the marketplace and have begun to offer "off-the-shelf" standardized software solutions. Currently, there are at least five providers that have built RTGS systems in more than one country.[17] The possibility of sharing development costs across customers and competition among providers has presumably lowered the cost of implementing RTGS and hence made it feasible for more countries to adopt.

The second point outlined above is known as a network spillover, while the third and fourth points are known as knowledge spillovers.[18] These are two types of indicators that we include in our analysis to capture interdependencies of central banks' RTGS adoption decisions.

Membership of International Organizations. To capture knowledge spillovers that arise from countries sharing information and jointly defining policy goals and standards in international organizations, we control for central banks being members of the Bank for International Settlements (BIS), countries being member states of the European Union, and countries being member states of the South African Development Council.

To the extent that membership in these organizations provides earlier access to knowledge about RTGS, we expect membership in these organizations to have a positive impact on the likelihood of a central bank adopting an RTGS system.

[17] These providers are LogicaCMG plc of the United Kingdom, CMA Small System AB of Sweden, the joint venture of Perago Ltd of South Africa and SIA SpA of Italy, Montran Corporation of the United States, and BCS Information Systems of Singapore.

[18] We are using the term *spillover* loosely here. The interdependencies in the decisions of central banks that we describe are only spillovers in a pure economic sense if, when making adoption decisions, central banks do not take into account the positive effects that their RTGS adoption decisions have on their counterparts. If central banks do not internalize these effects, the rate of adoption of RTGS around the world would be inefficiently low.

Bilateral Trade. To capture knowledge and network spillovers from the countries that interact most with each other economically, we calculate the trade-weighted fraction of countries' trading partners that have adopted an RTGS system.[19] This measure is very similar in spirit to the R&D spillover measure that Coe and Helpman (1995) use in their analysis and, just as in Coe and Helpman (1995), we expect it to have a positive effect on (RTGS) technology adoption.

5. Econometric Analysis

In this section, we outline our econometric approach. Let v_{it}^* denote the perceived value of the RTGS system for central bank i at time t, and let c_{it}^* denote the cost of adoption. In practice, we observe neither the perceived benefits nor the costs of adopting an RTGS system for any central bank. However, we assume that both v_{it}^* and c_{it}^* can be represented by a linear combination of a set of observable proxy variables. These are the variables described in section 4. We denote the vector with these variables as $\mathbf{x}_{it}$. Specifically, we assume

$$v_{it}^* = \mathbf{x}_{it}'\boldsymbol{\theta}_v + \varepsilon_{it}^v, \tag{2}$$

$$c_{it}^* = \mathbf{x}_{it}'\boldsymbol{\theta}_c + \varepsilon_{it}^c, \tag{3}$$

where $\boldsymbol{\theta}_v$ and $\boldsymbol{\theta}_c$ are vectors of unknown perceived marginal benefit and perceived marginal cost parameters. ε_{it}^v and ε_{it}^c are country- and time-specific random variables. The adoption time, τ_i, of RTGS for central bank i satisfies

$$\tau_i = \arg\min_t \left\{ t \,| y_{it}^* \geq 0 \right\}, \tag{4}$$

where

$$y_{it}^* = v_{it}^* - c_{it}^* = \mathbf{x}_{it}'(\boldsymbol{\theta}_v - \boldsymbol{\theta}_c) + \varepsilon_{it}^v - \varepsilon_{it}^c = \mathbf{x}_{it}'\boldsymbol{\theta} + \varepsilon_{it} \tag{5}$$

is the net benefit of adopting RTGS at time t. We do not observe this net benefit; it is a latent variable. What we do observe is whether an RTGS system is adopted by a central bank at time t in our sample period, $\{1, \ldots, T\}$.

[19] We limit ourselves to the ten largest trading partners for each country.

Let y_{it} be a binary variable that equals 1 if central bank i has adopted RTGS at time t and 0 otherwise. As the adoption process of RTGS is still ongoing, there are two types of central banks in our sample: (i) central banks that have already adopted—the uncensored part of the sample—and (ii) central banks that have not yet adopted—the censored part of our sample. For central banks that adopt RTGS at time $\tau_i \leq T$, we observe the binary variable $y_{it} = 0$ for all $t < \tau_i$ and $y_{it} = 1$ for $t \geq \tau_i$. This sequence of observations implies that the net benefit of adopting for central bank i was negative (i.e., $y^*_{it} < 0$) for $t = 1, \ldots, \tau_i - 1$ and positive (i.e., $y^*_{it} \geq 0$) for $t = \tau_i$. The assumed irreversibility of the RTGS adoption decision means that the observations for $t > \tau_i$ are of no empirical relevance for our analysis. For central banks in the censored part of our sample—i.e., those that have yet to adopt RTGS—we observe $y_{it} = 0$ for all $t = 1, \ldots, T$. This implies that the latent net benefit of RTGS adoption was negative for every period up to time T.

Following the approach of Allison (1982), we assume that ε_{it} is independently, logistically distributed—in our case across countries and over time. This allows us to write our model as a logistic hazard model and to relate the probability of RTGS adoption to the cross-country and time variation in the covariates $\mathbf{x}_{it}$.[20]

The probability that central bank i adopts an RTGS system at time $\tau_i = t$ equals

$$p(\tau_i = t) = \prod_{s=t}^{T} p(y_{is} = 1|Y_{s-1}) \cdot \prod_{u=2}^{t-1} p(y_{iu} = 0|Y_{u-1}) \cdot p(y_{i1} = 0), \quad (6)$$

where Y_{t-1} denotes the set of observations up to and including y_{it-1}. However, due to the irreversible nature of the adoption decision,

$$p(y_{is} = 1| \ldots, y_{iu} = 1, \ldots) = 1 \text{ for all } s > u. \quad (7)$$

Hence, the only observations that are relevant for calculating the probability are the ones before the central bank adopts and the one at the time that it adopts. From equations (6) and (7), we have

$$p(\tau_i = t) = p(y_{it} = 1|Y_{t-1}) \cdot \prod_{s=2}^{t-1} p(y_{is} = 0|Y_{s-1}) \cdot p(y_{i1} = 0). \quad (8)$$

[20] Our technology adoption analysis is similar to the retirement decision analysis in Ashenfelter and Card (2002).

In contrast, for central banks that do not adopt RTGS within our sample period (i.e., the censored part of the sample), we observe $y_{it} = 0$ for all t. We have

$$p(\tau_i > T) = \prod_{s=2}^{T} p(y_{is} = 0|Y_{s-1})p(y_{i1} = 0). \tag{9}$$

In practice, this means that, for our empirical analysis, we use data not only on central banks that have already implemented the system but also on central banks that have not yet done so. For the central banks that adopt, however, we only use information up to and including the time of adoption.

From equations (8) and (9), we can set up the likelihood function. Let the set of countries that have adopted RTGS by the end of the sample period be $A_T = \{i\,|\tau_i \leq T\}$. We have $p(y_{it} = 1|\mathbf{x}_{it}) = p(y^*_{it} > 0|\mathbf{x}_{it}) = p(\varepsilon_{it} > -\mathbf{x}'_{it}\boldsymbol{\theta})$. Assuming that $y_{i1}, \ldots, y_{iT}$ (conditional on $\mathbf{x}_{i1}, \ldots, \mathbf{x}_{iT}$) are independent and that ε_{it} has a logistic distribution $\Lambda(\varepsilon) = p(\varepsilon_{it} < \mathbf{x}'_{it}\boldsymbol{\theta})$, the likelihood function becomes

$$\begin{aligned} \mathcal{L}(\boldsymbol{\theta}; \tau_i) &= \prod_{i \in A_T} p(\tau_i = t|\mathbf{x}_{it}) \cdot \prod_{i \notin A_T} p(\tau_i > T|\mathbf{x}_{it}) \qquad (10) \\ &= \prod_{i \in A_T} [\prod_{t=1}^{\tau_i} p(y_{it} = 0|\mathbf{x}_{it})]p(y_{it} = 1|\mathbf{x}_{it}) \cdot \prod_{i \notin A_T} \prod_{t=1}^{T} p(y_{it} = 0|\mathbf{x}_{it}) \\ &= \prod_{i \in A_T} [\prod_{t=1}^{\tau_i} 1 - \Lambda(\mathbf{x}'_{it}\boldsymbol{\theta})]\Lambda(\mathbf{x}'_{it}\boldsymbol{\theta}) \cdot \prod_{i \notin A_T} \prod_{t=1}^{T} 1 - \Lambda(\mathbf{x}'_{it}\boldsymbol{\theta}). \end{aligned}$$

Our specification implies that we can only estimate $(\boldsymbol{\theta}_v - \boldsymbol{\theta}_c)$, which is the excess benefit from a particular variable in $\mathbf{x}_{i,t}$. However, for proxy variables that we know affect only costs or only benefits (or for which we can reasonably conjecture the same), the coefficient can be interpreted as the cost or benefit parameter.

The above likelihood function is that of a logit model. Since the probability of adoption can be interpreted as a hazard rate in terms of a duration model, our empirical analysis can be interpreted as a discrete-time duration model with logistic, time-varying hazard rates.

6. Results

Table 3 contains the estimated coefficients from the logistic-hazard-rate model. Our sample covers 108 central banks from 1970 through 2000. We take an incremental approach for our analysis of the causes of technology adoption of RTGS. We start by presenting the core results for the explanatory variables that matter consistently across our model specifications. These are (i) real GDP per capita, (ii) population, (iii) education, and (iv) the investment price. For all of these variables, the estimated coefficients are significant and of the expected sign for all model specifications for which we have 100 countries or more. That is, larger countries (as reflected by population) with a higher standard of living (as reflected by real GDP per capita) and a higher level of human capital (as reflected by education) seem to be more likely to adopt an RTGS system than others.

Moreover, countries with a relatively low price of capital also are more likely to adopt. Jones (1994) finds that capital prices, especially those of machines, help explain cross-country variation in growth rates of real GDP per capita. Our results suggest that this variation might have to do with the ways in which these capital prices affect technology adoption decisions. That is, if these prices seem to be significant in explaining variation in the adoption of a technology by the public sector, they most likely matter even more for the adoption of technologies by the private sector where the profit motive is an even more powerful incentive for adoption decisions.

In terms of our spillovers proxies, we find that bilateral trade relationships with countries that have adopted RTGS significantly increase the probability of adoption. This is very much in line with the evidence on international R&D spillovers, as in Coe and Helpman (1995). In terms of membership in international organizations, only membership of the South African Development Council seems to function as a significant catalyst of the RTGS adoption process. A country's membership in this organization significantly increases the per-period probability of adopting RTGS compared with its nonmember counterparts.

The financial development measures—i.e., those that proxy for financial markets benefits of RTGS—do not seem to explain much of the cross-country variation in the adoption of RTGS. The insignificance of these indicators is not due to their being highly correlated.

Table 3. Estimated Coefficients for Logit Adoption Model

Model	1	2	3	4	5	6	7	8
Observations	2,683	2,605	2,683	2,605	2,344	2,424	694	2,605
Central Banks	108	105	108	105	98	101	90	105
Log Real GDP per Capita	2.90**	2.55**	3.24**	2.91**	2.15**	2.26**	1.75**	2.66**
	0.40	0.42	0.46	0.52	0.48	0.46	0.65	0.44
Log Population	0.37**	0.34**	0.55**	0.48**	0.29**	0.29**	0.30**	0.41
	0.11	0.12	0.14	0.15	0.13	0.13	0.14	0.13
Education	0.04**	0.04**	0.04**	0.04**	0.04**	0.04**	0.07**	0.04**
	0.01	0.01	0.01	0.01	0.02	0.01	0.02	0.01
Log Investment Price	−1.20**	−0.98*	−0.98**	−0.83	−1.29**	−1.16**	−1.10	−0.92
	0.50	0.57	0.52	0.57	0.61	0.58	0.73	0.56
Bilateral Trade		1.86**		1.38*	2.02**	2.09**	2.49*	1.55*
		0.77		0.83	0.85	0.82	1.28	0.80
SADC			1.68**	1.42*				1.40*
			0.82	0.83				0.83
BIS Member			−0.40	−0.30				
			0.28	0.32				
New EU Member			1.15**	0.87				0.82
			0.54	0.56				0.56
Liquid Liabilities					−0.71			
					0.82			
Commercial–Central Bank Assets					1.03			
					1.66			
Private Credit					0.96	0.72		
					0.96	0.61		
Computers							0.01*	
							0.00	

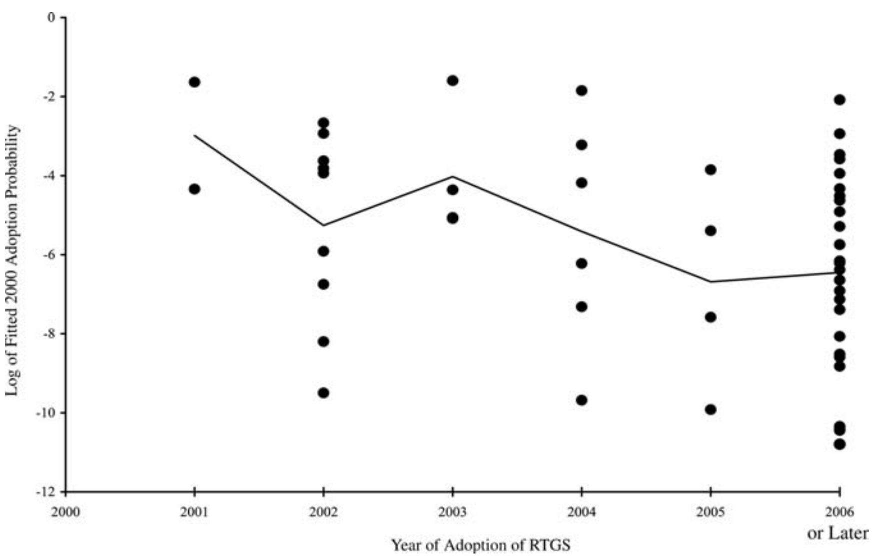
Note: ** denotes significance at the 5 percent level; * denotes significance at the 10 percent level (both two-sided).

Even when we only include private credit, as in column 7 of table 3, we do not find any significant effect of financial development on the probability of RTGS adoption beyond that which is already captured by including real GDP per capita among the regressors.

The adoption rate of personal computers per capita does seem to positively affect the probability of adoption of RTGS. This result, however, should be interpreted with caution because the small number of observations on personal computers per capita means that the coefficient is estimated with far fewer degrees of freedom than the other results.

To see how much the fitted cross-country variation in the adoption probability explains the cross-country adoption patterns, we present the results of an out-of-sample experiment in figure 6. This figure plots the log of the fitted RTGS adoption probability, based on model 8 from table 3, for the countries in the censored part of our sample against the actual adoption date, as well as the "2006 or later" date for countries that had not adopted RTGS by the end of 2005. The line is the average log of fitted probability of adoption in

Figure 6. Fitted 2000 Adoption Probabilities and Subsequent Adoption Dates



2000 for the countries that adopt in each subsequent year. As can be seen in the figure, later adopters had, on average, a lower fitted probability of RTGS adoption in 2000.

7. Conclusion

We study the adoption of real-time gross settlement systems across the world's 174 central banks. These systems are used to facilitate interbank payments. As Frame and White (2004) point out, the rapid rate of financial innovation over the past few decades is widely recognized as a stylized fact. A striking feature of the literature based on this fact, however, is the relative dearth of empirical studies. Ours can be interpreted as one such study. The study of RTGS in the context of technology adoption is interesting, since the decision to adopt RTGS is in large part based on the perception by a central bank of the public goods benefit of the system, rather than on the profit motive driving adoption in the private sector.

The adoption pattern of RTGS follows an S-shaped diffusion curve, similar to that observed for other technologies in technology adoption literature (e.g., Griliches 1957, Mansfield 1961, and Gort and Klepper 1982). The lags in the adoption of RTGS are long. In the last twenty years, the number of central banks using RTGS for their interbank payments has increased from three to ninety-three. This means that eighty-one central banks have not yet adopted RTGS and thus are not yet benefiting from the reduction in settlement risk provided by such a system. Estimation of the world adoption curve of RTGS suggests that it might take another fifteen years before RTGS technology is adopted by the remaining eighty-one central banks.

The probability that a country adopts RTGS in a given year increases significantly with the level of real GDP per capita. Moreover, countries with a lower relative price of capital and countries whose major trading partners have already adopted RTGS are also more likely adopters. This suggests that, beyond market forces reflected by real GDP and capital costs, spillovers seem to play a significant role in the adoption of this financial innovation. These spillovers seem to be transferred mainly through trade relationships.

These results suggest that the policy role of central banks is not only to coordinate the adoption of the RTGS system in their domestic market but also to internalize the externalities of this adoption decision by other central banks. International cooperation among central banks is one way to facilitate this internalization. To what extent the RTGS adoption pattern, as well as its determinants, is representative of central banks' technology decisions more broadly is a topic for further research.

Appendix. Data: Sources and Definitions

Countries and Time Span

Our data cover the 174 central banks and monetary authorities listed in the 2006 Morgan Stanley *Central Bank Directory*. The data span a time period from 1970 through 2005. The majority of the central banks in our sample serve only one country. However, the following five central banks encompass various countries: the Eastern Caribbean Central Bank, the Central Bank of West African States (BCEAO), the Central Bank of Central African States (BEAC), the Bank of France, and the Swiss National Bank.

Dependent Variable

RTGS Adoption: Dichotomous variable that indicates whether or not a central bank has implemented an RTGS system. We gathered information on each central bank's usage of RTGS, as well as the date of implementation, from publicly available web sites.

Explanatory Variables (Listed in Alphabetical Order)

Bilateral Trade Spillovers: Fraction of trade with top-ten trading partners that is conducted with partners that have implemented RTGS. Source: International Monetary Fund (2006) and Gleditsch (2002). Period: 1970–2005.

BIS Member: Dummy variable that indicates whether country is a member of the BIS. Source: www.correlatesofwar.org. Period: 1970–2005.

Commercial–Central Bank Assets: Assets of deposit-money banks divided by assets of deposit-money banks plus central bank assets. Source: Levine, Loayza, and Beck (2000). Period: 1970–2004.

Computers: Number of computers per 1,000 people. Source: World Bank (2006). Period: 1980–2003.

Deposits/GDP: Bank deposits as a fraction of GDP. Source: International Monetary Fund (2006). Period: 1970–2004.

Education: Average primary school attainment rate. Source: Barro and Lee (1993). Period: 1970–2000 (average over sample).

Investment Price: Capital goods price index. Source: Heston, Summers, and Aten (2002). Period: 1970–2000.

Liquid Liabilities: Liquid liabilities of the financial system (currency plus demand and interest-bearing liabilities of banks and nonbank financial intermediaries) divided by GDP. Source: Levine, Loayza, and Beck (2000). Period: 1970–2004.

New EU Member: Dummy variable that indicates whether country became a member of the European Union or applied after 1986. Source: www.wikipedia.com (keyword: Union). Period: 1970–2005.

Population: Source: World Bank (2006). Period: 1970–2004.

Private Credit: Credit by deposit-money banks and other financial institutions to the private sector divided by GDP. Source: Levine, Loayza, and Beck (2000). Period: 1970–2004.

Real GDP per Capita: Source: Heston, Summers, and Aten (2002). Period: 1970–2000.

SADC: Dummy variable that indicates whether a country is a member of the South African Development Council. Source: www.sadc.int. Period: 1970–2005.

Note: Not all series are available for every country and every year in our sample.

References

Akhavein, J., S. Frame, and L. White. 2005. "The Diffusion of Financial Innovations: An Examination of the Adoption of Small Business Credit Scoring by Large Banking Organizations." *Journal of Business* 78 (2): 577–96.

Allison, P. D. 1982. "Discrete-Time Methods for the Analysis of Event Histories." *Sociological Methodology*, ed. Samuel Leinhardt, 61–98. San Francisco: Jossey-Bass.

Angelini, P., G. Maresca, and D. Russo. 1996. “Systemic Risk in the Netting System.” *Journal of Banking and Finance* 20 (5): 853–68.

Ashenfelter, O., and D. Card. 2002. “Did the Elimination of Mandatory Retirement Affect Faculty Retirement?” *American Economic Review* 92 (4): 957–80.

Bank for International Settlements. 2005. “New Developments in Large-Value Payment Systems.” Publication No. 67, Committee on Payment and Settlement Systems.

Barro, R. J., and J. Lee. 1993. “International Comparisons of Educational Attainment.” NBER Working Paper No. 4349.

Basu, S., and D. N. Weil. 1998. “Appropriate Technology and Growth.” *Quarterly Journal of Economics* 113 (4): 1025–54.

Bech, M. Forthcoming. “The Diffusion of Real Time Gross Settlement.” In *The Future of Payment*s, ed. S. Millard and V. Saporta. London: Routledge.

Bech, M., L. Natorp, and B. Madsen. 2002. “Systemic Risk in the Danish Interbank Netting System.” Working Paper No. 8, Danmarks Nationalbank.

Blix, M., S. Daltung, and L. Heikensten. 2003.“On Central Bank Efficiency.” *Economic Review* (Sveriges Riksbank) 3: 81–93.

Brione, P. 2006. “Central Bank Staff Costs.” *Central Banking* XVI (2): 69–74.

Brynjolfsson, E., and L. M. Hitt. 2000. “Beyond Computation: Information Technology, Organizational Transformation and Business Performance.” *Journal of Economic Perspectives* 14 (4): 23–48.

Caselli, F., and W. J. Coleman. 2001. “Cross-Country Technology Diffusion: The Case of Computers.” *American Economic Review* 91 (2): 328–35.

Coe, D. T., and E. Helpman. 1995. “International R&D Spillovers.” *European Economic Review* 39 (5): 859–87.

Comín, D., and B. Hobijn. 2004. “Cross-Country Technology Adoption: Making the Theories Face the Facts.” *Journal of Monetary Economics* 51 (1): 38–83.

Frame, W. S., and L. J. White. 2004. “Empirical Studies of Financial Innovation: Lots of Talk, Little Action?” *Journal of Economic Literature* 42 (1): 116–44.

Gleditsch, K. S. 2002. “Expanded Trade and GDP Data.” *Journal of Conflict Resolution* 46 (5): 712–24.

Goolsbee, A., and P. Klenow. 2002. "Evidence on Learning and Network Externalities in the Diffusion of Home Computers." *Journal of Law & Economics* 45 (2): 317–44.

Gort, M., and S. Klepper. 1982. "Time Paths in the Diffusion of Product Innovations." *Economic Journal* 92 (367): 630–53.

Gowrisankaran, G., and J. Stavins. 2004. "Network Externalities and Technology Adoption: Lessons from Electronic Payments." *RAND Journal of Economics* 35 (Summer): 260–76.

Griliches, Z. 1957. "Hybrid Corn: An Exploration in the Economics of Technological Change." *Econometrica* 25 (4): 501.

Hannan, T. H., and J. M. McDowell. 1984. "The Determinants of Technology Adoption: The Case of the Banking Firm." *RAND Journal of Economics* 15 (3): 328–35.

Heston, A., R. Summers, and B. Aten. 2002. "Penn World Table Version 6.1." Center for International Comparisons at the University of Pennsylvania (CICUP).

Humphrey, D. B. 1986. "Payments Finality and Risk of Settlement Failure." *Technology and the Regulation of Financial Markets: Securities, Futures, and Banking*, ed. A. Saunders and L. J. White. Lexington, MA: D. C. Heath and Company.

International Monetary Fund. 2006. *International Financial Statistics (IFS)*. Online Database. Washington, DC: International Monetary Fund.

Jensen, R. 1982. "Adoption and Diffusion of an Innovation of Uncertain Profitability." *Journal of Economic Theory* 27 (1): 182–93.

Jones, C. I. 1994. "Economic Growth and the Relative Price of Capital." *Journal of Monetary Economics* 34 (3): 359–82.

———. 1995. "R&D-Based Models of Economic Growth." *Journal of Political Economy* 103 (4): 759–84.

Jovanovic, B., and S. Lach. 1989. "Entry, Exit, and Diffusion with Learning by Doing." *American Economic Review* 79 (4): 690–99.

Kahn, C. M., J. McAndrews, and W. Roberds. 2003. "Settlement Risk under Gross and Net Settlement." *Journal of Money, Credit, and Banking* 35 (4): 591–608.

Keller, W. 2004. "International Technology Diffusion." *Journal of Economic Literature* 42 (3): 752–82.

Lester, B., S. Millard, and M. Willison. 2006. "Optimal Settlement Rules for Payment Systems." Mimeo, Bank of England.

Levine, R., N. Loayza, and T. Beck. 2000. "Financial Intermediation and Growth: Causality and Causes." *Journal of Monetary Economics* 46 (1): 31–77.

Mansfield, E. 1961. "Technical Change and the Rate of Imitation." *Econometrica* 29 (4): 741–66.

McAndrews, J., and J. Trundle. 2001. "New Payment System Designs: Causes and Consequences." *Financial Stability Review* (Bank of England) 11 (December): 127–36.

Morgan Stanley. 2006. *Central Bank Directory*. London: Central Banking Publications.

Nelson, R. R., and E. S. Phelps. 1966. "Investment in Humans, Technological Diffusion, and Economic Growth." *American Economic Review* 56 (2): 69–75.

Richards, F. J. 1959. "A Flexible Growth Curve for Empirical Use." *Journal of Experimental Botany* 10 (2): 290–300.

Rogers, E. 1995. *Diffusion of Innovations*. 5th ed. New York: Free Press.

Selgin, G. 2004. "Wholesale Payments: Questioning the Market-Failure Hypothesis." *International Review of Law and Economics* 24 (3): 333–50.

Skinner, J., and D. Staiger. 2005. "Technology Adoption from Hybrid Corn to Beta Blockers." NBER Working Paper No. 11251.

World Bank. 2006. *World Development Indicators (WDI).* Online Database. Washington, DC: World Bank Group.

Fear of Floating and Social Welfare*

Demosthenes N. Tambakis
Faculty of Economics and Pembroke College, University of Cambridge

This paper studies the welfare implications of financial stability and inflation stabilization as distinct monetary policy objectives. Introducing asymmetric aversion to exchange rate depreciation in the Barro-Gordon model mitigates inflation bias due to credibility problems. The net welfare impact of fear of floating depends on the economy's recent track record, the credibility of monetary policy, and the central bank's discount factor. It is shown that fear of floating is more appropriate for financially fragile developing countries with imperfectly credible monetary policy than for advanced economies.

JEL Codes: E52, E58, F33.

1. Introduction

Following the emerging-market financial crises of 1997–99, consensus has arisen that financial stability is an important element in the conduct of monetary policy; see Goodhart and Illing (2002), Svensson (2002), and Svensson and Woodford (2003).[1] A central bank with explicit responsibility for financial stability would have a clearer mandate to respond to the buildup of financial imbalances even if monetary stability did not appear to be under threat. However, little work has been done on the welfare implications of financial stability as a distinct monetary policy objective, particularly in emerging-market economies.

---

*I am grateful to two anonymous referees and an editor for their valuable suggestions. I would also like to thank Andrew Filardo, Paul Masson, Nikola Tarashev, and seminar participants at Cambridge, Birkbeck, Stockholm, and the University of Crete for helpful comments on earlier drafts of this paper. The usual disclaimer applies. Author contact: Pembroke College, Cambridge CB2 1RF, United Kingdom. E-mail: dnt22@cam.ac.uk.

[1] Indeed, Borio, English, and Filardo (2003) report that, since the mid-1980s, the rapid pace of financial liberalization has led to more-frequent financial booms and busts in developed and developing countries alike.

The key premise of this paper is that a welfare assessment has to consider the link between financial stability concerns and the credibility deficit often facing policymakers in developing countries. We explore this link by extending the symmetric preferences of the benchmark Barro and Gordon (1983) discretionary policy model with one-sided aversion to exchange rate depreciation. The recent adoption by many developing countries of floating exchange rates coupled with an inflation target has reinstated the policy relevance of the Barro-Gordon framework, which had earlier lost its appeal to self-fulfilling models of the authorities' decision to defend a fixed exchange rate under speculative pressure; see Jeanne and Masson (2000) and Obstfeld (1994, 1996).[2] Further, the lack of precommitment in the Barro-Gordon model reflects a tendency by policymakers in developing countries to use inflation surprises to improve the government's fiscal position.

We are motivated by two stylized empirical observations. First, developing-country policymakers typically pursue macroeconomic stabilization against a background of significant *financial fragility*, involving a negatively skewed supply-shock distribution and substantial balance sheet mismatch. The latter occurs along both currency and maturity dimensions, as the financial sector's liabilities are predominantly dollar denominated and short term, while its assets are home-currency denominated and long term. Consequently, Kaminsky and Reinhart (1999) find that devaluations in financially fragile economies tend to be recessionary. Although initially restricted to the financial sector, a devaluation's aftershocks spread to other sectors and result in widespread corporate failure and unemployment.

Second, although many developing countries responded to the financial crises by adopting a floating exchange rate coupled with an inflation target—the latter serving as a nominal anchor—in practice there is extensive *fear of floating.* As documented by Calvo and Reinhart (2001, 2002), nominal interest rates in financially fragile developing economies are much more volatile than expected future

[2]Emerging markets adopting inflation targeting since the Asian financial crises include Brazil (June 1999), Colombia (September 1999), the Czech Republic (January 1998), Mexico (January 1999), Poland (October 1998), South Africa (February 2000), South Korea (January 1998), and Thailand (April 2000).

depreciation, leading to massive rejection of the risk-neutral version of uncovered interest parity. Indeed, post-crisis economies continue to actively manage their currencies so as to limit exchange rate fluctuations, and there is strong evidence of fear of floating for a wide cross-section of countries formally classified as “floaters” by the International Monetary Fund.[3]

Against that background, we assume that the policy weight attached to fear of floating—originating with financial fragility concerns—is independent of the weight on inflation stabilization. This contrasts with the literature on optimal contracts for central bankers (Walsh 1995, 2003). There, it is the coefficient on current inflation that measures the optimal penalty factor, and its magnitude is derived endogenously as a function of the structural and preference fundamentals. As a result, expected inflation bias can be eliminated. Further, whereas in Walsh (1995) only current inflation enters the loss function, our one-period loss function stresses the role of inflation *change* (or, with purchasing power parity [PPP], depreciation change) as the prime determinant of financial fragility. It follows that, as the effects of the current inflation choice are incorporated in next period’s payoff, the central bank’s discount factor—or degree of patience—matters for average policy outcomes.[4] We thus introduce one-period persistence into the reduced-form model and solve a dynamic problem for the central bank, assuming that its choice of inflation last period affects expected welfare this period.

The main results are as follows. First, asymmetric aversion to exchange rate change imparts deflation bias to the economy, mitigating excess inflation due to time inconsistency. Given the relative magnitude of the asymmetric-preference coefficient, the deflationary impact of fear of floating decreases with the central bank’s discount factor. Thus, to the extent that policymakers in financially fragile developing countries tend to be constrained by shorter time horizons than their counterparts in advanced economies, the resulting decline in inflation bias is bigger.

[3]See Calvo (2005), Ganapolsky (2003), Lahiri and Végh (2001), Levy-Yeyati and Sturzenegger (2005), McKinnon and Schnabl (2004), and Reinhart (2000).

[4]We are grateful to an anonymous referee for raising this point.

Second, we show that optimal policy outcomes under fear of floating do not always yield higher expected welfare than the symmetric Barro-Gordon benchmark. The net impact of fear of floating is sensitive to the economy's recent inflation (depreciation) record, the extent of the credibility problem in monetary policy, and the policymaker's discount factor. Specifically, under symmetric information the supply shocks are observed by the private sector; hence they directly affect welfare. Adverse supply shocks in the last period improve expected welfare with fear of floating for the current period, all else equal. Although adverse shocks are sufficient for fear of floating to outperform the symmetric loss function, they are necessary only if monetary policy is fully credible. In the more realistic developing-country case where credibility is weak, fear of floating can improve upon the Barro-Gordon benchmark even if the last supply shock was expansionary. In contrast, if monetary policy is fully credible and the last shock is favorable, the symmetric benchmark is preferred on average.

Ceteris paribus, we also find that higher discount factors—amounting to longer policy horizons for central bankers—tend to generate higher average welfare with fear of floating, particularly if monetary policy credibility is strong. Conversely, lower discount factors have the opposite effect unless credibility is weak. We argue that the conditionality implicit in this result may be consistent with the "new-environment" (post-crises) view of monetary policy put forward by Borio, English, and Filardo (2003), according to which central banks should place more weight on safeguarding financial stability also in developed economies, where credibility is not an issue.

Third, the model can shed light on the result of Lahiri and Végh (2001) that fear of floating delivers less-variable inflation than the symmetric benchmark. Curiously, this empirical regularity exists despite the fact that countries with less-variable inflation also tend to be subject to larger external shocks. Assuming that the asymmetric-preference coefficient follows an AR(1) process that covaries negatively with supply shocks, so that adverse supply shocks and fear of floating are positively correlated, we show that fear of floating delivers less-variable inflation if the policymaker's discount factor is above a certain threshold. Thus, a longer policymaking horizon pays off in terms of lower inflation variability.

The paper's reduced-form approach is common to research on asymmetric policy preferences for the United States and other advanced economies; see Nobay and Peel (2003) and Ruge-Murcia (2003a, 2003b), among others. In that literature, the rationale for extending the monetary policy loss function involves asymmetric preferences over macroeconomic stabilization: depending on the inflation outcome, recession aversion may exceed inflation aversion or vice versa.

The preference asymmetry in Nobay and Peel (2003) is nonlinear (linex) and more general than the one considered here. In our case, positive deviations from target contribute an extra loss that is linear in last period's change in inflation/depreciation. This maintains the key result that $k = 0$ does not remove inflation bias. On account of the extra term, the change in expected inflation is unambiguously negative, i.e., expected appreciation. Inflation bias is independent of the higher moments of shocks. However, actual inflation exhibits one-period path dependence, which may be more relevant for developing countries, as they face more-extreme shocks than advanced economies.

In that respect, the paper is also motivated by research on the impact of path dependence on optimal monetary policy choice; see Drazen and Masson (1994). Whereas the methodology of these authors is related to second-generation models of conditional escape clauses from a fixed exchange rate mechanism, our approach follows that of third-generation crisis models stressing the role of financial fragility; see Aghion, Bacchetta, and Banerjee (2004), Calvo (1998, 2005), and Chang and Velasco (2000, 2001).

Our findings can be seen to offer theoretical support for the recent empirical survey of Eichengreen and Razo-Garcia (2006). Using de facto exchange rate data from a large number of countries, these authors report that "heavily managed" floating exchange rates continue to be more popular for emerging markets and developing countries than for advanced economies.[5] Applying the transition matrix methodology of Masson and Ruge-Murcia (2005), they also forecast that the first two country groups will only gradually move away from fear of floating, and indeed the move will extend

[5]Eichengreen and Razo-Garcia's (2006) data set comprises 24 advanced economies, 32 emerging markets, and 131 developing countries from 1990 to 2004.

beyond a twenty-year horizon. Significantly, the strong persistence of intermediate, rather than "bipolar," exchange rate regimes is positively related to countries' reluctance to lift capital controls, as found also by von Hagen and Zhou (2006). Hence, to the extent that maintaining capital controls is reflecting policymakers' concerns about financial fragility, the present framework highlights the implications of such concerns for average social welfare.

The paper proceeds as follows. Section 2 reviews the one-period Barro-Gordon discretionary monetary policy model. Section 3 extends this benchmark with a value function incorporating asymmetric depreciation aversion, obtains the average policy outcomes, and derives expected social welfare. Section 4 compares the two welfare alternatives and discusses the policy implications for developing countries, and section 5 concludes.

2. The Barro-Gordon Benchmark

Let s, p, and p^* denote the logs of the nominal exchange rate and the home and foreign price levels, respectively. Assuming PPP and constant foreign prices implies $\Delta s_t = p_t - p_{t-1} = \pi_t$, so depreciation and inflation coincide. PPP effectively converts the policymaker's inflation target to a target depreciation rate; see Calvo and Reinhart (2001) and Ho and McCauley (2003) for evidence that exchange rate pass-through is significantly higher in developing countries than it is in industrialized countries.

The one-period quadratic-symmetric policy loss function is

$$L_t^{BG} = (y_t - y^*)^2 + \chi(\pi_t - \pi^*)^2, \tag{1}$$

where $\chi > 0$ is the symmetric-preference coefficient, i.e., the relative weight on inflation stabilization. The inflation and output-growth targets $\pi^* \geq 0$ and $y^* \geq 0$ are constant, without loss of generality. It is common to assume that equation (1) also describes social welfare, so in the sequel we interchangeably refer to the policymaker's and society's losses.

Output growth follows the short-run aggregate supply function

$$y_t = \overline{y} + \alpha(\pi_t - E_{t-1}\pi_t) + \varepsilon_t, \tag{2}$$

where $E_{t-1}\pi_t$ is determined at $t-1$, α is the economy's inverse sacrifice ratio, and ε_t is an i.i.d. $(0, \sigma_\varepsilon)$ aggregate supply shock independent of π_t. Importantly, the short-term output-growth target, y^*, can exceed the economy's long-term potential, $\overline{y}$, by $y^* - \overline{y} = k > 0$. The magnitude of k is inversely related to the credibility of monetary policy. In developing and emerging-market economies, in particular, $k > 0$ reflects policymakers' tendency of using inflation surprises to improve the government's fiscal position. Overreliance on the inflation tax lowers the real value of government debt and erodes public-sector wages. Thus, although in principle one can allow for $k = 0$, reflecting *prudent discretion* by the monetary authority (Blinder 2000), this is arguably unrealistic for developing countries, especially in the aftermath of financial crises.[6]

Minimizing (1) subject to (2) and taking expectations using all information available at $t-1$ yields the Barro and Gordon (1983) and Kydland and Prescott (1977) inflation bias result: the short-term expansionary motive delivers average inflation above target with no average output gain,

$$\text{bias}_t^{BG} = E_{t-1}\pi_t^{BG} - \pi^* = \frac{k\alpha}{\chi} > 0\,,\ E_{t-1}^{BG} y_t = \overline{y}, \qquad (3)$$

where BG denotes optimal policy outcomes in the Barro-Gordon model. Equilibrium inflation, output growth, and their variabilities are

$$\pi_t^{BG} = \pi^* + \frac{k\alpha}{\chi} - \frac{\alpha}{\alpha^2+\chi}\varepsilon_t\,,\ y_t^{BG} = \overline{y} + \frac{\chi}{\alpha^2+\chi}\varepsilon_t$$
$$var\pi_t^{BG} = \frac{\alpha^2}{(\alpha^2+\chi)^2}\sigma_\varepsilon^2\,,\ vary_t^{BG} = \frac{\chi^2}{(\alpha^2+\chi)^2}\sigma_\varepsilon^2. \qquad (4)$$

It follows that average welfare declines at the square of inflation bias,

$$\begin{aligned} E_{t-1}L_t^{BG} &= E_{t-1}[(y_t - y^*)^2 + \chi(\pi_t - \pi^*)^2] \\ &= vary_t^{BG} + \chi(\text{bias}^{BG})^2. \end{aligned}$$

[6]The expansionary motive associated with $k > 0$ may also reflect labor-market distortions and/or political business-cycle considerations relevant for advanced economies. See Walsh (2003) for a review of the sources of, and responses to, time inconsistency in monetary policy.

Substituting (3) and (4) into this expression yields

$$E_{t-1}L_t^{BG} = \frac{\chi}{\alpha^2+\chi}\sigma_\varepsilon^2 + \frac{k^2\alpha^2}{\chi} + 2k\alpha\pi^* + \chi\pi^{*2}. \quad (5)$$

Hence, when monetary policy is guided by the symmetric losses in (1), social welfare deteriorates with the expansionary motive, driving the case for overcoming time inconsistency using a reputational mechanism and/or commitment technology to set $k = 0$. Note that a more ambitious inflation target is also welfare improving, all else equal.

3. Monetary Policy with Fear of Floating

3.1 Asymmetric Aversion to Exchange Rate Change

In the presence of nominal wage rigidities, alternative microfoundations for the cost of exchange rate fluctuations turn on the negative impact of exchange rate changes on output and employment. A rationale for fear of floating then arises through the real costs of exchange rate variability, so fluctuations are costly regardless of the direction of movement; see, for example, Lahiri and Végh (2001). In this paper we assume, instead, that depreciation of the home currency involves a social cost independent of stabilization efforts. The extra cost is motivated by foreign currency exposure of the corporate sector and the resulting financial fragility.

As discussed in the introduction, monetary policy preferences may be asymmetric if a substantial component of the financial sector's liabilities is dollarized; exchange rate devaluations can then often be recessionary, and appreciations expansionary. In this context, it is often developing countries' *original sin*—defined as the de facto inability to borrow in their home currency—that underlies financial fragility and induces fear of floating.[7]

We thus propose capturing asymmetric aversion to exchange rate fluctuations using the one-period loss function

$$L_t^{FF} = (y_t - y^*)^2 + \chi(\pi_t - \pi^*)^2 + 2\varphi\Delta\pi_t\,,\ \varphi > 0, \quad (6)$$

[7] On the symbiotic relationship between financial fragility and original sin in developing countries, see the contributions in Eichengreen and Hausmann (2004).

where $\Delta\pi_t = \pi_t - \pi_{t-1}$ is the one-period change in inflation. Coefficient φ captures the policymaker's asymmetric aversion to exchange rate depreciation, while χ measures the (symmetric) weight on inflation versus output stabilization. Importantly, the strength of fear of floating is assumed independent of χ because the underlying financial fragility is taken as given. Thus, the relative magnitudes of φ and χ reflect the weight of fear of floating and inflation stabilization as independent monetary policy objectives.

Note that the linear term in loss function (6) will only add a constant to the first-order condition. This contrasts with the nonlinear (linex) preferences of Nobay and Peel (2003), where positive inflation deviations from target change the slope of the loss function. In their very flexible reduced form, the magnitude of excess inflation depends on higher moments of supply shocks. The authors then need to assume that actual inflation is conditionally normally distributed in order to get closed-form solutions for expected inflation. We argue that, despite the consequent greater generality, this assumption does not fit the experience of developing countries experiencing sharp devaluations.

The specification of loss function (6) is similar to that of the literature on optimal central bank contracts, introduced by Walsh (1995). There, the additional linear term is interpreted as a linear contract between the central bank and the government. However, the central bank is penalized for higher inflation, so it is the *level* and not the first-difference of inflation that enters the loss function. Walsh then shows there is an optimal level of the penalty factor that eliminates inflation bias, and that level is linear in actual inflation: $t(\pi_t) = t_0 - \frac{\alpha k}{\chi}\pi_t$. Thus, given π_t, the optimal linear contract penalizes the central bank relatively more the higher the value of k.[8]

By contrast, in our model the strength of fear of floating (φ) is exogenously determined by the underlying fragility of the banking and corporate sector, whose degree of foreign currency exposure and risk of devaluation due to sharp reversals on capital accounts—Calvo's "sudden stops"—are in principle both unrelated to the central bank's inflation aversion. Moreover, as it is the one-period *change* in inflation/depreciation that matters to current welfare, the

[8] In Nobay and Peel (2003) the variances of inflation and output also depend on $t(\cdot)$, the optimal penalty factor.

central bank takes into account the effects of its previous inflation choice on this period's payoff. Therefore, the one-period loss function L_t^{FF} from equation (6) enters recursively into the following value function $V(\cdot)$,

$$V(\pi_{t-1}) = \max_{\{\pi_t\}}[(y_t - y^*)^2 + \chi(\pi_t - \pi^*)^2 + 2\varphi(\pi_t - \pi_{t-1}) + \beta E_{t-1}V(\pi_t)], \tag{7}$$

where $\beta \in (0, 1)$ is the policy discount factor, assumed constant for simplicity.

3.2 Optimal Monetary Policy

We proceed to solve the central bank's dynamic optimization problem. Substituting linear Phillips curve (2) into value function (7) implies

$$V(\pi_{t-1}) = \max_{\{\pi_t\}}(\overline{y} + \alpha\pi_t - \alpha E_{t-1}\pi_t + \varepsilon_t - y^*)^2 + \chi(\pi_t - \pi^*)^2 + 2\varphi(\pi_t - \pi_{t-1}) + \beta E_{t-1}V(\pi_t). \tag{8}$$

Maximizing (8) with respect to π_t and applying the envelope theorem $V'(\pi_{t-1}) = -2\varphi$ yields the first-order condition

$$\alpha(\overline{y} + \alpha\pi_t - \alpha E_{t-1}\pi_t + \varepsilon_t - y^*) + \chi(\pi_t - \pi^*) = \varphi(\beta - 1). \tag{9}$$

Taking expectations at $t - 1$, the average policy outcomes, denoted FF, are just

$$\begin{aligned} \text{bias}_t^{FF} &= E_{t-1}\pi_t^{FF} - \pi^* = \frac{k\alpha - \varphi(1-\beta)}{\chi} < \text{bias}_t^{BG} \\ E_{t-1}y_t^{FF} &= E_{t-1}y_t^{BG} = \overline{y}. \end{aligned} \tag{10}$$

The deflationary impact of fear of floating is $-\varphi(1 - \beta)/\chi < 0$, mitigating excess inflation under the Barro-Gordon benchmark with no change to average output growth. Provided $\varphi > 0$, note that the reduction in average inflation decreases in the discount factor β. Thus, the deflationary contribution of asymmetric aversion to exchange rate depreciation declines with the central bank's rate of time preference.

Substituting (10) into (9) yields period-t equilibrium inflation and output growth:

$$\pi_t^{FF} = \pi^* + \frac{k\alpha - \varphi(1-\beta)}{\chi} - \frac{\alpha}{\alpha^2+\chi}\,\varepsilon_t$$
$$y_t^{FF} = \overline{y} + \frac{\chi}{\alpha^2+\chi}\varepsilon_t. \qquad (11)$$

Comparing equations (11) and (4) suggests that inflation and output variability are unchanged from the Barro-Gordon benchmark. That is because optimality condition (9) is still linear in the current supply shock. However, equilibrium inflation/depreciation bias is lower on account of fear of floating. As a result, the implications for average welfare are nontrivial.

3.3 Equilibrium Social Welfare

Expected welfare losses under fear of floating combine the output variability and squared inflation bias terms, due to symmetric losses, with the social cost of financial fragility:

$$E_{t-1}L_t^{FF} = vary_t^{FF} + \chi\left(\text{bias}_t^{FF}\right)^2 + 2\varphi E_{t-1}\Delta\pi_t^{FF}$$
$$= \frac{\chi}{\alpha^2+\chi}\sigma_\varepsilon^2 + \frac{1}{\chi}[k\alpha - \varphi(1-\beta)]^2 + \frac{2\varphi\alpha}{\alpha^2+\chi}\varepsilon_{t-1}. \qquad (12)$$

As output variability is unchanged under the alternative loss functions, their expected welfare differential reduces to

$$E_{t-1}\Delta L_t \equiv E_{t-1}L_t^{FF} - E_{t-1}L_t^{BG}$$
$$= \chi\left[\left(\text{bias}_t^{FF}\right)^2 - \left(\text{bias}_t^{BG}\right)^2\right] + 2\varphi E_{t-1}\Delta\pi_t^{FF}. \qquad (13)$$

If the above expression is positive (negative), fear of floating generates higher (lower) expected welfare losses than the symmetric benchmark. Substituting into (13) the inflation bias expressions, from (3) and (10), and the difference between expected inflation for period t and actual inflation at $t-1$ under fear of floating, from (11), yields

$$E_{t-1}\Delta L_t = \frac{\varphi(1-\beta)}{\chi}[\varphi(1-\beta) - 2k\alpha] + \frac{2\varphi\alpha}{\alpha^2+\chi}\varepsilon_{t-1}. \qquad (14)$$

Thus, placing more policy weight on inflation stabilization (χ) narrows the average welfare gap between the two loss functions. Given $\chi > 0$, the expected welfare gap is a function of the relative importance of asymmetric depreciation aversion, captured by φ; the latest shock realization ε_{t-1}; the policymaker's discount factor or "degree of patience" β; and the economy's sacrifice ratio $1/\alpha$. The dependence of average welfare on β reflects the one-period persistence built into the preference asymmetry.

The first two terms in (14) combine the welfare impact of fear of floating, imperfect credibility, and the policymaker's rate of time preference. Note that $-2k\alpha\varphi(1-\beta)/\chi < 0$ for all $k > 0$.[9] Therefore, less-credible monetary policy strengthens the case for fear of floating in the loss function, ceteris paribus. Conversely, building up credibility—for example, by legislating central bank independence into the constitution—weakens the case for fear of floating for all $\varphi > 0$.

Equation (14) yields a necessary and sufficient condition for fear of floating to outperform the Barro-Gordon benchmark in expectation:

$$E_{t-1}\Delta L_t < 0 \Leftrightarrow \frac{1-\beta}{\chi}[\varphi(1-\beta) - 2\alpha k] + \frac{2\alpha}{\alpha^2+\chi}\varepsilon_{t-1} < 0. \quad (15)$$

Inequality (15) implies the following upper bound for asymmetric-preference coefficient φ in order for fear of floating to be preferred:[10]

$$\varphi < \varphi^{\max} = \frac{2\alpha}{1-\beta}\left[k - \frac{\chi}{(\alpha^2+\chi)(1-\beta)}\ \varepsilon_{t-1}\right]. \quad (16)$$

The magnitude of $\varphi^{\max}$ then acts as a "welfare threshold" for the presence of fear of floating. Put differently, a necessary condition for fear of floating to arise is $\varphi^{\max} > 0$. In the next section, we examine the welfare link between credibility problems, fear of floating, and the economy's inflation record.

[9] From equations (5) and (12), expected welfare losses under the benchmark and fear of floating both include a term in k^2, so that cancels out of their difference.

[10] The Barro-Gordon benchmark does better on average if $\varphi > \varphi^{\max}$, and the policymaker is indifferent between the two welfare alternatives if $\varphi = \varphi^{\max}$.

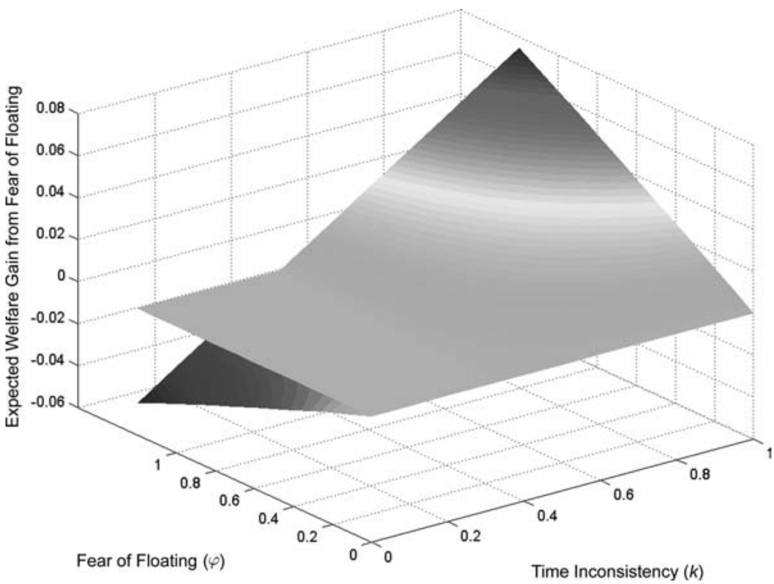
4. Policy Implications for Developing Countries

4.1 Comparative Welfare Evaluation

We first address the case of full credibility. When $k = 0$, (16) implies that any favorable shock at $t-1$ induces $\varphi^{\max} < 0$. The inequality is then violated for all $\varphi > 0$. This is represented graphically in figure 1, where a three-dimensional surface of the expected welfare gain from fear of floating is plotted over the Barro-Gordon benchmark—i.e., minus the expected loss differential in equation (14)—for an arbitrary positive (favorable) supply shock, $\varepsilon_{t-1} = 0.04$.[11]

The symmetric and asymmetric alternatives perform better, respectively, for negative and positive expected welfare gain

Figure 1. Average Welfare Comparison with Favorable Supply Shocks



[11]For illustration purposes, we fix $\alpha = \chi = 1$, $\beta = 0.95$, and $\sigma_\varepsilon = 0.12$, and let k and φ vary from 0 to 1 and 1.2 in steps of size 0.1. This generates a grid of 143 points for $[k, \varphi]$ over which we evaluate the expected welfare gain from fear of floating. The x-, y-, and z-axes represent k, φ, and $-E_{t-1}\Delta L_t$, respectively.

values on the z-axis. To facilitate the comparison, we also show the flat "zero plane" where the welfare alternatives are at par ($E_{t-1}\Delta L_t = 0$). The case $k = 0$ corresponds to points along the φ-axis. Note that the expected gain from fear of floating is always negative—the welfare surface lies below the zero plane—hence the Barro-Gordon benchmark is preferred.

Turning to the general case of imperfect credibility, $k > 0$, inequality (16) suggests policymakers then have a greater $\varphi^{\max}$ threshold. In figure 1, the expected gain from fear of floating grows smoothly with k; for the particular parameter and shock values, the welfare surface crosses the zero plane near $k = 0.4$. If credibility weakens further, then fear of floating outperforms the benchmark. When $k > 0$, figure 1 also shows that the welfare surface slopes up along the φ-axis. The expected gain from fear of floating rises with the asymmetric-preference coefficient; it is greatest when k and φ are both large. Thus, credibility problems help explain why developing countries are likely to have bigger $\varphi^{\max}$ values than industrialized countries. Faced with imperfect credibility, developing-country policymakers tend to be more reluctant to let their home currency depreciate than their counterparts in developed economies, where time inconsistency is not an issue and k is near 0.

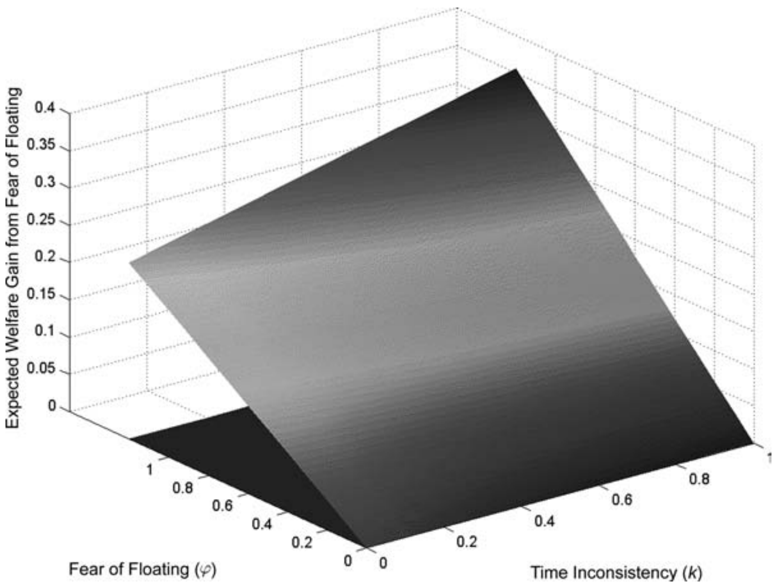
We next analyze the case of negative (adverse) supply shocks at $t - 1$. From the last term in expression (16), any $\varepsilon_{t-1} < 0$ results in higher $\varphi^{\max}$; hence fear of floating outperforms the benchmark for a wider range of shocks.[12] The welfare comparison for the same parameter values as above and $\varepsilon_{t-1} = -0.11$ is shown in figure 2.

The expected welfare gain from fear of floating is now in the range $[0, 0.35]$, compared to $[-0.06, 0.08]$ in figure 1. Indeed, the zero plane lies below the welfare surface at every point on the $[k, \varphi]$ grid, so the two welfare alternatives are at par only in the limiting case of no fear of floating ($\varphi = 0$).

The stronger case for asymmetric preferences relates to value function (7). Recall that any nonzero shock at $t-1$ is observed by the private sector, so the expected welfare gap for period t depends on the previous inflation rate. The additional losses due to the change in depreciation introduce path dependence to the model, and that

[12] An adverse shock realization at $t - 1$ is sufficient for $\varphi^{\max} > 0$ and, if $k = 0$, also necessary.

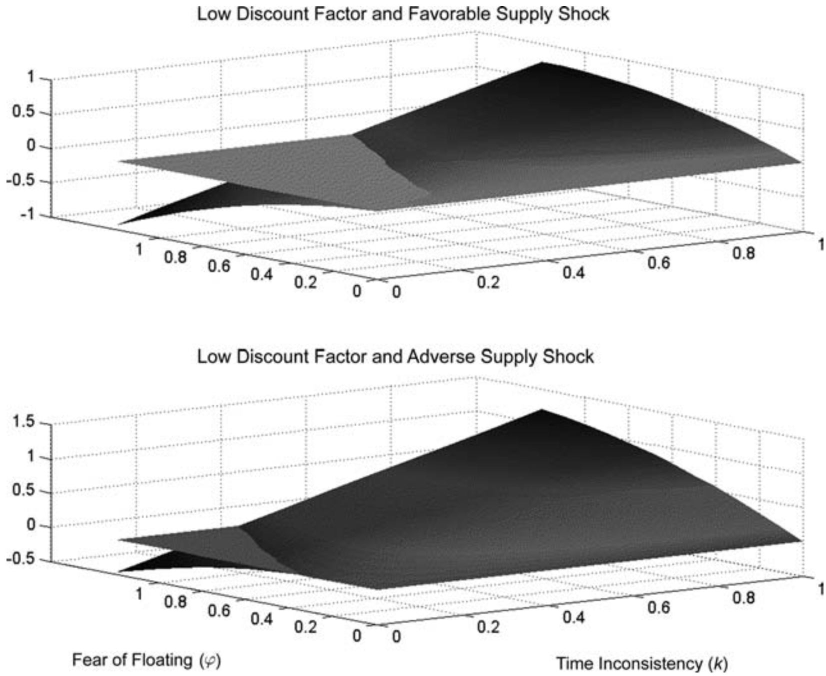
Figure 2. Average Welfare Comparison with Adverse Supply Shocks



is independent of the rate of time preference. Path dependence is asymmetric: adverse supply shocks improve average welfare under fear of floating, while favorable ones render it more costly. Intuitively, the underlying financial fragility becomes highly relevant, or *salient*, immediately following a severe financial crisis triggered by devaluation. Therefore, to the extent that developing and emerging-market economies tend to be characterized by negatively skewed shock distributions, fear-of-floating behavior is more appropriate from a welfare point of view.

Lastly, we assess the welfare impact of changes to the central bank's rate of time preference. Differentiating equation (14) with respect to β yields $\frac{\partial E_{t-1}\Delta L_t}{\partial \beta} = \frac{2\varphi}{\chi}[k\alpha - \varphi(1-\beta)]$. With imperfect credibility, and assuming β is not too small, this expression is positive, suggesting that $\varphi > 0$ becomes more costly with a higher discount factor. If k is at or close to 0, however, then $\frac{\partial E_{t-1}\Delta L_t}{\partial \beta} < 0$ for all $\beta \in (0,1)$. Ceteris paribus, higher β then implies that the expected welfare gain from fear of floating increases.

Figure 3. Average Welfare Comparison



These comparative statics are supported by figure 3, in which the behavior of $-E_{t-1}\Delta L_t$ is graphed against k and φ.

The top panel combines the favorable shock from figure 1 ($\varepsilon_{t-1} = 0.04$) with $\beta = 0.30$, without loss of generality.[13] Asymmetric preferences unambiguously lower average welfare for values of k smaller than about 0.2. However, if credibility worsens ($k > 0.2$), then fear of floating still outperforms the benchmark. This property is also highlighted in the lower panel of figure 3, which combines a large adverse shock and a low discount factor, $\varepsilon_{t-1} = -0.11$ and $\beta = 0.30$. Note that, compared to the top panel, less of the welfare surface now lies below the zero plane. Thus, the earlier implication that $\varphi > 0$ improves average welfare following adverse

[13]Welfare surfaces conditional on different β and ε_{t-1} combinations are available upon request.

supply shocks is robust to the discount factor, provided credibility is imperfect.

Equivalently, lower discount factors render fear of floating less appropriate than the Barro-Gordon benchmark, particularly if credibility is strong. If credibility problems are persistent, however, then fear of floating outperforms the benchmark even with a low discount factor. Interestingly, such conditioning of the average welfare performance on policymakers' progress on the credibility front is in line with Eichengreen and Razo-Garcia's (2006) projection that developing-country policymakers will only gradually abandon "intermediate" exchange rate regimes and float their currencies. These authors' forward-looking conclusion is that as emerging-market economies build up their institutions and develop liquid financial markets, they enter a virtuous circle (a "path to prudence") through which fear of floating will remain attractive.

In the context of industrialized countries—whose k values are arguably smaller than those of developing countries—the finding that fear of floating combined with large β improves average welfare is also related to the "new-environment" view of monetary policy (Borio, English, and Filardo 2003). According to that view, central banks in advanced economies need to place greater weight on financial (and exchange rate) imbalances when calibrating monetary policy; they may, consequently, also require a longer horizon for evaluating policy alternatives. Thus, proxying longer policymaking horizons with higher β values, this finding appears consistent with the "new-environment" view that financial stability may be an independent objective of monetary policy, also when credibility is perfect.

4.2 Fear of Floating and Inflation Variability

A key observation of Lahiri and Végh (2001) is that fear of floating results in less-variable inflation rates in countries that are subject to larger shocks. That is, instead of a monotonic relationship between nominal exchange rate variability and the size of supply shocks, the extra cost of currency depreciation appears to lower inflation variability in developing economies displaying fear-of-floating behavior.

If fear of floating is considered to be time varying, then our reduced-form framework can shed light on this stylized fact. To illustrate, assume that asymmetric-preference coefficient φ follows the stationary process

$$\varphi_t = \theta\varphi_{t-1} - \varepsilon_t, \tag{17}$$

where $\theta \in (0, 1)$ measures the persistence of φ_t, and ε_t is the period-t supply shock. Equation (17) implies the covariance between φ_t and ε_t is always negative: $cov(\varphi_t, \varepsilon_t) = -\sigma_\varepsilon^2 < 0$. Following the discussion in section 3.1, the intuition is that the underlying financial fragility deteriorates with adverse supply shocks. Hence, adverse shocks induce *more* fear of floating, while favorable shocks have the opposite effect.

Substituting (17) into equation (11), equilibrium inflation variability under fear of floating becomes

$$\begin{aligned} var\pi_t^{FF} &= \left[\frac{\alpha^2}{(\alpha^2+\chi)^2} + \frac{(1-\beta)^2}{\chi^2(1-\theta^2)}\right]\sigma_\varepsilon^2 + \frac{2\alpha(1-\beta)}{\chi(\alpha^2+\chi)}\, cov(\varphi_t, \varepsilon_t) \\ &= \left[\frac{\alpha^2}{(\alpha^2+\chi)^2} + \frac{(1-\beta)^2}{\chi^2(1-\theta^2)} - \frac{2\alpha(1-\beta)}{\chi(\alpha^2+\chi)}\right]\sigma_\varepsilon^2. \end{aligned} \tag{18}$$

Note that inflation variability increases with σ_ε^2 by the constant factor of proportionality in square brackets. Thus, in comparing inflation variability under fear of floating with its equilibrium value in the Barro-Gordon model, from equation (4), supply-shock variability cancels out, and the relative position of the two welfare alternatives will depend only on reduced-form parameters α, β, χ, and θ.

Comparing expressions (4) and (18), the asymmetric alternative delivers less-variable inflation than the Barro-Gordon benchmark, $var\pi_t^{FF} < var\pi_t^{BG}$, if and only if

$$\beta > \beta_{\min} = 1 - \frac{2\alpha\chi(1-\theta^2)}{\alpha^2+\chi}. \tag{19}$$

Inequality (19) suggests that the volatility comparison is driven by a lower bound for the central bank's discount factor. For values of β above (below) $\beta_{\min}$, fear of floating generates less (more) volatile inflation than the benchmark.[14] To build intuition for this result, from equation (18) note that bigger discount factors lower the contribution of σ_ε^2 on inflation variability at a quadratic rate but raise the contribution of $cov(\varphi_t, \varepsilon_t) = -\sigma_\varepsilon^2$ at a linear rate. The net impact of a longer policymaking horizon is then to lower equilibrium inflation variability under fear of floating. We tentatively conclude that the stylized fact of Lahiri and Végh (2001) is consistent with one-sided depreciation aversion, provided the central bank is sufficiently "patient."

5. Concluding Remarks

This paper presented an explicit welfare evaluation of the loss function of the Barro-Gordon discretionary model, on one hand, and monetary policy preferences displaying asymmetric aversion to exchange rate depreciation in addition to the "twin" objectives of inflation and output stabilization, on the other. Persistent fear-of-floating behavior by policymakers arises because financial fragility has adverse systemic spillovers. For emerging-market economies, in particular, there is growing consensus that output costs are significantly higher when financial crises coincide with currency crises.

It was found that average social welfare does not unambiguously improve when the central bank employs the asymmetric loss function. The expected welfare differential of fear of floating vis-à-vis the benchmark depends on the underlying financial fragility, the credibility of the monetary policy framework, the economy's recent inflation experience, and the policymaker's rate of time preference. Conditional on these influences, the results indicate that, while accounting for financial fragility can improve average social welfare for developing and developed economies alike, fear-of-floating behavior is better suited to the former than the latter group. Finally, the welfare impact of the discount factor and credibility concerns appears consistent with recent empirical work suggesting that de facto floating exchange rates are chosen by countries at intermediate stages

[14] From the right-hand side of (19) it is easy to check that $\frac{\partial \beta_{\min}}{\partial \theta}$ and $\frac{\partial \beta_{\min}}{\partial \chi}$ are both positive, while $\frac{\partial \beta_{\min}}{\partial \alpha} > 0$ for $\alpha^2 > \chi$ and negative otherwise.

of development adopting a gradualist approach to liberalizing their capital account.

References

Aghion, P., P. Bacchetta, and A. Banerjee. 2004. "A Corporate Balance-Sheet Approach to Currency Crises." *Journal of Economic Theory* 119 (1): 6–30.

Barro, R. J., and D. Gordon. 1983. "A Positive Theory of Monetary Policy in a Natural Rate Model." *Journal of Political Economy* 91 (4): 589–610.

Blinder, A. S. 2000. "Central Bank Credibility: Why Do We Care? How Do We Build It?" *American Economic Review* 90 (5): 1421–31.

Borio, C., W. English, and A. Filardo. 2003. "A Tale of Two Perspectives: Old or New Challenges for Monetary Policy?" BIS Working Paper No. 127.

Calvo, G. A. 1998. "Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops." *Journal of Applied Economics* 1: 35–54.

———. 2005. *Emerging Capital Markets in Turmoil*: *Bad Luck or Bad Policy?* Cambridge, MA: MIT Press.

Calvo, G., and C. Reinhart. 2001. "Fixing for Your Life." In *Brookings Trade Forum 2000*: *Policy Challenges in the Next Millenium*, ed. W. M. Collins and D. Rodrik. Washington, DC: Brookings Institution.

———. 2002. "Fear of Floating." *The Quarterly Journal of Economics* 117 (2): 379–408.

Chang, R., and A. Velasco. 2000. "Banks, Debt Maturity and Financial Crises." *Journal of International Economics* 51 (1): 169–94.

———. 2001. "Monetary Policy in a Dollarized Economy Where Balance Sheets Matter." *Journal of Development Economics* 66 (2): 445–64.

Drazen, A., and P. R. Masson. 1994. "Credibility of Policies versus Credibility of Policymakers." *The Quarterly Journal of Economics* 109 (3): 735–54.

Eichengreen, B., and R. Hausmann, eds. 2004. *Other People's Money: Debt Denomination and Financial Instability in Emerging Market Economies.* Chicago: University of Chicago Press.

Eichengreen, B., and R. Razo-Garcia. 2006. "The International Monetary System in the Last and Next 20 Years." *Economic Policy* 21 (47): 393–442.

Ganapolsky, E. 2003. "Optimal Fear of Floating: The Role of Currency Mismatches and Fiscal Constraints." Federal Reserve Bank of Atlanta Working Paper No. 2003-31 (October).

Goodhart, C., and G. Illing, eds. 2002. *Financial Crises, Contagion and the Lender of Last Resort: A Reader*. New York: Oxford University Press.

Ho, C., and R. McCauley. 2003. "Living with Flexible Exchange Rates: Issues and Recent Experience in Inflation Targeting Emerging Market Economies." BIS Working Paper No. 130.

Jeanne, O., and P. Masson. 2000. "Currency Crises, Sunspots and Markov-Switching Regimes." *Journal of International Economics* 50 (2): 327–50.

Kaminsky, G. L., and C. M. Reinhart. 1999. "The Twin Crises: The Causes of Banking and Balance-of-Payments Problems." *American Economic Review* 89 (3): 473–500.

Kydland, F. E., and E. C. Prescott. 1977. "Rules Rather than Discretion: The Inconsistency of Optimal Plans." *Journal of Political Economy* 85 (3): 473–91.

Lahiri, A., and C. Végh. 2001. "Living with the Fear of Floating: An Optimal Policy Perspective." In *Preventing Currency Crises in Emerging Markets*, ed. S. Edwards and J. Frankel. Chicago: University of Chicago Press for the National Bureau of Economic Research.

Levy-Yeyati, E., and F. Sturzenegger. 2005. "Classifying Exchange Rate Regimes: Deeds vs. Words." *European Economic Review* 49 (6): 1603–35.

Masson, P., and F. J. Ruge-Murcia. 2005. "Explaining the Transition between Exchange Rate Regimes." *The Scandinavian Journal of Economics* 107 (2): 261–78.

McKinnon, R., and G. Schnabl. 2004. "The East Asian Dollar Standard, Fear of Floating, and Original Sin." *Review of Development Economics* 8 (3): 331–60.

Nobay, R. A., and D. A. Peel. 2003. "Optimal Discretionary Monetary Policy in a Model of Asymmetric Central Bank Preferences." *The Economic Journal* 113 (489): 657–65.

Obstfeld, M. 1994. "The Logic of Currency Crises." *Cahiers Economiques et Monetaires* 43: 189–214.

———. 1996. "Models of Currency Crises with Self-Fulfilling Features." *European Economic Review* 40 (3–5): 1037–47.

Reinhart, C. M. 2000. "The Mirage of Floating Exchange Rates." *American Economic Review* 90 (2): 65–70.

Ruge-Murcia, F. J. 2003a. "Does the Barro-Gordon Model Explain the Behavior of U.S. Inflation? A Reexamination of the Empirical Evidence." *Journal of Monetary Economics* 50 (6): 1375–90.

———. 2003b. "Inflation Targeting under Asymmetric Preferences." *Journal of Money, Credit, and Banking* 35 (October): 763–85.

Svensson, L. E. O. 2002. "Monetary Policy and Real Stabilization." In *Rethinking Stabilization Policy*, 261–312. Jackson Hole, WY: Symposium sponsored by the Federal Reserve Bank of Kansas City.

Svensson, L. E. O., and M. Woodford. 2003. "Indicator Variables for Optimal Policy." *Journal of Monetary Economics* 50 (3): 691–720.

von Hagen, J., and J. Zhou. 2006. "The Interaction between Capital Controls and Exchange Rate Regimes: Evidence from Developing Countries." CEPR Discussion Paper No. 5537.

Walsh, C. 1995. "Optimal Contracts for Central Bankers." *American Economic Review* 85 (1): 150–67.

———. 2003. *Monetary Theory and Policy*. 2nd ed. Cambridge, MA: MIT Press.