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Monetary Policy Inertia or Persistent Shocks: A DSGE Analysis*

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In this paper, we propose a simple econometric framework to disentangle the respective roles of monetary policy inertia and persistent shocks in interest rate rules. We exploit the restrictions of a DSGE model that is confronted with a monetary SVAR. We show that, provided enough informative variables are included in the formal test, the data favor a monetary policy representation with modest inertia and highly serially correlated monetary shocks. To the contrary, when the procedure is based solely on the dynamic behavior of the nominal interest rate, no clear-cut conclusion can be reached about the correct representation of monetary policy.

JEL Codes: C52, E31, E32, E52.

1. Introduction

The purpose of this paper is to investigate whether the dynamics of the nominal interest rate are better described as featuring monetary policy inertia or as characterized by highly persistent factors or shocks. This paper deals with this long-debated issue by reconciling some of the earlier, inconclusive results based on single-equation

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estimates. Essentially, we show that a multivariate system brings useful information to answer this question. Using such an approach, we show that the data favor a representation of monetary policy with serially correlated shocks and modest policy inertia.

Over the recent years, there has been a renewed interest in modeling monetary policymaking in terms of simple rules. In this context, the Taylor rule has become the workhorse description of central bank behavior. Nevertheless, Taylor (1993) pointed out that one should not expect that policymakers "follow policy rules mechanically."[1] Instead, one should consider the Taylor rule as a "hypothetical but representative" description summarizing the complex process of monetary policy.

Applied monetary economists have followed this general guideline by specifying and estimating extended Taylor rules to better approximate the central bank policy. An important result obtained in the literature, as exemplified by Clarida, Galí, and Gertler (2000), is that the lagged interest rate is highly significant in the estimated policy rule, suggesting that the nominal interest rate exhibits a sizable degree of inertia.[2] It has been argued that this apparent inertia might result from deliberate *policy inertia* from the central bank, the latter enforcing a partial-adjustment process on its instrument. However, Rudebusch (2002, 2006) claimed that, if the central bank actually smoothes its policy, then future adjustments in the interest rate should be largely predictable. Unfortunately, Rudebusch (2002) showed that this is not supported by financial data (see also Söderlind, Söderström, and Vredin 2005).

An alternative explanation to the persistence of monetary policy is the presence of serially correlated shocks in the realizations of the interest rate. These shocks represent a set of special factors that cannot be systematically modeled by a simple, parsimonious interest rate rule such as an augmented Taylor rule. If these factors are persistent, the interest rate will display inertia.

[1]See also Taylor (1999) for different possible interpretations of this monetary policy rule.

[2]This result was also found by Amato and Laubach (2003), Kozicki (1999), Levin, Wieland, and Williams (1999), and Sack and Wieland (2000), among others.

While these two competing views entail very different conclusions about the behavior of central banks, aggregate data have been fairly silent as to which is the correct representation of actual policy. For instance, Rudebusch (2002) cannot distinguish between the two competing specifications in an estimated interest rate rule. English, Nelson, and Sack (2003) find that there is supportive evidence for both representations to be significant components of the Federal Reserve behavior. Castelnuovo (2003) suggests that both views of monetary policy are equally important to describe the central bank decisions.[3] This lack of a clear-cut conclusion may be due to a well-known problem of identification and multiple optima typically arising in models of partial adjustment with serially correlated shocks. The latter calls into question the use of a single equation—i.e., a Taylor rule taken in isolation—as a proper way to discriminate between the two competing views of monetary policy.

To eschew this identification problem, we propose to resort to a dynamic stochastic general equilibrium (DSGE) model to interpret the data and disentangle these two alternative views about monetary policy. An important and celebrated virtue of such models is that they can generate very different aggregate dynamics when subjected to different policy rules, and this holds not only for the nominal interest rate but also for a broader set of macroeconomic variables. We build on this property to assess which of the two views generates aggregate dynamics in accordance with the data.

In order to implement these ideas, we resort to a limited-information approach that allows us to exclusively focus on that portion of aggregate fluctuations due to monetary shocks in U.S. data. We first estimate a structural vector autoregression (SVAR) with short-run restrictions to identify monetary policy shocks and the implied impulse response functions (IRFs) of a set of aggregate variables. Second, we estimate the DSGE parameters that govern policy inertia and the amount of serial correlation in monetary shocks. These parameters are pinned down so that the DSGE model matches as well as possible the IRFs drawn from the SVAR.

[3] Apel and Jansson (2005) and Gerlach-Kristen (2004) find similar results using Kalman filtering to account for omitted unobserved factors in the interest rate rule.

When we consider the IRFs of output, inflation, wage inflation, the federal funds rate, and money growth, we are able to unambiguously discriminate between the two different representations. Our results suggest that the dynamics of these variables are better fitted by a scheme with moderate policy inertia and a high degree of serial correlation. Thus, the smoothness in the interest rate is mainly explained by persistent factors beyond the target level of the policy rule. In contrast, when we consider only the responses of the federal funds rate, we find that there is not enough information to discriminate between the two views. Therefore, we insist that in order to disentangle the relative importance of each regime, one should take into account informative features of the data. In our case, this role is devoted to the responses of inflation and wage inflation. We also investigate two practical differences between these two alternative views about monetary policy: (i) in terms of the responsiveness of the nominal interest rate to inflation and the output gap; (ii) in terms of how the economy responds under the two rules during a specific episode, namely, the Volcker disinflation. In both cases, our findings confirm that the persistent-shocks view provides a more accurate approximation of actual monetary policy.

The remainder of the paper is organized as follows. Section 2 presents the monetary policy rule and discusses identification problems when partial adjustment and serial correlation are included. Section 3 describes the main ingredients of the DSGE model used in our empirical exercise. Section 4 presents the econometric approach employed. Section 5 discusses the main estimation results. Practical differences between policy inertia and persistent shocks are reviewed in section 6. Finally, the last section offers some concluding comments.

2. Identification Problems with Monetary Policy Rules

Following recent studies that have estimated models of central bank behavior, we postulate a monetary policy rule of the form

$$\hat{\imath}_t^\star = a_\pi \hat{\pi}_t + a_y \hat{y}_t, \tag{1}$$

$$\hat{\imath}_t = \rho_i \hat{\imath}_{t-1} + (1 - \rho_i)\hat{\imath}_t^\star + e_t, \tag{2}$$

$$e_t = \rho_e e_{t-1} + \nu_t, \quad \nu_t \sim \text{iid}\big(0, \sigma_\nu^2\big). \tag{3}$$

Equation (1) specifies how the target level $\hat{\imath}_t^\star$ evolves in response to current inflation $\hat{\pi}_t$ and output $\hat{y}_t$. More precisely, a_π and a_y govern the sensitivity of the desired level of the nominal interest rate to the log-deviations of inflation and output, respectively.[4] If the actual nominal interest rate $\hat{\imath}_t$ were equal to $\hat{\imath}_t^\star$, this would correspond to the policy rule proposed by Taylor (1993). Instead, equation (2) allows for a partial adjustment of the nominal interest rate to its target level at rate ρ_i. In addition, the rule is hit by monetary shocks e_t. If the latter were i.i.d., this would correspond to a standard specification for an augmented Taylor rule. Instead, equation (3) specifies a parametric model of serial correlation in e_t that can potentially account for part of the actual persistence found in $\hat{\imath}_t$. These shocks may represent any contingent event the central bank faces when deciding the interest rate, such as credit crunches or financial crises (see Rudebusch 2002 or Taylor 1993). Moreover, the use of real-time data could also reinforce the apparent degree of serial correlation in policy shocks (Orphanides 2004). In addition, a persistent change of the inflation target can be interpreted as a serially correlated shock to monetary policy (see Smets and Wouters 2003, 2005).

The empirical literature on Taylor rules has had trouble reaching a clear-cut conclusion about the correct representation of monetary policy. Although there is no evidence that the partial-adjustment hypothesis is fully responsible for the significance of the lagged interest rate term, there is also no evidence supporting the total rejection of monetary policy inertia.[5] The absence of a clear-cut conclusion is in part due to a well-known problem of identification and multiple optima in the partial-adjustment model with serially correlated shocks (see, e.g., Blinder 1986, Griliches 1967, Harvey 1990, and McManus, Nankervis, and Savin 1994). Rational-expectation econometrics suffer from the same problems, especially when the

[4]Without loss of generality, we omit a constant term. Notice that, here, we assume that the Taylor rule penalizes the log-deviations of output rather than those of the output gap. In the DSGE model presented in the next section, it turns out that this distinction is irrelevant because the implied natural level of output is irresponsive to monetary shocks, which are the only shocks considered in the analysis. Thus, in this framework, the output gap exactly coincides with output. See Woodford (2003, 420).

[5]See Apel and Jansson (2005), Castelnuovo (2003), English, Nelson, and Sack (2003), Gerlach-Kristen (2004), and Rudebusch (2002).

framework conveys little information, as in Keenan (1988) or Sargent (1978).

To see this problem, let us consider our simple representation of monetary policy (1)–(3):

$$\hat{\imath}_t = (\rho_i + \rho_e)\hat{\imath}_{t-1} - \rho_i\rho_e\hat{\imath}_{t-2} + (1 - \rho_i)\left(\hat{\imath}^\star_t - \rho_e\hat{\imath}^\star_{t-1}\right) + \nu_t,$$

where the target $\hat{\imath}^\star_t$ is a linear function of shocks that hit the economy. We assume for simplicity a single shock, namely, the monetary shock ν_t:

$$\hat{\imath}^\star_t = \sum_{k=0}^{\infty} \eta_k \nu_{t-k},$$

where η_k is a complicated nonlinear function of the policy rule parameters, as well as other deep parameters. Suppose that η_k for $k = 0, \ldots, \infty$ are small and not sensitive to ρ_i and ρ_e. In this case, $\hat{\imath}^\star_t$ is essentially zero with a very small amount of variance. The policy function accordingly rewrites

$$\hat{\imath}_t \approx (\rho_i + \rho_e)\hat{\imath}_{t-1} - \rho_i\rho_e\hat{\imath}_{t-2} + \nu_t.$$

In this case, the parameters ρ_i and ρ_e are not identified in general. To see this, consider the reduced form associated with the approximate monetary policy

$$\hat{\imath}_t = \beta_1\hat{\imath}_{t-1} + \beta_2\hat{\imath}_{t-2} + \nu_t.$$

Provided that $\rho_i \neq \rho_e$, there does not exist a unique solution for ρ_i and ρ_e as a function of the reduced-form parameters β_1 and β_2. Indeed, as long as $\beta_2 \neq 0$, the solutions for ρ_i and ρ_e are given by $\rho_i = (\beta_1 \pm (\beta_1^2 + 4\beta_2)^{1/2})/2$ and $\rho_e = \beta_1 - \rho_i$, where $\beta_1^2 + 4\beta_2 = (\rho_i - \rho_e)^2 \geq 0$. This means that two sets of values for ρ_i and ρ_e are observationally equivalent. The first solution is associated with the monetary-policy-inertia view (ρ_i large and ρ_e small), whereas the second is related to the persistent-shocks view (ρ_i small and ρ_e large). When $\hat{\imath}^\star_t \approx 0$, we cannot distinguish between a highly inertial monetary policy with transitory shocks and a monetary policy with small partial adjustment and highly serially correlated shocks. In contrast, if $\rho_i = \rho_e$, the parameters are identified, but this configuration is inconclusive since it assigns the same weights to both views about monetary policy.

When $\hat{\imath}_t^\star$ is responsive to shocks, and thus is more volatile, this multiple-optima problem can potentially disappear, provided that η_k is highly sensitive to perturbations in ρ_i and ρ_e. However, nothing guarantees this in practice, so that estimating ρ_i and ρ_e by focusing on the nominal interest rate only might fail to reveal the correct information about monetary policy.

A way to eschew this problem is to consider additional variables and equations in the estimation stage. We argue that moving from a single-equation setup to a system of equations is likely to aid in discriminating between the two views of policy dynamics (see Rudesbusch and Wu 2004 for a similar approach). Our strategy to identify the policy parameters rests on the restrictions imposed by a DSGE model. When the policy rule parameters have strong effects on aggregate dynamics, this gives us an opportunity to properly identify ρ_i and ρ_e and to deliver clear-cut conclusions. The next section gives a brief overview of the model used in our empirical analysis.

3. The DSGE Model

We consider a standard New Keynesian model with price and wage stickiness,[6] along the lines of Galí and Rabanal (2005) and Giannoni and Woodford (2004). Since, later on, we will seek to compare this model with a monetary SVAR à la Christiano, Eichenbaum, and Evans (1996, 1999), it is important to make sure that both models embed the same timing restrictions. To achieve this, we assume that output, inflation, and wage inflation are decided prior to observing the monetary shock, as in Rotemberg and Woodford (1997, 1999).

The first equation is the New Keynesian Phillips curve:

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \mathrm{E}_{t-1}\left\{ \frac{(1-\alpha_p)(1-\beta\alpha_p)}{\alpha_p((1-\mu_p s_q)^{-1}(1+\theta_p \epsilon_{\mu_p}) + \theta_p \omega_p)} (\hat{w}_t + \omega_p \hat{y}_t) + \beta(\hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t) \right\}, \tag{4}$$

where E_{t-1} is the expectation operator conditional on information available to the firm when reoptimizing its price. $\hat{\pi}_t$, $\hat{y}_t$, and $\hat{w}_t$

[6] See the appendix for more details about the model.

are the log-deviations of inflation, output, and real wage, respectively. The parameter $\beta \in (0,1)$ is the subjective discount factor, $\gamma_p \in [0,1]$ is the degree of indexation of prices to the most recently available inflation measure, $\alpha_p \in [0,1)$ is the degree of nominal rigidity, $s_q \in (0,1)$ represents the share of material goods, $\theta_p > 0$ is the steady-state price elasticity of demand, $\mu_p > 1$ is the steady-state markup factor, ϵ_μ is the steady-state elasticity of the markup factor, and ω_p is the real-marginal-cost elasticity with respect to the level of production.

A second set of equations defines the IS and LM curves:

$$\begin{aligned} &\mathrm{E}_{t-1}\{\beta b(\hat{y}_{t+1} - b\hat{y}_t) - (\hat{y}_t - b\hat{y}_{t-1}) \\ &\quad + \sigma\chi(\hat{m}_t - \beta b\hat{m}_{t+1}) - \varphi^{-1}\hat{\lambda}_t\} = 0, \end{aligned} \tag{5}$$

$$\hat{\lambda}_t = \hat{\imath}_t + \mathrm{E}_t\{\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\}, \tag{6}$$

$$\hat{m}_t = \eta_y(\hat{y}_t - b\hat{y}_{t-1}) - \eta_i\hat{\imath}_t, \tag{7}$$

where $\hat{m}_t$, $\hat{\imath}_t$, and $\hat{\lambda}_t$ are the log-deviations of real balances, the nominal interest rate, and the representative household's marginal utility of wealth, respectively. The parameter $b \in [0,1)$ represents the degree of habit formation. The additional parameters σ, χ, φ, η_y, and η_i are deduced from the utility function. Notice that we enforce the implied constraints on these parameters when we calibrate the model. Equation (5) illustrates the role played by habits in consumption, which reinforces the backward dimension of the IS curve. Provided $\sigma\chi > 0$, this equation includes a real balance effect. Equation (6) is the standard Euler equation on bond holdings. Finally, equation (7) is the money-demand function. The difference in the information sets in equations (5) and (6) reflects the timing of decisions. Prior to observing the monetary policy shock, the household decides how much to consume and sets its nominal wage. The shock is then realized, and bond and money holdings decisions are taken.

The wage-setting equation is given by

$$\begin{aligned} \hat{\pi}_t^w - \gamma_w\hat{\pi}_{t-1} = \mathrm{E}_{t-1}\Bigg\{ &\frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\omega_w\theta_w)}(\omega_w\phi\hat{y}_t - \hat{\lambda}_t - \hat{w}_t) \\ &+\beta\big(\hat{\pi}_{t+1}^w - \gamma_w\hat{\pi}_t\big)\Bigg\}, \end{aligned} \tag{8}$$

where $\hat{\pi}_t^w$ is the log-deviation of wage inflation. The parameter $\gamma_w \in [0, 1]$ is the degree of wage indexation to the most recently available inflation measure, $\alpha_w \in [0, 1)$ is the degree of nominal wage rigidity, $\theta_w > 0$ is the wage elasticity of labor demand, $\omega_w > 0$ is the elasticity of the marginal disutility of labor, and $\phi > 1$ is the inverse elasticity of output with respect to the labor input. Finally, $\hat{\pi}_t$ and $\hat{\pi}_t^w$ are linked together through the relation

$$\hat{\pi}_t^w = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t. \tag{9}$$

The model is closed by postulating the monetary policy rule (1)–(3).

4. Econometric Approach

This section details our monetary SVAR and the implied IRFs used to estimate the DSGE model, and presents the minimum distance estimation (MDE) approach.

4.1 The Monetary SVAR

We start our analysis by characterizing the actual economy's response to a monetary policy shock. As is now standard, this is done by estimating a monetary SVAR in the lines of Christiano, Eichenbaum, and Evans (1996, 1999) so as to identify monetary policy shocks.[7] We consider a structural VAR of the form

$$A_0 Z_t = A_1 Z_{t-1} + \cdots + A_\ell Z_{t-\ell} + \eta_t,$$

where the data vector Z_t can be decomposed according to $Z_t = (Z'_{1,t}, \hat{\imath}_t, Z'_{2,t})'$. $Z_{1,t}$ is an $n_1 \times 1$ vector composed of variables whose current and past realizations are included in the information set available to the policymaker at t and that are assumed to be predetermined with respect to the monetary shock ϵ_t. $Z_{2,t}$ is an $n_2 \times 1$ vector containing variables that are allowed to respond contemporaneously to ϵ_t but whose value is unknown to monetary policy authorities at t. The lag length ℓ is determined by minimizing the Hannan-Quinn information criterion. In our empirical analysis, we found that $\ell = 4$.

[7] See also Christiano, Eichenbaum, and Evans (1997, 2005) and Rotemberg and Woodford (1997, 1999) for other examples of this identifying strategy.

4.2 Minimum Distance Estimation

Let ψ denote the whole set of model parameters. Let $\psi_2 = (\rho_i, \rho_e, \sigma_\nu)'$ and let ψ_1 denote the vector collecting all the remaining parameters, so that $\psi = (\psi_1^{'}, \psi_2^{'})'$. To implement our approach, it is important that ψ_1 be fixed, so that variations in the empirical performance of the DSGE model result only from changes in ψ_2, thus revealing information about the relevant specification of the monetary policy rule.

The policy parameters ψ_2 are estimated by minimizing a measure of the distance between the empirical responses of key aggregate variables and their model counterparts.[8] More precisely, we focus our attention on the responses of the vector X_t, regrouping the actual data that we are interested in. Here, X_t is a subset of Z_t. We define θ_j as the vector of responses of the variables in X_t to a monetary shock at horizon $j \geq 0$, as implied by the above SVAR.

Then, the object that we seek to match is $\theta = \text{vec}([\theta_0, \theta_1, \ldots, \theta_k])'$, where k is the selected horizon.[9] Then let $h(\cdot)$ denote the mapping from the structural parameters $\psi_2 = (\rho_i, \rho_e\, \sigma_\nu)'$ to the DSGE counterpart of θ. Our estimate of ψ_2 is obtained by minimizing

$$\mathcal{J}_T = (h(\psi_2) - \hat{\theta}_T)V_T(h(\psi_2) - \hat{\theta}_T)',$$

where $\hat{\theta}_T$ is an estimate of θ, T is the sample size, and V_T is a weighting matrix that we assume is the inverse of a matrix containing the asymptotic variances of each element of θ along its diagonal and zeros elsewhere. We make this particular choice for the weighting matrix to avoid singularity problems of the covariance matrix of IRFs. In addition, as suggested by Christiano, Eichenbaum, and Evans (2005), this choice of weighting matrix

[8]See Altig et al. (2004), Amato and Laubach (2003), Boivin and Giannoni (2006), Christiano, Eichenbaum, and Evans (2005), Giannoni and Woodford (2004), and Rotemberg and Woodford (1997). Following these studies, we implicitly assume that the SVAR is able to identify the structural monetary-policy reaction function, which can differ from the reaction function in the DSGE model (see Rudebusch 1998).

[9]Notice that we have to exclude from θ_0 the responses corresponding to the elements in X_t that belong to the information set available to the policymaker at t. It is important to emphasize that the DSGE model previously expounded embeds the same exclusion restrictions as the monetary SVAR.

ensures that the model-based IRFs lie as much as possible inside the confidence interval of the SVAR-based IRFs. Under the null hypothesis that the DSGE model is true, $\mathcal{J}_T$ asymptotically follows a chi-squared distribution with $\dim(\hat{\theta}_T) - \dim(\psi_2)$ degrees of freedom. We will use the statistic $\mathcal{J}_T$ as a discriminating criterion between the two representations of monetary policy. Additionally, we decompose $\mathcal{J}_T$ into components pertaining to each element of X_t. This decomposition provides a simple diagnostic tool that allows us to locate on which dimension the model succeeds or fails to replicate the IRFs implied by the SVAR.

5. Empirical Results

In this section, we first present our data and results drawn from our SVAR analysis. Second, we discuss the calibration of the model's parameters. Third, we present our estimation results. Finally, we provide a sensitivity analysis to calibration.

5.1 Data and SVAR

In addition to the federal funds rate, we use data from the nonfarm business (NFB) sector over the sample period 1960:Q1–2002:Q4.[10] The variables used for estimation are the linearly detrended logarithm of per capita GDP, $\hat{y}_t$; the growth rate of GDP's implicit price deflator, $\hat{\pi}_t$; and the growth rate of nominal hourly

[10] Arguably, this sample period might be characterized by significant changes in monetary policy. As a consequence, the assumption that monetary policy can be represented by a single Taylor rule is rather strong. Unfortunately, the estimated IRFs from the SVAR in the period 1985:Q1–2002:Q4 exhibit a number of pathologies. For example, output persistently rises after a contractionary monetary policy shock. In addition, the estimated IRFs are not precisely estimated, implying that estimating DSGE parameters so as to replicate these responses is meaningless. This is reminiscent of the point raised by Sims (1998) that SVARs estimated on short time series can produce very erratic IRFs. Thus we follow Christiano, Eichenbaum, and Evans (1996, 1999, 2005) and adopt a longer sample. In addition, Sims and Zha (2006) found more evidence in favor of stable dynamics with unstable disturbance variances than of clear changes in model dynamics. See also Leeper and Roush (2003) and Rudebusch and Wu (2004).

compensation, $\hat{\pi}_t^w$.[11] We also include two "information" variables in the SVAR model. First, though not formally justified by the theoretical model, the growth rate of the logarithm of the Commodity Research Bureau price index of sensitive commodities, $\hat{\pi}_t^c$, is included to mitigate the so-called price puzzle (see Christiano, Eichenbaum, and Evans 1996, 1999; Eichenbaum 1992; and Sims 1992). Second, the growth rate of M2, $\hat{\xi}_t$, is included to exploit information included in money growth.[12] To implement the identification strategy outlined above, we set $Z_{1,t} = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_t^w, \hat{\pi}_t^c)'$ and $Z_{2,t} = (\hat{\xi}_t)$. In addition, the variables of interest are $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_t^w, \hat{\imath}_t, \hat{\xi}_t)'$. The empirical responses of X_t are reported in figure 1, with $k = 30$. The plain line is our point estimates of the empirical responses of X_t, and the shaded areas indicate the asymptotic 95 percent confidence interval about the point estimates.

Though we focus on a different data set and a different sample period, our findings echo previous results reported by Christiano, Eichenbaum, and Evans (1996, 1997, 1999, 2005).[13] Output initially responds very little and then sharply drops, with an inverted hump pattern. Notice that the latter is precisely estimated. The response of inflation displays a persistent U-shaped profile, with a narrow confidence interval. Inflation's lowest response is reached several quarters (more than three years) after output has reached its trough. Moreover, inflation does not present a significant price puzzle in the very short run. The response of wage inflation is qualitatively similar, with a trough response slightly lagging that of inflation. As discussed in Woodford (2003), the delayed response of inflation is a key stylized fact that any monetary DSGE model should accurately mimic. The federal funds rate instantaneously increases and then gradually declines to the point where it will eventually cross the x axis before reverting back to its steady-state value. Finally, nominal money growth drops sharply and rapidly returns to its steady-state level.

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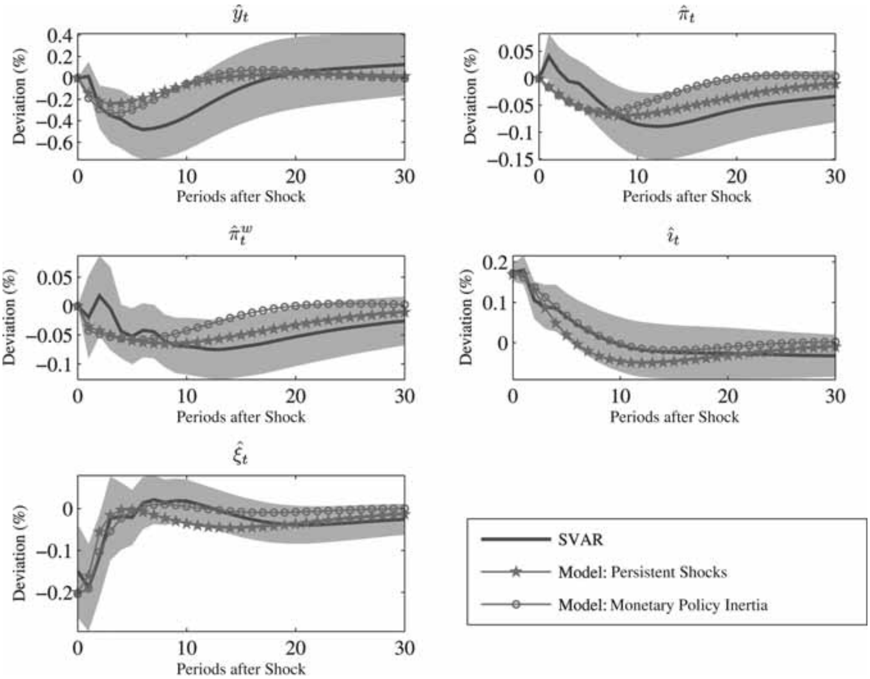
[11]The civilian non-institutional population over age sixteen is used as our measure of population. We also experimented with quadratically detrended or first-differenced output, without quantitatively altering our conclusions.

[12]The data are extracted from the Bureau of Labor Statistics web site, except for the federal funds rate and M2, which are obtained from the FREDII database.

[13]See also Bernanke and Mihov (1998), Leeper and Roush (2003), and Rotemberg and Woodford (1997, 1999) for similar IRF profiles.

Figure 1. IRFs to a Monetary Policy Shock (SVAR and DSGE Models)



5.2 Calibration

As explained above, parameters other than ψ_2 are calibrated prior to estimation. The rationale for doing this is that we want to make sure that the model's IRFs depend only on the particular specification of monetary policy. The calibration is reported in table 1.

Preferences. First, we set $\beta = 0.99$, implying a steady-state annualized real interest rate of 4 percent. The habit persistence parameter b is set to 0.75, lying in the range of available estimates based on aggregate data (see Boivin and Giannoni 2006 and Christiano, Eichenbaum, and Evans 2005). We then set $\sigma = 1 - b$, which implies intertemporal complementarities in consumption decisions (see Rotemberg and Woodford 1997).

As in Altig et al. (2004) and Christiano, Eichenbaum, and Evans (2005), the elasticity of marginal labor disutility, ω_w, is set to 1. The

Table 1. Calibrated Parameters

Parameters	**Interpretation**	**Value**
Preferences		
β	Subjective discount factor	0.99
b	Habit persistence	0.75
σ	Intertemporal elasticity of substitution $(= 1 - b)$	0.25
ω_w	Elasticity of marginal labor disutility	1.00
$\bar{v}$	Steady-state money velocity	1.36
η_y	Money-demand elasticity wrt $\hat{y}_t$	1.00
η_i	Money-demand elasticity wrt $\hat{\imath}_t$	1.18
Technology		
ϕ	Inverse of the elasticity of $\hat{y}_t$ wrt $\hat{n}_t$	1.33
ω_p	$\phi - 1$	0.33
s_q	Share of material goods	0.50
θ_p	Elasticity of demand for goods	6.00
μ_p	Markup $(= \theta_p/(\theta_p - 1))$	1.20
ϵ_μ	Markup elasticity	1.00
θ_w	Elasticity of demand for labor	21.00
μ_w	Markup $(= \theta_w/(\theta_w - 1))$	1.05
Price/Wage Setting		
γ_p	Price indexation	1.00
γ_w	Wage indexation	1.00
α_p	Probability of no price adjustment	0.66
α_w	Probability of no wage adjustment	0.66
Nominal-Interest-Rate Target Level		
a_π	Monetary policy reaction to $\hat{\pi}_t$	1.500
a_y	Monetary policy reaction to $\hat{y}_t$	0.125

money-demand function implied by our model is of the form (7). The parameter η_y governs the elasticity of real money demand to output. Following Woodford (2003), the latter is normalized to 1.

Calibrating the semi-elasticity η_i raises specific issues, especially so if the model has to reproduce the short-run behavior of money demand, as explained by Christiano, Eichenbaum, and Evans (2005). To pin down the value of η_i, we follow a different approach from theirs, yielding very similar results. From the SVAR and identified monetary shocks, we construct data series for real balances ($\tilde{m}_t$), real output ($\tilde{y}_t$), and the nominal interest rate ($\tilde{\imath}_t$) when only monetary shocks hit the SVAR. We then estimate a linear money-demand function using OLS. The estimated money demand takes the form[14]

$$\tilde{m}_t = 0.8571\tilde{m}_{t-1} + 0.1429\tilde{y}_t - 0.1072\tilde{y}_{t-1} - 1.1846\tilde{\imath}_t + \vartheta_t.$$

We use the estimated short-run semi-elasticity of money demand to the nominal interest rate (1.1846) to calibrate η_i. Notice that, in the course of estimation, we imposed $\eta_y = 1$ and took into account the calibrated value of b. The implied long-run semi-elasticity is slightly above 8, which is the value obtained by Chari, Kehoe, and McGrattan (2000), Lucas (1988), and Mankiw and Summers (1986). Consequently, our calibration of η_i must be interpreted as a way to account for the short-run response of money growth, as in Christiano, Eichenbaum, and Evans (2005).

Recall that the parameter χ governs the extent to which a real balance effect is present in our model. Under our calibration, we use the restriction $\chi = (1-\beta b)\eta_y/(\eta_i\bar{v})$. We calibrate the money velocity from actual data and obtain $\bar{v} = 1.36$. From these calibrated values, we obtain $\chi = 0.138$, implying a non-negligible real balance effect.

Technology. Here ϕ is the inverse of the elasticity of value added to labor input. We set $\phi = 1.333$, which corresponds to a steady-state share of labor income of 62.5 percent, after correcting for the markup. Assuming further that the production function is Cobb-Douglas, direct calculations yield $\omega_p = \phi - 1$. The share of material goods in gross output, s_q, is set to 50 percent,

[14]An important limit of our approach is that it assumes that OLS consistently estimates η_i. However, our estimate is not far from previous estimations. Moreover, we conduct a sensitivity analysis of our results to η_i (see section 5.4). Our findings are not qualitatively affected.

as in Basu (1995) and Rotemberg and Woodford (1995). Following Christiano, Eichenbaum, and Evans (2005) and Rotemberg and Woodford (1997), we set the markup on prices to 20 percent, i.e., $\mu_p = 1.20$. This implies an elasticity of demand for goods $\theta_p = 6$. The markup elasticity to relative demand, ϵ_μ, is set to 1, as in Bergin and Feenstra (2000) and Woodford (2003). Finally, we set θ_w to 21, as in Christiano, Eichenbaum, and Evans (2005), implying a wage markup of 5 percent.

Price/Wage Setting. Following Rotemberg and Woodford (1997), we set α_p to 0.66, implying an average spell of no price reoptimization of 2.5 quarters. This value is consistent with microeconomic evidence, e.g., Bils and Klenow (2004). We set $\gamma_p = 1$, as in Christiano, Eichenbaum, and Evans (2005). This value allows us to reinforce the backward dimension of inflation. Following Amato and Laubach (2003), we symmetrically set $\alpha_w = 0.66$. As in Christiano, Eichenbaum, and Evans (1995), we also set $\gamma_w = 1$.

Nominal-Interest-Rate Target Level. Following Taylor (1993), we set $a_\pi = 1.5$ and $a_y = 0.125$, since we focus on quarterly measures of y_t, π_t, and i_t. These values are approximately the same as those considered by Christiano, Eichenbaum, and Evans (2005) in their sensitivity analysis.

5.3 Estimation Results

The estimation results are reported in table 2 for different X_t and different restrictions on the policy rule parameters. In each case, we set the IRF horizon k to 30. The table is organized as follows: the left panel reports parameter estimates when $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_t^w, \hat{\imath}_t, \hat{\xi}_t)'$, i.e., when ψ_2 is selected so as to reproduce the responses of output, inflation, wage inflation, the federal funds rate, and money growth to a monetary policy shock; the right panel corresponds to the case where $X_t = \hat{\imath}_t$, i.e., when we exclusively focus on the federal funds rate's behavior. In each panel, we consider five cases, depending on the minimum value of $\mathcal{J}_T$ reached at convergence and on restrictions on ρ_i or ρ_e. More precisely, column 1 corresponds to the minimum value of $\mathcal{J}_T$ reached when using as an initial condition a large ρ_e and a small ρ_i. Conversely, column 2 corresponds to the case with a large ρ_i and a small ρ_e. Column 3 corresponds to the restriction $\rho_i = 0$, i.e., to a model with only serially correlated shocks and

Table 2. Estimation Results

Parameter	Based on $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_t^w, \hat{\imath}_t, \hat{\xi}_t)'$					Based on $X_t = (\hat{\imath}_t)$				
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**
ρ_i	0.2976	0.8986	0.0000	0.9026	0.7535	0.3018	0.8951	0.0000	0.9091	0.5606
	(0.061)	(0.009)	(*)	(0.007)	(0.001)	(0.079)	(0.049)	(*)	(0.035)	(0.002)
ρ_e	0.8740	0.0676	0.8900	0.0000		0.8207	0.0440	0.8383	0.0000	
	(0.006)	(0.092)	(0.007)	(*)		(0.012)	(0.135)	(0.016)	(*)	
σ_ν	0.1691	0.1720	0.1882	0.1760	0.1232	0.1731	0.1742	0.1898	0.1754	0.1679
	(0.010)	(0.010)	(0.009)	(0.009)	(0.005)	(0.011)	(0.011)	(0.010)	(0.010)	(0.006)
$\mathcal{J}$	145.66	245.54	162.64	246.15	246.52	15.78	14.36	26.97	14.48	21.63
	[62.94]	[0.00]	[28.17]	[0.00]	[0.00]	[96.86]	[98.44]	[57.32]	[98.86]	[83.51]
$\mathcal{J}_y$	40.22	40.76	43.80	40.25	43.39	—	—	—	—	—
$\mathcal{J}_\pi$	34.22	86.25	28.34	86.43	72.05	—	—	—	—	—
$\mathcal{J}_{\pi^w}$	17.03	75.55	12.74	75.73	56.64	—	—	—	—	—
$\mathcal{J}_i$	23.50	14.48	37.63	14.56	50.65	15.78	14.36	26.97	14.48	21.63
$\mathcal{J}_\xi$	30.70	28.48	40.12	29.22	23.79	—	—	—	—	—

Notes: Standard errors in parentheses, P-value in brackets. (1) Initialization with ρ_i small and ρ_e large; (2) initialization with ρ_e small and ρ_i large; (3) constrained case $\rho_i = 0$; (4) constrained case $\rho_e = 0$; (5) constrained case $\rho_e = \rho_i$. In columns (3) and (4), * denotes a standard error not available.

no policy inertia. Column 4 corresponds to the restriction $\rho_e = 0$, i.e., to a model with nominal-interest-rate inertia and i.i.d. shocks to monetary policy. Finally, column 5 reports the estimation outcome when imposing the constraint $\rho_i = \rho_e$, thus granting the same weight to both alternative views about monetary policy. The point estimates of ψ_2 are reported, with their standard errors in parentheses. The table also reports the value of $\mathcal{J}_T$ at convergence, with the associated P-value in brackets. Finally, with our choice of weighting matrix, we can further decompose the $\mathcal{J}_T$ statistic into various components pertaining to each element of X_t.

Let us first consider the case with $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_t^w, \hat{\imath}_t, \hat{\xi}_t)'$. In this context, we obtain a global minimum associated with the model with serially correlated monetary shocks. Additionally, the model successfully passes the overidentification test (see column 1). In contrast, the local minimum associated with a model with monetary policy inertia is blatantly rejected by the data (see column 2). The global-minimum-distance estimator yields $\rho_i = 0.30$ and $\rho_e = 0.87$. This suggests that the correct representation of monetary policy is a mix of serially correlated shocks and a modest degree of policy inertia, in the line of Rudebusch (2002, 2006). Notice that these two parameters are found to be significant. In addition, the data do not reject a model version imposing $\rho_i = 0$, while they reject the restriction $\rho_e = 0$ (see columns 3 and 4 in table 2). Notice that a quasi-likelihood ratio test would, however, reject the restriction of no monetary policy inertia (see columns 1 and 3).

To understand why the data reject the model with high monetary policy inertia, it is instructive to consider the decomposition of $\mathcal{J}_T$ according to the components of X_t. When comparing columns 1 and 2, we see that the two representations of monetary policy deliver very similar results when it comes to output, the nominal interest rate, and money growth. In other words, these three variables are weakly informative about the relevant form of monetary policy. What turns out to be really discriminating is the behavior of inflation and wage inflation. In this case, the DSGE model with policy inertia proves unable to mimic the delayed and persistent responses of these variables.

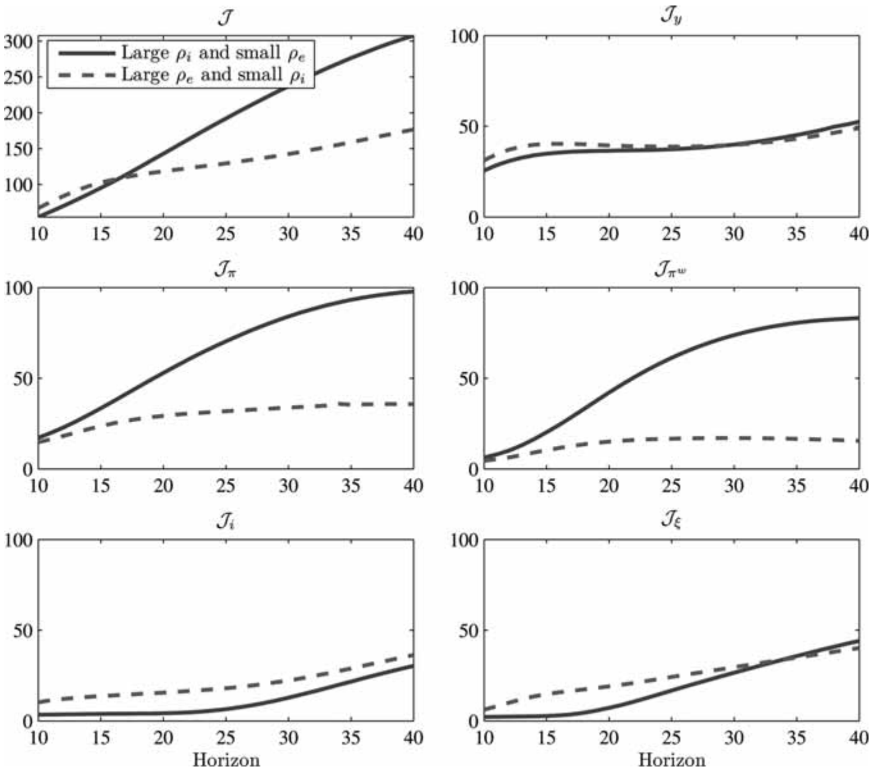
This failure is illustrated by comparing IRFs in figure 1. The lines marked with circles correspond to the DSGE point estimates with monetary policy inertia, whereas the lines marked with stars correspond to the model with persistent shocks. The dynamic responses of output, the federal funds rate, and money growth do not appear to be qualitatively affected by the specification of monetary policy. To the contrary, the model's IRFs of inflation and wage inflation sharply differ. The model with persistent shocks and moderate interest rate inertia successfully matches the essential features of the data. This is no longer the case when we consider a model with a large degree of interest rate inertia, especially so when it comes to inflation and wage inflation.

Column 5 in table 2 shows that the restriction $\rho_i = \rho_e$ is not supported by the data. Indeed, such a restriction deteriorates the model fit on virtually all dimensions, except maybe for money growth. Thus, a specification of monetary policy that grants the same weights to policy inertia and persistent shocks provides a fit that is substantially worse than the one with highly persistent shocks and moderate policy inertia.

Second, let us consider the case with $X_t = \hat{\imath}_t$. The latter is investigated as a simple way of illustrating the lack of information resulting from a quantitative assessment of our model based on a single variable. In some sense, this problem is reminiscent of the absence of clear-cut conclusions obtained in the literature focusing on a single policy rule equation; see Rudebusch (2002, 2006). Now we face the "multiple-optima" problem, since the two representations of monetary policy deliver very close objective functions at convergence. In addition, none are rejected by the data, so that they appear to be "observationally equivalent" in terms of the $\mathcal{J}_T$ statistic (see columns 1–5 in the right panel of table 2). This experiment illustrates that focusing only on the nominal interest rate does not yield a clear conclusion as to the relevant representation of monetary policy. What really matters is the aggregate dynamics (especially the dynamics of inflation and wage inflation) implied by the alternative specifications of monetary policy.

The previous results are obtained for a horizon $k = 30$. Under this assumption, we were able to discriminate between the two competing representations of monetary policy, because a model with large interest rate inertia fails to mimic the delayed U-shaped responses of inflation and wage inflation. To further illustrate the information contained in these hump-shaped patterns, we now vary the horizon k between 10 and 40. Figure 2 reports the $\mathcal{J}_T$ statistic as well as its

Figure 2. Decomposition of the $\mathcal{J}_T$ Statistic as a Function of the Time Horizon



decomposition according to the elements of X_t. In this exercise, we select $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_t^w, \hat{\imath}_t, \hat{\xi}_t)'$ and reestimate the policy parameters for each selected horizon. In each panel, the solid lines correspond to the value of the objective function $\mathcal{J}_T$ as well as its decomposition in the case of monetary policy inertia, while the dashed lines correspond to the case with persistent shocks. Let us first focus on the global test—i.e., the $\mathcal{J}_T$ statistic—in the upper-left panel. We see that for relatively short horizons ($k = 10, \ldots, 15$), the two representations of monetary policy yield comparable results. Clearly, focusing only on short-run responses does not allow us to discriminate between the two specifications. However, as soon as k is sufficiently large to include the delayed hump patterns of inflation and wage inflation (see the two middle graphs), the performances of the two competing versions start to dramatically diverge. In particular, the monetary-policy-inertia specification faces more and more troubles reproducing the data.

5.4 Sensitivity to Calibration

We check whether the previous findings crucially depend on our particular calibration. A simple way to assess the importance of our calibration is to redo our analysis, perturbing some key model parameters. Table 3 reports the outcome of this sensitivity analysis. We identify key parameters governing the dynamic behavior of our model relating to preferences, technology, price/wage setting, and the nominal-interest-rate target level. For each alternative parameter value, we reestimate the model and recompute the $\mathcal{J}_T$ statistic at convergence with $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_t^w, \hat{\imath}_t, \hat{\xi}_t)'$ or $X_t = (\hat{\imath}_t)$.

Preferences. The "Preferences" panel of table 3 reports the effect of shutting habit formation down (i.e., $b = 0$). When $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_t^w, \hat{\imath}_t, \hat{\xi}_t)'$, this has the obvious effect of dramatically worsening the model's performance. Notice that, in this case, the two representations are unambiguously rejected by the data. Following Giannoni and Woodford (2004), we drastically decrease the elasticity of labor supply, setting $\omega_w = 10$. In this case, the model's performances are always improved, but the model with policy inertia is still rejected. Finally, we increase the sensitivity of money demand to the nominal interest rate, i.e., $\eta_i = 3$. The model's performances with persistent shocks are affected, but the model still passes the overidentification test. In contrast, when we focus exclusively on the nominal interest rate ($X_t = (\hat{\imath}_t)$), neither of the alternative representations can be rejected. More importantly, we cannot discriminate between these two policies based on the $\mathcal{J}_T$ statistic. This means that while the estimated models cannot generically mimic the dynamic responses of inflation and wage inflation, focusing exclusively on $\hat{\imath}_t$ would lead us to incorrectly fail to reject any model versions. This is a further illustration of the need for considering the dynamic behavior of alternative variables to properly discriminate between the competing monetary policies.

Technology. In the "Technology" panel of table 3, we investigate the sensitivity of our results to perturbations on technology parameters. Following Galí and Rabanal (2004), we assume constant

Table 3. Sensitivity to Calibration

Initialization		Based on $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}^w_t, \hat{\imath}_t, \hat{\xi}_t)'$		Based on $X_t = (\hat{\imath}_t)$	
		$\rho_e \gg \rho_i$	$\rho_i \gg \rho_e$	$\rho_e \gg \rho_i$	$\rho_e \gg \rho_i$
Parameters	**Value**	$\mathcal{J}_T$			
Preferences					
b	0.00	396 [0.00]	528 [0.00]	25 [62.69]	#
ω_w	10.00	120 [97.00]	218 [0.04]	13 [99.25]	14 [99.02]
η_i	3.00	181 [5.56]	275 [0.00]	16 [97.20]	14 [98.48]
Technology					
ϕ	1.00	103 [99.99]	199 [0.65]	11 [99.89]	13 [99.25]
θ_p	11.00	110 [99.50]	211 [0.11]	25 [62.69]	13 [99.21]
θ_w	11.00	183 [4.33]	281 [0.00]	19 [88.62]	16 [96.47]
Price/Wage Setting					
γ_p	0.00	235 [0.00]	300 [0.00]	10 [99.93]	16 [96.47]
γ_w	0.00	266 [0.00]	308 [0.00]	22 [76.44]	19 [90.21]
α_p	0.00	356 [0.00]	#	20 [87.69]	#
α_w	0.00	481 [0.00]	499 [0.00]	24 [70.36]	#
Target Level					
a_π	3.00	133 [86.45]	264 [0.00]	15 [98.29]	16 [96.77]
a_y	0.50	147 [60.38]	216 [0.05]	18 [92.15]	14 [98.88]

Notes: The label $\rho_e \gg \rho_i$ refers to an initialization of the estimation with ρ_e larger than ρ_i. Symmetrically, the label $\rho_i \gg \rho_e$ refers to an initialization of the estimation with ρ_i larger than ρ_e. P-value in brackets. A # in the "$\rho_i \gg \rho_e$" panel refers to the corresponding figure in the "$\rho_e \gg \rho_i$" panel.

returns to scale in labor input, thus imposing $\phi = 1$. The model's performances are improved for both specifications of monetary policy. However, the policy inertia is again rejected. We also modify the markups on prices without affecting our results. To the contrary, when we increase the degree of market power on the labor market, we substantially reduce the model's ability to reproduce the IRFs of X_t. Under this assumption, both versions are rejected by the data. Once again, when we focus on $X_t = (\hat{\imath}_t)$, we fail to reject either of the two competing representations of monetary policy.

Price/Wage Setting. In the "Price/Wage Setting" panel of table 3, we experiment with altering the details of the price- and wage-setting side of the model. We first shut down the indexation to past inflation in either the price or wage equations ($\gamma_p = 0$ or $\gamma_w = 0$). In both cases, this dramatically worsens the model's fit, especially so when it comes to inflation and wage inflation. Recall that these two variables were crucial in helping us sort out which specification of monetary policy was supported by the data. Not surprisingly, in the present case, both versions are rejected. Second, we assume perfect flexibility of either prices or wages ($\alpha_p = 0$ or $\alpha_w = 0$). In both cases, the model is rejected. Contrary to the previous experiment, when we focus on $X_t = (\hat{\imath}_t)$, we cannot reject either of the two competing representations of monetary policy, which prove almost completely insensitive to such parameter perturbations. This illustrates once more the need for further information.

Nominal-Interest-Rate Target Level. Finally, in the "Target Level" panel of table 3, we experiment with the parameters governing the target level of the nominal interest rate, namely a_π and a_y. We set a_π to a larger value than considered by Taylor (1993), $a_\pi = 3$. When $X_t = (\hat{y}_t, \hat{\pi}_t, \hat{\pi}_t^w, \hat{\imath}_t, \hat{\xi}_t)'$, the discrepancy between the two alternative specifications of monetary policy widens, especially so when it comes to inflation and wage inflation. This results from the fact that increasing a_π increases the amount of information in the target level of the nominal interest rate. When it comes to a_y, the quantitative findings are left unaffected. Conversely, when we focus on $X_t = (\hat{\imath}_t)$, we fail to reject either of the two competing representations of monetary policy. This is more troubling than one would have expected. Indeed, increasing the volatility of the target can potentially eliminate the identification problem. This is not the case in practice. When we focus only on $\hat{\imath}_t$, the discriminating power of inflation and wage inflation is shut down, which keeps us from reaching a clear-cut conclusion.

6. Practical Differences between Policy Inertia and Persistent Shocks

This section presents two illustrations of the practical differences between policy inertia and persistent shocks. We first investigate their quantitative implications for policy rule estimation. Second, we perform forecasting exercises using the Volcker disinflation as a case study that can potentially reveal striking differences between the two views.

6.1 Implications for Monetary Policy

The previous exercise has allowed us to discriminate between two alternative representations of monetary policy. However, since all the parameters were calibrated, including the responsiveness of monetary policy to inflation and output, this exercise is necessarily silent on the consequences of a monetary policy misspecification. The question we ask now is the following: Would we get different estimates of the responsiveness of monetary policy to inflation and output in the case of monetary policy inertia and in the case of persistent shocks?

So as to answer this question, we reestimate our model under the two alternative representations and allow a_π and a_y to be freely estimated. Table 4 reports the estimation results. The first column reports results obtained with persistent shocks, while the second column corresponds to policy inertia. As before, the global minimum is obtained with highly serially correlated monetary shocks and a small degree of interest rate smoothing. Once again, the responses of inflation and wage inflation allow us to discriminate between the two competing views.

Our results also suggest that the two alternative views yield very contrasted findings relative to the reaction of monetary authorities to inflation and output. In the case of policy inertia, the latter is almost passive regarding inflation but highly reactive to output fluctuations. Under persistent shocks, we obtain a reverse configuration, suggesting a very aggressive monetary policy in response to inflation

Table 4. Estimation Results for the Complete Taylor Rule

Parameter	$\rho_e \gg \rho_i$	$\rho_i \gg \rho_e$
ρ_i	0.2773 (0.187)	0.8832 (0.233)
ρ_e	0.9501 (0.013)	0.4169 (0.223)
a_π	3.0815 (0.568)	1.0509 (0.809)
a_y	0.0000 (−)	0.7065 (2.152)
σ_ν	0.1670 (0.010)	0.1594 (0.010)
$\mathcal{J}$	129.43 [88.64]	173.64 [9.06]
$\mathcal{J}_y$	46.28	42.32
$\mathcal{J}_\pi$	26.56	51.13
$\mathcal{J}_{\pi^w}$	10.90	34.73
$\mathcal{J}_i$	20.87	24.43
$\mathcal{J}_\xi$	24.82	21.03

Notes: The label $\rho_e \gg \rho_i$ refers to an initialization of the estimation with ρ_e larger than ρ_i. Symmetrically, the label $\rho_i \gg \rho_e$ refers to an initialization of the estimation with ρ_i larger than ρ_e. Standard errors in parentheses, P-value in brackets.

and a zero concern for output fluctuations. The global minimum thus corresponds to a policy rule enforcing the Taylor principle.

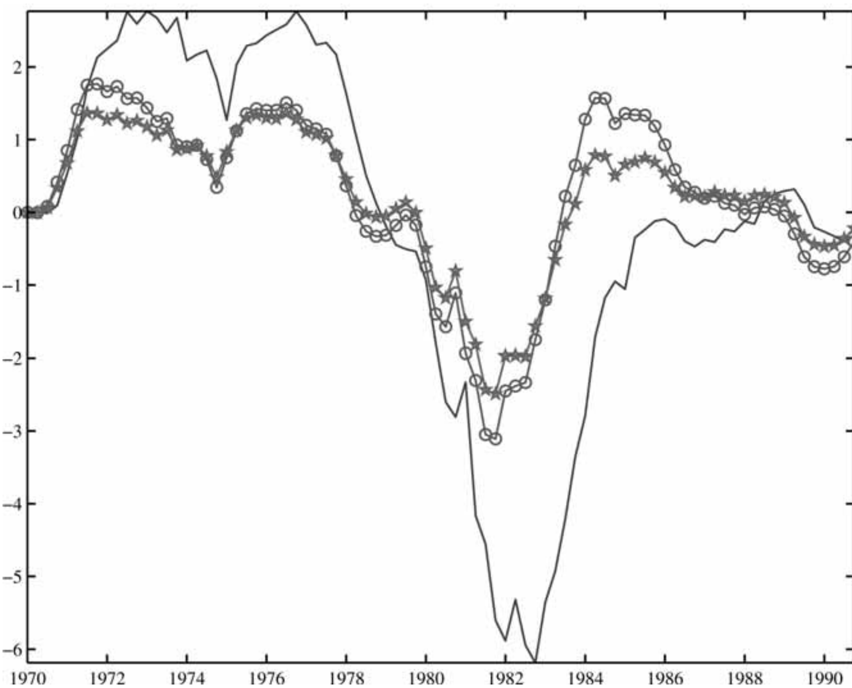
6.2 Inspecting the Monetary Policy Rules through the Lenses of the Volcker Disinflation

In this section, we compare the performances of the two alternative representations of monetary policy using the Volcker disinflation as an episode that can potentially reveal striking differences between these policy rules. This episode corresponds to what can be a priori viewed as a period of large contractionary monetary policy shocks.

Thus, comparing the model under the two rules with what actually happened during this episode constitutes a legitimate experiment.

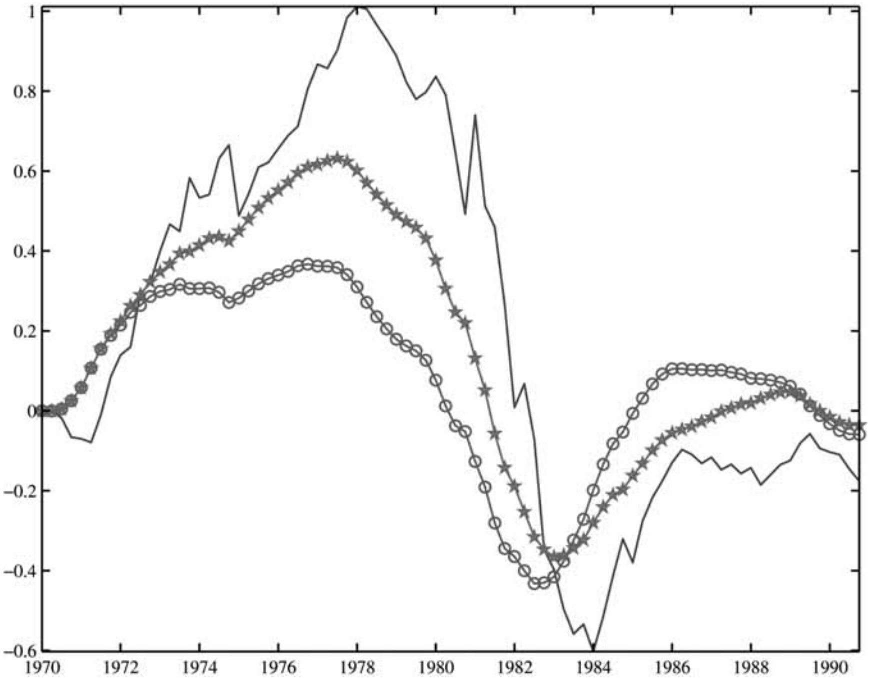
Since our limited-information approach exclusively relies on monetary policy shocks, we start by reconstructing historical data from the SVAR after having shut down all other shocks than monetary shocks. We then feed the identified monetary shocks in our DSGE model using either of the rules and compute artificial data. To compare the performances of the two rules on output and inflation, we focus on the sample period preceeding and succeeding the Volcker disinflation, 1970:Q1–1990:Q4. To make things comparable, we use the same initial conditions in 1970 either for the SVAR or for the model. The outcome of these comparisons is reported in figures 3

Figure 3. Historical Simulation of Output



Note: Plain line: SVAR. Line marked with circles: DSGE model with policy inertia. Line marked with stars: DSGE model with persistent shocks. All the data are demeaned prior to simulation.

Figure 4. Historical Simulation of Inflation



Note: Plain line: SVAR. Line marked with circles: DSGE model with policy inertia. Line marked with stars: DSGE model with persistent shocks. All the data are demeaned prior to simulation.

and 4 for output and inflation, respectively. In each case, the plain line corresponds to the SVAR-based historical data, while the lines marked with circles and stars correspond to the DSGE model with policy inertia and serially correlated shocks, respectively.[15]

Figure 3 confirms our previous findings: the two alternative policy rules have similar implications when it comes to output dynamics. As is clear from the picture, in both cases, the simulated samples are very similar, either in terms of persistence or volatility. To the contrary, as shown in figure 4, the two alternative rules have very

[15]Notice that this exercise contains the same information as our IRF-based assessment of the model's performance. What is interesting here is that it allows us to focus on a specific episode within our sample.

different implications in terms of inflation dynamics. Under the rule with policy inertia, inflation dynamics exhibit a smaller variance when compared to the SVAR-based counterpart. In addition, inflation drops too slowly during the Volcker disinflation and rises much too fast after 1983. Overall, this gives an inaccurate description of actual inflation dynamics during this particular episode. In contrast, when the rule features serially correlated shocks and a modest degree of policy inertia, the model is better suited to capture the large inflation peak of 1978 as well as the sharp decline following the disinflation. Additionally, the model does not predict a rapid rise in inflation after 1983, consistent with what the SVAR-based path suggests.

This exercise provides a confirmation that inflation dynamics contain more useful pieces of information than the dynamics of output for the purpose of disentangling the two alternative representations of monetary policy. The Volcker disinflation, taken as a case study, favors the persistent-shocks view as a practical approximation of actual monetary policy, as was to be expected from our previous quantitative investigation.

7. Conclusion

In this paper, we proposed a simple econometric framework to discriminate between two alternative representations of monetary policy. This approach draws heavily from the restrictions contained in the monetary DSGE model used in our empirical analysis. More precisely, thanks to these restrictions, different monetary policies can have radically different implications in terms of aggregate dynamics. Building on this well-known property of DSGE models, we are able to identify which policy rule best fits the data.

Our results are twofold. First, when the framework contains enough information, a policy rule with modest interest rate inertia and highly serially correlated shocks—which contrasts with most current implementations of monetary policy rules—satisfactorily matches the data. In particular, we found that the dynamics of inflation and wage inflation are particularly helpful for inferring the correct specification of monetary policy. However, output, the nominal interest rate, and the money growth rate do not contain very discriminating information. In addition, the inverted hump patterns displayed by the impulse responses of inflation and wage inflation are found to be particularly relevant for this purpose. Second, when the framework is not informative enough—i.e., when we focus on the sole dynamics of the federal funds rate—we are unable to discriminate between the two alternative monetary policy rules. These results highlight the low discriminating power of single-equation approaches. Overall, our results suggest that using extra macroeconomic information can help reach clear-cut conclusions as to the correct empirical representation of monetary policy rules. These two main findings are confirmed when we investigate practical differences between these two alternative monetary policy representations.

Appendix. Model Details

Production Side

A large number of competitive firms produce a homogeneous good that can be either consumed (y_t) or used as a material input in production (q_t). The overall aggregate demand is $d_t \equiv y_t + q_t$, and P_t is the associated nominal price. Following Kimball (1995) and Woodford (2003), the production function is of the form

$$\int_0^1 G\left(\frac{d_t(\varsigma)}{d_t}\right) \mathrm{d}\varsigma = 1, \tag{10}$$

where $d_t(\varsigma)$ denotes the input of intermediate goods $\varsigma \in [0,1]$, and the function G is increasing, strictly concave, and satisfies the normalization $G(1) = 1$. The representative final goods producer chooses $\{d_t(\varsigma), \varsigma \in [0,1]\}$ and d_t in order to maximize profits

$$\max_{\{d_t(\varsigma)\}} P_t d_t - \int_0^1 P_t(\varsigma) d_t(\varsigma) \mathrm{d}\varsigma,$$

subject to (10), where $P_t(\varsigma)$ is the nominal price of intermediate good ς. Monopolistic firms produce the intermediate goods $\varsigma \in [0,1]$. Each firm ς is the sole producer of intermediate good ς. Following Woodford (2003), we assume that monopolist ς produces good ς

with the inputs of aggregate labor $n_t(\varsigma)$ and material goods $q_t(\varsigma)$ according to the following production possibilities:

$$\min\left\{\frac{F(n_t(\varsigma))}{1-s_q}, \frac{q_t(\varsigma)}{s_q}\right\} \geq d_t(\varsigma),$$

where $F(\cdot)$ is an increasing and concave production function and s_q is the share of material goods in gross output. Let $\theta_p(z)$ denote the elasticity of demand for a producer of intermediate goods facing the relative demand $z = d_t(\varsigma)/d_t$. According to our specification, $\theta_p(z) \equiv -G'(z)/(zG''(z))$. This illustrates that intermediate-goods firms face a varying elasticity of demand for their output, implying a varying markup, which is denoted by $\mu_p(z) \equiv \theta_p(z)/(\theta_p(z)-1)$.

Following Calvo (1983), we assume that in each period of time and prior to observing the monetary policy shock, a monopolistic firm can reoptimize its price with probability $1-\alpha_p$, irrespective of the elapsed time since it last revised its price. As in Woodford (2003), if the firm cannot reoptimize its price, the latter is rescaled according to the simple revision rule $P_T(\varsigma) = (1+\delta^p_{t,T})P_t(\varsigma)$, where

$$1+\delta^p_{t,T} = \begin{cases} \prod_{j=t}^{T-1}(1+\pi)^{1-\gamma_p}(1+\pi_j)^{\gamma_p} & \text{if } T > t \\ 1 & \text{otherwise} \end{cases},$$

where $\pi_t = P_t/P_{t-1} - 1$ represents the inflation rate, π is the steady-state inflation rate, and $\gamma_p \in [0,1]$ measures the degree of indexation to the most recently available inflation measure. Let $P^\star_t(\varsigma)$ denote the price chosen in period t by monopolist ς if drawn to reoptimize. Then, firm ς chooses $P^\star_t(\varsigma)$ in order to maximize

$$\mathrm{E}_{t-1}\sum_{T=t}^{\infty}(\beta\alpha_p)^{T-t}\lambda_T\left\{\frac{\left(1+\delta^p_{t,T}\right)P^\star_t(\varsigma)}{P_T}d^\star_{t,T}(\varsigma) - S\left(d^\star_{t,T}(\varsigma)\right)\right\},$$

where λ_T is the representative household's marginal utility of wealth in period T; $\mathrm{E}_{t-1}\{\cdot\}$ is the expectation operator conditional on information available when the firm sets its price; $S(d_t(\varsigma))$ is the real cost of producing $d_t(\varsigma)$ units of good ς; and $d^\star_{t,T}(\varsigma)$, the demand for good ς at T if firm ς last reoptimized its price at t, obeys

$$G'\left(\frac{d^\star_{t,T}(\varsigma)}{d_T}\right) = \left(\frac{\left(1+\delta^p_{t,T}\right)P^\star_t(\varsigma)}{P_T}\int_0^1 \frac{d_t(u)}{d_t}G'\left(\frac{d_t(u)}{d_t}\right)\mathrm{d}u\right).$$

Standard manipulations yield the log-linear New Keynesian Phillips curve

$$\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1} = \mathrm{E}_{t-1}\{\kappa_p(\hat{w}_t + \omega_p \hat{y}_t) + \beta(\hat{\pi}_{t+1} - \gamma_p \hat{\pi}_t)\}, \tag{11}$$

with

$$\kappa_p \equiv \varkappa \frac{(1-\alpha_p)(1-\beta\alpha_p)}{\alpha_p}, \quad \varkappa \equiv \frac{1}{(1-\mu_p s_q)^{-1}(1+\theta_p \epsilon_\mu) + \theta_p \omega_p}.$$

In equation (11), $\hat{\pi}_t$ is the log-deviation of $1+\pi_t$; $\hat{y}_t$ and $\hat{w}_t$ are the log-deviations of y_t and w_t (real wage), respectively; $\theta_p \equiv \theta_p(1)$ is the steady-state elasticity of demand for a producer of intermediate goods; $\mu_p \equiv \mu_p(1)$ is the steady-state markup factor; and

$$\omega_p \equiv -\frac{F''(n)n}{F'(n)} \frac{F(n)}{F'(n)n}.$$

Here, $F(n)$, $F'(n)$, and $F''(n)$ denote the value of F and its first and second derivatives, evaluated at the steady-state value of n. Following Woodford (2003), we let ϵ_μ denote the elasticity of $\mu_p(z)$ in the neighborhood of $z=1$, i.e., $\epsilon_\mu = \mu_p'(1)/\mu_p(1)$.

Aggregate Labor Index and Households

Following Erceg, Henderson, and Levin (2000), we assume for convenience that a set of differentiated labor inputs, indexed by $\upsilon \in [0,1]$, are aggregated into a single labor index h_t by competitive firms, which will be referred to as labor intermediaries. They produce the aggregate labor input according to the following constant elasticity of substitution technology:

$$h_t = \left(\int_0^1 h_t(\upsilon)^{(\theta_w - 1)/\theta_w} \mathrm{d}\upsilon \right)^{\theta_w/(\theta_w - 1)},$$

where $\theta_w > 1$ is the elasticity of substitution between any two labor types. The associated aggregate nominal wage obeys

$$W_t = \left(\int_0^1 W_t(\upsilon)^{1-\theta_w} \mathrm{d}\upsilon \right)^{1/(1-\theta_w)},$$

where $W_t(\upsilon)$ denotes the nominal wage rate paid to type υ labor. The economy is inhabited by a continuum of differentiated households, indexed by $\upsilon \in [0,1]$. A typical household, say household υ, must select a sequence of consumptions and nominal money and bond holdings, as well as a nominal wage. The timing of events is as follows. Prior to observing the monetary policy shock, the household decides how much to consume and sets its nominal wage. The shock is then realized, and bond and money holdings decisions are taken. Household υ's goal in life is to maximize

$$\mathrm{E}_{\Phi_t} \sum_{T=t}^{\infty} \beta^{T-t}[U(c_T - bc_{T-1}, m_T) - V(h_T(\upsilon))],$$

where $\beta \in (0,1)$ is the subjective discount factor; $b \in (0,1)$ is the habit parameter; c_t is consumption; $m_t \equiv M_t/P_t$ denotes real cash balances at the end of the period, where M_t denotes nominal cash balances; and $h_t(\upsilon)$ denotes household υ's labor supply at period t. Here, E_{Φ_t} is a conditional expectation operator reflecting the particular information sets at the household's disposal when taking its decisions. Household υ maximizes its intertemporal utility subject to the sequence of constraints

$$P_t \mathrm{tax}_t + P_t c_t + M_t + \frac{B_t}{1+i_t} \leq W_t(\upsilon)h_t(\upsilon) + B_{t-1} + M_{t-1} + P_t \mathrm{div}_t,$$

where div_t denotes real profits redistributed by monopolistic firms; B_t denotes the nominal bonds acquired in period t and maturing in period $t+1$; i_t denotes the gross nominal interest rate; and tax_t is a lump-sum tax levied by the government. As in Woodford (2003), we assume that there is a satiation level $m^\star$ for real balances such that $U_m = 0$ for $m \geq m^\star$. Thus, when m_t reaches $m^\star$ from below, the transaction services of real cash balances yield lower and lower marginal utility. Let λ_t denote the Lagrange multiplier associated with the household's budget constraint. According to the timing of decisions embedded in Φ_t, the log-linearization of the first-order conditions associated with c_t, B_t, and M_t yields

$$\begin{aligned}\mathrm{E}_{t-1}\{&\beta b(\hat{c}_{t+1} - b\hat{c}_t) - (\hat{c}_t - b\hat{c}_{t-1}) \\ &+ \sigma\chi(\hat{m}_t - \beta b\hat{m}_{t+1}) - \varphi^{-1}\hat{\lambda}_t\} = 0, \end{aligned} \tag{12}$$

$$\hat{\lambda}_t = \hat{\imath}_t + \mathrm{E}_t\{\hat{\lambda}_{t+1} - \hat{\pi}_{t+1}\}, \tag{13}$$

$$\hat{m}_t = \eta_y(\hat{c}_t - b\hat{c}_{t-1}) - \eta_i\hat{\imath}_t, \tag{14}$$

where $\hat{c}_t$, $\hat{m}_t$, $\hat{\imath}_t$, and $\hat{\lambda}_t$ are the log-deviations of c_t, m_t, $1+i_t$, and λ_t, respectively, and where we defined the auxiliary parameters $\sigma^{-1} = -U_{cc}c/U_c$, $\chi = U_{cm}m/U_c$, $\varphi^{-1} = (1-\beta b)\sigma$, $\eta_y = -U_{mc}c/(U_{mm}m)$, and $\eta_i = -(1-\beta b)U_c/(U_{mm}m)$. Notice that $\chi = (1-\beta b)\eta_y/(\bar{v}\eta_i)$, where $\bar{v}$ is the steady-state value of c_t/m_t.

A typical household v acts as a monopolistic supplier of type v labor. It is assumed that at each point in time, and prior to observing the monetary policy shock, only a fraction $1-\alpha_w$ of the households can set a new wage, which will remain fixed until the next time period the household is drawn to reset its wage. The remaining households simply revise their wages according to the simple rule $W_T(v) = (1+\delta^w_{t,T})W_t(v)$, where

$$1+\delta^w_{t,T} = \begin{cases} \prod_{j=t}^{T-1}(1+\pi)^{1-\gamma_w}(1+\pi_j)^{\gamma_w} & \text{if } T>t \\ 1 & \text{otherwise} \end{cases},$$

where $\gamma_w \in [0,1]$ measures the degree of indexation to the most recently available inflation measure.

Let us now consider the wage-setting decision confronting a household drawn to reoptimize its nominal wage rate in period t, say household v. Let us define wage inflation $\pi^w_t \equiv W_t/W_{t-1} - 1$. Now, let $W^\star_t(v)$ denote the wage rate chosen in date t and $h^\star_{t,T}(v)$ denote hours worked in period T if household v last reoptimized its wage in period t, which obey the relationship

$$h^\star_{t,T}(v) = \left(\frac{(1+\delta^w_{t,T})W^\star_t(v)}{W_T}\right)^{-\theta_w} h_T.$$

$W^*_t(v)$ is then selected so as to maximize

$$\mathrm{E}_{t-1}\sum_{T=t}^{\infty}(\beta\alpha_w)^{T-t}\left\{\lambda_T\frac{(1+\delta^w_{t,T})W^\star_t(v)}{P_T}h^\star_{t,T}(v) - V\big(h^\star_{t,T}(v)\big)\right\}.$$

Log-linearizing the associated first-order condition yields

$$\hat{\pi}^w_t - \gamma_w\hat{\pi}_{t-1} = \mathrm{E}_{t-1}\big\{\kappa_w(\omega_w\hat{h}_t - \hat{\lambda}_t - \hat{w}_t) + \beta\big(\hat{\pi}^w_{t+1} - \gamma_w\hat{\pi}_t\big)\big\}, \tag{15}$$

where $\hat{\pi}_t^w$ is the log-deviation of $1 + \pi_t^w$ and where we defined the composite parameters

$$\kappa_w = \frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\omega_w\theta_w)}, \quad \omega_w = \frac{V_{hh}h}{V_h}.$$

Finally, $\hat{\pi}_t$ and $\hat{\pi}_t^w$ are linked together through the relation

$$\hat{\pi}_t^w = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t. \tag{16}$$

The model is closed by specifying the policy rule (1)–(3).

In equilibrium, it must be the case that $y_t = c_t$ and $h_t = n_t$. Furthermore, from the aggregate production function, it must also be the case that $\hat{n}_t = \phi\hat{y}_t$, where $\phi^{-1} = F'(n)n/F(n)$. Substituting these relations in the system composed of (11)–(16), augmented with equations (1)–(3), we obtain a rational-expectations system of linear equations, which we solve using standard methods.

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The Role of the Bias in Crafting Consensus: FOMC Decision Making in the Greenspan Era*

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We examine the role of the "bias" associated with a monetary policy directive—wording in the directive that concerns possible policy shifts in the period between one FOMC meeting and the next—in FOMC decision making in the Greenspan years. Previous studies have suggested that the bias provided the Chairman a tool for orchestrating Committee consensus. Our evidence shows that when the bias had meaningful implications for intermeeting funds rate changes (1987–92), it influenced voting by FOMC members. Biases both provoked and discouraged dissents, depending on the direction of the bias and the preferences of individual Committee members. When the bias did not have meaningful implications for intermeeting policy adjustments (1993–99), we find no evidence that it affected members' voting choices. Overall, our results are consistent with the view that FOMC members voted on the basis of a rational assessment of the policy content of proposed directives.

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> *An important function of the bias was (and perhaps still is) to aid the Chairman in reducing the number of dissenting votes and to allow members to have their views "count" without dissenting. For example, if the Chairman wanted to achieve a consensus on "no change" in the funds rates while limiting dissents he could offer a bias towards higher rates. Members who wanted a decision for higher rates might accept a bias instead of dissenting.*
>
> (Hoskins 1999, 3)

1. Introduction

From 1983 through 1999, monetary policy directives adopted by the Federal Open Market Committee (FOMC) included a statement of "bias," wording that purportedly described the likelihood of policy shifts in the period between one FOMC meeting and the next.[1] Although the stated purpose of the bias was to describe policy choices in a probabilistic manner, a second possible role has been emphasized by Thornton and Wheelock (2000) and Meade (2005), who argue that the wording of the bias was often framed to orchestrate Committee consensus.

An appropriately formulated policy bias might have encouraged consensus in two ways. First, if the bias were a meaningful indicator of upcoming policy moves, then it could have permitted the Committee to adopt a middle ground when some members preferred a policy shift and others did not. Such an action might have increased consensus and lowered the likelihood of dissent voting. Second, even if the bias were not a meaningful indicator of policy, it might have fostered consensus by offering conciliatory language. Suppose that the Committee decided to maintain the status quo funds rate, but

[1] Consider the following example, taken from the policy directive adopted at the FOMC meeting held on July 2–3, 1996: "In the implementation of policy for the immediate future, the Committee seeks to maintain the existing degree of pressure on reserve positions. In the context of the Committee's long-run objectives for price stability and sustainable economic growth, and giving careful consideration to economic, financial, and monetary developments, somewhat greater reserve restraint would or slightly lesser reserve restraint might be acceptable in the inter-meeting period." The distinction between "somewhat greater reserve restraint," which *would* be acceptable in the intermeeting period, and "slightly lesser reserve restraint," which *might* be acceptable in the intermeeting period, indicates that this directive is biased (or asymmetric) toward tightness.

a minority preferred a move. While the Committee might have had no intention of moving, it could have adopted a bias in an effort to placate the losers. Such an action, while not meaningful in terms of policy intent, might have reduced dissent if members were assuaged by the inclusive gesture from the majority.

The setting of the bias also had the potential to create discord. If a proposed policy had detractors on both sides (with some favoring ease and others tightness), any bias moving toward one group would have risked alienating the other group. Furthermore, if the Chairman used the bias to advance his agenda at the expense of other Committee members, he could have provoked additional dissent.

In this paper, we investigate the role that the bias played in the formulation of FOMC policy directives and in Committee voting on those directives during the 1987–99 portion of Alan Greenspan's tenure as Chairman of the FOMC. Specifically, we ask (i) whether a "favorable" bias reduced the probability of a member casting a dissenting vote, (ii) whether an "unfavorable" bias increased the probability of a member casting a dissenting vote, (iii) whether biases were usually set in a way that induced members to view them favorably, and (iv) whether the importance of the bias in voting choices reflected its importance as an indicator of policy.

Our findings throughout are consistent with the view that FOMC members voted in a rational, policy-oriented manner. From 1987 through 1992, the bias was a good indicator of upcoming intermeeting policy moves, and the setting of the bias also affected individual FOMC members' votes—biases favorable to an individual generated assents, and biases that were unfavorable generated dissents. After 1993, the bias was a less reliable predictor of intermeeting funds rate movements, and it simultaneously lost significance as a predictor of individuals' votes. These results cast doubt on the view that bias setting might have produced consensus without the offer of a meaningful policy concession. Given the pattern of adopted biases and dissenting votes observed in our sample period, it is not obvious that bias setting lowered the observed frequency of dissenting votes.

2. FOMC Decision Making

In the Greenspan era, FOMC meetings followed a routine agenda. The policymaking portion of the meeting began with a staff report on economic conditions and subsequent questioning and discussion by the Committee. This was followed by an "economics go-around" in which Committee members presented personal assessments of economic conditions. In the economics go-around, District Reserve Bank presidents generally reported on anecdotal regional information, while governors assessed national conditions. The staff then presented a report describing policy options, and this was followed by a "policy go-around" in which Committee members described their own policy preferences. Chairman Greenspan typically spoke first in the policy go-around and offered a policy proposal that provided a frame of reference for subsequent speakers. After the policy go-around, the Chairman proposed final policy specifications, including both a target funds rate and a setting for the bias, for a formal vote. Voting members could either "assent" or "dissent." As a practical matter, Greenspan's original proposal was usually adopted with broad support; only about 7 percent of all votes cast were dissents.

Members' voting choices presumably depend on how their own policy preferences compare with those adopted by the Committee, but members also value consensus and recognize that the Chairman's views are accorded greater weight than their own when preferences are aggregated (Chappell, McGregor, and Vermilyea 2004). Under these circumstances, members are likely to cast assenting votes if the Chairman's proposal is not too different from their own. Because the Chairman also values consensus, his proposals are likely to give weight to his perception of the central tendency of the Committee[2]—he might sometimes marginally sacrifice his own preference in order to achieve a consensual outcome.[3] An appropriately crafted bias

[2]Alan Greenspan has described his ability to divine the Committee's view this way: "I've been around this committee for a number of years and I think I can say that I pretty much know how every single member of this committee would come out under [any given hypothetical] event. In other words, I could take the vote myself if I had to and I bet I'd get it on the nose three times out of four. The reason for that is that I know where you're all coming from" (*FOMC Transcripts*, May 18, 1993, p. 54).

[3]Blinder (2004, 58–59) states that "if push ever comes to shove, the chairman knows that he lacks the de jure authority to force his committee members to accept his position. Rebellion is always possible, if the chairman is out of step with the committee. The strong desire for de facto consensus therefore enables the rest of the committee to serve as a kind of check on the chairman, who cannot easily pursue extreme policies, follow highly idiosyncratic procedures, or base policy on controversial theories."

could offer a way to make such a policy concession and, because of its importance on the margin, could limit dissent. Moreover, a bias could also give the Chairman discretion to act between meetings; however, if members prefer to constrain the Chairman, a bias that grants added discretion could provoke dissents.[4]

3. Data: Policies, Preferences, and Votes

In this section, we describe the data sources we have used in our analysis. Because votes on the policy directive provide the most visible and timely indication of the degree of consensus within the Committee, our analysis employs formal voting records reported in the meeting summaries published in the monthly *Federal Reserve Bulletin*.[5] We supplement the voting record with detailed indicators of members' policy preferences derived from transcripts of FOMC deliberations.[6] Because the transcripts describe the policies that members advocated before submitting formal votes, it is possible to link members' voting decisions to more-detailed expressions of their policy preferences.

In the course of the policy go-around in a Greenspan-era FOMC meeting, it was common for members to identify themselves with a specific target federal funds rate. Members often indicated agreement with the Chairman, but it was not unusual for members to advocate higher or lower rates, typically associating themselves with alternatives specified in the Bluebook or with 25- or 50-basis-point movements relative to the prevailing funds rate.[7] Whenever

[4]Meeting transcripts show that members did sometimes voice concerns with discretion exercised by the Chairman between meetings. Governor Wayne Angell once complained, "I vote with the majority and I end up losing. And, Governor Johnson, I just have to congratulate you . . . you voted in the minority and you've won! . . . I would like some assurance that we are not going to just keep doing this, Mr. Chairman" (*FOMC Transcripts*, February 9, 1988, p. 64).

[5]Before 1993, the meeting summaries were published under the title "Record of Policy Actions of the Federal Open Market Committee"; from 1993 through the end of our sample, they were published under the title "Minutes of the Federal Open Market Committee."

[6]FOMC meeting transcripts are available (after a five-year lag) on the Federal Reserve Board web site at www.federalreserve.gov/fomc/transcripts.

[7]The Bluebook prepared by the Federal Reserve Board staff for each FOMC meeting presents a set of policy scenarios for discussion as the Committee crafts a monetary policy directive.

individual members stated preferences in this manner, we recorded a preferred target funds rate for them.[8] For the 1987–99 period, members' desired target federal funds rates could be directly inferred from statements in the transcripts in 91.9 percent of all member-meeting observations; for each of these observations, we directly observe whether members' preferences differ from the Chairman's proposal and, if so, by how much. Less frequently, members described their preferences in qualitative terms. In these cases, we coded members as "leaning toward tightness," "leaning toward ease," or "assenting" relative to a benchmark funds rate. For example, a member might state a funds rate preference as "4.75 percent or a bit higher," which we would code as "leaning toward tightness" relative to the 4.75 percent benchmark.[9]

The meeting transcripts also provide information on members' desired bias settings. Members who voiced agreement with the Chairman's proposal on the funds rate also usually revealed a preferred bias setting; these preferences on the bias are recorded in our data set. Members who advocated a funds rate different from the Chairman's proposal typically did not state a preference about the bias.

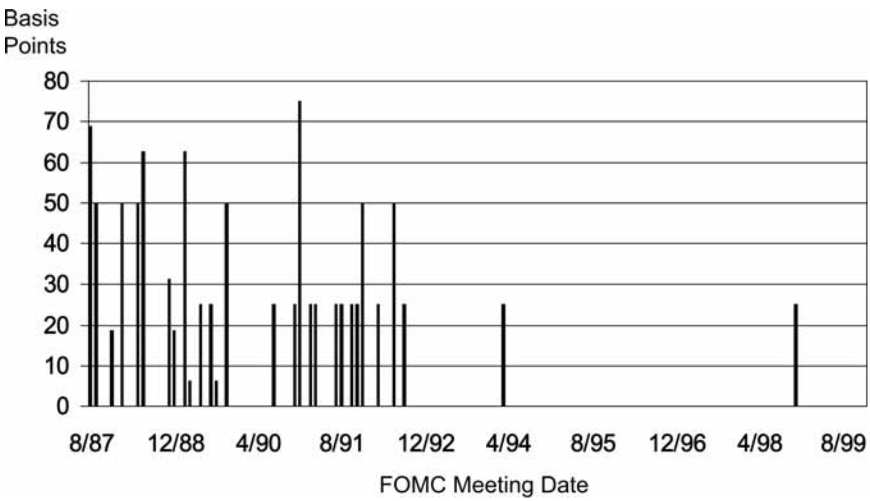
4. A Natural Experiment: The Changing Meaning of the Bias

While formal decision-making procedures were essentially unchanged over our sample period, an important change occurred in practice, as figure 1 illustrates. The figure plots intermeeting changes in the

[8]For most of the meetings in our sample, Bluebook policy scenarios reported target levels of the federal funds rate, and policy discussions were carried out with reference to funds rate targets. Early in the Greenspan era, policy scenarios reported both target levels of reserve borrowing and the funds rate targets associated with those borrowing levels. In the policy discussions, some members referred to borrowing targets, while other members referred to funds rate targets. For the former, we used the mapping between funds rates and borrowing targets provided in the Bluebook to code the implied target federal funds rate. See Chappell, McGregor, and Vermilyea (2005) for details.

[9]See Chappell, McGregor, and Vermilyea (2005, 57–69) for a detailed description of our coding procedures. The codings themselves for the 1987–96 period appear in appendix 5 of that work. We have extended our sample through 1999 for the analysis in this paper.

Figure 1. Absolute Difference Between the Committee's Adopted Federal Funds Rate and the Federal Funds Rate Prevailing at the Next Meeting



target federal funds rate (as absolute values) over the entire 1987–99 period. It reveals that intermeeting movements were very common in the 1987–92 interval but rare in the 1993–99 period. The absence of intermeeting movements in the latter part of the sample is not accompanied by a similar absence of asymmetric bias settings. From 1993 through 1999, nonsymmetric biases were adopted in twenty-four out of fifty-six meetings, but intermeeting movements took place only twice.[10] It appears that the bias was not a meaningful predictor of intermeeting rate adjustments in this period.

This change is confirmed by historical accounts. According to the *New York Times*, the decision-making process did change in the early 1990s. The FOMC began to set rates at its regularly scheduled meetings, and other members discouraged Greenspan from acting "on his own between meetings, as he often [had] in the past" (*New York Times*, April 8, 1994). Although there was no formal change in

[10]There have been only three intermeeting funds rate moves in the period since 1999 (up to the date of this writing in 2006). All three occurred in 2001, and all were approved by Committee votes in telephone conference call meetings.

rules, the *Times* reported that there was a new understanding that Greenspan did not challenge.[11] Statistical evidence also supports the hypothesis of a regime change. Lapp and Pearce (2000) have provided evidence confirming that the bias was a significant predictor of intermeeting funds rate movements over the period from August 1987 through December 1998. Using their model specifications and data, but restricting their sample to the post-1992 period, we find that the bias ceased to be a significant predictor of intermeeting rate changes.

This regime shift provides us with a useful natural experiment. If FOMC members' votes simply reflect a rational assessment of the policy content of the directive, then the bias should affect voting behavior in the early part of the sample but not in the later part of the sample. If voting choices respond to the bias in the later period, this would suggest that a conciliatory bias can assuage those in the minority even when it has no implications for intermeeting policy adjustments.

5. Empirical Analysis

We initially use our data to classify FOMC members' monetary policy positions into five categories based on a comparison of their preferences to the Chairman's proposed directives. Our categorical variables are described below in order of least to greatest difference of preference (our ordering assumes that target funds rates are more salient than biases).

D_0 *No Disagreement.* There are no revealed differences between the Chairman's proposed directive and the policy preferred by the member, either in terms of the target funds rate or the bias.

D_1 *No Disagreement on Rate/Disagreement on Bias.* There is no indicated disagreement between a member's desired funds rate

[11]The abandonment of intermeeting policy changes may have been related to the evolving procedural shift from borrowed reserves targeting to federal funds rate targeting. Intermeeting policy adjustments under a borrowed reserves targeting regime were probably less transparent and therefore less objectionable to Committee members; however, the shift to funds rate targeting was complete by 1989, so it clearly preceded the decision to limit intermeeting policy moves.

and the Chairman's proposed target, but there is disagreement on the bias.[12]

D_2 *Disagreement on Rate/Bias Is Favorable.* The member prefers a target rate that differs from that proposed by the Chairman, and the proposal has a bias that is favorable to the member. For example, a member might prefer a funds rate of 4.00 percent when the Chairman proposes 4.25 percent. If the Chairman's proposal incorporates a bias toward ease, this would be favorable to the member.

D_3 *Disagreement on Rate/No Bias.* The member prefers a target rate that differs from that proposed by the Chairman, and the directive is symmetric.

D_4 *Disagreement on Rate/Bias Is Unfavorable.* The member prefers a target rate that differs from that proposed by the Chairman, and the proposal has a bias that is unfavorable to the member. For example, a member might prefer a funds rate of 4.00 percent when the Chairman proposes 4.25 percent. If the Chairman's proposal incorporates a bias toward tightness, this would be unfavorable to the member.

Our analysis uses 1,005 member voting observations (excluding votes of the Chairman) over ninety-nine FOMC meetings held in the 1987–99 period.[13] In table 1, we report dissent frequencies (the number of dissenting votes as a fraction of the number of observations) for observations falling into each of the five categories defined by variables D_0 through D_4. The reported statistics for the full sample confirm our expectation that dissent frequencies will increase as

[12] The category "disagreement on the bias" could be further refined. Consider a member who favors a bias toward ease. This member would disagree with a symmetric directive or a bias favoring tightness, with the latter case implying more-severe disagreement. In our data set, we had just one observation of the latter type, so we did not create a categorical variable to distinguish it.

[13] The FOMC met 100 times in our sample period, but there is no transcript of the policy go-around for the meeting held on March 29, 1988. Because we do not have information on members' policy preferences for this meeting, we exclude it from our analysis.

Table 1. Dissent Frequency by Category of Disagreement with the Chairman's Proposal

	D_0	D_1	D_2	D_3	D_4
	Full Sample: 8/87 through 12/99				
Assent	689	121	69	43	10
Dissent	3	14	24	22	10
Dissent Frequency (percent)	0.43	10.37	25.81	33.85	50.00
	Early Years: 8/87 through 12/92				
Assent	252	74	45	15	8
Dissent	2	13	11	10	9
Dissent Frequency (percent)	0.79	14.94	19.64	40.00	52.94
	Late Years: 2/93 through 12/99				
Assent	437	47	24	28	2
Dissent	1	1	13	12	1
Dissent Frequency (percent)	0.23	2.08	35.14	30.00	33.33

we move through the categories from D_0 to D_4.[14] Dissent voting frequencies rise monotonically from 0.43 percent in category D_0 to 50 percent in category D_4. This pattern is also evident in the early 1987–92 subperiod, when the bias was known to be a good predictor of intermeeting funds rate movements.

However, for the 1993–99 period in which the bias was not a good predictor of intermeeting rate adjustments, the results are strikingly different. Dissent voting frequencies for observations in categories D_0 and D_1 are very low, implying that if a member agrees with the

[14]Note that three dissents occurred for observations where our preference coding did not reveal a disagreement. In one case, the dissent vote was motivated by a difference of opinion about the appropriate operating procedure rather than about the appropriate policy specifications. In the other two cases, the individuals' statements in the transcript were ambiguous, although the explanations the members later provided in the Committee's minutes were not. In such cases, our coding procedure requires us to infer "no revealed disagreement." It would be inappropriate for us to use the voting record, which provides our dependent variable, to assist in coding preferences, which are the basis for our explanatory variables.

funds rate target in the proposal, that member is unlikely to dissent. Moreover, the probability of dissent apparently does not depend on the setting of the bias. Dissent voting frequencies for observations in categories D_2, D_3, and D_4 are much higher and are approximately equal across the categories. These results imply that if a member disagrees with the funds rate target in the proposal, that member has a higher probability of dissenting; again, though, this probability does not depend on the setting of the bias. Therefore, the descriptive statistics strongly suggest that the setting of the bias affected dissent voting propensities before 1993 but not after.

We also examine dissent voting behavior in the framework of a logit model. Our simplest logit model specifies that an individual Committee member's probability of dissent is a function of the four categorical variables D_1, D_2, D_3, and D_4 (with the D_0 category captured by the intercept). Estimates reported in the first column of table 2 show a pattern consistent with that revealed by the simple dissent voting frequencies for the full sample. All coefficients are positive and significantly different from zero, indicating higher dissent probabilities in categories D_1 through D_4 than in the "complete agreement" category captured in the intercept.[15] Further, the coefficients are successively larger for categories arranged in order of increasing disagreement and, in most cases, pairwise comparisons of the coefficients produce differences that are statistically significant.[16]

The estimations for the 1987–92 sample mirror those reported for the full sample—coefficient patterns consistently imply that a stronger difference between a member's preference and the proposed policy increases the probability of a dissenting vote.[17] For the 1993–99 sample, the results differ notably. The D_1 coefficient is not significantly different from zero, implying that when members agree with

[15]Note that the coefficients for the intercept and D_4 in the full-sample estimation are identical in magnitude but opposite in sign. This result is coincidental. In our sample, the dissent voting frequency for observations in the D_4 category is exactly 50 percent, requiring that the intercept and D_4 coefficients sum to zero (in the logit model, the probability of dissent is given by $\frac{e^{\mathbf{X}\boldsymbol{\beta}}}{1+e^{\mathbf{X}\boldsymbol{\beta}}}$; for this probability to be 0.50, $\mathbf{X}\boldsymbol{\beta}$ must be 0).

[16]With five disagreement categories, there are ten possible pairwise comparisons, and eight of ten comparisons indicate significant differences at the 0.05 level or better.

[17]In the subsample, six of ten pairwise comparisons indicate significant differences at the 0.05 level or better.

Table 2. Logistic Regression: Probability of Dissenting as a Function of Categories of Disagreement

	Full Sample 8/87 through 12/99	**Early Years 8/87 through 12/92**	**Late Years 2/93 through 12/99**
Intercept	−5.437***	−4.836***	−6.080***
	(0.579)	(0.710)	(1.001)
D_1	3.280***	3.097***	2.230
	(0.644)	(0.771)	(1.423)
D_2	4.381***	3.427***	5.467***
	(0.625)	(0.785)	(1.059)
D_3	4.767***	4.430***	5.233***
	(0.635)	(0.819)	(1.059)
D_4	5.437***	4.954***	5.387***
	(0.731)	(0.860)	(1.582)
N	1,005	439	566
Tests of Additional Hypotheses (χ^2 Statistic)			
$D_2 = D_3$	1.193	3.598*	0.231
$D_2 = D_4$	4.354**	6.673***	0.004
$D_3 = D_4$	1.671	0.680	0.015

Note: The numbers in parentheses are standard errors. *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively.

the proposed rate, the bias has no effect on voting (recall that the D_0 category is captured in the intercept). Similarly, we are unable to reject the hypothesis that the coefficients of D_2, D_3, and D_4 are all equal, implying that when members disagree on the rate, alternative settings of the bias have no effect on dissent voting probabilities.

The estimates reported in table 2 use only categorical data to describe the relationship of a member's policy preference to the Chairman's policy proposal. In fact, we have continuous measures of both the proposed funds rate and a member's desired funds rate for 935 of the 1,005 observations in our data set (see table 3 for details).

Table 3. Difference Between Member's Desired Rate and Greenspan's Proposal

$\lvert R^*_{it} - R^*_t \rvert$	0.0	0.0313	0.0625	0.125	0.1875	0.25	0.5	Qual. Agree.[a]	Qual. Disagree.[b]
Frequency	817	1	11	10	3	65	28	10	60

[a] "Qualitative agreement" refers to situations in which an FOMC member did not explicitly state a funds rate preference and did not voice disagreement with Greenspan's proposal.
[b] "Qualitative disagreement" refers to situations in which an FOMC member voiced disagreement with Greenspan's proposal but did not explicitly state a funds rate preference.

Where it is possible to measure the quantitative extent of a member's reported difference with the proposed funds rate, it is desirable to include that information in the model. We therefore construct the following variables to add to the specification:

V_1 *Absolute Chairman-Member Deviation.* V_1 is equal to $\lvert R^*_{it} - R^*_t \rvert$, the absolute value of the deviation between the member's desired funds rate, R^*_{it}, and the Chairman's proposed funds rate, R^*_t, for cases where both R^*_{it} and R^*_t are directly observed. Otherwise, V_1 is equal to 0.

V_2 *Categorical Chairman-Member Deviation.* V_2 is a dummy variable equal to 1 if R^*_{it} is not directly observed but is known to differ from R^*_t based on categorical information. Otherwise, V_2 is equal to 0.

Estimates of this model are reported in table 4 for the full sample and for each of the two subsamples. The coefficients for the V_1 variable differ significantly from zero in each estimation, confirming that the propensity to dissent is influenced by the magnitude of policy differences. The V_2 coefficient is also positive, but it is significant only for the complete sample.

Conclusions about the impact of the bias derived from table 4 are consistent with those reported for the more parsimonious model in table 2. In the complete sample and the 1987–92 subperiod, the bias has predictable consequences for voting. Given agreement on the rate (i.e., both V_1 and V_2 equal 0), the significant positive D_1

Table 4. Logistic Regression: Probability of Dissenting as a Function of Indicators of Disagreement

	Full Sample 8/87 through 12/99	Early Years 8/87 through 12/92	Late Years 2/93 through 12/99
Intercept	−5.437***	−4.836***	−6.080***
	(0.579)	(0.710)	(1.001)
V_1	6.513***	8.303***	6.108*
	(1.689)	(2.365)	(3.227)
V_2	1.273*	1.405	1.327
	(0.670)	(1.070)	(1.125)
D_1	3.280***	3.097***	2.230
	(0.644)	(0.771)	(1.423)
D_2	2.522***	0.812	3.822**
	(0.879)	(1.262)	(1.497)
D_3	3.151***	2.613***	3.595**
	(0.801)	(1.007)	(1.444)
D_4	4.001***	3.219***	3.860**
	(0.836)	(1.026)	(1.776)
N	1,005	439	566
Tests of Additional Hypotheses (χ^2 Statistic)			
$D_2 = D_3$	2.471	6.052**	0.191
$D_2 = D_4$	6.607**	9.690***	0.001
$D_3 = D_4$	2.424	0.786	0.042

Note: The numbers in parentheses are standard errors. *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively.

coefficient implies that disagreement on the bias produces dissents. When there is disagreement on rates, the coefficients on D_2, D_3, and D_4 are consistently ordered, implying that favorable biases lower the propensity to dissent, while unfavorable biases raise it.[18]

[18] Pairwise differences are significant in one of three cases in the complete sample period and two of three cases in the early sample period.

The third column of table 4 reports results for the 1993–99 subperiod. The D_1 coefficient is no longer significantly different from zero, and we cannot reject the hypothesis that the coefficients of D_2, D_3, and D_4 are equal. As with the more parsimonious model, these results imply that the setting of the bias has no effect on dissent voting decisions in the later subperiod.

The incremental probability effects implied by our model provide additional insight into our findings. In the early period, if there is a 25-basis-point disagreement on the funds rate target, then the probability of dissent rises from 0.125 to 0.463 to 0.613 as we move through categories D_2, D_3, and D_4. Viewed another way, if we take a 25-basis-point rate disagreement and no bias as our base case, then we see that a move to a favorable bias reduces the probability of dissent by 0.338, while a move to an unfavorable bias increases the probability of dissent by only 0.150. Thus, when there is disagreement on the funds rate target, a favorable bias has a greater effect in reducing the likelihood of dissent than an unfavorable bias has in raising the likelihood of dissent. This finding is consistent with Meade's (2005) argument that favorable bias statements helped Greenspan obtain a consensus vote on the proposed monetary policy directive. Nevertheless, our results also show how the bias can provoke dissent not only when there is disagreement on the rate but also when there is agreement on the rate but a disagreement on the bias. In the early period, if we assume no difference in funds rate preferences ($V_1 = V_2 = 0$), then a disagreement on the bias increases the probability of dissent from 0.008 to 0.149.[19]

The results we have described are robust to a variety of specification changes. First, we note that the composition of the Committee changed over time. Three individuals (Wayne Angell, Lee Hoskins, and Martha Seger) frequently dissented prior to 1993, but all had left the Committee by early 1994. This suggests that the decline in the frequency of dissents in the later period might reflect the departure of these frequent dissenters rather than a change in the meaning of

[19]The incremental probability effects for the later period are much smaller. Given a 25-basis-point difference in rates, probabilities of dissent go from 0.184 to 0.155 to 0.189 as we move through the D_2, D_3, and D_4 categories. If we assume no difference in rates, then a disagreement on the bias increases the probability of dissent from 0.002 to 0.021 in the later period.

Table 5. Logistic Regression: Probability of Dissenting as a Function of Indicators of Disagreement—Robustness Check Removing Angell, Hoskins, and Seger from the Sample

	Full Sample 8/87 through 12/99	**Early Years 8/87 through 12/92**	**Late Years 2/93 through 12/99**
Intercept	−5.388***	−4.705***	−6.075***
	(0.579)	(0.710)	(1.001)
V_1	4.888**	9.756**	3.244
	(2.198)	(4.076)	(3.657)
V_2	1.210	3.074*	0.536
	(0.803)	(1.638)	(1.221)
D_1	3.267***	3.114***	2.247
	(0.654)	(0.784)	(1.423)
D_2	2.556**	−1.026	4.524***
	(0.999)	(1.944)	(1.578)
D_3	3.231***	1.872	4.312***
	(0.891)	(1.325)	(1.497)
D_4	3.639***	1.906	4.571**
	(0.979)	(1.384)	(1.827)
N	911	353	558
Tests of Additional Hypotheses (χ^2 Statistic)			
$D_2 = D_3$	2.105	6.137**	0.153
$D_2 = D_4$	2.122	5.772**	0.001
$D_3 = D_4$	0.335	0.001	0.040

Note: The numbers in parentheses are standard errors. *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively.

the bias. Table 5 replicates table 4 but eliminates all of the Angell, Hoskins, and Seger observations. The key results are unchanged—in the early period, the D_1 coefficient is positive and significant, and the coefficients on D_2, D_3, and D_4 are consistently ordered (with significant differences in two of three pairwise comparisons). In the later period, the D_1 coefficient is insignificant, as are all pairwise comparisons among the D_2, D_3, and D_4 coefficients.[20]

We have also investigated the sensitivity of results to the timing of the regime shift that resulted in the abandonment of frequent intermeeting funds rate moves. We initially assumed that the early period ended in December 1992, but our results are similar when we instead assume that the break occurred in December 1993 (figure 1 strongly suggests that the break occurred sometime in 1993). Table 6 presents the results for these estimations.

Another possible regime shift is the transition from borrowed reserves targeting to federal funds rate targeting. This transition had been completed by 1989, so as an additional robustness check, we reestimate our models, omitting 1987 and 1988 from the sample. Results presented in table 6 show that our conclusions are unaffected by this modification to our sample period.

Finally, our original specification assumed that the voting model is the same for governors and Reserve Bank presidents. In table 7, we have modified the specification to permit an intercept shift for governors. The relevant coefficient is significant only in the estimation for the 1993–99 subsample. The negative sign implies that, for given levels or categories of disagreement, governors were less likely to dissent than Bank presidents in that period. This result could reflect turnover in the Committee; however, all results regarding the impact of the bias on dissent voting are again robust to this change.

Overall, our results show that a policy-relevant bias can either provoke or limit dissent, depending on its setting vis-à-vis members' preferences. Somewhat surprisingly, we find that the bias was often set in a manner that might have provoked dissent. Over the ninety-nine Greenspan-era meetings in our sample, members advocated policy positions on both sides of the Chairman on thirty-five occasions, so some conflict was inevitable. In the 1987–92 subperiod when the bias had demonstrable policy content, there were notably more unfavorable bias settings (104 observations in the combined D_1 and D_4 categories) than favorable ones (56 observations in the D_2 category). In this period, Greenspan frequently used the bias

[20] Our results for the full sample and the early and late subsamples also hold if we use a model that accounts for member fixed effects.

Table 6. Logistic Regression: Probability of Dissenting as a Function of Indicators of Disagreement—Robustness Check for Alternative Sample Splits by Date

	Eliminating Borrowed Reserves Regime from Early Years 2/89 through 12/92	**Early Years with Alternative Sample Break Point 8/87 through 12/93**	**Late Years with Alternative Sample Break Point 2/94 through 12/99**
Intercept	−5.267*** (1.002)	−5.084*** (0.709)	−5.903*** (1.001)
V_1	5.881* (3.568)	8.491*** (2.197)	3.402 (3.691)
V_2	0.508 (1.515)	1.497 (0.916)	0.715 (1.256)
D_1	3.658*** (1.051)	3.367*** (0.766)	
D_2	2.287 (1.914)	1.144 (1.188)	4.356*** (1.592)
D_3	4.382*** (1.484)	2.898*** (0.991)	3.891** (1.517)
D_4	3.983*** (1.540)	3.421*** (1.003)	4.359** (1.831)
N	326	527	478
Tests of Additional Hypotheses (χ^2 Statistic)			
$D_2 = D_3$	4.839**	7.741***	0.654
$D_2 = D_4$	2.928*	9.521***	0.000
$D_3 = D_4$	0.223	0.640	0.128

Note: The numbers in parentheses are standard errors. *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively. When the sample is split after 1993 rather than after 1992, we have no dissents in the D_1 category during the later period (1994–99). Estimating the model for the later period requires us to omit this category from the specification.

to provide himself with justification for intermeeting moves when Committee sentiment for those moves was in doubt.[21] Although we

[21] Meade (2005) reports that a series of dissents by Governor John LaWare and Federal Reserve Bank of St. Louis President Thomas Melzer in 1991 and 1992 were probably motivated by Greenspan's reliance on the bias to justify intermeeting moves when the directive had not explicitly called for a change in the target rate.

Table 7. Logistic Regression: Probability of Dissenting as a Function of Indicators of Disagreement—Governors versus Reserve Bank Presidents

	Full Sample 8/87 through 12/99	Early Years 8/87 through 12/92	Late Years 2/93 through 12/99
Intercept	−5.338***	−4.933***	−5.638***
	(0.596)	(0.745)	(1.013)
V_1	6.484***	8.486***	8.351**
	(1.691)	(2.416)	(3.567)
V_2	1.254*	1.518	2.100*
	(0.672)	(1.105)	(1.226)
D_1	3.271***	3.107***	2.185
	(0.644)	(0.772)	(1.425)
D_2	2.506***	0.762	3.011*
	(0.880)	(1.273)	(1.567)
D_3	3.155***	2.567**	2.913*
	(0.803)	(1.013)	(1.508)
D_4	4.013***	3.159***	2.857
	(0.839)	(1.034)	(1.850)
Gov	−0.195	0.174	−1.128**
	(0.293)	(0.393)	(0.568)
N	1,005	439	566
Tests of Additional Hypotheses (χ^2 Statistic)			
$D_2 = D_3$	2.610	6.019**	0.034
$D_2 = D_4$	6.793***	9.538***	0.013
$D_3 = D_4$	2.461	0.751	0.002

Note: The numbers in parentheses are standard errors. *, **, and *** indicate statistical significance at the 0.10, 0.05, and 0.01 levels, respectively.

cannot draw unambiguous conclusions, it seems doubtful that bias settings reduced aggregate dissent frequencies.[22]

6. Conclusions

We have examined how the bias associated with monetary policy directives influenced dissent voting patterns of FOMC members in the Greenspan years. We find that the setting of the bias affected voting, but it did so only when the bias was a meaningful policy indicator for the upcoming intermeeting period. Specifically, bias settings affected voting choices in the 1987–92 period, when the bias was a good predictor of intermeeting movements in funds rates, but failed to do so in the 1993–99 period, when the bias was a poor predictor of intermeeting rate changes.

Our work encompasses and extends that of Meade (2005). We corroborate her finding that favorable biases helped to orchestrate consensus in FOMC decisions; however, our approach also incorporates an explicit analysis of how biases might have provoked dissent votes (a possibility considered only informally by Meade). Moreover, we have examined how the influence of the bias on Committee members' voting decisions changed over the course of the Greenspan era; specifically, we show that in the period when biases affected voting, they had the power to cause dissents as well as to diminish them, depending on whether the bias was viewed favorably or unfavorably by individual voters.

All of our results are compatible with the view that FOMC members based their votes on a rational assessment of the policy content of proposed directives—biases affected voting only when they meaningfully represented alternative policy options for the upcoming intermeeting period. Conversely, our results provide no support for the hypothesis that bias settings produced consensus with conciliatory language that lacked meaningful policy content. Biases were often set in a way that was "unfavorable" to individual Committee members; if biases were purely intended to provide symbolic

[22] Had there been no option for setting a bias, both the Chairman's proposals and members' stated preferences might have changed. As a consequence, we cannot draw unequivocal conclusions about how dissent frequencies might have differed in the absence of bias setting.

appeasement, this pattern would be difficult to explain. Ultimately, then, dissent voting frequencies depended on how adopted policies, including meaningful adopted biases, matched up with the preferences of individual Committee members.

If statements of bias after 1993 no longer conveyed any information about likely intermeeting policy moves, one might ask why the FOMC continued to adopt and report them until 1999. One possibility is simply that the Federal Reserve waits to acknowledge institutional change until it is certain that the change is permanent. It is also possible, though, that bias statements had value in terms of communicating the Committee's outlook, even if they lacked relevance for intermeeting rate adjustments.

We can find some support for this hypothesis. If we modify the Lapp and Pearce (2000) methodology to test whether an adopted bias aids in predicting the funds rate adopted at the *next* FOMC meeting (rather than a move *before* the next meeting), we find that it does, even over the 1994–99 period.[23] This suggests that the role of the bias changed—before 1994, it was a component of the policy choice for the current intermeeting period; afterward, it helped to communicate the Committee's forecast of the future course of policy over a longer horizon. Under this interpretation, our results show that FOMC members' votes were influenced by current policy choices but not by implied forecasts of future choices.[24]

In January 2000, the FOMC dropped the bias from the directive and instead began reporting a "balance-of-risks" statement. The latter statement provides an indication of whether the Committee's concerns are tilted toward inflationary pressures or economic weakness. Like the bias, it has been interpreted as an indicator of future countervailing policy moves, but with a time horizon that extends somewhat beyond the upcoming intermeeting period.[25] Our

[23]In a reaction-function specification, the lagged bias setting is significant in explaining the adopted target at better than the 0.01 level, after controlling for forecasts of macroeconomic conditions.

[24]In the interest of transparency, even FOMC members who anticipate that they will oppose a future policy move might favor giving the public an accurate indication of its likelihood.

[25]Preliminary evidence provided by Rasche and Thornton (2002) and Pakko (2005) suggests that the setting of the balance-of-risks statement does help to predict future monetary policy actions.

analysis is compatible with the view that by the time the bias was formally abandoned, its function already approximated that of the new balance-of-risks statement.

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Low Nominal Interest Rates: A Public Finance Perspective*

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This paper studies low-interest-rate policies from a public finance perspective. Two policy regimes are considered. In the first regime, the central bank is subordinate and its budget is integrated into the fiscal authority's budget constraint. In this case, monetary policy influences the revenue mainly through currency seigniorage. In the other regime, the central bank's budget is separated from that of the fiscal authority. Commitment to a low nominal interest rate forces the central bank to inject money when the primary deficit increases. Thus, even if the budgets are separated, the central bank's actions are constrained by the fiscal authority. Under a "passive" Taylor rule, a reduction in the nominal interest rate *lowers* the government revenue.

JEL Codes: E31, E43, E58, H63.

Monetary policy is, conceptually, institutionally and practically, a small but significant part of intertemporal public finance—its liquid corner. (Buiter 2005, C1)

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1. Introduction

This paper studies low-interest-rate policies from a public finance perspective. It studies how changes in the nominal interest rate influence government revenue. In this paper, two policy regimes are considered. In the first regime, the central bank is subordinate in that its budget is integrated into the fiscal authority's budget constraint. In such a situation, monetary policy influences the government's revenue through seigniorage—the revenue from printing money. In the other regime, the central bank's budget is separated from that of the fiscal authority. Monetary policy continues to influence the fiscal authority's revenue through interest payment on the public debt.

Figure 1 (A–C) presents the evolution of public debt, interest obligations on public debt, and the nominal interest rate in Japan since 1980.[1] The figures indicate that, though the government was heavily in debt, especially in the 1990s, the interest obligations on the debt declined over time during this decade. Is this a consequence of the low-interest-rate policies implemented by the Bank of Japan?[2] Or is it simply part of the risk-free rate puzzle?[3]

Motivated by this observation, this paper explores how monetary policy influences government revenue. For this purpose, this paper exploits Laffer curves to highlight precisely how the nominal interest rate and the inflation rate affect government revenue. Laffer curves have proven to be useful in the field of public finance and monetary economics from two dimensions. One is that the peak of a Laffer curve tells us the maximum sustainable level of budget deficit. The other is that the slope of a Laffer curve indicates whether revenue is increasing or decreasing in the variable under consideration.

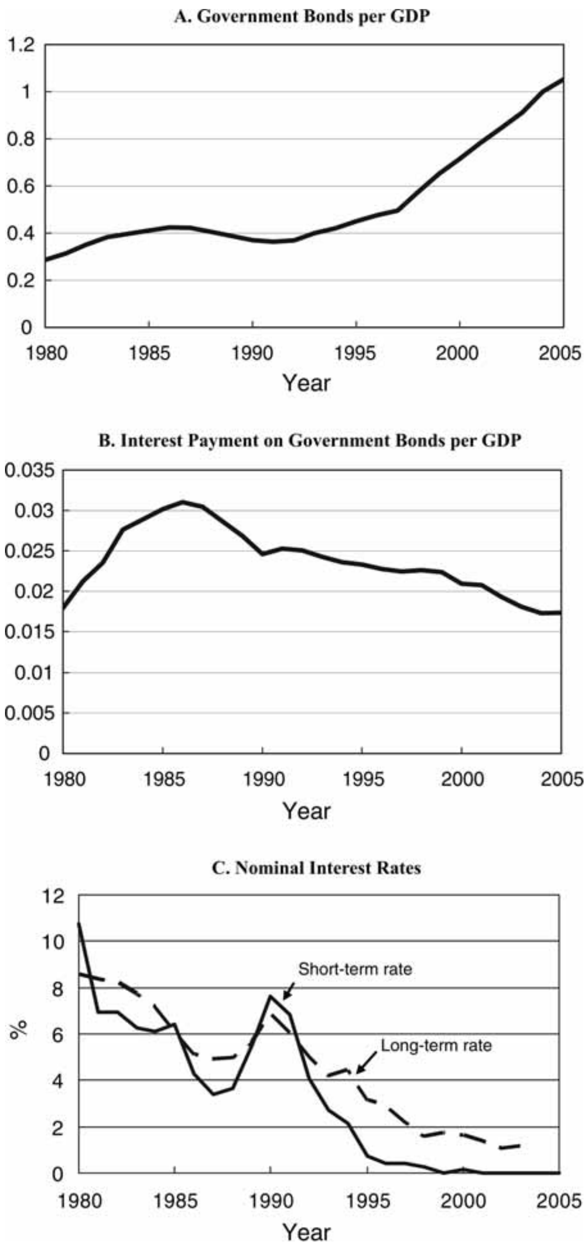
In monetary economics, the standard form of the Laffer curve relates the rate of inflation and total government revenue. I refer to it as the inflation Laffer curve. In addition to the standard Laffer curve, I introduce the nominal-interest-rate Laffer curve, which relates

[1]All data series, taken from the Ministry of Finance and the Bank of Japan, are annual data for the fiscal year.

[2]The zero-interest-rate policy in Japan ended on July 14, 2006. The Bank of Japan raised the target short-term rate to 0.25 percent.

[3]See Bullard and Russell (1999) for a discussion.

Figure 1. Evolution of Public Debt, Interest Obligations on Public Debt, and Nominal Interest Rate in Japan Since 1980



nominal interest rate and government revenue. It is downward sloping, suggesting that lower nominal interest rates are associated with higher total revenues for the government.

This paper also considers the Laffer curve when the central bank follows a Taylor rule, under which the nominal interest rate is an increasing function of the inflation rate. Generally, the Laffer curve under a Taylor rule is hump shaped. An interesting finding is that the slope of the Laffer curve at low nominal interest rates crucially depends on how "active" or "passive" monetary policy is. If monetary policy is sufficiently passive in the sense of Leeper (1991), then the Laffer curve slopes upward—higher inflation finances a larger deficit. If monetary policy is sufficiently active, then the Laffer curve slopes downward—a lower interest rate finances a larger deficit.

Sargent and Wallace (1981) made an important contribution by pointing out that the central bank loses one degree of freedom because its budget constraint is integrated into that of the fiscal authority. Since Sargent and Wallace (1981), the consolidated budget constraint has become the building block of monetary policy analysis. An important assumption of the analysis of Sargent and Wallace (1981) and their followers is that the central bank is subordinate to the fiscal authority in the sense that the central bank determines the growth rate of base money so as to ensure solvency of the government. However, contemporary central banking is best described as being independent, although the degree of independence differs across countries. As a result, the world economy appears to be in a state of low inflation.

Based on this observation, this paper considers, as an extension of the basic model, an environment in which the central bank is independent. Issues regarding central bank independence have gained much attention from many writers. The major theoretical approach to central bank independence is based on the model of time inconsistency, and it focuses on the inflationary bias that arises in various environments. A comprehensive study of this approach is found in Cukierman (1992), and a critical view is found in Blinder (1998).

The concept of central bank independence proposed in this paper is quite simple and easily integrated into any dynamic general equilibrium model. The key is that the budget constraint of the central bank is separated from that of the fiscal authority. Thus, the fiscal authority must finance its expenditures by tax and bonds. Money is injected directly into households via "helicopter drops." Interestingly, monetary policy continues to influence the fiscal authority's revenue through interest payment on the public debt. Under nominal-interest-rate targeting, an increase in the primary deficit raises the equilibrium inflation rate, because commitment to a particular bond price forces the central bank to inject more money into the economy. Under strict inflation targeting, the central bank does not need to maintain the bond price. However, there may be two steady states under inflation targeting. At the low-interest-rate equilibrium, low nominal interest rates are associated with *low* government revenues.

Intuitively, low-interest-rate policies raise government revenue by reducing interest obligations on past debt. In other words, the Laffer curve must be downward sloping. However, this paper finds that the slope of the Laffer curve under a passive Taylor rule may be *globally* upward sloping—a reduction in the nominal interest rate lowers government revenue by raising the real interest rate on bonds.

The organization of this paper is as follows. Section 2 provides a review of the literature. Section 3 describes the structure of the model. In sections 4, 5, and 6, I consider a policy regime in which the central bank's budget constraint is integrated into the government's budget. Section 4 assumes a nominal-interest-rate target, section 5 assumes a strict inflation target, and section 6 assumes a Taylor rule. Section 7 investigates an alternative policy regime in which the central bank's budget constraint is separated from that of the fiscal authority. That is, the central bank is "tough." Section 8 discusses the results and their implications for conducting monetary policy. Section 9 concludes. Appendices 1 and 2 present alternative versions of the model.

2. Related Literature

The traditional Laffer curve relates the tax rate and tax revenue. A tax Laffer curve appears in textbooks such as Barro (1997). Because they summarize how changes in a policy variable affect government revenue, Laffer curves are also useful in monetary policy analysis. An inflation Laffer curve relates the inflation rate and total seigniorage. Related papers are Sargent and Wallace (1981), Miller and Sargent (1984), Aiyagari and Gertler (1985), King and Plosser

(1985), Bhattacharya, Guzman, and Smith (1998), Espinosa-Vega and Russell (1998), Bhattacharya and Kudoh (2002), and Nikitin and Russell (2006). Review articles by Brunner (1986) and Sargent (1999) are also available. An interesting application of the Laffer curve analysis is found in Bhattacharya and Haslag (2003), who considered a "reserve-ratio Laffer curve" to study how the reserve requirement influences seigniorage. In addition to the standard inflation Laffer curve, this paper introduces the nominal-interest-rate Laffer curve, which relates the nominal interest rate and total revenue.

It is well known that the revenue from issuing bonds is positive if and only if the economy is dynamically inefficient, under which the output growth rate exceeds the real interest rate. The bond seigniorage is positive, because in such an economy, the government can roll over the debt forever—it enjoys a Ponzi game. Thus, whether or not an economy is dynamically efficient is an important issue. Darby (1984) argued that the U.S. economy is dynamically inefficient, while Abel et al. (1989) concluded that many OECD economies are dynamically efficient. Bullard and Russell (1999) argued that dynamically inefficient equilibria are empirically plausible. Chalk (2000) computed the maximum sustainable deficit, which is essentially finding the peak of the Laffer curve.

There is a large and growing body of literature on monetary policy rules. Clarida, Galí, and Gertler (1998, 2000) summarized recent results and presented some international estimates of monetary policy rules. Benhabib, Schmitt-Grohe, and Uribe (2001) and Carlstrom and Fuerst (2001) studied optimizing monetary models with Taylor-type feedback rules and clarified how monetary policy rules might cause self-fulfilling fluctuations. A contribution of this paper is to characterize the Laffer curve under a Taylor rule.

3. Subordinate Central Bank and Public Finance

3.1 Environment

Consider a pure exchange economy comprising an infinite sequence of two-period-lived overlapping generations, the initial old generation, and an infinitely lived government. Let $t = 1, 2, \ldots$ index time. At each date t, a new generation is born. The population is normalized to 1. Each young agent is endowed with y_t units of the consumption good, and the endowment grows at a gross rate of $n > 0$: $y_{t+1} = ny_t$. The price level of the consumption good at date t is p_t. Throughout this paper, I focus on steady-state equilibria, in which all per-output real variables are constant over time.

3.2 Consumers

In order to focus on the agents' portfolio choice, I assume that all individuals save their entire income. As a means of saving, agents may hold money M_t and government bonds B_t. In order to motivate the demand for money as a liquid asset, divide each period into two subperiods. The bonds are assumed to yield a gross nominal return of $I_{t+1} \geq 1$ in the next period. However, bonds cannot be liquidated until the second subperiod. Money, yielding no nominal interest rate, can be liquidated in the first subperiod. Related environments are found in Stiglitz (1970), Diamond and Dybvig (1983), and Dutta and Kapur (1998).

Each individual wishes to consume in both subperiods. Let c_{1t} and c_{2t} denote the consumption of the final good in the first and second subperiods by an old agent born at date t. The consumer's objective function is

$$\phi u(c_{1t}) + (1 - \phi)u(c_{2t}), \tag{1}$$

where ϕ captures the relative weight of utility between the two subperiods. Throughout, I use the following specification: $u(c) = [1 - \rho]^{-1}c^{1-\rho}$ with $\rho \neq 1$ and $\rho > 0$. Because the individual cannot liquidate bonds in the first subperiod, the agent faces a cash-in-advance constraint:

$$p_{t+1}c_{1t} \leq M_t, \tag{2}$$

which is binding for $I_{t+1} > 1$.

The budget constraint for each young individual is

$$M_t + B_t = p_t y_t - T_t, \tag{3}$$

where T_t is the amount of lump-sum tax. Similarly, the budget constraint for each old individual is

$$p_{t+1}c_{1t} + p_{t+1}c_{2t} = M_t + I_{t+1}B_t. \tag{4}$$

Thus, each young individual maximizes (1) subject to (2)–(4). It is easy to transform the problem into

$$\max_{M_t} \left\{ \phi \frac{[M_t/p_{t+1}]^{1-\rho}}{1-\rho} + (1-\phi) \frac{[(p_t y_t - T_t - M_t) I_{t+1}/p_{t+1}]^{1-\rho}}{1-\rho} \right\}.$$

Using the first-order condition, obtain the money-demand function:

$$M_t = \gamma(I_{t+1})[p_t y_t - T_t], \tag{5}$$

$$\gamma(I_{t+1}) \equiv \left[1 + \left(\frac{1-\phi}{\phi} \right)^{1/\rho} I_{t+1}^{1/\rho - 1} \right]^{-1}. \tag{6}$$

From (6),

$$\gamma'(I) = -\left[1 + \left(\frac{1-\phi}{\phi} \right)^{1/\rho} I^{1/\rho-1} \right]^{-2} \times \frac{1-\rho}{\rho} \left(\frac{1-\phi}{\phi} \right)^{1/\rho} I^{1/\rho-2}.$$

Thus, it is easy to establish the following.

LEMMA 1. *(i)* $\gamma'(I) < 0$ *holds for* $\rho \in (0,1)$*; (ii)* $\lim_{I\to\infty} \gamma(I) = 0$ *for* $\rho \in (0,1)$*; (iii)* $\gamma(1) = [1 + ((1-\phi)/\phi)^{1/\rho}]^{-1}$*; (iv)* $\lim_{I\to 1} \gamma'(I) = -[1 + ((1-\phi)/\phi)^{1/\rho}]^{-2}((1-\phi)/\phi)^{1/\rho}(1-\rho)/\rho$*; and (v) the interest elasticity of money demand satisfies*

$$-\frac{I\gamma'(I)}{\gamma(I)} = \frac{1-\rho}{\rho}[1-\gamma(I)]. \tag{7}$$

There are several other environments that induce this money-demand function. In Schreft and Smith (1997, 2000), for example, markets are spatially separated, and communication across the markets is limited. Thus, only money is universally accepted as a means of payment. “Relocation shock” similar to the liquidity preference shock of Diamond and Dybvig (1983) induces agents to hold a mix of money and interest-bearing assets. Financial intermediation arises to provide perfect risk sharing through demand-deposit contracts, and the deposit-demand function is of the form (5). It is also easy to verify that a class of money-in-the-utility function or cash-in-advance specification can generate the money-demand function (5).

The value of ρ captures the strength of the income effect of a change in I. Throughout, this paper focuses on the case in which $\rho \in (0,1)$—that is, when the income effect is relatively weak. Money demand is independent of the *real* interest rate because, by construction, the saving rate is constant and equal to 1. In general, the real interest rate influences the intertemporal allocation, while the nominal interest rate influences the composition of competing assets. In models such as Sargent and Wallace (1981), Bhattacharya, Guzman, and Smith (1998), Espinosa-Vega and Russell (1998), and Bhattacharya and Kudoh (2002), the rate of inflation affects the demand for money by influencing saving. The effect of inflation on money demand is important, especially when the focus of study is on *high* inflation. Because this paper is concerned mostly with *low* inflation, the absence of the channel does not seem very problematic. Appendix 1 presents an alternative environment in which aggregate money demand depends on the rate of inflation.

3.3 Government

This paper focuses on how the government finances its primary deficits. The consolidated government budget constraint is

$$G_t - T_t + I_t B_{t-1} = B_t + M_t - M_{t-1} \tag{8}$$

for $t \geq 2$ and $G_1 - T_1 = M_1 + B_1$ for $t = 1$, where it is assumed that $B_0 = 0$ and $M_0 \geq 0$. Money is supplied through the channel of open-market operations by the central bank. As is conventional, the fiscal authority determines the total government liability, while the central bank determines its composition. Throughout the paper, this policy regime is referred to as the SCB (subordinate central bank) regime. I assume that the government simply consumes G_t and that it does not affect the utility of any generation or production process at any date. Divide (8) by $p_t y_t$ to obtain

$$g_t - \tau_t = b_t - \frac{1}{\Pi_t n} I_t b_{t-1} + m_t - \frac{1}{\Pi_t n} m_{t-1}, \tag{9}$$

where $b_t = B_t/p_t y_t$, $\tau_t = T_t/p_t y_t$, $m_t = M_t/p_t y_t$, and $\Pi_t \equiv p_t/p_{t-1}$.

There is a recurrent theme regarding the monetary–fiscal policy regime. This paper assumes a Ricardian policy regime; that is, the government's budget constraint is satisfied at any price level. Thus, I consider (8) and (9) to be identities. In addition, this paper considers a policy regime called fiscal dominance, under which the fiscal authority "moves first" and commits to a level of primary deficit. In particular, I assume that g_t and τ_t are constant over time. Thus, $g - \tau$ is the permanent primary deficit that must be financed by money and bonds. Throughout, it is assumed that $g \in (0, 1)$ and $\tau \in (0, 1)$.

DEFINITION 1. *A monetary equilibrium in the SCB regime is a set of sequences for real allocations $\{m_t, b_t\}$ and relative prices $\{I_t, \Pi_t\}$ such that (i) each generation maximizes utility (1) subject to (2)–(4); (ii) the asset market clears; (iii) the government's budget constraint (9) is satisfied; (iv) $g_t = g$ and $\tau_t = \tau$ for all t; and (v) monetary policy determines I_t, Π_t, or a function that relates I_t and Π_t.*

In any equilibrium, $m_t = \gamma(I_{t+1})[1 - \tau]$, $b_t = [1 - \gamma(I_{t+1})][1 - \tau]$, and

$$\delta = 1 - \frac{\mu(I_t)}{n\Pi_t} \tag{10}$$

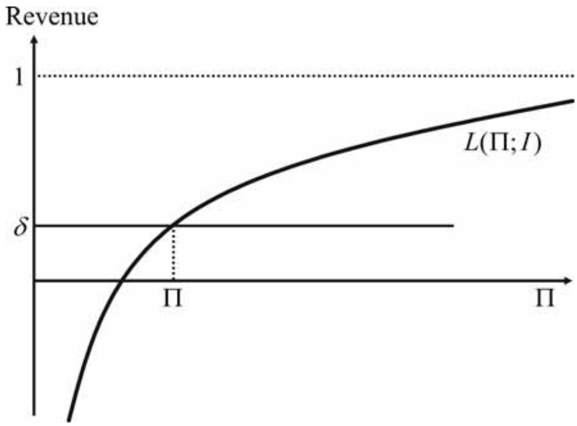
hold, where $\mu(I) \equiv I[1 - \gamma(I)] + \gamma(I) > 1$ and $\delta \equiv (g - \tau)/(1 - \tau)$. Under primary *surplus*, $\delta < 0$. Once monetary policy is specified, (10) determines the equilibrium.

4. Inflation Laffer Curve

4.1 Total Seigniorage, Currency Seigniorage, and Bond Seigniorage

Before investigating any equilibrium, I introduce Laffer curves to explore how government revenue is affected by inflation. First, consider a Laffer curve that relates inflation and total revenue. Suppose that the central bank targets the nominal interest rate such that $I_t = I$ for all t. The endogenous variable under nominal-interest-rate targeting is the rate of inflation. Because $M_{t+1}/M_t = \Pi n$ holds in the steady state, one could view that the central bank adjusts the base money growth rate in each period in order to maintain the target nominal interest rate.

Figure 2. Inflation Laffer Curve



Under nominal-interest-rate targeting, (10) reduces to

$$\delta = 1 - \frac{\mu(I)}{n\Pi_t} \equiv L(\Pi_t; I). \tag{11}$$

It is easy to verify that $\mu(1) = 1$ and $\mu'(I) = 1-\gamma(I)-(I-1)\gamma'(I) > 0$. The right-hand side of (11) is referred to as the inflation Laffer curve, while equation (11) determines the equilibrium inflation rate. For a given I, the inflation Laffer curve $L(\Pi_t; I)$ is increasing in the rate of inflation, as shown in figure 2. The Laffer curve analysis is useful because it tells us the maximum possible level of revenue for the government. It is easy to verify that as $\Pi_t \to \infty$, $L(\Pi_t; I) \to 1$. It is important to note here that the upward-sloping Laffer curve depends crucially on the money-demand function that is independent of the inflation rate. As shown in appendix 1, the inflation Laffer curve becomes hump shaped if the money demand decreases with the inflation rate.

Because fiscal dominance is assumed, the primary deficit δ determines the need for seigniorage, thereby determining the equilibrium rate of inflation that finances the deficit. The equilibrium rate of inflation changes as the primary deficit changes. As is standard, an increase in δ raises the equilibrium inflation rate, because the upward-sloping Laffer curve implies that a higher inflation finances a greater deficit.

Total government revenue comprises currency seigniorage and bond seigniorage. I now investigate whether these two sources of revenue increase with inflation. First, consider the revenue from printing money. Under nominal-interest-rate pegging, the currency seigniorage is given by

$$m_t - \frac{1}{\Pi_t n} m_{t-1} = \left(1 - \frac{1}{\Pi_t n}\right)\gamma(I)[1-\tau] \equiv CS(\Pi_t; I). \qquad (12)$$

It is easy to verify that $\partial CS/\partial \Pi_t > 0$. The reason for this is as follows. For a given level of the nominal interest rate, the inflation tax base is invariant, because in this economy inflation does not influence money demand, and an increase in the inflation rate raises the inflation tax rate. Thus, the revenue is increasing in Π. It is important to note here that the currency seigniorage can be positive even under deflation as long as the output growth rate is positive ($n > 1$). The maximum possible currency seigniorage is $\lim_{\Pi\to\infty} CS(\Pi; I) = \gamma(I)[1-\tau]$.

Consider the revenue from issuing bonds. The bond seigniorage is given by

$$b_t - \frac{I}{\Pi_t n} b_{t-1} = \left(1 - \frac{I}{\Pi_t n}\right)[1-\gamma(I)][1-\tau] \equiv BS(\Pi_t; I). \qquad (13)$$

The bond seigniorage is increasing in Π. This is because an increase in the rate of inflation reduces the real interest on bonds. In other words, inflation reduces the interest payment on the government's outstanding debt. The maximum possible bond seigniorage is $\lim_{\Pi\to\infty} BS(\Pi; I) = [1-\gamma(I)][1-\tau]$.

4.2 Equilibrium

Under nominal-interest-rate pegging, the equilibrium inflation rate is determined by (11). Comparative statics exercises reveal that output growth is disinflationary, the primary deficit is inflationary, and nominal interest rates and inflation are positively related. These results are easily established by (11) with figure 2. First, (11) implies $\partial L/\partial n > 0$: an increase in n shifts the Laffer curve upward. From figure 2, this means that Π must be reduced. Figure 2 also implies that an increase in δ raises Π. Finally, because

$\partial L/\partial I = -\mu'(I)/n\Pi < 0$ holds, an increase in I shifts the Laffer curve downward and Π increases. At this point, it is important to note that total government revenue *in equilibrium* is $g - \tau$ by construction. Some other properties of the equilibrium are presented below.

LEMMA 2. *In any equilibrium, $\Pi_t > 1/(1-\delta)n$ must hold.*

Proof. From (11), it is easy to derive the equilibrium inflation rate as $\Pi_t = \mu(I)/(1-\delta)n$. Note that $\mu(I) > 1$. Thus, $\Pi_t = \mu(I)/(1-\delta)n > 1/(1-\delta)n$.

COROLLARY 1. *Necessary conditions for long-run deflation are (i) $n > (1-\delta)^{-1}$ and (ii) the nominal interest rate is sufficiently low.*

Proof. The equilibrium inflation rate is given by $\Pi_t = \mu(I)/(1-\delta)n$. Thus, $\Pi_t < 1 \Longleftrightarrow \mu(I) < (1-\delta)n$. Because μ is increasing, I must be sufficiently low.

Because the rates of inflation and nominal interest are positively related in equilibrium, low-interest-rate policies are associated with low inflation and perhaps deflation. Because the primary deficit is inflationary, it must be sufficiently small for a deflationary equilibrium.

PROPOSITION 1. *The equilibrium currency seigniorage is positive in the case of a budget deficit.*

Proof. From (11), $\Pi_t n = \mu(I)/(1-\delta)$ holds in equilibrium. Substitute it into (12) to obtain the currency seigniorage as $(1-(1-\delta)/\mu(I))\gamma(I)[1-\tau]$. Because $\mu(I) > 1$, CS is positive under $\delta \in (0,1)$.

LEMMA 3. *The equilibrium bond seigniorage is positive if $\delta > (I-1)/I$.*

Proof. From (13), the equilibrium bond seigniorage is given by $(1 - I/\Pi n)[1-\gamma(I)][1-\tau]$, which is positive if $I/\Pi n < 1$. From the equilibrium condition, it is easy to show that $I/\Pi n < 1$ holds if and only if $(1-\delta)I < \mu(I)$. Because $\mu(I) > 1$ holds for any $I \geq 1$, a sufficient condition for $(1-\delta)I < \mu(I)$ is $(1-\delta)I < 1$.

5. Nominal-Interest-Rate Laffer Curve

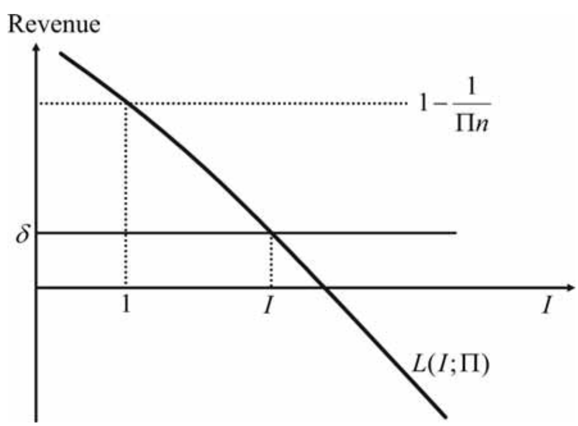
The primary purpose of this paper is to study whether and how low-interest-rate policies raise government revenue. For this purpose, this section studies another type of Laffer curve, which relates the nominal interest rate and total revenue, given the inflation rate.

Suppose that the central bank targets the rate of inflation by adjusting the nominal interest rate in each period. Such a policy rule is sometimes referred to as strict inflation targeting. Because $M_{t+1}/M_t = \Pi n$ holds in the steady state, strict inflation targeting is essentially the same as money growth targeting. Letting $\Pi_t = \Pi$, (11) can be rewritten as

$$\delta = 1 - \frac{\mu(I_t)}{n\Pi} \equiv L(I_t; \Pi), \tag{14}$$

where $L(.)$ is redefined. Because $\mu'(I) = 1 - \gamma(I) - (I-1)\gamma'(I) > 0$, it is evident that the nominal-interest-rate Laffer curve $L(I_t; \Pi)$ is a decreasing function of I. Thus, it is downward sloping, as shown in figure 3. In other words, a lower nominal interest rate can finance a greater deficit. The maximum possible revenue is given by $L(1; \Pi) = 1 - 1/\Pi n$. This implies that the target inflation rate must satisfy $\Pi n > 1$, or there is no equilibrium with budget

Figure 3. Nominal-Interest-Rate Laffer Curve



deficit. In other words, the target base money growth rate must be positive, or the government cannot run a permanent primary deficit.

Once the target inflation rate is set, equation (14) determines the equilibrium nominal interest rate. Given a level of inflation, a larger deficit raises the need for revenue. Because the nominal-interest-rate Laffer curve is downward sloping, the equilibrium nominal interest rate must be reduced in order to finance a larger deficit.

Consider the currency seigniorage, which satisfies $CS(I;\Pi) = (1 - 1/\Pi n)\gamma(I)[1-\tau]$. It is easy to verify $\partial CS/\partial I < 0$ as long as $\Pi n > 1$. The reason for this is as follows. For a given level of inflation, an increase in the nominal interest rate reduces the demand for money, which reduces the inflation tax base. The maximum possible level of currency seigniorage is $(1 - 1/\Pi n)\gamma(1)[1-\tau]$, at which the inflation tax base is maximized.

Consider the bond seigniorage, $BS(I;\Pi) = (1 - I/\Pi n)[1 - \gamma(I)][1-\tau]$, from which

$$[1-\tau]^{-1}\frac{\partial BS}{\partial I} = -\left(1 - \frac{I}{\Pi n}\right)\gamma'(I) - \frac{1-\gamma(I)}{\Pi n}.$$

The first term is negative for $I > \Pi n$ or $R > n$, under which the *level* of bond seigniorage is negative. The bond Laffer curve is decreasing in the region in which the economy is dynamically efficient. For $I < \Pi n$, the bond seigniorage is positive because the economy is dynamically inefficient, so the government can roll over its debt forever. In this region, the slope of the bond Laffer curve is ambiguous. The determinant is the interest elasticity of the money-demand function. The slope at the lower bound is

$$\begin{aligned}[1-\tau]^{-1}\left.\frac{\partial BS}{\partial I}\right|_{I=1} &= -\left(1 - \frac{1}{\Pi n}\right)\gamma'(1) - \frac{1-\gamma(1)}{\Pi n} \\ &= \frac{1-\gamma(1)}{\Pi n}\left[(\Pi n - 1)\frac{1-\rho}{\rho}\gamma(1) - 1\right],\end{aligned}$$

which is positive if and only if $(\Pi n - 1)(1-\rho)/\rho > 1 + [(1-\phi)/\phi]^{1/\rho}$. Because it is assumed that $\rho \in (0,1)$, this condition is rewritten as $\Pi n > 1 + \Phi$, where $\Phi \equiv \rho(1-\rho)^{-1}[1 + ((1-\phi)/\phi)^{1/\rho}] > 0$. Thus, the following result is obtained.

Proposition 2. *The bond seigniorage increases with the nominal interest rate near the lower bound of the nominal interest rate if and only if the target inflation rate satisfies* $\Pi > (1+\Phi)/n$.

6. Laffer Curve under Taylor Rules

There is a large body of recent research on Taylor rules. The theoretical literature mainly focuses on questions such as whether Taylor rules cause the indeterminacy of equilibrium. Indeterminacy implies that the economy is exposed to self-fulfilling fluctuations. Leading examples include Leeper (1991), Clarida, Galí, and Gertler (2000), and Carlstrom and Fuerst (2001). Although dynamic issues such as determinacy are quite important, these issues do not concern this paper, because many results are now established. A contribution of this paper is to perform Laffer curve analyses under a Taylor-type feedback rule.

Suppose that the central bank follows a Taylor-type feedback rule:

$$I_t = A\left(\frac{\Pi_t}{\Pi^*}\right)^{\beta}, \tag{15}$$

where $\Pi^* > 0$ is the implicit target level of the gross inflation rate, $A \geq 1$ is a scale parameter, and $\beta > 0$. According to Leeper (1991), monetary policy is said to be "active" if $\beta > 1$ and "passive" if $\beta < 1$. Solve (15) for Π_t to obtain $\Pi_t = \Pi^*(I_t/A)^{1/\beta} = \alpha\Pi^* I_t^{1/\beta}$, where $\alpha \equiv A^{-1/\beta}$. Substitute this into (10) to obtain

$$\delta = 1 - \frac{\mu(I)}{n\alpha\Pi^* I^{1/\beta}} \equiv L(I), \tag{16}$$

where $L(.)$ is redefined. The Laffer curve may be written in terms of I or Π. I describe total revenue as a function of I only for expositional reasons. The slope of the Laffer curve is given by

$$L'(I) = -\frac{\mu(I)}{n\alpha\Pi^* I^{1/\beta+1}}\left[\frac{\mu'(I)I}{\mu(I)} - \frac{1}{\beta}\right].$$

Thus, the Laffer curve slopes downward if and only if the interest elasticity of $\mu(I)$ is greater than $1/\beta$. Remember that $\mu(I) \equiv$

$I[1-\gamma(I)]+\gamma(I)$ and $\mu'(I) = 1-\gamma(I)-(I-1)\gamma'(I) > 0$. Thus, the slope is expressed as

$$L'(I) = \frac{\frac{1-\beta}{\beta}(1-\gamma(I)) + (I-1)\gamma'(I) + \frac{1}{\beta}\gamma(I)/I}{n\alpha\Pi^* I^{1/\beta}}. \tag{17}$$

Consider the slope near the lower bound. From (17),

$$\left.\frac{\partial L}{\partial I}\right|_{I=1} = \frac{1}{n\alpha\Pi^*}\left[\frac{1}{\beta} - (1-\gamma(1))\right].$$

It is now easy to verify that near the zero bound of the net nominal interest rate, the Laffer curve slopes downward if and only if $\beta > [1-\gamma(1)]^{-1}$. From (6), it is easy to establish the following.

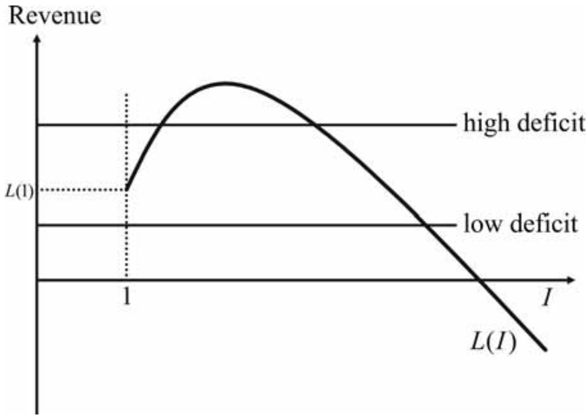
PROPOSITION 3. *The Laffer curve $L(I)$ slopes downward near the lower bound of the nominal interest rate if and only if $\beta > 1 + [(1-\phi)/\phi]^{-1/\rho}$.*

An important implication of proposition 8 is that low nominal interest rates are not necessarily helpful to the government's budget. The slope of the Laffer curve crucially depends on how "active" or "passive" the central bank is. In particular, for low nominal interest rates to be revenue increasing, the coefficient β must be sufficiently greater than unity: monetary policy must be sufficiently active.

Why is the activeness important in determining the slope of the Laffer curve? The key is the link between the nominal and real interest rates. Remember that $\Pi_t = \alpha\Pi^* I_t^{1/\beta}$ holds under the Taylor rule. Substitute this into the Fisher equation, $I_t = \Pi_t R_t$, to obtain $I_t^{1-1/\beta} = \alpha\Pi^* R_t$, from which it is easy to establish that $dR_t/dI_t > 0$ holds if and only if $\beta > 1$. In other words, a reduction in the nominal interest rate reduces the real interest rate if and only if monetary policy is active. Thus, a passive monetary policy ($\beta < 1$) is sufficient for the Laffer curve to be upward sloping near $I = 1$, because a reduction in the nominal interest rate *raises* the real interest rate on bonds. Thus, if a central bank follows a passive monetary policy in a disinflationary phase, the nominal interest rate is lowered, but the real interest rate rises and the government's revenue falls.

The Laffer curve suggests another issue regarding passive Taylor rules. Because the Laffer curve is generally hump shaped under a

Figure 4. Laffer Curve under a Passive Taylor Rule



Taylor rule, there arises a scope for multiple equilibria for a high primary deficit, as shown in figure 4. Many writers, such as Clarida, Galí, and Gertler (2000), have pointed out the self-fulfilling nature of passive Taylor rules. The Laffer curve suggests that, for a unique equilibrium, either (i) monetary policy is sufficiently active or (ii) the primary deficit δ is sufficiently small so that $L(1) > \delta$.

7. Central Bank Independence and Public Finance

7.1 Preliminaries

The preceding sections have investigated government revenue under a regime in which the central bank is subordinate to the fiscal authority in the sense that there is a single government budget constraint that integrates the central bank's budget. Thus, the fiscal authority determines the total government liability, while the central bank determines its composition. Although this modeling strategy is standard, it is not entirely consistent with modern, independent central banking. This section considers an alternative policy regime in which the central bank is independent in that its budget constraint is *separated* from that of the fiscal authority such that there is no direct interaction between monetary and fiscal policies.

Definition 2. *The central bank is independent if (i) the fiscal authority does not receive any revenue from it and (ii) it never purchases government bonds.*

Requirement (i) is insufficient for separating the monetary authority's budget from that of the fiscal authority, because if money is supplied by permanent open-market purchases of government bonds, then the two budget constraints are connected and only the consolidated budget constraint matters.

There exists a large body of literature, both theoretical and empirical, on central bank independence, and the definition of the term varies. According to Alesina and Summers (1993) and Fischer (1995), there are two important definitions: "political independence" and "economic independence." Political or goal independence is defined as the ability of central banks to set objectives such as price stability. Economic or instrument independence is defined as the ability to conduct monetary policy without restrictions. This paper defines central bank independence as the independence of the central bank's budget from that of the fiscal authority—this is a form of instrument independence.

Because open-market operations are ruled out, money in this economy is directly injected into the economy. The monetary authority's budget constraint is

$$H_t = M_t - M_{t-1}, \tag{18}$$

where H_t denotes the transfer to the household. This formulation implies that money is supplied via "helicopter drops." Divide (18) by $p_t y_t$ to obtain

$$h_t = m_t - \frac{1}{\Pi_t n} m_{t-1}, \tag{19}$$

where $h_t \equiv H_t / p_t y_t$. The steady-state money injection is positive if and only if $\Pi n > 1$.

Similarly, the fiscal authority's budget constraint is $G_t - T_t + I_t B_{t-1} = B_t$, from which

$$g - \tau = b_t - \frac{I_t}{\Pi_t n} b_{t-1}. \tag{20}$$

Note that under this policy regime, the fiscal authority must maintain its solvency on its own; there is no currency seigniorage available for the government. Thus, definition 9 specifies a "tough" central bank. In what follows, this policy regime is referred to as the CBI (central bank independence) regime.

Because money is injected directly into each household, the young individual's budget constraint is replaced with $M_t + B_t = p_t y_t - T_t + H_t$. Thus, the demands for money and bonds are given by $m_t = \gamma(I_{t+1})(1-\tau+h_t)$ and $b_t = [1-\gamma(I_{t+1})](1-\tau+h_t)$, respectively. Appendix 2 presents a brief sketch of an alternative version of the model in which the old generation receives the transfer.

Definition 3. *A monetary equilibrium in the CBI regime is a set of sequences for real allocations $\{m_t, b_t\}$ and relative prices $\{I_t, \Pi_t\}$ such that (i) each generation maximizes utility (1) subject to (2)–(4); (ii) the asset market clears; (iii) the government's budget constraints (19) and (20) are satisfied; (iv) $g_t = g$ and $\tau_t = \tau$ for all t; and (v) monetary policy determines I_t, Π_t, or a function that relates I_t and Π_t.*

7.2 Inflation Laffer Curve

Consider the case in which the central bank targets the nominal interest rate. From (20),

$$g - \tau = \left(1 - \frac{I}{\Pi n}\right)[1 - \gamma(I)](1 - \tau + h) \tag{21}$$

holds in a steady state. Solve (19) and $m = \gamma(I)(1 - \tau + h)$ for h as

$$h = \frac{\left(1 - \frac{1}{\Pi n}\right)\gamma(I)(1 - \tau)}{1 - \left(1 - \frac{1}{\Pi n}\right)\gamma(I)}.$$

Substitute it into (21) to obtain

$$\delta = \frac{(1 - I/\Pi n)[1 - \gamma(I)]}{1 - (1 - 1/\Pi n)\gamma(I)} = \frac{\Pi n - I}{\Pi n + \Gamma(I)} \equiv \Omega(\Pi; I), \tag{22}$$

where $\Gamma(I) \equiv \gamma(I)/(1 - \gamma(I))$. If the nominal interest rate is the policy parameter, then $\Omega(\Pi; I)$ defines the inflation Laffer curve. In contrast to the model considered in the preceding sections, the primary deficit must equal the bond seigniorage. It is evident that total revenue is positive if and only if $I < \Pi n$ or, equivalently, if $R < n$. Thus, in the CBI regime, the government can run a permanent primary deficit if and only if the economy is dynamically inefficient, under which the fiscal authority is able to roll over the debt forever.

Consider the shape of the inflation Laffer curve. It is easy to verify that the inflation Laffer curve slopes upward, because $\partial\Omega/\partial\Pi > 0$ for any Π. This is because for a given nominal interest rate, an increase in inflation reduces the real interest rate. Therefore, the interest payment on the government's outstanding debt is reduced. In addition, $\lim_{\Pi\to\infty} \Omega(\Pi; I) = 1$. It is now evident that the shape of the inflation Laffer curve should be similar to the one shown in figure 2.

The upward-sloping Laffer curve implies that there is a unique equilibrium. The level of the primary deficit determines the need for bond seigniorage, and the equilibrium rate of inflation is determined. As $\delta \equiv (g - \tau)/(1 - \tau)$ increases, the equilibrium inflation rate goes up. Although the analysis focuses on the case in which the bond seigniorage is positive, it is not necessary to rule out equilibria with $\delta < 0$, in which case the economy is dynamically efficient.

The effect of a change in the target nominal interest rate is more subtle. Rewrite (22) as $[\Pi n + \Gamma(I)]\delta = \Pi n - I$. Totally differentiate this expression to yield $d\Pi/dI = [\delta\Gamma'(I) + 1]/(1 - \delta)n$, where $\Gamma'(I) = \gamma'(I)/(1 - \gamma(I))^2 < 0$. Thus, $d\Pi/dI > 0$ holds if $\delta\Gamma'(I) + 1 > 0$. Using (7), rewrite this condition as

$$\frac{1 - \gamma(I)}{\gamma(I)} I > \frac{1 - \rho}{\rho}\delta, \tag{23}$$

which holds for sufficiently small values of δ and sufficiently large values of ρ. Thus, an increase in the target nominal interest rate raises the inflation rate if (23) is satisfied. The left-hand side of (23) increases with I, implying that the condition is more likely to be satisfied for large values of I. In other words, this condition becomes tighter when the central bank follows a low-interest-rate policy. Consider the case in which the nominal interest rate is near the lower bound. Use (6) to rewrite (23) to obtain the following.

PROPOSITION 4. *The target nominal interest rate and the equilibrium inflation rate are positively related near $I = 1$ if*

$$\left(\frac{1-\phi}{\phi}\right)^{1/\rho} > \frac{1-\rho}{\rho}\delta. \tag{24}$$

Remember that in the SCB regime, the target nominal interest rate and the equilibrium inflation rate are always positively related under nominal-interest-rate targeting. In the CBI regime, if the central bank follows a low-interest-rate policy, then the target nominal interest rate and equilibrium inflation rate are *negatively* related if δ is large, ρ is small, and ϕ is large. A large ϕ implies that households face greater liquidity needs.

7.3 Nominal-Interest-Rate Laffer Curve

Consider an economy in which the central bank targets inflation. From (22), redefine Ω as

$$\delta = \frac{\Pi n - I}{\Pi n + \Gamma(I)} \equiv \Omega(I; \Pi), \tag{25}$$

which is the nominal-interest-rate Laffer curve, given a target inflation rate. At $I = 1$, $\Omega(1; \Pi) = (\Pi n - 1)[\Pi n + (\phi/(1-\phi))^{1/\rho}]^{-1}$. The revenue is positive if and only if $I < \Pi n$ or, equivalently, if $R < n$. Thus, it is positive in the region $I \in [1, \Pi n)$. Consider the slope of the Laffer curve. From (25),

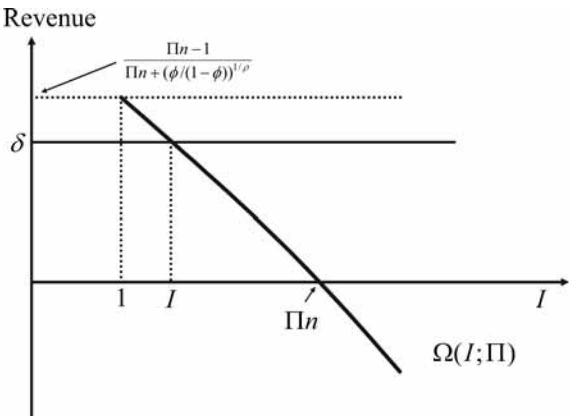
$$\left[1 + \frac{\Gamma(I)}{\Pi n}\right]^2 \Pi n \times \frac{\partial \Omega}{\partial I} = -\left[1 + \frac{\Gamma(I)}{\Pi n}\right] - \left[1 - \frac{I}{\Pi n}\right]\Gamma'(I). \tag{26}$$

Make use of (7) to rewrite the right-hand side of (26) as

$$-1 + \Gamma(I)\left\{\left(1 - \frac{I}{\Pi n}\right)\frac{1-\rho}{\rho I} - \frac{1}{\Pi n}\right\}. \tag{27}$$

The Laffer curve slopes downward if and only if this expression is negative. The question here is whether the Laffer curve slopes downward near the lower bound of the nominal interest rate. Let $I = 1$ in (27) to obtain the following.

Figure 5. Nominal-Interest-Rate Laffer Curve in the CBI Regime



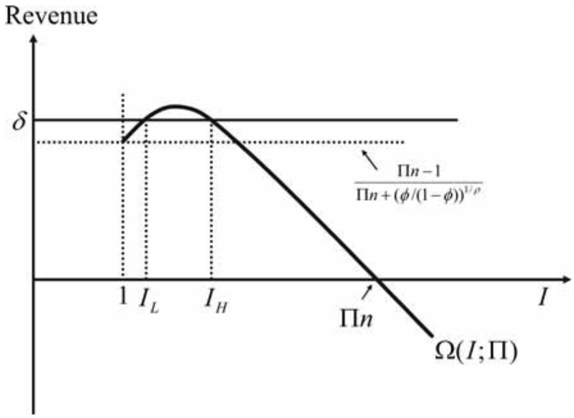
Proposition 5. *The nominal-interest-rate Laffer curve slopes downward near* $I = 1$ *if and only if*

$$1 - \frac{1}{\Pi n} < \rho + \rho\left(\frac{1-\phi}{\phi}\right)^{1/\rho}. \tag{28}$$

This condition is satisfied if (i) the target inflation rate is sufficiently low (i.e., the money injection is sufficiently small), (ii) the interest elasticity of money demand is high (i.e., ρ is sufficiently high), and (iii) the households' liquidity need is small (i.e., ϕ is sufficiently small). If these conditions are satisfied, then the Laffer curve is decreasing for all $I \geq 1$. In such a case, a unique equilibrium is obtained for each level of the primary deficit, as shown in figure 5. Because the Laffer curve attains its maximum at $\Omega(1;\Pi)$, equilibrium exists for $\delta \leq \Omega(1;\Pi)$.

On the other hand, if (28) is not satisfied, then the Laffer curve slopes upward near $I = 1$, as shown in figure 6. Such a case occurs if the target inflation rate is high, the interest elasticity of money demand is low, and the household's liquidity need is large. In this case, lower nominal interest rates are associated with *smaller* government revenues. In addition, if the primary deficit satisfies $\delta > \Omega(1;\Pi)$, then there arise *two* steady-state equilibria.

Figure 6. Multiple Equilibria in the CBI Regime



EXAMPLE 1. *Let the parameter values be* $\phi = 0.6$*,* $\rho = 0.1$*,* $n = 1.05$*, and* $\Pi = 1.08$*. The maximum total revenue is* 0.002309*. For* $\delta = 0.0023$*, there are two steady-state equilibria,* $I = 1.008601$ *and* $I = 1.027440$*.*

Remember that there is a unique equilibrium under strict inflation targeting if the central bank is subordinate to the fiscal authority. With a tough central bank that follows a strict inflation target, there is scope for multiple equilibria if the primary deficit is sufficiently large. Keeping the primary deficit low can ensure the uniqueness of equilibrium and a positive relation between the nominal interest rate and revenue.

7.4 Taylor Rules

Consider the feedback rule specified in (15) under the CBI regime. In this case, (22) is replaced with

$$\delta = \frac{n\alpha\Pi^* I^{1/\beta} - I}{n\alpha\Pi^* I^{1/\beta} + \Gamma(I)} \equiv \Omega(I), \tag{29}$$

where $\Omega(.)$ is redefined. At the lower bound of the nominal interest rate, $\Omega(1) = (n\alpha\Pi^* - 1)[n\alpha\Pi^* + (\phi/(1-\phi))^{1/\rho}]^{-1}$, which is positive as long as $n\alpha\Pi^* > 1$. The slope of the Laffer curve satisfies

$$\Omega'(I)=\frac{(\beta^{-1}-1)n\alpha\Pi^* I^{1/\beta}-(n\alpha\Pi^* I^{1/\beta}-I)\Gamma'(I)+(\beta^{-1}n\alpha\Pi^* I^{1/\beta-1}-1)\Gamma(I)}{[n\alpha\Pi^* I^{1/\beta}+\Gamma(I)]^2}.$$

Thus, it is easy to verify that $\Omega'(I) > 0$ holds if $\beta < 1$, $n\alpha\Pi^* > I^{1-1/\beta}$, and $n\alpha\Pi^* > \beta I^{1-1/\beta}$ are satisfied. The third condition is redundant under $\beta < 1$. Because $I^{1-1/\beta}$ is decreasing in I under $\beta < 1$, its maximum is 1. Thus, $\Omega'(I) > 0$ holds for all $I \geq 1$ if the implicit target Π^* satisfies $n\alpha\Pi^* > 1$. Similarly, $\Omega'(I) < 0$ holds if $\beta > 1$, $n\alpha\Pi^* < I^{1-1/\beta}$, and $n\alpha\Pi^* < \beta I^{1-1/\beta}$ are satisfied. With $\beta > 1$, the third condition is redundant. Because $I^{1-1/\beta}$ is increasing in I under $\beta > 1$, its minimum is 1. Thus, $\Omega'(I) < 0$ holds for all $I \geq 1$ if the implicit target Π^* satisfies $n\alpha\Pi^* < 1$. The following summarizes.

PROPOSITION 6. *Consider the CBI regime. Under a passive Taylor rule, the Laffer curve slopes upward for all $I \geq 1$ if the implicit target Π^* is sufficiently high; under an active Taylor rule, the Laffer curve slopes downward for all $I \geq 1$ if the implicit target Π^* is sufficiently low.*

Two comments are worth making. First, the implicit target level of the inflation rate Π^* influences the slope of the Laffer curve in the CBI regime; this is not true in the SCB regime, in which the slope of the Laffer curve depends on the shape of the money-demand function. Second, in the SCB regime, the Laffer curve under a Taylor rule can slope upward only near the zero bound of the nominal interest rate, while in the CBI regime the Laffer curve can be *globally* upward sloping.

The slope near the lower bound of the nominal interest rate is positive if and only if

$$\frac{1-\beta}{\beta}-\frac{n\alpha\Pi^*-1}{n\alpha\Pi^*}\Gamma'(1)+\left(\frac{1}{\beta}-\frac{1}{n\alpha\Pi^*}\right)\Gamma(1)>0. \tag{30}$$

Because $\Gamma'(I) < 0$, it is easy to verify that Ω slopes upward near $I = 1$ if $\beta < 1$. Applying (7), rewrite (30) to establish the following.

Proposition 7. *The Laffer curve slopes upward near the lower bound of the nominal interest rate if and only if*

$$\left(\frac{1-\phi}{\phi}\right)^{-1/\rho}\left[\frac{n\alpha\Pi^{*}-1}{n\alpha\Pi^{*}}\frac{1}{\rho}+\frac{1-\beta}{\beta}\right]+\frac{1-\beta}{\beta}>0.$$

Notice that this condition is satisfied for any $\beta \in (0,1)$. A passive monetary policy is sufficient for the Laffer curve to slope upward near the lower bound of the nominal interest rate. Under a passive monetary policy, a lower nominal interest rate is associated with a higher real interest rate on bonds. Thus, the Laffer curve slopes downward near the lower bound only when monetary policy is considerably active.

8. Discussion of the Results

In the SCB regime, the primary deficit is financed by bonds and money. The Laffer curve under nominal-interest-rate targeting is increasing in the inflation rate. This suggests that as the primary deficit increases, the central bank is forced to increase money growth, a standard result. Under strict inflation targeting, the Laffer curve is decreasing in the nominal interest rate, suggesting that the central bank is forced to cut the nominal interest rate if the primary deficit increases. Finally, under a Taylor rule, the Laffer curve is not necessarily decreasing in the nominal interest rate. In particular, the Laffer curve is increasing in the nominal interest rate near the lower bound of the nominal interest rate under a passive Taylor rule, because a reduction in the nominal interest rate raises the real interest rate on government bonds.

In the CBI regime, the central bank is "tough," and the deficit must be financed only by bonds. Under nominal-interest-rate targeting, the government's revenue continues to be increasing in inflation. As the primary deficit increases, the central bank *must* inject money into the economy in order to maintain the level of the nominal interest rate. In other words, the central bank must offset the increased demand for money by greater money injection. In this sense, the central bank is not fully independent of the fiscal policy when it has to peg the bond price.

Under strict inflation targeting in the CBI regime, the central bank has no obligation to raise revenue or maintain the bond price. In this case, the nominal-interest-rate Laffer curve is not always downward sloping. This contrasts with the Laffer curve under the SCB regime, which is unambiguously downward sloping. The slope of the nominal-interest-rate Laffer curve under the CBI regime is strongly influenced by the money-demand function. Near the lower bound of the nominal interest rate, the Laffer curve slopes upward if the interest elasticity of money demand is sufficiently small. Then, cutting the nominal interest rate *reduces* government revenue. In such a situation, the government must reduce the primary deficit if it wishes to lower the equilibrium nominal interest rate further.

Finally, consider the case in which the central bank follows a Taylor rule in the CBI regime. The key finding is that the slope of the Laffer curve crucially depends on how active or passive the central bank is. If the central bank follows a passive Taylor rule and the implicit target level of the inflation rate is sufficiently high, then cutting the nominal interest rate *lowers* government revenue for all nominal interest rates. In contrast, the Laffer curve under a passive Taylor rule in the SCB regime slopes upward only near the zero lower bound of the nominal interest rate. Under an active Taylor rule in the CBI regime, the central bank can ensure a downward-sloping Laffer curve by setting the implicit target to a sufficiently low level.

There are several findings regarding the link between targets and uniqueness of (long-run) equilibrium. In the SCB regime, unique equilibrium results under nominal-interest-rate targeting and strict inflation targeting.[4] Under Taylor rules, uniqueness is obtained if the monetary policy is sufficiently active. Uniqueness results even under a passive Taylor rule if the primary deficit is sufficiently small. In the CBI regime, unique equilibrium is obtained under nominal-interest-rate targeting. There may be two steady states under inflation targeting if the target inflation rate is high, the interest elasticity of money demand is low, and the household's liquidity need is large. Uniqueness prevails even in such an environment if the primary deficit is small enough. Finally, equilibrium is likely to be unique

[4]The results depend on the money-demand function that is independent of the inflation rate.

under Taylor rules in the CBI regime, because in many situations the Laffer curve is either upward sloping or downward sloping.

There are two main reasons why central banks around the globe follow low-interest-rate policies when the economy is in a state of stagnation. One is obviously to stimulate the economy by influencing the cost of funds for private firms. The other is to reduce the cost of funds for the fiscal authority that faces a greater need to spend and a smaller tax revenue. This suggests an interesting measure of central bank independence—the degree of central bank independence is low if a central bank hesitates to raise the nominal interest rate when the fiscal authority has accumulated a large public debt. Because monetary policy has a strong impact on the bond seigniorage even under the CBI regime, the fact that the central bank's budget is separated from the fiscal authority's budget is not enough to ensure instrument independence of a central bank. The key is whether a central bank can raise the nominal interest rate when it must.

9. Conclusion

This paper has studied various types of Laffer curves in order to clarify the link between monetary policy instruments and government revenue under different fiscal–monetary policy arrangements. Two aspects are particularly new. One is that Laffer curves are studied under a Taylor-type feedback rule. The other analysis that is new in this paper is that it has studied Laffer curves in which there is no currency seigniorage. Interestingly, monetary policy continues to influence government revenue through changing the interest obligation on the public debt.

There are several important cases that are left unexplored. One possible extension of the analysis is to consider a production economy, in which a change in the real interest rate influences output. Another possible extension is to consider nonindexed bonds. Studying the bond seigniorage with nonindexed bonds is important, because an unexpected change in monetary policy may have a greater impact on the bond seigniorage. Finally, it is certainly interesting to investigate the political pressure on the central bank to follow low-interest-rate policies. Politicians dislike high interest rates for the same reason that they dislike high tax rates. Such a direction of research may enhance our understanding of central bank independence.

Appendix 1. Inflation and Money Demand

Consider an alternative environment in which there are two types of households, type 1 and type 2. A type 1 household is the same as the one described in section 3.2. A type 2 household is assumed to consume both when young and when old. It is also assumed that a type 2 cannot participate in the bond market. These households are closely related to "poor" households in Sargent and Wallace (1981). Alternatively, one could assume that type 2 consumers are "early diers," in the context of Diamond and Dybvig (1983), who *must* consume in the first subperiod. In any case, a type 2 individual wishes to hold money as a means of saving. A type 2's problem is to choose the amount of saving s_t to maximize the lifetime utility. Thus,

$$\max_{s_t} U\left(\frac{p_t y_t - T_t - s_t}{p_t}, \frac{s_t}{p_{t+1}}\right),$$

where it is assumed that the utility function U is homothetic. Assuming interior solution, the maximization problem gives the standard real saving function: $S(p_t/p_{t+1}, y_t - T_t/p_t) = \eta(p_t/p_{t+1})[y_t - T_t/p_t]$. Note that homotheticity ensures that the saving rate η is a function of the real interest rate and independent of income.

Suppose that the fraction of type 2 household is θ. Then, the money-market clearing condition is $M_t = (1-\theta)\gamma(I_{t+1})[p_t y_t - T_t] + \theta\eta(p_t/p_{t+1})[p_t y_t - T_t]$. Divide this equation by $p_t y_t$ to obtain

$$m_t = [(1-\theta)\gamma(I_{t+1}) + \theta\eta(\Pi_{t+1}^{-1})](1-\tau). \tag{31}$$

An important implication of (31) is that aggregate money demand now depends on both the nominal interest rate and the inflation rate.

Substitute (31) and $b_t = (1-\theta)[1-\gamma(I_{t+1})][1-\tau]$ into (9) to obtain

$$\delta = (1-\theta)\left\{1 - \frac{\mu(I)}{\Pi n}\right\} + \theta\left\{1 - \frac{1}{\Pi n}\right\}\eta(\Pi^{-1}), \tag{32}$$

which implies that the inflation Laffer curve may have a downward-sloping region for high rates of inflation if $\eta' > 0$. As inflation increases, the real interest rate on money decreases, which in turn decreases real money demand by type 2 households.

Appendix 2. Transfer to the Old

Consider the CBI regime. The quantity of base money newly injected into the economy in period t is denoted by H_t. In section 7, it is assumed that the entire H_t is in the form of transfer to the young. The purpose of this section is to present an alternative environment in which a fraction of H_t is given to the old. In particular, let φH_t be the quantity of money transferred to the old, while $(1-\varphi)H_t$ is transferred to the young. Then, the household's budget constraints are replaced with $M_t+B_t = p_t y_t - T_t + (1-\varphi)H_t$ and $p_{t+1}c_{1t}+p_{t+1}c_{2t} = M_t + I_{t+1}B_t + \varphi H_{t+1}$. Then, the demands for money and bonds are $M_t = \gamma(I_{t+1})[p_t y_t - T_t + (1-\varphi)H_t] + \varphi H_{t+1}\gamma(I_{t+1})/I_{t+1}$ and $B_t = [1-\gamma(I_{t+1})][p_t y_t - T_t + (1-\varphi)H_t] - \varphi H_{t+1}\gamma(I_{t+1})/I_{t+1}$, respectively. Divide these expressions by $p_t y_t$ to obtain

$$m_t = \gamma(I_{t+1})[1-\tau+(1-\varphi)h_t] + \frac{\gamma(I_{t+1})}{I_{t+1}}\varphi h_{t+1}\Pi_{t+1}n, \tag{33}$$

$$b_t = [1-\gamma(I_{t+1})][1-\tau+(1-\varphi)h_t] - \frac{\gamma(I_{t+1})}{I_{t+1}}\varphi h_{t+1}\Pi_{t+1}n. \tag{34}$$

It is important to note here that real money demand now depends on the inflation rate if $\varphi > 0$. In particular, a higher inflation rate expands the demand for money, which is precisely the opposite of the model considered in appendix 1.

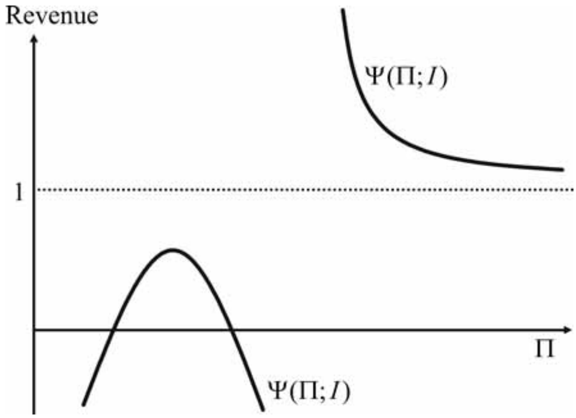
Focus on the steady state. Solve (33) and (19) for h as

$$h = \frac{(1-1/\Pi n)\gamma(I)[1-\tau]}{1-(1-1/\Pi n)\gamma(I)[(1-\varphi)+\varphi\Pi n/I]}.$$

Substitute it and (34) into (20) to obtain

$$\delta = \left(1-\frac{I}{\Pi n}\right)[1-\gamma(I)] + \frac{1-\varphi-\gamma(I)[(1-\varphi)+\varphi\Pi n/I]}{1-(1-1/\Pi n)\gamma(I)[(1-\varphi)+\varphi\Pi n/I]} \times \left(1-\frac{I}{\Pi n}\right)\left(1-\frac{1}{\Pi n}\right)\gamma(I).$$

Figure 7. Transfer to the Old



This coincides with (22) when $\varphi = 0$. Consider the case with $\varphi = 1$. Then,

$$\delta = \left(1 - \frac{I}{\Pi n}\right)[1 - \gamma(I)] - \frac{\gamma(I)(1 - I/\Pi n)(1 - 1/\Pi n)\gamma(I)\Pi n}{I - (1 - 1/\Pi n)\gamma(I)\Pi n}.$$

Simplify this expression to obtain

$$\delta = \left(1 - \frac{I}{\Pi n}\right)\left\{1 - \frac{\gamma(I)}{1 - (\Pi n - 1)\gamma(I)/I}\right\} \equiv \Psi(\Pi; I), \qquad (35)$$

which defines the inflation Laffer curve. The typical configuration of the inflation Laffer curve is depicted in figure 7. For relatively low levels of the inflation rate, the Laffer curve is hump shaped, and therefore there may be two equilibria. Beyond this region, there is no equilibrium with $\delta < 1$.

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Inflation Convergence and Divergence Within the European Monetary Union*

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We study the convergence properties of inflation rates among the countries of the European Monetary Union over the period 1980–2004. Given the Maastricht agreements and the adoption of the single currency, the sample can be naturally split into two parts, before and after the birth of the euro. We study convergence in the first subsample by means of unit-root tests on inflation differentials, arguing that for testing absolute convergence, a power gain is achieved if the Dickey-Fuller regressions are run without an intercept term. We find evidence for the convergence hypothesis over the period 1980–97 and a clear indication of the important role played by the Exchange Rate Mechanism (ERM) in strengthening the convergence process. We then investigate whether the second subsample is characterized by stable inflation rates across the European countries. Using stationarity tests on inflation differentials, we find evidence of diverging behavior. In particular, we can statistically detect two separate clusters, or stability clubs: (i) a lower-inflation group that comprises Germany, France, Belgium, Austria, and Finland and (ii) a higher-inflation one with Spain, the Netherlands, Greece, Portugal, and Ireland. Italy appears to form a cluster of its own, standing between the other two.

JEL Codes: C12, C22, C32, E31.

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1. Introduction

Inflation differentials among the countries of the European Monetary Union (EMU) have shown a tendency to increase after the introduction of the common currency. The cross-country standard deviation of European inflation rates reached its minimum in the second half of 1999, picked up in 2000, and remained relatively stable thereafter; the mean absolute differential between each country's inflation rate and the European average was around half a percentage point in 1999 and nearly doubled in 2003. While the slowdown of prices and the converging behavior of inflation rates were remarkable successes of the process that led to the adoption of the single currency, the subsequent dynamics of national rates of inflation has raised some concern. Persistent differences in (actual and expected) inflation among members of a monetary union may lead to disparities in real interest rates, given the common monetary policy. These diversities may be exacerbated by cyclical considerations: a country where economic activity is relatively subdued is likely to have weak inflationary pressures[1] and therefore experience a relatively high real interest rate; this in turn could add further to the divergence of inflation.[2] On the other hand, in the absence of exchange rate flexibility, inflation differentials may work as an adjustment mechanism:

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here are those of the authors, not the Bank of Italy. Andrew Harvey gratefully acknowledges the hospitality and financial support of the Research Department of the Bank of Italy. Author e-mails: Busetti: fabio.busetti@bancaditalia.it; Forni: lorenzo.forni@bancaditalia.it; Harvey: Andrew.Harvey@econ.cam.ac.uk; Venditti: fabrizio.venditti@bancaditalia.it.

[1]See European Central Bank (2003) for a concise overview on the relationship between price dynamics and the output gap in the euro area. In particular, it is argued that for the larger euro-area economies, a 1-percentage-point increase of the output gap leads to a rise of about 15 to 30 basis points in the annualized inflation rate.

[2]This argument should be qualified. If inflation differentials are due, for example, to administered prices, there is no reason to expect that the differences in real interest rate should lead to different incentives to investment. A similar argument holds if the differentials are due to different import prices or divergent wage growth while profit margins remain unchanged. Furthermore, von Hagen and Hoffman (2004) argue that, for firms selling in all euro-area markets, the relevant measure of real rate of interest is based on average euro-area inflation. This would somehow attenuate the effect of heterogeneity of inflation differentials on investment decisions across countries. Differences in real interest rates, however, would still affect other demand components, like private consumption.

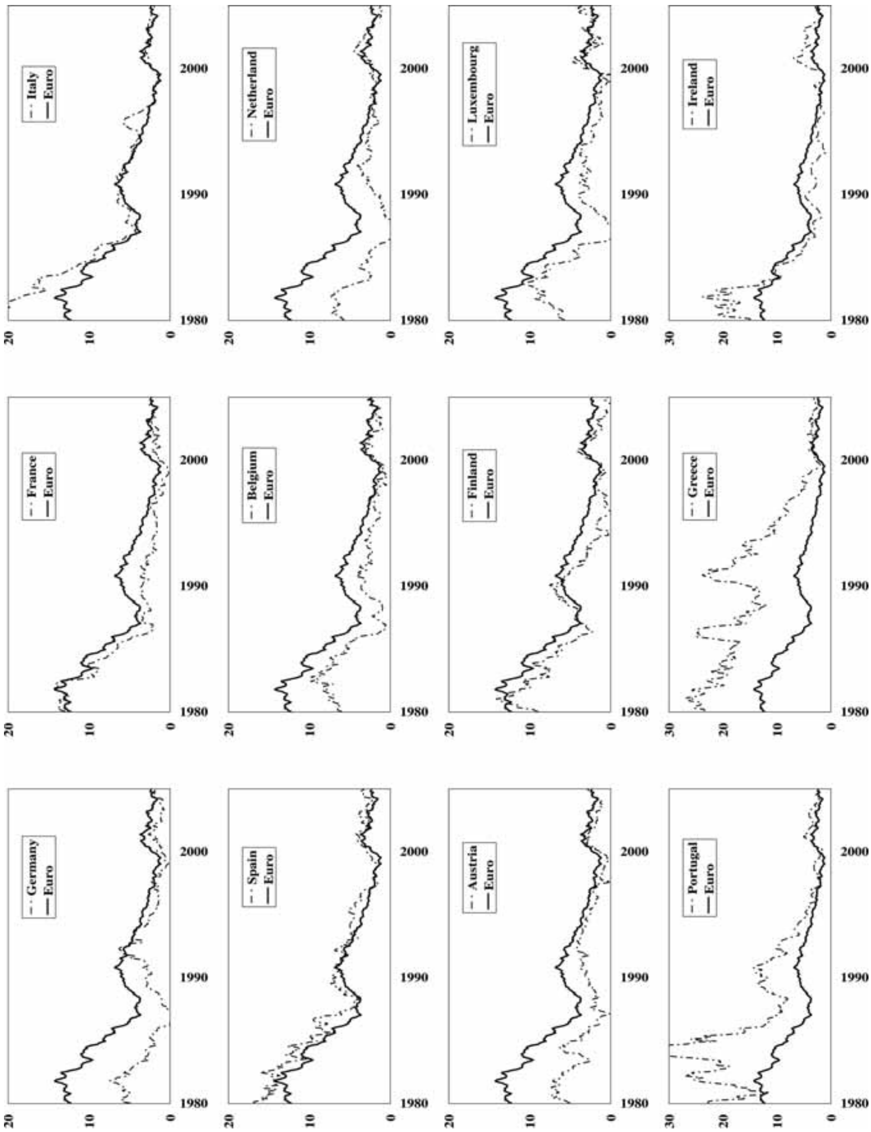
countries with higher productivity or lower wage growth than others would experience a depreciation of the real exchange rate and thus a gain in trade competitiveness. Overall, whether the expansionary effects associated with a real-interest-rate reduction or the contractionary ones induced by real-exchange-rate appreciation due to a positive inflation differential would dominate, and the horizon at which this might happen, is an empirical question. The answer will depend to a large extent on the magnitude of inflation differentials and on their persistence. However, part of the differences in inflation could also be due to country heterogeneities in the relative productivity growth of the tradable versus the nontradable sector (the so-called Balassa-Samuelson effect), and therefore they might last as long as these persist.

In this article we analyze the convergence properties of inflation rates of euro-area countries using monthly consumer price index (CPI) data from 1980 up to 2004. Given the Maastricht agreements and the adoption of the single currency, the sample can be naturally split into two parts, before and after the birth of the euro. We address two separate questions regarding the convergence properties of the inflation rates of the euro-area countries. The former is whether convergence actually occurred by 1997[3] and whether the Exchange Rate Mechanism (ERM) actually helped in accelerating the convergence process. The latter is whether inflation rates significantly drifted apart after the introduction of the single monetary policy.

Figure 1 shows the year-on-year rate of inflation for each country together with the euro-area average. It seems clear that some convergence process was in action since the early eighties at least until the beginning of the common monetary policy. We study convergence in the pre-euro subsample by unit-root tests on inflation differentials, arguing that the power of the tests is considerably increased if the Dickey-Fuller regressions are run without an intercept term. Overall, we are able to accept the convergence hypothesis and to show that

[3]One of the Maastricht criteria for joining the EMU required each country's inflation differential (with respect to the average of the three best performers) to be less than 1.5 percentage points in 1997. The three lowest inflation countries turned out to be Austria (1.2 percent), Ireland (1.2 percent), and France (1.3 percent).

Figure 1. Year-on-Year Rates of Inflation for European Countries and EMU Average, 1980–2004



the ERM played an important role in strengthening the convergence process.

Having obtained evidence in favor of convergence before the start of the common monetary policy, we investigate whether the second subsample is characterized by stable inflation rates across the member countries. The year-on-year inflation rates over the period 1998–2004 are graphed in figure 2, while figures 3 and 4 show the dispersion of inflation in terms of cross-country standard deviation and coefficient of variation. The tendency for inflation differentials to increase appears quite clear. The minimum for the standard deviation occurred in 1999; the coefficient of variation, on the other hand, reached its lowest levels in the biennium 2000–01 but edged up afterward. Using stationarity tests on the inflation differentials, we find evidence of diverging behavior. From our analysis we can statistically detect two separate clusters: (i) a low-inflation group that comprises Germany, France, Belgium, Austria, and Finland and (ii) a higher-inflation one with Spain, the Netherlands, Greece, Portugal, and Ireland. Italy appears to form a cluster of its own, standing between the other two. To the high-inflation cluster belong countries whose convergence process started rather late in the nineties (i.e., Portugal, Spain, and Greece[4]) and in which the move to stage 3 of EMU reduced considerably nominal interest rates (Ireland, Portugal, Spain, and, later, Greece), therefore contributing to sustained price dynamics.[5]

It is worth emphasizing that, since stationarity tests that do not allow for an intercept term are applied to inflation differentials, each cluster contains inflation rates that are found to be stationary around the same mean. Thus the evidence for divergence over the period 1998–2004 is in the sense that countries belonging to different clusters (or *stability clubs*) are characterized by inflation dynamics stable within their group but statistically different from other groups, where the difference may be due to either nonstationary

[4]For example, in 1995 (two years before qualifying for the euro) Portugal and Spain had, respectively, a 4.2 percent and 4.7 percent inflation rate, while Greece, which entered two years later, in 1997 recorded an inflation rate of 5.4 percent; these numbers must be compared with the 1997 threshold.

[5]Note that in Portugal, Spain, and Greece, inflation rates have been above the euro-area average since 1990. On the other hand, Ireland had negative inflation differentials (relative to the euro average) during most of the nineties.

Figure 2. Year-on-Year Rates of Inflation for European Countries and EMU Average, 1998–2004

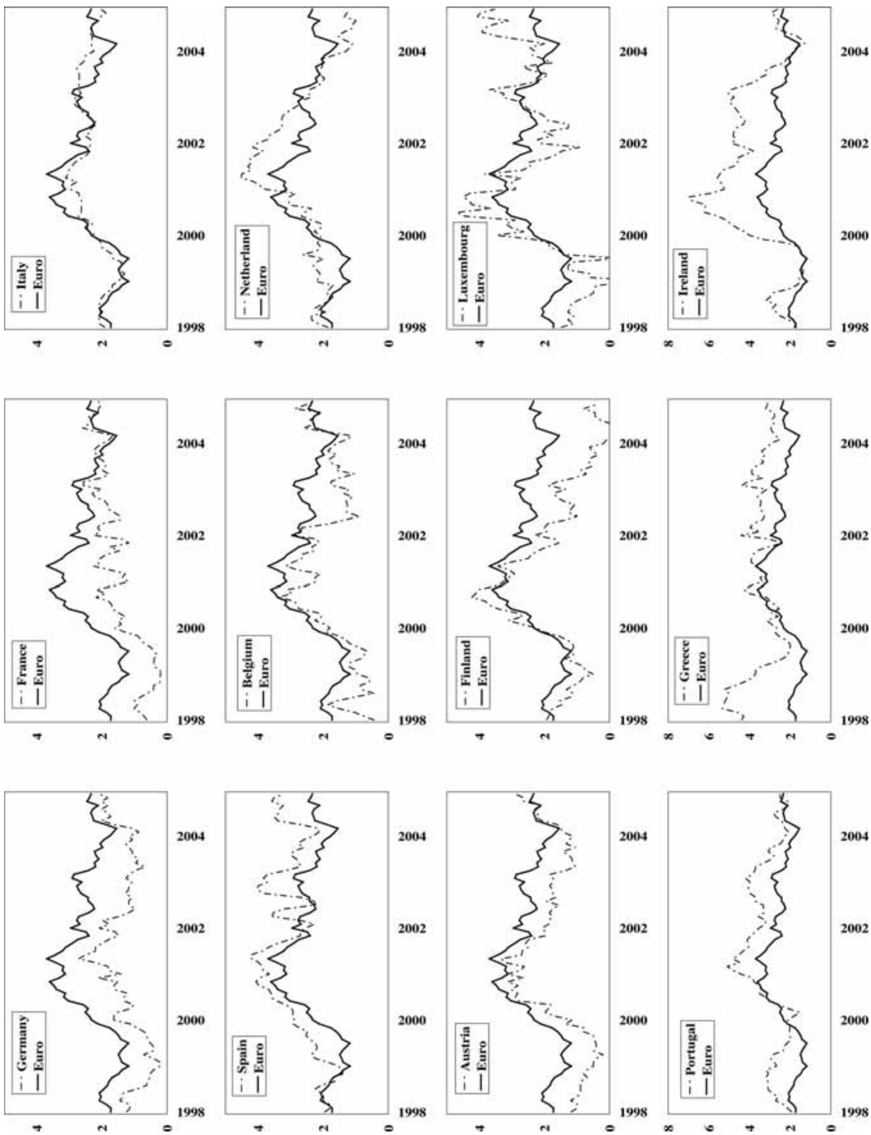
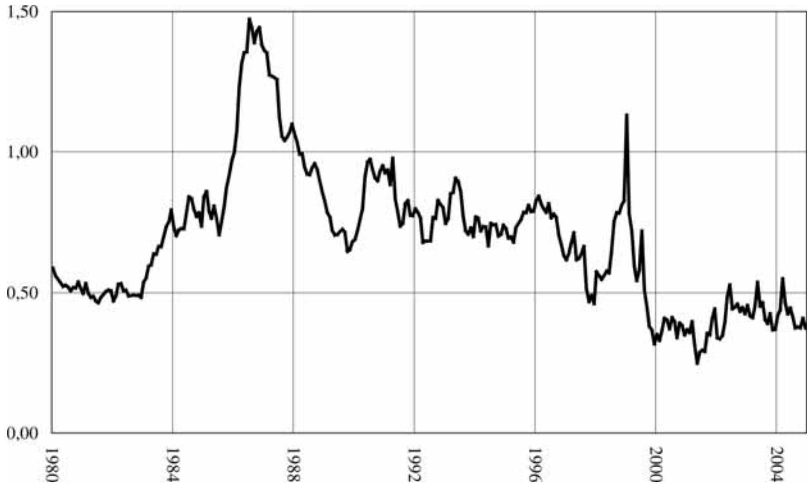


Figure 3. Cross-Sectional Standard Deviation of European Inflation Rates

behavior or to different underlying means (or both). In fact, our results suggest that differences in the underlying means may explain the divergence result.

The issue of inflation convergence within European countries has already been analyzed in several papers, mainly within the framework of unit-root and cointegration tests for panel data. Kocenda and Papell (1997) use panel unit-root tests and find evidence in favor of inflation convergence, in particular among the countries participating from the start of the ERM, and they argue that the convergence process was not substantially affected by the 1992/1993 ERM crises. Siklos and Wohar (1997) run cointegration tests for several European countries to obtain evidence for the presence of a single stochastic trend, a result that is consistent with the hypothesis of convergence. Holmes (2002) finds that inflation convergence was strongest during the years 1983–90, whereas the turbulence experienced within the ERM in the early nineties conferred some degree of macroeconomic independence to certain member countries. More recently, Beck and Weber (2003) have performed a beta and sigma convergence analysis of regional inflation data for the United States,

Figure 4. Cross-Sectional Coefficient of Variation of European Inflation Rates



Japan, and Europe over the period 1981–2001, showing that inflation dispersion among European regions is higher than in the United States or in Japan. Honohan and Lane (2003) argue that the increase in inflation differentials immediately after the start of the single monetary policy is partly due to the differential impact of the depreciation of the euro. Angeloni and Ehrmann (2004) estimate a stylized multicountry structural model for the euro area to analyze the response of inflation differentials to a number of different shocks. Their model suggests that the persistence of inflation differentials is mainly determined by the level of inflation persistence at the country level.

The paper is organized as follows. Section 2 provides the theoretical background: it describes our definition of convergence and stability and the testing methodology by means of unit-root and stationarity tests. Section 3 describes the results for the "convergence subperiod" (1980–97), while section 4 explores the issue of whether inflation rates have started drifting apart after the adoption of the single currency. Concluding remarks and a brief summary are contained in section 5.

2. Convergence and Stability of Inflation Rates: Definition and Tests

If $\pi_{t,i}$ denotes the series of inflation rate in country $i, i = 1, \ldots, n$, the convergence properties between countries i and j can be studied from the time-series properties of the inflation differential between them,

$$y_t^{i,j} = \pi_{t,i} - \pi_{t,j}, \quad i, j = 1, \ldots, n,$$

which we call the *contrast* between i and j. In order to simplify the notation, we drop the superscript i, j in the remainder of this section.

In the time-series literature on convergence, there is sometimes confusion on the role played by unit-root and stationarity tests for detecting convergence. The two types of tests are in fact meant for different purposes and cannot be arbitrarily interchanged. Unit-root tests are mostly useful for establishing whether two (or more) variables are in the process of converging, with a large part of the gap between them depending on the initial conditions. Stationarity tests, on the other hand, are the more appropriate tool for investigating whether the series have converged—that is, whether the difference between them is stable. In the presence of strong serial correlation—which typically characterizes converging paths—it is known that the actual size of stationarity tests is well above the significance level; see table 3 of Kwiatkoski et al. (1992), henceforth KPSS, and Müller (2005). It is therefore important to distinguish between convergence and stability, the former analyzed by testing the null hypothesis of unit root, the latter by testing the null of stationarity.[6] The subsections below formally describe how to test for convergence and stability and to detect stability clubs; see also Busetti, Fabiani, and Harvey (2006).

[6] In the present paper we also refer, with some abuse of terminology, to *divergence* as being associated with rejection of stationarity tests. However, this seems reasonable within our empirical investigation of inflation rates in the post-euro period, as inflation differentials were typically close to 0 at the beginning of the sample and tended to widen thereafter.

2.1 Convergence and Stability

A suitable model for convergence will be asymptotically stationary, satisfying the condition that

$$\lim_{\tau\to\infty} E(y_{t+\tau}|Y_t) = \alpha, \tag{1}$$

where Y_t denotes current and past observations. Convergence is said to be *absolute* if $\alpha = 0$; otherwise, it is *relative* (or conditional) (see, for example, Durlauf and Quah 1999). The simplest such convergence model is the AR(1) process

$$y_t - \alpha = \phi(y_{t-1} - \alpha) + \eta_t, \quad t = 1, \ldots, T, \tag{2}$$

where η_t's are martingale difference innovations and y_0 is a fixed initial condition. By rewriting (2) in error-correction form as

$$\Delta y_t = \gamma + (\phi - 1)y_{t-1} + \eta_t, \tag{3}$$

where $\gamma = \alpha(1 - \phi)$, it can be seen that the expected growth rate in the current period is a negative fraction of the gap in the two regions after allowing for a permanent difference, α. We can therefore test for convergence by a unit-root test—that is, a test of $H_0 : \phi = 1$ against $H_1 : \phi < 1$. The power of a unit-root test will depend on the initial conditions—that is, how far y_0 is from α.

For inflation differentials, the interest in most cases is in testing the hypothesis of absolute convergence. If α is known to be 0, the test based on the Dickey-Fuller t-statistic, denoted τ_0 when there is no constant, is known to perform well, with a high value of $|y_0|$ actually enhancing power. The test based on τ_0 is also more powerful, for detecting absolute convergence, than the popular GLS-based alternative of Elliott, Rothenberg, and Stock (1996). Monte Carlo experiments in Harvey and Bates (2003) and Busetti, Fabiani, and Harvey (2006) quantify the power properties of many unit-root tests for different initial conditions; the findings are in line with the arguments of Müller and Elliott (2003).

An AR(p) process provides a natural generalization of (2) that allows for richer dynamics, i.e.,

$$\Delta y_t = \gamma + (\phi - 1)y_{t-1} + \gamma_1 \Delta y_{t-1} + \gamma_{p-1}\Delta y_{t-p+1} + \eta_t, \tag{4}$$

parameterized in error-correction form, with $0 < \phi < 1$. The augmented Dickey-Fuller (ADF) test is based on such a regression. Again, a constant term should not be included if the hypothesis of interest is that of absolute convergence.

Regarding stability, we say that countries i and j *have converged* if the inflation differential y_t is a stationary process (with strictly positive and bounded long-run variance). Stationarity tests as proposed in Hobijn and Franses (2000), KPSS, and Busetti and Harvey (2007) are then the appropriate instrument for testing whether convergence has already taken place.

For the case of zero-mean stationarity (the most relevant for the analysis of inflation differentials), the test statistic should be computed without demeaning (or detrending) the series as in KPSS. Thus the stationarity test will reject for large values of

$$\xi_0 = \frac{\sum_{t=1}^{T} \left(\sum_{j=1}^{t} y_j \right)^2}{T^2 \widehat{\sigma}_{LR}^2}, \tag{5}$$

where $\widehat{\sigma}_{LR}^2$ is a nonparametric estimator of the long-run variance of y_t—that is,

$$\hat{\sigma}_{LR}^2 = \hat{\gamma}(0) + 2 \sum_{\tau=1}^{m} w(\tau, m) \hat{\gamma}(\tau), \tag{6}$$

with $w(\tau, m)$ being a weight function, such as the Bartlett window, $w(\tau, m) = 1 - |\tau|/(m+1)$, and $\widehat{\gamma}(\tau)$ the sample autocovariance of y_t at lag τ. The bandwidth parameter m must be such that, as $T \to \infty$, $m \to \infty$ and $m^2/T \to 0$; see Stock (1994). Under the null hypothesis of zero-mean stationarity of y_t, $\xi \xrightarrow{d} \int_0^1 W^2(r)dr$, where $W(r)$ is a standard Brownian motion process. The 5 percent and 1 percent critical values are 1.656 and 2.787, respectively; see Nyblom (1989).

If the null hypothesis of interest is stationarity around a nonzero mean, the stationarity test must be computed using the demeaned observations. We will denote the resulting statistic as ξ_1; the limiting distribution is now in terms of a Brownian bridge instead of Brownian motion, with critical values provided in KPSS. Note that the statistic ξ_0 will asymptotically diverge if the data are generated by a stationary process with a nonzero mean; see Busetti and Harvey (2007).

2.2 Multivariate Tests and Detection of Stability Clubs

If interest lies in studying stability and convergence across a group of countries, a multivariate test is appropriate. Let $\mathbf{x}_t$ be the $N = n-1$ vector of contrasts between each of n countries and a benchmark, e.g., $\mathbf{x}_t = (y_t^{1,n}, y_t^{2,n}, \ldots, y_t^{n-1,n})'$ if the benchmark is the n-th country. Multivariate tests of convergence can be obtained by a generalization of the Dickey-Fuller methodology as proposed in Abuaf and Jorion (1990) and Harvey and Bates (2003); see the working paper version Busetti et al. (2006) for details.

For investigating stability, a generalization of the KPSS test can be applied to $\mathbf{x}_t$ to test whether the n countries have converged. The statistic is

$$\xi_0(N) = Trace(\widehat{\mathbf{\Omega}}^{-1}\mathbf{C}), \tag{7}$$

where $\mathbf{C} = \sum_{t=1}^{T}(\sum_{j=1}^{t}\mathbf{x}_j)(\sum_{j=1}^{t}\mathbf{x}_j)'$ and $\widehat{\mathbf{\Omega}}$ is a nonparametric estimator of the long-run variance of $\mathbf{x}_t$ (obtained by a straightforward multivariate extension of (6)). Under the null hypothesis of zero-mean stationarity, $\xi_0(N) \xrightarrow{d} \sum_{i=1}^{n-1}\int_0^1 W_i(r)^2 dr$; critical values are provided in Nyblom (1989) and Hobijn and Franses (2000). The multivariate stationarity test is invariant to the benchmark country. Nonrejection of the null hypothesis would imply overall evidence of stability, in the sense that the n countries should have converged absolutely.[7]

Hobijn and Franses (2000) have proposed a clustering algorithm that utilizes a sequence of multivariate stationarity tests to identify *stability clubs*, where each club will be formed by series that are found to be stationary around the same mean. The algorithm, described in the appendix, is independent of the ordering of the series because of the invariance properties of the tests. However, it is not independent of the number of countries included in the sample, and including additional countries may alter the composition of clusters.

[7]If interest lies in relative convergence and stability, the demeaned observations, $\mathbf{x}_t - \overline{\mathbf{x}}$, must be used and the limiting distribution is different; see Hobijn and Franses (2000).

2.3 Power Gain from Testing without an Intercept

When the relevant hypotheses are absolute convergence and stability, to enhance power, unit-root and stationarity tests should be run without allowing for an intercept term. In the working paper version Busetti et al. (2006), the limiting local power function of the tests was used to compute the power gain of the tests without intercept. Furthermore, Busetti and Harvey (2007) show that ξ_0 (but not ξ_1) is also powerful and consistent against a stationary process with a nonzero mean; in this case the limiting power of ξ_0 is not much lower than that of the Wald t-test on the mean of the observations (and, similarly, the Wald t-test can be used to detect the presence of a random walk component).

3. Convergence of European Inflation Rates: January 1980–December 1997

In this section we analyze the convergence properties of European inflation rates in the pre-euro subsample (January 1980–December 1997). The data are the (monthly) log-differences of the national CPIs; the source is the Bank for International Settlements.[8] Seasonality was removed using the STAMP software of Koopman et al. (2000). In general, we would expect to see a clearer rejection of the unit-root hypothesis in the contrasts between countries that were part of the ERM.[9] The same data, but over the post-euro subsample, will be the object of the empirical investigation of the next section.

The results of the ADF tests on the pairwise contrasts are displayed in the left-hand panel of table 1, in the eight columns jointly labeled "Subsample 1: January 1980–December 1997." The first two columns report the ADF t-test statistic (obtained by the ADF

[8]Notice that while figures 1 and 2 show the year-on-year price changes, the tests are computed on the monthly rates of inflation.

[9]The ERM was established by the European Community in March 1979 as part of the European Monetary System (EMS) to reduce exchange rate variability among member countries. The system was reformed in 1993 to allow for wider fluctuation bands. Spain and Portugal joined in 1989 and 1992, respectively. Austria and Finland joined in 1995 and 1996, respectively, while Greece, although participating in the European Community since 1981, only entered the ERM in 1998.

regression without an intercept) and the estimated autoregressive parameter ϕ, respectively. The third column contains the outcome of the ADF test (whether the null hypothesis is rejected at the 1, 5, or 10 percent significance level or not rejected), while the fourth column shows the number of lags in the ADF regression selected according to the modified AIC criterion of Ng and Perron (2001). The following four columns refer to the ADF test with an intercept and are organized in the same way. The contrasts are ordered by countries according to their GDP—that is, Germany (DE), France (FR), Italy (IT), Spain (ES), the Netherlands (NL), Belgium (BE), Austria (AT), Greece (GR), Finland (FI), Ireland (IE), Portugal (PT), and Luxembourg (LU).

The first point to note is that the null hypothesis of no convergence is rejected much more frequently when the ADF regression is run without an intercept. This is a reflection of the power loss from testing with an unnecessary intercept term as noted in section 2.3. In particular, we find that, at the 10 percent significance level, τ_0 rejects the null hypothesis of no-convergence 58 percent of the time as opposed to only 23 percent for τ_1. In what follows, we only comment on the results for τ_0.

The results of table 1 have a clearer interpretation when we separate the European countries that joined the ERM since the beginning (Germany, France, Italy, the Netherlands, Belgium, Ireland, and Luxembourg) from the ones that joined at a later stage (Spain, Portugal, and Greece). To allow an even clearer interpretation, we include Austria and Finland in the first group of countries, even though they entered the ERM in 1995 and 1996, respectively. This can be justified by the fact that the fluctuations of the Austrian schilling have consistently been closely related to those of the German mark, while the movement in the Finnish currency significantly departed from those of the German mark only in the last four to five years of the sample. Notice that the nine countries in the first group are, in general, characterized by lower inflation than the others.

The evidence in favor of convergence in the low-inflation group is very strong: the ADF test rejects the null hypothesis at the 10 percent significance level for thirty-three out of thirty-six inflation differentials. On the other hand, in the contrasts that involve countries of the late-joining group (Spain, Portugal, and Greece), the null is

Table 1. Unit-Root Tests (Subsample 1) and Stationarity Tests (Subsample 2) on Pairwise Inflation Contrasts

	Subsample 1: January 1980–December 1997								Subsample 2: January 1998–December 2004			
	ADF—No Intercept				ADF—With Intercept				KPSS—No Intercept		KPSS—With Intercept	
	Statistic	ø	Reject	Lags	Statistic	ø	Reject	Lags	Statistic	Reject	Statistic	Reject
DE-FR	−1.76	0.91	10%	13	−1.50	0.91	–	13	0.47	–	0.25	–
DE-IT	−2.39	0.91	5%	14	−1.85	0.89	–	14	11.43	1%	0.16	–
DE-ES	−1.57	0.92	–	9	−1.49	0.87	–	13	17.14	1%	0.07	–
DE-NL	−2.26	0.54	5%	15	−2.36	0.49	–	15	6.44	1%	0.46	5%
DE-BE	−1.38	0.86	–	13	−1.35	0.85	–	14	2.91	1%	0.13	–
DE-AT	−2.69	0.48	1%	15	−2.82	0.40	10%	15	1.29	10%	0.13	–
DE-GR	−1.15	0.96	–	11	−1.24	0.87	–	14	18.61	1%	0.23	–
DE-FI	−1.83	0.84	10%	11	−1.75	0.82	–	13	0.77	–	0.45	10%
DE-IE	−2.14	0.89	5%	9	−2.03	0.88	–	9	8.65	1%	0.22	–
DE-PT	−1.28	0.95	–	12	−1.22	0.92	–	15	14.58	1%	0.19	–
DE-LU	−1.47	0.86	–	15	−1.49	0.85	–	15	1.76	5%	0.16	–
FR-IT	−2.01	0.88	5%	14	−8.34	0.48	1%	1	9.55	1%	0.60	5%
FR-ES	−1.36	0.88	–	15	−5.45	0.31	1%	5	13.44	1%	0.16	–
FR-NL	−1.67	0.91	10%	13	−1.29	0.91	–	13	3.58	1%	0.56	5%
FR-BE	−3.04	0.62	1%	13	−3.00	0.57	5%	13	1.27	10%	0.41	10%
FR-AT	−1.89	0.87	10%	15	−1.70	0.86	–	15	0.35	–	0.12	–
FR-GR	−0.92	0.96	–	14	−1.75	0.79	–	14	13.17	1%	0.43	10%

(continued)

Table 1 (continued). Unit-Root Tests (Subsample 1) and Stationarity Tests (Subsample 2) on Pairwise Inflation Contrasts

	Subsample 1: January 1980–December 1997								Subsample 2: January 1998–December 2004			
	ADF—No Intercept				ADF—With Intercept				KPSS—No Intercept		KPSS—With Intercept	
	Statistic	ø	Reject	Lags	Statistic	ø	Reject	Lags	Statistic	Reject	Statistic	Reject
FR-FI	−1.72	0.72	10%	14	−1.74	0.71	–	14	0.46	–	0.59	5%
FR-IE	−2.98	0.71	1%	9	−3.08	0.68	5%	9	7.53	1%	0.33	–
FR-PT	−0.97	0.96	–	15	−1.41	0.87	–	15	9.80	1%	0.34	10%
FR-LU	−2.56	0.72	5%	14	−2.57	0.68	–	14	1.51	10%	0.10	–
IT-ES	−4.01	0.34	1%	10	−3.97	0.34	1%	10	2.69	5%	0.14	–
IT-NL	−2.02	0.94	5%	6	−1.55	0.91	–	14	0.71	–	0.36	10%
IT-BE	−2.98	0.83	1%	9	−3.69	0.59	1%	9	1.45	10%	0.07	–
IT-AT	−2.26	0.91	5%	13	−2.07	0.87	–	13	2.43	5%	0.21	–
IT-GR	−0.87	0.95	–	14	−1.97	0.76	–	14	5.80	1%	0.16	–
IT-FI	−2.03	0.83	5%	14	−2.28	0.69	–	14	0.99	–	0.38	10%
IT-IE	−2.33	0.83	5%	13	−3.68	0.63	1%	8	3.62	1%	0.18	–
IT-PT	−1.68	0.89	10%	15	−2.58	0.77	10%	7	4.38	1%	0.11	–
IT-LU	−2.37	0.89	5%	8	−3.00	0.72	5%	14	0.23	–	0.24	–
ES-NL	−1.41	0.94	–	13	−1.18	0.90	–	10	0.21	–	0.40	10%
ES-BE	−1.97	0.85	5%	10	−2.78	0.50	10%	14	4.91	1%	0.10	–
ES-AT	−1.54	0.92	–	13	−1.54	0.84	–	13	11.93	1%	0.09	–

(continued)

Table 1 (continued). Unit-Root Tests (Subsample 1) and Stationarity Tests (Subsample 2) on Pairwise Inflation Contrasts

	Subsample 1: January 1980–December 1997								Subsample 2: January 1998–December 2004			
	ADF—No Intercept				ADF—With Intercept				KPSS—No Intercept		KPSS—With Intercept	
	Statistic	ø	Reject	Lags	Statistic	ø	Reject	Lags	Statistic	Reject	Statistic	Reject
ES-GR	−1.10	0.94	–	11	−2.04	0.69	–	14	0.73	–	0.19	–
ES-FI	−1.50	0.85	–	14	−2.33	0.56	–	14	3.42	1%	0.52	5%
ES-IE	−1.73	0.84	10%	15	−2.57	0.70	10%	15	1.38	10%	0.23	–
ES-PT	−1.54	0.90	–	13	−2.06	0.80	–	13	0.45	–	0.21	–
ES-LU	−1.63	0.88	10%	14	−2.36	0.62	–	14	2.18	5%	0.16	–
NL-BE	−1.48	0.83	–	12	−1.43	0.80	–	14	2.03	5%	0.25	–
NL-AT	−1.76	0.72	10%	15	−2.00	0.58	–	15	2.77	1%	0.48	5%
NL-GR	−1.12	0.96	–	14	−0.90	0.91	–	14	1.32	10%	0.20	–
NL-FI	−1.62	0.87	–	13	−1.42	0.85	–	14	2.64	5%	0.10	–
NL-IE	−2.26	0.90	5%	14	−1.95	0.90	–	14	2.03	5%	0.07	–
NL-PT	−1.09	0.96	–	15	−0.99	0.94	–	15	1.32	10%	0.23	–
NL-LU	−1.69	0.84	10%	14	−1.74	0.80	–	14	0.49	–	0.35	10%
BE-AT	−1.88	0.69	10%	14	−1.93	0.67	–	14	0.39	–	0.25	–
BE-GR	−1.26	0.95	–	11	−1.60	0.80	–	14	7.07	1%	0.12	–
BE-FI	−2.47	0.65	5%	15	−2.47	0.59	–	14	0.17	–	0.35	10%

(continued)

Table 1 (continued). Unit-Root Tests (Subsample 1) and Stationarity Tests (Subsample 2) on Pairwise Inflation Contrasts

	Subsample 1: January 1980–December 1997								Subsample 2: January 1998–December 2004			
	ADF—No Intercept				ADF—With Intercept				KPSS—No Intercept		KPSS—With Intercept	
	Statistic	ø	Reject	Lags	Statistic	ø	Reject	Lags	Statistic	Reject	Statistic	Reject
BE-IE	−3.20	0.76	1%	9	−3.17	0.73	5%	9	6.25	1%	0.18	–
BE-PT	−1.34	0.94	–	12	−1.45	0.88	–	10	4.98	1%	0.09	–
BE-LU	−3.07	0.14	1%	14	−3.06	0.14	5%	14	0.34	–	0.31	–
AT-GR	−1.15	0.96	–	14	−1.20	0.87	–	14	6.83	1%	0.25	–
AT-FI	−1.88	0.78	10%	14	−1.88	0.75	–	14	0.28	–	0.71	5%
AT-IE	−2.19	0.89	5%	14	−1.99	0.89	–	13	8.16	1%	0.32	–
AT-PT	−1.05	0.96	–	15	−1.00	0.93	–	15	7.55	1%	0.26	–
AT-LU	−2.08	0.74	5%	13	−2.02	0.74	–	14	0.88	–	0.12	–
GR-FI	−1.03	0.95	–	13	−2.00	0.72	–	11	4.63	1%	0.21	–
GRIE	−0.91	0.96	–	15	−1.63	0.83	–	14	0.33	–	0.18	–
GRPT	−2.17	0.78	5%	11	−2.49	0.70	–	11	0.36	–	0.14	–
GR-LU	−1.14	0.96	–	11	−1.60	0.81	–	14	2.06	5%	0.25	–
FI-IE	−2.49	0.75	5%	10	−2.45	0.74	–	10	12.33	1%	0.18	–
FI-PT	−1.23	0.93	–	12	−1.36	0.86	–	11	5.09	1%	0.24	–
FI-LU	−2.14	0.72	5%	14	−2.20	0.67	–	14	0.41	–	0.64	5%
IE-PT	−1.01	0.95	–	13	−1.59	0.86	–	13	0.75	–	0.12	–
IE-LU	−2.84	0.80	1%	8	−2.91	0.77	5%	8	3.89	1%	0.38	10%
PT-LU	−1.48	0.93	–	7	−1.36	0.88	–	15	1.69	5%	0.24	–

Table 2. Persistence Parameters in Pairwise Inflation Differentials

	Subsample 1: Jan. 1980–Dec. 1997			Subsample 2: Jan. 1998–Dec. 2004		
	Early ERM	**Late ERM**	**All**	**Low Club**	**High Club**	**All**
Minimum	0.14	0.34	0.14	0.04	0.26	0.02
Maximum	0.94	0.96	0.96	0.88	0.94	0.94
Median	0.83	0.94	0.88	0.39	0.78	0.67
Average	0.78	0.90	0.84	0.42	0.72	0.61

rejected at the 10 percent level of significance in only four out of thirty cases.

To complement the evidence documented in table 1, we report, in table 2, the minimum, maximum, median, and average estimates of the persistence parameter ϕ. The results that apply to the pre-euro period are contained in the three columns jointly labeled "Subsample 1: January 1980–December 1997." For the differentials among ERM members, the estimated persistence parameters range from 0.14 to 0.94, with the median being equal to 0.83; for monthly data this value corresponds to very fast convergence, as it implies a half life of 3.7 months. For the contrasts involving Spain, Portugal, and Greece, the estimated persistence parameter ranges from 0.34 to 0.96 with a median value of 0.94 (and median half life of 11.2 months). In line with previous findings—for example, Kocenda and Papell (1997)—these results appear to grant an active role to the ERM in speeding up inflation convergence among European countries. In particular, it appears that countries that were not part of the ERM suffered from inflation rates persistently higher than the average, while countries that never defected from the narrow ERM bands displayed stronger convergence with each other.[10]

[10]Following a reviewer's suggestion, we have checked whether inflation convergence over this subsample has been common to the major industrialized countries. In particular, we have run the tests extending the sample to include seven OECD countries (United States, Canada, United Kingdom, Japan, Denmark, Norway, and Sweden). The results suggest that convergence over the period of analysis has been an international phenomenon.

4. Stability, Divergence, and Clustering: January 1998–December 2004

The other empirical issue that we want to explore is whether inflation differentials have remained stable since 1998. For this purpose, the appropriate instruments are the univariate and multivariate stationarity tests, (5) and (7). We run the tests both without and with an intercept term (the two statistics being ξ_0 and ξ_1, respectively), bearing in mind that not only do they have different power properties but they also convey different information. The test with an intercept, in fact, will tend to reject the null hypothesis of stability when inflation differentials display unit-root behavior around a possibly nonzero mean, while without the intercept the test will tend to reject the null if either the differentials contain a unit root or they are stationary around a nonzero mean. Here a rejection of the ξ_0 stationarity test will be taken as evidence for *divergence*, since it implies that inflation differentials, typically very close to 0 at the start of the post-euro sample, tended to widen thereafter either in a unit-root fashion or by stabilizing around a nonzero mean.

The results of the stationarity tests on the pairwise contrasts are displayed in the right-hand panel of table 1, in the four columns jointly labeled "Subsample 2: January 1998–December 2004." The first two columns report the values of the statistic ξ_0 and the outcome of the stationarity test without an intercept (whether the null hypothesis is rejected at the 1, 5, or 10 percent significance level or it is not rejected); the third and fourth columns contain analogous information but for the test with the intercept ξ_1. In all cases the nonparametric spectral estimator of the long-run variance is computed for a bandwidth parameter $m = 8$ (that is, using autocovariances up to order 8), but the results are very similar for all values of bandwidth between 4 and 12.[11]

A first look at the right-hand panel of table 1 immediately tells us that the stationarity test without an intercept rejects the null hypothesis much more frequently (70 percent of the time) than the test with an intercept (27 percent). As already explained, this is

[11]For these series, it therefore appears that a value of $m = 4$ is sufficient to take care of most of the serial correlation and that the typical power loss induced by adding extra lags is probably negligible.

coherent not only with the lower power properties of the tests that include a redundant constant term but also with the case of inflation differentials that are stable around a nonzero mean. In subsequent discussion, we will focus on the results for ξ_0, since the main interest is to establish whether inflation differentials have converged absolutely (among all European countries or among subsets of them).

The table shows that there is no evidence for overall stability (around a zero mean) of inflation differentials. However, inflation rates appear to move homogeneously within groups of countries. In particular, the univariate tests of table 1 show that there is a high degree of stability among the inflation rates of Germany, France, Austria, and Finland, countries characterized by relatively low average inflation over the period 1998–2004 (ranging from 1.3 percent in Germany, in annual terms, to 1.8 percent in Austria). There is also a second group of countries—namely Spain, Portugal, Greece, and Ireland—where inflation rates are stable but fluctuate around higher levels (from 3.1 percent in Spain to 3.7 percent in Ireland).

We used the clustering algorithm of Hobijn and Franses (2000) to identify, in a formal way, the existence of stability clubs. Table 3 contains the results of the algorithm applied to the series of n largest countries of the European Monetary Union, with n ranging from 5 to 12. We start by considering Germany, France, Italy, Spain, and the Netherlands ($n = 5$), corresponding to around 85 percent of the euro-area GDP. We then progressively add Belgium, Austria, Greece, Finland, Portugal, Ireland, and Luxembourg.[12] The tests are computed without fitting an intercept.

Considering the twelve series together ($n = 12$), three stability clubs are found: (i) a lower-inflation group with Germany, France, Belgium, Austria, and Finland; (ii) a medium group with Italy, the Netherlands, and Luxembourg; and (iii) a higher-inflation club with Spain, Greece, Portugal, and Ireland. This outcome broadly confirms the finding of the analysis performed using univariate tests. However, if Luxembourg is excluded from the sample, so $n = 11$,

[12]The results are for the period January 1998–December 2004 and are obtained setting $p^* = 0.05$ and with a bandwidth parameter for spectral estimation set equal to 8. However, very similar output is obtained with the bandwidth ranging between 4 to 12 and with $p^* = 0.01$.

Table 3. Stability Clubs (January 1998–December 2004)

n	Identified Clusters
5	k_1 = Germany, France k_2 = Italy, Spain, the Netherlands
6	k_1 = Germany, France k_2 = Belgium k_3 = Italy, Spain, the Netherlands
7	k_1 = Germany, France, Belgium, Austria k_2 = Italy k_3 = Spain, the Netherlands
8	k_1 = Germany, France, Belgium, Austria k_2 = Italy k_3 = Spain, the Netherlands, Greece
9	k_1 = Germany, France, Belgium, Austria, Finland k_2 = Italy k_3 = Spain, the Netherlands, Greece
10	k_1 = Germany, France, Belgium, Austria, Finland k_2 = Italy k_3 = Spain, the Netherlands, Greece, Portugal
11	k_1 = Germany, France, Belgium, Austria, Finland k_2 = Italy k_3 = Spain, Greece, Portugal, Ireland, the Netherlands
12	k_1 = Germany, France, Belgium, Austria, Finland k_2 = Italy, the Netherlands, Luxembourg k_3 = Spain, Greece, Portugal, Ireland

the Netherlands is allocated to the higher-inflation club, and Italy forms a cluster of its own. In particular, Italy stands out by itself for $n = 7, 8, 9, 10, 11$, while in all cases except $n = 12$ the Netherlands belongs to the higher-inflation club. Figures 5 and 6 graph the average rates of inflation within clusters for $n = 12$ and $n = 11$: the patterns are very similar, but it is interesting to see that for $n = 12$ average inflation rates never cross.

Thus statistical evidence points toward divergence of inflation rates since the adoption of the euro. However, the persistence of

Figure 5. Inflation Clusters, EMU Countries

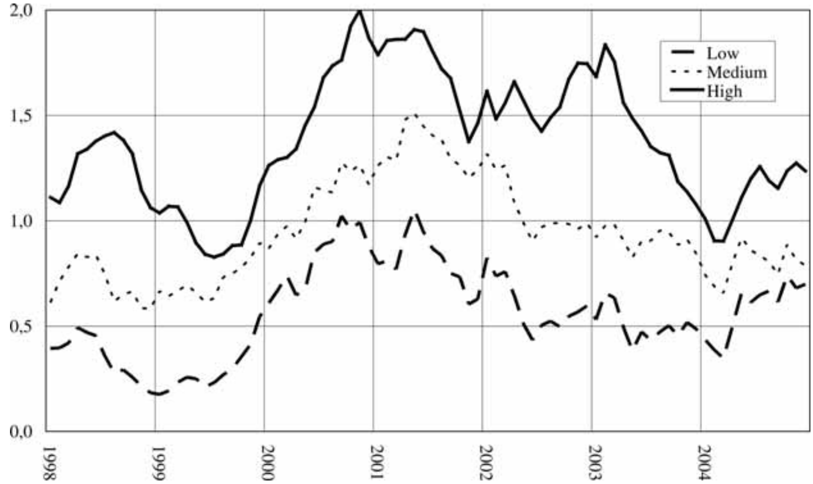
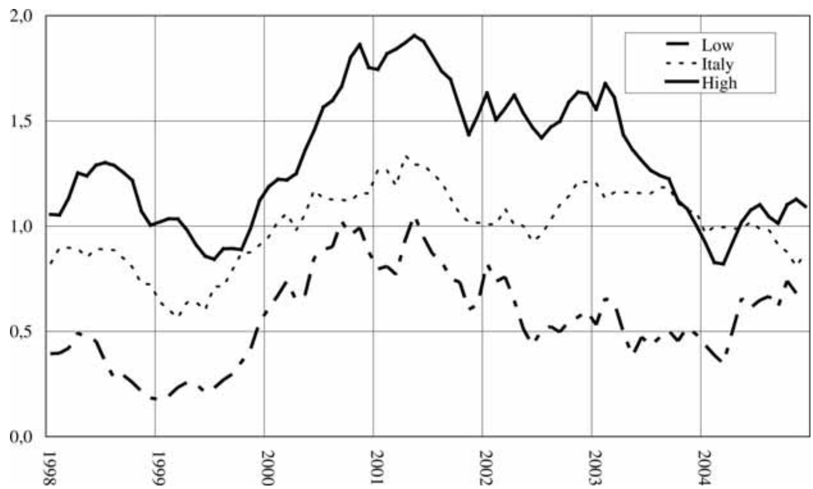


Figure 6. Inflation Clusters, EMU Countries Excluding Luxembourg



inflation differentials has fallen considerably with respect to previous years. The minimum, maximum, median, and average estimates of the persistence parameter, estimated in the post-euro subsample, are reported in the right-hand panel of table 2, in the three columns jointly labeled “Subsample 2: January 1998–December 2004.” The first column refers to the pairwise contrasts involving the countries belonging to the low-inflation club (obtained with $n = 11$); the second column considers the inflation differentials involving at least one country of the high-inflation club; the third column contains results for all pairwise contrasts. As expected, the estimates of the persistence parameter are lower for countries within the low-inflation club and, interestingly, they are also significantly lower than in the pre-euro subsample.

Finally, if we apply the multivariate stability test to the vector of all inflation differentials, we find that the null hypothesis is clearly rejected when testing without an intercept term, while it cannot be rejected (for any value of the bandwidth parameter) if an intercept term is included. This is consistent with the idea that while inflation rates within the EMU can be considered jointly stationary over the period 1998–2004, they appear to fluctuate around different means, forming two or possibly three stability clubs.

5. Concluding Remarks

We have used unit-root and stationarity tests to show that convergence of European inflation rates had occurred by the birth of the single currency. The Exchange Rate Mechanism seems to have helped convergence. However, inflation rates seem to have begun to diverge after 1998. In particular, we have been able to statistically detect two separate clusters, or stability clubs, over the period 1998–2004, characterized by relatively lower and relatively higher rates of inflation. Germany, France, Belgium, Austria, and Finland belong to the low-inflation club, while the higher-inflation group contains the Netherlands, Spain, Greece, Portugal, and Ireland. Italy appears to stand between the two groups.

Additional empirical results, pertinent to the post-euro subsample, were included in an earlier version of the paper, available upon request. By decomposing the changes in the deflators of GDP and final demand, we were able to assess the relative contributions of external factors (such as import prices) and internal factors (mainly wages and productivity) to the inflation differentials observed after

1998. We found that the clusters obtained using the final demand and the GDP deflators closely resemble those obtained in section 4 (based on consumer price indexes) and that these clusters are mainly driven by country differences in the development of per-capita compensations.

Overall, the evidence presented in this paper suggests that while the single monetary policy has, so far, successfully stabilized member countries' inflation rates, a certain degree of cross-country heterogeneity still pervades the euro area.

Appendix. The Clustering Algorithm for the Identification of Stability Clubs

Let k_i be a set of indexes of the series in cluster i, $i \leq n^*$, where $n^* \leq n$ is the number of clusters, and let p^* be a significance level for testing whether some series form a cluster. The algorithm has the following steps:

(i) Initialization: $k_i = \{i\}$, $i = 1, ..., n = n^*$. Each country is a cluster.

(ii) For all $i, j \leq n^*$, such that $i < j$, test whether $k_i \cup k_j$ form a cluster (by a multivariate stationarity test on the contrasts) and let $p^{i,j}$ be the resulting p-value of the test. If $p^{i,j} < p^*$ for all i, j then go to step (iv).

(iii) Replace cluster k_i by $k_i \cup k_j$ and drop cluster k_j, where i, j correspond to the maximum p-value of the previous test (i.e., the most likely cluster); replace the number of clusters n^* by $n^* - 1$. Go to step (ii).

(iv) The n^* clusters are the stability clubs.

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Interbank Exposures: An Empirical Examination of Contagion Risk in the Belgian Banking System*

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Robust (cross-border) interbank markets are important for the proper functioning of modern financial systems. However, a network of interbank exposures may lead to domino effects following the event of an initial bank failure. We investigate the evolution and determinants of contagion risk for the Belgian banking system over the period 1993–2002 using detailed information on aggregate interbank exposures of individual banks, large bilateral interbank exposures, and cross-border interbank exposures. The "structure" of the interbank market affects contagion risk. We find that a change from a complete structure (where all banks have symmetric links) toward a "multiple-money-center" structure (where money centers are symmetrically linked to otherwise disconnected banks) has decreased the risk and impact of contagion. In addition, an increase in the relative importance of cross-border interbank exposures has lowered local contagion risk. However, this reduction may have been compensated by an increase in contagion risk stemming from foreign banks.

JEL Codes: G20, G15.

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1. Introduction

A well-functioning and robust interbank market is an essential element of the integration of a financial system. However, although interbank markets strengthen financial integration, they also increase linkages within the banking sector. Interbank markets therefore may represent an important channel of contagion through which problems affecting one bank or one country may spread to other banks or other countries.

In this paper, we empirically address the implications of domestic as well as cross-border interbank linkages for interbank contagion risk. Contagion results from the materialization of two risks: first, the risk that at least one component of the system is hit by a shock (likelihood of a shock) and, second, the risk that this shock propagates through the system (potential impact of the shock). As the former can result from a variety of unexpected situations, we focus on the latter. In particular, we evaluate the potential damages that a chain reaction in the interbank market—i.e., a situation where the failure of one bank would lead to the default of one or more of its interbank creditors—could create. We undertake a stylized exercise—resembling a stress test—in which we simulate the consequences of nonrepayment of interbank loans of an individual bank on the capital of its bank lenders, and any further domino-like effects. In order to isolate the potential impact of contagion, we assume that the initial default is caused by a sudden, unexpected, and idiosyncratic shock. Recent history has shown that this kind of shock is not totally unlikely (see, for instance, the failure of Barings in the United

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Kingdom or Drexel Burnham Lambert in the United States) and may trigger a systemic crisis. Worries of a systemic crisis and domino effects induced, for instance, the bailout of Continental Illinois.[1]

Our empirical analysis considers contagion risk in the Belgian financial system. Why should the reader be interested in Belgium, which only covers a small part of the euro zone? The Belgian interbank market is an instructive case for several reasons.[2] First, it is very international, a feature that may become a key characteristic of many interbank markets in the future. In addition, the Belgian financial landscape contains a number of key players in the payment and securities settlement infrastructure (such as, e.g., Euroclear Bank or SWIFT). Second, the Belgian banking sector underwent a period of significant consolidation in the years 1997–2001. As a result, some large banks now have total assets that far exceed the GDP of the country, a situation that is typical for many other small countries (e.g., the Netherlands, Sweden, or Switzerland). Such countries then may face a potential too-big-to-save situation. However, only the analysis of the propagation channels of a crisis will ultimately determine its gravity. Third, the structure of the Belgian interbank market has changed over time: it has moved from a "complete" structure (where all banks have reciprocal links) toward a "multiple-money-center" structure (where a few "money-center banks" are linked together and linked to otherwise disconnected banks). These observations raise several interesting questions, which are also relevant for the analysis of contagion risk within and across financial systems of other countries. How has interbank contagion risk evolved over time? How important is the interbank market structure in explaining interbank contagion risk? To what extent could the failure of a foreign bank affect domestic banks through cross-border interbank exposures? How does contagion risk in Belgium compare with assessments for other countries? What measures can a regulator take to limit interbank contagion risk?

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[1]The Federal Reserve decided immediately to step in. Later, Paul Volcker, then Chairman of the Board of Governors of the Federal Reserve System, argued that "if [they] had not stepped in, the ultimate domino effect that so many people have feared for so long, would have occurred and wiped out the Western financial system" (Feltham 2004).

[2]By "Belgian interbank market," we refer here to the set of interbank exposures where at least one of the counterparties is a bank incorporated in Belgium.

Our analysis goes beyond the existing literature in several respects. First, we point out that it is important to take into account time variation in interbank linkages. In contrast to most existing studies, we make use of time-series data on interbank exposures; other papers focus only on a single point in time. This enables us to examine the evolution over the past decade of contagion risk associated with the failure of Belgian banks. We find that contagion risk due to domestic interbank defaults has varied significantly over time, according to a well-identifiable pattern. In particular, contagion risk increased over the period 1993–97, decreased afterward, and flattened out at a very low level at the end of the sample period (end of 2002).

Second, we investigate the determinants of contagion in an attempt to explain the evolution of contagion risk over time. Although historical events—such as the long-term capital management (LTCM) crisis or the default on Russian debt—could potentially account for the peak in contagion risk observed in 1997, we argue that changes in the structure of the Belgian interbank market and in the capitalization of Belgian banks are the main drivers behind this evolution. Theory suggests that market structure may play an important role in determining contagion risk in interbank markets (see, e.g., Allen and Gale 2000 or Freixas, Parigi, and Rochet 2000). To our knowledge, this is the first paper to empirically investigate the impact of interbank market structure on contagion risk, employing regression analysis that allows us to control for other variables in the conditioning set. We find that a move from a complete structure toward a "multiple-money-center" structure and an increase in concentration in the banking market lead to a decrease in domestic contagion. In addition, an increase in the proportion of cross-border interbank assets further decreases the risk and impact of domestic contagion. Increases in bank capitalization also have a first-order effect in reducing interbank contagion when the loss given default (LGD) is relatively low.

Third, we investigate the contagion risk stemming from interbank linkages with foreign banks, in addition to the risk associated with linkages between domestic banks. The sharp increase in the proportion of cross-border interbank assets for Belgian banks, combined with the decrease in the indicators of domestic contagion, indeed suggests that the potential contagion risk stemming from foreign interbank exposures has gained in importance. According to our simulations, the failure of some foreign banks could have a sizable effect on Belgian banks' assets, albeit only for high values of loss given default. Since large banks are more involved in international interbank markets than small banks, contagion effects triggered by foreign banks generate higher levels of contagion.

Fourth, in addition to running simulations for a range of (exogenous) levels of LGD, we also attempt, in a supplementary exercise, to endogenize banks' LGD. This allows LGD to vary across banks. These simulations still reveal an evolution over time of contagion risk; however, at any given point in time, we no longer observe a strong correlation between the average implied LGD across banks and the level of contagion, as reflected in the worst-case scenario. This is because the average LGD interacts with the other dimensions of the market structure, which remain determinant in the propagation of contagion. In addition, we find that for a given average LGD across banks, contagion risk is higher when there is more cross-sectional variation in LGD. Heterogeneity in LGD appears to exacerbate contagion risk.

Finally, in robustness tests, we analyze several alternative scenarios. For example, we show that netting of interbank exposures—the setoff of bilateral positions—may substantially reduce contagion risk. In addition, we test the potential effect of a coordination mechanism whereby the supervisor requires some banks to merge in order to reduce contagion effects. Although mergers are often used in practice as a mechanism for dealing with ailing banks, in our simulations mergers seldom decrease contagion.

The rest of the paper is organized as follows. Section 2 reviews the literature on interbank contagion risk. Section 3 introduces the data set, describes the methodology and contagion indicators, and presents the most important features of the Belgian interbank market. Section 4 discusses the results of the various simulation exercises. Section 5 presents the regression results of the impact of the interbank market structure on contagion risk. Section 6 concludes.

2. Literature Review on Interbank Market Contagion

In some circumstances, the failure of an individual bank may lead to a domino effect. This happens when the nonrepayment of interbank obligations by the failing bank jeopardizes the ability of its creditor banks to meet their obligations to their interbank creditors. Contagion occurs then "mechanically" through the direct interlinkages between banks. *Theory* shows that the extent to which a crisis is propagated through the system depends on the structure of interbank linkages. The market structure of interbank claims can take different forms. Allen and Gale (2000) distinguish three structures: (i) the "complete structure" where banks are symmetrically linked to all other banks, (ii) the "incomplete market structure" where banks are only linked to neighboring banks, and (iii) the "disconnected incomplete market structure" where two disconnected markets coexist. They show that complete structures are less prone to contagion than incomplete market structures, since with complete structures, the impact of a financial crisis in one bank is absorbed by a large number of banks. Freixas, Parigi, and Rochet (2000) introduce a fourth structure: the "money center." The money center is symmetrically linked to all the other banks, which are themselves not linked together. They show that, in some cases, the failure of a bank linked to the money center will not trigger the failure of the money center, but the failure of the money center itself may trigger failures of the linked banks. Our paper empirically investigates how the market structure of the interbank market influences contagion risk.

Current *empirical work* mainly focuses on interbank contagion within a national banking system. Two empirical approaches are implemented, each having its strengths and weaknesses. A first approach tries to isolate contagion from other shocks affecting the economy. It simulates the consequences of an individual bank failure given observed or estimated interbank exposures and looks at the potential domino effects, i.e., first-round and potential further-round effects. This approach was applied to (part of) banking systems in several countries and—although contagion indicators were more important in some countries than in others—delivered generally reassuring results (see Sheldon and Maurer 1998, Furfine 2003, Cifuentes 2004, Upper and Worms 2004, Wells 2004, or van Lelyveld and Liedorp 2006 for Switzerland, the United States, Chile, Germany, the United Kingdom, and the Netherlands, respectively[3]).

[3]Upper (2006) provides a comparative overview of these contributions.

However, all these studies look at contagion at one moment in time and generally focus on domestic contagion only. Our paper adds to this literature by considering a time series covering Belgian banks during ten years, allowing us to investigate how and why contagion risk evolved over time.[4] Furthermore, we also try to adapt the mechanics of the exercise to better reflect real-life features. This allows us to endogenize the LGD and to subsequently analyze the extent to which the results depend upon a standard assumption used in this literature, i.e., a fixed LGD. Finally, we investigate how the failure of foreign banks affects interbank contagion within the Belgian banking market. The latter issue becomes more important as cross-border exposures grow. Following our empirical analysis on the role of interbank market structure and cross-border exposures, Mistrulli (2005) documents that the Italian interbank market also moved from a "complete" structure toward a "multiple-money-center" structure. In contrast to our findings for Belgium, he reports that the importance of cross-border exposures has decreased and that the transition toward the multiple-money-center structure has increased contagion risk. While the conclusions for Italy are drawn on the basis of simulations, our regression analysis allows us to disentangle the impact of the different determinants of contagion.

A second approach to estimate contagion risk takes into account a larger variety of shocks. Müller (2003) combines a network and a simulation approach to assess the risk of contagion in the Swiss interbank market and takes into account credit and liquidity effects in bank contagion. Elsinger, Lehar, and Summer (2006) simulate the joint impact of interest rate shocks, exchange rate shocks, and stock market movements on interbank payment flows of Austrian banks. These states of the world determine the net value of the bank and the feasibility of interbank payments. They distinguish between insolvency due to correlated exposures and due to domino effects. Their simulations indicate that although the probability of contagious default is low compared to the total default probability, there are situations in which up to 75 percent of the defaults

[4] Guerrero-Gómez and Lopez-Gallo (2004) study a short time series for interbank contagion in Mexico (December 2002–August 2003) and find considerable variation of contagion in this short time window.

are due to contagion.[5] Instead of simulating interbank contagion, another method to take into account a larger variety of shock is to investigate banks' stock price behavior. Lehar (2005) estimates correlations between bank portfolios to compute different measures of systemic risk. Gropp and Vesala (2003) use the tail properties of distance to default to study contagion risk. They find the presence of both domestic and cross-border contagion within Europe, although domestic contagion seems to dominate cross-border. The advantage of this second approach is that it takes a systemwide view. However, as we want to focus on contagion risk and perform a stress test, starting from an individual bank failure may yield more insights in the evolution of risk over time, in the propagation mechanism and ultimate consequences of contagion risk. In addition, some of these techniques require time series of stock prices. Since few Belgian banks are publicly listed, this second approach appears inadequate to study the Belgian financial system.

3. Data, Methodology, and Structure of the Interbank Market

3.1 Data

The data stem from a confidential database (Schéma A) containing banks' balance sheet statements and a set of key financial figures collected for supervisory purposes at a monthly frequency. This database provides valuable information with respect to interbank positions:

- At an aggregate level, each bank reports its total interbank loans and deposits and provides breakdowns of these "aggregate positions" according to the type of loan or deposit, the geographical origin of the lender or the borrower (Belgium,

[5]Elsinger, Lehar, and Summer (2006) also use their simulation to compare two generated matrices of bilateral exposures representing a complete and an incomplete structure. They find more contagion when they use a complete market structure. Note finally another study in that second approach: Iyer and Peydro-Alcalde (2006) study a postmortem case to see how an idiosyncratic shock that affected an Indian bank was transmitted to the other Indian banks. Their study includes indirect effects through depositors' runs and media destabilizing effects.

one of the other EU members, or the rest of the world [RoW]), and the residual maturity of interbank loans or deposits. The aggregate positions used in this paper cover a period ranging from December 1992 to December 2002.

- At an individual bank level, banks report their "large exposures" to both domestic and foreign single obligors, including their interbank exposures (i.e., exposures exceeding 10 percent of their own funds). Reliable data on large exposures are only available from 2002:Q3 onward. We use a cross-section of data on large exposures to banks for December 2002.

Figures are reported on a firm basis; i.e., they include banks incorporated in Belgium (i.e., Belgian banks and Belgian subsidiaries of foreign banks) as well as their foreign branches, and consequently exclude Belgian branches of foreign banks or foreign subsidiaries of Belgian banks. The Belgian banking system, at the end of 2002, comprises 65 banks with total assets of €792 billion. The banking system is characterized by a high degree of concentration, since the four largest banks account for 85 percent of total assets of Belgian banks. This concentration results from several mergers over the period 1997–2001 and from an overall decrease in the number of banks, from 112 in 1992 to 65 in 2002.

The interbank market evolution in Belgium was partly determined by the overall evolution of money markets in Europe over the last decade. First, the establishment of the Economic and Monetary Union (EMU) radically changed the European financial landscape and allowed greater market integration. Baele et al. (2004) find that the euro-area money markets have reached a very advanced level of integration. This "near-perfect" integration fostered a higher internationalization of interbank transactions, also observable in the Belgian data. Second, the launch of the EMU required efficient cross-border payment systems. To this end, the 1997 implementation of TARGET (Trans-European Automated Real-time Gross settlement Express Transfer system) facilitated the integration of European money markets and the setting up of international bank exposures. In Belgium, the entry point to TARGET is the real-time gross settlement system ELLIPS (ELectronic Large value Interbank Payment System). ELLIPS is structured in two tiers, with direct and indirect participants. Direct participants must have an account with the central bank. At the end of 2002, there were seventeen direct participants and seventy-six indirect participants. In our data set, accounts that direct participants must have with the central bank are not considered interbank exposures. On the other hand, accounts between participants and subparticipants are considered interbank exposures. One might expect that the two-tier structure of payment systems and the subsequent access to international payment systems influence the structure of the resulting interbank linkages.

As shown in table 1, the interbank loans of Belgian banks represent a gross exposure of €176 billion at the end of 2002, while interbank deposits amount to €228 billion.[6] On both sides of the balance sheet, term and secured loans/deposits represent the largest portions of interbank positions. The current level of secured loans is the consequence of a shift in the strategy of Belgian banks in the beginning of the 1990s, probably nurtured by the monetary policy reform in Belgium in 1991, which stimulated the use of repos between Belgian banks. Over the period 1992–2002, interbank loans always account for 20 to 27 percent of total assets of Belgian banks, and interbank deposits account for 29 to 35 percent of their total liabilities.[7]

Another noteworthy characteristic of interbank positions of Belgian banks is their high degree of internationalization. At the end of 2002, less than 15 percent of interbank exposures of Belgian banks were to other Belgian banks. Hence, Belgian banks might be more sensitive to international bank failures than to domestic ones. Manna (2004) reports that the share of interbank deposits traded within the euro area on a cross-border basis increased from 20.6 percent in 1998 to 25.2 percent in 2002. Countries with large domestic markets

[6] In 2002, banks reported large exposures amounting to 79.5 percent of the domestic interbank loans and to 70.1 percent of the foreign interbank loans. They reported 109 large exposures to domestic banks and 226 large exposures to 135 different foreign banks. These exposures account for a total value of €126 billion. The average value of a domestic large exposure (€190 million) is lower than the average value of a foreign large exposure (€467 million).

[7] These figures are in line with EMU averages, although one can observe huge differences between some countries.

Table 1. Structure of Interbank Loans and Deposits of Belgian Banks

Interbank Loans	**Belgium**	**EMU**	**RoW**	**Total**
Demand Loans	603 *0.3%*	1,047 *0.6%*	2,017 *1.1%*	3,667 *2.1%*
Term Loans	10,909 *6.2%*	48,020 *27.2%*	22,816 *12.9%*	81,744 *46.3%*
Secured Loans	10,680 *6.1%*	32,623 *18.5%*	43,844 *24.8%*	87,147 *49.4%*
Other	3,788 *2.1%*	110 *0.1%*	16 *0.0%*	3,914 *2.2%*
Total	25,980 *14.7%*	81,799 *46.4%*	68,692 *38.9%*	176,472 *100.0%*
Interbank Deposits				
Sight Deposits	739 *0.3%*	2,892 *1.3%*	2,868 *1.3%*	6,499 *2.8%*
Term Deposits	16,771 *7.3%*	26,670 *11.7%*	80,927 *35.4%*	124,368 *54.4%*
Secured Deposits	15,308 *6.7%*	46,425 *20.3%*	35,894 *15.7%*	97,627 *42.7%*
Total	32,818 *14.4%*	75,988 *33.3%*	119,688 *52.4%*	228,494 *100.0%*

Source: National Bank of Belgium.
Note: Data are for December 2002, in € million, with percentages shown in italics.

currently exhibit a smaller share of cross-border activity.[8] In that respect, Belgium's high degree of cross-border interbank exposures could provide a good assessment of the future ingredients of national money markets and interbank linkages in other European countries.

[8]Manna (2004) reports that in 2002 the share of cross-border interbank deposits amounted to approximately 15 percent in Finland, France, and Germany; amounted to 30 percent in Italy, the Netherlands, and Spain; and exceeded 50 percent in Belgium and Portugal.

3.2 Methodology

The methodology, based on Upper and Worms (2004), aims at assessing the impact on the Belgian financial system of the sudden and unexpected default of each banking counterpart of Belgian banks. The test of contagion uses a $(N \times (N + M))$ matrix of interbank bilateral exposures, X, to study the propagation mechanisms of crises. The matrix of bilateral exposures summarizes the interbank exposures of Belgian banks toward the other $(N - 1)$ Belgian banks and the M foreign banks:

$$X = \left[\begin{array}{ccccc|ccc} x_{11} & \cdots & x_{1j} & \cdots & x_{1N} & w_{1N+1} & \cdots & w_{1M} \\ \vdots & \ddots & \vdots & \iddots & \vdots & \vdots & & \vdots \\ x_{i1} & \cdots & x_{ij} & \cdots & x_{iN} & \vdots & & \vdots \\ \vdots & \iddots & \vdots & \ddots & \vdots & \vdots & & \vdots \\ x_{N1} & \cdots & x_{Nj} & \cdots & x_{NN} & w_{NN+1} & \cdots & w_{NM} \end{array}\right]$$

with

$$\sum_{j=1}^{N} x_{ij} = a_i; \ \sum_{i=1}^{N} x_{ij} = l_j \quad \text{and} \quad \sum_{j=N+1}^{M} w_{ij} = fa_i,$$

where x_{ij} represents the gross exposure of the Belgian bank i to the Belgian bank j, w_{ij} represents the gross exposure of the Belgian bank i to the foreign bank j, a_i represents the domestic interbank assets of bank i, l_j represents the domestic interbank liabilities of bank j, and fa_i represents the foreign interbank assets of bank i.

The simulations successively study the impact of the failure of each of the N Belgian banks and each of the M foreign banks for a given LGD. The initial failure is assumed to cause an additional failure when the exposure of one bank to failed banks is large enough to offset its tier 1 capital. More specifically, bank i fails subsequently to other failures when

$$C_i - \sum_{j=1}^{N} \lambda_j \theta x_{ij} - \sum_{j=N+1}^{M} \lambda_j \theta w_{ij} < 0,$$

where C_i refers to the tier 1 capital of bank i, θ refers to the LGD, and λ_j is a dummy variable equal to 1 if bank j fails and 0 otherwise. The LGD is assumed to be constant and identical for all failed banks. We assume that in the event of bankruptcy there is no netting, so we use gross exposures x_{ij} and w_{ij} rather than net exposures $(x_{ij} - x_{ji})$. The initial default may cause several successive rounds of failures. The contagion stops when banks that failed during the last round do not cause any additional failures, i.e., when the system is again stable.

The matrix of bilateral exposures is (partly) unknown and, hence, must be inferred. The inference technique (hereafter called *aggregate exposures technique*) is based on the observed aggregates a_i and l_j, which only provide incomplete information on interbank exposures of Belgian banks to Belgian banks—namely, the column and row sums of the matrix X, i.e., the marginal distribution of the x_{ij}. Since this information is partial, we need to make an assumption on the distribution of the individual interbank exposures. Following other papers,[9] we assume that banks seek to maximize the dispersion of their interbank activities.[10] This kind of problem is easily solved with the RAS algorithm.[11] Details on the methodology can be found in Upper and Worms (2004). Since we unfortunately lack the necessary data to apply this methodology to foreign banks, we cannot infer a matrix of international bilateral exposures for Belgian banks. Large exposures are used in this case to estimate the w_{ij}.

Any inference technique, and the general contagion exercise, involves biases—some of which tend toward underestimation and others toward overestimation of contagion risk. The sources of underestimation of contagion risk include the measure of interbank exposures, which is based on interbank loans and deposits only and

[9]See Upper and Worms (2004), Wells (2004), and Elsinger, Lehar, and Summer (2006).

[10]In order to test the robustness of our results, we use two additional techniques. The first one (*large exposures technique*) consists of using the matrix of bilateral exposures based on large exposures only. The second one (*mixed technique*) mixes both approaches by incorporating large exposures in the matrix of bilateral exposures and by using the a_i and l_j, net of large exposures, to calculate the residual, unreported exposures. However, since time series of large exposures are not available, analyses over time are only based on the aggregate exposures technique.

[11]See, e.g., Blien and Graef (1997).

consequently does not include other interbank exposures, such as off-balance-sheet exposures. The distributional assumption of maximum dispersion of banks' interbank exposures also potentially leads to an underestimation of contagion risk, as there are fewer peaks in the distribution[12] (on the other hand, the distributional assumption also creates interbank linkages that do not exist and that are new ways for contagion propagation). Moreover, indirect effects of the failure of foreign banks are not taken into account, since we are unable to measure contagion between foreign banks. Our results may thus suffer from a potential censoring bias. Another source of underestimation is the fact that credit risk is the only source of interbank contagion; liquidity risks are ignored. Furthermore, we use a conservative definition of bank failure, as, in reality, banks may fail before their tier 1 capital is exhausted by interbank losses.[13] Finally, bank panics by depositors are assumed not to occur.[14] On the other hand, since banks are assumed not to be able to refinance or to raise additional capital, we overestimate contagion risk. We also assume that they are not able to anticipate crises and to subsequently reduce their interbank exposures. The absence of safety nets also tends to generate an overestimation bias. Another source of overestimation is the measure of interbank exposures that is on a firm basis and not on a consolidated basis.[15] The extent to which contagion risk will actually be underestimated or overestimated in our simulations will

[12]The distributional assumption also rules out the possibility of having interbank relationship lending. Cocco, Gomes, and Martins (2003) find evidence of lending relationships in the interbank market. Interbank lending relationships could help to mitigate the risk of contagion (as, for instance, monitoring could be more efficient) but could also give rise to very high peaks in the matrix of bilateral exposures.

[13]While the contagion algorithm assumes that a bank only fails once its interbank losses amount to at least its tier 1 capital, there are situations in which a bank may fail before it reaches this threshold. For instance, even small interbank losses could generate additional non-interbank losses (e.g., if the interbank losses trigger a bank run).

[14]Bank panics may occur following an individual bank's failure if depositors make inferences about systemic weakness based on observation of the individual failure (see Aghion, Bolton, and Dewatripont 2000).

[15]Although the use of data at a company level leads to the implicit assumption that cross-border intragroup exposures are between different banks, our actual simulations reveal few cases where such exposures cause "contagion." Assuming away intragroup contagion would be equivalent to making the assumption that the subsidiary will receive assistance from its parent company. However, facing a

obviously depend upon the importance of each of these sources. We deal with some of these potential biases in section 4.2.

Since we want to investigate extreme events, our main indicator of contagion over time is the worst-case scenario (WCS). It is defined as the scenario for which the percentage of total banking assets represented by banks losing their entire tier 1 capital due to contagion is largest. We also provide information on the next-to-worst-case scenario. For brevity, and as the results are in line with the WCS, we do not report the results for two other contagion indicators—i.e., the number of cases of contagion, which measures the likelihood of the occurrence of a contagion effect conditionally to a bank failure, and the number of rounds of contagion, which provides some information on the interbank market structure.[16]

3.3 Structure of the Belgian Interbank Market

Table 2 presents a matrix of bilateral exposures based on the aggregate technique. For presentation purposes, we grouped banks by size in five groups (designated G1–G5). Natural thresholds in the empirical bank-size distribution were used in order to determine groups' composition. G1 comprises the four banks whose assets exceed €99 billion, G2 comprises five banks with assets between €8 and €14 billion, G3 comprises seven banks with assets between €3 and €6 billion, G4 comprises fifteen banks with assets between €1 and €2.6 billion, and G5 comprises thirty-four banks with less than €700 million in assets. Recall that bilateral interbank positions are determined before the grouping procedure. Note also that EMU, RoW, and total interbank rows and columns are directly observed and are thus independent of distributional assumptions.

Most domestic interbank transactions seem to involve large banks. Indeed, positions between G1 banks and other banks exceed by far positions between G2–G5 banks. This structure has not always been prevalent in Belgium. Table 3 shows the evolution over time of the total amount G2–G5 cells can account for. The first row of

large shock, the parent company may not be in a situation in which such a rescue is possible. Therefore, we prefer to treat intragroup exposures similarly to other exposures. Using consolidated data would implicitly rule out the possibility for banking groups to close down an ailing subsidiary.

[16]Results can be found in Degryse and Nguyen (2004).

Table 2. Bilateral Interbank Exposure by Size Categories—December 2002

	% of Assets of Banking System	G1	G2	G3	G4	G5	EMU	RoW	Total Interbank Loans	Foreign Interbank Loans as % of Total Loans
G1	85.10%	15.1	1.0	0.9	2.8	1.1	70.6	64.0	155.4	86.6%
G2	6.80%	2.6	0.1	0.1	0.4	0.1	4.2	2.8	10.4	67.9%
G3	3.50%	2.1	0.1	0.1	0.3	0.1	5.1	0.2	8.0	66.1%
G4	3.40%	2.7	0.1	0.1	0.4	0.2	0.9	1.4	5.7	39.4%
G5	1.30%	1.9	0.1	0.1	0.3	0.1	1.0	0.3	3.8	35.8%
EMU		71.4	3.0	0.6	0.8	0.2				
RoW		111.7	3.3	1.5	2.6	0.6				
Total Interbank Deposits		207.4	7.7	3.4	7.5	2.5				
% of Foreign Interbank Deposits		88.3%	81.5%	61.3%	45.6%	32.4%				

Notes: Data are for December 2002, in € billion, except when expressed as percentages.
Domestic Exposures: Estimates of the matrix of bilateral exposures are based on the aggregate technique, which maximizes the distribution of total interbank loans and deposits.
Banks were grouped by size for expositional purposes. Natural thresholds in the empirical bank-size distribution were used in order to determine groups' composition. G1 comprises the four banks whose assets exceed €99 billion, G2 comprises five banks with assets between €8 and €14 billion, G3 comprises seven banks with assets between €3 and €6 billion, G4 comprises fifteen banks with assets between €1 and €2.6 billion, and G5 comprises thirty-four banks with less than €700 million in assets.
Foreign exposures are based on reported figures.

Table 3. Interbank Share of Nonlarge Banks

	1993:Q2	1994:Q2	1995:Q2	1996:Q2	1997:Q2	1998:Q2	1999:Q2	2000:Q2	2001:Q2	2002:Q2	2002:Q4
Maximum	68.1%	42.4%	48.2%	46.5%	53.6%	40.4%	33.5%	40.0%	40.4%	23.2%	25.8%
Aggregate Exposures Technique	36.4%	30.0%	32.0%	30.7%	35.4%	17.1%	14.6%	20.5%	18.5%	6.1%	8.1%

Note: Figures are for Q2 of each year. “Maximum” and “Aggregate Exposures Technique” represent the percentage of total aggregate exposures of Belgian banks' domestic interbank exposures accounted for by small and medium-sized banks. “Maximum” is based on the minimum of the total interbank loans and total interbank deposits of small and medium-sized banks. “Aggregate Exposures Technique” is computed on the basis of the aggregate exposures technique.

the table shows the maximum amount these cells can represent. This maximum is calculated independently from any distributional assumption. It is defined as the minimum between the sum of domestic interbank deposits of G2–G5 banks (i.e., the sum of the l_j of G2–G5 banks) and the sum of their domestic interbank loans (i.e., the sum of their a_i).[17] The second row of the table presents the calculated G2–G5 total in the aggregate exposures technique. Both series show a downward time trend. In 1993, the structure of the interbank market was similar to a complete structure where estimated exposures between G2–G5 banks represent 36 percent of the domestic market (and could not exceed 68 percent with any alternative distributional assumptions). However, the interbank positions between G2–G5 banks decrease drastically between 1993 and 2002 (it is estimated to 8.1 percent with the aggregate exposures technique and to 10 percent with the mixed technique). So, although we still assume a complete structure,[18] small and medium-sized banks do not seem to have significant exposures to each other in 2002. We observe the same time trend in the maximum. In fact, it mainly reflects the very high concentration of interbank positions involving large banks on both sides of the balance sheet.[19] The evolution over time of the matrix of bilateral exposures thus demonstrates that the aggregate exposures technique is able to capture changes in the market structure, despite the initial assumption of maximum entropy.

Although interbank activities with foreign banks are mainly concentrated in large banks (table 2), access to international interbank markets does not seem to be strictly limited to large banks only. Nevertheless, we observe that the proportion of foreign interbank loans or deposits tends to decrease with bank-size category. Possible explanations are that smaller banks may not reach the critical

[17] By definition, the sum of G2–G5 cells will never exceed the minimum of domestic interbank loans and domestic interbank deposits of these banks. In fact, taking the minimum even constitutes an overestimation of the total G2–G5, as it does not take into account constraints such as a null diagonal.

[18] Assuming a maximum dispersion of interbank activities is similar to assuming a complete structure of claims as described in Allen and Gale (2000).

[19] The concentration on the interbank market increased over the last decade. As far as interbank activities are concerned, the Herfindahl index currently exceeds 0.25, while the market share of the five main players reaches about 90 percent.

size and be internationally less known to tap into the international interbank markets. This would be in line with one of the scenarios presented in Freixas and Holthausen (2005), where large banks with a good international reputation act as correspondent banks for their domestic peers in order to overcome asymmetric information problems.

The few interbank positions between G2–G5 banks, combined with their decreasing share of international financing, suggest that large banks (G1) tend to operate as money centers à la Freixas, Parigi, and Rochet (2000). One important difference in relation to their structure is that several money centers would be linked together, as reflected by the substantial position between the G1 banks.[20] Thus, each large bank tends to function as a money center connected to the other money centers. The Belgian interbank market would thus be characterized by a multiple-money-center structure versus the single money center of Freixas, Parigi, and Rochet (2000).

4. Simulation Results

This section presents the results of the simulations. Section 4.1 discusses the impact of both domestic and foreign contagion in the simulation for 2002:Q4. In section 4.2, the evolution of contagion risk over time is investigated using the algorithm with standard assumptions, such as described in section 3 (4.2.1), but also with additional assumptions aiming at endogenizing the LGD (4.2.2) and aiming at modifying players' behavior as robustness tests (4.2.3).

4.1 Simulations for 2002:Q4: Domestic and Foreign Contagion

Table 4 presents the results of the contagion exercises. Results are reported for five different LGD rates (first column). The second column gives the number of scenarios that generate contagion. The third column presents the median scenario. The median scenario

[20] In unreported data, we find that large banks hold cross-deposits in other large banks.

gives the median value of the percentage of total banking assets represented by banks losing their tier 1 capital, across all of the scenarios where contagion occurs. The two following columns provide information about the state of the banking system in the next-to-worst-case scenario and in the WCS, respectively. For the latter, we display the percentage of assets represented by, and the number of, failing banks and banks losing, respectively, between 100 percent and 70 percent, between 70 percent and 40 percent, between 40 percent and 10 percent, or less than 10 percent of their tier 1 capital. Finally, the last column presents an indicator of risk associated with the "domino" generating the WCS—namely, the rating of the first domino. Since few Belgian banks are publicly listed, neither a rating nor any other market-based indicator is available for a large number of Belgian banks.[21]

Panel A of table 4 reports the result of the simulations for *domestic contagion*, i.e., where contagion is triggered by exposures toward a Belgian bank. In December 2002, there were sixty-five banks incorporated in Belgium, i.e., sixty-five potential sources of domestic contagion. The frequency of domestic contagion occurring is limited. Under the assumption of 100 percent LGD, only four out of these sixty-five banks' defaults cause the failure of at least another Belgian bank. The knock-on effects are also limited. In the median scenarios, the percentages of assets represented by banks losing their tier 1 capital are extremely low. In the WCS, which is always caused by the default of a large bank, simulations show that banks that would lose their tier 1 capital as a result of the interbank defaults never represent more than 3.8 percent of the total assets of Belgian banks. Thus, the default of a Belgian bank in the interbank market does not cause a large Belgian bank to lose its entire tier 1 capital.

[21] Accounting risk measures for Belgian banks, such as the level of tier 1 capital of the domino generating the WCS (as a percentage of its total assets) or the level of its losses for bad loans (as a percentage of its commercial loans), are imperfect measures of risk, as there could be specific reasons, not necessarily linked to risks, justifying special levels for these ratios for a given bank (e.g., a large diversified bank may have a lower capital ratio). Similarly, an apparently sound bank may fail because of fraud, risk concentration, etc. Since presenting such indicators of risk could be misleading, we prefer to present no risk indicator for Belgian banks.

Table 4. Contagion Exercise 2002:Q4

			Next-to-Worst-Case Scenario		Worst-Case Scenario (Excluding First Domino)										
			Banks Failing		Banks Failing		Banks Losing 100%–70% of Tier 1 Capital		Banks Losing 70%–40% of Tier 1 Capital		Banks Losing 40%–10% of Tier 1 Capital		Banks Losing Less Than 10% of Tier 1 Capital		Rating of the First Domino
LGD (%)	Number of Scenarios Where Contagion Occurs	Median Scenario Assuming Contagion. Percentages of Balance Sheet Assets Represented by Failing Banks (Excluding Assets of "First Domino")	% Balance Sheet Assets	# of Banks	% Balance Sheet Assets	# of Banks	% Balance Sheet Assets	# of Banks	% Balance Sheet Assets	# of Banks	% Balance Sheet Assets	# of Banks	% Balance Sheet Assets	# of Banks	
A. Domestic Exposures															
100	4	3.33%	3.75%	15	3.79%	17	0.09%	3	53.67%	17	2.29%	12	40.17%	16	n.a.
80	4	2.13%	3.75%	14	3.75%	15	0.03%	≤2	0.94%	9	55.01%	21	40.27%	18	n.a.
60	4	1.73%	3.04%	9	3.33%	11	0.42%	4	0.12%	5	55.67%	25	40.46%	20	n.a.
40	2	2.98%	2.91%	5	3.04%	9	0.29%	≤2	0.45%	6	55.23%	25	40.98%	23	n.a.
20	2	0.50%	0.50%	3	0.50%	3	0.00%	≤2	2.54%	6	1.31%	12	95.66%	44	n.a.

(continued)

Table 4 (continued). Contagion Exercise 2002:Q4

LGD (%)	Number of Scenarios Where Contagion Occurs	Median Scenario Assuming Contagion. Percentages of Balance Sheet Assets Represented by Failing Banks (Excluding Assets of "First Domino")	Next-to-Worst-Case Scenario		Worst-Case Scenario (Excluding First Domino)										
			Banks Failing		Banks Failing		Banks Losing 100%–70% of Tier 1 Capital		Banks Losing 70%–40% of Tier 1 Capital		Banks Losing 40%–10% of Tier 1 Capital		Banks Losing Less Than 10% of Tier 1 Capital		Rating of the First Domino
			% Balance Sheet Assets	# of Banks	% Balance Sheet Assets	# of Banks	% Balance Sheet Assets	# of Banks	% Balance Sheet Assets	# of Banks	% Balance Sheet Assets	# of Banks	% Balance Sheet Assets	# of Banks	
B. Foreign Exposures															
100	13	0.07%	20.01%	0.07	20.01%	7	0.00%	≤2	1.02%	5	67.36%	8	11.61%	45	AA−
80	9	0.04%	0.29%	≤2	19.97%	6	0.04%	≤2	0.44%	3	32.34%	6	47.21%	49	AA+
60	8	0.04%	0.08%	≤2	18.15%	4	1.82%	≤2	0.04%	≤2	32.78%	9	47.21%	49	AA+
40	3	0.08%	0.08%	≤2	18.08%	≤2	0.04%	≤2	1.89%	4	20.22%	7	59.77%	51	AA+
20	1	0.08%	0.00%	≤2	0.08%	≤2	0.00%	≤2	0.00%	≤2	0.00%	≤2	99.92%	64	A

Note: The table presents the results of the contagion exercises for December 2002. Results are reported for five different LGDs. The second column gives the number of scenarios that generate contagion. The third column presents the median scenario. The median scenario gives the median value, across all of the scenarios where contagion occurs, of the percentage of total banking assets represented by banks losing their tier 1 capital. The remaining columns provide some statistics on the state of the banking system in the next-to-worst-case and worst-case scenarios. The table presents the percentage of assets represented by, and the number of failing banks and banks losing, respectively, between 100% and 70%, between 70% and 40%, between 40% and 10%, or less than 10% of their tier 1 capital. Cells with two or fewer banks are marked "≤ 2" in order to make single bank identification impossible. The last column presents the rating of the first domino in the WCS. Panel A presents the results of the contagion exercises assuming that the first defaulter is a Belgian bank. It is based on a matrix of bilateral exposures estimated with the aggregate exposures technique. Each line is based on 65 different scenarios (i.e., the individual failure of each of the sixty-five Belgian banks). Panel B, simulating the default of a foreign bank, is based on a matrix of bilateral exposure estimated with the large exposures technique. It is based on 135 different scenarios.

In addition, losses decrease in parallel with the LGD.[22] Although the losses in the next-to-worst-case scenario are lower, they remain very close to the WCS outcome.

As results are very similar to the results of the aggregate exposures technique, and for the sake of brevity, we do not report the results based on the large exposures and on the mixed techniques.[23] This comparability across techniques validates our use of the aggregate exposures technique for the estimation of contagion risk over time.[24]

Panel B of table 4 displays the results for *foreign contagion.* We identify 135 foreign banking counterparts for Belgian banks. For a 100 percent LGD, the default of 1 large foreign bank can lead to

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[22] The statistical estimation of an LGD for Belgian banks is very difficult, since fortunately very few Belgian banks have failed in the last decades. Moreover, actual losses on a defaulting bank can prove very complicated to calculate, since they depend on the time horizon chosen. Altman and Kishore (1996) estimate average recovery rates on defaulting bonds of financial institutions (for the period 1978–95) to be about 36 percent. However, recovery rates vary by type of institution, from 68 percent for mortgage banks to 9 percent for savings institutions. Moreover, the LGD for bonds is probably very different from the LGD for comparable loans (which in our case comprise secured and unsecured assets). James (1991) estimates that losses average 30 percent of the failed bank's assets and that the direct expenses associated with bank closures average 10 percent of assets, making a total of about 40 percent. Seeing that more than 50 percent of interbank loans granted by Belgian banks are secured, it may therefore be realistic to assume a recovery rate of somewhere between 60 and 80 percent (i.e., an LGD between 40 and 20 percent). On the other hand, as domino effects may be considered instantaneous, one could also argue that the time pattern of recovery does not matter and that an LGD of 100 percent should be used to simulate liquidity shocks. Yet the time pattern of recovery may matter, depending on the maturity of the liabilities.

[23] They are, however, available on request.

[24] In a recent paper, Mistrulli (2005) compares the estimated and observed large exposures techniques for Italy and finds that they may differ depending on the level of LGD. In particular, he finds that the observed bilateral exposures generate higher contagion (as a share of total assets) for low LGDs, whereas the opposite holds for large LGDs. In general, it is unclear whether using estimated bilateral exposures leads to overestimation or underestimation. This should, however, not influence our results under the assumption that the potential biases remain constant over time. Note in addition that van Lelyveld and Liedorp (2006) conclude that "the entropy estimation using large exposure data as applied in many previous papers gives an adequate approximation of the actual linkages between banks. Hence this methodology does not seem to introduce a bias." However, they also find that entropy maximization leads to an overestimation of contagion risk.

the failure of 7 Belgian banks whose assets account for 20 percent of total Belgian bank assets. These numbers are considerably higher than those of our simulations with Belgian banks as first domino. The results for the WCS also indicate that, even for an LGD of 40 percent, the default of a foreign bank can have a significant impact on Belgian banks. Note, however, that large differences exist between the median and the worst-case scenarios. For an LGD of 100 percent, only three of the thirteen simulations that involved contagion entailed the failure of banks representing at least 10 percent of the total assets of the Belgian banking system. The next-to-worst-case scenario shows that, for reasonable LGD, contagion is not likely.[25] In addition, all of the foreign banks representing the first domino in the WCS are European banks, and all rank as investment grade, which suggests that actual interbank defaults by these banks, although possible, are not frequent.

Our contagion analysis cannot incorporate indirect effects of the failure of foreign banks, which may be important (i.e., failure of other foreign banks caused by the failure of a given foreign bank). One way to proxy for indirect effects is to simulate the impact of the combined default of several foreign banks coming from the same country. Belgian banks provide a breakdown of their aggregate interbank exposures (the fa_i) by EU countries. The data are available for the last five years. We make the assumption that x percent of the interbank exposures of Belgian banks to banks in a particular EU country are unrecoverable. We use the propagation mechanism explained earlier to measure the impact on the Belgian system. Unreported results show that with the exception of France, the Netherlands, and the United Kingdom, simulations involving defaults on other countries' interbank loans (including Germany and Luxembourg) do not result in significant contagion in the Belgian banking sector at the end of 2002.[26] For example, for an LGD of

[25] This finding was confirmed afterward by van Lelyveld and Liedorp (2006), who found that below an LGD of 75 percent, domestic and foreign contagion was unlikely.

[26] Although the results are quite stable over 1999–2002 (the period over which data are available), with France and the United Kingdom often representing major risks, other neighboring countries sometimes show a higher potential for contagion. These jumps in simulated country impact probably reflect larger interbank positions with those countries. We do not observe any significant increase

100 percent, a simulation of the failure of all German banks shows that Belgian banks losing their entire tier 1 capital represent less than 1 percent of total Belgian bank assets. When we use a lower LGD, only bank defaults in the United Kingdom would yield significant levels of contagion in Belgium. This in fact reflects the United Kingdom's role as a money center and the importance of UK banks as counterparts of Belgian and other European banks. Manna (2004) finds that London is indeed an important nexus for all EMU banks, as UK banks account for more than one-third of their cross-border interbank deposits.

4.2 Evolution over Time: Simulations of Domestic Contagion Based on Aggregate Exposures

4.2.1 Baseline Case

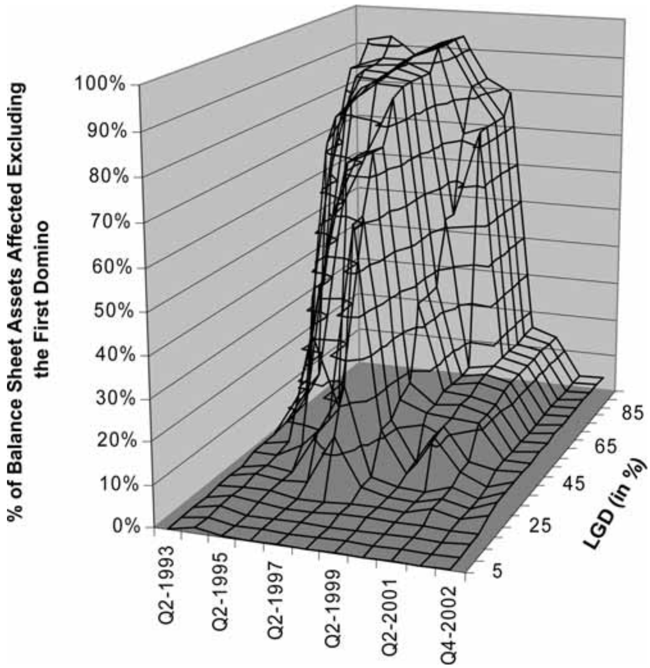
The simulations used to study the evolution of the domestic contagion risk over time cover the period 1992:Q4–2002:Q4. Figure 1 shows the behavior of the WCS over the period 1993–2002 for twenty different LGDs ranging from 5 percent to 100 percent, in steps of 5 percent. Thus, for each quarter, the number of scenarios tested amounts to twenty times the number of banks (between 65 and 112).[27]

Figure 1 shows that, over the last decade, the WCS has been subject to three major evolutions. Between 1993 and 1997, the WCS consistently worsens. Between 1997 and 1999, the WCS affects less of Belgian banking assets; i.e., there is a steep decrease between 1997 and 1999. Finally, between 1999 and 2002, the surface flattens and contagion remains limited, even with high LGDs. Thus, the degree of contagion generated in simulations with data for the last quarter of 2002 appears to be at a record low. These trends are particularly striking for an LGD of 60 percent. In this case, the percentage of total banking assets affected by contagion, excluding the first

in the cross-border contagion risk over 1999–2002. However, such an increase may have taken place earlier, in years in which internationalization of interbank exposures of Belgian banks substantially increased.

[27]For presentation purposes, figure 1 presents the results for Q2 only. Tests reported in subsection 5 show that the trends in the WCS presented in figure 1 are not sensitive to the quarter chosen.

Figure 1. Contagion Effect—Worst-Case Scenario: 1993–2002



Note: The graph presents the evolution of the worst-case scenario for twenty different LGDs over time, from 1993:Q2 to 2002:Q4. LGDs are shown as percentages. The results are based on contagion exercises using matrices of bilateral exposures estimated with the aggregate exposures technique.

domino, varies over the period from 86 percent to 3 percent. We also find that (i) the next-to-worst-case scenario is affected by the same structural changes as the WCS and (ii) the level of the next-to-worst-case scenario is similar to the level of the WCS (unreported).[28]

The WCS is frequently—but not always—generated by the default of a large bank. Actually, an analysis of banks initiating

[28]A potential concern is that the WCS is initiated by sound banks in a particular period and not-very-sound ones in other periods. We find that the bank triggering the WCS persistently belongs to the lowest quartile in terms of capitalization (unreported). This suggests that if the capital ratio is a good proxy for the likelihood of failure, the likelihood of failure of the bank initiating the WCS remained quite constant over time.

the WCS shows that different banks cause the WCS in different years, although some banks tend to do so more often than others. For instance, large banks generate the WCS more often than small banks. In addition, banks initiating the WCS are not only different from year to year but also, to a certain extent, within a given year, depending on the applied LGD. We also find that the default of a large bank is always directly preceded either by the default of another large bank or by the default of a medium-sized bank. Indeed, the tier 1 capital of large banks is never totally absorbed by the combined default of several small banks. However, the default of a small bank may trigger the failure of several small and medium-sized banks and, in turn, of a large bank. Note also that in some cases, no large bank fails, even in the WCS.[29]

The results on domestic contagion suggest that contagion risk in Belgium has evolved over time. Any attempt to compare our results with the results of simulations for other countries must therefore take the time dimension into consideration. A comparison with studies using the same methodology indicates that the simulated failure of a Belgian bank in December 1998 produced smaller contagion effects than the simulated failure of a German bank in the same period, at least for a high LGD (Upper and Worms 2004). Results for the United Kingdom (Wells 2004), which uses data for end 2000, show that the Belgian simulations produced a greater impact of contagion at the same time period. However, contagion occurred in a higher proportion of cases in the United Kingdom. Finally, the simulated impact of contagion for 2002:Q4 is similar in Belgium and in the Netherlands (van Lelyveld and Liedorp 2006). As results for Belgium are broadly similar to results for Germany, the Netherlands, and the United Kingdom at similar time periods, it is impossible to know whether dissimilarities between Germany, the Netherlands, and the United Kingdom are due to structural differences or to general time trends affecting several European countries. We investigate the determinants of domestic contagion in Belgium in section 5.

[29] Since the WCS is a very extreme outcome, we investigate, in unreported exercises, other measures of contagion, i.e., the variations in the percentage of banks intitiating contagion and the propagation mechanisms of contagion (number of rounds). The evolution over time of these indicators is similar to the evolution of the WCS.

4.2.2 Endogenous LGD

Our baseline simulations assume a fixed LGD for all banks. It is not obvious a priori that endogenizing the LGD would deliver additional results, especially as we already test very extreme LGD, ranging from 100 percent to 5 percent. Surprisingly, however, there are some indications that it may do so.

We take two complementary steps to endogenize the LGD. In both steps, the core of the endogenization process is that the LGD of a given bank depends upon the LGD of all the other banks to which it is linked. In a first step, we endogenize the LGD on interbank claims only and apply an exogenous LGD on other "remaining assets."[30] In a second step, we add some admittedly ad hoc assumptions to endogenize the recovery rate on the other remaining assets as well. The LGD is calculated for failing banks only, at each round of the algorithm, in order to assess the value of their remaining assets. The calculation of the LGD for a given bank does not determine whether a bank is bankrupted. It only assesses the value of its assets once it has been declared bankrupted, i.e., the value of the bankrupted bank for its creditors.

We start with the endogenization of the LGD on interbank claims. The LGD on interbank claims of bank i is defined as

$$\theta_i = \left[\frac{\sum_j(\theta_j x_{ij}) + \textit{remaining assets} * \textit{LGDs remaining assets}}{\textit{Total Assets} - \textit{shareholders' equity}}\right],$$

where θ_i is the LGD of bank i, x_{ij} is the gross interbank exposure of bank i to bank j, *remaining assets* represents all the other remaining assets of bank i, and *LGDs remaining assets* stands for the loss rate that bank i has to bear on its assets because of its default. Solving the system of equations for all failed banks simultaneously gives a different endogenous LGD for each failed bank.

We first distinguish between liquid and illiquid assets (partial endogenization). We assume a 0 percent LGD on liquid assets. We simulate different LGDs on the remaining illiquid assets. All simulations assume a 60 percent LGD on the first domino. Results are

[30] Note that by LGD on remaining assets, we mean the loss given the default of the bank to which these assets belong. The assets themselves, however, have not defaulted.

reported in panel A.1. of table 5. We present the results for two polar cases in which the LGD on illiquid assets is, respectively, equal to 100 percent and 0 percent, as well as for an intermediate LGD of 60 percent. The latter can be compared to the baseline simulation, which is based on a fixed 60 percent LGD. For each assumed LGD on illiquid assets, the first line presents the WCS, while the second line gives the average implied LGD in the WCS.

We conclude two things from the simulations. Firstly, although the level of and changes in the WCS are broadly similar to the results of simulations that assume a fixed LGD for all assets, the average implied LGD varies substantially within a given year. For instance, in 2002, the minimum LGD—assuming a 100 percent LGD on illiquid assets—was 8.8 percent, while its maximum was 76.5 percent. Thus, endogenizing only the LGD on interbank exposures already suffices to introduce a large heterogeneity between banks, even though it does not affect the general trends. Secondly, although the average implied LGD varies over time, we do not observe a strict correlation between the LGD and the WCS. Because the LGD interacts with other dimensions of the market structure, a higher average LGD does not necessarily generate a higher WCS.

Next, we try to endogenize the LGD on the "remaining assets" as well (labeled "Complete Endogenization"). Besides interbank loans, we distinguish five categories of assets:

(i) Liquid Assets: We assume a 100 percent recovery rate.

(ii) Customer Loans: We assume that the loss rate on a bank's loan portfolio is equal to the average residual maturity of its loan portfolio times its annual loan-loss provisions (as a percentage of its total loans). This amounts to 4 percent on average in 2002. The minimum is equal to 0 percent and the maximum to 35 percent.[31]

[31] As this is a broad measure of the expected losses of the loan portfolio in a going concern, it does not take into account losses resulting from the loss of information that could arise when the loan portfolio is sold. In an unreported test, we assume that the loss on the portfolio is an average between 30 percent and our estimates. The average LGD of commercial loans is indeed approximately 30 percent (see, e.g., Bank for International Settlements 2005). Remember, however, that in our simulations, loans are not in default. This does not qualitatively affect the results.

Table 5. Alternative Scenarios

	1993	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**
Baseline	3.3%	14.1%	58.5%	73.0%	86.4%	35.7%	13.6%	13.2%	11.5%	2.9%
A. Endogenous LGD										
A.1. Partial Endogenization										
LGD Illiquid 100%	3.8%	76.8%	85.9%	90.9%	91.9%	52.0%	13.6%	14.9%	13.3%	2.9%
Average LGD	*64.8%*	*82.5%*	*85.9%*	*85.0%*	*90.3%*	*86.6%*	*71.0%*	*78.5%*	*76.5%*	*59.3%*
LGD Illiquid 60%	2.5%	13.1%	54.3%	72.9%	75.6%	17.1%	12.4%	13.2%	11.5%	2.8%
Average LGD	*33.5%*	*26.5%*	*27.3%*	*37.9%*	*49.7%*	*43.9%*	*38.1%*	*43.9%*	*38.6%*	*41.0%*
LGD Illiquid 0%	1.9%	1.8%	8.5%	4.5%	10.7%	9.0%	11.6%	9.5%	10.8%	2.8%
Average LGD	*16.2%*	*20.2%*	*9.8%*	*10.0%*	*13.5%*	*14.4%*	*9.0%*	*10.1%*	*9.1%*	*8.7%*
A.2. Complete Endogenization										
LGD First Domino 100%	12.2%	12.5%	32.1%	29.4%	18.7%	15.3%	12.0%	29.1%	12.5%	0.8%
Average LGD	*19.2%*	*19.8%*	*22.1%*	*18.7%*	*20.1%*	*20.2%*	*19.6%*	*21.5%*	*20.4%*	*16.9%*
LGD First Domino 60%	6.2%	6.9%	13.5%	11.6%	17.0%	14.2%	10.4%	9.6%	10.8%	0.6%
Average LGD	*18.4%*	*16.3%*	*18.6%*	*16.7%*	*17.9%*	*18.2%*	*18.2%*	*18.6%*	*18.7%*	*17.9%*
LGD First Domino 5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Average LGD	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*

(continued)

Table 5 (continued). Alternative Scenarios

	1993	**1994**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**
Baseline	3.3%	14.1%	58.5%	73.0%	86.4%	35.7%	13.6%	13.2%	11.5%	2.9%
B. Players' Behavior										
Anticipation	NA	1.9%	29.6%	11.3%	55.9%	8.9%	3.1%	9.7%	8.6%	0.4%
TBTF	3.3%	4.2%	41.2%	10.2%	55.0%	2.4%	3.9%	2.4%	4.1%	0.0%
Coordination	3.3%	14.1%	58.5%	0.0%	0.0%	0.0%	13.6%	13.2%	11.5%	2.9%
Netting	0.7%	2.0%	1.1%	0.6%	0.6%	0.6%	3.3%	1.0%	4.1%	2.8%
C. Correlated Shocks										
Macro Shock	3.7%	75.9%	84.9%	88.6%	87.8%	35.9%	13.6%	14.9%	13.3%	2.9%
Two Failures	31.6%	72.5%	83.1%	84.8%	88.3%	36.1%	30.9%	14.9%	13.6%	3.4%
Three Failures	45.0%	74.0%	83.4%	85.3%	88.5%	55.7%	49.2%	14.9%	13.6%	3.7%
Four Failures	45.4%	75.6%	85.1%	85.3%	88.9%	70.3%	49.2%	14.9%	13.6%	3.7%

Note: Figures are for Q2 of each year. The table presents the evolution of the WCS under alternative assumptions. In the baseline case, a fixed LGD of 60 percent is assumed. Panel A presents results of simulation when the LGD is endogenous. For each assumption, we present both the WCS in the first line and the average implied LGD in the second line. In panel B, assumptions relating to banks' anticipation, too-big-to-fail mechanisms, coordination mechanisms, and netting are modified. In panel C, macro shocks are simulated.

(iii) Government Bonds: Similarly to Cifuentes, Ferrucci, and Shin (2005), we assume that failed banks liquidate their government bonds portfolio and that their value is inversely proportional to the supply of government bonds in the market. We apply a haircut of 1 percent on the portfolio each time the cumulated sales of government bonds by failed banks amount to 10 percent of the market, defined as the sum of government bonds held by Belgian banks. To ensure comparability, we assume that banks do not mark to market their bonds portfolio so that sound banks are not affected by this decrease. Hence, in our model, government bonds do not constitute an additional direct contagion channel.

(iv) Intangible Assets: We assume an LGD of 100 percent on intangible assets.

(v) Other Assets: We apply an arbitrary LGD of 30 percent on all the remaining assets. The latter is based on James (1991), who finds that loss on assets of failed banks amounts to 30 percent on average. These assets represent, on average, 17 percent of total assets in 2002.

In addition, we apply a fixed cost of bankruptcy amounting to 10 percent of total assets (see James 1991). We also take into account two kinds of privileged creditors—namely, the state and the employees of the bank. By subtracting claims of the latter from both the numerator and the denominator of the LGD ratio, we make the assumption that they are first served in the liquidation process. This increases the LGD applied on interbank claims. As we do not have any other information regarding the seniority of the remaining claims, we assume that the proceeds of the liquidation are shared proportionally.

In panel A.2. of table 5, we present the results using three different levels for the exogenous LGD applied to the first domino. Endogenizing the LGD decreases substantially the level of contagion. Yet, we still observe the same trend over time, with very low contagion indicators in 2002. The average implied LGD amounts to 19 percent. As indicated in section 4.1, this seems to be reasonable, although maybe conservative. A striking result is that the simulations implying an average endogenous LGD of around 20 percent result in more contagion than those with a fixed LGD of 20 percent, in which contagion was inexistent. This is partly due to two effects. Firstly, in two out of the three cases, the LGD of the first domino is assumed to be higher than 20 percent. Secondly, precisely because the LGD is endogenous, we observe heterogeneous LGD with sometimes high levels of losses, helping to propagate contagion. Thus, it is likely that the propagation of contagion is not only determined by the pattern of links defining the market structure but also by the relative strength of each counterpart to which banks are linked.

4.2.3 Robustness

In this subsection, we present some additional robustness checks related to the *behavior of market participants and market rules* and possible *correlated shocks*. The first one relates to *banks' expectations*. Banks may be able to (partly) *anticipate* a bank failure. In the simulation, we assume that banks are able to withdraw the short-term loans granted to all failed banks before the failure occurs. The residual maturity of more than 35 percent of interbank loans granted by Belgian banks at the end of 2002 does not exceed eight days. As we have information on each bank's aggregate short-term bilateral positions only, we assume that the maturity structure of interbank loans granted to each counterpart of a given bank is the same. The results are displayed in the first row of panel B of table 5. Although the WCS is lower than in the baseline case, its evolution over time remains very similar to the evolution of the WCS in the baseline case.

A second assumption relates to the potential presence of a *safety net*. Although interbank loans are not covered by explicit deposit insurance, issues like being "too big to fail" (TBTF) may introduce implicit deposit insurance. To proxy for this possibility, we assume that large Belgian banks would not be allowed to fail.[32] These banks would thus not create initial and additional contagion and could

[32]We define large banks as banks representing more than 10 percent of the total assets of Belgian banks. The TBTF policy is a working assumption made by the authors in order to test the sensitivity of the results. There is absolutely no certainty regarding the effective application of such a threshold or such a policy in case of a large bank failure.

even stop it. The results are displayed in the second row of panel B. Unsurprisingly, a TBTF policy reduces the WCS. Remarkably, however, our simulations indicate that contagion still propagates in 1995 and 1997, despite the safety net. In these two years, contagion effects are caused, in the first instance, by the successive failure of many small and medium-sized banks.

In the baseline simulations, banks do not have the opportunity to *coordinate* in order to avoid liquidation. Leitner (2004) develops a model in which liquid banks bail out illiquid banks because of the threat of contagion. To capture coordination, we will assume that banks may "merge" to avoid failure.[33] An important objection to this procedure is that, in reality, mergers are not observed at such short notice. However, one can view these "mergers" as alternatives to the bailout in Leitner (2004). We address coordination by starting from the WCS in the baseline case. We assume that neither banks nor the regulator know the full matrix of bilateral exposures. Banks only know their direct counterparts. After the initial shock on the first domino, banks observe their losses. At that moment, we assume that banks have time to start "merger discussions" with other banks in order to avoid liquidation.[34] Other banks will accept such a merger if, thanks to this operation, they avoid their own failure. As the matrix of bilateral exposures is unknown to participants, we assume that mergers are only possible between banks failing in the "second round" (subsequently to the first domino) and their direct counterparts, i.e., banks failing in the "third round."

[33] Suppose there are three banks: bank A, bank B (with a tier 1 capital of 2.4 and an exposure of 5 to bank A), and bank C (with a tier 1 capital of 1.9 and an exposure of 4 to bank B). The first domino is bank A. Merging bank B and bank C would give a bank with an exposure of 5 to bank A and a capital of 4.3. Assuming a 100 percent LGD, the failure of bank A triggers the failure of bank B, and the failure of bank B triggers the failure of bank C. Merging both banks would not have an impact on contagion, as the new bank would not be resilient to a loss of 5. With a 50 percent LGD, the failure of bank A triggers the failure of bank B and indirectly of bank C if banks do not coordinate. The merged entity, however, would be able to resist to a shock of 2.5, as it would present a tier 1 capital of 4.3. In this case, merging both banks is optimal, as it allows avoiding domino effects.

[34] Such a period could be due, for instance, to a lag between the failure of the first domino and the realization of losses, due to an arbitrary decision of the regulator, or due to bankruptcy procedures such as chapter 11.

We simulate the consequences of each possible merger involving one or more banks that would have failed in the second round and one or more banks that would have failed in the third round.[35] We identify the merger that minimizes the assets of the failing banks. The third row of panel B of table 5 presents the results assuming coordination. We observe that in some cases coordination would prevent contagion from taking place. Successful mergers involve relatively small banks, as the implied increase in the Herfindahl index never exceeds 58 points. This happens exactly in periods when the WCS affected a large proportion of total banking assets and when contagion was slow to propagate, affecting firstly small banks (i.e., 1996–98).

The baseline simulations started from a matrix of gross bilateral exposures. To the extent that legislation allows for bilateral setoff—*netting*—of interbank positions,[36] we performed contagion simulations based on "netted" matrices of domestic bilateral exposures (i.e., $x_{ij} - x_{ji}$). These simulations assume that all the interbank claims are covered by bilateral netting agreements. The results are displayed in the fourth row of panel B of table 5. Netting substantially reduces contagion toward very low levels and this for all years.[37] Furthermore, the WCS assuming netting becomes flat over the entire period 1992–2002, in contrast to the baseline case. However, our distributional assumption may partly drive the results, as

[35]The total number of mergers involving at least one bank that failed in the second round and one bank that failed in the third round is equal to the number of possible combinations of banks that failed in the second round times the number of possible combinations of banks that failed in the third round. For instance, in 1997, in the WCS, two banks fail in the second round and one in the third round. In total, there are thus three different possible mergers. The total number of potential mergers ranges from 3 in 1997 to 65,025 in 1999.

[36]The European Directive 2002/47/EC on financial collateral arrangements obliges all EU member states to recognize closeout netting arrangements. In the Belgian law, netting arrangements are accepted provided they have been concluded before the opening of the insolvency procedure. In case of bankruptcy, a claim that is not protected by a netting agreement is generally treated as a normal claim and is reimbursed, proportionally to the value of recovered assets, after privileged creditors have been served.

[37]Note, however, that netting may also present some drawbacks. For instance, Emmons (1995) shows that netting of interbank claims shifts the bank default risk away from interbank claimants toward nonbank creditors; i.e., the risk is transferred to the banks' creditors who are not included in the netting agreement.

we assume a complete matrix of bilateral exposures. In other words, we assume that each bank is both debtor and creditor of all the other Belgian banks. Bilateral netting with a given bank becomes effective once this bank is both debtor and creditor, which in practice may represent a limited number of cases.

While our baseline simulations assumed idiosyncratic initial shocks, the initial shock could also be common to several banks or the whole banking system. We address the impact of correlated shocks in two complementary ways. First, we simulate a macro shock in combination with an idiosyncratic shock. In order to simulate a macro shock, we assume that each bank loses 10 percent of its tier 1 capital. The results are displayed in the first row of panel C, table 5. The WCS remains relatively similar over the entire 1993–2002 period. Second, we simulate the consequences of multiple simultaneous failures (two, three, or four banks). The WCS results of each possible joint default of two, three, or four banks are shown in rows 2–4 of panel B. Although allowing for multiple failures increases the level of the WCS, its level remains very low in 2002.

5. Interbank Market Structure and Domestic Contagion

Because nearly all our contagion indicators tend to follow a regular time pattern, they are more likely to be caused by trends in the organization of the interbank market than by exceptional events. For instance, although the Russian crisis as well as the LTCM failure could have influenced the pattern of contagion in 1997, it is difficult to ascribe the whole evolution of contagion indicators over time to these two events. Rather, the combination of two main trends in the banking landscape could explain the changes in our simulation results over the period 1993–2002. First, the estimated matrix of bilateral exposures went through some structural changes. As described earlier, large banks now seem to show an increased tendency to operate as multiple money centers. Freixas, Parigi, and Rochet (2000) show that, for certain parameter values, a single-money-center structure could reduce the contagion risk, as banks at the periphery no longer trigger contagion. A multiple-money-center structure will also reduce contagion provided the exposures between banks at the center are such that they do not propagate contagion.[38] Second, following consolidation and international financial integration, (large) Belgian banks have further increased their cross-border interbank exposures.[39] Consequently, the bilateral interbank exposures between the large Belgian banks could be such that they would no longer propagate contagion.

In order to test for the respective impact of interbank market structure and internationalization on contagion risk, we estimate OLS regression models of the form

$$WCS_t = \beta_0 + \beta_1 LB_t + \beta_2 DOM_t + \beta_3 CAPDUMMY_t * LB_t + \beta_4 CAP_t + \sum_{i=5}^{9} \beta_i \, Control\ variable_{it} + u_t$$

for WCS, calculated employing several LGDs and using quarterly data from 1992:Q4 to 2002:Q4.[40]

Table 6 provides definitions and descriptive statistics for the variables employed in the regression.[41] *LB* captures the interbank market structure. It measures the domestic interbank exposures of large banks as a fraction of the total domestic exposures. In a money center, *LB* should be equal to 1 since small banks are not linked together and all interbank transactions transit through the money center. In a complete structure, we expect *LB* to be smaller, as small banks have

[38] Of course, the reverse causation, although unlikely, cannot be entirely ruled out. Banks may adopt a given market structure in reaction to a perceived increase in the risk of contagion. However, we believe that the interbank market structure is determined by business rationales rather than by systemic concerns.

[39] Although the share of international interbank loans has always been high for large banks, it has increased over the last decade. In December 1992, the interbank loans granted by large Belgian banks to foreign banks accounted for 79 percent of total interbank loans. This proportion reached 89 percent at the end of 2002.

[40] In order to isolate contagion from other simulated effects, we use the baseline WCS.

[41] For each variable, we performed Phillips-Perron tests to test for unit roots. The series appear stationary. We can reject the hypothesis of a unit root at a 10 percent level for all the dependent and explanatory variables, with exception of the WCS for an LGD of 80 percent and 60 percent, and *DOM* and *CAP*. Although we cannot formally reject the null hypothesis of unit roots for these series, there is a strong economic rationale to reject it, as they are, by construction, constrained between 0 and 1.

direct links. *DOM* is a proxy for the degree of internationalization and is defined as the total domestic interbank exposure of Belgian banks as a fraction of their total interbank exposures. A ratio equal to 1 would represent a "closed" system, relying only on the domestic interbank market. A ratio equal to 0 would represent a fully internationalized system. Our regression analysis also contains some control variables. Two variables are included to control for bank-capital cyclical patterns. *CAPdummy* aims at identifying periods in which large banks are well capitalized. It is a dummy variable equal to 1

Table 6. Definition of Explanatory Variables

Variable	Definition	Rationale	Min	Max	Median
Dependent Variables					
WCS100	Worst-case scenario assuming a fixed LGD of 100%		0.033	0.964	0.874
WCS80	Worst-case scenario assuming a fixed LGD of 80%		0.032	0.931	0.747
WCS60	Worst-case scenario assuming a fixed LGD of 60%		0.009	0.918	0.158
Variables Capturing the Hypotheses					
LB	Domestic interbank exposures of/to large banks as a percentage of the total domestic exposures	Proxies for the type of interbank market structure. In a money center, this ratio should be equal to 1 since small banks are not linked together. To the extent that the structure moves to a complete structure, this ratio decreases.	0.636	0.941	0.700
DOM	Domestic interbank exposures as a percentage of the total interbank exposures	This ratio indicates the level of internationalization of interbank positions. A ratio equal to 1 would represent a "closed" system relying only on the domestic interbank market. A ratio equal to 0 would represent a fully internationalized system.	0.147	0.373	0.297

(continued)

Table 6 (continued). Definition of Explanatory Variables

Variable	Definition	Rationale	Min	Max	Median
Variables Capturing Other Structural Changes					
CAP	Nonweighted average of the ratio tier 1 capital of Belgian banks on assets of Belgian banks	A higher capitalization of banks should increase their resiliency to shocks and decrease indicators of contagion.	0.075	0.109	0.089
CAPdummy	Dummy variable equal to 1 when the tier 1 capital ratio of large banks exceeds its long-term average and 0 otherwise	Used in combination with LB as an interaction variable measuring to what extent the money centers need to be well capitalized to reduce contagion.	0	1	
Variables Capturing Macroeconomic Evolution					
GDP	Quarterly GDP growth	Banks' profits should increase when the GDP growth is high, as the quality of their assets improves.	−0.041	0.058	0.017
INT	Term spread of the interbank interest rate (Bibor before 1999 and Euribor from 1999 onward)	The term spread of the interbank interest rate represents the difference between the one-year and the one-month interbank interest rate. A high spread will constitute a positive environment for banks whose interbank liabilities are short term and whose interbank assets are long term (which is, to a certain extent, the position of Belgian banks). A low spread, on the other hand, will constitute a negative environment for these banks.	−0.016	0.019	−0.002
Other Control Variables					
Q2, Q3, Q4	Dummy variables identifying quarters	Control for seasonal effects			

Note: The table presents the variables used in the regression analysis. The first column gives the name of the variable, the second column gives its definition, and the third column gives the rationale for including each of the variables in the analysis. The remaining three columns give, respectively, the minimum, maximum, and median value over the observation period.

when the average tier 1 capital ratio of large banks exceeds the long-term average of the ratio and 0 otherwise. *CAPdummy* is used in interaction with *LB*. This interaction variable captures the extent to which a change in the structure, combined with a higher capitalization of money centers, effectively reduces contagion. We also control for the leverage of banks (*CAP*). In addition, we control for the macroeconomic environment with the GDP growth rate (*GDP*) and the term spread of the interbank interest rate (*INT*), defined as the spread between the one-year and the one-month interbank interest rate.[42] Finally, we also introduce quarterly dummies (*Q2, Q3, Q4*) to control for potential seasonal effects.

Table 7 displays the results of our regression analysis. The three panels report the results for the levels of LGD at 100 percent, 80 percent, and 60 percent, respectively.[43] For each LGD, *LB* and *DOM* are significantly different from 0, and both have the expected sign. That is, a move toward a money-center structure (an increase in *LB*) and a higher internationalization (decrease in *DOM*) reduces the WCS. For example, a 10 percent increase in *LB* would lead to a decrease of 23 percent, 29 percent, and 14 percent of the WCS for the 100 percent, 80 percent, and 60 percent LGD, respectively. Similarly, a 10 percent decrease in *DOM* would lead to a decrease in the WCS of 38 percent, 41 percent, and 23 percent for the 100 percent, 80 percent, and 60 percent LGD, respectively. However, in some regressions, coefficients of *LB* and *DOM* are not significant when entered jointly, pointing to potential multicollinearity problems.[44] Mistrulli (2005) uses simulations keeping the tier 1 capital-to-asset

[42]We control for macroeconomic conditions, as they might affect the ability/willingness to take or grant interbank loans and might influence the behavior of interbank players.

[43]The results using the 40 percent LGD and the 20 percent LGD are less significant. This is not too surprising, as changes over time in the WCS are much more important for an LGD of 100 percent than for an LGD of 20 percent, where little or no contagion at all is observed.

[44]The correlation between the variables *LB* and *DOM* is -0.76. This high negative correlation is not too surprising. Indeed, an increase in *LB* goes together with an increase in concentration as large banks become more important. In small countries, a higher concentration may lead to a higher degree of internationalization. Technically, the relatively high correlation might prevent us from obtaining statistically significant results when including these variables jointly in a regression framework.

Table 7. Regression Results for the Worst-Case Scenario

Intercept	***LB***	***DOM***	***LB* CAPdummy***	***CAP***	***GDP***	***INT***	R^2	***DW***
LGD 100%								
2.69 (6.66)***	−2.28 (−4.69)***			−3.41 (−0.58)	2.03 (0.87)	0.88 (0.10)	0.61	1.26
1.65 (5.91)***	−0.97 (−2.85)**		−0.58 (−7.61)***	−0.83 (−0.23)	−3.25 (−0.63)	−0.22 (−0.15)	0.86	2.82
−0.01 (−0.01)		3.80 (6.82)***		−4.26 (−0.96)	6.06 (0.92)	1.74 (0.89)	0.73	1.35
0.56 (0.96)	−0.89 (−1.76)*	3.02 (4.33)***		−0.69 (−0.15)	1.59 (0.84)	1.79 (0.26)	0.76	1.62
1.23 (2.69)**	−0.78 (−2.05)**	0.78 (1.14)	−0.51 (−5.13)***	−0.45 (−0.13)	−0.05 (−0.04)	−2.50 (−0.48)	0.87	2.72
LGD 80%								
3.23 (8.47)***	−2.87 (−6.27)***			−4.92 (−0.89)	−1.04 (−0.47)	−9.98 (−1.24)	0.73	1.04
2.41 (7.36)***	−1.84 (−4.62)***		−0.46 (−5.17)***	−2.87 (−0.69)	−13.26 (−2.19)**	−2.83 (−1.67)	0.85	2.17
0.30 (0.54)		4.10 (6.79)***		−9.15 (−1.90)*	−1.26 (−0.18)	−1.15 (−0.54)	0.75	1.24
1.36 (2.35)**	−1.65 (−3.33)***	2.66 (3.89)***		−2.52 (−0.54)	−1.42 (−0.77)	−9.18 (−1.37)	0.81	1.49
1.83 (3.44)***	−1.57 (−3.57)***	1.07 (1.35)	−0.36 (−3.14)***	−2.35 (−0.57)	−2.59 (−1.54)	−12.23 (−2.03)*	0.86	2.11

(continued)

Table 7 (continued). Regression Results for the Worst-Case Scenario

Intercept	***LB***	***DOM***	***LB* CAPdummy***	***CAP***	***GDP***	***INT***	R^2	***DW***
LGD 60%								
2.58 (6.09)***	−1.36 (−2.66)**			−13.33 (−2.17)**	0.74 (0.30)	−10.28 (−1.15)	0.59	1.09
2.50 (5.10)***	−1.27 (−2.12)**		−0.04 (−0.30)	−13.15 (−2.11)**	−10.57 (−1.16)	0.58 (0.23)	0.59	1.11
0.91 (1.52)		2.35 (3.58)***		−13.43 (−2.57)**	−7.48 (−0.96)	0.53 (0.23)	0.64	1.41
1.21 (1.67)	−0.46 (−0.74)	1.95 (2.28)**		−11.57 (−1.99)*	0.46 (0.20)	−9.70 (−1.15)	0.65	1.31
−0.32 (−0.72)	0.53 (1.44)	2.42 (3.64)***	0.26 (2.73)**	−8.69 (−2.51)**	3.39 (2.40)**	11.22 (2.22)**	0.54	1.53

Note: Dependent variable is the worst-case scenario, measured as the percentage of total assets accounted for by failed banks (*WCS*). Explanatory variables are *GDP*, *INT*, *CAP*, *LB*, *DOM*, *LB*CAPdummy*, and three dummy variables for the quarters (not reported). Definitions are provided in table 6. Each cell displays the t-statistic for the OLS coefficient. For each OLS estimation, the R and the Durbin-Watson (*DW*) are given. The sample comprises forty-one observations (one per quarter between 1992:Q4 and 2002:Q2). The first panel of the table assumes an LGD of 100 percent, the second panel an LGD of 80 percent, and the third panel an LGD of 60 percent. Significance level of the t-tests: (***) at the 1 percent level, (**) at the 5 percent level, and (*) at the 10 percent level.

ratio at the sample mean. He reports that contagion has increased by moving from a complete structure toward a multiple-money-center structure. His analysis, however, does not control for the documented decrease in the degree of internationalization. As the Italian interbank market has become more domestic over time—in contrast to many other European countries—this reduction in internationalization may actually have caused the greater domestic contagion. In addition, because he keeps the tier 1 capital-to-asset ratio at the sample mean, he may fail to capture changes in bank capitalization that would constitute responses to higher contagion risk.

In regressions where both *LB* and *LB**CAPdummy* are used, both are negative and statistically significant (except for the 60 percent LGD where *LB**CAPdummy* is negative but not significantly different from 0). The mitigation effect of money centers is thus reinforced when money centers are well capitalized. *CAP*, the proxy for the capitalization of the whole banking system, also has a negative coefficient and is economically relevant. However, its coefficient is only statistically significant when the LGD is not too high. Thus, during the periods in which banks were holding more capital, contagion was less likely for lower LGDs. The impact of capitalization is also economically relevant: based on the first regression for an LGD of 60 percent, an increase in *CAP* from its lowest level to its highest level (i.e., from 0,075 to 0,109) reduces the WCS by 45 percent. In most cases, the unreported coefficients of the quarterly dummy variables are insignificant. The macroeconomic variables are also generally not significantly different from 0.

We investigate the *robustness* of our regression results by performing some additional tests. First, we employ instrumental variables for *LB* and *DOM*. We use instrumental variables to control for the fact that the same data set is used to generate simulations and to partially construct *LB* and *DOM*. As instrument for *LB*, we employ the Herfindahl index based on total assets (concentration in a money-center structure will tend to be higher than in a complete structure, as the money-center bank tends to be larger than banks at its periphery), and to instrument *DOM*, we compute an index of bank internationalization based on total assets. A second set of instruments uses lagged *LB* and *DOM*. Finally, we also test alternative specifications for the money-market structure such as the average of the ratio (exposure of bank i to small and medium-sized banks/exposure of bank i to large banks) over all small and medium-sized banks. The (unreported) results confirm our analysis.

The results hold when we run regressions for other characteristics of the distribution. For instance, the signs of the coefficients of the regression with the median value remain unchanged although, in some specifications, they are not statistically different from 0. In regressions where *LB* and *DOM* are taken separately, both coefficients remain significant for the 100 percent and 80 percent LGDs. A further issue is that our results may suffer from a potential censoring bias, as knock-on effects of foreign banks to Belgian banks are disregarded. We investigate this issue by using the WCS after one round (*WCS1r*).[45] Although the *WCS1r* suffers less from this censoring bias, it only measures the direct exposures of the banking sector to a given bank and, by construction, does not capture the whole contagion process. Therefore, if the market structure is an important driving factor of the second and further rounds of contagion, we may not observe any significant link between market structure and *WCS1r*. In unreported regressions, we find that *LB* and *DOM* are not significant, as *WCS1r* does not present sufficient heterogeneity. This paradoxically shows that the market structure strongly affects contagion propagation in second and further rounds.[46]

6. Concluding Remarks and Policy Implications

This paper exploits a unique time-series data set on interbank exposures in Belgium to study the determinants of interbank contagion. In our simulations, we track the consequences of nonrepayment of (a fraction of) interbank loans on the equity capital of other banks,

[45] *WCS1r* is defined as the maximum percentage of total banking assets accounted for by failing banks after one round of contagion.

[46] In unreported robustness exercises, we find that the results for the two other indicators of contagion, the percentage of banks initiating contagion and the "propagation mechanisms of contagion (rounds)," are similar to those of WCS. Namely, a change to a money-center structure leads to a decrease in the number of cases of contagion in the simulations. Although the proportion of banks capable of triggering contagion decreases when a multiple-money-market structure is adopted, the structure may become more risky if the probability of default of these banks, precisely because of the structural changes, increases.

including any further domino effects. The exercise provides insights on the potential impact of "stress" situations on the Belgian financial system, which may be representative for many other small countries due to the high degree of internationalization of its interbank market, the economic significance of its large banks, and the similarities in the structure of its interbank market.

We find that the risk of contagion due to domestic interbank defaults varies substantially over time: it increased over the period 1993–97, decreased afterward, and flattened out at a very low level at the end of the sample period (end of 2002). This is important, as existing studies focus on a single point in time. Our results reveal that the interbank market structure, the overall bank capitalization, and the degree of internationalization are important in explaining the time-series behavior of contagion. In Belgium, the structure of the interbank market has moved over time from a complete structure à la Allen and Gale (2000) toward a multiple-money-center structure. If large money centers are robust and can set off obligations against each of their counterparties, or if they are too big to fail, this move results in a de facto multilateral netting agreement for small banks. Simulations indicate that bilateral netting agreements dramatically reduce contagion indicators.

Interbank exposures between Belgian banks currently represent only 15 percent of total Belgian interbank exposures, suggesting that the potential contagion risk stemming from foreign interbank exposures is more important. We find that the failure of some foreign banks could have a sizable effect on Belgian banks' assets. Cross-country analyses could deliver additional results on the relationship between the interbank market structure and the risk of contagion, but to the best of our knowledge, there exists no database that covers interbank exposures in a region such as the European Union. In addition, since in some regions in the European Union, clear geographical segmentation remains, making the assumption of maximal dispersion of interbank loans and deposits would not be correct, while it is acceptable in a domestic context.

Existing methodologies assume a fixed LGD. When we endogenize the LGD, we find not only that the LGD interacts with other determinants of contagion such as the market structure but also that contagion effects are more important when cross-sectional variations in LGD are introduced than they are with the corresponding average

LGD. Assuming a fixed LGD may thus lead to an underestimation of contagion risk.

The findings of the paper highlight some specific regulatory issues. First, though the risk of contagion is currently low—the analysis shows that contagion is a low-frequency event—interbank exposures at some time periods may constitute a devastating contagion channel. This kind of event is particularly relevant for banking supervisors. As contagion risk evolves over time, supervisory practices should include not only a frequent monitoring of interbank large exposures but also a regular assessment of the interbank market structure. Yet, interbank contagion risk should not be monitored in isolation of other risks.

Second, to the extent that large money centers are resilient, we should not observe significant domestic contagion processes. Supervisory efforts to control propagation processes will thus be more successful if they are focused on large banks. In addition, although small banks may trigger some limited contagion effects, they do not cause a systemic crisis if large banks are resilient. Analyzing the different propagation channels will allow supervisors to distinguish nonsystemic contagion effects from real systemic crises.

Third, the default of some large foreign banks has the potential to trigger significant domino effects in Belgium. This suggests that it is important for regulators to monitor cross-border sources of interbank systemic risk. However, domestic regulators do not have any control over these banks. Fostering international regulatory cooperation is thus essential. To this extent, European initiatives such as the Committee of European Banking Supervisors or bilateral or multilateral memoranda of understanding agreed upon by regulators in different countries constitute significant progresses.

Finally, the current structure and characteristics of the Belgian interbank market reflect several changes that have taken place over the past decade. Integration of money markets at the European level, increased recourse by banks to secured interbank exposures, and several major mergers between Belgian banks have resulted in a trend toward market tiering and appear to have reshaped the risk of contagion. In the coming years, changes in the microstructure of interbank markets may further alter the structure of interbank markets, thus keeping alive the debate about interbank contagion risk.

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Is Moderate-to-High Inflation Inherently Unstable?*

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The data across time and countries suggest the level and variance of inflation are highly correlated. This paper examines the effect of trend inflation on the ability of the monetary authority to ensure a determinate equilibrium and inflation stability in a sticky-price model. Trend inflation increases the importance of future marginal costs for current price setters in a staggered-price-setting model. The greater importance of expectations makes it more difficult for the monetary authority to ensure stability in two senses. First, equilibrium determinacy is more difficult to achieve through reasonable specifications of nominal-interest-rate (Taylor) rules at moderate-to-high levels of inflation (e.g., at levels around 4 percent per year). In addition, the volatility of inflation induced by cost-push shocks is, all else equal, higher at higher rates of inflation under a reasonable specification of the nominal-interest-rate rule. If monetary policymakers have followed these types of policy rules in the past, these results may explain, in part, why moderate-to-high inflation is associated with inflation volatility.

JEL Codes: E3, E5.

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1. Introduction

The relationship between trend inflation and macroeconomic volatility is examined below in a standard New Keynesian sticky-price model.[1] Positive rates of trend inflation raise the importance of expected future marginal costs for current price setters in a staggered-price-setting model. The greater importance of expectations makes it more difficult for the monetary authority to ensure stability through simple policy rules. For example, much recent research has emphasized that monetary policymakers should raise real interest rates in response to an increase in inflation. In the context of a simple rule relating nominal interest rates to inflation (e.g., Taylor 1993), this requires that the coefficient on inflation exceed unity, a condition referred to as the Taylor principle. It is demonstrated below that the Taylor principle can be violated at fairly moderate rates of trend inflation. Equilibrium indeterminacy, and hence the possibility of sunspot fluctuations and increased macroeconomic instability, occurs for an increasingly large proportion of the range of policy settings when trend inflation rises to moderate levels (e.g., from 0 to 4 percent per year).[2]

This result may have implications for monetary policy practices and the interpretation of past events. With regard to practices, the results suggest that a focus on trend inflation in discussing policy rules is central, in that inflation stability requires that policymakers commit to low inflation and respond vigorously to inflation fluctuations around that trend level. With respect to past experience, Clarida, Galí, and Gertler (2000) have suggested that insufficient responsiveness of nominal interest rates to expected inflation in the 1970s—i.e., violations of the Taylor principle that allowed

[1] Most recent work on sticky-price models abstracts from positive trend inflation rates (e.g., Woodford 2003). Two exceptions are Bakhshi et al. (2002) and Ascari (2004). These authors illustrate how the equilibrium in a Calvo staggered-price-setting model (Calvo 1983) may not exist for high values of trend inflation, because the infinite sum on which the currently chosen nominal price is based may not converge for high rates of inflation.

[2] It should be emphasized that throughout this analysis the equilibria are always locally determinate and stable, or locally indeterminate. The term "instability" herein will often refer to indeterminacy and hence the possibility that sunspot shocks increase macroeconomic volatility. This is the same notion as in Clarida, Galí, and Gertler (2000).

real interest rates to fall with an increase in expected inflation—contributed to macroeconomic volatility. During that period, inflation in many countries was at least moderate. The results herein suggest that the level of inflation may have contributed to volatility, because moderate-to-high inflation is more difficult to stabilize (in two senses formalized below) when prices are rigid. This conclusion is also consistent with a substantial body of evidence showing a very strong correlation between the level and variance of inflation across countries and time (e.g., Kiley 2000)—if policymakers in that sample could be interpreted as following Taylor-type rules, as suggested in earlier work for Germany, Japan, and the United States (e.g., Clarida, Galí, and Gertler 1998, 2000).

The analysis herein will focus exclusively on the possible effects of trend inflation on economic volatility, and does not consider any other costs or benefits associated with moderate inflation. However, the finding of potentially pernicious effects of moderate-to-high inflation on economic volatility adds further weight to research suggesting that low inflation is desirable due to steady-state distortions to relative prices, interactions between trend inflation and nominal tax systems, and the classical costs associated with the area under the money-demand curve stemming from positive nominal interest rates.[3]

The next section briefly discusses the association between the level and variance of inflation. Section 3 presents a simple model illustrating the effect of trend inflation on price-setting behavior in a New Keynesian staggered-price-setting model. The model's implications for macroeconomic volatility are examined in section 4. The final section discusses necessary further work.

2. Inflation and Its Variance

There is a long history documenting a positive relationship between the level and variance of inflation. Okun (1971) and Taylor (1981) are classic examples. Both authors demonstrate through international

[3]The effects of trend inflation on various aspects of economic performance in New Keynesian models like that considered herein has attracted increased interest recently (e.g., Ascari 2004, Ascari and Ropele 2004, Blake and Fernandez-Corugedo 2006, and Cogley and Sbordone 2006).

Table 1. Basic Statistics for Consumer Price Inflation in the G-7

	1974–1985		1986–2000	
	Average	**Std. Deviation**	**Average**	**Std. Deviation**
Canada	8.5	2.3	2.7	1.4
France	10.7	2.5	2.3	1.1
Germany	4.3	1.7	1.9	1.3
Italy	16.0	3.6	4.9	1.7
Japan	6.7	5.5	0.7	1.2
United Kingdom	12.1	5.7	3.9	2.1
United States	6.9	2.5	2.8	1.1

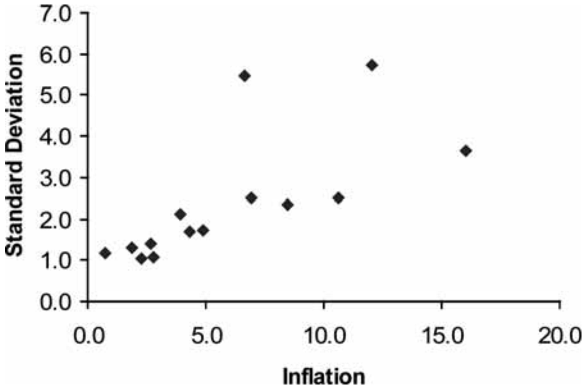
Note: Inflation is measured as the percent change in the annual average of the personal consumption deflator.

cross-sectional comparisons that high inflation is volatile inflation. Kiley (2000) recently reports a similar correlation across forty-three countries.

For our purposes, it is important to emphasize that this finding is not driven by inclusion of very high inflation economies; rather, it is true of the G-7 economies over the past thirty years, as well as during the moderate inflation conditions of the last fifteen years.[4] For example, table 1 presents average inflation rates, as measured by the annual percent change in the personal consumption deflator from the national accounts, and the standard deviation of inflation for the seven economies that make up the G-7 over two periods, 1974–85 and 1986–2000. The break between time periods was chosen to roughly correspond to the period after disinflation from the higher levels of the 1970s was completed in most countries and the period of increased macroeconomic stability identified in McConnell and Perez-Quiros (2000) and OECD (2002).

[4]We focus on low-to-high inflation, where high is defined as 10 percent per year. The positive association between very high inflation and its variance will not be considered, as very high rates of inflation make a New Keynesian sticky-price model implausible.

Figure 1. Scatter Plot of the Volatility of Inflation against Its Level in the G-7



In every country, the period of low inflation was also a period of more-stable inflation. This relationship is also apparent across countries: figure 1 presents the scatter plot of the fourteen country/time-period pairs for average inflation and its standard deviation. The simple correlation between the level and standard deviation of inflation is 0.7 in the fourteen country/time-period observations. This correlation is not driven by the high-inflation period—it remains 0.7 in the seven country-level observations over 1986–2000.

Finally, it is important to emphasize that the positive association between the level and variance of inflation is distinct from the positive association between inflation and the dispersion of relative prices at a point in time—which is also quite strong, as documented in, for example, Fischer (1981) and Stockton (1988). These earlier studies have demonstrated that relative price dispersion and the level of inflation are positively correlated largely because relative price shocks—particularly to food and energy—increase price dispersion and inflation in the short run. This literature has also found some positive effect of trend inflation on relative price dispersion; in other words, the causality runs in both directions, albeit more strongly from relative price shocks to dispersion and inflation than vice versa.

In contrast, the positive association between the level of inflation and its variance is not dominated by relative price shocks. Two factors suggest this interpretation: (i) the cross-country experience partially controls for energy price shocks (e.g., common global movements in oil prices and Okun's [1971] findings, which predate the 1970s' supply shocks), and (ii) the correlation between the level and variance is strong between 1986 and 2000, a period of greater stability in oil prices. Two hypotheses were offered in the earlier literature for the relationship between the level of inflation and its variance. Milton Friedman's Nobel lecture (1977) suggests that high inflation causes inefficient, and hence more variable, macroeconomic policies—perhaps reflecting a diminution of political consensus; much of Friedman's conjecture concerns inflation at the high end of the range we consider, and does not address the correlation between the rate of inflation and its variance at the moderate levels emphasized herein. Taylor (1981) suggests that accommodative monetary policies may lead to high inflation and greater variability in response to supply shocks. Our analysis will echo Taylor in emphasizing policy. But it will differ in demonstrating that moderate-to-high levels of inflation will potentially increase macroeconomic instability even when policymakers are not accommodative, because moderate-to-high inflation may both amplify the effects of intrinsic shocks and open up the possibility of self-fulfilling inflation fluctuations.

3. A Model

The model is a standard New Keynesian description of the macroeconomy, similar to the baseline case in Clarida, Galí, and Gertler (1999). The aggregate-demand side of the model—the IS curve and the monetary-policy reaction function—is not affected by trend inflation and hence its derivation from microeconomic behavior will not be discussed.

Focusing first on aggregate supply, the economy consists of a large number of (symmetric) monopolistically competitive firms producing intermediate goods that are aggregated with a constant-elasticity-of-substitution aggregation function to produce the final consumption good. The demand function facing firm j in period t is given by

$$D_{j,t}\left(\frac{X_{j,t}}{P_t}\right) = \left(\frac{X_{j,t}}{P_t}\right)^{-\theta} Y_t, \quad \theta > 1,$$

where $X_{j,t}$ is the price charged by the firm, P_t is the aggregate price level (defined below), and Y_t is aggregate demand. For simplicity, aggregate demand and consumption are used interchangeably, and the effects of investment by firms or consumer durable purchases are ignored (following, e.g., Woodford 2003).

Firms set nominal prices for two periods. We will fix this period of price rigidity across all the levels of inflation considered below. It is reasonable to suppose that the degree of price rigidity would vary with trend inflation, as demonstrated empirically by Kiley (2000). But the observed variation across countries and time has been for large differences in trend inflation, and we will consider more-modest variations in trend inflation. Future work may wish to reexamine the role of *endogenous* selection of price rigidity at different levels of trend inflation, along the lines pursued in Kiley (2000), for example. However, previous research suggests that the results herein will not be affected to a significant extent. Fischer (1981) notes that menu cost models will imply that the degree of price stickiness will decrease with the average inflation rate, but that this decrease will generally be partial in the sense that relative price dispersion increases with trend inflation; in the model below, the increase in relative price dispersion with trend inflation is the important mechanism driving our results. Recent research also suggests that, within calibrated dynamic general equilibrium models, the degree of price rigidity is not likely to be very sensitive to the average level of inflation for the differences across the G-7 since the 1970s (e.g., Klenow and Kryvtsov 2003).

In our model, there are two classes of firms, each with mass equal to one-half the total. The firms differ in that they alternate the period in which they adjust their price; i.e., price setting is staggered as in Taylor (1980). This sticky-price assumption preserves tractability and avoids some of the problems associated with the popular Calvo model.[5] The firm's profit

[5]A staggered-price-setting model with prices rigid for four periods, rather than two periods, was also examined. The results were very similar. However, the four-period model resulted in even more-complicated relationships between trend inflation and the coefficients in the aggregate supply relations of the model; as a result, the discussion focuses solely on the two-period model, where some intuition and very limited analytical results are more easily obtained. On the Calvo model, Kiley (2002a, 2002b) discusses a number of issues illustrating how this model fails to provide a good approximation to staggered-price-setting models with finite maximum lags.

maximization problem involves choosing the nominal price X_t that maximizes

$$\Lambda_t \left[\left(\frac{X_t}{P_t} \right)^{1-\theta} Y_t - \Gamma \left(\left(\frac{X_t}{P_t} \right)^{-\theta} Y_t \right) \right] + E \left\{ \Lambda_{t+1} \left[\left(\frac{X_t}{P_{t+1}} \right)^{1-\theta} Y_{t+1} - \Gamma \left(\left(\frac{X_t}{P_{t+1}} \right)^{-\theta} Y_{t+1} \right) \right] \middle| t \right\},$$

where $E\{.|t\}$ is the expectations operator conditional on period t information, Λ_t is the marginal utility of consumption for the firm's owners in period t (and hence the appropriate discount rate), and $\Gamma(.)$ is the firm's cost function.

Manipulating the first-order condition yields an expression for the optimal price

$$\frac{X_t}{P_t} = \frac{\theta}{\theta - 1} \frac{\Lambda_t MC_t \left(\frac{X_t}{P_t} \right)^{-\theta} Y_t + E \left\{ \Lambda_{t+1} MC_{t+1} \left(\frac{X_t}{P_{t+1}} \right)^{-\theta} Y_{t+1} | t \right\}}{\Lambda_t \left(\frac{X_t}{P_t} \right)^{-\theta} Y_t + E \left\{ \Lambda_{t+1} \Pi_{t+1}^{-1} \left(\frac{X_t}{P_{t+1}} \right)^{-\theta} Y_{t+1} | t \right\}}, \tag{1}$$

where Π_{t+1} is inflation (P_{t+1}/P_t). The real price equals a constant markup over the weighted average of marginal cost (MC) during the period for which the price is fixed, where the weights incorporate the effects of discounting and trend inflation.

The aggregate price level P_t is given by the standard equation

$$P_t = \left[\frac{1}{2} X_t^{1-\theta} + \frac{1}{2} X_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{2}$$

In order to complete the specification of the firm's problem, expressions for marginal cost and the discount rate are necessary. Assuming that households' preferences are separable in consumption and leisure, insurance markets are complete, and preferences are of the constant-relative-risk-aversion (CRRA) form with risk aversion equal to σ implies that the discount rate for period $t + j$ (Λ_{t+j}) is $\beta^{j-t} Y_{t+j}^{-\sigma}$. If the disutility from labor supply takes the typical power function form, household j's decision regarding its hours supply $(H_{j,t})$ is governed by the intratemporal optimality condition

$$\frac{W_{j,t}}{P_t} = H_{j,t}^{\phi} Y_t^{\sigma},$$

where $W_{j,t}/P_t$ is the real wage and $1/\phi$ is the labor-supply elasticity. Note that our assumption regarding wages has assumed that household j is attached to firm j (i.e., labor markets are sector specific); this is done to introduce a degree of "real rigidity" into the model, is equivalent to a yeoman-farmer specification (in its reduced-form implications, e.g., Woodford 2003), and has no effect on our qualitative results. Finally, the production function of firm j is given by

$$Y_{j,t} = H_{j,t}^{a},$$

implying that total costs for firm j in period t are

$$\Gamma(Y_{j,t}) = \frac{W_{j,t}}{P_t} H_{j,t} = \frac{W_{j,t}}{P_t} Y_{j,t}^{\frac{1}{a}}.$$

Substituting the demand curve in this expression, differentiating, and then substituting the real wage equation above yields marginal cost for a firm charging nominal price X_t in period k:

$$MC_{t+k} = \frac{1}{a}\left(\frac{X_t}{P_{t+k}}\right)^{-\omega\theta} Y_{t+k}^{\omega+\sigma}, \quad \omega = \frac{\phi}{a} + \frac{1}{a} - 1, \quad k = 0, 1. \quad (3)$$

The parameter ω represents the elasticity of marginal cost with respect to the firm's own output.

Inserting equation (3) into equation (1) along with the expression for the discount factor yields the solution for the optimal price chosen by a firm in period t:

$$\frac{X_t}{P_t} = \left[\frac{\theta}{\theta-1}\frac{1}{a}\frac{Y_t^{1+\omega} + E\{\beta\Pi_{t+1}^{\theta+\omega\theta}Y_{t+1}^{1+\omega}|t\}}{Y_t^{1-\sigma} + E\{\beta\Pi_{t+1}^{\theta-1}Y_{t+1}^{1-\sigma}|t\}}\right]^{\frac{1}{1+\omega\theta}}. \quad (4)$$

Log-linearizing equations (4) and (2) around the steady-state values of relative prices, output, and inflation ($P_{t+1}/P_t = \Pi$) yields (with lowercase letters denoting log-deviations from steady-state levels)

$$
\begin{aligned}
& x_t - p_t = d_1 y_t + E\{d_2 y_{t+1} + d_3 \pi_{t+1} | t\}, \\
& d_1 = \frac{1}{1+\omega\theta}\left[\frac{1+\omega}{1+\beta\Pi^{\theta+\omega\theta}} - \frac{1-\sigma}{1+\beta\Pi^{\theta-1}}\right], \\
& d_2 = \frac{1}{1+\omega\theta}\left[\frac{(1+\omega)\beta\Pi^{\theta+\omega\theta}}{1+\beta\Pi^{\theta+\omega\theta}} - \frac{(1-\sigma)\beta\Pi^{\theta-1}}{1+\beta\Pi^{\theta-1}}\right], \\
& d_3 = \frac{1}{1+\omega\theta}\left[\frac{(\theta+\omega\theta)\beta\Pi^{\theta+\omega\theta}}{1+\beta\Pi^{\theta+\omega\theta}} - \frac{(\theta-1)\beta\Pi^{\theta-1}}{1+\beta\Pi^{\theta-1}}\right]. \\
& \pi_t = \Pi^{1-\theta}[x_t - p_t] + x_{t-1} - p_{t-1}. \qquad (5)
\end{aligned}
$$

These expressions look cumbersome but should be familiar for the case where trend inflation equals 0 ($\Pi = 1$) and there is no discounting ($\beta = 1$); in that case, these equations simplify to

$$
\begin{aligned}
& x_t - p_t = \frac{1}{2}\frac{\omega+\sigma}{1+\omega\theta}[y_t + E\{y_{t+1}|t\}] + \frac{1}{2}E\{\pi_{t+1}|t\} \\
& \pi_t = x_t - p_t + x_{t-1} - p_{t-1},
\end{aligned}
$$

which are equivalent to the staggered-price-setting specification in Taylor (1980), Chari, Kehoe, and McGrattan (2000), and Kiley (2002a, 2002b). However, in the present case, trend inflation raises the importance of future output and inflation in decisions regarding the current price (i.e., d_2 and d_3 are increasing in trend inflation, as noted by Ascari 2000). This occurs because firms realize that demand for their product will be higher in the future, after inflation has eroded the real value of their nominal prices; hence, firms place a larger weight on future developments in setting current prices when inflation is higher. Note that these responses are related to the increase in relative price dispersion that accompanies higher inflation in the staggered-price-setting model, consistent with earlier empirical work and the emphasis in recent research (e.g., Woodford 2003) on the importance of this channel in staggered-price-setting models for aggregate welfare.

For the analysis of equilibrium determinacy, the set of stochastic disturbances affecting the economy can be ignored. But the analysis of volatility will require some set of exogenous disturbances. Both for simplicity and in line with earlier work (Clarida, Galí, and Gertler 1999), a cost-push shock is appended to the log-linearized equation for relative prices, yielding

$$x_t - p_t = d_1 y_t + E\{d_2 y_{t+1} + d_3 \pi_{t+1}|t\} + u_t, \tag{6}$$

where u is an i.i.d. disturbance term.

The remainder of the model follows the New Keynesian literature. The IS equation links the deviation of current output from its steady-state or potential value (y) to the real-interest-rate deviation (the nominal rate i minus future inflation) and future output:

$$y_t = E\{y_{t+1}|t\} - \frac{1}{\sigma}[i_t - E\{\pi_{t+1}|t\}]. \tag{7}$$

Microfoundations for this equation can be found in the consumption Euler equation (equation (7) is a log-linearized version of such an equation). As noted above, output replaces consumption for simplicity (reflecting its dominance in aggregate demand). A more thorough discussion can be found in Woodford (2003).

The aggregate-demand side of the model is closed with a specification of monetary policy, which follows a forward-looking (with respect to inflation) Taylor rule:

$$i(t) = \gamma_\pi E\{\pi_{t+1}|t\} + \gamma_y y_t. \tag{8}$$

This specification is relatively standard. A substantial body of earlier work (e.g., Clarida, Galí, and Gertler 2000; Bullard and Mitra 2002; and Woodford 2003) has demonstrated that equilibrium determinacy, and hence macroeconomic stability, can be achieved in this framework if the real interest rate increases with inflation ($\gamma_\pi > 1$). This property has been labeled the Taylor principle, following the influential work of Taylor (1993, 1999).[6] Two considerations drive our emphasis on forward-looking behavior. First, Clarida, Galí, and

[6] The condition for a determinate equilibrium depends on the response coefficients involving both inflation and output. Bullard and Mitra (2002) describe the Taylor principle in terms of the set of values for both response coefficients. The discussion herein focuses on the less formal notion emphasized by John Taylor in his contributions—i.e., that real interest rates should rise with inflation. We will return to the formal conditions ensuring determinacy in the next section.

Gertler (1998, 2000) and Orphanides (2002) both argue that monetary policy behavior in Germany, Japan, and the United States has been well described by this type of behavior. In addition, central banks that have adopted inflation targeting have placed increased focus on inflation expectations and have characterized their behavior in this regard as consistent with an equation like (8) (for example, the Reserve Bank of New Zealand models its own policy in this way [see page 39 of Black et al. 1997]; also see the summary of twenty countries' experience with inflation targeting in Schmidt-Hebbel and Tapias 2002). Of course, decisions in actual practice invariably include factors not included in the model. Section 4 will discuss the sensitivity of the results to alternative specifications of the monetary policy rule.

The model can be compactly expressed as a second-order expectational difference equation:

$$AE\{z_{t+1}|t\} + Bz_t + Cz_{t-1} = Du_t, \tag{9}$$

where z_t is a 4×1 vector containing the relative price set at t, inflation, output, and the nominal interest rate. A, B, C, and D are matrices containing structural coefficients. Equation (9) has a unique rational-expectations solution in which fluctuations are driven solely by the cost-push shock (u) when the number of roots of the matrix polynomial on the left-hand side that lie inside the unit circle equals the number of predetermined endogenous variables (one in this case, reflecting the lagged relative price in the inflation equation). When more than one of these roots lie inside the unit circle, rational-expectations solutions in which sunspots—nonfundamental shocks—can drive fluctuations are also possible; this multiplicity is termed indeterminacy herein.[7] Such indeterminacy is undesirable, as nonfundamental shocks could increase the variability of the economy—a notion pursued, for example, in Clarida, Galí, and Gertler (2000).

[7]Farmer (1993) provides a good introduction to indeterminacy and the possibility of sunspot equilibria in rational-expectations models of the sort discussed in this research.

4. Results on Equilibrium Determinacy and Volatility

4.1 Indeterminacy

We focus first on determinacy of equilibrium and return later to volatility under policy settings consistent with a unique equilibrium. Our results are derived from an extensive set of numerical exercises, in which the model is solved using the AIM algorithm originally developed by Anderson and Moore (for a recent presentation, see Anderson 2000).[8] We first assign a baseline set of parameter values to the model. Since the sticky-price model assumes that nominal prices are fixed for two periods, a period is assumed to correspond to one-half year. Table 2 presents values for most of the parameters. The discount factor (β) is set at 0.96 per year, implying a real interest rate of approximately 4 percent. The coefficient of relative risk aversion (σ) is set at $^1/_4$; while this value is quite low, Woodford (2003) justifies a low value by noting that the inverse of this parameter governs the interest sensitivity of aggregate demand and that this sensitivity is substantially higher than the intertemporal elasticity of substitution of consumption once investment in business capital and consumer durables—both absent from the model—are considered. The elasticity of output with respect to labor input (a) is set at $^2/_3$, approximately the value in U.S. data. The baseline setting for the markup of prices over marginal cost ($1/(\theta - 1)$) is 10 percent (Woodford 2003 typically uses a similar value, and other values of this parameter are discussed below). The labor-supply elasticity ($1/\phi$) equals 1 in the baseline calibration. This value for the labor-supply elasticity lies above traditional estimates (MaCurdy 1981, Altonji 1986, and Abowd and Card 1989) but below the common assumption in dynamic general equilibrium models that labor supply

[8] We have introduced a very simple model, and one might imagine that analytical results could be obtained. For example, it is straightforward to collapse the system of equations reported as equation (9) to a system that can be expressed as $AE\{z_{t+1}|t\} = z_t + Bu_t$, where z is a 2×1 vector containing relative prices $(x - p)$ and output (y). In this case, determinacy depends solely upon the eigenvalues of A, and relatively simple conditions on the trace and determinant of A can be checked to ensure determinacy. Unfortunately, the elements of A are extremely complicated functions of the structural parameters in the case where trend inflation is nonzero, making analytic results too difficult to obtain (at least for this author).

Table 2. Baseline Parameter Values

Parameter	Description	Value
β	Discount Factor	0.96
σ	Coefficient of Relative Risk Aversion	$^1/_4$
a	Elasticity of Y with Respect to H	$^2/_3$
$1/(\theta - 1)$	Markup (at Zero Inflation)	.10
$1/\phi$	Labor-Supply Elasticity	1

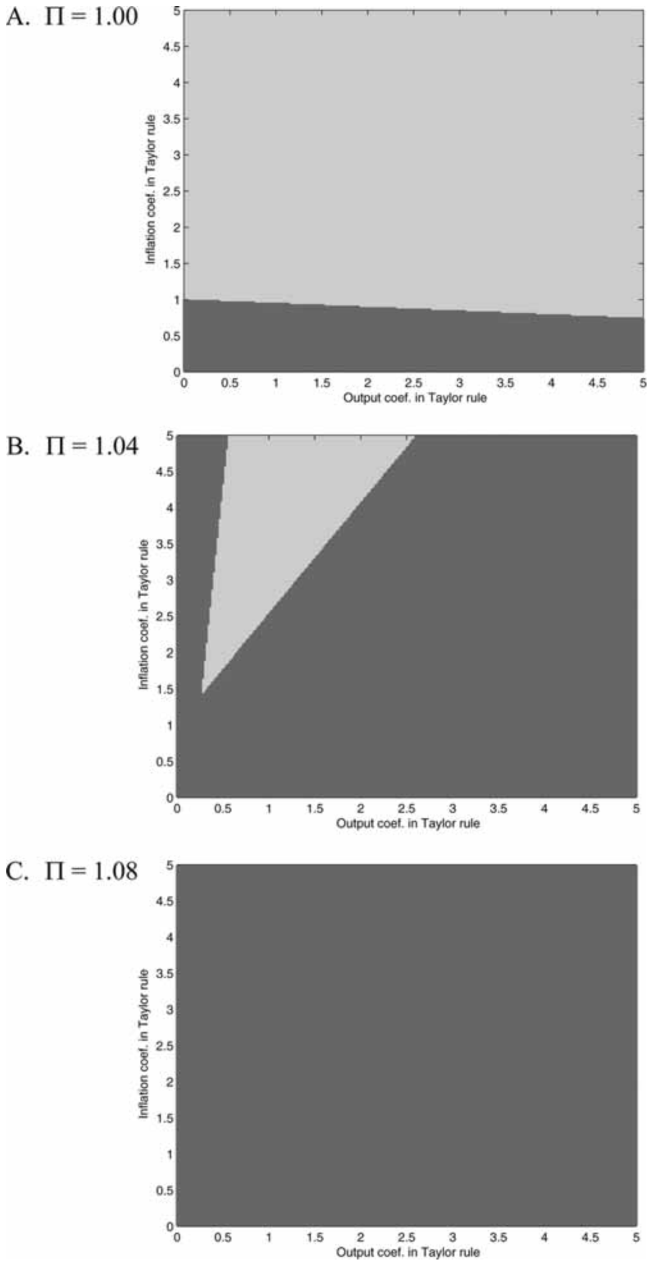
Note: The discount factor is expressed at an annual rate; as there are two periods in a year, β equals $\sqrt{0.96}$.

is perfectly elastic (an assumption associated with indivisible labor supply). This baseline value is near the recent estimates of Mulligan (1998), and alternatives are considered below.

The remaining parameter values are the trend inflation rate (Π) and the coefficients in the Taylor rule. Our experiments consider the possibility of indeterminacy for different values of these parameters. For each set of parameters, the indeterminacy of equilibrium is examined numerically, and the results for a range of inflation rates and policy settings are summarized in figure 2. The three panels of figure 2 correspond to trend inflation rates of 0 percent, 4 percent, and 8 percent, respectively. (The value for inflation refers to the steady-state percent change in the price level *at an annual rate.*) Each panel presents whether indeterminacy arises for coefficients in the Taylor rule ranging from 0 to 5 for both output and the one-period-ahead inflation forecast; indeterminacy is indicated by the dark-shaded region.

The results for a trend inflation rate of 0, shown in panel A, are closely related to the findings in the previous literature. In particular, indeterminacy, and hence the possibility of sunspot fluctuations, arises with trend inflation equal to 0 when nominal interest rates are insufficiently responsive to expected inflation. It is possible to derive an analytical characterization of the region in which a determinate equilibrium exists for the case of trend inflation equal to 0 by following the steps in Bullard and Mitra (2002) or Woodford (2003). A determinate equilibrium occurs when the coefficients in the nominal-interest-rate rule (assuming both coefficients are positive) satisfy the inequality $\gamma_\pi + \frac{1}{2}\frac{1-\beta}{1+\beta}\frac{1+\omega\theta}{\omega+\sigma}\gamma_y > 1$.

Figure 2. Equilibrium Determinacy for Different Trend Inflation Rates (Π)



This condition implies the loose version of the Taylor principle emphasized earlier—that nominal interest rates must respond to expected inflation with a coefficient exceeding unity—in the absence of discounting or when the coefficient on output in the rule equals 0. The restrictions on coefficients are very similar to those that arise in the Calvo pricing model.[9]

Panels B and C illustrate how this condition changes with trend inflation. For trend inflation of 4 percent, indeterminacy is possible for a much wider range of the parameter space: to ensure determinacy, the output response (γ_y) must be slightly positive (but not too large) and the inflation response (γ_π) must be greater than 1 (and by a significant amount for larger values of the output response). When trend inflation is 8 percent, no set of responses considered in the rule yields determinacy.

A comparison of the various panels of figure 2 also reveals that determinacy depends on both coefficients in the nominal-interest-rate rule in a complicated manner. For zero trend inflation (panel A), determinacy is ensured if the nominal interest rate is sufficiently responsive to expected inflation, and the degree of responsiveness required is slightly decreasing in the responsiveness of policy to the output gap. (This is consistent with the analytic expression presented earlier and is only a slightly different region than under the Calvo pricing model.) For positive trend inflation, the determinacy region shrinks from both sides (panel B), so that nominal interest rates can be neither too unresponsive nor too responsive to expected inflation for a given coefficient on the output gap.

4.2 Sensitivity to Important Structural Parameters

Unfortunately, the structural coefficients in the system represented by equation (9) are much more complex in the case of positive trend inflation, and analytic results could not be found. As mentioned earlier, higher trend inflation increases the role of expectations in the dynamic system. This appears to be the source that raises the

[9] As shown elsewhere (e.g., Woodford 2003), the coefficients in the interest rate rule with expected inflation as in equation (8) must satisfy another restriction to ensure determinacy (and this additional condition involves the response coefficients not being "too large"). This type of additional restriction does not arise in the staggered-price-setting model examined herein.

possibility of sunspot fluctuations. Other parameters influence how trend inflation impacts the coefficients on expectations and hence are important as well. (As noted above, other parameters are basically not important for determinacy when trend inflation is 0, as determinacy is ensured so long as $\gamma_\pi + \frac{1}{2}\frac{1-\beta}{1+\beta}\frac{1+\omega\theta}{\omega+\sigma}\gamma_y > 1$, which is always satisfied for γ_π greater than 1 and γ_y greater than 0.)

Two important parameters are the labor-supply elasticity and the elasticity of demand for a firm's product. Firms know that demand will be higher when inflation erodes their real price—i.e., in the future. The strength of this effect is driven by the demand elasticity; and low labor-supply elasticities imply that marginal cost is more sensitive to high demand, which implies that future profits are eroded to a greater extent by price rigidity with high inflation and lower labor-supply elasticities. Therefore, the demand elasticity and labor-supply elasticity play important roles in determining the importance of inflation for forward-looking behavior.

Figure 3 presents the indeterminacy regions for different values of the labor-supply elasticity, holding all other parameters at their baseline values and fixing trend inflation at 4 percent ($\Pi = 1.04$). Panel A reproduces the baseline results. Panel B shows that a low labor-supply elasticity ($^1/_4$, similar to estimates cited above) makes indeterminacy likely, while a high labor-supply elasticity (infinity, panel C) makes indeterminacy less of a problem for policymakers. Again, higher weight on expectations (in this case, through a lower labor-supply elasticity) makes indeterminacy a larger potential problem.

Figure 4 presents the results on indeterminacy for the baseline demand elasticity and a lower elasticity (consistent with a markup of 20 percent), again with trend inflation of 4 percent. A lower elasticity (higher markup) lowers the importance of future demand in price setting, and this shrinks the region of policy settings over which indeterminacy is a concern.

In summary, sunspot fluctuations are a possible concern when trend inflation is moderate, at least under the type of forward-looking Taylor rule that has been discussed as a reasonable characterization of behavior for some central banks. Even values of trend inflation of 4 percent per year substantially shrink the range of policy settings that deliver equilibrium determinacy. At trend inflation of 8 percent, no policy settings within the forward-looking Taylor-rule

Figure 3. Equilibrium Determinacy for Different Labor-Supply Elasticities ($1/\phi$)

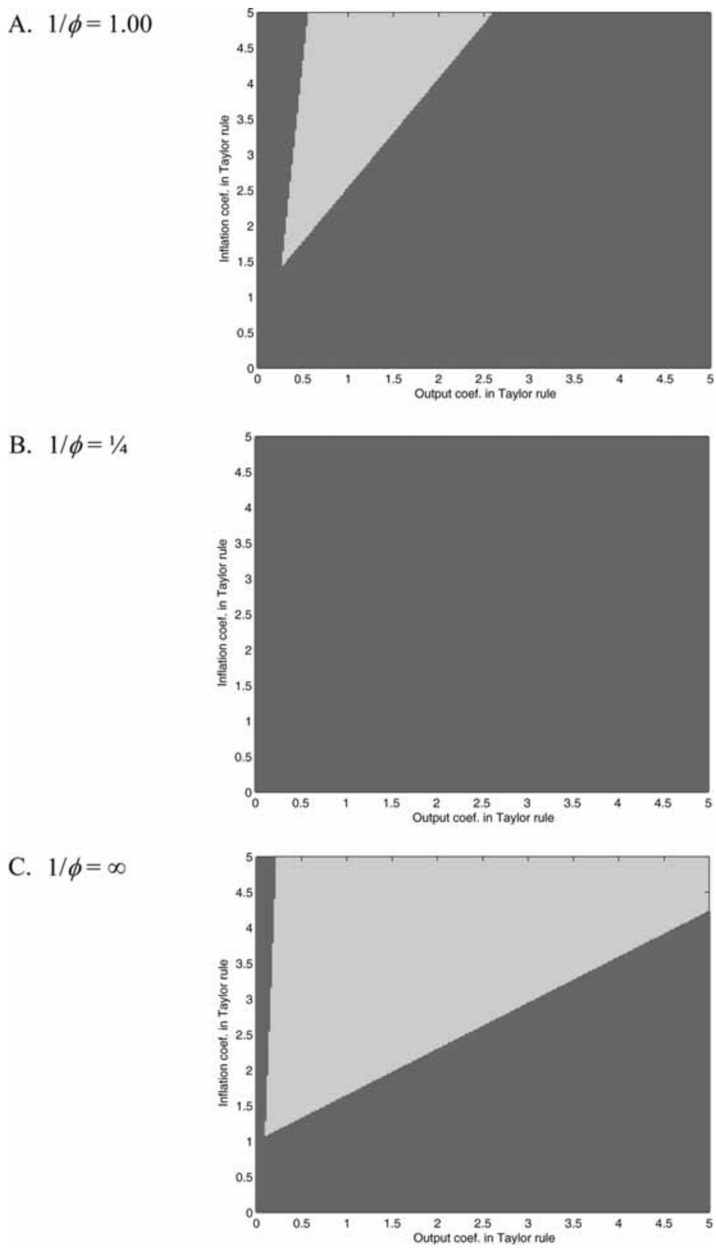
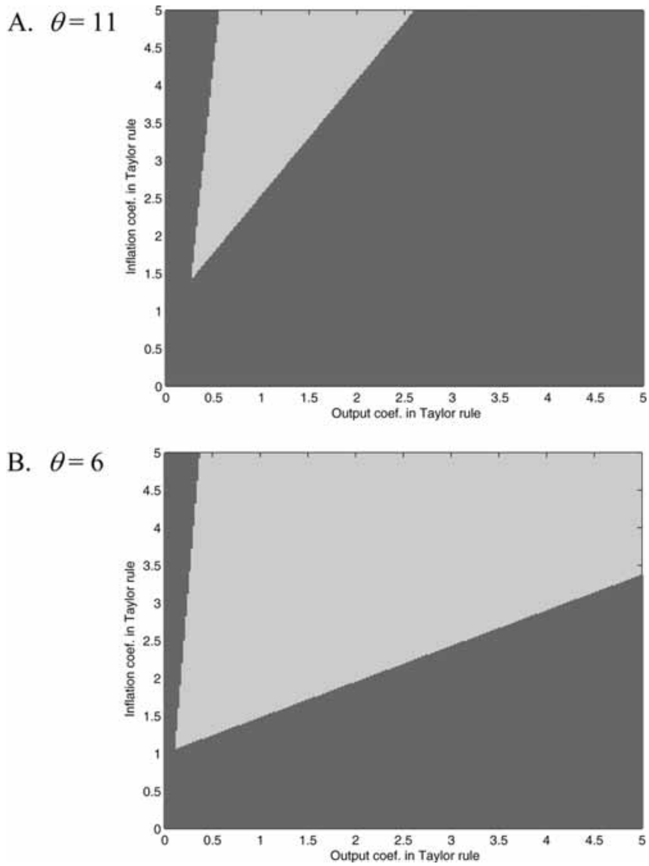


Figure 4. Equilibrium Determinacy for Different Demand Elasticities (θ)

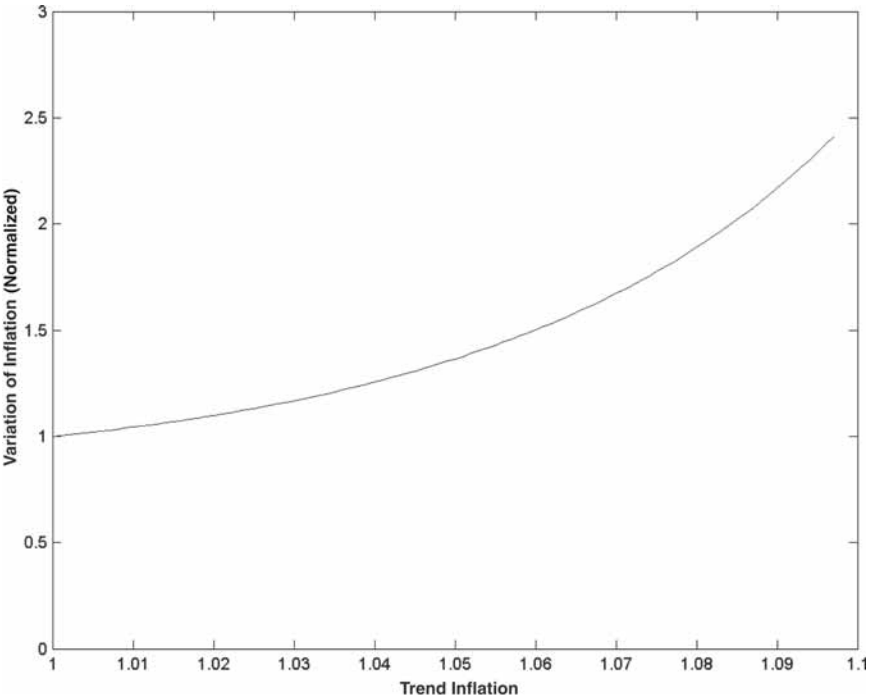


framework can ensure determinacy for the range of parameter values considered. Alternative policy rules are discussed in subsection 4.4 below.

4.3 Volatility in the Determinate Region

The previous section demonstrated that the increasing importance of forward-looking behavior with trend inflation in sticky-price models raises the potential problem of indeterminacy, which could increase

Figure 5. The Variance of Inflation from Cost-Push Shocks at Different Values of Trend Inflation



macroeconomic volatility through the possibility of sunspot fluctuations. Even within the parameter space consistent with a determinate equilibrium, greater forward-looking behavior could contribute to increased volatility in response to fundamental shocks.

The implications of trend inflation for the volatility of inflation in our model are examined numerically using a markup of 20 percent, an infinite labor-supply elasticity, and traditional values for the coefficients in the Taylor rule ($\gamma_\pi = 1.5, \gamma_y = 0.5$). Other parameters equal their values in table 2. (The high markup and high labor-supply elasticity were chosen to allow a substantial range for inflation within which the equilibrium is determinate; the qualitative results do not depend on the specific values chosen for these parameters.) Figure 5 presents the variances of inflation for different trend inflation rates, normalized to equal 1 at a 0 percent inflation rate.

The results show that inflation volatility rises with trend inflation when the monetary policy settings are held fixed. (Note that the variances are presented up to trend inflation somewhat below 10 percent, as equilibrium determinacy fails prior to that level of inflation.) This suggests that moderate trend inflation may contribute to macroeconomic instability both through its effect on the transmission of fundamental shocks and through the possibility of sunspot-induced volatility. (As a side note, output volatility increases as well in this case.)

Some intuition for the source of increased volatility can be seen from the form of the relative price equation and inflation equations (5). It is straightforward to show that the lags in equation system (9) can be eliminated by substituting the inflation equations into the relative price and nominal-interest-rate equations and recasting the system solely in terms of output, relative prices, the nominal interest, and the cost-push shock (e.g., a system like that in footnote 8, with no lags). With i.i.d. disturbances and no lags in the system, the coefficients on contemporaneous values for the variables are the sole determinants of equilibrium behavior (assuming a determinate equilibrium). The relative price equation after substitution of the inflation equation and ignoring date $t+1$ variables is given by

$$x_t - p_t = \left(\frac{1}{1-d_3}\right)(d_1 y_t + u_t). \tag{10}$$

As the coefficient d_3 is increasing in inflation, the variability in relative prices induced by cost-push shocks is increasing in trend inflation. This directly implies more-volatile inflation at higher trend inflation rates from equation (5). (Of course, the behavior of output is important as well, but the simple intuition holds.)

4.4 Robustness to Alternative Policy Rules

The analysis has held fixed the policy rule. An alternative to a forward-looking Taylor rule is the Taylor rule with contemporaneous inflation:

$$i(t) = \gamma_\pi \pi_t + \gamma_y y_t.$$

Figure 6. Equilibrium Determinacy under a Contemporaneous Taylor Rule

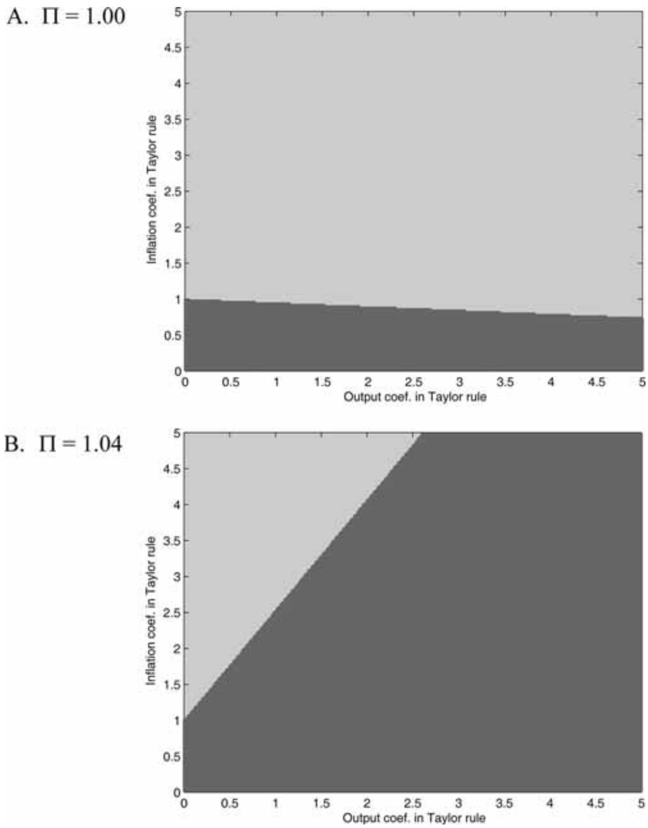


Figure 6 presents the indeterminacy regions for trend inflation rates of 0 and 4 percent at the baseline parameter values in table 2 using this alternative Taylor rule. While the indeterminacy region is slightly different, the picture is much the same as that with the forward-looking rule: trend inflation substantially increases the parameter space over which indeterminacy and sunspot fluctuations are a possible concern.

This examination of a slightly different form of the Taylor rule is only a tiny fraction of the very general set of rules that could be considered. A growing body of research has emphasized how different assumptions regarding policy rules can affect equilibrium determinacy.[10] An analysis of optimal rules under discretion and commitment by Ascari and Ropele (2004) and Blake and Fernandez-Corugedo (2006) has been performed for a Calvo sticky-price model since the initial drafts of this paper circulated. These authors illustrate that optimal rules are subject to the same effects as documented herein for the Taylor rule. While subsequent research may discover policy rules that perform well in the presence of trend inflation, the results herein remain important given the central role that Taylor-type rules have played in central bank practice according to a number of researchers (e.g., Clarida, Galí, and Gertler 1998, 2000 and Orphanides 2002).

4.5 Empirical Relevance

The analysis has illustrated that the impact of trend inflation on determinacy and volatility may be apparent at quite moderate levels of inflation, suggesting that equilibrium indeterminacy and the instability possible from sunspot fluctuations may be a serious concern for moderate trend inflation. This effect may be relevant for some historical episodes, in light of the different average rates of inflation witnessed in the G-7 since the 1970s.

It is clear from table 1 (in section 2) that average inflation rates in the earlier period were well within the range that can lead to equilibrium indeterminacy or affect macroeconomic volatility from fundamental shocks. This may suggest that the conclusion of Clarida, Galí, and Gertler (2000)—that the failure of policymakers to increase real interest rates in response to increases in expected inflation was a source of aggregate instability in the 1970s—should be reinterpreted. In fact, moderate-to-high inflation can contribute to instability under the Taylor-type rules currently in vogue in two ways—by potentially allowing for sunspot fluctuations and by boosting the volatility of inflation induced by fundamental shocks. Hence, it may have been the combination of the high level of inflation

[10]For example, Benhabib, Schmitt-Grohé, and Uribe (2002) and Carlstrom and Fuerst (2000) present alternative models in which forward-looking policymaking can lead to indeterminate equilibria; the latter authors emphasize the desirability of backward-looking policy, while the former authors highlight an important role for nominal-interest-rate inertia—i.e., a role for lagged interest rates in the policy rule.

and the policy actions attempting to stabilize fluctuations around that high level that contributed to macroeconomic volatility in the 1970s.[11]

We have focused on instability under monetary rules that have been suggested as summaries of the behavior of most inflation-targeting central banks (Black et al. 1997 and Schmidt-Hebbel and Tapias 2002). A substantial number of such inflation targeters, particularly in developing countries, pursue targets that are moderate to high. Lower target inflation rates may contribute to macroeconomic stability.

5. Conclusion

Evidence across time and countries suggests that moderate-to-high inflation tends to be less stable, but it has not been clear from previous work whether this is an intrinsic feature of such regimes. The analysis herein suggests that moderate-to-high inflation may contribute to instability of inflation in two ways. First, moderate-to-high inflation makes equilibrium indeterminacy more likely in a standard New Keynesian model with staggered price setting, raising the possibility of volatility induced by sunspot fluctuations. In addition, the variance of inflation induced by cost-push shocks is higher at higher trend inflation rates (even when the equilibrium is determinate) in the same model. These findings may provide some of the explanation for the evidence across time and countries on trend inflation and inflation volatility.

[11] This result may also be relevant in light of the work of Orphanides (2002), which finds that the Federal Reserve may have been following a forward-looking interest rate rule in the 1970s that satisfied the Taylor principle, but relied too heavily on output-gap estimates that suggested the economy was operating substantially below potential and hence pursued a policy that was excessively loose. To the extent such actions can be characterized as a medium-term shift in the inflation target (a reasonable description, as a persistently large and negative output-gap estimate inserted into a Taylor rule can be expressed algebraically as a rule with the correct output-gap estimate and a new, higher inflation target by simple substitution into equation (8)), Orphanides' conclusions, in conjunction with the result herein that moderate-to-high inflation targets can generate instability, are consistent with the notion that monetary policy contributed to macroeconomic volatility in the 1970s, as argued for different reasons by Clarida, Galí, and Gertler (2000).

Relatedly, these findings may suggest a reinterpretation of the 1970s. Clarida, Galí, and Gertler (2000) and Orphanides (2002) have agreed that monetary policy in the United States was well characterized by a Taylor-type rule at that time, but disagree with respect to whether the Taylor principle was followed and therefore whether volatility was increased by sunspot fluctuations. It has been shown herein that high rates of inflation contribute to the possibility of sunspot fluctuations and, within the model examined, amplify the effect of cost-push shocks on macroeconomic volatility. Given this, the vigorous responses of nominal interest rates to expected inflation called for by Clarida, Galí, and Gertler (2000) may not have been sufficient to deliver significantly increased stability in the 1970s until a commitment to low inflation had been put in place. This result is also relevant for central banks in countries with moderate-to-high inflation that have recently adopted (or are considering for the future) inflation targeting: stability under such regimes is only possible with low trend inflation rates in the New Keynesian model.

The analysis herein was kept as simple as possible to convey the main ideas. Extensions to more fully specified general equilibrium models may be useful, especially consideration of different assumptions regarding labor and product markets on the sensitivity of firms' price-setting behavior to future conditions. It is also important to note that the analysis herein assumed that firms' prices were rigid for two periods. Some recent work has assumed that firms index their nominal prices to inflation (e.g., Christiano, Eichenbaum, and Evans 2005). The main results of this paper do not generalize to that case, as the effect of inflation on relative prices is key to firms' increased sensitivity to future conditions. We do not view such indexation as a plausible characterization of price setting in the developed countries considered for several reasons (the first two were offered by Ascari and Ropele 2004). Survey evidence on price rigidity points to fixed nominal prices without indexation. Gray (1976) demonstrated that full indexation is unlikely to be optimal. And empirical findings such as those in Christiano, Eichenbaum, and Evans (2005)—which suggest a role for indexation across long samples of U.S. history—should be interpreted carefully, as their econometric work clearly mixes data from at least two different periods of trend inflation, as suggested above, while their theoretical model is a linearized approximation of a model around a single rate of trend inflation. The role of indexation found in some past empirical work may have reflected failure to account for a time-varying rate of trend inflation, as suggested in Cogley and Sbordone (2006). Kiley (2007) finds much greater support for a model without indexation in recent decades.

Finally, there is no evidence that the range of inflation experienced in the United States over the postwar period has been associated with differences in price rigidity sufficient to affect the analysis herein, indicating that the mechanisms identified in this study may be operative in similar economies. But the data suggest that nominal price rigidities are less important at much higher rates of inflation (e.g., Kiley 2000).

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