The Mystique of Central Bank Speak*

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Despite the recent trend toward greater transparency of monetary policy, in many respects mystique still prevails in central bank speak. This paper shows that the resulting perception of ambiguity could be desirable. Under the plausible assumption of imperfect common knowledge about the degree of central bank transparency, economic outcomes are affected by both the actual and perceived degree of transparency. It is shown that actual transparency is beneficial, while it may be useful to create the perception of opacity. The optimal communication strategy for the central bank is to provide clarity about the inflation target and to communicate information about the output target and supply shocks with perceived ambiguity. In this respect, the central bank benefits from sustaining transparency misperceptions, which helps to explain the mystique of central bank speak.

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Since I've become a central banker, I've learned to mumble with great incoherence. If I seem unduly clear to you, you must have misunderstood what I said.

(Alan Greenspan, as quoted in the *Wall Street Journal*, September 22, 1987)

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1. Introduction

Central banks have long been associated with secrecy. Even the recent trend toward greater transparency of monetary policy has not dispelled the mystique with which central bankers often speak. This paper provides an economic explanation for the role of oblique communication. Under the plausible assumption that there is imperfect common knowledge about the degree of transparency, economic outcomes are determined by both actual and perceived transparency. It is shown that it may be beneficial to combine actual transparency with perceived opacity. The optimal communication strategy for the central bank is to provide clarity about the inflation target but to provide information with perceived ambiguity about the output-gap target and supply shocks. Thus, the central bank benefits from sustaining transparency misperceptions, which helps to explain why transparency of monetary policy has not eliminated the mystique of central bank speak.

Intuitively, transparency is beneficial, as it reduces private-sector uncertainty. However, transparency can only be achieved through central bank communications that may upset market expectations. Since markets respond strongest to signals that are perceived to be clear, market volatility could be muted by creating a perception of ambiguity.

For both the central bank's inflation target and output target, transparency is shown to be optimal because it reduces erratic responses of market expectations. In addition, it is beneficial to create the perception of transparency regarding the inflation target (e.g., by publishing an explicit numeric target) because it aligns private-sector inflation expectations with the central bank's target. However, it is desirable to create the perception of ambiguity about the output-gap target since it makes it easier to reach the target without upsetting inflation expectations. Similarly, for supply shocks it is useful to combine maximum actual transparency with minimal perceived transparency.

In practice, many central banks have a quantitative inflation target, whereas opacity prevails for output (gap) targets (e.g., Geraats 2006). Furthermore, central bankers tend to be notorious for their "mumbling," as is illustrated by the introductory quote. Alan Greenspan, the former Chairman of the Board of Governors of the

Federal Reserve System, even used the term "constructive ambiguity" to describe his style of communication. This paper establishes that the perception of ambiguity could indeed be a constructive way to achieve transparency because it reduces volatility of market expectations.

This paper builds on two different strands of the transparency literature. There are several papers that model monetary uncertainty faced by the public by making a parameter in the central bank's objective function stochastic, completely abstracting from any communication of information (e.g., Sørensen 1991; Eijffinger, Hoeberichts, and Schaling 2000; Beetsma and Jensen 2003). Such monetary uncertainty directly increases the variability of economic outcomes, although it could also have indirect effects such as lower average inflation.[1] This "monetary uncertainty" literature provides an important argument in favor of transparency—namely, that it reduces private-sector uncertainty and economic volatility.

A second strand of the transparency literature explicitly models information transmission and incorporates the static effect that the information has on the formation of private-sector inflation expectations (e.g., Cukierman 2001; Hahn 2004).[2] In this "information approach," transparency could be detrimental because it leads to greater fluctuations in private-sector expectations and increases economic volatility. In a similar vein, Morris and Shin (2002) find that transparency could generate greater variability when agents disregard private information and rely on a sufficiently noisy public signal to coordinate their actions. A more comprehensive review of the transparency literature is provided in the survey by Geraats (2002).

Other interesting insights on central bank mystique are provided by Goodfriend (1986), who reviews the Federal Reserve's defense of secrecy in response to a Freedom of Information Act suit, including

[1] Sørensen (1991) provides an interesting example. However, it should be noted that many of the other indirect effects reported in this strand of the literature (including those in Eijffinger, Hoeberichts, and Schaling 2000) are spurious due to a biased specification of stochastic relative preferences (Geraats 2004).

[2] A third strand of the literature focuses on the dynamic effect of transparency on reputation (e.g., Faust and Svensson 2001; Jensen 2002; Geraats 2005). In this "reputation approach," transparency about central bank preferences reduces beneficial reputation effects, whereas transparency about economic shocks strengthens them.

the argument that disclosure of information could be prone to misinterpretation and cause inappropriate market reaction. In addition, Winkler (2002) discusses central bank communication and proposes to view transparency in terms of openness, clarity, honesty, and common understanding.

The present paper synthesizes the monetary uncertainty and information approaches. It allows for stochastic central bank preferences, and it features public signals that convey information about those preferences but could also generate undesirable market reactions.

The main innovation of this paper is that it relaxes the ubiquitous assumption of perfect common knowledge about the degree of transparency. This assumption requires perceived and actual stochastic distributions to be identical, which precludes an analysis of the role of transparency (mis)perceptions. Furthermore, in practice it is very difficult for the private sector to know how transparent the central bank actually is because the public cannot observe how much information the central bank withholds. Even if the private sector manages to perfectly predict monetary policy decisions, this need not imply complete transparency, since the forecasts may have been accurate despite asymmetric information about variables relevant for (future) policy decisions. So, it seems more realistic to allow for transparency misperceptions.

This paper deviates from the perfect-common-knowledge assumption by introducing asymmetric information about the degree of transparency. This allows for a discrepancy between actual transparency and private-sector perceptions of it. The result is that both the practice and perceptions of transparency matter for economic outcomes. It is shown that the drawbacks of transparency emphasized by the information approach stem not from the actual reduction of information asymmetries but from private-sector responses induced by transparency perceptions. So, it may be beneficial for perceived transparency to be less than actual transparency. To be precise, although it is best to have perfect actual and perceived transparency about the inflation target, for the output target and supply shocks it is desirable for the central bank to combine actual transparency with perceived opacity.

The remainder of the paper is organized as follows. The baseline model is presented in section 2. First, section 2.1 analyzes the case

with perfect common knowledge about the degree of transparency about the central bank's inflation and output targets. Subsequently, section 2.2 introduces imperfect common knowledge and investigates the role of transparency perceptions. It is shown in section 3 that the main conclusion of the paper—namely, that transparency misperceptions could be optimal—is robust to several extensions of the model, including different objective functions (section 3.1), transparency about supply shocks (section 3.2), and a New Keynesian Phillips curve (section 3.3). Two additional transparency issues are discussed in section 4. In particular, a more comprehensive theoretical measure of transparency is proposed (section 4.1), and various arguments related to monetary mystique are considered (section 4.2). Finally, section 5 concludes that there is an economic rationale for central bank communications that generate perceived opacity and sustain transparency misperceptions.

2. Model

The central bank has the objective function

$$U = -\frac{1}{2}\alpha(\pi - \theta)^2 - \frac{1}{2}(1-\alpha)(y-\kappa)^2, \tag{1}$$

where π denotes inflation, y the output gap, θ the central bank's inflation target, κ the central bank's output-gap target, and α the relative weight on inflation stabilization ($0 < \alpha < 1$). The inflation target θ and output-gap target κ are allowed to be stochastic with $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$ and $\kappa \sim N(\bar{\kappa}, \sigma_\kappa^2)$, and θ and κ independent. The assumption of stochastic shocks to central bank objectives is widespread in the transparency literature, starting with the seminal paper by Cukierman and Meltzer (1986). In addition, the monetary uncertainty approach relies on such preference shocks.[3] Nevertheless, the main result of the present paper also holds for deterministic central bank targets (see section 3.1).

[3]The reputation approach also hinges on uncertainty about central bank preferences (e.g., Faust and Svensson 2001 assume shocks to the central bank's output target).

The economy is described by the expectations-augmented Phillips curve

$$\pi = \pi^e + y + s, \tag{2}$$

where π^e denotes the inflation expectations of the private sector and s is a supply shock, which is assumed to be i.i.d. white noise with variance σ_s^2. For analytical convenience, the slope of the Phillips curve is normalized to one, but this does not affect any of the qualitative conclusions below. For simplicity, it is assumed that the central bank directly controls the output gap y.[4] It would be straightforward to extend the model with an aggregate demand equation that relates the output gap to an interest rate controlled by the central bank, but this would merely clutter the analytical expressions without affecting any of the qualitative results. Furthermore, the key findings of the model also hold for a New Keynesian Phillips curve with persistent supply shocks (see section 3.3).

There are two important information asymmetries between the central bank and the private sector. First, the private sector does not observe the central bank's inflation target θ and output-gap target κ. Instead, it receives the public signals

$$\xi_\theta = \theta + \varepsilon \tag{3}$$

$$\xi_\kappa = \kappa + \eta, \tag{4}$$

where ε and η are i.i.d. white noise, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$, and $\eta \sim N(0, \sigma_\eta^2)$. The noise ε and η stems from the difficulty the private sector has interpreting the central bank's fuzzy communication. When $\sigma_\varepsilon^2 = \sigma_\eta^2 = 0$, the signals ξ_θ and ξ_κ communicate θ and κ without any noise, so the information asymmetry is eliminated and there is perfect transparency about the central bank's targets.

The accuracy of the signals ξ_θ and ξ_κ is described by

$$\tau_\theta = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} \quad \text{and} \quad \tau_\kappa = \frac{\sigma_\kappa^2}{\sigma_\kappa^2 + \sigma_\eta^2}, \tag{5}$$

[4]Alternatively, one could assume a neomonetarist transmission mechanism in which the central bank controls inflation π and faces the Lucas supply equation $y = \pi - \pi^e - s$, but this leads to the same analytical results as for the Keynesian transmission mechanism in the model.

respectively, where $0 \leq \tau_\theta, \tau_\kappa \leq 1$. This measure of the *actual* degree of transparency follows Faust and Svensson (2002), who consider an announcement about a monetary control error. When the signals are completely accurate ($\sigma_\varepsilon^2 = \sigma_\eta^2 = 0$), there is perfect transparency ($\tau_\theta = \tau_\kappa = 1$) about the central bank's targets, which is defined as a situation of symmetric information between the central bank and the private sector. A shortcoming of the transparency measure in (5) is that a constant target ($\sigma_\theta^2 = 0$, $\sigma_\kappa^2 = 0$) implies minimal transparency ($\tau_\theta = 0$, $\tau_\kappa = 0$) regardless of the informativeness of the signal (ξ_θ, ξ_κ). This drawback disappears when private-sector perceptions are allowed to deviate from the actual stochastic distributions.[5]

The second information asymmetry is about the degrees of transparency τ_θ and τ_κ. The public is unsure how transparent the central bank really is. In particular, the public does not know the actual stochastic distributions of θ, κ, ε, and η. Instead, the public uses the perceived (or prior) distributions $\theta \sim N(\bar{\theta}, \tilde{\sigma}_\theta^2)$, $\kappa \sim N(\bar{\kappa}, \tilde{\sigma}_\kappa^2)$, $\varepsilon \sim N(0, \tilde{\sigma}_\varepsilon^2)$, and $\eta \sim N(0, \tilde{\sigma}_\eta^2)$. As a result, the *perceived* degrees of transparency are given by

$$\tilde{\tau}_\theta = \frac{\tilde{\sigma}_\theta^2}{\tilde{\sigma}_\theta^2 + \tilde{\sigma}_\varepsilon^2} \quad \text{and} \quad \tilde{\tau}_\kappa = \frac{\tilde{\sigma}_\kappa^2}{\tilde{\sigma}_\kappa^2 + \tilde{\sigma}_\eta^2}, \tag{6}$$

where $0 \leq \tilde{\tau}_\theta, \tilde{\tau}_\kappa \leq 1$. This (Bayesian) transparency measure does not depend on the actual variances σ_θ^2 and σ_κ^2, so it also applies when the central bank's targets θ and κ are deterministic. Furthermore, it describes transparency from the public's perspective, which makes it more relevant to understanding the behavior of the private sector.

The timing of events is as follows. First, the inflation target θ and output-gap target κ are realized but only observed by the central bank. Subsequently, the private sector receives the public signals ξ_θ and ξ_κ, which are used to rationally form private-sector inflation expectations π^e. Then, the supply shock s is realized and observed by the central bank. Finally, the central bank sets the output gap

[5]The transparency measure in (5) also has the peculiar feature that it is increasing in monetary uncertainty (σ_θ^2, σ_κ^2). This correctly reflects the relative accuracy of the signal (ξ_θ, ξ_κ), but it is an odd implication for a transparency measure. A more general measure of transparency that does not suffer from this shortcoming is presented in section 4.1.

y, and the level of inflation π is realized. This timing implies that monetary policy is conducted under discretion.

The central bank maximizes the expected value of its objective (1) with respect to y subject to the Phillips curve (2) and given private-sector inflation expectations π^e. This yields the optimal output gap

$$y = \alpha(\theta - \pi^e - s) + (1 - \alpha)\kappa. \tag{7}$$

The output gap is increasing in the central bank's inflation target θ and output-gap target κ as the central bank pursues expansionary policy to attempt to reach the targets. In addition, higher private-sector inflation expectations π^e cause the central bank to reduce the output gap to achieve price stability, and the same holds for a higher supply shock s. Substituting (7) into (2) produces the level of inflation

$$\pi = \alpha\theta + (1 - \alpha)(\pi^e + \kappa + s). \tag{8}$$

This gives rise to the standard result that inflation is increasing in the inflation target θ, the output-gap target κ, private-sector inflation expectations π^e, and the supply shock s.

To fully understand the role of the two information asymmetries in the formation of the private sector's inflation expectations, we assume in section 2.1 that the private sector only has asymmetric information about the central bank's inflation target θ and output-gap target κ, but has perfect common knowledge about the actual degrees of central bank transparency τ_θ and τ_κ. Then, in section 2.2, the assumption of asymmetric information about the degree of transparency is added and the role of transparency (mis)perceptions is analyzed.

2.1 Perfect Common Knowledge

The private sector has rational expectations, so it uses all available information, including the public signals ξ_θ and ξ_κ, to form its inflation expectations π^e. Taking expectations of (8) and solving for π^e gives

$$\pi^e = \mathrm{E}[\pi|\xi_\theta, \xi_\kappa] = \mathrm{E}[\theta|\xi_\theta] + \frac{1 - \alpha}{\alpha}\mathrm{E}[\kappa|\xi_\kappa], \tag{9}$$

using the fact that ξ_κ is uninformative about θ and ξ_θ is uninformative about κ. Private-sector inflation expectations depend on the private sector's expectations of the central bank's inflation target θ and output-gap target κ, which it attempts to infer from the public signals ξ_θ and ξ_κ. Using (3), (4), and (5),[6]

$$\mathrm{E}[\theta|\xi_\theta] = \bar{\theta} + \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}(\xi_\theta - \bar{\theta}) = (1 - \tau_\theta)\bar{\theta} + \tau_\theta \xi_\theta \tag{10}$$

$$\mathrm{E}[\kappa|\xi_\kappa] = \bar{\kappa} + \frac{\sigma_\kappa^2}{\sigma_\kappa^2 + \sigma_\eta^2}(\xi_\kappa - \bar{\kappa}) = (1 - \tau_\kappa)\bar{\kappa} + \tau_\kappa \xi_\kappa. \tag{11}$$

The private sector faces a signal extraction problem, and its expectation of θ (κ) equals a weighted average of its prior belief $\bar{\theta}$ ($\bar{\kappa}$) and the public signal ξ_θ (ξ_κ). For a higher degree of transparency τ_θ (τ_κ), the public signal ξ_θ (ξ_κ) is relatively more informative, so the private sector attaches greater weight to it. In the case of perfect transparency, $\tau_\theta = \tau_\kappa = 1$ and $\sigma_\varepsilon^2 = \sigma_\eta^2 = 0$, so the inflation target and output-gap target are perfectly inferred: $\mathrm{E}[\theta|\xi_\theta] = \xi_\theta = \theta$ and $\mathrm{E}[\kappa|\xi_\kappa] = \xi_\kappa = \kappa$. In the case of complete opacity ($\tau_\theta = \tau_\kappa = 0$), the private sector rationally ignores the signals so that $\mathrm{E}[\theta|\xi_\theta] = \bar{\theta}$ and $\mathrm{E}[\kappa|\xi_\kappa] = \bar{\kappa}$. Substituting (10) and (11) into (9) and using (3) and (4) gives

$$\pi^e = \bar{\theta} + \tau_\theta(\theta - \bar{\theta}) + \tau_\theta \varepsilon + \frac{1 - \alpha}{\alpha}[\bar{\kappa} + \tau_\kappa(\kappa - \bar{\kappa}) + \tau_\kappa \eta]. \tag{12}$$

The private sector's inflation expectations are determined by its prior expectations $\bar{\theta}$ and $\bar{\kappa}$ of the central bank's targets, the deviations of the central bank's targets from the private sector's priors, and the noise ε and η in the public signals. The latter shows how misinterpretation of monetary policy communications causes inappropriate market reaction. The variability of private-sector inflation expectations depends on the degrees of transparency. In particular,

$$\mathrm{Var}[\pi^e] = \tau_\theta \sigma_\theta^2 + \left(\frac{1 - \alpha}{\alpha}\right)^2 \tau_\kappa \sigma_\kappa^2,$$

[6]This uses the fact that for two jointly normally distributed variables x and z, $\mathrm{E}[x|z] = \mathrm{E}[x] + \frac{\mathrm{Cov}\{x,z\}}{\mathrm{Var}[z]}(z - \mathrm{E}[z])$.

using the fact that (5) implies $\sigma_\varepsilon^2 = \frac{1-\tau_\theta}{\tau_\theta}\sigma_\theta^2$ and $\sigma_\eta^2 = \frac{1-\tau_\kappa}{\tau_\kappa}\sigma_\kappa^2$. This shows that inflation expectations π^e are most stable when the central bank is least transparent ($\tau_\theta = \tau_\kappa = 0$). Intuitively, the complete lack of transparency makes the public signal so noisy that the public no longer relies on it and only uses its prior expectations.[7]

Substituting (12) into (7) and using (2) gives the levels of the output gap y and inflation π:

$$y = \alpha[(1-\tau_\theta)(\theta-\bar{\theta}) - \tau_\theta\varepsilon] + (1-\alpha)[(1-\tau_\kappa)(\kappa-\bar{\kappa}) - \tau_\kappa\eta] - \alpha s \tag{13}$$

$$\pi = \bar{\theta} + (\alpha + (1-\alpha)\tau_\theta)(\theta-\bar{\theta}) + (1-\alpha)\tau_\theta\varepsilon + \frac{1-\alpha}{\alpha}[\bar{\kappa} + (\alpha + (1-\alpha)\tau_\kappa)(\kappa-\bar{\kappa}) + (1-\alpha)\tau_\kappa\eta] + (1-\alpha)s. \tag{14}$$

The output gap and inflation depend on the central bank's targets θ and κ, the private sector's priors $\bar{\theta}$ and $\bar{\kappa}$, the signal noise ε and η, and the supply shock s. Although the degrees of transparency τ_θ and τ_κ influence the output gap and inflation, they have no effect on the expected values $\mathrm{E}[y]$ and $\mathrm{E}[\pi]$. In the case of perfect transparency ($\tau_\theta = \tau_\kappa = 1$, so $\varepsilon = \eta = 0$), the expressions simplify to $y = -\alpha s$ and $\pi = \theta + (1-\alpha)(\kappa + \alpha s)/\alpha$, which gives the familiar rational expectations outcome that the targets θ and κ only affect inflation and do not influence output.

The variability of the output gap and inflation is given by

$$\mathrm{Var}[y] = \alpha^2(1-\tau_\theta)\sigma_\theta^2 + (1-\alpha)^2(1-\tau_\kappa)\sigma_\kappa^2 + \alpha^2\sigma_s^2$$

$$\mathrm{Var}[\pi] = [\alpha^2 + (1-\alpha^2)\tau_\theta]\sigma_\theta^2 + \frac{(1-\alpha)^2}{\alpha^2}[\alpha^2 + (1-\alpha^2)\tau_\kappa]\sigma_\kappa^2 + (1-\alpha)^2\sigma_s^2,$$

where (5) is used to substitute for σ_ε^2 and σ_η^2. This shows that the output gap is most stable when the central bank is perfectly transparent ($\tau_\theta = \tau_\kappa = 1$). The reason is that greater transparency

[7]This case in which private-sector expectations do not incorporate any communications resembles the monetary uncertainty literature mentioned in section 1. It features deterministic private-sector inflation expectations π^e, and the degree of monetary uncertainty is described by σ_θ^2 and σ_κ^2.

makes private-sector inflation expectations more sensitive to the central bank's targets. For a change in the inflation target, the stronger response of private-sector inflation expectations means that a smaller adjustment of the output gap is required to reach the inflation target. For a change in the output-gap target, the output gap is adjusted by less because the larger shift in inflation expectations hampers inflation stabilization.[8] However, inflation is most stable when the central bank is least transparent ($\tau_\theta = \tau_\kappa = 0$). This is due to the greater stability of private-sector inflation expectations.

To determine the optimal degrees of transparency, substitute (8) and (7) into (1), use (12), and rearrange to get

$$\begin{aligned} U &= -\frac{1}{2}\alpha(1-\alpha)(\pi^e - \theta + \kappa + s)^2 \qquad (15)\\ &= -\frac{1}{2}\frac{1-\alpha}{\alpha}[\alpha(\tau_\theta - 1)(\theta - \bar{\theta}) + \alpha\tau_\theta\varepsilon + \bar{\kappa} + (\alpha + (1-\alpha)\tau_\kappa)(\kappa - \bar{\kappa})\\ &\quad + (1-\alpha)\tau_\kappa\eta + \alpha s]^2. \end{aligned}$$

When there is imperfect transparency about the inflation target ($\tau_\theta \neq 1$), the deviation between the actual target θ and the private sector's prior expectation $\bar{\theta}$ affects the level of U. The prior expectation $\bar{\kappa}$ also matters, unless there is perfect transparency about the output-gap target ($\tau_\kappa = 1$). So, the outcome is distorted when there is incomplete transparency.

Taking unconditional expectations of (15) and substituting for σ_ε^2 and σ_η^2 using (5) gives the ex ante expected central bank payoff

$$\mathrm{E}[U] = -\frac{1}{2}\frac{1-\alpha}{\alpha}\left[\alpha^2(1-\tau_\theta)\sigma_\theta^2 + \bar{\kappa}^2 + \left(\alpha^2 + (1-\alpha^2)\tau_\kappa\right)\sigma_\kappa^2 + \alpha^2\sigma_s^2\right].$$

As a result, it would be optimal to have maximum transparency about the inflation target ($\tau_\theta = 1$) and minimal transparency about the output-gap target ($\tau_\kappa = 0$). Although transparency about the inflation target increases the variance of inflation, this drawback is dominated by the benefits that transparency makes the output gap more stable and brings inflation closer to the inflation target. In

[8] For the neomonetarist transmission mechanism with a Lucas supply equation, the intuition is that greater transparency reduces inflation surprises, which makes the output gap more stable.

addition, opacity about the output-gap target makes the output gap more volatile, but this disadvantage is more than offset by the greater stability of inflation and the smaller deviation between the output gap and its target. The optimality of opacity about the output-gap target is similar in spirit to the result in the seminal paper by Cukierman and Meltzer (1986), where ambiguity about the output preference parameter allows the central bank to successfully stimulate output when it is most desirable. Cukierman and Meltzer (1986) assume that ambiguity is created through monetary control errors, whereas the present paper assumes perfect control over the monetary policy instrument but opacity caused by imperfect communications.

The following proposition summarizes the key results.

PROPOSITION 1. *When there is asymmetric information about the central bank's inflation target θ and output-gap target κ, and perfect common knowledge about the degree of central bank transparency τ_θ and τ_κ,*

(i) greater transparency (τ_θ and/or τ_κ) increases the variability of private-sector inflation expectations π^e and inflation π, but reduces the volatility of the output gap y; and

(ii) it is optimal to have maximum transparency about the inflation target ($\tau_\theta = 1$) and minimal transparency about the output target ($\tau_\kappa = 0$).

In section 2.2, the assumption of perfect common knowledge about the degree of transparency is relaxed, allowing for a difference between actual and perceived transparency.

2.2 Transparency Misperceptions

The assumption of perfect common knowledge about transparency has the critical drawback that private-sector perceptions are restricted to be determined by the actual volatilities σ_θ^2, σ_κ^2, σ_ε^2, and σ_η^2. This is problematic because it is hard for the private sector to establish how transparent the central bank actually is. For instance, what is the noise σ_η^2 associated with a central banker's speech? It could easily vary, which means that the public is unlikely to know

the level of transparency τ. So, it is realistic to allow for imperfect common knowledge about the degree of transparency. This has the virtue that it decouples private-sector perceptions of uncertainty from actual stochastic volatility.[9]

In contrast to section 2.1, assume now that the private sector does not know the actual stochastic distribution of the central bank's inflation target θ and output-gap target κ, and the noise ε and η. Instead, it uses the perceived (or prior) distributions $\theta \sim N(\bar{\theta}, \tilde{\sigma}_\theta^2)$, $\kappa \sim N(\bar{\kappa}, \tilde{\sigma}_\kappa^2)$, $\varepsilon \sim N(0, \tilde{\sigma}_\varepsilon^2)$, and $\eta \sim N(0, \tilde{\sigma}_\eta^2)$. This gives rise to the perceived degrees of transparency $\tilde{\tau}_\theta$ and $\tilde{\tau}_\kappa$ in (6).

Transparency perceptions do not affect the central bank's optimization problem, so (7) and (8) continue to hold. In addition, the private sector still receives the public signals (3) and (4), which it uses to rationally form its inflation expectations $\pi^e = \widetilde{\mathrm{E}}[\pi|\xi]$, where $\widetilde{\mathrm{E}}[.]$ denotes the private-sector expectation based on the perceived distributions of θ, κ, ε, and η. But the signal-extraction process is affected by private-sector perceptions. To be precise, (10) and (11) are replaced by

$$\widetilde{\mathrm{E}}[\theta|\xi_\theta] = (1 - \tilde{\tau}_\theta)\bar{\theta} + \tilde{\tau}_\theta \xi_\theta \tag{16}$$

$$\widetilde{\mathrm{E}}[\kappa|\xi_\kappa] = (1 - \tilde{\tau}_\kappa)\bar{\kappa} + \tilde{\tau}_\kappa \xi_\kappa. \tag{17}$$

So, with imperfect common knowledge about the degree of transparency, it is the perceived transparency $\tilde{\tau}_\theta$ and $\tilde{\tau}_\kappa$ that matters for the updating of private-sector expectations. As a result, private-sector inflation expectations now equal

$$\pi^e = \bar{\theta} + \tilde{\tau}_\theta(\theta - \bar{\theta}) + \tilde{\tau}_\theta \varepsilon + \frac{1-\alpha}{\alpha}[\bar{\kappa} + \tilde{\tau}_\kappa(\kappa - \bar{\kappa}) + \tilde{\tau}_\kappa \eta]. \tag{18}$$

The variability of private-sector inflation expectations depends on the perceived degrees of transparency $\tilde{\tau}_\theta$ and $\tilde{\tau}_\kappa$. But now there are two measures of variability: $\widetilde{\mathrm{Var}}[.]$ is based on the perceived stochastic distribution of θ, κ, ε, and η, and measures private-sector

[9] In a perceptive contribution, Hahn (2004) aims to analyze transparency about the central bank's relative preference weight α independently of the stochastic distribution of α. However, the private sector's ex ante distribution and the actual distribution of α are assumed to be the same, so there is no effective separation.

uncertainty (ex ante); and Var[.] is based on the actual stochastic distribution of θ, κ, ε, and η, and measures average volatility (ex post).

The perceived variance of private-sector inflation expectations equals

$$\widetilde{\mathrm{Var}}[\pi^e] = \tilde{\tau}_\theta \tilde{\sigma}_\theta^2 + \left(\frac{1-\alpha}{\alpha}\right)^2 \tilde{\tau}_\kappa \tilde{\sigma}_\kappa^2,$$

using the fact that (6) implies $\tilde{\sigma}_\varepsilon^2 = \frac{1-\tilde{\tau}_\theta}{\tilde{\tau}_\theta}\tilde{\sigma}_\theta^2$ and $\tilde{\sigma}_\eta^2 = \frac{1-\tilde{\tau}_\kappa}{\tilde{\tau}_\kappa}\tilde{\sigma}_\kappa^2$. This shows that private-sector uncertainty about inflation expectations is smallest when the central bank is perceived to be least transparent ($\tilde{\tau}_\theta = \tilde{\tau}_\kappa = 0$). The reason is that the perceived lack of transparency makes the public signals ξ_θ and ξ_κ unreliable, so the private sector only uses its prior expectations $\bar{\theta}$ and $\bar{\kappa}$.

The actual variance of private-sector inflation expectations equals

$$\mathrm{Var}[\pi^e] = \frac{\tilde{\tau}_\theta^2}{\tau_\theta}\sigma_\theta^2 + \left(\frac{1-\alpha}{\alpha}\right)^2 \frac{\tilde{\tau}_\kappa^2}{\tau_\kappa}\sigma_\kappa^2,$$

using the fact that (5) implies $\sigma_\varepsilon^2 = \frac{1-\tau_\theta}{\tau_\theta}\sigma_\theta^2$ and $\sigma_\eta^2 = \frac{1-\tau_\kappa}{\tau_\kappa}\sigma_\kappa^2$. This shows that the volatility of private-sector inflation expectations is increasing in perceived transparency $\tilde{\tau}_\theta$ and $\tilde{\tau}_\kappa$, and decreasing in actual transparency τ_θ and τ_κ. Intuitively, lower perceived transparency causes the private sector to rely less on the noisy public signals (ξ_θ and ξ_κ), and greater actual transparency reduces the variance of the noise (σ_ε^2 and σ_η^2), both making inflation expectations π^e less volatile.

Substituting (18) into (7) and using (2) gives the levels of the output gap y and inflation π for transparency perceptions $\tilde{\tau}$:

$$y = \alpha[(1-\tilde{\tau}_\theta)(\theta-\bar{\theta}) - \tilde{\tau}_\theta\varepsilon] + (1-\alpha)[(1-\tilde{\tau}_\kappa)(\kappa-\bar{\kappa}) - \tilde{\tau}_\kappa\eta] - \alpha s \tag{19}$$

$$\pi = \bar{\theta} + (\alpha + (1-\alpha)\tilde{\tau}_\theta)(\theta-\bar{\theta}) + (1-\alpha)\tilde{\tau}_\theta\varepsilon + \frac{1-\alpha}{\alpha}[\bar{\kappa} + (\alpha + (1-\alpha)\tilde{\tau}_\kappa)(\kappa-\bar{\kappa}) + (1-\alpha)\tilde{\tau}_\kappa\eta] + (1-\alpha)s. \tag{20}$$

These expressions are identical to their counterparts under common knowledge, (13) and (14), except that the actual degrees of

transparency τ_θ and τ_κ are replaced by the perceived degrees of transparency $\tilde{\tau}_\theta$ and $\tilde{\tau}_\kappa$. The same holds for $\widetilde{\text{Var}}[y]$ and $\widetilde{\text{Var}}[\pi]$ when σ_θ^2 and σ_κ^2 are also replaced by $\tilde{\sigma}_\theta^2$ and $\tilde{\sigma}_\kappa^2$, so the perceived variances only depend on private-sector perceptions. The actual variance is equal to

$$\text{Var}[y] = \alpha^2\left(1 - 2\tilde{\tau}_\theta + \frac{\tilde{\tau}_\theta^2}{\tau_\theta}\right)\sigma_\theta^2 + (1-\alpha)^2\left(1 - 2\tilde{\tau}_\kappa + \frac{\tilde{\tau}_\kappa^2}{\tau_\kappa}\right)\sigma_\kappa^2 + \alpha^2\sigma_s^2$$

$$\text{Var}[\pi] = \left[\alpha^2 + 2\alpha(1-\alpha)\tilde{\tau}_\theta + (1-\alpha)^2\frac{\tilde{\tau}_\theta^2}{\tau_\theta}\right]\sigma_\theta^2 + \frac{(1-\alpha)^2}{\alpha^2} \times \left[\alpha^2 + 2\alpha(1-\alpha)\tilde{\tau}_\kappa + (1-\alpha)^2\frac{\tilde{\tau}_\kappa^2}{\tau_\kappa}\right]\sigma_\kappa^2 + (1-\alpha)^2\sigma_s^2,$$

where (5) is used to substitute for σ_ε^2 and σ_η^2. The variability of the output gap and inflation depends on both the perceived and actual degrees of transparency. In the special case in which $\tilde{\tau}_\theta = \tau_\theta$ and $\tilde{\tau}_\kappa = \tau_\kappa$, the common-knowledge results in section 2.1 are obtained. With imperfect common knowledge, the volatility of the output gap is decreasing in actual transparency τ_θ and τ_κ, and is minimized for $\tilde{\tau}_\theta = \tau_\theta = 1$ and $\tilde{\tau}_\kappa = \tau_\kappa = 1$.[10] The variability of inflation is also decreasing in actual transparency τ_θ and τ_κ, but increasing in perceived transparency $\tilde{\tau}_\theta$ and $\tilde{\tau}_\kappa$. Intuitively, greater transparency corresponds to fewer inflation surprises and therefore more output-gap stability, whereas lower perceived and higher actual transparency reduces the volatility of private-sector expectations and thereby the variance of inflation.

To derive the optimal degrees of actual and perceived transparency, substitute (18) into (15) and rearrange to get

$$U = -\frac{1}{2}\frac{1-\alpha}{\alpha}[\alpha(\tilde{\tau}_\theta - 1)(\theta - \bar{\theta}) + \alpha\tilde{\tau}_\theta\varepsilon + \bar{\kappa} + (\alpha + (1-\alpha)\tilde{\tau}_\kappa)(\kappa - \bar{\kappa}) + (1-\alpha)\tilde{\tau}_\kappa\eta + \alpha s]^2. \quad (21)$$

This is identical to the expression under common knowledge, except that τ_θ and τ_κ are replaced by $\tilde{\tau}_\theta$ and $\tilde{\tau}_\kappa$, respectively. It shows that

[10] Formally, these results follow from differentiating Var[y] with respect to τ_θ, τ_κ, $\tilde{\tau}_\theta$, and $\tilde{\tau}_\kappa$.

in the presence of transparency misperceptions, it is the lack of perceived transparency that causes the prior expectations $\bar{\theta}$ and $\bar{\kappa}$ to exert their influence on the outcome, regardless of the stochastic distribution of the central bank targets.

Taking expectations using the distributions perceived by the private sector yields

$$\widetilde{\mathrm{E}}[U] = -\frac{1}{2}\frac{1-\alpha}{\alpha}\left[\alpha^2(1-\tilde{\tau}_\theta)\tilde{\sigma}_\theta^2 + \bar{\kappa}^2 + (\alpha^2 + (1-\alpha^2)\tilde{\tau}_\kappa)\tilde{\sigma}_\kappa^2 + \alpha^2\tilde{\sigma}_s^2\right].$$

This reflects the ex ante expectation based on private-sector perceptions. It is the same as the expression for $\mathrm{E}[U]$ under common knowledge after replacing τ with $\tilde{\tau}$ and σ^2 with $\tilde{\sigma}^2$.

Taking unconditional expectations based on the actual distributions and substituting for σ_ε^2 and σ_η^2 using (5) yields

$$\mathrm{E}[U] = -\frac{1}{2}\frac{1-\alpha}{\alpha}\left[\alpha^2\left(1 - 2\tilde{\tau}_\theta + \frac{\tilde{\tau}_\theta^2}{\tau_\theta}\right)\sigma_\theta^2 + \bar{\kappa}^2 + \left(\alpha^2 + 2\alpha(1-\alpha)\tilde{\tau}_\kappa + (1-\alpha)^2\frac{\tilde{\tau}_\kappa^2}{\tau_\kappa}\right)\sigma_\kappa^2 + \alpha^2\sigma_s^2\right].$$

This reflects the central bank's ex ante expectation, and it corresponds to the average ex post experience. It shows that $\mathrm{E}[U]$ is increasing in the actual degrees of transparency τ_θ and τ_κ, so that perfect transparency is optimal ($\tau_\theta = \tau_\kappa = 1$). In addition, $\mathrm{E}[U]$ is maximized for $\tilde{\tau}_\theta = \tau_\theta$ and $\tilde{\tau}_\kappa = 0$.[11] So, it is best to have complete perceived and actual transparency about the inflation target ($\tilde{\tau}_\theta = \tau_\theta = 1$) but maximum actual transparency ($\tau_\kappa = 1$) and minimal perceived transparency ($\tilde{\tau}_\kappa = 0$) about the output-gap target. Intuitively, it is desirable to have actual transparency about the central bank's targets because it avoids erratic reactions of private-sector expectations. Furthermore, it is beneficial to have perceived transparency about the inflation target so that private-sector inflation expectations are more responsive and become more closely aligned with the inflation target. However, perceived transparency

[11] Formally, $\partial\mathrm{E}[U]/\partial\tilde{\tau}_\theta = -\alpha(1-\alpha)\frac{\tilde{\tau}_\theta - \tau_\theta}{\alpha}\sigma_\theta^2$ and $\partial^2\mathrm{E}[U]/\partial\tilde{\tau}_\theta^2 < 0$ imply that $\tilde{\tau}_\theta = \tau_\theta$ is optimal, and $\partial\mathrm{E}[U]/\partial\tilde{\tau}_\kappa = -\frac{(\tau_\theta\alpha)^2}{\alpha}(\alpha + (1-\alpha)\frac{\tilde{\tau}_\kappa}{\tau_\kappa})\sigma_\kappa^2 < 0$ implies the corner solution $\tilde{\tau}_\kappa = 0$.

about the output-gap target is detrimental because the response of private-sector inflation expectations hampers the achievement of the inflation and output-gap targets.

This shows that the optimal communication strategy is different for the central bank's inflation and output-gap targets. It is best to be transparent and unambiguously clear about the inflation target. But for the output-gap target, it is desirable to provide information with perceived ambiguity.

The following proposition summarizes the results.

PROPOSITION 2. *When there is asymmetric information about the central bank's inflation target θ and output-gap target κ, and about the degree of central bank transparency τ_θ and τ_κ,*

(i) greater actual transparency (τ_θ and/or τ_κ) reduces the variability of private-sector inflation expectations π^e, inflation π, and the output gap y;

(ii) greater perceived transparency ($\tilde{\tau}_\theta$ and/or $\tilde{\tau}_\kappa$) increases the volatility of private-sector inflation expectations π^e and inflation π, whereas the output gap is most stable in the absence of transparency misperceptions ($\tilde{\tau}_\theta = \tau_\theta$ and $\tilde{\tau}_\kappa = \tau_\kappa$); and

(iii) it is optimal to have maximum actual and perceived transparency about the inflation target ($\tau_\theta = \tilde{\tau}_\theta = 1$) and maximum actual transparency but minimal perceived transparency about the output-gap target ($\tau_\kappa = 1$, $\tilde{\tau}_\kappa = 0$).

A comparison with proposition 1 reveals that the detrimental effects of transparency under common knowledge—namely, greater inflation volatility and the optimality of opacity about the output-gap target—are not due to the actual degree of transparency but to the private sector's perceptions of it. The fact that the public is actually better informed is beneficial, but the stronger response of private-sector expectations driven by greater perceived transparency leads to undesirable inflation volatility and makes it more difficult for the central bank to reach its inflation and output-gap targets.

3. Extensions

It is important to assess the robustness of the results above, so several extensions are analyzed in this section. In particular, it is shown that transparency misperceptions could also be optimal for different objective functions, including "conservative" central banks and deterministic central bank targets (section 3.1), for transparency about supply shocks (section 3.2), and for a New Keynesian Phillips curve (section 3.3).

3.1 Objective Functions

Propositions 1(i) and 2(ii) show that transparency (perceptions) could have different effects on inflation and output-gap variability, which may give the impression that the desirability of transparency depends on the weight attached to inflation versus output-gap stabilization. To explore this issue, suppose that the central bank's objective remains (1) but that social welfare is given by

$$W = -\frac{1}{2}\beta(\pi - \theta)^2 - \frac{1}{2}(1-\beta)(y-\kappa)^2, \tag{22}$$

where $0 < \beta < 1$. So, monetary policy has been delegated to a central bank with a different relative preference weight. For instance, $\alpha > \beta$ would amount to a "conservative" central bank that is more concerned about inflation stabilization than society (Rogoff 1985). Interestingly, the degrees of transparency given in propositions 1(ii) and 2(iii) that are optimal for the central bank are also socially optimal, regardless of the weight β. More precisely, both $\mathrm{E}[U]$ and $\mathrm{E}[W]$ are maximized for $\tau_\theta = 1$ and $\tau_\kappa = 0$ under common knowledge, and for $\tilde{\tau}_\theta = \tau_\theta = \tau_\kappa = 1$ and $\tilde{\tau}_\kappa = 0$ with transparency misperceptions.[12] The reason that β is immaterial is that social welfare is not determined by $\mathrm{Var}[y]$ and $\mathrm{Var}[\pi]$ but by $\mathrm{E}[(\pi-\theta)^2]$ and $\mathrm{E}[(y-\kappa)^2]$. The latter are always proportional when the central bank behaves optimally according to (7) and (8), so transparency affects them in the same way.

[12]To see this, substitute (7) and (8) into (22) and rearrange to get $W = -\frac{1}{2}(\beta(1-\alpha)^2 + (1-\beta)\alpha^2)(\pi^e - \theta + \kappa + s)^2$. This is directly proportional to (15) so that $\mathrm{E}[W]$ is maximized for the same degrees of transparency as $\mathrm{E}[U]$.

Suppose now that monetary policy is still delegated to a central bank that maximizes (1) but that the social welfare function equals

$$W = -\frac{1}{2}\beta(\pi - \bar{\theta})^2 - \frac{1}{2}(1-\beta)(y - \bar{\kappa})^2. \tag{23}$$

So, again, the central bank attaches a different weight to inflation stabilization. In addition, although the targets of the central bank (θ and κ) and society ($\bar{\theta}$ and $\bar{\kappa}$) are the same on average, they typically differ due to idiosyncratic shocks ($\theta \neq \bar{\theta}$ and $\kappa \neq \bar{\kappa}$). This variation on the basic model is analyzed in the appendix, section A.1. With perfect common knowledge, the degree of transparency that is socially optimal now depends on β. To be precise, $\tau_\theta = \tau_\kappa = 1$ is socially optimal for $\alpha^2 > \beta$, and $\tau_\theta = \tau_\kappa = 0$ for $\alpha^2 < \beta$. In other words, if the central bank is sufficiently conservative, the social optimum is transparency. Intuitively, if society cares a lot about output-gap stabilization, the benefit of greater output-gap stability under transparency outweighs the drawback of more inflation variability. This result is similar to Hahn (2004), who considers transparency about the central bank's relative preference weight α.

With imperfect common knowledge, perfect actual transparency about the central bank's targets ($\tau_\theta = \tau_\kappa = 1$) is socially optimal regardless of the value of β. The reason is that transparency avoids erratic movements of market expectations. Regarding perceived transparency, if the central bank is not conservative ($\alpha \leq \beta$), society benefits from complete perceived opacity ($\tilde{\tau}_\theta = \tilde{\tau}_\kappa = 0$). Furthermore, for any other β, the degree of perceived transparency in the social optimum is strictly positive but remains less than the degree of actual transparency ($0 < \tilde{\tau}_\theta < \tau_\theta$ and $0 < \tilde{\tau}_\kappa < \tau_\kappa$). Intuitively, the perception of opacity reduces the response of market expectations to noise in the signal and therefore limits volatility.

Another issue is whether the conclusions depend on the assumption that the central bank's inflation and output-gap targets follow a normal distribution. In particular, the expressions for $\mathrm{E}[U]$ in section 2 give the impression that the degrees of actual and perceived transparency τ and $\tilde{\tau}$ are immaterial when the targets θ and κ are deterministic ($\sigma_\theta^2 = \sigma_\kappa^2 = 0$). The case of constant central bank targets is more closely examined in the appendix, section A.2. This reveals that it is optimal to have complete perceived opacity about both targets ($\tilde{\tau}_\theta = \tilde{\tau}_\kappa = 0$) but maximum actual transparency in the

sense of minimally noisy signals ($\sigma_\varepsilon^2 = \sigma_\eta^2 = 0$). Intuitively, noisy signals lead to inflation and output-gap variability, but this effect is muted when the signals are perceived to be opaque so that the private sector pays less attention to them. So, again, it is desirable to have maximum actual transparency but to sustain transparency misperceptions such that perceived opacity exceeds actual opacity.

3.2 Transparency about Supply Shocks

Another interesting extension is to consider transparency about the supply shock s. In particular, suppose that the private sector receives a public signal of the supply shock before it forms its inflation expectations π^e. This is analyzed in the appendix, section A.3. In the case of perfect common knowledge, greater transparency τ_s about the supply shock s increases the volatility of both the output gap and inflation. Intuitively, greater transparency about the supply shock makes private-sector inflation expectations π^e more sensitive to the supply shock s, so the central bank increases the output-gap response to partially offset the increased volatility of inflation. Not surprisingly, minimal transparency about supply shocks ($\tau_s = 0$) is optimal. This result is consistent with Cukierman (2001), who compares limited ($\tau_s = 0$) and full ($\tau_s = 1$) transparency about the supply shock s in a model with a neomonetarist transmission mechanism.

With imperfect common knowledge about the degree of transparency τ_s, the variance of the output gap y and inflation π are both minimized for minimal perceived transparency ($\tilde{\tau}_s = 0$) and maximum actual transparency ($\tau_s = 1$). The intuition behind this result is familiar. Minimal perceived transparency mutes the response of private-sector expectations π^e to the supply shock s, which contributes to greater stability of the output gap and inflation. In addition, maximum actual transparency reduces the noise of the public signal, which makes inflation expectations more stable and thereby generates less volatility in the output gap and inflation. Not surprisingly, it is (socially) optimal to have minimal perceived and maximum actual transparency about supply shocks ($\tilde{\tau}_s = 0$ and $\tau_s = 1$).

So, the most effective communication strategy for supply shocks is to provide all the relevant information but to downplay its relevance. Perhaps this could explain why some central banks (e.g.,

the European Central Bank) stress that the quarterly macroeconomic forecasts they publish are staff forecasts that come without any endorsement by the monetary policymakers.

3.3 New Keynesian Phillips Curve

Finally, it is important to discuss to what extent the results extend to a New Keynesian Phillips curve. The baseline model assumes the expectations-augmented Phillips curve

$$\pi = \widetilde{\mathrm{E}}[\pi|\xi] + y + s,$$

where inflation expectations incorporate information from the public signal ξ about shocks affecting (current) inflation. With the New Keynesian Phillips curve

$$\pi_t = \widetilde{\mathrm{E}}_t[\pi_{t+1}|\xi_t] + y_t + s_t, \tag{24}$$

inflation expectations incorporate information from ξ_t about shocks affecting future inflation. If the shocks are (perceived to be) i.i.d., the signal ξ_t is (considered) uninformative about future shocks, and the (perceived) degree of transparency is immaterial. But for the more plausible case in which the shocks are (perceived to be) persistent, ξ_t is (considered) informative about both the current and the future shocks affecting inflation, so the effect on inflation expectations is similar for the expectations-augmented Phillips curve and the New Keynesian Phillips curve.

This is formally shown in the appendix, section A.4, for transparency about supply shocks when the central bank targets are deterministic.[13] In particular, the optimal (actual and perceived) transparency about supply shocks s_t is derived analytically for the New Keynesian Phillips curve (24) in an infinite-horizon model with discretionary monetary policy and commitment to a communication

[13] Allowing for asymmetric information about the central bank targets would greatly complicate the analysis with a New Keynesian Phillips curve. Even for the simple two-period model by Jensen (2002), no closed-form solution exists, and the optimal degree of transparency has to be computed numerically.

technology with a particular degree of (actual and perceived) transparency.[14] For i.i.d. supply shocks, transparency about the supply shock s_t does not affect economic outcomes, because information about current supply shocks has no effect on forward-looking inflation expectations. But for persistent supply shocks, the effect of (actual and perceived) transparency about supply shocks is qualitatively the same as for the model with the expectations-augmented Phillips curve. In the case of perfect common knowledge about the degree of transparency, greater transparency about the supply shock s_t increases the variability of inflation π_t and the output gap y_t, so minimal transparency is optimal. In the presence of transparency misperceptions, greater actual and smaller perceived transparency about the supply shock s_t reduce the variability of inflation π_t and the output gap y_t, so it is optimal to have maximum actual transparency but minimal perceived transparency. These results for the New Keynesian Phillips curve (24) are exactly the same as for the model with the expectations-augmented Phillips curve (2) in section 3.2. So, as long as shocks (are perceived to) have some persistence, the findings of the present paper remain relevant for a New Keynesian Phillips curve.

All these extensions of the baseline model show that the key findings of section 2 are robust: when the assumption of perfect common knowledge is relaxed, actual transparency is beneficial, and it is desirable to have a perceived degree of transparency that is no greater than the actual degree of transparency ($\tilde{\tau} \leq \tau$).[15]

4. Discussion

This section discusses two remaining issues. First, it addresses the limitation of τ as a measure of transparency and presents a more comprehensive alternative (section 4.1). In addition, various explanations for central bank mystique are discussed (section 4.2).

[14] In practice, communication commitments could stem from formal accountability requirements. For instance, many central banks are required to publish quarterly inflation reports and provide parliamentary testimony.

[15] Another extension would be to incorporate the reputation approach. Since reputation effects are based on the updating of private-sector inflation expectations, they would depend only on perceived transparency. So, actual transparency would remain desirable, and transparency perceptions would again play a key role.

4.1 Transparency Measures

Since the transparency measure in (5) suffers from some drawbacks, it is useful to reconsider it. Although τ describes the relative accuracy of the signal ξ, it is less suitable as a measure of central bank transparency because it is increasing in monetary uncertainty (σ_θ^2, σ_κ^2). In the literature, transparency typically refers to the absence of information asymmetries (e.g., Geraats 2002). So, transparency is decreasing in the extent to which the private sector faces asymmetric information. However, an increase in opacity due to greater variability of the central bank's targets has the awkward implication that it leads to a higher value of τ. This shows that (5) is not a good indicator of the degree of transparency.

Instead, it is useful to construct a more fundamental measure that is directly based on the definition of transparency. Focusing on the inflation target θ, the private sector has the prior $\bar{\theta}$, and symmetric information amounts to $\theta = \bar{\theta}$. The difference between θ and $\bar{\theta}$ gives an indication of the degree of asymmetric information. So, ex ante opacity can be described by $\mathrm{E}[(\theta - \bar{\theta})^2] = \sigma_\theta^2$, which is the monetary uncertainty measure used in one strand of the literature.

However, the private sector is able to use the public signal ξ_θ to update its prior $\bar{\theta}$, which leads to the posterior $\mathrm{E}[\theta|\xi_\theta]$ in (10). Taking into account the information conveyed by the signal, the appropriate measure of opacity becomes

$$\mathrm{E}[(\theta - \mathrm{E}[\theta|\xi_\theta])^2] = (1 - \tau_\theta)\sigma_\theta^2$$

after substituting (10), (3), and using (5) to substitute for σ_ε^2. This shows that opacity about θ is increasing in the amount of initial monetary uncertainty σ_θ^2 and decreasing in the relative accuracy τ_θ of the signal ξ_θ.

Taking the inverse of opacity and substituting (5) leads to the transparency measure

$$\gamma_\theta = \frac{1}{(1 - \tau_\theta)\sigma_\theta^2} = \frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2}.$$

This measure of (actual) transparency depends positively on the relative accuracy of the signal τ_θ and negatively on monetary uncertainty σ_θ^2. It has the intuitive property that transparency about

θ could be enhanced in two independent ways: (i) reduce the initial uncertainty (σ_θ^2) or (ii) reduce the noisiness of the signal (σ_ε^2). So, γ_θ has the desirable property that greater monetary uncertainty decreases transparency, which is in contrast to τ_θ.

Nevertheless, γ_θ still has the drawbacks that it depends on the actual stochastic distributions and implies infinite transparency if θ is deterministic ($\sigma_\theta^2 = 0$). These problems can be overcome by the following analogous measure of perceived transparency:

$$\tilde{\gamma}_\theta = \frac{1}{(1-\tilde{\tau}_\theta)\tilde{\sigma}_\theta^2} = \frac{1}{\tilde{\sigma}_\theta^2} + \frac{1}{\tilde{\sigma}_\varepsilon^2}.$$

If the private sector believes the target is deterministic ($\tilde{\sigma}_\theta^2 = 0$) and therefore known ($\theta = \bar{\theta}$), or if the private sector thinks that the public signal ξ_θ is completely accurate and has no noise ($\tilde{\sigma}_\varepsilon^2 = 0$), then the private sector has the perception of symmetric information about the inflation target θ, and perceived transparency $\tilde{\gamma}_\theta$ is infinite. On the other hand, complete perceived opacity ($\tilde{\gamma}_\theta = 0$) requires both an infinitely diffuse prior ($\tilde{\sigma}_\theta^2 \to \infty$) and the perception of an infinitely noisy signal ($\tilde{\sigma}_\varepsilon^2 \to \infty$).

The transparency measures γ_κ, $\tilde{\gamma}_\kappa$, γ_s, and $\tilde{\gamma}_s$ can be defined in a similar way. Although γ and $\tilde{\gamma}$ are better measures of the degree of asymmetric information, the economic effects of transparency are more easily understood in terms of the relative accuracy of the signal (τ, $\tilde{\tau}$) and the extent of monetary uncertainty (σ_θ^2, σ_κ^2, $\tilde{\sigma}_\theta^2$, $\tilde{\sigma}_\kappa^2$). The reason is that the relative signal accuracy need not have the same effect as initial monetary uncertainty. In particular, suppose there is common knowledge about all the variance parameters σ^2 and thereby about τ. Then, greater opacity through higher monetary uncertainty σ_θ^2, σ_κ^2, and σ_s^2 is always detrimental because it increases the variance of output and inflation, Var[y] and Var[π], and reduces E[U].[16] In contrast, greater opacity through a lower relative signal accuracy τ_κ or τ_s is beneficial and actually increases E[U].

Nevertheless, one of the main findings of the paper—namely, that actual transparency is beneficial in the presence of private-sector misperceptions—not only holds for the measure τ but also

[16]This holds not only ceteris paribus (i.e., for a constant τ_θ, τ_κ, and τ_s) but also for the total effects of σ_θ^2, σ_κ^2, and σ_s^2 on Var[y], Var[π], and E[U].

for the more general measure γ. To be precise, a decrease in initial monetary uncertainty (σ_θ^2, σ_κ^2, σ_s^2) and in signal noise (σ_ε^2, σ_η^2, σ_υ^2) are both beneficial because of a reduction in Var[y] and Var[π] and an increase in E[U].[17] As a result, the conclusion about the desirability of actual transparency remains robust even when a more comprehensive transparency measure is used.

4.2 Central Bank Mystique

Despite all the emphasis on transparency of monetary policy nowadays, central bankers still often speak with a remarkable lack of clarity. Although it is difficult to characterize "central bank speak," one insider described it as follows:

> *[Fed speak] is a language in which it is possible to speak, without ever saying anything.*
>
> (Mike Moskow, president of the Federal Reserve Bank of Chicago, December 7, 2002)

This paper shows that a central bank may try to give this impression of transparency while creating the perception of opacity. This could be achieved by avoiding the publication of precise, quantitative information and instead resorting to qualitative statements. For example, a numeric inflation target is likely to contribute to a high degree of perceived (and actual) transparency, whereas speeches that provide ambiguous perspectives could lower transparency perceptions.

It is worthwhile to note that the conclusions of this paper regarding the desirability of perceived opacity are independent of the public's prior expectation of the central bank's output-gap target, $\bar{\kappa}$. In particular, the results also hold for $\bar{\kappa} = 0$, in which case there is no average inflation bias, so the central bank has no systematic incentive to misrepresent its information. In that case, commitment to a truthful communication technology is perfectly credible. To the extent that this is not possible, there may be central bank "cheap talk" such that communication of central bank private information is only credible when it is imprecise (Stein 1989).

[17] This refers to the total effect, which is straightforward (though tedious) to compute by differentiating Var[y], Var[π], and E[U] after substituting for τ.

In addition, there may be institutional reasons for central banks to be vague. For example, a central bank without an explicit legal primary objective of price stability, such as the Federal Reserve, could be more reluctant to adopt a numeric inflation target because it may give the impression that it is neglecting its other objectives.

There could also be other reasons for oblique communications by central bankers. For instance, evasiveness could be used to limit accountability or hide incompetence. In addition, secretive central bankers receive more media attention, as their every word is scrutinized. Last but not least, vague communications could reflect the tremendous uncertainty faced by central bankers, which is often difficult to explicate.

The paper shows that under certain circumstances maximum perceived opacity is optimal. In principle, there are two ways to achieve this. The central bank could give the impression that the public signal ξ is infinitely noisy, so that $\tilde{\tau} = 0$. Alternatively, the central bank could remain silent and not communicate at all, so that $\xi \in \{\varnothing\}$ and $\pi^e = \mathrm{E}[\pi]$. In the latter case, the actual and perceived degrees of transparency always coincide: $\tau = \tilde{\tau} = 0$. In practice, few central bankers prefer to remain silent but, rather, engage in oblique speak. This still gives them the benefits of perceived opacity while allowing them to communicate relevant information to the private sector and achieve greater actual transparency.[18]

In practice, there are likely to be some feasibility constraints on the extent of transparency (e.g., Cukierman 2006). In particular, it may not be possible to achieve complete opacity or perfect transparency. Suppose that there are binding constraints on the degree of (actual and perceived) transparency such that $\tau_{MIN} \leq \tau \leq \tau_{MAX}$ and $\tilde{\tau}_{MIN} \leq \tilde{\tau} \leq \tilde{\tau}_{MAX}$. Then, an optimum of maximum actual transparency ($\tau = 1$) and minimal perceived transparency ($\tilde{\tau} = 0$) would not be achievable. In that case, the constrained optimum is maximum possible perceived opacity, $\tilde{\tau} = \tilde{\tau}_{MIN}$, and maximum attainable actual transparency, $\tau = \tau_{MAX}$.

[18] Another reason for not remaining completely silent is that most central banks face accountability requirements, such as testimony before parliament or the publication of inflation reports.

A key finding of the paper is that it tends to be desirable to have less perceived than actual transparency ($\tilde{\tau} < \tau$). The only exception is the inflation target θ, for which $\tilde{\tau}_\theta = \tau_\theta$ is preferred by the central bank but not necessarily by society. An important practical consideration is the extent to which it is possible to sustain systematic deviations between actual and perceived transparency. If all the parameters of the model were stable, it would be possible for the private sector to learn the degree of transparency τ over time.[19] For instance, inflation reports with consistently detailed information are likely to facilitate learning about the central bank's transparency τ. However, when the accuracy of communications is variable so that σ_ε^2, σ_η^2, and σ_υ^2 are unstable, τ_θ, τ_κ, and τ_s can never be learned.

As a result, it may be impossible to learn the degree of transparency, which is especially relevant for verbal communications such as speeches and testimonies. Their informativeness could easily vary from one occasion to another. So, there is no constant degree of transparency to be learned, and the private sector is left in limbo about how much weight to attach to a particular central bank communication.

Adding to potential ambiguity is the fact that speeches by central bankers are used for two purposes—not only to convey news (e.g., about the economic outlook) but also to educate the public (e.g., about the monetary policy strategy or the monetary transmission process). The choice of educational topics and the way in which they are explained could be deliberate.[20] So, financial market analysts are likely to scrutinize speeches to look for clues about (changes in) central bankers' perceptions. Since there is typically ambiguity about whether prima facie educational content also contains some new policy-relevant views, it is difficult to assess the precise informativeness of a central banker's speech.

[19] To see this, note that s and υ follow (ex post) from (2) and (25), so that σ_s^2, σ_υ^2, and τ_s could be learned over time. In addition, y, ξ_θ, and ξ_κ could be used to estimate $\mathrm{Var}[\xi_\theta]$, $\mathrm{Cov}\{y, \xi_\theta\}$, $\mathrm{Var}[\xi_\kappa]$, and $\mathrm{Cov}\{y, \xi_\kappa\}$, from which σ_θ^2, σ_ε^2, σ_κ^2, and σ_η^2 can be deduced. So, τ_θ and τ_κ would also be learnable.

[20] For instance, the speech on monetary policy and wage growth by Governor Svein Gjedrem of the Norwegian central bank in June 2002, which emphasized a positive empirical relation between interest rates and the growth in labor costs, was followed by an increase in the policy rate of 50 basis points in July 2002 to counter high wage growth.

In contrast to the fixed format of inflation reports and written policy statements, verbal communications tend to provide greater flexibility to convey information with a degree of transparency τ that is unknown to the private sector. Thus, speeches provide an important communication tool that is well suited to the dissemination of information with sustained transparency misperceptions.

An interesting finding of the present paper is that it could be beneficial to inhibit private-sector learning about the degree of transparency τ. Whenever $\tilde{\tau} = \tau$ is not optimal, it is actually desirable to have imperfect transparency about the actual degree of transparency. Central banks could exploit flexible communication tools such as speeches to hamper learning about τ and maintain advantageous transparency misperceptions.

Empirical evidence for the importance of speeches is provided by Reinhart and Sack (2006) for the Federal Reserve. Although speeches by members of the Federal Open Market Committee (FOMC) tend to have a very small average impact on market interest rates, their collective effect is sizable and second only to that of FOMC policy statements. This indicates that speeches are a major component of central bank communication.

Although it could be desirable to communicate with a sustained discrepancy between actual and perceived transparency, central bankers may not be equally skilled at it. Perhaps this is where part of the "art" of central banking comes in. A "maestro" like Alan Greenspan managed to effectively guide financial markets by means of statements that appeared to be open to multiple interpretations. He was (in)famous for his Delphic utterances.[21] The fact that his statements were perceived to be rife with ambiguity was constructive and prevented financial markets from reacting too strongly. In contrast, central bankers that speak with clarity appear to be more prone to criticism. For instance, the directness of the

[21]For example, Greenspan's befuddling speech to the Economic Club of New York on June 20, 1995, was summarized by the headline "Greenspan Hints Fed May Cut Interest Rates" in the *Washington Post* but by the headline "Doubts Voiced by Greenspan on a Rate Cut" in the *New York Times*.

first president of the European Central Bank, Wim Duisenberg, was often considered a liability.[22] Instead, central bankers tend to nurture speaking in guarded language that fosters transparency misperceptions, as is illustrated by Greenspan's quote at the beginning of this paper.

5. Conclusion

Central banks are transparent in many respects nowadays, but there is still considerable ambiguity in their communication. This paper shows that arcane statements by central bankers may serve an important purpose. They create the perception of opacity and make markets more cautious in their response to central bank communications, which reduces the volatility of private-sector expectations.

The paper models this mechanism by relaxing the strong assumption of perfect common knowledge about the degree of central bank transparency. In practice, there is considerable disagreement among researchers and market participants about how transparent central banks are. In addition, it would be difficult to verify the degree of transparency. So, it appears realistic to allow the actual and perceived degrees of transparency to differ from each other. This has the virtue that asymmetric information can be modeled regardless of the actual variability of parameters, thereby decoupling ex ante uncertainty and ex post volatility.

Moreover, the analysis of transparency perceptions of the private sector gives a better understanding of some of the disadvantages of transparency suggested in the literature. Although transparency is likely to reduce private-sector uncertainty, information disclosed by the central bank could alter private-sector expectations and give rise to greater economic volatility. However, this drawback appears to be entirely due to transparency perceptions. In particular, the paper shows that actual transparency is beneficial because it reduces the noisiness of communication, but perceived transparency

[22] To give an illustration, in response to a question about further interest rate cuts at a press conference after a cut of 50 basis points (on April 8, 1999), Duisenberg bluntly answered, "You be sure: this is it."

could be more problematic, as it makes markets more sensitive to (potentially noisy) information. This provides an economic rationale for transparent central bank communications that sustain transparency misperceptions. So, transparency about the degree of transparency may not be desirable. In particular, central banks may find it beneficial to disclose information under a veil of perceived ambiguity.

The paper shows that the central bank's optimal communication strategy is to be crystal clear about the inflation target but to be informative about the output-gap target and supply shocks through statements that are perceived to be opaque. In that respect, central bankers should speak, but with mystique.

Appendix

This appendix analyzes four extensions to the basic model that are discussed in section 3.

A.1 Alternative Social Welfare Function

This section computes the optimal degrees of transparency when the social welfare function equals (23). Substituting (7), (8), and (18) into (23) gives

$$\begin{aligned}
W &= -\frac{1}{2}\beta\{\alpha\theta + (1-\alpha)(\pi^e + \kappa + s) - \bar{\theta}\}^2 \\
&\quad - \frac{1}{2}(1-\beta)\{\alpha(\theta - \pi^e - s) + (1-\alpha)\kappa - \bar{\kappa}\}^2 \\
&= -\frac{1}{2}\beta\Big\{(\alpha + (1-\alpha)\tilde{\tau}_\theta)(\theta - \bar{\theta}) + (1-\alpha)\tilde{\tau}_\theta\varepsilon \\
&\quad + \frac{1-\alpha}{\alpha}[\bar{\kappa} + (\alpha + (1-\alpha)\tilde{\tau}_\kappa)(\kappa - \bar{\kappa}) + (1-\alpha)\tilde{\tau}_\kappa\eta + \alpha s]\Big\}^2 \\
&\quad - \frac{1}{2}(1-\beta)\{\alpha(1-\tilde{\tau}_\theta)(\theta - \bar{\theta}) - \alpha\tilde{\tau}_\theta\varepsilon + (1-\alpha)(1-\tilde{\tau}_\kappa)(\kappa - \bar{\kappa}) \\
&\quad - (1-\alpha)\tilde{\tau}_\kappa\eta - \bar{\kappa} - \alpha s\}^2.
\end{aligned}$$

Taking expectations and substituting for σ_ε^2 and σ_η^2 using (5) gives

$$\begin{aligned}\mathrm{E}[W] = &-\frac{1}{2}\beta\left\{\left(\alpha^2 + 2\alpha(1-\alpha)\tilde{\tau}_\theta + (1-\alpha)^2\frac{\tilde{\tau}_\theta^2}{\tau_\theta}\right)\sigma_\theta^2 + \frac{(1-\alpha)^2}{\alpha^2}\right.\\ &\left.\times\left[\bar{\kappa}^2 + \left(\alpha^2 + 2\alpha(1-\alpha)\tilde{\tau}_\kappa + (1-\alpha)^2\frac{\tilde{\tau}_\kappa^2}{\tau_\kappa}\right)\sigma_\kappa^2 + \alpha^2\sigma_s^2\right]\right\}\\ &-\frac{1}{2}(1-\beta)\left\{\alpha^2\left(1 - 2\tilde{\tau}_\theta + \frac{\tilde{\tau}_\theta^2}{\tau_\theta}\right)\sigma_\theta^2 + (1-\alpha)^2\right.\\ &\left.\times\left(1 - 2\tilde{\tau}_\kappa + \frac{\tilde{\tau}_\kappa^2}{\tau_\kappa}\right)\sigma_\kappa^2 + \bar{\kappa}^2 + \alpha^2\sigma_s^2\right\}.\end{aligned}$$

Since $\mathrm{E}[W]$ is increasing in τ_θ and τ_κ, it is socially optimal to have perfect actual transparency about the central bank's targets ($\tau_\theta = \tau_\kappa = 1$). Concerning perceived transparency, the first-order conditions $\partial\mathrm{E}[W]/d\tilde{\tau}_\theta = 0$ and $\partial\mathrm{E}[W]/d\tilde{\tau}_\kappa = 0$ yield

$$\begin{aligned}\tilde{\tau}_\theta &= \frac{\alpha(\alpha-\beta)}{\beta(1-\alpha)+\alpha(\alpha-\beta)}\tau_\theta\\ \tilde{\tau}_\kappa &= \frac{\alpha(\alpha-\beta)}{\beta(1-\alpha)+\alpha(\alpha-\beta)}\tau_\kappa,\end{aligned}$$

respectively. For $\alpha \geq \beta$, these are the socially optimal degrees of perceived transparency, since $\partial^2\mathrm{E}[W]/\partial\tilde{\tau}_\theta^2 < 0$ and $\partial^2\mathrm{E}[W]/\partial\tilde{\tau}_\kappa^2 < 0$. But for $\alpha < \beta$, the social optimum is the corner solution $\tilde{\tau}_\theta = \tilde{\tau}_\kappa = 0$. So, if the central bank is not conservative, society benefits from complete perceived opacity. Regardless of the value of β, in the social optimum the degree of perceived transparency is strictly less than the degree of actual transparency ($\tilde{\tau}_\theta < \tau_\theta$ and $\tilde{\tau}_\kappa < \tau_\kappa$).

In the case of common knowledge about the degree of transparency ($\tilde{\tau}_\theta = \tau_\theta$ and $\tilde{\tau}_\kappa = \tau_\kappa$),

$$\begin{aligned}\mathrm{E}[W] = &-\frac{1}{2}\beta\left\{(\alpha^2 + (1-\alpha^2)\tau_\theta)\sigma_\theta^2 + \frac{(1-\alpha)^2}{\alpha^2}\right.\\ &\left.\times\left[\bar{\kappa}^2 + (\alpha^2 + (1-\alpha^2)\tau_\kappa)\sigma_\kappa^2 + \alpha^2\sigma_s^2\right]\right\} - \frac{1}{2}(1-\beta)\\ &\times\left\{\alpha^2(1-\tau_\theta)\sigma_\theta^2 + \bar{\kappa}^2 + (1-\alpha)^2(1-\tau_\kappa)\sigma_\kappa^2 + \alpha^2\sigma_s^2\right\}.\end{aligned}$$

Differentiating yields

$$\frac{\partial \mathrm{E}[W]}{\partial \tau_\theta} = -\frac{1}{2}[\beta(1-\alpha^2) - (1-\beta)\alpha^2]\sigma_\theta^2 = -\frac{1}{2}(\beta - \alpha^2)\sigma_\theta^2$$

$$\frac{\partial \mathrm{E}[W]}{\partial \tau_\kappa} = -\frac{1}{2}\left[\beta\frac{(1-\alpha)^2}{\alpha^2}(1-\alpha^2) - (1-\beta)(1-\alpha)^2\right]\sigma_\kappa^2$$

$$= -\frac{1}{2}\left[\frac{\beta}{\alpha^2} - 1\right](1-\alpha)^2\sigma_\kappa^2.$$

Note that $\partial \mathrm{E}[W]/\partial \tau_\theta = \partial \mathrm{E}[W]/\partial \tau_\kappa = 0$ for $\beta = \alpha^2$, and $\mathrm{sgn}(\partial \mathrm{E}[W]/\partial \tau_\theta) = \mathrm{sgn}(\partial \mathrm{E}[W]/\partial \tau_\kappa) = \mathrm{sgn}(\alpha^2 - \beta)$. Hence, $\tau_\theta = \tau_\kappa = 1$ is socially optimal for $\alpha^2 > \beta$, and $\tau_\theta = \tau_\kappa = 0$ is socially optimal for $\alpha^2 < \beta$. So, if society attaches a sufficiently low weight to inflation stabilization or the central bank is sufficiently conservative, the social optimum is to have transparency about the central bank's targets.

The following statements summarize the results for the social welfare function (23):

- With perfect common knowledge about the degrees of transparency τ_θ and τ_κ, it is socially optimal to have maximum transparency about the central bank targets ($\tau_\theta = \tau_\kappa = 1$) for $\alpha^2 > \beta$ and minimal transparency ($\tau_\theta = \tau_\kappa = 0$) for $\alpha^2 < \beta$.
- With transparency misperceptions, it is socially optimal to have maximum actual transparency about the central bank's targets ($\tau_\theta = \tau_\kappa = 1$) regardless of α and β, some perceived opacity ($0 < \tilde{\tau}_\theta, \tilde{\tau}_\kappa < 1$) for $\alpha > \beta$, and maximum perceived opacity ($\tilde{\tau}_\theta = \tilde{\tau}_\kappa = 0$) for $\alpha \leq \beta$.

A.2 Constant Central Bank Targets

This section examines optimal transparency (mis)perceptions when the central bank's inflation target θ and output-gap target κ are constant. More precisely, the actual distributions of θ and κ are degenerate, but the private sector still faces asymmetric information about these targets and has the perceived (or prior) distributions $\theta \sim N(\bar{\theta}, \tilde{\sigma}_\theta^2)$, $\kappa \sim N(\bar{\kappa}, \tilde{\sigma}_\kappa^2)$. The optimal output gap and inflation still satisfy (7) and (8). In addition, private-sector expectations are

again given by (16), (17), and (18).[23] The difference with the model in section 2.2 is that the actual values of θ and κ are now deterministic, so $\theta = \bar{\theta}$, $\kappa = \bar{\kappa}$, and $\sigma_\theta^2 = \sigma_\kappa^2 = 0$. As a result, the actual variance of inflation expectations equals

$$\mathrm{Var}[\pi^e] = \tilde{\tau}_\theta^2 \sigma_\varepsilon^2 + \left(\frac{1-\alpha}{\alpha}\right)^2 \tilde{\tau}_\kappa^2 \sigma_\eta^2.$$

This shows that the volatility of inflation expectations is increasing in perceived transparency $\tilde{\tau}_\theta$ and $\tilde{\tau}_\kappa$, and in the noise variances σ_ε^2 and σ_η^2, so that it is essentially decreasing in actual transparency about θ and κ.

The levels of the output gap and inflation are still given by (19) and (20), but their actual variances now equal

$$\mathrm{Var}[y] = \alpha^2 \tilde{\tau}_\theta^2 \sigma_\varepsilon^2 + (1-\alpha)^2 \tilde{\tau}_\kappa^2 \sigma_\eta^2 + \alpha^2 \sigma_s^2$$

$$\mathrm{Var}[\pi] = (1-\alpha)^2 \tilde{\tau}_\theta^2 \sigma_\varepsilon^2 + \frac{(1-\alpha)^2}{\alpha^2}(1-\alpha)^2 \tilde{\tau}_\kappa^2 \sigma_\eta^2 + (1-\alpha)^2 \sigma_s^2.$$

So, the variability of the output gap and inflation are both increasing in perceived transparency $\tilde{\tau}_\theta$ and $\tilde{\tau}_\kappa$, and in the noise variances σ_ε^2 and σ_η^2. As a result, the output gap and inflation are more stable when there is greater perceived opacity about the inflation and output-gap targets, and greater transparency in the communications ξ_θ and ξ_κ.

Regarding welfare effects, (21) still holds, and taking unconditional expectations based on actual distributions yields

$$\mathrm{E}[U] = -\frac{1}{2}\frac{1-\alpha}{\alpha}\left[\alpha^2 \tilde{\tau}_\theta^2 \sigma_\varepsilon^2 + \bar{\kappa}^2 + (1-\alpha)^2 \tilde{\tau}_\kappa^2 \sigma_\eta^2 + \alpha^2 \sigma_s^2\right].$$

Clearly, the best outcome is obtained for maximum perceived opacity ($\tilde{\tau}_\theta = \tilde{\tau}_\kappa = 0$) and maximum actual transparency ($\sigma_\varepsilon^2 = \sigma_\eta^2 = 0$). So, again, it is optimal to have transparency misperceptions.

[23] Note that if the perceived distributions were not normal, (16) and (17) would still be the best linear predictors.

The same conclusion holds for the social welfare functions in (22) and (23). Concerning the latter, expected social welfare now equals

$$\mathrm{E}[W] = -\frac{1}{2}[(\beta(1-\alpha)^2 + (1-\beta)\alpha^2)] \times \left\{ \tilde{\tau}_\theta^2 \sigma_\varepsilon^2 + \frac{1}{\alpha^2}\bar{\kappa}^2 + \frac{(1-\alpha)^2}{\alpha^2}\tilde{\tau}_\kappa^2 \sigma_\eta^2 + \sigma_s^2 \right\}.$$

So, again, minimal perceived transparency ($\tilde{\tau}_\theta = \tilde{\tau}_\kappa = 0$) and maximum actual transparency ($\sigma_\varepsilon^2 = \sigma_\eta^2 = 0$) is optimal.

As a result, the conclusion that it is desirable to have transparency misperceptions does not depend on the assumption that the central bank targets θ and κ are stochastic, and it even holds when these targets are actually deterministic.

A.3 Transparency about Supply Shocks

This section analyzes the effect of transparency about the supply shock s, where $s \sim N(0, \sigma_s^2)$. In the model of section 2, transparency about the supply shock s is immaterial because s is only realized after the private sector has formed its inflation expectations π^e. Now suppose that the private sector receives a public signal ξ_s of the supply shock before it forms its inflation expectations π^e:

$$\xi_s = s + \upsilon, \tag{25}$$

where υ is i.i.d. white noise with $\upsilon \sim N(0, \sigma_\upsilon^2)$. Then, the actual degree of transparency about supply shocks is given by

$$\tau_s = \frac{\sigma_s^2}{\sigma_s^2 + \sigma_\upsilon^2}. \tag{26}$$

Similarly, the perceived degree of transparency about supply shocks is given by

$$\tilde{\tau}_s = \frac{\tilde{\sigma}_s^2}{\tilde{\sigma}_s^2 + \tilde{\sigma}_\upsilon^2}, \tag{27}$$

where $\tilde{\sigma}_s^2$ and $\tilde{\sigma}_\upsilon^2$ are the private-sector perceptions of the (prior) variance of s and υ, respectively.

Note that the optimal degrees of transparency about the inflation target θ and output-gap target κ in section 2 are independent of the

variability of the supply shock s. The reason is that σ_θ^2, σ_κ^2, and σ_s^2 enter separably in $\mathrm{E}[U]$, and θ, κ, and s are independent. Similarly, the optimal degree of transparency about the supply shock is independent of the variability of the inflation and output-gap targets. For simplicity, assume that the inflation target and output-gap target are deterministic and known to the private sector, so $\theta = \bar{\theta}$ and $\kappa = \bar{\kappa}$ with $\sigma_\theta^2 = \tilde{\sigma}_\theta^2 = \sigma_\kappa^2 = \tilde{\sigma}_\kappa^2 = 0$, which implies perfect (actual and perceived) transparency about the central bank's preferences. But now the private sector faces imperfect information about the supply shock s when it forms its inflation expectations. The central bank still maximizes (1) subject to (2) given π^e, which yields (7) and (8).

The results for imperfect common knowledge about the degree of transparency of supply shocks are derived first. Perfect common knowledge amounts to the special case in which there are no transparency misperceptions ($\tilde{\tau}_s = \tau_s$). Taking expectations of (8) and solving for π^e gives

$$\pi^e = \widetilde{\mathrm{E}}[\pi|\xi_s] = \bar{\theta} + \frac{1-\alpha}{\alpha}(\bar{\kappa} + \widetilde{\mathrm{E}}[s|\xi_s]).$$

Using (25) and (27),

$$\widetilde{\mathrm{E}}[s|\xi_s] = \frac{\tilde{\sigma}_s^2}{\tilde{\sigma}_s^2 + \tilde{\sigma}_\upsilon^2}\xi_s = \tilde{\tau}_s\xi_s.$$

Substituting into π^e and using (25) gives

$$\pi^e = \bar{\theta} + \frac{1-\alpha}{\alpha}(\bar{\kappa} + \tilde{\tau}_s s + \tilde{\tau}_s \upsilon). \tag{28}$$

Substituting this into (7) and (8) yields

$$y = -(\alpha + (1-\alpha)\tilde{\tau}_s)s - (1-\alpha)\tilde{\tau}_s\upsilon$$
$$\pi = \bar{\theta} + \frac{1-\alpha}{\alpha}[\bar{\kappa} + (\alpha + (1-\alpha)\tilde{\tau}_s)s + (1-\alpha)\tilde{\tau}_s\upsilon].$$

The variance of the output gap and inflation depend on the degree of transparency:

$$\mathrm{Var}[y] = \left[\alpha^2 + 2\alpha(1-\alpha)\tilde{\tau}_s + (1-\alpha)^2\frac{\tilde{\tau}_s^2}{\tau_s}\right]\sigma_s^2$$
$$\mathrm{Var}[\pi] = \frac{(1-\alpha)^2}{\alpha^2}\left[\alpha^2 + 2\alpha(1-\alpha)\tilde{\tau}_s + (1-\alpha)^2\frac{\tilde{\tau}_s^2}{\tau_s}\right]\sigma_s^2,$$

using the fact that (26) implies $\sigma_\upsilon^2 = \frac{1-\tau_s}{\tau_s}\sigma_s^2$. This shows that the variance of the output gap and inflation are decreasing in actual transparency τ_s and increasing in perceived transparency $\tilde{\tau}_s$.

Not surprisingly, perceived transparency about supply shocks is harmful, whereas actual transparency is beneficial. Formally, substitute (28) into (15) to get

$$U = -\frac{1}{2}\frac{1-\alpha}{\alpha}[\bar{\kappa} + (\alpha + (1-\alpha)\tilde{\tau}_s)s + (1-\alpha)\tilde{\tau}_s\upsilon]^2.$$

Taking unconditional expectations and substituting $\sigma_\varepsilon^2 = \frac{1-\tau_s}{\tau_s}\sigma_s^2$ gives the ex ante expected central bank payoff

$$\mathrm{E}[U] = -\frac{1}{2}\frac{1-\alpha}{\alpha}\left[\bar{\kappa}^2 + \left(\alpha^2 + 2\alpha(1-\alpha)\tilde{\tau}_s + (1-\alpha)^2\frac{\tilde{\tau}_s^2}{\tau_s}\right)\sigma_s^2\right].$$

As a result, for supply shocks it is optimal for the central bank to have maximum actual transparency ($\tau_s = 1$) and minimal perceived transparency ($\tilde{\tau}_s = 0$). Formally, this follows from $\partial\mathrm{E}[U]/\partial\tau_s > 0$ and $\partial\mathrm{E}[U]/\partial\tilde{\tau}_s < 0$.

The results under common knowledge are obtained by imposing the restriction that $\tilde{\tau}_s = \tau_s$. The variance of the output gap and inflation are equal to

$$\mathrm{Var}[y] = [\alpha^2 + (1-\alpha^2)\tau_s]\sigma_s^2$$
$$\mathrm{Var}[\pi] = \frac{(1-\alpha)^2}{\alpha^2}[\alpha^2 + (1-\alpha^2)\tau_s]\sigma_s^2.$$

This shows that greater transparency about the supply shock s increases the volatility of both the output gap and inflation.

This implies that transparency about supply shocks is detrimental. Formally,

$$\mathrm{E}[U] = -\frac{1}{2}\frac{1-\alpha}{\alpha}[\bar{\kappa}^2 + (\alpha^2 + (1-\alpha^2)\tau_s)\sigma_s^2].$$

Clearly, minimal transparency about supply shocks ($\tau_s = 0$) is optimal for the central bank. It is also socially optimal for the social welfare functions (22) and (23).

The following statements summarize the results concerning transparency about supply shocks s:

- With perfect common knowledge about the degree of transparency τ_s, greater transparency τ_s increases the variability of inflation π and the output gap y, and minimal transparency ($\tau_s = 0$) is optimal for the central bank and society.
- With transparency misperceptions, greater actual transparency τ_s and smaller perceived transparency $\tilde{\tau}_s$ reduce the variability of inflation π and the output gap y, and it is optimal for the central bank and society to have maximum actual transparency ($\tau_s = 1$) but minimal perceived transparency ($\tilde{\tau}_s = 0$).

A.4 Transparency about Supply Shocks with New Keynesian Phillips Curve

This section analyzes the effect of transparency about the supply shock s for an infinite-horizon model in which the central bank maximizes the expected value of

$$\bar{U} = (1-\delta)\sum_{t=1}^{\infty}\delta^{i-1}U_t,$$

where δ is the subjective intertemporal discount factor ($0 < \delta < 1$), and the objective function U_t is still given by (1). The central bank targets θ and κ are assumed to be deterministic and known to the private sector, just like in section A.3 of this appendix. But the expectations-augmented Phillips curve (2) is now replaced by the New Keynesian Phillips curve (24):

$$\pi_t = \widetilde{\mathrm{E}}_t[\pi_{t+1}|\xi_t] + y_t + s_t,$$

where $\widetilde{\mathrm{E}}_t[.]$ denotes the expectation of the private sector at the beginning of period t based on the stochastic distributions perceived by the private sector and (implicitly) conditional on all variables observed in periods $t-k$ for $k \in \{1, 2, \ldots\}$. The supply shock s_t is allowed to be persistent:

$$s_t = \rho s_{t-1} + \zeta_t, \tag{29}$$

where $0 \leq \rho < 1$ and ζ_t is i.i.d. white noise with $\zeta_t \sim N(0, \sigma_\zeta^2)$. So, $\mathrm{Var}[s_t] = \sigma_s^2 = \frac{1}{1-\rho^2}\sigma_\zeta^2$. The private sector observes a public signal ξ_t of the innovation ζ_t to the supply shock s_t:

$$\xi_t = \zeta_t + \upsilon_t, \tag{30}$$

where υ_t is i.i.d. white noise with $\upsilon_t \sim N(0, \sigma_\upsilon^2)$. So, $\mathrm{Var}[\xi_t] = \sigma_\zeta^2 + \sigma_\upsilon^2$. The degree of transparency about the supply shock innovation equals

$$\tau_\zeta = \frac{\sigma_\zeta^2}{\sigma_\zeta^2 + \sigma_\upsilon^2}. \tag{31}$$

Similarly, the perceived degree of transparency about the supply shock innovation is given by

$$\tilde{\tau}_\zeta = \frac{\tilde{\sigma}_\zeta^2}{\tilde{\sigma}_\zeta^2 + \tilde{\sigma}_\upsilon^2}, \tag{32}$$

where $\tilde{\sigma}_\zeta^2$ and $\tilde{\sigma}_\upsilon^2$ are the private-sector perceptions of the (prior) variance of ζ and υ, respectively.

The timing of events is as follows. At the beginning of each period, the private sector observes the public signal ξ_t of the innovation ζ_t to the supply shock s_t, and it forms its inflation expectations $\widetilde{\mathrm{E}}_t[\pi_{t+1}|\xi_t]$. Then, the supply shock innovation ζ_t, and thereby s_t, is realized. Finally, the central bank sets the output gap y_t, and the level of inflation π_t is realized. Monetary policy is still conducted under pure discretion. For simplicity, it is assumed that the central bank commits to a communication technology with a particular degree of (actual and perceived) transparency.

The private sector faces imperfect information about the supply shock s_t when it forms its inflation expectations. The information

set available to the private sector when it forms $\widetilde{\mathrm{E}}_t[\pi_{t+1}|\xi_t]$ includes the public signal ξ_t and the history of the output gap y_{t-k}, inflation π_{t-k}, and supply shocks s_{t-k}, for $k \in \{1, 2, \ldots\}$. Note that the supply shock s_t can always be inferred by the private sector at the end of period t from y_t, π_t, and $\widetilde{\mathrm{E}}_t[\pi_{t+1}|\xi_t]$ using the Phillips curve (24), even if the private sector cannot directly observe s_t.

In every period, the central bank faces the same infinite-horizon problem, but with a different state variable $\widetilde{\mathrm{E}}_t[\pi_{t+1}|\xi_t]$. Since the current policy decision y_t has no effect on future outcomes, the central bank simply sets y_t to maximize U_t every period subject to (24) and given $\widetilde{\mathrm{E}}_t[\pi_{t+1}|\xi_t]$.[24] This yields the optimal levels of the output gap and inflation under discretion:

$$y_t = \alpha(\theta - \widetilde{\mathrm{E}}_t[\pi_{t+1}|\xi_t] - s_t) + (1-\alpha)\kappa \tag{33}$$

$$\pi_t = \alpha\theta + (1-\alpha)(\widetilde{\mathrm{E}}_t[\pi_{t+1}|\xi_t] + \kappa + s_t). \tag{34}$$

These are similar to (7) and (8) in the baseline model.

Recursive substitution of (34) yields

$$\pi_t = \alpha\sum_{i=0}^{k}(1-\alpha)^i\theta + (1-\alpha)^{k+1}\widetilde{\mathrm{E}}_t[\pi_{t+k+1}|\xi_t] + (1-\alpha)\sum_{i=0}^{k}(1-\alpha)^i\kappa$$
$$+ (1-\alpha)\sum_{i=1}^{k}(1-\alpha)^i\widetilde{\mathrm{E}}_t[s_{t+i}|\xi_t] + (1-\alpha)s_t.$$

Taking the limit $\lim_{k\to\infty}\pi_t$, inflation reduces to

$$\pi_t = \theta + \frac{1-\alpha}{\alpha}\kappa + \frac{(1-\alpha)\rho}{1-(1-\alpha)\rho}s_{t-1} + (1-\alpha)\frac{1-(1-\alpha)\rho(1-\tilde{\tau}_\zeta)}{1-(1-\alpha)\rho}\zeta_t$$
$$+ \frac{(1-\alpha)^2\rho}{1-(1-\alpha)\rho}\tilde{\tau}_\zeta\upsilon_t, \tag{35}$$

[24] This no longer holds when there is asymmetric information about the central bank targets θ and/or κ. In that case, there is generally no closed-form solution for the output gap y_t and inflation π_t, and the optimal degree of transparency has to be computed numerically, as in the two-period model by Jensen (2002).

using (29), $\widetilde{\mathrm{E}}_t[s_{t+i}|\xi_t] = \rho^{i+1}s_{t-1} + \rho^i\widetilde{\mathrm{E}}_t[\zeta_t|\xi_t]$, (30), and $\widetilde{\mathrm{E}}_t[\zeta_t|\xi_t] = \frac{\tilde{\sigma}_\zeta^2}{\tilde{\sigma}_\zeta^2+\tilde{\sigma}_\upsilon^2}\xi_t = \tilde{\tau}_\zeta(\zeta_t + \upsilon_t)$. So, private-sector inflation expectations equal

$$\widetilde{\mathrm{E}}_t[\pi_{t+1}|\xi_t] = \theta + \frac{1-\alpha}{\alpha}\kappa + \frac{(1-\alpha)\rho}{1-(1-\alpha)\rho}[\rho s_{t-1} + \tilde{\tau}_\zeta(\zeta_t + \upsilon_t)]. \quad (36)$$

Substitute this into (33), use (29), and simplify to get

$$\begin{aligned} y_t = &-\frac{\alpha\rho}{1-(1-\alpha)\rho}s_{t-1} - \alpha\frac{1-(1-\alpha)\rho(1-\tilde{\tau}_\zeta)}{1-(1-\alpha)\rho}\zeta_t \\ &- \frac{\alpha(1-\alpha)\rho}{1-(1-\alpha)\rho}\tilde{\tau}_\zeta\upsilon_t. \end{aligned} \quad (37)$$

The variance of inflation and the output gap are equal to

$$\begin{aligned} \mathrm{Var}[\pi_t] = &\frac{(1-\alpha)^2}{(1-(1-\alpha)\rho)^2}\left[\frac{1}{1-\rho^2} - 2(1-\alpha)\rho(1-\tilde{\tau}_\zeta)\right. \\ &\left. + (1-\alpha)^2\rho^2\left(1 - 2\tilde{\tau}_\zeta + \frac{\tilde{\tau}_\zeta^2}{\tau_\zeta}\right)\right]\sigma_\zeta^2 \\ \mathrm{Var}[y_t] = &\frac{\alpha^2}{(1-(1-\alpha)\rho)^2}\left[\frac{1}{1-\rho^2} - 2(1-\alpha)\rho(1-\tilde{\tau}_\zeta)\right. \\ &\left. + (1-\alpha)^2\rho^2\left(1 - 2\tilde{\tau}_\zeta + \frac{\tilde{\tau}_\zeta^2}{\tau_\zeta}\right)\right]\sigma_\zeta^2, \end{aligned}$$

using the fact that (29) and (31) imply $\mathrm{Var}[s_t] = \frac{1}{1-\rho^2}\sigma_\zeta^2$ and $\sigma_\upsilon^2 = \frac{1-\tau_\zeta}{\tau_\zeta}\sigma_\zeta^2$. In the special case in which the supply shock s_t is i.i.d. ($\rho = 0$), the variance of inflation and the output gap do not depend on the (actual and perceived) degree of transparency (τ_ζ and $\tilde{\tau}_\zeta$). But, when the supply shock is persistent ($\rho \neq 0$), differentiating with respect to τ_ζ and $\tilde{\tau}_\zeta$ gives

$$\begin{aligned} \frac{\partial\mathrm{Var}[\pi_t]}{\partial\tau_\zeta} &= -\frac{(1-\alpha)^4\rho^2}{(1-(1-\alpha)\rho)^2}\frac{\tilde{\tau}_\zeta^2}{\tau_\zeta^2}\sigma_\zeta^2 < 0 \\ \frac{\partial\mathrm{Var}[\pi_t]}{\partial\tilde{\tau}_\zeta} &= \frac{2(1-\alpha)^3\rho}{(1-(1-\alpha)\rho)^2}\left[1 - (1-\alpha)\rho + (1-\alpha)\rho\frac{\tilde{\tau}_\zeta}{\tau_\zeta}\right]\sigma_\zeta^2 > 0. \end{aligned}$$

Similarly, $\partial \mathrm{Var}[y_t]/\partial \tau_\zeta < 0$ and $\partial \mathrm{Var}[y_t]/\partial \tilde{\tau}_\zeta > 0$. So, $\mathrm{Var}[\pi_t]$ and $\mathrm{Var}[y_t]$ are minimized for maximum actual transparency, $\tau_\zeta = 1$, and minimal perceived transparency, $\tilde{\tau}_\zeta = 0$.

Not surprisingly, for persistent supply shocks, actual transparency about supply shocks is beneficial, whereas perceived transparency is harmful. Formally, substituting (34) and (33) into (1) yields

$$U_t = -\frac{1}{2}\alpha(1-\alpha)(\widetilde{\mathrm{E}}_t[\pi_{t+1}|\xi_t] - \theta + \kappa + s_t)^2 \tag{38}$$

similar to (15). Substituting (36) and taking unconditional expectations,

$$\begin{aligned}
\mathrm{E}[U_t] &= -\frac{1}{2}\alpha(1-\alpha)\left[\frac{1}{\alpha^2}\kappa^2 + \frac{\rho^2}{(1-(1-\alpha)\rho)^2}\mathrm{E}[s_{t-1}^2]\right. \\
&\quad \left. + \frac{(1-(1-\alpha)\rho(1-\tilde{\tau}_\zeta))^2}{(1-(1-\alpha)\rho)^2}\sigma_\zeta^2 + \frac{(1-\alpha)^2\rho^2\tilde{\tau}_\zeta^2}{(1-(1-\alpha)\rho)^2}\sigma_\upsilon^2\right] \\
&= -\frac{1}{2}\alpha(1-\alpha)\left[\frac{1}{\alpha^2}\kappa^2\right. \\
&\quad \left. + \frac{\frac{1}{1-\rho^2} - 2(1-\alpha)\rho(1-\tilde{\tau}_\zeta) + (1-\alpha)^2\rho^2\left(1 - 2\tilde{\tau}_\zeta + \frac{\tilde{\tau}_\zeta^2}{\tau_\zeta}\right)}{(1-(1-\alpha)\rho)^2}\sigma_\zeta^2\right],
\end{aligned}$$

again using $\mathrm{E}[s_{t-1}^2] = \mathrm{Var}[s_t] = \frac{1}{1-\rho^2}\sigma_\zeta^2$ and $\sigma_\upsilon^2 = \frac{1-\tau_\zeta}{\tau_\zeta}\sigma_\zeta^2$. This shows that for i.i.d. supply shocks ($\rho = 0$), the degree of actual and perceived transparency τ_ζ and $\tilde{\tau}_\zeta$ is immaterial. For persistent supply shocks ($\rho \neq 0$), the optimal degrees of actual and perceived transparency follow from differentiating with respect to τ_ζ and $\tilde{\tau}_\zeta$, respectively:

$$\begin{aligned}
\frac{\partial \mathrm{E}[U_t]}{\partial \tau_\zeta} &= \frac{1}{2}\frac{\alpha(1-\alpha)^3\rho^2}{(1-(1-\alpha)\rho)^2}\frac{\tilde{\tau}_\zeta^2}{\tau_\zeta^2}\sigma_\zeta^2 > 0 \\
\frac{\partial \mathrm{E}[U_t]}{\partial \tilde{\tau}_\zeta} &= -\frac{\alpha(1-\alpha)^2\rho}{(1-(1-\alpha)\rho)^2}\left[1 - (1-\alpha)\rho + (1-\alpha)\rho\frac{\tilde{\tau}_\zeta}{\tau_\zeta}\right]\sigma_\zeta^2 < 0.
\end{aligned}$$

As a result, it is optimal to have maximum actual transparency ($\tau_\zeta = 1$) and minimal perceived transparency ($\tilde{\tau}_\zeta = 0$).

The special case of perfect common knowledge about the degree of transparency follows from imposing the restriction that $\tilde{\tau}_\zeta = \tau_\zeta$. This yields

$$\begin{aligned}\mathrm{Var}[\pi_t] &= \frac{(1-\alpha)^2}{(1-(1-\alpha)\rho)^2} \\ &\quad \times \left[\frac{1}{1-\rho^2} - (1-\alpha)\rho(2-(1-\alpha)\rho)(1-\tau_\zeta)\right]\sigma_\zeta^2 \\ \mathrm{Var}[y_t] &= \frac{\alpha^2}{(1-(1-\alpha)\rho)^2} \\ &\quad \times \left[\frac{1}{1-\rho^2} - (1-\alpha)\rho(2-(1-\alpha)\rho)(1-\tau_\zeta)\right]\sigma_\zeta^2.\end{aligned}$$

In the special case in which the supply shock s_t is i.i.d. ($\rho = 0$), the variance of inflation and the output gap are again independent of the degree of transparency τ_ζ. However, for persistent supply shocks ($\rho \neq 0$), the variance of inflation and the output gap are both increasing in actual transparency, so $\mathrm{Var}[\pi_t]$ and $\mathrm{Var}[y_t]$ are minimized for $\tau_\zeta = 0$.

It is straightforward to see that transparency about supply shocks is detrimental under perfect common knowledge ($\tilde{\tau}_s = \tau_s$). Formally,

$$\begin{aligned}\mathrm{E}[U_t] &= -\frac{1}{2}\alpha(1-\alpha) \\ &\quad \times \left[\frac{1}{\alpha^2}\kappa^2 + \frac{\frac{1}{1-\rho^2} - (1-\alpha)\rho(2-(1-\alpha)\rho)(1-\tau_\zeta)}{(1-(1-\alpha)\rho)^2}\sigma_\zeta^2\right].\end{aligned}$$

Again, the degree of transparency τ_ζ is immaterial when the supply shock is i.i.d. ($\rho = 0$). For persistent supply shocks ($\rho \neq 0$), $\partial \mathrm{E}[U_t]/\partial\tau_\zeta < 0$, so minimal transparency about supply shocks ($\tau_\zeta = 0$) is optimal in the absence of transparency misperceptions.

The following statements summarize the results concerning transparency about supply shocks s_t with a New Keynesian Phillips curve:

- In the special case in which the supply shocks s_t are i.i.d. ($\rho = 0$), the degree of actual transparency τ_ζ and perceived

transparency $\tilde{\tau}_\zeta$ have no effect on the variance of inflation π_t and the output gap y_t, and no effect on expected utility $\mathrm{E}[U_t]$.

- For persistent supply shocks ($\rho \neq 0$) and perfect common knowledge about the degree of transparency τ_ζ, greater transparency τ_ζ increases the variability of inflation π_t and the output gap y_t, and minimal transparency ($\tau_\zeta = 0$) is optimal.
- For persistent supply shocks ($\rho \neq 0$) and transparency misperceptions, greater actual transparency τ_ζ and smaller perceived transparency $\tilde{\tau}_\zeta$ reduce the variability of inflation π_t and the output gap y_t, and it is optimal to have maximum actual transparency ($\tau_\zeta = 1$) but minimal perceived transparency ($\tilde{\tau}_\zeta = 0$).

Clearly, when there is some persistence in the supply shock s_t, the qualitative results for the New Keynesian Phillips curve (24) are exactly the same as for the expectations-augmented Phillips curve (2) in the baseline model.

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