Firm-Specific Labor and Firm-Specific Capital: Implications for the Euro-Data New Phillips Curve*

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Standard GMM estimates of the New Phillips curve on euro-area data yield degrees of nominal rigidity that are not in accordance with recent microeconomic evidence. This paper studies whether similar conclusions are reached in a richer model where price setters face firm-specific capital and/or firm-specific labor. We find that combining these elements or considering firm-specific labor alone leads to statistically significant and economically reasonable estimates of the degree of nominal rigidity. In contrast, ignoring firm-specific labor yields estimates that are not supported by microeconomic evidence.

JEL Codes: E1, E3.

1. Introduction

In a set of influential papers, Galí and Gertler (1999) and Galí, Gertler, and López-Salido (2001) have shown that a hybrid New Keynesian Phillips curve (NKPC henceforth), based on the Calvo (1983) model, fit U.S. and European inflation data surprisingly well. Despite their attractiveness, these results have been criticized on the grounds that they imply unrealistic degrees of nominal rigidities.

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For example, Galí and Gertler (1999) report probabilities of price fixity for U.S. data almost always higher than 0.8, implying an average duration of price fixity higher than five quarters. Galí, Gertler, and López-Salido (2001) obtain an average price duration higher than ten quarters for euro-area data, based on the conventional NKPC.[1] Indeed, these results stand in sharp contrast with recent microeconomic evidence. For example, Bils and Klenow (2004), studying U.S. data, report that the average price duration is roughly two quarters. Dhyne et al. (2006) report an average price duration of ten months to four quarters on euro-area data. Thus, be it for U.S. or euro-area data, there seems to be a discrepancy between microeconomic data studies and macroeconomic estimates of the NKPC.

In response to these criticisms, a number of authors have argued in favor of including alternative supply-side refinements capable of diminishing the responsiveness of inflation to the real marginal cost. A fruitful direction of research, which has received attention in the recent literature, consists in rendering firms' real marginal cost increasing in their own output. In such a framework, when given the opportunity to reoptimize, a firm will change its price by a smaller amount than if its marginal cost were independent of its decisions. Everything else equal, this will translate into a smaller response of inflation to changes in the aggregate marginal cost.

Among the mechanisms working in this direction, firm-specific capital has been the subject of a significant number of recent papers, e.g., Altig et al. (2005), Christiano (2004), Eichenbaum and Fisher (2005), Sveen and Weinke (2004, 2005), and Woodford (2005). All suggest that this additional supply-side mechanism can have prominent effects on the dynamics of inflation and output in this kind of model.[2]

A key contribution to this literature is the paper by Eichenbaum and Fisher (2005). In that paper, the authors show that considering firm-specific capital and a variable demand elasticity allows an NKPC to better match postwar U.S. data. More precisely, they show

[1] However, in a specification allowing for firm-specific labor, they obtain much smaller average durations. See section 3 for similar results, though based on a slightly different econometric approach.

[2] See also de Walque, Smets, and Wouters (2004) for a similar analysis in the context of a model with Taylor (1980) contracts.

that the implied degree of nominal rigidity requested by the data is much smaller than obtained in conventional applications.

In this paper, we propose to estimate, via the generalized method of moments (GMM), NKPCs on euro data in the context of a refined supply-side environment, where firms face specific labor and capital markets. The parameters of interest are the probability of not reoptimizing a price and the degree of indexation to past inflation. We contrast the obtained estimates with those arising in (i) a model with aggregate markets for labor and capital, which yields the usual new Phillips curve; (ii) a model with fixed capital and firm-specific labor, as in Woodford's (2003) basic model; and (iii) a model with an aggregate labor market and firm-specific capital. All these specifications are observationally equivalent, yet they differ with respect to the implied degree of nominal rigidities.

Euro data aside, the present paper is a complementary study to Eichenbaum and Fisher (2005). Instead of assuming a variable demand elasticity, we explore the consequences of allowing for firm-specific labor in addition to firm-specific capital. As argued by Woodford (2003), allowing for firm-specific labor implies a higher degree of strategic complementarity between price setters, which, everything else equal, translates into a smaller partial elasticity of current inflation to the marginal cost. This in itself motivates the inclusion of firm-specific labor as an additional, possible channel of inflation persistence.

According to Woodford (2003), allowing for a variable demand elasticity and/or produced inputs strengthens the degree of strategic complementarity, but only to a marginal extent as long as labor is assumed to be firm specific. Specifically, the increase in the degree of strategic complementarity implied by the latter mechanism dwarfs that implied by a variable demand elasticity and/or the presence of produced inputs in price setters' production function.

Woodford (2005) shows that assuming firm-specific capital and capital adjustment costs plays a similar role, but it is unclear whether this additional mechanism contributes much to increasing the degree of strategic complementarity, compared to firm-specific labor. This is precisely the question under study in the present paper.

Obviously, there are many other supply-side refinements that can be included in a basic sticky-price model to help obtain sensible NKPC estimates. Recent research has followed this direction.

For example, in the context of euro-area data, McAdam and Willman (2004) consider a fairly disaggregated supply-side optimizing framework that allows them to treat possible nonstationarity in factor income shares and markups. Such a study is far beyond the scope of the present paper in which we seek to assess the usefulness of combining simple theoretical elements in order to obtain economically realistic parameter estimates of the euro-area NKPC.

Matheron and Maury (2004) is yet another paper investigating the robustness of Galí and Gertler (1999) conclusions to the inclusion of additional supply-side mechanisms. They show that when a sticky-price model features produced inputs and fixed production costs, the labor share is no longer an appropriate measure of the marginal cost. However, they demonstrate that this problem has no incidence on the message conveyed by Galí and Gertler (1999). In this paper, we abstract from these refinements and exclusively focus on firm-specific capital and/or firm-specific labor within the framework of a simple sticky-price model where the labor share is an appropriate measure of the marginal cost.

Our main results are as follows. First, considering firm-specific capital alone does not lead to reasonable degrees of nominal rigidities. This result is robust to various details of our empirical methodology. Overall, we find that this hypothesis barely improves the standard NKPC specification. Second, considering firm-specific labor yields very reasonable estimates of the degree of nominal rigidities. This result holds either with fixed capital or with firm-specific capital. Finally, abstracting from all these modeling refinements leads to statistically satisfying estimates of the NKPC, though at the cost of economically unacceptable average price durations.

The remainder of the paper is structured as follows. Section 2 details the sticky-price model with capital and labor both firm specific. The alternative specifications are introduced as particular cases of this general model. Section 3 lays out our GMM estimation procedure and then reports the results. The last section briefly concludes.

2. Model Specifications

In this section, we briefly sketch the reference model, which features capital and labor as both firm specific. All the details of the resolution are relegated in the appendix. The alternative specifications arise as specialized cases of the reference model. These are detailed in the following subsection.

2.1 A Sticky-Price Model with Capital and Labor Both Firm Specific

We consider a discrete-time economy populated by a large number of infinitely lived agents. The representative household's goal in life is to maximize

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[U(c_t) - \int_0^1 V(h_t(\varsigma)) \mathrm{d}\varsigma \right] \right\}, \tag{1}$$

where E_t is the expectation operator conditional on information available as of time t, β is the subjective discount factor, c_t is the consumption of final good, $h_t(\varsigma)$ is the supply of labor of type ς, $U(\cdot)$ is strictly concave and increasing, and $V(\cdot)$ is strictly convex and increasing.[3]

The household seeks to maximize (1) subject to the sequence of budget constraints

$$c_t \leq \int_0^1 w_t(\varsigma) h_t(\varsigma) \mathrm{d}\varsigma + \Phi_t, \tag{2}$$

where Φ_t is an aggregate variable summarizing all net sources of income other than wages (e.g., profits redistributed to the household, financial revenues, etc.), and $w_t(\varsigma)$ is the real wage paid to labor of type ς.

The final good y_t is produced by perfectly competitive firms according to the constant elasticity of substitution (CES) technology

$$y_t = \left(\int_0^1 y_t(\varsigma)^{(\theta-1)/\theta} \mathrm{d}\varsigma \right)^{\theta/(\theta-1)}, \quad \theta > 1, \tag{3}$$

where $y_t(\varsigma)$ is the input of intermediate good ς used in the production of y_t. The parameter θ is the elasticity of substitution between any

[3]Notice that to simplify our presentation, we have abstracted from preference and technology shocks.

two intermediate goods. The first-order condition associated with $y_t(\varsigma)$ is

$$y_t(\varsigma) = \left(\frac{P_t(\varsigma)}{P_t}\right)^{-\theta} y_t. \tag{4}$$

Notice that perfect competition ensures that the aggregate price level P_t must obey

$$P_t = \left(\int_0^1 P_t(\varsigma)^{1-\theta} \mathrm{d}\varsigma\right)^{1/(1-\theta)}. \tag{5}$$

Intermediate goods are produced by monopolistic firms, according to the constant-returns technology

$$y_t(\varsigma) = k_t(\varsigma) f\left(\frac{h_t(\varsigma)}{k_t(\varsigma)}\right), \tag{6}$$

where $f(x) = x^{1/\phi}$, $\phi > 1$. The capital stock $k_t(\varsigma)$ is assumed to be firm specific, i.e., we assume away the existence of an economy-wide capital market. Let $i_t(\varsigma)$ denote the real investment expenditures of firm ς in period t. Next-period's capital stock $k_{t+1}(\varsigma)$ and $i_t(\varsigma)$ are linked together through the relation

$$i_t(\varsigma) = k_t(\varsigma) I\left(\frac{k_{t+1}(\varsigma)}{k_t(\varsigma)}\right), \tag{7}$$

where $I(\cdot)$ is convex, with $I(1) = \delta \in (0,1)$, $I'(1) = 1$, and $I''(1) = \epsilon > 0$. Here, δ is the depreciation rate. Eichenbaum and Fisher (2005) demonstrate that, in the context of the accumulation technology (7), ϵ is linked to the elasticity of the investment-capital ratio with respect to Tobin's q, denoted by ϱ, through the relation $\varrho = 1/(\delta\epsilon)$. Following Eichenbaum and Fisher (2005), we assume that firm ς selects the capital stock $k_{t+1}(\varsigma)$ based on information available as of time $t-1$, i.e., $i_t(\varsigma)$ is in firm ς's information set at $t-1$.

Additionally, as in Woodford (2003), we assume that in each period, a random fraction $1-\alpha$ of intermediate goods producers get to revise their price. The remaining firms simply rescale their price according to the simple rule $P_T(\varsigma) = x_{t,T} P_t(\varsigma)$, where

$$x_{t,T} = \begin{cases} \prod_{j=t}^{T-1} \pi^{1-\gamma} \pi_j{}^{\gamma} & \text{if } \; T > t \\ 1 & \text{otherwise} \end{cases}. \tag{8}$$

Here, non-reoptimizing firms partially index their prices to past levels of inflation and steady-state inflation π.[4] More precisely, the parameter $\gamma \in [0,1]$ measures the degree of indexation to the most recently available inflation measure. This specification is an extension of the inflation indexation mechanism considered in Woodford (2003). While with the latter a hybrid New Phillips curve is only valid in the neighborhood of a zero-inflation steady state, the former enables us to consider strictly positive steady-state inflation rates. In the limiting case $\gamma = 1$, this specification reduces to that considered by Christiano, Eichenbaum, and Evans (2005).

Following Eichenbaum and Fisher (2005), we assume a delay in the implementation of a new price. The latter is chosen at $t-1$ and becomes effective at date t. Thus, if drawn to reoptimize in period $t-1$, firm ς will select its price $P_t^*(\varsigma)$ so as to maximize

$$\mathrm{E}_{t-1}^{\varsigma} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{\lambda_T}{\lambda_t} \left\{ \left(\frac{x_{t,T} P_t(\varsigma)}{P_T} \right)^{1-\theta} y_T - s_T(\varsigma) \left(\frac{x_{t,T} P_t(\varsigma)}{P_T} \right)^{-\theta} y_T \right\}, \tag{9}$$

where λ_t is the Lagrange multiplier on constraint (2), $s_t(\varsigma)$ is firm ς's marginal cost in period t, and $\mathrm{E}_t^{\varsigma}\{\cdot\}$ is an expectation operator specific to firm ς that integrates over those future states of the world in which firm ς has not reset its price since t.

Our notations emphasize the fact that firm ς's real marginal cost depends on ς. There are two origins to this fact: (i) labor is firm specific, so that the wage rate paid by firm ς depends on firm ς's output, and (ii) firm ς accumulates its own capital stock that, consequently, depends on present and past price decisions. These mechanisms contribute to make $s_t(\varsigma)$ an increasing function of $y_t(\varsigma)$.

[4]In the context of our setup, π is a free parameter that we calibrate to inflation's observed average value.

The above model implies the following NKPC:

$$\mathrm{E}_{t-1}\left\{\frac{\gamma}{1+\beta\gamma}\hat{\pi}_{t-1}+\frac{\beta}{1+\beta\gamma}\hat{\pi}_{t+1}+\zeta^{-1}\frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\beta\gamma)}\hat{s}_t-\hat{\pi}_t\right\}=0, \tag{10}$$

where a letter with a hat denotes the log-deviation of the associated variable, s_t is the average (economy-wide) real marginal cost, and

$$\zeta = 1 + (\phi(\nu+1)-1)\theta - \varkappa_0(\alpha,\beta,\delta,\epsilon,\theta,\phi,\nu),$$

where $\nu = V_{hh}h/V_h$ is the elasticity of labor's marginal disutility with respect to hours worked, evaluated in steady state, and $\varkappa_0(\cdot)$ is a positive, complicated function of the parameters listed, which we briefly describe in the appendix.[5]

2.2 Alternative Specifications

We consider three alternative specifications, which we briefly detail below.

2.2.1 Firm-Specific Labor and Fixed Capital

This specification is identical to that considered in Woodford (2003, chap. 3). We simply assume that there is no capital accumulation. The representative household still maximizes (1) subject to (2). Each firm possesses its own fixed stock of capital, and through a suitable normalization, they are assumed to operate the same technology given by

$$y_t(\varsigma) = f(h_t(\varsigma)).$$

Alternatively, one can interpret this specification as the limiting case in which capital adjustment costs are so important that firms simply prefer not to accumulate capital, i.e., the limiting case $\epsilon \to \infty$ (and $\delta = 0$).

Irrespective of the interpretation that one favors, what is important here is that firm ς faces a marginal cost that depends on ς.

[5] For further details, see also Christiano (2004), Eichenbaum and Fisher (2005), Sveen and Weinke (2004, 2005), and Woodford (2005).

This feature, as stressed out by Woodford (2003), generates a strong degree of strategic complementarity between price setters, which translates into a small elasticity of inflation with respect to the real marginal cost in the New Phillips curve.

This specification implies

$$\mathrm{E}_{t-1}\left\{\frac{\gamma}{1+\beta\gamma}\hat{\pi}_{t-1}+\frac{\beta}{1+\beta\gamma}\hat{\pi}_{t+1}+\zeta^{-1}\frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\beta\gamma)}\hat{s}_t-\hat{\pi}_t\right\}=0, \tag{11}$$

$$\zeta = 1+(\phi(\nu+1)-1)\theta,$$

which is very close to that derived by Woodford (2003). Additionally, Galí, Gertler, and López-Salido (2001) and Sbordone (2002) study a similar specification.

Notice that in the case when firms face specific labor and capital markets, the implied ζ is necessarily smaller than in the present specification, as long as $\varkappa_0(\alpha,\beta,\delta,\epsilon,\theta,\phi,\nu)$ is positive, which is the case in the estimation reported below. This means that the degree of strategic complementarity is smaller when firms face specific labor and capital markets than in the present specification. As a consequence, we should expect a higher degree of nominal rigidity when firms face specific labor and capital markets than under the assumption of fixed capital and specific labor. However, the hypotheses underlying specification (10) sound more realistic than assuming that capital is fixed over the business cycle. It remains to be seen whether the implied degree of nominal rigidity is within the reasonable range described in the introduction.

2.2.2 Firm-Specific Capital, Aggregate Labor Market

This is a simplified version of the model considered by Eichenbaum and Fisher (2005). In this setting, we assume that firms accumulate their own capital stock and rent labor services on an economy-wide market. In this case, the representative household is assumed to maximize

$$\mathrm{E}_0\left\{\sum_{t=0}^{\infty}\beta^t[U(c_t)-V(h_t)]\right\}, \tag{12}$$

subject to the constraint

$$c_t \leq w_t h_t + \Phi_t,$$

where w_t is the real aggregate wage and Φ_t represents other net sources of income.

Intermediate goods producers still operate technology (6) and accumulate physical capital according to (7). If drawn to reoptimize in period t, firm ς will select its price $P_t^*(\varsigma)$ so as to maximize (9). Again, this specification allows the real marginal cost of firm ς to depend on ς, thus rendering the elasticity of inflation with respect to s_t smaller than in the model with aggregate labor and capital markets. Notice, however, that firm ς's real marginal cost $s_t(\varsigma)$ depends on ς only through the specificity of $k_t(\varsigma)$. We then obtain the NKPC

$$\mathrm{E}_{t-1}\left\{\frac{\gamma}{1+\beta\gamma}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\gamma}\hat{\pi}_{t+1} + \zeta^{-1}\frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\beta\gamma)}\hat{s}_t - \hat{\pi}_t\right\} = 0, \tag{13}$$

$$\zeta = 1 + (\phi - 1)\theta - \varkappa_1(\alpha, \beta, \delta, \epsilon, \theta, \phi),$$

with $\varkappa_1(\alpha, \beta, \delta, \epsilon, \theta, \phi)$ another complicated function of the parameters listed, which we briefly describe in the appendix.

2.2.3 *Aggregate Labor and Capital Markets*

We now suppose that there exist aggregate capital and labor markets. Except for the automatic indexation that is considered here, this is the specification retained in Yun (1996). In this model, the representative household seeks to maximize (12) subject to the sequence of budget constraints

$$c_t + i_t \leq w_t h_t + \rho_t k_t + \Phi_t, \tag{14}$$

$$i_t = k_t I\left(\frac{k_{t+1}}{k_t}\right), \tag{15}$$

where w_t is the real wage rate, k_t is the stock of capital, which is now accumulated by the households, and ρ_t is the corresponding rental rate. Notice that in this case, neither h_t nor w_t depend on ς.

Intermediate goods producers still operate technology (6). However, since they have access to perfectly competitive inputs markets, and they operate with constant returns to scale, it can be shown that their marginal cost does not depend on ς.[6] In this case, they select $P_t^*(\varsigma)$ so as to maximize

$$\mathrm{E}_{t-1}^{\varsigma} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \frac{\lambda_T}{\lambda_t} \left\{ \left(\frac{x_{t,T} P_t(\varsigma)}{P_T} \right)^{1-\theta} y_T - s_T \left(\frac{x_{t,T} P_t(\varsigma)}{P_T} \right)^{-\theta} y_T \right\}. \tag{16}$$

This specification implies the standard NKPC

$$\mathrm{E}_{t-1} \left\{ \frac{\gamma}{1+\beta\gamma} \hat{\pi}_{t-1} + \frac{\beta}{1+\beta\gamma} \hat{\pi}_{t+1} + \frac{(1-\alpha\beta)(1-\alpha)}{\alpha(1+\beta\gamma)} \hat{s}_t - \hat{\pi}_t \right\} = 0. \tag{17}$$

3. Estimating the New Phillips Curves

3.1 Estimation Strategy

Our goal is now to estimate the new Phillips curves implied by the different model specifications. Following Galí and Gertler (1999) and Eichenbaum and Fisher (2005), we estimate these equations by resorting to Hansen's (1982) generalized method of moments (GMM). Equations (10), (11), (13), and (17) can be interpreted as orthogonality conditions, which lend themselves to instrumental variable estimation. In particular, each of these equations can be generically written as

$$\mathrm{E}_{t-1}\{u_{t+1}(\boldsymbol{\psi})\} = 0, \tag{18}$$

where we defined the parameter vector $\boldsymbol{\psi} = (\alpha, \gamma)'$. Recall that α is the probability of not reoptimizing prices in a given quarter and γ is the degree of price indexation to the most recently available inflation measure. These orthogonality conditions imply

$$\mathrm{E}\{\mathbf{f}_{t+1}(\boldsymbol{\psi})\} = 0, \quad \mathbf{f}_{t+1}(\boldsymbol{\psi}) \equiv u_{t+1}(\boldsymbol{\psi})\mathbf{Z}_{t-1} \tag{19}$$

for any ℓ-dimensional vector $\mathbf{Z}_t$ in the agents' time t information set. The fact that the vector of instruments belongs to date $t-1$'s

[6] See Chari, Kehoe, and McGrattan (2000) for a formal demonstration.

information set reflects our assumption regarding the timing of price decisions in the preceding section. Thus, our estimate of $\boldsymbol{\psi}$ is

$$\hat{\boldsymbol{\psi}} = \arg\min_{\boldsymbol{\psi}\in\boldsymbol{\Psi}} \mathbf{g}_T(\boldsymbol{\psi})'\mathbf{W}_T\mathbf{g}_T(\boldsymbol{\psi}), \tag{20}$$

where

$$\mathbf{g}_T(\boldsymbol{\psi}) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{f}_{t+1}(\boldsymbol{\psi}).$$

Here, $\boldsymbol{\Psi}$ is the set of admissible values for $\boldsymbol{\psi}$, T is the sample size, and $\mathbf{W}_T$ is a symmetric positive definite weighting matrix. For later reference, it is convenient to define $J_T(\boldsymbol{\psi}) = \mathbf{g}_T(\boldsymbol{\psi})'\mathbf{W}_T\mathbf{g}_T(\boldsymbol{\psi})$.

The optimal choice for the weighting matrix consists in choosing the inverse of the spectral density matrix at frequency zero of $\mathbf{f}_{t+1}(\boldsymbol{\psi}_0)$, where $\boldsymbol{\psi}_0$ is $\boldsymbol{\psi}$'s true value. Let $\mathbf{S}$ denote this matrix. As argued by Eichenbaum and Fisher (2005), the theory predicts that $\mathbf{f}_{t+1}(\boldsymbol{\psi}_0)$ has an MA(1) representation, so that $\mathbf{S}$ obeys

$$\mathbf{S} = \sum_{k=-1}^{1} \mathrm{E}\big\{[\mathbf{f}_{t+1+k}(\boldsymbol{\psi}_0)][\mathbf{f}_{t+1+k}(\boldsymbol{\psi}_0)]'\big\}. \tag{21}$$

We could thus obtain a consistent estimator of $\mathbf{S}$ by directly implementing the empirical counterpart of equation (21)—the truncated kernel estimate of $\mathbf{S}$. This method has the obvious advantage of being completely consistent with our model and could be interpreted as a strict test of the theory expounded before. One important drawback, though, is that it can eventually lead to a semidefinite weighting matrix that is not positive in small samples. In practice, we encountered this problem in our application.

Following West (1997), a simple remedy to this problem is to fit an MA(1) to the GMM residuals $u_t(\boldsymbol{\psi})$ and then use the estimated auxiliary parameters to compute the spectral density matrix of $\mathbf{f}_{t+1}(\boldsymbol{\psi})$ evaluated at frequency zero.[7] More precisely, the model implies that $u_t(\boldsymbol{\psi})$ admits the representation

$$u_t(\boldsymbol{\psi}) = \epsilon_t + \eta\epsilon_{t-1}.$$

[7]See Jondeau and Le Bihan (2005) and Sahuc (2004) for early implementations of this method in a similar context.

Fitting this model to u_t yields an estimate $\hat{\eta}$ of the MA parameter. Let us define $\hat{\mathbf{d}}_{t+1} = (\mathbf{Z}_{t-1} + \mathbf{Z}_t\hat{\eta})\hat{\epsilon}_t$, where $\hat{\epsilon}_t$ is the residual obtained after the first GMM step. Then, a heteroskedasticity- and autocorrelation-consistent estimate of $\mathbf{S}$ is given by

$$\hat{\mathbf{S}} = \frac{1}{T-1}\sum_{t=1}^{T-1} \hat{\mathbf{d}}_{t+1}\hat{\mathbf{d}}'_{t+1}.$$

This is our preferred method since it imposes all the restrictions implied by our model.[8] Notice that, based on simulation experiments, West (1997) concludes that his estimator is efficient precisely when the truncated kernel yields an estimate of $\mathbf{S}$ that is not semidefinite positive. As a complementary remedy, we also used the Newey and West (1987) (NW) consistent estimate with a one-lag window. This approach is close in spirit to that advocated by Eichenbaum and Fisher (2005), though the practical details differ.

Alternatively, if one does not completely believe in the model, allowing for more serial correlation in the GMM errors might be desirable; thus we also use the NW consistent estimate with a twelve-lag window, as in Galí and Gertler (1999).

The vector of instruments $\mathbf{Z}_t$ contains a constant as well as inflation, the labor share, the output gap,[9] and the short-term nominal interest rate. We have thus five moment conditions and two estimated parameters, so that $TJ_T(\psi)$ is asymptotically distributed as a $\chi^2(3)$. Following Eichenbaum and Fisher, and in contrast with former studies relying on the GMM, we use a very small set of instruments. This is not innocent, since it has been shown that in small samples, using too many instruments can generate substantial biases.

To conclude this section, it is important to emphasize that equations (10), (11), (13), and (17) are observationally equivalent in terms of the J statistic. Yet they differ with respect to the implied degree of nominal rigidities. Thus, the question under study is not so much whether these equations pass the overidentification test but

[8] An alternative to this strategy would be to implement the procedure described in Eichenbaum, Hansen, and Singleton (1988).

[9] The output gap is defined as linearly detrended, logged real output. We also considered HP-filtered output and quadratically detrended output and obtained qualitatively similar results. See the next section.

whether they imply an economically reasonable degree of price stickiness. To judge this, we resort to the empirical evidence reported in Dhyne et al. (2006).

3.2 Calibrated Parameters and Data

Since we only estimate α and γ, we must calibrate the remaining parameters. As is conventional in the literature, we set $\beta = 0.99$. The capital depreciation rate δ is set to 2.5 percent. The elasticity of demand θ is set to 10, as is conventional in the literature. We set $\phi = 0.54^{-1}$, thus allowing for a labor share equal to its euro-data empirical counterpart. Notice that we implicitly assume that profits are redistributed proportionately to factor income, so that $1/\phi$ is indeed the steady-state labor share.

The elasticity of capital adjustment costs ϵ is set to 3, as in Woodford (2005). This, with the value set for δ, implies an elasticity of the investment-capital ratio of 13.33. As argued by Eichenbaum and Fisher (2005), this value is consistent with microeconomic evidence reported by Gilchrist and Himmelberg (1995) when it comes to the United States.

Finally, we set $\nu = 1$, as in Altig et al. (2005) and Christiano, Eichenbaum, and Evans (2005). This implies a larger elasticity of labor supply than that estimated by Avouyi-Dovi and Matheron (2004) or Smets and Wouters (2003) on euro-area data. For our purpose, this choice is conservative because setting $\nu = 2$, a value consistent with euro-area data, would greatly reinforce the importance of labor specificity. In the following sections, we explore the sensitivity of our results to this key parameter.

The calibrated parameters are summarized in table 1. Alternatively, we also investigate the calibration considered by Eichenbaum and Fisher (2005). In their benchmark case, they set $\phi = 1.5$ (a labor share equal to 2/3) and $\theta = 11$ (a steady-state markup equal to 10 percent).

Table 1. Calibrated Parameters

ϕ	θ	ν	β	δ	ϵ
1.855	10.000	1.000	0.990	0.025	3.000

The data used in this paper are extracted from the area-wide database compiled by Fagan, Henry, and Mestre (2005). The mnemonics are as follows: real output is YER, nominal output is YEN, the aggregate nominal wage bill is WIN, the GDP deflator is YED, and the short-term nominal interest rate is STN. We construct the labor share as the ratio WIN/YEN. Finally, the inflation rate is computed as the first difference of the logarithm of YED.

3.3 Results

The estimation results are reported in table 2. The table reports estimates for α and γ, as well as the implied price duration. In addition, the table shows the estimated value of TJ_T and the associated p-value. Recall that the four equations considered in the present paper are observationally equivalent in terms of J_T. Similarly, each estimated equation has the same γ. Consequently, table 2 reports TJ_T and $\hat{\gamma}$ only for the first equation.

Following Eichenbaum and Fisher (2005), the 95 percent confidence interval for the average duration is computed as follows. We first determine the 95 percent confidence interval of $\hat{\alpha}$ and then simply transform the latter using the function $\alpha \mapsto (1-\alpha)^{-1}$. Consequently, the obtained confidence band is not symmetric. Notice that in some cases, $\hat{\alpha} + 1.96\text{std}(\hat{\alpha})$ is higher than 1. For these cases, the upper bound is simply denoted by NA, since an infinite price duration is theoretically possible.

The top panel shows results pertaining to our preferred estimation method, i.e., that consisting of fitting an MA(1) to the GMM residuals $u_t(\boldsymbol{\psi})$, following West (1997). In this estimation, we use linearly detrended, logged output as a measure of the output gap. According to the overidentification test, the model cannot be rejected at conventional confidence levels, with a p value of 9.6 percent. The first point to mention is that γ is small, though not very well estimated. Apparently, a small degree of price indexation is required to match the data.[10] Second, as expected, the degree

[10] We investigated a constrained version of our setup, imposing $\gamma = 0$ or $\gamma = 1$. The first constraint is not rejected by the data. From a quantitative point of view, there are no substantial differences between our benchmark estimation and this constrained estimation. To the contrary, the second constraint is not supported and leads to statistical rejection of the model.

Table 2. Benchmark Estimation Results

MA(1) *Fit to GMM Residuals*					
	$\hat{\alpha}$	$\hat{\gamma}$	**Duration**	TJ_T	p
	Linearly Detrended Output				
Eq. (10)	0.7458 (0.0824)	0.1430 (0.1786)	3.93 [2.41,10.79]	6.35	9.59
Eq. (11)	0.7284 (0.0775)	—	3.68 [2.36,8.36]	—	—
Eq. (13)	0.8508 (0.0561)	—	6.70 [3.86,25.54]	—	—
Eq. (17)	0.9454 (0.0192)	—	18.31 [10.85,58.58]	—	—
	HP-Filtered Output				
Eq. (10)	0.7788 (0.0883)	0.2801 (0.1727)	4.52 [2.54,20.78]	7.68	5.31
Eq. (11)	0.7594 (0.0833)	—	4.16 [2.48,12.93]	—	—
Eq. (13)	0.8730 (0.0589)	—	7.87 [4.12,87.64]	—	—
Eq. (17)	0.9529 (0.0198)	—	21.24 [11.64,121.06]	—	—
	Quadratically Detrended Output				
Eq. (10)	0.7480 (0.0795)	0.1775 (0.1653)	3.97 [2.45,10.41]	6.87	7.60
Eq. (11)	0.7305 (0.0748)	—	3.71 [2.40,8.14]	—	—
Eq. (13)	0.8523 (0.0541)	—	6.77 [3.94,24.03]	—	—
Eq. (17)	0.9459 (0.0184)	—	18.49 [11.08,55.70]	—	—

Notes: The standard errors of α and γ are in parentheses below the empirical estimates. The 95 percent confidence intervals of the average price durations are below the empirical estimates, in brackets.

of nominal rigidity varies a lot depending on the specification of the NKPC. The lowest estimate is obtained in the case when capital is fixed and labor is firm specific (equation (11)). In this case, $\alpha = 0.728$, implying an average price duration of 3.7 quarters, in accordance with microeconomic evidence. Allowing for firm-specific variable capital in addition somewhat increases the degree of nominal rigidity, though to a small extent (equation (10)). In this case, we obtain $\alpha = 0.746$, implying an average price duration of 3.9 quarters. Thus, the model remains in the admissible range, at least when it comes to euro-area data. Our estimates also imply average price durations smaller than those reported by Galí, Gertler, and López-Salido (2001, 2003), even when they allow for firm-specific labor. These durations are always higher than four quarters.

In contrast, when an aggregate labor market is assumed, either with or without firm-specific capital, the probability of not reoptimizing prices appears too high when compared to results reported by Dhyne et al. (2006). When capital is firm specific (equation (13)), we obtain an average price duration of 6.70 quarters; when there are aggregate markets for both capital and labor (equation (17)), this duration reaches the astonishing level of 18.31 quarters.

The middle panel of table 2 reports results obtained when we use the same estimation strategy but use HP-filtered logged output as our measure of the output gap.[11] In this case, the model still passes the overidentification test, with a smaller p-value. The results are less encouraging. The average price durations are always higher than four quarters. Notice, however, that when sampling uncertainty is taken into account, the estimates are still consistent with the European microeconomic evidence. The last panel reports results when we use quadratically detrended output as our measure of the output gap. The model still passes the overidentification test. In this case, we obtain average price durations that resemble those derived in the top panel of the table.

Overall, our results suggest that including firm-specific labor in the NKPC yields estimates of α that deliver both a good statistical fit and an economically plausible degree of nominal rigidity, at least when compared to microeconomic studies on the euro area (Dhyne et al. 2006). This conclusion is similar to that emphasized by Galí, Gertler, and López-Salido (2001), though reached with a slightly different econometric framework. Additionally, we conclude

[11] Our benchmark measure of the output gap might be questioned on the grounds that a linear trend might not be the appropriate measure of potential output. It is thus important to assess the robustness of our results to alternative definitions of this variable.

Table 3. Results with Eichenbaum and Fisher's (2005) Calibration

MA(1) *Fit to GMM Residuals*					
	$\hat{\alpha}$	$\hat{\gamma}$	**Duration**	TJ_T	p
	Linearly Detrended Output				
Eq. (10)	0.7641 (0.0755)	0.1430 (0.1786)	4.24 [2.60,11.38]	6.35	9.59
Eq. (11)	0.7509 (0.0725)	—	4.01 [2.56,9.34]	—	—
Eq. (13)	0.8763 (0.0472)	—	8.08 [4.62,32.07]	—	—
Eq. (17)	0.9454 (0.0192)	—	18.31 [10.85,58.58]	—	—

Notes: The standard errors of α and γ are in parentheses below the empirical estimates. The 95 percent confidence intervals of the average price durations are below the empirical estimates, in brackets.

that allowing for firm-specific capital in addition to firm-specific labor increases moderately the required degree of nominal rigidity necessary to match the data. However, this amount of nominal rigidity remains economically reasonable. In contrast, a model featuring firm-specific capital and aggregate labor implies too high a probability of not reoptimizing prices.

3.4 Sensitivity Analyses

As a first robustness check, we investigate the consequences of adopting the calibration considered by Eichenbaum and Fisher (2005). The results are reported in table 3. To this end, we set $\phi = 3/2$ and $\theta = 11$. The key difference between what is investigated here and the results reported by Eichenbaum and Fisher is that in our context, the parameter ν affects our estimates when it comes to equation (10) and equation (11). This is not the case in Eichenbaum and Fisher's paper, because they abstract from labor specificity. The figure to keep in mind is their estimate of $\alpha = 0.72$ (see their table 3), which they obtain under the assumption of a constant demand elasticity. This framework is equivalent to that underlying equation (13).

In this case, using euro-area data, we obtain $\alpha = 0.88$ when we resort to the same calibration as theirs. This illustrates an important fact. With our benchmark calibration (irrespective of the chosen estimation strategy), we always obtain larger average price durations than they do with U.S. data. Obviously, this can originate either from our calibration or from the data. The present exercise suggests that the data are the correct suspect. This is reminiscent of the contrast arising between the results reported by Bils and Klenow (2004) and by Dhyne et al. (2006). We investigate further the sensitivity of our results below.

Before doing so, we also investigate to what extent our conclusions depend on our choice of a weighting matrix. Table 4 reports results obtained when we use the Newey-West estimator instead of the West estimator. In this case, we fall back to our benchmark calibration. The top panel enforces a one-lag window ($L = 1$). We obtain qualitatively similar conclusions. In this case, the overidentification test is even more supportive of our model, with a p-value of almost 43 percent. The estimated value of γ is almost four times as small as in the top panel and still not statistically different from 0. The ordering of α remains the same, though the estimated values appear somewhat smaller (with higher standard errors also). Equation (11) implies the smallest degree of nominal rigidity, followed by equation (10). Equations (13) and (17) are ranked third and fourth. Once again, the estimated average price durations seem reasonable for the first two specifications and at odds with microeconomic evidence for the last two.

Finally, the bottom panel of table 4 reports results obtained when using the Newey-West estimator with a twelve-lag window ($L = 12$ in table 2). Once again, the overidentification test does not allow us to reject the model, with a p-value slightly above 20 percent. Overall, we obtain very similar results in qualitative terms. Notice, however, that the estimates of α are now substantially higher than in the previous exercises, i.e., higher than 0.8 for all four specifications. Consequently, we obtain average price durations between 5.6 and 30.5 quarters. If we were to take these results seriously, we would conclude that none of the specifications considered can simultaneously match the data and be in accordance with euro-area microeconomic evidence. However, these estimates are obtained using auxiliary assumptions that are not supported by the theory

Table 4. Alternative Estimation Results

	$\hat{\alpha}$	$\hat{\gamma}$	**Duration**	TJ_T	p
	Newey-West HAC, $L = 1$				
Eq. (10)	0.7188 (0.1177)	0.0385 (0.1681)	3.56 [1.95,19.75]	2.78	42.65
Eq. (11)	0.7030 (0.1107)	—	3.37 [1.95,19.75]	—	—
Eq. (13)	0.8322 (0.0815)	—	5.96 [3.05,125.26]	—	—
Eq. (17)	0.9390 (0.0283)	—	16.39 [8.59,178.56]	—	—
	Newey-West HAC, $L = 12$				
Eq. (10)	0.8446 (0.1314)	0.1201 (0.1555)	6.43 [2.42,NA]	4.61	20.27
Eq. (11)	0.8220 (0.1264)	—	5.62 [2.35,NA]	—	—
Eq. (13)	0.9160 (0.0836)	—	11.90 [4.03,NA]	—	—
Eq. (17)	0.9673 (0.0280)	—	30.54 [11.42,NA]	—	—

Notes: The standard errors of α and γ are in parentheses below the empirical estimates. The 95 percent confidence intervals of the average price durations are below the empirical estimates, in brackets.

expounded in the previous section. It is thus unclear whether they should be granted much attention.

To complete this sensitivity analysis, we investigate to what extent our results depend on parameters calibrated prior to estimation. We focus our attention on four key parameters, namely ϵ, θ, ν, and ϕ in the model underlying equation (10).[12] The results are reported in table 5. Overall, we find that when these parameters increase, the probability of not reoptimizing prices decreases. These results are fairly intuitive. When ϵ increases, it becomes more and more difficult to adjust the capital stock, so that in the limit, capital remains fixed. In this case, the model converges to the specification underlying equation (11). Notice that the differences between the results under $\epsilon = 100$ and those under $\epsilon = 3$ are not very big.

[12]Notice that the parameters θ, ν, and ϕ also affect specification (11).

Table 5. Sensitivity Analysis

		ϵ			θ		
		0.01	**1**	**100**	**6**	**11**	**21**
Eq. (10)	$\hat{\alpha}$	0.79 (0.07)	0.76 (0.08)	0.73 (0.08)	0.80 (0.07)	0.73 (0.08)	0.65 (0.10)
	D	4.81 [2.82,16.40]	4.14 [2.48,12.60]	3.70 [2.26,8.51]	5.01 [2.99,15.54]	3.76 [2.31,10.08]	2.83 [1.81,6.44]
Eq. (11)	$\hat{\alpha}$				0.78 (0.06)	0.72 (0.08)	0.63 (0.10)
	D				4.56 [2.88,10.97]	3.54 [2.28,7.94]	2.74 [1.81,5.62]
Eq. (13)	$\hat{\alpha}$	0.91 (0.04)	0.87 (0.05)	0.84 (0.05)	0.88 (0.05)	0.84 (0.06)	0.79 (0.07)
	D	11.73 [6.29,86.36]	7.48 [4.13,39.98]	6.01 [3.71,15.72]	8.48 [4.81,35.78]	6.40 [4.81,35.78]	4.69 [2.80,14.34]
		ν			ϕ		
		0.01	**2**	**10**	$\mathbf{0.7^{-1}}$	$\mathbf{0.6^{-1}}$	$\mathbf{0.5^{-1}}$
Eq. (10)	$\hat{\alpha}$	0.85 (0.06)	0.68 (0.10)	0.45 (0.12)	0.78 (0.07)	0.76 (0.08)	0.73 (0.08)
	D	6.64 [3.82,25.11]	3.14 [1.97,7.67]	1.82 [1.27,3.23]	4.55 [2.79,12.41]	4.17 [2.55,11.45]	3.78 [2.31,10.32]
Eq. (11)	$\hat{\alpha}$	0.83 (0.05)	0.66 (0.09)	0.44 (0.11)	0.77 (0.07)	0.74 (0.07)	0.72 (0.08)
	D	5.92 [3.68,15.14]	2.98 [1.95,6.32]	1.78 [1.27,2.98]	4.31 [2.73,10.22]	3.92 [2.50,9.05]	3.53 [2.27,10.32]
Eq. (13)	$\hat{\alpha}$				0.89 (0.04)	0.87 (0.08)	0.84 (0.06)
	D				8.97 [5.12,36.41]	7.47 [4.28,29.34]	6.25 [3.61,23.29]

Notes: The standard errors of α are in parentheses below the empirical estimates. D stands for average duration. The 95 percent confidence intervals of the average price durations are below the empirical estimates, in brackets. A blank cell indicates that the corresponding equation is not affected by the parameter under study.

This is reminiscent of Eichenbaum and Fisher (2005). The other three parameters contribute to increasing the degree of strategic complementarity between price setters, as discussed by Woodford (2003). It thus comes as no surprise that the required degree of nominal rigidity decreases when these parameters are set to higher values.

4. Conclusion

The recent literature has emphasized the importance of assuming firm-specific capital in optimizing models with nominal rigidity based on the Calvo (1983) specification. Following this literature, we have sought to assess the fruitfulness of this hypothesis when one is concerned solely with obtaining economically realistic estimates of the probability of not reoptimizing prices in NKPC based on euro-area data. By "economically realistic," we mean that this probability should imply an average price duration compatible with evidence drawn from microeconomic data (Dhyne et al. 2006).

Our objective was to compare the hypothesis of firm-specific capital with the complementary view that labor can also be firm specific. An important aspect of our analysis is our careful implementation of the generalized method of moments. First, following Eichenbaum and Fisher (2005), we select a very small number of instruments. Second, we exploit an important restriction implied by the theory, namely that the forecast error in the orthogonality condition should admit a first-order moving-average representation. Imposing this restriction during the course of the estimation has the merit of transparency.

We obtain two main conclusions. First, we confirm previous findings that allowing for firm-specific labor in a simple model with fixed capital yields a very reasonable estimate of the probability of not reoptimizing prices. The implied average price duration appears consistent with euro-area microeconomic evidence. Second, we found that allowing for firm-specific capital in addition to firm-specific labor results in a higher degree of nominal rigidity. This result is analytically derived by comparing equations (10) and (11). However, the implied probability of not reoptimizing remains acceptable when compared with microeconomic evidence. Thus, from the point of view of empirical realism, the model with labor and capital both firm specific fares well compared with the model with firm-specific labor only.

Appendix. Deriving the NKPC in the Benchmark Model

To make the paper completely self-contained, this appendix specializes the calculations in Woodford (2005) to the particular case under study here. Below, we outline, step by step, the calculations needed to obtain the NKPC.

Optimal Price Setting

The first step in deriving the New Keynesian Phillips curve is to obtain the first-order condition associated with the optimal choice of price. If drawn to reoptimize in period t, firm ς will select $P_t^*(\varsigma)$ so as to maximize

$$\mathrm{E}_{t-1}^{\varsigma}\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\frac{\lambda_T}{\lambda_t}\left\{\left(\frac{x_{t,T}P_t^*(\varsigma)}{P_T}\right)^{1-\theta}y_T - s_T(\varsigma)\left(\frac{x_{t,T}P_t^*(\varsigma)}{P_T}\right)^{-\theta}y_T\right\}.$$

Here, $s_t(\varsigma)$ is the Lagrange multiplier associated with firm ς's production constraint. This term corresponds to the real marginal cost of producing an additional unit of intermediate good ς. Notice that firm ς discounts future cash flows according to $\beta^{T-t}\lambda_T/\lambda_t$, where λ_t is the Lagrange multiplier associated with the budget constraint in the representative household's program.

The associated first-order condition writes

$$\mathrm{E}_{t-1}^{\varsigma}\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\frac{\lambda_T}{\lambda_t}y_T\left(\frac{x_{t,T}P_t(\varsigma)}{P_T}\right)^{-\theta}\left\{\frac{x_{t,T}}{\pi_{t,T}}\frac{P_t(\varsigma)}{P_t} - \frac{\theta}{\theta-1}s_T(\varsigma)\right\} = 0.$$

Expressed in log-linear terms, the above equation rewrites

$$\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\mathrm{E}_{t-1}^{\varsigma}\{\hat{p}_t^*(\varsigma) + \hat{x}_{t,T} - \hat{\pi}_{t,T} - \hat{s}_T(\varsigma)\} = 0. \qquad (22)$$

Here, a letter with a hat refers to the log-deviation of the associated variable. We would like to solve this equation for $\hat{p}_t^*(\varsigma)$. Doing so requires that we know the behavior of $\hat{s}_t(\varsigma)$.

Real Marginal Cost

In the context of the present model, an expression for the real marginal cost can be obtained as follows. The first-order condition associated with the optimal choice of labor by firm ς is

$$w_t(\varsigma) = s_t(\varsigma)\frac{1}{\phi}\left(\frac{h_t(\varsigma)}{k_t(\varsigma)}\right)^{\frac{1}{\phi}-1}.$$

At the same time, the optimal supply of labor of type ς by the representative household obeys

$$\lambda_t w_t(\varsigma) = V_h(h_t(\varsigma)).$$

Combining these expressions, and making use of the production function (equation (6)), we arrive at

$$s_t(\varsigma) = \frac{\phi}{\lambda_t} V_h \left(\left(\frac{y_t(\varsigma)}{k_t(\varsigma)} \right)^{\phi} k_t(\varsigma) \right) \left(\frac{y_t(\varsigma)}{k_t(\varsigma)} \right)^{\phi-1}.$$

Log-linearizing this expression yields

$$\hat{s}_t(\varsigma) = \omega \hat{y}_t(\varsigma) - (\omega - \nu)\hat{k}_t(\varsigma) - \hat{\lambda}_t, \tag{23}$$

where $\omega = \phi(\nu + 1) - 1$ and $\nu = V_{hh}h/V_h$. Using equation (4), we can eliminate $\hat{y}_t(i)$ and obtain

$$\hat{s}_t(\varsigma) = \omega \hat{y}_t - (\omega - \nu)\hat{k}_t(\varsigma) - \hat{\lambda}_t - \omega\theta\hat{p}_t(\varsigma),$$

where $\hat{p}_t(\varsigma)$ is the log-deviation of $P_t(\varsigma)/P_t$. Integrating the above equation yields

$$\hat{s}_t = \omega \hat{y}_t - (\omega - \nu)\hat{k}_t - \hat{\lambda}_t,$$

where $\hat{s}_t$ is interpreted as the average real marginal cost. Subtracting this equation from the previous one, we arrive at

$$\hat{s}_t(\varsigma) = \hat{s}_t - (\omega - \nu)\tilde{k}_t(\varsigma) - \omega\theta\hat{p}_t(\varsigma). \tag{24}$$

Here, $\tilde{k}_t(\varsigma)$ is the log-deviation of the ratio $k_t(\varsigma)/k_t$. Notice that in the case that $P_t(\varsigma)$ has not been reoptimized since period t, this equation rewrites

$$\hat{s}_T(\varsigma) = \hat{s}_T - (\omega - \nu)\tilde{k}_T(\varsigma) - \omega\theta[\hat{p}_t^*(\varsigma) + \hat{x}_{t,T} - \hat{\pi}_{t,T}]. \tag{25}$$

It is clear from either (24) or (25) that the real marginal cost depends on $\tilde{k}_t(\varsigma)$. Hence, the next step is to characterize the dynamic behavior of this variable.

Physical Capital Evolution

The first-order condition with respect to $k_{t+1}(\varsigma)$ is

$$I'\left(\frac{k_{t+1}(\varsigma)}{k_t(\varsigma)}\right) = \beta \mathrm{E}_{t-1}\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[s_{t+1}(\varsigma)\left(1-\frac{1}{\phi}\right)\left(\frac{h_{t+1}(\varsigma)}{k_{t+1}(\varsigma)}\right)^{\frac{1}{\phi}} + \frac{k_{t+2}(\varsigma)}{k_{t+1}(\varsigma)} I'\left(\frac{k_{t+2}(\varsigma)}{k_{t+1}(\varsigma)}\right) - I\left(\frac{k_{t+2}(\varsigma)}{k_{t+1}(\varsigma)}\right)\right]\right\}.$$

Recall that we have assumed that $I(1) = \delta$, $I'(1) = 1$, and $I''(1) = \epsilon > 0$. Log-linearizing the above equation yields

$$\begin{aligned}
&\epsilon(\hat{k}_{t+1}(\varsigma) - \hat{k}_t(\varsigma)) + \hat{\lambda}_t \\
&\quad = \mathrm{E}_{t-1}\big\{\hat{\lambda}_{t+1} + (1-\beta(1-\delta))\big[\hat{s}_{t+1}(\varsigma) + \hat{y}_{t+1}(\varsigma) - \hat{k}_{t+1}(\varsigma)\big] \\
&\qquad + \beta\epsilon(\hat{k}_{t+2}(\varsigma) - \hat{k}_{t+1}(\varsigma))\big\},
\end{aligned}$$

where we made use of equation (6) and of the steady-state relation

$$s\left(1-\frac{1}{\phi}\right)\left(\frac{h}{k}\right)^{\frac{1}{\phi}} = \frac{1-\beta(1-\delta)}{\beta}.$$

Using equation (23) to eliminate $\hat{s}_{t+1}(\varsigma)$, we finally arrive at

$$\begin{aligned}
&\epsilon(\hat{k}_{t+1}(\varsigma) - \hat{k}_t(\varsigma)) + \hat{\lambda}_t \\
&\quad = \mathrm{E}_{t-1}\big\{\beta(1-\delta)\hat{\lambda}_{t+1} + (1-\beta(1-\delta))(1+\omega)\hat{y}_{t+1}(\varsigma) \\
&\qquad - (1-\beta(1-\delta))(1+\omega-\nu)\hat{k}_{t+1}(\varsigma) + \beta\epsilon(\hat{k}_{t+2}(\varsigma) - \hat{k}_{t+1}(\varsigma))\big\}.
\end{aligned}$$

Integrating this equation over the set of intermediate goods-producing firms and subtracting the results from the above equation yields

$$\xi \mathrm{E}_{t-1}\{\hat{p}_{t+1}(\varsigma)\} = \mathrm{E}_{t-1}\big\{\beta\tilde{k}_{t+2}(\varsigma) - \tau\tilde{k}_{t+1}(\varsigma) + \tilde{k}_t(\varsigma)\big\}, \tag{26}$$

where we made use of equation (4) to eliminate $\hat{y}_{t+1}(\varsigma) - \hat{y}_t$ and we defined the auxiliary parameters

$$\begin{aligned}
\tau &= 1 + \beta + (1-\beta(1-\delta))(1+\omega-\nu)\epsilon^{-1}, \\
\xi &= \theta(1-\beta(1-\delta))\phi(1+\nu)\epsilon^{-1}.
\end{aligned}$$

Thus we are left with a serious problem: the optimal price depends on the real marginal cost. The latter is a function of $\tilde{k}_t(\varsigma)$, which in turn depends on next period's expected price. All this is an obvious consequence of capital specificity: we do not expect a firm that has reoptimized its price to select the same capital stock as a firm stuck with the same price as in the previous period.

Solving the Problem

Following Woodford (2005), we resort to the *undetermined coefficients method* to solve this problem. Let $\hat{p}_t^*$ denote the average value of $\hat{p}_t^*(\varsigma)$ across reoptimizing firms. We assume that

$$\hat{p}_t^*(\varsigma) = \hat{p}_t^* - \mu_{pk}\tilde{k}_t(\varsigma), \tag{27}$$

$$\tilde{k}_{t+1}(\varsigma) = \mu_{kk}\tilde{k}_t(\varsigma) - \mu_{kp}\mathrm{E}_{t-1}\{\hat{p}_t(\varsigma)\}, \tag{28}$$

where μ_{pk}, μ_{kk}, and μ_{kp} are the coefficients to be determined. Notice that the average value of $\tilde{k}_t(\varsigma)$ across reoptimizing firms is zero, since these firms are drawn with uniform probability over the entire set of firms.

First, let us use equation (28) to eliminate $\hat{k}_{t+2}(\varsigma)$ from (26). This yields

$$\xi\mathrm{E}_{t-1}\{\hat{p}_{t+1}(\varsigma)\} = \mathrm{E}_{t-1}\big\{(\beta\mu_{kk} - \tau)\tilde{k}_{t+1}(\varsigma) - \beta\mu_{kp}\hat{p}_{t+1}(\varsigma) + \tilde{k}_t(\varsigma)\big\}. \tag{29}$$

Now, notice that, according to the Calvo specification (augmented with the indexation mechanism considered here), the expected price of firm ς in $t+1$ conditional on information available as of time $t-1$ obeys

$$\mathrm{E}_{t-1}\{\hat{p}_{t+1}(\varsigma)\} = \alpha\mathrm{E}_{t-1}\{\hat{p}_t(\varsigma) + \pi_{t+1} - \gamma\pi_t\} + (1-\alpha)\mathrm{E}_{t-1}\{\hat{p}_{t+1}^*(\varsigma)\}.$$

Using equation (27), we then obtain

$$\begin{aligned}\mathrm{E}_{t-1}\{\hat{p}_{t+1}(\varsigma)\} = {} & \alpha\mathrm{E}_{t-1}\{\hat{p}_t(\varsigma) + \pi_{t+1} - \gamma\pi_t\} \\ & + (1-\alpha)\mathrm{E}_{t-1}\{\hat{p}_{t+1}^*\} - (1-\alpha)\mu_{pk}\tilde{k}_{t+1}(\varsigma).\end{aligned}$$

Then, resorting to the indexation rule and equation (5), we obtain

$$(1-\alpha)\hat{p}_t^* = \alpha(\hat{\pi}_t - \gamma\hat{\pi}_t). \tag{30}$$

Plugging this equation into the previous one, we obtain

$$\mathrm{E}_{t-1}\{\hat{p}_{t+1}(\varsigma)\} = \alpha \mathrm{E}_{t-1}\{\hat{p}_t(\varsigma)\} - (1-\alpha)\mu_{pk}\tilde{k}_{t+1}(\varsigma).$$

Using this equation and (28) in equation (29), we finally arrive at

$$\begin{aligned}&[\beta\mu_{kk} + \beta(1-\alpha)\mu_{kp}\mu_{pk} - \tau]\tilde{k}_{t+1}(\varsigma)\\ &\quad = [\beta\alpha\mu_{kp} + \xi(\alpha + (1-\alpha)\mu_{pk}\mu_{kp})]\mathrm{E}_{t-1}\{\hat{p}_t(\varsigma)\}\\ &\qquad - [1 + (1-\alpha)\mu_{pk}\mu_{kk}\xi]\tilde{k}_t(\varsigma).\end{aligned}$$

Comparing this relation with equation (28), we obtain two restrictions on the μ's:

$$\mu_{kk} = -\frac{1 + (1-\alpha)\mu_{pk}\mu_{kk}\xi}{\beta\mu_{kk} + \beta(1-\alpha)\mu_{kp}\mu_{pk} - \tau} \tag{31}$$

and

$$\mu_{kp} = -\frac{\beta\alpha\mu_{kp} + \xi(\alpha + (1-\alpha)\mu_{pk}\mu_{kp})}{\beta\mu_{kk} + \beta(1-\alpha)\mu_{kp}\mu_{pk} - \tau}. \tag{32}$$

To complete the solution, we need an extra constraint that should derive from the optimal price-setting equation (22)—which we have not used up to now. To do so, let us plug equation (25) into equation (22). After rearranging a little, this yields

$$\begin{aligned}\frac{1+\omega\theta}{1-\alpha\beta}\hat{p}^*_t(\varsigma) = \sum_{j=0}^{\infty}(\alpha\beta)^j \mathrm{E}^{\varsigma}_{t-1}\big\{&\hat{s}_{t+j} - (1+\omega\theta)(\hat{x}_{t,t+j} - \hat{\pi}_{t,t+j})\\ &- (\omega-\nu)\tilde{k}_{t+j}(\varsigma)\big\}.\end{aligned} \tag{33}$$

What complicates this expression is the presence of $\tilde{k}_{t+j}(\varsigma)$. Notice, however, that according to (28),

$$\mathrm{E}^{\varsigma}_{t-1}\{\tilde{k}_{t+j+1}(\varsigma)\} = \mu_{kk}\mathrm{E}^{\varsigma}_{t-1}\{\tilde{k}_{t+j}(\varsigma)\} - \mu_{kp}\mathrm{E}^{\varsigma}_{t-1}\{\hat{p}_{t+j}(\varsigma)\}, \quad j \geq 0.$$

Iterating over this equation yields

$$\mathrm{E}^{\varsigma}_{t-1}\{\tilde{k}_{t+j+1}(\varsigma)\} = \mu_{kk}^{j+1}\tilde{k}_t(\varsigma) - \mu_{kp}\sum_{i=0}^{j}\mu_{kk}^{j-i}\mathrm{E}^{\varsigma}_{t-1}\{\hat{p}_{t+i}(\varsigma)\}.$$

Thus

$$\sum_{j=0}^{\infty}(\alpha\beta)^j \mathrm{E}_{t-1}^{\varsigma}\{\tilde{k}_{t+j}(\varsigma)\}$$
$$= \frac{1}{1-\alpha\beta\mu_{kk}}\tilde{k}_t(\varsigma) - \mu_{kp}\alpha\beta\sum_{j=0}^{\infty}(\alpha\beta)^j\sum_{i=0}^{j}\mu_{kk}^{j-i}\mathrm{E}_{t-1}^{\varsigma}\{\hat{p}_{t+i}(\varsigma)\}.$$

Notice that

$$\sum_{j=0}^{\infty}(\alpha\beta)^j\sum_{i=0}^{j}\mu_{kk}^{j-i}\mathrm{E}_{t-1}^{\varsigma}\{\hat{p}_{t+i}(\varsigma)\}$$
$$= \frac{1}{1-\alpha\beta\mu_{kk}}\sum_{i=0}^{\infty}(\alpha\beta)^i\mathrm{E}_{t-1}^{\varsigma}\{\hat{p}_{t+i}(\varsigma)\},$$

thus

$$\sum_{j=0}^{\infty}(\alpha\beta)^j \mathrm{E}_{t-1}^{\varsigma}\{\tilde{k}_{t+j}(\varsigma)\}$$
$$= \frac{1}{1-\alpha\beta\mu_{kk}}\tilde{k}_t(\varsigma) - \frac{\mu_{kp}\alpha\beta}{1-\alpha\beta\mu_{kk}}\sum_{j=0}^{\infty}(\alpha\beta)^j\mathrm{E}_{t-1}^{\varsigma}\{\hat{p}_{t+j}(\varsigma)\}.$$

Since we are only considering states of nature where firm ς is not allowed to reset its price after t, it must be the case that

$$\mathrm{E}_{t-1}^{\varsigma}\{\hat{p}_{t+j}(\varsigma)\} = \hat{p}_t^*(\varsigma) + \mathrm{E}_{t-1}\{\hat{x}_{t,t+j} - \hat{\pi}_{t,t+j}\}, \quad j \geq 0.$$

(Recall our notational convention: $\hat{x}_{t,t} = \hat{\pi}_{t,t} = 0$.) Inserting this into the previous equation, we obtain

$$\sum_{j=0}^{\infty}(\alpha\beta)^j \mathrm{E}_{t-1}^{\varsigma}\{\tilde{k}_{t+j}(\varsigma)\}$$
$$= \frac{1}{1-\alpha\beta\mu_{kk}}\tilde{k}_t(\varsigma) - \frac{\mu_{kp}\alpha\beta}{(1-\alpha\beta\mu_{kk})(1-\alpha\beta)}\hat{p}_t^*(\varsigma)$$
$$- \frac{\mu_{kp}\alpha\beta}{1-\alpha\beta\mu_{kk}}\sum_{j=0}^{\infty}(\alpha\beta)^j\mathrm{E}_{t-1}\{\hat{x}_{t,t+j} - \hat{\pi}_{t,t+j}\}.$$

Plugging this relation into (33), we obtain

$$\zeta \hat{p}_t^*(\varsigma) = \sum_{j=0}^{\infty} (\alpha\beta)^j \mathrm{E}_{t-1}^{\varsigma}\{(1-\alpha\beta)[\hat{s}_{t+j} - \zeta(\hat{x}_{t,t+j} - \hat{\pi}_{t,t+j})]\} - \frac{(\omega-\nu)(1-\alpha\beta)}{1-\alpha\beta\mu_{kk}} \tilde{k}_t(\varsigma), \tag{34}$$

where

$$\zeta = 1 + \omega\theta - \frac{(\omega-\nu)\mu_{kp}\alpha\beta}{(1-\alpha\beta\mu_{kk})}. \tag{35}$$

Integrating over the set of reoptimizing firms, we obtain

$$\zeta \hat{p}_t^* = \sum_{j=0}^{\infty} (\alpha\beta)^j \mathrm{E}_{t-1}^{\varsigma}\{(1-\alpha\beta)[\hat{s}_{t+j} - \zeta(\hat{x}_{t,t+j} - \hat{\pi}_{t,t+j})]\}. \tag{36}$$

Using this and equation (27), we finally obtain

$$(1+\omega\theta)\mu_{pk}(1-\alpha\beta\mu_{kk}) - (\omega-\nu)\mu_{kp}\alpha\beta\mu_{pk} = (\omega-\nu)(1-\alpha\beta). \tag{37}$$

Equations (31), (32), and (37) define a nonlinear system in (μ_{kk}, μ_{kp}, μ_{pk}), the solution of which depends on $(\alpha, \beta, \delta, \epsilon, \theta, \phi, \nu)$. Once solved for $(\mu_{kk}, \mu_{kp}, \mu_{pk})$, this system implies

$$\varkappa_0(\alpha, \beta, \delta, \epsilon, \theta, \phi, \nu) = \frac{(\phi-1)(\nu+1)\mu_{kp}\alpha\beta}{(1-\alpha\beta\mu_{kk})}.$$

To obtain the NKPC, simply quasi-difference equation (33), and use equation (30) to eliminate $\hat{p}_t^*$.

In the case of firm-specific capital and aggregate labor markets, equation (37) rewrites

$$(1+(\phi-1)\theta)\mu_{pk}(1-\alpha\beta\mu_{kk}) - (\phi-1)\mu_{kp}\alpha\beta\mu_{pk} = (\phi-1)(1-\alpha\beta). \tag{38}$$

Equations (31) and (32) still hold, except that ξ and τ now obey

$$\xi = \theta(1-\beta(1-\delta))\phi\epsilon^{-1}, \quad \tau = 1 + \beta + (1-\beta(1-\delta))\phi\epsilon^{-1}.$$

In this case, we obtain

$$\zeta = 1 + (\phi - 1)\theta - \varkappa_1(\alpha, \beta, \delta, \epsilon, \theta, \phi),$$

with

$$\varkappa_1(\alpha, \beta, \delta, \epsilon, \theta, \phi) = \frac{(\phi - 1)\mu_{kp}\alpha\beta}{(1 - \alpha\beta\mu_{kk})}.$$

References

Altig, D., L. J. Christiano, M. Eichenbaum, and J. Linde. 2005. "Firm-Specific Capital, Nominal Rigidities and the Business Cycle." NBER Working Paper No. 11034.

Avouyi-Dovi, S., and J. Matheron. 2004. "Supply Shocks and Monetary Policy in an Estimated Sticky Price Model of the Euro Area." Banque de France Working Paper No. 126.

Bils, M., and P. J. Klenow. 2004. "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy* 112 (5): 947–85.

Calvo, G. A. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12 (3): 383–98.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan. 2000. "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?" *Econometrica* 68 (5): 1151–79.

Christiano, L. J. 2004. "Firm-Specific Capital and Aggregate Inflation Dynamics in Woodford's Model." Unpublished Manuscript, Northwestern University.

Christiano, L. J., M. Eichenbaum, and C. L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy* 113 (1): 1–45.

de Walque, G., F. Smets, and R. Wouters. 2004. "Price Setting in General Equilibrium: Alternative Specifications." Mimeo, European Central Bank.

Dhyne, E., L. J. Álvarez, H. Le Bihan, G. Veronese, D. Dias, J. Hoffman, N. Jonker, P. Lünnemann, F. Rumler, and J. Vilmunen. 2006. "Price Changes in the Euro Area and in the United States: Some Facts from Individual Consumer Price Data." *Journal of Economic Perspectives* 20 (2): 171–92.

Eichenbaum, M., and J. M. D. Fisher. 2005. "Evaluating the Calvo Model of Sticky Prices." NBER Working Paper No. 10617, revised version available on M. Eichenbaum's home page, www.faculty.econ.northwestern.edu/faculty/eichenbaum/.

Eichenbaum, M., L. P. Hansen, and K. J. Singleton. 1988. "A Time Series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty." *Quarterly Journal of Economics* 103 (1): 51–78.

Fagan, G., J. Henry, and R. Mestre. 2005. "An Area-Wide Model for the Euro Area." *Economic Modelling* 22 (1): 39–59.

Galí, J., and M. Gertler. 1999. "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics* 44 (2): 195–222.

Galí, J., M. Gertler, and J. D. López-Salido. 2001. "European Inflation Dynamics." *European Economic Review* 45 (7): 1237–70.

———. 2003. "Erratum to 'European inflation dynamics'." *European Economic Review* 47 (4): 759–60.

Gilchrist, S., and C. P. Himmelberg. 1995. "Evidence on the Role of Cash Flows for Investment." *Journal of Monetary Economics* 36 (3): 541–72.

Hansen, L. P. 1982. "Large Sample Properties of Generalized Method of Moments Estimators." *Econometrica* 50 (4): 1029–54.

Jondeau, E., and H. Le Bihan. 2005. "Testing for the New Keynesian Phillips Curve. Additional International Evidence." *Economic Modelling* 22 (3): 521–50.

Matheron, J., and T. P. Maury. 2004. "Supply Side Refinements and the New Phillips Curve." *Economics Letters* 82 (3): 391–96.

McAdam, P., and A. Willman. 2004. "Supply, Factor Shares and Inflation Persistence: Re-examining Euro-Area New Keynesian Phillips Curves." *Oxford Bulletin of Economics and Statistics* 66 (September): 637–70.

Newey, W. K., and K. D. West. 1987. "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix." *Econometrica* 55 (3): 703–708.

Sahuc, J. G. 2004. "Partial Indexation and Inflation Dynamics: What Do the Data Say?" *Applied Economics Letters* 11 (13): 827–32.

Sbordone, A. 2002. "Prices and Unit Labor Costs: A New Test of Price Stickiness." *Journal of Monetary Economics* 49 (2): 265–92.

Smets, F., and R. Wouters. 2003. "An Estimated Dynamic General Equilibrium Model of the Euro Area." *Journal of the European Economic Association* 1 (5): 1123–75.

Sveen, T., and L. Weinke. 2004. "Pitfalls in the Modelling of Forward-Looking Price Setting and Investment Behavior." Working Paper No. 2004-1, Norges Bank.

———. 2005. "New Perspectives on Capital and Sticky Prices." *Journal of Economic Theory* 123 (1): 21–39.

Taylor, J. 1980. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88 (1): 1–24.

West, K. D. 1997. "Another Heteroskedasticity- and Autocorrelation-Consistent Covariance Matrix Estimator." *Journal of Econometrics* 76 (1–2): 171–91.

Woodford, M. 2003. *Interest and Prices.* Princeton, NJ: Princeton University Press.

———. 2005. "Firm Specific Capital and the New Keynesian Phillips Curve." *International Journal of Central Banking* 1 (2): 1–46.

Yun, T. 1996. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics* 37 (2): 345–70.