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What Firms' Surveys Tell Us about Price-Setting Behavior in the Euro Area*

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This study investigates the pricing behavior of firms in the euro area on the basis of surveys conducted by nine Eurosystem national central banks, covering more than 11,000 firms. The results, consistent across countries, show that firms operate in monopolistically competitive markets, where prices are mostly set following markup rules and where price discrimination is common. Around one-third of firms follow mainly time-dependent pricing rules, while two-thirds allow for elements of state dependence. The majority of the firms take into account both past and expected economic developments in their pricing decisions. Price reviews happen with a low frequency, of about one to three times per year in most countries, but prices are actually changed even less. Hence, price stickiness arises at both stages of the price-setting process and is mainly driven by customer relationships—explicit and implicit contracts—and coordination failure. Firms adjust prices asymmetrically in response to shocks: while cost shocks have a greater impact when prices have to be raised than when they have to be reduced, a fall in demand is more likely to induce a price change than an increase in demand.

JEL Codes: E30, D40.

*This paper is based on the results of national studies conducted in the context of the Eurosystem Inflation Persistence Network (IPN). All of the authors belong to the national central banks that have been involved in Research Group 8 of the IPN ("Launching a Survey") except Bettina Landau, who is with the European Central Bank. The other members of Research Group 8, who are also the authors of national studies and whose contribution to this paper has been crucial, are Luis Álvarez, Luc Aucremanne, Josef Baumgartner, Angela Gattulli, Marco Hoeberichts, Patrick Lünnemann, Pedro Neves, Roland Ricart,

1. Introduction

In recent decades, a substantial amount of theoretical research devoted to improving the microeconomic foundations of macroeconomic behavior has shown that the nature of nominal rigidities plays a key role in determining the effects of different shocks on the economy. This theoretical research has made clear that a thorough understanding of the extent and causes of the sluggish adjustment of nominal prices is crucial to the design and conduct of monetary policy. In this respect, empirical work aimed at an improved characterization of the price-setting behavior of firms is of major interest for monetary policymaking. The objective of this paper is to deepen our understanding of the behavioral mechanisms underlying price setting by using a methodological approach—asking firms directly about how they set prices—that is particularly well suited for the purpose at hand.

Although the literature based on microdata has recently provided detailed descriptions of the periodicity and magnitude of price changes for a number of economies (on consumer prices see Bils and Klenow 2004 for the United States and Dyhne et al. 2005 for the euro area; on producer prices, see Álvarez et al. 2005 for the euro area), these quantitative characterizations of price dynamics are often not enough to understand the underlying rationale of the behavior of price setters. There are certain aspects of firms' pricing

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policies that can only be investigated on the basis of the qualitative information obtained from surveys. For example, firms' responses can provide valuable insights about the type of information set used in the review of prices. Furthermore, survey results allow us to investigate separately the two stages of the price-adjustment process (i.e., the review stage and the implementation stage), to empirically assess alternative theories on price stickiness, and to test whether the response of prices to shocks differs depending on the nature (costs/demand) or the sign of the disturbances. Finally, they are also useful in cross-checking the evidence obtained from quantitative databases.

The use of surveys to explore the price-setting behavior of firms was pioneered by the seminal work of Blinder (1991, 1994) and Blinder et al. (1998) for the United States.[1] This work has led to the conducting of similar surveys in other countries: Köhler (1996) in Germany; Hall, Walsh, and Yates (1997, 2000) in the United Kingdom; Apel, Friberg, and Hallsten (2005) in Sweden; and Amirault, Kwan, and Wilkinson (2004) in Canada.

Within this line of research, this paper provides an in-depth study of the price-setting mechanism in the euro area, based on the evidence obtained from surveys conducted in 2003 and 2004 in nine euro-area countries—Austria (AT), Belgium (BE), France (FR), Germany (DE), Italy (IT), Luxembourg (LU), the Netherlands (NL), Portugal (PT) and Spain (ES)—covering 94 percent of euro-area GDP.[2] The surveys display a sufficiently high degree of comparability across countries, despite the adoption of a decentralized approach in their design. Thus, a number of common characteristics can be observed in the full set of results from all of the countries. The country coverage and the high comparability of the

[1]For a very early example of this kind of work, see Hall and Hitch (1939).

[2]The detailed results for each country can be found in the country-specific studies conducted in the framework of the Eurosystem IPN: Kwapil, Baumgartner, and Scharler (2005) for Austria; Aucremanne and Druant (2005) for Belgium; Loupias and Ricart (2004) for France; Stahl (2005) for Germany; Fabiani, Gattulli, and Sabbatini (2004) for Italy; Lünnemann and Mathä (2006) for Luxembourg; Hoeberichts and Stokman (2006) for the Netherlands; Martins (2005) for Portugal; and Álvarez and Hernando (2005) for Spain. These studies can be downloaded from the European Central Bank website (ECB working papers).

national surveys imply that these common features may be regarded as "stylized facts" underlying the price-setting mechanism in the euro area. It is worth remarking that such features should be interpreted as referring mainly to producer prices in the manufacturing sector, which is overrepresented in most of the national samples. Nevertheless, results for the trade and services sectors, which are covered only in some countries, give indication about both producer and consumer prices.

One of our key findings is that firms engage in both time-dependent and state-dependent pricing strategies. Around one-third of the respondents indicate that they follow mainly time-dependent rules, while two-thirds use rules with state-dependent elements. With regard to the information set considered in the price-setting process, our results show that, while the majority of firms in the euro area take into account both past and expected developments, about one-third of them adopt a purely backward-looking behavior. Furthermore, our results are in line with the idea that price setting takes place in two stages: first, firms review their price to check whether it deviates from its optimal level; then, if this is the case, they decide whether to change the price or not. The surveys suggest that the modal number of price reviews ranges in most countries between one and three times per year, while the median firm in nearly all countries changes its price only once a year. Hence, there appear to be obstacles to price adjustment at both stages of price setting, although the main impediments seem to lie at the second stage. The respondents indicate that customer relationships are the main source of price rigidities. In particular, the fear of antagonizing customers with frequent price changes seems to be the most important explanation for price stickiness in the euro area. Finally, we find that prices react asymmetrically to shocks: they are more rigid upwardly in response to demand shocks and are more rigid downwardly in response to cost shocks.

The structure of the paper follows the different stages of the price-setting process. Section 2 reports the main characteristics of the national surveys. Section 3 deals with the price-reviewing stage and provides evidence on the time- or state-dependent nature of firms' pricing policies, the information set used, and the frequency of price reviews. Section 4 investigates how firms set prices, documents the frequency of price changes, and explores the empirical support of alternative theories on price stickiness. The factors underlying price setting are analyzed in section 5. Finally, section 6 concludes.

2. How the Surveys Were Carried Out

The national surveys were designed following a decentralized approach, and this explains some differences in the way they were carried out (for a detailed analysis of these differences, see Appendix B in Fabiani et al. 2005). The surveys were conducted either by phone, Internet, or traditional mail; a small number took the form of face-to-face interviews with one of the senior managers. The number of respondents in each country ranges from 333 to 2,008; altogether, more than 11,000 euro-area companies of different sizes in terms of number of employees were surveyed. All national surveys focus on the pricing behavior with respect to the firm's main product, which is found to account for 60 percent or more of the turnover of the respondents.

The sectoral coverage is limited to manufacturing in some countries, while in other countries, pricing strategies in construction, trade, and services are also investigated.[3] In general, there is some overrepresentation of the industrial sector in most of the national samples, which explains why the majority of companies (75 percent on average) indicate that they sell their main product predominantly to other firms. Overall, in spite of the differences in the sectoral coverage, there is quite a solid basis for comparing the industrial and the services sectors across countries.

The questionnaires differ with respect to the reference market, which in some countries (DE, FR, and LU) is the domestic market and in others (BE, ES, IT, and AT) is the main market (in the remaining countries it is not specified). However, reliable results for the euro area can be computed, since the majority of respondents, especially in the industrial sector, refer to their pricing strategies either in the domestic market or in the euro area.

It is worth emphasizing that the above minor differences across the national questionnaires represent an important value added of

[3]For details on the composition of the samples of the national surveys, see Fabiani et al. (2005). Detailed information on the sampling methods can be obtained from the country-specific studies (see footnote 2).

this research project compared to previous empirical literature. In particular, the common patterns detected across countries ("stylized facts") do not appear to depend on the particular way the national surveys were conducted, the number of questions asked, the precise wording and order of the questions, and the options within a particular question. Therefore, compared to other previous studies, the results for the euro area reported in this paper are characterized by a higher degree of robustness that is further strengthened by the fact that the nine national surveys were carried out under different business-cycle conditions. This consistency of findings across countries lessens to some extent the potential significance of the drawbacks traditionally attached to the use of surveys: first, the qualitative nature of the information gathered, which sometimes makes it difficult to ascertain the precise importance of a given statement; second, the lack of a time dimension, which means that they cannot be used to assess whether pricing patterns change over time; and, finally, the degree of uncertainty that surrounds the quality of the answers provided by the respondents.

3. Price Reviews

This section documents the main features of the first step of the price-adjustment process—the one in which firms evaluate the price they want to set, taking into account the information they have and checking whether it coincides with the price they currently charge.

3.1 Time-Dependent versus State-Dependent Pricing Rules

Individual firms do not continuously adjust their prices in response to all the relevant shocks in the economy. To model this fact, the theoretical literature considers mainly two types of pricing behavior: time-dependent pricing rules and state-dependent ones. According to the former, either with a deterministic (Taylor 1980) or a stochastic (Calvo 1983) process of price adjustment, firms review their prices periodically, i.e., the timing of the review is exogenous and does not depend on the state of the economy.

Firms following state-dependent rules review their prices whenever there is a large-enough shock. A standard justification for this type of discontinuous adjustment is the existence of a fixed cost of changing prices (see, for instance, Sheshinski and Weiss 1977; Caballero and Engel 1993; or Dotsey, King, and Wolman 1999). The existence of price-adjustment costs implies in state-dependent models that firms change their price only when the latter gets sufficiently "out of line" and, consequently, price reviews are likely to be a lot more frequent than price changes, as firms want to be aware of shocks in order to react as fast as possible. In time-dependent models, firms review—and change, if they find it optimal to do so—their price only on a periodic basis.

In the presence of shocks, time dependence might lead to stickier prices than state dependence. Hence, almost every national questionnaire investigates whether firms follow mainly time-dependent pricing rules, state-dependent pricing rules, or a combination of both. In this latter case, the idea is that firms can follow time-dependent rules as an implementation of state-dependent ones under a stable environment (as in Sheshinski and Weiss 1977) rather than purely time-dependent rules. To distinguish between these two groups, some national questionnaires asked firms whether they switch to state-dependent rules upon the occurrence of specific events.

Given that the firms following mainly time-dependent rules or both strategies are supposed, under certain assumptions, to introduce more rigidity in the price transmission mechanism than those following mainly state-dependent rules, our analysis focuses on cross-country comparisons of the share of mainly time-dependent firms (table 1, panel A) and of those that follow both types of rules (table 1, panel B).

In the euro area as a whole, 34 percent of the firms follow purely time-dependent rules; the share is roughly around 35–40 percent for six countries (FR, ES, IT, NL, AT, and PT) and below 30 percent for three countries (BE, DE, and LU). These results are in line with those obtained by Blinder et al. (1998), who report that in the United States 40 percent of the firms undertake meaningful periodic price reviews. Overall, the results are also rather similar to the figures reported by Apel, Friberg, and Hallsten (2005) for Sweden, where only 23 percent of the firms are found to follow time-dependent pricing rules when significant events occur. The evidence for the United Kingdom by Hall, Walsh, and Yates (2000), however, differs from the above-mentioned results, as 79 percent of the firms are found to be time dependent (10 percent follow both time- and state-dependent

Table 1. Firms' Price-Setting Rules (*Percentages*)[a]

	Total	Sector			Perceived Competition[d]			
		goods	trade	services	very low	low	high	very high
A. Firms Following Time-Dependent Rules								
BE	26	22	29	24	25	23	22	19
FR[b]	39	39	–	–	–	–	–	–
DE	26	26	–	–	27	21	25	33
ES	33	29	32	40	42	32	29	31
IT	40	40	35	45	37	35	51	19
LU	18	23	16	14	25	14	10	25
NL	36	26	34	40	35	36	35	36
AT	41	37	–	44	42	34	39	35
PT	35	32	–	63	47	42	38	25
Euro Area[c]	**34**	**32**						
B. Firms Following Both Time- and State-Dependent Rules								
BE	40	42	36	48	43	40	44	38
FR[b]	55	55	–	–	–	–	–	–
DE	55	55	–	–	51	64	58	45
ES	28	25	24	34	18	29	33	31
IT	46	45	62	26	45	53	43	40
LU	32	27	39	32	25	39	33	27
NL	18	19	21	16	12	18	16	24
AT	32	36	–	29	35	37	36	39
PT	19	23	–	17	14	19	22	28
Euro Area[c]	**46**	**46**						

[a]Share of respondents following time-dependent or both time- and state-dependent pricing rules. Figures for the third category, the share of firms following only state-dependent rules, are not shown, but they are the complement to 100 by column. The figures are not supposed to add up to 100 by row. The figures are rescaled excluding nonresponses.
[b]In the case of France, the issue has not been addressed directly; the information in the table has been estimated on the basis of the answers to other questions.
[c]Weighted average (GDP weights).
[d]As an indicator of the degree of competition, we use the degree of perceived competition defined as the importance firms attribute to competitors' prices in influencing a reduction in their own prices (unimportant, of minor importance, important, very important).

rules and 11 percent follow purely state-dependent ones). In the euro area, around two-thirds of the companies apply pricing strategies with some element of state dependence. Among these firms, those adopting a mixed strategy are predominant, except in four countries (ES, LU, NL, and PT).

Stylized Fact 1: Both time- and state-dependent pricing strategies are used by euro-area firms. Around one-third of the companies follow mainly time-dependent rules, while the remaining two-thirds adopt pricing rules with some element of state dependence.

3.2 Information Set Used in Price Reviews

The so-called New Keynesian Phillips curve (NKPC) models, which emphasize rational expectations and hence the existence of forward-looking price setters, are increasingly used for monetary policy analysis (see, for instance, Woodford 2003). Despite their theoretical success, however, these models generally fail to generate the sluggishness in price behavior that is empirically observed. Conversely, hybrid versions of the NKPC have been reported to provide a better representation of the observed price movements. In particular, price stickiness may stem from firms using some form of rule of thumb in setting their price (Galí and Gertler 1999; Galí, Gertler, and López-Salido 2001), from indexation schemes (Christiano, Eichenbaum, and Evans 2005), or from stickiness in gathering information (Mankiw and Reis 2002). In all these cases, deviations from fully optimizing behavior generate an additional source of sluggishness in the response of inflation to shocks. The information set used by companies when making their pricing decisions has, indeed, important implications for the speed of price adjustment in response to a broad range of disturbances.

Six national surveys (BE, ES, IT, LU, AT, and PT) provide data on the information set on which firms base their decisions when they review their prices. This is an important piece of evidence that reflects different degrees of optimality of price-setting strategies. Companies applying rules of thumb (for instance, changing prices by a fixed percentage, or following a CPI indexation rule) may end up charging a price that deviates substantially from the optimal one if a large shock occurs. In this sense, these companies behave nonoptimally. At the other extreme, price reviews are

Table 2. Information Set Used by Firms for Pricing Decisions (*Percentages*)[a]

	BE	ES	IT	LU	PT	AT	Euro Area[b]
Rule of Thumb	37	33	n.a.	30	25	n.a.	
Past/Present Context	29	39	32	26	33	37	**34**
Present/Future Context	34	28	68	44	42	12	**48**
Past, Present, and Future	n.a.	n.a	n.a.	n.a	n.a.	51	

[a]Rescaled figures excluding nonresponses.
[b]Weighted average (GDP weights). Note that the percentages for the euro area do not add up to 100, as different answer categories were used in the various countries.

addressed in an optimal way if companies use a wide set of indicators relevant for profit maximization, including expectations about the future economic environment.

On average, 48 percent of the firms in the euro area evaluate their prices on the basis of an information set that includes expectations about future economic conditions (table 2). There are some differences across countries in the share of forward-looking firms, which ranges from 28 percent in Spain to 68 percent in Italy.

A large fraction of firms, however, do not behave optimally, either due to backward-looking behavior or to the use of rules of thumb. About one-third of the firms take only historical data into account. For those surveys that included the rule-of-thumb option (such as indexation based on the consumer price index, a fixed percentage adaptation, etc.), the results indicate that this pricing method is adopted by 37 percent of firms in Belgium, 33 percent in Spain, 30 percent in Luxembourg, and 25 percent in Portugal.

Overall, the pattern of results reported in this section lends support to the recent wave of estimations of hybrid versions of the New Keynesian Phillips curve.

Information available for Spain and Luxembourg shows that smaller firms tend to be more backward looking than larger ones and, conversely, larger firms tend to attach more importance than smaller ones to expectations about future conditions when assessing their prices.

Stylized Fact 2: Around half of the firms review their prices taking into account a wide range of information, including both past and expected economic developments; one-third of them show a backward-looking behavior.

3.3 Frequency of Price Reviews

All national surveys contain a question about how often firms that follow time-dependent rules assess their prices. Typically, the respondents were given a choice among several categories (daily, weekly, monthly, quarterly, etc.).[4] Belgium, Luxembourg, and Spain opted for a slightly different formulation, asking whether the respondents review their prices more than once a year, once a year, or less than once a year; within these categories, respondents had to specify the number of times.[5]

Table 3 groups the results into three classes: the share of respondents that review their price (i) a maximum of three times a year, (ii) between four and eleven times a year, and (iii) at least twelve times a year. In all countries, the largest share of firms fall into the first category (57 percent for the euro area as a whole).

With respect to the median frequency of price reviews, countries can be classified into three groups: (i) in Belgium, Spain, and Italy, the median firm checks its price once a year, (ii) in France, the Netherlands, and Austria, reviews are carried out on a quarterly basis, and (iii) in Germany, Luxembourg, and Portugal, the frequency of price reviews falls somewhere between that of groups (i) and (ii).

In order to find regularities in the price-reviewing pattern in the euro area, we investigate whether firms' size, sector, and

[4]All those firms indicating that they carry out periodic price reviews and those applying time-dependent pricing rules in normal circumstances (and state-dependent ones in exceptional circumstances) were asked at what intervals they review their prices.

[5]As table 3 shows, Belgium and Spain report significantly higher shares (nearly 90 percent) of respondents indicating that they review their prices at most three times a year. This result suggests that the format of the answer categories might be relevant.

Table 3. Frequency of Price Reviews per Year (*Percentages*)[a]

	BE	DE	ES	FR	IT	LU	NL	AT	PT	Euro Area[b]
≥ 12	4	30	7	31	28	26	37	29	5	**26**
4–11	8	17	7	22	14	20	19	25	26	**17**
≤ 3	88	53	86	47	57	54	44	46	69	**57**
Median[c]	1	3	1	4	1	2	4	4	2	

[a]Rescaled figures excluding nonresponses.
[b]Weighted average (GDP weights).
[c]Median frequency of price reviews per year.

competitive environment have an effect on firms' behavior. We apply a Chi-square test to examine whether the distribution of frequencies is equal for each of the aforementioned characteristics. Firm size explains differences in Spain, France, Luxembourg, the Netherlands, and Austria. In all these countries except France, large firms review their prices more frequently than smaller ones. Similarly, Amirault, Kwan, and Wilkinson (2004) find that large firms change prices significantly more often than small or medium-sized firms. They argue that senior staff members at small firms have numerous tasks in addition to reviewing and adjusting prices and, consequently, managerial costs associated with the price-setting process might be particularly onerous for small firms.[6]

With regard to the degree of competition, firms facing higher competitive pressures review their prices more frequently. In seven out of the nine countries, firms indicating that competitors' prices have a very important effect on their own pricing decisions review their prices more often than other firms. The exceptions are Austria and Belgium, where the competitive environment does not give rise to any difference.

[6]They do not distinguish between the frequency of price reviews and the frequency of price changes, and their argument probably has more to do with the price-reviewing stage.

Finally, there are some interesting differences across sectors. The Chi-square test rejects the null hypothesis of equality across sectors in all seven countries for which this analysis is possible (in Germany and France, services are not covered) at the 10 percent significance level. In five countries (IT, LU, NL, AT, and PT), firms in the services sector review their prices significantly less frequently than firms operating in other sectors. Albeit not statistically significant, this tendency can also be observed in Belgium and Spain. In Spain, Luxembourg, and the Netherlands, firms in the trade sector carry out price reviews significantly more often than those in manufacturing and services. This is not the case for the other two countries that report results for trade (BE and IT).

Stylized Fact 3: In most countries the modal number of price reviews lies in the range of one to three times a year. Firms in the services sector review prices less frequently than firms in the other sectors. Firms facing high competitive pressures carry out price reviews more frequently.

There may be different reasons for the finding that price reviews happen with a relatively low frequency. On the one hand, the frequency could be related to the (potentially sporadic) arrival of information. In other words, it may not make sense for firms to review their prices more often, as no additional information is available.[7] On the other hand, there may be costs associated with price reviews. In the presence of informational costs, it may be optimal for firms to forego obtaining the most topical information instead of incurring the associated costs (see section 4.4).

4. Price Changes

This section focuses on the various aspects related to the implementation stage of the price-adjustment process, by documenting the frequency of actual price changes and the empirical support of alternative theories of price stickiness.

[7] Kashyap (1995) rejects this hypothesis. He observes different reviewing behavior also for products having similar cost and demand characteristics. However, if products are alike, then the arrival of the necessary information should also be correlated.

4.1 How Do Firms Set Prices?

4.1.1 Markup Pricing as a Dominant Strategy

A standard result in imperfectly competitive models is that, under quite general conditions, firms choose to charge a price that represents a markup over marginal cost and, therefore, have some room for not adjusting it when facing a variation in costs.[8] On the contrary, in the case of perfect competition, all firms belonging to the same market set their prices at a unique market-clearing level; there is no markup, and prices always equal marginal costs. Thus, price rigidities do not arise.

All questionnaires address the issue of how companies set their prices. In some cases (BE, ES, LU, NL, AT, and PT), firms were first asked to indicate whether they have an independent price-setting policy or whether their price is either regulated or set by the main company of the same group or dictated by the main customer. Firms with an independent policy were then asked to specify whether their price is set as a margin (markup) on costs, whether it depends on the price of their main competitor(s), or whether it is set according to other strategies. In the remaining countries, firms were directly requested to indicate their price-setting rule, choosing from among the above-mentioned options.

The option that the price is set as a margin applied to costs requires some clarification. First, whereas the theoretical literature refers to the concepts of markup and marginal costs, most businesspeople might not easily understand this terminology. In order to avoid confusion on the side of the respondents, the concept of markup has typically been translated into "profit margin," while the concept of marginal costs has been translated into a number of different expressions, which might slightly differ across the various questionnaires: "unit variable costs" (cost of labor and of other inputs); "(variable) unit costs"; "unit variable production costs"; and "variable production costs per unit." Second, all country questionnaires explore whether markup pricing is applied in general terms, except

[8] Within the models with imperfect competition, some assume time-varying markups, with important implications for business-cycle fluctuations. See sections 8 and 9 in Rotemberg and Woodford (1994) for an overview of different models with exogenously and endogenously determined time-varying markups.

in the cases of Belgium, Germany, and the Netherlands, where a distinction is made between constant and variable markup.

Table 4 summarizes the results by grouping the answers into three alternatives: "markup over costs," "price set according to competitors' prices," and "other." The results are in line with findings of similar studies for the United Kingdom and the United States. In the euro area, more than half of the firms fix their price as a markup (fixed or variable) over costs. At the two extremes we find Germany (73 percent) and France (40 percent). For those countries (Belgium, Germany, and the Netherlands) in which respondents could distinguish between constant and variable markup, the latter dominates.

Figure 1 shows a negative relationship between the share of firms following a markup rule and the degree of market competition. This finding, similar across countries, is consistent with the idea that in a highly competitive environment firms are essentially price takers and do not fix their prices as a markup over costs. It is, however, important to remark that the share of firms setting their price according to those of their main competitors is quite relevant (around 30 percent for the euro area as a whole), ranging from 38 percent in France to 13 percent in Portugal. Finally, for a minority of respondents, the price is set according to "other" rules. The share amounts to only 10 percent in Germany, while it rises to 26 percent in Italy, where it is particularly high in trade and services (49 percent and 40 percent, respectively), possibly due to the strict regulatory framework in such sectors.[9] The percentage of companies following "other" rules is also generally higher for large firms than it is for small ones.

Stylized Fact 4: Markup (constant or variable) pricing is the dominant price-setting practice adopted by firms in the euro area. However, the prices of around 30 percent of the firms are shaped by competitors' prices.

4.1.2 Price Discrimination

One of the main features characterizing the price-setting mechanism is the presence of some form of price discrimination aimed at

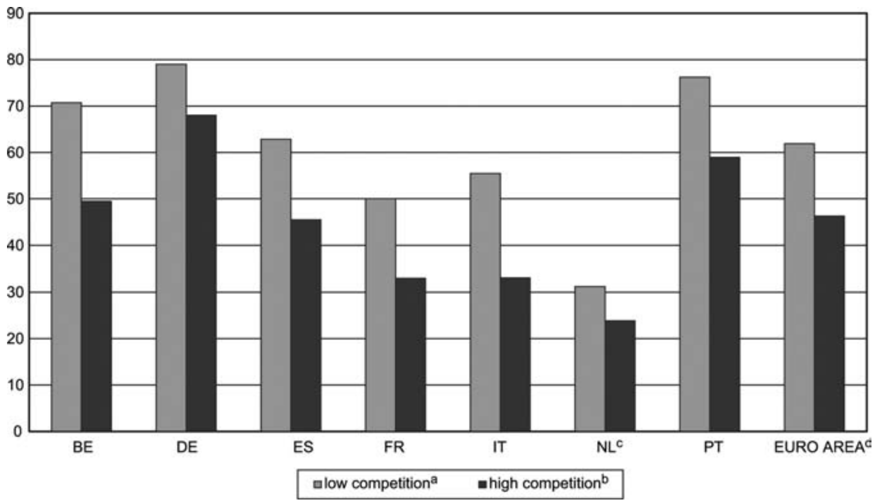
[9] Trade and services are not included in the German survey. If goods only are considered, the share falls to 19 percent in Italy.

Table 4. Price-Setting Rules (*Percentages*)[a]

	BE			DE			ES	FR	IT	NL			PT[c]	Euro Area[d]
	total	*const.*	*var.*[b]	*total*	*const.*	*var.*				*total*	*const.*	*var.*		
Markup														
Total	46	13	33	–	–	–	52	–	42	56	27	30	65	**54**
goods	49	14	35	73	4	69	55	40	48	63	28	34	67	**56**
trade	41	11	30	–	–	–	50	–	16	71	37	34	–	**37**
services	49	18	31	–	–	–	50	–	43	45	19	26	48	**46**
Competitors' Price														
Total	36			–			27	–	32	22			13	**27**
goods	40			17			24	38	33	19			13	**27**
trade	33			–			26	–	35	21			–	**30**
services	39			–			31	–	18	24			8	**24**
Other														
Total	18			–			21	–	26	21			23	**18**
goods	11			10			22	22	19	18			19	**17**
trade	26			–			23	–	49	8			–	**33**
services	12			–			20	–	40	31			44	**31**

[a]Rescaled figures excluding nonresponses.
[b]Firms adopting a markup rule and responding "important" or "very important" to at least one of the theories concerning countercyclical markups.
[c]In the case of Portugal, the issue was not addressed directly; the information reported in the table has been estimated on the basis of the answers to other questions.
[d]Weighted average (GDP weights).

Figure 1. Markup and Perceived Competition (*Percentages*)



[a]Mean share for a "very low" and "low" degree of perceived competition.

[b]Mean share for a "very high" and "high" degree of perceived competition.

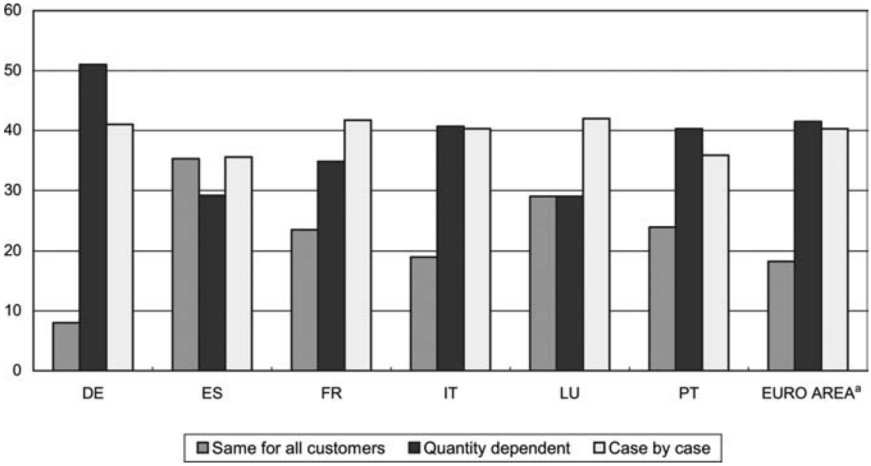
[c]For the Netherlands, the percentage of firms adopting a fixed markup is considered.

[d]Weighted average (GDP weights).

extracting a higher fraction of consumer surplus than the firm would be able to obtain if it charged a uniform price. Price discrimination may take many forms: the price of a product may vary according to the type of customer, the geographical area in which it is sold, the number of units purchased, or the specific time at which it is sold, to name but a few (see Tirole 1988, chap. 3).

The presence of some form of price discrimination is investigated in several of the national questionnaires. The findings presented in figure 2 strongly reject the use of a uniform pricing scheme as a general rule to describe the price-setting behavior of euro-area firms. In particular, the percentage of firms setting prices on a case-by-case basis or in accordance to the quantity of the product sold is, on average, around 80 percent in the euro area, ranging from 65 percent in Spain to 92 percent in Germany. In the other four countries (FR, IT, LU, and PT) the figure is around 75 percent.

Figure 2. Price Discrimination
(*Percentages*)



[a]Weighted average (GDP weights).

Stylized Fact 5: Price discrimination is common practice for euro-area firms.

More significant differences are found across sectors, although on this point the data is limited to only a few countries. In particular, uniform pricing is, as expected, more common in the trade sector, where the share of companies charging the same prices to all customers is around 55 percent in Italy and Spain and 44 percent in Luxembourg. The corresponding figures for the overall samples in these countries are 19 percent, 35 percent, and 29 percent, respectively. At the other extreme, the share of companies setting their prices on a case-by-case basis or according to the quantity sold is highest in manufacturing, which may explain the high numbers for Germany (figure 2).

4.1.3 Pricing to Market

The law of one price states that the price of a product must be the same across national markets. While the invoicing currency and hence exchange rate developments are certainly an issue in this context, the law of one price should even apply when adjusting for exchange rates. A substantial number of empirical studies, however, reject the validity of this law in the short run. A common explanation of departures from the law of one price is that transaction (arbitrage) costs between different geographical markets are high enough that firms can discriminate their prices across countries. In other words, when national markets are segmented by transportation costs or other barriers, exporting firms are able to set a different price in each market. "Pricing to market" is the usual term in the international trade literature for price discrimination across national markets.[10]

Given the open-economy nature of most euro-area economies, the price-setting behavior of exporters is a relevant issue. The surveys conducted in Belgium, Spain, Luxembourg, and Portugal include some specific questions, directed at firms operating in more than one market, which may provide valuable insights.

The questionnaires for Belgium, Spain, and Luxembourg inquire whether the price charged in different countries is the same or not. It turns out that around 50 percent of the exporting firms apply some form of pricing to market. As Aucremanne and Druant (2005) point out, this is a high proportion given that the exports of the countries considered are mostly directed toward the euro area, where a common currency is used. Price discrimination is even more frequent in the case of firms selling outside the euro area. In the Spanish sample, 60 percent of these companies charge different prices across non-euro-area countries. In the case of Portugal, the question is put differently. It only involves those firms exporting to countries outside the euro area, which are asked what would happen to the local price of their product in the selected country if the euro appreciated by 5 percent. For about 60 percent of the firms, the price would either remain unchanged or increase by less than 5 percent.

The questionnaires in the four countries also include a question on the importance of several factors in explaining differentiated prices across markets. Table 5, which reports the average scores assigned to the different factors, shows that the ranking is very similar in all four countries. Competitors' prices and transportation costs are the most relevant determinants; cyclical fluctuations in demand rank immediately below. Exchange rate developments and structural market conditions have only a moderate importance.

[10] See Obstfeld and Rogoff (1996) for a brief review of the empirical evidence on pricing to market.

Table 5. Importance of Factors in Differentiated Price Setting across Markets (*Mean Scores*)

	BE	ES	LU	PT
Price of Competitors	3.4	3.2	3.3	–
Transportation Costs and Other Factors	2.9	–	3.1	3.0
Cyclical Fluctuations in Demand	2.5	3.0	2.7	2.7
Structural Market Conditions	2.5	2.5	2.8	2.5
Exchange Rate of Payment Currency	2.4	2.2	1.8	2.5
Market Rules	2.1	–	2.7	2.8
Tax System	1.6	1.8	2.2	2.2

Not surprisingly, exchange rate movements receive a higher score from those firms exporting outside the euro area. Nevertheless, even for such firms, this factor is ranked below—at least in the Spanish sample—competitors' prices and demand. Finally, the local market tax system is generally singled out as the least relevant factor for explaining price differences across countries. As Aucremanne and Druant (2005) indicate, this factor is more important in consumer-oriented firms, for which differences in indirect taxation are presumably more significant.

Stylized Fact 6: Competitors' prices on the foreign market and transportation costs are the most relevant factors for pricing-to-market behavior.

4.2 How Often Do Firms Change Their Prices?

A rough measure of the degree of price stickiness is given by the number of price changes per year or, alternatively, by the average time elapsed between two consecutive price changes. Although the average duration of price spells is an essential ingredient in the calibration of dynamic stochastic general equilibrium (DSGE) models, which are widely used for monetary policy analysis (see, for instance, Woodford 2003 and Galí, López-Salido, and Vallés

2003), its empirical assessment has until very recently mainly relied on macroeconomic evidence.[11] In recent years, the availability of large-scale data sets of individual producer and consumer prices has strongly contributed to improve the measurement of the duration of price spells (see, for instance, Álvarez et al. 2005 and Dhyne et al. 2005). In this respect, survey results are useful for cross-checking the evidence obtained from these quantitative databases.

All national surveys, except that for Germany, contain a question on the number of price changes per year.[12] In particular, five questionnaires (BE, ES, LU, NL, and AT) inquire about the average number of price changes per year in recent years, and three of them (IT, FR, and PT) inquire about the number of price changes in a given year. Table 6 groups the results into four categories: (i) at least four price changes per year, (ii) two or three price changes per year, (iii) one price change per year, and (iv) less than one price change per year.[13]

The country results are very homogenous with the exception of Germany, where, as previously remarked, a different data source was used to obtain this specific information. On average, almost 40 percent of the firms in the euro area change their price once a year (the percentage share rises to 51 percent if Germany is excluded from the computation of the mean). In all countries except Germany and Luxembourg, approximately 70 percent of the respondents adjust their price a maximum of once a year.[14] On average, only around 34 percent of the firms change their price more frequently than once

[11]Smets and Wouters (2003) and Rabanal and Rubio-Ramírez (2005) are two recent examples of papers providing estimates of the average duration of prices.

[12]Since the German questionnaire does not contain a question on the number of price changes, the figures in table 6 concerning Germany are based on the number of months with price changes in 2003, reported by the same sample of firms as in the IFO business survey. The figures are quite different from those obtained for the rest of the countries, probably on account of the particularly low demand faced by German firms in that year.

[13]The categories are not the same as those used in table 3, since the number of price changes in most countries is considerably lower than that of price reviews.

[14]In Luxembourg this is largely due to the inclusion of the construction sector and its relative share in the responses (22 percent). Excluding this sector, as in the case of most other countries, would also result in a median of one price change per year.

Table 6. Frequency of Price Changes per Year (*Percentages*)[a]

	BE	DE	ES	FR	IT	LU	NL	AT	PT	Euro Area[b]
≥ 4	8	21	14	9	11	27	11	11	12	**14**
2–3	18	21	15	24	19	27	19	15	14	**20**
1	55	14	57	46	50	31	60	51	51	**39**
< 1	18	44	14	21	20	15	10	24	24	**27**
Median[c]	1	1	1	1	1	2	1	1	1	

[a]Rescaled figures excluding nonresponses.
[b]Weighted average (GDP weights).
[c]Median frequency of price changes per year.

a year. In all but one country, the median firm changes its price once a year.

These results are consistent with the frequency of producer price changes documented by Álvarez et al. (2005), on the basis of micro-PPI data for six euro-area countries. According to their findings, about 20 percent of individual producer prices are changed in a given month, which translates into a frequency of price change of about once a year. The survey results are also broadly in line with the evidence presented in Dhyne et al. (2005) who, on the basis of large-scale data sets of individual consumer price data for euro-area countries, find that the average duration of a price spell, based on a set of indirect estimators, ranges from four to five quarters.

With respect to comparable studies for non-euro-area countries, our results are in line with the findings of Apel, Friberg, and Hallsten (2005) for Sweden, where the modal number of actual price changes per year lies at the yearly frequency. However, the frequency estimated for euro-area firms is lower than that reported by Blinder et al. (1998) for the United States; Hall, Walsh, and Yates (1997) for the United Kingdom; and Amirault, Kwan, and Wilkinson (2004) for Canada (1.4, 2, and 4 price changes per year, respectively). The finding of a lower frequency of price adjustment in the euro area compared to the United States is consistent with the empirical evidence stemming from the analysis of microquantitative data (see

Dhyne et al. 2005) and from macro models (see Galí, Gertler, and López-Salido 2003).

As in the case of price reviews, the degree of competition faced by firms and the sector of activity help to explain differences in the frequency of price changes. With the exception of Austria and Portugal, in all the countries, firms that are subject to strong competitive pressures tend to change their prices significantly more often than those that do not face such pressures. In all the countries where the survey covers more than one sector, the Chi-square test for the equality of the distribution of price-change frequencies across sectors rejects the null hypothesis at the 5 percent level, pointing to significant differences. In five countries (BE, IT, LU, AT, and PT), firms in the services sector change their prices less frequently than those in other sectors; in four countries (ES, IT, LU, and NL), the frequency of price change is highest in the trade sector.

Stylized Fact 7: The median firm changes its price once a year. Prices are stickier in the services sector and more flexible in the trade sector. In most countries, firms facing strong competitive pressures adjust their prices more frequently.

4.3 The Relationship between Price Reviews and Price Changes

Taking into account only the companies that provided information concerning the frequency of price reviews and of price changes, all countries report that the former are conducted more frequently than the latter. Even with the categorized data used, at the euro-area level the share of firms changing their prices less than quarterly (maximum three times per year) is 86 percent, compared to 57 percent of firms reviewing their prices with the same frequency (table 7). Similar evidence is found in all but two countries.[15]

[15]In both Belgium and Spain, the frequency of price reviews is only slightly higher than that of price changes. As already mentioned, this might be partly explained by the format of the answer categories. In these two countries, firms are asked whether they review/change their prices more than once a year, once a year, or less than once a year. A substantial fraction of firms indicate that they review/change their prices once a year. If these questions had been formulated allowing for more answer categories, the fraction of firms declaring a yearly frequency of reviews/changes would have been lower.

Table 7. Comparison between Price Reviews and Price Changes per Year (*Percentages*)[a]

	BE	DE	ES	FR	IT	LU	NL	AT	PT	Euro Area[b]
Price Reviews ≤ 3	88	53	86	47	57	54	44	46	72	**57**
Price Changes ≤ 3	91	79	88	91	89	73	89	90	88	**86**

[a]Rescaled figures excluding nonresponses.
[b]Weighted average (GDP weights).

Stylized Fact 8: Price changes are less frequent than price reviews.

This finding stands in contrast to the assumption underlying the sticky-information model by Mankiw and Reis (2002), according to which prices are always changing, but price reviews are less frequent due to costly information or costs of reoptimization. The firms that cannot reoptimize their price in a given period simply follow old plans and outdated information to set prices. The finding also contradicts the assumption of lagged inflation indexation, which is assumed to be the price-setting rule of those firms that cannot reoptimize their price in a given period in Christiano, Eichenbaum, and Evans (2005), and which also implies that prices are always changing.

All in all, the evidence provided in this section is consistent with the notion that price adjustment takes place at two stages. First, the firms review their prices to check whether they are at the optimal level or need to be changed. As shown in section 3.3, they do this at discrete time intervals (the majority less than four times per year). Thus, some kind of stickiness can already be observed at the first stage of price setting. Once the review has taken place, firms may change their prices. However, they do so with a lower frequency than that of price reviews. One explanation of why prices are left unchanged may be that there is no reason to change them.[16] Alternatively, even though firms decided to incur the informational costs

[16]Although almost all national surveys address the issue of price reviews and price changes while referring to "normal conditions," in most cases it is not possible to control for the fact that the observed price behavior is in fact related to the occurrence of particular shocks, either of an idiosyncratic nature or of a common one.

of the price review, there may be other factors effectively preventing a desired price adjustment. Such factors are addressed in the next section.

4.4 Why Do Firms Hold Prices Constant?

The economic literature provides manifold explanations for sticky prices. As Blinder (1991) points out, however, it is difficult to evaluate how close the various theories come to the obstacles to changing prices encountered in the real world (one problem being observational equivalence). Thus, Blinder applied the interview method as a new way of examining the empirical relevance of different theories. He explained selected theories in face-to-face interviews with managers and assumed that they would recognize the line of reasoning if it came close to their way of thinking. All the national surveys on which this paper is based apply a similar method, presenting managers with different theories chosen according to their relevance in the economic literature, as well as their rankings in the surveys already conducted for other countries (Apel, Friberg, and Hallsten 2005; Blinder et al. 1998; Hall, Walsh, and Yates 1997). Before turning to the results, we summarize the most relevant theories.

Cost-Based Pricing. Inputs' costs are an important determinant in a firm's pricing decision. One line of reasoning based on this argument is that if costs do not change, prices will not change either. As products pass different production stages, a (demand or cost) shock somewhere in the production chain will take some time until it is propagated through the chain to finally reach consumers. Blanchard (1983) models production chains with n stages and assumes adjustment lags at each level of production. Even small lags in the adjustment process of a single firm can add up to long lags when taking into account the whole production chain.

Explicit Contracts. Firms have contractual arrangements with their customers, which may be in written form or orally agreed upon and in which they guarantee to offer a certain product at a specific price. An explanation of why firms engage in such agreements is that it is in their interest to build long-run customer relationships in order to stabilize their future sales. Customers, on the other hand, are attracted by a constant price because it makes their future costs more predictable and helps to minimize transaction costs (e.g., shopping time). Thus, customers might focus on the long-run average price rather than on the spot price. This is probably the most straightforward explanation of sticky prices. The idea that explicit contracts may be central for price stickiness was first introduced in the economic literature through wage contracts (e.g., Fischer 1977).

Implicit Contracts. This explanation is closely linked to the explicit contract theory but goes one step further. With implicit contracts, firms also want to build long-run customer relationships, and they try to win customer loyalty simply by changing prices as little as possible. This idea goes back to Okun (1981), who distinguishes between price increases due to cost shocks and those due to demand shocks. He argues that higher costs are an accepted rationale for rising prices, while increases in demand are viewed as unfair. Consequently, firms hold prices constant in the face of demand shocks, as they do not want to jeopardize customer relationships. They only adjust prices in response to cost shocks. The idea that consumers wish to buy from firms whose prices are “fair” is also applied by Rotemberg (2005).

Coordination Failure. This theory focuses on the interactions between firms as explanation for sticky prices. As in the case of explicit contracts, this idea was first introduced for the analysis of the labor market (e.g., Clower 1965; Leijonhufvud 1968). The argument is that the firm assumes that if it were to raise its price, it would lose customers, as no other firm would follow suit. On the other hand, if the firm were to decrease its price, it would not increase its market share, as all competitors would follow suit. After a shock a firm might, thus, want to change its price, but only if the other firms do the same. Without a coordinating mechanism that allows firms to move together, prices may remain fixed.

Menu Costs. Sheshinski and Weiss (1977) motivate the idea that the act of changing prices—printing and distributing new price lists—generates costs. Thus, a company facing these costs will change its prices less frequently than an otherwise identical firm without such costs. Akerlof and Yellen (1985) and Mankiw (1985) show that even “small” costs of changing prices can lead to nominal rigidities with “large” macroeconomic effects. In order to distinguish between different kinds of costs associated with price changes, we will use the term “menu cost” in a narrow sense and focus on the physical cost of changing prices.

Costly Information. Ball and Mankiw (1994) suggest a broader use of the term "menu costs," in the sense that it includes more than just the physical costs of changing prices. In particular, they argue that "the most important costs of price adjustment are the time and attention required of managers to gather the relevant information and to make and implement decisions" (p. 142). The distinction between these informational costs and physical menu costs enables us to investigate their relative importance in pricing decisions.

Temporary Shocks. When firms regard the shock they face as temporary, they may consider it appropriate to forego a price adjustment, as they expect the optimal new price to be short lived as well. It is not relevant whether the shock is indeed temporary or not, the main issue being how the firms assess the duration of the shock.

Change in Nonprice Factors. The price of a product is just one feature that can be adjusted in reaction to a changing environment. Firms can vary the delivery time, modify the quality of the product, or alter the level of service they offer in relation to the sale, to name but a few of the options that they have.

Judging Quality by Price. This line of reasoning reverses the argument used in the theory above addressing the issue of nonprice factors. The argument is that firms do not decrease the price of their product because customers might wrongly interpret the price decrease as a reduction in quality. Thus, they prefer to hold their nominal prices constant.

Pricing Thresholds. Firms may set their prices at psychologically attractive thresholds—for example, choosing €9.90 instead of €10.00. Attractive pricing strategies can cause price stickiness, because firms may postpone price adjustments in the face of small shocks, calling for small price changes until new events justify a large price change to the next pricing threshold.

All the national questionnaires asked the managers a question along the following lines: If there are reasons for changing the price of your main product, which of the following factors may well prevent an immediate price adjustment? The list following this question offered the above-mentioned theories, expressed in simple terms, as possible explanations. The respondents could indicate their degree of agreement with each theory, choosing from among four categories: unimportant (1), of minor importance (2), important (3), and very important (4), where the numbers in parentheses indicate the scores attached to each category.[17] Columns 1 to 9 in table 8 present the mean scores assigned by the firms in each country to the various theories. Column 10 reports the average of the country results, which is taken as an indication of the overall ranking for the euro area. Based on this ranking, two groups can be distinguished: the first group consists of those theories that have an average score well above 2, while the second group comprises the remaining theories. The last four columns of the table show the ranking of the same theories in the surveys by Blinder et al. (1998) for the United States; Apel, Friberg, and Hallsten (2005) for Sweden; Hall, Walsh, and Yates (1997) for the United Kingdom; and Amirault, Kwan, and Wilkinson (2004) for Canada.

The theory of "implicit contracts" receives the highest average score (2.7) and ranks first in five country studies. With an average score of 2.6, "explicit contracts" is the second most important explanation for sticky prices at the euro-area level (it ranks most important in four countries). The same average score is attributed to "cost-based pricing." Finally, with an average score of 2.4, "coordination failure" can also be regarded as a relevant factor behind price stickiness.

Implicit and explicit contracts are both based on the idea that firms want to establish long-run relationships with customers in order to make future sales more predictable. Their high score is consistent with the evidence presented in table 9, which shows that long-term relationships with customers are indeed a widespread phenomenon in the euro area. In this respect, Okun (1981) argues that price increases that are due to cost increases are viewed as fair by customers, while price increases that are due to a tight market are regarded as unfair. If this is the case—and the results suggest that managers indeed share this perception—it would be more likely that firms increase their prices in response to cost shocks than to demand shocks.

The theories ranked third and fourth are consistent with the price-setting strategies indicated by firms as the most common ones.

[17] In the Dutch questionnaire the scaling is more detailed (from 1 to 10). Results have been rescaled for comparability.

Table 8. The Importance of Theories Explaining Price Stickiness (*Mean Scores*)

	(1) BE	(2) DE	(3) ES	(4) FR	(5) IT	(6) LU	(7) NL	(8) AT	(9) PT	(10) Euro Area[a]	(11) US	(12) SW	(13) UK	(14) CA[b]
Implicit Contracts	2.5	–	2.6	2.2	–	2.7	2.7	3.0	3.1	**2.7**	4	1	5	2/7
Explicit Contracts	2.4	2.4	2.3	2.7	2.6	2.8	2.5	3.0	2.6	**2.6**	5	3	1	3
Cost-Based Pricing	2.4	–	–	2.5	–	2.7	–	2.6	2.7	**2.6**	2	2	2	1
Coordination Failure	2.2	2.2	2.4	3.0	2.6	2.1	2.2	2.3	2.8	**2.4**	1	4	3	5/8
Judging Quality by Price	1.9	–	1.8	–	–	2.2	2.4	1.9	2.3	**2.1**	12	–	10	–
Temporary Shocks	1.8	1.9	1.8	2.1	2.0	1.7	2.4	1.5	2.5	**2.0**	–	–	–	–
Nonprice Factors	1.7	–	1.3	–	–	1.9	1.9	1.7	–	**1.7**	3	–	8	4
Menu Costs	1.5	1.4	1.4	1.4	1.6	1.8	1.7	1.5	1.9	**1.6**	6	11	11	10
Costly Information	1.6	–	1.3	–	–	1.8	–	1.6	1.7	**1.6**	–	13	–	10
Pricing Thresholds	1.7	–	1.5	1.6	1.4	1.8	1.8	1.3	1.8	**1.6**	8	7	4	–

[a]Unweighted average of countries' scores. Columns 11 to 14 report the ranking of the theories in Blinder et al. (1998), Apel, Friberg, and Hallsten (2005), Hall, Walsh, and Yates (1997), and Amirault, Kwan, and Wilkinson (2004), respectively.
[b]In the column for Canada, two figures are reported for the implicit contracts and coordination failure theories, because in the Canadian questionnaire there are two different statements related to these theories.

Table 9. Firms' Relationships with Customers (*Percentages*)[a]

	BE	DE	ES	FR	IT	LU	NL	AT	PT	Euro Area[b]
Long Term	78	57	86	54	98	85	–	81	83	**70**
Occasional	22	43	14	46	2	15	–	19	17	**30**

[a]Rescaled figures excluding nonresponses. In the case of Belgium, France, and Italy, figures refer to relationships with other firms.
[b]Weighted average (GDP weights).

Cost-based pricing, which scores third, confirms the finding, reported in section 4.1.1, that the majority of firms set their price as a markup over costs. In this light, relatively stable costs and/or the sluggishness of the price response to cost changes are an important reason underlying price stickiness. The theory ranked fourth—coordination failure—relates instead to the interaction between firms on the same market. As shown in section 4.1.1, nearly 30 percent of the firms follow their competitors' prices when they set their own prices. Together with the fear of a lack in coordinating price movements, this provides a further explanation for price inertia—namely, that firms prefer not to change their prices as long as none of their competitors move first.

Each of the top four theories also ranks either first or second in the studies available for non-euro-area countries (Blinder et al. 1998; Apel, Friberg, and Hallsten 2005; Hall, Walsh, and Yates 1997; and Amirault, Kwan, and Wilkinson 2004). Moreover, the fact that the surveys for the euro area were conducted in different ways confirms that the findings do not depend on the survey method, on the particular wording used, or on the ordering of the answer categories.[18]

[18]The ordering of the theories differs considerably across the various questionnaires. For example, in the Dutch questionnaire, the theory of implicit contracts is the second answer category, while it appears in ninth place in the Austrian questionnaire. Nevertheless, in both country studies, the theory is regarded as the most important explanation. Overall, we do not find an association between the ordering of the answer categories and the scores given to the theories by respondents.

The importance attached to the various causes of price rigidity differs only slightly across sectors. In particular, in manufacturing and services the ranking is very similar to the one presented in table 8. There are small differences in the trade sector, in which explicit contracts are relatively less important, while, as expected, pricing thresholds are recognized as being slightly more relevant.

The remaining theories are, on average, not considered as important obstacles to price adjustment by euro-area firms. This group includes prominent candidates such as physical menu costs and costly information. Although they are frequently used explanations for price stickiness in the theoretical literature (e.g., Ball and Mankiw 1994), in practice they seem to be of minor importance for price setters.

The ranking attached to the various theories also provides some evidence on whether the factors preventing price adjustment have a greater bearing on the first or the second stage of the process itself, as discussed in section 3.3. In this respect, the evidence suggests that, for the majority of the firms, the main obstacles are not associated with the review stage but rather with the price change stage. In fact, the theory labeled "information costs"—i.e., costs associated with gathering and processing information for pricing decisions (stage 1 of price adjustment)—receives one of the lowest scores in all the surveys that included this category. A similar result is reported by Apel, Friberg, and Hallsten (2005) and Amirault, Kwan, and Wilkinson (2004).

Stylized Fact 9: Implicit and explicit contracts are the most relevant explanations for sticky prices, suggesting that price rigidities are associated with customers' preference for stable nominal prices. Other relevant factors rest on cost-based pricing and coordination failure. These results indicate that the main impediments to more-frequent price adjustment are associated with the price-change stage rather than with the price-review stage of the price-setting process.

Overall, these findings are in line with the results of Zbaracki et al. (2004), who report quantitative estimates of the different costs of price adjustments. They differentiate between costs of producing and distributing price sheets (what we call menu costs), managerial costs (information costs in our terminology), and customer costs. They conclude that while approximately one-quarter of the overall costs of changing prices are due to menu costs and information costs, three-quarters arise because customers dislike price changes.

5. Factors Driving Price Changes

5.1 Asymmetries of Price Reactions

The empirical literature provides evidence that price increases and price decreases do not occur with the same (conditional) probability. Dhyne et al. (2005) show that, for the euro area, price reductions are moderately less frequent than price increases: four out of ten price changes are decreases.[19] Analogous results are obtained by Lünnemann and Mathä (2005) using price index data. Asymmetries are also found with respect to the size of price changes, as average price increases tend to be smaller than average price decreases. The results for the United States are quite similar: Klenow and Kryvtsov (2004) report that 45 percent of all price changes are price reductions.

In order to analyze what drives price changes and whether there are asymmetries depending on the direction of the price adjustment, all national surveys included questions about factors underlying pricing decisions. Respondents were asked to assign scores between 1 (completely unimportant) and 4 (very important) to cost factors (labor costs, raw material costs, and financial costs) and market conditions (demand and competitors' prices) according to their importance in driving price adjustment. The question was posed separately for price increases and decreases.

The results are presented in table 10, which contains the mean scores for every factor in each country as well as the euro-area average score in the last column. The table shows that costs of raw material and labor costs are the most important factors driving prices upward. They receive an average score of 3.0 and rank first and second in every country. As for price decreases, competitors' prices (with an average score of 2.8) are the most widely recognized cause for downward price movements, followed by changes in demand

[19]Some country surveys (FR, NL, and PT) investigate the share of price increases and decreases, which turn out to be around 70 percent and 30 percent, respectively.

Table 10. The Importance of Different Factors Driving Price Changes (*Mean Scores*)

	BE	DE	ES	FR	IT	LU	NL	AT	PT	Euro Area[a]
A. Price Increases										
Labor Costs	2.9	2.7	2.7	2.5	2.9	3.5	2.7	3.4	3.3	**3.0**
Costs of Raw Materials	2.9	3.4	3.1	3.0	3.3	–	2.5	3.1	3.6	**3.1**
Financial Costs	2.2	1.9	1.8	–	2.3	3.0	2.1	1.9	2.5	**2.2**
Demand	2.2	2.2	2.4	2.0	2.4	2.3	2.3	1.9	2.5	**2.2**
Competitors' Price	2.5	2.1	2.5	2.3	2.6	2.4	2.5	2.0	2.7	**2.4**
B. Price Decreases										
Labor Costs	2.1	1.9	2.0	1.9	2.4	2.6	2.1	1.3	3.0	**2.1**
Costs of Raw Materials	2.3	2.8	2.6	2.6	2.9	–	2.0	2.2	3.3	**2.6**
Financial Costs	1.8	1.6	1.5	–	2.1	2.5	1.8	1.6	2.3	**1.9**
Demand	2.5	2.4	2.4	2.3	2.8	2.7	2.5	2.0	3.0	**2.5**
Competitors' Price	2.9	2.6	2.7	2.8	2.8	2.8	2.7	2.6	2.9	**2.8**

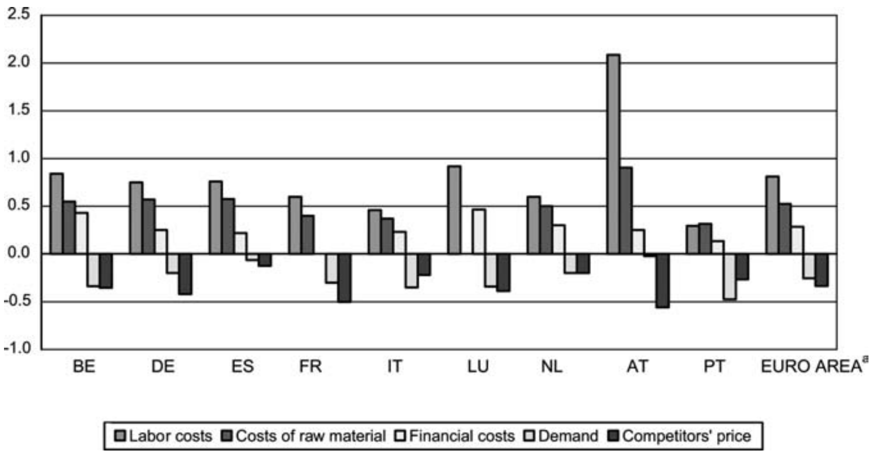
[a]Unweighted average of countries' scores.

conditions and costs of raw material. Financial costs do not seem to be relevant. These results hold for all sectors and are not sensitive to differences in the firms' size. The much-higher score received by cost changes in driving prices upward than downward could, in principle, be related to the fact that costs normally tend to increase. However, while this is likely to be the case for "wages," it does not hold necessarily for raw material prices, which exhibit a rather volatile pattern, mirroring movements in world demand and exchange rates.

To conclude, at the euro-area level, firms are more prompted to change their prices in response to shocks that lead to profit losses (rising costs of raw material and labor as well as a decrease in competitors' prices) than to shocks leading to profit gains (decreasing labor and financial costs as well as improving demand conditions and an increase in competitors' prices). Note that the results on the factors driving price movements do not seem to be sensitive to the economic outlook prevailing at the time the national surveys were conducted.

In order to present an even clearer picture on the asymmetries in the reasons underlying price increases and decreases, figure 3

Figure 3. Asymmetries in Price-Driving Factors (*Difference between Scores Regarding Price Rises and Price Decreases*)



[a]Unweighted average of countries' scores.

shows the difference of the reported scores for each factor. The results reveal a strikingly regular pattern of positive asymmetries for costs and negative asymmetries for market conditions.

Stylized Fact 10: Cost shocks are more relevant in driving prices upward than downward, while shocks to market conditions (changes in demand and competitors' prices) matter more for price decreases than increases.

Our findings about cost shocks are in line with the conclusions from Peltzman (2000), who provides evidence that, on average, prices respond faster to input price increases than to decreases and that the immediate response after a positive cost shock is at least twice the response to a negative one. The importance of implicit contracts as a cause of price stickiness, revealed in section 4.4, may provide a rationale for this asymmetry. If, as argued by Okun's "customer market" theory, customers view price increases due to cost increases as fair and price increases due to increased demand as unfair, firms should be more likely to increase their prices in response to cost shocks than to demand shocks, as they try to avoid jeopardizing customer relationships.

While Peltzman (2000) only focuses on the asymmetry with regard to cost shocks, we additionally find that demand shocks also affect prices asymmetrically. Negative demand changes are more likely to induce price adjustments than positive ones. Interpreting monetary policy shocks as demand shocks, we can compare our results with the discussion in the literature. Two classes of models can be identified, both implying asymmetric effects of money on output,[20] but with different implications about how nominal shocks affect prices. The first, based on the assumption of a convex aggregate supply curve (e.g., Ball and Romer 1989, 1990; Caballero and Engel 1992; Tsiddon 1993), imply that positive money-supply shocks have a larger effect on prices than negative ones. Conversely, the second class of models argue that positive money-supply shocks have a smaller effect on prices than negative ones. De Long and Summers

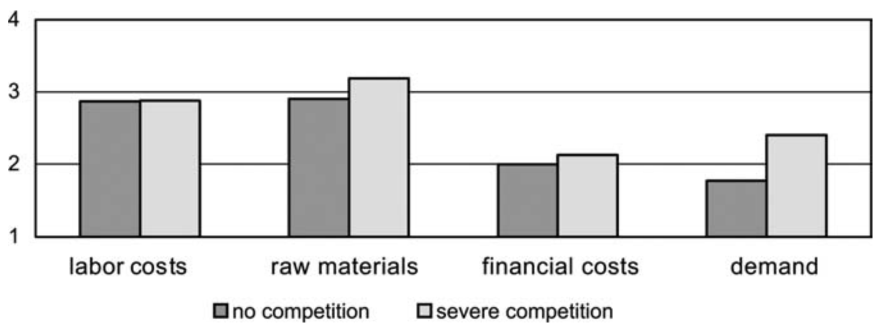
[20]Cover (1992) concludes that positive shocks in the money supply have no significant effect on output, whereas negative shocks reduce output. These results about the asymmetry of monetary policy shocks with regard to the sign of the shocks are also confirmed by other studies (e.g., Karras 1996 and De Long and Summers 1988).

(1988) associate this view with credit rationing in the monetary transmission mechanism. They argue that positive nominal shocks have a smaller effect on aggregate demand and thus also a smaller effect on prices (assuming a linear supply curve). Our results support this second type of model (see Karras 1996).

Finally, section 4.2 showed that in nearly all countries, firms facing stronger competitive pressures tend to change their prices significantly more often than those not subject to such pressures. Thus, we expect the degree of competition to matter also in shaping pricing behavior when it comes to the driving forces of price adjustment. The influence of competition is shown separately for price-raising shocks in figure 4 and price-decreasing shocks in figure 5. As the differences between countries are limited, we present average scores for the euro area for two types of firms, namely, those facing either severe or limited competition.

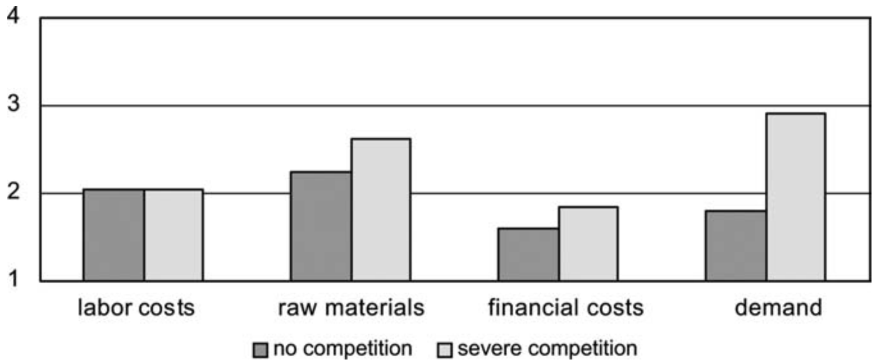
The degree of competition indeed matters. For firms in highly competitive markets, cost and demand factors are more important in driving price adjustment. The differences between the two groups are largest in the case of price-decreasing shocks, especially on the demand side. Interestingly, the results of the national surveys suggest that responses to labor cost shocks are more or less the same regardless of the degree of competition. An explanation might be that in most euro-area countries, wage adjustments are the outcome

Figure 4. Perceived Competition and Price-Raising Factors in the Euro Area (*Average Scores*)[a]



[a]Unweighted averages of countries' scores.

Figure 5. Perceived Competition and Price-Reducing Factors in the Euro Area (*Average Scores*)[a]



[a]Unweighted averages of countries' scores.

of yearly or half-yearly collective bargaining agreements at the national or sectoral level and, thus, all firms in a sector are equally affected.

Stylized Fact 11: Firms in highly competitive markets are more likely to respond to changes in underlying factors, especially in the case of demand shocks.

5.2 Price Adjustment after Shocks

Five countries (ES, FR, LU, AT, and PT) investigated further the issue of price reactions after shocks, focusing also on the time lag of the price response. Firms were asked whether they change their prices in reaction to a specific shock or not. In the case of a positive answer, they were requested to indicate the time (number of months) elapsed before the price change is implemented.

Table 11 presents the share of respondents who answered that they hold their prices constant in reaction to a specific shock. The results support the same conclusion about asymmetries as the previous section, though the issue is approached from a different angle: lower demand is more likely to lead to price adjustments than higher demand, while the opposite is true for cost shocks. Moreover, comparing the first and third columns, we observe that a larger share of firms adjust their price in reaction to increasing costs than to

Table 11. Speed of Price Adjustment after Different Kinds of Shocks (*Percentages*)[a]

	Higher Demand	Lower Demand	Higher Costs	Lower Costs
ES				
<1 month	18	21	15	13
1–3 months	17	21	18	18
>3 months	65	58	67	69
FR				
<1 month	35	37	34	31
1–3 months	34	35	27	29
>3 months	31	28	39	40
LU				
<1 month	34	42	47	40
1–3 months	24	31	25	28
>3 months	42	27	28	32
AT				
<1 month	4	3	2	2
1–3 months	51	71	65	61
>3 months	45	26	33	37
PT				
<1 month	22	28	24	23
1–3 months	31	32	27	33
>3 months	47	40	49	44

[a]Share of respondents who answered that they hold their prices constant for the number of months indicated in the table in reaction to a specific shock. Rescaled figures excluding nonresponses.

higher demand, which further corroborates what we found in previous sections.

The median firm changes its price one to three months after a shock in France, Luxembourg, Austria, and Portugal, while the median firm in Spain waits for more than three months—regardless of the sign and source of the shock. Thus, an adjustment process of one quarter in macro models for France, Luxembourg, Austria, and Portugal and of two or more quarters for Spain seems to be justified by these findings.

6. Conclusions

The responses collected from around 11,000 euro-area companies, mainly in the manufacturing sector, surveyed by nine central banks of the euro area shed light on important aspects of the price-setting behavior, which can hardly be assessed otherwise. Compared to previous similar empirical works, the distinguishing characteristic of the research project summarized in this paper is that the results are very consistent across countries: they are neither affected by differences in the national questionnaires (such as the different wording of the questions, their ordering, the possible answer categories, etc.), nor by the way in which the surveys were carried out, nor by the economic conditions prevailing in the countries at the time the surveys were conducted. The analogy of the results weakens the arguments traditionally raised against the use of surveys—in particular, concerning the qualitative nature of the information gathered, the lack of a time dimension, and the degree of uncertainty that surrounds the quality of the answers provided by the respondents.

Regarding the reviewing stage of the price-setting process, our evidence suggests that both time- and state-dependent pricing strategies are applied by firms in the euro area. Around one-third of the companies follow mainly time-dependent pricing rules, while the remaining two-thirds use pricing rules with some element of state dependence. Although the majority of firms take into account a wide range of information, including past and expected economic developments, about one-third adopt a purely backward-looking behavior. The pattern of results lends support to the recent wave of estimations of hybrid versions of the New Keynesian Phillips curve including past inflation in order to explain inflation developments.

Two pieces of evidence from our surveys suggest that the model of perfect competition with the law of one price does not seem to be the blueprint for most of the goods and service markets in the euro area. Firstly, markup pricing is the dominant price-setting strategy adopted by firms in the euro area, indicating that these firms have some form of market power and can set their prices above marginal costs. Secondly, price discrimination is a common practice. This suggests that models with monopolistic competition, like New Keynesian models, may be a better description for most goods and service markets than those models that assume perfect competition.

In most countries the modal number of price reviews lies in the range of one to three times per year, and in nearly all countries on which this report is based, the median firm changes its price once a year. The latter result is consistent with the evidence obtained by Álvarez et al. (2005) on the basis of micro-PPI data and is also largely in line with the findings on the frequency of price changes in euro-area consumer prices by Dhyne et al. (2005).

Among the structural characteristics explaining differences in the frequency of price adjustment, we find that companies operating in markets with severe competition review and adjust their prices more frequently. The degree of competitive pressures faced by firms indeed matters for pricing strategies. We provide evidence that the lower the level of competition, the more frequently firms use markup rules and the more likely they are to respond to changes in underlying factors (e.g., cost and demand factors potentially driving price changes).

Our results indicate that there are obstacles to price changes in the reviewing as well as the implementation stage of the price-setting process. However, in contrast to the suggestion of Ball and Mankiw (1994), informational costs, which are important at the reviewing stage of price setting, do not seem to be among the most important obstacles to price changes. The fear that a price adjustment could jeopardize customer relationships (expressed in the theories on implicit and explicit contracts) is found to be a much more important explanation for rigid prices. This finding is consistent with the results of Zbaracki et al. (2004), who conclude that one-quarter of the overall costs of changing prices is due to menu costs and information costs, while three-quarters are arising because customers dislike price changes. The implicit contract theory going back to Okun (1981), which was recognized as very relevant by our respondents, suggests that customers regard price increases in response to cost shocks as fairer than price adjustments in response to demand shocks. This finding ties in with Rotemberg (2005), who also argues that fairness is an important driving force in customers' buying decisions.

Finally, we provide evidence that firms adjust prices asymmetrically in response to shocks, depending on the source of the shock and the direction of the adjustment. Changes in costs are the main factor underlying price increases, whereas changes in market conditions (demand and competitors' prices) are the driving forces behind price reductions. Moreover, prices seem to be more flexible downward than upward in response to demand shocks, while the opposite result holds in the face of cost shocks.

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Intrinsic and Inherited Inflation Persistence*

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In the conventional view of inflation, the New Keynesian Phillips curve (NKPC) captures most of the persistence in inflation. The sources of persistence are twofold. First, the "driving process" for inflation is quite persistent, and the NKPC implies that inflation must "inherit" this persistence. Second, backward-looking or indexing behavior imparts some "intrinsic" persistence to inflation. This paper shows that, in practice, inflation in the NKPC inherits very little of the persistence of the driving process, and it is intrinsic persistence that constitutes the dominant source of persistence. The reasons are that, first, the coefficient on the driving process is small, and, second, the shock that disturbs the NKPC is large.

JEL Codes: E31, E52.

1. Introduction

The progression of price-setting models has a long and lively history. Beginning with A. W. Phillips (1958) and continuing with Lucas (1972), Fischer (1977), Gray (1978), Taylor (1980), Calvo (1983), Rotemberg (1983), Gordon (1985), Roberts (1995), Fuhrer and Moore (1995a), Galí and Gertler (1999), Erceg, Henderson, and Levin (2000), Galí, Gertler, and López-Salido (2001), Sbordone (2002), Mankiw and Reis (2002), Cogley and Sbordone (2005),

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Christiano, Eichenbaum, and Evans (2005), and countless others, the specifications have matured to include rational expectations, optimizing foundations, a more-persistent driving process (real marginal cost), and a variety of "frictions" that allow the models to mimic the gradual response of inflation to a variety of shocks.

In recent years, much of the development of Phillips curves has centered on two issues: (i) the emergence of real marginal cost (versus an output gap measure) as the preferred driving variable in the specification, on both theoretical and empirical grounds, and (ii) the incorporation of frictions into optimizing rational expectations models.[1] The frictions have included indexing (as in Christiano, Eichenbaum, and Evans 2005) and "rule-of-thumb" or "backward-looking" price setters (as in Galí and Gertler 1999). These frictions have been ad hoc, in that they are not microfounded. Still, the common view is that, after allowing for just a little friction, the baseline model works well. For example, in a fairly recent summary, Galí (2003, sec. 3.1) suggests that:

> The findings ... are ... quite encouraging for the baseline NKPC: while backward-looking behavior is often statistically significant, it appears to have limited quantitative importance. In other words, while the baseline pure forward-looking model is rejected on statistical grounds, it is still likely to be a reasonable first approximation to the inflation dynamics of both Europe and the U.S.

This view has been criticized by a number of authors from a variety of viewpoints. Representative papers include a recent paper by Rudd and Whelan (2006), who discuss a number of weaknesses of the NKPC, including the difficulty in developing a significant estimate of the coefficient on the driving process in the NKPC, and an older paper by Fuhrer (1997), who finds only weak empirical evidence of a forward-looking term in one simple version of the NKPC.

[1] There remains considerable debate with regard to the use of real marginal cost, widely proxied by labor's share of income, as the driving variable. Rudd and Whelan (2006) provide evidence that casts doubt on the empirical significance of marginal cost in forward-looking Phillips curves. A third development has been the inclusion of serially correlated shocks to the models. A model with serially correlated shocks is considered in section 3.

This paper will provide theoretical analysis and empirical evidence that largely contradicts the emerging consensus on price-setting models. It will show that, regardless of the persistence in the driving process, very little of that persistence is inherited by inflation in the conventional NKPC. This result runs counter to the common intuition that inflation in the NKPC directly inherits the persistence of the driving process, which, in the case of both real marginal cost and the output gap (or proxies thereof), is quite considerable. In fact, inflation does inherit some of the persistence of the driving process, but in the models commonly in use, the amount that it inherits is remarkably small.[2]

So how does this seemingly counterintuitive result arise? There are two reasons: (i) the coefficient on the driving variable in NKPCs is estimated to be very small, on the order of .001 to .05, and (ii) in addition to the shock that impels the driving process and thus indirectly influences inflation, there is another shock that disturbs the Phillips curve directly. The paper will show that the variance of that shock is large, generally at least as large as the shock driving real marginal cost or the output gap.

As the paper demonstrates below, those two facts together imply a very attenuated inheritance of the driving variable's persistence into the inflation process. A simple intuition for this result is as follows. Consider the purely forward-looking version of the NKPC displayed below.

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \gamma y_t + e_t \\ y_t &= \rho y_{t-1} + u_t \end{aligned} \tag{1}$$

If one iterates the top equation forward, one sees that inflation is simply a discounted sum of future y's plus the error term e_t, which is assumed to be iid for the moment.[3] If there were no other shock in the model—if e_t were identically zero—then inflation's dynamic properties would be solely determined by those of the driving process y.

[2]For the most part, this paper takes an agnostic view on the appropriate driving process. In the analytical sections, all that matters is that the driving process is persistent, which both leading candidates are. In the empirical sections, I examine cases in which marginal cost or the output gap is the assumed driving process. The results in this paper are generally insensitive to which driving variable is used.

[3]We consider the ramifications of a serially correlated shock below.

However, in the presence of a second shock, the intuition about inflation persistence changes. In that case, one can think of the simple forward-looking model as the sum of an AR(1) process y and an uncorrelated shock e. The persistence of the AR(1) process—summarized by its autocorrelation function—decays geometrically at rate ρ. The persistence of the shock process is rather uninteresting: its autocorrelation function equals one at lag zero and zero at all other lags. Which of these two processes dominates inflation's autocorrelation properties depends on two parameters: γ and the variance of e (relative to the variance of u). The larger the value of γ, the more the mix looks like the AR(1) process and the less it looks like white noise. The larger the variance of e relative to that of u, the more the process looks like white noise and the less it looks like an AR(1) process.

If γ is relatively small, and the variance of e relatively large, then the frictions added to the NKPC—the sources of "intrinsic persistence" in the model—will no longer be quantitatively unimportant but statistically significant additions. They will be of first-order importance to the model. But it also follows that the optimizing foundations, through which the forward-looking model with marginal cost as the driving process is motivated, become correspondingly less important for explaining inflation behavior. Thus, it becomes critical to understand *what* the inflation shock is and why the estimated coefficient on the driving process is so small.[4] This paper will provide only partial answers to these questions.

The paper demonstrates analytically the propositions about inherited persistence for the forward-looking model in section 2. It analyzes the case of the hybrid model in section 3. Section 4 considers some extensions, including a model with explicit monetary policy. It also considers the implications of possible recent changes in the persistence of inflation. Section 5 examines reduced-form properties in the data that will lead to structural models that embody a small γ and a relatively large variance of the inflation shock. Section 6 concludes.

[4]The same points are demonstrated for the Mankiw-Reis model of price setting in Fuhrer (2002). There, the presence of large "markup shocks," which are the equivalent of inflation shocks in the hybrid New Keynesian Phillips curve (HNKPC), similarly imply that inflation inherits very little of the driving variable's persistence.

2. The Purely Forward-Looking Model

Consider the canonical hybrid New Keynesian Phillips curve (HNKPC), which may be expressed as[5]

$$\begin{aligned} \pi_t &= (\beta - \mu) E_t \pi_{t+1} + \mu \pi_{t-1} + \gamma y_t + e_t \\ y_t &= \rho y_{t-1} + u_t \\ Var(e_t, u_t) &= \Sigma, \end{aligned} \tag{2}$$

where π denotes inflation, y is a driving variable (typically a proxy for real marginal cost or the output gap), μ and $(\beta - \mu)$ are the weights on past and expected inflation, and γ is the coefficient on the driving process.[6] The baseline case will assume that e, the "inflation shock," is a white-noise iid shock, although that assumption is relaxed below. The second equation specifies the simplest persistent process for the driving variable y, a first-order autoregression with autoregressive parameter ρ, which is set to 0.9 in all of the exercises below.[7] The covariance matrix of the error processes is denoted by Σ and will be assumed diagonal throughout. However, the relative sizes of the shock variances will be allowed to vary and will be shown to have important effects on inflation persistence.

---

[5] A related specification allows lagged inflation to Granger-cause the driving process y:

$$\begin{aligned} \pi_t &= (\beta - \mu) E_t \pi_{t+1} + \mu \pi_{t-1} + \gamma y_t + e_t \\ y_t &= \rho y_{t-1} + \delta \pi_{t-1} + u_t. \end{aligned}$$

Because this modification adds no intrinsic persistence to inflation, its implications for the autocorrelation properties of inflation are virtually identical to those of the model in equation (2), for any plausible value of δ. In addition, for the data employed in this paper, estimates of δ tend to be nearly zero and insignificantly different from zero.

[6] While I do not make this explicit here, one can map the coefficient γ into the underlying frequency of price adjustments, as in Woodford (2003, chap. 3, eq. 2.13) or Galí and Gertler (1999, eq. 16). As the fraction of prices that remain fixed each period increases, the coefficient on marginal cost declines. Thus a rise in γ implicitly corresponds to an increase in the frequency of price adjustment or, equivalently, to an increase in price flexibility.

[7] This value corresponds to estimates obtained later in the paper. The qualitative results in the paper are unchanged by a value for ρ up to 0.95.

2.1 The Analytical Autocorrelation Function for Inflation

The solution to the HNKPC model may be expressed as a vector first-order state-space system:

$$x_t = \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = A \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix} + S_0^{-1} \begin{bmatrix} e_t \\ u_t \end{bmatrix}, \tag{3}$$

where A is the matrix of reduced-form solution coefficients (see Anderson and Moore 1985), and S_0 is defined in Fuhrer and Moore (1995a). For this simple model with $\mu = 0$, A and S_0 are

$$A = \begin{bmatrix} 0 & \frac{\gamma\rho}{1-\rho\beta} \\ 0 & \rho \end{bmatrix}; S_0^{-1} = \begin{bmatrix} 1 & \frac{\gamma}{1-\rho\beta} \\ 0 & 1 \end{bmatrix}. \tag{4}$$

Note that the structure of A implies that the lagged inflation rate does not enter the solution for current inflation. The structure of S_0 implies that the relative effects of the two shocks on inflation will depend critically on γ.

Denote the k-period-ahead variance of x by V_k, where $\mathrm{V_k} = \mathrm{AV_{k-1}A'}$, with V_0 initialized to $\mathrm{S_0^{-1}\Sigma S_0^{-1\prime}}$. The unconditional variance of x, denoted V, is the convergent sum of the V_k. Then the correlation of the vector x_t with x_{t-k} can be computed recursively from $\Gamma_k = \mathrm{A}^{k-1}\Gamma_{k-1}$, with $\Gamma_0 = V$. Hence, the matrices that determine the autocorrelation properties of x are the transition matrix A and the unconditional variance matrix V.

Using these two matrices and the definition of the unconditional variance, we can show that the unconditional variance of inflation for the NKPC model is a linear combination of the variances of e and u:

$$V = Var \begin{bmatrix} \pi \\ y \end{bmatrix} = \begin{bmatrix} \frac{\left[\frac{\gamma}{1-\rho\beta}\right]^2}{1-\rho^2}\sigma_u^2 + \sigma_e^2 & \frac{\gamma\sigma_u^2}{(1-\rho\beta)(1-\rho^2)} \\ \frac{\gamma\sigma_u^2}{(1-\rho\beta)(1-\rho^2)} & \frac{\sigma_u^2}{1-\rho^2} \end{bmatrix}. \tag{5}$$

The first term in the unconditional variance of inflation is the unconditional variance of y scaled by $\left[\frac{\gamma}{1-\rho\beta}\right]^2$. The weight on the variance of y is strictly increasing in γ and ρ, so the larger the values of γ and ρ, the larger the relative influence of σ_u^2 and the smaller the relative influence of σ_e^2 in the variance of π.

The autocorrelations for y_t take the expected form for an AR(1) process. The autocorrelations for inflation at horizon i are denoted by Γ_i; they are[8]

$$\begin{aligned} \Gamma_i &= \tfrac{\rho^i \gamma^2}{a\sigma_e^2 + \gamma^2} \\ a &= (1-\rho^2)(1-\rho\beta)^2. \end{aligned} \tag{6}$$

Clearly, the autocorrelations decay at the rate ρ. The term $\frac{\gamma^2}{a\sigma_e^2+\gamma^2}$ sets the initial "level" of the autocorrelation function, with the rate of decay from the initial level dependent only on ρ. Thus, the difference between a persistent and a nonpersistent inflation rate in this model will hinge on how large the first autocorrelation is: do the autocorrelations jump down toward zero immediately, or do they decay from near one? As suggested above, the answer to this question must depend upon the extent to which y feeds into π (that is, how large γ is) and the relative size of the variances of the shock hitting the inflation equation and the shock hitting the driving process.

From here forward, for simplicity, the paper will normalize σ_u^2 to 1. It is important to remember, however, that wherever the algebra refers to σ_e^2, this should be understood as the *ratio* of the variances. With this simplification, it is straightforward to show that equation (6) implies

$$\frac{\partial \Gamma_1}{\partial \gamma} > 0; \frac{\partial \Gamma_1}{\partial \sigma_e^2} < 0. \tag{7}$$

That is, the smaller the influence of y on π, the smaller the initial autocorrelation. The larger the variance of e relative to u, the smaller the initial autocorrelation.

Note also that in the case in which the stochastic dimension of $[\pi, y]$ is 1—that is, for simplicity $\sigma_e^2 = 0$, so that the only shock in the system is u—then the autocorrelations of inflation take the simpler form

$$\Gamma_i = \rho^i.$$

[8]In an unpublished comment, Galí (2005) derives the solution to these models in the case of $e = 0$.

Table 1. Value of Γ_1 for Selected Values of σ_e^2 and γ

σ_e^2	γ .01	.03	.05	.1	.2
0	0.90	0.90	0.90	0.90	0.90
0.1	0.25	0.70	0.81	0.88	0.89
0.3	0.10	0.48	0.68	0.83	0.88
0.5	0.06	0.36	0.59	0.79	0.87
1	0.03	0.23	0.44	0.71	0.84
3	0.01	0.09	0.22	0.50	0.75
5	0.01	0.06	0.14	0.39	0.68
	$\varrho = 0.95$				
.5	0.29	0.76	0.87	0.93	0.94
3	0.06	0.37	0.61	0.83	0.92
	$\varrho = 0.95, \beta = 0.99$				
0.5	0.35	0.80	0.89	0.93	0.95
3.0	0.08	0.44	0.67	0.86	0.93

Not surprisingly, in this special case, inflation follows exactly the same AR(1) process as y.[9]

As it turns out, extreme values of σ_e^2 and/or γ are not required to imply a very small first-period autocorrelation for inflation, even when the autocorrelation of y is considerable. Table 1 displays the value of the first autocorrelation of inflation for various values of the ratio of variances σ_e^2 and the parameter on marginal cost γ. Because the autocorrelations following the first will die out geometrically at rate ρ, this first autocorrelation is a sufficient statistic for the entire function, once one knows ρ.[10]

[9]The autocovariances of π will still depend upon γ, but the autocorrelations for π are identical to those for y.

[10]Note that the empirical analysis in this paper employs annualized inflation rates, so the appropriate adjustment must be made to scale both γ and σ_e^2. More broadly, one must work very hard to obtain sizable and significant estimates of γ. Estimates on quarterly inflation rates range from 0.001 or below in Rudd and

As the table suggests, depending on the parameter configuration, the first autocorrelation of inflation can range from essentially zero for high values of σ_e^2 and low values of γ to ρ for the opposite. The relative size of the shock variances is not often reported in empirical studies, but given the evidence presented below, it will be unusual to find $\sigma_e^2 << 1$. In addition, it is quite widely known that estimated values of γ tend to be quite small. Thus, the most relevant sections of the table are the left-hand three columns. The table implies that most often, the first autocorrelation for inflation implied by the NKPC will be quite small, in the range .05 to 0.3, and quite often below 0.1.[11]

Therefore, the purely forward-looking version of the NKPC can only impart high persistence to inflation with an implausibly high estimate of γ or a very low relative variance ratio. Of course, when one includes a lag of inflation, as in the so-called "hybrid model" discussed in the next section, the interaction between lagged inflation and the forward-looking component of the model must be taken into account.[12]

3. The Hybrid Model

Now, consider the hybrid NKPC (HNKPC), which sets $\mu > 0$ in equation (2). The algebra becomes somewhat more complex (see the details in appendix 1), but much of the intuition from the simple NKPC remains. Larger values of σ_e^2 and γ will imply lesser and greater inheritance, respectively, of the persistence in the driving process. Now, however, the degree of "backward-looking" or indexing

Whelan (2005) to 0.037 in Galí and Gertler (1999). The GMM estimates presented below generally lie well below 0.01, with only one estimate on *annualized growth rates* exceeding 0.03, and none significantly different from zero. Thus, a γ of 0.03 is a quite generous annualized coefficient, given the number of near-zero estimates in the literature, and given the difficulty in developing significant estimates. The maximum likelihood estimates presented below, using annualized inflation rates and employing either the output gap or real marginal cost as the driving variable, develop estimates of γ of 0.011 and 0.001, respectively.

[11] Note that figure II in Fuhrer and Moore (1995a) displays the autocorrelation function implied by the Taylor (1980) nominal contract model, coupled with a persistent process for the output gap. That analysis displays the same qualitative result as those in this paper.

[12] Ireland (2004) also emphasizes the centrality of this shock, in his model a "cost-push" shock, in achieving data consistency.

behavior—the size of μ—becomes critical in determining the persistence of inflation implied by the model.

The key matrices A and S_0 for the state-space representation (equation [3]) of the hybrid model are

$$A = \begin{bmatrix} \lambda_s & \frac{\gamma\rho}{(\beta-\mu)(\lambda_\mathrm{b}-\rho)} \\ 0 & \rho \end{bmatrix}; \quad S_0^{-1} = \begin{bmatrix} \frac{\lambda_s}{\mu} & \frac{-\gamma\lambda_\mathrm{b}\lambda_\mathrm{s}}{\mu(\rho-\lambda_\mathrm{b})} \\ 0 & 1 \end{bmatrix}, \tag{8}$$

where λ_b and λ_s are the unstable and stable roots, respectively, of the system.[13] We can write the solution to the model as[14]

$$\begin{aligned} \pi_t &= \frac{\mu\pi_{t-1} - \frac{\gamma\lambda_b}{\rho-\lambda_b}y_t + e_t}{(\beta-\mu)\lambda_b} \\ &= \lambda_s\pi_{t-1} + \frac{\gamma\lambda_s}{\mu-(\beta-\mu)\rho\lambda_s}y_t + \frac{\lambda_s}{\mu}e_t. \end{aligned} \tag{9}$$

This representation shows that, as is common for simple second-order difference equations of this type, the coefficient on lagged inflation in the HNKPC solution is the stable root of the system. The stable root, in turn, is a function of the parameters μ and β; the dependence of the stable root and the first autocorrelation on μ is examined below.

The unconditional variance of inflation, denoted here by V_π, is again a weighted average of the underlying shock variances. The weights are given by

$$\begin{aligned} V_\pi = w_e\sigma_e^2 + w_u\sigma_u^2 &= \frac{\lambda_s^2}{\mu^2(1-\lambda_s^2)}\sigma_e^2 \\ &+ \frac{\lambda_s\gamma^2[2\mu^2\rho(1-\rho\lambda_s)+\lambda_s]}{(1-\rho^2)(1-\lambda_s^2)[\lambda_s\rho(\beta-\mu)-\mu]^2}\sigma_u^2. \end{aligned} \tag{10}$$

The stable root plays a key role in determining the contributions of the two conditional variances to the unconditional variance of

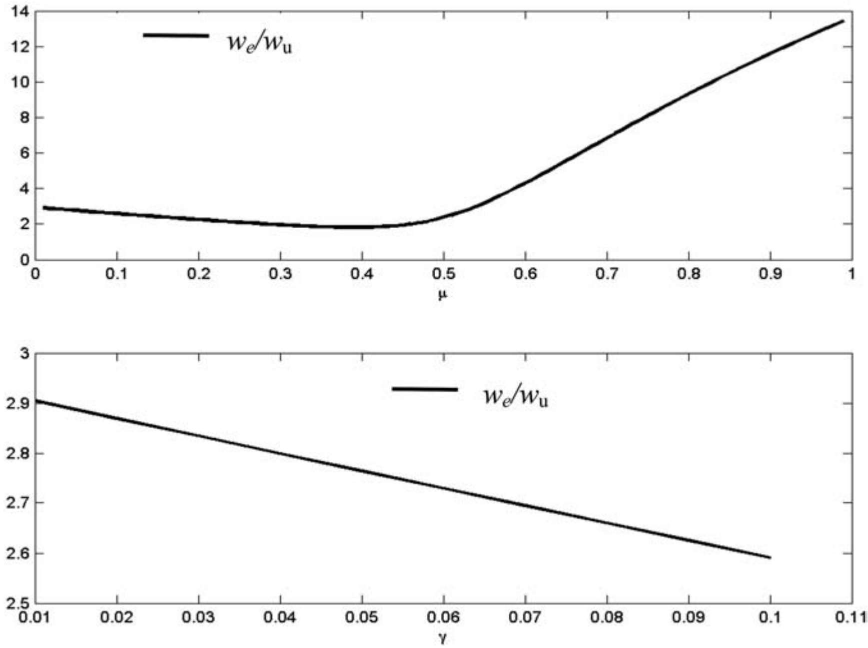
[13] The third root is always ρ. Appendix 1 shows that $\lambda_b\lambda_s = \frac{\mu}{\beta-\mu}$.

[14] This representation is a version of the familiar solution to the second-order difference equation,

$$\pi_t = \lambda_s\pi_{t-1} + \gamma f(\lambda_s, \lambda_b)\sum_{i=0}^{\infty}\lambda_b^i E_t y_{t+i} + \varepsilon_t,$$

in which the forward-sum term in the equation is solved for the sum of the t-period expectations of y_t.

Figure 1. Relative Weights of Shock Variances σ_e^2 and σ_u^2 in Inflation Variance



inflation. Figure 1 displays the variation in the ratio w_e/w_u as μ varies. The effect of μ on the contribution of σ_e^2 to V_π is not monotonic. Increasing μ from 0 to 0.4 slightly depresses the contribution of σ_e^2. But as μ increases from 0.4 to 0.9, the relative weight on σ_e^2 rises by a factor of six. Thus relatively modest differences in μ imply significant differences in the contributions of the two variances. As the lower panel of the figure indicates, the larger the value of γ, the smaller the relative contribution of σ_e^2 to V_π, but, in any case, the effect is relatively small.

The expression for the autocorrelations in the HNKPC is somewhat more complex than in the simple NKPC. Nonetheless, the autocorrelation function can be shown to decay approximately geometrically after the first few autocorrelations.[15] As a result,

[15] The rate of decay is slower than ρ for the first few autocorrelations and then converges to ρ as k gets large.

again, a critical question is, how large is the first autocorrelation? It can be expressed as

$$\Gamma_1 = \frac{a}{b\sigma_e^2 - c\rho\mu} d + \lambda_{\mathrm{s}}, \tag{11}$$

where $[a, b, c, d]$ are functions of the stable root λ_s (in turn a function of μ and β) and the underlying parameters $[\mu, \beta, \gamma, \sigma_e^2]$. As is the case for the purely forward-looking model above, it can be shown that Γ_1 is decreasing in σ_e^2. As will be shown below, the additive term in λ_s dominates Γ_1, and both λ_s and Γ_1 rise almost one-for-one with μ.

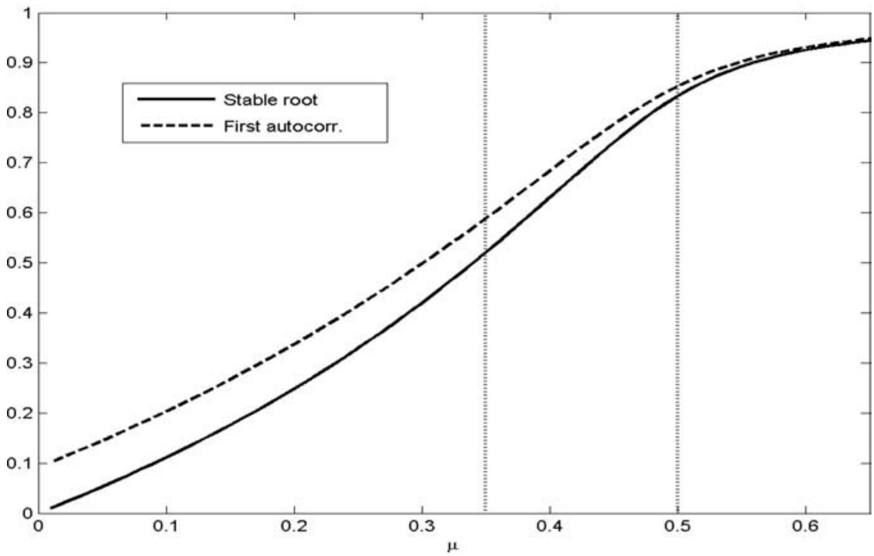
Table 2 shows the value of Γ_1 for an array of values for σ_e^2 and μ. The table illustrates that, for values of these parameters in the range commonly estimated, one obtains a relatively small first autocorrelation—0.6 or below. The bottom panel of the table shows the first eight autocorrelations of inflation when γ, σ_e^2, and μ are set to values consistent with parameter estimates in the literature. The autocorrelations die out quickly, and in the following section we will see that they die out significantly more quickly than those exhibited in the data.

Table 2. Value of Γ_1 for Selected Values of σ_e^2 and $\boldsymbol{\mu}$, Hybrid Model $\boldsymbol{\gamma = 0.03}$, $\boldsymbol{\beta = 0.98}$, $\boldsymbol{\rho = 0.9}$

	$\boldsymbol{\mu}$				
σ_e^2	.1	.3	.5	.7	.9
0	0.92	0.96	0.99	1.00	1.00
0.3	0.74	0.86	0.96	0.98	0.99
0.5	0.66	0.82	0.94	0.97	0.98
1	0.32	0.60	0.89	0.96	0.98
2	0.23	0.53	0.86	0.96	0.98
3	0.20	0.50	0.86	0.96	0.98
5	0.16	0.47	0.85	0.96	0.98

Autocorrelations for $\sigma_e^2 = 3$, $\gamma = .03$, $\boldsymbol{\mu} = .35$							
1	2	3	4	5	6	7	8
0.59	0.37	0.25	0.18	0.14	0.11	0.09	0.08

Figure 2. Effect of μ on Stable Root and First Autocorrelation of Hybrid Model



Note in the first row of table 2 that as σ_e^2 goes to zero, the autocorrelation of inflation is bounded below by ρ and rises quickly to one as μ increases.[16] The message of this table is that if one wishes to be roughly data consistent and to assume a relatively small fraction of backward-looking or rule-of-thumb price setters—say 0.3—one must motivate a relative variance that is close to zero. As will be shown below, such an estimate appears to be strongly at odds with the data.

Figure 2 illustrates the dominance of λ_s—and thus μ—in determining the first autocorrelation (see equations [9] and [11] above). The figure plots the stable root along with the first autocorrelation as μ rises from 0 to 0.65. The stable root rises from about 0.5 to almost 0.9 as μ varies from 0.35 to 0.5. Correspondingly, the first autocorrelation of inflation rises from about 0.55 to 0.85 over this range. From this figure, it is clear that μ is the critical determinant of the autocorrelation properties of inflation in the HNKPC and that small variations in μ will imply significant differences in the model's implications for the autocorrelation of inflation.

[16]Of course, with $\sigma_e^2 = 0$ and $\mu = 0$, the first autocorrelation is ρ.

3.1 How Much "Hybrid" Do We Need in the NKPC To Be Roughly Data Consistent?

Do commonly employed estimates of μ and γ, in conjunction with a data-consistent process for the driving variable, imply a data-consistent amount of persistence for inflation? Of course, it is difficult to know what the data imply about inherited versus intrinsic persistence—this requires structural identifying restrictions. But the reduced-form persistence of inflation is relatively simple to compute and provides a useful benchmark against which to judge the implications of the structural hybrid NKPC.

We begin with full-sample estimates of a simple three-variable vector autoregression in the inflation rate, the federal funds rate, and real marginal cost.[17] The full sample extends from 1966:Q1 to 2003:Q4. The autocorrelation for inflation that is implied by the VAR is derived in the same manner as described above for generic linear rational expectations models. Confidence intervals of 70 percent and 90 percent are displayed for the VAR's autocorrelation function, where the confidence intervals are computed by assuming that the vector of OLS estimates of the VAR parameters is drawn from a multivariate normal distribution.

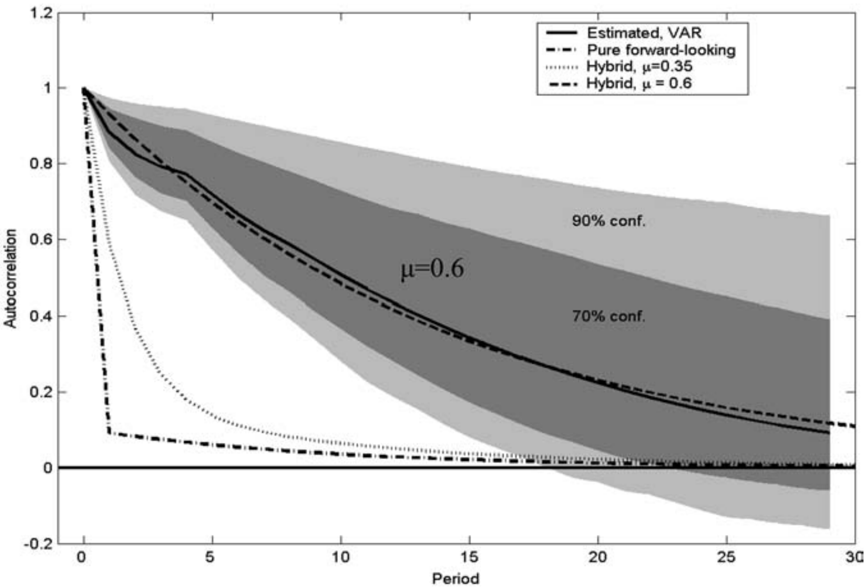
Figure 3 displays the theoretical autocorrelation for inflation implied by the pure and hybrid NKPCs at the parameter values indicated. As the figure suggests, at a somewhat generous estimate of $\gamma = 0.03$, and μ at the estimate for the United States developed in Galí, Gertler, and López-Salido (2001) ($\mu = 0$ for the pure NKPC), the implied autocorrelation for inflation lies outside the 90 percent confidence interval of the VAR's inflation autocorrelation for the first fifteen quarters, at which point the theoretical autocorrelation is essentially zero, and the VAR-based autocorrelation is insignificantly different from zero.[18] The heavy dashed line shows the implied

[17] See appendix 2 for variable definitions. Note that the inflation autocorrelations computed directly from the inflation data imply nearly identical patterns as those in the VAR, both in the full sample and in the post-1983 sample.

[18] Using a somewhat different methodology, Rudd and Whelan (2005) develop estimates of γ that are often an order of magnitude smaller than this.

A similar comparison that sets the relative variance of inflation to 1 produces essentially the same result. While the theoretical autocorrelations are shifted upward somewhat, they still lie completely outside the 90 percent confidence

Figure 3. Comparison of Theoretical ACFs with VAR (ulc), Full Sample (1966–2003) $\rho = 0.9, \gamma = 0.03, \mu = 0.35, \sigma_e = 3$



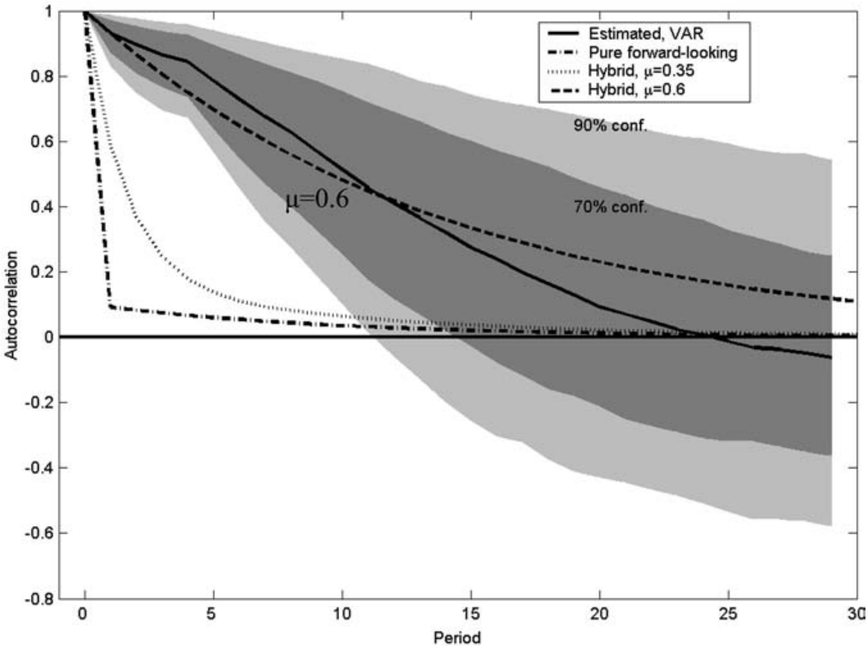
inflation autocorrelation with μ raised to 0.6. This parameter setting puts the theoretical hybrid autocorrelation in the middle of the distribution of estimates from the VAR.

As discussed below, it may be that the simple three-variable VAR misrepresents both the variance and autocorrelation of inflation, as it excludes the effects of large relative price movements for energy and non-oil imported goods. Figure 4 displays the same exercise for a five-variable VAR that includes the relative price of oil and the relative price of imported goods (again, see appendix 2 for details).

The inclusion of these variables does little to change the basic contours of the inflation autocorrelation, although the autocorrelations decay a bit more quickly toward zero in the five-variable VAR. Still, the qualitative conclusion remains: the pure and hybrid

intervals for the VAR. Raising γ by a factor of four (converting the highest estimates in Galí and Gertler 1999 to an annualized basis) similarly shifts the autocorrelations up, but they still lie outside the 90 percent confidence intervals.

Figure 4. Comparison of Theoretical ACFs with Five-Variable VAR (ulc), Full Sample (1966–2003) $\rho = 0.9, \gamma = 0.03, \mu = 0.6, \sigma_e = 3$

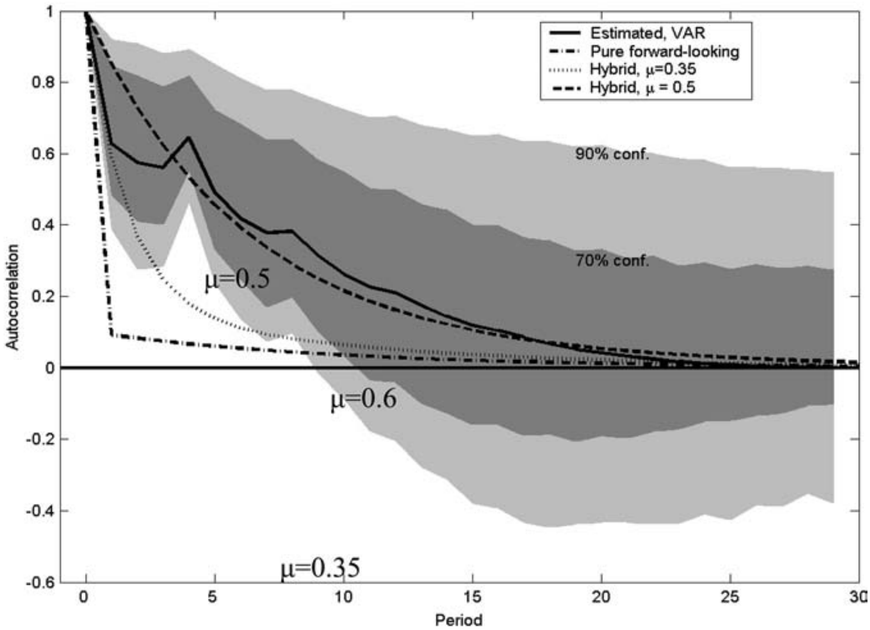


versions of the NKPC are unable to match the VAR's implications for the autocorrelation of inflation.

Recent work by Levin and Piger (2003) and O'Reilly and Whelan (2005) emphasizes the potential for time variation in the intercept for inflation, which may influence estimates of inflation persistence. Figure 5 addresses this concern, again estimating a three-variable VAR, but only over the period since mid-1984, a point that many have identified as a breakpoint for the volatility of macroeconomic time series, including output and inflation.[19] With these somewhat lower autocorrelations and wider confidence intervals, the hybrid model with $\mu = 0.35$ begins to skirt the now-wider 70 and 90 percent confidence intervals around the inflation autocorrelation. Now, a

[19]Choosing the breakpoint differently, say, to correspond to the change in the Chairman of the Board of Governors of the Federal Reserve System in July 1987, makes little difference to the conclusions drawn from the figure.

Figure 5. Comparison of Theoretical ACFs with VAR (ulc) (1984–2003) $\rho = 0.9, \gamma = 0.03, \mu = 0.35, \sigma_e = 3$

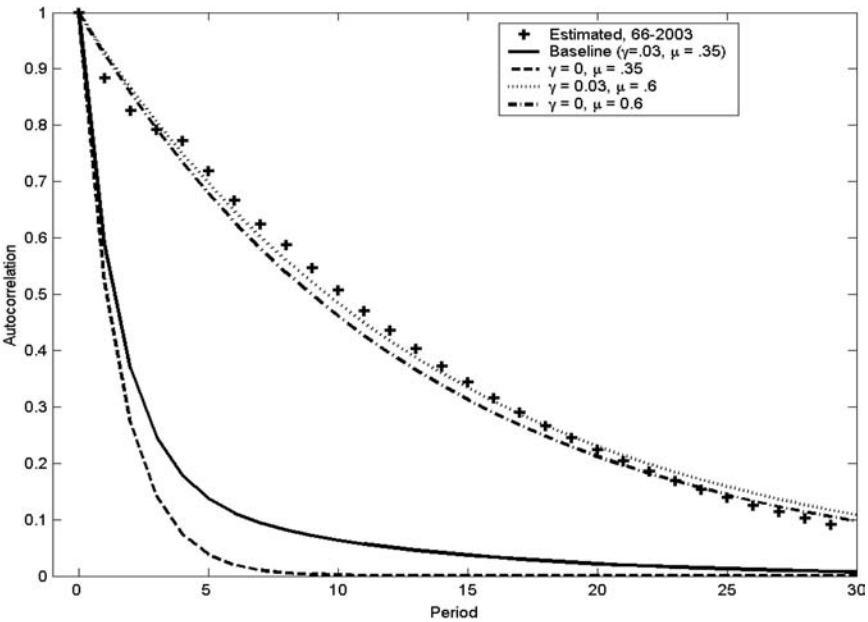


value of $\mu = 0.5$ implies an autocorrelation function squarely in the middle of the distribution of VAR autocorrelations. This computation emphasizes a point made above: the autocorrelation of inflation in the hybrid NKPC is very sensitive to relatively small changes in μ. The difference between $\mu = 0.35$ and $\mu = 0.5$ can move the implied inflation autocorrelation from outside the confidence intervals to the middle of the distribution.

3.2 How Much of the Persistence in the Hybrid Specification Comes from the Driving Variable?

While the analysis above demonstrates that the persistence of inflation in the HNKPC derives mostly from the lagged inflation term, the next exercise calibrates the remaining contribution of the driving variable. Figure 6 displays the theoretical autocorrelation functions for the hybrid model for pairs of parameter values. The pairs of

Figure 6. How Much Persistence from the Driving Variable?



lines highlight the contribution to persistence from the lag of inflation ($\mu = 0.35$, $\mu = 0.6$) versus the driving variable, through both contemporaneous and expected future effects ($\gamma = 0$, $\gamma = 0.03$). Of course, when $\gamma = 0$, the driving variable has no effect on inflation.

As the figure indicates, almost all of the persistence imparted to inflation in the hybrid specification arises from the effects of μ. For parameter values near the baseline chosen from the literature (the solid and dashed lines), the incremental difference between the autocorrelation for zero or nonzero γ is not zero, but it is quite small. For a value of μ that is data consistent (the dotted and dashed-dotted lines), the incremental difference is essentially zero. For a given degree of "backward-looking" behavior in the specification, the incremental addition from including the driving variable's persistence is very small.

3.3 Estimation of the Hybrid Model

A more direct way to compare the properties of the model with the data is to directly estimate the HNKPC specification. Maximum

Table 3. Maximum Likelihood Estimates of Specification (2)

Sample	μ	SE	γ	SE	σ_e^2/σ_u^2
Real Marginal Cost					
1960:4–2003:4	0.64	0.074	0.027	0.014	0.9
1987:3–2003:4	0.72	0.24	0.001	0.058	0.7
1992:1–2003:4	0.71	0.44	0.001	0.048	0.4
1960:4–1987:2	0.61	0.068	0.033	0.018	1.0
Output Gap (CBO Potential)					
1960:4–2003:4	0.87	0.28	0.055	0.050	1.4
1987:3–2003:4	0.67	0.17	0.011	0.028	1.1
1992:1–2003:4	0.62	0.13	0.0046	0.021	0.8
1960:4–1987:2	0.94	0.48	0.070	0.088	1.6

likelihood (ML) has been shown to have some attractive features for this class of Euler equation-based models (see, for example, Fuhrer and Rudebusch 2004 and Fuhrer and Olivei 2004). In this section, both ML and conventional GMM estimates of the specification are presented.

Table 3 displays the ML estimates for the specification in equation (2), using either the output gap or real marginal cost as the driving variable.[20] For this estimation, the sample is constrained to the Greenspan era, 1987:Q3 to the end of the sample. The discount rate is constrained to 0.98, and the remaining parameters are estimated freely. Two estimates of the standard error are presented, the first from the numerical Hessian of the optimization problem, and the second from the BHHH algorithm that uses only first-derivative information (Berndt et al. 1974). The table also displays the ratio of the estimated shock variances.

As the table indicates, ML yields estimates that are consistent with the informal calibrations in figures 3 and 5 above. The ML estimate of μ centers around 0.7 and is precisely estimated. It remains

[20] All data definitions appear in appendix 2. Note that the inflation data are annualized quarterly log changes.

difficult to estimate a significant γ, and the point estimates are generally quite small. Note that the larger estimates correspond to estimates in which μ is 0.8 or larger, dramatically reducing the importance of the forward-looking component of the model. Replicating the exercise in figure 6 around the ML estimates produces virtually identical results. At these estimates, the persistence inherited by inflation from the driving process is essentially nil. Note that for both driving variables, the estimate of ρ (not displayed in the table) is quite high, so in principle inflation could inherit considerable persistence from the driving variable. But in the HNKPC specification, it does not.

Table 4 summarizes GMM estimates for a variety of samples and instrument sets. The instrument sets vary from "bare bones" (three lags of inflation and marginal cost) to the "kitchen sink" (four lags of those two variables, plus an output gap, oil prices, and the federal funds rate). The baseline estimation sample spans the past forty-five years. To examine the stability of the estimates, the table provides results for subsets of those years that split at former Chairman of the Board of Governors Greenspan's term in mid-1987 and more recently.

The estimates of the "forward-looking" and "backward-looking" parameters vary considerably; in other work we address the difficulties in obtaining reliable estimates of these parameters via GMM as conventionally implemented (Fuhrer and Rudebusch 2004; Fuhrer and Olivei 2004). The basic results for estimating γ are similar to those for ML: in no case is the estimated parameter on real marginal cost significantly different from zero. Only one estimate exceeds 0.03, and in general the estimates center on about 0.005 for this annualized-change inflation data. Two of the J-tests reject at conventional levels of significance for instrument set 1. Lagging this instrument set one additional period raises the p-value for the J-statistic to .05 or above, leaving the parameter estimates and significance essentially unaffected. Thus it seems difficult to attribute the general result of a very small estimated γ to inadequate exogeneity of the instruments.[21] A small estimated γ, generally 0.01

[21]These estimates were run in Eviews version 5.0, using a fixed four-quarter Bartlett kernel, no prewhitening, and simultaneous iteration of the parameter estimates and the weight matrix. A constant is included in each instrument list.

Table 4. GMM Estimates of Hybrid Specification

Annualized Inflation Data

$\pi_t = \mu\pi_{t-1} + (\beta - \mu)E_t\pi_{t+1} + \gamma y_t$

Estimation Period	Instrument Set	Estimated μ	Estimated $(\beta - \mu)$	Estimated γ	p-value of t-statistic for γ	p-value of J-statistic
1960:4–2003:4	1	0.61	0.38	0.0052	0.84	0.27
1960:4–2003:4	2	0.62	0.37	0.0081	0.79	0.48
1960:4–2003:4	3	0.48	0.52	−0.0015	0.95	0.73
1960:4–2003:4	4	0.52	0.48	0.000	0.99	0.66
1987:3–2003:4	1	−0.0044	1.002	0.020	0.55	0.013
1987:3–2003:4	2	0.87	0.14	0.011	0.71	0.55
1987:3–2003:4	3	0.17	0.83	0.0092	0.74	0.71
1987:3–2003:4	4	0.66	0.34	0.0058	0.79	0.53
1992:1–2003:4	1	0.059	0.94	0.018	0.64	0.015
1992:1–2003:4	2	0.73	0.27	0.0086	0.77	0.81
1992:1–2003:4	3	0.34	0.67	0.032	0.27	0.75
1992:1–2003:4	4	0.45	0.54	−0.0015	0.95	0.62
1960:4–1987:2	1	0.58	0.42	−0.0035	0.93	0.16
1960:4–1987:2	2	0.59	0.40	0.0026	0.95	0.38
1960:4–1987:2	3	0.49	0.51	−0.0098	0.80	0.73
1960:4–1987:2	4	0.54	0.46	−0.013	0.74	0.66

Instrument sets:
1: Four lags of inflation, real unit labor cost, output gap
2: Three lags of inflation and real unit labor cost
3: Four lags of inflation, federal funds rate, real unit labor cost, relative price of oil
4: Three lags of inflation, federal funds rate, output gap, real unit labor cost, relative price of oil

or smaller on *annualized* inflation rates, is the norm, regardless of estimation method, sample period, or instrument set.

3.4 Autocorrelated Inflation Shocks

Many implementations of the NKPC, especially in fully articulated general equilibrium models such as Christiano, Eichenbaum, and Evans (2005), allow shocks to be autocorrelated, augmenting the behavioral dynamics of the model. This addition would obviously alter the model's implications for the autocorrelation of inflation. Appendix 1 presents the key matrices A and S_0 for the case of the purely forward-looking model augmented with a serially correlated shock e_t. Not surprisingly, in this version of the model, the autocorrelations of inflation depend almost entirely on the size of the autocorrelation coefficient for the inflation shock e_t.

That certain types of inflation shocks—"cost-push" shocks, for example, from large changes in relative prices—might be autocorrelated is not controversial. But how autocorrelated are such shocks, and how much persistence do they contribute to inflation in the United States? We can get a feel for the degree of serial correlation that might plausibly be added to e_t by examining the autocorrelation of the relative oil price and import price series used in the VARs above. Interestingly, the autocorrelation of the change in the relative oil price, both over the full sample and limited to the decade of the 1970s, is essentially zero. The autocorrelation of the change in relative import prices for the same two samples is about 0.5. Adding an autocorrelated shock with relatively low persistence to the pure forward-looking NKPC would not qualitatively change the conclusions about the model.[22]

4. Some Extensions

4.1 Adding Explicit Monetary Policy to the Model

There are good reasons to believe that the persistence of inflation should be affected by the systematic component of monetary

[22]For example, using the autocorrelations derived for the model with equation (22) in appendix 1, at the baseline parameter settings $\gamma = .03$, $\sigma_e^2 = 3$, $\rho = 0.9$, and $a = .5$, the first autocorrelation of inflation is 0.51, a bit lower than the autocorrelation for the hybrid model with $\mu = 0.35$ in figure 3.

policy. For example, Fuhrer and Moore (1995b) show that in a data-consistent, forward-looking model, policy rules that respond more or less aggressively to inflation and output imply corresponding changes in the persistence of output and inflation. Could the addition of inertial interest-rate policy save the purely forward-looking NKPC?

In the models examined below, changes in the systematic component of monetary policy *do* alter the properties of the driving process and of inflation. But the intuition from the discussion above remains: monetary policy affects inflation in this model through its effect on the current and expected values of the driving variable. While more inertial or aggressive monetary policy generally alters the persistence of the driving process, in the purely forward-looking model or in the hybrid model with modest μ, inflation is relatively unaffected by these changes.

To demonstrate this result, a simple inertial policy rule is added to the model without lagged inflation:

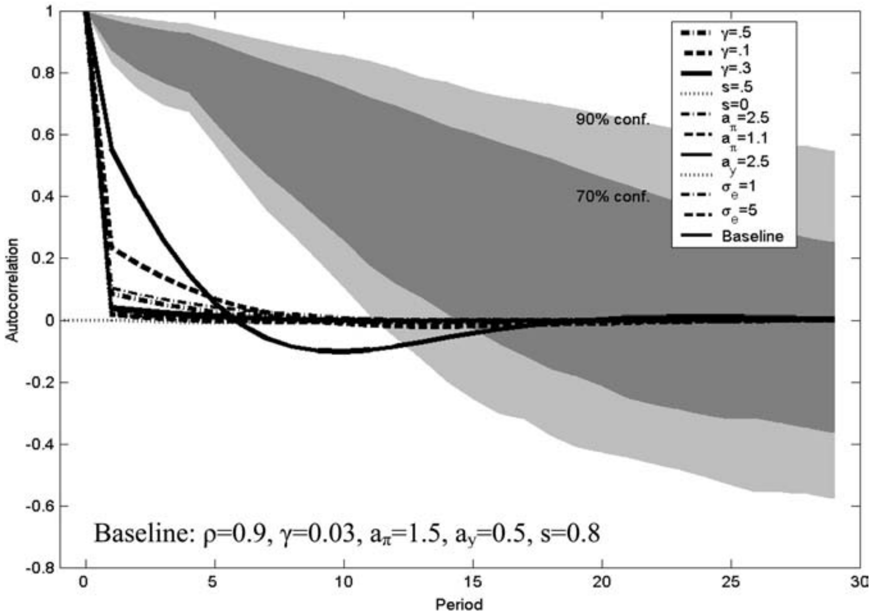
$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \gamma y_t + e_t \\ y_t &= \rho y_{t-1} - a(i_{t-1} - \pi_t) + u_t \\ i_t &= s i_{t-1} + (1-s)(a_\pi \pi_t + a_y y_t). \end{aligned} \tag{12}$$

Can the addition of inertial monetary policy qualitatively change the conclusions about inherited versus intrinsic persistence in inflation?

While the algebra becomes more tedious, numerical examples serve to illustrate the point well. As figures 7 and 8 demonstrate, without significant intrinsic persistence in the inflation process, the presence of inertial monetary policy does little to change the implications from the simpler model without monetary policy. Regardless of the size of γ, σ_e^2, or s, or the vigor with which monetary policy responds to inflation and output, and thus regardless of the persistence of y, inflation inherits quite little of the persistence of the driving process. When compared with the persistence implied by the full-sample VAR, the autocorrelations fall well outside the 90 percent confidence interval.

Figure 8 displays the results for the hybrid model. With relatively limited intrinsic persistence, the hybrid model cannot replicate the autocorrelation properties of inflation. Only setting $\mu = 0.6$ (the lighter dashed line) puts the autocorrelation into the confidence region for the full-sample VAR autocorrelation function. If we

Figure 7. Inflation Autocorrelation with Endogenous Monetary Policy: Forward-Looking Model, Various Parameter Settings

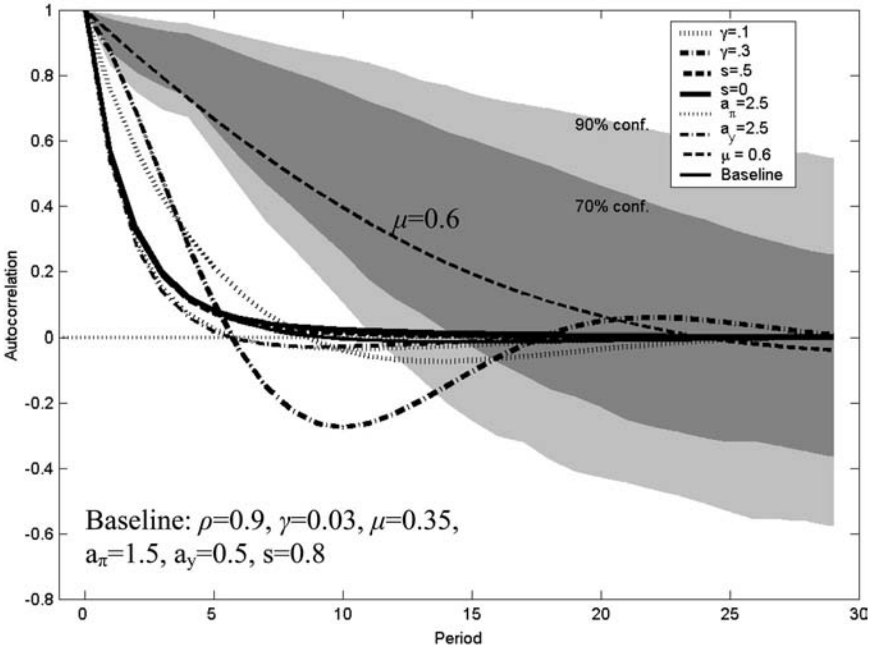


perform the same comparison for a VAR estimated beginning in 1984 (not shown), some of the cases lie between the 70th percentile and 90th percentile of the distribution. But qualitatively, the results are the same. A data-consistent representation of inflation, even with inertial monetary policy, requires a significant weight on lagged inflation.

4.2 Has the Persistence of the Driving Variables Changed over the Past Four Decades?

Figure 5 suggests that the estimated persistence of inflation may have declined over the past two decades. If so, is this the result of a decline in the persistence in the driving variable, which could be in turn the result of a change in monetary policy (or a change in any other factor that influences the reduced-form persistence of output or marginal cost)?

Figure 8. Inflation Autocorrelation with Endogenous Monetary Policy: Hybrid Model, Various Parameter Settings

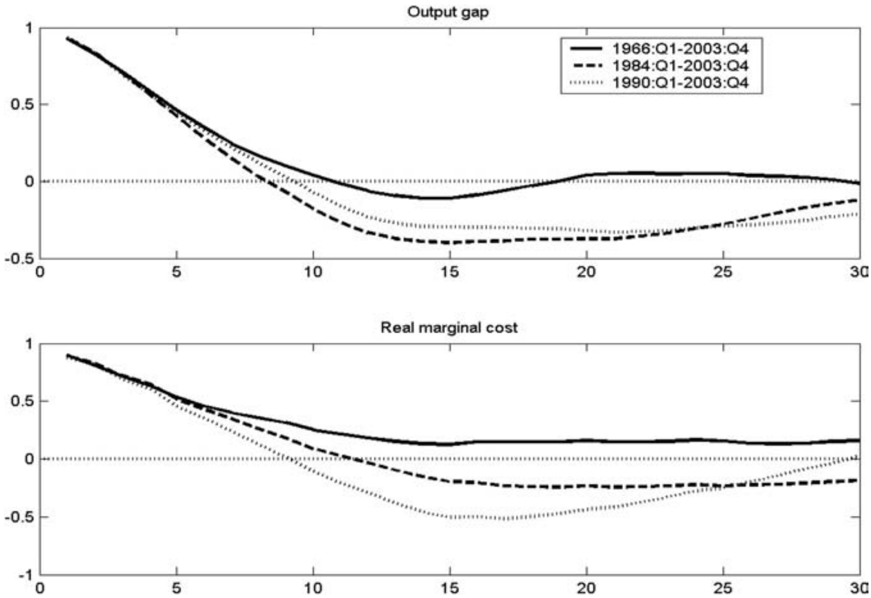


In short, the answer is no. Figure 9 displays a crude measure of persistence, the sample autocorrelations of an output gap, and a unit labor cost measure for three subsamples. While volatility of inflation and output have declined (as documented by many, including McConnell and Perez-Quiros 2000), the persistence of the key driving variables for the Phillips curve has remained just as it was in earlier decades.[23] This observation suggests that one must look not to monetary policy or other changes in the driving process, but to changes to the intrinsic persistence in inflation to explain recent declines in inflation persistence.[24]

[23]Of course, these sample autocorrelations implicitly allow for changes in the intercept of the series at the indicated breakpoints. This has been a significant element of the debate over the possibility of changes in inflation persistence in recent data.

[24]The sum of the lag coefficients in a univariate autoregression for these series varies from 0.89 to 0.91 for the three samples indicated.

Figure 9. Sample Autocorrelations for Output Gap and Unit Labor Cost, Various Subsamples



A slightly more sophisticated test of change in persistence may be obtained by performing an unknown (multiple) breakpoint test, using the methodology of Bai (1999). The test regression is

$$\Delta y_t = \alpha y_{t-1} + \sum_{i=1}^{k} \beta_i \Delta y_{t-i} + e_t, \tag{13}$$

where α is an estimate of minus one plus the sum of the lag coefficients in the univariate autoregression for y. The full sample begins in 1966:Q1 and ends in 2003:Q4. The smallest admissible subsample is set to 10 percent of the sample size, and the critical value for rejection of n breaks in favor of $n-1$ breaks is 0.05. A value of $k = 2$ appears to be sufficient for these two series, although the results do not depend importantly on the choice of k. The test for the output gap cannot reject a single break in favor of no breaks. The test for real unit labor cost finds a single break in 2000:Q1, at which point α is found to have increased from -0.08 to -0.04; that is, the sum of the lag coefficients for the level of unit labor costs has increased from 0.92 to 0.96, so that the persistence of real unit labor cost has *increased.*

The same test performed on the inflation series used in this paper develops two breakpoints, one in 1972:Q4 and one in 1981:Q1. The estimated value of α rises from 0.49 in the pre-1972 period to 0.69 in the 1972–81 period and to 0.77 in the post-1980 period. These estimates of persistence are lower than the full-sample estimate (0.93), perhaps because of shifts in the intercept, as suggested by the authors cited above.[25]

5. Empirical Evidence on the Size of the "Inflation Shock"

In this section, we examine empirical evidence bearing on the size of the shock to the inflation process. Properties of both unconstrained and constrained models of inflation are examined to explore further the source of this barrier to inheriting persistence in the NKPC.

5.1 What Is the Shock to Inflation?

In the model in which the driving variable is real marginal cost, many candidate interpretations of the shock—"supply shocks" such as large changes in the relative price of oil or non-oil imported goods, or shocks to trend productivity that shift the supply relation, or "markup" shocks of price over unit labor cost—are ruled out, as these are incorporated in the measure of marginal cost and thus should appear as part of the shocks in the driving process.[26] Such shocks may well be autocorrelated, but because they perturb only the driving process, they would still constitute a source of inherited, not intrinsic, persistence. There is no doubt that some measurement

[25]Note that an alternative interpretation of these results is that the lack of correspondence between changes in the persistence of inflation and the persistence of the driving process could mean that the NKPC model fits the recent data better than it does the data for the 1960s to 1980s.

[26]Shocks to the desired markup, which enters as an element in the nonlinear combination of parameters that premultiplies real marginal cost in the fully articulated NKPC, would show up as shocks to the inflation equation.

error distorts the measures of real marginal cost commonly used in the specification; if such error were autocorrelated, this would appear in the inflation shock. If the inflation shock were fairly small, measurement error might be a reasonable interpretation. The next section examines the size of the inflation shock.

5.2 How Big Is the Variance of the Shock to the Inflation Process?

Central to the discussion above about how much of inflation's persistence is inherited versus intrinsic is the size of σ_e^2, the variance of the inflation shock. Two approaches are used to measure the relative size of σ_e^2. The first looks at estimated variances from simple VARs, computing relative variances for the reduced-form errors. Of course, because of the well-known difficulties in associating reduced-form VAR errors with any underlying structural disturbance, this should only be done with some trepidation. Interestingly, the reduced-form errors are approximately orthogonal. This reduces somewhat the concern that the shock in the VAR's inflation equation is a linear combination of other underlying shocks.

The second approach employs the three structural models of inflation from sections 2, 3, and 4 (the NKPC, the HNKPC, and the HNKPC with explicit monetary policy, equations [1], [2], and [12], respectively). The U.S. data described in appendix 2 are used, solving each of the models for the structural (or pseudostructural) shocks for a variety of parameter values. Then, the ratio of the variance of the inflation shock to that of the driving variable is computed for each case. It is important to note that, in the first two cases, the identification of the driving process is suspect, as the simple AR(1) process likely serves as a reduced form for a more fully articulated aggregate demand relation and monetary policy rule. Only in the case of the HNKPC with explicit monetary policy can one claim to have identified underlying structural shocks.

As table 5 indicates, it is rather uncommon for the variance of inflation to be less than that of its driving process. For the VARs, the variance is about twice as large on average as the variance of the driving process. This finding is relatively invariant to the set of conditioning variables in the VAR. One might have assumed that partialling

Table 5. How Big Is the Inflation Shock?

VAR Specifications
Ratio of Reduced-Form Inflation Shock to Driving Variable Shock

Estimation Range	π, r, y	π, r, ulc	π, r, y, p^o,p^{no}	π, r, ulc, p^o,p^{no}
1966:Q1–2003	1.98	2.39	1.66	1.99
1984:Q3–2003	5.45	1.16	4.34	1.07

Structural Models
Ratio of Identified Inflation Shock to Driving Variable Shock

Parameter Sets	Pure Forward-Looking Model	Hybrid Model	Hybrid with Monetary Policy
1	7.41	1.08	1.17
2	8.67	0.68	0.59
3	8.93	1.31	1.22
4	4.55	3.68	1.04
5	–	6.16	–
6	–	16.20	–

Parameter Sets

	β	γ	μ	Γ	μ	γ	s
1	.98	.03	0.35	.03	0.35	.03	.8
2	.98	.01	0.50	.03	0.50	.03	.8
3	.50	.03	0.35	.01	0.35	.01	.8
4	.98	.03	0.15	.03	0.35	.03	.0
5			0.50	.30			
6			0.50	.50			

out the variation that arises from oil or non-oil import prices might significantly reduce the variance of the shock to inflation, but this is not the case.

In the "pure" NKPC model, the estimated variance is five to nine times greater than the driving process,[27] depending on the parameter values chosen. The hybrid model reduces the relative variance, as the presence of lagged inflation absorbs much of the autocorrelation that remains in the "pure" model's errors. Still, the variance of the identified inflation shock is on average about as large as that of the driving process. Adding explicit monetary policy leaves this conclusion unchanged.

While it is difficult to put a compelling economic interpretation on this shock, it is nearly impossible to relegate it to a small nuisance, perhaps attributable to the measurement error that no doubt plagues the standard proxy for real marginal cost. If the estimates above are of the right order of magnitude, there would have to be at least as large a variation in the measurement error as there is in the shock to the driving process. That seems implausible. Consequently, it appears that the "inflation shock" is central to the inflation process and central to the debate over how much of inflation's persistence is inherited versus intrinsic. What the inflation shock *is* remains an important challenge for inflation modeling.

6. Conclusions

Finding a data-consistent, optimizing, rational expectations model of price setting has been an important goal in macroeconomics for decades. An emerging consensus suggests that the New Keynesian Phillips curve, augmented by modest frictions of one flavor or another, is a good benchmark model for price setting in dynamic stochastic general equilibrium (DSGE) models usable for macroeconomic analysis. When the driving process is assumed to be real marginal cost, the parameter on the driving process can be estimated with the correct sign, and, in principle, inflation should inherit considerable persistence from this variable.

[27] Of course, some of this blowup in variance arises from the significant autocorrelation left in the inflation shock for this model.

This paper reaches conclusions that differ markedly from the prevailing wisdom. It suggests that:

- Using conventional parameter estimates, inflation in the hybrid NKPC inherits relatively little persistence from the driving process.
- In part, this lack of inherited persistence derives from the presence of a large inflation shock whose variance is typically between one-half and three times as large as the shock that perturbs the driving process.
- The lack of inherited persistence also derives from a rather small estimated coefficient on the driving process.
- The predominant source of inflation persistence in the NKPC is the lagged inflation term. The amount of persistence imparted by the lag is quite sensitive to the size of the lag, with significant differences in persistence implied by an increase in μ from 0.3 to 0.6.
- As several papers have noted, the persistence of inflation appears to have declined in recent years. If that is true, this paper suggests that the reason for that decline in persistence is unlikely to be related to a decline in the persistence of the driving process.[28] First, the standard candidates for the driving process have nearly the same persistence today as they did two decades ago. Second, to a first approximation, the NKPC as conventionally implemented does not allow important changes in the persistence of the driving process to affect the persistence of inflation.
- Because monetary policy in the standard models acts through its effect on output and marginal cost, it becomes more difficult to attribute recent changes in inflation persistence to changes in monetary policy. This does not necessarily imply that monetary policy has had no such effects, but it does suggest that the current crop of models will have difficulty in attributing such changes to monetary policy.

[28]There is considerable debate surrounding this observation, much of it methodological. Recent discussions at the European Central Bank's Inflation Persistence Network conference highlight the issues. See especially Session I at www.ecb.de/events/conferences/html/inflationpersistence.en.html.

These conclusions have other important implications for price modeling in DSGE models. They suggest that the optimizing foundations in the standard specifications are nearly unrelated to the dynamics observed in the data for inflation and real marginal cost. That is, lagged inflation is not a second-order add-on to the optimizing model; it *is* the model. One may motivate price-setting behavior from these optimizing foundations, but in practice, they tell us little about why inflation behaves the way it does.

The conclusions also imply that in order to understand inflation dynamics, we will need to identify the economic source of the large inflation shock in the specification. In turn, the findings in this paper imply either that this identified shock is itself highly autocorrelated, or that we require a microfounded mechanism that generates substantial intrinsic persistence in inflation.

Appendix 1. Algebraic Derivations

The Purely Forward-Looking Model

For the NKPC with $\mu = 0$, the matrix A in equation (3) takes a particularly simple form. It is the coefficient matrix in the reduced-form solution to the model, which may be expressed as[29]

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0 & \frac{\rho\gamma}{(1-\rho\beta)} \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix}. \tag{14}$$

We can use this solution to substitute for $E_t\pi_{t+1}$ in equation (2) to obtain the matrix S, which has partitions S_0 (the contemporaneous block) and S_1 (the lagged block):

$$S_0 \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = S_1 \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} e_t \\ u_t \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{-\gamma}{1-\rho\beta} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} e_t \\ u_t \end{bmatrix}. \tag{15}$$

Under the assumption that the covariance matrix of the errors is diagonal, and normalizing the variance of u to 1 and denoting the

[29]See Anderson and Moore (1985) for a derivation of the solution coefficient matrix.

variance of e by σ_e^2, we can derive the unconditional variance for the vector process as

$$V\begin{bmatrix}\pi_t\\ y_t\end{bmatrix}=\begin{bmatrix}\frac{\gamma^2}{(1-\rho\beta)^2(1-\rho^2)}+\sigma_e^2 & \frac{\gamma}{(1-\rho\beta)(1-\rho^2)}\\ \frac{\gamma}{(1-\rho\beta)(1-\rho^2)} & \frac{1}{1-\rho^2}\end{bmatrix}. \tag{16}$$

Then the autocovariances C_i and autocorrelations Γ_i may be derived from the recursive equations

$$\begin{aligned}C_i &= AC_{i-1}\\ \Gamma_i(j,k) &= \frac{C_i(j,k)}{\sqrt{V(j)V(k)}},\end{aligned} \tag{17}$$

where C_0 is initialized as V.

The Hybrid Model

Now the matrix A from the reduced-form perfect-foresight solution to the model may be expressed as

$$\begin{bmatrix}\pi_t\\ y_t\end{bmatrix}=\begin{bmatrix}\lambda_s & \frac{\gamma\rho}{(\beta-\mu)(\lambda_{\mathrm{b}}-\rho)}\\ 0 & \rho\end{bmatrix}\begin{bmatrix}\pi_{t-1}\\ y_{t-1}\end{bmatrix}, \tag{18}$$

where λ_s and λ_b are the "small" and "big" roots (or stable and explosive, with moduli less than and greater than 1, respectively) of the transition matrix for the model. It is important to note that λ_b and λ_s depend only on β and μ, and are independent of the parameters governing the y_t process or its interaction with π_t.

$$\begin{aligned}\lambda_{\mathrm{s}} &= \frac{1-\sqrt{(1-4\mu\beta+4\mu^2)}}{2(\beta-\mu)}\\ \lambda_{\mathrm{b}} &= \frac{1+\sqrt{(1-4\mu\beta+4\mu^2)}}{2(\beta-\mu)}\\ \lambda_{\mathrm{b}}\lambda_{\mathrm{s}} &= \frac{\mu}{\beta-\mu}\end{aligned} \tag{19}$$

We can use this solution to substitute for $E_t\pi_{t+1}$ in equation (2) to obtain

$$S_0\begin{bmatrix}\pi_t\\ y_t\end{bmatrix} = S_1\begin{bmatrix}\pi_{t-1}\\ y_{t-1}\end{bmatrix} + \begin{bmatrix}e_t\\ u_t\end{bmatrix}$$

$$\begin{bmatrix}\frac{\mu}{\lambda_s} & \frac{-\gamma\lambda_b}{\lambda_b-\rho}\\ 0 & 1\end{bmatrix}\begin{bmatrix}\pi_t\\ y_t\end{bmatrix} = \begin{bmatrix}\mu & 0\\ 0 & \rho\end{bmatrix}\begin{bmatrix}\pi_{t-1}\\ y_{t-1}\end{bmatrix} + \begin{bmatrix}e_t\\ u_t\end{bmatrix}. \tag{20}$$

We can derive the unconditional variance for the vector process as

$$V\begin{bmatrix}\pi_t\\ y_t\end{bmatrix} = \begin{bmatrix}\frac{V_y\frac{\gamma^2}{(\rho-\lambda_b)^2}\left[1+\frac{2\mu\rho}{(\beta-\mu)\lambda_b-\rho\mu}\right]+\frac{\sigma_e^2}{\lambda_b^2}}{(\beta-\mu)^2-\frac{\mu^2}{\lambda_b^2}} & \frac{\gamma\lambda_b}{(\rho-\lambda_b)[\lambda_b(\mu-\beta)+\rho\mu]}V_y\\ \frac{\gamma\lambda_b}{(\rho-\lambda_b)[\lambda_b(\mu-\beta)+\rho\mu]}V_y & \frac{1}{1-\rho^2}\end{bmatrix}, \tag{21}$$

where V_y is $V(y)$, i.e., $\frac{1}{1-\rho^2}$. Then the autocorrelations may be derived as above, using the transition matrix in equation (18).

The Forward-Looking Model with Autocorrelated Errors

The model is augmented to include an "inflation shock" that follows an AR(1) process:

$$\begin{aligned}\pi_t &= (\beta-\mu)E_t\pi_{t+1} + \mu\pi_{t-1} + \gamma y_t + e_t\\ y_t &= \rho y_{t-1} + u_t\\ e_t &= ae_{t-1} + \varepsilon_t.\end{aligned} \tag{22}$$

For this model, the key matrices are

$$A = \begin{bmatrix}0 & \frac{\rho\gamma}{(1-\rho\beta)} & \frac{a}{1-a\beta}\\ 0 & \rho & 0\\ 0 & 0 & a\end{bmatrix}; S_0^{-1} = \begin{bmatrix}1 & \frac{\gamma}{1-\rho\beta} & \frac{1}{1-a\beta}\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}, \tag{23}$$

and the unconditional variance for the vector process is

$$V = \begin{bmatrix}\frac{\gamma^2\sigma_u^2}{(1-\rho\beta)^2(1-\rho)^2} + \frac{\sigma_e^2}{(1-a\beta)^2(1-a)^2} & \frac{\gamma\sigma_u^2}{(1-\rho\beta)(1-\rho)^2} & \frac{\sigma_e^2}{(1-a\beta)(1-a)^2}\\ \frac{\gamma\sigma_u^2}{(1-\rho\beta)(1-\rho)^2} & \frac{\sigma_u^2}{(1-\rho)^2} & 0\\ \frac{\sigma_e^2}{(1-a\beta)(1-a)^2} & 0 & \frac{\sigma_e^2}{(1-a)^2}\end{bmatrix}, \tag{24}$$

and from these one can derive the autocorrelation function for inflation.[30] The first autocorrelation in this case is

$$\Gamma_1 = \frac{\rho(a^2-1)(1-a\beta)^2\gamma^2\sigma_u^2 + a(\rho^2-1)(1-\rho\beta)^2\sigma_e^2}{(a^2-1)(1-a\beta)^2\gamma^2\sigma_u^2 + (\rho^2-1)(1-\rho\beta)^2\sigma_e^2}$$
$$= \frac{\rho\delta_1\gamma^2\sigma_u^2 + a\delta_2\sigma_e^2}{\delta_1\gamma^2\sigma_u^2 + \delta_2\sigma_e^2}. \tag{25}$$

As the text in section 3 suggests, the autocorrelations are dominated by a, the autocorrelation parameter on the shock. In essence, this version of the model holds the same implications as the hybrid model: here, the correlation of the shock term does all the work in the model, whereas in the hybrid model, the lagged inflation term plays the same role.

Appendix 2. Variable Definitions

Inflation: 400 times the log change in the GDP chain-type price index.

Output Gap: 100 times the log difference between chain-weighted real GDP and the Congressional Budget Office's estimate of potential GDP.

Real Marginal Cost: Proxied by real unit labor costs, i.e., 100 times nominal unit labor costs (log of nonfarm compensation less the log of nonfarm output per hour) less the log of the implicit price deflator for the nonfarm business sector.

Relative Price of Oil: The log of West Texas intermediate oil price per barrel less the log of the GDP chain-type price index.

Relative Price of Imports: The log of the chain-type import price index less the log of the GDP price index.

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A Bayesian DSGE Model with Infinite-Horizon Learning: Do "Mechanical" Sources of Persistence Become Superfluous?*

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This paper estimates a monetary DSGE model with learning introduced from the primitive assumptions. The model nests infinite-horizon learning and features, such as habit formation in consumption and inflation indexation, that are essential for the model fit under rational expectations. I estimate the DSGE model by Bayesian methods, obtaining estimates of the main learning parameter, the constant gain, jointly with the deep parameters of the economy. The results show that relaxing the assumption of rational expectations in favor of learning may render mechanical sources of persistence superfluous. In particular, learning appears to be a crucial determinant of inflation inertia.

JEL Codes: C11, D84, E30, E50, E52.

1. Introduction

Recent dynamic stochastic general equilibrium (DSGE) models have proved successful in describing macroeconomic data. Smets and Wouters (2003, 2004, 2005) have provided the first example of a structural model that can compete in fit with unrestricted Bayesian VARs. Christiano, Eichenbaum, and Evans (2005), Giannoni and Woodford (2003), and Boivin and Giannoni (2005) have similarly developed models that approximate the impulse responses derived from VARs. The success of these papers stems from extending the

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simplest DSGE setup to include several features that help in generating endogenous persistence in their models. Modern DSGE models have increasingly followed their example. They typically incorporate habit formation in consumption, inflation and wage indexation, capital adjustment costs, and several autocorrelated disturbances. These additional sources of persistence, which we may view as “mechanical,” together with persistent structural shocks are essential for the empirical success of the models.

Milani (2004b), however, shows that allowing for a minimal deviation from the conventional assumption of rational expectations might lead one to reconsider the role of “mechanical” sources of persistence. In a model with subjective expectations and learning, in fact, the estimated degrees of habit formation in consumption and inflation indexation become negligible. Learning also improves the fit of a monetary DSGE model: the model with learning alone is preferred to the corresponding model with rational expectations, habits, and indexation.

Milani (2004b), following most of the adaptive learning literature (Evans and Honkapohja 2001, Bullard and Mitra 2002, among others), derives the model under rational expectations; he then introduces subjective expectations and learning only on the linearized equations found under rational expectations. But Preston (2005b) argues that introducing learning directly from the primitives of the model would lead to different laws of motion for inflation and output gap. The derived aggregate dynamics of the economy imply, in fact, that long-horizon expectations also matter. Preston (2005a) explains that decision rules that depend only on one-period-ahead expectations will generally not provide optimal decision rules under adaptive learning for the corresponding infinite-horizon decision problems. The problem arises from the use of a different conditional distribution with respect to which expectations are taken. For example, he shows that the Euler equation under one-period-ahead learning would not satisfy the intertemporal budget constraint and, therefore, will lead to suboptimal decisions.

In this paper, I follow Preston’s approach and build the model assuming subjective expectations from the primitives. I generalize Preston’s framework to allow for habit formation in consumption and inflation indexation in price setting. Since Milani (2004b) shows that inserting learning in an optimizing DSGE model may make typical sources of persistence redundant, it is therefore important to verify if the results also hold when more attention is paid to the microfoundations of the model under learning.

I therefore derive a simple monetary DSGE model that incorporates infinite-horizon learning and mechanical sources of persistence, such as habit formation and inflation indexation. I then estimate the model using Bayesian methods. The paper provides the first estimation in the literature of a DSGE model with infinite-horizon learning. The main learning parameter, the constant gain, is jointly estimated with the "deep" parameters of the economy. Estimation of the constant gain is crucial, for the empirical results often depend on the assumed gain, as shown in Milani (2004a), for example.

I find that infinite-horizon learning can generate substantial persistence in the model. When agents form subjective expectations and learn the relevant parameters, I find that the role of habit formation and indexation becomes smaller. Inflation indexation is superfluous. The persistence in inflation appears to be driven more by learning than by structural features such as indexation. Learning, in fact, substitutes for both indexation and a strong serial correlation in the exogenous cost-push shock. The results are less sharp for habit formation. In this case, the results depend on the assumed persistence in the aggregate demand disturbance. With a large autoregressive coefficient, habit formation becomes redundant. But with a small autoregressive coefficient, the model still needs a sizable coefficient on habits to match the data.

2. A Microfounded Model with Adaptive Learning

I derive the aggregate dynamics of the economy, introducing learning directly from the primitives of the model, as in Preston (2005b). This section generalizes Preston (2005b) by also incorporating habit formation in consumption and inflation indexation in price setting. In the model, agents know (i) their own preferences, (ii) the constraints they face, and (iii) how to solve their optimization problems. But they do not have any knowledge of other agents' preferences. Therefore, they are not able to infer the aggregate probability laws of the variables of interest, as they would be, instead, under rational expectations. To derive optimal decisions, agents need to form expectations about future macroeconomic variables. Here, I depart from the strong informational assumptions required by rational expectations, and I allow agents to form arbitrary subjective expectations.

2.1 Households' Optimal Consumption Decisions

The economy is populated by a continuum of households indexed by $i \in [0, 1]$. Each household i maximizes the expected discounted utility

$$\widehat{E}_t^i \left\{ \sum_{T=t}^{\infty} \beta^{T-t} \left[U\left(C_T^i - \eta C_{T-1}^i; \zeta_T\right) - \int_0^1 v\left(h_T^i(j); \zeta_T\right) dj \right] \right\}, \quad (1)$$

where $\widehat{E}_t^i$ indicates subjective expectations for household i. Households derive utility from the deviation of current consumption C_T^i from a stock of internal habits in consumption ηC_{T-1}^i, and they derive disutility from the hours of labor supplied $h_T^i(j)$. An aggregate shock ζ_T may affect the consumption-leisure decision in each period. The coefficient $0 < \beta < 1$ denotes the usual discount factor, while η measures the degree of habit formation in consumption. The consumption index C_T^i is the Dixit-Stiglitz CES aggregator of different goods, so that $C_t^i \equiv \left[\int_0^1 c_t^i(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$, and the associated price index is $P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$, where θ is the elasticity of substitution between differentiated goods. For simplicity, I assume homogeneous beliefs across agents (although this is *not known* to agents, who do not have any information about other agents' beliefs). As standard in the adaptive learning literature, the subjective expectations of individual agents obey the law of iterated expectations, $\widehat{E}_t^i \widehat{E}_{t+s}^i z = \widehat{E}_t^i z$ for any variable z.

I follow Preston (2005b) in assuming *incomplete* asset markets.[1] Agents can use a single one-period riskless asset to transfer wealth intertemporally. The flow budget constraint is given by

$$M_t^i + B_t^i \leq \left(1 + i_{t-1}^m\right) M_{t-1}^i + (1 + i_{t-1}) B_{t-1}^i + P_t Y_t^i - T_t - P_t C_t^i, \quad (2)$$

where M_t^i denotes end-of-period money holdings, B_t^i denotes end-of-period riskless bond holdings, i_t^m and i_t denote nominal interest rates

[1] This assumption limits the extent of information revelation from prices.

on money and bonds, and T_t denotes lump-sum taxes and transfers. Y_t^i is household's real income in period t, given by $\int_0^1 \left[w_t(j)h_t^i(j) + \Pi_t(j)\right] dj$, where $w_t(j)$ represents the wage received by the household for labor supplied in the production of good j and $\Pi_t(j)$ is the share of profits received from the sale of each firm's good j (households own an equal share of all the firms).

The intertemporal budget constraint (IBC) is

$$\widehat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} C_T^i = \omega_t^i + \widehat{E}_t^i \sum_{T=t}^{\infty} \beta^{T-t} Y_T^i, \tag{3}$$

where $\omega_t^i \equiv \frac{W_t^i}{P_t \overline{Y}}$ is the share of nominal wealth $(W_t^i \equiv (1+i_{t-1})B_{t-1}^i)$ as a fraction of nominal steady-state income. With habit formation, the first-order conditions become

$$\lambda_t^i = U_c\left(C_T^i - \eta C_{T-1}^i; \zeta_T\right) - \beta\eta \widehat{E}_t^i \left[U_c\left(C_{T+1}^i - \eta C_T^i; \zeta_{T+1}\right)\right] \tag{4}$$

$$\lambda_t^i = \beta \widehat{E}_t^i \left[\lambda_{t+1}^i (1+i_t) P_t / P_{t+1}\right], \tag{5}$$

where λ_t^i is the marginal utility of real income in period t. Substituting (4) into (5), and taking a log-linear approximation of the implied Euler equation, I obtain

$$\widetilde{C}_t^i = \widehat{E}_t^i \widetilde{C}_{t+1}^i - (1-\beta\eta)\sigma\left(i_t - \widehat{E}_t^i \pi_{t+1}\right) + g_t - \widehat{E}_t^i g_{t+1}, \tag{6}$$

where

$$\widetilde{C}_t^i = C_t^i - \eta C_{t-1}^i - \beta\eta \widehat{E}_t^i \left[C_{t+1}^i - \eta C_t^i\right], \tag{7}$$

and where $\sigma \equiv \frac{-U_c}{(\overline{C} U_{cc})} > 0$ represents the elasticity of intertemporal substitution of consumption in the absence of habit formation, and $g_t \equiv \frac{\sigma U_{c\zeta} \zeta_t}{U_c}$ is a preference shock. Solving (6) backwards, taking expectations, substituting into the modified IBC,[2] using $C_t = Y_t$,

[2]The modified IBC is found by substituting $C_t^i = \widetilde{C}_t^i + \eta C_{t-1}^i + \beta\eta \widehat{E}_t^i \left[C_{t+1}^i - \eta C_t^i\right]$ into the IBC.

and expressing everything in terms of the output gap $x_t \equiv Y_t - Y_t^n$ yields the aggregate demand equation[3]

$$\tilde{x}_t = \widehat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \big[(1-\beta)\tilde{x}_{T+1} - (1-\eta\beta)\sigma\big(i_T - \pi_{T+1} - r_T^n\big)\big], \quad (8)$$

where

$$\tilde{x}_t \equiv (x_t - \eta x_{t-1}) - \beta\eta \widehat{E}_t(x_{t+1} - \eta x_t),$$

and where Y_t^n is the natural rate of output (the equilibrium level of output under flexible prices) and $r_T^n \equiv \big[(1-\eta\beta)\sigma\big]^{-1}\big[\big(Y_{t+1}^n - g_{t+1}\big) - \big(Y_t^n - g_t\big)\big]$ is the flexible-price equilibrium real interest rate. Current output gap, therefore, depends on lagged and expected one-period-ahead output gap, on the ex ante real interest rate, and on long-horizon forecasts of future output gaps, real interest rates, and disturbances until the indefinite future.

2.2 Firms' Problem

I assume Calvo price setting. A fraction $0 < 1 - \alpha < 1$ of firms can set prices optimally in a given period t. The remaining α firms that are not allowed to optimize in t can still adjust their prices following the indexation rule proposed by Christiano, Eichenbaum, and Evans (2005):

$$\log p_t(i) = \log p_{t-1}(i) + \gamma \pi_{t-1}, \quad (9)$$

where the parameter $0 \leq \gamma \leq 1$ measures the degree of indexation to past inflation. The aggregate price index P_t evolves according to

$$P_t = \left[\alpha\left(P_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma}\right)^{1-\theta} + (1-\alpha)p_t^{*1-\theta}\right]^{\frac{1}{1-\theta}}. \quad (10)$$

[3]In the derivation, I also use $\int_i \omega_t^i di = 0$ from the bond market-clearing condition, and I integrate over the i households, using $C_t = \int_i C_t^i di$, $Y_t = \int_i Y_t^i di$ and $\widehat{E}_t[\cdot] \equiv \int_i \widehat{E}_t^i[\cdot] di$, which denotes average private-sector expectations.

Each firm i maximizes the expected present discounted value of future profits $\Pi_T^i(\cdot)$

$$\widehat{E}_t^i \left\{ \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left[\Pi_T^i \left(p_t^*(i) \left(\frac{P_{T-1}}{P_{t-1}} \right)^{\gamma} \right) \right] \right\}, \tag{11}$$

where a unit of income in date T is valued by the stochastic discount factor $Q_{t,T} = \beta^{T-t} \frac{P_t}{P_T} \frac{\lambda_T}{\lambda_t}$.

The first-order conditions for the problem are

$$\widehat{E}_t^i \left\{ \begin{array}{c} \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T Y_T P_T^{\theta} \left(\frac{P_{T-1}}{P_{t-1}} \right)^{\gamma(1-\theta)} \cdot \\ \left[p_t^*(i) - \mu P_T s \left(Y_T \left(\frac{\hat{p}_t^*(i)}{P_T} \right)^{-\theta} \left(\frac{P_{T-1}}{P_{t-1}} \right)^{-\gamma\theta}, Y_T; \tilde{\zeta}_T \right) \right] \end{array} \right\} = 0, \tag{12}$$

where $\mu = \theta/(\theta-1) > 1$, $\tilde{\zeta}_t$ is a vector of exogenous real disturbances incorporating both preference shocks ζ_t and technology shocks A_t, and where $s(\cdot)$ is firm i's real marginal cost function.

Log-linearization of the first-order condition yields

$$\begin{aligned} \hat{p}_t^*(i) = \widehat{E}_t^i \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \\ \times \left[\frac{1-\alpha\beta}{1+\omega\theta} \left(\omega Y_T - \lambda_T^i + \frac{v_{y\zeta}}{v_y} \zeta_T \right) + \alpha\beta(\pi_{T+1} - \gamma\pi_T) \right], \end{aligned} \tag{13}$$

where $\hat{p}_t^* \equiv \log\left(p_t^*/P_t\right)$ and $\omega \equiv v_{yy}\overline{Y}/v_y$ is the elasticity of the marginal disutility of producing output with respect to an increase in output.

From a log-linear approximation of the aggregate price index and integrating over the i firms, I can derive the aggregate supply relation

$$\begin{aligned} \tilde{\pi}_t = \xi_p\big(\omega x_t + [(1-\eta\beta)\sigma]^{-1}\tilde{x}_t\big) + \widehat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \\ \times \left[\alpha\beta\xi_p\big(\omega x_{T+1} + [(1-\eta\beta)\sigma]^{-1}\tilde{x}_{T+1}\big) + (1-\alpha)\beta\tilde{\pi}_{T+1} + u_T\right], \end{aligned} \tag{14}$$

where

$$\tilde{\pi}_t \equiv \pi_t - \gamma\pi_{t-1}$$
$$\tilde{x}_t \equiv (x_t - \eta x_{t-1}) - \beta\eta\widehat{E}(x_{t+1} - \eta x_t)$$
$$\xi_p = \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\omega\theta)}.$$

Current inflation therefore depends on lagged inflation; on current, lagged, and one-period-ahead output gaps; and on the long-horizon forecasts of future output gaps, inflation rates, and supply shocks. Deviations of the empirical output gap from the theoretically relevant gap will show up in the supply shock u_t.

2.3 Monetary Authority

I assume that the following Taylor rule with partial adjustment describes monetary policy in this economy:

$$i_t = \rho i_{t-1} + (1-\rho)[\chi_\pi \pi_t + \chi_x x_t] + \varepsilon_t,$$

where ρ denotes the degree of interest-rate smoothing, ψ_π and ψ_x are feedback coefficients, and ε_t accounts for unanticipated deviations from systematic monetary policy.

3. Infinite-Horizon Learning

With learning introduced as in Preston (2005a, 2005b), long-horizon expectations also matter. In the previous section, I have generalized Preston's framework to include habit formation and indexation. The model economy can be summarized as

$$\tilde{x}_t = \widehat{E}_t \sum_{T=t}^{\infty} \beta^{T-t}\big[(1-\beta)\tilde{x}_{T+1} - (1-\eta\beta)\sigma(i_T - \pi_{T+1} - r_T^n)\big] \quad (15)$$

$$\tilde{\pi}_t = \widehat{E}_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t}\big[\xi_p\big(\omega x_T + [(1-\eta\beta)\sigma]^{-1}\tilde{x}_T\big) + (1-\alpha)\beta\tilde{\pi}_{T+1} + u_T\big] \quad (16)$$

$$i_t = \rho i_{t-1} + (1-\rho)[\chi_\pi \pi_t + \chi_x x_t] + \varepsilon_t \tag{17}$$

$$r_t^n = \phi_r r_{t-1}^n + \nu_t^r \tag{18}$$

$$u_t = \phi_u u_{t-1} + \nu_t^u, \tag{19}$$

where $\tilde{x}_t$ and $\tilde{\pi}_t$ have the usual meaning. I have assumed that the disturbances r_t^n and u_t follow autoregressive processes. The shocks ε_t, ν_t^r, ν_t^u are i.i.d. normal with mean 0 and variance-covariance matrix Q.[4]

From (15) and (16), it is clear that economic agents need to form forecasts of macroeconomic variables until the indefinite future. I follow a number of papers in the adaptive learning literature (see Evans and Honkapohja 2001 for a comprehensive treatment) and assume that agents use simple linear economic models to form expectations. The agents have the following perceived law of motion (PLM):

$$Z_t = a_t + b_t Z_{t-1} + c_t r_t^n + d_t u_t + \varepsilon_t, \tag{20}$$

where $Z_t \equiv [\pi_t, x_t, i_t]'$ and a_t, b_t, c_t, d_t are coefficient vectors and matrices of appropriate dimensions. The PLM has the same structural form of the rational expectations solution of the system, i.e., it includes the same regressors that appear in the minimum state variable (MSV) solution under rational expectations. The agents, however, lack knowledge about the parameters of the model. Therefore, they use historical data to learn the parameters over time. As soon as they observe additional data, agents update their estimates of the parameter vector (a_t, b_t, c_t, d_t) through constant-gain learning, as described by the following formulas:

$$\hat{\phi}_t = \hat{\phi}_{t-1} + \bar{\mathbf{g}} R_{t-1}^{-1} X_t \big(Z_t - X_t' \hat{\phi}_{t-1}\big) \tag{21}$$

$$R_t = R_{t-1} + \bar{\mathbf{g}} \big(X_{t-1} X_{t-1}' - R_{t-1}\big), \tag{22}$$

where (21) describes the updating of the learning rule coefficients $\hat{\phi}_t = \big(a_t', vec(b_t, c_t, d_t)'\big)'$, and (22) describes the updating of the

[4]When learning is introduced on the linearized equations found under RE, the aggregate demand and supply equations become $\tilde{x}_t = \widehat{E}_t \tilde{x}_{t+1} - (1-\beta\eta)\sigma\big[i_t - \widehat{E}_t \pi_{t+1} - r_t^n\big]$ and $\tilde{\pi}_t = \xi_p\big[\omega x_t + [(1-\eta\beta)\sigma]^{-1}\tilde{x}_t\big] + \beta \widehat{E}_t \tilde{\pi}_{t+1} + u_t$. I refer the reader to Preston (2005a, 2005b) and Honkapohja, Mitra, and Evans (2003) for a discussion of the different approaches.

matrix of second moments R_t of the stacked regressors $X_t \equiv \{1, Z_{t-1}, u_t, r_t^n\}_0^{t-1}$. The parameter $\bar{\mathbf{g}}$ denotes the constant gain, which indicates the speed at which agents update their beliefs. From their PLM, and using the updated parameters through (21) and (22), agents can form expectations for any future horizon $T > t$ as

$$\begin{aligned}\widehat{E}_t Z_T = (I_5 - b_{t-1})^{-1}\big(I_5 - b_{t-1}^{T-t}\big)a_{t-1} + b_{t-1}^{T-t} E_t Z_t \\ + \phi_r r_t^n (\phi_r I_5 - b_{t-1})^{-1}\big(\phi_r^{T-t} I_5 - b_{t-1}^{T-t}\big)c_{t-1} \\ + \phi_u u_t (\phi_u I_5 - b_{t-1})^{-1}\big(\phi_u^{T-t} I_5 - b_{t-1}^{T-t}\big)d_{t-1},\end{aligned} \tag{23}$$

where I_5 is a 5×5 identity matrix.

4. Bayesian Estimation

The paper provides the first empirical analysis of a model with infinite-horizon learning. I estimate the system using Bayesian methods to fit the series for output gap, inflation, and the nominal interest rate. I use quarterly U.S. data for the period 1960:Q1 to 2004:Q2. Inflation is defined as the annualized quarterly rate of change of the GDP implicit price deflator, the output gap is defined as the log difference between GDP and potential GDP (Congressional Budget Office estimate), and I use the federal funds rate as the nominal interest rate.

The main learning parameter, the constant gain, is estimated *jointly* with the deep parameters of the economy. I can substitute the expectations formed as in (23) into (15) and (16) and rewrite the model in state-space form:

$$\begin{aligned}\xi_t &= A_t + F_t \xi_{t-1} + G_t w_t \\ Y_t &= H \xi_t,\end{aligned} \tag{24}$$

where $\xi_t = \big[x_t, \pi_t, i_t, u_t, r_t^n\big]$; $w_t \sim N(0, Q)$; H is a matrix of zeros and ones selecting observables from ξ_t; and A_t, F_t, G_t are time-varying matrices of coefficients, which are convolutions of structural parameters of the economy and agents' beliefs. Expression (24) represents the actual law of motion (ALM) of the economy: the ALM has the same structural form as the PLM, but possibly different parameter values. Having expressed the model as a linear

Gaussian system, I can evaluate the likelihood function using the Kalman filter. To derive the parameter estimates, I use a random-walk Metropolis-Hastings algorithm to generate draws from the posterior distribution.[5] I generate 300,000 draws with an initial burn-in of 60,000 draws. A similar estimation procedure has been used by several recent papers that focus on DSGE models under rational expectations (see An and Schorfheide 2006 for a first survey of this literature). This paper, instead, exploits similar techniques to provide the first estimation of a DSGE model with infinite-horizon learning.

I collect the structural parameters in the vector Ψ:

$$\Psi = \left\{\eta, \beta, \alpha, \sigma, \gamma, \xi_p, \omega, \rho, \chi_\pi, \chi_x, \phi_r, \phi_u, \sigma_\varepsilon, \sigma_r, \sigma_u, \sigma_{\varepsilon,r}, \sigma_{\varepsilon,u}, \sigma_{r,u}, \bar{\mathbf{g}}\right\}.$$

I fix some of the parameters: $\beta = 0.99$, $\xi_p = 0.0015$, and $\omega = 0.8975$ (ξ_p and ω are fixed at the values estimated in Giannoni and Woodford 2003 for the flexible wages case). I fix the autoregressive parameters ϕ_r and ϕ_u to 0.9 (I will also consider the case $\phi_r = \phi_u = 0.1$).

Table 1 presents information about the priors. The habit and indexation parameters η and γ are assumed to follow uniform distributions in the interval $[0, 1]$. The intertemporal elasticity of substitution coefficient σ follows a gamma distribution with mean 0.125 and standard deviation 0.09. I choose inverse gamma distributions for the standard deviations of the shocks. The constant-gain coefficient follows a gamma distribution with prior mean 0.031 and prior standard deviation 0.022.

I estimate the initial conditions for the learning algorithm using presample data for the 1954:Q3–1959:Q4 period. The evolution of agents' beliefs is shown in figures 1 and 2, together with the 95 percent probability bands. For example, we see that agents perceive inflation as more persistent starting in the second half of the 1970s until the first half of the 1980s (parameter b_{22}), and they perceive a smaller sensitivity of output to interest rates after 1980 (parameter b_{13}).

[5]More details about the estimation method can be found in Milani (2004b).

Table 1. Bayesian DSGE Model with Infinite-Horizon Learning: Prior Distributions, Posterior Estimates, and 95 Percent Probability Intervals

		Prior Distribution			
Description	Param.	Range	Distr.	Mean	95% Int.
Habit Formation	η	$[0, 1]$	U	.5	$[.025, .975]$
Discount Rate	β	.99	–	.99	–
Calvo Parameter	α	$[0, 1]$	U	.5	$[.025, .975]$
IES	σ	$\mathbb{R}^+$	G	.125	$[.015, .35]$
Infl. Indexation	γ	$[0, 1]$	U	.5	$[.025, .975]$
Fcn. of Price Stick.	ξ_p	.0015	–	.0015	–
Elasticity mc	ω	.8975	–	.8975	–
Interest-Rate Smooth	ρ	$[0, .97]$	U	.485	$[0.024, 0.946]$
Feedback to Infl.	χ_π	$\mathbb{R}$	N	1.5	$[1.01, 1.99]$
Feedback to Output	χ_x	$\mathbb{R}$	N	.5	$[.01, .99]$
Autocorr. r_t^n	ϕ_r	.9 or .1	–	–	–
Autocorr. u_t	ϕ_u	.9 or .1	–	–	–
Std. MP Shock	σ_ε	$\mathbb{R}^+$	IG	1	$[.34, 2.81]$
Std. r_t^n	σ_r	$\mathbb{R}^+$	IG	1	$[.34, 2.81]$
Std. u_t	σ_u	$\mathbb{R}^+$	IG	1	$[.34, 2.81]$
Cov ε_t, r_t^n	$\sigma_{\varepsilon,r}$	$[-.5, .5]$	U	0	$[-.475, .475]$
Cov ε_t, u_t	$\sigma_{\varepsilon,u}$	$[-.5, .5]$	U	0	$[-.475, .475]$
Cov r_t^n, u_t	$\sigma_{r,u}$	$[-.5, .5]$	U	0	$[-.475, .475]$
Constant Gain	$\bar{\mathbf{g}}$	$\mathbb{R}^+$	G	.031	$[.0038, .087]$

(*continued*)

Table 1 (continued). Bayesian DSGE Model with Infinite-Horizon Learning: Prior Distributions, Posterior Estimates, and 95 Percent Probability Intervals

		Posterior Distribution Constant-Gain Learning			
Description	Param.	Mean	95% Int.	Mean	95% Int.
Habit Formation	η	.113	[.14, .23]	.87	[.72, .97]
Discount Rate	β	.99	–	.99	–
Calvo Parameter	α	.992	[.97, .999]	.19	[.02, .42]
IES	σ	.067	[.04, .10]	.144	[.04, .36]
Infl. Indexation	γ	.009	[0, .033]	.216	[0, .77]
Fcn. of Price Stick.	ξ_p	.0015	–	.0015	–
Elasticity mc	ω	.8975	–	.8975	–
Interest-Rate Smooth	ρ	.91	[.87, .95]	.885	[.83, .93]
Feedback to Infl.	χ_π	1.523	[1.14, 1.92]	1.496	[1.14, 1.87]
Feedback to Output	χ_x	.681	[.30, 1.08]	.56	[.18, .97]
Autocorr. r_t^n	ϕ_r	.9	–	.1	–
Autocorr. u_t	ϕ_u	.9	–	.1	–
Std. MP Shock	σ_ε	.889	[.8, .99]	.89	[.8, .99]
Std. r_t^n	σ_r	.856	[.77, .95]	.82	[.74, .92]
Std. u_t	σ_u	1.56	[1.4, 1.73]	1.47	[1.18, 2.1]
Cov ε_t, r_t^n	$\sigma_{\varepsilon,r}$	.04	[−.09, .17]	−.08	[−.27, .06]
Cov ε_t, u_t	$\sigma_{\varepsilon,u}$	.315	[.17, .46]	.29	[.16, .43]
Cov r_t^n, u_t	$\sigma_{r,u}$	.03	[−.13, .18]	−.06	[−.23, .11]
Constant Gain	$\bar{\mathbf{g}}$	.006	[.0014, .01]	.017	[.007, .036]

(*continued*)

Table 1 (continued). Bayesian DSGE Model with Infinite-Horizon Learning: Prior Distributions, Posterior Estimates, and 95 Percent Probability Intervals

		Posterior Distribution		RLS Learning	
Description	Param.	Mean	95% Int.	Mean	95% Int.
Habit Formation	η	.059	[.002, .15]	.786	[.64, .92]
Discount Rate	β	.99	–	.99	–
Calvo Parameter	α	.911	[.80, .98]	.18	[.02, .43]
IES	σ	.077	[.04, .12]	.239	[.09, .44]
Infl. Indexation	γ	.01	[0, .038]	.134	[.005, .43]
Fcn. of Price Stick.	ξ_p	.0015	–	.0015	–
Elasticity mc	ω	.8975	–	.8975	–
Interest-Rate Smooth	ρ	.903	[.86, .94]	.877	[.83, .92]
Feedback to Infl.	χ_π	1.58	[1.21, 1.95]	1.63	[1.29, 1.99]
Feedback to Output	χ_x	.66	[.23, 1.04]	.54	[.19, .95]
Autocorr. r_t^n	ϕ_r	.9	–	.1	–
Autocorr. u_t	ϕ_u	.9	–	.1	–
Std. MP Shock	σ_ε	.887	[.8, .98]	.891	[.8, .99]
Std. r_t^n	σ_r	.844	[.76, .94]	.882	[.8, .98]
Std. u_t	σ_u	1.578	[1.42, 1.76]	1.36	[1.13, 1.72]
Cov ε_t, r_t^n	$\sigma_{\varepsilon,r}$	.04	[−.11, .17]	−.008	[−.15, .13]
Cov ε_t, u_t	$\sigma_{\varepsilon,u}$	.312	[.17, .47]	.32	[.17, .45]
Cov r_t^n, u_t	$\sigma_{r,u}$	−.025	[−.19, .12]	−.07	[−.23, .09]
Constant Gain	$\bar{\mathbf{g}}$	–	–	–	–

Figure 1. Agents' Time-Varying Beliefs 1960:Q1–2004:Q3 (Autoregressive Parameters = 0.9, CGL)

The learning rule is:

$$\begin{bmatrix} x_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} b_{11,t} & b_{12,t} & b_{13,t} \\ b_{21,t} & b_{22,t} & b_{23,t} \\ b_{31,t} & b_{32,t} & b_{33,t} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} c_{1,t} \\ c_{2,t} \\ c_{3,t} \end{bmatrix} r_t^n + \begin{bmatrix} d_{1,t} \\ d_{2,t} \\ d_{3,t} \end{bmatrix} u_t + \varepsilon_t.$$

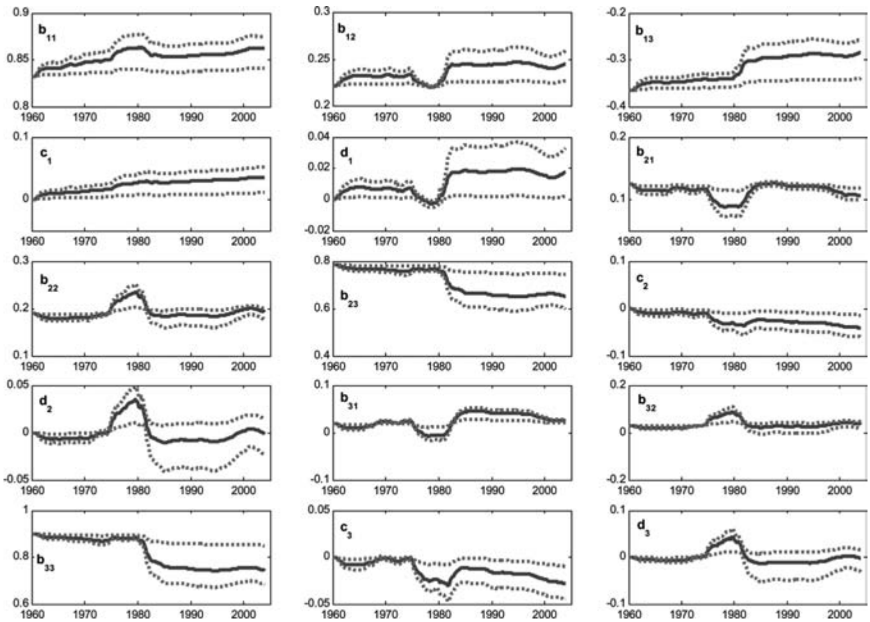
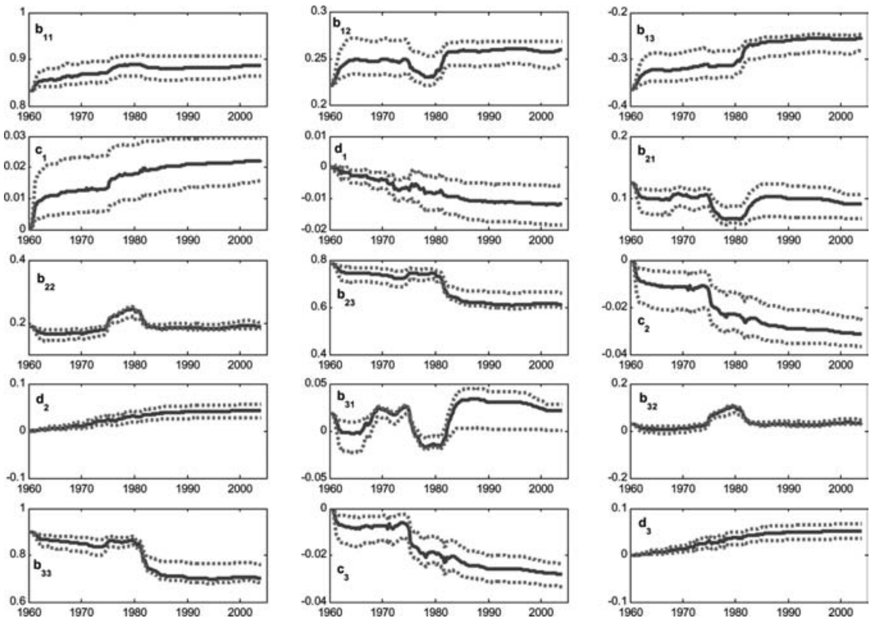


Table 1 presents the estimation results. First, I assume that the autoregressive coefficients regarding the disturbances r_t^n and u_t equal 0.9. I find very weak evidence of habit formation in consumption and no evidence of indexation in inflation. I estimate, in fact, η, the habit parameter, equal to 0.113, while I estimate γ, the inflation indexation parameter, equal to 0.009. The two parameters are tightly estimated: the 95 percent posterior probability intervals also remain close to zero. Therefore, infinite-horizon learning appears to account for the persistence in the data. Additional "mechanical" sources of persistence, which are essential under rational expectations, become superfluous under learning. Under infinite-horizon learning, however,

Figure 2. Agents' Time-Varying Beliefs 1960:Q1–2004:Q3 (Autoregressive Parameters = 0.1, CGL)

The learning rule is:

$$\begin{bmatrix} x_t \\ \pi_t \\ i_t \end{bmatrix} = \begin{bmatrix} b_{11,t} & b_{12,t} & b_{13,t} \\ b_{21,t} & b_{22,t} & b_{23,t} \\ b_{31,t} & b_{32,t} & b_{33,t} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} c_{1,t} \\ c_{2,t} \\ c_{3,t} \end{bmatrix} r_t^n + \begin{bmatrix} d_{1,t} \\ d_{2,t} \\ d_{3,t} \end{bmatrix} u_t + \varepsilon_t.$$



the estimate of α, the Calvo price-stickiness parameter, is unrealistic: I find α equal to 0.992, which implies an extreme degree of rigidity in prices. I obtain a value of 0.067 for the intertemporal elasticity of substitution parameter σ. The estimates for the monetary policy rule ballpark most estimates in the literature ($\rho = 0.91$, $\chi_\pi = 1.52$, and $\chi_x = 0.68$). A crucial parameter in the estimation is represented by the constant-gain parameter. The paper estimates the constant gain jointly with the deep parameters of the economy. I estimate the gain equal to 0.006. To get some intuition about this value, it may be useful to think about the gain as a rough indication of how many observations agents use to form their expectations. A gain equal to 0.006, therefore, mimics the situation of an econometrician running rolling-window regressions using a window with 1/0.006 observations (corresponding to 166.67 quarters of data, or 41.668 years). The estimated value implies substantially slower learning than that found in Milani (2004a, 2004b, 2005) assuming only one-period-ahead expectations.

Infinite-horizon learning, therefore, weakens the role of habits and indexation in a model where the disturbances are highly persistent. But can learning also substitute for the typically strongly autocorrelated structural disturbances? Here, I reestimate the model by fixing the autoregressive parameters ϕ_r and ϕ_u to 0.1. I obtain different results for habits and indexation. In the case of habits, the results seem to depend on the assumed persistence of the disturbances. When the assumed autocorrelation is low, a large degree of habit formation in consumption is still needed to fit the data (I find $\eta = 0.87$). The results are more favorable for inflation indexation. Even assuming a low autocorrelation of the disturbances, the estimated indexation is small (I estimate $\gamma = 0.21$). The results suggest that learning matters for inflation dynamics. A minimal deviation from rational expectations is sufficient to account for the persistence in inflation, so that both indexation and a strongly autocorrelated cost-push shock become redundant.

The estimated Calvo parameter α is now small: I find α equal to 0.19 in this case, suggesting a much smaller price rigidity. The results about α are therefore strongly dependent on the assumed autocorrelation and suggest difficulties in robustly identifying this parameter. I also obtain different results for the constant gain. Now, the gain coefficient equals 0.017. This estimate implies faster learning than in the previous case and is more similar to that found by Milani (2004a, 2004b, 2005). In general, various recent papers (Milani 2004a, 2004b, 2005; Orphanides and Williams 2005a, 2005b; Branch and Evans 2006) are starting to accumulate evidence that the most realistic values of the gain lie in the 0.01–0.05 range, with the majority of estimates around 0.02.

I also reestimate the model under recursive-least-squares (RLS) learning: this implies a decreasing gain equal to t^{-1}. The results substantially confirm what was found under constant-gain learning. When the autocorrelation of the shocks is large, I estimate $\eta = 0.059$ and $\gamma = 0.01$. The Calvo price-stickiness parameter is now slightly less extreme ($\alpha = 0.911$). When the autocorrelation is small, instead, I find $\eta = 0.786$ and $\gamma = 0.134$. The Calvo parameter is again reduced to 0.18.[6]

5. Conclusions

DSGE models under rational expectations typically need several additional sources of persistence to match macroeconomic data. In the paper, I have developed a model in which nonrational expectations and learning enter from the primitive assumptions. As in Preston (2005b), the aggregate dynamics of the economy imply that long-horizon expectations of future macroeconomic conditions matter for the current dynamics of output, inflation, and nominal interest rates. The model, therefore, nests infinite-horizon learning and some of the "mechanical" sources of persistence, such as habit formation and inflation indexation, that are essential under rational expectations. Once the assumption of rational expectations is relaxed in favor of learning, it becomes interesting to verify whether mechanical sources of persistence remain essential for the model fit.

I estimate the model using Bayesian methods. I obtain estimates of the main learning parameter, the constant gain, jointly with the other model parameters.

The results show that learning may render additional sources of persistence superfluous. Learning seems to represent the main cause of persistence in inflation: with learning, the estimated indexation is very close to zero. The results do not depend on the assumed autocorrelation of the shocks. Infinite-horizon learning generates sufficient persistence in inflation, so that it might be possible to avoid both indexation and serial correlation in the cost-push shock. The results are, instead, mixed for habit formation: learning and strongly autocorrelated shocks substantially weaken the evidence of habit formation. But the results in this case depend on the assumed autocorrelation. A low autocorrelation restores, in fact, a role to habit formation.

Overall, learning seems to provide a good description of the data. But the literature still needs to shed more light on the best way to model learning. In related research, I am comparing the estimates

[6] I do not report the results, but I have found that the model with constant-gain learning fits better than the model with RLS learning.

and fit of DSGE models under different learning mechanisms: one-period-ahead versus infinite-horizon learning, constant-gain versus recursive-least-squares learning, and different learning rule specifications. Moreover, as Preston (2004a, 2004b) shows, monetary policy rules may have very different properties under different learning mechanisms. A priority for future research, therefore, will consist of evaluating the robustness of policy rules to different assumptions about learning.

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Firm-Specific Production Factors in a DSGE Model with Taylor Price Setting*

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Using Bayesian likelihood methods, this paper estimates a dynamic stochastic general equilibrium model with Taylor contracts and firm-specific factors in the goods market on euro-area data. The paper shows how the introduction of firm-specific factors improves the empirical fit of the model and reduces the estimated contract length to a duration of four quarters, which is more consistent with the empirical evidence on average price durations in the euro area. However, in order to obtain this result, the estimated real rigidity is very large, either in the form of a very large constant elasticity of substitution between goods or in the form of an endogenous elasticity of substitution that is very sensitive to the relative price. Finally, the paper also investigates the implications of these estimates for the distribution of prices and quantities across the various goods sectors.

JEL Codes: E1–E3.

1. Introduction

Following the theoretical work of Yun (1996) and Woodford (2003), the New Keynesian Phillips curve, relating inflation to expected future inflation and the marginal cost, has become a popular tool

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for monetary policy analysis. Typically, the elasticity of inflation with respect to changes in the marginal cost is, however, estimated to be very small (e.g., Galí and Gertler 1999; Galí, Gertler, and López-Salido 2001; and Sbordone 2002). In models with constant-returns-to-scale technology, perfectly mobile production factors, and a constant elasticity of substitution between goods, such low estimates imply an implausibly high degree of nominal price stickiness. For example, Smets and Wouters (2003) find that, on average, nominal prices remain fixed for more than two years. This is not in line with existing microevidence that suggests that, on average, prices are sticky for around six months to one year.[1]

In response to these findings, a number of papers have investigated whether the introduction of additional real rigidities, such as frictions in the mobility of capital across firms, can address this apparent mismatch between the macro- and microestimates of the degree of nominal price stickiness. For example, Woodford (2005), Eichenbaum and Fisher (2004), and Altig et al. (2005) show how the introduction of firm-specific capital lowers the elasticity of prices with respect to the real marginal cost for a given degree of price stickiness. This paper focuses on the same issue. Using Bayesian likelihood methods as in Smets and Wouters (2003, 2005), it estimates a dynamic stochastic general equilibrium (DSGE) model with overlapping price and wage contracts as in Taylor (1980) and firm-specific factors in the monopolistically competitive goods market. With the exception of those two features, the specification of the DSGE model is the same as in Smets and Wouters (2005). As in

[1]See the evidence in Bils and Klenow (2004) for the United States and Altissimo, Ehrmann, and Smets (2005) for a summary of the Inflation Persistence Network (IPN) evidence on price stickiness in the euro area.

However, one should be careful with using the microevidence to interpret the macroestimates. Because of indexation and a positive steady-state inflation rate, all prices change all the time. However, only a small fraction of prices are set optimally. The alternative story for introducing a lagged inflation term in the Phillips curve based on the presence of rule-of-thumb price setters is more appealing from this perspective, as it does not imply that all prices change all the time. In that case, the comparison of the Calvo parameter with the microevidence makes more sense. As the reduced-form representations are almost identical, one could still argue that the estimated Calvo parameter is implausibly high.

Smets and Wouters (2005), the model is estimated on quarterly euro-area data from 1974:Q1 to 2002:Q2.[2] The reason for using Taylor contracts is twofold. First, while the simple Calvo model is analytically tractable, its derivation with firm-specific factors and endogenous capital accumulation is nontrivial and cannot be solved in closed form. This complicates the empirical estimation of the full model. The assumption of Taylor contracts facilitates the estimation of a fully specified linearized DSGE model that embeds the pricing decisions of monopolistically competitive price and wage setters and real rigidities such as firm-specific capital and/or labor. Second, the use of Taylor contracts in this DSGE setting makes it easier to analyze the distribution of prices and quantities across the various sectors. This analysis is important to check whether the introduction of real rigidities leads to a realistic distribution of prices and quantities (as in Altig et al. 2005). Our paper is most closely related to that of Coenen and Levin (2004), which also investigates the relative importance of real and nominal rigidities in a world with Taylor contracts. However, the Coenen and Levin (2004) paper focuses on Germany and does not specify the full structural model. Finally, in contrast to most of the papers mentioned above, our paper also analyzes the implications of firm-specific labor markets.

In the rest of this paper, we proceed in several steps. First, we estimate the Taylor contracting specification of the Smets-Wouters DSGE model under the assumption that firms are price takers in the factor markets, i.e., the labor and capital markets, and hence all firms face the same flat marginal cost curve. We compare this specification with the analogous Calvo model and find that the length of the Taylor contracts in the goods market needs to be extremely long (about five years) in order to match the data as well as the Calvo scheme. Though striking, this result is consistent with Dixon and Kara (2006), who show how to compare the mean duration of contracts in both time-dependent price-setting models. In this section, we also show that the standard way of introducing markup shocks in the Calvo model does not work very well with Taylor-type price

[2] As shown in Smets and Wouters (2005), estimation results are quite similar for U.S. data.

setting, and we propose a different way of introducing price-markup shocks.

Next, we reestimate the Taylor contracting models with firm-specific capital and/or firm-specific labor and analyze the impact of these assumptions on the empirical performance of the DSGE model and on the estimated contract length in the goods market. Our main findings are twofold. First, in line with the previous literature, we find that introducing firm-specific capital does lead to a fall in the estimated Taylor contract length in the goods market to a more reasonable length of four quarters. However, the elasticity of substitution between goods of the various price-setting cohorts is estimated to be improbably high. Furthermore, the corresponding price markup is estimated to be smaller than the fixed cost, implying negative profits in steady state. Enforcing a steady-state zero-profit condition leads to a significant deterioration of the empirical fit. At the same time, the estimated elasticity of substitution remains very large. Moving from the traditional Dixit-Stiglitz aggregator toward Kimball's (1995) generalized aggregator helps to solve both problems. In that case, the curvature parameter is estimated to be high, which is a sign that real rigidities are at work, but both the estimated elasticity of substitution and the cost of imposing the above-mentioned zero-profit constraint are sharply reduced. These results are in line with Eichenbaum and Fisher (2004), Coenen and Levin (2004), and Altig et al. (2005). In this context, we also investigate the implications of the various models for the firm-specific supply and pricing decisions, which are straightforward to perform in a Taylor contracting framework.

Finally, we also analyze the impact on empirical performance of introducing firm-specific labor markets. Here the results are less promising in terms of reducing the estimated degree of nominal price stickiness. The reason is that firm-specific labor markets only dampen the price impact of a change in demand for a given degree of nominal price stickiness if the firm-specific labor markets are flexible and the firm-specific wage is responding strongly to changes in the demand for labor. Such wage flexibility is, however, incompatible with the empirical properties of aggregate wage behavior.

The rest of the paper is structured as follows. First, section 2 reviews the estimated DSGE model of Smets and Wouters (2005) and introduces Taylor-type contracting in goods and labor markets.

Next, section 3 explores the impact of introducing firm-specific production factors. The concluding remarks are in section 4.

2. Taylor and Calvo Price Setting with Mobile Production Factors

In this section, we compare the empirical performance of the Taylor price-setting model with the Calvo model estimated in Smets and Wouters (2005), maintaining the assumption of mobile production factors across firms. We first briefly review the Calvo model of Smets and Wouters (2005) and the alternative Taylor specification. Then, we compare the estimates of both models on euro-area data.

2.1 The Smets-Wouters Model with Calvo and Taylor Price Setting

The Smets-Wouters (2005) model contains many frictions that affect both nominal and real decisions of households and firms. Households maximize a nonseparable utility function with two arguments (goods and labor effort) over an infinite life horizon. Consumption appears in the utility function relative to a time-varying external habit variable. Labor is differentiated, so that there is some monopoly power over wages, which results in an explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo (1983). Households rent capital services to firms and decide how much capital to accumulate, taking into account capital adjustment costs.

The main focus in this paper is on the firms' price setting. In the Calvo specification, a continuum of firms produces differentiated goods, decides on labor and capital inputs, and sets prices. Following Calvo (1983), every period, only a fraction $(1 - \xi_p)$ of firms in the monopolistic competitive sector are allowed to reoptimize their price. This fraction is constant over time. Moreover, those firms that are not allowed to reoptimize index their prices to the past inflation rate and the time-varying inflation target of the central bank. An additional important assumption is that all firms are price takers in the factor markets for labor and capital and thus face the same marginal cost. The marginal costs depend on wages, the rental rate of capital, and productivity.

As shown in Smets and Wouters (2005), this leads to the following linearized *inflation equation*:

$$\hat{\pi}_t - \bar{\pi}_t = \frac{\beta}{1+\beta\gamma_p}(E_t\hat{\pi}_{t+1} - \bar{\pi}_t) + \frac{\gamma_p}{1+\beta\gamma_p}(\hat{\pi}_{t-1} - \bar{\pi}_t) + \frac{1}{1+\beta\gamma_p}\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\hat{s}_t + \eta_t^p \quad (1)$$

$$\hat{s}_t = \alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - \varepsilon_t^a - (1-\alpha)\gamma t. \quad (2)$$

Parameters α and β are, respectively, the capital share and the household's discount factor. The deviation of inflation $\hat{\pi}_t$ from the target inflation rate $\bar{\pi}_t$ depends on past and expected future inflation deviations and on the current marginal cost (which itself is a function of the rental rate on capital $\hat{r}_t^k$, the real wage $\hat{w}_t$, and the productivity process) that is composed of a deterministic trend in labor efficiency γt and a stochastic component ε_t^a, which is assumed to follow a first-order autoregressive process: $\varepsilon_t^a = \rho_a\varepsilon_{t-1}^a + \eta_t^a$, where η_t^a is an i.i.d.-normal productivity shock. Finally, η_t^p is an i.i.d.-normal price-markup shock. When the degree of indexation to past inflation is zero ($\gamma_p = 0$), this equation reverts to the standard purely forward-looking New Keynesian Phillips curve. When all prices are flexible ($\xi_p = 0$) and the price-markup shock is zero, this equation reduces to the normal condition that, in a flexible-price economy, the real marginal cost is constant.

In the Taylor specification, firms set prices for a fixed number of periods, and price setting is staggered over the duration of the contract, i.e., the number of firms adjusting their price is the same every period.[3] The explicit modeling of the different cohorts in the Taylor model facilitates the introduction of firm-specific capital and labor in the next section, as no aggregation across cohorts is required. It also has the advantage that the cohort-specific output and price levels are directly available, which is important for checking whether

[3]See Coenen and Levin (2004) and Dixon and Kara (2005) for a generalization of the standard Taylor contracting model where different firms may set prices for different lengths of time. See also chapter 2 in Taylor (1993).

the dispersion of output and prices across price-setting cohorts is realistic.

In order to be able to compare the Taylor price-setting model with the Calvo model estimated in Smets and Wouters (2005), we maintain the assumption of partial indexation to lagged inflation and the inflation objective. As discussed in Whelan (2004) and Coenen and Levin (2004), the staggered Taylor contracting model gives rise to the following linearized equations for the newly set optimal price and the general price index:

$$\hat{p}_t^* = \frac{1}{\sum_{i=0}^{n_p-1}\beta^i}\left[\sum_{i=0}^{n_p-1}\beta^i(\hat{s}_{t+i}+\hat{p}_{t+i})\right.$$
$$\left. -\sum_{i=0}^{n_p-2}\left((\gamma_p\hat{\pi}_{t+i}+(1-\gamma_p)\bar{\pi}_{t+i+1})\sum_{q=i+1}^{n_p-1}\beta^q\right)\right]+d\varepsilon_t^p \tag{3}$$

$$\hat{p}_t = \frac{1}{n_p}\sum_{i=0}^{n_p-1}\left(\hat{p}_{t-i}^*+\sum_{q=0}^{i-1}(\gamma_p\hat{\pi}_{t-1-q}+(1-\gamma_p)\bar{\pi}_{t-q})\right)+(1-d)\varepsilon_t^p, \tag{4}$$

where n_p is the duration of the contract, d is a binary parameter ($d \in \{0,1\}$), and $\varepsilon_t^p = \rho_t^p\varepsilon_{t-1}^p + \eta_t^p$, with η_t^p an i.i.d. shock. We experiment with two ways of introducing the price-markup shocks in the Taylor contracting model. The first method ($d = 1$) is fully analogous with the Calvo model. We assume a time-varying markup in the optimal price-setting equation, which introduces a shock in the linearized price-setting equation (3) as shown above. The second method ($d = 0$) is somewhat more ad hoc. It consists of introducing a shock in the aggregate price equation (4).[4]

[4]This could be justified as a relative price shock to a flexible-price sector that is not explicitly modeled. Of course, such a shortcut ignores the general equilibrium implications (e.g., in terms of labor and capital reallocations).

Similarly, we introduce Taylor contracting in the wage-setting process. This leads to the following linearized equations for the newly set optimal wage and the average wage:

$$\hat{w}_t^* = \frac{1}{\sum_{i=0}^{n_w-1} \beta^i} \Bigg[\sum_{i=0}^{n_w-1} \beta^i \left(\sigma_l \hat{l}_{i,t+i} + \frac{1}{1-h}(\hat{c}_{t+i} - h\hat{c}_{t+i-1}) - \varepsilon_{t+i}^l \right) + \sum_{i=1}^{n_w-1} \left((\hat{\pi}_{t+i} - \gamma_w \hat{\pi}_{t+i-1} - (1-\gamma_w)\bar{\pi}_{t+i}) \sum_{q=i}^{n_w-1} \beta^q \right) \Bigg] + d\varepsilon_t^w \tag{5}$$

$$\hat{w}_t = \frac{1}{n_w} \left[\sum_{i=0}^{n_w-1} \hat{w}_{i,t} + \hat{p}_{t-i} \right] - \hat{p}_t + (1-d)\varepsilon_t^w \tag{6}$$

with

$$\hat{w}_{i,t} = \hat{w}_{t-i}^* + \sum_{q=0}^{i-1} (\gamma_w \hat{\pi}_{t-1-q} + (1-\gamma_w)\bar{\pi}_{t-q}) \tag{7}$$

$$\hat{l}_{i,t+i} = \hat{l}_{t+i} - \frac{1+\lambda_w}{\lambda_w} [\hat{w}_{i,t+i} + \hat{p}_t - (\hat{w}_{t+i} + \hat{p}_{t+i})], \tag{8}$$

where n_w is the duration of the wage contract; σ_l represents the inverse elasticity of work effort with respect to real wage; $\hat{l}_t$ is the labor demand described in equation (23) (cf. appendix 2); $\hat{l}_{i,t}$ is the demand for the labor supplied at nominal wage $\hat{w}_{i,t}$ by the households who reoptimized their wage i periods ago; h is the habit parameter; $\hat{c}_t$ is consumption; $\varepsilon_t^l = \rho_t^l \varepsilon_{t-1}^l + \eta_t^l$, with η_t^l an i.i.d. shock to the labor supply; γ_w is the degree of indexation to the lagged wage growth rate; and $\varepsilon_t^w = \rho_t^w \varepsilon_{t-1}^w + \eta_t^w$, *with* η_t^w an i.i.d. wage-markup shock. Finally, λ_w is the wage markup. Note that, as we did for price shocks, wage shocks have been introduced in two different ways.

The rest of the linearized DSGE model is summarized in appendix 2. In sum, irrespective of the pricing specification, the Smets-Wouters (2005) model determines nine endogenous variables: inflation, the real wage, capital, the value of capital, investment, consumption, the short-term nominal interest rate, the rental rate on capital, and hours worked. The stochastic behavior is driven by ten exogenous shocks. Five shocks arise from technology and preference parameters: the total factor productivity shock, the investment-specific technology shock, the preference shock, the labor-supply shock, and the government-spending shock. Those shocks are assumed to follow an autoregressive process of order one. Three shocks can be interpreted as "cost-push" shocks: the price-markup shock, the wage-markup shock, and the equity-premium shock. Those are assumed to follow a white-noise process. And, finally, there are two monetary policy shocks: a permanent inflation target shock and a temporary interest rate shock.

Before discussing the estimation results, it is worth highlighting two issues. First, Dixon and Kara (2006) have argued that a proper comparison of the degree of price stickiness in the Taylor and Calvo model should be based on the average age of the running contracts, rather than on the average frequency of price changes. As is well known, in a Calvo pricing model the average age of the running contracts is computed as

$$(1-\xi_p)\sum_{i=0}^{\infty}\xi_p^i\cdot(i+1)=\frac{1}{1-\xi_p},$$

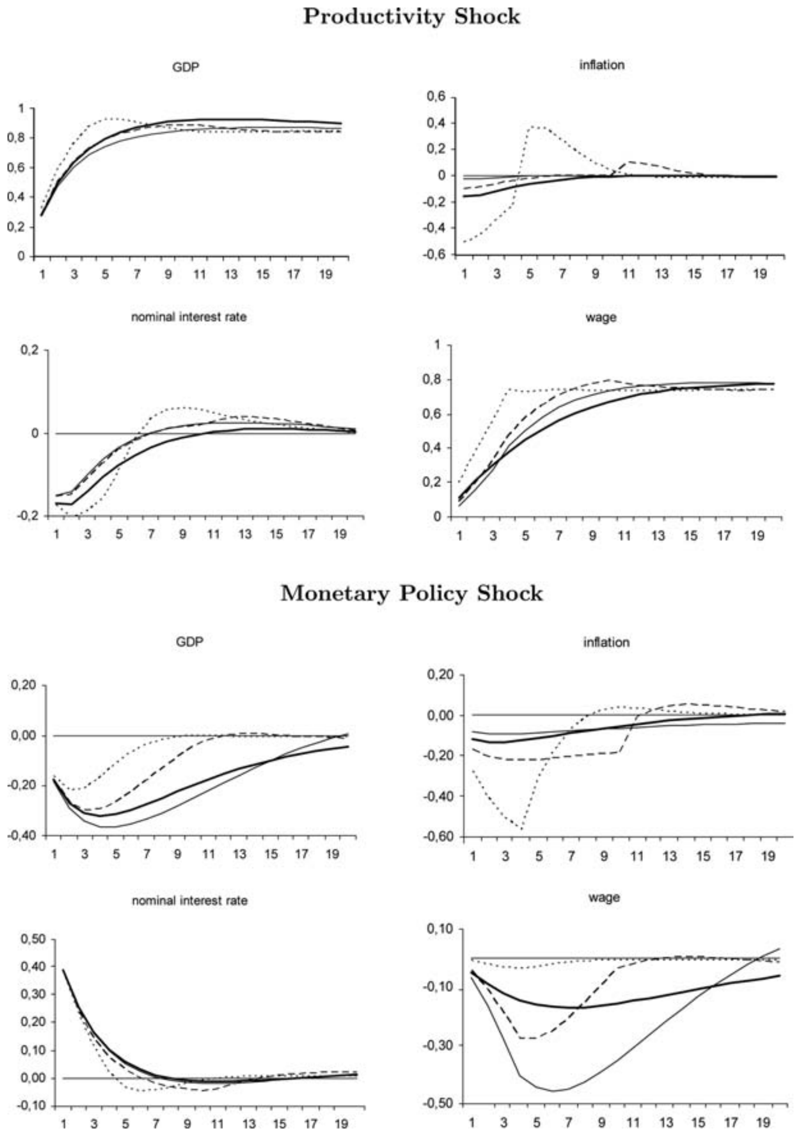
while the corresponding statistic for Taylor contracts is given by

$$\frac{1}{n_p}\sum_{i=1}^{n_p} i=\frac{n_p+1}{2}.$$

Thus, in order to produce the same average contract age as the one implied by a Calvo parameter ξ_p, the Taylor-contract length needs to be $\frac{1+\xi_p}{1-\xi_p}$ periods. The Calvo parameter $\xi_p = 0.9$ estimated in section 2 above therefore implies a long Taylor-contract length of nineteen quarters.

Figure 1 confirms the Dixon and Kara (2006) analysis by comparing the impulse responses to, respectively, a productivity and a monetary policy shock in the baseline Calvo model and four-, ten-, and twenty-quarter Taylor contracting, keeping the other parameters fixed at those estimated for the baseline Calvo model. In this figure the wage contract length n_w is fixed at four quarters. As the duration of the Taylor contract lengthens, the impulse responses appear to approach the outcome under the Calvo model. One needs a very long duration (about twenty quarters) in order to come close to the

Figure 1. Selected Impulse Responses: Calvo versus Taylor Contracts (Baseline Parameters)



Legend: Bold black line: baseline (Calvo) model; full line: 20-quarter Taylor price contract; dashed line: 10-quarter Taylor price contract; dotted line: 4-quarter Taylor price contract.

Calvo model. With shorter Taylor contracts, typically the inflation response becomes larger in size but also less persistent. Conversely, the output and real wage responses are closer to the flexible-price outcome. For example, in response to a monetary policy shock, the response of output is considerably smaller. Moreover, with shorter Taylor contracts, the inflation response changes sign quite abruptly after the length of the contract. This feature is absent in the Calvo specification. As discussed in Whelan (2004), in reduced-form inflation equations, the reversal of the inflation response after the contract length is captured by a negative coefficient on lagged inflation once current and expected future marginal costs are taken into account.

A second issue relates to the way in which the price shocks are introduced. As shown in figure 2, the two ways of introducing price (resp. wage) shocks discussed above generate very different short-run dynamics in response to such shocks. The right-hand panels of figure 2 shows that introducing a persistent shock in the GDP deflator equation (i.e., $d = 0$) allows the Taylor contracting model to mimic most closely the response to a markup shock in the baseline Calvo specification.[5]

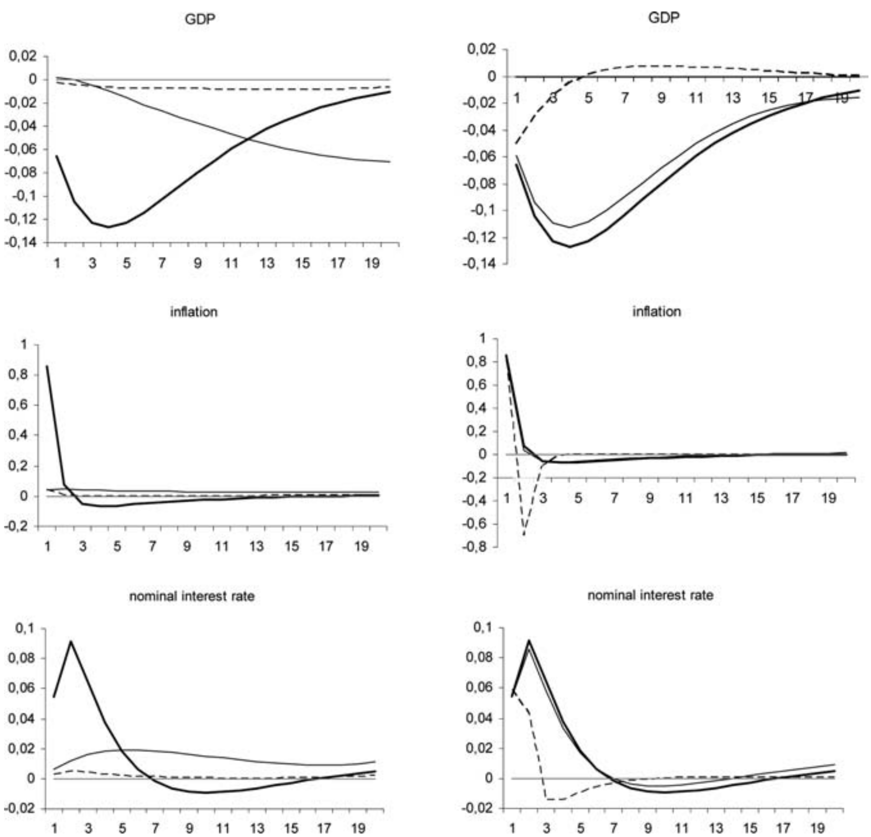
2.2 Estimation Results

We now turn to the main estimation results. The full set of results as well as a description of the euro-area data set and the assumed prior distributions can be found in the appendices. A number of results are worth highlighting. First, we confirm the findings of Smets and Wouters (2005) regarding the Calvo specification. The degree of indexation is rather limited, while the degree of Calvo price stickiness is very large: each period, 89 percent of the firms do not reoptimize their price setting. The average age of the price contract is therefore more than two years (9.1 quarters). Second, as illustrated by figure 3, which plots the log data density of the estimated Taylor model as a function of the contract length, the contract length that maximizes the predictive performance of the Taylor model is nineteen quarters. This again confirms the analysis of Dixon and

[5]The same exercise could actually be run for a wage shock. Since it leads to similar conclusions, we do not reproduce it here.

Figure 2. Impulse Response to a Price Shock in the 20-Quarter Taylor Model for Different Specifications of the Price Shock (Baseline Parameters)

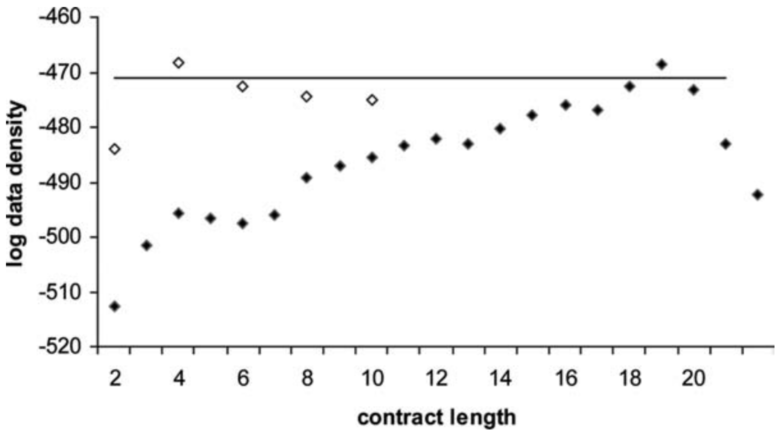
***d* = 1: Price Shock in (3)** **_d_ = 0: Price Shock in (4)**



Legend: Bold black line: baseline (Calvo) model; black line: 20-quarter Taylor contract with persistent price shock; dashed line: 20-quarter Taylor contract with i.i.d. price shock.

Kara (2006) discussed above. A Calvo parameter of 0.9 implies an average length of the contracts of about nineteen quarters. Third, we confirm (results not shown) that, in line with the impulse responses shown in figure 2, the specification with the persistent price shock in the GDP price equation ($d = 0$) does best in terms of empirical

Figure 3. Log Data Density for Taylor Contracting Models with Different Lengths



Legend: The black line represents the log data density in the baseline Calvo model; black diamonds denote the Taylor model with mobile production factors and a persistent price shock in the GDP price; white diamonds denote the same model but with nonmobile capital, zero profits, and endogenous markup (see section 4.4).

performance. For example, the log data density of the estimated model with ten-quarter Taylor contracts improves by ninety points relative to the specification with a persistent price shock in the optimal price-setting equation. Similar improvements are found for other contract lengths. Moreover, the empirical performance also improves significantly by allowing for persistence in the price shocks.[6]

Table 1 compares some of the estimated parameters across various Taylor models and the Calvo model. While most of the other parameters are estimated to be very similar, it is noteworthy that the estimated degree of indexation rises quite significantly as the assumed Taylor contracts become shorter. Possibly, this reflects the need to overcome the negative dependence on past inflation in the standard Taylor contract. Next we turn to the introduction of firm-specific factors in the Taylor model.

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[6]Similar findings have been found for various specifications of the wage shock. For that reason, we consider a persistent wage shock in the average wage equation for all the estimations performed in the rest of the paper.

Table 1. Comparing the Calvo Model with Taylor Contracting Models

	Calvo	4-Q Tayl.	8-Q Tayl.	10-Q Tayl.	19-Q Tayl.
Log Data Densities					
	−471.113	−495.566	−489.174	−485.483	−468.469
Selection of Estimated Parameter Outcomes					
ρ_a	0.991	0.980	0.982	0.962	0.983
	(0.006)	(0.007)	(0.006)	(0.006)	(0.006)
σ_a	0.653	0.615	0.682	0.619	0.622
	(0.093)	(0.068)	(0.085)	(0.076)	(0.085)
ρ_p	0	0.995	0.995	0.912	0.934
	(–)	(0.004)	(0.004)	(0.016)	(0.018)
σ_p	0.207	0.406	0.323	0.277	0.229
	(0.019)	(0.030)	(0.023)	(0.020)	(0.016)
ρ_w	0	0.973	0.966	0.881	0.955
	(–)	(0.012)	(0.014)	(0.017)	(0.012)
σ_w	0.250	0.4386	0.453	0.461	0.454
	(0.021)	(0.031)	(0.034)	(0.031)	(0.035)
γ_w	0.388	0.313	0.397	0.351	0.460
	(0.197)	(0.166)	(0.205)	(0.206)	(0.188)
γ_p	0.178	0.859	0.463	0.436	0.273
	(0.096)	(0.150)	(0.130)	(0.116)	(0.074)
A	9.1 Q	2.5 Q	4.5 Q	5.5 Q	10 Q

Note: ρ_a, ρ_p, and ρ_w are the persistence parameters associated with the productivity, the price, and the wage shock, respectively; σ_a, σ_p, and σ_w are the standard error of the productivity, the price, and the wage shock, respectively; γ_w and γ_p are, respectively, the wage and price indexation parameters; A is the average age of the price contract.

3. Firm-Specific Production Factors and Taylor Contracts

3.1 Modeling Firm-Specific Factors

So far the model includes all kinds of adjustment costs such as those related to the accumulation of new capital, to changes in prices and wages, and to changes in capacity utilization, but shifting capital or labor from one firm to another is assumed to be costless (see Danthine and Donaldson 2002). The latter assumption is clearly not fully realistic. In this section we instead assume that production factors are firm specific, i.e., the cost of moving them across firms is extremely high. Although this is also an extreme assumption, it may be more realistic. The objective is to investigate the implications of introducing this additional real rigidity on the estimated degree of nominal price stickiness and the overall empirical performance of the Taylor contracting model. As shown in Coenen and Levin (2004) for the Taylor model and Woodford (2003, 2005), Eichenbaum and Fisher (2004), and Altig et al. (2005) for the Calvo model, the introduction of firm-specific capital reduces the sensitivity of inflation with respect to its driving variables. Similarly, Woodford (2003, 2005) shows that firm-specific labor may also help reduce price variations and may lead to higher inflation persistence.

In the case of firm-specific factors, the key equations of the linearized model governing the decision of a firm belonging to the cohort j (with $j \in [1, n_p]$), which reoptimizes its price in period t, are given by

$$\hat{p}_t^*(j) = \frac{1}{\sum_{i=0}^{n_p-1} \beta^i} \left[\sum_{i=0}^{n_p-1} \beta^i (\hat{s}_{t+i}(j) + \hat{p}_{t+i}) - \sum_{i=0}^{n_p-2} \left((\gamma_p \hat{\pi}_{t+i} + (1-\gamma_p)\bar{\pi}_{t+i+1}) \sum_{q=i+1}^{n_p-1} \beta^q \right) \right] \quad (3b)$$

$$\hat{p}_t = \frac{1}{n_p} \sum_{i=0}^{n_p-1} \hat{p}_t(j-i) + \varepsilon_t^p \quad (4b)$$

$$\hat{s}_{t+i}(j) = \alpha\hat{\rho}_{t+i}(j) + (1-\alpha)\hat{w}_{t+i}(j) - \hat{\varepsilon}^a_{t+i} - (1-\alpha)\gamma t \tag{9}$$

$$\hat{Y}_{t+i}(j) = \hat{Y}_{t+i} - \frac{1+\lambda_p}{\lambda_p}(\hat{p}_{t+i}(j) - \hat{p}_{t+i}) \tag{10}$$

$$\hat{p}_{t+i}(j) = \hat{p}^*_t(j) + \sum_{q=0}^{i-1}(\gamma_p\bar{\pi}_{t-1-q} + (1-\gamma_p)\bar{\pi}_{t-q}) \tag{11}$$

with

$$\frac{\partial\hat{\rho}_{t+i}(j)}{\partial\hat{Y}_{t+i}(j)} > 0 \quad \text{and} \quad \frac{\partial\hat{w}_{t+i}(j)}{\partial\hat{Y}_{t+i}(j)} > 0, \tag{12}$$

where $\hat{\rho}_t(j)$ is the "shadow rental rate of capital services,"[7] and λ_p is the price markup so that $\frac{1+\lambda_p}{\lambda_p}$ is the elasticity of substitution between goods. The main difference with equations (3) and (4) is that the introduction of firm-specific factors implies that firms no longer share the same marginal cost. Instead, a firm's marginal cost and its optimal price will depend on the demand for its output. A higher demand for its output implies that the firm will have a higher demand for the firm-specific input factors, which in turn will lead to a rise in the firm-specific wage costs and capital rental rate. Because this demand will be affected by the pricing behavior of the firm's competitors, the optimal price will also depend on the pricing decisions of the competitors.

The net effect of this interaction will be to dampen the price effects of various shocks. Consider, for example, an unexpected demand expansion. Compared to the case of homogenous marginal costs across firms, the first price mover will increase its price by less because, everything else being equal, the associated fall in the relative demand for its goods leads to a fall in its relative marginal cost. This, in turn, reduces the incentive to raise prices. This relative marginal cost effect is absent when factors are mobile across firms and, as a result, firms face the same marginal cost irrespective of their output levels. From this example it is clear that the

[7]Indeed, we left aside the assumption of a rental market for capital services. Each firm builds its own capital stock. The "shadow rental rate of capital services"is the rental rate of capital services such that the firm would hire the same quantity of capital services in an economy with a market for capital services as it does in the economy with firm-specific capital.

extent to which variations in firm-specific marginal costs will reduce the amplitude of price variations will depend on the combination of two elasticities: (i) the elasticity of substitution between the goods produced by the firm and those produced by its competitors, which will govern how sensitive relative demand for a firm's goods is to changes in its relative price (see equation [4b]); (ii) the elasticity of the individual firm's marginal cost with respect to changes in the demand for its products (see equation [4b]). With a Cobb-Douglas production function, the latter elasticity will mainly depend on the elasticity of the supply of the factors with respect to changes in the factor prices. In brief, the combination of a steep firm-specific marginal cost curve and high demand elasticity will maximize the relative marginal cost effect and minimize the price effects, thereby reducing the need for a high estimated degree of nominal price stickiness.

Before turning to a quantitative analysis of these effects in the next sections, it is worth examining in somewhat more detail the determinants of the partial derivatives in equation (12) in each of the two factor markets (capital and labor). Consider first firm-specific capital. Given the one-period time-to-build assumption in capital accumulation, the firm-specific capital stock is given within the quarter. As a result, when the demand faced by the firm increases, production can only be adjusted by either increasing the labor/capital ratio or by increasing the rate of capital utilization. Both actions will tend to increase the cost of capital services. It is, however, also clear that when the firm can increase the utilization of capital at a constant marginal cost, the effect of an increase in demand on the cost of capital will be zero. In this case, the supply of capital services is infinitely elastic at a rental price that equals the marginal cost of changing capital utilization and, as a result, the first elasticity in equation (12) will be zero. In the estimations reported below, the marginal cost of changing capital utilization is indeed high, so that in effect there is nearly no possibility to change capital utilization. Over time, the firm can adjust its capital stock subject to adjustment costs. This implies that the firm's marginal cost depends on its capital stock, which itself depends on previous pricing and investment decisions of the firm. As a result, the capital stock, the value of capital, and investment will also be firm specific. In the case of a Calvo model, Woodford (2005) and Christiano (2004) show

how the linearized model can still be solved in terms of aggregate variables, without solving for the whole distribution of the capital stock over the different firms. This linearization is, however, complicated and remains model specific. With staggered Taylor contracts, it is straightforward to model the cohorts of firms characterized by the same price separately. The key linearized equations governing the investment decision for a firm belonging to the jth cohort are then

$$\hat{K}_t(j) = (1-\tau)\hat{K}_{t-1}(j) + \tau\hat{I}_{t-1}(j) + \tau\varepsilon_{t-1}^I \tag{13}$$

$$\hat{I}_t(j) = \frac{1}{1+\beta}\hat{I}_{t-1}(j) + \frac{\beta}{1+\beta}E_t\hat{I}_{t+1}(j) + \frac{1/\varphi}{1+\beta}\hat{Q}_t(j) + \varepsilon_t^I \tag{14}$$

$$\begin{aligned}\hat{Q}_t(j) = {} & -(\hat{R}_t - \hat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+\bar{\rho}}E_t\hat{Q}_{t+1}(j) \\ & + \frac{\bar{\rho}}{1-\tau+\bar{\rho}}E_t\hat{\rho}_{t+1}(j) + \eta_t^Q,\end{aligned} \tag{15}$$

where $\hat{K}_t(j)$, $\hat{I}_t(j)$, and $\hat{Q}_t(j)$ are, respectively, the capital stock, investment, and the Tobin's Q for each of the firms belonging to the jth price-setting cohort. Parameter τ is the depreciation rate of capital, and $\bar{\rho}$ is the shadow rental rate of capital discussed above, so that $\beta = 1/(1-\tau+\bar{\rho})$. Parameter φ depends on the investment adjustment-cost function.[8]

Consider next firm-specific monopolistic competitive labor markets. In this case each firm requires a specific type of labor that cannot be used in other firms. Moreover, within each firm-specific labor market, we allow for Taylor-type staggered wage setting. The following linearized equations display how a worker belonging to the fth wage-setting cohort (with $f \in [1, n_w]$) optimizes its wage in

[8]As in the baseline model, there are two aggregate investment shocks: ε_t^I, which is an investment technology shock, and η_t^Q, which is meant to capture stochastic variations in the external finance premium. The first one is assumed to follow an AR(1) process with an i.i.d.-normal error term and the second is assumed to be i.i.d.-normal distributed.

period t for the labor it rents to the firms of the jth price-setting cohort (with $j \in [1, n_p]$):

$$\begin{aligned} \hat{w}_t^*(f,j) = & \frac{1}{\sum_{i=0}^{n_w-1} \beta^i} \\ & \times \left[\sum_{i=0}^{n_w-1} \beta^i \left(\sigma_l \hat{l}_{t+i}(f,j) + \frac{1}{1-h}(\hat{c}_{t+i} - h\hat{c}_{t+i-1}) - \varepsilon_{t+i}^l \right) \right. \\ & \left. + \sum_{i=1}^{n_w-1} \left((\hat{\pi}_{t+i} - \gamma_w \hat{\pi}_{t+i-1} - (1-\gamma_w)\bar{\pi}_{t+i}) \sum_{q=i}^{n_w-1} \beta^q \right) \right] \end{aligned} \tag{5b}$$

$$\hat{w}_t(j) = \frac{1}{n_w} \left[\sum_{i=0}^{n_w-1} \hat{w}_t(f-i,j) + \hat{p}_{t-i} \right] - \hat{p}_t + \varepsilon_t^w \tag{6b}$$

$$\hat{w}_{t+i}(f,j) = \hat{w}_t^*(f,j) + \sum_{q=0}^{i-1} (\gamma_w \hat{\pi}_{t-1-q} + (1-\gamma_w)\bar{\pi}_{t-q}) \tag{7b}$$

$$\hat{l}_{t+i}(f,j) = \hat{l}_{t+i}(j) - \frac{1+\lambda_w}{\lambda_w}(\hat{w}_{t+i}(f,j) + \hat{p}_t - (\hat{w}_{t+i}(j) + \hat{p}_{t+i})) \tag{8b}$$

$$\hat{l}_t(j) = -\hat{w}_t(j) + (1+\psi)\hat{\rho}_t(j) + \hat{K}_{t-1}(j). \tag{23b}$$

It directly appears from these equations that there is now a labor market for each cohort of firms. Contrarily to the homogeneous labor setting, the labor demand of (cohort of) firm(s) j (equation [23b]) directly affects the optimal wage chosen by the worker f (equation [5b]) and, consequently, the cohort-specific average wage (equation [6b]). When $\gamma_w = 0$, real wages do not depend on the lagged inflation rate.[9]

Due to the staggered wage setting, it is not so simple to see how changes in the demand for the firm's output will affect the firm-specific wage cost (equation [12]). A number of intuitive statements can, however, be made. First, higher wage stickiness as captured by the length of the typical wage contract will tend to reduce the

[9]Parameter ψ is the inverse of the elasticity of the capital-utilization cost function.

response of wages to demand. As a result, high wage stickiness is likely to reduce the impact of firm-specific labor markets on the estimated degree of nominal price stickiness. In contrast, with flexible wages, the relative wage effect may be quite substantial, contributing to large changes in relative marginal cost of the firm and thereby dampening the relative price effects discussed above. Second, this effect is likely to be larger the higher the demand elasticity of labor (as captured by a lower labor-market markup parameter) and the higher the elasticity of labor supply. Concerning the latter, if labor supply is infinitely elastic, wages will again tend to be very sticky and, as a result, relative wage costs will not respond very much to changes in relative demand, even in the case of firm-specific labor markets.

3.2 Alternative Models

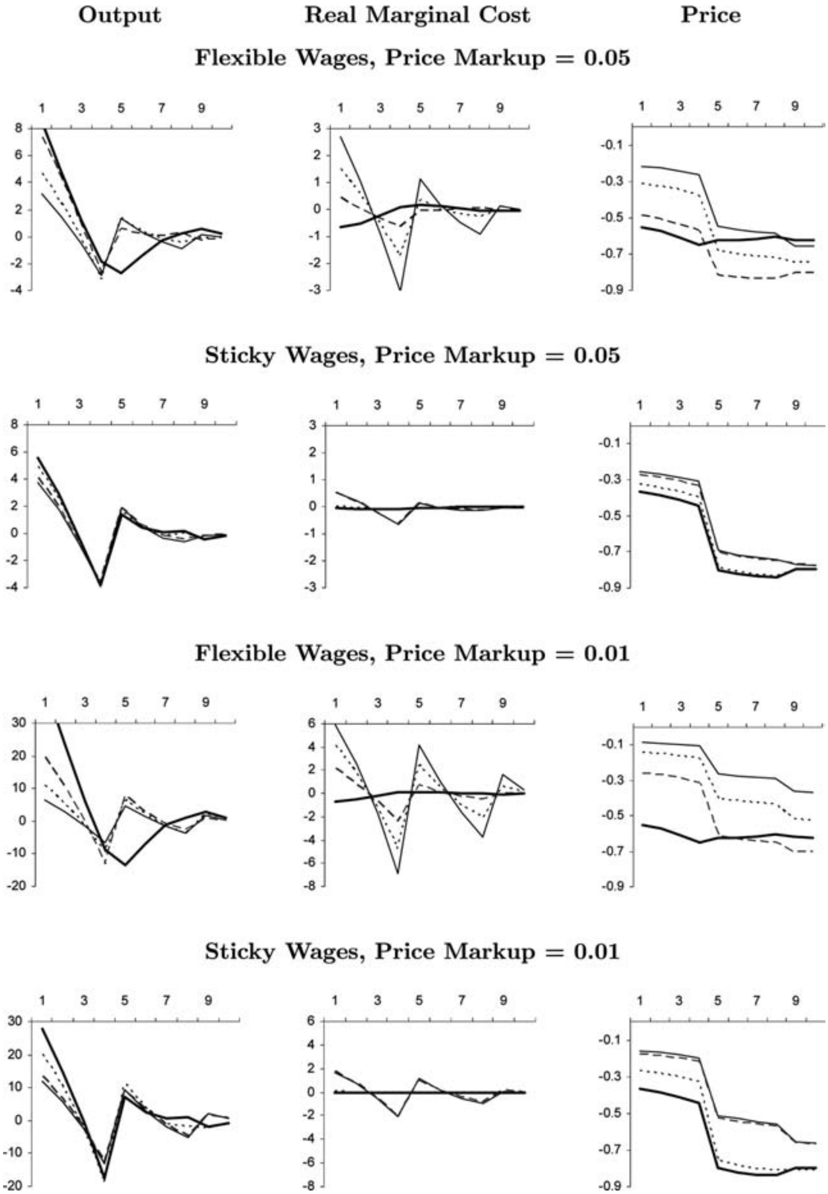
In this section we illustrate the discussion above by displaying how the output, the marginal cost, and the price of the first price-setting cohort respond to a monetary policy shock. We compare the benchmark model with mobile production factors (hereafter denoted MKL) with the following three models:

1. A model with homogeneous capital and firm-specific labor market (hereafter denoted NML).
2. A model with firm-specific capital and homogeneous labor (hereafter denoted NMK).
3. A model with firm-specific capital and labor (hereafter denoted NMKL).

Moreover, for each of those models we consider four cases corresponding to flexible and sticky wages and low (0.01) and high (0.05) markups in the goods market.[10] Figure 4 shows the responses of

[10]This corresponds to demand elasticities of 21 and 101, respectively. The latter is the one estimated by Altig et al. (2005). Furthermore, one needs rather high substitution elasticities to observe significant differences between the homogeneous marginal cost model and its firm-specific production-factors counterparts. So, for demand elasticities below 10, there is nearly no difference between the MKL model and the NML, NMK, and NMKL ones. This indicates again the importance of a very elastic demand curve.

Figure 4. The Effect of a Monetary Policy Shock on Output, Marginal Cost, and Price of the First Cohort in the Three Considered Models



Legend: Bold black line: MKL; black line: NMKL; dashed line: NMK; dotted line: NML the cohort that is allowed to change its price in the period of the monetary policy shock. In this figure we assume that the length of the price and wage contracts is four quarters. The rest of the parameters are those estimated for the benchmark Taylor model (MKL) with the corresponding contract length. Responses are displayed for the first ten quarters following the shock, i.e., prices are reoptimized three times by the considered cohort in the time span considered, at periods 1, 5, and 9.

Several points are worth noting. First, introducing firm-specific factors always reduces the initial impact on prices and output, while it increases the impact on the marginal cost. As discussed above, with firm-specific production factors, price-setting firms internalize the fact that large price responses lead to large variations in marginal costs and therefore lower their initial price response. Second, the introduction of firm-specific factors increases the persistence of price changes, in particular, when wages are flexible. While in the case of mobile production factors with flexible wages, the initial price decrease is partially reversed after four quarters; prices continue to decrease five and nine quarters after the initial shock when factors are firm specific. Third, in the case with mobile factors (MKL—the bold black curve in figure 4), it is clear that prices and marginal cost are not affected by changes in the demand elasticity, while the firm's output is very much affected. On the contrary, for all the models with at least one nonmobile production factor, price responses decrease, while marginal cost variations increase with a higher demand elasticity.

Finally, as long as wages are considered to be flexible, a firm-specific labor market is the device that leads to the largest reactions in marginal cost. It is also worth noting that the combination of a firm-specific labor market and firm-specific capital brings more reaction in the marginal cost than the respective effect of each assumption separately.[11] However, as soon as wages become sticky, firm-specific labor markets do not generate much more variability in marginal cost. In this case, it is striking that the responses of the NMK and NMKL models get very close to each other.

[11] This is actually much in line with the findings of Matheron (2005) in a Calvo price-flexible wage setting with firm-specific capital and labor.

3.3 Estimation

In this section we reestimate the model with firm-specific production factors to investigate the effects on the empirical performance of the model. Sbordone (2002) and Galí, Gertler, and López-Salido (2001) show that considering capital as a fixed factor that cannot be moved across firms does indeed reduce the estimated degree of nominal price stickiness in U.S. data. In particular, it reduces the implied duration of nominal contracts from an implausibly high number of more than two years to a duration of typically less than a year. Altig et al. (2005) reach the same conclusion in a richer setup where firms endogenously determine their capital stock. In this section, we extend this analysis to the case of firm-specific labor markets and test whether similar results are obtained in the context of Taylor contracts.

Table 2 reports the log data densities of the three models considered above and their flexible/sticky wages variants for various price-contract lengths. A higher log data density implies a better empirical fit in terms of the model's one-step-ahead prediction performance.

The following findings are noteworthy. First, in almost all cases, the data prefer the sticky-wage version over the flexible-wage version.

Table 2. Log Data Densities for the Three Models Considered and Their Variants

	2-Q Taylor	4-Q Taylor	6-Q Taylor	8-Q Taylor
Flexible Wages				
NML	–520.21	–481.86	–492.87	–490.16
NMKL	–484.92	–479.56	–481.87	–485.23
NMK	–486.50	–480.68	–482.16	–481.97
Sticky Wages (4-Quarter Taylor Contract)				
NML	–512.50	–490.19	–484.72	–480.54
NMKL	–484.46	–466.10	–475.80	–477.23
NMK	–479.11	–464.92	–473.17	–474.30

This is not surprising, as sticky wages are better able to capture the empirical persistence in wage developments. In what follows, we therefore focus on the sticky-wage models. Second, with sticky wages, the data prefer the model with firm-specific capital but mobile labor. The introduction of firm-specific labor markets does not help the empirical fit of the model. The main reason for this result is that, as argued before, in order for firm-specific labor markets to help in explaining price and inflation persistence, one needs a strong response of wages to changes in demand. But this is in contrast to the observed persistence in wage developments. On the other hand, as we do not observe the rental rate of capital, no such empirical constraint is relevant for the introduction of firm-specific capital. Finally, introducing firm-specific capital does indeed reduce the contract length that fits the data best. While the log data density is maximized at a contract length of nineteen quarters in the case of homogeneous production factors, it is maximized at only four quarters when capital cannot move across firms. This is clearly displayed in figure 3 (even though it is shown for a variant model with endogenous markup developed in section 4.4 below). As clarified by Dixon and Kara (2006), this is equivalent in terms of price duration to a Calvo probability of not reoptimizing equal to 0.6. This confirms the findings of Galí, Gertler, and López-Salido (2001) and Altig et al. (2005). Moreover, it turns out that the four-quarter Taylor contracting model with firm-specific capital performs as well as the nineteen-quarter Taylor contracting model with mobile capital.

In line with these results, in the rest of the paper we will focus on the model with firm-specific capital, homogeneous labor, and sticky wages. Table 3 presents a selection of the parameters estimated for this model with various contract lengths. Note that, in comparison to the case with homogeneous production factors, we also estimate the elasticity of substitution between the goods of the various cohorts. A number of findings are worth noting. First, allowing for firm-specific capital leads to a drop in the estimated degree of indexation to past inflation in the goods sector. In comparison with results displayed in table 1, in this case the parameter drops back to the low level estimated for the Calvo model and does not appear to be significantly different from zero. Second, as discussed in Coenen and Levin (2004), one advantage of the Taylor price setting is that the price-markup parameter is identified and therefore can

Table 3. A Selection of Estimated Parameters for the Taylor Contract Models with Firm-Specific Capital (NMK)

	2-Q Taylor	**4-Q Taylor**	**6-Q Taylor**	**8-Q Taylor**
σ_p	0.216	0.225	0.232	0.230
	(0.016)	(0.016)	(0.019)	(0.017)
ρ_p	0.997	0.979	0.863	0.802
	(0.002)	(0.029)	(0.124)	(0.085)
$1 + \phi$	1.616	1.515	1.522	1.520
	(0.093)	(0.138)	(0.111)	(0.100)
λ_p	0.0008	0.004	0.008	0.016
	(0.0003)	(0.0015)	(0.003)	(0.006)
γ_p	0.067	0.093	0.149	0.220
	(0.070)	(0.077)	(0.094)	(0.102)
γ_w	0.403	0.463	0.547	0.436
	(0.195)	(0.210)	(0.232)	(0.231)

Note: ρ_p is the persistence parameter associated to the price shock; σ_p is the standard error of the price shock; γ_w and γ_p are, respectively, the wage and price indexation parameters; ϕ is the share of the fixed cost; λ_p is the price markup.

be estimated. In contrast, with Calvo price setting, the model with firm-specific factors is observationally equivalent to its counterpart with homogeneous production factors. Table 3 shows that one needs a very high elasticity of substitution (or a low markup) to match the Calvo model in terms of empirical performance. It is also interesting to note that the estimated price markup increases with the length of the price contract, showing the substitutability between nominal and real rigidities. Finally, the persistence parameter of the price shock significantly decreases with the length of the price contract.

For the four-quarter price-contract model, the estimated parameter for the price markup is 0.004, which implies an extremely high elasticity of substitution of about 250. This clearly indicates that one needs large real rigidities in order to compensate for the reduction in

Table 4. Estimated Models with Constrained and/or Endogenous Demand Elasticity (Some Selected Parameters)

	$\phi = \lambda_p$ and $\epsilon = 0$	$\phi = \lambda_p$ and $\epsilon \neq 0$
Log Data Density	−479.671	−468.344
σ_p	0.208	0.178
	(0.015)	(0.013)
ρ_p	0.829	0.539
	(0.086)	(0.056)
σ_a	1.099	0.650
	(0.153)	(0.088)
ρ_a	0.960	0.981
	(0.011)	(0.007)
$\lambda_p = \phi$	0.006	0.489
	(0.001)	(0.128)
$\frac{\epsilon}{1+\epsilon}$	0	0.986
	–	(0.004)

Note: ρ_a and ρ_p are the persistence parameters associated with the productivity and the price shock, respectively; σ_a and σ_p are the standard error of the productivity and the price shock, respectively; γ_w and γ_p are, respectively, the wage and price indexation parameters; ϕ is the share of the fixed cost; λ_p and λ_w are, respectively, the price and the wage markup; ϵ is the curvature parameter.

price stickiness. However, this implies that the estimated fixed cost in production ($1 + \phi$ stands at 1.515) very much exceeds the profit margin, implying negative profits in steady state.

In order to circumvent this problem, one may simply impose the zero-profit condition in steady state. The estimation result obtained for the four-quarter price-contract model is displayed in the first column of table 4. The empirical cost of imposing the constraint is rather high, about 15 in log data density. Furthermore, the estimated demand elasticity remains very high at about 167. Note also that the constraint leads to a much larger estimated standard error of the productivity shock.

3.4 Endogenous Price Markup

Following Eichenbaum and Fisher (2004) and Coenen and Levin (2004), we can consider a model with an endogenous markup, whereby the optimal markup is a function of the relative price as in Kimball (1995). Replacing the Dixit-Stiglitz aggregator with the homogeneous-degree-one aggregator considered by Kimball (1995), the linearized optimal price equation (3b) becomes

$$\hat{p}_t^*(j) = \frac{1}{\sum_{i=0}^{n_p-1}\beta^i}\left[\frac{1}{1+\lambda_p\cdot\epsilon}\sum_{i=0}^{n_p-1}\beta^i\hat{s}_{t+i}(j) + \sum_{i=0}^{n_p-1}\beta^i\hat{p}_{t+i} - \sum_{i=0}^{n_p-2}\left((\gamma_p\hat{\bar{\pi}}_{t+i} + (1-\gamma_p)\bar{\pi}_{t+i+1})\sum_{q=i+1}^{n_p-1}\beta^q\right)\right], \tag{16}$$

where ϵ represents the deviation from the steady-state demand elasticity following a change in the relative price, while λ_p is the steady-state markup:[12]

$$\epsilon = \frac{\partial\left(\frac{1+\lambda_p(z)}{\lambda_p(z)}\right)}{\partial p^*}\cdot\frac{p^*}{\frac{1+\lambda_p(z)}{\lambda_p(z)}}\Bigg|_{z=1} \tag{17}$$

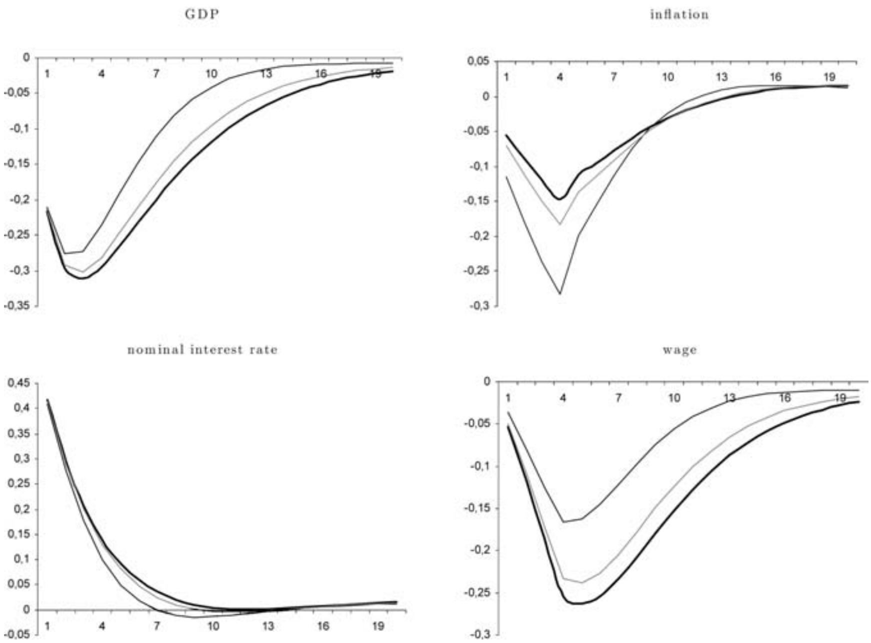
This elasticity plays the same role as the elasticity of substitution: the larger it is, the less the optimal price is sensitive to changes in the marginal cost. In this sense, having $\epsilon > 0$ can help to reduce the estimate for the demand elasticity to a more realistic level.

In order to illustrate this mechanism, figure 5 displays the reactions of aggregate output, inflation, the real wage, and the interest rate after a monetary policy shock for a model with an endogenous price markup. As benchmark, we use the four-quarter price-contract model with constant price markup estimated in table 3

[12] Of course, the Dixit-Stiglitz aggregator corresponds to the case where ϵ is equal to zero.

Figure 5. Assessing the Substitutability between the Steady State Demand Elasticity and the Curvature Parameter

Monetary Policy Shock



Legend: Bold black line: estimated 4-quarter NMK model with fixed markup; black line: $\lambda_p = 0.5$ and $\epsilon = 20$; gray line: $\lambda_p = 0.5$ and $\epsilon = 60$.

and we compare it with the model integrating both the zero-profit constraint and the endogenous price markup. For the latter model, we use the parameters estimated for the benchmark, except for the steady-state markup, λ_p, which is fixed at 0.5, while different values are used for the curvature parameter ϵ: 20 and 60. It is clear from figure 5 that an endogenous price markup that is very sensitive to the relative price can produce the same effect on aggregate variables as a very small constant price markup.

The next step is to reestimate the NMK model with four-quarter price and wage Taylor contracts but adding the modifications discussed above, i.e., imposing the price markup to equate the share of the fixed cost ($\phi = \lambda_p$) and allowing ϵ to be different from zero.

The results are displayed in column 2 of table 4. When the share of the fixed cost is forced to equate the markup, shifting from a final good production function with a constant price markup to one with a price markup declining in the relative price, the estimated steady-state price markup becomes much larger, implying a demand elasticity of about 3. This helps to reduce the cost of the constraint, and the log data density is improved by 11. The very high estimated curvature parameter ϵ (about 70) reveals the need for real rigidities.

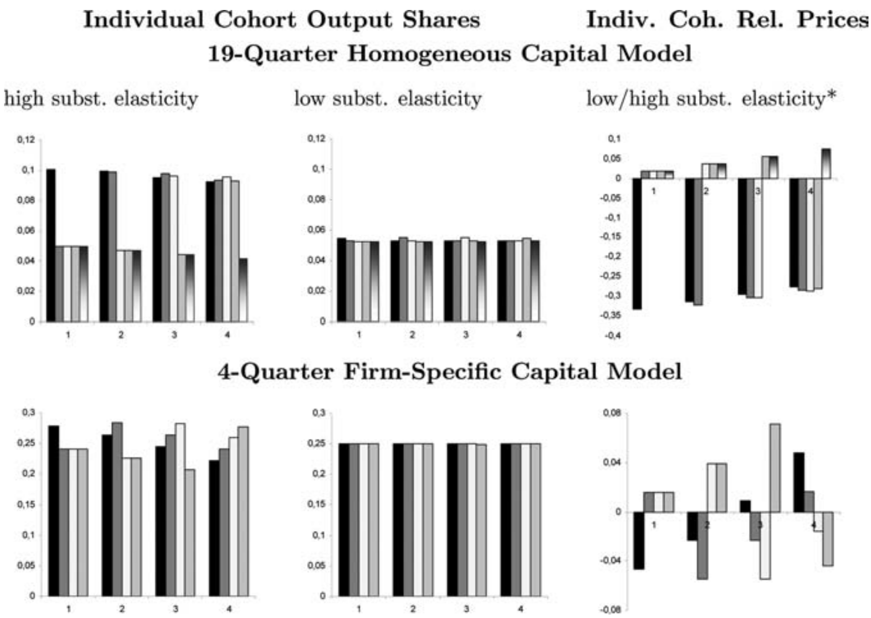
3.5 Comparing Models

Based on the log data density of the estimated models, we are not able to discriminate between the model with homogeneous capital and very long price contracts and the model with firm-specific capital, endogenous markup, and short price contracts. We are then somewhat in the same position as Altig et al. (2005), who have to compare two models that are observationally equivalent from a macroeconomic point of view. These authors reject the model with homogeneous capital for two reasons. First, it implies a price stickiness not in line with microevidence, and second, it generates too-high volatility in cohort-specific output shares. In this section we compare the various models in terms of their implied behavior of cohort-specific output shares and relative prices. The latter allows us to confront the models also with the microevidence on firms' price setting, which finds that price changes are typically large.

Figure 6 compares the evolution of the output share (as a percentage deviation from the steady state) of the first four cohorts of firms during the first four periods following a monetary policy shock and the corresponding relative price changes. We run this comparison for four models: (i) the four-quarter Taylor contracting model with nonmobile capital and a high elasticity of substitution (table 3, column 2); (ii) its variant with constrained elasticity of substitution and endogenous markup (table 4, column 2); (iii) the nineteen-quarter Taylor contracting model with mobile capital and an elasticity of substitution equal to 250; and (iv) the same model with a substitution elasticity of 3.

First, focusing on the evolution of the relative prices in these models, we observe that relative prices vary much more across cohorts in the homogeneous factor model than in the model with

Figure 6. Output Shares and Relative Prices for the First Four Periods after a Monetary Policy Shock in Homogeneous and Firm-Specific Capital Models



Legend: Columns from left to right are for cohort 1 to cohort 4. Column 5 is for the fifteen cohorts that have not yet had the opportunity to reoptimize their price.
*See footnote 18.

firm-specific capital.[13] There are two reasons for such a higher volatility: (i) the fact that the marginal cost is independent of firm-specific output and (ii) the length of the price contract, which implies that only a small fraction of firms can actually change their price. The corollary of this high relative price variability is a much larger variability in the market shares of firms in the model with

[13]Note that the relative prices are displayed only for the model with firm-specific capital and endogenous markup and for the model with mobile capital. Indeed, in the case of mobile capital, the relative prices are not influenced by the subsitution elasticity. For the two models with firm-specific capital, the numbers for relative prices are extremely close, and showing them twice would prove redundant.

homogeneous capital and a high substitution elasticity. In that case, the first cohort to reset optimally its price nearly doubles its share in production. Even though this result is less extreme than the one presented in Altig et al. (2005),[14] such a high variability in output shares following a monetary policy shock is empirically implausible. However, reducing the huge elasticity of substitution to the level consistent with a zero-profit condition, we observe that the variability of the market share becomes quite small in both models, which weakens the argument made by Altig et al. (2005) in favor of the model with firm-specific capital. Furthermore, it is also clear from figure 6 that the model with firm-specific capital fails to reproduce the large price changes observed at the microlevel.

To conclude this section, the introduction of firm-specific capital helps to reconcile the macro models with the microevidence concerning the frequency of the price changes. However, the mechanism for this achievement is entirely based on a very strong reaction of the marginal cost to output changes, which implies very small relative price variations. Such small relative price changes are incompatible with the microevidence, which typically finds that the average size of price changes is quite large.

4. Conclusions

In this paper we have introduced firm-specific production factors in a model with price and wage Taylor contracts. For this type of exercise, Taylor contracts present a twofold advantage over Calvo-type contracts: (i) firm-specific production factors can be introduced and handled explicitly and (ii) the individual firm variables can be analyzed explicitly. This allows a comparison of the implications of the various assumptions concerning the firm-specificity of production factors not only for aggregate variables but also for cross-firm variability.

Our main results are threefold. First, in line with existing literature, we show that introducing firm-specific capital reduces the estimated duration of price contracts from an implausible nineteen

[14]In their model, with their estimated parameters, at the fourth period after the monetary policy shock, 57 percent of the firms produce 180 percent of the global output, leaving the remaining firms with a negative output.

quarters to an empirically more plausible four quarters. Firm-specific production factors make the marginal costs of individual firms steep and very reactive to output changes. Since individual firms' output depends on their relative prices, firms will hesitate to make large price adjustments. Second, introducing firm-specific labor markets does not help in improving the empirical performance of the model. The main reason is that observed wages are sticky, and therefore large variations in firm-specific wages, which help in generating steep marginal costs, are empirically implausible. Overall, it thus appears that rigidities in the reallocation of capital across firms rather than rigidities in the labor market are a more plausible real friction for reducing the estimated degree of nominal price stickiness. Third, in order to obtain this outcome, one needs a very high demand elasticity, implying implausibly large variations in the demand faced by the firms throughout the length of the contract. Imposing the zero-profit condition drastically reduces the estimated demand elasticity and leads to a corresponding reduction in the volatility of output across firms. However, in this case, the need for important real rigidities becomes evident through a high estimated curvature of the demand curve.

To compare the respective merits of the models with mobile production factors (flat marginal cost) and firm-specific production factors (increasing marginal cost), it is important to remember the main findings emerging from microdata on firms' pricing behavior: price changes are at the same time frequent and large (cf. Bils and Klenow 2004; Angeloni et al. 2006). The model with flat marginal costs does lead to large price changes but requires a high degree of nominal stickiness to reproduce inflation persistence. The introduction of firm-specific marginal cost does lead to less nominal stickiness but implies small relative price variations across firms. It thus seems that, so far, neither model can simultaneously satisfy both stylized facts. Altig et al. (2005) favor the model with firm-specific marginal cost on the basis of the argument that it produces less-extreme variations in output shares after an exogenous shock. We have, however, shown that this outcome relies heavily on the price-contract length and on the very large demand elasticity. Introducing additional curvature in the demand function as in Kimball (1995) significantly reduces the variability of output shares in the model with flat marginal costs. Overall, we therefore conclude that other elements such as the presence of firm-specific shocks will have to be introduced to match all the important microstylized facts. Further research on the relationship between prices, output, and marginal costs at the firm level would be very useful in this respect.

Finally, note that in this paper and in contrast to Coenen and Levin (2004), we did not allow for heterogeneity in the contract length. Such heterogeneity is another important stylized fact of the microdata. Moreover, such heterogeneity could help explain the tension between the finding of macropersistence and microflexibility to the extent that the presence of sectors with long price durations can have a disproportionately large effect on the aggregate inflation behavior (Dixon and Kara 2005). Further research along these lines would be worthwhile.

Appendix 1. Data Appendix

All data are taken from the Area Wide Model (AWM) database from the European Central Bank (see Fagan, Henry, and Mestre 2005). Investment includes both private and public investment expenditures. The sample contains data from 1970:Q2 to 2002:Q2, and the first fifteen quarters are used to initialize the Kalman filter. Real variables are deflated with their own deflator. Inflation is calculated as the first difference of the log GDP deflator. In the absence of data on hours worked, we use total employment data for the euro area. As explained in Smets and Wouters (2003), we therefore use for the euro-area model an auxiliary observation equation linking labor services in the model and observed employment based on a Calvo mechanism for the hiring decision of firms. The series are updated for the most recent period using growth rates for the corresponding series published in the ECB's *Monthly Bulletin*. Consumption, investment, GDP, wages, and hours/employment are expressed in 100 times the log. The interest rate and inflation rate are expressed on a quarterly basis corresponding with their appearance in the model (in the graphs the series are translated on an annual basis).

Appendix 2. Model Appendix

This appendix describes the other linearized equations of the Smets-Wouters model (2003, 2004).

Indexation of nominal wages results in the following *real wage equation*:

$$
\begin{aligned}
\hat{w}_t = {} & \frac{\beta}{1+\beta} E_t \hat{w}_{t+1} + \frac{1}{1+\beta} \hat{w}_{t-1} + \frac{\beta}{1+\beta} (E_t \hat{\pi}_{t+1} - \bar{\pi}_t) \\
& - \frac{1+\beta\gamma_w}{1+\beta} (\hat{\pi}_t - \bar{\pi}_t) + \frac{\gamma_w}{1+\beta} (\hat{\pi}_{t-1} - \bar{\pi}_t) \\
& - \frac{1}{1+\beta} \frac{(1-\beta\xi_w)(1-\xi_w)}{\left(1 + \frac{(1+\lambda_w)\sigma_l}{\lambda_w}\right)\xi_w} \\
& \times \left[\hat{w}_t - \sigma_l \hat{l}_t - \frac{1}{1-h}(\hat{c}_t - h\hat{c}_{t-1}) + \varepsilon_t^l \right] + \eta_t^w. \qquad (18)
\end{aligned}
$$

The real wage $\hat{w}_t$ is a function of expected and past real wages and the expected, current, and past inflation rate where the relative weight depends on the degree of indexation γ_w to lagged inflation of the nonoptimized wages. When $\gamma_w = 0$, real wages do not depend on the lagged inflation rate. There is a negative effect of the deviation of the actual real wage from the wage that would prevail in a flexible labor market. The size of this effect will be greater, the smaller the degree of wage stickiness (ξ_w), the lower the demand elasticity for labor (higher markup λ_w), and the lower the inverse elasticity of labor supply (σ_l) or the flatter the labor supply curve. ε_t^l is a preference shock representing a shock to the labor supply and is assumed to follow a first-order autoregressive process with an i.i.d.-normal error term: $\varepsilon_t^l = \rho_l \varepsilon_{t-1}^l + \eta_t^l$. In contrast, η_t^w is assumed to be an i.i.d.-normal wage-markup shock.

The dynamics of *aggregate consumption* are given by

$$
\begin{aligned}
\hat{c}_t = {} & \frac{h}{1+h} \hat{c}_{t-1} + \frac{1}{1+h} E_t \hat{c}_{t+1} + \frac{\sigma_c - 1}{\sigma_c (1+\lambda_w)(1+h)} (\hat{l}_t - E_t \hat{l}_{t+1}) \\
& - \frac{1-h}{(1+h)\sigma_c} (\hat{R}_t - E_t \hat{\pi}_{t+1} + \varepsilon_t^b). \qquad (19)
\end{aligned}
$$

Consumption $\hat{c}_t$ depends on the ex ante real interest rate $(\hat{R}_t - E_t \hat{\pi}_{t+1})$ and, with external habit formation, on a weighted average of past and expected future consumption. When $h = 0$, only the traditional forward-looking term is maintained. In addition, due to the nonseparability of the utility function, consumption will also depend on expected employment growth $(E_t\hat{l}_{t+1} - \hat{l}_t)$. When the elasticity of intertemporal substitution (for constant labor) is smaller than one ($\sigma_c > 1$), consumption and labor supply are complements. Finally, ε_t^b, represents a preference shock affecting the discount rate that determines the intertemporal substitution decisions of households. This shock is assumed to follow a first-order autoregressive process with an i.i.d.-normal error term: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$.

The *investment equation* is given by

$$\hat{I}_t = \frac{1}{1+\beta}\hat{I}_{t-1} + \frac{\beta}{1+\beta}E_t\hat{I}_{t+1} + \frac{1/\varphi}{1+\beta}\hat{Q}_t + \varepsilon_t^I, \tag{20}$$

where $\varphi = \bar{S}''$ depends on the adjustment-cost function (S) and β is the discount factor applied by the households. As discussed in Christiano, Eichenbaum, and Evans (2005), modeling the capital adjustment costs as a function of the change in investment rather than its level introduces additional dynamics in the investment equation, which is useful in capturing the hump-shaped response of investment to various shocks, including monetary policy shocks. A positive shock to the investment-specific technology, ε_t^I, increases investment in the same way as an increase in the value of the existing capital stock $\hat{Q}_t$. This investment shock is also assumed to follow a first-order autoregressive process with an i.i.d.-normal error term: $\varepsilon_t^I = \rho_I \varepsilon_{t-1}^I + \eta_t^I$.

The corresponding *Q equation* is given by

$$\hat{Q}_t = -(\hat{R}_t - \hat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+\bar{r}^k}E_t\hat{Q}_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}E_t\hat{r}_{t+1}^k + \eta_t^Q, \tag{21}$$

where τ stands for the depreciation rate and $\bar{r}^k$ for the rental rate of capital so that $\beta = 1/(1-\tau+\bar{r}^k)$. The current value of the capital stock depends negatively on the ex ante real interest rate and positively on its expected future value and the expected rental rate. The introduction of a shock to the required rate of return on equity investment, η_t^Q, is meant as a shortcut to capture changes in the cost of capital that may be due to stochastic variations in the external finance premium. We assume that this equity premium shock follows an i.i.d.-normal process. In a fully fledged model, the production of capital goods and the associated investment process could be modeled in a separate sector. In such a case, imperfect information between the capital-producing borrowers and the financial intermediaries could give rise to a stochastic external finance premium. Here, we implicitly assume that the deviation between the two returns can be captured by a stochastic shock, whereas the steady-state distortion due to such informational frictions is zero.

The *capital accumulation equation* becomes a function not only of the flow of investment but also of the relative efficiency of these investment expenditures as captured by the investment-specific technology shock:

$$\hat{K}_t = (1-\tau)\hat{K}_{t-1} + \tau\hat{I}_{t-1} + \tau\varepsilon^I_{t-1}. \tag{22}$$

The equalization of marginal cost implies that, for a given installed capital stock, *labor demand* depends negatively on the real wage (with a unit elasticity) and positively on the rental rate of capital:

$$\hat{l}_t = -\hat{w}_t + (1+\psi)\hat{r}^k_t + \hat{K}_{t-1}, \tag{23}$$

where $\psi = \frac{\psi'(1)}{\psi''(1)}$ is the inverse of the elasticity of the capital-utilization cost function.

The *goods market equilibrium condition* can be written as

$$\hat{Y}_t = (1-\tau k_y - g_y)\hat{c}_t + \tau k_y\hat{I}_t + g_y\varepsilon^g_t + +k_y\frac{1-\beta(1-\tau)}{\beta}\psi\hat{r}^k_t \tag{24a}$$

$$= \phi\big[\alpha(\hat{K}_{t-1} + \psi\hat{r}^k_t) + (1-\alpha)(\hat{l}_t + \gamma t\big] - (\phi-1)\gamma t, \tag{24b}$$

where k_y is the steady-state capital-output ratio, g_y is the steady-state government-spending-output ratio, and ϕ is one plus the share of the fixed cost in production. We assume that the government-spending shock follows a first-order autoregressive process with an i.i.d.-normal error term: $\varepsilon^g_t = \rho_g\varepsilon^g_{t-1} + \eta^g_t$.

Finally, the model is closed by adding the following empirical *monetary policy reaction function*:

$$\begin{aligned}\hat{R}_t = \bar{\pi}_t &+ \rho(\hat{R}_{t-1} - \bar{\pi}_{t-1}) \\ &+ (1-\rho)\left[r_\pi(\hat{\pi}_{t-1} - \bar{\pi}_{t-1}) + r_Y(\hat{Y}_{t-1} - \hat{Y}^p_{t-1})\right] \qquad (25) \\ &+ r_{\Delta\pi}\left[(\hat{\pi}_t - \bar{\pi}_t) - (\hat{\pi}_{t-1} - \bar{\pi}_{t-1})\right] \\ &+ r_{\Delta Y}\left[(\hat{Y}_t - \hat{Y}^p_t) - (\hat{Y}_{t-1} - \hat{Y}^p_{t-1})\right] + \eta^R_t. \qquad (26)\end{aligned}$$

The monetary authorities follow a generalized Taylor rule by gradually responding to deviations of lagged inflation from an inflation objective and the lagged output gap defined as the difference between actual and potential output. Consistently with the DSGE model, potential output is defined as the level of output that would prevail under flexible price and wages in the absence of the three "cost-push" shocks. The parameter ρ captures the degree of interest rate smoothing. In addition, there is also a short-run feedback from the current changes in inflation and the output gap. Finally, we assume that there are two monetary policy shocks: one is a temporary i.i.d.-normal interest rate shock (η^R_t) also denoted a monetary policy shock; the other is a permanent shock to the inflation objective ($\bar{\pi}_t$), which is assumed to follow a nonstationary process ($\bar{\pi}_t = \bar{\pi}_{t-1} + \eta^\pi_t$). The dynamic specification of the reaction function is such that changes in the inflation objective are immediately and without cost reflected in actual inflation and the interest rate if there is no exogenous persistence in the inflation process.

Appendix 3. Description of the Priors

Some parameters are fixed. They are principally parameters related to the steady-state values of the state variables. The discount factor β is calibrated at 0.99, corresponding with an annual steady-state real interest rate of 4 percent. The depreciation rate τ is set at 0.025, so that the annual capital depreciation is equal to 10 percent. The steady-state share of capital income is fixed at $\alpha = 0.24$. The share of steady-state consumption in total output is assumed equal to 0.65, and the share of steady-state investment is assumed equal to 0.17.

The priors on the other parameters are displayed in tables 5–8 in the next appendix. The first column is the description of the parameter, the second column shows the prior distribution, and the next two columns give, respectively, the prior mean and standard error. Most of the priors are the same as in Smets and Wouters (2003). However, there is an important difference to note regarding the capital-utilization adjustment-cost parameter (ψ). Instead of estimating $\frac{1}{\psi}$ with a prior [normal 0.2 0.075], we now estimate $cz = \frac{\psi}{1+\psi}$ with a prior [beta 0.5 0.25], which actually corresponds to a much looser prior since it allows for values of the elasticity of the capital utilization cost function between 0.1 and 10. Some new parameters appear: the price and wage markups, which are given a rather loose prior of [beta 0.25 0.15], and the curvature parameter, which is estimated via $eps = \frac{\epsilon}{1+\epsilon}$ with a prior of [beta 0.85 0.1]. The latter allows for values of parameter ϵ between 1.5 and 100.

For the rest, as in Smets and Wouters (2003), the persistence parameters are given a normal prior distribution with a mean of 0.85 and a standard error of 0.10. The variance of the shocks is assumed to follow an inverted gamma distribution with two degrees of freedom.

Appendix 4. Parameter Estimates for the Main Models

The Metropolis-Hastings algorithm has been run with 250,000 draws. Convergence is assessed with the help of cumsum graphs and using the Brooks and Gelman (1998) uni- and multivariate tests performed by the Dynare software.

Table 5. Baseline Calvo Model (Table 1, First Column)

Marginal Likelihood:
Laplace Approximation: −471.113
Modified Harmonic Mean: −470.407

	Prior Distribution			Estimated Posterior Mode and Mean				Posterior Sample Based				
	Type	Mean	St. Error	Mode	St. Error	Mean	St. Error	5%	10%	50%	90%	95%
St. Dev. of the Shocks												
Productivity Shock	Inv. Gamma	0.250	2 d.f.	0.654	0.094	0.672	0.098	0.533	0.556	0.661	0.802	0.848
Inflation Obj. Shock	Inv. Gamma	0.050	2 d.f.	0.109	0.014	0.113	0.015	0.090	0.095	0.113	0.132	0.138
Cons. Pref. Shock	Inv. Gamma	0.250	2 d.f.	0.194	0.044	0.215	0.051	0.147	0.158	0.207	0.282	0.311
Gov. Spend. Shock	Inv. Gamma	0.250	2 d.f.	0.346	0.023	0.350	0.023	0.315	0.322	0.349	0.380	0.389
Lab. Supl. Shock	Inv. Gamma	0.250	2 d.f.	1.846	0.499	1.985	0.510	1.285	1.394	1.913	2.675	2.925
Investment Shock	Inv. Gamma	0.250	2 d.f.	0.228	0.046	0.232	0.049	0.163	0.175	0.226	0.295	0.319
Interest Rate Shock	Inv. Gamma	0.250	2 d.f.	0.142	0.018	0.144	0.017	0.118	0.123	0.144	0.166	0.174
Equity Premium Shock	Inv. Gamma	0.250	2 d.f.	0.564	0.052	0.565	0.058	0.471	0.491	0.563	0.639	0.663
Price Shock	Inv. Gamma	0.250	2 d.f.	0.207	0.019	0.211	0.020	0.182	0.188	0.210	0.236	0.245
Wage Shock	Inv. Gamma	0.250	2 d.f.	0.209	0.021	0.255	0.024	0.218	0.226	0.254	0.287	0.297
Persistence Parameters												
Productivity Shock	Beta	0.850	0.100	0.991	0.007	0.990	0.007	0.978	0.982	0.992	0.997	0.998
Cons. Pref. Shock	Beta	0.850	0.100	0.890	0.020	0.896	0.023	0.856	0.866	0.897	0.924	0.931
Gov. Spend. Shock	Beta	0.850	0.100	0.994	0.006	0.984	0.011	0.963	0.969	0.987	0.996	0.997
Lab. Supl. Shock	Beta	0.850	0.100	0.979	0.008	0.978	0.009	0.963	0.967	0.979	0.989	0.991
Investment Shock	Beta	0.850	0.100	0.995	0.005	0.988	0.009	0.970	0.976	0.990	0.997	0.998

(*continued*)

Table 5. (continued). Baseline Calvo Model (Table 1, First Column)

Marginal Likelihood:
Laplace Approximation: −471.113
Modified Harmonic Mean: −470.407

	Prior Distribution			Estimated Posterior Mode and Mean				Posterior Sample Based				
	Type	Mean	St. Error	Mode	St. Error	Mean	St. Error	5%	10%	50%	90%	95%
Miscellaneous												
Invest. Adj. Cost	Normal	4.000	1.500	5.501	1.014	5.765	1.031	4.159	4.470	5.710	7.131	7.551
Hsehold. Rel. Risk Aversion	Normal	1.000	0.375	2.254	0.309	2.109	0.307	1.597	1.707	2.112	2.508	2.620
Consumption Habit	Beta	0.700	0.100	0.483	0.053	0.502	0.051	0.419	0.438	0.502	0.567	0.585
Labor Utility	Normal	2.000	0.750	1.323	0.869	1.397	0.700	0.393	0.518	1.331	2.353	2.655
Calvo Employment	Beta	0.500	0.100	0.654	0.046	0.654	0.043	0.581	0.598	0.656	0.709	0.723
Calvo Wage	Beta	0.750	0.050	0.712	0.046	0.699	0.049	0.620	0.637	0.700	0.758	0.777
Calvo Price	Beta	0.750	0.050	0.891	0.014	0.890	0.012	0.870	0.874	0.889	0.905	0.910
Indexation Wage	Beta	0.500	0.250	0.389	0.197	0.381	0.183	0.098	0.146	0.369	0.627	0.704
Indexation Price	Beta	0.500	0.250	0.178	0.096	0.184	0.087	0.052	0.075	0.177	0.303	0.339
Cap. Util. Adj. Cost	Beta	0.500	0.250	0.815	0.105	0.850	0.078	0.711	0.745	0.856	0.949	0.967
Fixed Cost	Normal	1.250	0.125	1.715	0.104	1.740	0.104	1.561	1.604	1.743	1.869	1.905
Trend	Normal	0.400	0.025	0.331	0.027	0.324	0.023	0.288	0.295	0.323	0.354	0.363
Policy Rule Parameters												
r Inflation	Normal	1.500	0.100	1.510	0.102	1.529	0.100	1.364	1.399	1.528	1.658	1.694
r d(inflation)	Normal	0.300	0.100	0.101	0.049	0.115	0.047	0.037	0.053	0.115	0.177	0.193
r Lagged Interest Rate	Beta	0.750	0.050	0.901	0.017	0.895	0.018	0.863	0.871	0.896	0.918	0.924
r Output	Beta	0.125	0.050	0.069	0.034	0.092	0.038	0.038	0.046	0.087	0.145	0.162
r d(output)	Beta	0.063	0.050	0.127	0.034	0.132	0.034	0.078	0.090	0.130	0.176	0.191

Table 6. MK Model, 19-Quarter Price Contract (Table 1, Last Column)

Marginal Likelihood:
Laplace Approximation: −468.469
Modified Harmonic Mean: −467.496

	Prior Distribution			Estimated Posterior Mode and Mean				Posterior Sample Based				
	Type	Mean	St. Error	Mode	St. Error	Mean	St. Error	5%	10%	50%	90%	95%
St. Dev. of the Shocks												
Productivity Shock	Inv. Gamma	0.250	2 d.f.	0.622	0.085	0.636	0.091	0.508	0.531	0.625	0.758	0.805
Inflation Obj. Shock	Inv. Gamma	0.050	2 d.f.	0.104	0.017	0.110	0.018	0.083	0.088	0.109	0.134	0.141
Cons. Pref. Shock	Inv. Gamma	0.250	2 d.f.	0.162	0.028	0.188	0.038	0.136	0.144	0.182	0.237	0.254
Gov. Spend. Shock	Inv. Gamma	0.250	2 d.f.	0.346	0.023	0.349	0.024	0.312	0.320	0.348	0.381	0.390
Lab. Supl. Shock	Inv. Gamma	0.250	2 d.f.	0.324	0.173	0.694	0.393	0.232	0.275	0.628	1.174	1.367
Investment Shock	Inv. Gamma	0.250	2 d.f.	0.205	0.039	0.203	0.041	0.146	0.155	0.198	0.259	0.280
Interest Rate Shock	Inv. Gamma	0.250	2 d.f.	0.158	0.017	0.157	0.020	0.128	0.134	0.156	0.182	0.192
Equity Premium Shock	Inv. Gamma	0.250	2 d.f.	0.557	0.052	0.563	0.059	0.469	0.489	0.562	0.638	0.660
Price Shock	Inv. Gamma	0.250	2 d.f.	0.229	0.016	0.233	0.018	0.206	0.211	0.231	0.256	0.263
Wage Shock	Inv. Gamma	0.250	2 d.f.	0.454	0.035	0.459	0.037	0.404	0.414	0.455	0.509	0.525
Persistence Parameters												
Productivity Shock	Beta	0.850	0.100	0.983	0.006	0.981	0.007	0.969	0.972	0.982	0.990	0.992
Cons. Pref. Shock	Beta	0.850	0.100	0.907	0.017	0.900	0.021	0.866	0.874	0.902	0.925	0.930
Gov. Spend. Shock	Beta	0.850	0.100	0.990	0.009	0.983	0.010	0.963	0.968	0.985	0.995	0.997
Lab. Supl. Shock	Beta	0.850	0.100	0.904	0.067	0.888	0.084	0.713	0.778	0.911	0.969	0.976
Investment Shock	Beta	0.850	0.100	0.993	0.005	0.983	0.015	0.947	0.965	0.987	0.995	0.996
Price Shock	Beta	0.850	0.100	0.934	0.018	0.932	0.021	0.896	0.906	0.933	0.957	0.964
Wage Shock	Beta	0.850	0.100	0.955	0.013	0.950	0.017	0.917	0.928	0.953	0.968	0.972

(continued)

Table 6. (continued). MK Model, 19-Quarter Price Contract (Table 1, Last Column)

Marginal Likelihood:
Laplace Approximation: −468.469
Modified Harmonic Mean: −467.496

	Prior Distribution			Estimated Posterior Mode and Mean				Posterior Sample Based				
	Type	Mean	St. Error	Mode	St. Error	Mean	St. Error	5%	10%	50%	90%	95%
Miscellaneous												
Invest. Adj. Cost	Normal	4.000	1.500	6.543	1.032	6.396	1.036	4.736	5.084	6.364	7.770	8.155
Hsehold. Rel. Risk Aversion	Normal	1.000	0.375	2.085	0.274	1.986	0.282	1.522	1.622	1.992	2.337	2.450
Consumption Habit	Beta	0.700	0.100	0.340	0.049	0.376	0.054	0.291	0.308	0.375	0.445	0.468
Labor Utility	Normal	2.000	0.750	0.495	0.334	0.701	0.335	0.236	0.309	0.662	1.149	1.317
Calvo Employment	Beta	0.500	0.100	0.645	0.043	0.639	0.042	0.568	0.585	0.640	0.692	0.707
Indexation Wage	Beta	0.500	0.250	0.461	0.188	0.470	0.189	0.163	0.226	0.464	0.727	0.795
Indexation Price	Beta	0.500	0.250	0.273	0.074	0.274	0.078	0.146	0.173	0.276	0.372	0.398
Cap. Util. Adj. Cost	Beta	0.500	0.250	0.825	0.080	0.819	0.080	0.680	0.712	0.824	0.921	0.946
Fixed Cost	Normal	1.250	0.125	1.573	0.099	1.577	0.098	1.421	1.452	1.574	1.707	1.743
Wage Markup	Beta	0.250	0.150	0.206	0.123	0.279	0.124	0.105	0.132	0.264	0.445	0.512
Trend	Normal	0.400	0.025	0.394	0.023	0.391	0.022	0.354	0.362	0.390	0.419	0.427
Policy Rule Parameters												
r Inflation	Normal	1.500	0.100	1.562	0.084	1.575	0.082	1.443	1.470	1.574	1.683	1.714
r d(inflation)	Normal	0.300	0.100	0.197	0.046	0.200	0.048	0.123	0.139	0.199	0.263	0.281
r Lagged Interest Rate	Beta	0.750	0.050	0.869	0.018	0.862	0.020	0.828	0.836	0.864	0.887	0.893
r Output	Beta	0.125	0.050	0.094	0.026	0.101	0.028	0.058	0.066	0.099	0.139	0.151
r d(output)	Beta	0.063	0.050	0.185	0.048	0.193	0.052	0.112	0.128	0.190	0.260	0.282

Table 7. NMK Model, 4-Quarter Price Contract, $\phi \neq \lambda_p$ and $\varepsilon = 0$ (Table 3, Column 2)

Marginal Likelihood:
Laplace Approximation: −464.920
Modified Harmonic Mean: −463.902

	Prior Distribution			Estimated Posterior Mode and Mean				Posterior Sample Based				
	Type	Mean	St. Error	Mode	St. Error	Mean	St. Error	5%	10%	50%	90%	95%
St. Dev. of the Shocks												
Productivity Shock	Inv. Gamma	0.250	2 d.f.	0.655	0.092	0.680	0.092	0.544	0.569	0.672	0.803	0.845
Inflation Obj. Shock	Inv. Gamma	0.050	2 d.f.	0.129	0.016	0.133	0.016	0.107	0.112	0.132	0.154	0.160
Cons. Pref. Shock	Inv. Gamma	0.250	2 d.f.	0.138	0.026	0.164	0.032	0.120	0.127	0.159	0.207	0.224
Gov. Spend. Shock	Inv. Gamma	0.250	2 d.f.	0.347	0.023	0.351	0.023	0.316	0.323	0.350	0.380	0.389
Lab. Supl. Shock	Inv. Gamma	0.250	2 d.f.	0.284	0.112	0.512	0.331	0.204	0.231	0.414	0.898	1.069
Investment Shock	Inv. Gamma	0.250	2 d.f.	0.254	0.049	0.247	0.048	0.177	0.190	0.243	0.310	0.333
Interest Rate Shock	Inv. Gamma	0.250	2 d.f.	0.131	0.015	0.135	0.015	0.112	0.117	0.135	0.155	0.161
Equity Premium Shock	Inv. Gamma	0.250	2 d.f.	0.537	0.053	0.538	0.057	0.445	0.465	0.537	0.612	0.634
Price Shock	Inv. Gamma	0.250	2 d.f.	0.225	0.016	0.227	0.018	0.199	0.205	0.225	0.250	0.257
Wage Shock	Inv. Gamma	0.250	2 d.f.	0.441	0.035	0.449	0.035	0.395	0.406	0.447	0.495	0.512
Persistence Parameters												
Productivity Shock	Beta	0.850	0.100	0.979	0.009	0.979	0.008	0.964	0.968	0.979	0.988	0.990
Cons. Pref. Shock	Beta	0.850	0.100	0.922	0.019	0.914	0.016	0.885	0.892	0.915	0.934	0.938
Gov. Spend. Shock	Beta	0.850	0.100	0.992	0.009	0.984	0.010	0.965	0.971	0.986	0.995	0.997
Lab. Supl. Shock	Beta	0.850	0.100	0.882	0.087	0.855	0.098	0.668	0.718	0.874	0.965	0.976
Investment Shock	Beta	0.850	0.100	0.997	0.003	0.991	0.007	0.977	0.982	0.993	0.998	0.999
Price Shock	Beta	0.850	0.100	0.979	0.029	0.947	0.045	0.851	0.878	0.961	0.987	0.991
Wage Shock	Beta	0.850	0.100	0.959	0.011	0.954	0.014	0.929	0.937	0.956	0.970	0.974

(continued)

Table 7. (continued). NMK Model, 4-Quarter Price Contract, $\phi \neq \lambda_p$ and $\varepsilon = 0$ (Table 3, Column 2)

Marginal Likelihood:
Laplace Approximation: −468.469
Modified Harmonic Mean: −467.496

	Prior Distribution			Estimated Posterior Mode and Mean				Posterior Sample Based				
	Type	**Mean**	**St. Error**	**Mode**	**St. Error**	**Mean**	**St. Error**	**5%**	**10%**	**50%**	**90%**	**95%**
Miscellaneous												
Invest. Adj. Cost	Normal	4.000	1.500	6.261	1.029	6.221	1.025	4.620	4.930	6.177	7.585	7.986
Hsehold. Rel. Risk Aversion	Normal	1.000	0.375	2.083	0.285	1.956	0.282	1.485	1.594	1.960	2.311	2.413
Consumption Habit	Beta	0.700	0.100	0.348	0.048	0.388	0.055	0.302	0.320	0.387	0.459	0.483
Labor Utility	Normal	2.000	0.750	0.892	0.648	1.267	0.597	0.459	0.583	1.179	2.070	2.382
Calvo Employment	Beta	0.500	0.100	0.650	0.043	0.650	0.038	0.585	0.602	0.652	0.698	0.709
Indexation Wage	Beta	0.500	0.250	0.463	0.210	0.511	0.191	0.190	0.257	0.513	0.764	0.827
Indexation Price	Beta	0.500	0.250	0.093	0.077	0.113	0.065	0.024	0.035	0.103	0.201	0.233
Cap. Util. Adj. Cost	Beta	0.500	0.250	0.834	0.113	0.867	0.070	0.744	0.772	0.873	0.955	0.971
Fixed Cost	Normal	1.250	0.125	1.515	0.138	1.482	0.104	1.313	1.349	1.482	1.616	1.654
Price Markup	Beta	0.250	0.150	0.004	0.002	0.005	0.001	0.003	0.003	0.005	0.007	0.007
Wage Markup	Beta	0.250	0.150	0.280	0.139	0.345	0.125	0.163	0.196	0.334	0.513	0.576
Curvature Parameter	Beta	0.850	0.100									
Trend	Normal	0.400	0.025	0.398	0.023	0.400	0.023	0.364	0.371	0.400	0.429	0.437
Policy Rule Parameters												
r Inflation	Normal	1.500	0.100	1.536	0.083	1.556	0.081	1.429	1.454	1.553	1.661	1.695
r d(inflation)	Normal	0.300	0.100	0.172	0.045	0.183	0.046	0.107	0.124	0.183	0.242	0.259
r Lagged Interest Rate	Beta	0.750	0.050	0.868	0.017	0.861	0.018	0.829	0.837	0.862	0.883	0.889
r Output	Beta	0.125	0.050	0.114	0.027	0.106	0.027	0.066	0.074	0.104	0.142	0.153
r d(output)	Beta	0.063	0.050	0.114	0.035	0.120	0.036	0.064	0.075	0.119	0.168	0.183

Table 8. NMK Model, 4-Quarter Price Contract, $\phi = \lambda_p$ and $\varepsilon \neq 0$ (Table 4, Column 2)

Marginal Likelihood:
Laplace Approximation: −468.344
Modified Harmonic Mean: −467.130

	Prior Distribution			Estimated Posterior Mode and Mean				Posterior Sample Based				
	Type	Mean	St. Error	Mode	St. Error	Mean	St. Error	5%	10%	50%	90%	95%
St. Dev. of the Shocks												
Productivity Shock	Inv. Gamma	0.250	2 d.f.	0.650	0.088	0.659	0.089	0.528	0.552	0.651	0.778	0.820
Inflation Obj. Shock	Inv. Gamma	0.050	2 d.f.	0.130	0.017	0.130	0.017	0.102	0.108	0.129	0.152	0.159
Cons. Pref. Shock	Inv. Gamma	0.250	2 d.f.	0.144	0.024	0.168	0.033	0.124	0.132	0.163	0.211	0.229
Gov. Spend. Shock	Inv. Gamma	0.250	2 d.f.	0.347	0.023	0.350	0.023	0.314	0.321	0.349	0.380	0.389
Lab. Supl. Shock	Inv. Gamma	0.250	2 d.f.	0.286	0.115	0.511	0.289	0.205	0.234	0.420	0.934	1.125
Investment Shock	Inv. Gamma	0.250	2 d.f.	0.250	0.048	0.249	0.051	0.177	0.189	0.243	0.316	0.342
Interest Rate Shock	Inv. Gamma	0.250	2 d.f.	0.130	0.015	0.137	0.016	0.112	0.117	0.136	0.158	0.165
Equity Premium Shock	Inv. Gamma	0.250	2 d.f.	0.538	0.054	0.536	0.057	0.442	0.463	0.535	0.608	0.628
Price Shock	Inv. Gamma	0.250	2 d.f.	0.178	0.013	0.184	0.014	0.162	0.166	0.183	0.202	0.207
Wage Shock	Inv. Gamma	0.250	2 d.f.	0.437	0.033	0.448	0.035	0.395	0.406	0.446	0.493	0.507
Persistence Parameters												
Productivity Shock	Beta	0.850	0.100	0.981	0.007	0.978	0.008	0.964	0.968	0.979	0.988	0.990
Cons. Pref. Shock	Beta	0.850	0.100	0.917	0.013	0.912	0.017	0.882	0.890	0.913	0.931	0.936
Gov. Spend. Shock	Beta	0.850	0.100	0.994	0.006	0.983	0.013	0.959	0.967	0.986	0.996	0.997
Lab. Supl. Shock	Beta	0.850	0.100	0.892	0.093	0.850	0.101	0.656	0.707	0.870	0.963	0.975
Investment Shock	Beta	0.850	0.100	0.996	0.004	0.990	0.008	0.975	0.980	0.992	0.998	0.998
Price Shock	Beta	0.850	0.100	0.539	0.056	0.544	0.060	0.443	0.467	0.547	0.617	0.639
Wage Shock	Beta	0.850	0.100	0.956	0.013	0.954	0.015	0.927	0.935	0.956	0.971	0.975

(continued)

Table 8. (continued). NMK Model, 4-Quarter Price Contract, $\phi = \lambda_p$ and $\varepsilon \neq 0$ (Table 4, Column 2)

Marginal Likelihood:
Laplace Approximation: −468.344
Modified Harmonic Mean: −467.130

	Prior Distribution			Estimated Posterior Mode and Mean				Posterior Sample Based				
	Type	Mean	St. Error	Mode	St. Error	Mean	St. Error	5%	10%	50%	90%	95%
Miscellaneous												
Invest. Adj. Cost	Normal	4.000	1.500	6.327	1.032	6.376	1.034	4.753	5.078	6.335	7.733	8.140
Hsehold. Rel. Risk Aversion	Normal	1.000	0.375	2.111	0.261	1.983	0.283	1.510	1.618	1.987	2.344	2.447
Consumption Habit	Beta	0.700	0.100	0.356	0.049	0.388	0.053	0.303	0.321	0.386	0.458	0.480
Labor Utility	Normal	2.000	0.750	1.124	0.614	1.250	0.578	0.440	0.565	1.178	2.033	2.313
Calvo Employment	Beta	0.500	0.100	0.643	0.040	0.641	0.039	0.574	0.590	0.643	0.690	0.702
Indexation Wage	Beta	0.500	0.250	0.562	0.210	0.533	0.193	0.205	0.274	0.537	0.788	0.846
Indexation Price	Beta	0.500	0.250	0.121	0.096	0.156	0.085	0.034	0.052	0.146	0.274	0.313
Cap. Util. Adj. Cost	Beta	0.500	0.250	0.812	0.080	0.844	0.073	0.719	0.747	0.848	0.938	0.958
Fixed Cost	Normal	1.250	0.125									
Price Markup	Beta	0.250	0.150	0.489	0.098	0.530	0.100	0.370	0.403	0.526	0.660	0.701
Wage Markup	Beta	0.250	0.150	0.288	0.117	0.332	0.122	0.152	0.184	0.319	0.495	0.551
Curvature Parameter	Beta	0.850	0.100	0.986	0.004	0.984	0.006	0.973	0.977	0.984	0.990	0.991
Trend	Normal	0.400	0.025	0.399	0.022	0.395	0.022	0.359	0.367	0.395	0.424	0.432
Policy Rule Parameters												
r Inflation	Normal	1.500	0.100	1.535	0.081	1.552	0.080	1.424	1.452	1.551	1.655	1.686
r d(inflation)	Normal	0.300	0.100	0.175	0.044	0.185	0.046	0.110	0.126	0.184	0.243	0.260
r Lagged Interest Rate	Beta	0.750	0.050	0.866	0.017	0.862	0.018	0.830	0.838	0.863	0.884	0.890
r Output	Beta	0.125	0.050	0.115	0.026	0.112	0.027	0.070	0.078	0.110	0.147	0.158
r d(output)	Beta	0.063	0.050	0.117	0.034	0.128	0.037	0.070	0.081	0.126	0.176	0.192

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U.S. Wage and Price Dynamics: A Limited-Information Approach*

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This paper analyzes the dynamics of prices and wages using a limited-information approach to estimation. I estimate a two-equation model for the determination of prices and wages derived from an optimization-based dynamic model, where both goods and labor markets are monopolistically competitive, prices and wages can be reoptimized only at random intervals, and, when not reoptimized, can be partially adjusted to previous-period aggregate inflation. The estimation procedure is a *two-step* minimum-distance estimation, which exploits the restrictions that the model imposes on a time-series representation of the data. In the first step I estimate an *unrestricted* autoregressive representation of the variables of interest. In the second step, I express the model solution in the form of a *constrained* autoregressive representation of the data and define the distance between unconstrained and constrained representations as a function of the structural parameters that characterize the joint dynamics of inflation and labor share. This function summarizes the cross-equation restrictions between the model and the time-series representations of the data: I then estimate the parameters of interest by minimizing a quadratic function of that distance. I find that the estimated dynamics of prices and wages track actual dynamics quite well, and that the estimated parameters are consistent with the observed length of nominal contracts.

JEL Codes: E32, C32, C52.

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1. Introduction

This paper is an empirical analysis of the dynamics of wages and prices implied by a model of monopolistic competition in goods and labor markets, with sluggish adjustment of prices and wages. The objective of the paper is to investigate the link between real and nominal variables predicted by an optimization-based model, without specifying the whole general equilibrium structure.

I build on previous work that has shown that inflation fluctuations are fairly consistent with the predictions of an optimizing model of staggered price setting, if one takes as given the evolution of marginal cost.[1] I take the analysis one step further, endogenizing the determination of nominal wages, to provide an empirical analysis of the joint dynamics of wages and prices and their interaction with aggregate real variables. Allowing sluggish adjustment of both wages and prices, I also seek to shed light on whether the source of the inertia that appears to characterize nominal variables rests more on the price or on the wage-adjustment mechanism.

I analyze a generalized version of the discrete-time model of price and wage setting studied by Erceg, Henderson, and Levin (2000).[2] Specifically, I assume that monopolistically competitive goods-producing firms set their prices to maximize the discounted expected value of their future profits and reoptimize prices only at

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[1] This is argued by Galí and Gertler (1999), Galí, Gertler, and López-Salido (2005), and Sbordone (2002) for the United States; and Galí, Gertler, and López-Salido (2001), Batini, Jackson, and Nickell (2005), and Gagnon and Khan (2005) for European countries and Canada. The robustness of these estimates has been variously discussed: among the criticisms, see Rudd and Whelan (2005), Kurmann (2005), and Lindé (2005).

[2] This way of modeling the wage and price sector is now widely used in empirical DSGE models; see, for example, Amato and Laubach (2003), Christiano, Eichenbaum, and Evans (2005), Altig et al. (2002), and Smets and Wouters (2003, 2005). A comprehensive exposition of such a model can be found in Woodford (2003, ch. 3).

random intervals. Similarly, monopolistically competitive suppliers of differentiated labor services can reoptimize their wages only at random intervals. On the other hand, I assume that both firms and workers, when not allowed to reoptimize, can adjust their prices to past inflation.

Sluggish price and wage adjustments of this kind, following Calvo (1983) modeling, are often introduced in general equilibrium models of business cycle to build in a channel of persistence of monetary policy effects. Estimating the price/wage block within a completely specified general equilibrium model requires further specifications, such as the nature of capital accumulation, the details of fiscal and monetary policy, and the stochastic properties of the shocks. Some papers do so by adopting a full-information approach to estimation using maximum likelihood methods;[3] others rely on the identification of a single shock and estimate the model parameters by matching theoretical and empirical impulse response functions to that shock.[4]

The strategy I propose here aims instead at estimating the dynamics of wages and prices implied by this model without specifying a whole general equilibrium structure. I compare the equilibrium paths of wages and prices derived from the optimizing model to the paths described by an unrestricted vector autoregression model. Under the null hypothesis that the theoretical model is a correct representation of the stochastic process generating the data, the restrictions that the model solution imposes on the parameters of the time-series model should hold exactly. I propose to use these restrictions to construct a two-step distance estimator for the parameters of the structural model.

This approach follows directly from Campbell and Shiller's (1987) analysis, where they suggested testing the present-value model of stock prices by testing the restrictions that it imposes on a bivariate time-series representation of dividends growth and the price/dividend ratio. The model analyzed here also involves two

[3]For small models, the pioneering work using maximum likelihood estimation is Ireland (1997). Smets and Wouters (2003, 2005) have introduced the use of Bayesian techniques in the estimation of medium-scale models.

[4]See, for example, Amato and Laubach (2003), Christiano, Eichenbaum, and Evans (2005), and Altig et al. (2002).

present-value relationships. In the price equation, after solving out inflation expectations, price inflation depends upon the present discounted value of expected future deviations of marginal costs from the price level. Similarly, after solving forward wage expectations in the wage equation, wage growth depends upon the present discounted value of expected future deviations of the marginal rate of substitution from the real wage. The joint model therefore imposes testable restrictions on a multivariate time-series representation of wages and prices.

My estimation approach proceeds as follows. I derive the (approximate) equilibrium conditions for price and wage setting from the optimization-based model and write them in the form of two expectational difference equations in inflation and labor share. I then estimate a multivariate time-series model to describe the evolution of all the variables that matter in the determination of inflation and labor share. Combining the structural equations and the estimated time-series model, I solve for the paths of inflation and labor share as functions of exogenous and predetermined variables. This solution represents a *restricted* autoregressive representation for inflation and labor share, where the parameters are combinations of the structural parameters *and* the parameters of the unrestricted time-series process. I then recover the restrictions imposed by the theoretical model by comparing the coefficients of the restricted and the unrestricted autoregressive representations. These implied restrictions can be interpreted as a measure of the distance between the model and the time-series representation: the structural parameters are estimated as those that minimize a quadratic form of this distance.

The estimator I propose is therefore a two-step distance estimator: the first step involves the estimation of the time-series model, and the second, taking as given those estimated parameters, minimizes the distance function.

Two important issues are involved in the implementation of the proposed empirical strategy. First, the data need a preliminary transformation so that the stationary variables that define the equilibrium conditions of the model have a measurable counterpart. To handle the presence of a stochastic trend in the time series considered, I use a multivariate approach based on the estimated unrestricted vector autoregression representation: the specification of the VAR is therefore central to both steps of the estimation procedure.

The second issue is modeling the marginal rate of substitution, which is the real wage that would prevail in a competitive market, absent wage rigidities; throughout the paper I refer to the marginal rate of substitution as the flexible-wage equilibrium real wage. The expression for this equilibrium wage depends upon the assumptions that one makes about household preferences; without adopting specific functional forms for preferences, I discuss in turn the form that the flexible-wage equilibrium real wage would take under different assumptions.

The rest of the paper is organized as follows. In section 2, I lay out the elements of the optimization model for the determination of the path of price and wage inflation. In section 3, I characterize the model solution; in section 4, I describe the two-step estimator, relating it to similar estimation approaches used in business-cycle literature. Section 5 discusses how to model the flexible-wage equilibrium real wage, while section 6 presents the estimation of the time-series model and discusses the treatment of the trend. Results are presented and discussed in section 7. After a brief discussion of robustness checks in section 8, section 9 concludes.

2. Wage and Price Dynamics with Backward Indexation

The model is based on Erceg, Henderson, and Levin (2000), but allows partial indexation of both wages and prices to lagged inflation.[5] Since the basic structure of this model is quite well known in the literature, the exposition below is kept to a minimum[6] and targeted to illustrate the coefficients to be estimated.

2.1 Staggered Price Setting with Partial Indexation

At any point in time, a fraction $(1-\alpha_p)$ of the firms choose a price X_{pt} that maximizes the expected discounted sum of the firms' profits

$$E_t\Sigma_j\alpha_p^jQ_{t,t+j}(X_{pt}\Psi_{tj}Y_{t+j}(i)-\mathsf{C}(Y_{t+j}(i)), \tag{1}$$

[5]Full backward indexation was first introduced in Christiano, Eichenbaum, and Evans (2005). The generalized model with partial backward indexation is detailed in Woodford (2003, ch. 3).

[6]Details of some derivations are provided in the appendix.

where $Q_{t,t+j}$ is a nominal discount factor between time t and $t+j$; $Y_t(i)$ is the level of output of firm i; $\mathsf{C}(Y_{t+j}(i))$ is the total cost of production at $t+j$ of the firms that optimally set prices at t; and

$$\Psi_{tj} = \begin{cases} 1 & j = 0 \\ \Pi_{k=0}^{j-1} \pi_{t+k}^{\varrho_p} & j \geq 1 \end{cases}. \tag{2}$$

The coefficient $\varrho_p \epsilon [0,1]$ indicates the degree of indexation to past inflation of the prices that are not reoptimized.

The demand for goods of producer i is

$$Y_{t+j}(i) = \left(\frac{X_{pt}\Psi_{tj}}{P_{t+j}} \right)^{-\theta_p} Y_{t+j}, \tag{3}$$

where $\theta_p > 1$ denotes the Dixit-Stiglitz elasticity of substitution among differentiated goods, and the aggregate price level is

$$P_t = \left[(1-\alpha_p) X_{pt}^{1-\theta_p} + \alpha_p (\pi_{t-1}^{\varrho_p} P_{t-1})^{1-\theta_p} \right]^{\frac{1}{1-\theta_p}}. \tag{4}$$

The first-order condition for this problem can be expressed as

$$E_t \Sigma_j \alpha_p^j Q_{t,t+j} \left\{ Y_{t+j} P_{t+j}^{\theta_p} \Psi_{tj}{}^{1-\theta_p} \left(X_{pt} - \frac{\theta_p}{\theta_p - 1} S_{t+j,t}(i) \Psi_{tj}^{-1} \right) \right\} = 0,$$

where $S_{t+j,t}(i)$ is nominal marginal cost at $t+j$ of the firms that set optimal price at time t. Dividing this expression by P_t, and using (2), one gets

$$E_t \Sigma_j \alpha_p^j Q_{t,t+j} \left\{ Y_{t+j} P_{t+j}^{\theta_p} \Psi_{tj}{}^{1-\theta_p} \right.$$
$$\left. \times \left(x_{pt} - \frac{\theta_p}{\theta_p - 1} s_{t+j}(i) \left(\Pi_{k=1}^{j} \pi_{t+k} \right) \left(\Pi_{k=0}^{j-1} \pi_{t+k}^{\varrho_p} \right)^{-1} \right) \right\} = 0,$$

where x_{pt} is the relative price of the firms that set optimal price at t, and $s_{t+j,t}(i)$ is their real marginal cost at time $t+j$. A log-linearization of this expression around a steady state with zero inflation gives

$$\hat{x}_{pt} = (1-\alpha_p\beta) \Sigma_{j=0}^{\infty} (\alpha_p\beta)^j E_t \left(\hat{s}_{t+j,t} + \Sigma_{k=1}^{j} \hat{\pi}_{t+k} - \varrho_p \Sigma_{k=0}^{j-1} \hat{\pi}_{t+k} \right), \tag{5}$$

where hat variables are log-deviations from steady-state values.[7] Under the hypothesis that capital is not instantaneously reallocated across firms, $s_{t+j,t}$ is, in general, different from the average marginal cost at time $t+j$, s_{t+j}, so that

$$\hat{s}_{t+j,t} = \hat{s}_{t+j} - \theta_p \omega \left(\hat{x}_{pt} - \left(\Sigma_{k=1}^{j} \hat{\pi}_{t+k} - \varrho_p \Sigma_{k=0}^{j-1} \hat{\pi}_{t+k} \right) \right), \tag{6}$$

where ω is the output elasticity of real marginal cost for the individual firm.[8] Therefore, substituting (6) in (5), one obtains

$$(1+\theta_p\omega)\hat{x}_{pt} = (1-\alpha_p\beta)\Sigma_{j=0}^{\infty}(\alpha_p\beta)^j \times E_t \left(\hat{s}_{t+j} + (1+\theta_p\omega) \left(\Sigma_{k=1}^{j} \hat{\pi}_{t+k} - \varrho_p \Sigma_{k=0}^{j-1} \hat{\pi}_{t+k} \right) \right). \tag{7}$$

Similarly, dividing (4) by P_t and log-linearizing, one gets

$$\hat{x}_{pt} = \frac{\alpha_p}{1-\alpha_p}(\hat{\pi}_t - \varrho_p \hat{\pi}_{t-1}). \tag{8}$$

Finally, combining (7) and (8),

$$\hat{\pi}_t - \varrho_p \hat{\pi}_{t-1} = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p(1+\theta_p\omega)} \Sigma_{j=0}^{\infty}(\alpha_p\beta)^j \times E_t \left(\hat{s}_{t+j} + (1+\theta_p\omega) \left(\Sigma_{k=1}^{j} \hat{\pi}_{t+k} - \varrho_p \Sigma_{k=0}^{j-1} \hat{\pi}_{t+k} \right) \right), \tag{9}$$

which is equivalently written as[9]

$$\hat{\pi}_t - \varrho_p \hat{\pi}_{t-1} = \zeta \hat{s}_t + \beta E_t(\hat{\pi}_{t+1} - \varrho_p \hat{\pi}_t),$$

[7] I denote by β the steady-state value of the discount factor and suppress the index i on variables chosen by the firms that are changing prices, since all those firms solve the same optimization problem.

[8] Note that when the production function takes the Cobb-Douglas form, for example, $\omega = a/(1-a)$, where $(1-a)$ is the output elasticity with respect to labor.

[9] This result is obtained by forwarding (9) one period, multiplying it by β, and subtracting the resulting expression from (9).

where I set $\zeta = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p(1+\theta_p\omega)}$. This equation describes the evolution of inflation as a function of past inflation, expected future inflation, and real marginal costs; compared to the standard Calvo model, where $\varrho_p = 0$, this expression contains a backward-looking component that many have argued is a necessary component to fit the inertia of inflation data. This can be seen by rewriting (9) as:

$$\hat{\pi}_t = \frac{\varrho_p}{1+\varrho_p\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\varrho_p\beta}E_t\hat{\pi}_{t+1} + \frac{\zeta}{1+\varrho_p\beta}\hat{s}_t. \tag{10}$$

At the other extreme of complete indexation ($\varrho_p = 1$)—considered, for example, in Christiano, Eichenbaum, and Evans (2005)—the model predicts that the *growth rate* of inflation depends upon real marginal costs and the expected future growth rate of inflation. In this case, coefficients on past and future inflation sum to 1, and, for β close to 1, they are approximately the same. For low levels of indexation, instead, the coefficient on past inflation is significantly smaller than the one on future inflation.[10]

2.2 Staggered Wage Setting with Partial Indexation

Similarly to the firms, households are assumed to set their price (for leisure) in a monopolistically competitive way, analogous to the price model. Each household (indexed by i) offers a differentiated type of labor services to the firms and stipulates wage contracts in nominal terms: at the stipulated wage $W_t(i)$ they supply as many hours as are demanded. Unlike Erceg, Henderson, and Levin (2000), however, I allow preferences to be nonseparable in consumption and leisure.[11]

Total labor employed by any firm j is an aggregation of individual differentiated hours $h_t(i)$

$$H_t^j = \left[\int_0^1 h_t(i)^{(\theta_w-1)/\theta_w} di\right]^{\theta_w/(\theta_w-1)}, \tag{11}$$

[10] An equation of similar form is obtained with a slightly different set of assumptions by Galí and Gertler (1999). They assume that part of the firms that reset their price are not forward looking, but adopt instead "rule-of-thumb" price setting.

[11] Although I do not specify at this point the functional form of preferences, I assume here that they are time separable, and the momentary utility is defined on current values of consumption and leisure.

where θ_w is the Dixit-Stiglitz elasticity of substitution among differentiated labor services ($\theta_w > 1$). The wage index is an aggregate of individual wages, defined as

$$W_t = \left[\int_0^1 W_t(i)^{1-\theta_w} di \right]^{1/(1-\theta_w)}.$$

The demand function for labor services of household i from firm j is[12]

$$h_t^j(i) = (W_t(i)/W_t)^{-\theta_w} H_t^j, \tag{12}$$

which, aggregated across firms, gives the total demand of labor hours $h_t(i)$ equal to

$$h_t(i) = (W_t(i)/W_t)^{-\theta_w} H_t, \tag{13}$$

where $H_t = \left[\int_0^1 H_t^j dj \right]$.

At each point in time, only a fraction $(1 - \alpha_w)$ of the households can set a new wage, which I denote by X_{wt}, independently of the past history of wage changes.[13] The expected time between wage changes is therefore $\frac{1}{1-\alpha_w}$. I also assume, as in Erceg, Henderson, and Levin (2000), that households have access to a complete set of state-contingent contracts; in this way, although workers that work different amounts of time have different consumption paths, in equilibrium they have the same marginal utility of consumption.

Finally, for wages that are not reoptimized, I allow indexation to previous-period inflation: specifically, for $\varrho_w \epsilon \ [0, 1]$, the wage of a household l that cannot reoptimize at t evolves as

$$W_t(l) = \pi_{t-1}^{\varrho_w} W_{t-1}(l).$$

This hypothesis implies that wages reset at time t are expected to grow during the contract period according to

$$X_{wt+j} = X_{wt} \Psi_{tj}^w, \text{ where } \Psi_{tj}^w = \begin{cases} 1 & \text{if } j = 0 \\ \prod_{k=0}^{j-1} \pi_{t+k}^{\varrho_w} & \text{if } j \geq 1 \end{cases}. \tag{14}$$

[12]This demand is obtained by solving firm j's problem of allocating a given wage payment among the differentiated labor services, i.e., the problem of maximizing (11) for a given level of total wages to be paid.

[13]As for the price case, varying α_w between 0 and 1, the model allows various degrees of wage inertia, from perfect wage flexibility ($\alpha_w = 0$) to complete nominal wage rigidity ($\alpha_w \longrightarrow 1$).

The aggregate wage at any time t is an average of the wage set by the optimizing workers, X_{wt}, and the one set by those who do not optimize:

$$W_t = \left[(1-\alpha_w)(X_{wt})^{1-\theta_w} + \alpha_w(\pi_{t-1}^{\varrho_w} W_{t-1})^{1-\theta_w}\right]^{\frac{1}{1-\theta_w}}. \tag{15}$$

The wage-setting problem is defined as the choice of the wage X_{wt} that maximizes the expected stream of discounted utility from the new wage; this is defined as the difference between the gain (measured in terms of the marginal utility of consumption) derived from the hours worked at the new wage and the disutility of working the number of hours associated with the new wage. The objective function is then

$$E_t\left\{\Sigma_{j=0}^{\infty}(\beta\alpha_w)^j\left[\frac{\Lambda_{t+j,t}^c}{P_{t+j}}\left(X_{wt}\Psi_{tj}^w h_{t+j,t} - P_{t+j}C_{t+j,t}\right)\right.\right.$$
$$\left.\left. + U(C_{t+j,t}, h_{t+j,t})\right]\right\}, \tag{16}$$

where $\Lambda_{t+j,t}^c$ is the marginal utility of consumption at $t+j$ of workers that optimize at t, and $h_{t+j,t}$ is hours worked at $t+j$ at the wage set at time t. Given (14), the latter evolves as

$$h_{t+j,t} = \left(\frac{X_{wt}\Psi_{tj}^w}{W_{t+j}}\right)^{-\theta_w} H_{t+j}. \tag{17}$$

The first-order condition for this problem can be written as

$$E_t\left\{\Sigma_{j=0}^{\infty}(\beta\alpha_w)^j\left(\frac{X_{wt}\Psi_{tj}^w}{W_{t+j}}\right)^{-\theta_w} H_{t+j}\left[\frac{X_{wt}\Psi_{tj}^w}{P_{t+j}} - \frac{\theta_w}{\theta_w-1}v_{t+j,t}\right]\right\} = 0, \tag{18}$$

where $v_{t+j,t}$ is the marginal rate of substitution between consumption and leisure at date $t+j$, when the level of hours is $h_{t+j,t}$. A log-linear approximation of this equation is[14]

$$\hat{\pi}_t^w - \varrho_w\hat{\pi}_{t-1} = \gamma(\hat{v}_t - \hat{\omega}_t) + \beta\left(E_t\hat{\pi}_{t+1}^w - \varrho_w\hat{\pi}_t\right), \tag{19}$$

[14]See the derivation in the first section of the appendix.

where $\gamma = \frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\theta_w\chi)}$, and the parameter χ reflects the degree of nonseparability in preferences.[15]

2.3 A Complete Model

The dynamics of wages and prices are then described by the two log-linearized equilibrium conditions (10) and (19). Because the approximations are taken around a point with zero wage and price inflation, $\hat{\pi}_t = \pi_t \equiv \Delta p_t$, and $\hat{\pi}_t^w = \pi_t^w \equiv \Delta w_t$. Furthermore, $\hat{s}_t = w_t - p_t - q_t$, since real wage $(w_t - p_t)$ and labor productivity (q_t) share the same stochastic trend.[16] Similarly, $\hat{v}_t - \hat{\omega}_t = v_t - (w_t - p_t)$, since marginal rate of substitution and real wage also share the same stochastic trend.

Equations (10) and (19) can then be rewritten as

$$\pi_t = \frac{\varrho_p}{1+\varrho_p\beta}\Delta p_{t-1} + \frac{\beta}{1+\varrho_p\beta}E_t\Delta p_{t+1} + \frac{\zeta}{1+\varrho_p\beta}((w_t - q_t) - p_t) + u_{pt} \tag{20}$$

$$\pi_t^w = \varrho_w\Delta p_{t-1} + \beta E_t(\Delta w_{t+1} - \varrho_w\Delta p_t) + \gamma(v_t - (w_t - p_t)) + u_{wt}. \tag{21}$$

These equations show that the dynamics of prices and wages are driven by two *gaps*: the excess of unit labor costs over price (the real marginal cost) and the excess of the "equilibrium" real wage over the actual wage. The two parameters ζ and γ, defined quite symmetrically as $\zeta = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p(1+\theta_p\omega)}$ and $\gamma = \frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\theta_w\chi)}$, measure the degree of gradual adjustment of prices and wages to these gaps. These parameters, in turn, depend upon the parameters that determine the frequency of price and wage adjustments—respectively, α_p

[15] $\chi = \frac{-\Lambda_h^c H}{\Lambda_c^c C}\eta_c + \eta_h$, where η_c and η_h are, respectively, the elasticity of the marginal rate of substitution with respect to consumption and with respect to hours, evaluated at the steady state. Λ_c^c and Λ_h^c are derivatives of the marginal utility of consumption Λ^c with respect to consumption and with respect to hours, also evaluated at steady state. Note that when preferences are separable in consumption and leisure, $\Lambda_h^c = 0$.

[16] Note that I am also assuming valid conditions under which marginal cost is proportional to unit labor cost.

and α_w; the degree of substitutability between differentiated goods θ_p and that between differentiated labor services θ_w; the elasticity of firms' marginal costs with respect to their own output ω; and the degree of nonseparability in households' preferences, χ.

I have included an error term in each equation: these terms may pick up unobservable markup variations or allow for other possible misspecifications. I assume that the error terms are mutually uncorrelated, serially uncorrelated: $E(u_{it}u'_{jt-k}) = 0$ for $i, j = p, w$, and $k \neq 0$, and unforecastable, given the information set.

Equations (20) and (21) show the interdependence of wages and prices and their dependence upon the evolution of productivity and the other real variables that determine the evolution of the flexible-wage equilibrium real wage. In a fully specified model, this evolution would be described by similar structural relations. Here, instead, I focus on the restrictions that these equilibrium conditions impose on *any* general model that includes sluggish price and wage adjustment of the form described, independently of the specific form that the other structural relationships may take.

I proceed as follows: I assume that the evolution of the variables that determine the path of wages and prices can be summarized by a covariance stationary m-dimensional process X_t:

$$X_t = \Phi_1 X_{t-1} + \cdots + \Phi_p X_{t-p} + \varepsilon_t \tag{22}$$

(for some lag p to be determined empirically), where $E(\varepsilon_t) = 0$, and $E(\varepsilon_t \varepsilon'_\tau) = \Omega$ for $\tau = t$ and 0 otherwise. This vector includes, in addition to wages and prices, labor productivity q and the determinants of the flexible-wage equilibrium real wage v. Letting $Z_t = [X_t X_{t-1} \ldots X_{t-p+1}]'$, (22) can be represented as a first-order autoregressive process:

$$Z_t = AZ_{t-1} + Q\varepsilon_t, \tag{23}$$

where

$$A_{(mp\times mp)} = \begin{bmatrix} \Phi_1 & \Phi_2 & \cdots & \Phi_{p-1} & \Phi_p \\ I & \underline{0} & \cdots & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & & I & \underline{0} \end{bmatrix}, Q = \begin{bmatrix} I_{m\times m} \\ \underline{0}_{m(p-1)\times m} \end{bmatrix}.$$

The system of equations (20) and (21) places a set of restrictions on the parameters of the process (23). The nature of these restrictions can be recovered as follows: if one considers the joint process of (20), (21), and (23), one can solve for equilibrium processes $\{w_t, p_t\}$, given stochastic processes for $\{v_t, q_t\}$ and initial conditions $\{w_{-1}, p_{-1}\}$. This solution can be expressed as a particular restricted reduced-form representation for the vector Z_t,

$$Z_t = A^R Z_{t-1} + \widetilde{\varepsilon}_t,$$

with $A^R = G(\psi, A)$. ψ is the vector of the structural parameters of interest (defined below), and the function G incorporates the restrictions that the theoretical model imposes on the parameters of the time-series representation. The estimation procedure that I present in the next section is based on minimizing the distance between the restricted and the unrestricted representations of the relevant components of vector Z_t (i.e., the relevant elements of matrices A and A^R).

Before discussing my implementation of this estimation procedure, I will present a further transformation of equations (20) and (21) from equations in price and wage inflation into equations for price inflation and labor share (that is, real wage adjusted for productivity).[17] I will also derive the specific form of the restrictions that define the distance function used for the estimation of the structural parameters.

In what follows, I'll make use of the following identities:

$$q_t = q_{t-1} + \Delta q_t \tag{24}$$

$$w_t - p_t = w_{t-1} - p_{t-1} + \Delta w_t - \Delta p_t \tag{25}$$

and of an expression that defines the theoretical model for the flexible-wage equilibrium real wage:

$$v_t = q_t + \Xi Z_t. \tag{26}$$

The elements of the matrix Ξ depend upon assumptions about the long-run trend driving the time series and the specification of the

[17]As it will become clear later, this transformation is suggested by the properties of the time series of wage and productivity. The transformed structural equations have, therefore, the same form of their corresponding unrestricted representation in the process Z_t.

unrestricted representation (23). The crucial assumption that delivers (26) is that productivity, real wage, output, and consumption are all driven by a single stochastic trend, while hours are trend stationary. The specification of the vector X_t, the choice of the lag length p, and the form of the vector of coefficients Ξ are discussed later.

3. Model Solution

To rewrite equations (20) and (21) as a system in inflation and labor share $s_t \equiv w_t - p_t - q_t$, I first rearrange equation (20) as

$$E_t \Delta p_{t+1} = \frac{1 + \varrho_p \beta}{\beta} \Delta p_t - \frac{\varrho_p}{\beta} \Delta p_{t-1} - \frac{\zeta}{\beta}(w_t - p_t - q_t) + \widetilde{u}_{pt}, \quad (27)$$

where $\widetilde{u}_{pt} = (1 + \varrho_p \beta)\beta^{-1} u_{pt}$. Then I substitute (26) in (21) and rearrange it to get

$$\begin{aligned} E_t \Delta w_{t+1} = &\frac{1}{\beta} \Delta w_t + \varrho_w \Delta p_t - \frac{\varrho_w}{\beta} \Delta p_{t-1} \\ &+ \frac{\gamma}{\beta}(w_t - p_t - q_t) - \frac{\gamma}{\beta} \Xi Z_t + \widetilde{u}_{wt}, \end{aligned} \quad (28)$$

where $\widetilde{u}_{wt} = \beta^{-1} u_{wt}$. Subtracting (27) and $E_t \Delta q_{t+1}$ from (28), I derive $E_t \Delta s_{t+1} \equiv E_t(\Delta w_{t+1} - \Delta p_{t+1} - \Delta q_{t+1})$ as

$$\begin{aligned} E_t(s_{t+1} - s_t) = &\frac{1}{\beta} \Delta w_t + \left(\varrho_w - \varrho_p - \frac{1}{\beta}\right) \Delta p_t + \left(\frac{\varrho_p - \varrho_w}{\beta}\right) \Delta p_{t-1} \\ &+ \left(\frac{\gamma + \zeta}{\beta}\right) s_t - \frac{\gamma}{\beta} \Xi Z_t - E_t \Delta q_{t+1} + \nu_t, \end{aligned} \quad (29)$$

where ν_t is a composite error term.[18]

As I explain below, productivity growth Δq_t is an element of the vector X_t so that, by (23),

$$E_t \Delta q_{t+1} = e_q' A Z_t, \quad (30)$$

[18] $\nu_t = 1/\beta(u_{wt} - (1 + \varrho_p \beta) u_{pt})$.

where the selection vector e_q' has a 1 in correspondence to productivity growth and 0 elsewhere. Combining the terms in s_t and using (30), equation (29) becomes

$$E_t s_{t+1} = (\varrho_w - \varrho_p)\Delta p_t + \left(\frac{1+\beta+\gamma+\zeta}{\beta}\right) s_t + \left(\frac{\varrho_p - \varrho_w}{\beta}\right)\Delta p_{t-1} - \frac{1}{\beta} s_{t-1} - \left(\frac{\gamma}{\beta}\Xi - \frac{1}{\beta}e_q' + e_q' A\right) Z_t + \nu_t. \tag{31}$$

I now define a vector y_t as

$$y_t = [\pi_t \ s_t \ \pi_{t-1} \ s_{t-1}\ ]' \tag{32}$$

and let $Y_{t+1} = [y_{t+1} \ Z_{t+1}]'$. The system of equations composed of (27), (31), and (23) can then be written as

$$E_t Y_{t+1} = MY_t + Nu_t, \tag{33}$$

where $u_t = [u_{pt} \ u_{wt}]'$, and the matrices M (of dim. $(4+mp)$) and N are partitioned as follows:

$$M = \begin{bmatrix} M_{yy} & M_{yZ} \\ 0 & A \end{bmatrix}, \quad N = \begin{bmatrix} N_1 \\ \underline{0} \end{bmatrix}.$$

The (4×4) block M_{yy} describes the interaction of the structural variables; the $(4\times mp)$ block M_{yZ} describes the dependence of structural variables upon the exogenous block.[19] If the matrix M has exactly two unstable eigenvalues, the system of equations (33) has a unique solution, which can be expressed in autoregressive form as

$$Y_t = GY_{t-1} + F\upsilon_t, \tag{34}$$

where the matrices G and F depend upon the vector of structural parameters ψ and the parameters of the unrestricted VAR process, the elements of A; the error term is $\upsilon_t = (u_t', \varepsilon_t')'$. The solution for

[19]The matrix N_1 is $\begin{pmatrix} \beta^{-1}(1+\varrho_p\beta) & 0 \\ -\beta^{-1}(1+\varrho_p\beta) & \beta^{-1} \end{pmatrix}$.

the endogenous variables π_t and s_t is the upper block of (34), which can be expressed as

$$\pi_t \equiv y_{1t} = g^{\pi}(\psi, A)Y_{t-1} + f^{\pi}\upsilon_t = g_y^{\pi} y_{t-1} + g_Z^{\pi} Z_{t-1} + f^{\pi}\upsilon_t \quad (35)$$

$$s_t \equiv y_{2t} = g^{s}(\psi, A)Y_{t-1} + f^{s}\upsilon_t = g_y^{s} y_{t-1} + g_Z^{s} Z_{t-1} + f^{s}\upsilon_t, \quad (36)$$

where g^i and f^i (for $i = \pi, s$) denote the row of the matrices G and F corresponding to variable i.

4. Approach to Estimation

Since both inflation and labor share are elements of the unrestricted process (22), they can be expressed as elements of Z_t, with appropriate definitions of selection vectors e'_π and e'_s:

$$\pi_t = e'_\pi Z_t \quad \text{and} \quad s_t = e'_s Z_t. \quad (37)$$

Similarly, the components of vector y_{t-1}, which includes lagged inflation and labor share, can be expressed in terms of elements of the vector Z_{t-1}, by way of an appropriate selection matrix $\Upsilon : y_{t-1} = \Upsilon Z_{t-1}$. Using this definition, and substituting (37) in (35) and (36), I get

$$e'_\pi Z_t - g_y^{\pi} \Upsilon Z_{t-1} - g_Z^{\pi} Z_{t-1} = f^{\pi}\upsilon_t \quad (38)$$

$$e'_s Z_t - g_y^{s} \Upsilon Z_{t-1} - g_Z^{s} Z_{t-1} = f^{s}\upsilon_t. \quad (39)$$

Finally, projecting both sides of (38) and (39) onto the information set Z_{t-1} and observing that, by assumption, $E(\upsilon_t|Z_{t-1}) = 0$, and also $E(Z_t|Z_{t-1}) = AZ_{t-1}$, I obtain

$$e'_\pi A Z_{t-1} - g_y^{\pi} \Upsilon Z_{t-1} - g_Z^{\pi} Z_{t-1} = \underline{0}$$

$$e'_s A Z_{t-1} - g_y^{s} \Upsilon Z_{t-1} - g_Z^{s} Z_{t-1} = \underline{0}.$$

Since these equalities must hold for every t, it follows that

$$e'_\pi A - g_y^{\pi} \Upsilon - g_Z^{\pi} = \underline{0} \quad (40)$$

$$e'_s A - g_y^{s} \Upsilon - g_Z^{s} = \underline{0}. \quad (41)$$

Expressions (40) and (41) form a set of $2 \times mp$ restrictions on the parameters of the unrestricted process (23), which must hold if the model is true. The structural parameters can then be estimated as those values that most likely make these restrictions hold.

The estimation strategy proceeds in two steps. First, I estimate an unrestricted VAR in all the variables of interest, to obtain a consistent estimate $\hat{A}$ of the autoregressive matrix A. In the second step, taking as given the estimated matrix $\hat{A}$, and stacking the restrictions (40) and (41) in a vector function $\digamma(\psi, A) = 0$, I choose the structural parameters ψ to make the empirical value of the function $\digamma$ as close as possible to its theoretical value of zero; namely, I choose

$$\hat{\psi} = \arg\min \digamma(\psi, \hat{A})' W^{-1} \digamma(\psi, \hat{A}) \tag{42}$$

for an appropriate choice of the weighting matrix W.[20]

The proposed estimator can be interpreted as a minimum-distance estimator, in application of the approach that Campbell and Shiller (1987) proposed for the empirical evaluation of present-value models. I have in fact interpreted the restrictions that define the function $\digamma$ as measuring the "distance" between the restricted and unrestricted representations of the data.[21] This estimator is close in spirit to another distance estimator used in the business-cycle literature, based on matching empirical and theoretical impulse response functions to specific structural shocks.[22] That estimator, as the one proposed here, uses an auxiliary VAR model in the first stage to characterize the dynamics of the data; then it minimizes the distance between the dynamic response to identified exogenous shocks estimated in the data and the response predicted by the theoretical model. Unlike the estimator based on matching impulse response

[20]As weighting matrix, I use a diagonal matrix with the variance of the estimated parameters A along the diagonal. This choice downweights the parameters that are estimated with greater uncertainty.

[21]In my previous applications of a similar two-step minimum-distance estimation, the objective function had the form of an (unweighted) distance between "model" and data (Sbordone 2002).

[22]Rotemberg and Woodford (1997) were the first to propose to estimate the structural parameters of a small monetary model by matching the model's predicted responses to a monetary policy shock to the responses estimated in an identified VAR model. This type of estimator has since been applied in several monetary models of business cycle by, among others, Amato and Laubach (2003), Boivin and Giannoni (2005), and Christiano, Eichenbaum, and Evans (2005). It has been applied to match the responses to both technology and monetary shocks by Altig et al. (2002) and Edge, Laubach, and Williams (2003).

functions, the one proposed here doesn't rely on further identification restrictions—those necessary to recover the structural shocks from the VAR innovations. Instead, it exploits the specific restrictions that the VAR specification imposes on the solution of the structural model and tries to match the dynamic evolution of the endogenous variables implied by the theoretical model with their evolution as described by the data.

Finally, although the distance restrictions are not moments conditions, this estimator is similar to a GMM estimator whose instruments are the variables of the time-series representation. However, such an estimator is usually applied to orthogonality conditions that proxy the future values of the endogenous variables, as opposed to solving the expectational equations.[23]

5. Modeling the Flexible-Wage Equilibrium Real Wage

A crucial step in implementing the empirical strategy discussed is the specification of the flexible-wage equilibrium real wage. Relationship (26) expresses the theoretical link between the flexible-wage equilibrium real wage (which I denoted by v_t) and real variables in Z_t that are not determined by the two structural equations. Therefore, the expression for the parameter vector Ξ incorporates hypotheses about the determinants of the cyclical components of the marginal rate of substitution, together with hypotheses about the evolution of its trend component.

The real wage v_t is the equilibrium wage that solves the household optimization problem under flexible wages: it is therefore equal to the ratio of the marginal disutility of working Λ_t^h and the marginal utility of consumption Λ_t^c. If there is no time dependence in the momentary utility function, these marginal utilities depend only upon current values of consumption and hours,[24] and a log-linearized expression for v_t is

$$\hat{v}_t = \eta_c \hat{c}_t + \eta_h \hat{h}_t, \tag{43}$$

[23]See my discussion of this point in Sbordone (2005).

[24]With time dependence, for example, if one allows habit persistence in consumption, the marginal rate of substitution depends also on past and future expected values of consumption and hours.

where the coefficients η_i are elasticities. Since "hat" variables are deviations from steady state, which are defined after appropriate transformations of the variables to remove their (possibly stochastic) trends, their natural empirical counterparts are cyclical components defined as deviations from estimated trends. Their derivation is explained in the next section.

6. The Time-Series Model

The second crucial step of the empirical methodology that I described is the specification of the *unrestricted* joint dynamics of the variables that appear as endogenous and forcing variables in the structural equations (20) and (21). These variables are inflation, labor share, labor productivity, and, following the discussion of the previous section, consumption and hours of work, which determine the evolution of the flexible-wage equilibrium real wage.

The first order of problems is choosing a transformation of the data consistent with the hypotheses built into the model. The time series of productivity, real wage, consumption, and output all contain a unit root, but it appears that the consumption-output ratio and the ratio of real wage to labor productivity are stationary. Hours, in turn, appear stationary around a deterministic trend. One can then assume that there is only one common stochastic trend to drive the long-run behavior of the series considered.

The hypothesis of a single stochastic trend in the data is consistent with the assumption built into the model that the economy is driven by a single source of nonstationarity.[25] As in the model, stationary variables used in estimation are then defined as deviation from this single stochastic trend. I handle the nonstationarity in the same multivariate context that I use for the time-series representation and apply the Beveridge-Nelson (1981) detrending method. The vector X_t of (22) is specified as

$$X_t = [\Delta q_t \quad h_t \quad cy_t \quad \pi_t \quad s_t]', \tag{44}$$

[25]This is a stochastic process Θ_t, which I model as a logarithmic random walk. In the model, nonstationary variables such as consumption and real wage are transformed by dividing through this process.

where Δq_t is labor productivity growth, h_t is an index of hours, cy_t is the consumption output ratio, s_t is the share of labor in total output, and inflation is the rate of growth of the implicit GDP deflator.[26]

I use the fact that any difference stationary series can be decomposed in a random-walk component (the stochastic trend) and a stationary component. I identify the single common stochastic trend in vector X_t with the random-walk component of labor productivity, which is in turn defined as the current value of productivity plus all expected future productivity growth.[27] Formally, letting q_t denote labor productivity, its trend is defined as

$$q_t^T = \lim_{k \to \infty} E_t(q_{t+k} - k\mu_q) = q_t + \sum_{j=1}^{\infty} E_t(\Delta q_{t+j} - \mu_q), \tag{45}$$

where $\mu_q = E(\Delta q)$. The stationary, or cyclical, component of productivity is then defined as the deviation of the series from its stochastic trend. The assumption of stationary labor share in the VAR in turn implies that the trend in real wage is the same as the trend in productivity, and the stationarity of the consumption-output ratio, together with the stationarity of hours (which corresponds to the ratio of output to productivity), implies that consumption shares the same trend as productivity.

The cyclical variables that appear in the theoretical model can be constructed as deviations from their respective trends.[28] From the joint representation of the series in (23), the s-step-ahead forecasts that define the trend are easily computed, for each variable i in vector X, as

$$E_t X_{i,t+s} = e_i' E_t Z_{t+s} = e_i' A^s Z_t. \tag{46}$$

[26]Unless otherwise indicated, lowercase letters denote natural logs.

[27]The rationale is that, if productivity growth is expected to be higher than average in the future, then labor productivity today is below trend; vice versa, if productivity growth is expected to be below average, then productivity today is above trend.

[28]The theoretical model has implications only for the co-movement of the stationary components of real wage, consumption, and hours. The specific detrending procedure followed here intends to reflect closely the assumption about the nature of the trend assumed in the theoretical model.

These forecasts underlie the derivation of the vector of parameters Ξ in the expression for the real wage v_t in (26).[29] The specification of Ξ completes the specification of the system (33) used for the estimation of the structural parameters ψ.

Using (46), the trend in productivity defined in (45) is

$$q_t^T = q_t + e_q'[I - A]^{-1} A Z_t. \tag{47}$$

The cyclical component of consumption is derived using the fact that the output-productivity ratio and the consumption-output ratio are stationary so that output, productivity, and consumption share the same stochastic trend. Writing $c_t = (c_t - y_t) + (y_t - q_t) + q_t$, I obtain that

$$c_t^{cyc} = c_t - c_t^T = e_{cy}' Z_t + e_h' Z_t - e_q'[I - A]^{-1} A Z_t, \tag{48}$$

where I have also used the fact that hours are stationary, so that cyclical hours h_t^{cyc} are simply the appropriate component of vector Z_t.

7. Results

7.1 VAR Specification

In the estimation I use quarterly data from 1952:Q1 to 2002:Q1, with data for 1951:Q2–1951:Q4 as initial values. Productivity, output, wages, prices, and hours are for the nonfarm business sector of the economy.[30] Nominal wage is hourly compensation, and real wage is nominal wage divided by the implicit GDP deflator. Consumption is the aggregate of nondurables and services.[31] I fit a VAR with three lags[32] to the vector X_t defined in (44) and estimate the common trend as the trend in productivity defined in (47). As discussed above, productivity, real wage, and consumption share the

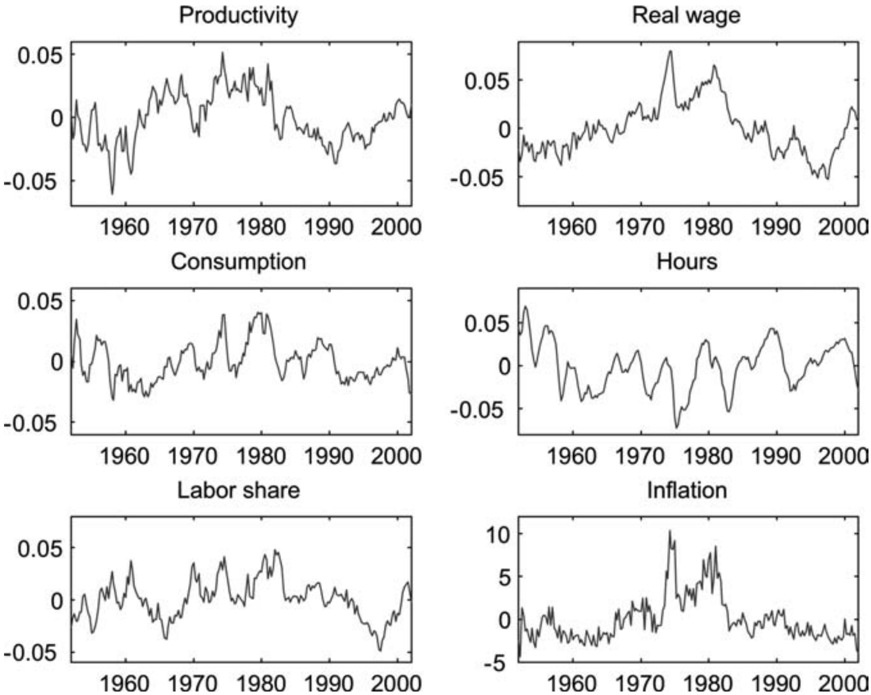
[29] The derivation of Ξ as a function of the exogenous variables in vector Z is detailed in the "Empirical Implementation" section of the appendix.

[30] The time series are downloaded from the Federal Reserve Economic Data (FRED) database at the Federal Reserve Bank of St. Louis.

[31] All variables are in deviation from the mean, and hours are linearly detrended. I also remove, prior to estimation, a moderate deterministic trend that appears in the consumption-output ratio and the labor share.

[32] The optimal lag length is chosen with the Akaike criterion.

Figure 1. Real Variables: Cyclical Components (Inflation: Deviation from Mean, Annualized)



same stochastic trend, while hours have a deterministic trend. Subtracting the appropriate trends from the actual real series, I derive the series' cyclical components, which I plot in figure 1. For inflation, the figure plots its deviation from a constant mean, annualized.

My objective is to compare the cyclical pattern of inflation and real wage to the pattern predicted by the theoretical model. As written, the model has implications for the dynamic behavior of inflation and labor share: given the behavior of productivity, the predicted path of real wages is then recovered from the estimated path of the labor share.

7.2 Estimation of Structural Parameters

Recall that the parameter vector is

$$\psi = (\beta, \varrho_p, \varrho_w, \eta_c, \eta_h, \zeta, \gamma)',$$

Table 1. Parameter Estimates—Baseline VAR (1952:Q1–2002:Q1)

β	ϱ_p	ϱ_w	η_c	η_h	ζ	γ
.967	.226	.058	2.41	−.891	.0255	.040
(.007)	(.041)	(.039)	(.537)	(.312)	(.007)	(.014)
Related Statistics						
$corr(\pi, \pi^m)$	.905					
$corr(s, s^m)$	.798				$Q = 38.42$	
$corr(\omega, \omega^m)$	.908				[p-value: .139]	

where β is a discount factor; ϱ_p and ϱ_w are indexation parameters, respectively, for price and wage setting; η_c and η_h are elasticities of the marginal rate of substitution with respect to consumption and hours of work; and ζ and γ are measures of the inertia in the price and wage settings. The last two parameters are nonlinear combinations of other structural parameters that are not separately identified: the frequency of price and wage adjustments and the structure of technology and preferences. However, calibrating some of these parameters, we can draw some inference on which values of the frequency of price and wage adjustments are consistent with the estimated values of ζ and γ.

Table 1 reports parameter estimates, standard errors (in parentheses),[33] and correlation of the theoretical paths of inflation, labor share, and real wage (denoted with superscript m) with their observed counterparts.

[33]To compute standard errors, I use the empirical distribution of the parameter matrix A to generate N samples A_i ($i = 1, \ldots, N$): for each of these, I estimate a vector of structural parameters $\hat{\psi}_i$. I then compute the sample variance of $\hat{\psi}$ and report the square root of its main diagonal elements as standard errors. For each estimated vector $\hat{\psi}_i$, I also compute the value of the distance function $\digamma_i$ and its covariance matrix $\Sigma_\digamma$; the Wald statistic reported in the table is $Q = \digamma(\hat{\psi})'\Sigma_\digamma \digamma(\hat{\psi})$, where $\digamma(\hat{\psi})$ is the value of the distance evaluated at the optimal value of ψ. It can be read as a test of the model restrictions.

Most of the estimated parameters are statistically significant. The parameters of the inflation model are consistent with several of the empirical results in the New Keynesian Phillips curve (NKPC) literature. First, there is a modest role for a backward-looking component in inflation dynamics: the indexation parameter ϱ_p is significantly different from zero, but the implied weight on the backward-looking component ($\varrho_p/(1+\beta\varrho_p) \simeq .18$) is quantitatively much smaller than the weight on the forward-looking component ($\beta/(1+\beta\varrho_p) \simeq .79$). Secondly, the size of the coefficient on the labor share, as it will be discussed below, is consistent with other estimates of price inertia in the literature.

In the labor share equation, the parameter of wage indexation ϱ_w is much smaller than 1, the value imposed in Christiano, Eichenbaum, and Evans (2005), and more in the range estimated by Smets and Wouters (2003) for the euro area. Finally, the value of the statistic Q indicates that the restrictions that the model imposes on the parameters of A cannot be rejected.

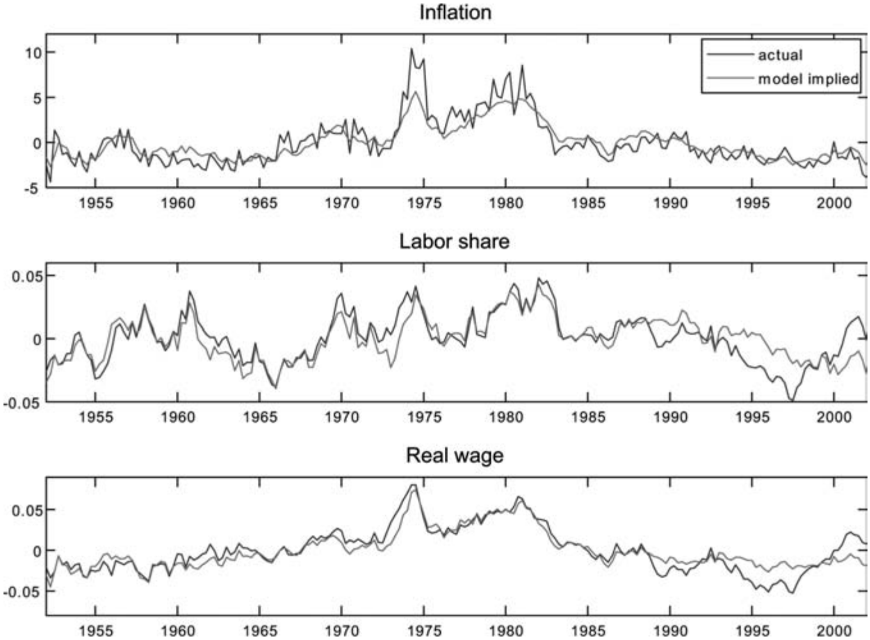
Figure 2 compares actual inflation, labor share, and real wage (namely, the cyclical components of these series as portrayed in figure 1) to the paths of inflation, labor share, and real wage constructed recursively from the model solution evaluated at the estimated parameters—labeled "model implied" in the figure.[34] These paths seem to capture well the underlying dynamics of the actual series: on these accounts, the model of wage and price inflation described seems to fit the data quite well.

Furthermore, the model is able to match the dynamic correlation between inflation and output. As noted in the literature,[35] output leads inflation in the data: the cyclical component of output, variously measured, is positively correlated with future inflation, with the highest value at about three quarters ahead. Purely forward-looking NKPCs driven by the output gap, when this is measured as deviation from a deterministic trend, are unable to reproduce such a result: output gap typically lags inflation in such a

[34] The "model implied" paths of inflation and labor share are directly computed from expressions (35) and (36); the path of real wage is recovered from that of the labor share by adding productivity.

[35] See, for example, the discussion of "reverse dynamic" cross-correlation in Taylor (1999).

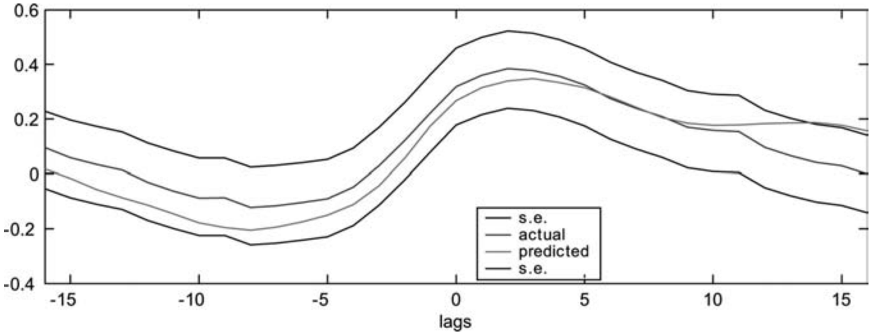
Figure 2. Inflation, Labor Share, and Real Wage: Actual versus Model Implied



model.[36] Output-inflation correlations are shown in figure 3. The figure compares the dynamic correlation of output gap and actual inflation (the line labeled "actual") with the dynamic correlation of output gap and the inflation series generated by the estimated model (the line labeled "predicted"). The output-gap measure used to compute these correlations is, consistently with the estimated time-series model, the deviation of output from the estimated stochastic trend. As the figure shows, output leads inflation both in the model and in the data, and actual and predicted dynamic correlations peak at about the same time. This provides further evidence that the model succeeds in capturing the main dynamics of inflation.

[36]See evidence presented in Sbordone (2001) or Galí and Gertler (1999). More recently, Guerrieri (2006) argued that the Fuhrer and Moore (1995) relative price contract is better able to reproduce this dynamic correlation than a standard n-period Taylor (1980) contract.

Figure 3. Dynamic Cross-Correlations: Output Gap (t) versus Inflation (t + k)



7.3 Implied Degree of Nominal Rigidities

The parameters that measure the degree of price and wage inertia are significantly different from zero, but they do not give a direct estimate of the frequency of price and wage adjustments. In the Calvo model, the frequency of price and wage adjustment is driven by the probability of changing prices or wages at any point in time, measured respectively by α_p and α_w. In order to infer those parameters from the estimated values of ζ and γ, some further hypotheses are needed. From the definition of $\zeta = \frac{(1-\alpha_p)(1-\alpha_p\beta)}{\alpha_p(1+\theta_p\omega)}$, to draw inference on α_p, one has to make some assumption about the degree of substitution among differentiated goods θ_p and the elasticity of real marginal cost to output for the individual firm, ω. On the upper part of table 2, I report the implied degree of inertia (measured as the average time between price changes, measured in months) under two different assumptions about these two parameters. For the parameter ω I consider two benchmark values, .33 and .54;[37] for θ_p, which is related to the steady-state markup μ^* by $\mu^* = \theta_p/(\theta_p - 1)$, I consider values that imply a low (20 percent) and a high (60 percent) steady-state markup, two benchmark

[37]As mentioned before, in the case of a Cobb-Douglas technology, $\omega = a/(1-a)$, where a is the output elasticity with respect to capital. The two values assumed for ω correspond, therefore, to an output elasticity with respect to capital of .25 and .35, respectively.

Table 2. Implied Degrees of Nominal Rigidity

Average Time between Price Changes (Months)			
	Low Markup ($\mu^{p*} = 1.2$)	High Markup ($\mu^{p*} = 1.6$)	
$\omega = .33$	12.4	15.1	
$\omega = .54$	10.7	13.6	
Average Time between Wage Changes (Months)			
	Low Wage Markup ($\mu^{w*} = 1.1$)	Mid Wage Markup ($\mu^{w*} = 1.3$)	High Wage Markup ($\mu^{w*} = 1.5$)
Low Nonsep.	13.4	12.3	16.1
Mid Nonsep.	8.6	11.4	12.5
High Nonsep.	5.8	7.6	8.4

values often used in the literature.[38] As the table shows, the average duration of prices ranges from a little more than three quarters to about five quarters, depending on these assumptions.

The bottom part of the table shows the implied degree of wage inertia, computed in a similar manner. Here the inertia is summarized by $\gamma = \frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\theta_w\chi)}$; in order to make inference on α_w, some assumption must be made about the value of the parameters θ_w and, therefore, about the value of the steady-state wage markup and about the degree of nonseparability between consumption and leisure in preferences, which determines the size of the parameter χ. In the table I consider different values for the steady-state markup and different degrees of nonseparability.[39] For low degrees of nonseparability, the average duration of wage contracts is similar to those of prices, while it is shorter for highly nonseparable preferences.

That preferences should be nonseparable in consumption and leisure is an implication of the negative sign of the elasticity of the

[38] Values of μ^* above 1.5 are, for example, estimated by Hall (1988) on a large number of U.S. manufacturing industries.

[39] I show in the appendix (in the section titled "Inference on Wage Rigidity") that the degree of nonseparability can be parameterized by calibrating the value of the intertemporal elasticity of substitution in consumption and the share of labor income in consumption.

marginal rate of substitution with respect to hours.[40] While most of the business-cycle literature adopts a separable preference specification, empirical evidence on significant nonseparability in preferences has been found, most recently, by Basu and Kimball (2000). Moreover, within the class of preferences that are consistent with balanced growth, a negative elasticity of the marginal rate of substitution with respect to hours can be obtained in a generalized indivisible labor model, as shown in King and Rebelo (1999). The interpretation of the large elasticity η_c is more problematic and requires further investigation. As we will see below, however, a modification in the specification of the time-series model reduces its size. Another possibility to be explored, which is left to future research, is that this parameter is overestimated for an omitted variable problem in the wage equation, as would be the case if preferences were time dependent.

8. Some Robustness Analysis

The inference presented on the structural parameters relies on the inference in the first step of the procedure: the estimation of the time-series model. I made a number of assumptions to model the VAR: the choice of variables was suggested by the need to limit its dimension, but the inclusion of additional variables could potentially improve the forecast of the driving forces of the structural equations. I modeled only one stochastic trend in the data, to mimic the trend assumption of the theoretical model; but the data may be consistent with other assumptions about the number of common stochastic trends. Finally, the VAR structure has been modeled as time invariant, while many recent analyses suggest that changes in policy regime have determined drifts over time in the reduced-form representation of the relation between nominal and real variables.[41]

While some of these issues are pursued in separate research,[42] in table 3 I present the results of alternative estimates to shed

[40] This can be shown by expressing the two elasticities of the marginal rate of substitution η_c and η_h in terms of the Frish elasticities of consumption and labor supply (see Sbordone 2001).

[41] See, for example, Boivin and Giannoni (2005) and Cogley and Sargent (2001, 2005).

[42] Cogley and Sbordone (2005) extend the two-step estimation procedure to the case of a small-scale first-stage VAR with drifting parameters.

Table 3. Parameter Estimates—Augmented VAR (1954:Q3–2002:Q1)*

β	ϱ_p	ϱ_w	η_c	η_h	ζ	γ
.967 .(0027)	.154 (.027)	.001 (.071)	2.74 (.581)	−.71 (.319)	.018 (.009)	.033 (.034)
Related Statistics						
$corr(\pi, \pi^m)$	.897					
$corr(s, s^m)$	.782					$Q = 36.44$
$corr(\omega, \omega^m)$	.903					[p-value: .194]
Average Time between Price Changes (Months)						
	Low Markup			High Markup		
$\omega = .54$	13.0			16.3		
Average Time between Wage Changes (Months)						
	Low Markup			High Markup		
Low Nonsep.	6.63			9.81		
High Nonsep.	5.70			8.26		

*The shorter sample is due to the federal funds rate data being available only from 1954:Q3.

some light on how sensitive the results presented so far are to the inclusion of additional variables in the time-series model. Specifically, I augment the baseline VAR with the federal funds rate: although the corresponding equation in the VAR is not meant to represent a policy rule, the introduction of the federal funds rate can be thought of as representing the reduced-form effect of monetary policy on inflation and the real variables of the system. The drawback of including an additional variable in the VAR, though, is an increase in uncertainty when the relative parameters are not tightly estimated.

Table 3 reports the second-stage parameter estimates and the implied nominal rigidity. The results are qualitatively similar to the previous ones, but the lower estimates of the inertia parameters imply a higher degree of nominal rigidity, especially for prices.

9. Conclusion

In this paper I estimate the joint dynamics of U.S. prices and wages using a partial-information approach. I derive the implied price and wage inflations from an optimization-based model of staggered price and wage contracts with random duration and then implement a two-step minimum-distance estimation of the structural parameters. In the first step, I estimate an unrestricted time-series representation for the variables of interest and derive the restrictions that the model solution imposes on this representation. In the second step, I use these restrictions to define a distance function to be minimized for the estimation of the structural parameters. This methodology allows me to investigate the dynamics of prices and wages without having to make all the additional assumptions required to close the model and to characterize its entire stochastic structure.

I find that a generalized version of the Calvo mechanism of random intervals between price and wage adjustments fits the data quite well, that there is some backward-looking component in inflation, and that the average duration of both contracts is around a year. The robustness of these results to the specification of the first stage of the proposed estimation procedure is to be further explored.

Appendix

Derivation of Equation (19)[43]

Under the hypothesis that there is a single stochastic trend driving long-run growth, say Θ_t, with $\gamma_{\Theta_t} = \Theta_t/\Theta_{t-1}$ an i.i.d. process, one can define stationary variables $x_{wt} \equiv \frac{X_{wt}}{W_t}$, $\pi_t^w \equiv \frac{W_t}{W_{t-1}}$, $\widetilde{\omega}_t = \frac{W_t}{\Theta_t P_t}$, and $\widetilde{v}_t = \frac{v_t}{\Theta_t}$. Then, using the fact that $\frac{X_{wt}}{W_{t+j}} = \frac{X_{wt}}{W_t}\frac{W_t}{W_{t+j}}$ and $\frac{X_{wt}}{P_{t+j}} = \frac{X_{wt}}{W_{t+j}}\frac{W_{t+j}}{P_{t+j}}$, equation (18) can be written as

$$E_t\left\{\Sigma_{j=0}^{\infty}(\beta\alpha_w)^j\left(x_{wt}\Psi_{tj}^w\Pi_{k=1}^j(\pi_{t+k}^w)^{-1}\right)^{-\theta_w}\right.$$
$$\left.\times\ H_{t+j}\left[x_{wt}\Psi_{tj}^w\widetilde{\omega}_{t+j}\Pi_{k=1}^j(\pi_{t+k}^w)^{-1} - \frac{\theta_w}{\theta_w - 1}\widetilde{v}_{t+j,t}\right]\right\} = 0,$$

[43] This derivation follows Sbordone (2001).

so that a log-linearization around steady-state values $x_w^*, \pi^*, \pi^{w*}, \omega^*$, v^* gives

$$\Sigma_{j=0}^{\infty}(\beta\alpha_w)^j \left(\hat{x}_{wt} + \varrho_w \Sigma_{k=0}^{j-1}\hat{\pi}_{t+k} - \Sigma_{k=1}^{j}\hat{\pi}_{t+k}^{w} + \hat{\omega}_{t+j}\right)$$
$$= \Sigma_{j=0}^{\infty}(\beta\alpha_w)^j \ E_t(\hat{v}_{t+j,t}),$$

or

$$\hat{x}_{wt} = (1-\beta\alpha_w)\Sigma_{j=0}^{\infty}(\beta\alpha_w)^j \ E_t \times \left(\hat{v}_{t+j,t} - \hat{\omega}_{t+j} - \varrho_w \Sigma_{k=0}^{j-1}\hat{\pi}_{t+k} + \Sigma_{k=1}^{j}\hat{\pi}_{t+k}^{w}\right). \tag{49}$$

To express $\hat{v}_{t+j,t}$ in terms of the average marginal rate of substitution, I write

$$v_{t+j,t} \equiv \frac{\Lambda^h}{\Lambda^c}(c_{t+j,t}, h_{t+j,t}) = \frac{\frac{\Lambda^h}{\Lambda^c}(c_{t+j,t}, h_{t+j,t})}{\frac{\Lambda^h}{\Lambda^c}(c_{t+j}, h_{t+j})}\left(\frac{\Lambda^h}{\Lambda^c}(c_{t+j}, h_{t+j})\right), \tag{50}$$

where $c_t = C_t/\Theta_t$, and Λ^h denotes the marginal disutility of work. Therefore, a log-linearization of (50) gives

$$\hat{v}_{t+j,t} = \eta_c(\hat{c}_{t+j,t} - \hat{c}_{t+j}) + \eta_h(\hat{h}_{t+j,t} - \hat{h}_{t+j}) + \hat{v}_{t+j}, \tag{51}$$

where η_x $(x = c, h)$ indicates the elasticity of the marginal rate of substitution between leisure and consumption with respect to x, evaluated at the steady state. By the assumption that changes in consumption occur in a way that maintains the marginal utility of consumption equal across households, $\hat{c}_{t+j,t}$ and $\hat{c}_{t+j}$ are, respectively, functions of $\hat{h}_{t+j,t}$ and $\hat{h}_{t+j}$. Moreover, from (17) it follows that

$$\hat{h}_{t+j,t} - \hat{h}_{t+j} = -\theta_w\left(\hat{x}_{wt} + \varrho_w \Sigma_{k=0}^{j-1}\hat{\pi}_{t+k} - \Sigma_{k=1}^{j}\hat{\pi}_{t+k}^{w}\right).$$

Substituting this result in (51), I get

$$\hat{v}_{t+j,t} = -\chi\theta_w\left(\hat{x}_{wt} + \varrho_w \Sigma_{k=0}^{j-1}\hat{\pi}_{t+k} - \Sigma_{k=1}^{j}\hat{\pi}_{t+k}^{w}\right) + \hat{v}_{t+j}, \tag{52}$$

where I defined $\chi = \frac{-\Lambda_h^c}{\Lambda_c^c}\eta_c + \eta_h$, and where Λ_i^c indicates the derivative of the marginal utility of consumption with respect to argument i.

In (15), dividing both sides by W_t and log-linearizing, I obtain

$$\hat{x}_{wt} = \frac{\alpha_w}{1-\alpha_w}(\hat{\pi}_t^w - \varrho_w \hat{\pi}_{t-1}). \tag{53}$$

Substituting (53) and (52) into (49), I obtain

$$\begin{aligned}(\hat{\pi}_t^w - \varrho_w \hat{\pi}_{t-1}) = \gamma \Sigma_{j=0}^{\infty} (\beta\alpha_w)^j \ E_t\Big(\hat{v}_{t+j} - \hat{\omega}_{t+j} \\ + (1+\chi\theta_w)\left(\Sigma_{k=1}^{j} \hat{\pi}_{t+k}^w - \varrho_w \Sigma_{k=0}^{j-1} \hat{\pi}_{t+k}\right)\Big),\end{aligned} \tag{54}$$

where $\gamma = \frac{(1-\alpha_w)(1-\beta\alpha_w)}{\alpha_w(1+\theta_w\chi)}$.

Finally, forwarding (54) one period, premultiplying it by $\beta\alpha_w$, and subtracting the resulting expression from (54), I obtain the wage equation (19) in the text.

Empirical Implementation

To compute the solution, I cast the model in the following canonical form:

$$Y_{t+1} = MY_t + \Psi u_{t+1} + \Pi \eta_{yt+1}, \tag{55}$$

where $\eta_{y,t+1} = y_{t+1} - E_t y_{t+1}$ are expectational errors.

The definitions of the vector Y_t and of the matrix M are as in the text, and the matrices Ψ and Π are

$$\Psi = \begin{bmatrix} N_1 & 0 \\ \underline{0} & 0 \\ \underline{0} & Q \end{bmatrix} \text{ and } \Pi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \underline{0} & \underline{0} \end{bmatrix}.$$

Furthermore,

$$M_{yy} = \begin{bmatrix} \frac{1+\varrho_p\beta}{\beta} & -\frac{\zeta}{\beta} & -\frac{\varrho_p}{\beta} & 0 \\ \varrho_w - \varrho_p & \frac{1+\beta+\gamma+\zeta}{\beta} & \frac{\varrho_p - \varrho_w}{\beta} & -\frac{1}{\beta} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$M_{yZ} = \begin{bmatrix} \underline{0} \\ -\left(\frac{\gamma}{\beta}\Xi - \frac{1}{\beta}e_q' + e_q' A\right) \\ \underline{0} \\ \underline{0} \end{bmatrix}.$$

As indicated in the text, the vector Ξ depends on the chosen specification of preferences and on the assumptions about trend.

Since $v_t = v_t^T + v_t^{cyc} = q_t^T + v_t^{cyc}$, from the definition of the trend in productivity (47), it follows that

$$v_t = q_t + e_q'[I-A]^{-1}A + \eta_c c_t^{cyc} + \eta_h h_t^{cyc},$$

and the vector Ξ is therefore defined as

$$\begin{aligned}\Xi &= e_q'[I-A]^{-1}A + \eta_c\left(e_{cy}' + e_h' - e_q'[I-A]^{-1}A\right) + \eta_h e_h' \\ &= (1-\eta_c)e_q'[I-A]^{-1}A + \left[\eta_c(e_{cy}' + e_h') + \eta_h e_h'\right].\end{aligned}$$

The parameters of interest in this expression are the elasticities η_c and η_h, which are estimated together with the adjustment parameters of the wage and price equations.

Inference on Wage Rigidity

To translate the estimate of the "inertia" parameter γ into an estimate of the degree of wage rigidity, I need to parameterize χ, which is

$$\chi = \frac{-\Lambda_h^c H}{\Lambda_c^c C}\eta_c + \eta_h. \tag{56}$$

I first consider a slight transformation of this expression:[44]

$$\chi = \frac{-\Lambda_h^c \Lambda^c}{\Lambda_c^c \Lambda^h}\left(\frac{\Lambda^h H}{\Lambda^c C}\right)\eta_c + \eta_h \tag{57}$$

and then write the expression for η_c as

$$\begin{aligned}\eta_c &= -\frac{\Lambda_c^c C}{\Lambda^c} + \frac{\Lambda_h^c C}{\Lambda^h} = \sigma + \frac{\Lambda_h^c C}{\Lambda^h} \\ &= \sigma + \frac{\Lambda_h^c}{\Lambda_c^c}\left(\frac{\Lambda_c^c C}{\Lambda^c}\right)\frac{\Lambda^c}{\Lambda^h} = \sigma\left(1 - \frac{\Lambda_h^c}{\Lambda_c^c}\frac{\Lambda^c}{\Lambda^h}\right),\end{aligned} \tag{58}$$

[44]A more detailed discussion of this parameterization is in Sbordone (2001).

where, with conventional notation, I indicate with σ the inverse of the intertemporal elasticity of substitution in consumption. Expression (58) implies that

$$\frac{\Lambda_h^c}{\Lambda_c^c}\frac{\Lambda^c}{\Lambda^h} = \frac{\sigma - \eta_c}{\sigma};$$

substituting this result in (57), I obtain

$$\chi = \left(\frac{\sigma - \eta_c}{\sigma} * \tau\right)\eta_c + \eta_h.$$

Therefore, given the estimated η_c and η_h, one can determine the value of χ for any value that one wishes to assign to σ and to the ratio wH/C, which I have denoted by τ. The computations in table 2 are based on three different assumptions about the value of the intertemporal elasticity of substitution in consumption (corresponding to $\sigma = 4, 5$, or 10) and the value of $\tau = 1$. Every value of σ implies, in turn, a different degree of nonseparability in preferences.

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Monetary Policy and Inflation Dynamics*

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Since the early 1980s, the U.S. economy has changed in some important ways: inflation now rises considerably less when unemployment is low, and the volatility of output and inflation have fallen sharply. This paper examines whether changes in monetary policy can account for these changes in the economy. The results suggest that changes in monetary policy can account for most or all of the change in the inflation-unemployment relationship. In addition, changes in policy can explain a large proportion of the reduction in the volatility of the output gap.

JEL Codes: E31, E32, E52, E61.

1. Introduction

In this paper, I assess the extent to which shifts in monetary policy can account for an important change in the relationship between unemployment and inflation in the United States: it appears that, in a simple reduced-form Phillips curve relationship between changes in inflation and the unemployment rate, the estimated coefficient on unemployment has been considerably smaller since the early 1980s than it was earlier (Atkeson and Ohanian 2001; Staiger, Stock, and Watson 2001). In addition, I look at the ability of monetary policy to account for changes in the reduction in the volatility of output

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and inflation that also dates from the early 1980s (McConnell and Perez-Quiros 2000).

The notion that monetary policy should affect inflation dynamics is an old one, dating at least to Friedman's dictum that inflation is always a monetary phenomenon (1968). In his famous "Critique," Lucas (1975) showed how changes in monetary policy could, in principle, affect inflation dynamics. However, Lucas considered only very stylized monetary policies. Here, I explore the effects of more realistic changes in policy on inflation dynamics.

I consider several ways in which U.S. monetary policy may have changed. First, monetary policy *may have become more reactive to output and inflation fluctuations* around the early 1980s (Clarida, Galí, and Gertler 2000). In addition, monetary policy *may have become more predictable*, implying smaller shocks to a simple monetary-policy reaction function. Finally, Orphanides et al. (2000) argue that *policymaker estimates of potential output may have become more accurate.* Such improvements in estimates of potential output would constitute a change in monetary policy, as policy would be made on the basis of more accurate information.

I consider the effects of changes in policy on expectations formation, holding fixed the other behavioral relationships in the economy. Although other relationships are unchanged, changes in policy can nonetheless affect the reduced-form relationship between inflation and economic activity by reducing the signal content of economic slack for future inflation. For example, if monetary policy acts more aggressively to stabilize the economy, then any given deviation in output from potential will contain less of a signal of future inflation. Similarly, a reduction in the persistence of potential output mismeasurement would mean that an increase in output resulting from a misestimate of potential output will not portend as much inflation because it is not expected to last as long.

I examine the predictions of these changes in policy for inflation dynamics and the economy's volatility using stochastic simulations of two macroeconomic models. One is a simple model composed of three equations—for inflation, the federal funds rate, and the output gap. The other is the Federal Reserve's large-scale FRB/US model. An advantage of looking at both models is that they represent points near the extremes of the range of complexity among models currently employed in policy analysis.[1]

Ball, Mankiw, and Romer (1988) have argued that changes in monetary policy may lead to changes in the frequency of price adjustment and, thus, changes in the parameters of the price-adjustment processes taken as structural here. In particular, they argue that the lower and more-stable inflation that has marked the post-1982 period is likely to lead to less-frequent price adjustment. The Ball-Mankiw-Romer conjecture could thus provide an alternative explanation for the reduction in the slope of the reduced-form Phillips curve. In a recent empirical study, however, Boivin and Giannoni (forthcoming) examined the sources of changes in the effects of monetary policy surprises on the economy. They found that the main source of changes in the effects of policy shocks was changes in the parameters of the policy reaction function rather than in the structural parameters of the economy, providing empirical support for the modeling strategy adopted here.

To summarize the results briefly, changes in monetary policy can account for most or all of the reduction in the slope of the reduced-form Phillips curve. Changes in policy can also account for a large portion of the reduction in the volatility of output *gap*, where the output gap is the percent difference between actual output and a measure of trend or potential output. However, as in other recent work (Stock and Watson 2002; Ahmed, Levin, and Wilson 2004), changes in policy account for a smaller proportion of changes in output *growth.* The ability to explain the reduction in inflation volatility is mixed: in the small-scale model, it is possible to explain all of the reduction in inflation volatility, whereas in FRB/US, the changes in policy predict only a small reduction in volatility. Finally, monetary policy's ability to account for changes in the economy is enhanced when changes in monetary policy are broadened to include improvements in the measurement of potential GDP.

[1]Rudebusch (2005) has also looked at the impact of changes in monetary policy on the slope of the Phillips curve. He also finds that changes in monetary policy can have an economically important effect on the estimated slope of the Phillips curve. He notes, however, that such a shift may be difficult to detect econometrically.

Table 1. The U.S. Economy's Changing Volatility (Standard Deviations, Percentage Points)

	GDP Growth[a]	Core Inflation[a]	Unem-ployment Rate	Output Gap, FRB/US	Output Gap, CBO
1984:Q1–2002:Q4	2.22	1.18	1.09	2.08	1.54
1960:Q1–1983:Q4	4.32	2.56	1.77	3.57	3.17
1960:Q1–1979:Q4	3.98	2.42	1.36	2.71	2.61

[a]Quarterly percent change, annual rate.

2. The Changing Economy

2.1 Volatility of Output and Inflation

Table 1 presents standard deviations of the annualized rate of quarterly GDP growth, core inflation (as measured by the annualized quarterly percent change in the price index for personal consumption expenditures other than food and energy), the civilian unemployment rate, and two measures of the output gap. The table compares standard deviations from two early periods—1960–79 and 1960–83—with a more recent period, 1984–2002.

A number of observations suggest this choice of sample periods. First, shortly after arriving as Federal Reserve Chairman in 1979, Paul Volcker initiated a major shift in U.S. monetary policy and, as noted in the introduction, the empirical evidence suggests that U.S. monetary policy shifted at about that time, or shortly thereafter. Second, as discussed in the next subsection, Atkeson and Ohanian (2001) find an important shift in the relationship between inflation and real economy activity dating from this period. Finally, as others have noted (McConnell and Perez-Quiros 2000; Blanchard and Simon 2001), the U.S. economy has been much less volatile since 1983: the standard deviation of GDP growth has fallen by almost half, and that of core inflation by a bit more than half. As discussed in McConnell and Perez-Quiros (2000), the drop in GDP growth volatility is statistically significant. As shown in the table, there has also been a drop in the volatility of the unemployment rate, although it is somewhat less sharp and more dependent on the sample period: relative to the 1960–79 period, the standard deviation of the unemployment rate has fallen by 20 percent, but relative to the period ending in 1983, the decline is almost 40 percent.

The table also shows results for two measures of the output gap—one from the Federal Reserve's FRB/US model and one from the Congressional Budget Office (CBO).[2] For the FRB/US gap measure, the 1984–2002 standard deviation is 23 percent less than in the 1960–79 period and 42 percent less than in the 1960–83 period. The declines in volatility are sharper for the CBO output gap, with a decline in standard deviation of 41 percent since the 1960–79 period and 51 percent since the 1960–83 period.

2.2 The Slope of the Reduced-Form Phillips Curve

Figure 1 plots the over-the-year change in the four-quarter core PCE inflation rate against a four-quarter moving average of the unemployment rate. The panel on the left shows the scatter plot over the 1960–83 period; the panel on the right shows the scatter plot over the 1984–2002 period. Each panel includes a regression line; the slope coefficients are shown in the first column of table 2. The regression run is:

$$(p_t - p_{t-4}) - (p_{t-4} - p_{t-8}) = \gamma_0 + \gamma_1(\Sigma_{i=0,3}\ UR_{t-i})/4, \qquad (1)$$

where $(p_t - p_{t-4})$ indicates the four-quarter percent change in core PCE prices and UR is the civilian unemployment rate. As can be seen in the table, the slope coefficient of this reduced-form Phillips curve falls by nearly half between either of the earlier periods and the post-1983 period. Atkeson and Ohanian (2001) have also noted a sharp drop in the slope of a similar reduced-form relationship, as have Staiger, Stock, and Watson (2001, figure 1.1).

Columns 2 and 3 of table 2 show the change in the slope coefficient in equation (1), using the FRB/US and CBO output gaps,

[2]The output gap is defined as the percentage deviation of real GDP from an estimate of potential GDP. The FRB/US estimate of potential GDP is production-function based, where the inputs are the current capital stock and estimates of structural multifactor productivity (MFP) and structural labor input. Structural MFP is estimated using Kalman-filter methods. Structural labor input also uses Kalman-filter estimates of trends for the workweek and labor force participation as well as other sources. CBO's estimate of potential GDP is described in CBO (2001).

Figure 1. Change over the Year in Four-Quarter Core PCE Inflation vs. Unemployment

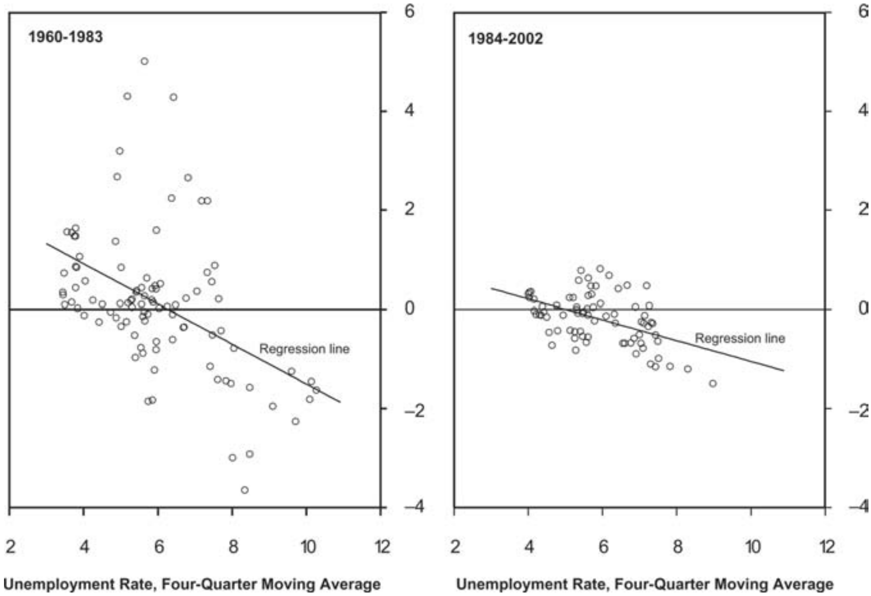


Table 2. Evidence of a Shift in the Slope of the Reduced-Form Phillips Curve, Simple Model (Equation 1)

	Unemployment Rate	FRB/US Output Gap	CBO Output Gap
1984:Q1–2002:Q4	−.201	.092	.159
	(.056)	(.032)	(.038)
1960:Q1–1983:Q4	−.389	.154	.207
	(.085)	(.050)	(.043)
1960:Q1–1979:Q4	−.378	.130	.180
	(.155)	(.095)	(.080)

Note: Standard errors, shown in parentheses, are adjusted for serial correlation of equation residuals using the Newey-West procedure.

Table 3. Evidence of a Shift in the Slope of the Reduced-Form Phillips Curve, More Complex Model (Equation 2)

	Unemployment Rate	FRB/US Output Gap	CBO Output Gap
Slope Coefficient (γ_1)			
1984:Q1–2002:Q4	−.098 (.069)	.051 (.036)	.091 (.048)
1961:Q1–1983:Q4	−.154 (.052)	.066 (.026)	.084 (.028)
1961:Q1–1979:Q4	−.157 (.075)	.056 (.038)	.076 (.038)
Coefficient on First Inflation Lag (γ_2)			
1984:Q1–2002:Q4	.29 (.12)	.29 (.12)	.28 (.12)
1961:Q1–1983:Q4	1.02 (.10)	1.05 (.11)	1.02 (.10)
1961:Q1–1979:Q4	1.01 (.11)	1.04 (.12)	1.02 (.10)

respectively, in lieu of the unemployment rate. Results using the output gap provide a useful robustness check. In addition, the simple three-equation model used below includes the output gap rather than the unemployment rate. The reduction in the Phillips-curve slope is smaller using the output gap: for the FRB/US output gap, the reduction is between 30 percent and 40 percent, depending on the reference period, whereas for the CBO output gap, the reduction is only 12 percent to 23 percent. (As might be expected given typical Okun's law relationships, the coefficients on the output gap are about half the size of the coefficients in the corresponding equations using the unemployment rate—and, of course, they have the opposite sign.)

In table 3, I look at an alternative specification of the reduced-form Phillips curve, in which the quarterly change in inflation is regressed on three lags of itself and the level of the unemployment rate:

$$\begin{aligned}\Delta p_t = \gamma_0 + \gamma_1 UR_t + \gamma_2 \Delta p_{t-1} + \gamma_3 \Delta p_{t-2} \\ + \gamma_4 \Delta p_{t-3} + (1 - \gamma_2 - \gamma_3 - \gamma_4)\Delta p_{t-4},\end{aligned} \qquad (2)$$

where Δp_t indicates the (annualized) one-quarter percent change in the core PCE price index. As in equation (1), the coefficients on lagged inflation are constrained to sum to one. I discuss the evidence for this restriction in section 2.3. For the unemployment rate, the results are qualitatively similar to those in table 2, although the magnitude of the reduction in the slope is a bit less, as the coefficient falls by 35 percent to 40 percent. In this regression, there is also a notable drop-off in the statistical significance of the slope coefficient, with the t-ratio falling to 1.4 in the post-1983 sample, from levels of around 2 or 3 in the earlier samples.

For the estimates with the output gap in columns 2 and 3, the slope coefficients now change little between the early and late samples—indeed, for the CBO output gap, the coefficient even rises. However, in equation (2), the slope coefficient no longer summarizes the effect of unemployment on inflation, because the pattern of the coefficients on lagged inflation also matters. As can be seen in the bottom three rows of the table, there was an important shift in these coefficients, with the coefficient on the first lag dropping from around 1 in the early samples to a bit less than 0.3 in the post-1983 sample. This change means that, in the later sample, an initial shock to unemployment will have a much smaller effect on inflation in the following quarter than was the case in the earlier period. If the impact of unemployment on inflation is adjusted for this change in lag pattern, then the estimates in table 3 suggest that there has been a sharp reduction of the impact of the output gap on inflation, of between 50 percent and 67 percent.[3] Because the slope coefficient in the simple model of equation (1) provides a single summary statistic for the change in the inflation dynamics, I will focus on changes in this coefficient in my work below.[4]

[3] In particular, I compute a "sacrifice ratio," which is the loss in output or unemployment required to obtain a permanent reduction of 1 percentage point in inflation.

[4] The working-paper version (Roberts 2004) includes additional reduced-form results with more control variables, including food and energy prices, productivity, and changes in the natural rate of unemployment. Estimates of the drop in the sacrifice ratio vary from 15 percent to 70 percent. Nonetheless, the results presented in tables 2 and 3 are representative of the range of estimates.

2.3 Has U.S. Inflation Stabilized?

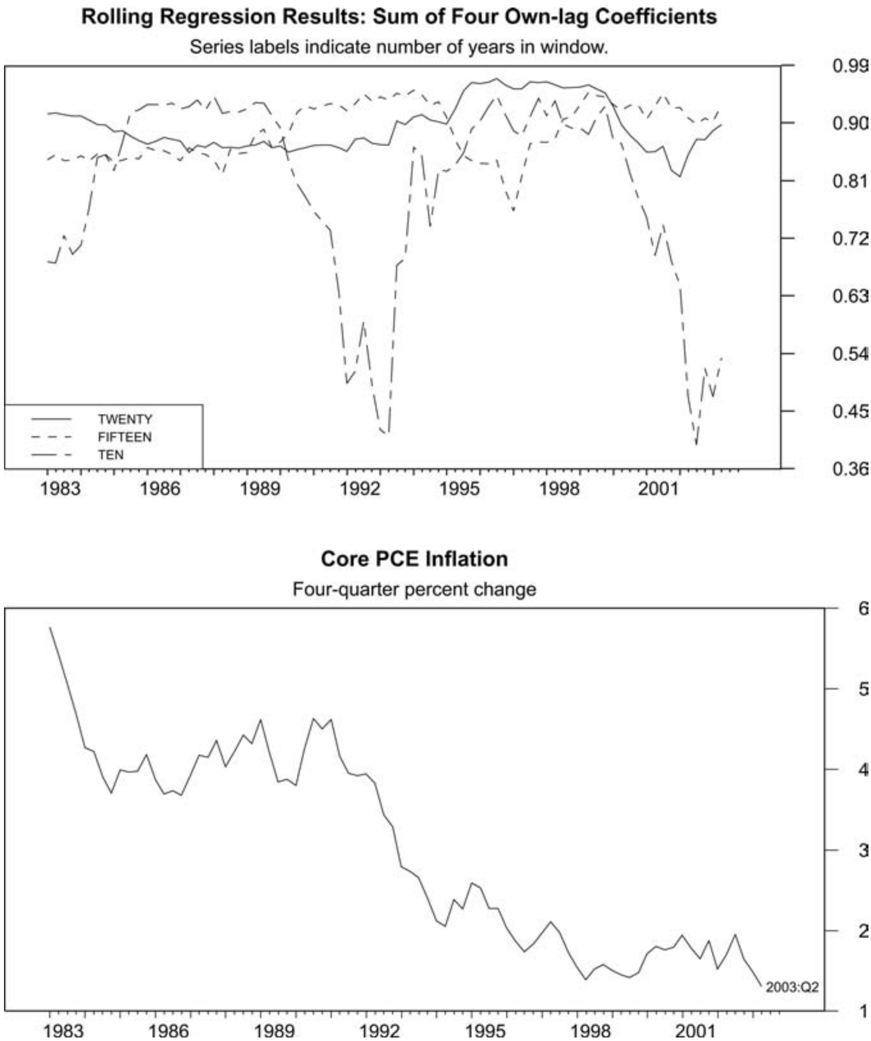
In the preceding subsection, it was assumed that the sum of coefficients on lagged inflation in the reduced-form Phillips curves remained equal to one. Of course, it is possible to imagine that if a central bank had managed to stabilize the inflation rate, inflation would no longer have a unit root, and the sum of lagged coefficients in the reduced-form Phillips curve would no longer equal one. Ball (2000) argues that, prior to World War I, inflation was roughly stable in the United States, and the sum of lagged coefficients in reduced-form Phillips curves was less than one; Gordon (1980) makes a similar point.

It is not yet clear if inflation stability is once again a reality for the United States. Figure 2 plots the sum of lagged inflation coefficients from a rolling regression of U.S. core PCE inflation on four lags of itself, using windows of ten, fifteen, and twenty years. With a twenty-year window, the sum of lagged coefficients remains near 0.9 at the end of the sample, about where it was twenty years earlier. With a fifteen-year window, the sum is more variable, but here, too, it ends the sample at a high level. Using a ten-year window, there is more evidence that the persistence of inflation has fallen, as the sum of lagged coefficients drops to around 0.5.

The inflation data in the bottom panel helps explain these results. Inflation has moved down over the post-1983 period: inflation as measured by core PCE prices averaged 4 percent from 1984 to 1987 but only $1\frac{1}{2}$ percent over the 1998–2002 period. In the most recent ten-year period, inflation has moved in a relatively narrow range, consistent with the small coefficient sum estimated over this period.

While the results with the ten-year window suggest that the United States may have entered a period of inflation stability, the evidence from the wider windows is less conclusive. Of course, a longer time series generally provides more convincing evidence than a shorter one. On net, the evidence would seem to suggest that inflation has remained highly persistent in the United States over the 1984–2002 period. I will return to the issue of inflation stability in section 7.

Figure 2. Has Inflation Stabilized?



3. Changing Monetary Policy

3.1 Changes in the Reaction Function

One way to characterize the implementation of monetary policy is with a "dynamic Taylor rule" of the form:

$$\begin{aligned} ff_t = \rho ff_{t-1} + (1-\rho)\{r^* + (p_t - p_{t-4}) + \alpha\ xgap_t \\ + \beta[(p_t - p_{t-4}) - \pi^*_t]\} + \epsilon_t, \end{aligned} \tag{3}$$

where ff is the federal funds rate, r^* is the equilibrium real interest rate, $(p_t - p_{t-4})$ is the four-quarter inflation rate, $xgap$ is the GDP gap, and π^* is the target inflation rate. A number of studies have found that such a rule characterizes monetary policy after 1983 quite well. Among others, these studies include Clarida, Galí, and Gertler (CGG, 2000) and English, Nelson, and Sack (ENS, 2003).

While the dynamic Taylor rule appears to be a good characterization of policy over the past two decades, its performance prior to 1980 is less impressive. For example, CGG find, in a similar model, very small estimates of the inflation parameter β—indeed, their point estimates put β at less than zero over the 1960–79 period. In this case, real interest rates will fail to rise when inflation is above target, which, as CGG discuss, can lead to an unstable inflation rate.

CGG emphasize the increase in the value of β as indicating an important shift in monetary policy in the early 1980s. They also provide evidence that policy has become more responsive to output fluctuations, reporting a large increase in α. Taylor (1999) and Stock and Watson (2002) also find a large increase in the coefficient on output in a similar monetary policy rule.

Table 4 summarizes the assumptions about monetary policy coefficients used in the simulations below. One set of parameters—labeled "aggressive"—is similar to the estimates of ENS for recent U.S. monetary policy. In the less-aggressive policy settings, the response to output is assumed to be half that in the aggressive setting, while the response to inflation is intended to be a minimal response that is consistent with stability.[5]

[5] The "least" aggressive policy in column 1 is the base case; the "less" aggressive alternative in column 2 is used in cases where the policy in column 1 leads to numerical solution problems.

Table 4. Alternative Monetary Policy Rules

	Least Aggressive Policy	Less-Aggressive Policy	Aggressive Policy
ρ	0.7	0.7	0.7
α	0.5	0.5	1.0
β	0.0001	0.1	0.5

The reaction function in equation (3) includes an error term. One interpretation of such "shocks to monetary policy" is that they constitute changes to the objectives of monetary policy that are not fully captured by a simple econometric specification. Such an interpretation is perhaps most straightforward in a setting in which the long-run inflation objective of the central bank is not firmly established, as may have been the case for the United States in the pre-1980 period. In such a context, shocks to the reaction function could correspond to changes in the inflation target. Another interpretation—that the shocks represent errors in the estimation of the right-hand-side variables of the model—will be taken up shortly.

Table 5 presents some evidence that the variability of the error term in the reaction function has fallen. The first column presents the unconditional standard deviation of the change in the quarterly average funds rate, which falls by between 40 percent and 55 percent, depending on the early reference period. The second column reports the standard error of the residuals from simple reduced-form models of the funds rate, in which the funds rate is regressed on four lags

Table 5. Volatility of the Federal Funds Rate (Standard Deviations, Percentage Points)

	Change in Funds Rate	Residuals from Reduced-Form Model
1984:Q1–2002:Q4	.56	.38
1960:Q1–1983:Q4	1.27	1.16
1960:Q1–1979:Q4	.92	.70

of itself, the current value and four lags of the FRB/US output gap, and the current value and four lags of quarterly core PCE inflation. This residual is considerably less variable in the post-1983 period, with the standard deviation falling by between 45 percent and 67 percent. In the simulations in sections 5 and 6, I will consider a reduction in the standard deviation of the shock to a monetary-policy reaction function like equation (3) of a bit more than half, from 1.0 to 0.47.

3.2 Improvements in Output Gap Estimation

As noted in the introduction, Orphanides et al. (2000) have suggested a specific interpretation for the error term—namely, that it reflects measurement error in the output gap. In particular, suppose that the monetary authorities operate under the reaction function:

$$\begin{aligned} ff_t = \rho ff_{t-1} + (1-\rho)\{r^* + (p_t - p_{t-4}) \\ + \alpha(xgap_t + noise_t) + \beta[(p_t - p_{t-4}) - \pi_t^*]\}, \end{aligned} \tag{4}$$

where,

$$noise_t = \phi \; noise_{t-1} + \epsilon_t. \tag{5}$$

Here, *noise* has the interpretation of measurement error in the output gap. Orphanides et al. (2000) estimate the time-series process for ex post errors in the output gap by comparing real-time estimates of the output gap with the best available estimates at the end of their sample. They find that there was an important shift in the time-series properties of the measurement error in the output gap. In particular, for the period 1980–94, the serial correlation of output gap mismeasurement is 0.84, considerably smaller than the 0.96 serial correlation they find when they extend their sample back to 1966.[6]

[6]It is reasonable to suppose that recent revisions to potential output will be smaller than revisions in the more-distant past, owing simply to the passage of time: estimates in the middle of the sample will be more accurate because future data as well as past data can be used to inform the estimate. To get some notion of the potential importance of this effect, I ran a Monte Carlo experiment on a Kalman-filter model of trend output. I found that the reduction in revisions at the end of the sample was much smaller than what Orphanides et al. (2000) document. Hence, the reduction in revisions in Orphanides et al. appears too large to be explained by the simple passage of time.

The later period examined by Orphanides et al.—1980–1994—is earlier than the post-1983 period that has been characterized by reduced volatility and reduced responsiveness of inflation to the unemployment rate. It would thus be of interest to have an estimate of such errors for a more-recent period. The paper by English, Nelson, and Sack (2003) suggests an indirect method of obtaining such an estimate. They estimate a monetary-policy reaction function similar to equation (3), but with a serially correlated error term. When estimated using current-vintage data, such a model can be given an interpretation in terms of the reaction function with noisy output gap measurement in equations (4) and (5). This can be seen by rewriting equation (4) as

$$\begin{aligned} ff_t = \rho ff_{t-1} + (1-\rho)\{r^* + (p_t - p_{t-4}) \\ + \alpha\ xgap_t + \beta[(p_t - p_{t-4}) - \pi_t^*]\} + u_t, \end{aligned} \tag{6}$$

where $u_t \equiv \alpha(1-\rho)\ noise_t$, and is thus an AR(1) error process because, as noted in equation (5), $noise_t$ is an AR(1) process.

The results of ENS suggest that the noise process had a root of 0.7 over the 1987–2001 period. Estimates of a model similar to theirs suggest that the standard deviation of the shock to the noise process was 1.2 percentage points—about the same as Orphanides et al. (2000) found for both their overall and post-1979 samples.[7]

The preceding discussion suggests two extreme noise processes—the "worst-case" process identified by Orphanides et al. (2000), with a serial correlation parameter of 0.96, and the process implicit in the serial correlation process of the error term from a reaction function estimated with recent data, where $\phi = 0.70$. I will also consider an intermediate case, with $\phi = 0.92$, which can be thought of as a less-extreme version of the Orphanides et al. worst case. Because there is little evidence for a shift in the standard deviation of the shock to the noise process, I assume the same value for all three processes, 1.10. These assumptions are summarized in table 6.

[7]The standard errors of the shocks to the estimated processes were similar in the two samples of Orphanides et al.: 1.09 and 0.97 percentage points in the longer and shorter samples, respectively.

Table 6. Alternative Assumptions about Gap Estimation Errors (Standard Deviations, Percentage Points)

	Serial Correlation ϕ	Impact Standard Deviation	Unconditional Standard Deviation
Worst Case	.96	1.1	3.9
Intermediate	.92	1.1	2.8
Recent Past	.70	1.1	1.5

3.3 Specification of the Inflation Target

As discussed in section 2.3, movements in inflation appear to have remained persistent in the 1983–2002 period. One reason for such persistence may be that the implicit inflation objective varied over this period. A specification that allows for inflation objectives to drift in response to actual events is

$$\pi_t^* = \mu\pi_{t-1}^* + (1-\mu)\Delta p_t, \tag{7}$$

where, as before, Δp_t represents annualized inflation. In equation (7), a fixed inflation target can be specified by setting $\mu = 1$. If $\mu < 1$, however, then the inflation target will be affected by past inflation experience, and inflation will possess a unit root. In most of the simulations that follow, I assume $\mu = 0.9$. In simulations of the FRB/US model, this value of μ allows the model to capture the historical relationship between economic slack and persistent changes in inflation. The results are not greatly affected by small changes in this parameter.

4. Models

I examine the implications of changes in the conduct of monetary policy for output and inflation variability using two models. One is a variant of the three-equation macroeconomic model that has been used in many recent analyses of monetary policy (see, for example, Fuhrer and Moore 1995; Rotemberg and Woodford 1997; Levin, Wieland, and Williams 1999; and Rudebusch 2005). One appeal of the three-equation model is that it can be thought of as including the minimal number of variables needed to model the monetary policy process: the monetary-policy reaction function is combined with equations for its independent variables—inflation and the output gap. In addition, the model's small size makes it straightforward to vary model parameters. The model is described more fully shortly.

The other model I use is the Federal Reserve's large-scale FRB/US model. FRB/US is described in detail in Brayton et al. (1997) and Reifschneider, Tetlow, and Williams (1999). Among the key features of the FRB/US model are the following: the underlying structure is optimization based, decisions of agents depend on explicit expectations of future variables, and the structural parameters of the model are estimated. More-specific features of the model are discussed at the end of this section.

In both the three-equation model and FRB/US, economic agents are assumed to be at least somewhat forward looking, and they form model-consistent expectations of future outcomes. As a consequence, their expectations will be functions of the monetary policy rule in the model. In this way, these models are—at least to some extent—robust to the Lucas critique, which argues that agents' expectations should change when the policy environment changes.

In addition to the monetary-policy reaction function described in section 3, the three-equation model also includes a New Keynesian Phillips curve and a simple "IS curve" that relates the current output gap to its lagged level and to the real short-term interest rate. The New Keynesian Phillips curve is

$$\Delta p_t = E_t \Delta p_{t+1} + \kappa \ xgap_t + \eta_t, \tag{8}$$

where η_t is an error term representing shocks to inflation. The microeconomic underpinnings of such a model are discussed in various places—see, for example, Roberts (1995). Because equation (8) can be thought of as having an explicit structural interpretation, it will be referred to as the "structural Phillips curve," in contrast to "reduced-form Phillips curves" such as equations (1) and (2).

One shortcoming of the New Keynesian Phillips curve under rational expectations is that it does a poor job of fitting some key macroeconomic facts (Fuhrer and Moore 1995). A number of suggestions have been made for addressing its empirical shortcomings.

Some recent work has focused on the possibility that inflation expectations are less than perfectly rational (Mankiw and Reis 2002). One way of specifying inflation expectations that are less than perfectly rational is

$$E_t \Delta p_{t+1} = \omega M_t \Delta p_{t+1} + (1-\omega)\Delta p_{t-1}, \tag{9}$$

where the operator M indicates rational or "mathematical" expectations. An interpretation of this specification is that only a fraction ω of agents use rational expectations, while the remainder use last period's inflation rate as a simple rule of thumb for forecasting inflation. Substituting equation (9) into equation (8) yields

$$\Delta p_t = \omega M_t \Delta p_{t+1} + (1-\omega)\Delta p_{t-1} + \kappa\ xgap_t + \eta_t. \tag{10}$$

Fuhrer and Moore (1995) and Christiano, Eichenbaum, and Evans (CEE, 2005) provide alternative microeconomic interpretations of equation (10). Fuhrer and Moore assume that agents are concerned with relative real wages. CEE argue that in some periods, agents fully reoptimize their inflation expectations, whereas in others, they simply move their wage or price along with last period's aggregate wage or price inflation. In their model, wages and prices are reset each period and thus are only sticky for a very brief period. The only question is how much information is used in changing those wages and prices.

The theoretical models of both Fuhrer and Moore and CEE suggest that $\omega = 1/2$. The results of Boivin and Giannoni (forthcoming) provide empirical support for $\omega = 1/2$. I will therefore assume ω is about $1/2$ in the simulations below.[8] I discuss other aspects of the calibration choice in section 6.

The IS curve is

$$xgap_t = \theta_1 xgap_{t-1} + (1-\theta_1)E_t\ xgap_{t+1} - \theta_2(r_{t-2} - r^*) + \nu_t, \tag{11}$$

[8] To be precise, I assume $\omega = 0.475$. I choose a value slightly less than $1/2$ because with larger values, the model with an evolving inflation target often proved unstable—technically, it had too many large roots—when expectations formation was strongly forward looking. This result suggests that with an evolving inflation target, stability is affected by the degree of forward-looking behavior. Technical issues aside, this result suggests an alternative reason why the economy may be highly volatile when there is not a firm commitment to an inflation target.

where r is the real federal funds rate and ν_t is a random shock to aggregate demand. As in equations (3) and (4), r^* is the equilibrium real federal funds rate, which is assumed to be constant. One way to interpret the equation's error term, ν_t, however, is as a variation in r^*. Rotemberg and Woodford (1997) discuss how an IS curve with $\theta_1 = 0$ can be derived from household optimizing behavior. Amato and Laubach (2004) show how habit persistence can lead to a specification with lagged as well as future output. These papers also show how the equilibrium real interest rate, r^*, is related to the underlying preference parameters of households. The lagged effect of interest rates on output can be justified by planning lags; Rotemberg and Woodford (1997) assume a similar lag. Again, details of the calibration are provided in section 6.

Stock and Watson (2002) also examine the effects of changes in monetary policy using a small macroeconomic model. Their model also has a reduced-form output equation, a model of inflation with explicitly forward-looking elements, and a monetary-policy reaction function; in addition, they include an equation for commodity prices. While Stock and Watson's model includes several explicitly calibrated parameters, it also includes a number of lag variables for which parameters are not reported, making a close comparison of the models difficult.

The three-equation model is limited in the detail it can provide. For example, it is specified in terms of the output gap rather than overall output. Thus, for this model, only the variability of the output gap, and not output growth as well, can be reported. The more-elaborate FRB/US model includes estimates of output growth as well as output gap, which will facilitate comparisons with earlier work that only reports results for output growth. Also, in the three-equation model, inflation is a function directly of the output gap, whereas in most structural models, prices should be related to marginal cost. As discussed in Brayton et al. (1997), however, in the FRB/US model, inflation is modeled as ultimately moving with marginal cost, subject to adjustment costs. Finally, the three-equation model cannot show the implications of a shock to trend productivity; the only supply shock in the model is the shock to the Phillips curve. In the current version of the FRB/US model, multifactor productivity is explicitly modeled as a stochastic trend. Hence, the stochastic simulations of the FRB/US model include technology shocks.[9]

5. Impulse Responses

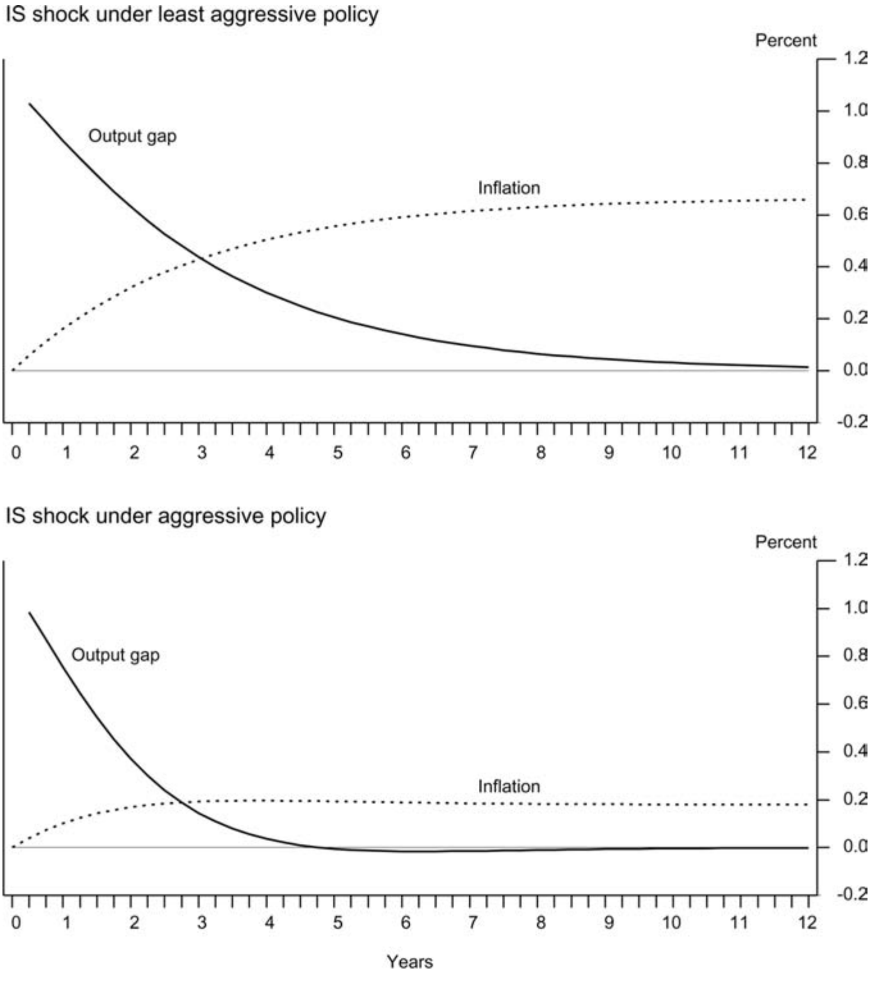
This section examines how the output and inflation effects of shocks to the model economy change under different monetary policies. Figure 3 shows the effects of a shock to the IS curve on output and inflation using the three-equation model discussed in the previous section. The top panel shows the effects of the shock under the least aggressive monetary policy, while the bottom panel shows the effects under the aggressive policy. In both panels, the shock initially raises output and inflation. Because the inflation target is affected by past inflation, there is a permanent increase in inflation in both panels. However, under the aggressive policy in the bottom panel, output returns more rapidly to its preshock value, and the long-run increase in inflation is much smaller. Moreover, one to two years after the initial shock, the increase in inflation is notably smaller relative to output, suggesting a smaller reduced-form relationship between these variables under the aggressive policy.

Figure 4 looks at the effects of a reduction in the persistence of output gap estimation errors, holding fixed the responsiveness of monetary policy at the aggressive level. The top panel considers the worst-case estimate for error persistence ($\phi = 0.96$), while the bottom panel shows the recent-past case ($\phi = 0.7$). As can be seen, an initial 1-percentage-point estimation error has much-larger and more-persistent effects on output and inflation in the top panel than in the bottom panel. Moreover, the impact of the shock on inflation over the first couple of years is much larger relative to the impact on inflation in the top panel, again suggesting a larger reduced-form slope.[10]

[9]Footnote 2 contains additional information about the supply side of the FRB/US model.

[10]Some notion of how the relationship between output and inflation changes between the top and bottom panels of figures 3 and 4 can be gleaned from changes in the sacrifice ratio, which can be thought of as the integral of the output gap divided by the permanent change in inflation. In figure 3, the sacrifice ratio is 4.6 for the simulation in the top panel and 7.7 in the bottom panel. Hence, under

Figure 3. Implications of a More-Aggressive Monetary Policy for the Effects of an IS Shock Three-Equation Model



the aggressive policy in the bottom panel, any given change in inflation is associated with a larger output gap, consistent with the expectation that an aggressive policy will limit the inflation consequences of any given movement in the output gap. In figure 4, the sacrifice ratio is 3.1 in the top panel and 4.7 in the bottom panel, again consistent with the idea that better monetary policy limits the responsiveness of inflation to output gaps.

Figure 4. Effects of an Output Gap Estimation Error Three-Equation Model

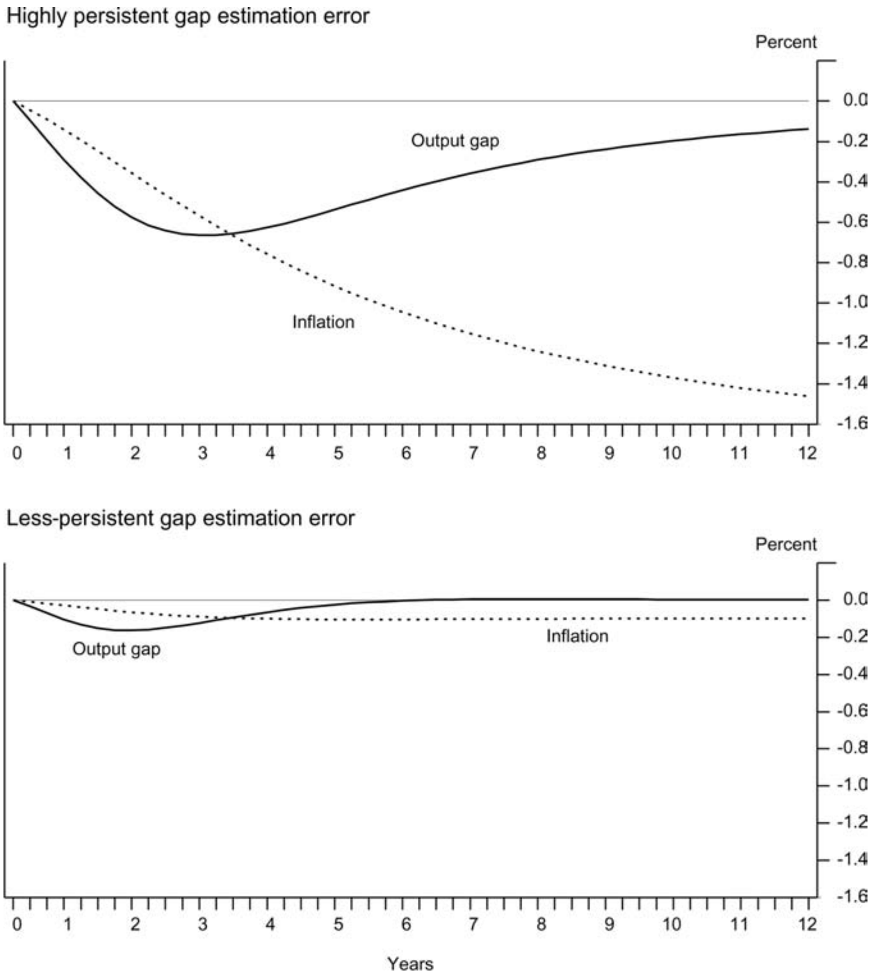
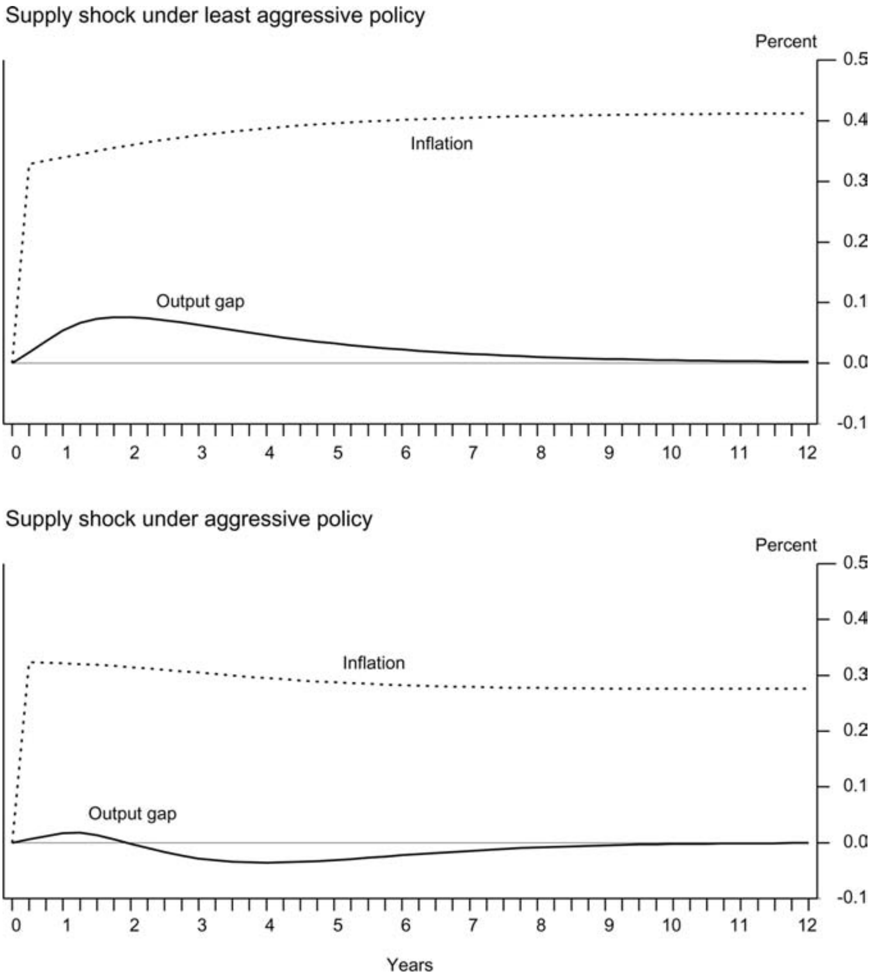


Figure 5 shows the effects of a shock to the Phillips curve itself. The two panels once again show the effects of the shock under alternative assumptions about how aggressively monetary policy reacts. Looking first at the top panel, the shock leads to an immediate increase in inflation and, because the inflation target is assumed to be affected by past inflation in this simulation, inflation remains

Figure 5. Implications of a More-Aggressive Monetary Policy for the Effects of a Supply Shock Three-Equation Model



permanently higher. The output gap rises in this case, as the weak—and gradual—response of monetary policy to the higher inflation leads initially to a decline in the real interest rate and thus stimulates aggregate demand. As a result, inflation rises a bit more than in response to the initial shock. Under the aggressive policy in the bottom panel, however, real interest rates eventually rise relative to baseline, so there is a period of negative output gaps. The eventual increase in inflation is thus smaller than under the least aggressive policy, although, once again, because the inflation target is allowed to be endogenous, inflation is permanently higher.

Of course, the simulations in figures 3 through 5 show only the effects of individual shocks. By contrast, the empirical reduced-form Phillips-curve coefficients such as those discussed in section 2 reflect the effects of all shocks. A convenient way to consider the joint effect of a number of shocks on the reduced-form coefficients is stochastic simulation, the subject of the next section.

6. Stochastic Simulations

6.1 Results with the Three-Equation Model

In this subsection, I use the three-equation model to assess how changes in monetary policy affect volatility and the relationship between inflation and unemployment.

To calibrate the IS curve of the model, I first chose a weight on future output, θ_1, of $1/2$, similar to the degree of forward looking assumed in the structural Phillips curve discussed above. I then chose the IS-curve slope, θ_2, so as to match the effect of an identified monetary policy shock on output in a VAR estimated over the 1960–2002 period, which implied $\theta_2 = 0.1$. I chose the slope of the structural Phillips curve and the standard errors of the shocks to the IS and structural Phillips curves so as to approximate the volatility of output, the volatility of inflation, and the slope of the simple reduced-form Phillips curve. This exercise resulted in standard deviations of the IS and Phillips-curve shocks of 0.55 percentage point and 0.17 percentage point, respectively, and a structural Phillips-curve slope of 0.005. Note that the model has been calibrated so that the volatility of the residuals is representative of the low-volatility period for the U.S. economy. The IS and Phillips-curve parameter estimates are in broad agreement with empirical estimates of New Keynesian models, such as Boivin and Giannoni (forthcoming).

To carry out the stochastic simulations, I first solve the model under model-consistent expectations. I then use this solution to generate simulated data by taking random draws from the distribution of the model residuals. For each draw, a time series of 160 quarters is created. To reduce the influence of starting values, the first eighty quarters are discarded, and sample statistics are computed using the last eighty quarters. The shocks are drawn from a normal distribution.

The standard deviation of output growth, the output gap, and inflation are calculated for each draw, and the summary statistics are averaged over the draws. Similarly, the slope of the reduced-form Phillips curve is estimated for each draw. I present the slope coefficients from two models. One is the simple reduced-form Phillips curve of equation (1), modified to use the output gap rather than the unemployment rate:

$$(p_t - p_{t-4}) - (p_{t-4} - p_{t-8}) = \gamma_0 + \gamma_1 \ (\Sigma_{i=0,3} \, xgap_{t-i})/4. \tag{12}$$

Presenting results for this simple model has the advantage that the results are directly comparable to the simple relations illustrated in figure 1; also, the results focus attention (and econometric power) on the slope coefficient. I also consider the slightly more elaborate reduced-form Phillips curve:

$$\begin{aligned} \Delta p_t = \gamma_0 + \gamma_1 \, xgap_t + \gamma_2 \Delta p_{t-1} \\ + \gamma_3 \Delta p_{t-2} + \gamma_4 \Delta p_{t-3} + \gamma_5 \Delta p_{t-4}. \end{aligned} \tag{13}$$

Equation (13) is similar to equation (2) except that, here, the sum of the lagged inflation coefficients is not constrained to sum to one. Estimates of this equation can thus allow for the possibility that the changes in policy under consideration may have reduced the sum of lag coefficients.[11]

In table 7, I consider the effects of changes in the coefficients of the reaction function and changes in the volatility of a simple i.i.d. error term added to the reaction function. These changes in policy are similar to those that Stock and Watson (2002) have considered. I later turn to the possibility that the serial correlation of errors in the measurement of the output gap may have fallen. Initially, to

[11]The reported coefficient standard errors for equation (12) are based on the simulated distributions and so are not affected by the serial correlation induced by the overlapping left-hand-side variable.

Table 7. The Effects of Changes in Monetary Policy on Volatility and the Slope of the Reduced-Form Phillips Curve: Three-Equation Model

	(1)	(2)	(3)	(4)
Inflation Target: M-policy Shock: α and β:	Evolving S.D. = 1.0 0.5 and 0.0001	Evolving S.D. = 1.0 1.0 and 0.5	Evolving S.D. = 0.47 1.0 and 0.5	Fixed S.D. = 0.47 1.0 and 0.5
Volatility:				
S.D. (Gap)	2.32	1.82	1.79	1.84
S.D. (Inflation)	2.36	1.20	1.19	1.02
Phillips Curves:				
Simple; Slope	.213	.139	.139	.146
	(.073)	(.084)	(.086)	(.085)
With Lags; Slope	.062	.041	.041	.040
	(.024)	(.026)	(.026)	(.026)
Sum Lag Coefs.	.98	.94	.94	.93
	(.03)	(.06)	(.06)	(.06)
Inflation,	.97	.94	.94	.92
Largest Root	(.03)	(.05)	(.05)	(.05)

Notes: Numbers in parentheses are standard errors.
"Sum lag coefs." indicates the sum of the coefficients on lagged inflation.
Based on stochastic simulations with 5,000 draws.

isolate the effects of these changes in policy from the possibility that target inflation has become better anchored, I assume that target inflation is updated using equation (7) with a parameter $\mu = 0.9$.

Comparing columns 1 and 2 shows the effects of moving to a more-aggressive monetary policy. This policy shift leads to an important reduction in the volatility of both the output gap and inflation: the standard deviation of the gap falls by about one-quarter, while the standard deviation of inflation falls by half. The reduction in the volatility of the output gap is at the low end of the range of the historical decline between early and later periods, while the reduction in inflation volatility is in line with that seen historically.[12]

The slope of the simple reduced-form Phillips curve falls by about one-third, in line with historical reductions in the Phillips-curve slope as measured with the output gap, although somewhat short of the reductions in the coefficient on the unemployment rate. Also, the t-statistic on the slope of the simple Phillips curve falls from 2.9 to 1.6, suggesting that these changes in monetary policy may also have affected the apparent statistical robustness of the simple Phillips-curve relationship. The more-elaborate Phillips curve shows a similar reduction in slope, while the sum of coefficients on lagged inflation remains high.[13]

Column 3 introduces an additional change in monetary policy, a reduction in the volatility of the shock to policy. In this model, this additional change has very little effect on the results: the volatility of the output gap and inflation fall somewhat more, but the estimated Phillips-curve slopes are very similar to those in column 2.

Column 4 looks at the implications of the switch to a fixed inflation target, under the assumption of an aggressive monetary policy and small shocks to the reaction function, as in column 3. This shift in policy has only small effects on the results: the volatility of the output gap actually rises a bit, while that of inflation falls by about 15 percent. The slope of the reduced-form Phillips curve rises a bit. Perhaps surprisingly, the persistence of inflation falls only slightly, and inflation remains highly persistent. In section 7, I return to the question of how the behavior of the economy might change if the central bank were to adopt a strict inflation target.

[12]It is worth noting that this improvement in both output and inflation volatility is not at variance with Taylor's (1979) well-known volatility trade-off. Taylor's trade-off described the choice among alternative *optimal* policies, whereas the policy changes I am examining here represent a shift from policies that are well outside the optimality frontier toward policies that are closer to that frontier.

[13]The change in policy between columns 1 and 2 of table 7 involves an increase in the reaction-function coefficients on both the output gap and inflation. If only the coefficient on inflation is increased, there is a sharp reduction in inflation volatility (to a standard deviation of 1.5 percent) but a proportionately smaller reduction in output gap volatility (to a standard deviation of 2.1 percent). The slope of the simple reduced-form Phillips curve falls to 0.173, near the midpoint of the estimates shown in columns 1 and 2, suggesting that both elements of the more-aggressive policy contribute to the reduction in the Phillips-curve slope.

Table 8. The Effects of Changes in the Persistence of Potential Output Errors on Volatility and the Slope of the Reduced-Form Phillips Curve: Three-Equation Model

	(1)	(2)	(3)	(4)	(5)
Error Persistence: Inflation Target: α and β:	0.96 Evolving 0.5 & 0.1	0.96 Fixed 1.0 & 0.5	0.92 Evolving 0.5 & 0.1	0.7 Evolving 1.0 & 0.5	0.7 Fixed 1.0 & 0.5
Volatility:					
S.D. (Gap)	3.51	2.64	2.68	1.82	1.88
S.D. (Inflation)	6.71	2.37	3.31	1.22	1.05
Phillips Curves:					
Simple; Slope	.289	.221	.239	.145	.150
	(.066)	(.068)	(.070)	(.085)	(.082)
With Lags: Slope	.069	.059	.064	.043	.043
	(.024)	(.022)	(.023)	(.026)	(.026)
Sum Lag Coefs.	.99	.97	.98	.94	.93
	(.02)	(.03)	(.03)	(.06)	(.06)
Inflation,	.99	.97	.98	.94	.93
Largest Root	(.02)	(.03)	(.03)	(.05)	(.05)

Notes: Numbers in parentheses are standard errors.
"Sum lag coefs." indicates the sum of the coefficients on lagged inflation.
Based on stochastic simulations with 5,000 draws.

Table 8 considers the model with persistent gap errors, along the lines suggested by Orphanides et al. (2000). Column 1 shows Orphanides et al.'s "worst-case scenario," in which output-gap estimation errors have a quarterly autocorrelation of 0.96. In column 1, a weak response of monetary policy to output and inflation errors is assumed. In this case, the volatility of the output gap is at the high end of historical estimates, while the volatility of inflation is far greater than was the case historically. In columns 2 and 3, I therefore consider two ways of reducing the influence of persistent output gap errors. In column 2, I assume the more-aggressive policy reaction-function parameters along with a fixed inflation target.[14] With this policy, the volatility of the output gap is about in line with historical values for the 1960–79 period, and inflation is much closer to the historical range. The slope of the simple reduced-form Phillips curve is just above the historical range. Inflation is highly persistent, with a root of 0.97. These results are consistent with the argument made in Orphanides (2001) that poor estimation of potential output, rather than weak response of monetary policy to output and inflation, was responsible for volatile and persistent inflation in the pre-1984 period.

In column 3, I consider an alternative in which the monetary policy reaction remains weak, but output gap errors are somewhat less persistent than in columns 1 and 2. This adjustment cuts the standard deviation of inflation by more than half, bringing it closer to the range that was seen historically. Both columns 2 and 3 provide plausible candidate characterizations of the high-volatility period.

Column 4 considers the implications of a reduction in the persistence of the shock to the reaction function, so that $\phi = 0.7$, under the assumption of an aggressive monetary policy. As can be seen by comparing column 4 with column 2, reducing the persistence of the shock to the reaction function while holding the parameters of the reaction function fixed has important effects on the volatility of output and the slope of the Phillips curve. The standard deviation of the output gap falls by 30 percent, and the volatility of inflation falls by almost half. The slope of the simple Phillips curve falls by about one-third. As in table 7, there is a marked reduction in the statistical significance of the Phillips-curve slope, as the t-ratio falls from more than 3 to just 1.7. Inflation remains highly persistent, with an autoregressive root of 0.94.

Comparing column 4 with column 3 gives an alternative view of the change in monetary policy—namely, that it represented a combination of more-aggressive policy and better estimation of potential output. The story on output volatility is about the same as for column 2. The reduction in the slope of the simple reduced-form Phillips curve is greater in this case, at around one-half.

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[14] A fixed inflation target is assumed because of numerical problems with the solution under an evolving target. Inflation is nonetheless highly persistent in this case.

The final column of table 8 adds the assumption of a fixed inflation target. The volatility of inflation falls somewhat further, while that of the output gap rises a bit. Estimates of the slope of the Phillips curve are little changed. As in table 7, the adoption of a fixed inflation target has surprisingly little effect on the persistence of inflation in this model.

6.2 Results with FRB/US

In this section, I work with the Federal Reserve's FRB/US model of the U.S. economy. In the simulations, I solve a linearized version of the FRB/US model under model-consistent expectations. The draws for the stochastic simulations are taken from a multivariate normal distribution using the variance-covariance matrix of residuals from the FRB/US model estimated over the 1983–2001 period. Hence, the volatility of the residuals is representative of the low-volatility period for the U.S. economy. Because the FRB/US model includes the unemployment rate, these reduced-form Phillips-curve results are based on this variable, as in equations (1) and (2).

Columns 1, 2, and 3 of table 9 consider the effects of first increasing the parameters of the reaction function and then reducing the volatility of the (not serially correlated) shock to the reaction function. As in table 7, it is primarily the change in the reaction-function parameters that affects the volatility of output; there is only a small further reduction from reducing the volatility of the reaction-function shock. Between column 1 and column 3, the standard deviation of the GDP *gap* falls by about 30 percent, somewhat more than with the three-equation model. However, the standard deviation of GDP *growth* falls by only about 10 percent. The reduction in the volatility of the output gap is in the range of the historical decline between the 1960–79 and 1983–2002 periods. But the reduction in GDP growth volatility is considerably smaller than what actually occurred, a finding similar to that of Stock and Watson (2002).

Increasing the reaction-function parameters leads to a reduction of only about 10 percent in the slope of the simple reduced-form Phillips curve; the slope of the more-elaborate Phillips curve, however, falls by more than 40 percent. Inflation is highly persistent in both columns 1 and 2, with an autoregressive root around 0.9.

Table 9. The Effects of Changes in Monetary Policy on Volatility and the Slope of the Reduced-Form Phillips Curve: FRB/US

	(1)	(2)	(3)	(4)
Inflation Target: M-policy Shock: α and β:	Evolving S.D. = 1.0 0.5 and 0.0001	Evolving S.D. = 1.0 1.0 and 0.5	Evolving S.D. = .47 1.0 and 0.5	Fixed S.D. = .47 1.0 and 0.5
Volatility:				
S.D. (Gap)	2.4	1.8	1.7	1.9
S.D. (GDP Growth)	3.5	3.2	3.2	3.1
S.D. (Inflation)	2.8	3.2	3.2	1.6
Phillips Curves:				
Simple; Slope	−.186	−.176	−.129	−.081
	(.279)	(.394)	(.409)	(.278)
With Lags; Slope	−.130	−.074	−.054	−.081
	(.179)	(.198)	(.205)	(.181)
Sum Lag Coefs.	.84	.89	.89	.71
	(.12)	(.10)	(.10)	(.20)
Inflation,	.89	.92	.92	.77
Largest Root	(.07)	(.07)	(.07)	(.11)

Notes: Numbers in parentheses are standard errors.
"Sum lag coefs." indicates the sum of the coefficients on lagged inflation.
Based on stochastic simulations with 2,000 draws.

With the reduction in the volatility of the monetary policy shock in column 3, the slope of the simple Phillips curve relative to column 1 is now about one-third smaller. The slope of the more-elaborate Phillips curve also declines further, and the total reduction is now almost 60 percent. As in the three-equation model, these changes in monetary policy can account for most or all of the reduction in the slope of the reduced-form Phillips curve.

Column 4 looks at the implications of the switch to a fixed inflation target, under the assumption of an aggressive monetary policy

Table 10. The Effects of Changes in the Persistence of Potential Output Errors on Volatility and the Slope of the Reduced-Form Phillips Curve: FRB/US

	(1)	**(2)**	**(3)**	**(4)**	**(5)**
Error Persistence: Inflation Target: α and β:	0.96 Evolving 0.5 & 0.1	0.96 Fixed 1.0 & 0.5	0.90 Evolving 0.5 & 0.1	0.70 Evolving 1.0 & 0.5	0.70 Fixed 1.0 & 0.5
Volatility:					
S.D. (Gap)	6.1	3.4	3.5	1.8	1.9
S.D. (GDP Growth)	5.8	4.0	4.1	3.2	3.1
S.D. (Inflation)	6.3	3.6	3.7	3.3	1.7
Phillips Curves:					
Simple; Slope	−.47	−.50	−.33	−.17	−.102
	(.16)	(.20)	(.22)	(.41)	(.26)
With Lags; Slope	−.37	−.35	−.22	−.070	−.093
	(.18)	(.16)	(.17)	(.21)	(.18)
Sum Lag Coefs.	.83	.84	.84	.89	.71
	(.09)	(.09)	(.11)	(.11)	(.20)
Inflation,	.94	.92	.92	.92	.77
Largest Root	(.04)	(.06)	(.05)	(.07)	(.11)

Notes: Numbers in parentheses are standard errors.
"Sum lag coefs." indicates the sum of the coefficients on lagged inflation.
Based on stochastic simulations with 2,000 draws.

and small shocks to the reaction function, as in column 3. In contrast to the three-equation model, there is now a large reduction in inflation persistence with the switch to a fixed inflation target, a result perhaps more in line with prior expectations.

Table 10 considers the implications of reduced serial correlation in output-gap estimation errors as well as changes in the responsiveness of policy. Assuming the "worst-case" output-gap estimation errors along with a weak response of monetary policy to output and inflation (column 1) leads to volatilities of output and inflation that are far greater than was the case historically. For inflation, this result is similar to that in table 8; for the output gap, the excess volatility is much greater. Columns 2 and 3 repeat the two solutions to this excess volatility used with the three-equation model: (i) assuming a more-aggressive policy and (ii) assuming somewhat smaller persistence of estimation errors. Both solutions reduce the volatility of output and inflation, and both lead to plausible characterizations of the earlier period.

Column 4 considers the implications of a reduction in the persistence of the shock to the reaction function, so that $\phi = 0.7$, under the assumption of an aggressive monetary policy. As can be seen by comparing column 4 with column 2, reducing the persistence of the shock to the reaction function while holding the parameters of the reaction function fixed has large effects on the volatility of the output gap and the slope of the Phillips curve. The standard deviation of the output gap falls by almost half, at the high end of the range of the historical decline. However, the reduction in the standard deviation of output *growth* is only about 20 percent. The slope of the simple Phillips curve falls by two-thirds—even more than the declines that have been seen historically. Inflation remains highly persistent, with an autoregressive root of 0.92. The comparison of column 4 with column 3 yields similar results for output volatility. The reduction in the slope of the simple reduced-form Phillips curve is less sharp in this case, but it is still around 40 percent.

While the FRB/US model predicts reductions in output gap volatility and the slope of the Phillips curve that are consistent with historical changes, it does not suggest that monetary policy had much to do with the reduction in inflation volatility. Looking across tables 9 and 10, changes in policy lead to reductions of the standard deviation of inflation of at most 10 percent, well short of the historical reductions.

The final column of table 10 adds the assumption of a fixed inflation target. As might be expected, the persistence of inflation drops further, as does the slope of the simple Phillips curve. The standard deviation of inflation also drops and is now in the historical range. However, this reduction in inflation volatility occurs only when the persistence of inflation is considerably lower than was the case for the 1983–2002 period.

7. How Might the Economy Behave under a Fixed Inflation Target?

As figure 2 suggested, the evidence for a drop in inflation persistence is, thus far, inconclusive. However, estimates limited to the past decade are suggestive that inflation may have become more stable. In this section, I use the three-equation model to consider how the behavior of the economy may change in a regime of inflation stability.

Expectations formation is central to the question of how inflation dynamics are likely to change in a regime of inflation stability. In the simulations considered so far, equation (9) has been used as the model of expectations formation. In equation (9), "nonoptimizing" agents rely on past inflation as an indicator of future inflation. Similarly, under CEE's interpretation of the model, agents find it useful to use lagged inflation to index prices in periods when they do not reoptimize. It is most useful to use lagged inflation as a predictor of future inflation when inflation is highly persistent; if inflation were not persistent, lagged inflation would not be a good indicator of future inflation. It is reasonable to assume, therefore, that if inflation were to be stabilized, agents would change their inflation forecasting rules. Here, I consider one characterization of how agents might change the way they set expectations as inflation dynamics change. In particular, I consider the following generalization of equation (9):

$$E_t \, \Delta p_{t+1} = \omega M_t \, \Delta p_{t+1} + (1 - \omega)\lambda \Delta p_{t-1}, \tag{14}$$

where the parameter λ is chosen so as to give the best "univariate" forecast of inflation. (For simplicity, the implicit inflation target is assumed to be zero.) Thus, in equation (9), $\lambda = 1$ was the best univariate forecast under the assumption—which has heretofore been close to accurate—that inflation had a unit root.

Suppose that the central bank adopts a fixed inflation target. According to column 4 of table 7, in the three-equation model with $\lambda = 1$, this change would result in an inflation process with a root of 0.92. We can then ask what would happen if agents adopted a univariate inflation forecast with $\lambda = 0.92$. A simulation under this assumption shows that inflation will have a root of 0.79. But then, agents will want to update their forecasting rule to be consistent with this new assessment of the serial correlation of inflation. Doing so, however, further reduces the persistence of inflation. Following this process to its logical conclusion suggests that a "fixed point" for univariate expectations formation is approximately zero autocorrelation (actually, the fixed point is $\lambda = 0.02$).

According to this model, then, the long-run consequences of a policy of a fixed inflation target is inflation that is not only stationary but actually uncorrelated. Of course, this evolution is based on a particular assumption about expectations formation. But there is some historical precedent for such an outcome. Ball (2000) argues that in the 1879–1914 period, when the United States was on the gold standard, the "best univariate forecast" for inflation was zero: under the gold standard, inflation was not expected to persist. Ball (2000) and Gordon (1980) argue that allowing perceived inflation dynamics to change with the policy regime can go a long way to allowing the expectations-augmented Phillips curve—which works well in the second half of the twentieth century—to account for the properties of inflation under the gold standard.

If a central bank were to adopt a fixed inflation target, how quickly might a transition to stable inflation take place? Based on simulations with the baseline three-equation model, it could take a while for agents to catch on. In table 7, for example, inflation is predicted to have an autoregressive root of 0.92 even under a fixed inflation target. In the twenty-year sample underlying these simulations, the t-ratio of the hypothesis that this coefficient is still one is only 1.6, well short of Dickey-Fuller critical values. So even if agents with univariate expectations were good time-series econometricians, they may see little need to change the way they form their expectations. FRB/US, however, is more sanguine: switching to a fixed inflation target leads to a large reduction in the persistence of inflation, and the largest root of inflation falls to 0.77 (table 9, column 4, and table 10, column 5). If this were the case, the transition to stable inflation could occur more rapidly.

8. Conclusions

Can the changes in monetary policy that took place in the United States in the years after 1979 account for the subsequent changes in inflation dynamics? Overall, the evidence presented here suggests that the answer is yes: the monetary policy changes I consider predict large declines in the slope of the reduced-form relationship between the change in inflation and the unemployment rate, holding fixed the structural parameters underlying inflation behavior. This result holds both in the large-scale FRB/US model and in a small New Keynesian-style model.

These changes in policy also have implications for the volatility of output and inflation—which also changed in the early 1980s. The results for inflation volatility were mixed: in the small model, changes in monetary policy can account for most or all of the reduction in the standard deviation of inflation. By contrast, in FRB/US, these monetary policy changes predict only a small reduction in inflation volatility.

The paper considered two alternative views of the change in monetary policy. One view is that the responsiveness of monetary policy to output and inflation increased, along the lines suggested by Clarida, Galí, and Gertler (2000). I also considered the implications of an alternative view of the change in the monetary policy process suggested by Orphanides et al. (2000)—that policymakers may have improved their methods for estimating potential GDP. This alternative view strengthens the ability of monetary policy changes to explain changes in the economy, implying greater reductions in volatility and in the slope of the reduced-form Phillips curve.

As in other recent work, I find that changes in monetary policy can explain only a small fraction of the reduction in the standard deviation of the *growth rate* of output. However, I find that changes in monetary policy can explain most or all of the reduction in the standard deviation of the output *gap*; such effects are especially strong in the FRB/US model. There are a number of possible explanations for this result. One possibility is that improvements in monetary policy can account for a large proportion of the reduction in aggregate demand volatility—and thus can account for the reduction in the volatility of the output gap, which abstracts from shocks to aggregate supply. As suggested by McConnell and Perez-Quiros (2000), another possibility is that improvements in inventory management are also an important source of the reduction in output volatility. Because inventory investment is not very persistent, improvements in inventory management would have a disproportionate effect on output growth relative to the output gap. Sorting through these possibilities would be an interesting topic for future research.

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