



INTERNATIONAL JOURNAL OF CENTRAL BANKING

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Lars E. O. Svensson

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Preface

In July 2004, the Bank for International Settlements (BIS), the European Central Bank, and each of the Group of Ten* (G-10) central banks announced their plans to support the development of a new publication focused on central bank theory and practice. Other central banks were invited to participate in this joint project and will join as sponsoring institutions later this year.

From its initiation, the sponsors were committed to ensuring that the *International Journal of Central Banking* (IJCB) offer articles of high analytical quality for a professional audience. The primary objectives of the IJCB are to widely disseminate the best policy-relevant and applied research on central banking and to promote communication among researchers both inside and outside of central banks.

Federal Reserve Vice Chairman Roger W. Ferguson, Jr. first proposed the idea of such a journal and discussed the concept with several BIS colleagues and with Federal Reserve Board Governor Ben S. Bernanke, who agreed to serve as the initial managing editor. Bank for International Settlements General Manager Malcolm Knight and Bank of England Chief Economist Charles Bean provided important early support for the journal project. Bean currently serves as chairman of the journal's governing body, which is comprised of representatives from the sponsoring institutions.

Journal editors include Governor Bernanke, London School of Economics professor Dr. Hyun Song Shin, European Central Bank economist Dr. Frank Smets, University of Tokyo Graduate School of Economics professor Dr. Kazuo Ueda, and Dr. Michael Woodford, John Bates Clark Professor of Political Economy at Columbia University. A distinguished group of associate editors coordinate solicitation and review of articles across a range of disciplines reflecting the missions of central banks around the world. While featuring policy-relevant articles on any aspect of the theory and practice of central banking, the publication has a special emphasis on research bearing on monetary and financial stability.

*The G-10 central banks are the Bank of Canada, Bank of England, Bank of France, Bank of Italy, Bank of Japan, Deutsche Bundesbank, Federal Reserve Board, National Bank of Belgium, Netherlands Bank, Sveriges Riksbank, and Swiss National Bank.

Monetary Policy with Judgment: Forecast Targeting*

Lars E.O. Svensson
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“Forecast targeting,” forward-looking monetary policy that uses central-bank judgment to construct optimal policy projections of the target variables and the instrument rate, may perform substantially better than monetary policy that disregards judgment and follows a given instrument rule. This is demonstrated in a few examples for two empirical models of the U.S. economy, one forward looking and one backward looking. A complicated infinite-horizon central-bank projection model of the economy can be closely approximated by a simple finite system of linear equations, which is easily solved for the optimal policy projections. Optimal policy projections corresponding to the optimal policy under commitment in a timeless perspective can easily be constructed. The whole projection path of the instrument rate is more important than the current instrument setting. The resulting reduced-form reaction function for the current instrument rate is a very complex function of all inputs in the monetary-policy decision process, including the central bank’s judgment. It cannot be summarized as a simple reaction function such as a Taylor rule. Fortunately, it need not be made explicit.

JEL Codes: E42, E52, E58.

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On a general level, this paper is motivated by a desire to provide a better theory of modern monetary policy, both from a descriptive and a normative point of view, than much of the current literature on monetary policy. The current literature to a large extent applies a one-line modeling of monetary policy, such as when the instrument rate is assumed to be a given function of a few variables—for instance, “monetary policy is assumed to follow a Taylor rule.”

I believe that the theory that I develop here is better from a *descriptive* point of view, since it takes into account some crucial aspects of monetary policy decisions, such as the collection, processing, and analysis of large amounts of data, the construction of projections of the target variables, the use of considerable amounts of judgment, and the desire to achieve (mostly) relatively specific objectives. The modern monetary policy process I have in mind can be concisely described as “forecast targeting,” meaning “setting the instrument rate such that the forecasts of the target variables look good,” where “look good” refers to the objectives of monetary policy, such as a given target for inflation and a zero target for the output gap.¹ I believe this view of the monetary policy process is also helpful from a *normative* point of view, for instance, in evaluating the performance of and suggesting improvements to existing monetary policy.²

On a more specific level, this paper is motivated by a desire to demonstrate the crucial and beneficial role of *judgment*—information, knowledge, and views outside the scope of a particular model—in modern monetary policy and, in particular, to demonstrate that the appropriate use of good judgment can dramatically improve monetary policy performance, even when compared to

¹Bernanke (2004) discusses and compares forecast targeting (which he refers to as “forecast-based policies”) and simple instrument rules (which he refers to as “simple feedback policies”). He states that “the Federal Reserve relies primarily on the forecast-based approach for making policy” and cites Greenspan’s (2004) speech entitled “Risk and Uncertainty in Monetary Policy” as evidence. He also notes “that not only have most central banks chosen to rely most heavily on forecast-based policies but also that the results, at least in recent years, have generally been quite good, as most economies have enjoyed low inflation and overall economic stability.”

² See Svensson (2001a) and Svensson et al. (2002) for examples of evaluations of monetary policy in New Zealand and Norway, respectively, with this perspective.

policy that is optimal in all respects except for incorporating judgment.³ As will be explained in detail below, judgment will be represented as the central bank's conditional mean estimate of arbitrary multidimensional stochastic "deviations"—"add factors"—to the model equations.⁴ I also wish to demonstrate the benefits of regarding the whole projection paths of the target variables rather than forecasts at some specific horizon, such as eight quarters, as the relevant objects in the monetary policy decision process. In particular, I believe that it is important to emphasize the whole projection of future instrument rates rather than just the current instrument rate. Furthermore, the modern view of the transmission mechanism of monetary policy emphasizes that monetary policy actions have effects on the economy and the central bank's target variables almost exclusively through the private-sector expectations of the future paths of inflation, output, and interest rates that these actions give rise to; therefore, monetary policy is really the management of private-sector expectations (Woodford 2003a). From this follows that effective implementation of monetary policy requires the effective communication to the private sector of the central bank's preferred projections, including the instrument-rate projection. The most obvious communication of these projections is to explicitly announce and motivate them. Finally, I want to demonstrate the benefits of the approximation of inherently rather complex infinite-horizon central-bank projection models of the economy to much simpler finite-horizon projection models that are much easier to use but still arbitrarily close approximations to the infinite-horizon models.

The decision process of modern monetary policy has several distinct characteristics (see Brash 2000, Sims 2002, and Svensson 2001a):

³ Svensson (2003) also emphasizes the role of judgment in monetary policy but does not provide any direct comparison of the performance of monetary policy with and without judgment.

⁴ Svensson and Tetlow (2005) show how central-bank judgment can be extracted according to the method of optimal policy projections (OPP). This is a method to provide advice on optimal monetary policy while taking policymakers' judgment into account. An early version of the method was developed by Robert Tetlow for a mostly backward-looking variant of the Federal Reserve Board's FRB/US model. The resulting projections have been referred to (somewhat misleadingly) at the Federal Reserve Board as "policymaker perfect-foresight projections." The paper demonstrates the usefulness of OPP with a few example projections for two Greenbook forecasts and the FRB/US model.

1. Large amounts of data about the state of the economy and the rest of the world, including private-sector expectations and plans, are collected, processed, and analyzed before each major decision.
2. Because of lags in the transmission process, monetary policy actions affect the economy with a lag. For this reason alone, good monetary policy must be forward looking, aim to influence the future state of the economy, and therefore rely on forecasts—projections. Central-bank staff and policymakers make projections of the future development of a number of exogenous variables, such as foreign developments, import supply, export demand, fiscal policy, productivity growth, and so forth. They also construct projections of a number of endogenous variables, quantities and prices, under alternative assumptions, including alternative assumptions about the future path of instrument rates. The policymakers are presented with projections of the most important variables, including target variables such as inflation and output, often under alternative assumptions about exogenous variables and, in particular, the instrument rate (such as the instrument rate being constant, following market expectations, following some arbitrary reaction function, or being optimal relative to a specific objective function).
3. Throughout this process, a considerable amount of judgment is applied to assumptions and projections. Projections and monetary policy decisions cannot rely on models and simple observable data alone. All models are drastic simplifications of the economy, and data give a very imperfect view of the state of the economy. Therefore, judgmental adjustments in both the use of models and the interpretation of their results—adjustments due to information, knowledge, and views outside the scope of any particular model—are a necessary and essential component in modern monetary policy.
4. Based on this large amount of information and analysis, the policymakers decide on a current instrument rate, such that the corresponding projections of the target variables look good relative to the central bank's objectives. Since the projections of the target variables depend insignificantly on the current

instrument-rate setting and mainly on the whole path of future instrument rates, the policymakers, explicitly or implicitly, actually choose an instrument-rate projection—an instrument-rate plan—and the current instrument-rate decision can be seen as the first element of that plan.

5. Finally, the current instrument rate is announced and implemented. In many cases, the corresponding projections for inflation and output or the output gap are also announced. In a few cases, an instrument-rate projection is announced as well.⁵

This process makes the current instrument-rate decision a very complex function of the large amounts of data and judgment that have entered into the process. I believe that it is not very helpful to summarize this function as a simple reaction function such as a Taylor rule. Furthermore, the resulting complex reaction function is a *reduced form*, which depends on the central bank's objectives, its view of the transmission mechanism of monetary policy, the data the central bank has collected, and the judgment it has exercised. It is the endogenous complex result of a complex process. In no way is this reaction function *structural*, in the sense of being invariant to the central bank's view of the transmission mechanism and private-sector behavior, or the amount of information and judgmental adjustments. Still, much current literature treats monetary policy as characterized by a given reaction function that is essentially structural and invariant to changes in the model of the economy. Treating the reaction function as a reduced form is a first step in a sensible theory of monetary policy. But, fortunately, this complex reduced-form reaction function need not be made explicit. It is actually not needed in the modern monetary policy process.

However, there is a convenient, more structural representation of monetary policy, namely in the form of a *targeting rule*, as advocated recently in some detail in Svensson and Woodford (2005) and Svensson (2003) and earlier (more generally) in Svensson (1999). An *optimal* targeting rule is a first-order condition for optimal monetary policy. It corresponds to the standard efficiency condition of equality between the marginal rates of substitution and the marginal rates of

⁵The Reserve Bank of New Zealand has published an instrument-rate projection for many years. The Bank of Norway is increasingly providing more information about the future path of the instrument rate.

transformation between the target variables, the former given by the monetary policy loss function, the latter given by the transmission mechanism of monetary policy. An optimal targeting rule is invariant to everything else in the model, including additive judgment and the stochastic properties of additive shocks. Thus, it is a compact, robust, and structural representation of monetary policy, and much more robust than the optimal reaction function. A *simple* targeting rule can potentially be a practical representation of robust monetary policy, a robust monetary policy that performs reasonably well under different circumstances.⁶

Optimal targeting rules remain a practical way of representing optimal monetary policy in the small models usually applied for academic monetary policy analysis. However, for the larger and higher-dimensional operational macromodels used by many central banks in constructing projections, the optimal targeting rule becomes more complex and arguably less practical as a representation of optimal monetary policy. In this paper, it is demonstrated that optimal policy projections, the projections corresponding to optimal policy under commitment in a timeless perspective, can easily be derived directly with simple numerical methods, without reference to any optimal targeting rule.⁷ For practical optimal monetary policy, policymakers actually need not know the optimal targeting rule. They also need not know any reaction function. They only need to ponder the graphs of the projections of the target variables that are generated in the policy process and choose the projections of the target variables and the instrument rate that look best relative to the central bank's objectives.

The paper is organized as follows. Section 1 lays out a reasonably general infinite-horizon model of the transmission mechanism and the central bank's objectives; defines projections, judgment, and optimal policy projections; and specifies how the optimal policy can be implemented and what information the private sector needs from the

⁶ McCallum and Nelson (2004) have recently criticized the advocacy of targeting rules in Svensson (2003). Svensson (2004) rebuts this criticism and gives references to a rapidly growing literature that applies targeting rules to monetary-policy analysis. Walsh (forthcoming) shows a case of equivalence between targeting rules and robust control.

⁷ Nevertheless, a general form of an optimal targeting rule is derived in appendix F (available at www.ijcb.org), for the finite-horizon approximation of the projection model.

central bank. The section also presents a simple model of judgment, when the deviation is a version of a finite-order moving average. Then judgment can be seen as the accumulation of information over time and allows for a recursive but high-dimensional representation of the dynamics of the deviation and judgment. Finally, the section represents the optimal policy projections as the solution to a somewhat complex system of difference equations, while taking judgment into account. It also makes the point that, fortunately, the complex reduced-form reaction function need not be made explicit. Section 2 presents a convenient finite-horizon model for the construction of optimal policy projections, for both forward- and backward-looking models. This finite-horizon model can be written as a simple finite system of linear equations. Nevertheless, it is an exact or arbitrarily close approximation to the infinite-horizon model and is easily solved for the optimal policy projections taking judgment into account. Section 3 discusses and specifies monetary policy that disregards judgment and follows different instrument rules, such as variants of the Taylor rule or more complex instrument rules that are optimal in the absence of judgment. Section 4 gives examples of and compares monetary policy with and without judgment for two different empirical models of the U.S. economy: the backward-looking model of Rudebusch and Svensson (1999) and the forward-looking New Keynesian model of Lindé (2002). In these examples, monetary policy with judgment results in substantially better performance than monetary policy without judgment. This is also the case when monetary policy without judgment is represented as a Taylor rule where the instrument rate responds to forward-looking variables that incorporate private-sector judgment (although, as emphasized below, there are serious principal and practical problems in implementing such an instrument rule). Section 5 presents conclusions.

A separate and extensive appendix contains numerous technical details.⁸ These include a general solution of the policy problem and the related system of difference equations with forward-looking variables when the deviation is an arbitrary stochastic process; a specification of the model when the deviation and judgment are finite-order moving-average processes and the application of the practical method of Marcet and Marimon (1998) to that case; the precise

⁸ The appendix is available at www.ijcb.org.

mathematical structure of the finite-horizon approximation model, including the optimal targeting rule; and details on the empirical backward- and forward-looking models.

1. A Model of the Policy Problem with Judgment

Consider the following linear model of an economy with a private sector and a central bank,⁹ in a form that allows for both predetermined and forward-looking variables as well as judgment:

$$\begin{bmatrix} X_{t+1} \\ Cx_{t+1|t} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bi_t + \begin{bmatrix} z_{t+1} \\ 0 \end{bmatrix}. \quad (1)$$

Here, X_t is a (column) n_X -vector of *predetermined* variables (one of these may be unity to conveniently incorporate constants in the model) in period t ; x_t is an n_x -vector of *forward-looking* variables; i_t is an n_i -vector of central-bank *instruments* (the forward-looking variables and the instruments are the *nonpredetermined* variables); z_t is an exogenous n_X -vector stochastic process and called the *deviation* in period t ; and A , B , and C are matrices of the appropriate dimension. For any variable q_t , I let $q_{t+\tau|t}$ denote private-sector expectations of the realization in period $t + \tau$ of $q_{t+\tau}$ conditional on private-sector information available in period t . I assume that the private sector has rational expectations, given its information.

For increased generality, the model is formulated in terms of an arbitrary number of instruments, n_i . In most practical applications, monetary policy can be seen as having only one instrument—a short interest rate, the instrument rate—so then $n_i = 1$.

The upper block of (1) provides n_X equations determining the n_X -vector X_{t+1} in period $t + 1$ for given X_t , x_t , i_t , and z_{t+1} ,

$$X_{t+1} = A_{11}X_t + A_{12}x_t + B_1i_t + z_{t+1} \quad (2)$$

where A and B are partitioned conformably with X_t and x_t as

$$A \equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B \equiv \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}. \quad (3)$$

⁹ For simplicity, there is no explicit fiscal authority in the model, but such an authority and its behavior can be included in the model (1).

The realization of the deviation and the predetermined variables in each period occurs and is observed by the private sector and the central bank in the beginning of the period (the realization of z_{t+1} can be inferred from X_{t+1} , X_t , x_t and i_t and [2]).¹⁰

The lower block of (1) provides n_x equations determining the n_x -vector x_t in period t for given $x_{t+1|t}$, X_t , and i_t ,

$$x_t = A_{22}^{-1}(Cx_{t+1|t} - A_{21}X_t - B_2i_t); \quad (4)$$

I hence assume that the $n_x \times n_x$ submatrix A_{22} is invertible. The realization of X_t is observed by the private sector and the central bank in the beginning of period t ; the central bank then sets the instruments, i_t ; after observing the instruments, the private sector forms its expectations, $x_{t+1|t}$; and this finally determines the forward-looking variables x_t .

To assume that the deviation appears only in the upper block of (1) is not restrictive. Suppose that I have a model where the deviation appears in both blocks:

$$\begin{bmatrix} X_{t+1}^o \\ Cx_{t+1|t} \end{bmatrix} = \begin{bmatrix} A_{11}^o & A_{12}^o \\ A_{21}^o & A_{22}^o \end{bmatrix} \begin{bmatrix} X_t^o \\ x_t \end{bmatrix} + \begin{bmatrix} B_1^o \\ B_2^o \end{bmatrix} i_t + \begin{bmatrix} z_{1,t+1}^o \\ z_{2,t}^o \end{bmatrix}.$$

By adding the vector z_t^o to the predetermined variables, I can always form a new model of the form (1), where

$$X_t \equiv \begin{bmatrix} X_t^o \\ z_{2t}^o \end{bmatrix}, \quad A \equiv \begin{bmatrix} A_{11}^o & 0 & A_{12}^o \\ 0 & 0 & 0 \\ A_{21}^o & 0 & A_{22}^o \end{bmatrix}, \quad B \equiv \begin{bmatrix} B_1^o \\ 0 \\ B_2^o \end{bmatrix},$$

$$z_t \equiv \begin{bmatrix} z_{1t}^o \\ z_2^o \end{bmatrix},$$

and there is no deviation in the lower block.

As in Svensson (2003), the deviation represents additional determinants—determinants outside the model—of the variables in

¹⁰See Svensson and Woodford (2003) for an analysis of optimal policy in a model with forward-looking variables when the current state of the economy is imperfectly observed and inferred from observed indicators.

the economy, the difference between the actual value of a variable and the value predicted by the model. It can be interpreted as model perturbations, as in the literature on robust control.¹¹ The central bank's mean estimate of future deviations will be identified with the central bank's *judgment*. It represents the unavoidable judgment always applied in modern monetary policy. Any existing model is always an approximation of the true model of the economy, and monetary policymakers always find it necessary to make some judgmental adjustments to the results of any given model. Such judgmental adjustments could refer to future fiscal policy, productivity, consumption, investment, international trade, foreign-exchange and other risk premia, raw-material prices, private-sector expectations, and so forth. The "add factors" applied to model equations in central-bank projections (Reifschneider, Stockton, and Wilcox 1997) are an example of central-bank judgment. Given this interpretation of judgment and the deviation z_{t+1} , it would be completely misleading to make a simplifying assumption such as the deviation being a simple autoregressive process. In that case, it could just be incorporated among the predetermined variables. Thus, I will refrain from such an assumption and instead leave the dynamic properties of z_{t+1} unspecified, except in a special case when the deviation is a version of a finite-order moving-average process. Generally, the focus will be on the central bank's judgment of the whole sequence of future deviations.

More precisely, let the infinite-dimensional period- t random vector $\zeta^t \equiv (z'_{t+1}, z'_{t+2}, \dots)'$ (where $'$ denotes the transpose) denote the vector of the (in period t) unknown random vectors $z_{t+1}, z_{t+2}, \dots$. Let the central bank's beliefs in period t about the random vector ζ^t be represented by the infinite-dimensional probability distribution Φ_t with distribution function $\Phi_t(\zeta^t)$. The probability distribution Φ_t may itself be time-varying and stochastic. The central bank is assumed to know the matrices A, B, C, D , and W and the discount factor δ (D, W , and δ refer to the central bank's objectives and are defined below). The private sector is assumed to know the matrices A, B , and C , but may or may not know the central bank's objectives

¹¹See, for instance, Hansen and Sargent (1998). However, that literature deals with the more complex case when the model perturbations are endogenous and chosen by nature to correspond to a worst-case scenario.

(that is, D , W , and δ). The private sector may or may not have the same beliefs about the future deviations as the central bank.

Let Y_t be an n_Y -vector of *target* variables. For simplicity, these target variables are measured as the difference from a fixed n_Y -vector Y^* of *target levels*. This vector of target levels is held fixed throughout this paper. In order to examine the consequences of shifting target levels, one only needs to replace Y^t by $Y^t - Y^*$ throughout the paper. Let the target variables be given by

$$Y_t = D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} \quad (5)$$

where D is an $n_Y \times (n_X + n_x + n_i)$ matrix.

Let the central bank's intertemporal loss function in period t be

$$E_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau} \equiv \int \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau} d\Phi_t(\zeta^t) \quad (6)$$

where $0 < \delta \leq 1$ is a discount factor, L_t is the period loss given by

$$L_t = \frac{1}{2} Y_t' W Y_t, \quad (7)$$

and W is a symmetric positive semidefinite $n_Y \times n_Y$ matrix. That is, in period t the central bank wants to minimize the expected discounted sum of current and future losses, where the expectation E_t is with respect to the distribution Φ_t .

Since this is a linear model with a quadratic loss function and the random deviations enter additively, the conditions for certainty equivalence are satisfied. Then, as shown in detail in appendix A, the optimal policy in period t need only consider central-bank mean forecasts—*projections*—of all variables, including the infinite-dimensional mean forecast, z^t , of the random vector ζ^t ,

$$z^t \equiv E_t \zeta^t \equiv \int \zeta^t d\Phi_t(\zeta^t).$$

The central-bank projection in period t of the realization of the deviation in period $t+\tau$, $z_{t+\tau,t}$, is denoted $z^t \equiv (z'_{t+1,t}, z'_{t+2,t}, \dots)'$.

The projection z^t is identified with the central bank's *judgment*. Under the assumed certainty equivalence, the projection z^t is, for optimal policy, a sufficient statistic for the distribution Φ_t . Although there is genuine uncertainty about the future random deviations, ζ^t , the only thing that matters for policy is the mean, the judgment, z^t . The second and higher moments of ζ^t —the variance, skew, kurtosis, and so forth—do not matter for policy.¹² The judgment can itself be seen as an exogenous infinite-dimensional random vector that is realized in the beginning of each period and summarizes the central bank's relevant information in that period about expected future deviations.

Let $q^t \equiv (q'_{t,t}, q'_{t+1,t}, \dots)'$ denote a central-bank projection in period t for any vector of variables $q_{t+\tau}$ ($\tau \geq 0$) (with the exception of ε^t and $\varepsilon_{t+\tau,t}$, to be introduced below), a central-bank mean forecast conditional on central-bank information in period t . (Thus, for variables other than the deviation, the projection also includes the current value, $q_{t,t} = q_t$.) The central bank then constructs various projections of the endogenous variables to be used in its decision process. These projections of endogenous variables may be conditional on various assumptions. In order to keep private-sector expectations and central-bank projections conceptually distinct, I denote the former by $q_{t+\tau|t}$ and the latter by $q_{t+\tau,t}$ for any variable q_t .

For a given judgment, z^t , the projection model of the central bank for the projections (X^t, x^t, i^t, Y^t) in period t —the model the central bank uses in the decision process to consider alternative projections—will be

$$\begin{bmatrix} X_{t+\tau+1,t} \\ Cx_{t+\tau+1,t} \end{bmatrix} = A \begin{bmatrix} X_{t+\tau,t} \\ x_{t+\tau,t} \end{bmatrix} + Bi_{t+\tau,t} + \begin{bmatrix} z_{t+\tau+1,t} \\ 0 \end{bmatrix}, \quad (8)$$

$$Y_{t+\tau,t} = D \begin{bmatrix} X_{t+\tau,t} \\ x_{t+\tau,t} \\ i_{t+\tau,t} \end{bmatrix} \quad (9)$$

for $\tau \geq 0$, where $X_{t,t}$ satisfies

$$X_{t,t} = X_t, \quad (10)$$

¹² The variance of the future deviations will add a term to the intertemporal loss, but that term is independent of policy.

since the realization of the predetermined variables is assumed to be observed in the beginning of period t .

In order to introduce more compact notation, let the $(n_X + n_x + n_i)$ -vector $s_t \equiv (X'_t, x'_t, i'_t)'$ denote the state of the economy in period t , and let the vector $s_{t+\tau,t} \equiv (X'_{t+\tau,t}, x'_{t+\tau,t}, i'_{t+\tau,t})'$ denote the projection in period t of the state of the economy in period $t + \tau$. Finally, let the infinite-dimensional vector $s^t \equiv (s'_{t,t}, s'_{t+1,t}, s'_{t+2,t}, \dots)'$ denote a projection in period t of the (current and future) states of the economy. By (9), I can write the projection of the target variables in a compact way, as a linear function of the projection of the states of the economy, as

$$Y^t = \tilde{D}s^t \quad (11)$$

where $\tilde{D}$ is an infinite-dimensional block-diagonal matrix with the $\tau + 1$ -th diagonal block equal to D for $\tau \geq 0$.

The *set of feasible projections of the states of the economy* in period t , $\mathcal{S}_t$, can now be defined as the set of projections s^t that satisfy (8)–(10) for given X_t and z^t .

The intertemporal loss function (6) with (7) induces an intertemporal loss function for the target-variable projection,¹³

$$\mathcal{L}(Y^t) \equiv \sum_{\tau=0}^{\infty} \delta^\tau Y'_{t+\tau,t} W Y_{t+\tau,t}. \quad (12)$$

The *policy problem* in period t is to find the optimal policy projection $(\hat{s}^t, \hat{Y}^t)$, the projection that minimizes (12) over the set of feasible projections of the states of the economy—that is, subject to (8)–(11) for $\tau \geq 0$ for given X_t and z^t . More compactly,

$$\hat{s}^t = \arg \min_{s^t \in \mathcal{S}_t} \mathcal{L}(\tilde{D}s^t), \quad (13)$$

$\hat{Y}^t \equiv \tilde{D}\hat{s}^t$; and the optimal policy projection $(\hat{X}^t, \hat{x}^t, \hat{i}^t)$ of the predetermined variables, forward-looking variables, and instruments can be extracted from $\hat{s}^t$.

¹³ Note that $\min \mathbb{E}_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau} = \min \{ \mathcal{L}(Y^t) + \mathbb{E}_t \sum_{\tau=0}^{\infty} \delta^\tau (Y_{t+\tau} - Y_{t+\tau,t})' W (Y_{t+\tau} - Y_{t+\tau,t}) \} = \min \{ \mathcal{L}(Y^t) + \sum_{\tau=0}^{\infty} \delta^\tau \text{trace}(W \text{Cov}_t Y_{t+\tau}) \}$. By certainty equivalence, $\text{Cov}_t Y_{t+\tau} \equiv \mathbb{E}_t (Y_{t+\tau} - Y_{t+\tau,t})(Y_{t+\tau} - Y_{t+\tau,t})' \equiv \int (Y_{t+\tau} - Y_{t+\tau,t})(Y_{t+\tau} - Y_{t+\tau,t})' d\Phi_t(\zeta^t)$ is independent of policy, so minimizing (12) in period t implies the same policy as minimizing (6) in period t . Furthermore, note that, since $\text{trace}(W \text{Cov}_t Y_{t+\tau})$ will normally be strictly positive, (6) will normally converge only for $\delta < 1$, whereas (12) will normally converge also for $\delta = 1$.

The policy problem will be further specified below to correspond to commitment in a “timeless perspective,” in order to avoid any time-consistency problems (see Woodford 2003b and Svensson and Woodford 2005).

1.1 *Implementation and What Information the Private Sector Needs*

The *implementation* of the optimal policy in period t involves announcing the optimal policy projection and setting the instruments in period t equal to the first element of the instrument projection,

$$i_t = \hat{i}_{t,t}.$$

In period $t + 1$, conditional on new realizations of the predetermined variables, X_{t+1} , and the judgment, z^{t+1} , a new optimal policy projection, $(\hat{X}^{t+1}, \hat{x}^{t+1}, \hat{z}^{t+1}, \hat{Y}^{t+1})$, is found and announced together with a new instrument setting,

$$i_{t+1} = \hat{i}_{t+1,t+1}.$$

In a forward-looking model, the private sector (including policymakers other than the central bank) will need to know at least parts of the aggregate projections $\hat{X}^t$, $\hat{x}^t$, and $\hat{z}^t$, in order to make decisions consistent with these and make the rational-expectations equilibrium in the economy correspond to the central bank’s optimal policy projection. If the private sector knows the matrices A , B , C , D , and W and the discount factor δ and has the same judgment z^t as the central bank, it can in principle compute the optimal policy projection itself—assuming that it has the same computational capacity as the central bank.

However, the private sector actually needs to know less. An assumption maintained throughout this paper is that the private sector knows the model (1), in the sense of knowing the matrices A , B , and C . Furthermore, it observes X_t (determined by X_{t-1} , x_{t-1} , and i_{t-1} in period $t - 1$ and the realization of z_t in the beginning of period t according to [2]) in the beginning period t , then observes $i_t = \hat{i}_{t,t}$ set by the central bank, thereafter forms one-period-ahead expectations $x_{t+1|t}$, and finally determines (and thereby knows) x_t ; after this, period t ends. In order to make decisions in period t consistent with the optimal policy projection—that is, decisions resulting

in $x_t = \hat{x}_{t,t}$ from (4)—the private sector needs to be able to form expectations $x_{t+1|t} = \hat{x}_{t+1,t}$. The most direct way is if the central bank announces $\hat{x}_{t+1,t}$ and the private sector believes the announcement. Formally, $\hat{x}_{t+1,t}$ is the minimum additional information the private sector needs. However, the central bank may have to provide the whole optimal policy projection, and also motivate the underlying judgment, in order to demonstrate that the optimal policy projections are *internally consistent* with the model (1). In particular, the private sector may not believe $\hat{x}_{t+1,t}$ unless it is apparently consistent with the whole projection $\hat{i}^t$. Furthermore, the private sector will need to know the central bank's loss function— D , W , and δ —in order to judge whether the projections announced are really optimal relative to the central bank's loss function and thereby *incentive-compatible*, *credible*. Only then may the central bank be able to *convince* the private sector to form expectations according to the optimal policy projection.¹⁴ Indeed, the private sector completely trusting the central bank's isolated announcement of $\hat{x}_{t+1,t}$ could invite misleading announcements, given the time-consistency problem discussed below.¹⁵

1.2 Judgment as a Finite-Order Moving Average

Consider the special case when the deviation is a version of a moving-average stochastic process with a given finite order $T > 0$ (where T could be relatively large):

$$z_{t+1} = \varepsilon_{t+1} + \sum_{j=1}^T \varepsilon_{t+1,t+1-j} \quad (14)$$

where $\tilde{\varepsilon}_t \equiv (\varepsilon'_t, \varepsilon'^t)' \equiv (\varepsilon'_t, \varepsilon'_{t+1,t}, \dots, \varepsilon'_{t+T,t})'$ is a zero-mean iid random $(T+1)n_X$ -vector realized in the beginning of period t and called the

¹⁴ Being explicit about the loss function and announcing the optimal policy projection also seem to take care of the criticism of real-world inflation targeting expressed by Faust and Henderson (2004).

¹⁵ See Geraats (2002) for such examples.

In a much noted contribution, Morris and Shin (2002) and Amato, Morris, and Shin (2002) have emphasized the possibility that public information may be bad and reduce social welfare by crowding out private information. Svensson (2005) scrutinizes this result and shows that, in the model considered by Morris and Shin, public information actually increases social welfare for reasonable parameters.

innovation in period t .¹⁶ For $T = 0$, z_{t+1} is a simple iid disturbance. For $T > 0$, the deviation is a version of a moving-average process.

It follows that the central-bank judgment $z_{t+\tau,t}$ ($\tau \geq 1$) is also a finite-order moving average and satisfies

$$\begin{aligned} z_{t+\tau,t} &\equiv E_t z_{t+\tau} = \sum_{j=\tau}^T \varepsilon_{t+\tau,t+\tau-j} = \varepsilon_{t+\tau,t} \\ &+ \sum_{j=\tau+1}^T \varepsilon_{t+\tau,t+\tau-j} = \varepsilon_{t+\tau,t} + z_{t+\tau,t-1}. \end{aligned}$$

Hence, $\varepsilon_{t+\tau,t} = z_{t+\tau,t} - z_{t+\tau,t-1}$ can be interpreted as the innovation in period t to the previous judgment $z_{t+\tau,t-1}$, the new information the central bank receives in period t about the realization of $z_{t+\tau}$ in period $t + \tau$. Hence, the judgment $z_{t+\tau,t}$ in period t is the sum of current and previous information about $z_{t+\tau}$. For horizons larger than T , the central-bank judgment is constant and, without loss of generality, equal to zero:

$$z_{t+\tau,t} = 0 \quad (\tau > T). \quad (15)$$

The dynamics of the deviation z_t and the judgment z^{t+1} can then be written as

$$\begin{bmatrix} z_{t+1} \\ z^{t+1} \end{bmatrix} = A_z \begin{bmatrix} z_t \\ z^t \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon^{t+1} \end{bmatrix} \quad (16)$$

where the $(T+1)n_X \times (T+1)n_X$ matrix A_z is defined as

$$A_z \equiv \begin{bmatrix} 0_{n_X \times n_X} & I_{n_X} & 0_{n_X \times (T-1)n_X} \\ 0_{(T-1)n_X \times n_X} & 0_{(T-1)n_X \times n_X} & I_{(T-1)n_X} \\ 0_{n_X \times n_X} & 0_{n_X \times n_X} & 0_{n_X \times (T-1)n_X} \end{bmatrix}$$

where $0_{m \times n}$ and I_m denote an $m \times n$ zero matrix and an $n \times n$ identity matrix, respectively. Thus, the dynamics of the deviation and the judgment have a convenient linear and recursive representation.

¹⁶ Note that $\varepsilon^t \equiv (\varepsilon'_{t+1,t}, \varepsilon'_{t+2,t}, \dots, \varepsilon'_{t+T,t})'$ here denotes a random vector realized in the beginning of period t and not the projection in period t of the random variables $\varepsilon_{t+1}, \varepsilon_{t+2}, \dots, \varepsilon_{t+T}$. That projection is always zero under the above assumption of ε^t being a zero-mean iid random variable.

The modeling of the dynamics of the deviation, z_t , and the additive judgment, z^t , in (16) allows for a relatively flexible accumulation of information about future deviations. Whereas the stochastic process for the deviation is not a simple Markov process in terms of itself, but a finite-order moving-average process, it can be written as a higher-dimensional AR(1) process. The restriction imposed is that the innovation is zero-mean and iid across periods. There is no restriction of the variance and covariance of the elements of $\tilde{\varepsilon}_t$ within the period. It follows that, for instance, $\varepsilon_{t+\tau,t}$ may have a variance that is decreasing in τ , corresponding to a situation where there is less information about the mean projection of deviations further into the future; by assumption, there is no specific information about the deviation for $\tau > T$. For given t , there may be serial correlation of $\varepsilon_{t+\tau,t}$ across τ , corresponding to new information about serially correlated future deviations.

1.3 Representing Optimal Policy Projections

Without the judgment terms (or, alternatively, with the deviation being an iid zero-mean process or an autoregressive process with iid shocks), the above infinite-horizon linear-quadratic problem with forward-looking variables is a well-known problem, examined and solved in Backus and Driffill (1986), Currie and Levine (1993), and Söderlind (1999). The traditional way to find a solution to this problem is to derive the first-order conditions for an optimum and combine the first-order conditions with the model (1) to form a system of difference equations with an infinite horizon. The solution can then also be expressed as a difference equation. Furthermore, Marcet and Marimon (1998) have shown a new practical way of reformulating the problem with forward-looking variables as a recursive saddle-point problem (see appendix D).

A new element here is the solution with the judgment. For the case when the deviation is a finite-order moving average, the dynamics of the deviation and the judgment, (16), can be incorporated with (1), the vector of predetermined variables can be expanded to include z^t , and the standard solution can be applied directly.¹⁷ The details for this case are provided in appendices C and D. When the

¹⁷ Since z_t is incorporated in X_t , one does not need to add z_t as a separate predetermined variable.

judgment is a realization of an infinite-dimensional random vector, the standard solution has to be modified to take that into account. The details of that solution in the form of a difference equation are explained in appendices A and B. Here I shall first report the solution in the form of an infinite-horizon difference equation and later develop a very convenient finite-horizon version of the model.

Under the assumption of optimization under commitment, one way to describe the optimal policy projection is by the following difference equations,

$$\begin{bmatrix} \hat{x}_{t+\tau,t} \\ \hat{l}_{t+\tau,t} \end{bmatrix} = F \begin{bmatrix} \hat{X}_{t+\tau,t} \\ z^{t+\tau,t} \\ \Xi_{t+\tau-1,t} \end{bmatrix}, \quad (17)$$

$$\begin{bmatrix} \hat{X}_{t+\tau+1,t} \\ \Xi_{t+\tau,t} \end{bmatrix} = M \begin{bmatrix} \hat{X}_{t+\tau,t} \\ z^{t+\tau,t} \\ \Xi_{t+\tau-1,t} \end{bmatrix} \quad (18)$$

for $\tau \geq 0$, where $\hat{X}_{t,t} = X_t$. When the deviation is a finite-order moving average and the judgment is finite-dimensional, $z^{t+\tau,t}$ denotes the Tn_X -vector $(z'_{t+\tau+1,t}, z'_{t+\tau+2,t}, \dots, z'_{t+\tau+T,t})'$, where $z_{t+\tau+j,t} = 0$ for $j + \tau > T$. When the judgment is infinite-dimensional, $z^{t+\tau,t}$ denotes the infinite-dimensional vector $(z'_{t+\tau+1,t}, z'_{t+\tau+2,t}, \dots)'$. In the former case, F and M are finite-dimensional matrices. In the latter case, F and M include a linear operator R on $z^{t+\tau,t}$ (an infinite-dimensional matrix) of the form $\sum_{j=0}^{\infty} R_j z_{t+1+\tau+j,t}$, where $\{R_j\}_{j=0}^{\infty}$ is a sequence of matrices. The matrices F , M , and $\{R_j\}_{j=0}^{\infty}$ depend on A , B , C , D , W , and δ , but they are independent of the second and higher moments of the deviation. The n_X -vector $\Xi_{t+\tau,t}$ consists of the Lagrange multipliers of the lower block of (8), the block determining the projection of the forward-looking variables.

As discussed in appendix A, the value of the initial Lagrange multiplier, $\Xi_{t-1,t}$, is zero, if there is commitment from scratch in period t —that is, disregarding any previous commitments. This reflects a time-consistency problem when there is reoptimization and recommitment in later periods, as is inherently the case in practical monetary policy. Instead, I assume that the optimization is under

commitment in a timeless perspective. Then, if the optimization, and reoptimization, under commitment in a timeless perspective started in an earlier period and has occurred since then, the initial value of the Lagrange multiplier satisfies

$$\Xi_{t-1,t} = \Xi_{t-1,t-1} \quad (19)$$

where $\Xi_{t-1,t-1}$ denotes the Lagrange multiplier of the lower block of (8) for the determination of $x_{t-1,t-1}$ in the decision problem in period $t-1$. The dependence of the optimal policy projection in period t on this Lagrange multiplier from the decision problem in the previous period makes the optimal policy projection depend on previous projections and illustrates the history dependence of optimal policy under commitment in a forward-looking model shown in Backus and Driffill (1986) and Currie and Levine (1993) and especially examined and emphasized in Woodford (2003b).

It follows from (17)–(19) and (11) that the optimal policy projection of the states of the economy, the target variables, and the instruments will be linear functions of X_t , z^t , and $\Xi_{t-1,t-1}$, which can be written in a compact way as

$$\begin{aligned} \hat{s}^t &= H \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix}, & \hat{Y}^t &= \tilde{D}H \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix}, \\ \hat{i}^t &= H_i \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix} \end{aligned} \quad (20)$$

where H is an appropriately formed infinite-dimensional matrix and H_i is an infinite-dimensional submatrix of H consisting of the rows corresponding to the instruments. In particular, the instrument setting in period t will be given by

$$i_t = \hat{i}_{t,t} = h \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix} \quad (21)$$

where the finite- or infinite-dimensional matrix h consists of the n_i first rows of the matrix H_i .

As explained in Svensson and Woodford (2005), a simple way of imposing the timeless perspective is to add a term to the intertemporal loss function (12),

$$\mathcal{L}(Y^t) + \Xi_{t-1,t-1} \frac{1}{\delta} C x_{t,t}. \quad (22)$$

In the policy problem in period $t - 1$, $\Xi_{t-1,t-1} C$ can be interpreted as the marginal loss in period $t - 1$ of a change in the one-period-ahead projection of the forward-looking variables, $x_{t,t-1}$. The time-consistency problem arises from disregarding that marginal loss in the policy problem in period t . Adding the corresponding term to the loss function in period t as in (22) handles the time-consistency problem, and the optimal policy under commitment in the timeless perspective will result from minimizing (22) subject to (8)–(10) for given X_t , z^t , and $\Xi_{t-1,t-1}$.¹⁸ Since $x_{t,t}$ is an element of the projection s^t , the optimal policy projection $\hat{s}^t$ is then defined as

$$\hat{s}^t = \arg \min_{s^t \in \mathcal{S}_t} \left\{ \mathcal{L}(\tilde{D}s^t) + \Xi_{t-1,t-1} \frac{1}{\delta} C x_{t,t} \right\} \quad (23)$$

for given X_t , z^t , and $\Xi_{t-1,t-1}$.

From (18) it follows that the Lagrange multiplier $\Xi_{t,t}$, to be used in the decision problem in period $t + 1$, will be given by

$$\Xi_{t,t} = H_{\Xi} \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix} \quad (24)$$

where H_{Ξ} is a finite- or infinite-dimensional matrix.

¹⁸ Alternatively, as discussed in Giannoni and Woodford (2002) and Svensson and Woodford (2005), one can impose the constraint

$$x_{t,t} = F_x \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix}$$

where F in (17) is suitably partitioned. In the present context, it is more practical to add the term to the intertemporal loss function as in (22).

Let the *set of feasible target-variable projections* in period t , $\mathcal{Y}_t$, be defined as the set of target-variable projections satisfying (11) for projections s^t in the set S_t for given X_t and z^t . In the special case where the forward-looking variables, x_t , happen to be target variables and elements in Y_t , so $x_{t,t}$ is an element of Y^t , the optimal target-variable projection, $\hat{Y}^t$, can be defined as the target-variable projection Y^t that minimizes (22) on the set $\mathcal{Y}_t$, for given X_t , z^t , and $\Xi_{t-1,t-1}$,

$$\hat{Y}^t = \arg \min_{Y^t \in \mathcal{Y}_t} \left\{ \mathcal{L}(Y^t) + \Xi_{t-1,t-1} \frac{1}{\delta} C x_{t,t} \right\}.$$

However, in the more general case when some or all forward-looking variables are not target variables, $x_{t,t}$ is not an element of Y^t , and the optimal policy projection has to be found by optimization over the set $\mathcal{S}_t$, as in (23).

1.4 Backward-Looking Model

In a backward-looking model, there are no forward-looking variables: $n_x = 0$. There is no lower block in (1) and (8), and there are no forward-looking variables in (5) and (9). There are no projections of forward-looking variables and Lagrange multipliers in (17), (18), (20), and (21). There is no time-consistency problem and no need to consider commitment in a timeless perspective.

Hence, for a backward-looking model, the optimal target-variable projection can always be found by minimizing (12) over the set of feasible target-variable projections,

$$\hat{Y}^t = \arg \min_{Y^t \in \mathcal{Y}_t} \mathcal{L}(Y^t),$$

for given X_t and z^t .

1.5 The Complex Reduced-Form Reaction Function Need Not Be Made Explicit

The compact notation for the determination of the period- t instrument i_t in (21) and the Lagrange multiplier $\Xi_{t,t}$ in (24) may have given the impression that optimal monetary policy is just a matter

of calculating the finite- or infinite-dimensional matrices h and H_{Ξ} once and for all; then, in each period, first observe X_t , form z^t , and recall $\Xi_{t-1,t-1}$ from last period's decision; then simply compute, announce, and implement i_t from (21); and finally compute $\Xi_{t,t}$ to be used in next period's decision.

This is a misleading impression, though. First, h and H_{Ξ} are indeed high- or infinite-dimensional and therefore difficult to grasp and interpret. Second, z^t is also high- or infinite-dimensional. It is difficult to conceive of policymakers or even staff pondering pages and pages, or computer screens and computer screens, of huge arrays of numbers in small print, arguing and debating about adjustments of the numbers of z^t , such as the numbers in rows 220–250 and 335–385. Third, no central bank (certainly no central bank that I have any more thorough information about) behaves in that way, and is ever likely to behave that way. Instead, the practical presentation of information and options to policymakers is always in the form of multiple graphs, modest-size tables, and modest amounts of text.

Fourth, the intertemporal loss function $\mathcal{L}(Y^t)$ has the projections of the target variables as its argument. What matters for the construction of the target variables is the whole projection path of the instruments, not the current instrument setting. The obvious conclusion is that the relevant objects of importance in the decision process are the whole projection paths of the target variables and the instruments, not the current instrument setting or projections of the target variables at some fixed horizon. These projection paths are most conveniently illustrated as graphs. Indeed, graphs of projections are prominent in the existing monetary policy reports where projections are reported. The analytical techniques discussed in this paper should predominantly be seen as techniques for computer-generated graphs of whole projection paths. Pondering such graphs is an essential part of the monetary policy decision process. Importantly, policymakers need not know the underlying detailed high- or infinite-dimensional matrices behind the construction of those graphs. Therefore, the complex reduced-form reaction functions embedded in these matrices need not be made explicit.

Fifth, in the discussion in section 1.1, there was no reference to the reaction function, only to the optimal policy projection. Given X_t and i_t , the private sector needs to be able to form the expectations $x_{t+1|t}$ in order to make decisions in period t . The minimum for this

is the central bank's announcement of $\hat{x}_{t+1,t}$. In order to make that announcement credible, the central bank may have to announce the complete optimal policy projection and motivate its judgment. But it does not need to announce any reaction function. In principle, given the reaction function, the private sector could combine the reaction function with the model and solve for the optimal policy projection, but that is an overwhelmingly complicated and roundabout way.

2. A Finite-Horizon Projection Model

Regardless of whether the judgment is finite- or infinite-dimensional (that is, whether [15] holds or not), the problem of minimizing the intertemporal loss function is an infinite-horizon problem. From a practical and computational point of view, it is convenient to transform the infinite-horizon policy problem above to a finite-horizon one. When the judgment satisfies (15), this can be done in a simple and approximate, but arbitrarily close to exact, way for the forward-looking model, and in a simple and exact way for the backward-looking model. The finite-horizon model also makes it very easy to incorporate any arbitrary constraints on the projections—for instance, a particular form of the instrument projection, such as a constant instrument for some periods. Then, all the relevant projection paths are computed in one simple step.

2.1 Forward-Looking Model

Suppose that the estimate of the deviation is constant beyond a fixed horizon T . Without loss of generality, assume that the constant is zero.¹⁹ That is, I assume (15).

Start by writing the projection model (8) and (10) for $\tau = 0, \dots, T-1$ as

$$X_{t,t} = X_t, \quad (25)$$

¹⁹ If the estimate of the deviation from horizon T on is constant but nonzero, it can be incorporated among other constants in the model. If the estimate of the deviation from horizon T on is not constant but follows an autoregressive process (for instance, if it is assumed to gradually approach a constant), it can be incorporated among the predetermined variables.

$$-\tilde{A}s_{t+\tau,t} + \begin{bmatrix} X_{t+\tau+1,t} \\ Cx_{t+\tau+1,t} \end{bmatrix} = \begin{bmatrix} z_{t+\tau+1,t} \\ 0 \end{bmatrix} \quad (\tau = 0, \dots, T-1) \quad (26)$$

where $\tilde{A}$ is the $(n_X + n_x) \times (n_X + n_x + n_i)$ matrix defined by $\tilde{A} \equiv [A \ B]$. The first n_X equations of the last block of $n_X + n_x$ equations in (26) determine $X_{t+T,t}$ for given $X_{t+T-1,t}$, $x_{t+T-1,t}$, $i_{t+T-1,t}$, and $z_{t+T,t}$. The last n_x equations of that block are

$$-A_{21}X_{t+T-1,t} - A_{22}x_{t+T-1,t} - B_2i_{t+T-1,t} + Cx_{t+T,t} = 0.$$

They determine $x_{t+T-1,t}$ for given $X_{t+T-1,t}$ and $i_{t+T-1,t}$, and, importantly, for given $x_{t+T,t}$. A problem is that n_x equations determining $x_{t+T,t}$ are lacking. I will assume that $x_{t+T,t}$ is determined by the assumption that $x_{t+T+1,t}$ is equal to its steady-state level. That is, I assume that the optimal policy projection has the property that, for (15), it approaches a steady state when $T \rightarrow \infty$. This is true for the models and loss functions considered here. Without loss of generality, I assume that the steady-state values for the forward-looking variables are zero,

$$x_{t+T+1,t} = 0. \quad (27)$$

From this it follows that $X_{t+T,t}$, $x_{t+T,t}$, and $i_{t+T,t}$ must satisfy

$$-A_{21}X_{t+T,t} - A_{22}x_{t+T,t} - B_2i_{t+T,t} = 0, \quad (28)$$

which gives me the desired n_x equations for $x_{t+T,t}$.

Let s^t , the projection of the states of the economy, now denote the *finite-dimensional* $(T+1) \times (n_X + n_x + n_i)$ -vector $s^t \equiv (s'_{t,t}, s'_{t+1,t}, \dots, s'_{t+T,t})'$. Similarly, let all projections q^t for $q = X, x, i$ and Y now denote the finite-dimensional vector $q^t \equiv (q'_{t,t}, q'_{t+1,t}, \dots, q'_{t+T,t})'$. Finally, let z^t be the Tn_X -vector $z^t \equiv (z'_{t+1,t}, z'_{t+2,t}, \dots, z'_{t+T,t})'$ (recall that z^t does not include the component z_t).

The finite-horizon projection model for the projection of the states of the economy, s^t , then consists of (25), (26), and (28). It can be written compactly as

$$Gs^t = g^t \quad (29)$$

where G is the $(T+1)(n_X + n_x) \times (T+1)(n_X + n_x + n_i)$ matrix formed from the matrices on the left side of (25), (26), and (28), and

g^t is a $(T+1)(n_X + n_x)$ -vector formed from the right side of (25), (26), and (28) as $g^t \equiv (X'_t, z'_{t+1,t}, 0', z'_{t+2,t}, 0', \dots, z'_{t+T,t}, 0', 0')'$ (where zeros denote zero vectors of appropriate dimension).

Since Y^t now denotes the finite-dimensional $(T+1)n_Y$ -vector $Y^t \equiv (Y'_{t,t}, Y'_{t+1,t}, \dots, Y'_{t+T,t})'$, I can write

$$Y^t = \tilde{D}s^t \quad (30)$$

where $\tilde{D}$ now denotes a finite-dimensional $(T+1)n_Y \times (T+1)(n_X + n_x + n_i)$ block-diagonal matrix with the matrix D in each diagonal block.

The set of feasible projections, $\mathcal{S}_t$, is then defined as the finite-dimensional set of s^t that satisfy (29) and (30) for a given g^t —that is, for a given X_t and z^t .

It remains to specify the intertemporal loss function in the forward-looking model in the finite-horizon case. In the forward-looking model, under assumption (15), the minimum loss from the horizon $T+1$ on depends on the projection of the predetermined variables for period $t+T+1$, $X_{t+T+1,t}$, and the Lagrange multipliers $\Xi_{t+T,t}$ according to the quadratic form

$$\frac{1}{2}\delta^{T+1}[X'_{t+T+1,t} \quad \Xi'_{t+T,t}]V \begin{bmatrix} X_{t+T+1,t} \\ \Xi_{t+T,t} \end{bmatrix}$$

where V is a symmetric positive semidefinite matrix that depends on the matrices A , B , C , D , and W and the discount factor δ (see appendix A). It follows from (18) and (15) that this quadratic form can be written as a function $X_{t+T,t}$ and $\Xi_{t+T-1,t}$ as²⁰

$$\frac{1}{2}\delta^{T+1}[X'_{t+T,t} \quad \Xi'_{t+T-1,t}]M'VM \begin{bmatrix} X_{t+T,t} \\ \Xi_{t+T-1,t} \end{bmatrix}. \quad (31)$$

In principle, I could use (18) to keep track of $\Xi_{t+T-1,t}$. However, a simpler way is to extend the horizon T so far that $X_{t+T,t}$ and $\Xi_{t+T-1,t}$ are arbitrarily close to their steady-state levels. Without loss of generality, I assume that the steady-state levels are zero, in

²⁰ The matrix M appearing in (31) is the matrix M in (18) with the columns corresponding to z^t deleted.

which case the above quadratic form is zero, and the loss from horizon T can be disregarded. Checking that $X_{t+T,t}$ is close to zero is straightforward; I will show a practical way to check that $\Xi_{t+T-1,t}$ is also close to zero.²¹

Under this assumption, it follows from (9), (12), and (30) that the intertemporal loss function can be written as a function of s^t as the finite-dimensional quadratic form

$$\frac{1}{2}s^{t'}\Omega s^t \quad (32)$$

where Ω is a symmetric positive semidefinite block-diagonal $(T+1)(n_X + n_x + n_i)$ matrix with its $(\tau+1)$ -th diagonal block being $\delta^\tau D'WD$ for $0 \leq \tau \leq T$. However, in order to impose the timeless perspective, as explained in section 1.3, I need to add the term

$$\Xi_{t-1,t-1} \frac{1}{\delta} C x_{t,t}$$

to the loss function, where $\Xi_{t-1,t-1}$ is the relevant Lagrange multiplier from the policy problem in period $t-1$. This term can be written $\omega'_{t-1}s^t$, with the appropriate definition of the $(T+1)(n_X + n_x + n_i)$ -vector ω_{t-1} as $\omega_{t-1} \equiv (0, {}^t0', (\Xi_{t-1,t-1} \frac{1}{\delta} C)', {}^t0', \dots, {}^t0')'$ (where the zeros denote zero vectors of appropriate dimension). Thus, the intertemporal loss function with the added term is

$$\frac{1}{2}s^{t'}\Omega s^t + \omega'_{t-1}s_t. \quad (33)$$

Then, the policy problem is to find the optimal policy projection $\hat{s}^t$ that minimizes (33) subject to (29). The Lagrangian for this problem is

$$\frac{1}{2}s^{t'}\Omega s^t + \omega'_{t-1}s_t + \Lambda^{t'}(Gs^t - g^t) \quad (34)$$

where Λ^t is the $(T+1)(n_X + n_x)$ -vector of Lagrange multipliers of (29). The first-order condition is

$$s^{t'}\Omega + \omega'_{t-1} + \Lambda^{t'}G = 0.$$

²¹ Appendix E presents an iterative numerical procedure that will provide a projection arbitrarily close to the optimal policy projection without requiring such a long horizon that $X_{t+T,t}$ and $\Xi_{t+T-1,t}$ are close to their steady-state levels.

Combining this with (29) gives the linear equation system

$$\begin{bmatrix} G & 0 \\ \Omega & G' \end{bmatrix} \begin{bmatrix} s^t \\ \Lambda^t \end{bmatrix} = \begin{bmatrix} g^t \\ -\omega_{t-1} \end{bmatrix}. \quad (35)$$

The optimal policy projection $\hat{s}^t$ and Lagrange multiplier Λ^t are then given by the simple matrix inversion²²

$$\begin{bmatrix} \hat{s}^t \\ \Lambda^t \end{bmatrix} = \begin{bmatrix} G & 0 \\ \Omega & G' \end{bmatrix}^{-1} \begin{bmatrix} g^t \\ -\omega_{t-1} \end{bmatrix}. \quad (36)$$

The optimal target-variable projection then follows from (30). The optimal policy projection is a linear function of X_t , z^t , and $\Xi_{t-1,t-1}$, and it can be written compactly as in section 1.3:

$$\hat{s}^t = H \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix}, \quad \hat{Y}^t = \tilde{D}H \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix},$$

$$\hat{i}^t = H_i \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix},$$

except that the matrices H and H_i and the vector z^t now are finite-dimensional. The matrices can be directly extracted from (36). The period- t instrument setting can be written

$$i_t = \hat{i}_{t,t} = h \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix} \quad (37)$$

where the finite-dimensional matrix h consists of the first n_i rows of the matrix H_i . Under assumption (15) and a sufficiently long

²² Numerically, it is faster to solve the system of linear equations (35) by other methods than first inverting the left-side matrix (see Judd 1998).

horizon T , the finite-horizon projections here are arbitrary close to the optimal infinite-horizon policy projections for $\tau = 0, \dots, T$ in section 1.3.

The Lagrange multiplier Λ^t can be written $\Lambda^t \equiv (\frac{1}{\delta}\xi'_{t,t}, \xi'_{t+1,t}, \Xi'_{t,t}, \delta\xi'_{t+2,t}, \delta\Xi'_{t+1,t}, \dots, \delta^T\xi'_{t+T,t}, \delta^T\Xi'_{t+T-1,t})'$, where $\xi_{t+\tau,t}$ is the vector of Lagrange multipliers for the block of equations in (25), (26), and (28) determining $X_{t+\tau,t}$ and $\Xi_{t+\tau-1,t}$ is the vector of Lagrange multipliers for the block of equations determining $x_{t+\tau-1,t}$. Hence, extraction of $\Xi_{t+T-1,t}$ from Λ^t allows me to check that the assumption made above of $\Xi_{t+T-1,t}$ being close to zero is satisfied. If the assumption is not satisfied, the horizon T can be extended until the assumption is satisfied.²³ Furthermore, $\Xi_{t,t}$ can be extracted from Λ^t in order to form the vector ω_t to be used in the loss function for the policy problem in period $t+1$, to ensure the timeless perspective. The Lagrange multiplier needed in the loss function in period $t+1$, $\Xi_{t,t}$, can be written

$$\Xi_{t,t} = H_{\Xi} \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix} \quad (38)$$

where the finite-dimensional matrix H_{Ξ} can be extracted from (36).

Again, as noted above in section 1.3, in spite of the compact notation for the instrument i_t and Lagrange multiplier $\Xi_{t,t}$ in (37) and (38), these analytical techniques should predominantly be seen as techniques for computer-generated graphs to be pondered by the policymakers, and the matrices never need be made explicit to the policymakers. Although the matrices are now formally finite-dimensional, they are still high-dimensional and somewhat difficult to interpret.

2.2 Backward-Looking Model

Make the same assumption (15) as for the forward-looking model. The projection in period t of the state of the economy in period $t+\tau$, $s_{t+\tau,t}$, is now defined as the $(n_X + n_i)$ -vector $s_{t+\tau,t} \equiv (X'_{t+\tau,t}, i'_{t+\tau,t})'$

²³ In practice, the horizon T is extended until the optimal projection $\hat{s}^t$ is insensitive to variations in T .

for $\tau \geq 0$, in which case I can write, for the backward-looking model,

$$X_{t+\tau+1,t} = \tilde{A}s_{t+\tau,t} \quad (\tau \geq T). \quad (39)$$

The projection model with horizon T can now be written

$$X_{t,t} = X_t, \quad (40)$$

$$-\tilde{A}s_{t+\tau,t} + X_{t+\tau+1,t} = z_{t+\tau+1,t} \quad (0 \leq \tau \leq T-1) \quad (41)$$

where X_t and z^t are given. The projection of the state of the economy, s^t , is now a $(T+1)(n_X + n_i)$ -vector. Then the projection model can be written as (29), where G is a $(T+1)n_X \times (T+1)(n_X + n_i)$ matrix formed from the left side of (40) and (41), and g^t is a $(T+1)n_X$ -vector formed from the right side of (40) and (41) as $g^t \equiv (X'_t, z'^t)'$.

It is a standard result for a linear-quadratic backward-looking model that the minimum loss from the horizon $T+1$ on depends on the projection of the predetermined variables for period $t+T+1$, $X_{t+T+1,t}$, according to the quadratic form

$$\frac{1}{2}\delta^{T+1}X'_{t+T+1,t}VX_{t+T+1,t} \quad (42)$$

where V is a symmetric positive semidefinite matrix that depends on the matrices A , B , D , and W and the discount factor δ (see appendix A). I could now (as for the forward-looking model) assume that the predetermined variables approach a steady-state level for large T , without loss of generality assume that the steady-state level is zero, and extend the horizon T so far that the predetermined variables are arbitrarily close to zero and the loss from period T on is arbitrarily close to zero. I could then form the finite-horizon loss function as for the forward-looking model, and this together with (40) and (41) would form the finite-horizon model, which would be an arbitrarily close approximation to the infinite-horizon model for sufficiently large T .

However, the absence of the time-consistency problem and the need to keep track of the Lagrange multiplier $\Xi_{t+T-1,t}$ allows a simple approach, which is exact also for small T , as long as assumption (15) holds for that T . I follow this approach here.

From (39) it follows that the quadratic form (42) can be written as a function of $s_{t+T,t}$ as

$$\frac{1}{2}\delta^{T+1}s'_{t+T,t}\tilde{A}'V\tilde{A}s_{t+T,t}.$$

The finite-horizon intertemporal loss function can then be written

$$\frac{1}{2} \sum_{\tau=0}^T \delta^\tau s'_{t+\tau,t} D' W D s_{t+\tau,t} + \frac{1}{2} \delta^{T+1} s'_{t+T,t} \tilde{A}' V \tilde{A} s_{t+T,t}.$$

The intertemporal loss function can now be written more compactly as the quadratic form (32), where Ω now is a symmetric positive-semidefinite block-diagonal $(T+1)(n_X + n_i)$ -matrix, whose $(\tau+1)$ -th diagonal block is $\delta^\tau D' W D$ for $0 \leq \tau \leq T-1$ and whose $(T+1)$ -th diagonal block now is $\delta^T (D' W D + \delta \tilde{A}' V \tilde{A})$. Thus, it differs from the matrix Ω for the forward-looking model by the addition of that last term, $\frac{1}{2} \delta \tilde{A}' V \tilde{A}$.

The finite-horizon policy problem is now to find the optimal policy projection $\hat{s}^t$ that minimizes (32) subject to (29), for given g^t , that is, for given X_t and z^t . The corresponding optimal target-variable projection $\hat{Y}^t$ then follows from (30).

The Lagrangian for this problem is

$$\frac{1}{2} s^{t'} \Omega s^t + \Lambda^{t'} (G s^t - g^t)$$

where Λ^t is a vector of Lagrange multipliers for (29). The first-order condition is

$$s^{t'} \Omega + \Lambda^{t'} G = 0.$$

Combining this with (29) gives the linear equation system

$$\begin{bmatrix} G & 0 \\ \Omega & G' \end{bmatrix} \begin{bmatrix} s^t \\ \Lambda^t \end{bmatrix} = \begin{bmatrix} g^t \\ 0 \end{bmatrix}.$$

The optimal policy projection $\hat{s}^t$ is then given by the simple matrix inversion,

$$\begin{bmatrix} \hat{s}^t \\ \Lambda^t \end{bmatrix} = \begin{bmatrix} G & 0 \\ \Omega & G' \end{bmatrix}^{-1} \begin{bmatrix} g^t \\ 0 \end{bmatrix}. \quad (43)$$

The optimal target-variable projection then follows from (30).

The optimal policy projection is obviously a linear function of X_t and z^t , and I can write

$$\hat{s}^t = H \begin{bmatrix} X_t \\ z^t \end{bmatrix}, \quad \hat{Y}^t = \tilde{D} H \begin{bmatrix} X_t \\ z^t \end{bmatrix}, \quad \hat{i}^t = H_i \begin{bmatrix} X_t \\ z^t \end{bmatrix}$$

where the finite-dimensional matrices H and H_i can be extracted from (43). The instrument setting for period t can be written

$$i_t = \hat{i}_{t,t} = h \begin{bmatrix} X_t \\ z^t \end{bmatrix} \quad (44)$$

where the finite-dimensional matrix h consists of the first n_i rows of the matrix H_i .

2.3 Other Considerations

A finite-horizon projection model has several advantages. One is that it is very easy to incorporate any restrictions on the projections. Any equality restriction on X^t , x^t , i^t , or Y^t can be written

$$\bar{R}s^t = \bar{s}^t \quad (45)$$

where the number of rows of the matrix $\bar{R}$ and the dimension of the given vector $\bar{s}^t$ equal the number of restrictions. This makes it easy to incorporate any restriction on the instrument projection—for instance, a restriction that it shall be constant or of a particular shape for some periods.

Transforming the model into a finite system of equations may be particularly practical from a computational point of view for a nonlinear model. It may then also be easy to handle commitment in a timeless perspective for a nonlinear model.

3. Monetary Policy Without Judgment

Modern monetary policy inherently relies, to a large extent, on judgment. Previous sections of this paper have attempted to model this dependence on judgment in a simple but specific way. This section attempts to specify monetary policy without judgment, in order to compare monetary policy with and without judgment.

There are several alternatives in modeling monetary policy without judgment. Above, monetary policy with judgment has been modeled as forecast targeting, finding an instrument projection such that the corresponding projection of the target variables minimizes a loss function. This procedure uses all information available to the central

bank, including central-bank judgment. This results in a complex reduced-form reaction function, which fortunately never needs to be made explicit. When modeling monetary policy without judgment, however, the most obvious route is to consider monetary policy as a more mechanical process—a commitment to a particular reaction function, a *commitment to a particular instrument rule*.²⁴

Instrument rules can be divided into two categories: explicit instrument rules and implicit instrument rules (Svensson and Woodford 2005; Svensson 2003). An *explicit instrument rule* is a reaction function where the instrument responds to predetermined variables only. Its implementation then consists of the central bank observing the predetermined variables in the beginning of the period, and then calculating, announcing, and setting the instrument according to this instrument rule. The implementation obviously requires that the relevant predetermined variables must be observed by the central bank, but since the predetermined variables in a particular period are independent of the instrument setting in that period, no further complications arise. An *implicit* instrument rule is a relation between the current instrument and some of the current forward-looking variables. Then, since the forward-looking variables depend on the instrument setting, the instrument and the forward-looking variables are simultaneously determined. Thus, an implicit instrument rule is actually an *equilibrium condition*, a relation that holds in equilibrium. The implementation of an implicit instrument rule (that is, how to get to the desired equilibrium) is not trivial but a complex issue. This fact has largely been overlooked in the literature, except, for instance, in Svensson and Woodford (2005) and Svensson (2003).

Here, I shall discuss how monetary policy without judgment can be modeled as a commitment to an instrument rule—either an explicit instrument rule or an implicit instrument rule—with some discussion of the implementation of the latter. I will start from the reduced-form reaction function for the instrument setting that follows from the forecast targeting modeled above. For simplicity, I now consider the realistic case when there is only one instrument, the instrument rate, so $n_i = 1$.

²⁴ This is an approach that a large part of the literature has taken—for instance, most papers in the conference volume Taylor (1999). The approach is criticized in Svensson (2003) and (2004).

3.1 Explicit Instrument Rules

The construction of the optimal instrument projection, $\hat{i}^t$, in the forward-looking model results in an optimal reduced-form reaction function for the current instrument setting, (37) (or [21]), which is repeated here as

$$i_t = h_X X_t + h_z z^t + h_{\Xi} \Xi_{t-1,t-1} \quad (46)$$

where the row vector h is partitioned conformably with X_t , z^t , and $\Xi_{t-1,t-1}$ as $h \equiv [h_X \ h_z \ h_{\Xi}]$ (h is now a row vector, since $n_i = 1$). The optimal reaction function implies a particular instrument response to the predetermined variables, X_t ; the judgment, z^t ; and the Lagrange multiplier for the equations for $x_{t-1,t-1}$ in the policy problem in period $t - 1$, $\Xi_{t-1,t-1}$. The discussion here refers to the forward-looking model; for the backward-looking model, I can simply delete the term $h_{\Xi} \Xi_{t-1,t-1}$.

I want to model the central bank setting its instrument in a mechanical way, via a commitment to a particular instrument rule, while disregarding judgment. Considering (46) as a potential instrument rule, it is natural that disregarding judgment means that the central bank behaves as if it believes that $z^t = 0$ and hence disregards the judgment term in (46), $h_z z^t$. Thus, *monetary policy without judgment is modeled as the central bank erroneously believing that the expected future deviations equal zero*—for instance, the central bank believing that z_t is a zero-mean iid process. A first possibility is then that the central bank also disregards the term with the Lagrange multiplier from the previous policy problem. For one thing, if the central bank did set the instrument mechanically in period $t - 1$ rather than by explicit optimization, it may not be aware of the Lagrange multiplier and its value. This leaves responding to the current predetermined variables only,

$$i_t = h_X X_t. \quad (47)$$

Such a policy would be inefficient for two reasons, even if the response coefficients to X_t are those of the optimal reaction function (46). First, it disregards judgment, the term $h_z z^t$. Second, it also disregards the response to lagged predetermined variables implied by the response to the Lagrange multiplier, $\Xi_{t-1,t-1}$, as indicated in (19). Indeed, optimization under discretion would result in a

reduced-form reaction function where the instrument responds only to the current predetermined variables.²⁵

The response to the Lagrange multipliers in (46) implies a response to lagged predetermined variables. Disregarding judgment, the Lagrange multiplier in (38) (or [24]) follows

$$\begin{aligned}\Xi_{t-1,t-1} &= H_{\Xi X} X_{t-1} + H_{\Xi\Xi} \Xi_{t-2,t-2} \\ &= \sum_{j=0}^{\infty} H_{\Xi\Xi}^j H_{\Xi X} X_{t-1-j}\end{aligned}$$

where H_{Ξ} is partitioned conformably with X_t , z^t , and $\Xi_{t-1,t-1}$ as $H_{\Xi} \equiv [H_{\Xi X} \ H_{\Xi z} \ H_{\Xi\Xi}]$. A second possibility for policy disregarding judgment is then that the policy responds with the optimal coefficients h_{Ξ} to these lagged predetermined variables, resulting in the reaction function

$$i_t = h_X X_t + h_{\Xi} \sum_{j=0}^{\infty} H_{\Xi\Xi}^j H_{\Xi X} X_{t-1-j}. \quad (48)$$

This would seem to be an instrument rule corresponding to a rather sophisticated policy, commitment to the reaction function resulting from optimal policy under commitment in a timeless perspective, while disregarding judgment.²⁶

²⁵ The resulting reduced-form reaction function resulting from optimization under discretion would have different coefficients than the optimal h_X . Because of this, and because of the missing response to lagged predetermined variables, the response is suboptimal and results in so-called stabilization bias relative to the commitment policy (Svensson and Woodford 2005).

Given a particular restricted class of instrument rules—for instance, simple instrument rules with only a few arguments—one can find the optimal instrument rule in that restricted class (see appendix G). The optimal instrument rule in a restricted class will depend on the stochastic properties of the disturbances to the economy. Many papers of Taylor (1999) provide examples of such optimal restricted instrument rules.

²⁶ Note that only if h_{Ξ} is invertible can this be written on the “instrument smoothing” form

$$\begin{aligned}i_t &= h_X X_t + h_{\Xi} H_{\Xi X} X_{t-1} + h_{\Xi} H_{\Xi\Xi} h_{\Xi}^{-1} \sum_{j=0}^{\infty} H_{\Xi\Xi}^j H_{\Xi X} X_{t-1-j} \\ &= h_X X_t + h_{\Xi} H_{\Xi X} X_{t-1} + h_{\Xi} H_{\Xi\Xi} h_{\Xi}^{-1} (i_{t-1} - h_X X_{t-1}) \\ &= h_X X_t + h_{\Xi} (H_{\Xi X} - H_{\Xi\Xi} h_{\Xi}^{-1} h_X) X_{t-1} + h_{\Xi} H_{\Xi\Xi} h_{\Xi}^{-1} i_{t-1}.\end{aligned}$$

3.2 *Implicit Instrument Rules*

Another apparent possibility would be an implicit instrument rule, where the instrument responds to the forward-looking variables, x_t . This might seem advantageous, since, in a rational-expectations equilibrium, the forward-looking variables might be affected by private-sector expectations of future deviations. Then, by responding to forward-looking variables, the central bank might indirectly take judgment into account—although, in this case, private-sector judgment. Thus, one might want to consider an ad hoc implicit instrument rule of the form

$$i_t = f_X X_t + f_x x_t \quad (49)$$

where f_X and f_x are row vectors of given response coefficients.

As mentioned above, however, there is a specific problem with the central bank responding to forward-looking variables, something largely overlooked in the literature (except, for instance, Svensson and Woodford 2005 and Svensson 2003). Since the forward-looking variables depend on the central bank's instrument setting, a simultaneity problem arises. The central bank cannot observe the forward-looking variables before it sets the instrument, and the private sector needs to observe the instrument setting before it determines the forward-looking variables. A relation such as (49) is actually an equilibrium condition, where i_t and x_t are simultaneously determined. The implementation of such an equilibrium condition is not straightforward.

A sophisticated way to implement (49) is for the central bank to construct projections (X^t, x^t, i^t) that satisfy (49). The central bank can amend the relation

$$i_{t+\tau,t} = f_X X_{t+\tau,t} + f_x x_{t+\tau,t} \quad (50)$$

for $\tau \geq 0$ to its projection model (29) (or [28]). It can then solve for the projection $(\tilde{X}^t, \tilde{x}^t, \tilde{i}^t)$ for given X_t , *disregarding the judgment* (setting $z_{t+\tau,t} = 0$ for $\tau \geq 1$). These projections will by construction

If there is only one instrument, h_{Ξ} is invertible only if $\Xi_{t-1,t-1}$ is a scalar—that is, if there is only one forward-looking variable. The point is that the optimal reaction function under commitment usually cannot be written as an instrument rule involving current predetermined variable and the lagged instrument, unless i_{t-1} happens to be one of the predetermined variables.

satisfy (50). The corresponding instrument setting, $i_t = \tilde{i}_{t,t}$, will then be a linear function of X_t ,

$$i_t = \tilde{f}_X X_t. \quad (51)$$

The central bank can then set this instrument level according to the reduced-form reaction function (51).²⁷

However, if the private sector understands that the central bank is effectively implementing the reaction function (51); has rational expectations of future variables; and, in particular, has expectations of future nonzero deviations, $z_{t+\tau|t} \neq 0$ ($\tau \geq 1$); the resulting market-determined forward-looking variables, x_t , will deviate from the central-bank projection, $\tilde{x}_{t,t}$. Thus, although the instrument setting will satisfy

$$i_t = f_X X_t + f_x \tilde{x}_{t,t}$$

for the central-bank projection $\tilde{x}_{t,t}$, it will *not* satisfy (49) for the market-determined forward-looking variables x_t .

For relation (49) to be satisfied for the market-determined forward-looking variables, the central bank has to amend the relation (50) for $\tau \geq 0$ to its production model (29) (or [28]) and solve

²⁷ In the context of the finite-horizon projection model, relation (50) can be written as $(T+1)n_i$ equations,

$$\bar{R}s^t = 0,$$

where $\bar{R}$ is an $(T+1)n_i \times (T+1)(n_X + n_x + n_i)$ matrix. Combining these with (29) for $z^t = 0$ gives an equation system

$$\begin{bmatrix} G \\ \bar{R} \end{bmatrix} s^t = \begin{bmatrix} \tilde{g}^t \\ 0 \end{bmatrix}$$

where $\tilde{g}^t$ is the $(T+1)(n_X + n_x)$ -vector $(X_t', 0)'$. Under the assumption that the matrix on the left side has full rank, s^t is given by

$$\tilde{s}^t \equiv \begin{bmatrix} G \\ \bar{R} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{g}^t \\ 0 \end{bmatrix}. \quad (52)$$

This results in

$$\tilde{x}_{t,t} = \tilde{H}_{x0} X_t$$

where the matrix $\tilde{H}_{x0}$ can be extracted from the right side of (52) and

$$\tilde{i}_{t,t} = f_X X_t + f_x \tilde{x}_{t,t} = (f_X + f_x \tilde{H}_{x0}) X_t \equiv \tilde{f}_X X_t.$$

for the projection (X^t, x^t, x^t) for given X_t , *taking the judgment into account*.²⁸ This results in

$$\begin{aligned} x_{t,t} &= \tilde{H}_{xX0}X_t + \tilde{H}_{xz0}z^t \\ i_{t,t} &= f_X X_t + f_x x_{t,t} = (f_X + f_x \tilde{H}_{xX0})X_t + f_x \tilde{H}_{xz0}z^t \\ &\equiv \tilde{f}_X X_t + \tilde{f}_z z^t. \end{aligned}$$

Thus, the resulting reduced-form reaction function is

$$i_t = \tilde{f}_X X_t + \tilde{f}_z z^t.$$

If the private sector understands that this is the reaction function followed by the central bank and has rational expectations corresponding to the same judgment, the market-determined forward-looking variables, x_t , will equal the central-bank projection, $x_{t,t}$, and the relation (49) will be satisfied in equilibrium.

This is, of course, an example of a central bank explicitly taking judgment into account, not an example of a central bank disregarding judgment. But instead of finding the optimal policy projection, $(\hat{X}^t, \hat{x}^t, \hat{i}^t, \hat{Y}^t)$, that minimizes its loss function, it finds the projection (X^t, x^t, i^t, Y^t) that satisfies the ad hoc relation (49). Since the latter is no easier than the former and, in particular, suboptimal, this behavior seems a bit far-fetched.

Thus, I can model monetary policy without judgment as following either the explicit instrument rule (47), where both judgment

²⁸ The equation system is then

$$\begin{bmatrix} G \\ R \end{bmatrix} s^t = \begin{bmatrix} g^t \\ 0 \end{bmatrix}$$

where g^t is the $(T+1)(n_X + n_x)$ -vector $(X'_t, z'_{t,t}, 0', z'_{t+1,t}, 0', \dots, z'_{t+T,t}, 0', 0')'$ specified in section 2.1. Under the assumption that the matrix on the left side has full rank, s^t is given by

$$s^t = \begin{bmatrix} G \\ R \end{bmatrix}^{-1} \begin{bmatrix} g^t \\ 0 \end{bmatrix}. \quad (53)$$

This results in

$$x_{t,t} = \tilde{H}_{xX0}X_t + \tilde{H}_{xz0}z^t$$

where $\tilde{H}_{xX0}$ and $\tilde{H}_{xz0}$ can be extracted from the right side of (53) and

$$i_{t,t} = f_X X_t + f_x x_{t,t} = (f_X + f_x \tilde{H}_{xX0})X_t + f_x \tilde{H}_{xz0}z^t \equiv \tilde{f}_X X_t + \tilde{f}_z z^t.$$

and lagged predetermined variables are ignored, or the more sophisticated explicit instrument rule (48), or perhaps some intermediate case of (48) where the summation is over a finite past period. Using the implicit instrument rule (49) is somewhat problematic, since its implementation is complex and open to alternative very different interpretations, with very different resulting equilibria.

3.3 Taylor Rules

In the literature, a number of simple ad hoc instrument rules have been used to discuss and evaluate monetary policy. The most common are variants of the Taylor rule.²⁹ One variant of the Taylor rule with instrument smoothing (meaning, in this context, a response to the lagged instrument rate) is

$$i_t = (1 - f_i)(f_\pi \pi_t + f_y y_t) + f_i i_{t-1}$$

where π_t denotes a measure of the difference of inflation from a given inflation target; y_t denotes a measure of the output gap; f_π and f_y are given positive coefficients that can be interpreted as the long-run response to inflation and the output gap, respectively; and the coefficient f_i satisfies $0 \leq f_i \leq 1$. If inflation and the output gap are predetermined, this is an explicit instrument rule, and its implementation only requires that the central bank can observe or estimate current inflation and the output gap. If inflation and/or the output gap are forward-looking variables, this is an implicit instrument rule, where the instrument and the forward-looking variable are simultaneously determined. As noted above, such an instrument rule is somewhat problematic and its implementation may need to be further specified.

One variant of the Taylor rule, a so-called forecast-based or forward-looking Taylor rule, can be written

$$i_t = (1 - f_i)[f_\pi \pi_{t+J,t} + f_y y_{t+K,t}] + f_i i_{t-1}$$

where $\pi_{t+J,t}$ denotes a projection of the difference of inflation from an inflation target at horizon $J \geq 0$ and $y_{t+K,t}$ denotes a projection of the output gap at horizon $K \geq 0$, where at least one of J or K is

²⁹ Kozicki (1999) provides a discussion of the usefulness of Taylor rules.

positive. Such an instrument rule is an explicit or an implicit instrument rule depending on how the projections are constructed. If the projections are constructed with information that is predetermined in period t , the projections are predetermined and the instrument rule is explicit. If the projections are constructed with information that includes simultaneously determined forward-looking variables, the instrument rule is implicit and hence an equilibrium condition. Again, the implementation of such an instrument rule is not trivial and open to alternative interpretations.³⁰

4. Examples

In this section, I discuss examples of monetary policy with and without judgment in two different empirical models of the U.S. economy: a backward-looking model due to Rudebusch and Svensson (1999) and a forward-looking model due to Lindé (2002).

4.1 Backward-Looking Model

The backward-looking empirical model of Rudebusch and Svensson (1999) has two equations (with estimates rounded to two decimal points):

$$\pi_{t+1} = 0.70 \pi_t - 0.10 \pi_{t-1} + 0.28 \pi_{t-2} + 0.12 \pi_{t-3} + 0.14 y_t + z_{\pi,t+1} \quad (54)$$

$$y_{t+1} = 1.16 y_t - 0.25 y_{t-1} - 0.10 \left(\frac{1}{4} \sum_{j=0}^3 i_{t-j} - \frac{1}{4} \sum_{j=0}^3 \pi_{t-j} \right) + z_{y,t+1}. \quad (55)$$

The period is a quarter, π_t is quarterly gross domestic product inflation measured in percentage points at an annual rate, y_t is the output gap measured in percentage points, and i_t is the quarterly average of the federal funds rate, measured in percentage points at an annual rate. All variables are measured as differences from their means, their steady-state levels. The deviations $z_{\pi,t+1}$ and $z_{y,t+1}$ for inflation and the output gap have been substituted for the shocks

³⁰ Svensson (2001c) discusses additional serious problems with forecast-based instrument rules.

of the original Rudebusch-Svensson model. The predetermined variables are $(\pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_t, y_{t-1}, i_{t-1}, i_{t-2}, i_{t-3})$. See appendix H for details.

The target variables are inflation, the output gap, and the first-difference of the federal funds rate. The period loss function is

$$L_t = \frac{1}{2} [\pi_t^2 + \lambda y_t^2 + \nu(i_t - i_{t-1})^2] \quad (56)$$

where π_t is measured as the difference from the inflation target, which is equal to the steady-state level. The discount factor, δ , and the relative weights on the output-gap stabilization, λ , and interest-rate smoothing, ν , are set so $\delta = 1$, $\lambda = 1$, and $\nu = 0.2$.

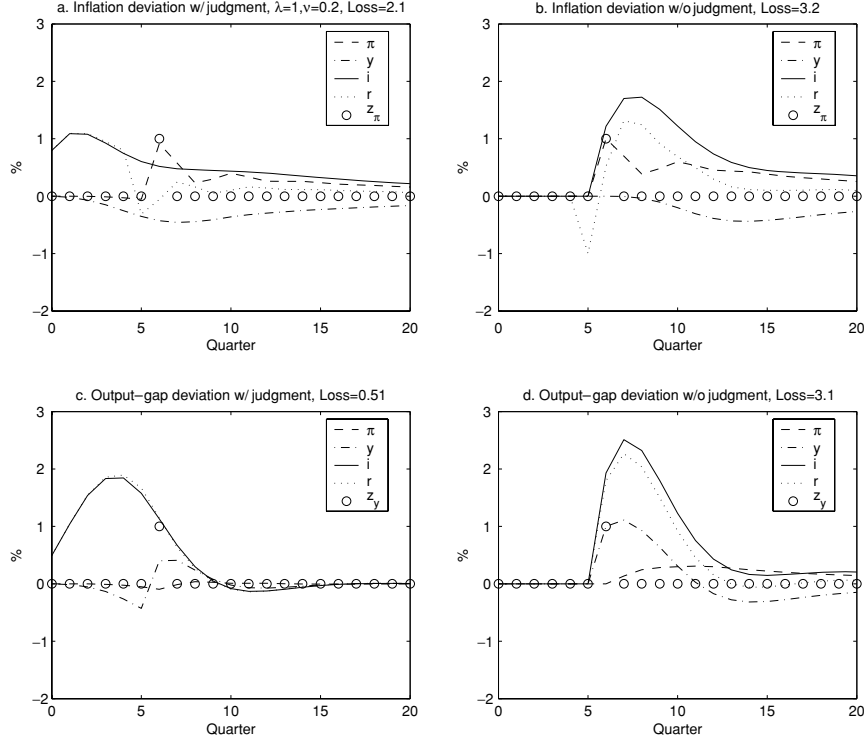
Let me emphasize that there may be considerable uncertainty about the future deviations, ζ^t , in this case $\{z_{\pi,t+\tau}, z_{y,t+\tau}\}_{\tau=1}^{\infty}$. Consider a simple case, when the distribution Φ_t is such that there is uncertainty only about $z_{\pi,t+\tau}$ and only for a finite number of quarters, $1 \leq \tau \leq T$. Then, I can take ζ^t to be the random T -vector $\zeta^t = (z_{\pi,t+1}, \dots, z_{\pi,t+T})$. Suppose furthermore that there are only four possible events with realizations $\zeta^t(j)$ ($j = 1, 2, 3, 4$), and that these are as follows:

1. With probability 0.4, $\zeta^t(1) = (0, 0, \dots)'$, no deviation.
2. With probability 0.3, $\zeta^t(2) = (0, 1.0, 1.0, 0.0, 0.0, 0, \dots)'$, a short sequence of large “cost-push” shocks.
3. With probability 0.2, $\zeta^t(3) = (0, 0.2, 0.2, 0.2, 0.2, 0, \dots)'$, a long sequence of small cost-push shocks.
4. With probability 0.1, $\zeta^t(4) = (0, 1.0, 1.0, 1.0, 1.0, 0, \dots)'$, a long sequence of large cost-push shocks.

The resulting judgment is the mean of the future deviations, the T -vector $z^t = 0.4\zeta^t(1)' + 0.3\zeta^t(2)' + 0.2\zeta^t(3)' + 0.1\zeta^t(4)' = (0, 0.44, 0.44, 0.14, 0.14, 0, \dots)'$.

Note that the same judgment arises if the probabilities are the same for the four events but the first event is that all components $\tau = 1, \dots, T$ of ζ^t have independent uniform distributions between -1 and 1 ; the second event is that all components have the same distributions as for the first event except that components $\tau = 2$

**Figure 1. Monetary Policy with and without Judgment:
Backward-Looking Model**



and 3 have independent uniform distributions between 0 and 2; the third event is that all components have the same distributions as for the first event except that components $\tau = 2, 3, 4$, and 5 have independent uniform distributions between -0.8 and 1.2 ; and the fourth event is that all components have the same distribution as for the first event except that component $\tau = 2, 3, 4$, and 5 have independent uniform distributions between 0 and 2. Thus, behind a given judgment vector can be a distribution Φ_t involving considerable uncertainty. Still, only the mean of that distribution matters.

Figure 1 shows a situation where the predetermined variables, inflation and the output gap, and the deviations are assumed to have been equal to their steady-state levels, zero, up to quarter 0. Furthermore, in previous quarters, the central bank's judgment, z^t ($t < 0$), has been zero: The central bank's expected future inflation

and output-gap deviations have been zero (although possibly with large variances).

In panel a, the central bank's judgment in quarter 0, z^0 , changes from that in previous quarters, such that the central bank's expected inflation deviation equals 1 percentage point for quarter 6, whereas it is still zero in all other quarters; the expected output-gap deviation is still zero for all quarters.³¹ Again, behind these means may be a distribution Φ_0 corresponding to considerable uncertainty. The expected inflation deviation, denoted z_π , is marked by circles with no connecting line. The panel shows the optimal policy projection in quarter 0, (π^0, y^0, i^0, r^0) , of inflation, π (the dashed line); the output gap, y (the dashed-dotted line); the instrument rate, i (the solid line); and the short real interest rate, r (the dotted line).³²

Panel a has two main interpretations. The first interpretation is that the panel just shows the judgment z^0 and the optimal policy projection $(\hat{\pi}^0, \hat{y}^0, \hat{i}^0, \hat{r}^0)$ in quarter $t = 0$ for the future quarters $\tau \geq 1$ and thereby the realization of z_0 , $\hat{\pi}_0$, $\hat{y}_0$, $\hat{i}_0$, and $\hat{r}_0$ in quarter 0. Conditional on the actual realization of π_1 and y_1 (in turn, depending on the realization of $z_{\pi 1}$ and $z_{y 1}$) and the realization of z^1 in quarter $t = 1$, a new optimal policy projection $(\hat{\pi}^1, \hat{y}^1, \hat{i}^0, \hat{r}^0)$ has to be plotted in quarter 1 for future quarters $\tau \geq 2$, and so forth for $t = 2, 3, \dots$

The second interpretation is that the panel shows the probability-zero event that the future realizations of the random deviation z_t for $t \geq 1$ are exactly equal to the judgment z^0 in quarter 0. That is, the realizations of $z_{\pi t}$ for $t \geq 1$ are zero, except in quarter 6 when it is 1 percentage point, and the realizations of $z_{y t}$ for $t \geq 1$ are all zero. In this interpretation, the panel also shows the optimal policy projection $(\hat{\pi}^t, \hat{y}^t, \hat{i}^t, \hat{r}^t)$ for each quarter $t \geq 1$. These optimal policy projections are then simply the continuation of the optimal policy projection of the previous quarter. Furthermore, the actual future realization of inflation, the output gap, the instrument rate, and the real interest rate are then equal to the previous optimal policy projections.

Thus, in the first interpretation, panel a just shows a particular realization of the judgment z^0 and the corresponding optimal policy projection $(\hat{\pi}^0, \hat{y}^0, \hat{i}^0, \hat{r}^0)$. In the second interpretation, panel a in

³¹ In terms of the modeling of the deviation as a moving average process in section 1.2, panel a shows the impulse response to ε^0 .

³² The short real interest rate is defined as $r_t \equiv i_t - \pi_{t+1|t}$.

addition shows a time series of a particular realization of the future deviation—namely the realization that is exactly equal to the judgment in quarter 0—as well as the resulting realizations over time of inflation, the output gap, the instrument rate, and the real interest rate. Clearly, the probability of the future realizations of the deviation being exactly equal to the previous judgment is generally zero.

Panel a shows that when the central bank expects a 1 percentage point inflation deviation in quarter 6, it chooses an optimal instrument-rate projection such that the instrument rate is raised to about 1 percentage point during the first few quarters and then gradually lowered back to its steady-state level. As a result, the projected output gap gradually falls to about -0.5 percentage points in quarter 7 and then very gradually rises back towards zero. The inflation projection shows inflation falling slightly before it is hit by the inflation-deviation in quarter 6, then rising to almost 1 percentage point, and finally falling back towards its steady-state level after quarter 6. Thus, the optimal policy projection is a clear example of preemptive monetary policy: the instrument rate is raised and the output gap is reduced well before the expected inflation-deviation shock, so as to efficiently control inflation and bring it back to target after the shock. The optimal policy projection in quarter 0 results in an intertemporal loss of 2.1 units.³³ In the second interpretation, when panel a shows the actual realization of the deviation, the accumulated realized loss over time, discounted to quarter 0, is also the same 2.1 units (since $\delta = 1$, the discounting does not affect the loss).

Panel b shows the time series of the variables for the same particular realization of the future deviations when the inflation deviation equals 1 percentage point in quarter 6 and is zero elsewhere. However, in this panel, the central bank in each quarter disregards judgment, while still responding optimally to the predetermined variables. That is, the central bank responds in the same way to the predetermined variables as for the optimal policy, but it does not respond to any expected future deviation. It behaves as if it believes

³³ Given how the target variables are measured, with the loss function (56) and $\delta = 1$, an expected difference of inflation from target of one (two) annualized percentage point(s) for a single quarter gives rise to an intertemporal loss of one (four) units.

that the deviation is a serially uncorrelated zero-mean process, so its expected future deviations are zero. This corresponds to a commitment to the instrument rule (47) (recall that there is no optimal response to Lagrange multipliers or lagged predetermined variables for the backward-looking model). The central bank then keeps the instrument rate at its steady-state level through quarter 5. Accordingly, inflation and the output gap stay at the steady-state levels through quarter 5. In quarter 6, the inflation shock hits and inflation jumps to 1 percentage point, while the predetermined output gap still stays at zero. In this situation, once the inflation shock has hit, the optimal monetary policy response is to raise the instrument rate substantially, to more than 1.5 percentage points above the steady-state level during the following few quarters. This reduces the output gap to almost -0.5 percentage points during the next eight to nine quarters. The instrument rate is gradually lowered back to the steady-state level, and inflation and the output gap return to their steady-state levels very slowly. The absence of any preemption requires a larger instrument-rate response when the shock occurs, the output-gap is nevertheless reduced with a considerable lag, and inflation stays above target for a long time. The resulting intertemporal loss is 3.2 units, 1.1 units higher than when monetary policy relies on judgment.

Panel c is analogous to panel a, except that it shows a situation when the central bank's judgment in quarter 0, z^0 , is such that the central bank expects an output-gap deviation of 1 percentage point in quarter 6, whereas no other deviations are expected. The expected output-gap deviation, z_y , is denoted by circle markers with no connecting line. Again, panel c has two interpretations: the first is that the panel just shows the judgment and optimal policy projection in quarter 0; the second is that it also shows the time series of the variables if the future realizations of the output-gap deviation are exactly equal to the judgment in quarter 0. For this expected output-gap deviation, the optimal policy projection shows a substantial increase in the instrument rate to almost 2 percentage points above the steady-state level in quarters 3–5 and then a rather quick reduction back to the steady-state level. As a result, the output-gap projection shows output falling by almost -0.5 percentage points before the expected output-gap deviation hits, after which it rises to less than 0.5 percentage points

and then relatively quickly comes back to the steady-state level. The resulting movements in the inflation projection are small. A modest loss of 0.51 units results from this preemptive optimal policy projection.

Panel d is analogous to panel b, except that it shows the time series of the variables for the particular realization of the output-gap deviation when it equals 1 percentage point in quarter 6 and is zero in the other quarter. The central bank disregards judgment and only responds to current and lagged inflation and the output gap (although, again, optimally so, according to the instrument rule [47]). Then the central bank keeps the instrument rate at its steady-state level until the output-gap shock hits in quarter 6. Once the shock has hit, it is optimal to raise the instrument rate even more, to more than 2 percentage points for a few quarters, before it is lowered back to the steady-state level. The output gap stays up around 1 percentage point for several quarters. This causes inflation to rise and only very slowly return to target. The output gap has to undershoot the steady-state level significantly in order to bring inflation back. Clearly, inflation and the output gap deviate substantially more than when the central bank uses its judgment. The resulting intertemporal loss is 3.1 units, 2.6 units higher than the loss for the optimal policy projection with judgment.

This example shows a substantial difference between monetary policy with and without judgment, with substantial differences in the development of the target variables and corresponding intertemporal losses.

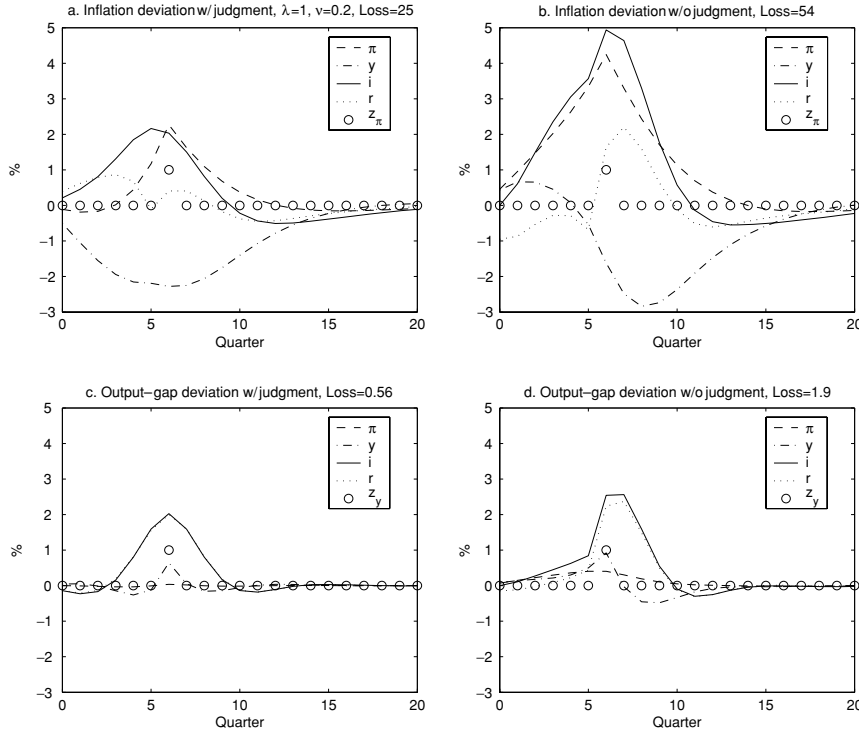
4.2 *Forward-Looking Model*

The forward-looking New Keynesian model of Lindé (2002) has two equations. I use the following parameter estimates:

$$\begin{aligned}\pi_t &= 0.457 \pi_{t+1|t} + (1 - 0.457) \pi_{t-1} + 0.048 y_t + z_{\pi t}, \\ y_t &= 0.425 y_{t+1|t} + (1 - 0.425) y_{t-1} - 0.156 (i_t - \pi_{t+1|t}) + z_{yt}.\end{aligned}$$

The variables are measured as for the backward-looking model. The predetermined variables are $(\pi_{t-1}, y_{t-1}, i_{t-1}, z_{\pi t}, z_{yt})$ (the lagged instrument rate enters because it enters into the loss function, and the two deviations are entered in order to write the model on the form

**Figure 2. Monetary Policy with and without Judgment:
Forward-Looking Model**



[1]; see section 1). The forward-looking variables are (π_t, y_t) . See appendix I for details. The loss function and its parameters used in the experiment below are the same as for the backward-looking model.³⁴

Figure 2, panels a–d, shows the same experiments as figure 1, but for the forward-looking model. Thus, before quarter 0, the variables are equal to their steady-state levels, zero, and the central bank does not expect any future inflation and output-gap deviations.

In panel a, in quarter 0, while the inflation and the output-gap deviations in that quarter are still zero, the central bank's judgment,

³⁴ I find it very unrealistic to consider inflation and output in the current quarter as forward-looking variables. I believe it makes more sense to have current inflation and the output gap predetermined, and to have one-quarter-ahead inflation, output-gap, and instrument-rate plans be determined by the model above. Such a variant of the New Keynesian model is used in Svensson and Woodford (2005) and Svensson (2003).

z^0 , changes, such that the central bank expects an inflation deviation equal to 1 percentage point in quarter 6, whereas it still expects zero inflation deviations for all other quarters and zero output-gap deviations for all quarters. Again, behind these expected deviations could be a probability distribution Φ_0 corresponding to substantial uncertainty. As with figure 1, panel a in figure 2 has two interpretations. In the first interpretation, it just shows the judgment z^0 and the optimal policy projection $(\hat{\pi}^0, \hat{y}^0, \hat{i}^0, \hat{r}^0)$. In the second interpretation, it shows the time series of inflation, the output-gap, the instrument rate, and the real interest rate, for the particular realizations of the future deviations that are exactly equal to the central bank's judgment in quarter 0. In this interpretation, I also assume that the private sector has sufficient information—cf. the discussion in section 1.1—to form expectations consistent with the optimal policy projection.

The optimal policy projection in panel a shows that the central bank plans to raise the instrument rate to about 2 percentage points above the steady-state level in the quarters before and including the time of the inflation shock. This makes the output-gap projection fall to more than -2 percentage points at the time of the expected inflation deviation. The inflation projection rises before and up to the expected inflation deviation, because private-sector expectations are forward looking and consistent with the optimal inflation projection. After the expected inflation deviation, the instrument rate, the output gap, and inflation are projected to return to their steady-state levels. Again, there is a considerable amount of preemption in the optimal policy with judgment, with a projected positive real interest rate and negative output gap before the expected inflation deviation. A substantial intertemporal loss of 25 units results from the optimal policy projection.

Panel b shows the realizations over time of these variables when the realization of the inflation deviation is equal to 1 percentage point in quarter 6 and zero in other quarters and the central bank in each quarter disregards judgment while still responding optimally to current and lagged inflation and output gap. In this case, the central bank is assumed to respond optimally to both the predetermined variables *and* the lagged predetermined variables, as if the central bank had committed itself to the optimal policy under commitment while ignoring its judgment. Hence, the central bank behaves

according to instrument rule (48) and responds optimally to the current deviation but expects zero future deviations. However, the private sector is assumed to have rational expectations of the future inflation shock. These expectations will increase inflation to more than 4 percentage points at the time of the inflation shock. The central bank's optimal response to current and predetermined variables induces it to raise the instrument rate in line with inflation, but it is nevertheless behind the curve in the sense that the real interest rate becomes negative and the output gap becomes positive in the first few quarters. The central bank's response eventually leads to a high positive real interest rate, a negative output gap, and a fall in inflation. In comparison with panel a, inflation rises earlier and more, and the output gap falls later, than under the optimal monetary policy with judgment. The intertemporal loss is 54 units, a substantial increase of 29 units above the loss for monetary policy with judgment.

Panel c shows the situation where the central bank's judgment in quarter 0 is such that it expects an output-gap deviation of 1 percentage point in quarter 6 and otherwise zero deviations. The optimal policy projection, taking this judgment into account, is to raise the instrument rate before the expected output-gap deviation, which moderates the expected impact on the output gap. The inflation projection remains very flat, and the projections of the real interest rate and the instrument rate are almost identical. The resulting intertemporal loss is small, 0.56 units.

Panel d shows the realizations over time of the variables in the situation where the realization of the output-gap deviation is 1 percentage point in quarter 6 and zero in other quarters and the central bank disregards judgment and only responds to current and lagged predetermined variables, although again optimally so, corresponding to the instrument rule (48). In comparison with the second interpretation of panel c, when the panel shows the actual realization of the variables for the same realization of the deviations, the central bank ends up raising the instrument rate later and more, and there is more movement in both the output gap and inflation. The intertemporal loss is 1.9 units, 1.3 units above the loss for optimal policy under judgment.

Again, there are substantial differences between monetary policy with and without judgment and corresponding intertemporal losses.

4.2.1 *Taylor Rules*

I also examine two variants of Taylor rules for the forward-looking model, an explicit instrument rule for which the instrument rate responds to lagged inflation and the output gap,

$$i_t = 1.5 \pi_{t-1} + 0.5 y_{t-1},$$

and an implicit instrument rule for which the instrument rate responds to the forward-looking current inflation and output,

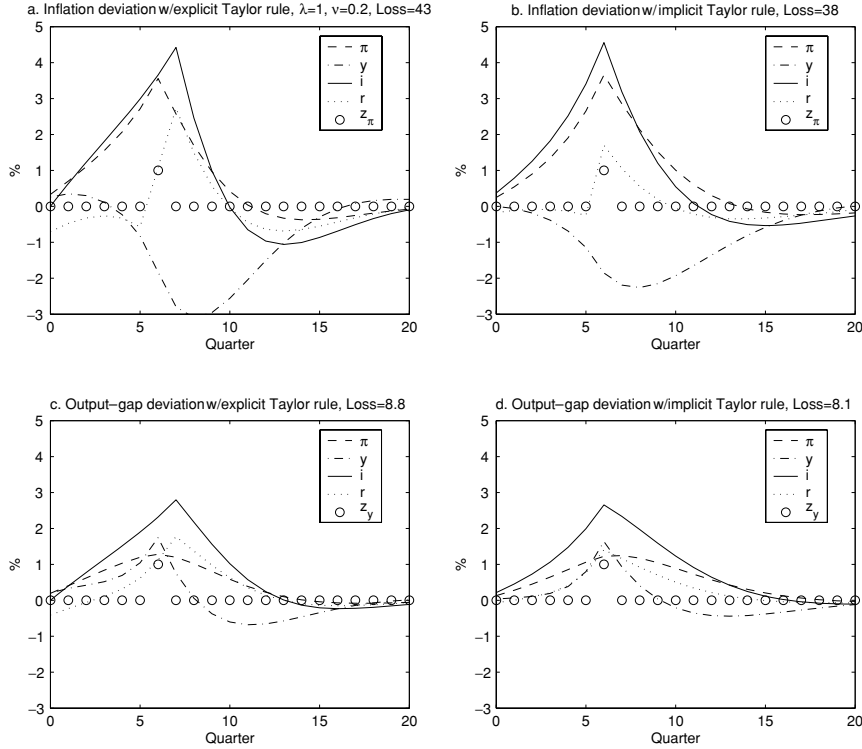
$$i_t = 1.5 \pi_t + 0.5 y_t.$$

As noted in section 3, the implementation of an implicit instrument rule is somewhat complex. I disregard these complications here, and just assume that it is somehow implemented. Figure 3 shows the realizations over time of the variables when the central bank implements the two Taylor rules for the two cases of either an inflation deviation or an output-gap deviation only in quarter 6.

Panels a and b show the result of the explicit and implicit Taylor rule, respectively, when there is an inflation deviation in quarter 6 and the private sector has rational expectations of that deviation. The resulting intertemporal losses are substantial, 43 and 38 units, respectively—18 and 13 units, respectively, above the loss for optimal monetary policy with judgment, 25 units. Interestingly, the intertemporal loss with either of the two Taylor rules is less than the policy without judgment that responds optimally to current and lagged predetermined variables, panel b in figure 2, which has an intertemporal loss of 54 units. One possible interpretation of this is that history dependence in the form of responding to the Lagrange multipliers is not always advantageous, when these multipliers do not take into account the expected future deviations. The loss for the implicit Taylor rule is lower than for the explicit one. One interpretation is that the implicit Taylor rule takes private-sector expectations better into account, and therefore indirectly takes the expected future deviation better into account.

Panels c and d show the result of the two Taylor rules when there is an output-gap deviation in quarter 6. Here, the intertemporal loss is substantially higher than the small loss for monetary policy without judgment in panel d of figure 2. In this case, the optimal

**Figure 3. Monetary Policy with Taylor Rules:
Forward-Looking Model**



response to current and lagged predetermined variables does much better than the two Taylor rules.

I conclude that the two Taylor rules perform considerably worse than the optimal policy with judgment, especially when there are expected future output-gap deviations.

5. Conclusions

The decision process of modern monetary policy that can be called “forecast targeting”—finding a projection of the current and future instrument rate such that the projection paths of the target variables “look good” relative to the central bank’s objectives—is formalized in this paper as a technique that provides projections of the

instrument rate and the target variables that minimize an intertemporal loss function. The paper shows how this technique can easily incorporate central-bank judgment, a necessary ingredient in modern monetary policy. In two empirical models of the U.S. economy, a few examples are shown in which forecast targeting that incorporates judgment provides significantly better monetary policy performance than a policy that follows an instrument rule and disregards judgment. The paper shows how the policy problem, normally treated as an infinite-horizon problem, can be reformulated as a convenient finite-horizon decision problem, which is either an exact or a very close approximation to the infinite-horizon problem. This approximation makes the policy problem much easier to handle numerically. The paper also shows how the time-consistency problem can be easily managed and the resulting projections made to be optimal under commitment in a timeless perspective. In particular, the paper shows that it is not necessary to be explicit about the underlying complex reduced-form reaction function of monetary policy. The policymakers only need to ponder the projections of the target variables and the instrument rate under alternative assumptions, and these projections can be presented as graphs.

Several of the ideas and techniques presented here are already applied to various extents by different central banks. I hope the presentation here will be useful for attempts to apply them more extensively and systematically.

If policymakers hesitate to make the parameters of their loss function explicit (for instance, the weight on output-gap stabilization relative to inflation stabilization), the techniques presented here can still be very useful. For instance, the policymakers can ask the staff to provide optimal policy projections of the target variables for a range of loss-function parameters. These projections then provide one way to illustrate the available trade-offs among the target variables, the set of feasible projections of the target variables from which the policymakers should choose their optimal policy projection.

The framework used here is one where mean projections are sufficient for optimal decisions, what can be called *mean* forecast targeting, which is sufficient under the assumptions that result in certainty equivalence. If these assumptions are not satisfied, the principal ideas in this paper can be extended to a situation when the projections are probability *distributions* rather than means, and the intertemporal

losses can be computed by numerical integration over those distributions. This I have previously called *distribution* forecast targeting (Svensson 2001b). The details in such an undertaking remain to be completed, and the practical differences between mean and distribution forecast targeting remain to be clarified. Svensson and Williams (2005) examine distribution forecast targeting in a situation where genuine model uncertainty implies that certainty equivalence does not hold.

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Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements*

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We investigate the effects of U.S. monetary policy on asset prices using a high-frequency event-study analysis. We test whether these effects are adequately captured by a single factor—changes in the federal funds rate target—and find that they are not. Instead, we find that two factors are required. These factors have a structural interpretation as a “current federal funds rate target” factor and a “future path of policy” factor, with the latter closely associated with Federal Open Market Committee statements. We measure the effects of these two factors on bond yields and stock prices using a new intraday data set going back to 1990. According to our estimates, both monetary policy actions and statements have important but differing effects on asset prices, with statements having a much greater impact on longer-term Treasury yields.

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The Federal Reserve's announcement following its January 28, 2004, policy meeting led to one of the largest reactions in the Treasury market on record, with two- and five-year yields jumping 20 and 25 basis points (bp) respectively in the half-hour surrounding the announcement—the largest movements around any Federal Open Market Committee (FOMC) announcement over the fourteen years for which we have data. Even more remarkably, this outsized reaction was spurred not by what the FOMC *did*, but rather by what it *said*: indeed, the decision to leave the current federal funds rate unchanged was completely anticipated by financial markets, but the FOMC's decision to drop the phrase “policy accommodation can be maintained for a considerable period” from its accompanying statement and replace it with “the Committee believes it can be patient in removing its policy accommodation” was read by financial markets as indicating that the FOMC would begin tightening policy sooner than previously expected.¹ On this date, then, treating the monetary policy action as a 0 bp surprise change in the current federal funds rate target would be missing the whole story.

In this paper, we investigate the extent to which this observation is true more generally: are the effects of monetary policy announcements on asset prices adequately characterized by a single factor, namely the surprise component of the change in the current federal funds rate target? We perform a test of this hypothesis using the rank test of Cragg and Donald (1997) and strongly reject the hypothesis of a single factor. By contrast, we do not reject the hypothesis that the effects of monetary policy on asset prices are characterized by two factors. By performing a suitable rotation of these unobserved factors, we show that they can be given a structural interpretation as a “current federal funds rate target” factor, corresponding to surprise changes in the current federal funds rate target, and a “future path of policy” factor, corresponding to changes in futures rates out to horizons of one year that are independent of changes in the current funds rate target. We show that this latter (“path”) factor has typically been associated with significant changes in FOMC statements, such as the January 28 example above. In this way, we generalize and

¹For example, the front page of *The Wall Street Journal* reported the following morning that “investors interpreted the omission of ‘considerable period’ as a signal that the Fed is closer to raising interest rates than many thought” (Ip 2004).

improve the single-factor analysis of the effects of monetary policy on asset prices performed by earlier authors, such as Cook and Hahn (1989), Kuttner (2001), Cochrane and Piazzesi (2002), Rigobon and Sack (2004), Ellingsen and Soderstrom (2003), and Bernanke and Kuttner (forthcoming).

To measure the effects of monetary policy actions and statements on asset prices, we construct a new data set that captures changes in asset prices in a thirty-minute window and in a one-hour window bracketing every FOMC announcement going back to 1990 (we use the term *announcement* to refer to any means by which a policy decision was communicated to financial markets, including open market operations as well as explicit press releases). The use of intraday data allows us to better isolate the response of asset prices to monetary policy announcements, since we can be almost certain that no other economic news was released within such a brief interval of time. The gains from using intraday data are most apparent in the early 1990s, when on a number of occasions the FOMC eased policy just a few hours after a weak employment report released earlier in the day.

In estimating the separate effects of monetary policy actions and statements, we are not suggesting that FOMC statements represent a policy tool that is completely independent of the federal funds rate target. In particular, FOMC statements likely exert their effects on financial markets through their influence on financial market expectations of *future* policy actions. Nonetheless, the two-factor approach we employ in this paper greatly adds to our understanding of the response of asset prices to monetary policy announcements—for example, we find that *75 to 90 percent* of the explainable variation in five- and ten-year Treasury yields in response to monetary policy announcements is due to the path factor (associated with statements) rather than to changes in the federal funds rate target. We interpret this result as indicating not that policy actions are secondary so much as that their influence comes earlier, when investors build in expectations of those actions in response to FOMC statements (and perhaps other events, such as speeches and testimony by FOMC members).

Our findings have important implications for the conduct of monetary policy in a low-inflation environment. In particular, the fact that FOMC statements have such significant effects on long-term yields suggests that the FOMC may be able to credibly commit to future plans for the federal funds rate. Thus, even when

faced with a low or zero nominal funds rate, our results indicate that the FOMC may be able to effectively communicate to the markets its intention to keep the federal funds rate low for an extended period, thereby lowering longer-term interest rates and stimulating economic growth.²

The remainder of the paper proceeds as follows. Section 1 provides a detailed study of surprises in the federal funds rate target and measures asset price responses to these surprises. Section 2 tests whether there are additional dimensions underlying monetary policy announcements, finds that one additional dimension is required, shows that this additional factor is closely associated with FOMC statements, and estimates the response of asset prices to this additional factor. Section 3 concludes. The appendix provides technical details regarding our factor test and the estimation of the unobserved factors, and a data appendix (only available online at www.ijcb.org) provides a complete listing of our monetary policy announcement dates, times, and federal funds rate and statement surprise measures extending back to 1990.

1. The Effects of Changes in the Federal Funds Rate Target on Asset Prices

1.1 Methodology

We begin our analysis in the framework of one-dimensional measures of monetary policy surprises that has been used in the existing literature. To measure the effects of unexpected monetary policy actions (changes in the federal funds rate) on asset prices, we rely on the following regression, which has been frequently estimated in the literature:

$$\Delta y_t = \alpha + \beta \Delta x_t + \varepsilon_t \quad (1)$$

where Δx_t denotes the surprise component of the change in the federal funds rate target announced by the FOMC, Δy_t denotes the change in a bond yield or stock market index over an interval that

²Reifschneider and Williams (2000) and Eggertsson and Woodford (2003) provide theoretical support for the argument that a central bank is relatively unconstrained by the zero nominal bound so long as it can credibly commit to future policy actions. Bernanke, Reinhart, and Sack (2004) investigate the potential effectiveness of monetary policy at the zero nominal bound empirically.

brackets the monetary policy announcement, and ε_t is a stochastic error term that captures the effects of other factors that influence the asset in question.

We use a high-frequency event-study analysis to estimate equation (1). One generally cannot estimate equation (1) using monthly or quarterly data due to simultaneous equations and omitted variables bias. In particular, the change in monetary policy could actually be a *response* of monetary policy to the change in the asset price that took place earlier in the month or quarter, due to the direct effects of stock market wealth on the economic outlook or due to the signal that term spreads provide about future economic activity and inflation.³ Alternatively, both the change in monetary policy and the change in the asset price could be responding to important macroeconomic news (captured by ε_t) that was released earlier in the period, such as an employment report.⁴ In either case, the classical regression assumption that ε_t is orthogonal to Δx_t is violated.⁵

These problems can be mitigated by using higher-frequency data to shrink the time period around the policy decision. Kuttner (2001), for example, uses daily data to measure changes in Treasury yields and the surprise component of FOMC monetary policy announcements. Cochrane and Piazzesi (2002) and Ellingsen and

³Rigobon and Sack (2003) discuss reasons why asset prices might be expected to feed back into monetary policy. Using a heteroskedasticity-based identification procedure, they estimate a statistically and economically significant response of monetary policy to the stock market, and this response has the expected positive sign. Using their heteroskedasticity-based procedure, Rigobon and Sack (2004) also estimate the size of the endogeneity and omitted variables problems in estimating equation (1).

⁴Asset prices can respond to macroeconomic news for many reasons other than changes in monetary policy expectations: First, information about economic output is likely to influence expectations of corporate earnings and dividends, hence stock prices; second, information about the government budget or current account deficit would, assuming home bias in savings and imperfect Ricardian equivalence, affect bond yields; third, investors' appetite for risk can change in response to economic developments, to name a few examples.

⁵One way to partially account for these issues is to estimate a VAR for the asset price, the policy instrument, and other relevant macroeconomic variables, as in Leeper, Sims, and Zha (1996) and Evans and Marshall (1998). The problem with this approach is that the recursive identifying restrictions typically employed are not plausible for fast-moving financial market variables. (Two exceptions to this rule are the heteroskedasticity-based identification procedure used by Rigobon and Sack [2003, 2004] and the high-frequency identification procedure used by Cochrane and Piazzesi [2002] and Faust, Swanson, and Wright [2004a].)

Soderstrom (2003) perform variations on this analysis, and Bernanke and Kuttner (forthcoming) apply the method to measure the effects of monetary policy announcements on the stock market. However, as noted by Rudebusch (1998) and Bernanke and Kuttner (forthcoming), simultaneity in equation (1) is still a potential problem even at daily frequency because, for a time, the FOMC often changed its target for the federal funds rate just hours after (and in response to) the Bureau of Labor Statistics' employment report release. As a result, event-study regressions using daily data in part capture the endogenous response of asset prices and monetary policy to the information that was released earlier in the day, as well as noise from other financial market developments that took place throughout the day.

We address this potential problem by estimating regression (1) using intraday data to measure both the funds rate surprise Δx_t and the change in the asset price Δy_t . By measuring these changes in a sufficiently narrow window of time around the monetary policy announcement, we can be sure that the FOMC decision was in no way influenced by asset price movements or other macroeconomic news over that interval. In addition, by shrinking the event-study window down to an hour or less, it becomes much less likely that any other significant events took place within this narrow window that might have influenced asset prices, thereby increasing the precision of our estimates.

1.2 Dates and Times of Monetary Policy Announcements

To perform the above analysis using intraday data, we first put together a complete list of dates and times of monetary policy announcements from January 1990 through December 2004. In February 1994, the FOMC began issuing a press release after every meeting and every change in policy, and thus the announcement dates and times are simply those of the corresponding press releases.⁶ Prior to 1994, the FOMC did not explicitly announce changes in its target for the federal funds rate, but such changes were implicitly

⁶Since 1995, these press releases have occurred at about 2:15 p.m., after regularly scheduled FOMC meetings; press releases for intermeeting policy moves and FOMC decisions in 1994 were released at varying times throughout the day. We obtained all of these dates and times from the office of the secretary of the Federal Reserve Board.

communicated to financial markets through the size and type of open market operation. Thus, prior to 1994, the date and time of a monetary policy announcement are typically those of the next open market operation following the FOMC decision.⁷

The dates, times, and methods of communication of FOMC monetary policy announcements are reported in table 1 of the online data appendix. Note that this listing includes not just dates on which the FOMC actually changed the federal funds rate, but also dates on which there was an FOMC meeting followed by no change in policy, since in some cases the FOMC's decision not to change policy surprised financial markets and led to movements in asset prices. The rightmost column of this table reports other major macroeconomic data releases that took place on each date, before the monetary policy announcement. From the table, we can see that eight monetary policy announcements occurred on the date of an employment report release, seven announcements occurred on the date of a gross domestic product release, nine occurred on the date of a consumer price index release, and five on the date of a producer price index release, to name just a few.

1.3 *The Surprise Component of Federal Funds Rate Changes*

For each monetary policy announcement, we measure the surprise component of the change in the federal funds rate target using federal funds futures. We use the surprise component of monetary policy announcements in estimating regression (1) because changes in policy that are expected by financial markets should have little or

⁷On a few occasions between 1990 and 1994, the FOMC issued, prior to the open market operation, a press release announcing a change in the *discount* rate offered to depository institutions, and market participants correctly inferred from the press release a corresponding change in the target federal funds rate as well. On those occasions, we set the time of the monetary policy announcement to the time of the discount rate change press release. Open market operations over this period were conducted at 11:30 a.m. every day. There are a few dates on which volatility in the federal funds market prevented the Open Market Trading Desk from successfully communicating the FOMC's intentions for the funds rate the first morning after the FOMC's decision (see Kuttner 2003). On these dates, we regard the announcement as having taken place on whichever morning the Credit Markets column of *The Wall Street Journal* regarded as a clear signal of the Federal Reserve's intentions, as reported by Kuttner (2003). This is more of an issue prior to 1990 than for our sample period in this paper.

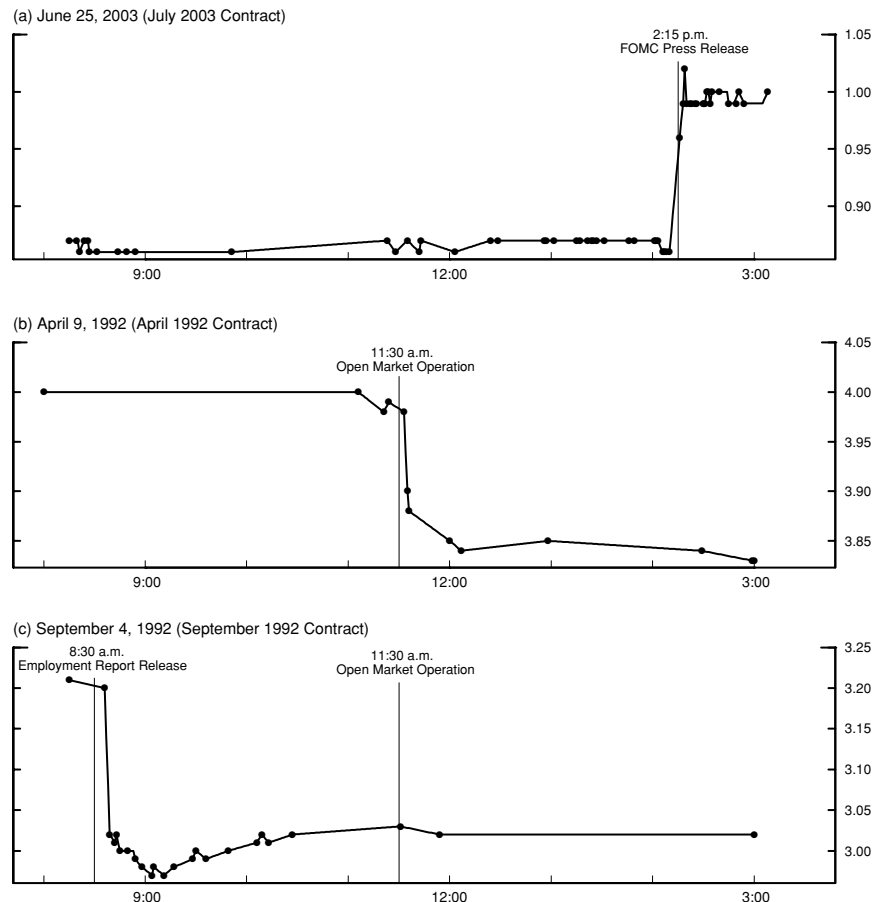
no effect on asset prices, a hypothesis that is confirmed by Kuttner (2001). Thus, using the raw changes in the federal funds rate target as the right-hand-side variable Δx_t would impart an errors-in-variables bias to our estimates of β to the extent that the monetary policy decisions were correctly anticipated by financial markets.

Federal funds futures have traded at the Chicago Board of Trade exchange since October 1988 and settle based on the average effective federal funds rate that is realized for the calendar month specified in the contract. Thus, daily changes in the current-month futures rate largely reflect revisions to the market's expectations for the federal funds rate over the remainder of the month. As described in the appendix, the change in the current month's contract rate on the day of an FOMC announcement can be scaled up to account for the timing of the announcement within the month, and thereby measure the surprise component of the FOMC's announcement for the federal funds rate. For the present paper, we acquired tick-by-tick data on all federal funds futures contract trades from January 1990 to the present from Genesis Financial Technologies. To provide a sense of the quality of this data and its advantages, figure 1 graphs the data on three illustrative dates:

1. June 25, 2003, was the date of a regularly scheduled FOMC meeting (see panel a). Trades were intermittent throughout the day until just before and just after the FOMC's press release at 2:15 p.m. At that time, the FOMC announced that it was lowering its target for the federal funds rate from 1.25 percent to 1 percent. According to surveys and press reports both before and after the policy announcement, many market participants had been expecting the FOMC to ease policy by 50 bp at the meeting. Thus, this decision is characterized as a 13 bp tightening surprise under our measure. This example illustrates two key points: First, financial markets seem to fully adjust to the policy action within just a few minutes—i.e., there is no evidence of learning or sluggish adjustment to the target rate decision going on after about 2:20 p.m. in this example. Second, the federal funds rate surprise is not necessarily in the same direction as the federal funds rate action itself.

2. April 9, 1992, was the date of an intermeeting monetary policy move (see panel b). The FOMC reduced its target for the federal funds rate from 4 percent to 3.75 percent that morning, but given that this date precedes 1994, the FOMC did not issue a press release

Figure 1. Intraday Trading in Federal Funds Futures Contracts



about its change in policy to the public. As can be seen in the figure, trading in federal funds futures was thin until shortly before the open market operation at 11:30 a.m. At that time, the Open Market Trading Desk injected a significant quantity of reserves into the market, and market participants correctly inferred from this that the FOMC had changed its target for the funds rate, causing the futures rate to move quickly to the new target rate.⁸

⁸The federal funds futures contract rate falls to 3.85 percent after the announcement rather than the new funds rate target of 3.75 percent because nine

3. September 4, 1992, witnessed the release of a very weak employment report at 8:30 a.m. (see panel c). In response to that report, investors significantly revised downward their expectations for the federal funds rate, pushing the futures rate down sharply. Sometime after the poor data release, the FOMC decided to reduce its target for the federal funds rate from 3.25 percent to 3 percent. Again, because there was no press release, the FOMC's decision became known to the markets at 11:30 a.m., the time of the open market operation. In contrast to panel b, however, the FOMC's decision for the funds rate on this date was essentially completely anticipated by the time it was signaled to the market—indeed, our intraday measure of the funds rate surprise (reported in table 2 of the online data appendix and discussed below) is 0 bp. By contrast, the daily measure of the funds rate surprise is -22 bp, because it incorporates the *endogenous* policy response to the weak employment report. In this case, we would not want to use the daily measure of the funds rate surprise in an event-study regression, because it would suffer from the omitted variables problem discussed earlier (in that the employment report itself has sizable effects on stock prices, bond yields, and monetary policy expectations).

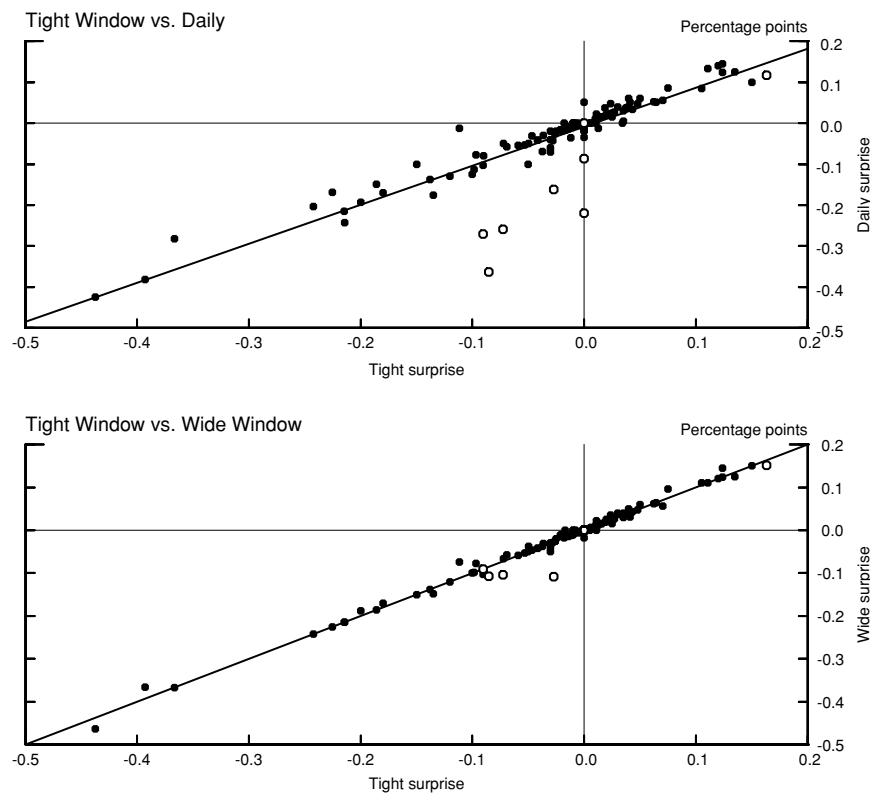
To focus on the monetary policy decision itself, we compute policy surprises by looking at changes in the futures rate in narrow windows around the FOMC announcements. More specifically, online data appendix table 2 reports two intraday measures, a “tight” window and a “wide” window, which begin ten (fifteen) minutes prior to the monetary policy announcement and end twenty (forty-five) minutes after the policy announcement, respectively, for the period from January 1990 through December 2004.⁹ For comparison, the table also reports a “daily” window that begins with the financial market close the day before the policy announcement and ends with the financial market close the day of the policy announcement. On most of the days in our sample, the two intraday measures are quite

days of the month have already elapsed with an average federal funds rate of 4 percent, which will result in a month-average funds rate for April of 3.85 percent.

⁹When there is no federal funds futures trade exactly at the beginning of the specified window, we use the most recent price. When there is no trade exactly at the end of the specified window, we use the next available trade price. Federal funds futures trading is often sparse early in our sample period, but becomes significantly more dense around the times of macroeconomic data releases and monetary policy announcements.

similar to the daily measure: the average absolute difference between the daily and two intraday surprise measures is only about 2 bp, and the difference is 0 bp on many days. There are a few days, however, on which the differences between the intraday measures and the daily measure are quite large: for example, there are five observations for which the discrepancy between the tight surprise and the daily surprise exceeds 10 bp. Each of those observations took place before 1994 on the day of an employment report release, as in the example of September 4, 1992, discussed previously. Figure 2 makes this point graphically: as can be seen in the top panel, the tight window

Figure 2. Measures of Surprises in the Federal Funds Rate Target



Note: Hollow circles denote days of employment report releases.

and daily window surprises are in very close agreement on all but a handful of dates, almost all of which correspond to days on which the FOMC was responding to an employment report release (depicted by the hollow circles). By contrast, in the bottom panel of the figure, we see that the two intraday measures are in very close agreement on all dates in our sample.

We draw two conclusions from these observations. First, the FOMC decision accounted for the vast majority of the movement in the federal funds futures rate on all of the non-employment-report days in our sample. This is perhaps surprising, given the large number of other data releases that also coincided with monetary policy announcements in data appendix table 1, and is in itself an important finding: it shows that for samples that exclude employment report dates, or samples that begin in 1994, the surprise component of monetary policy announcements can be measured very well using just daily data. Second, FOMC actions were priced into the federal funds futures market almost immediately—quickly enough to be completely captured by our tight, thirty-minute window—consistent with the examples discussed above. Thus, we can feel comfortable focusing on the analysis using our tight window of thirty minutes, although we will report results using the wider one-hour window as well.

1.4 The Effect of Federal Funds Rate Changes on Asset Prices

Table 1 presents our results for regression equation (1) estimated using intraday data on bond yields and stock prices.¹⁰ The independent variable is the surprise component of the change in the federal funds rate target just described, and the dependent variable is the change in the financial variable measured over the same window. We present results for the tight (thirty-minute), wide (one-hour), and daily windows described above.

Our results for stock prices imply that, on average, a surprise 25 bp tightening in the federal funds rate leads to a little more than

¹⁰We obtained tick-by-tick Treasury yield data back to June 1991 for on-the-run Treasury securities from GovPX, a consortium of interdealer brokers that accounted for a large portion of trading volume in Treasury securities over our sample. For equity prices, we obtained five-minute intraday quotes on the S&P 500 index back to the mid-1980s, which are available from a variety of sources.

Table 1. Response of Asset Prices to Changes in the Federal Funds Rate

	Tight Window		Wide Window		Daily	
	<i>Constant</i> (<i>std. err.</i>)	<i>MP Surprise</i> (<i>std. err.</i>)	<i>Constant</i> (<i>std. err.</i>)	<i>MP Surprise</i> (<i>std. err.</i>)	<i>Constant</i> (<i>std. err.</i>)	<i>MP Surprise</i> (<i>std. err.</i>)
<i>R</i> ²	<i>R</i> ²	<i>R</i> ²	<i>R</i> ²	<i>R</i> ²	<i>R</i> ²	<i>R</i> ²
<i>S&P 500</i>	−0.095*** (0.034)	−4.032*** (1.110)	−0.059 (0.042)	−4.531*** (1.069)	0.171* (0.090)	−3.961*** (1.560)
<i>Three-Month Bill</i>	−0.005** (0.002)	0.537*** (0.040)	−0.007** (0.003)	0.583*** (0.038)	−0.004 (0.004)	0.669*** (0.087)
<i>Six-Month Bill</i>	−0.005 (0.003)	0.522*** (0.057)	−0.007** (0.003)	0.570*** (0.053)	−0.008 (0.005)	0.627*** (0.083)
<i>Two-Year Note</i>	−0.002 (0.004)	0.455*** (0.087)	−0.001 (0.006)	0.475*** (0.092)	−0.006 (0.007)	0.429*** (0.117)
<i>Five-Year Note</i>	−0.000 (0.004)	0.264*** (0.081)	0.000 (0.005)	0.267*** (0.100)	−0.007 (0.007)	0.318** (0.142)
<i>Ten-Year Note</i>	−0.001 (0.003)	0.125** (0.058)	0.000 (0.005)	0.129 (0.080)	−0.006 (0.006)	0.166 (0.124)
<i>Five-Year Forward Rate Five Years Ahead</i>	−0.001 (0.003)	−0.087* (0.047)	0.001 (0.004)	−0.061 (0.061)	−0.006 (0.006)	−0.042 (0.114)

Note: Sample is all monetary policy announcements from July 1991 through December 2004 (January 1990–December 2004 for S&P 500). Target factor and path factor are defined in the main text. Heteroskedasticity-consistent standard errors reported in parentheses. *, **, and *** denote significance at 10 percent, 5 percent, and 1 percent, respectively. See text for details.

a 1 percent fall in the S&P 500, and these estimates are highly significant. The estimated coefficients do not differ greatly across the intraday and daily regressions, although the effects of the omitted employment report variable can be seen clearly in the scatter plots in figure 3: the handful of days on which the policy decision followed an employment report, shown by the hollow points, do not appear unusual when the intraday data are used, but in the daily data they stand out as large policy easings that yielded no gains in equity prices, most likely due to the negative direct influence of the weak employment reports.

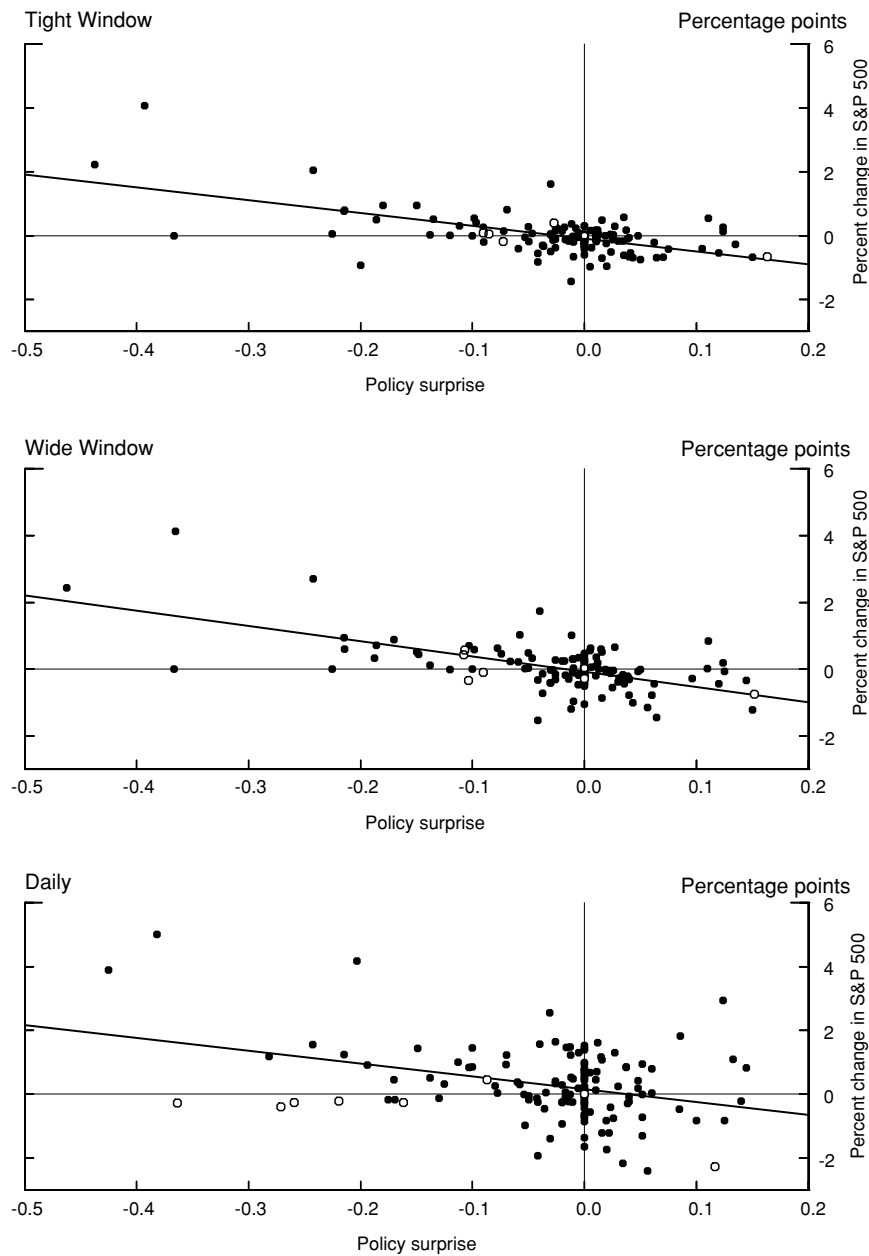
The most striking feature of figure 3, however, is the increase in tightness of the relationship as we move from daily to intraday data. By eliminating the effects of employment reports and other news that occurred on the days of monetary policy announcements, the relationship between monetary policy actions and equity prices becomes much clearer in the figure. This advantage also stands out in the regression results in table 1, in terms of the much greater precision of the coefficient estimates and a tripling of R^2 from .12 to .36.

Intraday data yield additional benefits for our Treasury yield regressions. As shown in figure 4, employment report days (the hollow points) stand out in the daily data as very large funds rate surprises and large changes in the three-month Treasury-bill rate in the same direction, reflecting the fact that the employment report has a very large influence on both the FOMC decision and the short end of the Treasury yield curve.¹¹ By contrast, those days do not stand out at all when the intraday windows are used. Moreover, employment report surprises appear to have a larger effect on the T-bill rate compared to policy surprises. Because of this, the estimated coefficient in the regression is biased upward if daily data are used. As with equity prices, the response of the T-bill rate to monetary policy actions is also estimated much more precisely using intraday data, reflected in the much smaller standard errors (about half the size of the daily measures) and the much higher R^2 (.80 versus .56).

The differences in the coefficients between the intraday regressions and the daily regression become much smaller at horizons of

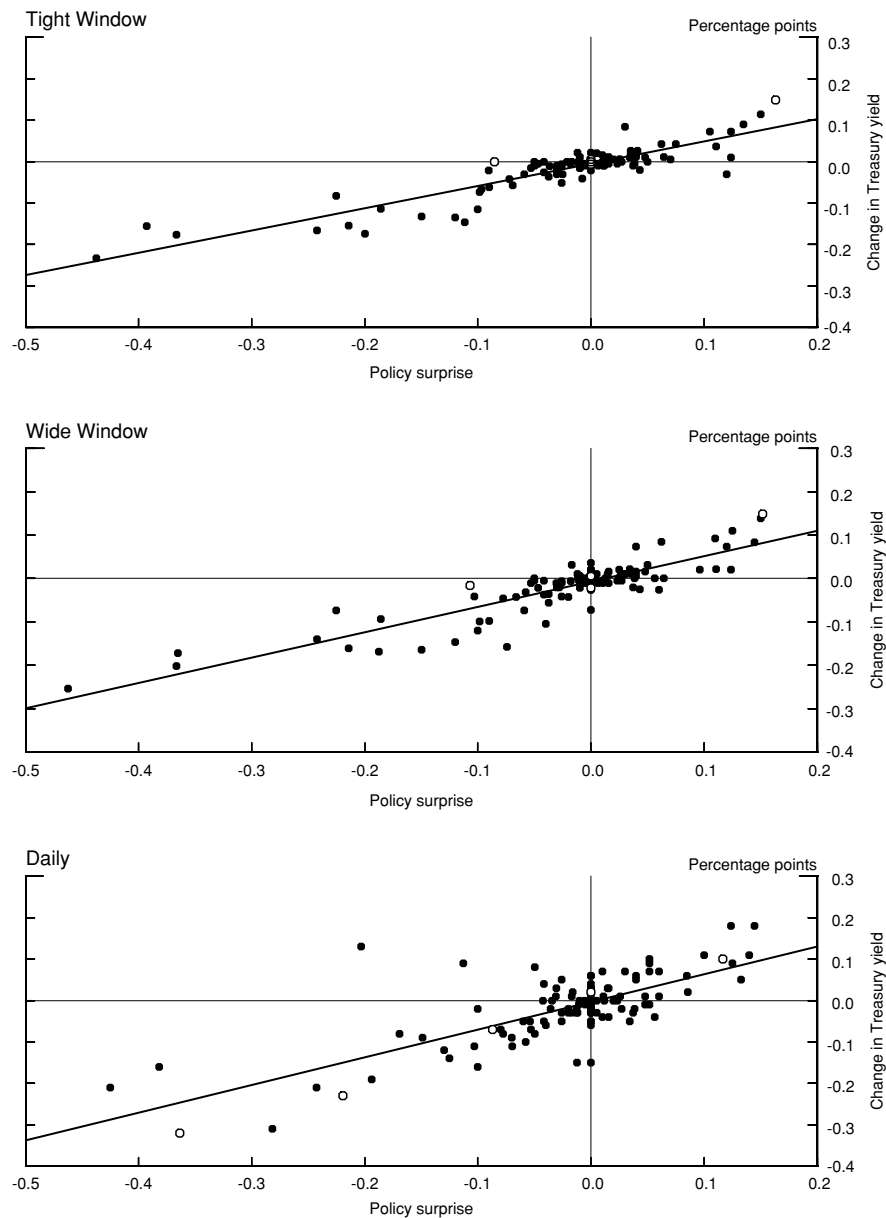
¹¹Recall that our intraday Treasury data extend back only to June 1991, so our Treasury yield regressions and graphs contain eighteen fewer observations (and only five employment report dates instead of eight).

Figure 3. Response of S&P 500 to Monetary Policy Surprises



Note: Hollow circles denote days of employment report releases.

Figure 4. Response of Three-Month Treasury Yield to Monetary Policy Surprises



Note: Hollow circles denote days of employment report releases.

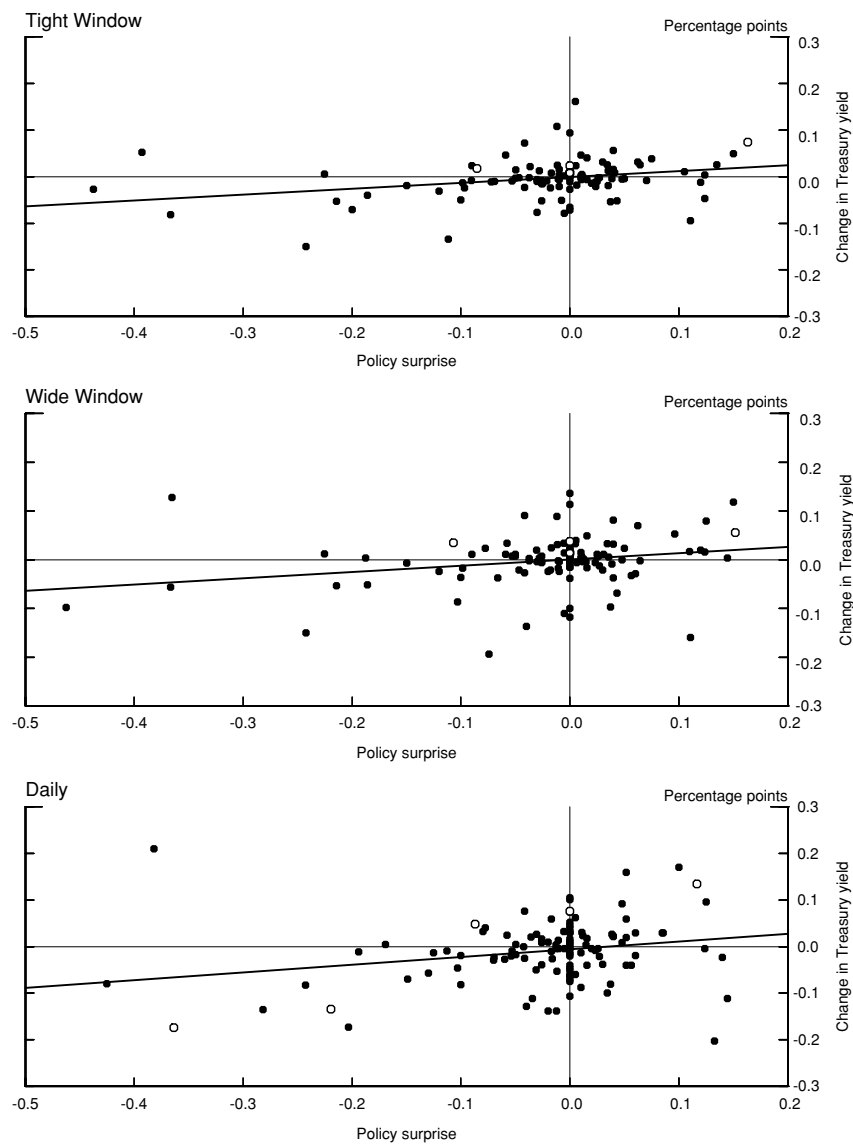
two years or more, but this observation is somewhat misleading, as a single outlier, January 3, 2001, pulls the daily estimate toward the intraday one (shown in figure 5).¹² Without that one observation in the upper left, the daily regression would estimate a significant response of the ten-year rate to the FOMC announcement of 0.28 rather than the statistically insignificant 0.17 that is estimated including the outlier. By contrast, using the intraday data, the estimated coefficient changes only very slightly (from 0.13 to 0.17) when we exclude that observation. As before, the precision of our estimates also improves dramatically using intraday data, with the standard errors being about half as large. This makes the response of the ten-year Treasury yield—which is statistically indistinguishable from zero using daily data—significantly greater than zero (albeit small) using our tight-window data.

The response of the term structure can also be expressed in terms of forward rates. We compute the five-year forward Treasury rate beginning five years ahead from five- and ten-year Treasury yields using the Shiller-Campbell-Schoenholtz (1983) approximation. The estimated response of the forward rate to the policy surprise is *negative* over this sample. This is consistent with the findings of Gürkaynak, Sack, and Swanson (2003) that far-ahead forward rates typically move *inversely* with the monetary policy surprise, although our estimates here are not highly statistically significant, partly because we cannot compute intraday forward rate changes that begin more than five years ahead. Gürkaynak, Sack, and Swanson (2003) present their findings in terms of the nine-year-ahead one-year forward rate, which is probably a better measure of far-ahead forward rates.¹³

¹²On that day, although the FOMC unexpectedly eased policy, which would normally be associated with a fall in Treasury yields, market participants reportedly became much more optimistic about the economic outlook as a result, leading to a huge rally in equity markets (including an astounding 14 percent rise in the NASDAQ that afternoon) and a large *upward* shift in Treasury yields. This outlier poses particular problems for a researcher using only daily data, since it is unclear whether the observation should be dropped or retained, and the empirical results are highly sensitive to this choice; by contrast, using the intraday data, the observation poses no problems.

¹³We only have intraday data for the on-the-run five- and ten-year Treasury notes, which gives us the forward rate from five to ten years ahead, but not for any finer subintervals. The five-year-ahead five-year rate may not capture movements in far-ahead forward rates as well as the nine-year-ahead one-year rate.

Figure 5. Response of Ten-Year Treasury Yield to Monetary Policy Surprises



Note: Hollow circles denote days of employment report releases.

2. The Effects of FOMC Statements on Asset Prices

2.1 *Testing for Additional Dimensions of Monetary Policy Announcements*

The preceding section assumed that the effects of FOMC announcements on asset prices are completely (or at least adequately) described by the surprise component of the change in the federal funds rate target. Although this assumption is standard in the existing literature, the example of January 28, 2004, given earlier raises serious questions regarding its validity. In this section, we ask whether the example of January 28 was a one-time occurrence or a manifestation of something much more systematic. If the latter, how many additional dimensions are required to adequately characterize monetary policy announcements?¹⁴

The above questions essentially ask how many latent factors underlie the response of asset prices to monetary policy announcements. Let X denote the $T \times n$ matrix, with rows corresponding to monetary policy announcements, columns corresponding to asset prices, and each element of X reporting the change in the corresponding asset price in a thirty-minute window around the corresponding announcement. Writing X in the form

$$X = F\Lambda + \eta \quad (2)$$

where F is a $T \times k$ matrix of unobserved factors (with $k < n$), Λ is a $k \times n$ matrix of factor loadings, and η is a $T \times n$ matrix of white noise disturbances, we wish to know how many factors (columns of F) are required to adequately describe X . The hypothesis that a single factor (for example, the surprise component of changes in the federal funds rate) is sufficient is a statement that there exists a $T \times 1$ vector F and constants λ_i , $i = 1, \dots, k$, such that the matrix X is described by $F \times [\lambda_1, \dots, \lambda_k]$ up to white noise.

This restriction on the structure of the data X can be tested using the matrix rank test of Cragg and Donald (1997). In brief, the null hypothesis that X is described by k_0 common factors can

¹⁴One can imagine that several dimensions might be required, corresponding to information about the future path of interest rates, policymakers' objectives and preferences, and perhaps the future paths of output and inflation, if the FOMC possesses private information about these latter two variables.

be tested against the alternative that X is described by $k > k_0$ factors by measuring the minimum distance between $\text{Cov}(X)$ and the covariance matrices of all possible factor models (2) with k_0 factors. This distance, after a suitable normalization, has a limiting χ^2 distribution with $(n - k_0)(n - k_0 + 1)/2 - n$ degrees of freedom. Additional details of the method are provided in the appendix.

Table 2 reports the results of this test applied to two sets of assets: one that includes all of our Treasury yields and stock prices, and another that includes only federal funds futures and eurodollar futures with one year or less to expiration.¹⁵ We consider the latter set of assets in this table because they will turn out to be very useful in the structural interpretation of the factors, below.

As can be seen in table 2, the hypothesis that the response of asset prices to monetary policy announcements is characterized by zero or by one common factor is clearly rejected. An important and direct corollary of this finding is that surprise changes in the federal funds rate alone are not sufficient to describe the response of asset prices to monetary policy announcements, in contrast to one of the standard assumptions in the literature. However, as is also clear in table 2, we find that two factors are sufficient—tests of this hypothesis are not rejected at even the 10 percent level. Given our strong rejection of zero or one factor, this result is perhaps somewhat surprising—it implies that, despite the potentially unlimited complexity of monetary policy statements and hence announcements, financial markets nonetheless have reacted as if there is essentially only one additional degree of information beyond the surprise change in the federal funds rate target. In the remainder of this section, we turn to estimating and interpreting this additional dimension of U.S. monetary policy and estimating its effect on Treasury yields and stock prices.

¹⁵Treasury yields include the three-month, six-month, two-year, five-year, and ten-year yields. Recall that we only have intraday data for these yields beginning in July 1991, which explains the smaller number of observations for these securities in table 2. For futures rates, we use five contracts that pin down the expected path of the federal funds rate over the next year without overlapping: the current-month and three-month-ahead federal funds futures contracts (with a scale factor to account for the timing of FOMC meetings within the month) and the two-, three-, and four-quarter-ahead eurodollar futures contracts. See the appendix for details. Avoiding overlap is desirable because very similar assets will tend to covary strongly, producing an additional factor even if that variation is orthogonal to all of the other assets in X .

Table 2. Tests of Number of Factors Characterizing Monetary Policy Announcements

	Treasury Yields and Stock Prices				Futures Rates with ≤ 1 Year to Expiration			
H_0 : Number of Factors Equals	Wald Statistic	χ^2 Degrees of Freedom	p -value	Number of Obs.	Wald Statistic	χ^2 Degrees of Freedom	p -value	Number of Obs.
0	46.72	15	.00004	120	36.61	10	.00007	138
1	21.41	9	.011	120	17.19	5	.004	138
2	4.36	4	.360	120	1.06	1	.304	138

Note: Test is from Cragg and Donald (1997) and tests the null hypothesis of N_{H0} factors against the alternative of $N > N_{H0}$ factors. Sample: January 1990–December 2004 (July 1991–December 2004 for Treasuries). Treasury yields comprise three-month, six-month, two-year, five-year, and ten-year yields, stock prices the S&P 500. Futures rates comprise one- and three-month-ahead federal funds futures rates (with scale adjustment for timing of FOMC meetings within the month) and two-, three-, and four-quarter-ahead eurodollar futures rates.

2.2 *Estimation of the Two Factors*

We estimate the unobserved factor matrix F using the standard method of principal components applied to our data matrix X .¹⁶ This procedure decomposes the matrix X into a set of orthogonal vectors F_i , $i = 1, \dots, n$, where F_1 is the (length- T) vector that has maximum explanatory power for X , F_2 is the vector that has maximum explanatory power for the residuals of X after projecting each column on F_1 , and so on. Based on the results of our factor rank test, above, we will restrict attention to the first two factors (F_1 and F_2) estimated by this procedure. Details are provided in the appendix.

As mentioned above, we estimate the unobserved factors F_1 and F_2 using the set of federal funds futures and eurodollar futures rates with one year or less to expiration that characterize the expected path of the funds rate over the upcoming year.¹⁷ Estimates of the unobserved factors using Treasury yields and stock prices are quite similar; the reason for focusing on the shorter-term futures rates for estimation is that it makes the structural interpretation of the two factors, and interpretation of the effects of the two factors on bond yields and stock prices, more clear.

2.3 *A Structural Interpretation of the Two Factors*

The two factors F_1 and F_2 explain a maximal fraction of the variance of X , but do not have a structural interpretation. For example, both factors are correlated with the surprises in the current federal funds rate target, so we cannot interpret one factor as the change in the federal funds rate target and the other factor as some other dimension of monetary policy. To address this deficiency and allow for a more structural interpretation of the factors, we perform a rotation of F_1 and F_2 to yield two new factors, which we call Z_1 and

¹⁶The primary alternative to principal components is Kalman filtering, which is optimal under the assumption of normally distributed residuals. However, the normality assumption may not fit our data very well.

¹⁷Gürkaynak, Sack, and Swanson (2002) show that these rates are the best financial market predictors of the federal funds rate at horizons out to a year. As one familiar with factor decompositions of the yield curve might expect, we find that F_1 and F_2 look very much like “level” and “slope” factors from that literature, although our sample differs from that literature in that we focus exclusively on the brief intervals around monetary policy announcements.

Z_2 , which are still orthogonal and explain the matrix X to exactly the same extent that F_1 and F_2 did, but for which the second factor (Z_2) has no effect on the current federal funds rate. In other words, we define

$$Z = FU$$

in such a way that U is a 2×2 orthogonal matrix and the second column of Z is a vector that is associated on average with *no change in the current-month federal funds futures rate*. Exact details of this rotation are reported in the appendix. As a result of this transformation, we can regard the unexpected change in the current target for the federal funds rate as being driven exclusively by Z_1 (plus a small amount of white noise), and Z_2 represents all other aspects of FOMC announcements that move futures rates for the upcoming year *without* changing the *current* federal funds rate. The factor Z_2 therefore includes any information (besides the decision for the current target rate) that affects the expected path for monetary policy over the upcoming year. Accordingly, we refer to Z_1 and Z_2 as the “target” factor and the “path” factor, respectively. The estimated values for the target and path factors for each monetary policy announcement in our sample are reported in table 3 of the online data appendix.

Note that the target factor (Z_1) defined in this way should be similar to—but not exactly equal to—the measure of federal funds target surprises we presented in the previous section and reported in data appendix table 2.¹⁸ To check the tightness of the relationship between these two measures, we regressed our target surprises on the factor Z_1 (results not reported) and found that Z_1 is in fact very close to our previous measure, with a correlation of over 95 percent (R^2 of .91). Thus, to further facilitate interpretation of Z_1

¹⁸The two measures generally are not identical because the factor estimation procedure strips out white noise from the data. Thus, Z_1 will tend to exclude part of the “timing” component of federal funds rate surprises—i.e., changes in the funds rate target that are only a surprise to the extent that they occur at the present meeting instead of the next meeting. (Gürkaynak, Sack, and Swanson [2002] and Gürkaynak [2005] discuss “timing” surprises in more detail.) To the extent that assets other than the current-month federal funds futures rate ignore these timing surprises, the factor estimation procedure will assign this component to white noise rather than to Z_1 . In this respect, Z_1 is a *better* measure of the important component of funds rate surprises than is the standard measure based on the current-month futures rate.

as the surprise change in the funds rate target, we normalize its scale so that a change of .01 in Z_1 corresponds to a surprise of 1 bp in the federal funds rate target. Similarly, to facilitate interpretation of the second factor, we normalize its scale so that the effect of Z_2 on the four-quarter-ahead eurodollar futures rate is exactly the same as the effect of Z_1 on the four-quarter-ahead eurodollar futures rate, about 55 bp.¹⁹

2.4 The Association Between the Path Factor and FOMC Statements

The path factor Z_2 corresponds to all aspects of FOMC announcements that move futures rates for the upcoming year without changing the current federal funds rate; thus, it is to some extent a residual that is subject to various interpretations. In this section, we present three pieces of evidence that show a close correlation between the path factor and FOMC statements: first, a time series plot that allows comparison of pre-1994 to post-1994 (and also statement to nonstatement) monetary policy announcements; second, a regression test of the path factor on a dummy variable indicating the presence of an FOMC statement; and third, a case study of the largest path factor observations in our sample. In addition, in section 2.5 below, we compare our estimated effects of the path factor on asset prices to the effects of the FOMC minutes release on January 4, 2005, which, because of new procedures adopted by the FOMC, has many of the features of an FOMC statement release with no change in the funds rate target.

In figure 6, we plot the target and the path factors over time, with dates on which there was an FOMC statement plotted as solid bars and those on which there was no FOMC statement (dates prior to 1994 and dates from 1994 to 1999 on which there was no change in the federal funds rate target) plotted as hollow bars. As can be seen in the figure, there have been many large realizations of the path factor in recent years, while realizations of the path factor prior to 1994 (and on nonstatement days in general) were typically very small.

¹⁹These scale normalizations are performed for the July 1991–December 2004 sample, the period for which we have data on U.S. Treasury yields.

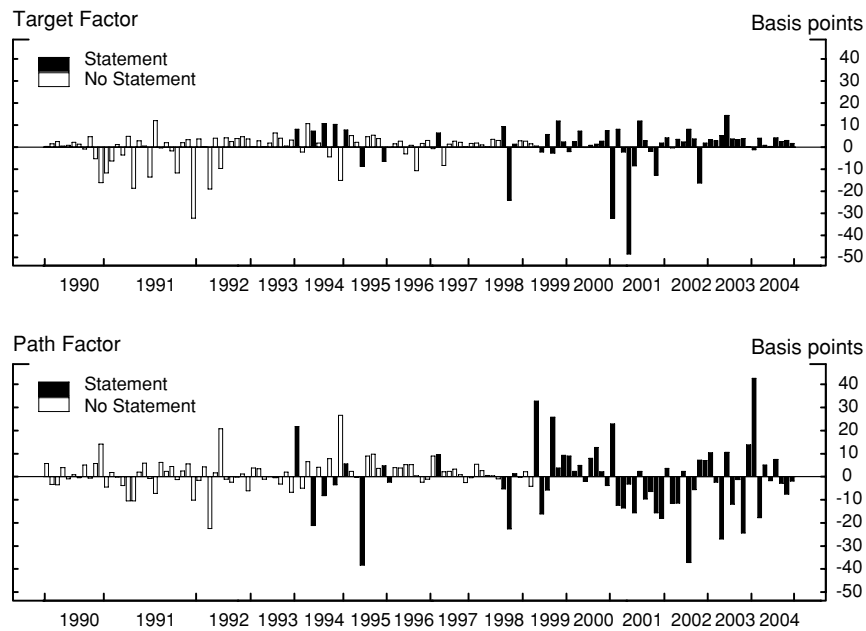
Figure 6. Monetary Policy Surprises as Two Factors

Table 3 verifies this observation econometrically by regressing the absolute value of the path factor on a constant and a dummy variable that takes on the value one for all dates on which there was an FOMC statement. The coefficient on the dummy variable is positive and highly statistically significant, allowing us to strongly

Table 3. Estimated Effects of Policy Statements on the Size of the Path Factor

Dependent Variable	Constant (std. err.)	Statement (std. err.)	R ²	Number of Obs.
Abs(Z ₂)	0.044*** (0.005)	0.070*** (0.014)	.18	138
Note: Regression of absolute value of path factor on a constant and dummy variable indicating presence of an FOMC statement. Heteroskedasticity-consistent standard errors reported in parentheses. *** denotes significance at 1 percent level.				

reject the hypothesis that FOMC statements and the path factor are unrelated.²⁰

In table 4, we report details of the ten largest path factor observations over our sample. Nine of these ten observations (and twenty-one of the top twenty-five) correspond to dates on which an FOMC statement accompanied the federal funds rate target decision; moreover, financial market commentary on these dates (as reported in the Credit Markets column of *The Wall Street Journal* the following day) typically directly attributed the move in the bond market to the specifics of the FOMC statement. Nonetheless, there are a few exceptions to the correspondence between the path factor and FOMC statements, which represent significant changes in financial market expectations regarding the future path of policy that were driven by events other than FOMC statements. For example, on December 20, 1994, market participants were reportedly nervous about inflationary pressures arising from output growth overshooting potential (the subject of a speech by Vice Chairman Blinder just a few days before), and the failure of the FOMC to move at the December meeting was reportedly regarded by some participants as perhaps requiring greater tightening down the road (Pesek and Young 1994), leading to larger-than-usual changes in expectations for the future path of policy.²¹

An interesting feature of the path factor is that—in contrast to the target factor—there is evidence that financial markets may take some time to digest its implications. For example, if we regress a wide (one-hour) window measure of the path factor Z_2 on the tight (thirty-minute) window measure of the factor (both calculated as above), we get an R^2 of .83. By contrast, if we perform the same exercise on the target surprise factor Z_1 , we get a much higher R^2 of .98 (which is consistent with figures 1 and 2). A natural interpretation

²⁰This finding is similar to a result in Kohn and Sack (2004), who used a simple regression to control for the effects of target surprises, with the residual intended to capture the effects of all other variables, including FOMC statements. They found that the variance of this residual was much higher on days that the FOMC released statements.

²¹We also checked the financial press for each of these dates to see whether the strong response of the path factor on dates without statements could possibly be due to the release of other important news in the same thirty-minute window as the monetary policy announcements. We found no such confounding news releases.

Table 4. Ten Largest Observations of the Path Factor

Date	Z_1 (Target Factor)	Z_2 (Path Factor)	Statement	Financial Market Commentary
Jan. 28, 2004	-1.1	42.7	✓	Statement drops commitment to keep policy unchanged for “a considerable period,” bringing forward expectations of future tightenings
Jul. 6, 1995	-8.7	-38.4	✓	First easing after long (seventeen-month) series of tightenings raises expectations of further easings; statement notes that inflationary pressures have receded
Aug. 13, 2002	8.1	-37.2	✓	Statement announces balance of risks has shifted from neutral to economic weakness
May 18, 1999	0.5	32.8	✓	Statement announces change in policy bias going forward from neutral to tightening
May 6, 2003	5.2	-27.0	✓	Statement announces balance of risks now dominated by risk of “an unwelcome substantial fall in inflation”
Dec. 20, 1994	-15.1	26.6		Surprise that FOMC not tightening considering recent comments by Blinder on “overshooting”; some fear Fed may have to tighten more in 1995 as a result
Oct. 5, 1999	-2.7	25.8	✓	Statement announces change in policy bias going forward from neutral to tightening
Oct. 28, 2003	3.9	-24.4	✓	Statement leaves the “considerable period” commitment unchanged, pushing back expectations of future tightenings
Jan. 3, 2001	-32.3	22.8	✓	Large surprise intermeeting ease reportedly causes financial markets to mark down probability of a recession; Fed is perceived as being “ahead of the curve” and as needing to ease less down the road as a result
Oct. 15, 1998	-24.0	-22.6	✓	First intermeeting move since 1994 and statement pointing to “unsettled conditions in financial markets... restraining aggregate demand” increases expectations of further easings

of this finding is that changes in the federal funds rate target itself are immediately and clearly observable to all financial market participants within minutes of the announcement while, by contrast, FOMC statements about the policy and economic outlook typically require time to digest and are subject to a great deal of uncertainty with respect to how they are interpreted by other financial market participants, so that the process of assimilating the information contained in the statements is not instantaneous. Nonetheless, we continue to emphasize our tight window responses in the analysis below because most of the policy information is incorporated within that window and having a narrower window reduces the amount of noise in our left-hand-side variables, increasing the precision of our estimates.²²

2.5 The Response of Asset Prices to the Target Factor and Path Factor

We now estimate the effects of each of these two dimensions of monetary policy announcements on asset prices. For each monetary policy announcement from January 1990 through December 2004, we have estimates of the target factor and path factor components of the announcement and we observe the change in Treasury yields and stock prices in a narrow window bracketing the announcement (the same left-hand-side variables as in section 1).²³ For each Treasury yield or stock prices, we run the regression

$$\Delta y_t = \alpha + \beta Z_{1,t} + \gamma Z_{2,t} + \varepsilon_t. \quad (3)$$

Results are reported in table 5.²⁴

²²Robustness tests using the wide window data were in line with the results reported below.

²³Recall that, due to data availability, we only observe intraday Treasury responses beginning in July 1991.

²⁴Because the regressors in table 5 are generated rather than directly observed, we have also computed the standard errors by bootstrapping to capture the additional sampling error associated with the first-stage factor estimation procedure. The statistical significance of all our results was essentially identical whether we used the bootstrapped or asymptotic standard errors. Bootstrap percentiles were based on 1,000 repetitions; in each repetition, we sampled 138 new observations from the data with replacement, took the two largest principal components from the synthetic futures data, rotated and normalized them, and regressed the

Table 5. Response of Asset Prices to Target and Path Factors

	One Factor		Two Factors		
	<i>Constant</i> (<i>std. err.</i>)	<i>Target Factor</i> (<i>std. err.</i>)	<i>Constant</i> (<i>std. err.</i>)	<i>Target Factor</i> (<i>std. err.</i>)	<i>Path Factor</i> (<i>std. err.</i>)
<i>MP Surprise</i>	−0.021*** (0.003)	1.000*** (0.047)	−0.021*** (0.003)	1.000*** (0.048)	0.001 (0.026)
<i>One-Year-Ahead Eurodollar Future</i>	−0.018*** (0.006)	0.555*** (0.076)	−0.017*** (0.001)	0.551*** (0.017)	0.551*** (0.014)
<i>S&P 500</i>	−0.008 (0.041)	−4.283*** (1.083)	−0.008 (0.040)	−4.283*** (1.144)	−0.966 (0.594)
<i>Two-Year Note</i>	−0.011** (0.005)	0.485*** (0.080)	−0.011*** (0.002)	0.482*** (0.032)	0.411*** (0.023)
<i>Five-Year Note</i>	−0.006 (0.005)	0.279*** (0.078)	−0.006** (0.002)	0.276*** (0.044)	0.369*** (0.035)
<i>Ten-Year Note</i>	−0.004 (0.004)	0.130** (0.059)	−0.004* (0.002)	0.128*** (0.039)	0.283*** (0.025)
<i>Five-Year Forward Rate Five Years Ahead</i>	0.001 (0.003)	−0.098** (0.049)	0.001 (0.003)	−0.099** (0.047)	0.157*** (0.028)
Note: Sample is all monetary policy announcements from July 1991–December 2004 (January 1990–December 2004 for S&P 500). Target factor and path factor are defined in the main text. Heteroskedasticity-consistent standard errors reported in parentheses. *, **, and *** denote significance at 10 percent, 5 percent, and 1 percent, respectively. See text for details.					

As we would expect from the close correspondence between our target factor and the funds rate target surprises in section 1 (and from the orthogonality of Z_2 to Z_1), the estimated coefficients on the target factor (Z_1) are very similar to those we estimated previously in table 1. In particular, we estimate that a 1 percentage point surprise tightening in the federal funds rate leads, on average, to a 4.3 percent decline in the S&P 500 and increases of 49, 28, and 13 bp in two-, five-, and ten-year Treasury yields, respectively.

The novel feature of table 5, however, is our estimates of the effects of the path factor (Z_2) on asset prices. As can be seen in the table, the effect of this factor on the one-year-ahead eurodollar future rate is the same as the effect of the target factor, by definition.²⁵ However, the path factor has effects on the other financial variables that differ considerably from the target factor. In particular, the path factor has a much greater impact on the long end of the yield curve, with a 1 percentage point innovation to the factor causing responses of 37 and 28 bp in five- and ten-year Treasury yields, respectively. Thus, FOMC statements that move one-year-ahead policy expectations appear to have much greater effects on the long end of the yield curve than do changes in the federal funds target rate itself, *even when they generate the same size movement in one-year-ahead interest rates*. Moreover, as can be seen by comparing the R^2 statistics from the one- and two-factor regressions, the large majority of variation in long-term Treasury yields seems to be due to statements rather than funds rate changes, with two-thirds of the explainable variation in two-year yields, three-fourths of the explainable variation in five-year yields, and nine-tenths of the explainable variation in ten-year yields attributable to the path factor rather than to changes in the federal funds rate target. Thus, by focusing only on the FOMC's decisions regarding the current federal funds rate target, previous authors have been missing by far the most important component of monetary policy decisions, especially

synthetic changes in asset prices on the synthetically generated Z_1 or Z_1 and Z_2 . The bootstrapped standard errors are available from the authors upon request.

²⁵The path factor is orthogonal to the monetary policy surprise (and the target factor) over our full sample, January 1990–December 2004. The coefficient of the MP Surprise on the path factor in table 5 is not exactly zero because it is reported for the July 1991–December 2004 sample, for comparability to the Treasury yield responses.

in recent years when target funds rate changes have often been well-anticipated.

By contrast, the effect of changes in the path factor on the stock market appears to be smaller than the effect of changes in the funds rate target, amounting to only about -1 percent for a 1 percentage point innovation. Given the yield curve findings above, this result is somewhat surprising: for longer-maturity Treasury securities, policy-makers' statements seem to have much larger effects than changes in the current federal funds rate, but the effect of statements on stock prices, which also have very long durations, is smaller.

One possible explanation for this pattern is that the statements that seem to drive the path factor lead, to a greater extent, to positive revisions in investors' assessment of the future path of output and inflation, consistent with a story first formalized by Romer and Romer (2000).²⁶ A large positive realization of the path factor, for example, might be related to a statement suggesting that the FOMC sees greater output or inflation going forward than markets had been expecting. Gürkaynak, Sack, and Swanson (forthcoming) show that long-term yields move substantially and positively in response to positive surprises in macroeconomic data releases for output and inflation; thus, if in fact FOMC statements do reveal information about the future course of these variables, the strong response of long-term yields to the path factor in this paper would be completely consistent with those earlier results. Moreover, to the extent that financial markets revise upward their forecasts of output (and hence earnings and dividends) in response to positive path factor surprises, then the tendency for stocks to fall in response would be muted by the upwardly revised economic outlook.

As a final note, the events of January 4, 2005, provide us with an interesting out-of-sample test of our results. At 2:00 p.m. that day, the FOMC for the first time released the minutes for its previous meeting under a new, accelerated schedule—three weeks *before* (as opposed to a few days after) the next FOMC meeting. This gives

²⁶This story has been questioned on the basis of a pure target factor analysis by Faust, Swanson, and Wright (2004b). Those authors find that surprise tightenings in what we call the target factor convey essentially *no* positive information about the future path of output or inflation. However, as in all of the previous literature, those authors did not consider changes in what we call the path factor, which our results suggest could be more informative about these variables.

the minutes release on that date many of the features of an FOMC statement: for example (and in contrast to speeches or testimony by individual FOMC members), the minutes report the thinking and deliberation of the entire FOMC and have been voted on and approved by the whole committee; moreover, the minutes now provide additional information about the most recent FOMC meeting. In the thirty-minute window around the minutes release on January 4, one-year-ahead eurodollar futures rose 6.5 bp, as the minutes were taken to imply greater risks of inflation and funds rate tightening than markets had previously been expecting. Two-, five-, and ten-year Treasury yields increased 4.9, 5.0, and 4.1 bp, respectively—very much in line with our estimates of the effects of the path factor in table 5, which would predict changes of 4.8, 4.4, and 3.3 bp for these securities, respectively. The S&P 500 declined by .49 percent, somewhat larger than the .11 percent decline predicted by table 5, and the five-to-ten-year forward rate rose by 3.0 bp, a bit more than the 1.9 bp predicted by our estimates. Although we cannot rule out the possibility that markets view minutes and FOMC statements as differing in important ways, all of these results are broadly in line with our estimates of the effects of FOMC statements, suggesting that our estimation and identification procedures have been reasonably effective.

3. Conclusions

Do central bank actions speak louder than words? We find that the answer to this question is a qualified “no.” In particular, we find that viewing the effects of FOMC announcements on financial markets as driven by a single factor—changes in the federal funds rate target—is inadequate. Instead, we find that a second policy factor—one not associated with the current federal funds rate decision of the FOMC but instead with statements that it releases—accounted for more than three-fourths of the explainable variation in the movements of five- and ten-year Treasury yields around FOMC meetings.

We emphasize that our findings do not imply that FOMC statements represent an *independent* policy tool. In particular, FOMC statements likely exert their effects on financial markets through their influence on financial market expectations of *future* policy actions. Viewed in this light, our results do not indicate that policy

actions are secondary so much as that their influence comes earlier—when investors build in expectations of those actions in response to FOMC statements (and perhaps other events, such as speeches and testimony by FOMC members).

Our findings have important implications for the literature on the effects of monetary policy on asset markets. Indeed, we have shown that previous studies estimating the effects of changes in the federal funds rate on bond yields and stock prices have been missing most of the story. This is especially true in recent years, when FOMC decisions regarding the target for the funds rate have rarely been a surprise and, instead, changes in the wording of FOMC statements typically have been the major driver of financial market responses.

More broadly, our finding that FOMC statements have such significant effects on asset prices suggests that the FOMC has the ability to conduct policy with a substantial degree of commitment to a state-contingent, or conditional, path for the funds rate several quarters or even years into the future. This finding has important implications for the conduct of monetary policy in a low-inflation environment—in particular, even when faced with a low or zero nominal funds rate, our results directly support the theoretical analysis of Reifschneider and Williams (2000) and Eggertsson and Woodford (2003) that the FOMC is largely unhindered in its ability to conduct policy, because it has the ability to manipulate financial market expectations of *future* policy actions and thereby longer-term interest rates and the economy more generally.

Appendix. Factor Model Rank Test and Estimation

Testing the Number of Factors

Let X denote a $T \times n$ matrix of data generated by a factor model:

$$X = F\Lambda + \eta, \quad (\text{A1})$$

where F is a $T \times k$ matrix of unobserved factors (with $k < n$), Λ is a $k \times n$ matrix of factor loadings, and η is a $T \times n$ matrix of white noise disturbances. We wish to test the hypothesis that X was generated by k_0 factors against the alternative that X was generated by $k > k_0$ factors. We do this by computing the distance between

the covariance matrix of the data, X , and the set of all covariance matrices corresponding to factor models (A1) with k_0 factors.

The covariance matrix Σ_X of the data, X , has elements given by $\hat{\sigma}_{jk} = (1/T) \sum_i (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)$. To compute the Wald statistic, below, we also need to know the sampling uncertainty surrounding the $\hat{\sigma}_{jk}$. This is given by:

$$\begin{aligned} \text{Cov}(\hat{\sigma}_{jk}, \hat{\sigma}_{lm}) = (1/T^2) \sum_i [(x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k) - \hat{\sigma}_{jk}] \\ \times [(x_{il} - \bar{x}_l)(x_{im} - \bar{x}_m) - \hat{\sigma}_{lm}]. \end{aligned} \quad (\text{A2})$$

The covariance matrix of the factor model (A1) is given by $\Lambda' \Lambda + \Sigma_\eta$. To compute the distance between the two, we stack the elements of Σ_X and $\Lambda' \Lambda + \Sigma_\eta$ into vectors and normalize the difference, $\text{vech}(\Sigma_X) - \text{vech}(\Lambda' \Lambda + \Sigma_\eta)$, by $\text{Cov}(\text{vech}(\Sigma_X))$, whose elements are given by (A2):

$$W = [\text{vech}(\Sigma_X) - \text{vech}(\Lambda' \Lambda + \Sigma_\eta)]' \hat{\Omega}^{-1} [\text{vech}(\Sigma_X) - \text{vech}(\Lambda' \Lambda + \Sigma_\eta)], \quad (\text{A3})$$

where $\hat{\Omega} = \text{Cov}(\text{vech}(\Sigma_X))$. We search over all possible factor loadings Λ (nk_0 parameters) and white noise variances Σ_η (n parameters) to find the minimum distance (A3). Under the null hypothesis, this is a Wald statistic with limiting χ^2 distribution on $n(n+1)/2 - (nk_0 + n) + k_0(k_0 - 1)/2$ degrees of freedom (the last term comes from the fact that there are $k_0(k_0 - 1)/2$ possible rotations of the unobserved factors F , and hence Λ , which all yield the same observable model). The Matlab code that performs this factor test is available from the authors upon request.

Factor Estimation

To estimate the unobserved factors F , let X be the matrix with 138 rows corresponding to monetary policy announcements and five columns corresponding to futures contracts with one year or less to maturity. The third through fifth columns of X are the changes in price of the second, third, and fourth eurodollar futures contracts, which have 1.5, 2.5, and 3.5 quarters to expiration on average (eurodollar futures have expiration dates that lie about two weeks before

the end of each quarter).²⁷ The first two columns of X are essentially the changes in the current-month and three-month-ahead federal funds futures contracts, but contain a scaling adjustment to account for the timing of FOMC meetings within those months, as follows.

Federal funds futures have a payout that is based on the average effective federal funds rate that prevails over the calendar month specified in the contract. Thus, immediately before an FOMC meeting, at time $t - \Delta t$, the implied rate from the current-month federal funds future contract, $ff1$, is largely a weighted average of the federal funds rate that has prevailed so far in the month, r_0 , and the rate that is expected to prevail for the remainder of the month, r_1 :²⁸

$$ff1_{t-\Delta t} = \frac{d1}{D1}r_0 + \frac{D1-d1}{D1}E_{t-\Delta t}(r_1) + \rho1_{t-\Delta t} \quad (A4)$$

where $d1$ denotes the day of the FOMC meeting, $D1$ is the number of days in the month, and $\rho1$ denotes any term or risk premium that may be present in the contract. Then, by leading this equation to time t (twenty minutes after the policy announcement) and differencing, the surprise component of the change in the federal funds rate target, which we call $mp1$, is given by²⁹

$$mp1_t = (ff1_t - ff1_{t-\Delta t}) \frac{D1}{D1 - d1}. \quad (A5)$$

Note that to interpret (A5) as the surprise change in monetary policy expectations, we need to assume that the change in the risk

²⁷Thus, the second eurodollar futures contract can have as little as one quarter plus one day to expiration and as much as two quarters to expiration, with an average horizon of 1.5 quarters over our sample. On expiration, eurodollar futures settle based on the spot ninety-day eurodollar rate, which is closely tied to expectations for the federal funds rate over the subsequent ninety-day period. Thus, these three eurodollar futures contracts are related to federal funds rate expectations from 1.5–2.5, 2.5–3.5, and 3.5–4.5 quarters ahead, respectively.

²⁸For simplicity, assume that federal funds rate is always equal to the target rate set by the FOMC, so that we do not have to differentiate between the target and actual rates. This has no impact on the surprise measures due to the differencing involved.

²⁹Kuttner (2001) also uses this method. For FOMC meetings that occur very late in the month (i.e., in the last seven days of the month), we use the unscaled change in the *next-month* federal funds futures contract to avoid multiplying by a very large scale factor in (A5), which could unduly magnify changes in bid-ask spreads or other factors, since federal funds futures are only priced to the nearest half basis point.

premium ρ in this narrow window of time is small in comparison to the change in expectations itself. Piazzesi and Swanson (2004) provide some evidence that this assumption is not inconsistent with the data.

We can apply a similar procedure to measure the change in expectations about r_2 , the federal funds rate target that will prevail after the *second* FOMC meeting from now. Let $ff2$ denote the federal funds futures rate for the month containing the second FOMC meeting (typically the three-month-ahead contract). Then

$$ff2_{t-\Delta t} = \frac{d2}{D2} E_{t-\Delta t}(r_1) + \frac{D2-d2}{D2} E_{t-\Delta t}(r_2) + \rho2_{t-\Delta t}, \quad (A6)$$

where $d2$ and $D2$ are the day of that FOMC meeting and the number of days in the month containing that FOMC meeting, respectively, and $\rho2$ denotes any risk premium in the contract. By leading this equation to time t and differencing, the change in expectations for that second FOMC meeting, which we call $mp2$, is given by

$$mp2_t = \left[(ff2_t - ff2_{t-\Delta t}) - \frac{d2}{D2} mp1_t \right] \frac{D2}{D2-d2}. \quad (A7)$$

Additional details can be obtained from various sources.³⁰

As described in section 2, we decompose X into its principal components after normalizing each column to have zero mean and unit variance. We let F_1 and F_2 denote the first two principal components of X , and normalize each of them to have unit variance.

To allow for a more structural interpretation of these unobserved factors, we rotate them so that the first factor corresponds to surprise changes in the current federal funds rate target and the second factor corresponds to moves in interest rate expectations over the coming year that are not driven by changes in the current funds rate. In other words, we define a 138×2 matrix Z by

$$Z = FU, \quad (A8)$$

³⁰Gürkaynak (2005) discusses measuring policy expectations and surprises at horizons farther ahead than the current meeting; Gürkaynak, Sack, and Swanson (2002) show that federal funds futures are the best financial market predictors of the federal funds rate with the smallest average term premium; Piazzesi and Swanson (2004) estimate to what extent risk premia in these markets vary over time; and Kuttner (2001) discusses the construction and some uses of $mp1$.

where

$$U = \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{bmatrix},$$

and where U is identified by four restrictions. First, the columns of U are normalized to have unit length (which normalizes Z_1 and Z_2 to have unit variance). Second, the new factors Z_1 and Z_2 should remain orthogonal to each other:

$$E(Z_1 Z_2) = \alpha_1 \beta_1 + \alpha_2 \beta_2 = 0. \quad (\text{A9})$$

Lastly, we impose the restriction that Z_2 does not influence the current policy surprise, $mp1$, as follows. Let γ_1 and γ_2 denote the (known) loadings of $mp1$ on F_1 and F_2 , respectively. Since

$$F_1 = \frac{1}{\alpha_1 \beta_2 - \alpha_2 \beta_1} [\beta_2 Z_1 - \alpha_2 Z_2], \quad (\text{A10})$$

$$F_2 = \frac{1}{\alpha_1 \beta_2 - \alpha_2 \beta_1} [\alpha_1 Z_2 - \beta_1 Z_1], \quad (\text{A11})$$

it follows that:

$$\gamma_2 \alpha_1 - \gamma_1 \alpha_2 = 0,$$

which is the final restriction. It is then easy to solve for the unique matrix U satisfying these restrictions.

Finally, as noted in section 2, we rescale Z_1 and Z_2 so that Z_1 moves the current policy surprise $mp1$ one-for-one and Z_2 has the same magnitude effect on the year-ahead eurodollar futures rate as Z_1 has on that rate (about 55.1 bp).

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Monetary Policy with Judgment: Forecast Targeting*

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“Forecast targeting,” forward-looking monetary policy that uses central-bank judgment to construct optimal policy projections of the target variables and the instrument rate, may perform substantially better than monetary policy that disregards judgment and follows a given instrument rule. This is demonstrated in a few examples for two empirical models of the U.S. economy, one forward looking and one backward looking. A complicated infinite-horizon central-bank projection model of the economy can be closely approximated by a simple finite system of linear equations, which is easily solved for the optimal policy projections. Optimal policy projections corresponding to the optimal policy under commitment in a timeless perspective can easily be constructed. The whole projection path of the instrument rate is more important than the current instrument setting. The resulting reduced-form reaction function for the current instrument rate is a very complex function of all inputs in the monetary-policy decision process, including the central bank’s judgment. It cannot be summarized as a simple reaction function such as a Taylor rule. Fortunately, it need not be made explicit.

JEL Codes: E42, E52, E58.

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On a general level, this paper is motivated by a desire to provide a better theory of modern monetary policy, both from a descriptive and a normative point of view, than much of the current literature on monetary policy. The current literature to a large extent applies a one-line modeling of monetary policy, such as when the instrument rate is assumed to be a given function of a few variables—for instance, “monetary policy is assumed to follow a Taylor rule.”

I believe that the theory that I develop here is better from a *descriptive* point of view, since it takes into account some crucial aspects of monetary policy decisions, such as the collection, processing, and analysis of large amounts of data, the construction of projections of the target variables, the use of considerable amounts of judgment, and the desire to achieve (mostly) relatively specific objectives. The modern monetary policy process I have in mind can be concisely described as “forecast targeting,” meaning “setting the instrument rate such that the forecasts of the target variables look good,” where “look good” refers to the objectives of monetary policy, such as a given target for inflation and a zero target for the output gap.¹ I believe this view of the monetary policy process is also helpful from a *normative* point of view, for instance, in evaluating the performance of and suggesting improvements to existing monetary policy.²

On a more specific level, this paper is motivated by a desire to demonstrate the crucial and beneficial role of *judgment*—information, knowledge, and views outside the scope of a particular model—in modern monetary policy and, in particular, to demonstrate that the appropriate use of good judgment can dramatically improve monetary policy performance, even when compared to

¹Bernanke (2004) discusses and compares forecast targeting (which he refers to as “forecast-based policies”) and simple instrument rules (which he refers to as “simple feedback policies”). He states that “the Federal Reserve relies primarily on the forecast-based approach for making policy” and cites Greenspan’s (2004) speech entitled “Risk and Uncertainty in Monetary Policy” as evidence. He also notes “that not only have most central banks chosen to rely most heavily on forecast-based policies but also that the results, at least in recent years, have generally been quite good, as most economies have enjoyed low inflation and overall economic stability.”

² See Svensson (2001a) and Svensson et al. (2002) for examples of evaluations of monetary policy in New Zealand and Norway, respectively, with this perspective.

policy that is optimal in all respects except for incorporating judgment.³ As will be explained in detail below, judgment will be represented as the central bank's conditional mean estimate of arbitrary multidimensional stochastic "deviations"—"add factors"—to the model equations.⁴ I also wish to demonstrate the benefits of regarding the whole projection paths of the target variables rather than forecasts at some specific horizon, such as eight quarters, as the relevant objects in the monetary policy decision process. In particular, I believe that it is important to emphasize the whole projection of future instrument rates rather than just the current instrument rate. Furthermore, the modern view of the transmission mechanism of monetary policy emphasizes that monetary policy actions have effects on the economy and the central bank's target variables almost exclusively through the private-sector expectations of the future paths of inflation, output, and interest rates that these actions give rise to; therefore, monetary policy is really the management of private-sector expectations (Woodford 2003a). From this follows that effective implementation of monetary policy requires the effective communication to the private sector of the central bank's preferred projections, including the instrument-rate projection. The most obvious communication of these projections is to explicitly announce and motivate them. Finally, I want to demonstrate the benefits of the approximation of inherently rather complex infinite-horizon central-bank projection models of the economy to much simpler finite-horizon projection models that are much easier to use but still arbitrarily close approximations to the infinite-horizon models.

The decision process of modern monetary policy has several distinct characteristics (see Brash 2000, Sims 2002, and Svensson 2001a):

³ Svensson (2003) also emphasizes the role of judgment in monetary policy but does not provide any direct comparison of the performance of monetary policy with and without judgment.

⁴ Svensson and Tetlow (2005) show how central-bank judgment can be extracted according to the method of optimal policy projections (OPP). This is a method to provide advice on optimal monetary policy while taking policymakers' judgment into account. An early version of the method was developed by Robert Tetlow for a mostly backward-looking variant of the Federal Reserve Board's FRB/US model. The resulting projections have been referred to (somewhat misleadingly) at the Federal Reserve Board as "policymaker perfect-foresight projections." The paper demonstrates the usefulness of OPP with a few example projections for two Greenbook forecasts and the FRB/US model.

1. Large amounts of data about the state of the economy and the rest of the world, including private-sector expectations and plans, are collected, processed, and analyzed before each major decision.
2. Because of lags in the transmission process, monetary policy actions affect the economy with a lag. For this reason alone, good monetary policy must be forward looking, aim to influence the future state of the economy, and therefore rely on forecasts—projections. Central-bank staff and policymakers make projections of the future development of a number of exogenous variables, such as foreign developments, import supply, export demand, fiscal policy, productivity growth, and so forth. They also construct projections of a number of endogenous variables, quantities and prices, under alternative assumptions, including alternative assumptions about the future path of instrument rates. The policymakers are presented with projections of the most important variables, including target variables such as inflation and output, often under alternative assumptions about exogenous variables and, in particular, the instrument rate (such as the instrument rate being constant, following market expectations, following some arbitrary reaction function, or being optimal relative to a specific objective function).
3. Throughout this process, a considerable amount of judgment is applied to assumptions and projections. Projections and monetary policy decisions cannot rely on models and simple observable data alone. All models are drastic simplifications of the economy, and data give a very imperfect view of the state of the economy. Therefore, judgmental adjustments in both the use of models and the interpretation of their results—adjustments due to information, knowledge, and views outside the scope of any particular model—are a necessary and essential component in modern monetary policy.
4. Based on this large amount of information and analysis, the policymakers decide on a current instrument rate, such that the corresponding projections of the target variables look good relative to the central bank's objectives. Since the projections of the target variables depend insignificantly on the current

instrument-rate setting and mainly on the whole path of future instrument rates, the policymakers, explicitly or implicitly, actually choose an instrument-rate projection—an instrument-rate plan—and the current instrument-rate decision can be seen as the first element of that plan.

5. Finally, the current instrument rate is announced and implemented. In many cases, the corresponding projections for inflation and output or the output gap are also announced. In a few cases, an instrument-rate projection is announced as well.⁵

This process makes the current instrument-rate decision a very complex function of the large amounts of data and judgment that have entered into the process. I believe that it is not very helpful to summarize this function as a simple reaction function such as a Taylor rule. Furthermore, the resulting complex reaction function is a *reduced form*, which depends on the central bank's objectives, its view of the transmission mechanism of monetary policy, the data the central bank has collected, and the judgment it has exercised. It is the endogenous complex result of a complex process. In no way is this reaction function *structural*, in the sense of being invariant to the central bank's view of the transmission mechanism and private-sector behavior, or the amount of information and judgmental adjustments. Still, much current literature treats monetary policy as characterized by a given reaction function that is essentially structural and invariant to changes in the model of the economy. Treating the reaction function as a reduced form is a first step in a sensible theory of monetary policy. But, fortunately, this complex reduced-form reaction function need not be made explicit. It is actually not needed in the modern monetary policy process.

However, there is a convenient, more structural representation of monetary policy, namely in the form of a *targeting rule*, as advocated recently in some detail in Svensson and Woodford (2005) and Svensson (2003) and earlier (more generally) in Svensson (1999). An *optimal* targeting rule is a first-order condition for optimal monetary policy. It corresponds to the standard efficiency condition of equality between the marginal rates of substitution and the marginal rates of

⁵The Reserve Bank of New Zealand has published an instrument-rate projection for many years. The Bank of Norway is increasingly providing more information about the future path of the instrument rate.

transformation between the target variables, the former given by the monetary policy loss function, the latter given by the transmission mechanism of monetary policy. An optimal targeting rule is invariant to everything else in the model, including additive judgment and the stochastic properties of additive shocks. Thus, it is a compact, robust, and structural representation of monetary policy, and much more robust than the optimal reaction function. A *simple* targeting rule can potentially be a practical representation of robust monetary policy, a robust monetary policy that performs reasonably well under different circumstances.⁶

Optimal targeting rules remain a practical way of representing optimal monetary policy in the small models usually applied for academic monetary policy analysis. However, for the larger and higher-dimensional operational macromodels used by many central banks in constructing projections, the optimal targeting rule becomes more complex and arguably less practical as a representation of optimal monetary policy. In this paper, it is demonstrated that optimal policy projections, the projections corresponding to optimal policy under commitment in a timeless perspective, can easily be derived directly with simple numerical methods, without reference to any optimal targeting rule.⁷ For practical optimal monetary policy, policymakers actually need not know the optimal targeting rule. They also need not know any reaction function. They only need to ponder the graphs of the projections of the target variables that are generated in the policy process and choose the projections of the target variables and the instrument rate that look best relative to the central bank's objectives.

The paper is organized as follows. Section 1 lays out a reasonably general infinite-horizon model of the transmission mechanism and the central bank's objectives; defines projections, judgment, and optimal policy projections; and specifies how the optimal policy can be implemented and what information the private sector needs from the

⁶ McCallum and Nelson (2004) have recently criticized the advocacy of targeting rules in Svensson (2003). Svensson (2004) rebuts this criticism and gives references to a rapidly growing literature that applies targeting rules to monetary-policy analysis. Walsh (forthcoming) shows a case of equivalence between targeting rules and robust control.

⁷ Nevertheless, a general form of an optimal targeting rule is derived in appendix F (available at www.ijcb.org), for the finite-horizon approximation of the projection model.

central bank. The section also presents a simple model of judgment, when the deviation is a version of a finite-order moving average. Then judgment can be seen as the accumulation of information over time and allows for a recursive but high-dimensional representation of the dynamics of the deviation and judgment. Finally, the section represents the optimal policy projections as the solution to a somewhat complex system of difference equations, while taking judgment into account. It also makes the point that, fortunately, the complex reduced-form reaction function need not be made explicit. Section 2 presents a convenient finite-horizon model for the construction of optimal policy projections, for both forward- and backward-looking models. This finite-horizon model can be written as a simple finite system of linear equations. Nevertheless, it is an exact or arbitrarily close approximation to the infinite-horizon model and is easily solved for the optimal policy projections taking judgment into account. Section 3 discusses and specifies monetary policy that disregards judgment and follows different instrument rules, such as variants of the Taylor rule or more complex instrument rules that are optimal in the absence of judgment. Section 4 gives examples of and compares monetary policy with and without judgment for two different empirical models of the U.S. economy: the backward-looking model of Rudebusch and Svensson (1999) and the forward-looking New Keynesian model of Lindé (2002). In these examples, monetary policy with judgment results in substantially better performance than monetary policy without judgment. This is also the case when monetary policy without judgment is represented as a Taylor rule where the instrument rate responds to forward-looking variables that incorporate private-sector judgment (although, as emphasized below, there are serious principal and practical problems in implementing such an instrument rule). Section 5 presents conclusions.

A separate and extensive appendix contains numerous technical details.⁸ These include a general solution of the policy problem and the related system of difference equations with forward-looking variables when the deviation is an arbitrary stochastic process; a specification of the model when the deviation and judgment are finite-order moving-average processes and the application of the practical method of Marcet and Marimon (1998) to that case; the precise

⁸ The appendix is available at www.ijcb.org.

mathematical structure of the finite-horizon approximation model, including the optimal targeting rule; and details on the empirical backward- and forward-looking models.

1. A Model of the Policy Problem with Judgment

Consider the following linear model of an economy with a private sector and a central bank,⁹ in a form that allows for both predetermined and forward-looking variables as well as judgment:

$$\begin{bmatrix} X_{t+1} \\ Cx_{t+1|t} \end{bmatrix} = A \begin{bmatrix} X_t \\ x_t \end{bmatrix} + Bi_t + \begin{bmatrix} z_{t+1} \\ 0 \end{bmatrix}. \quad (1)$$

Here, X_t is a (column) n_X -vector of *predetermined* variables (one of these may be unity to conveniently incorporate constants in the model) in period t ; x_t is an n_x -vector of *forward-looking* variables; i_t is an n_i -vector of central-bank *instruments* (the forward-looking variables and the instruments are the *nonpredetermined* variables); z_t is an exogenous n_X -vector stochastic process and called the *deviation* in period t ; and A , B , and C are matrices of the appropriate dimension. For any variable q_t , I let $q_{t+\tau|t}$ denote private-sector expectations of the realization in period $t + \tau$ of $q_{t+\tau}$ conditional on private-sector information available in period t . I assume that the private sector has rational expectations, given its information.

For increased generality, the model is formulated in terms of an arbitrary number of instruments, n_i . In most practical applications, monetary policy can be seen as having only one instrument—a short interest rate, the instrument rate—so then $n_i = 1$.

The upper block of (1) provides n_X equations determining the n_X -vector X_{t+1} in period $t + 1$ for given X_t , x_t , i_t , and z_{t+1} ,

$$X_{t+1} = A_{11}X_t + A_{12}x_t + B_1i_t + z_{t+1} \quad (2)$$

where A and B are partitioned conformably with X_t and x_t as

$$A \equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B \equiv \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}. \quad (3)$$

⁹ For simplicity, there is no explicit fiscal authority in the model, but such an authority and its behavior can be included in the model (1).

The realization of the deviation and the predetermined variables in each period occurs and is observed by the private sector and the central bank in the beginning of the period (the realization of z_{t+1} can be inferred from X_{t+1} , X_t , x_t and i_t and [2]).¹⁰

The lower block of (1) provides n_x equations determining the n_x -vector x_t in period t for given $x_{t+1|t}$, X_t , and i_t ,

$$x_t = A_{22}^{-1}(Cx_{t+1|t} - A_{21}X_t - B_2i_t); \quad (4)$$

I hence assume that the $n_x \times n_x$ submatrix A_{22} is invertible. The realization of X_t is observed by the private sector and the central bank in the beginning of period t ; the central bank then sets the instruments, i_t ; after observing the instruments, the private sector forms its expectations, $x_{t+1|t}$; and this finally determines the forward-looking variables x_t .

To assume that the deviation appears only in the upper block of (1) is not restrictive. Suppose that I have a model where the deviation appears in both blocks:

$$\begin{bmatrix} X_{t+1}^o \\ Cx_{t+1|t} \end{bmatrix} = \begin{bmatrix} A_{11}^o & A_{12}^o \\ A_{21}^o & A_{22}^o \end{bmatrix} \begin{bmatrix} X_t^o \\ x_t \end{bmatrix} + \begin{bmatrix} B_1^o \\ B_2^o \end{bmatrix} i_t + \begin{bmatrix} z_{1,t+1}^o \\ z_{2,t}^o \end{bmatrix}.$$

By adding the vector z_t^o to the predetermined variables, I can always form a new model of the form (1), where

$$X_t \equiv \begin{bmatrix} X_t^o \\ z_{2t}^o \end{bmatrix}, \quad A \equiv \begin{bmatrix} A_{11}^o & 0 & A_{12}^o \\ 0 & 0 & 0 \\ A_{21}^o & 0 & A_{22}^o \end{bmatrix}, \quad B \equiv \begin{bmatrix} B_1^o \\ 0 \\ B_2^o \end{bmatrix},$$

$$z_t \equiv \begin{bmatrix} z_{1t}^o \\ z_2^o \end{bmatrix},$$

and there is no deviation in the lower block.

As in Svensson (2003), the deviation represents additional determinants—determinants outside the model—of the variables in

¹⁰See Svensson and Woodford (2003) for an analysis of optimal policy in a model with forward-looking variables when the current state of the economy is imperfectly observed and inferred from observed indicators.

the economy, the difference between the actual value of a variable and the value predicted by the model. It can be interpreted as model perturbations, as in the literature on robust control.¹¹ The central bank's mean estimate of future deviations will be identified with the central bank's *judgment*. It represents the unavoidable judgment always applied in modern monetary policy. Any existing model is always an approximation of the true model of the economy, and monetary policymakers always find it necessary to make some judgmental adjustments to the results of any given model. Such judgmental adjustments could refer to future fiscal policy, productivity, consumption, investment, international trade, foreign-exchange and other risk premia, raw-material prices, private-sector expectations, and so forth. The "add factors" applied to model equations in central-bank projections (Reifschneider, Stockton, and Wilcox 1997) are an example of central-bank judgment. Given this interpretation of judgment and the deviation z_{t+1} , it would be completely misleading to make a simplifying assumption such as the deviation being a simple autoregressive process. In that case, it could just be incorporated among the predetermined variables. Thus, I will refrain from such an assumption and instead leave the dynamic properties of z_{t+1} unspecified, except in a special case when the deviation is a version of a finite-order moving-average process. Generally, the focus will be on the central bank's judgment of the whole sequence of future deviations.

More precisely, let the infinite-dimensional period- t random vector $\zeta^t \equiv (z'_{t+1}, z'_{t+2}, \dots)'$ (where $'$ denotes the transpose) denote the vector of the (in period t) unknown random vectors $z_{t+1}, z_{t+2}, \dots$. Let the central bank's beliefs in period t about the random vector ζ^t be represented by the infinite-dimensional probability distribution Φ_t with distribution function $\Phi_t(\zeta^t)$. The probability distribution Φ_t may itself be time-varying and stochastic. The central bank is assumed to know the matrices A, B, C, D , and W and the discount factor δ (D, W , and δ refer to the central bank's objectives and are defined below). The private sector is assumed to know the matrices A, B , and C , but may or may not know the central bank's objectives

¹¹See, for instance, Hansen and Sargent (1998). However, that literature deals with the more complex case when the model perturbations are endogenous and chosen by nature to correspond to a worst-case scenario.

(that is, D , W , and δ). The private sector may or may not have the same beliefs about the future deviations as the central bank.

Let Y_t be an n_Y -vector of *target* variables. For simplicity, these target variables are measured as the difference from a fixed n_Y -vector Y^* of *target levels*. This vector of target levels is held fixed throughout this paper. In order to examine the consequences of shifting target levels, one only needs to replace Y^t by $Y^t - Y^*$ throughout the paper. Let the target variables be given by

$$Y_t = D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} \quad (5)$$

where D is an $n_Y \times (n_X + n_x + n_i)$ matrix.

Let the central bank's intertemporal loss function in period t be

$$E_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau} \equiv \int \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau} d\Phi_t(\zeta^t) \quad (6)$$

where $0 < \delta \leq 1$ is a discount factor, L_t is the period loss given by

$$L_t = \frac{1}{2} Y_t' W Y_t, \quad (7)$$

and W is a symmetric positive semidefinite $n_Y \times n_Y$ matrix. That is, in period t the central bank wants to minimize the expected discounted sum of current and future losses, where the expectation E_t is with respect to the distribution Φ_t .

Since this is a linear model with a quadratic loss function and the random deviations enter additively, the conditions for certainty equivalence are satisfied. Then, as shown in detail in appendix A, the optimal policy in period t need only consider central-bank mean forecasts—*projections*—of all variables, including the infinite-dimensional mean forecast, z^t , of the random vector ζ^t ,

$$z^t \equiv E_t \zeta^t \equiv \int \zeta^t d\Phi_t(\zeta^t).$$

The central-bank projection in period t of the realization of the deviation in period $t+\tau$, $z_{t+\tau,t}$, is denoted $z^t \equiv (z'_{t+1,t}, z'_{t+2,t}, \dots)'$.

The projection z^t is identified with the central bank's *judgment*. Under the assumed certainty equivalence, the projection z^t is, for optimal policy, a sufficient statistic for the distribution Φ_t . Although there is genuine uncertainty about the future random deviations, ζ^t , the only thing that matters for policy is the mean, the judgment, z^t . The second and higher moments of ζ^t —the variance, skew, kurtosis, and so forth—do not matter for policy.¹² The judgment can itself be seen as an exogenous infinite-dimensional random vector that is realized in the beginning of each period and summarizes the central bank's relevant information in that period about expected future deviations.

Let $q^t \equiv (q'_{t,t}, q'_{t+1,t}, \dots)'$ denote a central-bank projection in period t for any vector of variables $q_{t+\tau}$ ($\tau \geq 0$) (with the exception of ε^t and $\varepsilon_{t+\tau,t}$, to be introduced below), a central-bank mean forecast conditional on central-bank information in period t . (Thus, for variables other than the deviation, the projection also includes the current value, $q_{t,t} = q_t$.) The central bank then constructs various projections of the endogenous variables to be used in its decision process. These projections of endogenous variables may be conditional on various assumptions. In order to keep private-sector expectations and central-bank projections conceptually distinct, I denote the former by $q_{t+\tau|t}$ and the latter by $q_{t+\tau,t}$ for any variable q_t .

For a given judgment, z^t , the projection model of the central bank for the projections (X^t, x^t, i^t, Y^t) in period t —the model the central bank uses in the decision process to consider alternative projections—will be

$$\begin{bmatrix} X_{t+\tau+1,t} \\ Cx_{t+\tau+1,t} \end{bmatrix} = A \begin{bmatrix} X_{t+\tau,t} \\ x_{t+\tau,t} \end{bmatrix} + Bi_{t+\tau,t} + \begin{bmatrix} z_{t+\tau+1,t} \\ 0 \end{bmatrix}, \quad (8)$$

$$Y_{t+\tau,t} = D \begin{bmatrix} X_{t+\tau,t} \\ x_{t+\tau,t} \\ i_{t+\tau,t} \end{bmatrix} \quad (9)$$

for $\tau \geq 0$, where $X_{t,t}$ satisfies

$$X_{t,t} = X_t, \quad (10)$$

¹² The variance of the future deviations will add a term to the intertemporal loss, but that term is independent of policy.

since the realization of the predetermined variables is assumed to be observed in the beginning of period t .

In order to introduce more compact notation, let the $(n_X + n_x + n_i)$ -vector $s_t \equiv (X'_t, x'_t, i'_t)'$ denote the state of the economy in period t , and let the vector $s_{t+\tau,t} \equiv (X'_{t+\tau,t}, x'_{t+\tau,t}, i'_{t+\tau,t})'$ denote the projection in period t of the state of the economy in period $t + \tau$. Finally, let the infinite-dimensional vector $s^t \equiv (s'_{t,t}, s'_{t+1,t}, s'_{t+2,t}, \dots)'$ denote a projection in period t of the (current and future) states of the economy. By (9), I can write the projection of the target variables in a compact way, as a linear function of the projection of the states of the economy, as

$$Y^t = \tilde{D}s^t \quad (11)$$

where $\tilde{D}$ is an infinite-dimensional block-diagonal matrix with the $\tau + 1$ -th diagonal block equal to D for $\tau \geq 0$.

The *set of feasible projections of the states of the economy* in period t , $\mathcal{S}_t$, can now be defined as the set of projections s^t that satisfy (8)–(10) for given X_t and z^t .

The intertemporal loss function (6) with (7) induces an intertemporal loss function for the target-variable projection,¹³

$$\mathcal{L}(Y^t) \equiv \sum_{\tau=0}^{\infty} \delta^\tau Y'_{t+\tau,t} W Y_{t+\tau,t}. \quad (12)$$

The *policy problem* in period t is to find the optimal policy projection $(\hat{s}^t, \hat{Y}^t)$, the projection that minimizes (12) over the set of feasible projections of the states of the economy—that is, subject to (8)–(11) for $\tau \geq 0$ for given X_t and z^t . More compactly,

$$\hat{s}^t = \arg \min_{s^t \in \mathcal{S}_t} \mathcal{L}(\tilde{D}s^t), \quad (13)$$

$\hat{Y}^t \equiv \tilde{D}\hat{s}^t$; and the optimal policy projection $(\hat{X}^t, \hat{x}^t, \hat{i}^t)$ of the predetermined variables, forward-looking variables, and instruments can be extracted from $\hat{s}^t$.

¹³ Note that $\min \mathbb{E}_t \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau} = \min \{ \mathcal{L}(Y^t) + \mathbb{E}_t \sum_{\tau=0}^{\infty} \delta^\tau (Y_{t+\tau} - Y_{t+\tau,t})' W (Y_{t+\tau} - Y_{t+\tau,t}) \} = \min \{ \mathcal{L}(Y^t) + \sum_{\tau=0}^{\infty} \delta^\tau \text{trace}(W \text{Cov}_t Y_{t+\tau}) \}$. By certainty equivalence, $\text{Cov}_t Y_{t+\tau} \equiv \mathbb{E}_t (Y_{t+\tau} - Y_{t+\tau,t})(Y_{t+\tau} - Y_{t+\tau,t})' \equiv \int (Y_{t+\tau} - Y_{t+\tau,t})(Y_{t+\tau} - Y_{t+\tau,t})' d\Phi_t(\zeta^t)$ is independent of policy, so minimizing (12) in period t implies the same policy as minimizing (6) in period t . Furthermore, note that, since $\text{trace}(W \text{Cov}_t Y_{t+\tau})$ will normally be strictly positive, (6) will normally converge only for $\delta < 1$, whereas (12) will normally converge also for $\delta = 1$.

The policy problem will be further specified below to correspond to commitment in a “timeless perspective,” in order to avoid any time-consistency problems (see Woodford 2003b and Svensson and Woodford 2005).

1.1 *Implementation and What Information the Private Sector Needs*

The *implementation* of the optimal policy in period t involves announcing the optimal policy projection and setting the instruments in period t equal to the first element of the instrument projection,

$$i_t = \hat{i}_{t,t}.$$

In period $t + 1$, conditional on new realizations of the predetermined variables, X_{t+1} , and the judgment, z^{t+1} , a new optimal policy projection, $(\hat{X}^{t+1}, \hat{x}^{t+1}, \hat{z}^{t+1}, \hat{Y}^{t+1})$, is found and announced together with a new instrument setting,

$$i_{t+1} = \hat{i}_{t+1,t+1}.$$

In a forward-looking model, the private sector (including policymakers other than the central bank) will need to know at least parts of the aggregate projections $\hat{X}^t$, $\hat{x}^t$, and $\hat{z}^t$, in order to make decisions consistent with these and make the rational-expectations equilibrium in the economy correspond to the central bank’s optimal policy projection. If the private sector knows the matrices A , B , C , D , and W and the discount factor δ and has the same judgment z^t as the central bank, it can in principle compute the optimal policy projection itself—assuming that it has the same computational capacity as the central bank.

However, the private sector actually needs to know less. An assumption maintained throughout this paper is that the private sector knows the model (1), in the sense of knowing the matrices A , B , and C . Furthermore, it observes X_t (determined by X_{t-1} , x_{t-1} , and i_{t-1} in period $t - 1$ and the realization of z_t in the beginning of period t according to [2]) in the beginning period t , then observes $i_t = \hat{i}_{t,t}$ set by the central bank, thereafter forms one-period-ahead expectations $x_{t+1|t}$, and finally determines (and thereby knows) x_t ; after this, period t ends. In order to make decisions in period t consistent with the optimal policy projection—that is, decisions resulting

in $x_t = \hat{x}_{t,t}$ from (4)—the private sector needs to be able to form expectations $x_{t+1|t} = \hat{x}_{t+1,t}$. The most direct way is if the central bank announces $\hat{x}_{t+1,t}$ and the private sector believes the announcement. Formally, $\hat{x}_{t+1,t}$ is the minimum additional information the private sector needs. However, the central bank may have to provide the whole optimal policy projection, and also motivate the underlying judgment, in order to demonstrate that the optimal policy projections are *internally consistent* with the model (1). In particular, the private sector may not believe $\hat{x}_{t+1,t}$ unless it is apparently consistent with the whole projection $\hat{i}^t$. Furthermore, the private sector will need to know the central bank's loss function— D , W , and δ —in order to judge whether the projections announced are really optimal relative to the central bank's loss function and thereby *incentive-compatible*, *credible*. Only then may the central bank be able to *convince* the private sector to form expectations according to the optimal policy projection.¹⁴ Indeed, the private sector completely trusting the central bank's isolated announcement of $\hat{x}_{t+1,t}$ could invite misleading announcements, given the time-consistency problem discussed below.¹⁵

1.2 Judgment as a Finite-Order Moving Average

Consider the special case when the deviation is a version of a moving-average stochastic process with a given finite order $T > 0$ (where T could be relatively large):

$$z_{t+1} = \varepsilon_{t+1} + \sum_{j=1}^T \varepsilon_{t+1,t+1-j} \quad (14)$$

where $\tilde{\varepsilon}_t \equiv (\varepsilon'_t, \varepsilon'^t)' \equiv (\varepsilon'_t, \varepsilon'_{t+1,t}, \dots, \varepsilon'_{t+T,t})'$ is a zero-mean iid random $(T+1)n_X$ -vector realized in the beginning of period t and called the

¹⁴ Being explicit about the loss function and announcing the optimal policy projection also seem to take care of the criticism of real-world inflation targeting expressed by Faust and Henderson (2004).

¹⁵ See Geraats (2002) for such examples.

In a much noted contribution, Morris and Shin (2002) and Amato, Morris, and Shin (2002) have emphasized the possibility that public information may be bad and reduce social welfare by crowding out private information. Svensson (2005) scrutinizes this result and shows that, in the model considered by Morris and Shin, public information actually increases social welfare for reasonable parameters.

innovation in period t .¹⁶ For $T = 0$, z_{t+1} is a simple iid disturbance. For $T > 0$, the deviation is a version of a moving-average process.

It follows that the central-bank judgment $z_{t+\tau,t}$ ($\tau \geq 1$) is also a finite-order moving average and satisfies

$$\begin{aligned} z_{t+\tau,t} &\equiv E_t z_{t+\tau} = \sum_{j=\tau}^T \varepsilon_{t+\tau,t+\tau-j} = \varepsilon_{t+\tau,t} \\ &+ \sum_{j=\tau+1}^T \varepsilon_{t+\tau,t+\tau-j} = \varepsilon_{t+\tau,t} + z_{t+\tau,t-1}. \end{aligned}$$

Hence, $\varepsilon_{t+\tau,t} = z_{t+\tau,t} - z_{t+\tau,t-1}$ can be interpreted as the innovation in period t to the previous judgment $z_{t+\tau,t-1}$, the new information the central bank receives in period t about the realization of $z_{t+\tau}$ in period $t + \tau$. Hence, the judgment $z_{t+\tau,t}$ in period t is the sum of current and previous information about $z_{t+\tau}$. For horizons larger than T , the central-bank judgment is constant and, without loss of generality, equal to zero:

$$z_{t+\tau,t} = 0 \quad (\tau > T). \quad (15)$$

The dynamics of the deviation z_t and the judgment z^{t+1} can then be written as

$$\begin{bmatrix} z_{t+1} \\ z^{t+1} \end{bmatrix} = A_z \begin{bmatrix} z_t \\ z^t \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon^{t+1} \end{bmatrix} \quad (16)$$

where the $(T+1)n_X \times (T+1)n_X$ matrix A_z is defined as

$$A_z \equiv \begin{bmatrix} 0_{n_X \times n_X} & I_{n_X} & 0_{n_X \times (T-1)n_X} \\ 0_{(T-1)n_X \times n_X} & 0_{(T-1)n_X \times n_X} & I_{(T-1)n_X} \\ 0_{n_X \times n_X} & 0_{n_X \times n_X} & 0_{n_X \times (T-1)n_X} \end{bmatrix}$$

where $0_{m \times n}$ and I_m denote an $m \times n$ zero matrix and an $n \times n$ identity matrix, respectively. Thus, the dynamics of the deviation and the judgment have a convenient linear and recursive representation.

¹⁶ Note that $\varepsilon^t \equiv (\varepsilon'_{t+1,t}, \varepsilon'_{t+2,t}, \dots, \varepsilon'_{t+T,t})'$ here denotes a random vector realized in the beginning of period t and not the projection in period t of the random variables $\varepsilon_{t+1}, \varepsilon_{t+2}, \dots, \varepsilon_{t+T}$. That projection is always zero under the above assumption of ε^t being a zero-mean iid random variable.

The modeling of the dynamics of the deviation, z_t , and the additive judgment, z^t , in (16) allows for a relatively flexible accumulation of information about future deviations. Whereas the stochastic process for the deviation is not a simple Markov process in terms of itself, but a finite-order moving-average process, it can be written as a higher-dimensional AR(1) process. The restriction imposed is that the innovation is zero-mean and iid across periods. There is no restriction of the variance and covariance of the elements of $\tilde{\varepsilon}_t$ within the period. It follows that, for instance, $\varepsilon_{t+\tau,t}$ may have a variance that is decreasing in τ , corresponding to a situation where there is less information about the mean projection of deviations further into the future; by assumption, there is no specific information about the deviation for $\tau > T$. For given t , there may be serial correlation of $\varepsilon_{t+\tau,t}$ across τ , corresponding to new information about serially correlated future deviations.

1.3 Representing Optimal Policy Projections

Without the judgment terms (or, alternatively, with the deviation being an iid zero-mean process or an autoregressive process with iid shocks), the above infinite-horizon linear-quadratic problem with forward-looking variables is a well-known problem, examined and solved in Backus and Driffill (1986), Currie and Levine (1993), and Söderlind (1999). The traditional way to find a solution to this problem is to derive the first-order conditions for an optimum and combine the first-order conditions with the model (1) to form a system of difference equations with an infinite horizon. The solution can then also be expressed as a difference equation. Furthermore, Marcet and Marimon (1998) have shown a new practical way of reformulating the problem with forward-looking variables as a recursive saddle-point problem (see appendix D).

A new element here is the solution with the judgment. For the case when the deviation is a finite-order moving average, the dynamics of the deviation and the judgment, (16), can be incorporated with (1), the vector of predetermined variables can be expanded to include z^t , and the standard solution can be applied directly.¹⁷ The details for this case are provided in appendices C and D. When the

¹⁷ Since z_t is incorporated in X_t , one does not need to add z_t as a separate predetermined variable.

judgment is a realization of an infinite-dimensional random vector, the standard solution has to be modified to take that into account. The details of that solution in the form of a difference equation are explained in appendices A and B. Here I shall first report the solution in the form of an infinite-horizon difference equation and later develop a very convenient finite-horizon version of the model.

Under the assumption of optimization under commitment, one way to describe the optimal policy projection is by the following difference equations,

$$\begin{bmatrix} \hat{x}_{t+\tau,t} \\ \hat{l}_{t+\tau,t} \end{bmatrix} = F \begin{bmatrix} \hat{X}_{t+\tau,t} \\ z^{t+\tau,t} \\ \Xi_{t+\tau-1,t} \end{bmatrix}, \quad (17)$$

$$\begin{bmatrix} \hat{X}_{t+\tau+1,t} \\ \Xi_{t+\tau,t} \end{bmatrix} = M \begin{bmatrix} \hat{X}_{t+\tau,t} \\ z^{t+\tau,t} \\ \Xi_{t+\tau-1,t} \end{bmatrix} \quad (18)$$

for $\tau \geq 0$, where $\hat{X}_{t,t} = X_t$. When the deviation is a finite-order moving average and the judgment is finite-dimensional, $z^{t+\tau,t}$ denotes the Tn_X -vector $(z'_{t+\tau+1,t}, z'_{t+\tau+2,t}, \dots, z'_{t+\tau+T,t})'$, where $z_{t+\tau+j,t} = 0$ for $j + \tau > T$. When the judgment is infinite-dimensional, $z^{t+\tau,t}$ denotes the infinite-dimensional vector $(z'_{t+\tau+1,t}, z'_{t+\tau+2,t}, \dots)'$. In the former case, F and M are finite-dimensional matrices. In the latter case, F and M include a linear operator R on $z^{t+\tau,t}$ (an infinite-dimensional matrix) of the form $\sum_{j=0}^{\infty} R_j z_{t+1+\tau+j,t}$, where $\{R_j\}_{j=0}^{\infty}$ is a sequence of matrices. The matrices F , M , and $\{R_j\}_{j=0}^{\infty}$ depend on A , B , C , D , W , and δ , but they are independent of the second and higher moments of the deviation. The n_X -vector $\Xi_{t+\tau,t}$ consists of the Lagrange multipliers of the lower block of (8), the block determining the projection of the forward-looking variables.

As discussed in appendix A, the value of the initial Lagrange multiplier, $\Xi_{t-1,t}$, is zero, if there is commitment from scratch in period t —that is, disregarding any previous commitments. This reflects a time-consistency problem when there is reoptimization and recommitment in later periods, as is inherently the case in practical monetary policy. Instead, I assume that the optimization is under

commitment in a timeless perspective. Then, if the optimization, and reoptimization, under commitment in a timeless perspective started in an earlier period and has occurred since then, the initial value of the Lagrange multiplier satisfies

$$\Xi_{t-1,t} = \Xi_{t-1,t-1} \quad (19)$$

where $\Xi_{t-1,t-1}$ denotes the Lagrange multiplier of the lower block of (8) for the determination of $x_{t-1,t-1}$ in the decision problem in period $t-1$. The dependence of the optimal policy projection in period t on this Lagrange multiplier from the decision problem in the previous period makes the optimal policy projection depend on previous projections and illustrates the history dependence of optimal policy under commitment in a forward-looking model shown in Backus and Driffill (1986) and Currie and Levine (1993) and especially examined and emphasized in Woodford (2003b).

It follows from (17)–(19) and (11) that the optimal policy projection of the states of the economy, the target variables, and the instruments will be linear functions of X_t , z^t , and $\Xi_{t-1,t-1}$, which can be written in a compact way as

$$\begin{aligned} \hat{s}^t &= H \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix}, & \hat{Y}^t &= \tilde{D}H \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix}, \\ \hat{i}^t &= H_i \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix} \end{aligned} \quad (20)$$

where H is an appropriately formed infinite-dimensional matrix and H_i is an infinite-dimensional submatrix of H consisting of the rows corresponding to the instruments. In particular, the instrument setting in period t will be given by

$$i_t = \hat{i}_{t,t} = h \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix} \quad (21)$$

where the finite- or infinite-dimensional matrix h consists of the n_i first rows of the matrix H_i .

As explained in Svensson and Woodford (2005), a simple way of imposing the timeless perspective is to add a term to the intertemporal loss function (12),

$$\mathcal{L}(Y^t) + \Xi_{t-1,t-1} \frac{1}{\delta} C x_{t,t}. \quad (22)$$

In the policy problem in period $t - 1$, $\Xi_{t-1,t-1} C$ can be interpreted as the marginal loss in period $t - 1$ of a change in the one-period-ahead projection of the forward-looking variables, $x_{t,t-1}$. The time-consistency problem arises from disregarding that marginal loss in the policy problem in period t . Adding the corresponding term to the loss function in period t as in (22) handles the time-consistency problem, and the optimal policy under commitment in the timeless perspective will result from minimizing (22) subject to (8)–(10) for given X_t , z^t , and $\Xi_{t-1,t-1}$.¹⁸ Since $x_{t,t}$ is an element of the projection s^t , the optimal policy projection $\hat{s}^t$ is then defined as

$$\hat{s}^t = \arg \min_{s^t \in \mathcal{S}_t} \left\{ \mathcal{L}(\tilde{D}s^t) + \Xi_{t-1,t-1} \frac{1}{\delta} C x_{t,t} \right\} \quad (23)$$

for given X_t , z^t , and $\Xi_{t-1,t-1}$.

From (18) it follows that the Lagrange multiplier $\Xi_{t,t}$, to be used in the decision problem in period $t + 1$, will be given by

$$\Xi_{t,t} = H_{\Xi} \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix} \quad (24)$$

where H_{Ξ} is a finite- or infinite-dimensional matrix.

¹⁸ Alternatively, as discussed in Giannoni and Woodford (2002) and Svensson and Woodford (2005), one can impose the constraint

$$x_{t,t} = F_x \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix}$$

where F in (17) is suitably partitioned. In the present context, it is more practical to add the term to the intertemporal loss function as in (22).

Let the *set of feasible target-variable projections* in period t , $\mathcal{Y}_t$, be defined as the set of target-variable projections satisfying (11) for projections s^t in the set S_t for given X_t and z^t . In the special case where the forward-looking variables, x_t , happen to be target variables and elements in Y_t , so $x_{t,t}$ is an element of Y^t , the optimal target-variable projection, $\hat{Y}^t$, can be defined as the target-variable projection Y^t that minimizes (22) on the set $\mathcal{Y}_t$, for given X_t , z^t , and $\Xi_{t-1,t-1}$,

$$\hat{Y}^t = \arg \min_{Y^t \in \mathcal{Y}_t} \left\{ \mathcal{L}(Y^t) + \Xi_{t-1,t-1} \frac{1}{\delta} C x_{t,t} \right\}.$$

However, in the more general case when some or all forward-looking variables are not target variables, $x_{t,t}$ is not an element of Y^t , and the optimal policy projection has to be found by optimization over the set $\mathcal{S}_t$, as in (23).

1.4 Backward-Looking Model

In a backward-looking model, there are no forward-looking variables: $n_x = 0$. There is no lower block in (1) and (8), and there are no forward-looking variables in (5) and (9). There are no projections of forward-looking variables and Lagrange multipliers in (17), (18), (20), and (21). There is no time-consistency problem and no need to consider commitment in a timeless perspective.

Hence, for a backward-looking model, the optimal target-variable projection can always be found by minimizing (12) over the set of feasible target-variable projections,

$$\hat{Y}^t = \arg \min_{Y^t \in \mathcal{Y}_t} \mathcal{L}(Y^t),$$

for given X_t and z^t .

1.5 The Complex Reduced-Form Reaction Function Need Not Be Made Explicit

The compact notation for the determination of the period- t instrument i_t in (21) and the Lagrange multiplier $\Xi_{t,t}$ in (24) may have given the impression that optimal monetary policy is just a matter

of calculating the finite- or infinite-dimensional matrices h and H_{Ξ} once and for all; then, in each period, first observe X_t , form z^t , and recall $\Xi_{t-1,t-1}$ from last period's decision; then simply compute, announce, and implement i_t from (21); and finally compute $\Xi_{t,t}$ to be used in next period's decision.

This is a misleading impression, though. First, h and H_{Ξ} are indeed high- or infinite-dimensional and therefore difficult to grasp and interpret. Second, z^t is also high- or infinite-dimensional. It is difficult to conceive of policymakers or even staff pondering pages and pages, or computer screens and computer screens, of huge arrays of numbers in small print, arguing and debating about adjustments of the numbers of z^t , such as the numbers in rows 220–250 and 335–385. Third, no central bank (certainly no central bank that I have any more thorough information about) behaves in that way, and is ever likely to behave that way. Instead, the practical presentation of information and options to policymakers is always in the form of multiple graphs, modest-size tables, and modest amounts of text.

Fourth, the intertemporal loss function $\mathcal{L}(Y^t)$ has the projections of the target variables as its argument. What matters for the construction of the target variables is the whole projection path of the instruments, not the current instrument setting. The obvious conclusion is that the relevant objects of importance in the decision process are the whole projection paths of the target variables and the instruments, not the current instrument setting or projections of the target variables at some fixed horizon. These projection paths are most conveniently illustrated as graphs. Indeed, graphs of projections are prominent in the existing monetary policy reports where projections are reported. The analytical techniques discussed in this paper should predominantly be seen as techniques for computer-generated graphs of whole projection paths. Pondering such graphs is an essential part of the monetary policy decision process. Importantly, policymakers need not know the underlying detailed high- or infinite-dimensional matrices behind the construction of those graphs. Therefore, the complex reduced-form reaction functions embedded in these matrices need not be made explicit.

Fifth, in the discussion in section 1.1, there was no reference to the reaction function, only to the optimal policy projection. Given X_t and i_t , the private sector needs to be able to form the expectations $x_{t+1|t}$ in order to make decisions in period t . The minimum for this

is the central bank's announcement of $\hat{x}_{t+1,t}$. In order to make that announcement credible, the central bank may have to announce the complete optimal policy projection and motivate its judgment. But it does not need to announce any reaction function. In principle, given the reaction function, the private sector could combine the reaction function with the model and solve for the optimal policy projection, but that is an overwhelmingly complicated and roundabout way.

2. A Finite-Horizon Projection Model

Regardless of whether the judgment is finite- or infinite-dimensional (that is, whether [15] holds or not), the problem of minimizing the intertemporal loss function is an infinite-horizon problem. From a practical and computational point of view, it is convenient to transform the infinite-horizon policy problem above to a finite-horizon one. When the judgment satisfies (15), this can be done in a simple and approximate, but arbitrarily close to exact, way for the forward-looking model, and in a simple and exact way for the backward-looking model. The finite-horizon model also makes it very easy to incorporate any arbitrary constraints on the projections—for instance, a particular form of the instrument projection, such as a constant instrument for some periods. Then, all the relevant projection paths are computed in one simple step.

2.1 Forward-Looking Model

Suppose that the estimate of the deviation is constant beyond a fixed horizon T . Without loss of generality, assume that the constant is zero.¹⁹ That is, I assume (15).

Start by writing the projection model (8) and (10) for $\tau = 0, \dots, T-1$ as

$$X_{t,t} = X_t, \quad (25)$$

¹⁹ If the estimate of the deviation from horizon T on is constant but nonzero, it can be incorporated among other constants in the model. If the estimate of the deviation from horizon T on is not constant but follows an autoregressive process (for instance, if it is assumed to gradually approach a constant), it can be incorporated among the predetermined variables.

$$-\tilde{A}s_{t+\tau,t} + \begin{bmatrix} X_{t+\tau+1,t} \\ Cx_{t+\tau+1,t} \end{bmatrix} = \begin{bmatrix} z_{t+\tau+1,t} \\ 0 \end{bmatrix} \quad (\tau = 0, \dots, T-1) \quad (26)$$

where $\tilde{A}$ is the $(n_X + n_x) \times (n_X + n_x + n_i)$ matrix defined by $\tilde{A} \equiv [A \ B]$. The first n_X equations of the last block of $n_X + n_x$ equations in (26) determine $X_{t+T,t}$ for given $X_{t+T-1,t}$, $x_{t+T-1,t}$, $i_{t+T-1,t}$, and $z_{t+T,t}$. The last n_x equations of that block are

$$-A_{21}X_{t+T-1,t} - A_{22}x_{t+T-1,t} - B_2i_{t+T-1,t} + Cx_{t+T,t} = 0.$$

They determine $x_{t+T-1,t}$ for given $X_{t+T-1,t}$ and $i_{t+T-1,t}$, and, importantly, for given $x_{t+T,t}$. A problem is that n_x equations determining $x_{t+T,t}$ are lacking. I will assume that $x_{t+T,t}$ is determined by the assumption that $x_{t+T+1,t}$ is equal to its steady-state level. That is, I assume that the optimal policy projection has the property that, for (15), it approaches a steady state when $T \rightarrow \infty$. This is true for the models and loss functions considered here. Without loss of generality, I assume that the steady-state values for the forward-looking variables are zero,

$$x_{t+T+1,t} = 0. \quad (27)$$

From this it follows that $X_{t+T,t}$, $x_{t+T,t}$, and $i_{t+T,t}$ must satisfy

$$-A_{21}X_{t+T,t} - A_{22}x_{t+T,t} - B_2i_{t+T,t} = 0, \quad (28)$$

which gives me the desired n_x equations for $x_{t+T,t}$.

Let s^t , the projection of the states of the economy, now denote the *finite-dimensional* $(T+1) \times (n_X + n_x + n_i)$ -vector $s^t \equiv (s'_{t,t}, s'_{t+1,t}, \dots, s'_{t+T,t})'$. Similarly, let all projections q^t for $q = X, x, i$ and Y now denote the finite-dimensional vector $q^t \equiv (q'_{t,t}, q'_{t+1,t}, \dots, q'_{t+T,t})'$. Finally, let z^t be the Tn_X -vector $z^t \equiv (z'_{t+1,t}, z'_{t+2,t}, \dots, z'_{t+T,t})'$ (recall that z^t does not include the component z_t).

The finite-horizon projection model for the projection of the states of the economy, s^t , then consists of (25), (26), and (28). It can be written compactly as

$$Gs^t = g^t \quad (29)$$

where G is the $(T+1)(n_X + n_x) \times (T+1)(n_X + n_x + n_i)$ matrix formed from the matrices on the left side of (25), (26), and (28), and

g^t is a $(T+1)(n_X + n_x)$ -vector formed from the right side of (25), (26), and (28) as $g^t \equiv (X'_t, z'_{t+1,t}, 0', z'_{t+2,t}, 0', \dots, z'_{t+T,t}, 0', 0')'$ (where zeros denote zero vectors of appropriate dimension).

Since Y^t now denotes the finite-dimensional $(T+1)n_Y$ -vector $Y^t \equiv (Y'_{t,t}, Y'_{t+1,t}, \dots, Y'_{t+T,t})'$, I can write

$$Y^t = \tilde{D}s^t \quad (30)$$

where $\tilde{D}$ now denotes a finite-dimensional $(T+1)n_Y \times (T+1)(n_X + n_x + n_i)$ block-diagonal matrix with the matrix D in each diagonal block.

The set of feasible projections, $\mathcal{S}_t$, is then defined as the finite-dimensional set of s^t that satisfy (29) and (30) for a given g^t —that is, for a given X_t and z^t .

It remains to specify the intertemporal loss function in the forward-looking model in the finite-horizon case. In the forward-looking model, under assumption (15), the minimum loss from the horizon $T+1$ on depends on the projection of the predetermined variables for period $t+T+1$, $X_{t+T+1,t}$, and the Lagrange multipliers $\Xi_{t+T,t}$ according to the quadratic form

$$\frac{1}{2}\delta^{T+1}[X'_{t+T+1,t} \quad \Xi'_{t+T,t}]V \begin{bmatrix} X_{t+T+1,t} \\ \Xi_{t+T,t} \end{bmatrix}$$

where V is a symmetric positive semidefinite matrix that depends on the matrices A , B , C , D , and W and the discount factor δ (see appendix A). It follows from (18) and (15) that this quadratic form can be written as a function $X_{t+T,t}$ and $\Xi_{t+T-1,t}$ as²⁰

$$\frac{1}{2}\delta^{T+1}[X'_{t+T,t} \quad \Xi'_{t+T-1,t}]M'VM \begin{bmatrix} X_{t+T,t} \\ \Xi_{t+T-1,t} \end{bmatrix}. \quad (31)$$

In principle, I could use (18) to keep track of $\Xi_{t+T-1,t}$. However, a simpler way is to extend the horizon T so far that $X_{t+T,t}$ and $\Xi_{t+T-1,t}$ are arbitrarily close to their steady-state levels. Without loss of generality, I assume that the steady-state levels are zero, in

²⁰ The matrix M appearing in (31) is the matrix M in (18) with the columns corresponding to z^t deleted.

which case the above quadratic form is zero, and the loss from horizon T can be disregarded. Checking that $X_{t+T,t}$ is close to zero is straightforward; I will show a practical way to check that $\Xi_{t+T-1,t}$ is also close to zero.²¹

Under this assumption, it follows from (9), (12), and (30) that the intertemporal loss function can be written as a function of s^t as the finite-dimensional quadratic form

$$\frac{1}{2}s^{t'}\Omega s^t \quad (32)$$

where Ω is a symmetric positive semidefinite block-diagonal $(T+1)(n_X + n_x + n_i)$ matrix with its $(\tau+1)$ -th diagonal block being $\delta^\tau D'WD$ for $0 \leq \tau \leq T$. However, in order to impose the timeless perspective, as explained in section 1.3, I need to add the term

$$\Xi_{t-1,t-1} \frac{1}{\delta} C x_{t,t}$$

to the loss function, where $\Xi_{t-1,t-1}$ is the relevant Lagrange multiplier from the policy problem in period $t-1$. This term can be written $\omega'_{t-1}s^t$, with the appropriate definition of the $(T+1)(n_X + n_x + n_i)$ -vector ω_{t-1} as $\omega_{t-1} \equiv (0, {}^t0', (\Xi_{t-1,t-1} \frac{1}{\delta} C) ', {}^t0', \dots, {}^t0')'$ (where the zeros denote zero vectors of appropriate dimension). Thus, the intertemporal loss function with the added term is

$$\frac{1}{2}s^{t'}\Omega s^t + \omega'_{t-1}s_t. \quad (33)$$

Then, the policy problem is to find the optimal policy projection $\hat{s}^t$ that minimizes (33) subject to (29). The Lagrangian for this problem is

$$\frac{1}{2}s^{t'}\Omega s^t + \omega'_{t-1}s_t + \Lambda^{t'}(Gs^t - g^t) \quad (34)$$

where Λ^t is the $(T+1)(n_X + n_x)$ -vector of Lagrange multipliers of (29). The first-order condition is

$$s^{t'}\Omega + \omega'_{t-1} + \Lambda^{t'}G = 0.$$

²¹ Appendix E presents an iterative numerical procedure that will provide a projection arbitrarily close to the optimal policy projection without requiring such a long horizon that $X_{t+T,t}$ and $\Xi_{t+T-1,t}$ are close to their steady-state levels.

Combining this with (29) gives the linear equation system

$$\begin{bmatrix} G & 0 \\ \Omega & G' \end{bmatrix} \begin{bmatrix} s^t \\ \Lambda^t \end{bmatrix} = \begin{bmatrix} g^t \\ -\omega_{t-1} \end{bmatrix}. \quad (35)$$

The optimal policy projection $\hat{s}^t$ and Lagrange multiplier Λ^t are then given by the simple matrix inversion²²

$$\begin{bmatrix} \hat{s}^t \\ \Lambda^t \end{bmatrix} = \begin{bmatrix} G & 0 \\ \Omega & G' \end{bmatrix}^{-1} \begin{bmatrix} g^t \\ -\omega_{t-1} \end{bmatrix}. \quad (36)$$

The optimal target-variable projection then follows from (30). The optimal policy projection is a linear function of X_t , z^t , and $\Xi_{t-1,t-1}$, and it can be written compactly as in section 1.3:

$$\hat{s}^t = H \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix}, \quad \hat{Y}^t = \tilde{D}H \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix},$$

$$\hat{i}^t = H_i \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix},$$

except that the matrices H and H_i and the vector z^t now are finite-dimensional. The matrices can be directly extracted from (36). The period- t instrument setting can be written

$$i_t = \hat{i}_{t,t} = h \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix} \quad (37)$$

where the finite-dimensional matrix h consists of the first n_i rows of the matrix H_i . Under assumption (15) and a sufficiently long

²² Numerically, it is faster to solve the system of linear equations (35) by other methods than first inverting the left-side matrix (see Judd 1998).

horizon T , the finite-horizon projections here are arbitrary close to the optimal infinite-horizon policy projections for $\tau = 0, \dots, T$ in section 1.3.

The Lagrange multiplier Λ^t can be written $\Lambda^t \equiv (\frac{1}{\delta}\xi'_{t,t}, \xi'_{t+1,t}, \Xi'_{t,t}, \delta\xi'_{t+2,t}, \delta\Xi'_{t+1,t}, \dots, \delta^T\xi'_{t+T,t}, \delta^T\Xi'_{t+T-1,t})'$, where $\xi_{t+\tau,t}$ is the vector of Lagrange multipliers for the block of equations in (25), (26), and (28) determining $X_{t+\tau,t}$ and $\Xi_{t+\tau-1,t}$ is the vector of Lagrange multipliers for the block of equations determining $x_{t+\tau-1,t}$. Hence, extraction of $\Xi_{t+T-1,t}$ from Λ^t allows me to check that the assumption made above of $\Xi_{t+T-1,t}$ being close to zero is satisfied. If the assumption is not satisfied, the horizon T can be extended until the assumption is satisfied.²³ Furthermore, $\Xi_{t,t}$ can be extracted from Λ^t in order to form the vector ω_t to be used in the loss function for the policy problem in period $t+1$, to ensure the timeless perspective. The Lagrange multiplier needed in the loss function in period $t+1$, $\Xi_{t,t}$, can be written

$$\Xi_{t,t} = H_{\Xi} \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix} \quad (38)$$

where the finite-dimensional matrix H_{Ξ} can be extracted from (36).

Again, as noted above in section 1.3, in spite of the compact notation for the instrument i_t and Lagrange multiplier $\Xi_{t,t}$ in (37) and (38), these analytical techniques should predominantly be seen as techniques for computer-generated graphs to be pondered by the policymakers, and the matrices never need be made explicit to the policymakers. Although the matrices are now formally finite-dimensional, they are still high-dimensional and somewhat difficult to interpret.

2.2 Backward-Looking Model

Make the same assumption (15) as for the forward-looking model. The projection in period t of the state of the economy in period $t+\tau$, $s_{t+\tau,t}$, is now defined as the $(n_X + n_i)$ -vector $s_{t+\tau,t} \equiv (X'_{t+\tau,t}, i'_{t+\tau,t})'$

²³ In practice, the horizon T is extended until the optimal projection $\hat{s}^t$ is insensitive to variations in T .

for $\tau \geq 0$, in which case I can write, for the backward-looking model,

$$X_{t+\tau+1,t} = \tilde{A}s_{t+\tau,t} \quad (\tau \geq T). \quad (39)$$

The projection model with horizon T can now be written

$$X_{t,t} = X_t, \quad (40)$$

$$-\tilde{A}s_{t+\tau,t} + X_{t+\tau+1,t} = z_{t+\tau+1,t} \quad (0 \leq \tau \leq T-1) \quad (41)$$

where X_t and z^t are given. The projection of the state of the economy, s^t , is now a $(T+1)(n_X + n_i)$ -vector. Then the projection model can be written as (29), where G is a $(T+1)n_X \times (T+1)(n_X + n_i)$ matrix formed from the left side of (40) and (41), and g^t is a $(T+1)n_X$ -vector formed from the right side of (40) and (41) as $g^t \equiv (X'_t, z'^t)'$.

It is a standard result for a linear-quadratic backward-looking model that the minimum loss from the horizon $T+1$ on depends on the projection of the predetermined variables for period $t+T+1$, $X_{t+T+1,t}$, according to the quadratic form

$$\frac{1}{2}\delta^{T+1}X'_{t+T+1,t}VX_{t+T+1,t} \quad (42)$$

where V is a symmetric positive semidefinite matrix that depends on the matrices A , B , D , and W and the discount factor δ (see appendix A). I could now (as for the forward-looking model) assume that the predetermined variables approach a steady-state level for large T , without loss of generality assume that the steady-state level is zero, and extend the horizon T so far that the predetermined variables are arbitrarily close to zero and the loss from period T on is arbitrarily close to zero. I could then form the finite-horizon loss function as for the forward-looking model, and this together with (40) and (41) would form the finite-horizon model, which would be an arbitrarily close approximation to the infinite-horizon model for sufficiently large T .

However, the absence of the time-consistency problem and the need to keep track of the Lagrange multiplier $\Xi_{t+T-1,t}$ allows a simple approach, which is exact also for small T , as long as assumption (15) holds for that T . I follow this approach here.

From (39) it follows that the quadratic form (42) can be written as a function of $s_{t+T,t}$ as

$$\frac{1}{2}\delta^{T+1}s'_{t+T,t}\tilde{A}'V\tilde{A}s_{t+T,t}.$$

The finite-horizon intertemporal loss function can then be written

$$\frac{1}{2} \sum_{\tau=0}^T \delta^\tau s'_{t+\tau,t} D' W D s_{t+\tau,t} + \frac{1}{2} \delta^{T+1} s'_{t+T,t} \tilde{A}' V \tilde{A} s_{t+T,t}.$$

The intertemporal loss function can now be written more compactly as the quadratic form (32), where Ω now is a symmetric positive-semidefinite block-diagonal $(T+1)(n_X + n_i)$ -matrix, whose $(\tau+1)$ -th diagonal block is $\delta^\tau D' W D$ for $0 \leq \tau \leq T-1$ and whose $(T+1)$ -th diagonal block now is $\delta^T (D' W D + \delta \tilde{A}' V \tilde{A})$. Thus, it differs from the matrix Ω for the forward-looking model by the addition of that last term, $\frac{1}{2} \delta \tilde{A}' V \tilde{A}$.

The finite-horizon policy problem is now to find the optimal policy projection $\hat{s}^t$ that minimizes (32) subject to (29), for given g^t , that is, for given X_t and z^t . The corresponding optimal target-variable projection $\hat{Y}^t$ then follows from (30).

The Lagrangian for this problem is

$$\frac{1}{2} s^{t'} \Omega s^t + \Lambda^{t'} (G s^t - g^t)$$

where Λ^t is a vector of Lagrange multipliers for (29). The first-order condition is

$$s^{t'} \Omega + \Lambda^{t'} G = 0.$$

Combining this with (29) gives the linear equation system

$$\begin{bmatrix} G & 0 \\ \Omega & G' \end{bmatrix} \begin{bmatrix} s^t \\ \Lambda^t \end{bmatrix} = \begin{bmatrix} g^t \\ 0 \end{bmatrix}.$$

The optimal policy projection $\hat{s}^t$ is then given by the simple matrix inversion,

$$\begin{bmatrix} \hat{s}^t \\ \Lambda^t \end{bmatrix} = \begin{bmatrix} G & 0 \\ \Omega & G' \end{bmatrix}^{-1} \begin{bmatrix} g^t \\ 0 \end{bmatrix}. \quad (43)$$

The optimal target-variable projection then follows from (30).

The optimal policy projection is obviously a linear function of X_t and z^t , and I can write

$$\hat{s}^t = H \begin{bmatrix} X_t \\ z^t \end{bmatrix}, \quad \hat{Y}^t = \tilde{D} H \begin{bmatrix} X_t \\ z^t \end{bmatrix}, \quad \hat{i}^t = H_i \begin{bmatrix} X_t \\ z^t \end{bmatrix}$$

where the finite-dimensional matrices H and H_i can be extracted from (43). The instrument setting for period t can be written

$$i_t = \hat{i}_{t,t} = h \begin{bmatrix} X_t \\ z^t \end{bmatrix} \quad (44)$$

where the finite-dimensional matrix h consists of the first n_i rows of the matrix H_i .

2.3 Other Considerations

A finite-horizon projection model has several advantages. One is that it is very easy to incorporate any restrictions on the projections. Any equality restriction on X^t , x^t , i^t , or Y^t can be written

$$\bar{R}s^t = \bar{s}^t \quad (45)$$

where the number of rows of the matrix $\bar{R}$ and the dimension of the given vector $\bar{s}^t$ equal the number of restrictions. This makes it easy to incorporate any restriction on the instrument projection—for instance, a restriction that it shall be constant or of a particular shape for some periods.

Transforming the model into a finite system of equations may be particularly practical from a computational point of view for a nonlinear model. It may then also be easy to handle commitment in a timeless perspective for a nonlinear model.

3. Monetary Policy Without Judgment

Modern monetary policy inherently relies, to a large extent, on judgment. Previous sections of this paper have attempted to model this dependence on judgment in a simple but specific way. This section attempts to specify monetary policy without judgment, in order to compare monetary policy with and without judgment.

There are several alternatives in modeling monetary policy without judgment. Above, monetary policy with judgment has been modeled as forecast targeting, finding an instrument projection such that the corresponding projection of the target variables minimizes a loss function. This procedure uses all information available to the central

bank, including central-bank judgment. This results in a complex reduced-form reaction function, which fortunately never needs to be made explicit. When modeling monetary policy without judgment, however, the most obvious route is to consider monetary policy as a more mechanical process—a commitment to a particular reaction function, a *commitment to a particular instrument rule*.²⁴

Instrument rules can be divided into two categories: explicit instrument rules and implicit instrument rules (Svensson and Woodford 2005; Svensson 2003). An *explicit instrument rule* is a reaction function where the instrument responds to predetermined variables only. Its implementation then consists of the central bank observing the predetermined variables in the beginning of the period, and then calculating, announcing, and setting the instrument according to this instrument rule. The implementation obviously requires that the relevant predetermined variables must be observed by the central bank, but since the predetermined variables in a particular period are independent of the instrument setting in that period, no further complications arise. An *implicit* instrument rule is a relation between the current instrument and some of the current forward-looking variables. Then, since the forward-looking variables depend on the instrument setting, the instrument and the forward-looking variables are simultaneously determined. Thus, an implicit instrument rule is actually an *equilibrium condition*, a relation that holds in equilibrium. The implementation of an implicit instrument rule (that is, how to get to the desired equilibrium) is not trivial but a complex issue. This fact has largely been overlooked in the literature, except, for instance, in Svensson and Woodford (2005) and Svensson (2003).

Here, I shall discuss how monetary policy without judgment can be modeled as a commitment to an instrument rule—either an explicit instrument rule or an implicit instrument rule—with some discussion of the implementation of the latter. I will start from the reduced-form reaction function for the instrument setting that follows from the forecast targeting modeled above. For simplicity, I now consider the realistic case when there is only one instrument, the instrument rate, so $n_i = 1$.

²⁴ This is an approach that a large part of the literature has taken—for instance, most papers in the conference volume Taylor (1999). The approach is criticized in Svensson (2003) and (2004).

3.1 Explicit Instrument Rules

The construction of the optimal instrument projection, $\hat{i}^t$, in the forward-looking model results in an optimal reduced-form reaction function for the current instrument setting, (37) (or [21]), which is repeated here as

$$i_t = h_X X_t + h_z z^t + h_{\Xi} \Xi_{t-1,t-1} \quad (46)$$

where the row vector h is partitioned conformably with X_t , z^t , and $\Xi_{t-1,t-1}$ as $h \equiv [h_X \ h_z \ h_{\Xi}]$ (h is now a row vector, since $n_i = 1$). The optimal reaction function implies a particular instrument response to the predetermined variables, X_t ; the judgment, z^t ; and the Lagrange multiplier for the equations for $x_{t-1,t-1}$ in the policy problem in period $t - 1$, $\Xi_{t-1,t-1}$. The discussion here refers to the forward-looking model; for the backward-looking model, I can simply delete the term $h_{\Xi} \Xi_{t-1,t-1}$.

I want to model the central bank setting its instrument in a mechanical way, via a commitment to a particular instrument rule, while disregarding judgment. Considering (46) as a potential instrument rule, it is natural that disregarding judgment means that the central bank behaves as if it believes that $z^t = 0$ and hence disregards the judgment term in (46), $h_z z^t$. Thus, *monetary policy without judgment is modeled as the central bank erroneously believing that the expected future deviations equal zero*—for instance, the central bank believing that z_t is a zero-mean iid process. A first possibility is then that the central bank also disregards the term with the Lagrange multiplier from the previous policy problem. For one thing, if the central bank did set the instrument mechanically in period $t - 1$ rather than by explicit optimization, it may not be aware of the Lagrange multiplier and its value. This leaves responding to the current predetermined variables only,

$$i_t = h_X X_t. \quad (47)$$

Such a policy would be inefficient for two reasons, even if the response coefficients to X_t are those of the optimal reaction function (46). First, it disregards judgment, the term $h_z z^t$. Second, it also disregards the response to lagged predetermined variables implied by the response to the Lagrange multiplier, $\Xi_{t-1,t-1}$, as indicated in (19). Indeed, optimization under discretion would result in a

reduced-form reaction function where the instrument responds only to the current predetermined variables.²⁵

The response to the Lagrange multipliers in (46) implies a response to lagged predetermined variables. Disregarding judgment, the Lagrange multiplier in (38) (or [24]) follows

$$\begin{aligned}\Xi_{t-1,t-1} &= H_{\Xi X} X_{t-1} + H_{\Xi\Xi} \Xi_{t-2,t-2} \\ &= \sum_{j=0}^{\infty} H_{\Xi\Xi}^j H_{\Xi X} X_{t-1-j}\end{aligned}$$

where H_{Ξ} is partitioned conformably with X_t , z^t , and $\Xi_{t-1,t-1}$ as $H_{\Xi} \equiv [H_{\Xi X} \ H_{\Xi z} \ H_{\Xi\Xi}]$. A second possibility for policy disregarding judgment is then that the policy responds with the optimal coefficients h_{Ξ} to these lagged predetermined variables, resulting in the reaction function

$$i_t = h_X X_t + h_{\Xi} \sum_{j=0}^{\infty} H_{\Xi\Xi}^j H_{\Xi X} X_{t-1-j}. \quad (48)$$

This would seem to be an instrument rule corresponding to a rather sophisticated policy, commitment to the reaction function resulting from optimal policy under commitment in a timeless perspective, while disregarding judgment.²⁶

²⁵ The resulting reduced-form reaction function resulting from optimization under discretion would have different coefficients than the optimal h_X . Because of this, and because of the missing response to lagged predetermined variables, the response is suboptimal and results in so-called stabilization bias relative to the commitment policy (Svensson and Woodford 2005).

Given a particular restricted class of instrument rules—for instance, simple instrument rules with only a few arguments—one can find the optimal instrument rule in that restricted class (see appendix G). The optimal instrument rule in a restricted class will depend on the stochastic properties of the disturbances to the economy. Many papers of Taylor (1999) provide examples of such optimal restricted instrument rules.

²⁶ Note that only if h_{Ξ} is invertible can this be written on the “instrument smoothing” form

$$\begin{aligned}i_t &= h_X X_t + h_{\Xi} H_{\Xi X} X_{t-1} + h_{\Xi} H_{\Xi\Xi} h_{\Xi}^{-1} \sum_{j=0}^{\infty} H_{\Xi\Xi}^j H_{\Xi X} X_{t-1-j} \\ &= h_X X_t + h_{\Xi} H_{\Xi X} X_{t-1} + h_{\Xi} H_{\Xi\Xi} h_{\Xi}^{-1} (i_{t-1} - h_X X_{t-1}) \\ &= h_X X_t + h_{\Xi} (H_{\Xi X} - H_{\Xi\Xi} h_{\Xi}^{-1} h_X) X_{t-1} + h_{\Xi} H_{\Xi\Xi} h_{\Xi}^{-1} i_{t-1}.\end{aligned}$$

3.2 *Implicit Instrument Rules*

Another apparent possibility would be an implicit instrument rule, where the instrument responds to the forward-looking variables, x_t . This might seem advantageous, since, in a rational-expectations equilibrium, the forward-looking variables might be affected by private-sector expectations of future deviations. Then, by responding to forward-looking variables, the central bank might indirectly take judgment into account—although, in this case, private-sector judgment. Thus, one might want to consider an ad hoc implicit instrument rule of the form

$$i_t = f_X X_t + f_x x_t \quad (49)$$

where f_X and f_x are row vectors of given response coefficients.

As mentioned above, however, there is a specific problem with the central bank responding to forward-looking variables, something largely overlooked in the literature (except, for instance, Svensson and Woodford 2005 and Svensson 2003). Since the forward-looking variables depend on the central bank's instrument setting, a simultaneity problem arises. The central bank cannot observe the forward-looking variables before it sets the instrument, and the private sector needs to observe the instrument setting before it determines the forward-looking variables. A relation such as (49) is actually an equilibrium condition, where i_t and x_t are simultaneously determined. The implementation of such an equilibrium condition is not straightforward.

A sophisticated way to implement (49) is for the central bank to construct projections (X^t, x^t, i^t) that satisfy (49). The central bank can amend the relation

$$i_{t+\tau,t} = f_X X_{t+\tau,t} + f_x x_{t+\tau,t} \quad (50)$$

for $\tau \geq 0$ to its projection model (29) (or [28]). It can then solve for the projection $(\tilde{X}^t, \tilde{x}^t, \tilde{i}^t)$ for given X_t , *disregarding the judgment* (setting $z_{t+\tau,t} = 0$ for $\tau \geq 1$). These projections will by construction

If there is only one instrument, h_{Ξ} is invertible only if $\Xi_{t-1,t-1}$ is a scalar—that is, if there is only one forward-looking variable. The point is that the optimal reaction function under commitment usually cannot be written as an instrument rule involving current predetermined variable and the lagged instrument, unless i_{t-1} happens to be one of the predetermined variables.

satisfy (50). The corresponding instrument setting, $i_t = \tilde{i}_{t,t}$, will then be a linear function of X_t ,

$$i_t = \tilde{f}_X X_t. \quad (51)$$

The central bank can then set this instrument level according to the reduced-form reaction function (51).²⁷

However, if the private sector understands that the central bank is effectively implementing the reaction function (51); has rational expectations of future variables; and, in particular, has expectations of future nonzero deviations, $z_{t+\tau|t} \neq 0$ ($\tau \geq 1$); the resulting market-determined forward-looking variables, x_t , will deviate from the central-bank projection, $\tilde{x}_{t,t}$. Thus, although the instrument setting will satisfy

$$i_t = f_X X_t + f_x \tilde{x}_{t,t}$$

for the central-bank projection $\tilde{x}_{t,t}$, it will *not* satisfy (49) for the market-determined forward-looking variables x_t .

For relation (49) to be satisfied for the market-determined forward-looking variables, the central bank has to amend the relation (50) for $\tau \geq 0$ to its production model (29) (or [28]) and solve

²⁷ In the context of the finite-horizon projection model, relation (50) can be written as $(T+1)n_i$ equations,

$$\bar{R}s^t = 0,$$

where $\bar{R}$ is an $(T+1)n_i \times (T+1)(n_X + n_x + n_i)$ matrix. Combining these with (29) for $z^t = 0$ gives an equation system

$$\begin{bmatrix} G \\ \bar{R} \end{bmatrix} s^t = \begin{bmatrix} \tilde{g}^t \\ 0 \end{bmatrix}$$

where $\tilde{g}^t$ is the $(T+1)(n_X + n_x)$ -vector $(X_t', 0')$. Under the assumption that the matrix on the left side has full rank, s^t is given by

$$\tilde{s}^t \equiv \begin{bmatrix} G \\ \bar{R} \end{bmatrix}^{-1} \begin{bmatrix} \tilde{g}^t \\ 0 \end{bmatrix}. \quad (52)$$

This results in

$$\tilde{x}_{t,t} = \tilde{H}_{x0} X_t$$

where the matrix $\tilde{H}_{x0}$ can be extracted from the right side of (52) and

$$\tilde{i}_{t,t} = f_X X_t + f_x \tilde{x}_{t,t} = (f_X + f_x \tilde{H}_{x0}) X_t \equiv \tilde{f}_X X_t.$$

for the projection (X^t, x^t, x^t) for given X_t , *taking the judgment into account*.²⁸ This results in

$$\begin{aligned} x_{t,t} &= \tilde{H}_{xX0}X_t + \tilde{H}_{xz0}z^t \\ i_{t,t} &= f_X X_t + f_x x_{t,t} = (f_X + f_x \tilde{H}_{xX0})X_t + f_x \tilde{H}_{xz0}z^t \\ &\equiv \tilde{f}_X X_t + \tilde{f}_z z^t. \end{aligned}$$

Thus, the resulting reduced-form reaction function is

$$i_t = \tilde{f}_X X_t + \tilde{f}_z z^t.$$

If the private sector understands that this is the reaction function followed by the central bank and has rational expectations corresponding to the same judgment, the market-determined forward-looking variables, x_t , will equal the central-bank projection, $x_{t,t}$, and the relation (49) will be satisfied in equilibrium.

This is, of course, an example of a central bank explicitly taking judgment into account, not an example of a central bank disregarding judgment. But instead of finding the optimal policy projection, $(\hat{X}^t, \hat{x}^t, \hat{i}^t, \hat{Y}^t)$, that minimizes its loss function, it finds the projection (X^t, x^t, i^t, Y^t) that satisfies the ad hoc relation (49). Since the latter is no easier than the former and, in particular, suboptimal, this behavior seems a bit far-fetched.

Thus, I can model monetary policy without judgment as following either the explicit instrument rule (47), where both judgment

²⁸ The equation system is then

$$\begin{bmatrix} G \\ R \end{bmatrix} s^t = \begin{bmatrix} g^t \\ 0 \end{bmatrix}$$

where g^t is the $(T+1)(n_X + n_x)$ -vector $(X'_t, z'_{t,t}, 0', z'_{t+1,t}, 0', \dots, z'_{t+T,t}, 0', 0')'$ specified in section 2.1. Under the assumption that the matrix on the left side has full rank, s^t is given by

$$s^t = \begin{bmatrix} G \\ R \end{bmatrix}^{-1} \begin{bmatrix} g^t \\ 0 \end{bmatrix}. \quad (53)$$

This results in

$$x_{t,t} = \tilde{H}_{xX0}X_t + \tilde{H}_{xz0}z^t$$

where $\tilde{H}_{xX0}$ and $\tilde{H}_{xz0}$ can be extracted from the right side of (53) and

$$i_{t,t} = f_X X_t + f_x x_{t,t} = (f_X + f_x \tilde{H}_{xX0})X_t + f_x \tilde{H}_{xz0}z^t \equiv \tilde{f}_X X_t + \tilde{f}_z z^t.$$

and lagged predetermined variables are ignored, or the more sophisticated explicit instrument rule (48), or perhaps some intermediate case of (48) where the summation is over a finite past period. Using the implicit instrument rule (49) is somewhat problematic, since its implementation is complex and open to alternative very different interpretations, with very different resulting equilibria.

3.3 Taylor Rules

In the literature, a number of simple ad hoc instrument rules have been used to discuss and evaluate monetary policy. The most common are variants of the Taylor rule.²⁹ One variant of the Taylor rule with instrument smoothing (meaning, in this context, a response to the lagged instrument rate) is

$$i_t = (1 - f_i)(f_\pi \pi_t + f_y y_t) + f_i i_{t-1}$$

where π_t denotes a measure of the difference of inflation from a given inflation target; y_t denotes a measure of the output gap; f_π and f_y are given positive coefficients that can be interpreted as the long-run response to inflation and the output gap, respectively; and the coefficient f_i satisfies $0 \leq f_i \leq 1$. If inflation and the output gap are predetermined, this is an explicit instrument rule, and its implementation only requires that the central bank can observe or estimate current inflation and the output gap. If inflation and/or the output gap are forward-looking variables, this is an implicit instrument rule, where the instrument and the forward-looking variable are simultaneously determined. As noted above, such an instrument rule is somewhat problematic and its implementation may need to be further specified.

One variant of the Taylor rule, a so-called forecast-based or forward-looking Taylor rule, can be written

$$i_t = (1 - f_i)[f_\pi \pi_{t+J,t} + f_y y_{t+K,t}] + f_i i_{t-1}$$

where $\pi_{t+J,t}$ denotes a projection of the difference of inflation from an inflation target at horizon $J \geq 0$ and $y_{t+K,t}$ denotes a projection of the output gap at horizon $K \geq 0$, where at least one of J or K is

²⁹ Kozicki (1999) provides a discussion of the usefulness of Taylor rules.

positive. Such an instrument rule is an explicit or an implicit instrument rule depending on how the projections are constructed. If the projections are constructed with information that is predetermined in period t , the projections are predetermined and the instrument rule is explicit. If the projections are constructed with information that includes simultaneously determined forward-looking variables, the instrument rule is implicit and hence an equilibrium condition. Again, the implementation of such an instrument rule is not trivial and open to alternative interpretations.³⁰

4. Examples

In this section, I discuss examples of monetary policy with and without judgment in two different empirical models of the U.S. economy: a backward-looking model due to Rudebusch and Svensson (1999) and a forward-looking model due to Lindé (2002).

4.1 Backward-Looking Model

The backward-looking empirical model of Rudebusch and Svensson (1999) has two equations (with estimates rounded to two decimal points):

$$\pi_{t+1} = 0.70 \pi_t - 0.10 \pi_{t-1} + 0.28 \pi_{t-2} + 0.12 \pi_{t-3} + 0.14 y_t + z_{\pi,t+1} \quad (54)$$

$$y_{t+1} = 1.16 y_t - 0.25 y_{t-1} - 0.10 \left(\frac{1}{4} \sum_{j=0}^3 i_{t-j} - \frac{1}{4} \sum_{j=0}^3 \pi_{t-j} \right) + z_{y,t+1}. \quad (55)$$

The period is a quarter, π_t is quarterly gross domestic product inflation measured in percentage points at an annual rate, y_t is the output gap measured in percentage points, and i_t is the quarterly average of the federal funds rate, measured in percentage points at an annual rate. All variables are measured as differences from their means, their steady-state levels. The deviations $z_{\pi,t+1}$ and $z_{y,t+1}$ for inflation and the output gap have been substituted for the shocks

³⁰ Svensson (2001c) discusses additional serious problems with forecast-based instrument rules.

of the original Rudebusch-Svensson model. The predetermined variables are $(\pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_t, y_{t-1}, i_{t-1}, i_{t-2}, i_{t-3})$. See appendix H for details.

The target variables are inflation, the output gap, and the first-difference of the federal funds rate. The period loss function is

$$L_t = \frac{1}{2} [\pi_t^2 + \lambda y_t^2 + \nu (i_t - i_{t-1})^2] \quad (56)$$

where π_t is measured as the difference from the inflation target, which is equal to the steady-state level. The discount factor, δ , and the relative weights on the output-gap stabilization, λ , and interest-rate smoothing, ν , are set so $\delta = 1$, $\lambda = 1$, and $\nu = 0.2$.

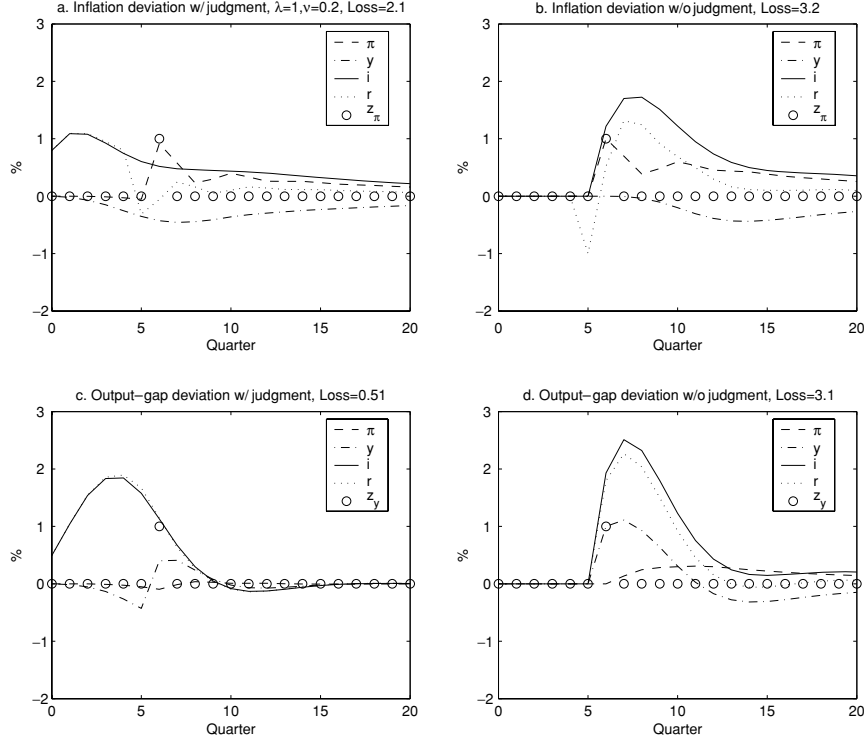
Let me emphasize that there may be considerable uncertainty about the future deviations, ζ^t , in this case $\{z_{\pi,t+\tau}, z_{y,t+\tau}\}_{\tau=1}^{\infty}$. Consider a simple case, when the distribution Φ_t is such that there is uncertainty only about $z_{\pi,t+\tau}$ and only for a finite number of quarters, $1 \leq \tau \leq T$. Then, I can take ζ^t to be the random T -vector $\zeta^t = (z_{\pi,t+1}, \dots, z_{\pi,t+T})$. Suppose furthermore that there are only four possible events with realizations $\zeta^t(j)$ ($j = 1, 2, 3, 4$), and that these are as follows:

1. With probability 0.4, $\zeta^t(1) = (0, 0, \dots)'$, no deviation.
2. With probability 0.3, $\zeta^t(2) = (0, 1.0, 1.0, 0.0, 0.0, 0, \dots)'$, a short sequence of large “cost-push” shocks.
3. With probability 0.2, $\zeta^t(3) = (0, 0.2, 0.2, 0.2, 0.2, 0, \dots)'$, a long sequence of small cost-push shocks.
4. With probability 0.1, $\zeta^t(4) = (0, 1.0, 1.0, 1.0, 1.0, 0, \dots)'$, a long sequence of large cost-push shocks.

The resulting judgment is the mean of the future deviations, the T -vector $z^t = 0.4\zeta^t(1)' + 0.3\zeta^t(2)' + 0.2\zeta^t(3)' + 0.1\zeta^t(4)' = (0, 0.44, 0.44, 0.14, 0.14, 0, \dots)'$.

Note that the same judgment arises if the probabilities are the same for the four events but the first event is that all components $\tau = 1, \dots, T$ of ζ^t have independent uniform distributions between -1 and 1 ; the second event is that all components have the same distributions as for the first event except that components $\tau = 2$

**Figure 1. Monetary Policy with and without Judgment:
Backward-Looking Model**



and 3 have independent uniform distributions between 0 and 2; the third event is that all components have the same distributions as for the first event except that components $\tau = 2, 3, 4$, and 5 have independent uniform distributions between -0.8 and 1.2 ; and the fourth event is that all components have the same distribution as for the first event except that component $\tau = 2, 3, 4$, and 5 have independent uniform distributions between 0 and 2. Thus, behind a given judgment vector can be a distribution Φ_t involving considerable uncertainty. Still, only the mean of that distribution matters.

Figure 1 shows a situation where the predetermined variables, inflation and the output gap, and the deviations are assumed to have been equal to their steady-state levels, zero, up to quarter 0. Furthermore, in previous quarters, the central bank's judgment, z^t ($t < 0$), has been zero: The central bank's expected future inflation

and output-gap deviations have been zero (although possibly with large variances).

In panel a, the central bank's judgment in quarter 0, z^0 , changes from that in previous quarters, such that the central bank's expected inflation deviation equals 1 percentage point for quarter 6, whereas it is still zero in all other quarters; the expected output-gap deviation is still zero for all quarters.³¹ Again, behind these means may be a distribution Φ_0 corresponding to considerable uncertainty. The expected inflation deviation, denoted z_π , is marked by circles with no connecting line. The panel shows the optimal policy projection in quarter 0, (π^0, y^0, i^0, r^0) , of inflation, π (the dashed line); the output gap, y (the dashed-dotted line); the instrument rate, i (the solid line); and the short real interest rate, r (the dotted line).³²

Panel a has two main interpretations. The first interpretation is that the panel just shows the judgment z^0 and the optimal policy projection $(\hat{\pi}^0, \hat{y}^0, \hat{i}^0, \hat{r}^0)$ in quarter $t = 0$ for the future quarters $\tau \geq 1$ and thereby the realization of z_0 , $\hat{\pi}_0$, $\hat{y}_0$, $\hat{i}_0$, and $\hat{r}_0$ in quarter 0. Conditional on the actual realization of π_1 and y_1 (in turn, depending on the realization of $z_{\pi 1}$ and $z_{y 1}$) and the realization of z^1 in quarter $t = 1$, a new optimal policy projection $(\hat{\pi}^1, \hat{y}^1, \hat{i}^0, \hat{r}^0)$ has to be plotted in quarter 1 for future quarters $\tau \geq 2$, and so forth for $t = 2, 3, \dots$

The second interpretation is that the panel shows the probability-zero event that the future realizations of the random deviation z_t for $t \geq 1$ are exactly equal to the judgment z^0 in quarter 0. That is, the realizations of $z_{\pi t}$ for $t \geq 1$ are zero, except in quarter 6 when it is 1 percentage point, and the realizations of $z_{y t}$ for $t \geq 1$ are all zero. In this interpretation, the panel also shows the optimal policy projection $(\hat{\pi}^t, \hat{y}^t, \hat{i}^t, \hat{r}^t)$ for each quarter $t \geq 1$. These optimal policy projections are then simply the continuation of the optimal policy projection of the previous quarter. Furthermore, the actual future realization of inflation, the output gap, the instrument rate, and the real interest rate are then equal to the previous optimal policy projections.

Thus, in the first interpretation, panel a just shows a particular realization of the judgment z^0 and the corresponding optimal policy projection $(\hat{\pi}^0, \hat{y}^0, \hat{i}^0, \hat{r}^0)$. In the second interpretation, panel a in

³¹ In terms of the modeling of the deviation as a moving average process in section 1.2, panel a shows the impulse response to ε^0 .

³² The short real interest rate is defined as $r_t \equiv i_t - \pi_{t+1|t}$.

addition shows a time series of a particular realization of the future deviation—namely the realization that is exactly equal to the judgment in quarter 0—as well as the resulting realizations over time of inflation, the output gap, the instrument rate, and the real interest rate. Clearly, the probability of the future realizations of the deviation being exactly equal to the previous judgment is generally zero.

Panel a shows that when the central bank expects a 1 percentage point inflation deviation in quarter 6, it chooses an optimal instrument-rate projection such that the instrument rate is raised to about 1 percentage point during the first few quarters and then gradually lowered back to its steady-state level. As a result, the projected output gap gradually falls to about -0.5 percentage points in quarter 7 and then very gradually rises back towards zero. The inflation projection shows inflation falling slightly before it is hit by the inflation-deviation in quarter 6, then rising to almost 1 percentage point, and finally falling back towards its steady-state level after quarter 6. Thus, the optimal policy projection is a clear example of preemptive monetary policy: the instrument rate is raised and the output gap is reduced well before the expected inflation-deviation shock, so as to efficiently control inflation and bring it back to target after the shock. The optimal policy projection in quarter 0 results in an intertemporal loss of 2.1 units.³³ In the second interpretation, when panel a shows the actual realization of the deviation, the accumulated realized loss over time, discounted to quarter 0, is also the same 2.1 units (since $\delta = 1$, the discounting does not affect the loss).

Panel b shows the time series of the variables for the same particular realization of the future deviations when the inflation deviation equals 1 percentage point in quarter 6 and is zero elsewhere. However, in this panel, the central bank in each quarter disregards judgment, while still responding optimally to the predetermined variables. That is, the central bank responds in the same way to the predetermined variables as for the optimal policy, but it does not respond to any expected future deviation. It behaves as if it believes

³³ Given how the target variables are measured, with the loss function (56) and $\delta = 1$, an expected difference of inflation from target of one (two) annualized percentage point(s) for a single quarter gives rise to an intertemporal loss of one (four) units.

that the deviation is a serially uncorrelated zero-mean process, so its expected future deviations are zero. This corresponds to a commitment to the instrument rule (47) (recall that there is no optimal response to Lagrange multipliers or lagged predetermined variables for the backward-looking model). The central bank then keeps the instrument rate at its steady-state level through quarter 5. Accordingly, inflation and the output gap stay at the steady-state levels through quarter 5. In quarter 6, the inflation shock hits and inflation jumps to 1 percentage point, while the predetermined output gap still stays at zero. In this situation, once the inflation shock has hit, the optimal monetary policy response is to raise the instrument rate substantially, to more than 1.5 percentage points above the steady-state level during the following few quarters. This reduces the output gap to almost -0.5 percentage points during the next eight to nine quarters. The instrument rate is gradually lowered back to the steady-state level, and inflation and the output gap return to their steady-state levels very slowly. The absence of any preemption requires a larger instrument-rate response when the shock occurs, the output-gap is nevertheless reduced with a considerable lag, and inflation stays above target for a long time. The resulting intertemporal loss is 3.2 units, 1.1 units higher than when monetary policy relies on judgment.

Panel c is analogous to panel a, except that it shows a situation when the central bank's judgment in quarter 0, z^0 , is such that the central bank expects an output-gap deviation of 1 percentage point in quarter 6, whereas no other deviations are expected. The expected output-gap deviation, z_y , is denoted by circle markers with no connecting line. Again, panel c has two interpretations: the first is that the panel just shows the judgment and optimal policy projection in quarter 0; the second is that it also shows the time series of the variables if the future realizations of the output-gap deviation are exactly equal to the judgment in quarter 0. For this expected output-gap deviation, the optimal policy projection shows a substantial increase in the instrument rate to almost 2 percentage points above the steady-state level in quarters 3–5 and then a rather quick reduction back to the steady-state level. As a result, the output-gap projection shows output falling by almost -0.5 percentage points before the expected output-gap deviation hits, after which it rises to less than 0.5 percentage points

and then relatively quickly comes back to the steady-state level. The resulting movements in the inflation projection are small. A modest loss of 0.51 units results from this preemptive optimal policy projection.

Panel d is analogous to panel b, except that it shows the time series of the variables for the particular realization of the output-gap deviation when it equals 1 percentage point in quarter 6 and is zero in the other quarter. The central bank disregards judgment and only responds to current and lagged inflation and the output gap (although, again, optimally so, according to the instrument rule [47]). Then the central bank keeps the instrument rate at its steady-state level until the output-gap shock hits in quarter 6. Once the shock has hit, it is optimal to raise the instrument rate even more, to more than 2 percentage points for a few quarters, before it is lowered back to the steady-state level. The output gap stays up around 1 percentage point for several quarters. This causes inflation to rise and only very slowly return to target. The output gap has to undershoot the steady-state level significantly in order to bring inflation back. Clearly, inflation and the output gap deviate substantially more than when the central bank uses its judgment. The resulting intertemporal loss is 3.1 units, 2.6 units higher than the loss for the optimal policy projection with judgment.

This example shows a substantial difference between monetary policy with and without judgment, with substantial differences in the development of the target variables and corresponding intertemporal losses.

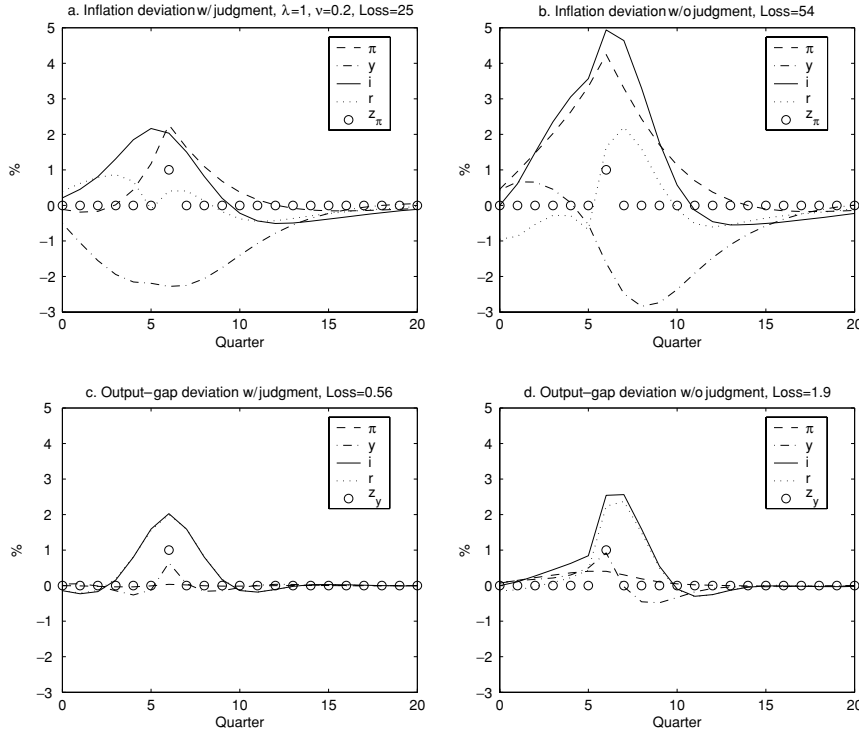
4.2 *Forward-Looking Model*

The forward-looking New Keynesian model of Lindé (2002) has two equations. I use the following parameter estimates:

$$\begin{aligned}\pi_t &= 0.457 \pi_{t+1|t} + (1 - 0.457) \pi_{t-1} + 0.048 y_t + z_{\pi t}, \\ y_t &= 0.425 y_{t+1|t} + (1 - 0.425) y_{t-1} - 0.156 (i_t - \pi_{t+1|t}) + z_{yt}.\end{aligned}$$

The variables are measured as for the backward-looking model. The predetermined variables are $(\pi_{t-1}, y_{t-1}, i_{t-1}, z_{\pi t}, z_{yt})$ (the lagged instrument rate enters because it enters into the loss function, and the two deviations are entered in order to write the model on the form

**Figure 2. Monetary Policy with and without Judgment:
Forward-Looking Model**



[1]; see section 1). The forward-looking variables are (π_t, y_t) . See appendix I for details. The loss function and its parameters used in the experiment below are the same as for the backward-looking model.³⁴

Figure 2, panels a–d, shows the same experiments as figure 1, but for the forward-looking model. Thus, before quarter 0, the variables are equal to their steady-state levels, zero, and the central bank does not expect any future inflation and output-gap deviations.

In panel a, in quarter 0, while the inflation and the output-gap deviations in that quarter are still zero, the central bank's judgment,

³⁴ I find it very unrealistic to consider inflation and output in the current quarter as forward-looking variables. I believe it makes more sense to have current inflation and the output gap predetermined, and to have one-quarter-ahead inflation, output-gap, and instrument-rate plans be determined by the model above. Such a variant of the New Keynesian model is used in Svensson and Woodford (2005) and Svensson (2003).

z^0 , changes, such that the central bank expects an inflation deviation equal to 1 percentage point in quarter 6, whereas it still expects zero inflation deviations for all other quarters and zero output-gap deviations for all quarters. Again, behind these expected deviations could be a probability distribution Φ_0 corresponding to substantial uncertainty. As with figure 1, panel a in figure 2 has two interpretations. In the first interpretation, it just shows the judgment z^0 and the optimal policy projection $(\hat{\pi}^0, \hat{y}^0, \hat{i}^0, \hat{r}^0)$. In the second interpretation, it shows the time series of inflation, the output-gap, the instrument rate, and the real interest rate, for the particular realizations of the future deviations that are exactly equal to the central bank's judgment in quarter 0. In this interpretation, I also assume that the private sector has sufficient information—cf. the discussion in section 1.1—to form expectations consistent with the optimal policy projection.

The optimal policy projection in panel a shows that the central bank plans to raise the instrument rate to about 2 percentage points above the steady-state level in the quarters before and including the time of the inflation shock. This makes the output-gap projection fall to more than -2 percentage points at the time of the expected inflation deviation. The inflation projection rises before and up to the expected inflation deviation, because private-sector expectations are forward looking and consistent with the optimal inflation projection. After the expected inflation deviation, the instrument rate, the output gap, and inflation are projected to return to their steady-state levels. Again, there is a considerable amount of preemption in the optimal policy with judgment, with a projected positive real interest rate and negative output gap before the expected inflation deviation. A substantial intertemporal loss of 25 units results from the optimal policy projection.

Panel b shows the realizations over time of these variables when the realization of the inflation deviation is equal to 1 percentage point in quarter 6 and zero in other quarters and the central bank in each quarter disregards judgment while still responding optimally to current and lagged inflation and output gap. In this case, the central bank is assumed to respond optimally to both the predetermined variables *and* the lagged predetermined variables, as if the central bank had committed itself to the optimal policy under commitment while ignoring its judgment. Hence, the central bank behaves

according to instrument rule (48) and responds optimally to the current deviation but expects zero future deviations. However, the private sector is assumed to have rational expectations of the future inflation shock. These expectations will increase inflation to more than 4 percentage points at the time of the inflation shock. The central bank's optimal response to current and predetermined variables induces it to raise the instrument rate in line with inflation, but it is nevertheless behind the curve in the sense that the real interest rate becomes negative and the output gap becomes positive in the first few quarters. The central bank's response eventually leads to a high positive real interest rate, a negative output gap, and a fall in inflation. In comparison with panel a, inflation rises earlier and more, and the output gap falls later, than under the optimal monetary policy with judgment. The intertemporal loss is 54 units, a substantial increase of 29 units above the loss for monetary policy with judgment.

Panel c shows the situation where the central bank's judgment in quarter 0 is such that it expects an output-gap deviation of 1 percentage point in quarter 6 and otherwise zero deviations. The optimal policy projection, taking this judgment into account, is to raise the instrument rate before the expected output-gap deviation, which moderates the expected impact on the output gap. The inflation projection remains very flat, and the projections of the real interest rate and the instrument rate are almost identical. The resulting intertemporal loss is small, 0.56 units.

Panel d shows the realizations over time of the variables in the situation where the realization of the output-gap deviation is 1 percentage point in quarter 6 and zero in other quarters and the central bank disregards judgment and only responds to current and lagged predetermined variables, although again optimally so, corresponding to the instrument rule (48). In comparison with the second interpretation of panel c, when the panel shows the actual realization of the variables for the same realization of the deviations, the central bank ends up raising the instrument rate later and more, and there is more movement in both the output gap and inflation. The intertemporal loss is 1.9 units, 1.3 units above the loss for optimal policy under judgment.

Again, there are substantial differences between monetary policy with and without judgment and corresponding intertemporal losses.

4.2.1 *Taylor Rules*

I also examine two variants of Taylor rules for the forward-looking model, an explicit instrument rule for which the instrument rate responds to lagged inflation and the output gap,

$$i_t = 1.5 \pi_{t-1} + 0.5 y_{t-1},$$

and an implicit instrument rule for which the instrument rate responds to the forward-looking current inflation and output,

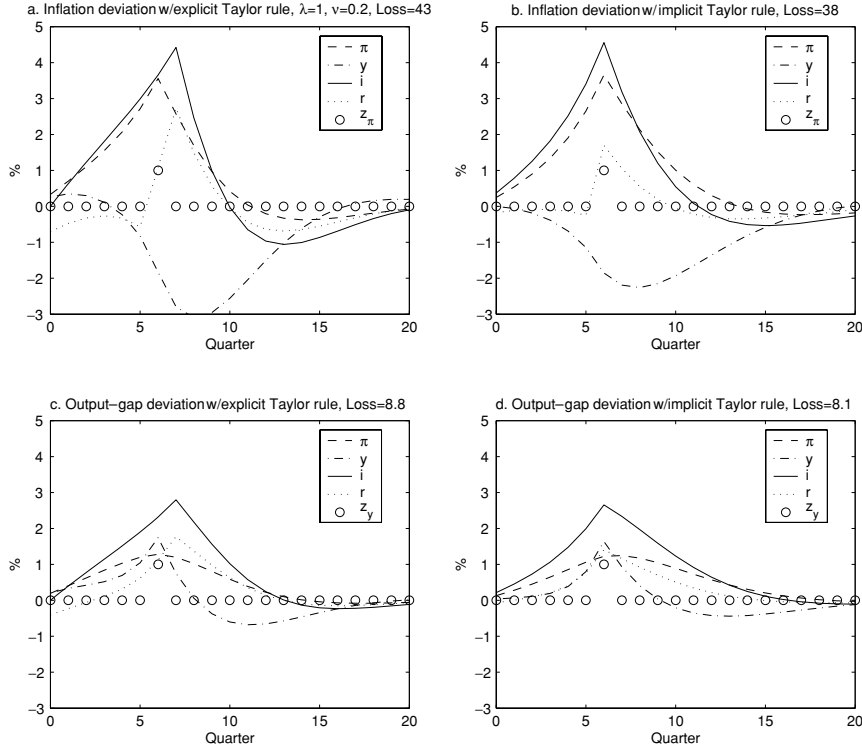
$$i_t = 1.5 \pi_t + 0.5 y_t.$$

As noted in section 3, the implementation of an implicit instrument rule is somewhat complex. I disregard these complications here, and just assume that it is somehow implemented. Figure 3 shows the realizations over time of the variables when the central bank implements the two Taylor rules for the two cases of either an inflation deviation or an output-gap deviation only in quarter 6.

Panels a and b show the result of the explicit and implicit Taylor rule, respectively, when there is an inflation deviation in quarter 6 and the private sector has rational expectations of that deviation. The resulting intertemporal losses are substantial, 43 and 38 units, respectively—18 and 13 units, respectively, above the loss for optimal monetary policy with judgment, 25 units. Interestingly, the intertemporal loss with either of the two Taylor rules is less than the policy without judgment that responds optimally to current and lagged predetermined variables, panel b in figure 2, which has an intertemporal loss of 54 units. One possible interpretation of this is that history dependence in the form of responding to the Lagrange multipliers is not always advantageous, when these multipliers do not take into account the expected future deviations. The loss for the implicit Taylor rule is lower than for the explicit one. One interpretation is that the implicit Taylor rule takes private-sector expectations better into account, and therefore indirectly takes the expected future deviation better into account.

Panels c and d show the result of the two Taylor rules when there is an output-gap deviation in quarter 6. Here, the intertemporal loss is substantially higher than the small loss for monetary policy without judgment in panel d of figure 2. In this case, the optimal

**Figure 3. Monetary Policy with Taylor Rules:
Forward-Looking Model**



response to current and lagged predetermined variables does much better than the two Taylor rules.

I conclude that the two Taylor rules perform considerably worse than the optimal policy with judgment, especially when there are expected future output-gap deviations.

5. Conclusions

The decision process of modern monetary policy that can be called “forecast targeting”—finding a projection of the current and future instrument rate such that the projection paths of the target variables “look good” relative to the central bank’s objectives—is formalized in this paper as a technique that provides projections of the

instrument rate and the target variables that minimize an intertemporal loss function. The paper shows how this technique can easily incorporate central-bank judgment, a necessary ingredient in modern monetary policy. In two empirical models of the U.S. economy, a few examples are shown in which forecast targeting that incorporates judgment provides significantly better monetary policy performance than a policy that follows an instrument rule and disregards judgment. The paper shows how the policy problem, normally treated as an infinite-horizon problem, can be reformulated as a convenient finite-horizon decision problem, which is either an exact or a very close approximation to the infinite-horizon problem. This approximation makes the policy problem much easier to handle numerically. The paper also shows how the time-consistency problem can be easily managed and the resulting projections made to be optimal under commitment in a timeless perspective. In particular, the paper shows that it is not necessary to be explicit about the underlying complex reduced-form reaction function of monetary policy. The policymakers only need to ponder the projections of the target variables and the instrument rate under alternative assumptions, and these projections can be presented as graphs.

Several of the ideas and techniques presented here are already applied to various extents by different central banks. I hope the presentation here will be useful for attempts to apply them more extensively and systematically.

If policymakers hesitate to make the parameters of their loss function explicit (for instance, the weight on output-gap stabilization relative to inflation stabilization), the techniques presented here can still be very useful. For instance, the policymakers can ask the staff to provide optimal policy projections of the target variables for a range of loss-function parameters. These projections then provide one way to illustrate the available trade-offs among the target variables, the set of feasible projections of the target variables from which the policymakers should choose their optimal policy projection.

The framework used here is one where mean projections are sufficient for optimal decisions, what can be called *mean* forecast targeting, which is sufficient under the assumptions that result in certainty equivalence. If these assumptions are not satisfied, the principal ideas in this paper can be extended to a situation when the projections are probability *distributions* rather than means, and the intertemporal

losses can be computed by numerical integration over those distributions. This I have previously called *distribution* forecast targeting (Svensson 2001b). The details in such an undertaking remain to be completed, and the practical differences between mean and distribution forecast targeting remain to be clarified. Svensson and Williams (2005) examine distribution forecast targeting in a situation where genuine model uncertainty implies that certainty equivalence does not hold.

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Appendix to Monetary Policy with Judgment: Forecast Targeting

A. Optimal policy under commitment with the deviation being an arbitrary stochastic process

Let the model equations for $t \geq 0$ be (2.1). A common special case is when the matrix $C = I$, but in general C need not be invertible. This system can be written

$$\tilde{C} \begin{bmatrix} X_{t+1} \\ E_t x_{t+1} \\ E_t i_{t+1} \end{bmatrix} = \tilde{A} \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} + \begin{bmatrix} z_{t+1} \\ 0 \end{bmatrix}, \quad (\text{A.1})$$

where $E_t q_{t+\tau} \equiv \int q_{t+\tau} d\Phi_t(\zeta^t)$ for any variable $q_{t+\tau}$ ($\tau \geq 0$), the matrices $\tilde{A}$ and $\tilde{C}$ are of dimension $(n_X + n_x) \times (n_X + n_x + n_i)$ and given by

$$\tilde{A} \equiv \begin{bmatrix} A & B \end{bmatrix} \equiv \begin{bmatrix} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \end{bmatrix}, \quad \tilde{C} \equiv \begin{bmatrix} I & 0 & 0 \\ 0 & C & 0 \end{bmatrix}.$$

where A and B are partitioned according to (2.3).

The target variables are defined by (2.5). The intertemporal loss function in period 0 is

$$E_0 \sum_{t=0}^{\infty} \delta^t L_t,$$

where the period loss function, (2.7), can be written as

$$L_t = \frac{1}{2} \begin{bmatrix} X'_t & x'_t & i'_t \end{bmatrix} D' W D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}.$$

Consider minimizing this intertemporal loss function under once-and-for-all commitment in period $t = 0$, for given $X_0 = \bar{X}_0$. For convergence, when the variance of z_{t+1} is nonzero, I need $0 < \delta < 1$.

Variants of this problem are solved in Backus and Driffill [2], Currie and Levine [5], and Söderlind [20], when the deviation is an iid shock. The focus here is on the case when the deviation is an arbitrary stochastic process.

Construct the Lagrangian,

$$\begin{aligned} \mathcal{L}_0 &= E_0 \sum_{t=0}^{\infty} \delta^t \left\{ L_t + \begin{bmatrix} \xi'_{t+1} & \Xi'_t \end{bmatrix} \left(\tilde{C} \begin{bmatrix} X_{t+1} \\ E_t x_{t+1} \\ E_t i_{t+1} \end{bmatrix} - \tilde{A} \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} - \begin{bmatrix} z_{t+1} \\ 0 \end{bmatrix} \right) \right\} \\ &\quad + \xi'_0 (X_0 - \bar{X}_0) / \delta \\ &= E_0 \sum_{t=0}^{\infty} \delta^t \left\{ L_t + \begin{bmatrix} \xi'_{t+1} & \Xi'_t \end{bmatrix} \left(\tilde{C} \begin{bmatrix} X_{t+1} \\ x_{t+1} \\ i_{t+1} \end{bmatrix} - \tilde{A} \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} - \begin{bmatrix} z_{t+1} \\ 0 \end{bmatrix} \right) \right\} \\ &\quad + \xi'_0 (X_0 - \bar{X}_0) / \delta, \end{aligned}$$

where ξ_{t+1} and Ξ_t are vectors of n_X and n_x Lagrange multipliers of the upper and lower block, respectively, of (A.1). The law of iterated expectations has been used in the second equality, $E_0 E_t = E_0$ for $t \geq 0$. Note that Ξ_t is dated to emphasize that it depends on information available in period t .

For $t \geq 1$, the first-order conditions with respect to X_t , x_t and i_t can be written

$$\begin{bmatrix} X'_t & x'_t & i'_t \end{bmatrix} D'WD + \begin{bmatrix} \xi'_t & \Xi'_{t-1} \end{bmatrix} \frac{1}{\delta} \tilde{C} - \begin{bmatrix} E_t \xi'_{t+1} & \Xi'_t \end{bmatrix} \tilde{A} = 0. \quad (\text{A.2})$$

For $t = 0$, the first-order condition with respect to X_0 , x_0 , and i_0 can be written

$$\begin{bmatrix} X'_t & x'_t & i'_t \end{bmatrix} D'WD + \begin{bmatrix} \xi'_t & 0 \end{bmatrix} \frac{1}{\delta} \tilde{C} - \begin{bmatrix} E_t \xi'_{t+1} & \Xi'_t \end{bmatrix} \tilde{A} = 0, \quad (\text{A.3})$$

where $X_0 = \bar{X}_0$. In comparison with (A.2), a vector of zeros enters in place of Ξ_{-1} , since there is no constraint corresponding to the lower block of (A.1) for $t = -1$. By including a fictitious vector of Lagrange multipliers, Ξ_{-1} , equal to zero,

$$\Xi_{-1} = 0, \quad (\text{A.4})$$

in (A.3), I can write the first-order conditions more compactly as (A.2) for $t \geq 0$ and (A.4).

The system of difference equations (A.2) has $n_X + n_x + n_i$ equations. The first n_X equations can be associated with the Lagrange multipliers ξ_t . Indeed, $-\xi_t/\delta$ can be interpreted as the total marginal losses in period t of the predetermined variables X_t (for $t = 0$, with given X_0 , the equations determine ξ_0). They are forward-looking variables: Lagrange multipliers for predetermined variables are always forward-looking, whereas the Lagrange multipliers for the (equations for the) forward-looking variables are predetermined. The middle n_x equations are associated with the Lagrange multipliers Ξ_t . Indeed, $\Xi_t A_{22}$ can be interpreted as the total marginal losses in period t of the forward-looking variables, x_t . Also, $\Xi_t C$ can be seen as the marginal loss in period t of expectations $E_t x_{t+1}$ of the forward-looking variables. The last n_i equations are the first-order equations for the vector of instruments. In the special case when the lower right $n_i \times n_i$ submatrix of $D'WD$ is of full rank, the instruments can be solved in terms of the other variables and eliminated from (A.2), leaving the first $n_X + n_x$ equations involving the Lagrange multipliers and the predetermined and forward-looking variables only.

Rewrite the $n_X + n_x + n_i$ first-order conditions as

$$\tilde{A}' \begin{bmatrix} E_t \xi_{t+1} \\ \Xi_t \end{bmatrix} = D'WD \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} + \frac{1}{\delta} \tilde{C}' \begin{bmatrix} \xi_t \\ \Xi_{t-1} \end{bmatrix}. \quad (\text{A.5})$$

They can be combined with the model equations (A.1) to get a system of $2(n_X + n_x) + n_i$ difference equations for $t \geq 0$,

$$\begin{bmatrix} \tilde{C} & 0 \\ 0 & \tilde{A}' \end{bmatrix} \begin{bmatrix} X_{t+1} \\ \text{E}_t x_{t+1} \\ \text{E}_t i_{t+1} \\ \text{E}_t \xi_{t+1} \\ \Xi_t \end{bmatrix} = \begin{bmatrix} \tilde{A} & 0 \\ D'WD & \frac{1}{\delta}\tilde{C}' \end{bmatrix} \begin{bmatrix} X_t \\ x_t \\ i_t \\ \xi_t \\ \Xi_{t-1} \end{bmatrix} + \begin{bmatrix} z_{t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (\text{A.6})$$

Here, X_t and Ξ_t are predetermined variables, and x_t , i_t , and ξ_t are non-predetermined variables.

This can be rearranged as the system

$$\mathcal{C} \begin{bmatrix} y_{1,t+1} \\ \text{E}_t y_{2,t+1} \end{bmatrix} = \mathcal{M} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} + \begin{bmatrix} z_{t+1} \\ 0 \\ 0 \end{bmatrix},$$

where

$$\mathcal{C} \equiv \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 \\ 0 & A'_{21} & 0 & 0 & A'_{11} \\ 0 & A'_{22} & 0 & 0 & A'_{12} \\ 0 & B'_2 & 0 & 0 & B'_1 \end{bmatrix}, \quad (\text{A.7})$$

$$y_{1t} \equiv \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix}, \quad y_{2t} \equiv \begin{bmatrix} x_t \\ i_t \\ \xi_t \end{bmatrix}.$$

Thus, y_{1t} is a vector of $m_1 \equiv n_X + n_x$ predetermined variables, and y_{2t} is a vector of $m_2 \equiv n_x + n_i + n_X$ non-predetermined variables.

Under suitable assumptions (see appendix B), such a system has a unique solution, which can be written

$$y_{2t} = F_1 y_{1t} + Z_t \quad (\text{A.8})$$

$$y_{1,t+1} = M_1 y_{1t} + N \text{E}_t Z_{t+1} + P \text{E}_t z_{t+1} + \begin{bmatrix} z_{t+1} - \text{E}_t z_{t+1} \\ 0 \end{bmatrix}, \quad (\text{A.9})$$

where Z_t is an m_2 -dimensional stochastic process given by

$$Z_t \equiv \sum_{\tau=0}^{\infty} R_{\tau} \text{E}_t z_{t+1+\tau} \equiv R z^t, \quad (\text{A.10})$$

where I can interpret R as a linear operator on $z^t \equiv \text{E}_t(z'_{t+1}, z'_{t+2}, \dots)'$.

In terms of the original variables, the solution for $t \geq 0$, given X_0 and $\Xi_{-1} = 0$, can be written

$$\begin{aligned} \begin{bmatrix} x_t \\ i_t \\ \xi_t \end{bmatrix} &= F_1 \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix} + Rz^t \\ &\equiv F \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1} \end{bmatrix}, \end{aligned} \quad (\text{A.11})$$

$$\begin{aligned} \begin{bmatrix} X_{t+1} \\ \Xi_t \end{bmatrix} &= M_1 \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix} + NRE_t z^{t+1} + PE_t z_{t+1} + \begin{bmatrix} z_{t+1} - E_t z_{t+1} \\ 0 \end{bmatrix} \\ &\equiv M \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1} \end{bmatrix} + \begin{bmatrix} z_{t+1} - E_t z_{t+1} \\ 0 \end{bmatrix}, \end{aligned} \quad (\text{A.12})$$

where F and M are linear operators. The details of the solution are derived in appendix B. The matrices F_1 , M_1 , N , P , and $\{R_\tau\}_{\tau=0}^\infty$ —and thereby the linear operators M and F —depend on A , B , C , D , W , and δ , but are independent of the second and higher moments of the exogenous stochastic process $\{z_t\}_{t=1}^\infty$. This demonstrates the certainty equivalence of the commitment solution.³⁷

If the commitment is once and for all and starts in period 0, $\Xi_{-1} = 0$. Commitment in a timeless perspective can be seen as corresponding to a situation where the lower block of (A.12) is restricted to apply also for previous periods. Then, Ξ_{t-1} is determined by

$$\begin{aligned} \Xi_{t-1} &= M_{121}X_{t-1} + M_{122}\Xi_{t-2} + N_2E_{t-1}Z_t + P_2E_{t-1}z_t \\ &= \sum_{\tau=0}^{\infty} M_{122}^\tau (M_{121}X_{t-1-\tau} + N_2E_{t-1-\tau}Z_{t-\tau} + P_2E_{t-1-\tau}z_{t-\tau}), \end{aligned}$$

where M_1 , N , and P are partitioned conformably with X_t and Ξ_{t-1} .

Alternatively, the commitment in a timeless perspective can be generated as optimization under commitment or discretion with a term added to the intertemporal loss function in period 0,

$$E_0 \sum_{t=0}^{\infty} \delta^t L_t + \Xi_{-1} \frac{1}{\delta} Cx_0,$$

where Ξ_{-1} is the Lagrange multiplier for the block of forward-looking equations from the optimization in period -1 (see Svensson and Woodford [30] and Svensson [25]).

In the standard case, when z_t is a vector of iid zero-mean shocks, I have $E_t z_{t+1} \equiv 0$, $Z_t \equiv E_t Z_{t+1} \equiv 0$, and $z^t \equiv 0$. Thus, the terms involving Z_t in (A.11) and (A.12) vanish.³⁸ Consequently,

³⁷ The middle block of (A.11) is the optimal *explicit instrument rule* for this problem, the instrument written as a function of predetermined and exogenous variables. Eliminating the Lagrange multipliers from (A.2) results in the optimal *targeting rule* for this problem, a consolidated optimal first-order condition for the target variables. See Svensson [25] on instrument and targeting rules, as well as the lecture notes Svensson [A7].

³⁸ In the case when $\{z_t\}$ is an autoregressive process and can be written $z_{t+1} = \Psi z_t + \varepsilon_{t+1}$, where Ψ is a matrix and ε_t an iid zero-mean process, z_t can simply be included among the predetermined variable.

the effect of z_t being an arbitrary exogenous stochastic process shows up only in the addition of the terms involving Z_t and the corresponding matrices N , P , and $\{R_\tau\}_{\tau=0}^\infty$. Then, I can set $M \equiv M_1$ and $F \equiv F_1$, and

$$y_{1,t+1} = My_{1t} + z_{t+1}.$$

Let Σ denote the variance-covariance matrix of the iid shocks z_{t+1} . Define the matrices $\bar{D}$ and $\bar{W}$ according to

$$\begin{aligned} Y_t &= D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix} = D \begin{bmatrix} I & 0 \\ F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} y_{1t} \equiv \bar{D}y_{1t}, \\ L_t &= \frac{1}{2}Y_t'WY_t = \frac{1}{2}y_{1t}'\bar{D}'W\bar{D}y_{1t} \equiv \frac{1}{2}y_{1t}'\bar{W}y_{1t}, \end{aligned}$$

where $\bar{W}$ is symmetric and positive semidefinite. Then twice the minimum loss in period t will satisfy

$$\begin{aligned} y_{1t}'Vy_{1t} + w &= \text{E}_t \sum_{\tau=0}^{\infty} \delta^\tau y_{1,t+\tau}'\bar{W}y_{1,t+\tau} \\ &= y_{1t}'\bar{W}y_{1t} + \text{E}_t \sum_{\tau=1}^{\infty} \delta^\tau y_{1,t+\tau}'\bar{W}y_{1,t+\tau} \\ &= y_{1t}'\bar{W}y_{1t} + \delta \text{E}_t \text{E}_{t+1} \sum_{\tau=0}^{\infty} \delta^\tau y_{1,t+1+\tau}'\bar{W}y_{1,t+1+\tau} \\ &= y_{1t}'\bar{W}y_{1t} + \delta \text{E}_t (y_{1,t+1}'Vy_{1,t+1} + w) \\ &= y_{1t}'\bar{W}y_{1t} + \delta (y_{1t}'M'VM y_{1t} + \text{E}_t z_{1,t+1}'V z_{1,t+1} + w) \\ &= y_{1t}'\bar{W}y_{1t} + \delta y_{1t}'M'VM y_{1t} + \delta \text{trace}(V\Sigma) + \delta w. \end{aligned}$$

It follows that

$$w = \frac{\delta}{1-\delta} \text{trace}(V\Sigma),$$

and that the matrix V satisfies the Lyapunov equation

$$V = \bar{W} + \delta M'VM. \tag{A.13}$$

It follows that when $\text{trace}(V\Sigma)$ is nonzero, I must have $\delta < 1$ for the existence of a finite w .

I can use the relations $\text{vec}(A+B) = \text{vec}(A) + \text{vec}(B)$ and $\text{vec}(ABC) = (C' \otimes A) \text{vec}(B)$ on (A.13) (where $\text{vec}(A)$ denotes the vector of stacked column vectors of the matrix A , and $\otimes$ denotes the Kronecker product) which results in

$$\begin{aligned} \text{vec}(V) &= \text{vec}(\bar{W}) + \delta \text{vec}(M'VM) \\ &= \text{vec}(\bar{W}) + \delta (M' \otimes M') \text{vec}(V). \end{aligned}$$

Solving for $\text{vec}(V)$ gives

$$\text{vec}(V) = [I - \delta (M' \otimes M')]^{-1} \text{vec}(\bar{W}). \quad (\text{A.14})$$

A.1. No forward-looking variables

If there are no forward-looking variables, so $n_x = 0$, I have

$$\tilde{C} \begin{bmatrix} X_{t+1} \\ E_t i_{t+1} \end{bmatrix} = \tilde{A} \begin{bmatrix} X_t \\ i_t \end{bmatrix} + z_{t+1}, \quad (\text{A.15})$$

where the matrices $\tilde{A}$ and $\tilde{C}$ are of dimension $n_X \times (n_X + n_i)$ and given by

$$\tilde{A} \equiv \begin{bmatrix} A & B \end{bmatrix}, \quad \tilde{C} \equiv \begin{bmatrix} I & 0 \end{bmatrix}.$$

The period loss function is

$$L_t = \frac{1}{2} Y_t' W Y_t \equiv \frac{1}{2} \begin{bmatrix} X_t' & i_t' \end{bmatrix} D' W D \begin{bmatrix} X_t \\ i_t \end{bmatrix}.$$

The $n_X + n_i$ first-order conditions can be written

$$\tilde{A}' E_t \xi_{t+1} = D' W D \begin{bmatrix} X_t \\ i_t \end{bmatrix} + \frac{1}{\delta} \tilde{C}' \xi_t. \quad (\text{A.16})$$

Combined with the model equations, I get a system of $2n_X + n_i$ difference equations for $t \geq 0$,

$$\begin{bmatrix} \tilde{C} & 0 \\ 0 & \tilde{A}' \end{bmatrix} \begin{bmatrix} X_{t+1} \\ \frac{E_t i_{t+1}}{E_t \xi_{t+1}} \end{bmatrix} = \begin{bmatrix} \tilde{A} & 0 \\ D' W D & \frac{1}{\delta} \tilde{C}' \end{bmatrix} \begin{bmatrix} X_t \\ \frac{i_t}{\xi_t} \end{bmatrix} + \begin{bmatrix} \frac{z_{t+1}}{0} \end{bmatrix}.$$

Here, X_t are predetermined variables, and i_t and ξ_t are non-predetermined variables.

Under suitable assumptions, this system will have a unique solution for $t \geq 0$, given X_0 , which can be written

$$\begin{aligned} \begin{bmatrix} i_t \\ \xi_t \end{bmatrix} &= F_1 X_t + R z^t, \\ X_{t+1} &= M_1 X_t + N_0 R z^t + z_{t+1}. \end{aligned}$$

When there are no forward-looking variables, X_{t+1} is directly determined by X_t , i_t , and z_{t+1} according to (2.1), so M_1 and N_0 are determined by A , B , and F_1 as

$$\begin{aligned} M_1 &\equiv A + B F_i, \\ N_0 &\equiv [B \ 0], \end{aligned}$$

where

$$F_1 \equiv \begin{bmatrix} F_i \\ F_\xi \end{bmatrix}$$

is partitioned conformably with i_t and ξ_t . In comparison with the general solution of (A.9), for the backward-looking case,

$$N_0 R z^t \equiv N R E_t z^{t+1} + (P - I) E_t z_{t+1}.$$

B. The solution of a system of difference equations with the deviation

In order to understand the term in the solution (A.10) and (A.11) that corresponds to the deviation, consider the system

$$\mathcal{C} \begin{bmatrix} y_{1,t+1} \\ E_t y_{2,t+1} \end{bmatrix} = \mathcal{M} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} + \begin{bmatrix} \theta_{t+1} \\ 0 \end{bmatrix} \quad (\text{B.1})$$

for $t \geq 0$; where y_{1t} is a vector of m_1 predetermined variables ($y_{1t} \equiv (X'_t, \Xi'_{t-1})'$ and $m_1 = n_X + n_x$ in the previous section); y_{2t} is a vector of m_2 non-predetermined variables ($y_{2t} \equiv (x'_t, i'_t, \xi'_t)'$ and $m_2 = n_x + n_i + n_X$ in the previous section); θ_t is an m_1 -vector of stochastic processes ($\theta_t \equiv (z'_t, 0)'$ in the previous section); and y_{10} is given.

By defining the m_2 -vector of endogenous expectation errors, η_t , as

$$\eta_t \equiv y_{2t} - E_t y_{2t},$$

(B.1) can be written in the form used in Sims [A6],

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi \theta_t + \Pi \eta_t,$$

where $y_t \equiv (y'_{1t}, y'_{2t})'$. Sims shows that, under suitable assumptions, this system has a unique solution of the form

$$y_t = \Theta_1 y_{t-1} + \Theta_0 \theta_t + \Theta_y \sum_{\tau=0}^{\infty} \Theta_f^\tau \Theta_\theta E_t \theta_{t+1+\tau},$$

where Θ_0 and Θ_1 are real matrices, Θ_y , Θ_f , and Θ_θ are complex matrices, and $\Theta_y \Theta_f^\tau \Theta_\theta$ for any integer $\tau \geq 0$ is a real matrix. These matrices can be calculated by his Matlab program Gensys, available at www.princeton.edu/~sims. An advantage with Sims's approach is that one need not keep track of what variables are predetermined or nonpredetermined. An arguable disadvantage is that the determination of the expectational errors is somewhat complex.

Here, I prefer to keep close track of what variables are predetermined and nonpredetermined and therefore choose to derive the solution to (B.1) following a route closer to Klein [A4] than Sims [A6], but going beyond Klein in, as Sims, explicitly treating the case of θ_t being an arbitrary

stochastic process rather than an autoregressive process. The solution will then be of the form

$$\begin{aligned} y_{2t} &= F_1 y_{1t} + Z_t, \\ y_{1,t+1} &= M_1 y_{1t} + N E_t Z_{t+1} + P E_t \theta_{t+1} + (\theta_{t+1} - E_t \theta_{t+1}), \\ Z_t &\equiv \sum_{\tau=0}^{\infty} R_{\tau} E_t \theta_{t+1+\tau}, \end{aligned}$$

where F_1 , M_1 , N , P , and R_{τ} are real matrices to be determined.

Take the expectation conditional on information in period t and write the system as

$$\mathcal{C} \begin{bmatrix} E_t y_{1,t+1} \\ E_t y_{2,t+1} \end{bmatrix} = \mathcal{M} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} + \begin{bmatrix} E_t \theta_{t+1} \\ 0 \end{bmatrix}. \quad (\text{B.2})$$

Following Klein [A4], Sims [A6], and Söderlind [20], I use the generalized Schur decomposition.

This decomposition results in the square complex matrices Q , S , T , and Z such that

$$\mathcal{C} = Q' S Z', \quad (\text{B.3})$$

$$\mathcal{M} = Q' T Z', \quad (\text{B.4})$$

where Z' for a complex matrix denotes the complex conjugate transpose of Z (the transpose of the complex conjugate of Z).³⁹ The matrices Q and Z are unitary ($Q'Q = Z'Z = I$), and S and T are upper triangular (see Golub and van Loan [A2]). The decomposition is furthermore ordered so the block consisting of the stable generalized eigenvalues (the j th diagonal element of T divided by the j th diagonal element of S , $\lambda_j \equiv t_{jj}/s_{jj}$) comes first and the block of unstable generalized eigenvalues comes last.⁴⁰

More precisely, I assume the *saddle-point property* emphasized by Blanchard and Kahn [A1]: The number of eigenvalues with modulus larger than unity equals the number of nonpredetermined variables. Thus, I assume that $|\lambda_j| > 1$ for $m_1 + 1 \leq j \leq m_1 + m_2$ and $|\lambda_j| < 1$ for $1 \leq j \leq m_1$ (for an exogenous predetermined variable with a unit root, I can actually allow $|\lambda_j| = 1$ for some $1 \leq j \leq m_1$).

Define

$$\begin{bmatrix} \tilde{y}_{1t} \\ \tilde{y}_{2t} \end{bmatrix} \equiv Z' \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}. \quad (\text{B.5})$$

I can interpret $\tilde{y}_{1t}$ as a complex vector of m_1 transformed predetermined variables and $\tilde{y}_{2t}$ as a complex vector of m_2 transformed non-predetermined variables. Premultiply the system (B.2) by

³⁹ Let the elements of the complex matrix Z be denoted $z_{jk} \equiv \text{Re}(z_{jk}) + i \text{Im}(z_{jk})$. Then the complex conjugate of the matrix Z is the matrix of elements $\bar{z}_{jk} \equiv \text{Re}(z_{jk}) - i \text{Im}(z_{jk})$.

⁴⁰ The sorting of the eigenvalues is often done by two programs written by Sims and available at www.princeton.edu/~sims, Qzdiv and Qzswitch.

Q and use (B.3)-(B.5) to write it as

$$\begin{bmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{bmatrix} \begin{bmatrix} E_t \tilde{y}_{1,t+1} \\ E_t \tilde{y}_{2,t+1} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{bmatrix} \begin{bmatrix} \tilde{y}_{1t} \\ \tilde{y}_{2t} \end{bmatrix} + \begin{bmatrix} Q_{11} \\ Q_{21} \end{bmatrix} E_t \theta_{t+1}, \quad (\text{B.6})$$

where S , T , and Q have been partitioned conformably with $\tilde{y}_{1t}$ and $\tilde{y}_{2t}$.

Consider the lower block of (B.6),

$$S_{22} E_t \tilde{y}_{2,t+1} = T_{22} \tilde{y}_{2t} + Q_{21} E_t \theta_{t+1}. \quad (\text{B.7})$$

Since the diagonal terms of S_{22} and T_{22} (s_{jj} and t_{jj} for $m_1 + 1 \leq j \leq m_1 + m_2$) satisfy $|t_{jj}/s_{jj}| > 1$, the diagonal terms of T_{22} are nonzero, the determinant of T_{22} is nonzero, and T_{22} is invertible. Note that S_{22} may not be invertible. I can then solve for $\tilde{y}_{2t}$ as

$$\tilde{y}_{2t} = J E_t \tilde{y}_{2,t+1} + K E_t \theta_{t+1} \quad (\text{B.8})$$

$$= \sum_{\tau=0}^{\infty} J^\tau K E_t \theta_{t+1+\tau} \quad (\text{B.9})$$

for $t \geq 0$, where the complex matrices J and K ($m_2 \times m_2$ and $m_2 \times m_1$, respectively) are given by

$$J \equiv T_{22}^{-1} S_{22}, \quad (\text{B.10})$$

$$K \equiv -T_{22}^{-1} Q_{21}. \quad (\text{B.11})$$

Here, I have exploited that the modulus of the diagonal terms of $T_{22}^{-1} S_{22}$ is less than one. I also assume that $E_t \tilde{y}_{2,t+\tau}$ and $E_t \theta_{t+\tau}$ are sufficiently bounded. Then $J^\tau E_t \tilde{y}_{2,t+\tau} \rightarrow 0$ when $\tau \rightarrow \infty$, and the infinite sum on the right side converges. Note that J may not be invertible, since S_{22} may not be invertible.

I have, by (B.5),

$$y_{1t} = Z_{11} \tilde{y}_{1t} + Z_{12} \tilde{y}_{2t}, \quad (\text{B.12})$$

$$y_{2t} = Z_{21} \tilde{y}_{1t} + Z_{22} \tilde{y}_{2t}, \quad (\text{B.13})$$

where

$$Z \equiv \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \quad (\text{B.14})$$

is partitioned conformably with y_{1t} and y_{2t} . Under the assumption of the saddle-point property, Z_{11} is square. I furthermore assume that Z_{11} is *invertible*. Then I can solve for $\tilde{y}_{1t}$ in (B.12),

$$\tilde{y}_{1t} = Z_{11}^{-1} y_{1t} - Z_{11}^{-1} Z_{12} \tilde{y}_{2t}, \quad (\text{B.15})$$

and use this in (B.13) to get

$$y_{2t} = F_1 y_{1t} + H \tilde{y}_{2t}, \quad (\text{B.16})$$

where the real $m_2 \times m_1$ matrix F_1 and the complex $m_2 \times m_2$ matrix H are given by

$$F_1 \equiv Z_{21} Z_{11}^{-1}, \quad (\text{B.17})$$

$$H \equiv Z_{22} - Z_{21} Z_{11}^{-1} Z_{12}. \quad (\text{B.18})$$

I will show below that H is invertible.

By (B.9) and (B.16), I can then write the solution of y_{2t} as

$$y_{2t} = F_1 y_{1t} + Z_t, \quad (\text{B.19})$$

where Z_t is a real exogenous m_2 -vector stochastic process (not to be confused with the unitary matrix Z in the Schur decomposition) given by

$$Z_t \equiv H \tilde{y}_{2t} \equiv \sum_{\tau=0}^{\infty} R_{\tau} E_t \theta_{t+1+\tau}, \quad (\text{B.20})$$

$$R_{\tau} \equiv H J^{\tau} K \quad (\tau \geq 0), \quad (\text{B.21})$$

where the matrices R_{τ} are real.

I note that the complex conjugate transpose of Z , Z' , satisfies

$$Z' \equiv \begin{bmatrix} Z'_{11} & Z'_{21} \\ Z'_{12} & Z'_{22} \end{bmatrix}, \quad (\text{B.22})$$

where the submatrices are given by (B.14). Since $Z'Z = ZZ' = I$, I have

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} Z'_{11} & Z'_{21} \\ Z'_{12} & Z'_{22} \end{bmatrix} = \begin{bmatrix} Z_{11}Z'_{11} + Z_{12}Z'_{12} & Z_{11}Z'_{21} + Z_{12}Z'_{22} \\ Z_{21}Z'_{11} + Z_{22}Z'_{12} & Z_{21}Z'_{21} + Z_{22}Z'_{22} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}, \quad (\text{B.23})$$

By (B.22), I can write

$$\tilde{y}_{2t} = Z'_{12} y_{1t} + Z'_{22} y_{2t}.$$

Using this in (B.16) gives

$$\begin{aligned} y_{2t} &= F_1 y_{1t} + H(Z'_{12} y_{1t} + Z'_{22} y_{2t}) \\ &= (F_1 + H Z'_{12}) y_{1t} + H Z'_{22} y_{2t}. \end{aligned}$$

It follows that

$$F_1 + H Z'_{12} = 0, \quad (\text{B.24})$$

$$H Z'_{22} = I. \quad (\text{B.25})$$

I can also show (B.24) by using (B.23),

$$\begin{aligned}
Z_{21}Z_{11}^{-1} + (Z_{22} - Z_{21}Z_{11}^{-1}Z_{12})Z'_{12} &= Z_{21}Z_{11}^{-1} + Z_{22}Z'_{12} - Z_{21}Z_{11}^{-1}Z_{12}Z'_{12} \\
&= Z_{21}Z_{11}^{-1} + Z_{22}Z'_{12} - Z_{21}Z_{11}^{-1}(I - Z_{11}Z'_{11}) \\
&= Z_{21}Z_{11}^{-1} + Z_{22}Z'_{12} - Z_{21}Z_{11}^{-1} + Z_{21}Z'_{11} \\
&= 0.
\end{aligned}$$

Similarly, I can show (B.25) by

$$\begin{aligned}
(Z_{22} - Z_{21}Z_{11}^{-1}Z_{12})Z'_{22} &= Z_{22}Z'_{22} - Z_{21}Z_{11}^{-1}Z_{12}Z'_{22} \\
&= Z_{22}Z'_{22} - Z_{21}Z_{11}^{-1}(-Z_{11}Z'_{21}) \\
&= Z_{22}Z'_{22} + Z_{21}Z'_{21} \\
&= I.
\end{aligned}$$

It follows from (B.25) that H is invertible and that its inverse is given by

$$H^{-1} = Z'_{22}. \quad (\text{B.26})$$

It remains to find a solution for $y_{1,t+1}$. The upper block of (B.6) is

$$S_{11}E_t\tilde{y}_{1,t+1} + S_{12}E_t\tilde{y}_{2,t+1} = T_{11}\tilde{y}_{1t} + T_{12}\tilde{y}_{2t} + Q_{11}E_t\theta_{t+1}.$$

Since the diagonal terms of S_{11} and T_{11} satisfy $|t_{jj}/s_{jj}| < 1$, all diagonal terms of S_{11} must be nonzero, so the determinant of S_{11} is nonzero, and S_{11} is invertible. I can then solve for $E_t\tilde{y}_{1,t+1}$ as

$$E_t\tilde{y}_{1,t+1} = S_{11}^{-1}(T_{11}\tilde{y}_{1t} + T_{12}\tilde{y}_{2t}) - S_{11}^{-1}S_{12}E_t\tilde{y}_{2,t+1} + S_{11}^{-1}Q_{11}E_t\theta_{t+1}.$$

By (B.12),

$$\begin{aligned}
\mathbf{E}_t y_{1,t+1} &= Z_{11} \mathbf{E}_t \tilde{y}_{1,t+1} + Z_{12} \mathbf{E}_t \tilde{y}_{2,t+1} \\
&= Z_{11} [S_{11}^{-1} (T_{11} \tilde{y}_{1t} + T_{12} \tilde{y}_{2t}) - S_{11}^{-1} S_{12} \mathbf{E}_t \tilde{y}_{2,t+1} + S_{11}^{-1} Q_{11} \mathbf{E}_t \theta_{t+1}] + Z_{12} \mathbf{E}_t \tilde{y}_{2,t+1} \\
&= Z_{11} S_{11}^{-1} T_{11} \tilde{y}_{1t} + Z_{11} S_{11}^{-1} T_{12} \tilde{y}_{2t} + (Z_{12} - Z_{11} S_{11}^{-1} S_{12}) \mathbf{E}_t \tilde{y}_{2,t+1} + Z_{11} S_{11}^{-1} Q_{11} \mathbf{E}_t \theta_{t+1} \\
&= Z_{11} S_{11}^{-1} T_{11} (Z_{11}^{-1} y_{1t} - Z_{11}^{-1} Z_{12} \tilde{y}_{2t}) + Z_{11} S_{11}^{-1} T_{12} \tilde{y}_{2t} + (Z_{12} - Z_{11} S_{11}^{-1} S_{12}) \mathbf{E}_t \tilde{y}_{2,t+1} \\
&\quad + Z_{11} S_{11}^{-1} Q_{11} \mathbf{E}_t \theta_{t+1} \\
&= Z_{11} S_{11}^{-1} T_{11} Z_{11}^{-1} y_{1t} + Z_{11} S_{11}^{-1} (T_{12} - T_{11} Z_{11}^{-1} Z_{12}) \tilde{y}_{2t} \\
&\quad + (Z_{12} - Z_{11} S_{11}^{-1} S_{12}) \mathbf{E}_t \tilde{y}_{2,t+1} + Z_{11} S_{11}^{-1} Q_{11} \mathbf{E}_t \theta_{t+1} \\
&= Z_{11} S_{11}^{-1} T_{11} Z_{11}^{-1} y_{1t} + Z_{11} S_{11}^{-1} (T_{12} - T_{11} Z_{11}^{-1} Z_{12}) (J \mathbf{E}_t \tilde{y}_{2,t+1} + K \mathbf{E}_t \theta_{t+1}) \\
&\quad + (Z_{12} - Z_{11} S_{11}^{-1} S_{12}) \mathbf{E}_t \tilde{y}_{2,t+1} + Z_{11} S_{11}^{-1} Q_{11} \mathbf{E}_t \theta_{t+1} \\
&= Z_{11} S_{11}^{-1} T_{11} Z_{11}^{-1} y_{1t} \\
&\quad + [Z_{11} S_{11}^{-1} (T_{12} - T_{11} Z_{11}^{-1} Z_{12}) J + (Z_{12} - Z_{11} S_{11}^{-1} S_{12})] \mathbf{E}_t \tilde{y}_{2,t+1} \\
&\quad + Z_{11} S_{11}^{-1} [Q_{11} + (T_{12} - T_{11} Z_{11}^{-1} Z_{12}) K] \mathbf{E}_t \theta_{t+1}, \tag{B.27}
\end{aligned}$$

where I have used (B.15) and (B.8).

It follows that I can use (B.27), (B.20), and (B.26) and write the solution as

$$y_{1,t+1} = M y_{1t} + N \mathbf{E}_t Z_{t+1} + P \mathbf{E}_t \theta_{t+1} + (\theta_{t+1} - \mathbf{E}_t \theta_{t+1}), \tag{B.28}$$

where the real matrices M , N , and P are given by

$$M \equiv Z_{11} S_{11}^{-1} T_{11} Z_{11}^{-1}, \tag{B.29}$$

$$N \equiv [Z_{11} S_{11}^{-1} (T_{12} - T_{11} Z_{11}^{-1} Z_{12}) J + (Z_{12} - Z_{11} S_{11}^{-1} S_{12})] Z'_{22}, \tag{B.30}$$

$$P \equiv Z_{11} S_{11}^{-1} [Q_{11} + (T_{12} - T_{11} Z_{11}^{-1} Z_{12}) K]. \tag{B.31}$$

Thus, the solution to the system (B.1) is given by (B.19) and (B.28) for $t \geq 0$. This results in the solution (A.11)-(A.12) above, where the matrix P in (A.12) is the submatrix of the first n_X rows of the matrix P in (B.31) (since $\theta_{t+1} \equiv (z'_{t+1}, 0')'$).

C. The model when judgment is a finite-order moving average

When the deviation is a finite-order moving-average process and the dynamics of the deviation and judgment is described by (2.16), the model can be written as

$$\begin{bmatrix} X_{t+1} \\ z^{t+1} \\ Cx_{t+1|t} \end{bmatrix} = \bar{A} \begin{bmatrix} X_t \\ z^t \\ x_t \end{bmatrix} + \bar{B}i_t + \begin{bmatrix} \varepsilon_{t+1} \\ \varepsilon^{t+1} \\ 0 \end{bmatrix}, \quad (\text{C.1})$$

where the matrices $\bar{A}$ and $\bar{B}$ are given by

$$\bar{A} \equiv \begin{bmatrix} A_{11} & A_{z12} & A_{12} \\ 0 & A_{z22} & 0 \\ A_{21} & 0 & A_{22} \end{bmatrix}, \quad \bar{B} \equiv \begin{bmatrix} B_1 \\ 0 \\ B_2 \end{bmatrix},$$

the matrix A_z is partitioned conformably with z_t and z^t as

$$A_z \equiv \begin{bmatrix} 0 & A_{z12} \\ 0 & A_{z22} \end{bmatrix},$$

and $\tilde{\varepsilon}_t \equiv (\varepsilon'_t, \varepsilon^{t'})'$ is zero-mean and iid. Thus, this results in the standard forward-looking linear-quadratic model, with the predetermined variables being X_t and z^t . The optimal policy projection can then be described as (2.17) and (2.18), where F and M are finite-dimensional matrices. The intertemporal loss for the optimal policy projection can then be written as

$$\frac{1}{2} \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix}' V \begin{bmatrix} X_t \\ z^t \\ \Xi_{t-1,t-1} \end{bmatrix},$$

where the matrix V is the solution to the Lyapunov equation,

$$V = \bar{W} + \delta M' V M,$$

the symmetric and positive semidefinite matrix $\bar{W}$ is defined by

$$\bar{W} = \begin{bmatrix} I & 0 & 0 \\ F_x & & \\ F_i & & \end{bmatrix}' D' W D \begin{bmatrix} I & 0 & 0 \\ F_x & & \\ F_i & & \end{bmatrix},$$

and the matrix F is partitioned conformably with x_t and i_t as

$$F \equiv \begin{bmatrix} F_x \\ F_i \end{bmatrix}.$$

D. The Marcet-Marimon method to solve the linear-quadratic optimization problem with forward-looking variables

Let $\bar{X}_t \equiv (X_t, z^t)$ and write the model (C.1) as

$$\bar{X}_{t+1} = \bar{A}_{11}\bar{X}_t + \bar{A}_{12}x_t + \bar{B}_1i_t + \tilde{\varepsilon}_{t+1}, \quad (\text{D.1})$$

$$CE_tx_{t+1} = \bar{A}_{21}\bar{X}_t + \bar{A}_{22}x_t + \bar{B}_2i_t. \quad (\text{D.2})$$

Write the period loss function as

$$L_t = \frac{1}{2} \begin{bmatrix} \bar{X}_t \\ x_t \\ i_t \end{bmatrix}' W^0 \begin{bmatrix} \bar{X}_t \\ x_t \\ i_t \end{bmatrix}, \quad (\text{D.3})$$

where the symmetric positive semidefinite matrix W^0 is defined by

$$\begin{bmatrix} \bar{X}_t \\ x_t \\ i_t \end{bmatrix}' W^0 \begin{bmatrix} \bar{X}_t \\ x_t \\ i_t \end{bmatrix} \equiv \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}' D' W D \begin{bmatrix} X_t \\ x_t \\ i_t \end{bmatrix}.$$

Consider the problem in period 0,

$$\min_{\{i_t\}_{t \geq 0}} E_0 \sum_{t=0}^{\infty} \delta^t L_t, \quad (\text{D.4})$$

subject to (D.1), (D.2) and X_0 given. The minimization is taken to be under commitment.

Marcet and Marimon [14] show that this problem can be reformulated as a recursive saddlepoint problem,

$$\max_{\{\gamma_t\}_{t \geq 0}} \min_{\{x_t, i_t\}_{t \geq 0}} E_0 \sum_{t=0}^{\infty} \delta^t \tilde{L}_t, \quad (\text{D.5})$$

where the modified period loss function satisfies

$$\begin{aligned} \tilde{L}_t &\equiv \tilde{L}(\bar{X}_t, \Xi_{t-1}; x_t, i_t, \gamma_t) \\ &\equiv L_t + L_t^1 \\ &\equiv L_t + \gamma_t'(-\bar{A}_{21}\bar{X}_t - \bar{A}_{22}x_t - \bar{B}_2i_t) + \frac{1}{\delta}\Xi_{t-1}'Cx_t, \end{aligned}$$

and the optimization is subject to (D.1), to

$$\Xi_t = \gamma_t, \quad (\text{D.6})$$

and to X_0 and $\Xi_{-1} = 0$ given. The value function for the saddlepoint problem, starting in any period t , satisfies

$$\tilde{V}(\bar{X}_t, \Xi_{t-1}) \equiv \max_{\gamma_t} \min_{(x_t, i_t)} \{ \tilde{L}(\bar{X}_t, \Xi_{t-1}; x_t, i_t, \gamma_t) + \delta E_t \tilde{V}(\bar{X}_{t+1}, \Xi_t) \},$$

subject to (D.1) and (D.6).

Define

$$\tilde{X}_t \equiv \begin{bmatrix} \bar{X}_t \\ \Xi_{t-1} \end{bmatrix}, \quad \tilde{i}_t \equiv \begin{bmatrix} x_t \\ i_t \\ \gamma_t \end{bmatrix},$$

and define $\bar{W}$, $\tilde{A}$, $\tilde{B}$, and $\tilde{C}$ such that

$$\tilde{L}_t \equiv \frac{1}{2} \begin{bmatrix} \tilde{X}_t \\ \tilde{i}_t \end{bmatrix}' \bar{W} \begin{bmatrix} \tilde{X}_t \\ \tilde{i}_t \end{bmatrix}, \quad (\text{D.7})$$

$$\tilde{X}_{t+1} = \tilde{A}\tilde{X}_t + \tilde{B}\tilde{i}_t + \tilde{C}\tilde{\varepsilon}_{t+1}. \quad (\text{D.8})$$

The problem (D.5) subject to (D.8) and given $\tilde{X}_t$ is isomorphic to a standard backward-looking linear-quadratic problem, except being a saddlepoint problem. However, the saddlepoint aspect does not affect the first-order conditions. It is easy to show that the first-order conditions of the saddlepoint problem are identical to those of the original problem, (D.4) subject to (D.1) and (D.2).

The value function for the saddlepoint problem is quadratic,

$$\tilde{V}(\tilde{X}_t) \equiv \frac{1}{2}(\tilde{X}_t' \tilde{V} \tilde{X}_t + \tilde{w}),$$

where $\tilde{V}$ solves the Riccati equation,

$$\tilde{V} = Q + \delta \tilde{A}' \tilde{V} \tilde{A} - (\delta \tilde{B}' \tilde{V} \tilde{A} + N')' (\delta \tilde{B}' \tilde{V} \tilde{B} + R)^{-1} (\delta \tilde{B}' \tilde{V} \tilde{A} + N'),$$

where

$$\bar{W} \equiv \begin{bmatrix} Q & N \\ N' & R \end{bmatrix},$$

is partitioned conformably with $\tilde{X}_t$ and $\tilde{i}_t$.

The optimal reaction function for the saddlepoint problem is linear,

$$\tilde{i}_t = F \tilde{X}_t \equiv \begin{bmatrix} F_x \\ F_i \\ F_\gamma \end{bmatrix} \tilde{X}_t,$$

where F is partitioned conformably with x_t , i_t , and γ_t and satisfies

$$F \equiv -(\delta \tilde{B}' \tilde{V} \tilde{B} + R)^{-1} (\delta \tilde{B}' \tilde{V} \tilde{A} + N').$$

This reaction function is the optimal reaction function for the original problem. Optimization in a timeless perspective in period t corresponds to taking Ξ_{t-1} from the previous period's decision problem as given, also in period 0.

The equilibrium dynamics will be given by

$$\begin{aligned}\tilde{X}_{t+1} &= M\tilde{X}_t + \tilde{C}\varepsilon_{t+1}, \\ x_t &= F_x\tilde{X}_t, \\ i_t &= F_i\tilde{X}_t, \\ L_t &= \frac{1}{2}\tilde{X}_t'\tilde{W}\tilde{X}_t,\end{aligned}$$

where

$$\begin{aligned}M &\equiv \tilde{A} + \tilde{B}\tilde{F}, \\ \tilde{W} &\equiv \begin{bmatrix} I & 0 \\ F_x & \\ F_i & \end{bmatrix}' W^0 \begin{bmatrix} I & 0 \\ F_x & \\ F_i & \end{bmatrix}.\end{aligned}$$

The value function for the saddlepoint problem can be decomposed according to

$$\frac{1}{2}(\tilde{X}_t'\tilde{V}\tilde{X}_t + \tilde{w}) \equiv \frac{1}{2}(\tilde{X}_t'V\tilde{X}_t + w) + \frac{1}{2}(\tilde{X}_t'V^1\tilde{X}_t + w^1),$$

where

$$\frac{1}{2}(\tilde{X}_t'V\tilde{X}_t + w) \equiv \mathbb{E}_t \sum_{\tau=0}^{\infty} \delta^{\tau-t} \frac{1}{2}\tilde{X}_{t+\tau}'\tilde{W}\tilde{X}_{t+\tau},$$

is the value function for the original problem starting in period t with $\tilde{X}_t \equiv (X_t', \Xi_{t-1}')'$ given. The matrix V will satisfy the Lyapunov equation,

$$V = \tilde{W} + \delta M'VM,$$

and, when $\delta < 1$, the constant w will satisfy

$$w = \frac{\delta}{1-\delta} \text{tr}(\tilde{C}'V\tilde{C}\Sigma_{\tilde{\varepsilon}\tilde{\varepsilon}}),$$

where $\Sigma_{\tilde{\varepsilon}\tilde{\varepsilon}}$ is the covariance matrix for $\tilde{\varepsilon}_t$.

E. An alternative finite-horizon numerical procedure for forward-looking models

In the finite-horizon model in section 3.1, there is an obvious alternative numerical procedure that will provide a projection arbitrarily close to the optimal policy projection without requiring such a long horizon that $X_{t+T,t}$ and $\Xi_{t+T-1,t}$ are close to their steady-state levels. It requires iterations, though.

Assume that iteration $j-1$ has resulted in $\Xi_{t+T-1,t}^{(j-1)}$. Start iteration j by using (2.17) and (2.18) to replace (3.3) by

$$x_{t+T+1,t} = F_x M_1 \begin{bmatrix} X_{t+T,t} \\ \Xi_{t+T-1,t}^{(j-1)} \end{bmatrix},$$

where the matrices F_1 and M_1 are defined by

$$F \begin{bmatrix} X_t \\ 0 \\ \Xi_{t-1} \end{bmatrix} \equiv F_1 \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix}, \quad M \begin{bmatrix} X_t \\ 0 \\ \Xi_{t-1} \end{bmatrix} \equiv M_1 \begin{bmatrix} X_t \\ \Xi_{t-1} \end{bmatrix},$$

and F_1 is partitioned conformably with x_t and i_t as

$$F_1 \equiv \begin{bmatrix} F_x \\ F_i \end{bmatrix}.$$

Consequently, replace (3.4) by

$$-A_{21}X_{t+T,t} - A_{22}x_{t+T,t} - B_2i_{t+T,t} + CF_x M_1 \begin{bmatrix} X_{t+T,t} \\ \Xi_{t+T-1,t}^{(j-1)} \end{bmatrix} = 0. \quad (\text{E.1})$$

Use (3.1), (3.2), and (E.1) to construct G and g^t (the left submatrix of the matrix $CF_x M_1$ will enter the last block of G and the product of the right submatrix and $\Xi_{t+T-1,t}^{(j-1)}$ will enter the last block of g^t). Furthermore, add the term (3.7) with $\Xi_{t+T-1,t} = \Xi_{t+T-1,t}^{(j-1)}$ to the loss function (that is, modify the diagonal block of Ω that corresponds to $X_{t+T,t}$ and add a linear term that corresponds to the cross products of $X_{t+T,t}$ and $\Xi_{t+T-1,t}^{(j-1)}$). Find the optimal policy projection $\hat{s}^{t(j)}$ and Lagrange multiplier $\Lambda^{t(j)}$ via the analogue of (3.12). This ends iteration j and results in $\Xi_{t+T-1,t}^{(j)}$. Continue until $\Xi_{t+T-1,t}^{(j)}$ has converged.

Obviously this alternative procedure does not require that $X_{t+T,t}$ and $\Xi_{t+T-1,t}$ are close to their steady-state levels. Which procedure is fastest will depend on the number of variables in the problem and the rate of convergence towards the steady state of the optimal policy projection.

F. The feasible set of projections of the states of the economy, the feasible set of projections of the target variables, and the optimal targeting rule

In the finite-horizon projection model in section 3.1, the feasible set of projections in period t of the states of the economy, $\mathcal{S}_t$, is the set of projections s^t that satisfy (3.5), repeated here as

$$Gs^t = g^t. \quad (\text{F.1})$$

That is, $\mathcal{S}_t$ is the set of solutions to (F.1) for given G and g^t . Define $n \equiv (T+1)(n_X + n_x + n_i)$, $m \equiv (T+1)(n_X + n_x) < n$, and $p \equiv (T+1)n_i \equiv n - m$. Note that G is $m \times n$, s^t is $n \times 1$, and g^t is $m \times 1$. Assume that G is of rank m .

Since G is of rank m , the set of solutions to (F.1) is a linear manifold of R^n of dimension $p \equiv n - m$.⁴¹ It can be written as the set of projections s^t that satisfy

$$s^t = G^+g^t + (I - G^+G)\xi \quad (\text{F.2})$$

for any $\xi \in R^n$ (see Harville [A2, chapters 11 and 20]). Here, the $n \times m$ matrix G^+ is the Moore-Penrose inverse of G . When G is $m \times n$ and of rank m , the Moore-Penrose inverse is given by

$$G^+ = G'(GG')^{-1}$$

(note that GG' is $m \times m$, of rank m , and hence invertible). Then, $G^+G = G'(GG')^{-1}G$ is a projection matrix that projects vectors in R^n on the m -dimensional column space of the $n \times m$ matrix G' , the transpose of G .⁴² Denote the column space of G' by $\mathcal{C}(G')$. For any ξ in R^n , the vector $G^+G\xi$ lies in $\mathcal{C}(G')$. Then $I - G^+G$ is a projection matrix that projects vectors in R^n off the column space of G' , that is, on the p -dimensional subspace of R^n orthogonal to $\mathcal{C}(G')$, the orthogonal complement of $\mathcal{C}(G')$ (relative to R^n), denoted $\mathcal{C}^\perp(G')$. Hence, the solution set $\mathcal{S}_t$ consist of $\mathcal{C}^\perp(G')$ shifted away from the origin by the vector G^+g^t ,

$$\mathcal{S}_t = \{G^+g^t\} + \mathcal{C}^\perp(G').$$

Furthermore, the vector G^+g^t is the s^t of minimum norm that satisfies (F.1). Then, G^+g^t is orthogonal to the solution set $\mathcal{S}_t$ and lies in the column space of G , $\mathcal{C}(G')$.⁴³

Figure F.1 provides an illustration of the above, when $n = 2$ and $m = p = 1$. The linear manifold $\mathcal{S}_t$, the set of feasible projections of the states of the economy, s^t , is shown as the negatively sloped line through the point $s^t = G^+g^t$. The column space $\mathcal{C}(G')$ is the positively sloped line through the origin. The linear manifold $\mathcal{S}_t$ is orthogonal to the column space. The orthogonal complement of the column space, $\mathcal{C}^\perp(G')$, is the negatively sloped line through the origin. The linear manifold is the orthogonal complement shifted away from the origin to the point G^+g^t . Furthermore, the point G^+g^t is the point in the linear manifold with the shortest distance to the origin.

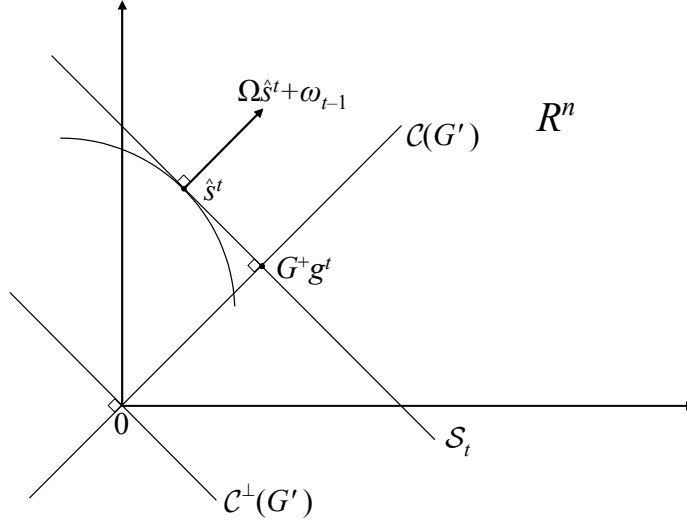
Let $G^\perp$ denote a $p \times n$ matrix with p linearly independent rows, each of which is orthogonal to the m rows of G . Then $\mathcal{C}^\perp(G') = \mathcal{C}(G^\perp)$, where the latter expression denotes the column space of

⁴¹ Let V be a linear space. A subset S of V is a linear manifold of V (also called a linear variety of V), if there is a v in V such that the set $S - \{v\} \equiv \{s - v | s \in S\}$ is a subspace of V . The dimension of S is the dimension of $S - \{v\}$. Hence, a linear manifold is a subspace that has possibly been shifted away from the origin (in the above case by the vector v).

⁴² In this section, the word “projection” is used not only to refer to mean forecasts but also, depending on the context, to refer to mathematical projections in linear space.

⁴³ A vector is orthogonal to a linear manifold if it is orthogonal to the corresponding subspace.

Figure F.1: The set of feasible projections of the state of the economy, $\mathcal{S}_t$



$G^{\perp'}$, and $\mathcal{S}_t$ can be written as the set of projections s^t that satisfy

$$s^t = G^+g^t + G^{\perp'}\xi$$

for any $\xi \in R^n$.

The projection of the target variables, Y^t , is a linear function of the projection of the states of the economy according to (3.6), repeated here as

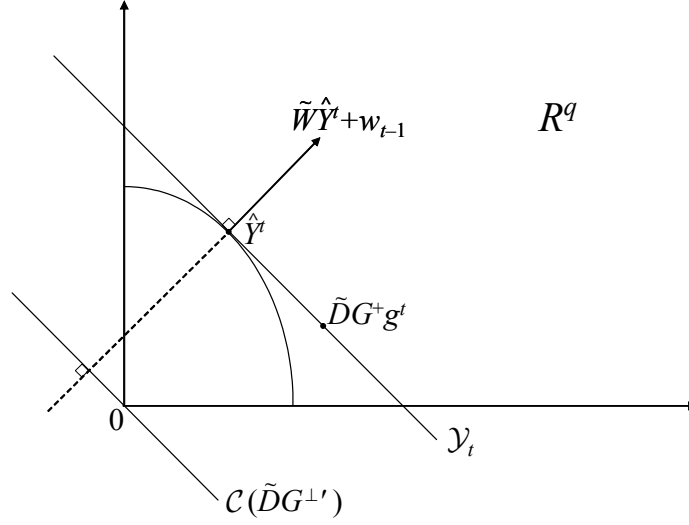
$$Y^t = \tilde{D}s^t. \quad (\text{F.3})$$

Let $q \equiv (T+1)n_Y \leq n$, note that Y^t is $q \times 1$ and $\tilde{D}$ is $q \times n$, and take $\tilde{D}$ to be of rank q . It follows that the set of feasible projections of the target variables, $\mathcal{Y}_t$, consists of the set of projections Y^t that satisfy

$$Y^t = \tilde{D}G^+g^t + \tilde{D}G^{\perp'}\xi$$

for any ξ in R^n . This is a linear manifold of R^q of dimension at most $\min(p, q)$. If I take as the normal case that the number of target variables is at least as large as the number of instruments, $n_Y \geq n_i$ (typically, there are at least two target variables, inflation and the output gap, but only one instrument, the instrument rate), I have $q \geq p$, and the set of feasible projections of the target variables, $\mathcal{Y}_t$, is a linear manifold of R^q of dimension at most $p \leq q$. The matrix $\tilde{D}$ simply maps the p -dimensional linear manifold $\mathcal{S}_t$ of R^n into the at most p -dimensional linear manifold $\mathcal{Y}_t$ of R^q .

Figure F.2: The set of feasible projections of the target variables, $\mathcal{Y}_t$



It follows that $\mathcal{Y}_t$ is the at most p -dimensional column space $\mathcal{C}(\tilde{D}G^{\perp'})$ in R^q shifted away from the origin by the vector $\tilde{D}G^+g^t$,

$$\mathcal{Y}_t = \{\tilde{D}G^+g^t\} + \mathcal{C}(\tilde{D}G^{\perp'}).$$

Figure F.2 provides an illustration of the above, when $q = 2$ and $p = 1$. The linear manifold $\mathcal{Y}_t$, the set of feasible projections of the target variables, Y^t , is shown as the negatively sloped line through the point $Y^t = \tilde{D}G^+g^t$. The column space of the matrix $\tilde{D}G^{\perp'}$, $\mathcal{C}(\tilde{D}G^{\perp'})$, is shown as the negative sloped line through the origin. The linear manifold $\mathcal{Y}_t$ is this column space shifted away from the origin to the point $\tilde{D}G^+g^t$.

F.1. An optimal targeting rule for the forward-looking model

Consider the first-order condition for optimal policy under commitment in a timeless perspective in the forward-looking model, (3.10), rewritten here as

$$\Omega s^t + \omega_{t-1} + G' \Lambda^t = 0 \tag{F.4}$$

The *optimal targeting rule* is the first-order condition in terms of Y^t when the Lagrange multiplier has been eliminated.

Let me interpret the first-order condition in terms of s^t , eliminate the Lagrange multiplier, and interpret the resulting targeting rule. Note that Ω is $n \times n$, s^t and ω_{t-1} are $n \times 1$, G' is $n \times m$ and of rank m , and Λ^t is $m \times 1$.

Write the first-order condition as

$$\Omega s^t + \omega_{t-1} = G'(-\Lambda^t). \quad (\text{F.5})$$

The term $\Omega s^t + \omega_{t-1}$ on the left side is the gradient of the loss function with respect to s^t , a vector in R^n . The condition (F.5) can be interpreted as stating that the gradient of the loss function is an element of the m -dimensional column space of the $n \times m$ matrix G' , $\mathcal{C}(G')$, with $-\Lambda^t$ providing the coefficients of the corresponding linear combination of the column vectors of G' . This is equivalent to the tangency of the loss function's iso-loss surface in R^n with the feasible set of projections, $\mathcal{S}_t$. The gradient of the loss function is orthogonal to the iso-loss surface. Tangency of the iso-loss surface with $\mathcal{S}_t$ is then equivalent to the gradient being orthogonal to $\mathcal{S}_t$. The subspace orthogonal to $\mathcal{S}_t$ is $\mathcal{C}(G')$, as noted above.

This is illustrated in figure F.1 when $n = 2$ and $m = p = 1$. The curve shows part of the iso-loss surface of the loss function that is tangential to the linear manifold $\mathcal{S}_t$. The tangency occurs at the optimal policy projection, $\hat{s}^t$. The gradient of the loss function at that point, $\Omega \hat{s}^t + \omega_{t-1}$, is shown as the vector pointing northeast from that point. Tangency between the iso-loss surface and the linear manifold is equivalent to the gradient being orthogonal to the linear manifold, or the gradient being an element in the column space, $\mathcal{C}(G')$.

In order to eliminate the Lagrange multipliers, premultiply (F.5) by G ,⁴⁴

$$G(\Omega s^t + \omega_{t-1}) = GG'(-\Lambda^t). \quad (\text{F.6})$$

Exploit that GG' is $m \times m$, of rank m , and hence invertible, and solve for $-\Lambda^t$,

$$-\Lambda^t = (GG')^{-1}G(\Omega s^t + \omega_{t-1}). \quad (\text{F.7})$$

(The matrix $(GG')^{-1}G$ is actually the Moore-Penrose inverse of G' , G'^+ , where G' is $n \times m$ with rank m .) Substitution of Λ^t in (F.4) gives

$$M(\Omega s^t + \omega_{t-1}) = 0, \quad (\text{F.8})$$

where M is the $n \times n$ matrix (not to be confused with the matrix denoted M in other sections of this paper)

$$M \equiv I - G'(GG')^{-1}G = I - G^+G.$$

⁴⁴ One might ask why multiplying with the matrix G with rank $m < n$ rather than a matrix with full rank n does not lose any information of (F.5). More formally, let $G^\perp$ be a $p \times n$ matrix whose p rows are linearly independent and orthogonal to the m rows of G . That is, the column space of $G^{\perp'}$ is the space in R^n orthogonal to the column space of G' . Then the $n \times n$ matrix $\begin{bmatrix} G \\ G^\perp \end{bmatrix}$ is of full rank. Multiplying (F.5) by this matrix leads to the m equations of (F.6) and p additional trivial equations of zero equals zero, since we know that the left and right sides of (F.5) lie in the column space of G' .

As noted above, M is the projection matrix that projects vectors in R^n on the p -dimensional orthogonal complement of the column space of G' , $\mathcal{C}^\perp(G')$. Hence, (F.8) states that the projection on $\mathcal{C}^\perp(G')$ of the gradient of the loss function is zero. Of course, this follows directly from the observation above that the gradient lies in $\mathcal{C}(G')$.

In any case, the optimal targeting rule in terms of s^t is equivalent to the statement that the gradient is orthogonal to the feasible set of projections of the states of the economy, $\mathcal{S}_t$, which can be expressed algebraically as (F.8).

However, (F.8) involves n equations, but only p independent equations. It is hence desirable to condense (F.8) to only p equations. The projection matrix M is a symmetric idempotent matrix of rank p . Then its spectrum consists of p eigenvalues equal to one and m eigenvalues equal to zero, and it can be decomposed as

$$M = Q \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix} Q' \equiv \begin{bmatrix} Q_p & Q_m \end{bmatrix} \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} Q'_p \\ Q'_m \end{bmatrix} \equiv Q_p Q'_p.$$

Here Q is the orthonormal $n \times n$ matrix whose columns are the eigenvectors of M , I_p is the $p \times p$ identity matrix, and Q_p is the $n \times p$ matrix whose columns are the p eigenvectors corresponding to the p nonzero eigenvalues. Then, pre-multiplying (F.8) by Q' gives the p nontrivial equations,

$$Q'_p(\Omega s^t + \omega_{t-1}) = 0, \tag{F.9}$$

and m trivial equations of zero equals zero.

Furthermore, (F.9) is expressed in terms of the projection of the states of the economy, s^t . In order to express it in terms of the projection of the target variables, Y^t , note that, by the definition of Ω for the forward-looking model in section 3.1,

$$\Omega s^t \equiv \tilde{D}' \tilde{W} \tilde{D} s^t \equiv \tilde{D}' \tilde{W} Y^t,$$

where $\tilde{W}$ is a symmetric positive semidefinite block-diagonal $(T+1)n_Y$ matrix with the $(\tau+1)$ -th diagonal block being $\delta^\tau W$ for $0 \leq \tau \leq T$. Hence, I can write (F.9) as involving only the target variables and, through the vector ω_{t-1} , the Lagrange multiplier $\Xi_{t-1,t-1}$ from the optimization in period $t-1$,

$$Q'_p[\tilde{D}' \tilde{W} Y^t + \omega_{t-1}] = 0. \tag{F.10}$$

This is one concise form of the targeting rule. The history-dependence of the optimal policy under commitment in a timeless perspective enters via $\Xi_{t-1,t-1}$.

By combining (F.9) with (3.5), I get

$$\begin{bmatrix} G \\ Q'_p \Omega \end{bmatrix} s^t = \begin{bmatrix} g^t \\ -Q'_p \omega_{t-1} \end{bmatrix},$$

and

$$\begin{aligned} \hat{s}^t &= \begin{bmatrix} G \\ Q'_p \Omega \end{bmatrix}^{-1} \begin{bmatrix} g^t \\ -Q'_p \omega_{t-1} \end{bmatrix} \equiv H \begin{bmatrix} X_t \\ \Xi_{t-1,t-1} \\ z^t \end{bmatrix}, \\ \hat{Y}^t &= \tilde{D} \hat{s}^t = \tilde{D} H \begin{bmatrix} X_t \\ \Xi_{t-1,t-1} \\ z^t \end{bmatrix}. \end{aligned} \tag{F.11}$$

From (F.7) and (F.11), I can extract

$$\Xi_{t,t} = H \Xi \begin{bmatrix} X_t \\ \Xi_{t-1,t-1} \\ z^t \end{bmatrix},$$

to be used in the intertemporal loss function for the decision problem in period $t + 1$.

If the forward-looking variables, x_t , are target variables—elements of Y_t —the intertemporal loss function with the added term can be written

$$\frac{1}{2} Y^{t'} \tilde{W} Y^t + w'_{t-1} Y^t,$$

where w_{t-1} is a q -vector whose only nonzero elements contain the vector $(\Xi_{t-1,t-1} \frac{1}{\delta} C)'$ such that $w'_{t-1} Y^t \equiv \Xi_{t-1,t-1} \frac{1}{\delta} C x_{t,t}$. Then, the optimal targeting rule can be expressed as the gradient, $\tilde{W} Y^t + w_{t-1}$, being orthogonal to the linear manifold $\mathcal{Y}_t$. Suppose $\mathcal{Y}_t$ is of dimension p , and let $F \equiv \tilde{D} G^{\perp'}$ (not to be confused with the matrix denoted F in other sections of the paper). The projection matrix that projects vectors in R^q on the p -dimensional subspace $\mathcal{Y}_t - \{\tilde{D} G^+ g^t\}$ is then $F(F'F)^{-1}F'$, so the condition that the gradient is orthogonal to the linear manifold $\mathcal{Y}^t$ can be written as the p equations.

$$F(F'F)^{-1}F' \tilde{W}(\Omega Y^t + w_{t-1}) = 0.$$

This is the optimal targeting rule for this case.

This case is illustrated in figure F.2. The curve in the figure shows a part of the iso-loss surface of the loss function that is tangential to the linear manifold $\mathcal{Y}_t$. The tangency point is the optimal policy projection of the target variables, $\hat{Y}^t$. The gradient of the loss function at that point, $\tilde{W} \hat{Y}^t + w_{t-1}$, is shown as the vector at that point that points northeast. It is orthogonal to the linear manifold.

Svensson [25] interprets optimal targeting rules in terms of the equality between the marginal rates of transformation and marginal rates of substitution between the target variables. A vector

of marginal rates of transformation between the target variables is a vector in the column space $\mathcal{C}(\tilde{D}G^{\perp'})$, the subspace associated with $\mathcal{Y}_t$. A vector of marginal rates of substitution between the target variables is a vector in the tangent space of the intertemporal loss function, the subspace orthogonal to the gradient of the loss function. Equality between the marginal rates of transformation and substitution is equivalent to the gradient being orthogonal to $\mathcal{Y}_t$, that is, the iso-loss surface being tangential to $\mathcal{Y}_t$.

G. An optimal restricted instrument rule

Add to the model (2.1) an explicit instrument rules of the form

$$i_t = f_X X_t, \quad (\text{G.1})$$

where the $n_i \times n_X$ matrix f_X is restricted to be an element $f_X \in \mathcal{F}$ of a given class $\mathcal{F}$ of instrument rules. Assume that the deviation z_t is an iid zero-mean process with variance-covariance matrix Σ . Let the loss function in period t be

$$\lim_{\delta \rightarrow 1} \mathbb{E}_t \sum_{\tau=0}^{\infty} (1 - \delta) \delta^\tau L_{t+\tau} = \mathbb{E}[L_t],$$

where L_t is given by (2.7). By appendix A, for a given instrument rule f_X , the conditional loss in period t is, for a given δ ($0 < \delta < 1$), given by

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} (1 - \delta) \delta^\tau L_{t+\tau} = \frac{1}{2} \{ (1 - \delta) X_t' V(f_X, \delta) X_t + \delta \text{trace}[V(f_X, \delta) \Sigma] \},$$

where $V(f_X, \delta)$ is a symmetric positive semidefinite $n_X \times n_X$ matrix that depends on A , B , C , D , W , f_X , and δ . It follows that

$$\mathbb{E}[L_t] = \frac{1}{2} \text{trace}[V(f_X, 1) \Sigma].$$

The optimal restricted instrument rule, $\hat{f}_X$, is then given by

$$\hat{f}_X = \arg \min_{f_X \in \mathcal{F}} \frac{1}{2} \text{trace}[V(f_X, 1) \Sigma].$$

It depends on the class $\mathcal{F}$ and the variance-covariance matrix Σ , in addition to A , B , C , D , and W .

Note that there is little point in considering implicit instrument rules here,

$$i_t = f_X X_t + f_x x_t. \quad (\text{G.2})$$

For any such implicit instrument rule $f \equiv [f_X \ f_x]$ for which a unique equilibrium exists,

$$x_t = g(f)X_t,$$

where the matrix $g(f)$ depends on f . Then,

$$i_t = [f_X + f_x g(f)]X_t \equiv \tilde{f}_X(f)X_t.$$

That is, for each implicit instrument rule f for which there is a unique equilibrium, there is a unique explicit instrument rule $\tilde{f}_X(f)$ consistent with that equilibrium. Furthermore, for any explicit instrument rule f_X in (G.1) for which there is a unique equilibrium, there is a *continuum* of implicit instrument rules consistent with that equilibrium. For any given instrument rule f_X for which there exists a unique equilibrium, $x_t = g(f_X)X_t$, where the matrix $g(f_X)$ depends on f_X . For any arbitrary $n_i \times n_x$ matrix f_x , I can then write

$$i_t = f_X X_t + f_x [x_t - g(f_X)X_t] = [f_X - f_x g(f_X)]X_t + f_x x_t \equiv \tilde{f}_X(f_X, f_x)X_t + f_x x_t.$$

The only reason for considering implicit instrument rules rather than an explicit instrument rule in this context (when the deviation is an iid zero-mean shock) is when an explicit instrument rule has a determinacy problem—multiple equilibria—in which case one may be able to find a corresponding implicit instrument rule for which there is a unique equilibrium. Svensson and Woodford [30] examine such issues further.

H. An empirical backward-looking model

The two equations of the model of Rudebusch and Svensson [18] are

$$\pi_{t+1} = \alpha_{\pi 1}\pi_t + \alpha_{\pi 2}\pi_{t-1} + \alpha_{\pi 3}\pi_{t-2} + \alpha_{\pi 4}\pi_{t-3} + \alpha_y y_t + z_{\pi, t+1} \quad (\text{H.1})$$

$$y_{t+1} = \beta_{y1}y_t + \beta_{y2}y_{t-1} - \beta_r \left(\frac{1}{4}\sum_{j=0}^3 i_{t-j} - \frac{1}{4}\sum_{j=0}^3 \pi_{t-j} \right) + z_{y, t+1}, \quad (\text{H.2})$$

where π_t is quarterly inflation in the GDP chain-weighted price index (p_t) in percentage points at an annual rate, i.e., $400(\ln p - \ln p_{t-1})$; i_t is the quarterly average federal funds rate in percentage points at an annual rate; y_t is the relative gap between actual real GDP (q_t) and potential GDP (q_t^*) in percentage points, i.e., $100(q_t - q_t^*)/q_t^*$. These five variables were de-measured prior to estimation, so no constants appear in the equations.

The estimated parameters, using the sample period 1961:1 to 1996:2, are shown in table H.1.

Table H.1

$\alpha_{\pi 1}$	$\alpha_{\pi 2}$	$\alpha_{\pi 3}$	$\alpha_{\pi 4}$	α_y	β_{y1}	β_{y2}	β_r
0.70	-0.10	0.28	0.12	0.14	1.16	-0.25	0.10
(0.08)	(0.10)	(0.10)	(0.08)	(0.03)	(0.08)	(0.08)	(0.03)

The hypothesis that the sum of the lag coefficients of inflation equals one has a p -value of .16, so this restriction was imposed in the estimation.⁴⁵

The state-space form can be written

$$\begin{bmatrix} \pi_{t+1} \\ \pi_t \\ \pi_{t-1} \\ \pi_{t-2} \\ y_{t+1} \\ y_t \\ i_t \\ i_{t-1} \\ i_{t-2} \end{bmatrix} = \begin{bmatrix} \sum_{j=1}^4 \alpha_{\pi j} e_j + \alpha_y e_5 \\ e_1 \\ e_2 \\ e_3 \\ \beta_r e_{1:4} + \beta_{y1} e_5 + \beta_{y2} e_6 - \beta_r e_{7:9} \\ e_5 \\ e_0 \\ e_7 \\ e_8 \end{bmatrix} \begin{bmatrix} \pi_t \\ \pi_{t-1} \\ \pi_{t-2} \\ \pi_{t-3} \\ y_t \\ y_{t-1} \\ i_{t-1} \\ i_{t-2} \\ i_{t-3} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\frac{\beta_r}{4} \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} i_t + \begin{bmatrix} z_{\pi,t+1} \\ 0 \\ 0 \\ 0 \\ z_{y,t+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where e_j ($j = 0, 1, \dots, 9$) denotes a 1×9 row vector, for $j = 0$ with all elements equal to zero, for $j = 1, \dots, 9$ with element j equal to unity and all other elements equal to zero; and where $e_{j:k}$ ($j < k$) denotes a 1×9 row vector with elements $j, j+1, \dots, k$ equal to $\frac{1}{4}$ and all other elements equal to zero. The predetermined variables are $\pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, y_t, y_{t-1}, i_{t-1}, i_{t-2}, i_{t-3}$, and i_{t-3} . There are no forward-looking variables.

For a loss function (5.3) with $\delta = 1$, $\lambda = 1$, and $\nu = 0.2$, and the case where z_t is an iid zero-mean shock; the optimal reaction function (2.21) is (the coefficients are rounded to two decimal points),

$$i_t = 1.22 \pi_t + 0.43 \pi_{t-1} + 0.53 \pi_{t-2} + 0.18 \pi_{t-3} + 1.93 y_t - 0.49 y_{t-1} + 0.36 i_{t-1} - 0.09 i_{t-2} - 0.05 i_{t-3}.$$

I. An empirical forward-looking model

An empirical New Keynesian model estimated by Lindé [13] is

$$\begin{aligned} \pi_t &= \omega_f \pi_{t+1|t} + (1 - \omega_f) \pi_{t-1} + \gamma y_t + z_{\pi t}, \\ y_t &= \beta_f y_{t+1|t} + (1 - \beta_f) (\beta_{y1} y_{t-1} + \beta_{y2} y_{t-2} + \beta_{y3} y_{t-3} + \beta_{y4} y_{t-4}) - \beta_r (i_t - \pi_{t+1|t}) + z_{yt}, \end{aligned}$$

where the restriction $\sum_{j=1}^4 \beta_{yj} = 1$ is imposed. The estimated coefficients are (Table 6a in Lindé [13], non-farm business output) are shown in table I.1.

⁴⁵ This p -value was obtained by simulating the above inflation equation 1000 times and ranking the sum of coefficients from the unrestricted Phillips curve estimated from the actual data (i.e., 0.969) in the set of unrestricted sums estimated from the simulated data. This is in the spirit of Rudebusch [A5]. For comparison, the simple t -test gives a p -value of 0.42.

Table I.1

ω_f	γ	β_f	β_r	β_{y1}	β_{y2}	β_{y3}
0.457	0.048	0.425	0.156	1.310	-0.229	-0.011
(0.065)	(0.007)	(0.027)	(0.016)	(0.174)	(0.279)	(0.037)

For simplicity, I set $\beta_{y1} = 1$, $\beta_{y2} = \beta_{y3} = \beta_{y4} = 0$. Then the state-space form can be written as

$$\begin{bmatrix} \pi_t \\ y_t \\ i_t \\ z_{\pi,t+1} \\ z_{y,t+1} \\ \omega_f \pi_{t+1|t} \\ \beta_r \pi_{t+1|t} + \beta_f y_{t+1|t} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(1-\omega_f) & 0 & 0 & -1 & 0 & 1 & -\gamma \\ 0 & -(1-\beta_f) & 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ y_{t-1} \\ i_{t-1} \\ z_{\pi t} \\ z_{y t} \\ \pi_t \\ y_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \beta_r \end{bmatrix} i_t + \begin{bmatrix} 0 \\ 0 \\ 0 \\ z_{\pi,t+1} \\ z_{y,t+1} \\ 0 \\ 0 \end{bmatrix}.$$

The predetermined variables are π_{t-1} , y_{t-1} , i_{t-1} , $z_{\pi t}$, and $z_{y t}$, and the forward-looking variables are π_t and y_t .

For a loss function (5.3) with $\delta = 1$, $\lambda = 1$, and $\nu = 0.2$, and the case where z_t is an iid zero-mean shock; the optimal reaction function (2.21) is (the coefficients are rounded to two decimal points),

$$i_t = 0.58 \pi_{t-1} + 0.80 y_{t-1} + 0.41 i_{t-1} + 1.06 z_{\pi t} + 1.38 z_{y t} + 0.02 \Xi_{\pi,t-1,t-1} + 0.20 \Xi_{y,t-1,t-1},$$

where $\Xi_{\pi,t-1,t-1}$ and $\Xi_{y,t-1,t-1}$ are the Lagrange multipliers for the two equations for the forward-looking variables in the decision problem in period $t - 1$.

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Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements*

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We investigate the effects of U.S. monetary policy on asset prices using a high-frequency event-study analysis. We test whether these effects are adequately captured by a single factor—changes in the federal funds rate target—and find that they are not. Instead, we find that two factors are required. These factors have a structural interpretation as a “current federal funds rate target” factor and a “future path of policy” factor, with the latter closely associated with Federal Open Market Committee statements. We measure the effects of these two factors on bond yields and stock prices using a new intraday data set going back to 1990. According to our estimates, both monetary policy actions and statements have important but differing effects on asset prices, with statements having a much greater impact on longer-term Treasury yields.

JEL Codes: E52, E58, E43, G14.

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The Federal Reserve's announcement following its January 28, 2004, policy meeting led to one of the largest reactions in the Treasury market on record, with two- and five-year yields jumping 20 and 25 basis points (bp) respectively in the half-hour surrounding the announcement—the largest movements around any Federal Open Market Committee (FOMC) announcement over the fourteen years for which we have data. Even more remarkably, this outsized reaction was spurred not by what the FOMC *did*, but rather by what it *said*: indeed, the decision to leave the current federal funds rate unchanged was completely anticipated by financial markets, but the FOMC's decision to drop the phrase “policy accommodation can be maintained for a considerable period” from its accompanying statement and replace it with “the Committee believes it can be patient in removing its policy accommodation” was read by financial markets as indicating that the FOMC would begin tightening policy sooner than previously expected.¹ On this date, then, treating the monetary policy action as a 0 bp surprise change in the current federal funds rate target would be missing the whole story.

In this paper, we investigate the extent to which this observation is true more generally: are the effects of monetary policy announcements on asset prices adequately characterized by a single factor, namely the surprise component of the change in the current federal funds rate target? We perform a test of this hypothesis using the rank test of Cragg and Donald (1997) and strongly reject the hypothesis of a single factor. By contrast, we do not reject the hypothesis that the effects of monetary policy on asset prices are characterized by two factors. By performing a suitable rotation of these unobserved factors, we show that they can be given a structural interpretation as a “current federal funds rate target” factor, corresponding to surprise changes in the current federal funds rate target, and a “future path of policy” factor, corresponding to changes in futures rates out to horizons of one year that are independent of changes in the current funds rate target. We show that this latter (“path”) factor has typically been associated with significant changes in FOMC statements, such as the January 28 example above. In this way, we generalize and

¹For example, the front page of *The Wall Street Journal* reported the following morning that “investors interpreted the omission of ‘considerable period’ as a signal that the Fed is closer to raising interest rates than many thought” (Ip 2004).

improve the single-factor analysis of the effects of monetary policy on asset prices performed by earlier authors, such as Cook and Hahn (1989), Kuttner (2001), Cochrane and Piazzesi (2002), Rigobon and Sack (2004), Ellingsen and Soderstrom (2003), and Bernanke and Kuttner (forthcoming).

To measure the effects of monetary policy actions and statements on asset prices, we construct a new data set that captures changes in asset prices in a thirty-minute window and in a one-hour window bracketing every FOMC announcement going back to 1990 (we use the term *announcement* to refer to any means by which a policy decision was communicated to financial markets, including open market operations as well as explicit press releases). The use of intraday data allows us to better isolate the response of asset prices to monetary policy announcements, since we can be almost certain that no other economic news was released within such a brief interval of time. The gains from using intraday data are most apparent in the early 1990s, when on a number of occasions the FOMC eased policy just a few hours after a weak employment report released earlier in the day.

In estimating the separate effects of monetary policy actions and statements, we are not suggesting that FOMC statements represent a policy tool that is completely independent of the federal funds rate target. In particular, FOMC statements likely exert their effects on financial markets through their influence on financial market expectations of *future* policy actions. Nonetheless, the two-factor approach we employ in this paper greatly adds to our understanding of the response of asset prices to monetary policy announcements—for example, we find that *75 to 90 percent* of the explainable variation in five- and ten-year Treasury yields in response to monetary policy announcements is due to the path factor (associated with statements) rather than to changes in the federal funds rate target. We interpret this result as indicating not that policy actions are secondary so much as that their influence comes earlier, when investors build in expectations of those actions in response to FOMC statements (and perhaps other events, such as speeches and testimony by FOMC members).

Our findings have important implications for the conduct of monetary policy in a low-inflation environment. In particular, the fact that FOMC statements have such significant effects on long-term yields suggests that the FOMC may be able to credibly commit to future plans for the federal funds rate. Thus, even when

faced with a low or zero nominal funds rate, our results indicate that the FOMC may be able to effectively communicate to the markets its intention to keep the federal funds rate low for an extended period, thereby lowering longer-term interest rates and stimulating economic growth.²

The remainder of the paper proceeds as follows. Section 1 provides a detailed study of surprises in the federal funds rate target and measures asset price responses to these surprises. Section 2 tests whether there are additional dimensions underlying monetary policy announcements, finds that one additional dimension is required, shows that this additional factor is closely associated with FOMC statements, and estimates the response of asset prices to this additional factor. Section 3 concludes. The appendix provides technical details regarding our factor test and the estimation of the unobserved factors, and a data appendix (only available online at www.ijcb.org) provides a complete listing of our monetary policy announcement dates, times, and federal funds rate and statement surprise measures extending back to 1990.

1. The Effects of Changes in the Federal Funds Rate Target on Asset Prices

1.1 Methodology

We begin our analysis in the framework of one-dimensional measures of monetary policy surprises that has been used in the existing literature. To measure the effects of unexpected monetary policy actions (changes in the federal funds rate) on asset prices, we rely on the following regression, which has been frequently estimated in the literature:

$$\Delta y_t = \alpha + \beta \Delta x_t + \varepsilon_t \quad (1)$$

where Δx_t denotes the surprise component of the change in the federal funds rate target announced by the FOMC, Δy_t denotes the change in a bond yield or stock market index over an interval that

²Reifschneider and Williams (2000) and Eggertsson and Woodford (2003) provide theoretical support for the argument that a central bank is relatively unconstrained by the zero nominal bound so long as it can credibly commit to future policy actions. Bernanke, Reinhart, and Sack (2004) investigate the potential effectiveness of monetary policy at the zero nominal bound empirically.

brackets the monetary policy announcement, and ε_t is a stochastic error term that captures the effects of other factors that influence the asset in question.

We use a high-frequency event-study analysis to estimate equation (1). One generally cannot estimate equation (1) using monthly or quarterly data due to simultaneous equations and omitted variables bias. In particular, the change in monetary policy could actually be a *response* of monetary policy to the change in the asset price that took place earlier in the month or quarter, due to the direct effects of stock market wealth on the economic outlook or due to the signal that term spreads provide about future economic activity and inflation.³ Alternatively, both the change in monetary policy and the change in the asset price could be responding to important macroeconomic news (captured by ε_t) that was released earlier in the period, such as an employment report.⁴ In either case, the classical regression assumption that ε_t is orthogonal to Δx_t is violated.⁵

These problems can be mitigated by using higher-frequency data to shrink the time period around the policy decision. Kuttner (2001), for example, uses daily data to measure changes in Treasury yields and the surprise component of FOMC monetary policy announcements. Cochrane and Piazzesi (2002) and Ellingsen and

³Rigobon and Sack (2003) discuss reasons why asset prices might be expected to feed back into monetary policy. Using a heteroskedasticity-based identification procedure, they estimate a statistically and economically significant response of monetary policy to the stock market, and this response has the expected positive sign. Using their heteroskedasticity-based procedure, Rigobon and Sack (2004) also estimate the size of the endogeneity and omitted variables problems in estimating equation (1).

⁴Asset prices can respond to macroeconomic news for many reasons other than changes in monetary policy expectations: First, information about economic output is likely to influence expectations of corporate earnings and dividends, hence stock prices; second, information about the government budget or current account deficit would, assuming home bias in savings and imperfect Ricardian equivalence, affect bond yields; third, investors' appetite for risk can change in response to economic developments, to name a few examples.

⁵One way to partially account for these issues is to estimate a VAR for the asset price, the policy instrument, and other relevant macroeconomic variables, as in Leeper, Sims, and Zha (1996) and Evans and Marshall (1998). The problem with this approach is that the recursive identifying restrictions typically employed are not plausible for fast-moving financial market variables. (Two exceptions to this rule are the heteroskedasticity-based identification procedure used by Rigobon and Sack [2003, 2004] and the high-frequency identification procedure used by Cochrane and Piazzesi [2002] and Faust, Swanson, and Wright [2004a].)

Soderstrom (2003) perform variations on this analysis, and Bernanke and Kuttner (forthcoming) apply the method to measure the effects of monetary policy announcements on the stock market. However, as noted by Rudebusch (1998) and Bernanke and Kuttner (forthcoming), simultaneity in equation (1) is still a potential problem even at daily frequency because, for a time, the FOMC often changed its target for the federal funds rate just hours after (and in response to) the Bureau of Labor Statistics' employment report release. As a result, event-study regressions using daily data in part capture the endogenous response of asset prices and monetary policy to the information that was released earlier in the day, as well as noise from other financial market developments that took place throughout the day.

We address this potential problem by estimating regression (1) using intraday data to measure both the funds rate surprise Δx_t and the change in the asset price Δy_t . By measuring these changes in a sufficiently narrow window of time around the monetary policy announcement, we can be sure that the FOMC decision was in no way influenced by asset price movements or other macroeconomic news over that interval. In addition, by shrinking the event-study window down to an hour or less, it becomes much less likely that any other significant events took place within this narrow window that might have influenced asset prices, thereby increasing the precision of our estimates.

1.2 Dates and Times of Monetary Policy Announcements

To perform the above analysis using intraday data, we first put together a complete list of dates and times of monetary policy announcements from January 1990 through December 2004. In February 1994, the FOMC began issuing a press release after every meeting and every change in policy, and thus the announcement dates and times are simply those of the corresponding press releases.⁶ Prior to 1994, the FOMC did not explicitly announce changes in its target for the federal funds rate, but such changes were implicitly

⁶Since 1995, these press releases have occurred at about 2:15 p.m., after regularly scheduled FOMC meetings; press releases for intermeeting policy moves and FOMC decisions in 1994 were released at varying times throughout the day. We obtained all of these dates and times from the office of the secretary of the Federal Reserve Board.

communicated to financial markets through the size and type of open market operation. Thus, prior to 1994, the date and time of a monetary policy announcement are typically those of the next open market operation following the FOMC decision.⁷

The dates, times, and methods of communication of FOMC monetary policy announcements are reported in table 1 of the online data appendix. Note that this listing includes not just dates on which the FOMC actually changed the federal funds rate, but also dates on which there was an FOMC meeting followed by no change in policy, since in some cases the FOMC's decision not to change policy surprised financial markets and led to movements in asset prices. The rightmost column of this table reports other major macroeconomic data releases that took place on each date, before the monetary policy announcement. From the table, we can see that eight monetary policy announcements occurred on the date of an employment report release, seven announcements occurred on the date of a gross domestic product release, nine occurred on the date of a consumer price index release, and five on the date of a producer price index release, to name just a few.

1.3 The Surprise Component of Federal Funds Rate Changes

For each monetary policy announcement, we measure the surprise component of the change in the federal funds rate target using federal funds futures. We use the surprise component of monetary policy announcements in estimating regression (1) because changes in policy that are expected by financial markets should have little or

⁷On a few occasions between 1990 and 1994, the FOMC issued, prior to the open market operation, a press release announcing a change in the *discount* rate offered to depository institutions, and market participants correctly inferred from the press release a corresponding change in the target federal funds rate as well. On those occasions, we set the time of the monetary policy announcement to the time of the discount rate change press release. Open market operations over this period were conducted at 11:30 a.m. every day. There are a few dates on which volatility in the federal funds market prevented the Open Market Trading Desk from successfully communicating the FOMC's intentions for the funds rate the first morning after the FOMC's decision (see Kuttner 2003). On these dates, we regard the announcement as having taken place on whichever morning the Credit Markets column of *The Wall Street Journal* regarded as a clear signal of the Federal Reserve's intentions, as reported by Kuttner (2003). This is more of an issue prior to 1990 than for our sample period in this paper.

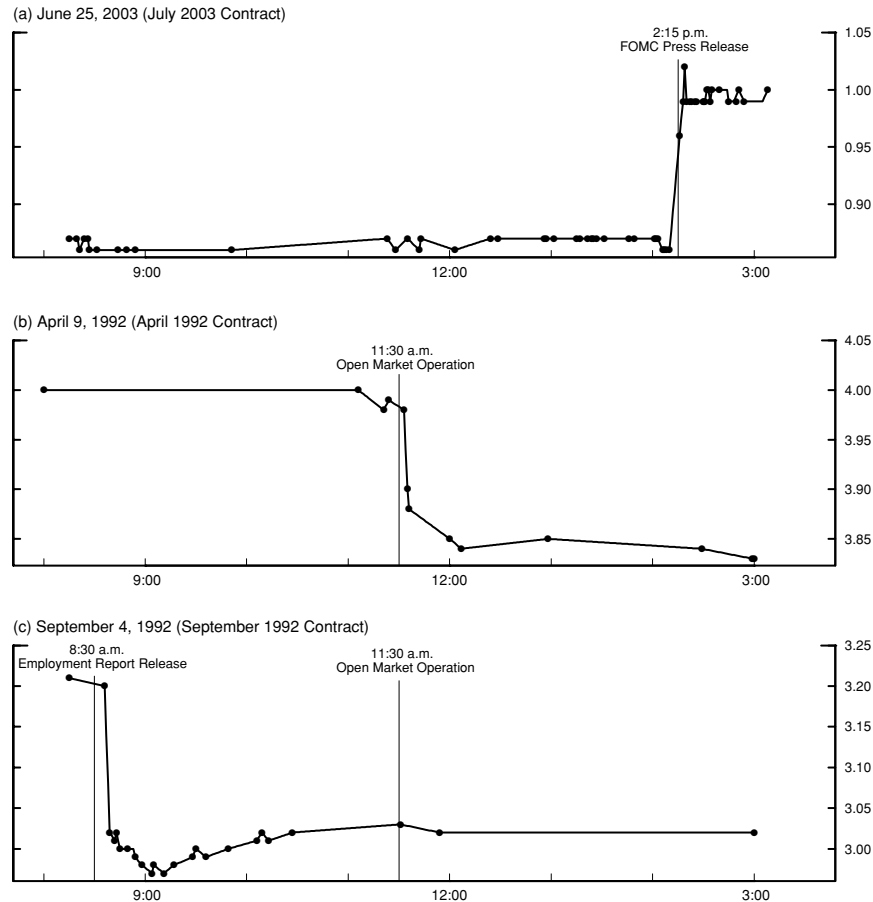
no effect on asset prices, a hypothesis that is confirmed by Kuttner (2001). Thus, using the raw changes in the federal funds rate target as the right-hand-side variable Δx_t would impart an errors-in-variables bias to our estimates of β to the extent that the monetary policy decisions were correctly anticipated by financial markets.

Federal funds futures have traded at the Chicago Board of Trade exchange since October 1988 and settle based on the average effective federal funds rate that is realized for the calendar month specified in the contract. Thus, daily changes in the current-month futures rate largely reflect revisions to the market's expectations for the federal funds rate over the remainder of the month. As described in the appendix, the change in the current month's contract rate on the day of an FOMC announcement can be scaled up to account for the timing of the announcement within the month, and thereby measure the surprise component of the FOMC's announcement for the federal funds rate. For the present paper, we acquired tick-by-tick data on all federal funds futures contract trades from January 1990 to the present from Genesis Financial Technologies. To provide a sense of the quality of this data and its advantages, figure 1 graphs the data on three illustrative dates:

1. June 25, 2003, was the date of a regularly scheduled FOMC meeting (see panel a). Trades were intermittent throughout the day until just before and just after the FOMC's press release at 2:15 p.m. At that time, the FOMC announced that it was lowering its target for the federal funds rate from 1.25 percent to 1 percent. According to surveys and press reports both before and after the policy announcement, many market participants had been expecting the FOMC to ease policy by 50 bp at the meeting. Thus, this decision is characterized as a 13 bp tightening surprise under our measure. This example illustrates two key points: First, financial markets seem to fully adjust to the policy action within just a few minutes—i.e., there is no evidence of learning or sluggish adjustment to the target rate decision going on after about 2:20 p.m. in this example. Second, the federal funds rate surprise is not necessarily in the same direction as the federal funds rate action itself.

2. April 9, 1992, was the date of an intermeeting monetary policy move (see panel b). The FOMC reduced its target for the federal funds rate from 4 percent to 3.75 percent that morning, but given that this date precedes 1994, the FOMC did not issue a press release

Figure 1. Intraday Trading in Federal Funds Futures Contracts



about its change in policy to the public. As can be seen in the figure, trading in federal funds futures was thin until shortly before the open market operation at 11:30 a.m. At that time, the Open Market Trading Desk injected a significant quantity of reserves into the market, and market participants correctly inferred from this that the FOMC had changed its target for the funds rate, causing the futures rate to move quickly to the new target rate.⁸

⁸The federal funds futures contract rate falls to 3.85 percent after the announcement rather than the new funds rate target of 3.75 percent because nine

3. September 4, 1992, witnessed the release of a very weak employment report at 8:30 a.m. (see panel c). In response to that report, investors significantly revised downward their expectations for the federal funds rate, pushing the futures rate down sharply. Sometime after the poor data release, the FOMC decided to reduce its target for the federal funds rate from 3.25 percent to 3 percent. Again, because there was no press release, the FOMC's decision became known to the markets at 11:30 a.m., the time of the open market operation. In contrast to panel b, however, the FOMC's decision for the funds rate on this date was essentially completely anticipated by the time it was signaled to the market—indeed, our intraday measure of the funds rate surprise (reported in table 2 of the online data appendix and discussed below) is 0 bp. By contrast, the daily measure of the funds rate surprise is -22 bp, because it incorporates the *endogenous* policy response to the weak employment report. In this case, we would not want to use the daily measure of the funds rate surprise in an event-study regression, because it would suffer from the omitted variables problem discussed earlier (in that the employment report itself has sizable effects on stock prices, bond yields, and monetary policy expectations).

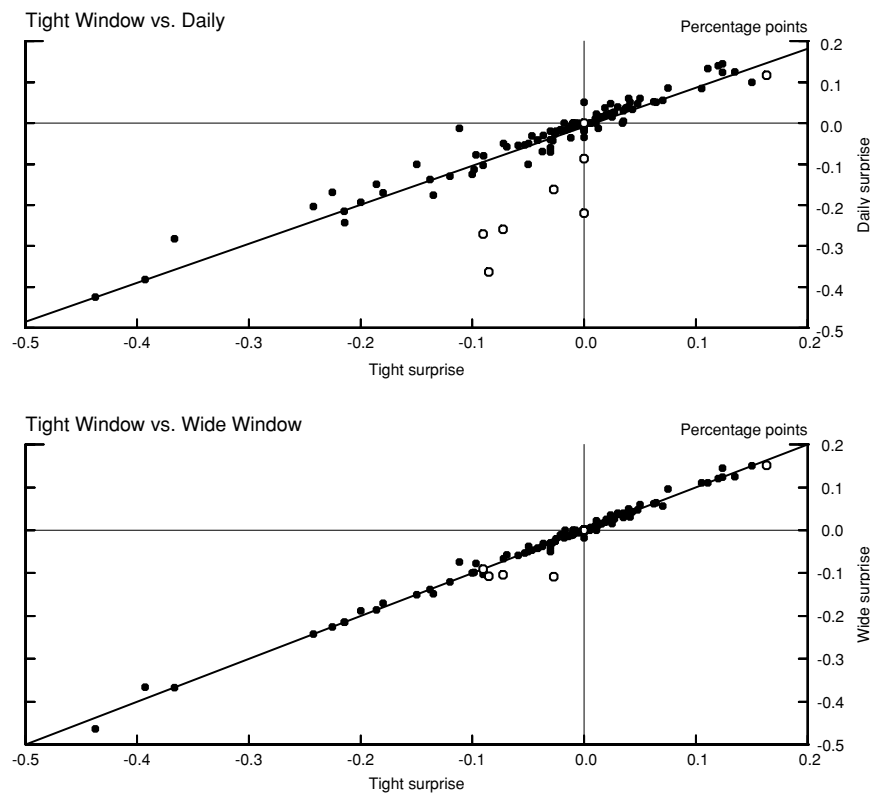
To focus on the monetary policy decision itself, we compute policy surprises by looking at changes in the futures rate in narrow windows around the FOMC announcements. More specifically, online data appendix table 2 reports two intraday measures, a “tight” window and a “wide” window, which begin ten (fifteen) minutes prior to the monetary policy announcement and end twenty (forty-five) minutes after the policy announcement, respectively, for the period from January 1990 through December 2004.⁹ For comparison, the table also reports a “daily” window that begins with the financial market close the day before the policy announcement and ends with the financial market close the day of the policy announcement. On most of the days in our sample, the two intraday measures are quite

days of the month have already elapsed with an average federal funds rate of 4 percent, which will result in a month-average funds rate for April of 3.85 percent.

⁹When there is no federal funds futures trade exactly at the beginning of the specified window, we use the most recent price. When there is no trade exactly at the end of the specified window, we use the next available trade price. Federal funds futures trading is often sparse early in our sample period, but becomes significantly more dense around the times of macroeconomic data releases and monetary policy announcements.

similar to the daily measure: the average absolute difference between the daily and two intraday surprise measures is only about 2 bp, and the difference is 0 bp on many days. There are a few days, however, on which the differences between the intraday measures and the daily measure are quite large: for example, there are five observations for which the discrepancy between the tight surprise and the daily surprise exceeds 10 bp. Each of those observations took place before 1994 on the day of an employment report release, as in the example of September 4, 1992, discussed previously. Figure 2 makes this point graphically: as can be seen in the top panel, the tight window

Figure 2. Measures of Surprises in the Federal Funds Rate Target



Note: Hollow circles denote days of employment report releases.

and daily window surprises are in very close agreement on all but a handful of dates, almost all of which correspond to days on which the FOMC was responding to an employment report release (depicted by the hollow circles). By contrast, in the bottom panel of the figure, we see that the two intraday measures are in very close agreement on all dates in our sample.

We draw two conclusions from these observations. First, the FOMC decision accounted for the vast majority of the movement in the federal funds futures rate on all of the non-employment-report days in our sample. This is perhaps surprising, given the large number of other data releases that also coincided with monetary policy announcements in data appendix table 1, and is in itself an important finding: it shows that for samples that exclude employment report dates, or samples that begin in 1994, the surprise component of monetary policy announcements can be measured very well using just daily data. Second, FOMC actions were priced into the federal funds futures market almost immediately—quickly enough to be completely captured by our tight, thirty-minute window—consistent with the examples discussed above. Thus, we can feel comfortable focusing on the analysis using our tight window of thirty minutes, although we will report results using the wider one-hour window as well.

1.4 The Effect of Federal Funds Rate Changes on Asset Prices

Table 1 presents our results for regression equation (1) estimated using intraday data on bond yields and stock prices.¹⁰ The independent variable is the surprise component of the change in the federal funds rate target just described, and the dependent variable is the change in the financial variable measured over the same window. We present results for the tight (thirty-minute), wide (one-hour), and daily windows described above.

Our results for stock prices imply that, on average, a surprise 25 bp tightening in the federal funds rate leads to a little more than

¹⁰We obtained tick-by-tick Treasury yield data back to June 1991 for on-the-run Treasury securities from GovPX, a consortium of interdealer brokers that accounted for a large portion of trading volume in Treasury securities over our sample. For equity prices, we obtained five-minute intraday quotes on the S&P 500 index back to the mid-1980s, which are available from a variety of sources.

Table 1. Response of Asset Prices to Changes in the Federal Funds Rate

	Tight Window		Wide Window		Daily	
	<i>Constant</i>	<i>MP Surprise</i>	<i>Constant</i>	<i>MP Surprise</i>	<i>Constant</i>	<i>MP Surprise</i>
	(<i>std. err.</i>)	(<i>std. err.</i>)	(<i>std. err.</i>)	(<i>std. err.</i>)	(<i>std. err.</i>)	(<i>std. err.</i>)
<i>R</i> ²	<i>R</i> ²	<i>R</i> ²	<i>R</i> ²	<i>R</i> ²	<i>R</i> ²	<i>R</i> ²
<i>S&P 500</i>	−0.095*** (0.034)	−4.032*** (1.110)	−0.059 (0.042)	−4.531*** (1.069)	0.171* (0.090)	−3.961*** (1.560)
<i>Three-Month Bill</i>	−0.005** (0.002)	0.537*** (0.040)	−0.007** (0.003)	0.583*** (0.038)	−0.004 (0.004)	0.669*** (0.087)
<i>Six-Month Bill</i>	−0.005 (0.003)	0.522*** (0.057)	−0.007** (0.003)	0.570*** (0.053)	−0.008 (0.005)	0.627*** (0.083)
<i>Two-Year Note</i>	−0.002 (0.004)	0.455*** (0.087)	−0.001 (0.006)	0.475*** (0.092)	−0.006 (0.007)	0.429*** (0.117)
<i>Five-Year Note</i>	−0.000 (0.004)	0.264*** (0.081)	0.000 (0.005)	0.267*** (0.100)	−0.007 (0.007)	0.318** (0.142)
<i>Ten-Year Note</i>	−0.001 (0.003)	0.125** (0.058)	0.000 (0.005)	0.129 (0.080)	−0.006 (0.006)	0.166 (0.124)
<i>Five-Year Forward Rate Five Years Ahead</i>	−0.001 (0.003)	−0.087* (0.047)	0.001 (0.004)	−0.061 (0.061)	−0.006 (0.006)	−0.042 (0.114)

Note: Sample is all monetary policy announcements from July 1991 through December 2004 (January 1990–December 2004 for S&P 500). Target factor and path factor are defined in the main text. Heteroskedasticity-consistent standard errors reported in parentheses. *, **, and *** denote significance at 10 percent, 5 percent, and 1 percent, respectively. See text for details.

a 1 percent fall in the S&P 500, and these estimates are highly significant. The estimated coefficients do not differ greatly across the intraday and daily regressions, although the effects of the omitted employment report variable can be seen clearly in the scatter plots in figure 3: the handful of days on which the policy decision followed an employment report, shown by the hollow points, do not appear unusual when the intraday data are used, but in the daily data they stand out as large policy easings that yielded no gains in equity prices, most likely due to the negative direct influence of the weak employment reports.

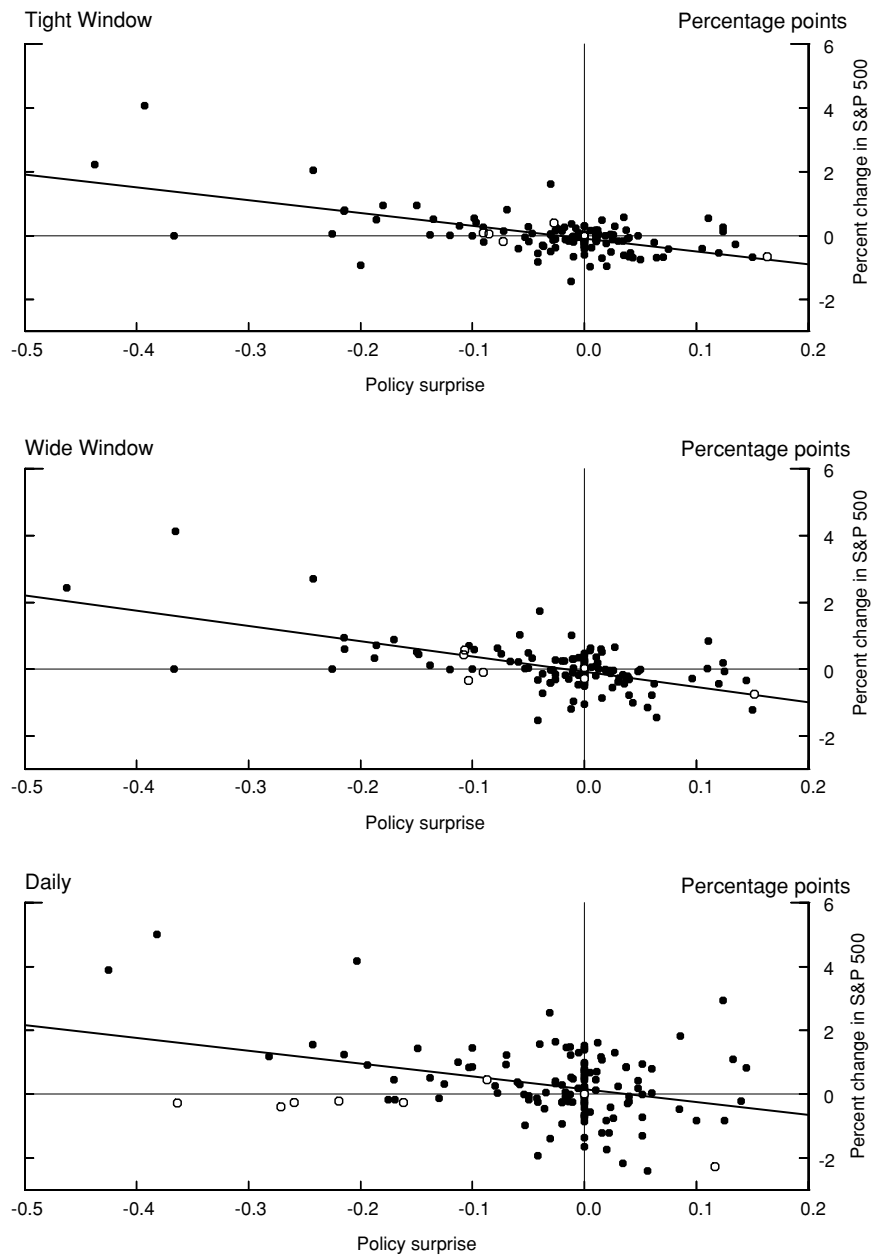
The most striking feature of figure 3, however, is the increase in tightness of the relationship as we move from daily to intraday data. By eliminating the effects of employment reports and other news that occurred on the days of monetary policy announcements, the relationship between monetary policy actions and equity prices becomes much clearer in the figure. This advantage also stands out in the regression results in table 1, in terms of the much greater precision of the coefficient estimates and a tripling of R^2 from .12 to .36.

Intraday data yield additional benefits for our Treasury yield regressions. As shown in figure 4, employment report days (the hollow points) stand out in the daily data as very large funds rate surprises and large changes in the three-month Treasury-bill rate in the same direction, reflecting the fact that the employment report has a very large influence on both the FOMC decision and the short end of the Treasury yield curve.¹¹ By contrast, those days do not stand out at all when the intraday windows are used. Moreover, employment report surprises appear to have a larger effect on the T-bill rate compared to policy surprises. Because of this, the estimated coefficient in the regression is biased upward if daily data are used. As with equity prices, the response of the T-bill rate to monetary policy actions is also estimated much more precisely using intraday data, reflected in the much smaller standard errors (about half the size of the daily measures) and the much higher R^2 (.80 versus .56).

The differences in the coefficients between the intraday regressions and the daily regression become much smaller at horizons of

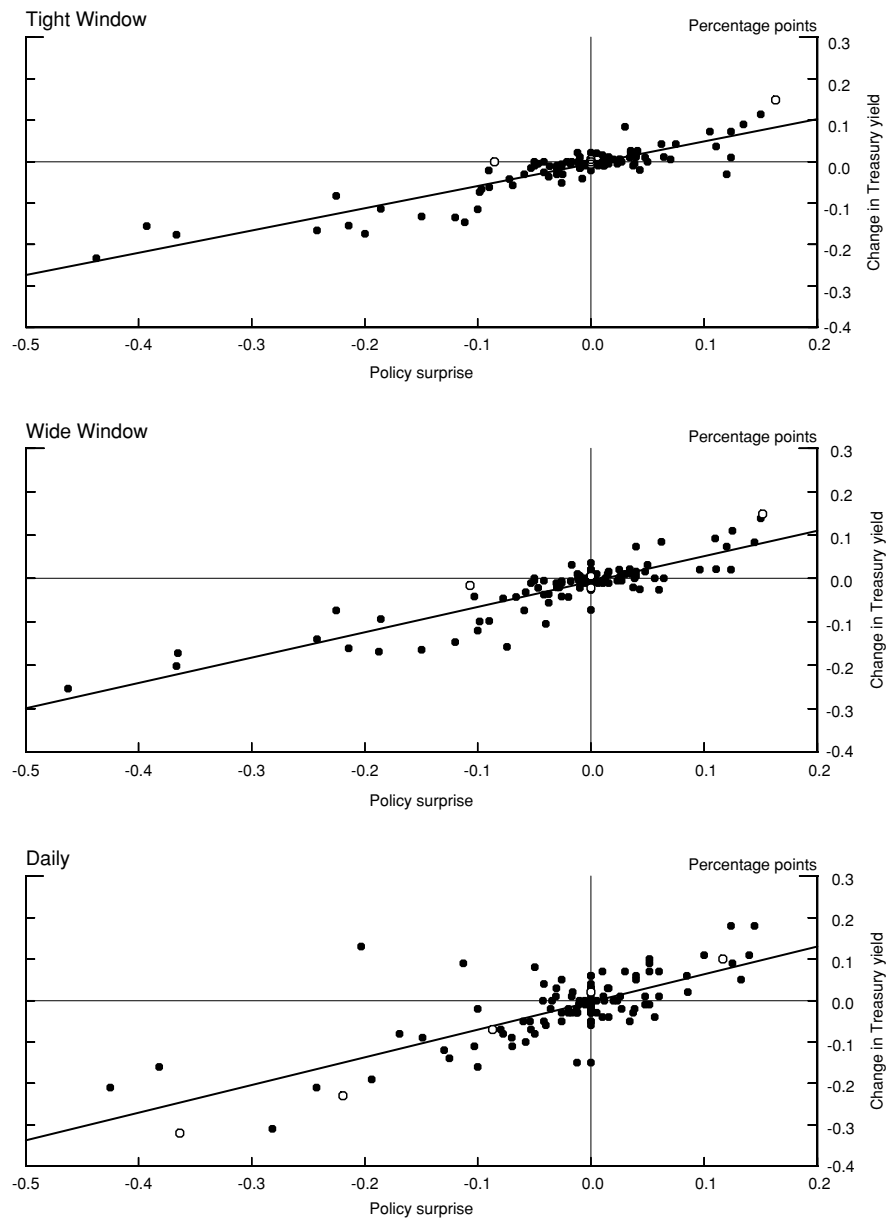
¹¹Recall that our intraday Treasury data extend back only to June 1991, so our Treasury yield regressions and graphs contain eighteen fewer observations (and only five employment report dates instead of eight).

Figure 3. Response of S&P 500 to Monetary Policy Surprises



Note: Hollow circles denote days of employment report releases.

Figure 4. Response of Three-Month Treasury Yield to Monetary Policy Surprises



Note: Hollow circles denote days of employment report releases.

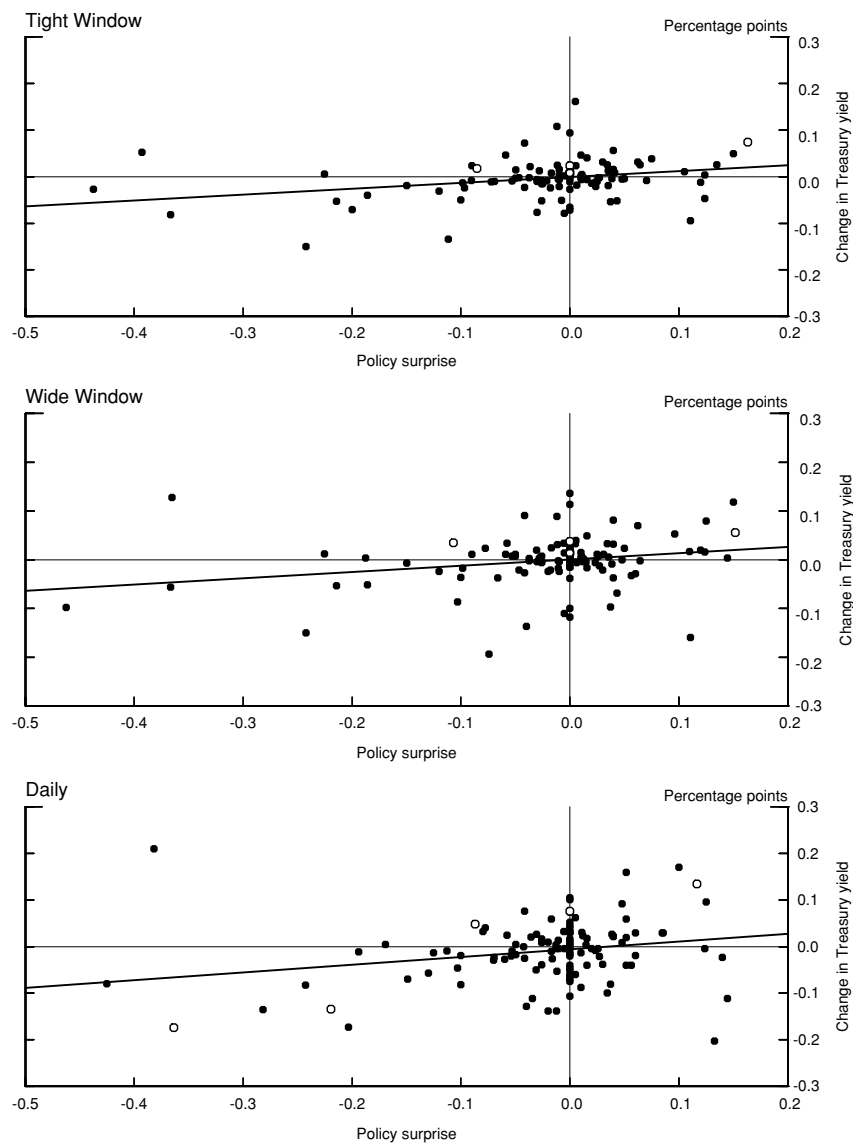
two years or more, but this observation is somewhat misleading, as a single outlier, January 3, 2001, pulls the daily estimate toward the intraday one (shown in figure 5).¹² Without that one observation in the upper left, the daily regression would estimate a significant response of the ten-year rate to the FOMC announcement of 0.28 rather than the statistically insignificant 0.17 that is estimated including the outlier. By contrast, using the intraday data, the estimated coefficient changes only very slightly (from 0.13 to 0.17) when we exclude that observation. As before, the precision of our estimates also improves dramatically using intraday data, with the standard errors being about half as large. This makes the response of the ten-year Treasury yield—which is statistically indistinguishable from zero using daily data—significantly greater than zero (albeit small) using our tight-window data.

The response of the term structure can also be expressed in terms of forward rates. We compute the five-year forward Treasury rate beginning five years ahead from five- and ten-year Treasury yields using the Shiller-Campbell-Schoenholtz (1983) approximation. The estimated response of the forward rate to the policy surprise is *negative* over this sample. This is consistent with the findings of Gürkaynak, Sack, and Swanson (2003) that far-ahead forward rates typically move *inversely* with the monetary policy surprise, although our estimates here are not highly statistically significant, partly because we cannot compute intraday forward rate changes that begin more than five years ahead. Gürkaynak, Sack, and Swanson (2003) present their findings in terms of the nine-year-ahead one-year forward rate, which is probably a better measure of far-ahead forward rates.¹³

¹²On that day, although the FOMC unexpectedly eased policy, which would normally be associated with a fall in Treasury yields, market participants reportedly became much more optimistic about the economic outlook as a result, leading to a huge rally in equity markets (including an astounding 14 percent rise in the NASDAQ that afternoon) and a large *upward* shift in Treasury yields. This outlier poses particular problems for a researcher using only daily data, since it is unclear whether the observation should be dropped or retained, and the empirical results are highly sensitive to this choice; by contrast, using the intraday data, the observation poses no problems.

¹³We only have intraday data for the on-the-run five- and ten-year Treasury notes, which gives us the forward rate from five to ten years ahead, but not for any finer subintervals. The five-year-ahead five-year rate may not capture movements in far-ahead forward rates as well as the nine-year-ahead one-year rate.

Figure 5. Response of Ten-Year Treasury Yield to Monetary Policy Surprises



Note: Hollow circles denote days of employment report releases.

2. The Effects of FOMC Statements on Asset Prices

2.1 *Testing for Additional Dimensions of Monetary Policy Announcements*

The preceding section assumed that the effects of FOMC announcements on asset prices are completely (or at least adequately) described by the surprise component of the change in the federal funds rate target. Although this assumption is standard in the existing literature, the example of January 28, 2004, given earlier raises serious questions regarding its validity. In this section, we ask whether the example of January 28 was a one-time occurrence or a manifestation of something much more systematic. If the latter, how many additional dimensions are required to adequately characterize monetary policy announcements?¹⁴

The above questions essentially ask how many latent factors underlie the response of asset prices to monetary policy announcements. Let X denote the $T \times n$ matrix, with rows corresponding to monetary policy announcements, columns corresponding to asset prices, and each element of X reporting the change in the corresponding asset price in a thirty-minute window around the corresponding announcement. Writing X in the form

$$X = F\Lambda + \eta \quad (2)$$

where F is a $T \times k$ matrix of unobserved factors (with $k < n$), Λ is a $k \times n$ matrix of factor loadings, and η is a $T \times n$ matrix of white noise disturbances, we wish to know how many factors (columns of F) are required to adequately describe X . The hypothesis that a single factor (for example, the surprise component of changes in the federal funds rate) is sufficient is a statement that there exists a $T \times 1$ vector F and constants λ_i , $i = 1, \dots, k$, such that the matrix X is described by $F \times [\lambda_1, \dots, \lambda_k]$ up to white noise.

This restriction on the structure of the data X can be tested using the matrix rank test of Cragg and Donald (1997). In brief, the null hypothesis that X is described by k_0 common factors can

¹⁴One can imagine that several dimensions might be required, corresponding to information about the future path of interest rates, policymakers' objectives and preferences, and perhaps the future paths of output and inflation, if the FOMC possesses private information about these latter two variables.

be tested against the alternative that X is described by $k > k_0$ factors by measuring the minimum distance between $\text{Cov}(X)$ and the covariance matrices of all possible factor models (2) with k_0 factors. This distance, after a suitable normalization, has a limiting χ^2 distribution with $(n - k_0)(n - k_0 + 1)/2 - n$ degrees of freedom. Additional details of the method are provided in the appendix.

Table 2 reports the results of this test applied to two sets of assets: one that includes all of our Treasury yields and stock prices, and another that includes only federal funds futures and eurodollar futures with one year or less to expiration.¹⁵ We consider the latter set of assets in this table because they will turn out to be very useful in the structural interpretation of the factors, below.

As can be seen in table 2, the hypothesis that the response of asset prices to monetary policy announcements is characterized by zero or by one common factor is clearly rejected. An important and direct corollary of this finding is that surprise changes in the federal funds rate alone are not sufficient to describe the response of asset prices to monetary policy announcements, in contrast to one of the standard assumptions in the literature. However, as is also clear in table 2, we find that two factors are sufficient—tests of this hypothesis are not rejected at even the 10 percent level. Given our strong rejection of zero or one factor, this result is perhaps somewhat surprising—it implies that, despite the potentially unlimited complexity of monetary policy statements and hence announcements, financial markets nonetheless have reacted as if there is essentially only one additional degree of information beyond the surprise change in the federal funds rate target. In the remainder of this section, we turn to estimating and interpreting this additional dimension of U.S. monetary policy and estimating its effect on Treasury yields and stock prices.

¹⁵Treasury yields include the three-month, six-month, two-year, five-year, and ten-year yields. Recall that we only have intraday data for these yields beginning in July 1991, which explains the smaller number of observations for these securities in table 2. For futures rates, we use five contracts that pin down the expected path of the federal funds rate over the next year without overlapping: the current-month and three-month-ahead federal funds futures contracts (with a scale factor to account for the timing of FOMC meetings within the month) and the two-, three-, and four-quarter-ahead eurodollar futures contracts. See the appendix for details. Avoiding overlap is desirable because very similar assets will tend to covary strongly, producing an additional factor even if that variation is orthogonal to all of the other assets in X .

Table 2. Tests of Number of Factors Characterizing Monetary Policy Announcements

		Treasury Yields and Stock Prices				Futures Rates with ≤ 1 Year to Expiration			
H_0 : Number of Factors Equals	Wald Statistic	χ^2 Degrees of Freedom	p -value	Number of Obs.	Wald Statistic	χ^2 Degrees of Freedom	p -value	Number of Obs.	
0	46.72	15	.00004	120	36.61	10	.00007	138	
1	21.41	9	.011	120	17.19	5	.004	138	
2	4.36	4	.360	120	1.06	1	.304	138	

Note: Test is from Cragg and Donald (1997) and tests the null hypothesis of N_{H0} factors against the alternative of $N > N_{H0}$ factors. Sample: January 1990–December 2004 (July 1991–December 2004 for Treasuries). Treasury yields comprise three-month, six-month, two-year, five-year, and ten-year yields, stock prices the S&P 500. Futures rates comprise one- and three-month-ahead federal funds futures rates (with scale adjustment for timing of FOMC meetings within the month) and two-, three-, and four-quarter-ahead eurodollar futures rates.

2.2 *Estimation of the Two Factors*

We estimate the unobserved factor matrix F using the standard method of principal components applied to our data matrix X .¹⁶ This procedure decomposes the matrix X into a set of orthogonal vectors F_i , $i = 1, \dots, n$, where F_1 is the (length- T) vector that has maximum explanatory power for X , F_2 is the vector that has maximum explanatory power for the residuals of X after projecting each column on F_1 , and so on. Based on the results of our factor rank test, above, we will restrict attention to the first two factors (F_1 and F_2) estimated by this procedure. Details are provided in the appendix.

As mentioned above, we estimate the unobserved factors F_1 and F_2 using the set of federal funds futures and eurodollar futures rates with one year or less to expiration that characterize the expected path of the funds rate over the upcoming year.¹⁷ Estimates of the unobserved factors using Treasury yields and stock prices are quite similar; the reason for focusing on the shorter-term futures rates for estimation is that it makes the structural interpretation of the two factors, and interpretation of the effects of the two factors on bond yields and stock prices, more clear.

2.3 *A Structural Interpretation of the Two Factors*

The two factors F_1 and F_2 explain a maximal fraction of the variance of X , but do not have a structural interpretation. For example, both factors are correlated with the surprises in the current federal funds rate target, so we cannot interpret one factor as the change in the federal funds rate target and the other factor as some other dimension of monetary policy. To address this deficiency and allow for a more structural interpretation of the factors, we perform a rotation of F_1 and F_2 to yield two new factors, which we call Z_1 and

¹⁶The primary alternative to principal components is Kalman filtering, which is optimal under the assumption of normally distributed residuals. However, the normality assumption may not fit our data very well.

¹⁷Gürkaynak, Sack, and Swanson (2002) show that these rates are the best financial market predictors of the federal funds rate at horizons out to a year. As one familiar with factor decompositions of the yield curve might expect, we find that F_1 and F_2 look very much like “level” and “slope” factors from that literature, although our sample differs from that literature in that we focus exclusively on the brief intervals around monetary policy announcements.

Z_2 , which are still orthogonal and explain the matrix X to exactly the same extent that F_1 and F_2 did, but for which the second factor (Z_2) has no effect on the current federal funds rate. In other words, we define

$$Z = FU$$

in such a way that U is a 2×2 orthogonal matrix and the second column of Z is a vector that is associated on average with *no change in the current-month federal funds futures rate*. Exact details of this rotation are reported in the appendix. As a result of this transformation, we can regard the unexpected change in the current target for the federal funds rate as being driven exclusively by Z_1 (plus a small amount of white noise), and Z_2 represents all other aspects of FOMC announcements that move futures rates for the upcoming year *without* changing the *current* federal funds rate. The factor Z_2 therefore includes any information (besides the decision for the current target rate) that affects the expected path for monetary policy over the upcoming year. Accordingly, we refer to Z_1 and Z_2 as the “target” factor and the “path” factor, respectively. The estimated values for the target and path factors for each monetary policy announcement in our sample are reported in table 3 of the online data appendix.

Note that the target factor (Z_1) defined in this way should be similar to—but not exactly equal to—the measure of federal funds target surprises we presented in the previous section and reported in data appendix table 2.¹⁸ To check the tightness of the relationship between these two measures, we regressed our target surprises on the factor Z_1 (results not reported) and found that Z_1 is in fact very close to our previous measure, with a correlation of over 95 percent (R^2 of .91). Thus, to further facilitate interpretation of Z_1

¹⁸The two measures generally are not identical because the factor estimation procedure strips out white noise from the data. Thus, Z_1 will tend to exclude part of the “timing” component of federal funds rate surprises—i.e., changes in the funds rate target that are only a surprise to the extent that they occur at the present meeting instead of the next meeting. (Gürkaynak, Sack, and Swanson [2002] and Gürkaynak [2005] discuss “timing” surprises in more detail.) To the extent that assets other than the current-month federal funds futures rate ignore these timing surprises, the factor estimation procedure will assign this component to white noise rather than to Z_1 . In this respect, Z_1 is a *better* measure of the important component of funds rate surprises than is the standard measure based on the current-month futures rate.

as the surprise change in the funds rate target, we normalize its scale so that a change of .01 in Z_1 corresponds to a surprise of 1 bp in the federal funds rate target. Similarly, to facilitate interpretation of the second factor, we normalize its scale so that the effect of Z_2 on the four-quarter-ahead eurodollar futures rate is exactly the same as the effect of Z_1 on the four-quarter-ahead eurodollar futures rate, about 55 bp.¹⁹

2.4 The Association Between the Path Factor and FOMC Statements

The path factor Z_2 corresponds to all aspects of FOMC announcements that move futures rates for the upcoming year without changing the current federal funds rate; thus, it is to some extent a residual that is subject to various interpretations. In this section, we present three pieces of evidence that show a close correlation between the path factor and FOMC statements: first, a time series plot that allows comparison of pre-1994 to post-1994 (and also statement to nonstatement) monetary policy announcements; second, a regression test of the path factor on a dummy variable indicating the presence of an FOMC statement; and third, a case study of the largest path factor observations in our sample. In addition, in section 2.5 below, we compare our estimated effects of the path factor on asset prices to the effects of the FOMC minutes release on January 4, 2005, which, because of new procedures adopted by the FOMC, has many of the features of an FOMC statement release with no change in the funds rate target.

In figure 6, we plot the target and the path factors over time, with dates on which there was an FOMC statement plotted as solid bars and those on which there was no FOMC statement (dates prior to 1994 and dates from 1994 to 1999 on which there was no change in the federal funds rate target) plotted as hollow bars. As can be seen in the figure, there have been many large realizations of the path factor in recent years, while realizations of the path factor prior to 1994 (and on nonstatement days in general) were typically very small.

¹⁹These scale normalizations are performed for the July 1991–December 2004 sample, the period for which we have data on U.S. Treasury yields.

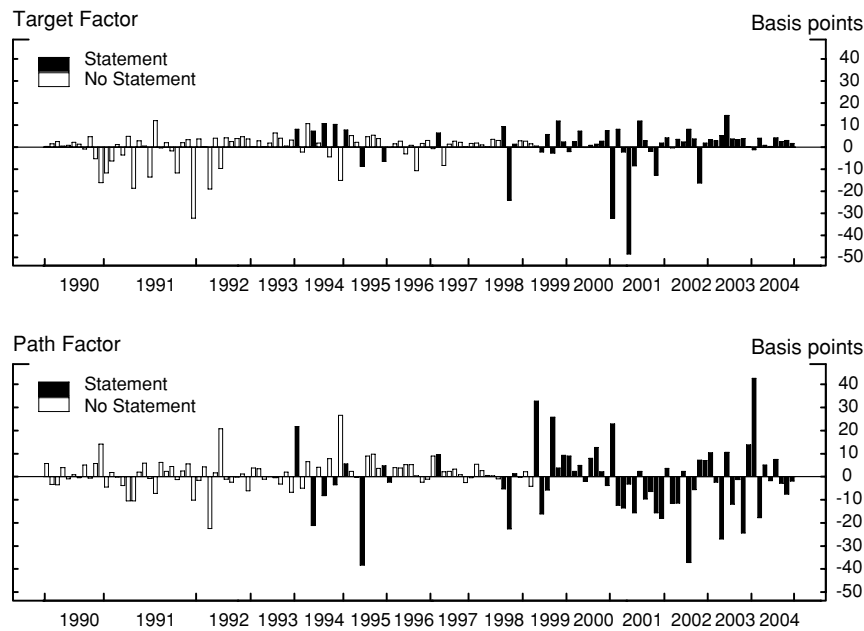
Figure 6. Monetary Policy Surprises as Two Factors

Table 3 verifies this observation econometrically by regressing the absolute value of the path factor on a constant and a dummy variable that takes on the value one for all dates on which there was an FOMC statement. The coefficient on the dummy variable is positive and highly statistically significant, allowing us to strongly

Table 3. Estimated Effects of Policy Statements on the Size of the Path Factor

Dependent Variable	Constant (std. err.)	Statement (std. err.)	R ²	Number of Obs.
Abs(Z ₂)	0.044*** (0.005)	0.070*** (0.014)	.18	138
Note: Regression of absolute value of path factor on a constant and dummy variable indicating presence of an FOMC statement. Heteroskedasticity-consistent standard errors reported in parentheses. *** denotes significance at 1 percent level.				

reject the hypothesis that FOMC statements and the path factor are unrelated.²⁰

In table 4, we report details of the ten largest path factor observations over our sample. Nine of these ten observations (and twenty-one of the top twenty-five) correspond to dates on which an FOMC statement accompanied the federal funds rate target decision; moreover, financial market commentary on these dates (as reported in the Credit Markets column of *The Wall Street Journal* the following day) typically directly attributed the move in the bond market to the specifics of the FOMC statement. Nonetheless, there are a few exceptions to the correspondence between the path factor and FOMC statements, which represent significant changes in financial market expectations regarding the future path of policy that were driven by events other than FOMC statements. For example, on December 20, 1994, market participants were reportedly nervous about inflationary pressures arising from output growth overshooting potential (the subject of a speech by Vice Chairman Blinder just a few days before), and the failure of the FOMC to move at the December meeting was reportedly regarded by some participants as perhaps requiring greater tightening down the road (Pesek and Young 1994), leading to larger-than-usual changes in expectations for the future path of policy.²¹

An interesting feature of the path factor is that—in contrast to the target factor—there is evidence that financial markets may take some time to digest its implications. For example, if we regress a wide (one-hour) window measure of the path factor Z_2 on the tight (thirty-minute) window measure of the factor (both calculated as above), we get an R^2 of .83. By contrast, if we perform the same exercise on the target surprise factor Z_1 , we get a much higher R^2 of .98 (which is consistent with figures 1 and 2). A natural interpretation

²⁰This finding is similar to a result in Kohn and Sack (2004), who used a simple regression to control for the effects of target surprises, with the residual intended to capture the effects of all other variables, including FOMC statements. They found that the variance of this residual was much higher on days that the FOMC released statements.

²¹We also checked the financial press for each of these dates to see whether the strong response of the path factor on dates without statements could possibly be due to the release of other important news in the same thirty-minute window as the monetary policy announcements. We found no such confounding news releases.

Table 4. Ten Largest Observations of the Path Factor

Date	Z_1 (Target Factor)	Z_2 (Path Factor)	Statement	Financial Market Commentary
Jan. 28, 2004	-1.1	42.7	✓	Statement drops commitment to keep policy unchanged for “a considerable period,” bringing forward expectations of future tightenings
Jul. 6, 1995	-8.7	-38.4	✓	First easing after long (seventeen-month) series of tightenings raises expectations of further easings; statement notes that inflationary pressures have receded
Aug. 13, 2002	8.1	-37.2	✓	Statement announces balance of risks has shifted from neutral to economic weakness
May 18, 1999	0.5	32.8	✓	Statement announces change in policy bias going forward from neutral to tightening
May 6, 2003	5.2	-27.0	✓	Statement announces balance of risks now dominated by risk of “an unwelcome substantial fall in inflation”
Dec. 20, 1994	-15.1	26.6		Surprise that FOMC not tightening considering recent comments by Blinder on “overshooting”; some fear Fed may have to tighten more in 1995 as a result
Oct. 5, 1999	-2.7	25.8	✓	Statement announces change in policy bias going forward from neutral to tightening
Oct. 28, 2003	3.9	-24.4	✓	Statement leaves the “considerable period” commitment unchanged, pushing back expectations of future tightenings
Jan. 3, 2001	-32.3	22.8	✓	Large surprise intermeeting ease reportedly causes financial markets to mark down probability of a recession; Fed is perceived as being “ahead of the curve” and as needing to ease less down the road as a result
Oct. 15, 1998	-24.0	-22.6	✓	First intermeeting move since 1994 and statement pointing to “unsettled conditions in financial markets... restraining aggregate demand” increases expectations of further easings

of this finding is that changes in the federal funds rate target itself are immediately and clearly observable to all financial market participants within minutes of the announcement while, by contrast, FOMC statements about the policy and economic outlook typically require time to digest and are subject to a great deal of uncertainty with respect to how they are interpreted by other financial market participants, so that the process of assimilating the information contained in the statements is not instantaneous. Nonetheless, we continue to emphasize our tight window responses in the analysis below because most of the policy information is incorporated within that window and having a narrower window reduces the amount of noise in our left-hand-side variables, increasing the precision of our estimates.²²

2.5 The Response of Asset Prices to the Target Factor and Path Factor

We now estimate the effects of each of these two dimensions of monetary policy announcements on asset prices. For each monetary policy announcement from January 1990 through December 2004, we have estimates of the target factor and path factor components of the announcement and we observe the change in Treasury yields and stock prices in a narrow window bracketing the announcement (the same left-hand-side variables as in section 1).²³ For each Treasury yield or stock prices, we run the regression

$$\Delta y_t = \alpha + \beta Z_{1,t} + \gamma Z_{2,t} + \varepsilon_t. \quad (3)$$

Results are reported in table 5.²⁴

²²Robustness tests using the wide window data were in line with the results reported below.

²³Recall that, due to data availability, we only observe intraday Treasury responses beginning in July 1991.

²⁴Because the regressors in table 5 are generated rather than directly observed, we have also computed the standard errors by bootstrapping to capture the additional sampling error associated with the first-stage factor estimation procedure. The statistical significance of all our results was essentially identical whether we used the bootstrapped or asymptotic standard errors. Bootstrap percentiles were based on 1,000 repetitions; in each repetition, we sampled 138 new observations from the data with replacement, took the two largest principal components from the synthetic futures data, rotated and normalized them, and regressed the

Table 5. Response of Asset Prices to Target and Path Factors

	One Factor		Two Factors		R^2
	<i>Constant</i> (<i>std. err.</i>)	<i>Target Factor</i> (<i>std. err.</i>)	<i>Constant</i> (<i>std. err.</i>)	<i>Target Factor</i> (<i>std. err.</i>)	<i>Path Factor</i> (<i>std. err.</i>)
<i>MP Surprise</i>	-0.021*** (0.003)	1.000*** (0.047)	-0.021*** (0.003)	1.000*** (0.048)	0.001 (0.026)
<i>One-Year-Ahead Eurodollar Future</i>	-0.018*** (0.006)	0.555*** (0.076)	-0.017*** (0.001)	0.551*** (0.017)	0.551*** (0.014)
<i>S&P 500</i>	-0.008 (0.041)	-4.283*** (1.083)	-0.008 (0.040)	-4.283*** (1.144)	-0.966 (0.594)
<i>Two-Year Note</i>	-0.011** (0.005)	0.485*** (0.080)	-0.011*** (0.002)	0.482*** (0.032)	0.411*** (0.023)
<i>Five-Year Note</i>	-0.006 (0.005)	0.279*** (0.078)	-0.006** (0.002)	0.276*** (0.044)	0.369*** (0.035)
<i>Ten-Year Note</i>	-0.004 (0.004)	0.130** (0.059)	-0.004* (0.002)	0.128*** (0.039)	0.283*** (0.025)
<i>Five-Year Forward Rate Five Years Ahead</i>	0.001 (0.003)	-0.098** (0.049)	0.001 (0.003)	-0.099** (0.047)	0.157*** (0.028)

Note: Sample is all monetary policy announcements from July 1991–December 2004 (January 1990–December 2004 for S&P 500). Target factor and path factor are defined in the main text. Heteroskedasticity-consistent standard errors reported in parentheses. *, **, and *** denote significance at 10 percent, 5 percent, and 1 percent, respectively. See text for details.

As we would expect from the close correspondence between our target factor and the funds rate target surprises in section 1 (and from the orthogonality of Z_2 to Z_1), the estimated coefficients on the target factor (Z_1) are very similar to those we estimated previously in table 1. In particular, we estimate that a 1 percentage point surprise tightening in the federal funds rate leads, on average, to a 4.3 percent decline in the S&P 500 and increases of 49, 28, and 13 bp in two-, five-, and ten-year Treasury yields, respectively.

The novel feature of table 5, however, is our estimates of the effects of the path factor (Z_2) on asset prices. As can be seen in the table, the effect of this factor on the one-year-ahead eurodollar future rate is the same as the effect of the target factor, by definition.²⁵ However, the path factor has effects on the other financial variables that differ considerably from the target factor. In particular, the path factor has a much greater impact on the long end of the yield curve, with a 1 percentage point innovation to the factor causing responses of 37 and 28 bp in five- and ten-year Treasury yields, respectively. Thus, FOMC statements that move one-year-ahead policy expectations appear to have much greater effects on the long end of the yield curve than do changes in the federal funds target rate itself, *even when they generate the same size movement in one-year-ahead interest rates*. Moreover, as can be seen by comparing the R^2 statistics from the one- and two-factor regressions, the large majority of variation in long-term Treasury yields seems to be due to statements rather than funds rate changes, with two-thirds of the explainable variation in two-year yields, three-fourths of the explainable variation in five-year yields, and nine-tenths of the explainable variation in ten-year yields attributable to the path factor rather than to changes in the federal funds rate target. Thus, by focusing only on the FOMC's decisions regarding the current federal funds rate target, previous authors have been missing by far the most important component of monetary policy decisions, especially

synthetic changes in asset prices on the synthetically generated Z_1 or Z_1 and Z_2 . The bootstrapped standard errors are available from the authors upon request.

²⁵The path factor is orthogonal to the monetary policy surprise (and the target factor) over our full sample, January 1990–December 2004. The coefficient of the MP Surprise on the path factor in table 5 is not exactly zero because it is reported for the July 1991–December 2004 sample, for comparability to the Treasury yield responses.

in recent years when target funds rate changes have often been well-anticipated.

By contrast, the effect of changes in the path factor on the stock market appears to be smaller than the effect of changes in the funds rate target, amounting to only about -1 percent for a 1 percentage point innovation. Given the yield curve findings above, this result is somewhat surprising: for longer-maturity Treasury securities, policy-makers' statements seem to have much larger effects than changes in the current federal funds rate, but the effect of statements on stock prices, which also have very long durations, is smaller.

One possible explanation for this pattern is that the statements that seem to drive the path factor lead, to a greater extent, to positive revisions in investors' assessment of the future path of output and inflation, consistent with a story first formalized by Romer and Romer (2000).²⁶ A large positive realization of the path factor, for example, might be related to a statement suggesting that the FOMC sees greater output or inflation going forward than markets had been expecting. Gürkaynak, Sack, and Swanson (forthcoming) show that long-term yields move substantially and positively in response to positive surprises in macroeconomic data releases for output and inflation; thus, if in fact FOMC statements do reveal information about the future course of these variables, the strong response of long-term yields to the path factor in this paper would be completely consistent with those earlier results. Moreover, to the extent that financial markets revise upward their forecasts of output (and hence earnings and dividends) in response to positive path factor surprises, then the tendency for stocks to fall in response would be muted by the upwardly revised economic outlook.

As a final note, the events of January 4, 2005, provide us with an interesting out-of-sample test of our results. At 2:00 p.m. that day, the FOMC for the first time released the minutes for its previous meeting under a new, accelerated schedule—three weeks *before* (as opposed to a few days after) the next FOMC meeting. This gives

²⁶This story has been questioned on the basis of a pure target factor analysis by Faust, Swanson, and Wright (2004b). Those authors find that surprise tightenings in what we call the target factor convey essentially *no* positive information about the future path of output or inflation. However, as in all of the previous literature, those authors did not consider changes in what we call the path factor, which our results suggest could be more informative about these variables.

the minutes release on that date many of the features of an FOMC statement: for example (and in contrast to speeches or testimony by individual FOMC members), the minutes report the thinking and deliberation of the entire FOMC and have been voted on and approved by the whole committee; moreover, the minutes now provide additional information about the most recent FOMC meeting. In the thirty-minute window around the minutes release on January 4, one-year-ahead eurodollar futures rose 6.5 bp, as the minutes were taken to imply greater risks of inflation and funds rate tightening than markets had previously been expecting. Two-, five-, and ten-year Treasury yields increased 4.9, 5.0, and 4.1 bp, respectively—very much in line with our estimates of the effects of the path factor in table 5, which would predict changes of 4.8, 4.4, and 3.3 bp for these securities, respectively. The S&P 500 declined by .49 percent, somewhat larger than the .11 percent decline predicted by table 5, and the five-to-ten-year forward rate rose by 3.0 bp, a bit more than the 1.9 bp predicted by our estimates. Although we cannot rule out the possibility that markets view minutes and FOMC statements as differing in important ways, all of these results are broadly in line with our estimates of the effects of FOMC statements, suggesting that our estimation and identification procedures have been reasonably effective.

3. Conclusions

Do central bank actions speak louder than words? We find that the answer to this question is a qualified “no.” In particular, we find that viewing the effects of FOMC announcements on financial markets as driven by a single factor—changes in the federal funds rate target—is inadequate. Instead, we find that a second policy factor—one not associated with the current federal funds rate decision of the FOMC but instead with statements that it releases—accounted for more than three-fourths of the explainable variation in the movements of five- and ten-year Treasury yields around FOMC meetings.

We emphasize that our findings do not imply that FOMC statements represent an *independent* policy tool. In particular, FOMC statements likely exert their effects on financial markets through their influence on financial market expectations of *future* policy actions. Viewed in this light, our results do not indicate that policy

actions are secondary so much as that their influence comes earlier—when investors build in expectations of those actions in response to FOMC statements (and perhaps other events, such as speeches and testimony by FOMC members).

Our findings have important implications for the literature on the effects of monetary policy on asset markets. Indeed, we have shown that previous studies estimating the effects of changes in the federal funds rate on bond yields and stock prices have been missing most of the story. This is especially true in recent years, when FOMC decisions regarding the target for the funds rate have rarely been a surprise and, instead, changes in the wording of FOMC statements typically have been the major driver of financial market responses.

More broadly, our finding that FOMC statements have such significant effects on asset prices suggests that the FOMC has the ability to conduct policy with a substantial degree of commitment to a state-contingent, or conditional, path for the funds rate several quarters or even years into the future. This finding has important implications for the conduct of monetary policy in a low-inflation environment—in particular, even when faced with a low or zero nominal funds rate, our results directly support the theoretical analysis of Reifschneider and Williams (2000) and Eggertsson and Woodford (2003) that the FOMC is largely unhindered in its ability to conduct policy, because it has the ability to manipulate financial market expectations of *future* policy actions and thereby longer-term interest rates and the economy more generally.

Appendix. Factor Model Rank Test and Estimation

Testing the Number of Factors

Let X denote a $T \times n$ matrix of data generated by a factor model:

$$X = F\Lambda + \eta, \quad (\text{A1})$$

where F is a $T \times k$ matrix of unobserved factors (with $k < n$), Λ is a $k \times n$ matrix of factor loadings, and η is a $T \times n$ matrix of white noise disturbances. We wish to test the hypothesis that X was generated by k_0 factors against the alternative that X was generated by $k > k_0$ factors. We do this by computing the distance between

the covariance matrix of the data, X , and the set of all covariance matrices corresponding to factor models (A1) with k_0 factors.

The covariance matrix Σ_X of the data, X , has elements given by $\hat{\sigma}_{jk} = (1/T) \sum_i (x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)$. To compute the Wald statistic, below, we also need to know the sampling uncertainty surrounding the $\hat{\sigma}_{jk}$. This is given by:

$$\begin{aligned} \text{Cov}(\hat{\sigma}_{jk}, \hat{\sigma}_{lm}) = (1/T^2) \sum_i [(x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k) - \hat{\sigma}_{jk}] \\ \times [(x_{il} - \bar{x}_l)(x_{im} - \bar{x}_m) - \hat{\sigma}_{lm}]. \end{aligned} \quad (\text{A2})$$

The covariance matrix of the factor model (A1) is given by $\Lambda' \Lambda + \Sigma_\eta$. To compute the distance between the two, we stack the elements of Σ_X and $\Lambda' \Lambda + \Sigma_\eta$ into vectors and normalize the difference, $\text{vech}(\Sigma_X) - \text{vech}(\Lambda' \Lambda + \Sigma_\eta)$, by $\text{Cov}(\text{vech}(\Sigma_X))$, whose elements are given by (A2):

$$W = [\text{vech}(\Sigma_X) - \text{vech}(\Lambda' \Lambda + \Sigma_\eta)]' \hat{\Omega}^{-1} [\text{vech}(\Sigma_X) - \text{vech}(\Lambda' \Lambda + \Sigma_\eta)], \quad (\text{A3})$$

where $\hat{\Omega} = \text{Cov}(\text{vech}(\Sigma_X))$. We search over all possible factor loadings Λ (nk_0 parameters) and white noise variances Σ_η (n parameters) to find the minimum distance (A3). Under the null hypothesis, this is a Wald statistic with limiting χ^2 distribution on $n(n+1)/2 - (nk_0 + n) + k_0(k_0 - 1)/2$ degrees of freedom (the last term comes from the fact that there are $k_0(k_0 - 1)/2$ possible rotations of the unobserved factors F , and hence Λ , which all yield the same observable model). The Matlab code that performs this factor test is available from the authors upon request.

Factor Estimation

To estimate the unobserved factors F , let X be the matrix with 138 rows corresponding to monetary policy announcements and five columns corresponding to futures contracts with one year or less to maturity. The third through fifth columns of X are the changes in price of the second, third, and fourth eurodollar futures contracts, which have 1.5, 2.5, and 3.5 quarters to expiration on average (eurodollar futures have expiration dates that lie about two weeks before

the end of each quarter).²⁷ The first two columns of X are essentially the changes in the current-month and three-month-ahead federal funds futures contracts, but contain a scaling adjustment to account for the timing of FOMC meetings within those months, as follows.

Federal funds futures have a payout that is based on the average effective federal funds rate that prevails over the calendar month specified in the contract. Thus, immediately before an FOMC meeting, at time $t - \Delta t$, the implied rate from the current-month federal funds future contract, $ff1$, is largely a weighted average of the federal funds rate that has prevailed so far in the month, r_0 , and the rate that is expected to prevail for the remainder of the month, r_1 :²⁸

$$ff1_{t-\Delta t} = \frac{d1}{D1}r_0 + \frac{D1-d1}{D1}E_{t-\Delta t}(r_1) + \rho1_{t-\Delta t} \quad (A4)$$

where $d1$ denotes the day of the FOMC meeting, $D1$ is the number of days in the month, and $\rho1$ denotes any term or risk premium that may be present in the contract. Then, by leading this equation to time t (twenty minutes after the policy announcement) and differencing, the surprise component of the change in the federal funds rate target, which we call $mp1$, is given by²⁹

$$mp1_t = (ff1_t - ff1_{t-\Delta t}) \frac{D1}{D1 - d1}. \quad (A5)$$

Note that to interpret (A5) as the surprise change in monetary policy expectations, we need to assume that the change in the risk

²⁷Thus, the second eurodollar futures contract can have as little as one quarter plus one day to expiration and as much as two quarters to expiration, with an average horizon of 1.5 quarters over our sample. On expiration, eurodollar futures settle based on the spot ninety-day eurodollar rate, which is closely tied to expectations for the federal funds rate over the subsequent ninety-day period. Thus, these three eurodollar futures contracts are related to federal funds rate expectations from 1.5–2.5, 2.5–3.5, and 3.5–4.5 quarters ahead, respectively.

²⁸For simplicity, assume that federal funds rate is always equal to the target rate set by the FOMC, so that we do not have to differentiate between the target and actual rates. This has no impact on the surprise measures due to the differencing involved.

²⁹Kuttner (2001) also uses this method. For FOMC meetings that occur very late in the month (i.e., in the last seven days of the month), we use the unscaled change in the *next-month* federal funds futures contract to avoid multiplying by a very large scale factor in (A5), which could unduly magnify changes in bid-ask spreads or other factors, since federal funds futures are only priced to the nearest half basis point.

premium ρ in this narrow window of time is small in comparison to the change in expectations itself. Piazzesi and Swanson (2004) provide some evidence that this assumption is not inconsistent with the data.

We can apply a similar procedure to measure the change in expectations about r_2 , the federal funds rate target that will prevail after the *second* FOMC meeting from now. Let $ff2$ denote the federal funds futures rate for the month containing the second FOMC meeting (typically the three-month-ahead contract). Then

$$ff2_{t-\Delta t} = \frac{d2}{D2} E_{t-\Delta t}(r_1) + \frac{D2-d2}{D2} E_{t-\Delta t}(r_2) + \rho2_{t-\Delta t}, \quad (A6)$$

where $d2$ and $D2$ are the day of that FOMC meeting and the number of days in the month containing that FOMC meeting, respectively, and $\rho2$ denotes any risk premium in the contract. By leading this equation to time t and differencing, the change in expectations for that second FOMC meeting, which we call $mp2$, is given by

$$mp2_t = \left[(ff2_t - ff2_{t-\Delta t}) - \frac{d2}{D2} mp1_t \right] \frac{D2}{D2-d2}. \quad (A7)$$

Additional details can be obtained from various sources.³⁰

As described in section 2, we decompose X into its principal components after normalizing each column to have zero mean and unit variance. We let F_1 and F_2 denote the first two principal components of X , and normalize each of them to have unit variance.

To allow for a more structural interpretation of these unobserved factors, we rotate them so that the first factor corresponds to surprise changes in the current federal funds rate target and the second factor corresponds to moves in interest rate expectations over the coming year that are not driven by changes in the current funds rate. In other words, we define a 138×2 matrix Z by

$$Z = FU, \quad (A8)$$

³⁰Gürkaynak (2005) discusses measuring policy expectations and surprises at horizons farther ahead than the current meeting; Gürkaynak, Sack, and Swanson (2002) show that federal funds futures are the best financial market predictors of the federal funds rate with the smallest average term premium; Piazzesi and Swanson (2004) estimate to what extent risk premia in these markets vary over time; and Kuttner (2001) discusses the construction and some uses of $mp1$.

where

$$U = \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \end{bmatrix},$$

and where U is identified by four restrictions. First, the columns of U are normalized to have unit length (which normalizes Z_1 and Z_2 to have unit variance). Second, the new factors Z_1 and Z_2 should remain orthogonal to each other:

$$E(Z_1 Z_2) = \alpha_1 \beta_1 + \alpha_2 \beta_2 = 0. \quad (\text{A9})$$

Lastly, we impose the restriction that Z_2 does not influence the current policy surprise, $mp1$, as follows. Let γ_1 and γ_2 denote the (known) loadings of $mp1$ on F_1 and F_2 , respectively. Since

$$F_1 = \frac{1}{\alpha_1 \beta_2 - \alpha_2 \beta_1} [\beta_2 Z_1 - \alpha_2 Z_2], \quad (\text{A10})$$

$$F_2 = \frac{1}{\alpha_1 \beta_2 - \alpha_2 \beta_1} [\alpha_1 Z_2 - \beta_1 Z_1], \quad (\text{A11})$$

it follows that:

$$\gamma_2 \alpha_1 - \gamma_1 \alpha_2 = 0,$$

which is the final restriction. It is then easy to solve for the unique matrix U satisfying these restrictions.

Finally, as noted in section 2, we rescale Z_1 and Z_2 so that Z_1 moves the current policy surprise $mp1$ one-for-one and Z_2 has the same magnitude effect on the year-ahead eurodollar futures rate as Z_1 has on that rate (about 55.1 bp).

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Data Appendix Table 1
Monetary Policy Announcement Dates and Times

Date	Time	Method of Announcement	Intermeeting Move?	Other Macroeconomic Data Releases That Day
8-Feb-90	11:30am	Open Market Operation		
28-Mar-90	11:30am	Open Market Operation		GNP and NIPAs, New Home Sales
16-May-90	11:30am	Open Market Operation		CPI, Housing Starts and Permits
5-Jul-90	11:30am	Open Market Operation		Auto Sales
13-Jul-90	11:30am	Open Market Operation	✓	PPI, Retail Sales
22-Aug-90	11:30am	Open Market Operation		
3-Oct-90	11:30am	Open Market Operation		Auto Sales, Factory Orders
29-Oct-90	11:30am	Open Market Operation	✓	
14-Nov-90	11:30am	Open Market Operation		Retail Sales
7-Dec-90	11:30am	Open Market Operation	✓	Employment Report
18-Dec-90	3:30pm	Discount Rate Change Press Release		CPI, Merchandise Trade
8-Jan-91	11:30am	Open Market Operation	✓	
1-Feb-91	9:15am	Discount Rate Change Press Release	✓	Employment Report, NAPM Survey
7-Feb-91	11:30am	Open Market Operation		
8-Mar-91	11:30am	Open Market Operation	✓	Employment Report
27-Mar-91	11:30am	Open Market Operation		GNP and NIPAs
30-Apr-91	9:30am	Discount Rate Change Press Release	✓	Consumer Confidence, ECI, Factory Orders
15-May-91	11:30am	Open Market Operation		Business Inventories
5-Jul-91	11:30am	Open Market Operation		Employment Report
6-Aug-91	11:30am	Open Market Operation	✓	
21-Aug-91	11:30am	Open Market Operation		U.S. Budget Deficit
13-Sep-91	9:10am	Discount Rate Change Press Release	✓	CPI, Retail Sales
2-Oct-91	11:30am	Open Market Operation		New Home Sales
30-Oct-91	11:30am	Open Market Operation	✓	New Home Sales, PCE, Personal Income
6-Nov-91	8:45am	Discount Rate Change Press Release		
6-Dec-91	11:30am	Open Market Operation	✓	Employment Report
18-Dec-91	11:30am	Open Market Operation		
20-Dec-91	8:30am	Discount Rate Change Press Release	✓	GDP and NIPAs, U.S. Budget Deficit
6-Feb-92	11:30am	Open Market Operation		Factory Orders
1-Apr-92	11:30am	Open Market Operation		NAPM Survey
9-Apr-92	11:30am	Open Market Operation	✓	PPI
20-May-92	11:30am	Open Market Operation		Merchandise Trade
2-Jul-92	9:15am	Discount Rate Change Press Release	✓	Employment Report, Factory Orders
19-Aug-92	11:30am	Open Market Operation		Merchandise Trade
4-Sep-92	11:30am	Open Market Operation	✓	Employment Report
7-Oct-92	11:30am	Open Market Operation		
18-Nov-92	11:30am	Open Market Operation		Merchandise Trade
23-Dec-92	11:30am	Open Market Operation		Consumer Confidence, Durable Goods Orders, PCE
4-Feb-93	11:30am	Open Market Operation		Factory Orders
24-Mar-93	11:30am	Open Market Operation		Durable Goods Orders
19-May-93	11:30am	Open Market Operation		Merchandise Trade
8-Jul-93	11:30am	Open Market Operation		
18-Aug-93	11:30am	Open Market Operation		
22-Sep-93	11:30am	Open Market Operation		U.S. Budget Deficit
17-Nov-93	11:30am	Open Market Operation		Housing Starts and Permits
22-Dec-93	11:30am	Open Market Operation		GDP and NIPAs

Date	Time	Method of Announcement	Intermeeting Move?	Other Macroeconomic Data Releases That Day
4-Feb-94	11:05am	Post-Meeting Press Release	✓	Employment Report
22-Mar-94	2:20pm	Post-Meeting Press Release		Merchandise Trade
18-Apr-94	10:06am	Post-Meeting Press Release		
17-May-94	2:26pm	Post-Meeting Press Release		Housing Starts and Permits
6-Jul-94	2:18pm	Post-Meeting Press Release		Auto Sales
16-Aug-94	1:18pm	Post-Meeting Press Release		Housing Starts and Permits
27-Sep-94	2:18pm	Post-Meeting Press Release		Consumer Confidence
15-Nov-94	2:20pm	Post-Meeting Press Release		Ind Production, Cap Utilization, International Trade
20-Dec-94	2:17pm	Post-Meeting Press Release		International Trade
1-Feb-95	2:15pm	Post-Meeting Press Release		Leading Indicators, Auto Sales
28-Mar-95	2:15pm	Post-Meeting Press Release		Consumer Confidence
23-May-95	2:15pm	Post-Meeting Press Release		
6-Jul-95	2:15pm	Post-Meeting Press Release		Leading Indicators, Auto Sales
22-Aug-95	2:15pm	Post-Meeting Press Release		
26-Sep-95	2:15pm	Post-Meeting Press Release		Consumer Confidence
15-Nov-95	2:15pm	Post-Meeting Press Release		CPI, Ind. Prod., Cap. Util, Business Inventories
19-Dec-95	2:15pm	Post-Meeting Press Release		GDP and NIPAs, Housing Starts and Permits
31-Jan-96	2:15pm	Post-Meeting Press Release		PPI, Purchasing Managers Survey
26-Mar-96	11:39am	Post-Meeting Press Release		Consumer Confidence
21-May-96	2:15pm	Post-Meeting Press Release		U.S. Budget Deficit
3-Jul-96	2:15pm	Post-Meeting Press Release		Factory Orders, Auto Sales
20-Aug-96	2:15pm	Post-Meeting Press Release		International Trade
24-Sep-96	2:15pm	Post-Meeting Press Release		Consumer Confidence
13-Nov-96	2:15pm	Post-Meeting Press Release		PPI
17-Dec-96	2:15pm	Post-Meeting Press Release		Housing Starts and Permits
5-Feb-97	2:15pm	Post-Meeting Press Release		Factory Orders, Auto Sales
25-Mar-97	2:15pm	Post-Meeting Press Release		Consumer Confidence, Existing Home Sales
20-May-97	2:15pm	Post-Meeting Press Release		
2-Jul-97	2:15pm	Post-Meeting Press Release		Factory Orders
19-Aug-97	2:15pm	Post-Meeting Press Release		Housing Starts and Permits
30-Sep-97	2:15pm	Post-Meeting Press Release	✓	Consumer Confidence, New Home Sales
12-Nov-97	2:15pm	Post-Meeting Press Release		
16-Dec-97	2:15pm	Post-Meeting Press Release		CPI, Housing Starts and Permits
4-Feb-98	2:12pm	Post-Meeting Press Release		
31-Mar-98	2:15pm	Post-Meeting Press Release		Consumer Confidence
19-May-98	2:15pm	Post-Meeting Press Release		Housing Starts and Permits
1-Jul-98	2:15pm	Post-Meeting Press Release		NAPM Survey, Leading Indicators, Auto Sales
18-Aug-98	2:15pm	Post-Meeting Press Release		CPI, International Trade
29-Sep-98	2:15pm	Post-Meeting Press Release		Consumer Confidence
15-Oct-98	3:15pm	Intermeeting Press Release		PPI, Business Inventories
17-Nov-98	2:15pm	Post-Meeting Press Release		CPI, Business Inventories
22-Dec-98	2:15pm	Post-Meeting Press Release		
3-Feb-99	2:12pm	Post-Meeting Press Release		Auto Sales
30-Mar-99	2:12pm	Post-Meeting Press Release		Consumer Confidence
18-May-99	2:11pm	Post-Meeting Press Release		Housing Starts and Permits
30-Jun-99	2:15pm	Post-Meeting Press Release		Leading Indicators
24-Aug-99	2:15pm	Post-Meeting Press Release		
5-Oct-99	2:12pm	Post-Meeting Press Release		Leading Indicators
16-Nov-99	2:15pm	Post-Meeting Press Release		Industrial Production, Capacity Utilization

Date	Time	Method of Announcement	Intermeeting Move?	Other Macroeconomic Data Releases That Day
21-Dec-99	2:15pm	Post-Meeting Press Release		U.S. Budget Deficit
2-Feb-00	2:15pm	Post-Meeting Press Release		New Home Sales, Leading Indicators, Auto Sales
21-Mar-00	2:15pm	Post-Meeting Press Release		International Trade
16-May-00	2:15pm	Post-Meeting Press Release		CPI, Housing Starts and Permits
28-Jun-00	2:15pm	Post-Meeting Press Release		Durable Goods Sales
22-Aug-00	2:15pm	Post-Meeting Press Release		
3-Oct-00	2:12pm	Post-Meeting Press Release		New Home Sales, Leading Indicators
15-Nov-00	2:12pm	Post-Meeting Press Release		Ind. Prod., Cap. Util., Business Inventories
19-Dec-00	2:15pm	Post-Meeting Press Release		International Trade
3-Jan-01	1:13pm	Intermeeting Press Release	✓	
31-Jan-01	2:15pm	Post-Meeting Press Release		GDP and NIPAs, New Home Sales
20-Mar-01	2:15pm	Post-Meeting Press Release		International Trade, U.S. Budget Deficit
18-Apr-01	10:54am	Intermeeting Press Release	✓	Leading Indicators, International Trade
15-May-01	2:15pm	Post-Meeting Press Release		
27-Jun-01	2:12pm	Post-Meeting Press Release		
21-Aug-01	2:15pm	Post-Meeting Press Release		
17-Sep-01	8:20am	Intermeeting Press Release	✓	
2-Oct-01	2:15pm	Post-Meeting Press Release		
6-Nov-01	2:20pm	Post-Meeting Press Release		
11-Dec-01	2:15pm	Post-Meeting Press Release		
30-Jan-02	2:15pm	Post-Meeting Press Release		GDP and NIPAs
19-Mar-02	2:15pm	Post-Meeting Press Release		International Trade
7-May-02	2:15pm	Post-Meeting Press Release		
26-Jun-02	2:15pm	Post-Meeting Press Release		Durable Goods Orders, New Home Sales
13-Aug-02	2:15pm	Post-Meeting Press Release		Retail Sales
24-Sep-02	2:15pm	Post-Meeting Press Release		Consumer Confidence
6-Nov-02	2:15pm	Post-Meeting Press Release		
10-Dec-02	2:15pm	Post-Meeting Press Release		
29-Jan-03	2:15pm	Post-Meeting Press Release		
18-Mar-03	2:15pm	Post-Meeting Press Release		Housing Starts and Permits
6-May-03	2:15pm	Post-Meeting Press Release		
25-Jun-03	2:15pm	Post-Meeting Press Release		Drbl Goods Ords, New Home Sales, Exist Home Sales
12-Aug-03	2:15pm	Post-Meeting Press Release		
16-Sep-03	2:15pm	Post-Meeting Press Release		CPI
28-Oct-03	2:15pm	Post-Meeting Press Release		Durable Goods Orders, Consumer Confidence
9-Dec-03	2:15pm	Post-Meeting Press Release		
28-Jan-04	2:15pm	Post-Meeting Press Release		Durable Goods Orders, New Home Sales
16-Mar-04	2:15pm	Post-Meeting Press Release		Housing Starts and Permits
4-May-04	2:15pm	Post-Meeting Press Release		Factory Orders
30-Jun-04	2:15pm	Post-Meeting Press Release		
10-Aug-04	2:15pm	Post-Meeting Press Release		
21-Sep-04	2:15pm	Post-Meeting Press Release		Housing Starts and Permits
10-Nov-04	2:15pm	Post-Meeting Press Release		U.S. Budget, International Trade
14-Dec-04	2:15pm	Post-Meeting Press Release		Industrial Prod., Capacity Util., International Trade

Data Appendix Table 2
Monetary Policy Surprises

Date	---Monetary Policy Surprise (bp)---			---Differences (bp)---		Intermeeting Move?	Employment Report?
	Tight Window (30 min.)	Wide Window (1 hour)	Daily Window (1 day)	Daily - Tight	Wide - Tight		
8-Feb-90	-1.4	-1.4	-1.4	0.0	0.0		
28-Mar-90	0.0	0.0	0.0	0.0	0.0		
16-May-90	0.0	0.0	0.0	0.0	0.0		
5-Jul-90	0.0	0.0	0.0	0.0	0.0		
13-Jul-90	-13.8	-13.8	-13.8	0.0	0.0	✓	
22-Aug-90	0.0	0.0	0.0	0.0	0.0		
3-Oct-90	1.1	2.2	2.2	1.1	1.1		
29-Oct-90	-3.0	-3.0	-2.0	1.0	0.0	✓	
14-Nov-90	1.9	1.9	3.8	1.9	0.0		
7-Dec-90	-9.0	-9.0	-27.1	-18.1	0.0	✓	✓
18-Dec-90	-21.5	-21.5	-21.5	0.0	0.0		
8-Jan-91	-13.5	-14.8	-17.5	-4.0	-1.4	✓	
1-Feb-91	-7.3	-10.4	-25.9	-18.7	-3.1	✓	✓
7-Feb-91	0.0	0.0	0.0	0.0	0.0		
8-Mar-91	-2.7	-10.8	-16.2	-13.5	-8.1	✓	✓
27-Mar-91	-2.0	-1.0	-2.0	0.0	1.0		
30-Apr-91	-18.0	-17.0	-17.0	1.0	1.0	✓	
15-May-91	1.9	1.9	1.9	0.0	0.0		
5-Jul-91	0.0	0.0	0.0	0.0	0.0		✓
6-Aug-91	-18.6	-18.6	-14.9	3.7	0.0	✓	
21-Aug-91	12.4	12.4	12.4	0.0	0.0		
13-Sep-91	-5.3	-5.3	-5.3	0.0	0.0	✓	
2-Oct-91	-1.1	-1.1	-1.1	0.0	0.0		
30-Oct-91	-3.0	-5.0	-6.0	-3.0	-2.0	✓	
6-Nov-91	-10.0	-10.0	-12.5	-2.5	0.0		
6-Dec-91	0.0	0.0	-8.7	-8.7	0.0	✓	✓
18-Dec-91	4.8	4.8	4.8	0.0	0.0		
20-Dec-91	-36.6	-36.6	-28.2	8.5	0.0	✓	
6-Feb-92	1.3	1.3	-1.3	-2.5	0.0		
1-Apr-92	1.0	1.0	1.0	0.0	0.0		
9-Apr-92	-21.4	-21.4	-24.3	-2.9	0.0	✓	
20-May-92	0.0	0.0	0.0	0.0	0.0		
2-Jul-92	-8.6	-10.7	-36.3	-27.8	-2.1		✓
19-Aug-92	2.6	2.6	2.6	0.0	0.0		
4-Sep-92	0.0	0.0	-21.9	-21.9	0.0	✓	✓
7-Oct-92	0.0	0.0	5.2	5.2	0.0		
18-Nov-92	-5.0	-5.0	-10.0	-5.0	0.0		
23-Dec-92	3.9	3.9	3.9	0.0	0.0		
4-Feb-93	0.0	0.0	-1.2	-1.2	0.0		
24-Mar-93	0.0	0.0	0.0	0.0	0.0		
19-May-93	-2.6	-2.6	-2.6	0.0	0.0		
8-Jul-93	2.7	2.7	2.7	0.0	0.0		
18-Aug-93	0.0	0.0	0.0	0.0	0.0		
22-Sep-93	0.0	0.0	0.0	0.0	0.0		
17-Nov-93	2.3	2.3	2.3	0.0	0.0		
22-Dec-93	0.0	0.0	0.0	0.0	0.0		

Date	---Monetary Policy Surprise (bp)---			---Differences (bp)---			
	Tight Window (30 min.)	Wide Window (1 hour)	Daily Window (1 day)	Daily - Tight	Wide - Tight	Intermeeting Move?	Employment Report?
4-Feb-94	16.3	15.2	11.7	-4.7	-1.2		✓
22-Mar-94	0.0	0.0	-3.4	-3.4	0.0		
18-Apr-94	15.0	15.0	10.0	-5.0	0.0	✓	
17-May-94	11.1	11.1	13.3	2.2	0.0		
6-Jul-94	-5.0	-3.7	-5.0	0.0	1.2		
16-Aug-94	12.4	14.5	14.5	2.1	2.1		
27-Sep-94	-9.0	-9.0	-8.0	1.0	0.0		
15-Nov-94	12.0	12.0	14.0	2.0	0.0		
20-Dec-94	-22.6	-22.6	-16.9	5.6	0.0		
1-Feb-95	6.2	6.2	5.2	-1.0	0.0		
28-Mar-95	-1.0	0.0	0.0	1.0	1.0		
23-May-95	0.0	0.0	0.0	0.0	0.0		
6-Jul-95	-11.2	-7.4	-1.2	9.9	3.7		
22-Aug-95	3.4	3.4	0.0	-3.4	0.0		
26-Sep-95	3.0	4.0	4.0	1.0	1.0		
15-Nov-95	4.0	5.0	6.0	2.0	1.0		
19-Dec-95	-9.0	-10.3	-10.3	-1.3	-1.3		
31-Jan-96	-3.0	-3.0	-7.0	-4.0	0.0		
26-Mar-96	1.0	1.0	1.0	0.0	0.0		
21-May-96	0.0	0.0	0.0	0.0	0.0		
3-Jul-96	-7.2	-6.6	-5.0	2.2	0.6		
20-Aug-96	-2.8	-2.8	-4.2	-1.4	0.0		
24-Sep-96	-12.0	-12.0	-13.0	-1.0	0.0		
13-Nov-96	-1.8	-1.8	0.0	1.8	0.0		
17-Dec-96	1.1	0.0	1.1	0.0	-1.1		
5-Feb-97	-3.7	-3.0	-3.0	0.6	0.6		
25-Mar-97	4.0	4.0	4.0	0.0	0.0		
20-May-97	-9.9	-9.9	-11.3	-1.4	0.0		
2-Jul-97	-2.1	-1.1	-1.6	0.5	1.1		
19-Aug-97	0.0	0.0	-1.3	-1.3	0.0		
30-Sep-97	0.0	0.0	0.0	0.0	0.0		
12-Nov-97	-4.2	-4.2	-4.2	0.0	0.0		
16-Dec-97	0.0	0.0	-1.0	-1.0	0.0		
4-Feb-98	0.0	0.0	0.0	0.0	0.0		
31-Mar-98	-1.0	-1.0	0.0	1.0	0.0		
19-May-98	-2.6	-2.6	-2.6	0.0	0.0		
1-Jul-98	-0.5	-0.5	-0.5	0.0	0.0		
18-Aug-98	1.2	1.2	1.2	0.0	0.0		
29-Sep-98	5.0	6.0	6.0	1.0	1.0		
15-Oct-98	-24.2	-24.2	-20.3	3.9	0.0	✓	
17-Nov-98	-6.9	-5.8	-5.8	1.2	1.2		
22-Dec-98	0.0	-1.7	-1.7	-1.7	-1.7		
3-Feb-99	0.6	0.6	0.0	-0.6	0.0		
30-Mar-99	-1.0	0.0	0.0	1.0	1.0		
18-May-99	-1.2	-1.2	-3.6	-2.4	0.0		
30-Jun-99	-3.0	-4.0	-4.0	-1.0	-1.0		
24-Aug-99	3.5	3.0	3.0	-0.5	-0.5		
5-Oct-99	-4.2	-4.2	-4.2	0.0	0.0		
16-Nov-99	7.5	9.6	8.6	1.1	2.1		

Date	---Monetary Policy Surprise (bp)---			---Differences (bp)---		Intermeeting Move?	Employment Report?
	Tight Window (30 min.)	Wide Window (1 hour)	Daily Window (1 day)	Daily - Tight	Wide - Tight		
21-Dec-99	1.6	1.6	1.6	0.0	0.0		
2-Feb-00	-5.9	-5.9	-5.4	0.5	0.0		
21-Mar-00	-4.7	-4.7	-3.1	1.6	0.0		
16-May-00	4.1	3.1	5.2	1.0	-1.0		
28-Jun-00	-2.5	-2.0	-2.0	0.5	0.5		
22-Aug-00	-1.7	0.0	-1.7	0.0	1.7		
3-Oct-00	0.0	-0.6	0.0	0.0	-0.6		
15-Nov-00	-1.0	-1.0	0.0	1.0	0.0		
19-Dec-00	6.5	6.5	5.2	-1.3	0.0		
3-Jan-01	-39.3	-36.5	-38.2	1.1	2.8	✓	
31-Jan-01	3.5	4.0	0.5	-3.0	0.5		
20-Mar-01	7.1	5.6	5.6	-1.4	-1.4		
18-Apr-01	-43.8	-46.3	-42.5	1.3	-2.5	✓	
15-May-01	-9.7	-7.8	-7.8	1.9	1.9		
27-Jun-01	10.5	11.0	8.5	-2.0	0.5		
21-Aug-01	1.6	1.6	1.6	0.0	0.0		
17-Sep-01	omitted	omitted	omitted	omitted	omitted	✓	
2-Oct-01	-3.7	-3.7	-7.0	-3.2	0.0		
6-Nov-01	-15.0	-15.0	-10.0	5.0	0.0		
11-Dec-01	-0.8	0.0	0.0	0.8	0.8		
30-Jan-02	2.5	1.5	1.5	-1.0	-1.0		
19-Mar-02	-2.6	-2.6	-2.6	0.0	0.0		
7-May-02	0.7	0.7	0.0	-0.7	0.0		
26-Jun-02	0.0	0.0	-2.0	-2.0	0.0		
13-Aug-02	4.3	4.3	3.4	-0.9	0.0		
24-Sep-02	2.0	2.5	2.0	0.0	0.5		
6-Nov-02	-20.0	-18.8	-19.4	0.6	1.3		
10-Dec-02	0.0	0.0	0.0	0.0	0.0		
29-Jan-03	1.0	0.5	0.5	-0.5	-0.5		
18-Mar-03	2.4	3.6	4.8	2.4	1.2		
6-May-03	3.7	3.7	3.7	0.0	0.0		
25-Jun-03	13.5	12.5	12.5	-1.0	-1.0		
12-Aug-03	0.0	0.0	0.0	0.0	0.0		
16-Sep-03	1.1	1.1	0.0	-1.1	0.0		
28-Oct-03	-0.5	-0.5	0.0	0.5	0.0		
9-Dec-03	0.0	0.0	0.0	0.0	0.0		
28-Jan-04	0.5	0.0	0.0	-0.5	-0.5		
16-Mar-04	0.0	0.0	0.0	0.0	0.0		
4-May-04	-1.2	-1.2	-0.6	0.6	0.0		
30-Jun-04	-0.5	-1.5	-1.0	-0.5	-1.0		
10-Aug-04	0.7	1.5	2.2	1.5	0.7		
21-Sep-04	0.0	0.0	1.7	1.7	0.0		
10-Nov-04	-0.8	0.0	0.0	0.8	0.8		
14-Dec-04	-0.9	0.0	0.0	0.9	0.9		

Data Appendix Table 3
Target and Path Factors

Date	Target Factor	Path Factor	FOMC Statement?	Date	Target Factor	Path Factor	FOMC Statement?	Date	Target Factor	Path Factor	FOMC Statement?
8-Feb-90	0.3	5.8		16-Aug-94	10.7	-8.3	✓	15-Nov-00	2.7	2.2	✓
28-Mar-90	1.5	-3.3		27-Sep-94	-4.3	7.9		19-Dec-00	7.5	-3.9	✓
16-May-90	2.5	-3.4		15-Nov-94	10.3	-3.4	✓	3-Jan-01	-32.3	22.8	✓
5-Jul-90	0.4	3.9		20-Dec-94	-15.1	26.6		31-Jan-01	8.2	-12.5	✓
13-Jul-90	0.9	-1.0		1-Feb-95	7.8	5.5	✓	20-Mar-01	-2.3	-13.5	✓
22-Aug-90	2.3	0.8		28-Mar-95	5.3	2.2		18-Apr-01	-48.5	-3.2	✓
3-Oct-90	1.3	-0.4		23-May-95	2.2	-0.3		15-May-01	-8.6	-15.7	✓
29-Oct-90	-0.9	5.0		6-Jul-95	-8.7	-38.4	✓	27-Jun-01	11.8	2.3	✓
14-Nov-90	4.7	-0.6		22-Aug-95	4.7	9.0		21-Aug-01	2.9	-9.7	✓
7-Dec-90	-5.3	5.7		26-Sep-95	5.5	9.7		2-Oct-01	-1.8	-6.4	✓
18-Dec-90	-16.1	14.1		15-Nov-95	3.9	3.6		6-Nov-01	-12.7	-15.7	✓
8-Jan-91	-11.7	-4.6		19-Dec-95	-6.5	4.8	✓	11-Dec-01	1.9	-18.1	✓
1-Feb-91	-6.2	1.8		31-Jan-96	-0.2	-2.4	✓	30-Jan-02	4.2	3.6	✓
7-Feb-91	1.3	-0.2		26-Mar-96	1.6	4.0		19-Mar-02	-0.3	-11.7	✓
8-Mar-91	-3.5	-3.8		21-May-96	2.7	3.8		7-May-02	3.5	-11.4	✓
27-Mar-91	4.9	-10.5		3-Jul-96	-3.0	5.3		26-Jun-02	2.4	2.3	✓
30-Apr-91	-18.7	-10.5		20-Aug-96	0.8	5.2		13-Aug-02	8.1	-37.2	✓
15-May-91	3.0	2.0		24-Sep-96	-10.6	0.4		24-Sep-02	3.8	-5.6	✓
5-Jul-91	0.5	5.8		13-Nov-96	1.6	-2.4		6-Nov-02	-16.3	7.2	✓
6-Aug-91	-13.5	-0.8		17-Dec-96	3.1	-1.0		10-Dec-02	1.9	7.0	✓
21-Aug-91	12.0	-7.2		5-Feb-97	-0.6	9.0		29-Jan-03	3.5	10.4	✓
13-Sep-91	-0.4	6.2		25-Mar-97	6.5	9.6	✓	18-Mar-03	2.9	-2.4	✓
2-Oct-91	2.1	2.2		20-May-97	-8.3	2.1		6-May-03	5.2	-27.0	✓
30-Oct-91	-1.6	4.3		2-Jul-97	1.4	2.3		25-Jun-03	14.4	10.6	✓
6-Nov-91	-11.7	-1.3		19-Aug-97	2.6	3.3		12-Aug-03	3.7	-11.9	✓
6-Dec-91	2.1	2.6		30-Sep-97	2.1	0.8		16-Sep-03	3.4	-1.3	✓
18-Dec-91	3.4	5.6		12-Nov-97	0.2	-2.6		28-Oct-03	3.9	-24.4	✓
20-Dec-91	-32.2	-10.2		16-Dec-97	1.8	-0.4		9-Dec-03	0.2	13.9	✓
6-Feb-92	3.7	-1.6		4-Feb-98	1.9	5.4		28-Jan-04	-1.1	42.7	✓
1-Apr-92	0.1	4.3		31-Mar-98	1.1	2.6		16-Mar-04	4.1	-17.8	✓
9-Apr-92	-19.0	-22.5		19-May-98	0.2	0.5		4-May-04	0.9	5.1	✓
20-May-92	4.1	1.6		1-Jul-98	3.6	0.4		30-Jun-04	0.4	-1.7	✓
2-Jul-92	-9.7	20.8		18-Aug-98	3.1	-0.9		10-Aug-04	4.3	7.5	✓
19-Aug-92	4.2	-1.0		29-Sep-98	9.4	-5.3	✓	21-Sep-04	2.5	-2.9	✓
4-Sep-92	2.6	-2.3		15-Oct-98	-24.0	-22.6	✓	10-Nov-04	3.0	-7.6	✓
7-Oct-92	3.8	-0.1		17-Nov-98	1.3	1.4	✓	14-Dec-04	1.7	-1.8	✓
18-Nov-92	4.8	1.2		22-Dec-98	2.9	-0.3					
23-Dec-92	3.8	-6.2		3-Feb-99	2.7	2.2					
4-Feb-93	0.0	3.8		30-Mar-99	1.5	-4.1					
24-Mar-93	2.9	3.5		18-May-99	0.5	32.8	✓				
19-May-93	0.0	-1.2		30-Jun-99	-2.1	-16.2	✓				
8-Jul-93	1.8	0.1		24-Aug-99	5.7	-5.8	✓				
18-Aug-93	6.4	-0.5		5-Oct-99	-2.7	25.8	✓				
22-Sep-93	4.0	-3.2		16-Nov-99	11.9	3.8	✓				
17-Nov-93	0.6	1.9		21-Dec-99	2.4	9.3	✓				
22-Dec-93	3.2	-6.7		2-Feb-00	-2.0	9.0	✓				
4-Feb-94	8.1	21.8	✓	21-Mar-00	2.5	2.3	✓				
22-Mar-94	-2.2	-4.9		16-May-00	7.4	4.9	✓				
18-Apr-94	10.7	6.6		28-Jun-00	-0.2	-2.1	✓				
17-May-94	7.2	-21.2	✓	22-Aug-00	0.8	8.1	✓				
6-Jul-94	1.8	4.0		3-Oct-00	1.4	12.7	✓				

The Performance and Robustness of Interest-Rate Rules in Models of the Euro Area*

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In this paper, we examine the performance and robustness of optimized interest-rate rules in four models of the euro area that differ considerably in terms of size, degree of aggregation, relevance of forward-looking behavioral elements, and adherence to microfoundations. Our findings are broadly consistent with results documented for models of the U.S. economy: backward-looking models require relatively more aggressive policies with, at most, moderate inertia; rules that are optimized for such models tend to perform reasonably well in forward-looking models, while the reverse is not necessarily true; and, hence, the operating characteristics of robust rules (i.e., rules that perform satisfactorily in all models) are heavily weighted towards those required by backward-looking models.
JEL Codes: E31, E52, E58, E61

*The paper summarizes the results of a project aimed at evaluating the performance of monetary policy rules in a number of small- to medium-scale models developed by Eurosystem staff that have been regularly used for policy analysis. Helpful comments by Keith Küster, Sergio Nicoletti Altimari, Athanasios Orphanides, Glenn Rudebusch, Frank Smets, Volker Wieland, John Williams, Raf Wouters; by participants at the ECB conference on “Monetary Policy and Imperfect Knowledge”; and by an anonymous referee are greatly appreciated. The opinions expressed are those of the authors and do not necessarily reflect views of the European Central Bank or the Banca d'Italia. Any remaining errors are the sole responsibility of the authors. Correspondence: Coenen: Directorate General Research, European Central Bank, e-mail: gunter.coenen@ecb.int, homepage: www.guentercoenen.com; Adalid: Directorate General Research, European Central Bank, e-mail: ramon.adalid_lozano@ecb.int; McAdam: Directorate General Research, European Central Bank, e-mail: peter.mcadam@ecb.int; Siviero: Economic Research Department, Banca d'Italia, e-mail: stefano.siviero@bancaditalia.it.

The design of robust monetary policy rules has attracted considerable interest in recent macroeconomic research. This interest largely reflects the increased awareness on the part of policymakers and academics alike that, when evaluating the possible consequences of alternative monetary policies, the existing uncertainties about the structure of the economy need to be given due account. As a result, a large body of research has been oriented toward identifying the characteristics of monetary policy rules that perform “reasonably well” across a range of potentially nonnested and competing macroeconomic models.¹ While this line of research has primarily focused on models of the U.S. economy, no systematic study has yet been conducted for the euro area.²

In an attempt to fill this gap, we concentrate on the euro area and examine the performance and robustness of monetary policy rules using a rather diverse set of macroeconomic models of the euro area economy. Such an examination seems particularly relevant as the single monetary policy of the European Central Bank (ECB) has to focus on the euro area as a whole, being a new and still relatively unexplored economic entity, and with models of the euro area having been developed only very recently. Specifically, our analysis relies on four models of the euro area economy that have been built by staff of the Eurosystem in recent years. These models differ considerably in terms of size, degree of aggregation, relevance of forward-looking behavioral elements, and adherence to microfoundations. They thus cover a wide range of features that are *a priori* considered to be of high relevance in the context of evaluating the robustness of monetary policy rules.³ One of our main goals is to ascertain which of the various features with respect to which these models differ are of importance when designing rules suitable for model-based evaluations of monetary policy, and which features are, by contrast, arguably less important.

¹For an early claim in this respect see McCallum (1988).

²Earlier studies of the performance of interest-rate rules across alternative models of the U.S. economy are provided in Bryant, Hooper, and Mann (1993) and Taylor (1999). More recent studies have been undertaken by Levin, Wieland, and Williams (2003) and Levin and Williams (2003). The study of Côté et al. (2002) focuses on models of the Canadian economy.

³Indeed, Svensson (2003) argues that models used to analyze simple monetary policy rules, such as the Taylor rule, were too similar in structure (i.e., of the New Keynesian variety) to constitute a true test of their robustness.

In terms of methodology, our analysis builds on recent work by Levin, Wieland, and Williams (1999, 2003) and Levin and Williams (2003).⁴ This methodology involves implementing simple reaction functions describing the response of the short-term nominal interest rate to inflation and the output gap, either observed or forecast, and then optimizing over the respective response coefficients. The performance of these optimized interest-rate rules is then evaluated across alternative, potentially competing models with regard to their ability to stabilize inflation and output around their targets, while avoiding undue fluctuations in the nominal interest rate itself.

Of course, variables other than inflation and the output gap may enter the interest-rate rule, such as the exchange rate or monetary and financial-market indicators. However, the literature tends to suggest that including information variables of this kind in the policy rule yields relatively modest gains in model-based evaluations because they are typically highly correlated with the interest rate itself or closely related with the measures of inflation and the output gap entering the rule (although this result may not hold in sufficiently complex models). Furthermore, simple rules with a feedback to a small set of variables are arguably more robust to model misspecification and uncertainty than rules based on a larger set of variables, which might overfit specific model characteristics.

From an institutional viewpoint, the advantage of simple interest-rate rules is clearly their transparency and, thus, the ease with which they may be communicated to and monitored by the outside world. While it is unlikely that monetary policymakers will follow their literal execution, such rules may nonetheless be a useful benchmark for assessing the actual conduct of monetary policy. Likewise, from an empirical point of view, simple interest-rate rules seem to match the data well for the United States as well as a number of European countries (see Clarida, Galí, and Gertler 1998). More recently, evidence for the euro area as a whole has been provided by Gerdesmeier and Roffia (2004).

⁴There are alternative approaches to analyzing the consequences of uncertainty about the structure of the economy (see, e.g., Giannoni 2002; Giannoni and Woodford 2002; Hansen and Sargent 2002; Onatski and Stock 2002; Onatski and Williams 2003; and Tetlow and von zur Muehlen 2001). This coexistence of alternative approaches reflects the fact that there has not yet emerged a consensus on how to address the issue of model uncertainty.

The findings of our analysis are broadly consistent with the results documented for models of the U.S. economy: backward-looking models require relatively more aggressive policies with, at most, moderate inertia; rules that are optimized for such models tend to perform reasonably well in forward-looking models, while the reverse is not necessarily true; and, hence, the operating characteristics of robust rules (i.e., rules that perform satisfactorily in all models) are heavily weighted toward those required by backward-looking models. In the course of collecting these results, we highlight a number of model features (notably, the degree of forward-lookingness) that play a crucial role in shaping the operating characteristics of optimized interest-rate rules. This in turn suggests that future empirical research that aims at casting additional light on these features may enhance the reliability and usefulness of interest-rate rules for model-based evaluations of monetary policy.

The remainder of this paper is organized as follows. Section 1 outlines the set of euro area models used in our analysis and illustrates the implied differences in inflation and output-gap dynamics in response to a monetary policy shock. Section 2 briefly describes the methodology used for evaluating the performance of interest-rate rules and provides a set of optimized benchmark rules for each of the euro area models under examination. Section 3 evaluates the robustness of these benchmark rules when there is uncertainty about the true structure of the euro area economy, as represented by the coexistence of possibly competing models, while section 4 identifies the operating characteristics of rules that attain satisfactory outcomes across the set of models used. Section 5 reports additional sensitivity analysis, and section 6 concludes.

1. The Models of the Euro Area

To examine the performance of monetary policy rules across a range of potentially competing models of the euro area economy, we utilize a set of four macroeconomic models that differ considerably in terms of variable and country coverage, relevance of forward-looking behavioral elements, and adherence to microfoundations. All models are estimated with aggregate euro area-wide data except one, which separately models the three largest euro area economies. Two of the models are entirely or at least predominantly backward looking,

while forward-looking elements abound in the other two. All models are consistent with basic economic principles; however, there is only one in our set of models that is strictly based on the assumption of optimizing agents. Hence, the models under examination cover a fairly broad range of different modeling strategies.

In the remainder of this section, we briefly present the four models of the euro area and illustrate the implied differences in inflation and output-gap dynamics in response to a monetary policy shock.

1.1 An Outline of the Euro Area Models

1.1.1 The Coenen-Wieland Model

The Coenen-Wieland (CW) model (see Coenen and Wieland 2000) is a small-scale model of aggregate supply and aggregate demand that is designed to capture the broad characteristics of inflation and output dynamics in the euro area. Since its development, the model has been mainly used as a laboratory for evaluating the performance of alternative monetary policy strategies.

The supply side of the model incorporates price and wage staggering, with wage setters negotiating long-term nominal wage contracts with reference to past contracts that are still in effect and future contracts that will be negotiated over the life of the current contract. If wage setters expect the output gap to be positive, they adjust the current contract wages upward, and vice versa. Consequently, inflation depends on its own leads and lags, excess-demand conditions, and transitory contract wage shocks—the latter representing “cost-push” shocks. There are two versions of the supply side that feature distinct types of staggered wage contracts: the nominal wage contracting specification due to Taylor (1980) and the relative real wage contracting specification by Fuhrer and Moore (1995).⁵ The two specifications differ with respect to the degree of inflation persistence that they induce, with Fuhrer-Moore-type contracts giving

⁵While the two types of specifications represent rules for price and wage setting that are not explicitly derived from a framework with optimizing agents, they need not be necessarily inconsistent with such a framework. More recently, Taylor-type contracts have been analyzed within more fully fleshed-out dynamic general equilibrium models (see, e.g., Chari, Kehoe, and McGrattan 2000 or King and Wolman 1999). Fuhrer-Moore-type contracts, however, have typically been criticized for lacking microfoundations.

more weight to past inflation developments. In this paper, the version with Taylor-type contracts is used.⁶

A simple aggregate demand relationship relates the output gap (measured as the deviation of actual output from a smooth trend) to several lags of itself, the ex-ante long-term real interest rate, and a transitory demand shock. The long-term real rate is determined jointly by a term-structure relationship and the Fisher equation. The short-term nominal interest rate is the instrument of monetary policy, and changes in the latter affect aggregate demand through the impact on the ex-ante long-term real interest rate.

1.1.2 The Smets-Wouters Model

The Smets-Wouters (SW) model (see Smets and Wouters 2003) is an extended version of the standard New Keynesian dynamic stochastic general equilibrium (DSGE) model with sticky prices and wages. The model is estimated by Bayesian techniques using seven euro area macroeconomic time series: real gross domestic product (GDP), consumption, investment, employment, real wages, inflation, and the nominal short-term interest rate.

The model features three types of economic agents: households, firms, and the monetary policy authority. Households maximize a utility function with two arguments (goods and leisure) over an infinite life horizon. Consumption appears in the utility function relative to a time-varying external habit-formation variable. Labor is differentiated over households, so that there is some monopoly power over wages, which results in an explicit wage equation and allows for the introduction of sticky nominal wages à la Calvo (1983). Households also rent capital services to firms and decide how much capital to accumulate given certain capital adjustment costs. As the rental price of capital goes up, the capital stock can be used more intensively according to some cost schedule. Firms produce differentiated goods, decide on labor and capital inputs, and set prices, again à la Calvo (1983). The Calvo model in both wage and price setting is augmented

⁶See Coenen (2003) for an evaluation of the performance of monetary policy rules across the two variants with Taylor- and Fuhrer-Moore-type contracts, respectively. This paper shows that, when faced with uncertainty about the prevailing degree of inflation persistence, a cautious monetary policymaker is well-advised to design interest-rate policies under the assumption that the inflation process is characterized by a high degree of persistence—as induced by Fuhrer-Moore-type contracts.

by the assumption that those prices and wages that cannot be freely set are partially indexed to past inflation. Prices are therefore set as a function of current and expected real marginal cost, but are also influenced by past inflation. Real marginal cost depends on wages and the rental rate of capital. The short-term nominal interest rate is the instrument of monetary policy.⁷

The stochastic behavior of the model is driven by ten exogenous shocks: five shocks arising from technology and preferences, three cost-push shocks, and two monetary policy shocks. The first set of shocks is assumed to follow first-order autoregressive processes, whereas the second set is assumed to follow serially uncorrelated processes. Consistent with the DSGE setup, potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of cost-push shocks.

1.1.3 *The Area-Wide Model*

The Area-Wide Model (AWM; see Fagan, Henry, and Mestre 2001) is a medium-size structural macroeconomic model that treats the euro area as a single economy.⁸ It has a long-run neoclassical equilibrium with a vertical Phillips curve, but with some short-run frictions in price and wage setting and factor demands. Consequently, activity is demand-determined in the short run but supply-determined in the long run. In the latter, employment converges to a level consistent with the exogenously given level of equilibrium unemployment, and factor demands are consistent with the solution of the firms' profit-maximization problem. Stock-flow adjustments are accounted for by, for example, the inclusion of a wealth term in consumption. At present, the treatment of expectations in the model is quite limited. With the exception of the exchange rate (determined by uncovered interest parity) and the long-term nominal interest rate (modeled by a term-structure relationship), the model embodies backward-looking expectations.

⁷Extending the study of Coenen (2003), Angeloni, Coenen, and Smets (2003) utilize the SW model to analyze the design of monetary policy when the monetary policymaker is uncertain about the degree of nominal as well as real persistence. The results of this analysis confirm the conclusions of the earlier study.

⁸For an examination of optimal monetary policy in the context of the AWM alone, see Dieppe, Küster, and McAdam (2004).

As to the mechanisms through which monetary policy affects the economy, aggregate demand in the AWM is presently influenced only by short-term real interest rates. Long-term nominal rates determine the government's debt service but do not explicitly enter investment decisions. The expectations channel in principle allows monetary policy to influence inflation via wage- and price-setting behavior. In addition to these influences, further effects enter through the exchange rate. Apart from an indirect effect of exchange rates on domestic demand, there is also a direct exchange-rate effect on consumer price inflation through the price for imported goods. The output gap is defined as the ratio of actual output to potential output, which is based on an aggregate Cobb-Douglas production function with constant returns to scale and Hicks-neutral technical progress. For this, trend total factor productivity has been estimated within-sample by applying the Hodrick-Prescott filter to the Solow residual derived from the production function.

1.1.4 A Disaggregate Model of the Euro Area

The Disaggregate Model (DM) of the euro area used in Angelini et al. (2002) and Monteforte and Siviero (2002) is a multicountry version of the simple backward-looking two-equation model in Rudebusch and Svensson (1999). It consists of an aggregate supply equation and an aggregate demand equation for each of the three largest economies in the euro area: Germany, France, and Italy, which jointly account for over 70 percent of euro area GDP. The first equation, interpretable as a Phillips curve, determines inflation in each country as a function of lagged inflation and the output gap, as well as inflation "imported" from the other two countries. The sum of the coefficients on lagged and imported inflation is constrained to equal unity (a restriction accepted by the data), so that a long-run vertical Phillips curve exists for all countries. The second equation represents an IS curve and relates the output gap of each country (modeled as the deviation of actual output from a smooth trend) to its own lagged values, the short-term real interest rate, and the output gaps in the other two countries (a design meant to capture the trade linkages among euro area economies). Euro area inflation and the aggregate output gap are determined via identities as weighted averages of the corresponding individual country variables. Finally,

the area-wide short-term nominal interest rate is the instrument of monetary policy.

As the model allows for simultaneous cross-country linkages, it was estimated with the three-stage least-squares (3SLS) method using data on inflation, output, and nominal interest rates. Not all cross-country terms are significant in all equations. This results in a clear causal pattern, with the German economy affecting the other two comparatively more, and with the Italian economy being essentially recursive. Furthermore, the estimation results indicate that there exists a significant degree of heterogeneity among the three economies included in the model.⁹ Accordingly, the results in Angelini et al. (2002) suggest that monetary policy effectiveness may be considerably enhanced if country-specific information is used.

1.2 Differences in Monetary Policy Transmission

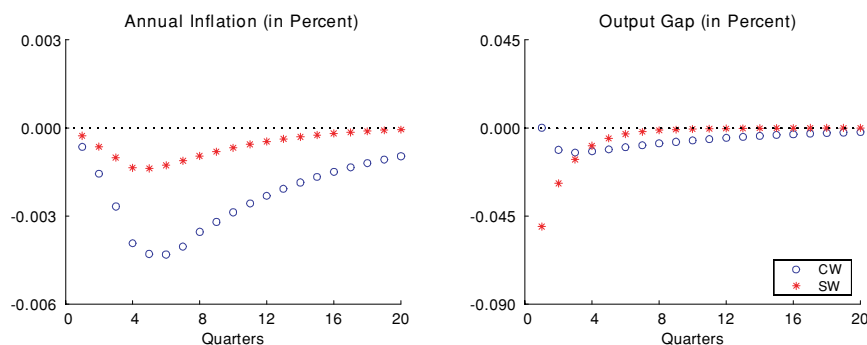
Figure 1 provides a comparison of the effects of an unexpected one-quarter tightening of monetary policy by 100 basis points in the four different models of the euro area, with the monetary policymaker following the interest-rate rule proposed by Taylor (1993) thereafter. Panel A of the figure depicts the dynamic responses of annual inflation and the output gap for the largely forward-looking models (CW and SW), while panel B shows the responses for the predominantly backward-looking models (AWM and DM).

Qualitatively, the tightening of policy has the same consequences in the four models. As the short-term nominal interest rate rises unexpectedly, demand falls short of potential and inflation falls below the monetary policymaker's target, with the dynamic adjustment being drawn out lastingly. Quantitatively, however, the responses exhibit some noticeable differences. Most importantly, the disinflation effect is considerably larger and more persistent in the backward-looking models with the timing of the peak effect on inflation noticeably delayed relative to that on the output gap. By contrast, in the

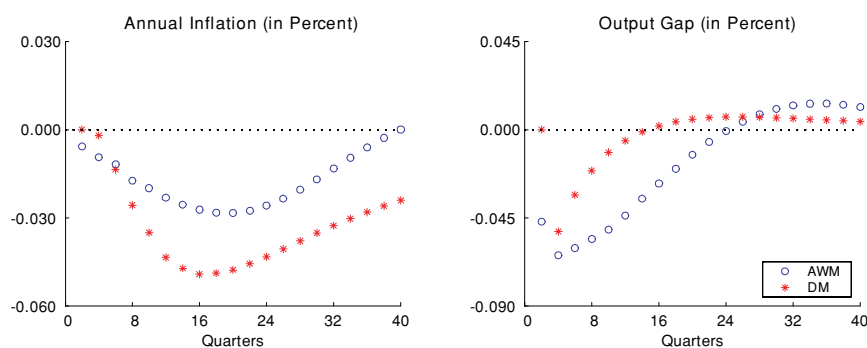
⁹Monteforte and Siviero (2002) also report estimates of an aggregate companion of the DM (referred to as Aggregate Model, AM, of the euro area), consisting of just two equations. Both statistical and economic criteria indicate that the DM is to be preferred to the AM.

Figure 1. Responses to an Interest-Rate Shock (100 Basis Points) under Taylor's Rule

A. The Forward-Looking Euro Area Models



B. The Backward-Looking Euro Area Models



forward-looking models, the timing of the peak effect on inflation is much closer to that on the output gap, with the reversion to base of both inflation and the output gap being more rapid and smoother. Differences in the responses of the output gap, notably in the initial periods, largely reflect differences in the employed output-gap concepts. Overall, this suggests that the degree of forward-lookingness is of utmost relevance for explaining the differences in inflation and output-gap dynamics across models.

Based on the documented patterns of the dynamic responses to the monetary policy shock under Taylor's rule, we summarize that

a given interest-rate rule may perform quite differently in terms of inflation and output-gap stabilization, depending on the characteristics of the particular euro area model used. Hence, it is evident why monetary policymakers should be concerned about the design of the interest-rate rule to be used in model-based analyses.

2. Evaluating the Performance of Interest-Rate Rules

We now proceed to describe the methodology that we will use to evaluate the stabilization performance of alternative monetary policies across our set of models of the euro area economy. Our starting point is an evaluation of simple interest-rate rules that respond to outcomes or forecasts of annual inflation and the output gap and allow for inertia due to dependence on the lagged short-term nominal interest rate.

2.1 The Methodology

Following the approach in Levin, Wieland, and Williams (2003), we consider a three-parameter family of simple interest-rate rules,

$$i_t = \rho i_{t-1} + (1 - \rho) (r^* + E_t [\tilde{\pi}_{t+\theta}]) + \alpha E_t [\tilde{\pi}_{t+\theta} - \pi^*] + \beta E_t [y_{t+\kappa}],$$

where i_t denotes the short-term nominal interest rate, r^* is the equilibrium real interest rate, $\tilde{\pi}_t = p_t - p_{t-4}$ is the annual inflation rate, π^* denotes the monetary policymaker's inflation target, and y_t is the output gap.¹⁰ Under rational expectations, the operator $E_t[\cdot]$ indicates the model-consistent forecast of a particular variable, using information available in period t .¹¹ The integer parameters θ and κ denote the length of the forecast horizons for inflation and the output gap, respectively. This specification accommodates forecast-based rules (with forecast horizons $\theta, \kappa > 0$) as well as outcome-based

¹⁰Even in the case where the multicountry model DM is used, both inflation and the output gap are to be interpreted as area-wide variables, reflecting the assumption that the policymaker is concerned with area-wide developments.

¹¹We employ the AIM algorithm of Anderson and Moore (1985), which uses the Blanchard and Kahn (1980) method for solving linear rational expectations models, to compute model-consistent expectations.

rules ($\theta = \kappa = 0$) and simplifies to the one proposed by Taylor (1993) if $\theta = \kappa = 0$, $\rho = 0$ and $\alpha = \beta = 0.5$.¹²

For fixed inflation and output-gap forecast horizons θ and κ , the above family of interest-rate rules is defined by the response coefficients ρ , α , and β . The coefficients α and β represent the policymaker's short-term reaction to inflation in deviation from target and the output gap, respectively, while ρ determines the inertia of the interest-rate response, commonly interpreted as the desired degree of policy "smoothing." The latter plays an important role for model-based evaluations of interest-rate rules. In particular, the degree to which the optimal policy in any given model embodies smoothing tends to depend largely on the expectation formation mechanism embedded in that model. Typically, if expectations are backward looking, values of ρ at or above unity can perform poorly since they may engender undampened oscillations in the model economy due to instrument instability (see, e.g., Rudebusch and Svensson 1999 and Batini and Nelson 2001). By contrast, models with largely forward-looking expectations, notably small optimizing New Keynesian models, tend to favor a comparatively high degree of smoothing because the inertial adjustment of the short-term interest rate enables the policymaker to steer expectations and thereby to stabilize the economy more effectively.

In our evaluation of the stabilization performance of variants of the above family of interest-rate rules, we assume that the monetary policymaker has a standard loss function equal to the weighted sum of the unconditional variances of inflation, the output gap, and changes in the short-term nominal interest rate,

$$\mathcal{L} = \text{Var}[\pi_t] + \lambda \text{Var}[y_t] + \mu \text{Var}[\Delta i_t].$$

Here, inflation is measured by the annualized one-quarter inflation rate, $\pi_t = 4(p_t - p_{t-1})$. The weight $\lambda \geq 0$ refers to the policymaker's preference for reducing output variability relative to inflation variability, and the weight $\mu \geq 0$ on the variability of changes in the

¹²In the special case with $\rho = 1$, the rule represents a first-difference rule, a class of rules that Levin, Wieland, and Williams (2003) advocate as being robust when examining the performance of simple interest-rate rules across a set of distinct models of the U.S. economy—none of them, however, being fully backward-looking. Orphanides and Williams (2002) emphasize that first-difference rules are also robust to misperceptions about the equilibrium real interest rate r^* .

short-term nominal interest rate, $\Delta i_t = i_t - i_{t-1}$, reflects a desire to avoid undue fluctuations in the nominal interest rate itself.¹³ Establishing this loss function is consistent with the assumption that the policymaker aims at stabilizing inflation around the inflation target π^* and actual output around potential, with the concern regarding excessive interest-rate variability typically justified by financial stability considerations.¹⁴

For fixed inflation and output-gap forecast horizons θ and κ , the three-parameter family of interest-rate rules defined above is optimized by minimizing the policymaker's loss function $\mathcal{L}$ with respect to the coefficients ρ , α , and β . In this context, we repeatedly need to compute the unconditional variances of the models' endogenous variables for a given interest-rate rule. In preparation for these computations, we first identify the series of historical shocks that would be consistent with the alternative models under rational expectations. Based on the covariance matrix of the historical shocks, it is then possible to calculate the unconditional covariance matrix of the endogenous variables for any given interest-rate rule by applying standard methods to the reduced-form solution of the model including that rule.¹⁵

In the subsequent analysis, we will consider four alternative values for the relative weight on output-gap variability, namely $\lambda = 0, 1/3, 1$, and 3 . Regarding the weight on the variability of interest-rate changes, we will concentrate the analysis on a fixed value of $\mu = 0.10$. We shall briefly discuss additional results for a lower weight of $\mu = 0.01$ in the sensitivity analysis in section 5. There, we will also report on results for interest-rate rules that only

¹³For an explicit derivation of the policymaker's loss function $\mathcal{L}$ from quadratic intertemporal preferences, the reader is referred to Rudebusch and Svensson (1999). In Svensson's terminology, the case of $\lambda = \mu = 0$ corresponds to "strict" inflation targeting, while "flexible" inflation targeting is characterized by $\lambda, \mu > 0$ (see Svensson 1999).

¹⁴It is recognized that it would be beneficial to use a welfare criterion derived as an approximation of the representative agent's utility function (see, e.g., Rotemberg and Woodford 1997). The weights in this approximate welfare criterion would be functions of the parameters of the structural model itself. However, to the extent that the models used in this paper, with the exception of the SW model, are lacking microfoundations, a well-defined welfare criterion does not exist.

¹⁵The exception is the SW model, for which the covariance matrix of shocks is estimated jointly with the model's structural parameters.

allow for a response to lagged inflation and the lagged output gap; that is, with $\theta = \kappa = -1$. Such rules have been proposed as a proxy for actual policymaking in “real time” (see McCallum 1988).

2.2 *The Performance of Optimized Benchmark Rules*

As benchmarks for evaluating the performance of simple interest-rate rules across our set of models, we focus on two types of rules: outcome-based rules with $\theta = \kappa = 0$, and forecast-based rules that relate the interest rate to the one-year-ahead forecast of inflation and the contemporaneous output gap with $\theta = 4$ and $\kappa = 0$.

Table 1 reports the optimized response coefficients of the two types of rules for the four different models of the euro area economy and for alternative values of the preference parameter λ , together with an indication of their stabilization performance. The latter is measured by the value of the policymaker’s loss function yielded under the rule optimized for a particular model $\mathcal{L}$ and alternatively, in relative terms, as the percentage-point difference of the loss under the optimized rule from the loss under the fully optimal policy for that model, $\% \Delta \mathcal{L}$.¹⁶ Panel A in table 1 indicates the results for the largely forward-looking models (CW and SW), while panel B shows the results for the predominantly backward-looking models (AWM and DM).

We start with the results for the forward-looking models in panel A and observe that, regardless of the policymaker’s preference for output stabilization, the optimized rules are characterized by a substantial degree of interest-rate smoothing, as captured by the high coefficient on the lagged interest rate, ρ . Interestingly, with a low weight on output stabilization and, in particular, with the inflation forecast horizon extending one year into the future, the magnitude of ρ tends to exceed unity, notably for the SW model. This feature is commonly referred to as “super-inertia” in the interest rate. Not surprisingly, as the weight on output stabilization increases, the coefficient on the output gap, β , rises while the coefficient on inflation,

¹⁶For the CW and SW models, as well as the AWM, the fully optimal policy corresponds to the optimal policy under commitment. See Finan and Tetlow (1999) for details on computing the optimal policy under commitment for large rational expectations models using AIM. Details regarding the unconditional variability of the individual target variables underlying the calculation of the loss function, $\mathcal{L}$, can be found in table A of the appendix.

Table 1. The Stabilization Performance of Optimized Interest-Rate Rules

A. The Forward-Looking Euro Area Models													
θ	κ	λ	CW						SW				
			ρ	α	β	$\mathcal{L}$	$\% \Delta \mathcal{L}$	ρ	α	β	$\mathcal{L}$	$\% \Delta \mathcal{L}$	
0	0	0	0.97	0.81	0.10	1.8	2	1.03	0.81	0.08	1.0	3	
		1/3	0.81	0.49	0.86	2.5	5	1.00	0.64	1.53	1.3	8	
		1	0.79	0.30	1.55	3.2	4	0.99	0.43	3.05	1.5	6	
		3	0.77	0.23	2.50	4.7	3	0.92	0.11	5.87	1.7	5	
4	0	0	1.59	4.36	-0.25	1.9	5	1.96	6.59	-0.05	1.0	6	
		1/3	0.83	1.18	0.84	2.6	6	1.01	1.19	1.50	1.3	9	
		1	0.79	0.76	1.55	3.2	4	1.00	0.79	2.99	1.5	7	
		3	0.77	0.59	2.49	4.6	3	0.99	0.53	5.64	1.7	5	

B. The Backward-Looking Euro Area Models													
θ	κ	λ	AWM						DM				
			ρ	α	β	$\mathcal{L}$	$\% \Delta \mathcal{L}$	ρ	α	β	$\mathcal{L}$	$\% \Delta \mathcal{L}$	
0	0	0	0.44	1.14	1.13	1.3	26	0.31	5.81	3.65	6.0	54	
		1/3	0.39	0.93	2.37	1.9	20	0.28	5.78	3.89	6.9	53	
		1	0.44	0.65	3.69	2.6	18	0.25	5.71	4.26	8.5	51	
		3	0.50	0.40	5.79	3.8	15	0.19	5.59	5.01	12.5	47	
4	0	0	0.47	2.22	0.55	1.2	15	0.55	3.99	0.56	4.6	20	
		1/3	0.37	2.47	1.67	1.8	13	0.51	4.06	0.74	5.4	21	
		1	0.39	2.49	3.00	2.5	12	0.46	4.18	1.05	6.9	23	
		3	0.42	2.38	5.12	3.7	11	0.36	4.39	1.75	10.6	24	

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ), for each preference parameter (λ) and for each model, this table indicates the optimized interest-rate response coefficients (ρ , α , and β), the value of the policymaker's loss function ($\mathcal{L}$), and the percentage-point difference of the latter from the loss under the fully optimal policy ($\%\Delta\mathcal{L}$).

α , falls. As regards the stabilization performance of the optimized interest-rate rules, we observe that the implied values of the policymaker's loss function are typically somewhat larger in the CW model, possibly reflecting a less dominant role for the expectations channel in transmitting monetary policy to aggregate demand. At the same time, for both models the performance of monetary policy deteriorates fairly little when the policymaker follows a relatively

simple optimized interest-rate rule rather than the fully optimal policy. The value of the policymaker's loss function never rises by more than 9 percentage points. We also observe that there is no discernible stabilization gain from following forecast-based as opposed to outcome-based rules.

Turning to the backward-looking models in panel B, a number of differences are noteworthy. First, the optimized interest-rate rules embody only a mild degree of smoothing, with ρ varying between 0.4 and 0.5 for the AWM and between 0.2 and 0.6 for the DM. This discrepancy largely reflects the fact that expectations, notably those determining long-term interest rates, do not play a dominant role in either model. Thus, there is no scope for increasing the effectiveness of monetary policy by an inertial adjustment of the short-term rate. Second, the optimized response coefficients α and β are typically quite a bit larger than for the forward-looking models, in particular for the DM. This mirrors the fact that, in backward-looking models, inflation and the output gap are much harder to stabilize and, as a result, the policymaker has to respond more aggressively to any signs of rising inflation and cumulating output gaps. Third, when compared with the outcomes under the fully optimal policies, the stabilization performance of simple interest-rate rules deteriorates quite substantially, albeit less strongly for the AWM. In the extreme case when the policymaker does not attach any weight to output stabilization, the loss under the optimized outcome-based rule exceeds the loss associated with the fully optimal policy by more than 50 percentage points for the DM, while the loss differs by about 25 percentage points for the AWM.¹⁷ In the case of the AWM, the relatively poor performance of simple optimized rules is likely due to the model's fairly high degree of structural detail and its rather complex dynamics (see Finan and Tetlow [1999] for a similar observation based on the Federal Reserve Board's FRB/US model).¹⁸ Similarly,

¹⁷Evidently, while the values of the loss function obtained for the AWM are comparable with those yielded in the forward-looking models, the losses for the DM are quite a bit larger.

¹⁸Giannoni and Woodford (2002) show that the fully optimal policy for a model, regardless of its degree of structural detail, can be represented as a generalized Taylor rule, in which the relation between the interest-rate instrument and the other target variables includes leads and lags of the target variables. Therefore, the relatively poor performance of the simple interest-rate rules examined here ought to be mitigated by the inclusion of additional leads and lags of the

the DM departs from the rest of the models to the extent that it is the only model in which the three largest euro area economies are modeled separately. Fourth, contrary to the results for the forward-looking models, there are substantial gains in performance from using forecast-based rather than outcome-based rules, with the maximum gain equal to 11 percentage points for the AWM and 34 percentage points for the DM (in the case of $\lambda = 0$). We attribute these improvements in performance to the information-encompassing nature of forecast-based rules, with the inflation forecast incorporating information about incipient risks to future inflation arising from a possibly larger set of underlying determinants (see Batini and Haldane 1999) and the transmission lag of monetary policy (see Batini and Nelson 2001).

3. Evaluating the Robustness of Interest-Rate Rules

In the previous section, we implicitly assumed that the policymaker knows the “true” model of the euro area economy when optimizing variants of the three-parameter family of interest-rate rules for any given model. While the optimized rules typically succeeded in stabilizing inflation and output satisfactorily for that given model, we now proceed to analyze to what extent the optimized rules are robust to model uncertainty, in the sense of also performing reasonably well across the other, possibly competing models of the euro area.

3.1 *The Performance of Optimized Rules Across Models*

Table 2 summarizes our findings regarding the stabilization performance of the optimized benchmark rules documented in table 1 above across our set of models. Here, performance is measured by the value of the policymaker’s loss function, $\mathcal{L}$, and by the percentage-point difference of the latter from the loss under the fully optimal policy, $\% \Delta \mathcal{L}$, when the rule optimized for any particular model m is evaluated in the possibly competing model $n \neq m$. Panel A in table 2 reports the findings for the rules optimized for

inflation rate, the output gap, and the interest rate. In fact, Williams (2003) shows for the FRB/US model that moderately more complicated Taylor-type interest-rate rules that respond to a slightly enhanced number of lags of the target variables yield noticeable, albeit fairly small, stabilization gains over optimized three-parameter rules.

Table 2. The Robustness of Optimized Interest-Rate Rules in Models of the Euro Area

A. Rules Optimized for the Forward-Looking Euro Area Models															
			Rules Optimized for CW						Rules Optimized for SW						
			$\mathcal{L}$			$\% \Delta \mathcal{L}$			$\mathcal{L}$			$\% \Delta \mathcal{L}$			
θ	κ	λ	SW	AWM	DM	SW	AWM	DM	CW	AWM	DM	CW	AWM	DM	
0	0	0	1.0	∞	∞	6	∞	∞	1.9	∞	∞	3	∞	∞	
		1/3	1.4	2.5	35.1	15	56	684	2.7	4.8	23.2	13	202	419	
		1	1.6	3.4	465.6	18	54	8228	3.7	4.6	59.8	23	110	969	
		3	2.0	5.2	∞	25	58	∞	6.2	6.1	∞	38	85	∞	
4	0	0	1.0	∞	∞	9	∞	∞	1.9	∞	∞	6	∞	∞	
		1/3	1.4	2.4	11.4	15	52	155	2.8	5.2	∞	14	232	∞	
		1	1.6	3.2	23.3	17	45	318	3.7	4.9	67.4	23	122	1106	
		3	2.0	5.0	73.3	25	51	763	6.4	5.7	∞	43	72	∞	

B. Rules Optimized for the Backward-Looking Euro Area Models															
			Rules Optimized for AWM						Rules Optimized for DM						
			$\mathcal{L}$			$\% \Delta \mathcal{L}$			$\mathcal{L}$			$\% \Delta \mathcal{L}$			
θ	κ	λ	CW	SW	DM	CW	SW	DM	CW	SW	AWM	CW	SW	AWM	
0	0	0	2.2	1.3	48.6	20	36	1157	3.4	1.4	2.0	87	56	94	
		1/3	2.7	1.5	222.2	11	24	4859	3.8	1.7	2.3	58	39	43	
		1	3.5	1.7	∞	14	23	∞	4.7	2.2	2.7	53	55	24	
		3	ME	2.0	∞	ME	23	∞	6.8	3.2	3.9	52	97	17	
4	0	0	2.3	1.3	11.8	25	40	205	2.5	1.3	1.3	40	37	27	
		1/3	2.7	1.6	15.2	13	30	238	2.8	1.7	2.1	16	35	30	
		1	3.4	1.8	20.2	13	31	261	3.7	2.3	3.1	20	64	41	
		3	5.2	2.1	34.3	16	31	303	5.8	3.6	5.2	29	120	56	

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ), for each preference parameter (λ) and for each model, this table indicates the value of the policymaker's loss function ($\mathcal{L}$) and the percentage-point difference of the latter from the loss under the fully optimal policy (% $\Delta\mathcal{L}$), when the rule optimized for model m is evaluated in model $n \neq m$. The notation "ME" indicates that the implemented rule yields multiple equilibria; the notation " ∞ " indicates that the implemented rule results in instability.

the forward-looking models, while panel B documents those for the backward-looking ones.

Beginning with the rules optimized for the forward-looking models in panel A, we observe that rules that are optimized for the CW model also perform satisfactorily when evaluated in the SW model,

and vice versa. For example, with $\theta = \kappa = 0$ and $\lambda = 1/3$, implementing the CW-based rule in the SW model leads to an increase in the loss function of about 15 percentage points, relative to the loss associated with the fully optimal policy for the SW model. When compared to the performance of the interest-rate rule optimized for the SW model itself (see table 1), the loss increases by about 7 percentage points. Similarly, when implementing the SW-based rule in the CW model, the increase in the loss function amounts to 13 and 8 percentage points, depending on the benchmark for comparison. By contrast, when evaluating the CW- and SW-based rules in the backward-looking models, the performance of monetary policy tends to deteriorate quite substantially. For the AWM, the loss increases by about 50 percentage points relative to the fully optimal policy when CW-based rules are implemented and by more than 100 percentage points on average for SW-based rules. The deterioration is found to be particularly dramatic if the policymaker puts zero weight on output stabilization since the CW- and SW-based rules do not succeed in stabilizing the AWM any longer. Finally, when evaluated in the DM, the performance of CW- and SW-based rules is even worse, generating even higher increases in relative losses and resulting in instability more often.

Turning to the rules optimized for the backward-looking models in panel B, we observe that the AWM-based rules typically result in reasonable stabilization outcomes when evaluated in the CW or the SW model. The exception is the outcome-based rule for $\lambda = 3$, which yields multiple equilibria when implemented in the CW model. By contrast, the AWM-based rules do not perform satisfactorily in the DM and may occasionally even generate instability. Finally, the DM-based rules result in a substantial deterioration in the performance of monetary policy when evaluated in any of the three other models, although the deterioration exceeds 100 percentage points only in exceptional cases. Interestingly, for forecast-based rules, the deterioration is found to be somewhat more benign.

Based on these results, we conclude that simple interest-rate rules that are designed for the predominantly backward-looking euro area models tend to perform reasonably well in the largely forward-looking models, while the reverse is not necessarily true.¹⁹ The

¹⁹It is interesting to note that the aggregate companion of the DM (AM; see footnote 9) demands policies that are closer to those obtained for the AWM (the

fault-tolerance analysis undertaken in the next section will cast some further light on the reasons underlying these findings.

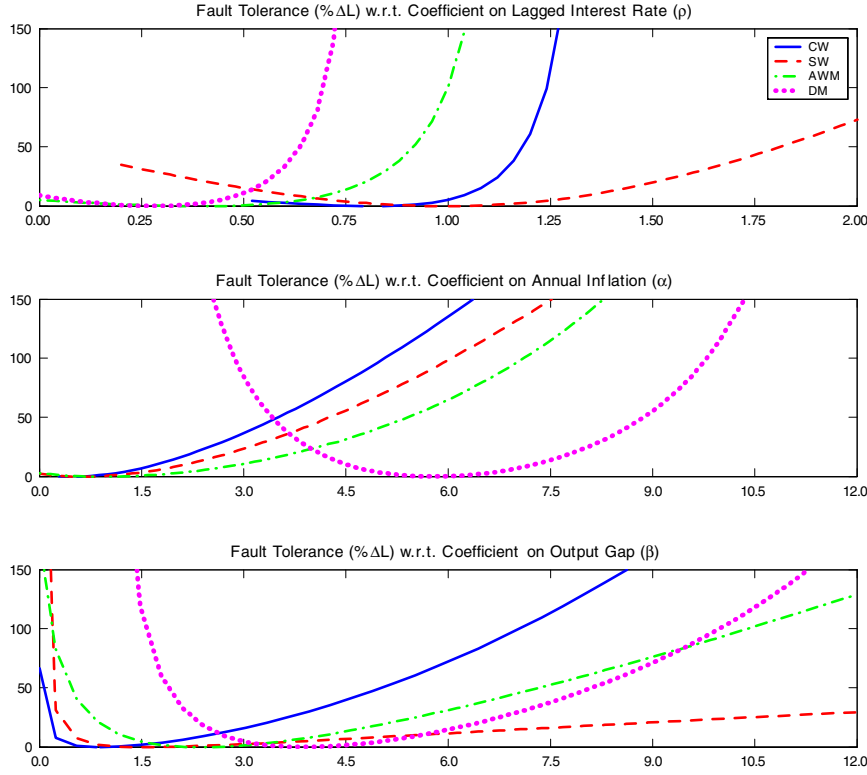
3.2 *A Fault-Tolerance Analysis of Optimized Rules*

Fault-tolerance analysis of optimized interest-rate rules, as proposed by Levin and Williams (2003), is deemed to provide useful insights into the reasons that underlie our earlier findings. Fault-tolerance analysis is a concept borrowed from engineering and involves, in the present context, appraising the increase in the loss function that results when a single parameter of an optimized interest-rate rule is varied, holding the other parameters constant at their optimized values. A highly *fault tolerant* model is one for which the parameters of the rule may vary over a relatively broad range of values without resulting in a large deterioration of its performance. By contrast, an intolerant model would be a model whose performance deteriorates dramatically as soon as one deviates even modestly from some optimized parameter value (i.e., the loss function exhibits strong curvature with respect to suboptimal variations in some parameter). Clearly, if one is dealing with a set of relatively tolerant models, there is a fair chance to find a robust policy rule. If all models are intolerant, then a robust rule may not exist.

Figure 2 depicts the fault tolerances of our four models for the case $\theta = \kappa = 0$ and $\lambda = 1/3$ (i.e., for the outcome-based rules obtained with a moderate weight on output stabilization).²⁰ Each curve shows the percentage-point change in the policymaker's loss function under the optimized rule as a single parameter is varied, with its minimum of zero attained at the optimized value itself. As can be seen in the figure, while any single parameter may be varied over a relatively broad range of values without deteriorating dramatically the performance of the individual model concerned, there are no obvious overlapping regions of high *mutual* fault tolerance for all four models under examination and for all three policy-rule parameters at the same time.

other predominantly backward-looking model), suggesting that the choice of the degree of aggregation is also a relevant factor in shaping the results.

²⁰For other values of the preference parameter λ , the results are qualitatively similar.

Figure 2. Fault-Tolerance Analysis of Outcome-Based Interest-Rate Rules

Note: For $\lambda = 1/3$ and for each model, the figure indicates the percentage-point change in the policymaker's loss function ($\%\Delta\mathcal{L}$) under the optimized outcome-based interest-rate rule ($\theta = \kappa = 0$) as a single parameter (ρ , α , or β) of the optimized rule is varied, holding the other two parameters fixed at their respective optimized values.

Regarding the fault tolerances with respect to the smoothing coefficient ρ (displayed in the upper panel of the figure), the two forward-looking models perform best when ρ is close to unity, while the performance for both the AWM and DM deteriorates quite markedly in this region, eventually resulting in instrument instability for the DM. In contrast, the AWM prefers a ρ coefficient of somewhat below 0.5 and the DM an even lower coefficient of about 0.25. Values

of ρ in this region, however, tend to yield indeterminate equilibria for the forward-looking models, notably for the CW model. With respect to the response coefficient α (depicted in the middle panel), the forward-looking models and the AWM seem mutually tolerant to variations in α in the region of close to 0 to 2.5. The DM, however, behaves very differently in its optimal prescription for α , demanding a significantly higher value of about 6, but with the curvature of the loss function in that region being modest. By contrast, as shown in the lower panel, a comfortable region of relatively high mutual tolerance seems to exist with respect to variations in the response coefficient β in the range of 2 to 4. While the DM favors a strong response to the output gap in the range of 2 to 7.5, the three other models perform satisfactorily as a group for coefficients in the range of 0.5 to 4, with the CW model constraining the upper bound of this range.

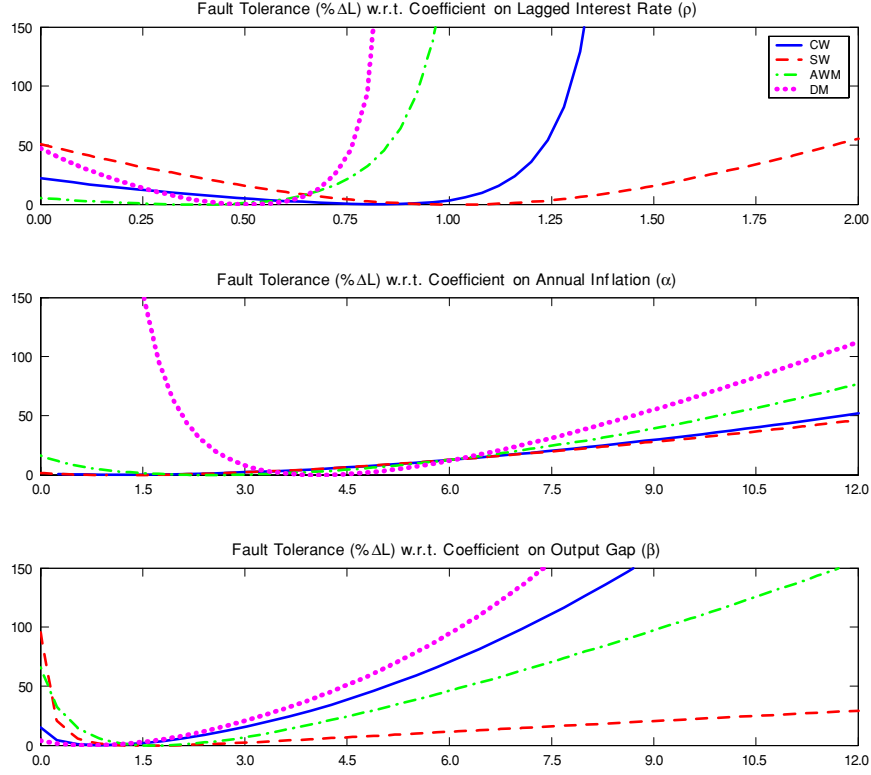
Interestingly, as shown in figure 3, the four models exhibit considerably larger regions of mutual fault tolerance when the policymaker follows a forecast-based rule with $\theta = 4$ and $\kappa = 0$. In this case, variations in ρ in the range of 0.4 to 0.7 result in a reasonable performance of all four models. Similarly, there are regions of mutual tolerance with respect to variations in α and β , the regions being centered at 3.5 and 1, respectively.

4. Designing Robust Interest-Rate Rules

The fault-tolerance analysis in the previous section has provided an indication under which circumstances a robust interest-rate rule might exist for our set of euro area models. However, to the extent that fault-tolerance analysis rests on suboptimal variations in a single policy-rule parameter, holding fixed the other parameters at their optimized values, we finally proceed to use a more formal approach that allows taking into account the interaction among all policy-rule parameters to identify the operating characteristics of interest-rate rules that are likely to yield satisfactory outcomes across our models.

In search of such robust rules we follow the Bayesian approach outlined in Levin, Wieland, and Williams (1999, 2003) and optimize the response coefficients of the three-parameter family of interest-rate rules across our four models simultaneously by

Figure 3. Fault-Tolerance Analysis of Forecast-Based Interest-Rate Rules



Note: For $\lambda = 1/3$ and for each model, the figure indicates the percentage-point change in the policymaker's loss function ($\%\Delta\mathcal{L}$) under the optimized forecast-based interest-rate rule ($\theta = 4$, $\kappa = 0$) as a single parameter (ρ , α , or β) of the optimized rule is varied, holding the other two parameters fixed at their respective optimized values.

minimizing a weighted average of the loss functions associated with the individual models,

$$\bar{\mathcal{L}} = \sum_{m \in \mathcal{M}} \omega_m \mathcal{L}_m,$$

where ω_m denotes the weight attached to any given model $m \in \mathcal{M} \subseteq \{\text{CW}, \text{SW}, \text{AWM}, \text{DM}\}$ with $\omega_m > 0$ and $\sum \omega_m = 1$. For $\omega_m = 1/|\mathcal{M}|$, the average loss function corresponds to the expected

Table 3. The Stabilization Performance of Bayesian Robust Interest-Rate Rules

θ	κ	λ	ρ	α	β	$\mathcal{L}_m$ generated in				% $\Delta\mathcal{L}_m$ generated in			
						CW	SW	AWM	DM	CW	SW	AWM	DM
A. Optimization Across All Models													
0	0	0	0.54	3.38	2.28	2.6	1.3	1.6	6.4	46	38	58	64
		1/3	0.58	3.22	2.71	3.1	1.5	2.3	7.2	28	22	46	60
		1	0.58	3.21	3.33	3.9	1.8	3.2	8.8	28	30	45	58
		3	0.54	3.33	4.48	5.8	2.4	4.8	13.2	29	50	47	56
4	0	0	0.60	3.28	0.49	2.3	1.2	1.3	4.7	26	33	25	22
		1/3	0.62	3.27	1.16	2.7	1.5	2.0	5.7	11	22	25	28
		1	0.60	3.46	1.90	3.4	1.8	2.7	7.5	10	29	25	34
		3	0.54	3.76	3.16	5.0	2.3	4.1	12.1	11	42	24	43
B. Optimization Across All Aggregate Models													
0	0	0	0.71	0.76	0.55	2.1	1.1	1.4		14	24	33	
		1/3	0.63	0.64	1.72	2.6	1.4	2.0		8	17	24	
		1	0.64	0.42	2.78	3.3	1.6	2.7		8	17	22	
		3	0.64	0.37	4.37	5.0	1.9	4.0		10	18	20	
4	0	0	0.72	1.26	0.39	2.2	1.2	1.2		20	29	20	
		1/3	0.64	1.40	1.46	2.6	1.5	1.9		8	19	17	
		1	0.62	1.38	2.56	3.3	1.7	2.6		8	19	17	
		3	0.62	1.42	4.17	4.9	2.0	3.8		9	20	16	

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ) and for each preference parameter (λ), this table indicates the jointly optimized interest-rate response coefficients (ρ , α , and β), the contribution of the individual model m to the policymaker's loss function ($\mathcal{L}_m$), and the percentage-point difference of this contribution from the loss under the fully optimal policy ($\%\Delta\mathcal{L}_m$).

loss function when the policymaker has uniform prior beliefs as to which model in $\mathcal{M}$ is a plausible representation of the euro area economy.

Panel A of table 3 reports the response coefficients of outcome- and forecast-based rules that are optimized across all four models simultaneously (assuming uniform prior beliefs) and indicates the

performance of these rules yielded in the individual models. Here, performance is measured as the contributions of the individual models to the value of the policymaker's overall loss function, $\mathcal{L}_m$, and as the percentage-point difference of these contributions from the losses under the fully optimal policies, $\% \Delta \mathcal{L}_m$.

Starting with the outcome-based rules with $\theta = \kappa = 0$, it turns out that the jointly optimized (that is, *Bayesian robust*) policies are heavily weighted toward those demanded by the DM, with the optimized interest-rate response coefficients close to those implied by the DM alone (see table 1). Specifically, the Bayesian robust policies prescribe a degree of interest-rate smoothing in the range of 0.5 to 0.6, while the responses to inflation and the output gap are rather aggressive. The response to inflation is relatively stable at around 3, while the response to the output gap varies quite a bit, namely in the range of 2 to 4, depending on the weight given to output stabilization. In light of the fault-tolerance analysis reported above, this outcome is not really surprising, since, in the absence of regions of high mutual tolerance with respect to some parameter, the least tolerant model is supposed to be most influential in shaping the operating characteristics of the robust policy with respect to that parameter.²¹ Nevertheless, the robust policies perform reasonably well in all four models, notably if a sensible weight is given to output-gap stabilization, as can be seen when comparing the outcomes under the robust policies with the performance measures reported in table 1. In fact, the increase in the reported losses never exceeds 50 percentage points, even if the sole policy objective is to stabilize inflation (i.e., for $\lambda = 0$), with the deterioration in performance obviously smallest for the DM due to its influential role in shaping the operating characteristics of the robust policies. This finding is broadly consistent with the results documented in Levin and Williams (2003) for a set of models of the U.S. economy including the backward-looking model of Rudebusch and Svensson (1999). Also in this study, the contours of the robust policies are found to be

²¹In fact, the DM is rather intolerant to nontrivial deviations from the optimized coefficients implied by the DM alone, as can be seen in figure 2. Furthermore, the decisive role of the DM in our Bayesian analysis, after all, relates to the fact that the baseline level of the loss for the DM is relatively high when compared with the baseline losses for the other models.

heavily weighted towards those demanded by the backward-looking model.

Turning to the forecast-based rules with $\theta = 4$ and $\kappa = 0$, the performance of Bayesian robust policies is found to be even more satisfactory across models. Yet again, this finding is not surprising in light of the fault-tolerance analysis above indicating the existence of regions of relatively high mutual tolerance for all three parameters. However, the optimized parameters of the robust rules appear to be largely influenced by the DM again.

To the extent that the robust policies for the full set of models are heavily weighted toward the policies demanded by the DM, panel B of table 3 also reports results obtained when optimizing across all aggregate models, but excluding the DM. In this case, the robust policies are characterized by a uniformly higher degree of interest-rate smoothing (in a range of 0.6 to 0.7) and, overall, by less aggressive responses to inflation and the output gap. As expected, the performance of the robust policies designed for the two forward-looking models (CW and SW) and the AWM alone appears more favorable across these models when compared to the performance under the robust rules designed for the full set of models.

5. Sensitivity Analysis

Now we briefly summarize some additional sensitivity analysis regarding the results presented above. First, we consider the implications of changing the weight μ on the variability of interest-rate changes in the policymaker's loss function. For the preceding analysis we have chosen a weight of $\mu = 0.1$, which has been widely employed in policy evaluation exercises like ours. As shown in table B-1 in the appendix, with a weight of $\mu = 0.01$ on interest-rate variability, the relative stabilization performance of optimized rules is not significantly affected when compared to the baseline results reported in table 1 above. However, with the variability in interest-rate changes penalized less, there are a number of noticeable differences with regard to the pattern of the optimized coefficients. First, the response coefficients to inflation and the output gap are a good bit larger; second, for the backward-looking models, the optimal degree of interest-rate smoothing is found to be even lower. However,

despite these changes in the operating characteristics of the optimized rules, we observe—by comparing the relative performance measures reported in table B-2 with those in table 2—that the robustness properties of the optimized rules are broadly unaffected if the weight on interest-rate variability is lowered to $\mu = 0.01$. Finally, as documented in table B-3, jointly optimized interest-rate rules are yet again found to be weighted toward the DM, but with the relatively favorable performance of robust forecast-based rules being less clear-cut.

Regarding the sensitivity of our results with respect to using a real-time information set with $\theta = \kappa = -1$, table C-1 shows that the performance of real-time-based rules tends to deteriorate relative to that of outcome-based rules. For the more backward-looking models, this deterioration is rather pronounced (up to 69 and 34 percentage points for the AWM and the DM, respectively, depending on the weight given to output stabilization). Similarly, as shown in table C-2, the robustness of real-time-based rules tends to be inferior to that of outcome-based ones, resulting in a somewhat larger deterioration of performance across models. Finally, as indicated in table C-3, the performance of robust real-time-based rules is slightly inferior across models as well.

6. Conclusions

In this paper, we examined the performance and robustness of optimized interest-rate rules in four models of the euro area that differ considerably in terms of size, degree of aggregation, relevance of forward-looking behavioral elements, and adherence to micro-foundations. Based on our examination, we conclude that simple interest-rate rules that are optimized for a given model may perform satisfactorily in models with a rather similar structure but may result in a dramatic deterioration of performance in models with markedly different features. In particular, rules that are designed for largely forward-looking models tend to perform poorly in predominantly backward-looking models, although the reverse is not necessarily true. In view of this asymmetry, we find that the operating characteristics of interest-rate rules that are optimized across our set of models simultaneously are heavily weighted toward

those required by the backward-looking models. Nevertheless, the Bayesian robust policies identified in such a way perform reasonably well in all four models, notably if a sensible weight is given to output stabilization. Especially, we find that a forecast-based rule that relates the short-term interest rate to the one-year-ahead forecast of inflation and the contemporaneous output gap and (importantly) that allows for only a moderate degree of inertia, attains reasonable outcomes.

While other model features such as variable and country coverage and adherence to microfoundations are apparently of relevance as well, the nature of the expectation formation mechanism embedded in the various models seems to be of key importance for explaining our results. This in turn suggests that future research that aims at casting light on the empirical relevance of forward-looking behavioral elements in macroeconomic models may enhance the reliability and usefulness of interest-rate rules for model-based evaluations of monetary policy. Of course, in the case that a policy rule prescribed to set the interest rate in response to forecasts of future inflation, we assumed in our analysis that these forecasts happened to be consistent with the structure of the particular model in which the performance of the forecast-based rule was evaluated. Similarly, the measure of the output gap used when evaluating the rule was consistent with the output-gap concept employed in that particular model. To the extent that the monetary policymaker faces uncertainty regarding the reliability of the inflation forecast itself or the correct measurement of the output gap, these additional sources of uncertainty may heighten the risks associated with relying too heavily on a rule optimized for any particular model.²² Extensions of our study along these directions are left for future research.

²²Indeed, the analysis in Coenen (2003) shows that erroneously relying on the false forecasting model may result in a significant deterioration of the stabilization performance of a forecast-based rule, even when the rule itself is designed under the assumption that the structure of the economy is known. The consequences of relying on alternative, possibly inconsistent output-gap concepts is analyzed in Smets and Wouters (2002).

Appendix

Table A. Detailed Results for the Optimized Interest-Rate Rules

A. The Forward-Looking Euro Area Models										
θ	κ	λ	CW				SW			
			σ_π	σ_y	$\sigma_{\Delta i}$	$\mathcal{L}$	σ_π	σ_y	$\sigma_{\Delta i}$	$\mathcal{L}$
0	0	0	1.33	2.30	0.80	1.83	0.93	2.47	0.47	0.95
		1/3	1.45	1.07	0.82	2.54	1.06	0.60	0.87	1.33
		1	1.48	0.91	1.26	3.17	1.08	0.40	1.21	1.48
		3	1.48	0.83	1.93	4.65	1.09	0.28	1.68	1.71
4	0	0	1.35	2.09	0.79	1.89	0.94	2.44	0.51	0.98
		1/3	1.47	1.02	0.83	2.58	1.07	0.58	0.87	1.34
		1	1.48	0.90	1.26	3.18	1.08	0.40	1.21	1.48
		3	1.49	0.83	1.91	4.65	1.09	0.29	1.64	1.70

B. The Backward-Looking Euro Area Models										
θ	κ	λ	AWM				DM			
			σ_π	σ_y	$\sigma_{\Delta i}$	$\mathcal{L}$	σ_π	σ_y	$\sigma_{\Delta i}$	$\mathcal{L}$
0	0	0	1.09	1.57	1.04	1.30	2.15	1.68	3.69	5.97
		1/3	1.07	1.17	1.74	1.90	2.17	1.60	3.62	6.86
		1	1.07	0.91	2.50	2.59	2.22	1.50	3.59	8.45
		3	1.07	0.69	3.52	3.79	2.36	1.36	3.68	12.48
4	0	0	1.04	1.59	1.01	1.19	1.89	1.58	3.29	4.64
		1/3	1.02	1.16	1.69	1.78	1.90	1.51	3.24	5.44
		1	1.03	0.90	2.43	2.46	1.95	1.43	3.23	6.87
		3	1.04	0.69	3.42	3.65	2.07	1.31	3.34	10.56

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ), for each preference parameter (λ), and for each model, this table indicates the unconditional standard deviations of the target variables (σ_π , σ_y , and $\sigma_{\Delta i}$) and the value of the policymaker's loss function ($\mathcal{L}$).

Table B-1. The Stabilization Performance of Optimized Interest-Rate Rules Generated with a Lower Weight of $\mu = 0.01$ on Interest-Rate Variability

A. The Forward-Looking Euro Area Models												
θ	κ	λ	CW					SW				
			ρ	α	β	$\mathcal{L}$	$\% \Delta \mathcal{L}$	ρ	α	β	$\mathcal{L}$	$\% \Delta \mathcal{L}$
0	0	0	0.98	2.98	0.25	1.7	1	1.03	1.71	0.16	0.9	2
		1/3	0.75	1.26	2.29	2.4	5	0.99	1.92	5.25	1.2	9
		1	0.73	0.74	4.09	2.9	4	1.02	0.83	11.22	1.3	6
		3	0.69	0.44	6.54	4.0	3	1.00	0.84	20.93	1.3	4
4	0	0	2.75	29.26	−0.24	1.7	5	2.32	18.26	−0.06	0.9	4
		1/3	0.79	3.30	2.14	2.5	7	1.01	3.40	4.73	1.2	9
		1	0.73	1.96	4.14	2.9	4	1.00	2.49	10.32	1.3	7
		3	0.68	1.33	6.62	4.0	3	1.00	1.72	20.42	1.3	4
B. The Backward-Looking Euro Area Models												
θ	κ	λ	AWM					DM				
			ρ	α	β	$\mathcal{L}$	$\% \Delta \mathcal{L}$	ρ	α	β	$\mathcal{L}$	$\% \Delta \mathcal{L}$
0	0	0	0.17	3.05	3.00	1.1	24	0.11	13.83	8.33	3.8	41
		1/3	0.42	2.03	6.57	1.4	12	0.02	13.24	9.11	4.8	42
		1	0.51	1.37	9.71	1.6	9	0.00	11.88	9.65	6.4	42
		3	0.55	0.84	14.55	2.0	6	0.00	9.94	10.22	10.4	41
4	0	0	0.23	6.76	1.16	1.0	11	0.39	10.58	1.53	2.9	9
		1/3	0.30	5.57	4.69	1.3	5	0.27	10.43	2.30	3.8	12
		1	0.37	4.53	8.08	1.6	4	0.15	10.28	3.33	5.2	16
		3	0.44	3.53	13.10	2.0	4	0.00	9.83	5.12	8.8	19

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ), for each preference parameter (λ), and for each model, this table indicates the optimized interest-rate response coefficients (ρ , α , and β), the value of the policymaker's loss function ($\mathcal{L}$), and the percentage-point difference of the latter from the loss under the fully optimal policy ($\% \Delta \mathcal{L}$).

Table B-2. The Robustness of Optimized Interest-Rate Rules Generated with a Lower Weight of $\mu = 0.01$ on Interest-Rate Variability

A. Rules Optimized for the Forward-Looking Euro Area Models															
			Rules Optimized for CW						Rules Optimized for SW						
			$\mathcal{L}$			$\% \Delta \mathcal{L}$			$\mathcal{L}$			$\% \Delta \mathcal{L}$			
θ	κ	λ	SW	AWM	DM	SW	AWM	DM	CW	AWM	DM	CW	AWM	DM	
0	0	0	0.9	∞	∞	3	∞	∞	1.7	∞	∞	4	∞	∞	
		1/3	1.3	1.7	17.6	12	36	420	2.6	1.7	∞	13	34	∞	
		1	1.4	1.9	97.4	13	29	2046	3.6	1.9	∞	31	29	∞	
		3	1.5	2.5	∞	17	30	∞	6.5	2.2	∞	67	16	∞	
4	0	0	0.9	∞	∞	4	∞	∞	1.7	∞	∞	6	∞	∞	
		1/3	1.3	1.6	27.3	12	30	704	2.6	1.8	∞	13	43	∞	
		1	1.3	1.8	30.3	12	21	567	3.5	1.8	∞	25	23	∞	
		3	1.5	2.4	90.2	16	23	1125	6.3	2.3	∞	63	18	∞	
B. Rules Optimized for the Backward-Looking Euro Area Models															
			Rules Optimized for AWM						Rules Optimized for DM						
			$\mathcal{L}$			$\% \Delta \mathcal{L}$			$\mathcal{L}$			$\% \Delta \mathcal{L}$			
θ	κ	λ	CW	SW	DM	CW	SW	DM	CW	SW	AWM	CW	SW	AWM	
0	0	0	2.1	1.2	17.6	28	30	556	2.6	1.2	1.3	57	29	44	
		1/3	2.5	1.3	52.7	10	12	1454	3.0	1.4	1.5	31	26	19	
		1	3.1	1.3	199.4	11	10	4291	3.7	2.8	1.7	33	47	14	
		3	4.5	1.4	∞	15	9	∞	5.4	2.5	2.2	39	92	17	
4	0	0	1.9	1.2	24.8	18	33	824	2.1	1.2	1.1	27	30	15	
		1/3	2.6	1.3	32.0	12	16	842	3.3	1.4	1.5	41	26	21	
		1	3.0	1.4	60.7	9	13	1237	3.3	1.7	2.0	17	45	33	
		3	4.3	1.4	342.5	10	12	4551	4.9	2.4	3.0	26	88	54	

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ), for each preference parameter (λ), and for each model, this table indicates the value of the policymaker's loss function ($\mathcal{L}$) and the percentage-point difference of the latter from the loss under the fully optimal policy (% $\Delta\mathcal{L}$), when the rule optimized for model m is evaluated in model $n \neq m$. The notation " ∞ " indicates that the implemented rule results in instability.

Table B-3. The Stabilization Performance of Bayesian Robust Interest-Rate Rules Generated with a Lower Weight of $\mu = 0.01$ on Interest-Rate Variability

θ	κ	λ	ρ	α	β	$\mathcal{L}_m$ generated in				% $\Delta\mathcal{L}_m$ generated in			
						CW	SW	AWM	DM	CW	SW	AWM	DM
A. Optimization Across All Models													
0	0	0	0.42	8.54	5.81	2.3	1.1	1.2	3.9	39	25	32	46
		1/3	0.39	7.86	6.69	2.7	1.3	1.6	5.0	17	18	27	47
		1	0.33	7.63	8.02	3.3	1.6	2.0	6.7	20	31	36	49
		3	0.21	7.54	10.31	4.9	2.0	2.9	11.0	26	60	53	49
4	0	0	0.47	8.62	1.19	2.2	1.2	1.1	3.9	33	28	14	45
		1/3	0.45	8.14	3.17	2.5	1.3	1.4	4.7	10	17	13	38
		1	0.38	8.31	4.93	3.0	1.5	1.7	6.2	10	23	16	37
		3	0.26	8.56	7.56	4.3	1.8	2.3	10.3	12	39	22	39
B. Optimization Across All Aggregate Models													
0	0	0	0.56	2.73	1.53	2.0	1.1	1.2		21	21	32	
		1/3	0.59	1.74	4.78	2.5	1.2	1.4		7	11	14	
		1	0.63	1.04	7.03	3.0	1.3	1.7		6	10	12	
		3	0.61	0.65	10.37	4.1	1.4	2.1		6	11	10	
4	0	0	0.64	4.78	0.81	2.1	1.1	1.1		29	25	16	
		1/3	0.57	3.77	4.07	2.5	1.3	1.3		8	13	7	
		1	0.59	3.09	6.63	3.0	1.3	1.6		6	11	7	
		3	0.58	2.66	10.17	4.1	1.4	2.1		6	12	7	

Note: For each choice of the inflation and output-gap forecast horizons (θ and κ) and for each preference parameter (λ), this table indicates the jointly optimized interest-rate response coefficients (ρ , α , and β), the contribution of the individual model m to the policymaker's loss function ($\mathcal{L}_m$), and the percentage-point difference of this contribution from the loss under the fully optimal policy ($\%\Delta\mathcal{L}_m$).

Table C-1. The Stabilization Performance of Optimized Interest-Rate Rules Based on a Real-Time Information Set with $\theta = \kappa = -1$

A. The Forward-Looking Euro Area Models										
λ	CW					SW				
	ρ	α	β	$\mathcal{L}$	$\%\Delta\mathcal{L}$	ρ	α	β	$\mathcal{L}$	$\%\Delta\mathcal{L}$
0	0.95	0.79	0.11	1.8	2	1.02	0.79	0.08	1.0	3
1/3	0.76	0.48	0.91	2.6	6	0.98	0.56	1.38	1.4	11
1	0.74	0.29	1.58	3.2	5	0.96	0.35	2.62	1.6	12
3	0.72	0.28	2.43	4.8	6	0.98	0.00	3.44	1.9	20
B. The Backward-Looking Euro Area Models										
λ	AWM					DM				
	ρ	α	β	$\mathcal{L}$	$\%\Delta\mathcal{L}$	ρ	α	β	$\mathcal{L}$	$\%\Delta\mathcal{L}$
0	0.42	0.99	1.11	1.4	34	0.17	6.07	4.26	7.3	88
1/3	0.27	0.82	2.16	2.2	40	0.13	6.06	4.46	8.3	85
1	0.26	0.54	3.19	3.4	55	0.07	6.02	4.77	10.1	81
3	0.25	0.34	4.60	6.1	84	0.00	5.79	5.30	14.8	74

Note: For each preference parameter (λ) and for each model, this table indicates the optimized interest-rate response coefficients (ρ , α , and β), the value of the policymaker's loss function ($\mathcal{L}$), and the percentage-point difference of the latter from the loss under the fully optimal policy ($\%\Delta\mathcal{L}$).

Table C-2. The Robustness of Optimized Interest-Rate Rules Based on a Real-Time Information Set with $\theta = \kappa = -1$

A. Rules Optimized for the Forward-Looking Euro Area Models												
λ	Rules Optimized for CW						Rules Optimized for SW					
	$\mathcal{L}$			$\% \Delta \mathcal{L}$			$\% \mathcal{L}$			$\% \Delta \mathcal{L}$		
	SW	AWM	DM	SW	AWM	DM	CW	AWM	DM	CW	AWM	DM
0	1.0	∞	∞	6	∞	∞	1.9	∞	∞	4	∞	∞
1/3	1.4	2.8	66.7	18	80	1388	2.8	13.7	68.0	14	766	1417
1	1.7	4.5	∞	24	103	∞	3.8	21.4	∞	23	872	∞
3	2.2	8.5	∞	38	157	∞	ME	∞	∞	ME	∞	∞
B. Rules Optimized for the Backward-Looking Euro Area Models												
λ	Rules Optimized for AWM						Rules Optimized for DM					
	$\mathcal{L}$			$\% \Delta \mathcal{L}$			$\mathcal{L}$			$\% \Delta \mathcal{L}$		
	CW	SW	DM	CW	SW	DM	CW	SW	AWM	CW	SW	AWM
0	2.2	1.3	94.5	20	36	2345	3.5	1.6	2.5	93	71	140
1/3	2.7	1.6	∞	11	29	∞	3.9	1.9	3.2	61	50	106
1	ME	1.9	∞	ME	38	∞	4.8	2.3	4.6	59	67	110
3	ME	2.6	∞	ME	58	∞	7.1	3.5	8.0	57	115	143

Note: For each preference parameter (λ) and for each model, this table indicates the value of the policymaker's loss function ($\mathcal{L}$) and the percentage-point difference of the latter from the loss under the fully optimal policy ($\% \Delta \mathcal{L}$), when the rule optimized for model m is evaluated in model $n \neq m$. The notation "ME" indicates that the implemented rule yields multiple equilibria; the notation " ∞ " indicates that the implemented rule results in instability.

Table C-3. The Stabilization Performance of Bayesian Robust Interest-Rate Rules Based on a Real-Time Information Set with $\theta = \kappa = -1$

λ	ρ	α	β	$\mathcal{L}_m$ generated in				% $\Delta\mathcal{L}_m$ generated in			
				CW	SW	AWM	DM	CW	SW	AWM	DM
A. Optimization Across All Models											
0	0.41	3.62	2.52	2.7	1.3	1.9	7.8	48	44	81	103
1/3	0.41	3.57	2.88	3.2	1.6	2.8	8.7	31	30	80	95
1	0.39	3.62	3.46	4.1	2.0	4.3	10.6	33	44	95	89
3	0.32	3.75	4.54	6.2	2.9	7.4	15.5	37	76	125	82
B. Optimization Across All Aggregate Models											
0	0.65	0.78	0.61	2.1	1.2	1.4		14	26	40	
1/3	0.51	0.64	1.70	2.6	1.5	2.3		8	22	44	
1	0.47	0.57	2.68	3.3	1.8	3.5		9	29	60	
3	0.41	0.59	4.10	5.1	2.4	6.2		13	45	88	

Note: For each preference parameter (λ), this table indicates the jointly optimized interest-rate response coefficients (ρ , α , and β), the contribution of the individual model m to the policymaker's loss function ($\mathcal{L}_m$), and the percentage-point difference of this contribution from the loss under the fully optimal policy ($\%\Delta\mathcal{L}_m$).

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Monetary Policy Neglect and the Great Inflation in Canada, Australia, and New Zealand*

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This paper studies the Great Inflation in Canada, Australia, and New Zealand. Newspaper coverage and policymakers' statements are used to analyze the views on the inflation process that led to the 1970s macroeconomic policies, and the different movement in each country away from 1970s views. I argue that to understand the course of policy in each country, it is crucial to use the monetary policy neglect hypothesis, which claims that the Great Inflation occurred because policymakers delegated inflation control to nonmonetary devices. This hypothesis helps explain why, unlike Canada, Australia and New Zealand continued to suffer high inflation in the mid-1980s. The delayed disinflation in these countries reflected the continuing importance accorded to nonmonetary views of inflation.

JEL Codes: E31, E52, E58, E64

Christiano and Gust (2000, 21) observe that the Great Inflation of the 1970s took place in many countries and note the desirability of “understanding why it happened and what can be done to prevent

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it happening again.” In practice, however, examination of the U.S. experience has dominated the literature on the Great Inflation. This paper instead looks at three other countries that experienced the Great Inflation: Canada, Australia, and New Zealand. The record of these countries has been useful in studying the international experience with inflation targeting (Bernanke et al. 1999; Debelle 1996) and earlier developments in these countries can be useful in discriminating between different explanations for the Great Inflation.

As discussed below, several different hypotheses have been advanced in the literature on the Great Inflation to explain the over-expansionary monetary policy of the 1970s. While differing in detail, most of the different explanations—such as those emphasizing inflation bias by policymakers (e.g., Ireland 1999) or attempts to exploit Phillips curve trade-offs (e.g., Sargent 1999)—take for granted that the monetary authorities understand that their own actions, by expanding or contracting aggregate demand, have a decisive effect on the inflation rate. By contrast, the prevalent features of policymakers’ statements in the United States (Romer and Romer 2002; Nelson 2004) and the United Kingdom (Nelson and Nikolov 2004) are the use of special-factors or “cost-push” explanations of inflation behavior and the promotion of nonmonetary devices (such as wage and price controls) as the means of controlling inflation. In light of this, Nelson and Nikolov (2004) argue that the monetary policy neglect hypothesis explains the Great Inflation: policymakers had a misguided view of the nature of inflation, and the Great Inflation occurred because inflation control was delegated to instruments other than monetary policy.

This paper presents further evidence for the monetary policy neglect hypothesis by showing that it accounts for policy developments during the Great Inflation in Canada, Australia, and New Zealand. I examine policymakers’ statements and the record of their actions during the Great Inflation to draw out their views on the forces driving inflation. For the United States, Romer and Romer (2002) obtained information of this kind from Federal Open Market Committee (FOMC) minutes. No strictly comparable material exists for the three countries studied here, largely because their central banks were not independent during the 1970s. Instead, I document policymakers’ views by examining newspaper reports as well as speeches of senior government figures. In addition to supporting the neglect

hypothesis, this material helps rule out several other hypotheses that have been advanced to explain the Great Inflation.

This paper proceeds as follows. Section 1 gives an overview of the Great Inflation in each country. Section 2 outlines the monetary policy neglect hypothesis and discusses several alternative hypotheses. Sections 3–5 examine the policy record during the Great Inflation in Canada, Australia, and New Zealand. Section 6 concludes.

1. An Overview of the Great Inflation

Figures 1–3 plot four-quarter consumer price index inflation for Canada, Australia, and New Zealand. The increase in Canada's inflation rate from its 1960s values to its 1981 peak is interrupted by declines in 1970–71 and after 1975. Only the first of these declines,

Figure 1. Four-Quarter CPI Inflation: Canada

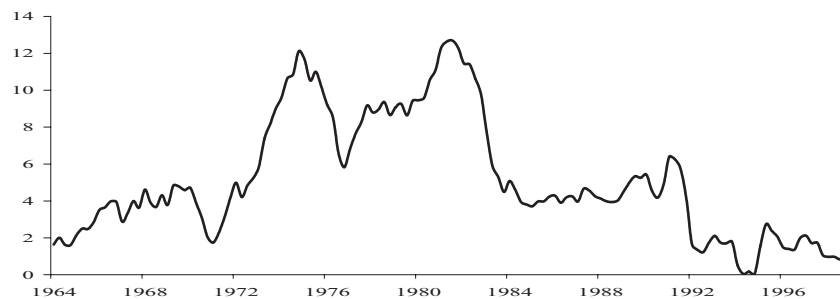


Figure 2. Four-Quarter CPI Inflation: Australia

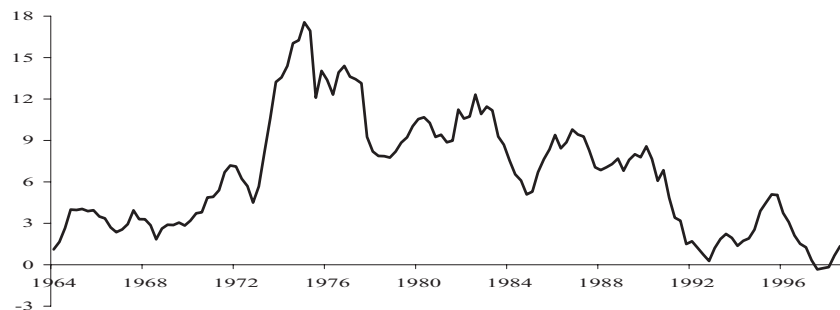
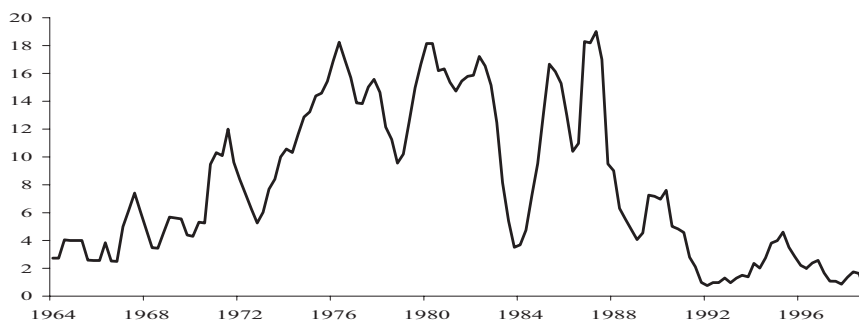


Figure 3. Four-Quarter CPI Inflation: New Zealand

however, is genuine, as the second period was characterized by compulsory price controls (see section 3). Inflation shifts decisively to single digits in late 1982. Australia also has lower inflation in the 1980s than the 1970s, but the improvement is marred by rebounds in 1985–86 and 1988–90, and in no quarter over 1973–90 does the rate fall below 5 percent. New Zealand inflation performance is poor in both the 1970s and 1980s; the temporary improvement in inflation over 1982–84 is wholly artificial, reflecting the imposition of a wage-price freeze over that period (see section 5).

Figures 4–6 compare actual short-term interest-rate choices with prescriptions for each year from a Taylor (1993) rule.¹ In these charts, the target inflation rate is 2.5 percent and the steady-state real rate has been imposed at 4 percent (3 percent for Canada). These differ from the choice of 2 percent for both in Taylor (1993), but are justified by the different experience of these countries from the United States. The imposed 2.5 percent inflation target lies inside the target ranges currently being pursued in all three countries and, as discussed below, also corresponds to the concept of price stability prevailing among Australian and Canadian policymakers in the 1970s. Real interest rates in Australia and New Zealand have averaged above 3 percent under inflation targeting and have also tended to be higher than in the United States or Canada. This suggests that their average real rate compatible with price stability is at least

¹The interest rate for Canada and Australia is the nominal Treasury bill rate; for New Zealand, it is the nominal short-term rate series of Orden and Fisher (1993).

Figure 4. Short-Term Interest Rate and Taylor Rule Prescriptions: Canada

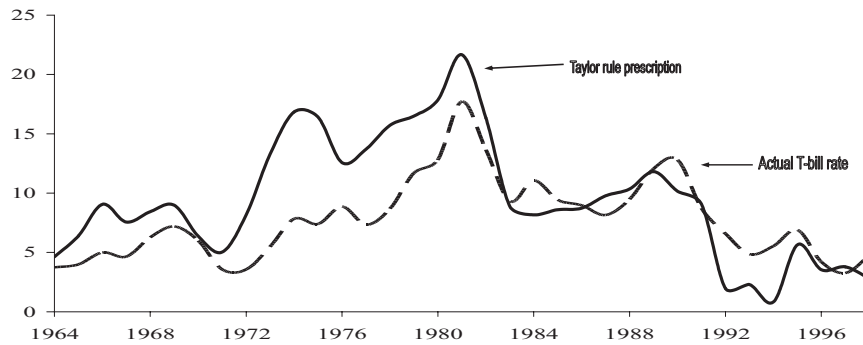


Figure 5. Short-Term Interest Rate and Taylor Rule Prescriptions: Australia

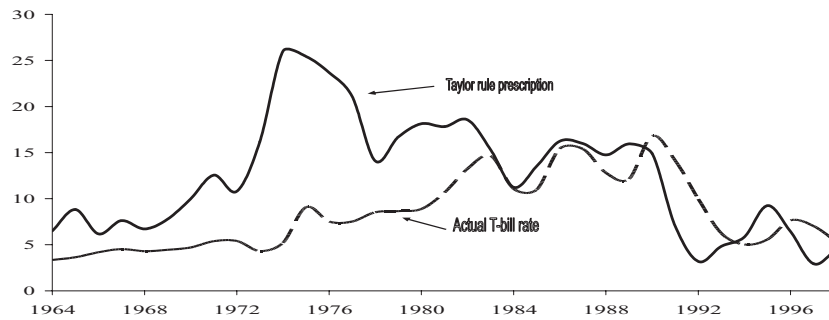
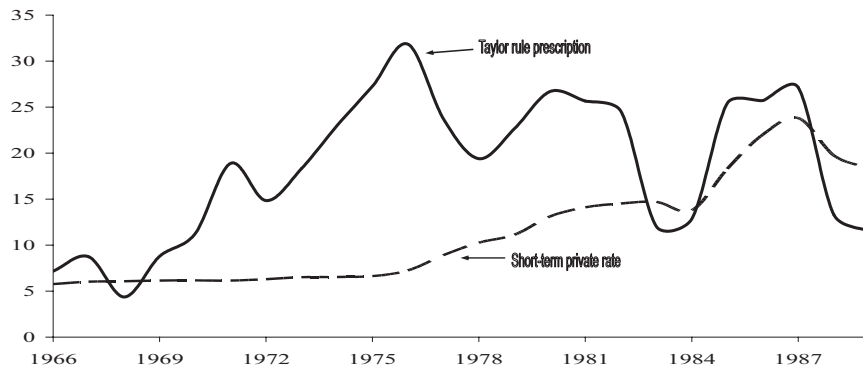


Figure 6. Short-Term Interest Rate and Taylor Rule Prescriptions: New Zealand



3 percent, while the global fall in real rates in the 1990s suggests that a still higher steady-state real rate should be assumed for the period studied here.

The Great Inflation in each country tends to correspond with large deviations from the Taylor rule prescription. One caveat is that the reliability of the rule prescriptions depends on the quality of the data used as inputs. Just as price controls in New Zealand artificially suppressed inflation over 1982–84, the use of the distorted inflation data in the Taylor rule produces a spurious dip in the interest-rate prescriptions.

A related caveat about data quality is that the rule prescriptions in figures 4–6 use detrended output computed from final data. As Orphanides (2003) shows for the United States, such revised estimates may differ substantially from policymakers' perception in real time of the position of output relative to potential. Figures 7–9 plot detrended output for the 1970s for each country, using final data,² against estimates of the output gap during the 1970s. For Australia and Canada, historical estimates of the output gap were reported by the Organisation for Economic Co-Operation and Development (OECD) (1973). The 1977 Canadian output gap in figure 7 is an approximation of the series used in McCracken et al. (1977).³ For Australia and New Zealand, the 1977 gap is obtained by detrending the log of the 1977 *IFS*⁴ vintage of real gross domestic product (GDP) data, with the trend estimated on annual data for 1957–73.

²Final data on detrended output in figures 7–9 are linear-detrended annual GDP data, also used in figures 4–6. The linear trends assume breaks in 1974 (as well as 1995 for Canada and Australia). As figure 7 shows, for Canada, the resulting series has the implausible property of being negative virtually throughout the 1970s. An alternative (quadratic) detrending method instead yields a series that is positive throughout the 1970s. The high inflation and money growth outcomes in Canada suggest that the actual output gap was generally positive rather than negative.

³This approximation is obtained first by using the annual data on real GDP for Canada in *IFS* (June 1977). A potential GDP series for Canada, designed to match that used by McCracken et al., was then deduced from the information given in McCracken et al. (1977, 319) and Laidler (1978), i.e., 5 percent annual growth in potential output, and only one year (1973) in the 1970s during which output exceeded potential.

⁴Articles from newspapers and other periodicals are cited in the text by their acronym and date. Acronyms are given in appendix 1, while a bibliographical appendix (appendix 2) gives full details for each article.

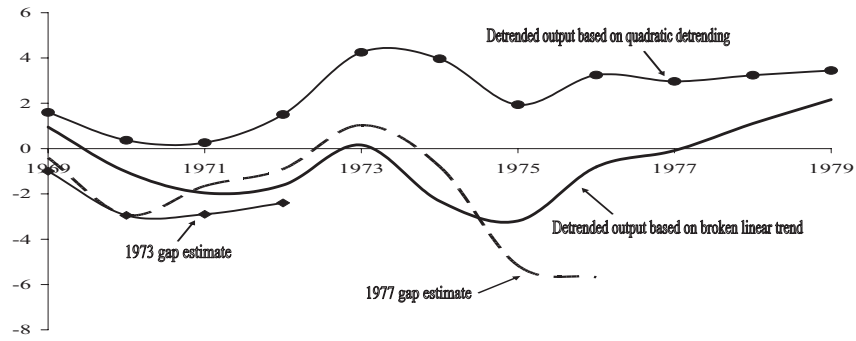
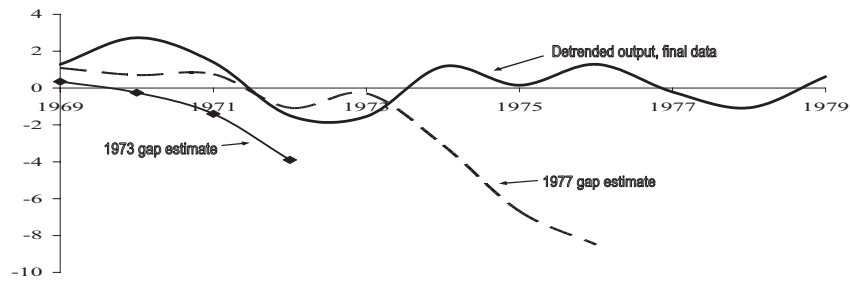
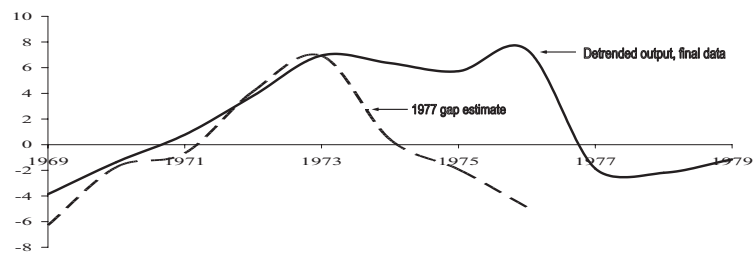
Figure 7. Output Gap Estimates: Canada**Figure 8. Output Gap Estimates: Australia****Figure 9. Output Gap Estimates: New Zealand**

Table 1. Average Annual Output Growth Rates*

Canada		Australia		New Zealand	
1962–1973	1974–1993	1961–1973	1974–1993	1961–1973	1974–1993
5.3	2.7	4.8	2.8	4.9	1.0
*For Canada and Australia, output is defined as real GDP; for New Zealand, output is defined as nominal GDP divided by the CPI.					

This is intended as an approximation of how policymakers in 1977 estimated potential GDP.⁵

Comparison of these 1973 estimates with today's estimates of detrended output suggests that the initial numbers overstated the weakness of demand in the early 1970s. In addition, like the United States, the three countries studied here experienced slower growth in output after 1973 (table 1). The detrended output series plotted in figures 7–9 take this break in trend into account, but gap estimates up to 1977 did not. As a result, the 1977 gap estimates reported in figures 7–9 are far more negative in the late 1970s than the final estimates of output relative to trend. As we will see below, the nature of these errors did much to move policymakers away from orthodox approaches to analyzing inflation.

2. The Monetary Policy Neglect Hypothesis

Using the standard New Keynesian Phillips curve, the behavior of inflation may be written

$$\pi_t = b_1 + b_2 E_t \sum_{i=0}^{\infty} b_3^i (y_{t+i} - y_{t+i}^*) + b_4 (u_t - E[u_t]) \quad (1)$$

where π_t is inflation, y_t is log output, y_t^* is log potential output, u_t is an exogenous disturbance, and the b_i are positive coefficients with b_3 near unity. As in Nelson (2004), the alternative views of inflation in 1970s debates—monetary versus nonmonetary—may be expressed as restrictions on equation (1).

A nonmonetary explanation of inflation attributes the 1970s inflation to persistent movements in the u_t term in equation (1)—often

⁵The post-1973 slowdown in potential was not noted officially in New Zealand until 1978 (see section 5). Recognition was slow too in Australia, with Neville's (1981, 50) assumed lower bound on post-1973 potential growth being 4.1 percent.

termed “cost-push” fluctuations—as well as to shifts in the mean of the u_t process (which thus produced shifts in the constant term, b_1). This explanation contends that excess demand did not matter for inflation, as it maintains that the coefficient b_2 goes to zero when the output gap ($y_t - y_t^*$) is negative. Therefore, according to this explanation, output falling below potential does not produce any downward pressure on inflation. Exponents of the nonmonetary explanation of inflation in the 1970s frequently conceded that a role for excess demand factors in driving inflation arises when output is above potential,⁶ but this was a minor qualification, as they believed that the output gap was rarely positive during the 1970s.

A monetary explanation of inflation, on the other hand, contends that b_2 is well above zero for all values of the output gap, and that the shifts in inflation during the 1970s are attributable to excessive aggregate demand (protracted periods of positive output gaps) created by the monetary authorities. The explanation concedes a role for the u_t term in equation (1) in producing one-time movements in the price level, but not sustained movements in inflation. Together with $b_2 > 0$, the monetary explanation therefore imposes the restrictions that y_t is interest-elastic (so excessive aggregate demand can be prevented by monetary policy), and that u_t has a zero mean (implying that $E[u_t]$ does not enter the expression for b_1) and is white noise (so only excess demand and the constant term enter the expression for expected future inflation).⁷

The monetary policy neglect hypothesis claims that the monetary explanation is the correct one, but that policymakers in the 1970s subscribed to the nonmonetary explanation. They therefore relied on wage and price controls, rather than monetary policy, to control inflation.

The hypothesis thus has two components: about *inflation* (the validity of the monetary view) and about *policy behavior* (the claim that policymakers embraced a nonmonetary view of inflation). In section 2.1 below, I consider the applicability of the monetary view of inflation to the three countries studied here. Before that, it is useful to discuss several alternative hypotheses about policy behavior in the Great Inflation and why they do not fit the facts for these countries.

⁶That is, they subscribed to an asymmetric version of (1), where b_2 turns positive only when the output gap is positive.

⁷The white-noise restriction additionally implies $b_4 = 1$ in equation (1).

2.1 Consciously Inflationary Policies

One class of explanations for the Great Inflation attributes to policymakers a conscious choice of inflationary policies. For example, the time-consistency explanation, pursued for the 1970s inflation by Ireland (1999), postulates that policymakers had an objective of a positive output gap. While rejecting the time-consistency story, DeLong (1997) argues that political pressures made it necessary to set unemployment targets below the natural rate. But as we will see, policymakers in Canada, Australia, and New Zealand repeatedly stated their output goal as potential, not some level beyond their estimate of potential. This goal also featured in their internal discussions: for example, a Treasury submission to cabinet in Australia in 1973 stated that “the capacity of supply” should govern policy on demand management (*ACR*, October 5, 1973).

Nor does these countries’ experience suggest a conscious targeting of high inflation rates, either because of feared costs of low-inflation equilibria (as in Christiano and Gust 2000) or low estimates of the cost of high inflation. Inflation was regarded as costly—for example, Prime Minister Whitlam of Australia said in 1974, “Let me be frank: inflation costs jobs” (*TA*, October 22, 1974)—and the authorities’ recourse to wage or price controls in each country was a drastic (though misguided) attempt to escape high inflation. The concept of price stability seems to have been similar to that in the later inflation targeting period: for example, Prime Minister Pierre Trudeau said in 1970 that “inflation no longer exists in Canada” when the four-quarter rate fell into the 2–3 percent range (*TST*, December 24, 1970), while the Australian Treasury in 1973 characterized the costs of inflation as occurring when inflation rose above the 3 percent barrier (*ACR*, October 5, 1973).

2.2 Convergence to a Vertical Phillips Curve Model

Sargent (1999) characterizes policymakers as entering the Great Inflation with a nonvertical long-run Phillips curve model and as converging to a vertical model as they observed the stagflation of the 1970s. This account does not describe either the initial conditions or the learning process in the three countries studied here. As of 1970, none had a nonvertical Phillips curve guiding policy;

rather, Canada's authorities actually subscribed to a long-run vertical Phillips curve, while the other two countries had a cost-push view of inflation. Moreover, in all three countries, the 1970s outcomes convinced the authorities of the validity of the cost-push view of inflation, rather than moving them to a more orthodox framework. The reason is that the Sargent story requires policymakers to observe that the output gap returns to zero despite larger doses of inflation, whereas 1970s policymakers thought they observed combinations of sustained high inflation and large negative output gaps. The cost-push view of inflation was therefore appealing because it rationalized arbitrary inflation/output gap combinations. Consequently, support for the conventional approach to inflation collapsed in Canada, while in Australia and New Zealand, support for the cost-push view of inflation hardened. For example, Prime Minister Muldoon of New Zealand said in 1977, "In this economy, unemployment and inflation are not related; I'm convinced of that" (*EP*, October 14, 1977), offering this as the basis for why his expansion of demand would not stimulate inflation, while former Prime Minister Whitlam of Australia said that inflation-control methods based on demand management didn't "cater for this new situation" where "there was both unemployment and inflation" (*MES*, January 6, 1977).

2.3 *Unfavorable Sacrifice Ratios*

Taylor (1997) suggests that 1970s policymakers understood that the Phillips curve was vertical in the long run but believed that the output-gap elasticity in the Phillips curve had diminished, so that an unfavorable sacrifice ratio discouraged policymakers from disinflating. According to this hypothesis, a disinflation via monetary restraint was regarded as too costly, whereas the neglect hypothesis instead says that policymakers believed that it was infeasible. As close as these hypotheses are, the neglect hypothesis is preferable, as it better rationalizes policymakers' views on both the effects of monetary restriction and on their menu of policy options.

The sacrifice-ratio story does not rationalize policymakers' actual views on the effects of monetary restriction because it attributes to them the understanding that as monetary policy is made progressively tighter, inflationary pressure is reduced—that is, that b_2 in equation (1) is low but strictly positive. By contrast, 1970s

policymakers often subscribed to the view that tightening *worsens* inflation—either via an interest-cost-push channel or through a unit-cost-push effect (i.e., the positive effect on unit costs of a reduction in aggregate demand and thus output). We will see that this view was influential in Canada in the lead-up to price controls in 1975 and was prominent in New Zealand and Australian public debate until 1984 and 1985, respectively. From a monetary perspective on inflation, this view is inappropriate because it emphasizes partial-equilibrium effects instead of the general-equilibrium effect (i.e., the dependence of total costs on excess demand). But from a pure cost-push perspective (i.e., $b_2 = 0$ in equation [1]), the view can be rationalized because the general-equilibrium effect is absent.

The sacrifice-ratio story also does not rationalize policymakers' accounts of their menu of choices during the Great Inflation. A low b_2 in the Phillips curve still leaves a continuous relationship between aggregate demand control and inflation, and so standard messages still hold: greater inflation control requires tighter monetary policy; direct wage and price controls merely suppress the symptoms of inflation. By contrast, policymakers in the Great Inflation felt that the cost-push nature of inflation gave them a "free lunch" whereby successful direct measures could eliminate the causes of inflation while leaving them to stimulate aggregate demand to eliminate the output gap. This free-lunch view not only guided policy during periods of mandatory wage and price controls in these countries, but also underpinned policymakers' support for wage/tax trade-offs. For example, New Zealand's Prime Minister Muldoon said in 1981, "The most important single action to reduce inflation would be a wage/tax trade-off" (*EP*, November 4, 1981), while Australian governments both in late 1974 and early 1988 advanced wage/tax trade-offs as means of breaking inflation, even though they had undertaken major easings of monetary policy. Because it posits the compatibility of monetary expansion and inflation reduction, this free-lunch aspect of policymakers' views is inconsistent with the sacrifice-ratio hypothesis, but is consistent with the neglect hypothesis's account of their views.⁸

⁸Note that the free-lunch view cannot be brought within the prism of the standard monetary view of inflation by interpreting it as support for measures to improve aggregate supply. Starting from a position of a negative output gap, a successful supply-side measure reduces inflation but worsens the output gap by adding to unused potential. Monetary expansion alongside the supply-side

2.4 Output Gap Misperceptions

Orphanides (2003) attributes the Great Inflation to policymakers' responses to estimates of the output gap that overestimated the amount of slack in the economy. As figures 7–9 suggest, output gap misperceptions were certainly part of the experience of the three countries studied here. The hypothesis advanced in this paper instead takes issue with the position that output gap mismeasurement was the *primary* source of policy error. While Orphanides (2003) argues that actual monetary policy in the 1970s had a modern inflation-oriented outlook, the monetary policy neglect hypothesis argues that policymakers had an incorrect (nonmonetary) model of inflation, leading to an assignment of inflation control to nonmonetary devices. These conceptual errors were compounded by output gap mismeasurement which, by giving the appearance that negative gaps coexisted with high inflation, ratified the authorities' adherence to cost-push views of inflation. The discussion in sections 3–5 will identify specific episodes where misperceptions of the output gap and policymakers' cost-push inflation theories reinforced one another.

2.5 The Nature of Inflation

Since the monetary policy neglect hypothesis rests on the contention that the monetary view of inflation is valid, it is worthwhile to discuss the implications of this view of inflation and its applicability to the three countries studied here.

The above characterization of the monetary view of inflation as corresponding to $b_2 > 0$ with u_t white noise means that price-level shocks can shift current inflation even without monetary accommodation, but cannot become entrenched in the expected inflation rate in the absence of accommodation. *No matter how exogenous* the shock to a specific price or wage, standard theory suggests that the response of *aggregate* prices over time will be *endogenous*: a sudden price movement may initially jolt the aggregate inflation rate

improvement involves sacrificing some of the improvement in inflation. In addition, the principal rationale for incomes policies (such as wage-price controls or wage/tax trade-offs) was that they directly removed cost-push pressures, not that they increased potential output.

upward, but prices cannot continue to rise at the new higher rate if the monetary authority does not allow faster rates of growth of aggregate demand. For example, the effect of an oil shock on the expected future inflation rate can be neutralized if the authorities allow the path of real aggregate demand to match the new path of potential output. This framework can also accommodate the openness of the three economies studied here by treating imports as an intermediate good and thus mattering for potential output in the Phillips curve.

The wage-push view of inflation has sometimes been defended for Australia and New Zealand on the grounds that independent government bodies (arbitration courts) determined minimum wages and some aspects of the relative wage structure. This institutional feature does not overturn the monetary view of inflation in principle, because the courts' decisions directly affected current growth in a portion of wages in the economy, while in the optimizing analysis underlying (1), inflation depends on the whole expected path of marginal cost (see Woodford 2003), which will be endogenous even if some period- t cost increases are exogenous. Moreover, arbitration court decisions in practice often ratified where market forces were pushing wages. For example, the mandated increases in wages in 1973–74, often cited as an exogenous event, occurred in the wake of a booming economy and extreme monetary expansion. Indeed, actual wage growth in 1973–74 in Australia exceeded the mandated growth rates by wide margins (Gruen, Pagan, and Thompson 1999, app. A). This period therefore fits into Bernanke's (2004) description of a case where "[s]eemingly unexplained or autonomous movements in wages and prices... may in fact have been the result of earlier monetary policy actions."

With this background to the monetary policy neglect hypothesis in mind, let us now consider in detail the record of each country during the Great Inflation, starting with Canada.

3. Canada

Canadian monetary policymakers ended the 1960s with a disinflationary strategy informed by several modern insights. Not only did they subscribe to a conventional view of the inflationary process, they recognized and explicitly endorsed the hypothesis that the

long-run Phillips curve was vertical. Louis Rasminsky, the governor of the Bank of Canada, did so in early 1969 when he described the inflation/unemployment trade-off as “an illusion” because “people...develop an expectation of continuing inflation and adjust their behavior accordingly” (*TGM*, February 21, 1969). In addition, Prime Minister Pierre Trudeau himself stated in 1970 that it had “been shown that this trade-off is a short-run phenomenon, and that sacrificing price stability will not by itself aid in improving a country’s long-run employment situation” (*CPD*, October 9, 1970, p. 33). The latter statement was especially notable because it came from the executive branch of government which, in Canada as well as the two other countries studied in this paper, had effective control over monetary policy throughout the 1970s.

By 1975, however, the Canadian authorities had moved from this conventional position to an adherence to a nonmonetary view of inflation, and imposed compulsory wage and price controls. The transition to this state of affairs is due to two factors. First, a strong body of opinion in Canada advocating cost-push positions existed from the beginning of the 1970s. Second, errors in official estimates of the output gap led to estimated gap/inflation combinations that were a challenge for the conventional theory of inflation to explain. Both these factors grew in importance over the period of conventional policy, 1969–71.

3.1 1969–71: *Orthodox Beginnings*

Over 1969 and 1970, the Canadian authorities gave primacy to aggregate demand restraint in reducing inflation. Prime Minister Trudeau indicated that “our policies to slow down the economy...will be slackened when the inflationary psychology is broken in this country” (*CPD*, April 17, 1970, p. 5984). Moreover, policymakers indicated that they would not be perturbed if the period of monetary restriction led to an interim period of simultaneous rising unemployment and continuing high inflation. Governor Rasminsky said that it would “not be too surprising, given the lags involved, if there is a temporary, rather discouraging period when we seem to be getting the worst of both worlds” (*FP*, November 1, 1969).

Several outside commentators resorted instead to cost-push explanations of Canada’s macroeconomic behavior, most persistently

the *Toronto Star* newspaper, both on its editorial page and through its economics columnist, Dian Cohen. Cohen's view was that it was "very hard to maintain that our current inflation is the result of excessive demand," and that the lesson from the U.S. experience for Canada was that monetary tightening "invariably resulted in massive unemployment without the desired price stabilization" (*TST*, February 2, 1970). Restrictive monetary policy worsened inflation in her analysis via the unit-cost-push effect; fiscal tightening could also be counterproductive because "as taxes go up so, ultimately, do prices" (*TST*, December 14, 1970).⁹ Cohen combined these positions with interest-push and wage-push views to conclude that "the chief sources of inflation are rising taxes, rising interest-rate costs, rising wages and salaries—and even rising unemployment" (*TST*, March 1, 1971) and that "economic policies that have traditionally been thought to curb inflation actually aggravate and encourage it" (*TST*, March 27, 1972). The *Toronto Star* editorials concurred that Canada's problem was "'cost-push' inflation [which] occurs when corporations and organized labor take out more than they're putting in" (*TST*, August 21, 1971) and advocated a wage-price freeze followed by a system of direct controls—which it called "the only weapon left against inflation" (*TST*, February 10, 1972).

The government was initially in a good position to resist these criticisms, as its restrictive policy brought inflation below 3 percent by the end of 1970 without resort to controls. However, as figure 7 shows, the government overestimated the amount of slack it had created in producing this favorable result. Consequently, it switched to a considerably easier policy in 1970. Even in late 1971, the OECD judged that "an expansionary economic policy still appears appropriate, given the existing amount of slack in the Canadian economy" (*WFP*, December 28, 1971). In fact, however, the government had already injected so much stimulus that inflation was on the way back up.

⁹Similarly, Don McGillivray in the *Financial Times of Canada* wrote that the "[e]vidence that taxes are a cause of inflation" was "piling up" (*FTC*, December 4, 1972), while Lubor Zink in the *Toronto Sun* asserted that "every increase in tax rates is soon reflected in higher production costs and consumer prices" (*TSN*, March 8, 1974).

3.2 1972–74: *Blaming International Factors*

The resurgence of inflation in 1972 converted the Canadian authorities to cost-push analysis and nonmonetary remedies. They would deny, in line with the output-gap estimates available at the time, that Canada had an excess-demand problem. Instead, they would blame international factors—what Prime Minister Trudeau called “world-wide economic pressures largely beyond our control” (*TSN*, March 6, 1974)—for provoking inflation in Canada. According to this argument, the revival of inflation in Canada arose from its pressure on the prices of world commodities, due to excess demand outside Canada. Import-price inflation and cost-of-living adjustments of wages had then driven up CPI inflation within Canada. The policy response over 1972–74 consisted of actions to affect specific prices, as well as tax cuts. Monetary policy, on the other hand, was eased to fight the perceived unemployment problem.

Finance Minister Turner foreshadowed the government’s reliance on commodity-price-push as the explanation for Canada’s inflation when in early 1972 he noted that the country’s inflation performance had recently deteriorated, but “this is partly attributable to special factors relating to food” (*CPD*, February 24, 1972, p. 215). His diagnosis was similar two years later, when he contended that “the sharp increase in oil prices plus the rise in commodity and food prices constitutes the primary thrust of inflation in this country” (*CPD*, January 7, 1974, p. 9100).

In September 1973, Turner listed the specific measures the government had taken against inflation: sales tax cuts, tariff reductions, and controls on the export of agricultural products (*CPD*, September 13, 1973, p. 6513). Similar measures were announced in his May 1974 budget. These actions were intended to offset the effects of world inflation on the CPI, by reducing the extent of the commodity price increase in Canada and by acting on the prices of other CPI components.

The government’s acknowledgment of a contribution of developments within Canada to inflation was limited. Turner saw the problem as “a complicated interaction between world demand-pull inflationary forces and the reaction in Canada of cost-push forces,” such as wage earners demanding compensation for price increases (*CPD*, January 7, 1974, p. 9100). The authorities saw income tax cuts as

the means of arresting spirals of this type, by increasing disposable income without wage increases (*CPD*, February 19, 1973, p. 1435). Prime Minister Trudeau also indicated that the government would increase antitrust regulation to deal with “the cause of certain types of cost-push inflation... [W]e’re dealing first with the monopoly of business because that is the more dangerous one” (*TST*, September 20, 1973).

To accompany these nonmonetary measures, Turner said that “monetary policy... [will] continue to be expansionary” (*CPD*, April 10, 1973, p. 3132). In 1974, he rejected “deflation of demand by severe measures of fiscal and monetary restraint” as a solution to inflation (*CPD*, May 6, 1974, p. 2078) and instead described his strategy as “breaking the inflationary spiral without impeding the continued strong growth of production, employment and income” (*TGM*, June 8, 1974). Endorsing the unit-cost-push argument, he claimed that restrictive demand policies abroad were worsening inflation because they resulted in lower production (*CPD*, October 9, 1974, p. 260).

The evidence in figure 4 suggests that, contrary to beliefs at the time, loose Canadian monetary policy was responsible for the return of inflation. As noted above, the easing had started with the expansionary measures of 1970. Neither excess demand abroad nor the rise in commodity prices actually provides grounds for believing that Canada could have sustained inflation without monetary stimulus. Canada’s exchange rate had floated in 1970, providing the conditions under which international shocks should impact relative prices, but not the ongoing inflation rate, within Canada. The authorities did not take advantage of those conditions. Instead, their cost-push interpretation of inflation encouraged them to continue monetary expansion.

3.3 1974–75: Drift to Controls

In late 1974 the government began to nominate wage-push as the source of inflation. In his November budget speech, Turner claimed that “the response to earlier international price increases has provoked domestic forces of cost-push inflation in Canada” (*CPD*, November 18, 1974, p. 1421). Mechanically, the government’s changed diagnosis arose from the fact that commodity price growth had receded, while double-digit CPI inflation continued.

Inflation—according to the cost-push view—thus became “domestic” in character. A more standard analysis, however, would dispute that inflation’s character had changed—rather, excessive demand conditions remained the problem throughout. These conditions account for why high inflation continued even after commodity price pressures eased.

The government responded by arranging talks on a voluntary incomes policy (*CPD*, November 18, 1974, p. 1423). Following the collapse of these talks in May 1975, Prime Minister Trudeau said that the change in the character of inflation from international to wage-push meant that the case against compulsory wage and price controls had weakened (*TST*, May 23, 1975). On October 13, 1975, Trudeau announced a three-year control program.

3.4 1975–78: Monetary Policy in the Controls Period

In conjunction with the imposition of the controls, Finance Minister Macdonald announced that the Bank of Canada would pursue targets for M1 growth, on the grounds that it was “essential for the success of any prices and incomes policy to avoid too rapid a rate of growth of demand and the development of excess pressure on resources” (*TGM*, October 15, 1975). The stage, therefore, seemed set for a restrictive monetary policy to accompany price controls, thus providing a counterexample to the generalization that periods of control are accompanied by monetary ease.

In practice, however, such a counterexample did *not* emerge: while policy did tighten over 1976, nominal interest rates fell and currency growth picked up during 1977. The M1 targets were met throughout, but hitting these targets did not restrain other measures of money growth or require increases in nominal interest rates, because households reduced their M1 holdings relative to other deposit categories. Consistent with the easing, inflation revived over 1978–81.

Much of the easing was conscious and reflected the lingering influence of cost-push views—i.e., that Canada’s inflation was of a nonmonetary character and that, therefore, the case for restraining aggregate demand was not compelling. In 1976, Finance Minister Macdonald reaffirmed the government’s nonmonetary diagnosis of inflation, stating: “It is frequently argued that... excessive increases

in the money supply have been the major cause of inflation in Canada... But I do not believe they are the primary cause of the inflation we have experienced in this country" (*TGM*, August 19, 1976).

Indeed, as they watched inflation fall during the controls period, policymakers appear to have had too much confidence that the decline represented the genuine removal of inflationary pressure. Just eight months into the controls, Bank of Canada Governor Bouey stated that it was "within the realm of possibility that receding inflation will permit a gradual moderation of money supply growth without the need for significantly higher interest rates than we have at present," and that "in an atmosphere of growing confidence that inflation was being brought under control... interest rates could over time be expected to begin declining" (*TGM*, June 24, 1976). This message would have been unexceptional during a period of no price controls. But in a period where an observed decline in inflation was virtually inevitable because of the controls, the message would have helped encourage expectations of interest-rate cuts to accompany the decline. By early 1977, the Bank of Canada had indeed cut short-term nominal interest rates down to levels below those prevailing at the start of price controls.

In addition, the post-1973 slowdown in potential growth had become a new source of output-gap mismeasurement. Contemporary observers of the Canadian economy initially mistook much of this slowdown as reflecting weakness in demand—a misconception paralleling the U.S. situation, studied by Orphanides (2003). As figure 7 shows, the OECD's 1977 estimates of Canada's output gap were far more negative than now seem appropriate.¹⁰ Subscribers to a monetary view of inflation, such as Laidler (1978), questioned these gap estimates on the grounds that they were inconsistent with Canada's severe inflation. Cost-push adherents, on the other hand, felt that severely negative output gaps and high inflation were compatible.

While cost-push views do appear to have discouraged the authorities from implementing a sufficiently tight monetary policy over 1975–78, such views did lead to a genuine disinflation beginning in

¹⁰ As late as March 1977, a Canadian financial columnist described 5 percent as "Canada's long-term average rate of real growth," and characterized the Bank of Canada's monetary targets as guided by that rate (*FTC*, March 14, 1977).

1978. Just as in the United States in the same year (see Nelson 2004, sec. 5), a fear based on a cost-push scenario—a wage-price spiral triggered by exchange-rate depreciation—triggered a monetary tightening. The deputy governor of the Bank of Canada explained that the bank in 1978 was “raising domestic interest rates repeatedly” out of concern that “the direct impact on Canada’s price level of further substantial exchange [rate] depreciation could...set off a renewed outbreak of serious wage inflation” (Freeman 1981, 200). Trudeau himself claimed in October 1978 that “inflation is coming in from outside, very simply because of the high cost of things we import, and very simply because of the lowered value of the [Canadian] dollar” (*CPD*, October 12, 1978, p. 33). Finance Minister Chrétien blamed the revival of inflation on “the rise in food prices and the fall in the dollar” and endorsed the fear that these would trigger “a fresh outbreak of large, leap-frogging wage and price increases” (*CPD*, November 16, 1978, p. 1200).

According to an orthodox (monetary) view of inflation, this import-price-push view was invalid—depreciation could not trigger a new inflation spiral unless the depreciation was itself a symptom of excessive demand created by the government. Despite its misguided character, however, the import-price-push view was important, both in Canada and the United States, in shifting monetary policymakers to a tighter position in late 1978. The tighter policy eventually produced dividends in lower inflation: four-quarter CPI inflation peaked in Canada in mid-1981 and fell below 5 percent by the end of 1983.

4. Australia

Australia never had compulsory price controls of the type imposed in Canada, but in other respects was more immersed in a nonmonetary tradition of inflation control that lingered until the 1990s.

4.1 1971–72: Blaming Wage-Push

In the fourth quarter of 1970, annualized CPI inflation was 7.7 percent and the four-quarter rate reached 4.9 percent; both were the highest rates in Australia since the mid-1950s. The inflation breakout occurred during the term of office of the government headed

by Liberal Party leader John Gorton.¹¹ In response, Prime Minister Gorton gave a televised address to the nation on January 29, 1971, in which he blamed inflation on wage-push. Gorton ruled out “taking monetary and fiscal measures at this time,” stating that his government did not “believe the present situation requires a lift in interest rates, already high” (*SMH*, January 30, 1971). Three weeks later, Gorton reaffirmed that “the real problem... is cost-push inflation,” and maintained that the “biggest single influence now which can prevent inflation is a conscious and firm effort on the part of wage-fixing tribunals” (*APD*, February 18, 1971, p. 314). The cabinet minister with responsibility for labor market issues, B. M. Snedden, maintained that “excess demand does not presently exist” and agreed that the “answer to the present problems is that there should be more restraints on wages” (*APD*, February 18, 1971, pp. 307, 308).¹²

For some months in late 1971, the government (now led by William McMahon) did pursue measures to rein in demand. This did not represent an abandonment of its wage-push diagnosis, but instead reflected a fear that wage increases could lead to demand pressure. Inflation did fall in 1972, but in the meantime, the apparent failure of inflation to respond to demand restriction validated the government’s earlier diagnosis of autonomous wage-push. Prime Minister McMahon denied responsibility: “It is obvious that the rise in the consumer price index is due mainly to cost increases brought about by very large wage increases. It is plainly absurd for the Leader of the Opposition to attribute the rise to the Government” (*DM*, October 22, 1971).

The severity of the government’s demand restriction was overestimated by observers, with the Institute of Applied Economic and Social Research claiming: “At no other time during the past eight years has there been such a high percentage of idle capacity” (*TA*,

¹¹In Australia, the principal conservative party is known as the Liberal Party, with the Australian Labor Party its main electoral opponent. In the period studied here, the Liberal Party headed a government until December 1972, and was followed by Labor for two terms (1972–75). The Liberals then headed a government for three terms from 1975 to 1983, after which Labor served four terms over 1983–96.

¹²Snedden had also put on record the government’s support for the view—an important part of the cost-push position—that demand did not exert upward pressure on prices until full employment was reached (Department of Labour and National Service 1970, 17).

December 29, 1971). Arguments that demand stimulus would reduce inflation became prevalent, with business columnist Warren Beeby judging that “[i]n the present economic climate, a reduction in interest rates is more likely to act against inflation than feed it. It would relax some of the cost pressures, stimulate demand, and get industry moving again...” (*SA*, September 19, 1971). A commercial bank advised: “Greater spending would stimulate an expansion of production [and] reduce unit costs...” (*TA*, November 20, 1971). Over 1972 the government eased monetary policy and, late in its term, was still being described as “leaning heavily on the argument that wage increases cause inflation” (*TA*, August 3, 1972).

4.2 1972–73: Monetary Tightening and Push for Controls

Despite its cost-push explanation of 1972’s inflation, the Treasury was aware that the undervalued dollar and accompanying rapid money growth meant the prospect of excessive demand in 1973. In light of these concerns, the Labor government of Gough Whitlam, elected in December 1972, revalued the Australian dollar and introduced foreign exchange controls, both of which worked in the direction of reducing money growth. To this extent, the government was now taking conventional monetary measures against inflation. A submission to cabinet by the Assistant Treasurer in October 1973 reveals, however, that the authorities’ views remained unorthodox in important respects (*ACR*, October 5, 1973). This document classified inflation, which was now moving into double digits, as falling into three categories: imported inflation, which it characterized as working not only through the effect of balance-of-payments surpluses on the money supply, but also “through rising import prices; cost inflation; and demand inflation, occurring when “demand...run[s] beyond the capacity of supply.” This classification scheme amounted to a cost-push approach to inflation, supplemented by an acknowledgment that the output gap matters for inflation when the gap is positive. By contrast, a monetary view of inflation would not categorize cost and imported inflation as distinct from demand inflation, but as simply routes through which excessive demand is transmitted to the inflation rate.

The government’s classification of inflation into three distinct types is also reflected in the three main actions it took as

anti-inflationary policy in the second half of 1973: tariff cuts; a monetary tightening; and proposed wage and price controls.

The government announced a 25 percent tariff cut in July 1973 with the statement: "The justification for the general reduction of tariffs is the excessive rate of inflation which now prevails... The tariff changes will have a direct impact on import prices of about the same magnitude as an Australian revaluation of slightly less than 6%" (*TA*, July 19, 1973). This equivalence rests on the observation that both actions withdraw import-price-push pressure on prices. But from a monetary view of inflation, revaluation and tariff cuts are not equivalent. Revaluation drains aggregate demand and reduces pressure on prices by creating a balance-of-payments deterioration and thereby producing monetary contraction. A tariff cut, by contrast, need have no such effects. Prime Minister Whitlam would adhere to the import-price-push view well after he left office, maintaining in a 1977 television interview that "the biggest component of inflation at the moment in Australia is Australia's excessive protectionism" (*MC*, September 19, 1977).

Action was taken to reduce money growth with a 5 percent revaluation in September 1973, accompanied by an announcement by the Prime Minister of "a sharp rise [about 2 percent]... in interest yields... an essential precondition if inflation is to be countered at all" (*SMH*, September 10, 1973). The reference to monetary restraint as a "precondition" for inflation control again left open the possibility that some inflation was of a nonmonetary variety.

The third measure against inflation by the government during 1973 was an effort to introduce compulsory wage and price controls. Since it lacked the constitutional powers to impose wage and price controls, the government held a referendum in December 1973 to obtain those powers. The referendum proposals were defeated on December 8, 1973.

4.3 1974–75: Abandoning Monetary Solutions

As of late 1973, the Australian authorities were pursuing a largely orthodox economic package—employing monetary restriction rather than price controls—against inflation. That policy would eventually produce dividends in the form of falling inflation in mid-1975. By then, however, the government had become disillusioned with

monetary restraint as a cure for inflation and had embraced a non-monetary approach.

This shift was largely a response to 1974's stagflation. While the strict monetary policy of 1973–74 did cut deeply into aggregate demand, it happened to coincide with a shift to a lower path of potential output, due to permanent shifts in the labor market and the post-1973 productivity slowdown. The government incorrectly perceived a severely negative output gap. It was not until June 1975 that Treasurer Cairns publicly acknowledged that “the minimum level of unemployment will be considerably higher than in the past decade” (*TA*, June 5, 1975), while the slowdown in potential growth took years to recognize. With aggregate supply actually falling nearly as much as aggregate demand, the fall in inflation was not only delayed but modest. The apparent emergence of prolonged inflation alongside a deep output gap led Prime Minister Whitlam himself to adopt a strict cost-push diagnosis of inflation. “I have to be quite frank with you,” he said in a televised address to the nation, invoking import-price-push, “inflation will not be wholly beaten until there is a worldwide solution” (*DT*, August 27, 1974). In October 1974 he placed emphasis on wage-push as the source of inflation: “There should be no doubt that this severe inflation will continue if there are excessive wage claims” (*TA*, October 22, 1974).

A mini-budget delivered by Whitlam in November 1974 outlined the government's change in strategy. He now rejected aggregate demand limitation as an essential ingredient in inflation control, instead claiming “[w]hat is needed is a stimulus to the economy which increases demand, whilst at the same time abating cost pressures” (*APD*, November 12, 1974, p. 3360). To this end, he had approved “a very substantial relaxation of monetary policy,” while his statement announced cuts in personal income taxes. He argued that the tax cut would both stimulate demand and serve as a direct “attack on inflation, by reducing wage pressures” (*APD*, November 12, 1974, p. 3361), since disposable incomes could now rise without wage increases. The government was therefore pursuing a voluntary incomes policy based on a wage/tax-push view of inflation.

Not only did the government no longer view demand restraint as an essential part of an anti-inflation program, it now contended that monetary restriction actually promoted inflation. Treasurer Cairns said that tight monetary policy in 1974 had made unemployment

and inflation worse because it had been introduced when excess demand was ceasing to be a problem (*TA*, June 5, 1975), an argument he based on interest-cost-push and unit-cost-push views of inflation (*MEH*, September 23, 1975).

4.4 1976–79: Sending Mixed Signals

The Liberal government that took office in late 1975 ostensibly accepted an important role for monetary policy in the control of inflation, with steps to reduce money growth announced in January 1976 and targets for growth in M3 commenced in March 1976. In practice, however, inflation in the double-digit range was a problem for the whole life of the government. The government made little progress in reducing inflation after its first two years in office, and with the lag between monetary policy actions and inflation, the initial reduction is best attributed to the reining in of money growth that occurred in 1973–74 under Whitlam. It was not until 1980 that the government's monetary policy became more genuinely anti-inflationary, and—again due to lags—this did not deliver a substantial fall in inflation until after the government lost office in 1983.

In part, the government's failure to achieve a major monetary tightening earlier than 1980 is attributable to its flawed execution of monetary targeting. In contrast to Whitlam's tightening in 1973, open market sales and increases in short-term interest rates were typically eschewed in favor of heavy use of reserve-requirement changes. Indeed, the January 1976 package of measures to reduce money growth was accompanied by an announced *reduction* in official short-term rates (*AFR*, January 23, 1976). The reliance on quantitative controls is reflected in the fact that the compulsory cash-reserve requirement was changed eighteen times over 1976–79. Such actions should not have been regarded as effective, because with an unchanged policy rate, a higher reserve requirement leads to the extra monetary base being created by the central bank, with no downward pressure on deposit growth or aggregate demand. An additional error was that the authorities falsely believed that budget deficit reductions made an automatic contribution to restricting money growth. Indeed, *The Economist* characterized the government's strategy in 1978 as using deficit control to reduce money growth “without a rise

in interest rates" (*TE*, August 19, 1978)—this at a time when real interest rates were close to zero.

The government sent further mixed signals on inflation control because its adoption of monetary targeting was by no means accompanied by abandonment of the cost-push analysis of inflation. Both Prime Minister Fraser and Treasurer Lynch believed that wage-tribunal decisions could add directly to inflation, with Fraser saying they contributed to "this vicious spiral of higher prices, higher money wages, higher costs, and yet more price increases" (*DT*, February 14, 1976) and Lynch arguing that "excessive wage and salary claims remain a direct impetus to more inflation" (*DM*, February 20, 1976). In fact, the government unsuccessfully attempted in April-May 1977 to arrange a wage-price freeze, a measure Fraser claimed would "break the back of inflation" (*TE*, April 16, 1977).

This shows that the government's misguided means of pursuing monetary targets were not just a technical error—they were founded in the cost-push approach. Low real interest rates should have signaled to the government that it had not actually tightened monetary policy substantially. But the above statements show that the authorities believed that wage-price spirals could occur without monetary accommodation, and that a price freeze could break inflationary expectations. They could therefore attribute low real interest rates to autonomous movements in expected inflation, to be remedied by direct intervention in wage and price setting rather than by higher nominal interest rates.

As figure 8 indicates, another impediment to tighter policy was a still erroneous picture about the state of demand. In October 1977, *The Economist* expressed the widespread view that "the Australian economy is operating at well below capacity" (*TE*, October 29, 1977), an assessment that reflected the continued failure to recognize the post-1973 slowdown in potential GDP.

4.5 1980–82: Moving to a Stricter Rule

Around 1980, the Australian authorities do appear to have shifted to a more genuinely anti-inflationary stance. The real ex-post Treasury bill rate was 2.4 percent in the first ten quarters of the 1980s, higher than in any ten-quarter period since 1962–64; furthermore, discrepancies between actual interest rates and Taylor rule prescriptions

became less striking (figure 5). It therefore appears appropriate to characterize policy from 1980 as the adoption of a more restrictive interest-rate rule. This change may have been a response to criticism in public debate of quantitative controls and calls for a more market-oriented monetary policy (see Guttman 2004). The tighter policy also probably reflected the recognition that Australia's slower growth after 1973 was supply-side in character. The more inflation-oriented monetary policy helped deliver better inflation performance after 1982. This improvement was marred by two major revivals, in 1985–86 and 1988–90. Both occasions of higher inflation reflect lapses by the authorities back into nonmonetary strategies against inflation.

4.6 1982–84: Lapse into Monetary Policy Neglect

The early 1980s monetary tightening in Australia was initially felt only in recession, while wage and price inflation increased, reflecting delayed responses to past monetary ease. By late 1982, the Fraser government had reverted to a more stimulative policy on aggregate demand, and in October 1982, with four-quarter CPI inflation having reached 12.3 percent, Prime Minister Fraser announced his support for a six-month wage freeze (*DM*, October 26, 1982). The claimed rationale for such a freeze was that it would simultaneously reduce inflation and unemployment, and when the government formally advanced the plan, it estimated that the freeze would reduce inflation by six percentage points (*AFR*, December 17, 1982). The wage freeze was agreed to by the state governments and wage tribunals, and became effective in December 1982. Thus, at the end of 1982, Australia was pursuing a stricter incomes policy than it ever had in the 1970s.

The Hawke government that came into office in March 1983 eventually brought inflation below 5 percent, though not until 1991. In the intervening years, it deployed a number of nonmonetary strategies against inflation and experienced the usual disappointments with these approaches. The centerpiece of these nonmonetary tools was the "Accord," an agreement on wage growth between the government and labor unions. Early in the government's term, Treasurer Keating indicated that this incomes policy would not be used as a substitute for aggregate demand policies against inflation; this, however, proved a short-lived position (Kelly 1992, 66), and Keating was reported in September 1983 as believing that Australia's

wage-fixation system made inflation difficult to control via demand weapons (*DM*, September 8, 1983). The government also continued an expansionary monetary policy and reaffirmed its view of incomes policy as a weapon that could help reduce the reliance on demand management. In August 1983, Prime Minister Hawke made it clear that he saw incomes policy as an alternative to the “traditional instruments of tighter monetary and fiscal policy to restrain inflation” (*TA*, September 1, 1983).

In January 1985, the government faced a decision on whether to tighten monetary policy after growth in the M3 aggregate persistently exceeded the target rate. It was widely accepted that M3 behavior reflected distortions from financial innovation, but other monetary aggregates less subject to distortion also suggested easy money conditions, while real interest rates—though well above 1970s values—had been slightly below U.S. levels during most of 1984, even as Australia had more serious inflation. The government’s decision to drop the monetary target, and not tighten monetary policy at all, was applauded by the *Sydney Morning Herald* in an editorial: “[M]onetary policy is anything but loose. Any further tightening of monetary policy. . . would be more likely to fuel inflation than control it. . . [because with] the Accord, the Government has promised to deliver employment and economic growth” (*SMH*, January 30, 1985). Thus, three years after monetary policy had produced disinflation in major economies, the argument that monetary tightening worsened inflation was still being prominently advanced in Australian policy debate.

4.7 1985–91: Discarding the Nonmonetary Framework

The position that monetary policy was already tight, and that further tightening would relaunch inflation, was not borne out by developments in the rest of 1985. Faced with a resurgence of inflation, the government tightened monetary policy beginning in February 1985. The fact that monetary policy was tightened when inflation rose, despite the continuing prevalence of cost-push views among policymakers, reflects the import-price push diagnosis of the inflation outbreak. Even from a cost-push perspective, monetary tightening is the appropriate response to rising inflation if the rise is believed to have come from exchange-rate depreciation.

In contrast to the fears expressed earlier in 1985, monetary tightening did not produce a wage-price spiral. Price inflation receded in 1987–88, reflecting a delayed reaction to the 1985–86 tightening, while wage inflation was lower still, in part reflecting continuing attempts to use control of wages as an anti-inflationary weapon. To this end, the government negotiated over 1985–86 a trade-off between nominal wage increases and tax cuts.

Contrary to their aim, the 1980s incomes policies in Australia appear to have contributed little or nothing to the control of inflation. Price inflation in the 1980s behaved dissimilarly to wage inflation; as one of Keating's subsequent advisers observed, incomes policy appeared "extremely successful in restraining the growth of wages but not of inflation" (Edwards 1996, 282). Policymakers attributed this failure first to import-price-push and then to profit-push. In fact, however, it appears that it was the absence of monetary restraint that really made the difference, as price inflation in the 1980s responded quite predictably to prior monetary developments.¹³

Monetary policy setting in the 1980s was made more complicated by the global rise in real interest rates. But by the mid-1980s the experience of disinflating countries should have made it clear to the Australian authorities that the average real rate consistent with price stability was higher than in, say, the 1960s. Overconfidence in incomes policy, not uncertainty about real rates, explains why Treasurer Keating predicted in May 1988 that "inflation [is] now heading towards 5 percent or lower" (*APD*, May 25, 1988, p. 3013), despite nominal interest rates having been cut by 6 percentage points (and real rates by around 3 points) over the course of 1987.

Satisfied that inflation control had been successfully delegated to other devices, the authorities thus permitted monetary ease over 1987–88, an easing reflected in a surge in inflation over 1988–90. In addition, 1988 brought a new series of nonmonetary initiatives against inflation. Many of them were introduced in the August

¹³For the period of strictest wage control, 1983–89, the correlation of CPI inflation and growth in average nominal wages is -0.22 ; the maximum positive correlation of inflation with prior wage growth is 0.42 (inflation and wage growth one year earlier). By contrast, the maximum correlation of inflation and money (currency) growth is 0.61 (inflation and money growth two years earlier). Another measure of monetary ease—the excess of the Taylor rule prescriptions in figure 5 over actual short rates—leads inflation over 1983–89 by one year, with a correlation of 0.85 .

1988 budget, as a newspaper report recorded: “The Budget’s anti-inflationary strategy hinges on lower indirect taxes, mainly for beer drinkers, and a delay in personal tax cuts. . . [T]he size of the [income tax] cuts will depend on the second wage/tax trade-off. . .” (*TA*, August 24, 1988). Reflecting on these measures, Treasurer Keating remarked, “Having succeeded in bringing down the inflation rate of 11 per cent under the Liberal Party to 5 per cent in 1985, and having watched inflation rise to 10 per cent as a result of a big depreciation of the exchange rate, we are now succeeding in bringing it back to the 5 per cent area whence it came in 1985... Inflation is declining” (*APD*, November 30, 1988, p. 3547). But inflation actually rose from its 1988 Q1 trough of 6.9 percent, ending the 1980s at a four-quarter rate of 7.8 percent.

A major monetary policy tightening also occurred in 1988. The tightening itself was motivated by balance-of-payments rather than inflation considerations. The fall in inflation that it produced, however, transformed the views of policymakers and observers about the role of monetary policy in inflation control. As late as 1990, the governor of the Reserve Bank rejected central-bank inflation targeting as infeasible in Australia, and cited the need for other tools such as wages policy (*AFR*, October 18, 1990). When inflation fell below 5 percent in early 1991—clearly a response to the period of monetary restraint—Treasurer Keating spoke publicly about low inflation as the criterion by which macroeconomic policy should be judged (Edwards 1996, 405). Gruen and Stevens (2000, 52) record that in the 1990s, “the main insight of two centuries of monetary economics. . . that monetary policy ultimately determined inflation” convinced the authorities that nonmonetary approaches to inflation control should be abandoned in favor of central-bank inflation targeting.

5. New Zealand

In the 1970s and 1980s, decisions on monetary policy were the responsibility of the New Zealand Treasury, headed by the Minister of Finance. In May 1970, Finance Minister Robert Muldoon rejected private estimates that inflation in New Zealand, which stood above 4 percent, might reach 9 percent later in the year. Muldoon noted that New Zealand’s experience was of lower inflation than in Japan, the United States, Canada, and the United Kingdom (*EP*, May 6,

1970). Building on this comparison, Muldoon said that “countries which fail to hold inflation will suffer in comparison with those which are successful. This is the test...” (*EP*, July 21, 1970). Over the following fifteen years, during which Muldoon totaled eleven more years as Finance Minister, New Zealand performed poorly by this test. In 1985, New Zealand’s CPI averaged 5.7 times its 1970 value, compared to 2.7 in Japan, 2.8 in the United States, 3.1 in Canada, 4.1 in Australia, and 5.0 in the United Kingdom. The deterioration reflected repeated application by the authorities of the policy combination associated with monetary policy neglect—direct wage-price controls accompanied by monetary expansion.

5.1 1970–72: Rival Spiral Theories

The government had predicted in mid-1970 that inflation for the full year would be a little over 4 percent (*EP*, May 29, 1970). By the end of 1970, four-quarter CPI inflation was instead moving into double digits. Muldoon cited “a wage/price spiral caused... by the stronger and more militant unions” (*NZPD*, June 25, 1970, p. 1298), while the Reserve Bank of New Zealand similarly judged that “[t]he ‘cost-push’ type of inflation now seems to be dominant” (*RBNZ*, April 1970). The government imposed a three-month wage-price freeze in November 1970 and negotiated with unions for a voluntary wages policy to follow the freeze. After the negotiations failed, the government introduced a compulsory policy accompanied by continued monitoring of prices.

The opposition, headed by Labour Party leader Norman Kirk, had its own cost-push diagnosis of inflation. Kirk rejected the wage-push explanation on the grounds that “wages and salaries were increased because costs and prices had increased” (*NZPD*, June 15, 1971, p. 853). Instead he blamed a “cost-tax-price spiral” initiated by government tax increases (*NZPD*, June 15, 1971, p. 854). The government and opposition thus had rival “spiral” theories of inflation—a wage-price spiral versus a cost-tax-price spiral—neither of them consistent with a monetary explanation.

Kirk also endorsed the unit-cost-push argument that demand restriction worsened inflation: “Stagnation is a cause of inflation... [by] compelling factories to run at less than full capacity so that the unit cost remains high but the volume of output falls... If we want

to cure inflation, we must first increase production” (*NZPD*, June 14, 1972, pp. 116–17). Even the premise of Kirk’s argument—that demand was weak in 1972—has no support in revised data (figure 9).

By 1972, however, the government shared Kirk’s view that the output gap had gone negative. This perceived deterioration had occurred alongside continued double-digit inflation through most of 1971, so the government’s cost-push diagnosis was reinforced. The Reserve Bank of New Zealand claimed that in “a period of ‘stagflation’...orthodox monetary policy is of subsidiary importance” (*RBNZ*, April 1972). Similarly, Muldoon claimed that recent overseas attempts to fight inflation with monetary restriction had raised unemployment without reducing inflation (*NZPD*, June 10, 1971, p. 785).

In light of these judgments, the government in March 1972 increased its range of direct controls—imposing stricter controls over firms’ price decisions, freezes on prices of government output, and limits on dividend income (*NZPD*, June 22, 1972, pp. 391–92). The government also undertook monetary and fiscal expansion. In line with the cost-push view, it believed that pressure on inflation from demand only began once the economy reached potential, which Muldoon said was far off, given the “unused capacity in the economy” (*NZPD*, June 22, 1972, p. 409).

The timing of Muldoon’s policy loosening was far from appropriate as, like Australia, New Zealand had entered a period of tremendous increases in the monetary base due to large inflows of foreign capital and export earnings. Muldoon’s monetary easing was a response to the more restrained pre-1972 demand environment, which the external developments had already replaced with boom conditions. The earlier period of weaker demand, as well as the controls imposed by the government, nevertheless secured a fall of inflation to just above 5 percent by the end of 1972.

5.2 1972–75: Labour Government

The National Party administration was defeated at the election of November 1972 and was succeeded by a Labour government led by Norman Kirk. The government was immediately faced with a revival of inflation, which in the first quarter of 1973 returned to double-digit

annualized rates. The authorities took a strict cost-push interpretation of the increase. Presenting the government's 1973 budget, Finance Minister Rowling acknowledged that "inflation has become the number one problem" (*NZPD*, June 14, 1973, p. 1353), but insisted that "short-term economic management must keep the economy running close to full capacity" (p. 1349). To this end, the government actually cut the Treasury bill rate by 50 basis points to 2 percent in May 1973 (*RBNZ*, April 1974). The measures Rowling outlined to "contain the inflationary pressures which are apparent" (*NZPD*, June 14, 1973, p. 1349) consisted of interventions in the markets for particular goods.

Prime Minister Kirk returned from an overseas visit in August 1973 claiming inflation was a mystery: "So far, nobody has been able to say what is causing inflation, let alone suggest a remedy for it" (*EP*, August 16, 1973). His own account of New Zealand's inflation was that "living costs have risen because of the price of meat" (*NZPD*, September 1, 1973, p. 3422). In addition, he blamed import-price-push: "we are also feeling the effect of inflation in the Northern Hemisphere through a sharp and very severe rise in the prices of our imports." The import-price-push analysis would produce the main monetary tightening of the year—the exchange-rate revaluation in September 1973.

New Zealand's announcement of a 10 percent revaluation was synchronized with the revaluation taking place in Australia. However, in contrast to the statement by Prime Minister Whitlam in Australia, which had mentioned the need to curb money growth, Kirk's rationale for the revaluation concentrated on the import-price-reducing aspects of the action. "The increasing cost of imports and the higher prices of meat and other goods which we export have been major causes of the rising cost of living in New Zealand... A major threat to cost and price stability that we face now is the surge of inflation reaching New Zealand from abroad," he told a press conference. He argued that the revaluation would be anti-inflationary via "a reduction... in the cost of imported goods and of those export commodities whose prices tend to be determined in overseas markets... [with] further benefits as these price effects work their way through the national cost structure" (*CP*, September 10, 1973). Other than a brief reference to the strength of New Zealand's overseas reserves, the implications of the revaluation for

monetary and aggregate demand conditions went unmentioned by Kirk, even though, from the perspective of a monetary view of inflation, these were the most important aspects of the policy action.

Consistent with the government's rejection of excess demand as a source of inflation, Kirk's revaluation announcement was not accompanied by interest-rate increases; rather, the accompanying measures were a range of new controls on the prices and export of agricultural goods (*NZH*, September 10, 1973). The four-quarter rate of CPI inflation ended 1973 at 10 percent, nearly double its value a year earlier. Finance Minister Rowling attributed the increase in the inflation rate during 1973 to "three major factors: meat prices, governed by overseas realizations; fruit and vegetable prices, governed by the weather in this country; and import prices, governed by other people's costs. None of these things comes very directly within the control of the Government" (*NZPD*, June 5, 1974, p. 1642).

Later in 1974, the authorities turned to blaming domestic wage-push for inflation. The government (now led by Rowling after Kirk's death) introduced new controls on wage growth and in 1975 negotiated wage agreements with labor unions (*RBNZ*, April 1975). Finance Minister Tizard claimed that "an effective incomes policy is crucial in the current economic situation... a relaxation of controls on wage bargaining is directly dependent on a moderation of our present inflation" (*NZPD*, May 22, 1975, p. 1295). Prime Minister Rowling rejected demand contraction on the grounds that it had been tried overseas and had caused unemployment with little effect on inflation. Instead, he described his task as to "ensure that there is the least possible disruption to sustained economic growth and employment" (*EP*, December 3, 1974).

5.3 1976–79: *Misleading Austerity*

Following the November 1975 election, the National Party returned to office, now led by Robert Muldoon who, in addition to becoming Prime Minister, resumed his former position of Finance Minister. Throughout his term in office, Muldoon maintained cost-push views of inflation. He nevertheless consciously pursued a policy of zero real GDP growth in 1976; the rationale for this strategy was to improve the current account balance rather than inflation (*CP*, January 13, 1982). The government announced a tightening in March 1976, with

a 100 basis point increase in Treasury bill rates to 4 percent (*RBNZ*, April 1976). With inflation at over 15 percent, this still implied highly negative real rates. Real rates were nevertheless less negative than they had been in 1975, and this, along with the slowdown of money growth that had occurred after 1973, produced somewhat more restrained monetary conditions in the mid-1970s than had prevailed earlier in the decade.

A slowdown in the economy did occur over 1976–77, and this slowdown, in turn, accounts for the decline in inflation to around 10 percent in late 1978. The reason why inflation did not fall further is that in terms of *excess* demand, the measures to restrict demand were nowhere as austere as thought at the time, because potential output growth had undergone a particularly severe slowdown (table 1). As in other countries, the authorities were slow to recognize the reduced growth of supply; and, again as in other countries, in the meantime they observed protracted periods of high inflation and negative gaps, seemingly confirming cost-push views of inflation. It was not until 1978 that the Reserve Bank of New Zealand made it clear that “New Zealand’s slow growth reflects fundamental structural factors” (*RBNZ*, April 1978).

Growing recognition of the productivity slowdown had no effect on the Muldoon government’s adherence to nonmonetary views on inflation. Muldoon imposed a four-month freeze on prices in late 1976; when inflation continued after the freeze’s removal, he claimed, “Two main factors are maintaining the present inflationary momentum. These are import prices and wages” (*NZPD*, July 21, 1977, p. 1525). Giving little role to monetary policy in his anti-inflation strategy, Muldoon allowed policy to ease during 1978, and by the end of the 1970s, inflation was close to its 1970s peak of 17 percent.

5.4 1980–84: Reaffirming Nonmonetary Approaches

In January 1980, while several major economies were assigning a central role to monetary policy in fighting inflation, the Muldoon government continued to adhere to the view that inflation was largely beyond the reach of macroeconomic policy. “New Zealand is, of course, still vulnerable to overseas influences, particularly oil prices, and to domestic factors, such as wage increases,” Muldoon said. “What

happens to prices over the coming year will be determined largely by these two factors" (*CP*, January 14, 1980). Shortly afterward, four-quarter CPI inflation reached a peacetime peak of over 18 percent, a rate which Muldoon claimed was "almost entirely" due to the second oil shock (*AST*, October 2, 1981).

The wage-push diagnosis of inflation led the Muldoon government to attempt to negotiate a wage/tax trade-off with trade unions over 1981–82.¹⁴ When these negotiations broke down, Muldoon imposed a one-year wage-price freeze. Muldoon's justification for the freeze cited developments in major OECD countries. "Rates of inflation in most of these countries are declining... These countries are major suppliers of products to New Zealand... The price of oil is expected to decline in real terms throughout the remainder of this year... All of these things give us a unique opportunity to reduce New Zealand's rate of inflation..." (*NZH*, June 23, 1982).

The political reaction to the controls announcement was a watershed, reflecting the increased role since 1980 for conventional economic analysis in New Zealand debate. Opposition leader Rowling gave an orthodox critique of Muldoon's freeze:

"The wages and prices freeze will not work... After all, in the year A.D. 301, a little before the Minister of Finance's time, but not before the policies adopted by the present Government—that dates the policies—the Emperor Diocletian introduced a price freeze with the death penalty for non-compliance, and it still did not work. There is no way in which it will work in 1982, any more than it [c]ould in A.D. 301, because such a proposition resolves absolutely nothing." (*NZPD*, July 20, 1982, p. 1199).

Similarly, Rowling's successor as Labour Party leader, David Lange, said that New Zealand's trading partners had reduced inflation by tightening monetary policy, while Muldoon's government had undertaken no such attack on the causes of inflation (*CP*, July 11, 1983).

¹⁴The Reserve Bank governor also voiced support for an incomes policy, stating: "I don't believe inflation, which is well-entrenched and is being encouraged by the close linkage between costs and prices, can be overcome solely by tight monetary and fiscal policies" (*NZH*, February 13, 1982).

For his part, Muldoon in his 1983 budget claimed that “monetary restraint” had accompanied the wage-price freeze (*NZPD*, July 28, 1983, p. 891). In fact, however, the government followed an expansionary policy. Muldoon made it clear that he had no intention of reining in demand during the freeze: “I would think that we should be able to keep the general level of economic activity high enough that there should be no big increase in unemployment” (*CP*, December 28, 1982). In July 1983, Muldoon reduced the interest rate on three-month Treasury bills from 12 percent to 7.8 percent (*AST*, July 30, 1983). His rationale made it clear that his concept of “monetary restraint” involved making real rates positive by suppressing inflation, not by increasing nominal rates: with inflation down to 4 percent, he said, an 8 percent nominal return was “pretty reasonable” (*EP*, October 13, 1983).

The evolution of macroeconomic policies during the freeze thus confirmed a fear expressed by Don Brash, later head of the central bank, but in 1982 a newspaper columnist. At the time of the freeze’s imposition, Brash worried that the worst side effect of the freeze would be that it would “fool people into believing that something is actually being achieved in the reduction of inflation,” at the same time that the government did nothing to reduce monetary growth (*AST*, June 23, 1982). The appearance of success proceeded as the four-quarter CPI inflation rate fell into single digits in mid-1983, troughing at 3.5 percent at the end of the year, by which time the freeze had been extended to February 1984. A member of Muldoon’s cabinet, Foreign Affairs Minister Cooper, declared victory: the government, he said, was the first in the history of Western civilization to have cured inflation successfully via a wage-price freeze (*CP*, September 17, 1983).

The illusory nature of this improvement was revealed in July 1984, when CPI data for the first post-freeze quarter revealed that prices had jumped by 2.2 percent (*CP*, July 10, 1984). Shortly afterwards, the Muldoon government suffered electoral defeat.

5.5 1984–90: From Price Controls to Inflation Targets

Immediately upon taking office, the Labour government tightened monetary policy. Short-term interest rates increased sharply and in 1985 took positive real values for the first time (excluding the freeze

period) since 1969. Finance Minister Douglas justified the interest-rate increases on the grounds that they would “assist in keeping both inflationary pressure and inflationary expectations down” (*CP*, August 3, 1984), a contrary judgment to that of defeated Prime Minister Muldoon, who deplored the changes on interest-cost-push grounds (*CP*, July 19, 1984). Given the lagged response of inflation to monetary policy actions, the monetary policy tightening begun in 1984 did not prevent higher inflation over 1985–87—a rise reflecting the earlier period of monetary ease, the removal of remaining price controls, and the impact effect of higher indirect taxation. Inflation did, however, fall into single digits at the end of 1987, and the four-quarter rate fell below 5 percent at the end of 1988. Also in 1988, the government appointed Don Brash as governor of the Reserve Bank of New Zealand. Brash’s view of the record of controls was negative: “Literally hundreds of governments, of all political persuasions and in all parts of the world, have tried to reduce inflation by direct controls. There is no case where controls alone succeeded” (*AST*, June 23, 1982).¹⁵ Over 1989–90 the authorities cemented the delegation of inflation control in New Zealand to monetary policy through their much-discussed launch of inflation targeting (see, e.g., Debelle 1996, 60–62; Bernanke et al. 1999, chap. 5).

6. Conclusion

The analysis in this paper indicates that the course of macroeconomic policy during the Great Inflation in Canada, Australia, and New Zealand shared four common features.

First, policymakers (at least from 1971) viewed inflation as resulting from factors beyond their control, not as a consequence of their monetary policy decisions. This rules out explanations of the Great Inflation that characterize the monetary authority as consciously choosing the inflation rate to achieve certain outcomes for real variables, or as part of an optimization exercise. Such explanations include those used in the time-consistency literature and also accounts that rely on a government belief in a permanent trade-off between unemployment and inflation. The record of policymakers’

¹⁵Similarly, Brash judged that other nonmonetary devices such as wage/tax trade-offs had “a negligible impact on inflation” (*AST*, June 17, 1982).

views is, however, consistent with the monetary policy neglect hypothesis.

Second, nonmonetary actions against inflation were accompanied by monetary expansion, not restraint. Occasionally policymakers verbally acknowledged the need for monetary restriction to complement incomes policies, but in practice they did not follow such a combination. In particular, while recognizing that real interest rates had become abnormally low, they felt that positive real rates could be restored by direct actions against the expected-inflation component of nominal rates. On other occasions, they were quite explicit that they intended to stimulate or maintain aggregate demand while delegating inflation control to other devices.

Third, output-gap mismeasurement reinforced the tendency for monetary policy neglect. The belief that inflation behavior was proceeding in spite of negative output gaps motivated the shift to nonmonetary theories of inflation; and once nonmonetary devices against inflation had been deployed, the desire to eliminate output gaps justified monetary expansion.

Fourth, the monetary policy regime change that produced disinflation did not always coincide with a rejection of the nonmonetary view of the inflation process. In New Zealand, the adoption of firmer monetary policies did coincide with the election of a government that took a monetary view of inflation. In Australia and Canada, the monetary changes were triggered either by cost-push considerations (i.e., fear of exchange rate depreciation in Canada) or factors besides inflation (i.e., balance-of-payments considerations in Australia). The disinflations that followed, however, converted policymakers in these countries to a monetary view of inflation, an important step toward their adoption of an inflation-targeting framework.

Appendix 1. Abbreviations for Periodicals Cited in Text

Canada:

CPD—*Canadian Parliamentary Debates* (House of Commons)

FP—*Financial Post*

FTC—*Financial Times of Canada*

IFS—*International Financial Statistics* (Washington, DC)

TGM—*The Globe and Mail* (Toronto)

TSN—*Toronto Sun*
TST—*Toronto Star*
WFP—*Winnipeg Free Press*

Australia:

ACR—*Australian Cabinet Records* (National Archives, Canberra)
AFR—*Australian Financial Review*
APD—*Australian Parliamentary Debates* (House of Representatives)
DM—*Daily Mirror* (Sydney)
DT—*Daily Telegraph* (Sydney)
MC—*Monday Conference* (television program; transcripts)
MEH—*Melbourne Herald*
MES—*Melbourne Sun*
SA—*Sunday Australian*
SMH—*Sydney Morning Herald*
TA—*The Australian*
TE—*The Economist* (London, United Kingdom)

New Zealand:

AST—*Auckland Star*
CP—*Christchurch Press*
EP—*Evening Post* (Wellington)
NZH—*New Zealand Herald* (Auckland)
NZPD—*New Zealand Parliamentary Debates*
RBNZ—*Reserve Bank of New Zealand Annual Report*

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Committees Versus Individuals: An Experimental Analysis of Monetary Policy Decision Making*

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We report the results of an experimental analysis of monetary policy decision making under uncertainty. A large sample of economics students played a simple monetary policy game, both as individuals and in committees of five players. Our findings—that groups make better decisions than individuals—accord with previous work by Blinder and Morgan. We also attempt to establish why this is so. Some of the improvement is related to the ability of committees to strip out the effect of bad play, but there is a significant additional improvement, which we associate with players learning from each other's interest rate decisions.

JEL Codes: C91, C92, E5

On May 6, 1997, the Monetary Policy Committee (MPC) of the Bank of England was established and granted operational independence in setting short-term interest rates to achieve the government's inflation target of 2.5 percent. This new framework replaced the previous system of a single individual—the Chancellor of the Exchequer—deciding on the appropriate level of U.K. base rates.

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Why delegate monetary policy to a committee? A wide-ranging survey undertaken by Fry et al. (1999) found that seventy-nine central banks out of a sample of eighty-eight use some form of committee structure when setting monetary policy. So by weight of numbers, it appears that setting interest rates by committee is thought to be superior. And the intuitive argument that committees make better decisions than individuals—because they allow decision makers to pool judgment—also seems plausible.

With the exception of Gerlach-Kristen (2001) and Sibert (2003), the theoretical economics literature does not have much to say about the consequences of delegating responsibility to a committee. The hypothesis that groups make better monetary policy decisions is difficult to test, due to a lack of comparable empirical data. This problem motivated Blinder and Morgan (2000) to adopt a different approach: carrying out a “laboratory experiment” on a large sample of Princeton University students to test whether groups do indeed make monetary policy decisions differently.

In an experiment, the researcher can isolate the relative performance of individual and group behavior, controlling for differences in the abilities, incentives, and preferences of the decision makers, and of the environment in which they work. The main drawback is that it is artificial—it is not possible to replicate exactly the complexities of real world policymaking in the context of a simple experiment.

Psychologists have studied group behavior for many years. A series of experiments conducted by Stoner (1961) and Myers (1982) suggested that a group should not be expected to replicate the average of the individuals who compose it. Groups often polarize the initial tendency because participants are more likely to generate arguments in favor of the position initially favored by the majority of group members. And because of an innate desire to compare themselves favorably with their peers, individuals may often take increasingly extreme positions in favor of the group proposition, leading to what Stoner called a “risky shift.”

Janis’ (1972) study of U.S. foreign policymaking also found that where groups were highly cohesive and dominated by a strong leader, it was possible to make decisions that the majority did not agree with, especially when operating under time pressure. The best way to avoid this was by a frank and open exchange of views at the beginning of the meeting. Hall’s (1971) experiments also supported

this hypothesis, finding that for complex tasks, the best performing groups were often those who were least consensual in the early stages of discussion.

But overall, the evidence suggests that, provided that groups do not reach decisions too easily or too quickly, their performance should be at least as good as the average of their individual members. This hypothesis was supported by the Blinder and Morgan (2000) experiment, where committees made substantially better decisions than individuals, although, contrary to expectations, groups were not more inertial.

This paper describes a new experiment using students from the London School of Economics (LSE). Like Blinder and Morgan (2000), we find that committees make better decisions than individuals, but we try to explore in more detail why this is the case. Our experiment suggests that committees do more than just eliminate the poor decisions of a minority of members, also allowing members to learn by observing the behavior of others. The experiment also explicitly tested whether the ability to exchange information through discussion improved performance; perhaps surprisingly, it did not.

Another question is, why do committee members' votes differ? One possibility could be that participants came to the experiment with different prior beliefs about the nature of the (unknown) model of the economy. So we designed a questionnaire to help establish these "priors," and asked participants to complete the same questionnaire again at the end of the experiment, in order to see how much they had learned about the model they were using.

The rest of the paper is organized as follows. Section 1 describes the economic model used, the structure of the experiment, how the priors questionnaire was used, the information flows and incentives facing players, and the data set. Section 2 discusses the results on learning over time, about both the model and how to play the game. Section 3 describes the scores in the individual and committee games and tries to explore why committee play was superior—including a discussion of why the ability to discuss did not improve committee performance. Section 4 looks at the relationship between activism and committee performance. Section 5 concludes.

1. The Experiment

1.1 The Model

We asked participants to act as monetary policymakers by attempting to “control” a simple macroeconomic model subject to shocks. We used a standard small-scale macro model of the type widely used for policy analysis in modern macroeconomics (see, e.g., Fuhrer and Moore 1995) and similar to that in Blinder and Morgan (2000). Where possible, it is calibrated to match U.K. macroeconomic data (see Bank of England 2000) and is shown in equations (1) and (2) below:

$$y_t - y^* = 0.8(y_{t-1} - y^*) - 0.5(R_t - \pi_t - r^*) + \bar{g} + \eta_t \quad (1)$$

$$\pi_t = 0.7\pi_{t-1} + 0.3\pi_{t-2} + 0.2(y_t - y^*) + \nu_t \quad (2)$$

where y_t is log output; y^* is the natural level of output, arbitrarily calibrated to 5; π_t is inflation; R_t is the nominal interest rate; and r^* is the neutral *real* interest rate (calibrated to 3 percent per annum). In equation (2) the coefficients on lagged inflation sum to one, implying that although a short-run trade-off between output and inflation exists, the Phillips curve is vertical in the long run.

Equation (1) is subject to two types of shock. The first, $\bar{g}$, which can be thought of as a permanent change in the equilibrium real interest rate, takes the value $+/-0.5$ and occurs once, and with equal likelihood, in any one of the first five periods in each round. This type of shock is attractive because it does not affect the inflation-output trade-off and therefore the ability of the score function outlined in equation (4) below to capture participants’ performance adequately. Both the second shock in equation (1)— η_t —and ν_t in equation (2) are white noise, corresponding to a random draw from a normal distribution $\sim N(0, 0.01)$ in each period.

The monetary authority’s decision rule for the short-term interest rate—as decided by the participants of the experiment—closes the model.

1.2 Priors

An intriguing feature of Blinder and Morgan’s (2000) results was that committee members frequently disagreed about their decisions,

despite having identical loss functions and the same information set. But even without observing such differences in voting—whether experimentally or in real life—it seems entirely plausible that committee members can think differently about how to respond to shocks that are only indirectly observed via the response of the endogenous variables. This may be especially true of a committee where members have diverse backgrounds and specialties.

We posit that the differences of opinion observed in the Blinder and Morgan experiment reflected different subjective judgments about the structure of the (unknown) model. So at the beginning of our experiment, we asked the players to fill in a questionnaire that attempted to reveal their priors (see appendix 2 for details). A set of “correct” answers to this priors questionnaire would yield both the parameters of the underlying model and the structure of the optimal rule.

During the experiment, players should learn about the structure of the economy—just like real world policymakers—by observing the response of inflation and output to changes in interest rates, updating their priors, and changing their perception of the “correct” model accordingly. We attempted to capture the extent of this learning by asking participants to fill in the same questionnaire again at the end, in light of what they had learned during the experiment.

One way to calibrate the extent of learning is to compare both the “before” and “after” questionnaire responses with the parameters of the underlying model and the associated optimal rule. However, in our experiment, the optimal rule for the real interest rate, r_t , does not correspond to a continuous function. So instead, we calculated an approximation to the optimal rule, under full information and the assumption that the scoring function (see equation [4] below) is a linear quadratic.¹ This benchmark rule, derived in appendix 1, is given by

$$r_t = 1.6y_{t-1} + 0.27\pi_{t-1} + 0.115\pi_{t-2} + 2\bar{g}. \quad (3)$$

¹Svensson and Woodford (2000) demonstrate that, in a backward-looking model, under the assumption of a quadratic loss function, optimal policy under partial information is the same as under full information because of the principal of certainty equivalence. However, equation (3) should only be thought of as an indicative benchmark for optimal policy setting.

1.3 *Information Flows and Incentives for Players*

Players received a clear mandate at the beginning of the experiment:² their objective was to maximize a “score” function that penalized deviations of output and inflation from their target values of 5 percent and 2.5 percent, respectively:

$$Score(t) = 100 - 40|Output(t) - 5| - 40|Inflation(t) - 2.5| \quad (4)$$

As in Blinder and Morgan (2000), we chose a linear rather than quadratic loss function so that players could easily translate their (average) score into a final payoff. And at the end of the game, the participants were paid in pounds according to the following (known) formula:

$$Payoff = 10 + \text{Average Score}/10 \quad (5)$$

where the maximum payoff was £20 for a perfect score and was bounded from below at £10. In practice, most students earned around £15–£16. We also offered top prizes of £100 for the best individual score and £100 to be shared equally among the best committee.³

Just like actual policymakers, participants did not know with certainty the exact structure of the economy, but they were told that the representative model was linear, learnable, and broadly characteristic of the U.K. economy. There was also uncertainty about the nature of the shocks hitting the economy. Players were informed that “... a structural change occurs at some point during each game. The key to playing successfully is to identify when the change has occurred and how best to respond to it.”

They were also told that the economy was subject to other shocks in each period of play. In Blinder and Morgan (2000), subjects were told the probability laws governing the occurrence of the structural shock, but we believe that our specification makes game play more typical of real-world policymaking, where central bankers are unlikely to face shocks with a known distribution or size. The relative

²A copy of the oral and written instructions can be found in the Bank of England working paper version at www.bankofengland.co.uk.

³These bonus payments were instigated in order to try to ensure that players had an incentive not to exchange information with future participants outside the laboratory.

sizes of the three shocks were calibrated after testing the model on subjects within the Bank of England.

Some manipulation of equation (3) shows that a positive $\bar{g}$ shock corresponds to a 1 percent increase in the neutral real interest rate to 4 percent, and vice versa for a negative shock. So, for example, if players do not react to an upward shift in r^* , they risk accelerating inflation, and the model can quickly become unstable because of the unit root in inflation built into the Phillips curve. Players must therefore extract the signal from the noise and change their behavior accordingly in order to maximize their score.

1.4 Outline of the Experiment

To analyze the effect of individual versus committee decision making, we structured the experiment so that participants played the game under a number of different decision-making structures. The sequencing of the experiment is shown in table 1.

Table 1. The Structure of the Monetary Policy Experiment

Read instructions sheet		
Complete priors questionnaire		
Practice rounds	No score recorded	
Stage 1 (rounds 1–4)	Played as individuals	
Stage 2 (rounds 5–8)	Played as a group	(i): No discussion (ii): With discussion
Stage 3 (rounds 9–12)	Played as a group	(i): With discussion (ii): No discussion
Stage 4 (rounds 13–16)	Played as individuals	
Complete priors questionnaire again, using knowledge gained from participating in experiment		
Students are paid according to their average score across the four stages		

After entering the laboratory, participants were allocated randomly into groups of five; given a standard, short, oral briefing; and asked to read a set of instructions. Each player then filled in the priors questionnaire and, following that, was given about ten minutes to practice on their own with the actual version of the game used in the experiment before starting to play “for real.”

The experiment itself comprised four stages. Each stage consisted of four rounds, with each round containing ten periods of play in which participants had to decide on what interest rate to set. Players were scored according to equation (4), with the overall score for each round being the average across the ten periods.

In the first stage, the participants acted as individual policymakers, playing separate games on separate computers for four rounds. Each experiment began at round 1, period $t = 1$, with inflation and output near the steady-state equilibrium ($y = 5$, $\pi = 2.5$).⁴ In each period, inflation and output were observed with a one-period lag, and participants would then decide on the appropriate level of interest rates and enter this into the computer. The game then proceeded to the next period ($t = 2$). The computer displayed output and inflation outturns for period 1, the score for that period, the cumulative score so far, and the previous interest-rate decision(s). The same decision problem was repeated until the game reached $t = 10$. At this point, players were told their average score for round 1, the game was reset, and play continued, being repeated for another three rounds.

In stage 2—beginning in round 5—the group acted as a committee, with each member entering his or her own vote on their computer as before. But this time, in each period, the computer calculated and then implemented the group’s median vote—as a proxy for a majority-voting rule⁵—and participants observed this committee decision, as well as the response of output and inflation to it. They also saw the (unattributed) votes of their fellow committee members and the committee score for the period and the round so

⁴The first observation at time $t = 0$ would always be the steady state perturbed by a random shock to each equation of the model.

⁵Not all monetary policy committees make interest rate decisions in this way. But in practical terms, one appealing feature of our rule is its ease of implementation during the “no discussion” committee stages, when an alternative rule, such as unanimity, would be difficult to implement.

far. Again, each round lasted for ten periods, and stage 2 finished at the end of round 8.

The committee phase was played over two stages—stage 2 and 3 as shown in table 1 above—each of which corresponded to a distinct scenario. The order of these two stages was randomized across committees in order to control for learning. Under scenario (i), discussion among members of the committee was not allowed in stage 2. The five players observed the same information in each period—the level of output and inflation of the previous period(s) as well as the history of interest rate decisions and scores—and entered their votes while sitting at separate computers, without talking to their fellow players. Participants were then allowed to discuss their decisions in stage 3, and again, the computer would set the median interest rate of the group.⁶ This discussion was not constrained, and in practice could take many forms. Under scenario (ii), stages 2 and 3 were reversed.

Stage 4 (rounds 13–16) served as another control, to ensure that the comparison between individual and committee play was not biased by the fact that participants had had four (or more) individual rounds to learn before entering the committee stage. By returning to individual play at the end of the experiment, it was possible to verify that not all of the improvement in scores during the committee stages (rounds 5–12) was an extension of the learning trend observed in rounds 1–4.

When the experiment was over, participants were asked to fill in the questionnaire again in light of what they had learned about the economy from playing the game.⁷ Each experiment lasted between 90 and 150 minutes.

1.5 *The Data*

The experiment was conducted on ten evenings between November 12 and December 11, 2001. Participation in the experiment was voluntary. The sample was independent of the authors: participants were students at the London School of Economics. A total of 170

⁶Participants were again asked to sit at their own computers to enter their votes: during testing we observed that if the committee gathered around one computer, this created a bias towards the decision of a chairperson who entered the votes.

⁷The wording at the beginning of the priors questionnaire was changed to reflect this focus.

students participated in thirty-four independent experiments,⁸ giving a cross-section of thirty-four committees with sixteen time series observations for each. All participating students had taken at least one undergraduate-level economics course.

2. Learning Results

The main focus of the experiment was to provide evidence on the differences between group and individual policymaking; this is discussed in section 3 below. But because the nature of the experiment is one of decision making under uncertainty, we begin by discussing the results on learning about both the model and how to play the game.

2.1 *Learning About the Model*

Players' answers to the priors questionnaire give some insight into their initial beliefs about the structure of the economy. Participants were also asked to fill in the questionnaire again at the end in light of what they had learned during the experiment, and from this we can judge whether their beliefs had converged on the actual parameters of the underlying model. Summary statistics for the questionnaire responses are shown in table 2.

For all questions apart from question 1 (Q1), the dispersion of responses to the final questionnaire is lower, although this decline is not significant. For each question, more than half of the participants improved in the final questionnaire.

The mean square error (MSE) of responses for each individual question was also calculated with reference to the underlying model and benchmark rule. We can use paired sample tests to examine the changes in players' responses after participating in the experiment. Across all questions, the MSE of responses fell from 0.17 in the initial questionnaire to 0.15 at the end of the experiment, which is significant at the 1 percent level ($t = 3.4$). We can also examine whether this learning is concentrated on certain aspects of the model.

The biggest improvements were in answers to Q2 and Q3. This shows that participants learned most about the lags in the

⁸No student was allowed to play the game more than once.

Table 2. Summary Statistics for the Distribution of Answers to the Priors Questions

Question Number	Correct Answer	Mean Response Before	Mean Response After	MSE of Responses Before	MSE of Responses After	Percentage of Players Who Improve	t-Statistic (Difference in MSE)	Wilcoxon Signed Rank Test
1	0	0.626	0.591	0.426	0.403	59%	0.74	<u>-0.92</u>
2	0.1	0.427	0.298	0.140	0.057	87%	6.74*	<u>-7.16*</u>
3	0.8	0.438	0.569	0.171	0.094	81%	5.38*	<u>-5.59*</u>
4	0.8	0.555	0.550	0.105	0.109	62%	-0.37	-0.22
5	0.5	0.499	0.649	0.037	0.058	53%	-4.43*	<u>-4.13*</u>
6	0.7	0.582	0.586	0.055	0.053	68%	0.33	<u>-0.68</u>
7	0.2	0.485	0.502	0.126	0.129	63%	-0.29	<u>-0.19</u>
8	0	0.490	0.531	0.310	0.320	68%	-0.51	-0.13
Over All Questions	n/a	n/a	n/a	0.170	0.150	n/a	3.4*	<u>-3.63</u>

Note: Starred values indicate statistical significance for a two-tailed test at the 1 percent level. Underlined entries indicate where the Wilcoxon signed rank test is supportive of the hypothesis that the MSE of responses to the questionnaire has fallen during the experiment. Starred values indicate a significant difference (in either direction).

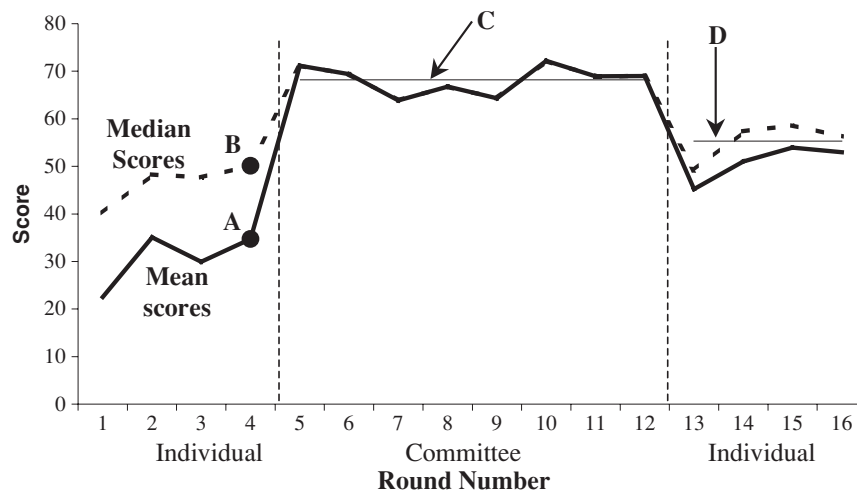
transmission mechanism of monetary policy (Q2) and the weight they should attach to deviations of output from trend in their rule (Q3). The responses to Q5, which considers the responsiveness of output to interest rate changes, are interesting: this is the only question where responses got significantly worse. On average, players started the experiment with surprisingly accurate beliefs about this parameter, but after playing the game they believed that output was more sensitive to interest rate changes than it actually is. There are several reasons why this might be the case: first, the short-run policy multiplier is larger for output than inflation, and second, output responds much more to the structural shock in the short run than inflation.

For the other five questions, beliefs changed by much less, and t-tests show that the changes in means are not significant. Participants did not learn much, if anything, about the appropriate degree of interest rate smoothing and the parameters of the model (Q1 and Q4–Q8, respectively). But this may not be too surprising: although participants played the game a number of times, each individual round lasted only ten periods and may have been too short for players to deduce much about the long-run properties of the model. We cross-checked these results with Wilcoxon signed rank tests, which support the conclusions of the t-tests and are also reported in table 2.

Which players learned most? There is a positive and significant (at the 1 percent level, $t = 10.7$) correlation between players whose MSE was largest at the beginning and those who learned most about the parameters of the model. There is also a positive and significant (at the 1 percent level, $t = 3.66$) correlation between the fall in the MSE of responses to the priors questionnaire and performance in rounds 13–16. So those who learned most about the model during the game—whether through their own experience or by playing with others—did better in the final set of individual rounds.

2.2 Learning About Playing the Game

Did players actually get better at playing the game over time? The solid line in figure 1 shows the mean score attained by the 170 individuals in each of rounds 1–4 and 13–16, and the mean score achieved

Figure 1. Scores for Players Over Time

by the thirty-four committees during rounds 5–12.⁹ There are three striking features:

1. the significant upward trend in the results over time
2. the large rise in scores when players moved to committee decision making
3. the large downward move in scores when participants returned to playing as individuals

Individuals' scores were higher in round 16 than in round 1: the mean score rose from 23 to 53. This increase is significant at the 1 percent level ($t = 5.12$), providing evidence of a significant learning effect during the game.

Within the individual rounds, there was also some evidence of learning. The average individual score was twelve points higher in round 4 than in round 1 and was eight points higher in round 16 than in round 13. Both differences in mean tests are significant at the 1 percent level for a one-tailed test ($t = 2.38$ and

⁹It is not possible to calculate an individual score for each person in the committee rounds, as the evolution of the round and therefore the score are determined by the decision of the committee as a whole.

$t = 2.61$). These results suggest that it is possible to get better at playing the game, even when not exchanging information with others.¹⁰

If we rank the players in each committee by their initial performance, we find—perhaps unsurprisingly—that the worst players improved most. Although even the best players in each committee improved somewhat, the improvement in scores was only significant for the worst two players in each committee. This is consistent with the view that some players begin the game with a totally incorrect model in their head, and so their decisions attract a very low score initially relative to others with more accurate priors. As they learn that their priors do not accord with the truth—through playing the game and observing their scores—they update their beliefs and their performance improves accordingly.

These findings contrast with those of Blinder and Morgan (2000), where there was less evidence of learning. One reason for this might be that our model is slightly simpler. For example, participants have to learn fewer parameters in our game: five as compared with seven in the Blinder and Morgan experiment.

3. Groups Versus Individuals

We found strong evidence that decisions made by committees were superior to those made by individuals. The average score during the committee rounds was nearly two-thirds better (68 compared with 41), and significantly higher ($t = 7.4$) than the average for the individual rounds.

We can also use our benchmark rule to calibrate the size of this improvement. The average score from simulating the game under this rule was 85, much higher than the *best* individual player's score (71), but only slightly better than the *best* committee (83). On average, moving from individual decision making to a committee structure closed nearly two-thirds of this "policy gap."

¹⁰Because we use a "within subject" rather than a "between subject" experimental design, it is difficult to draw firm conclusions about how much learning in rounds 13–16 is related to earlier group feedback. However, the upward trend over the final four individual rounds is consistent with at least *some* learning being related to individual play.

3.1 Explaining the Improvement

How do we explain the improvement in performance of committees over individuals? There are (at least) two distinct, competing hypotheses that can be used to explain why committee decisions are superior:

Hypothesis 1: A committee with majority voting can neutralize the impact of some members playing badly in any given game.

Hypothesis 2: Committees allow members to improve performance by sharing information and learning from each other.

We can use figure 1 to give a visual representation of the contribution of these two hypotheses. We first took the scores of the five players in each committee during the individual rounds and calculated that of the median player (the third best performer in each committee across all the individual rounds). So the dashed line in figure 1 represents the mean of the median players' scores in each of rounds 1–4 and 13–16. The solid line is the mean score across all 170 players in the individual rounds, and the mean score across the thirty-four committees during each of rounds 5–12. Line C is the mean score across rounds 5–12 taken together, and line D is the average of the median players' scores over rounds 13–16 taken together.

The overall improvement in performance—generated by setting interest rates by committee—is taken to be $C - A$: the difference between the mean individual score in round 4 and the mean committee score across rounds 5–12.

Figure 1 decomposes this improvement into two distinct components. One component is the difference between the score of the mean and median player in the individual rounds (represented by the distance $B - A$ in figure 1), which should be equal to the adverse effect of a minority of poor performers on the mean individual score. This is therefore the extent of improvement under hypothesis 1 described above. This portion of the difference in means is significant at the 1 percent level ($t = 3.7$), so we cannot reject hypothesis 1. The contribution of hypothesis 2 should be represented by the second component—the residual, or $C - B$

(the portion of the committee improvement not explained by the move to majority voting). This difference is also significant at the 1 percent level ($t = 2.8$), so we cannot reject hypothesis 2 either.

The mean committee score (68) was also higher ($t = 1.51$, significant at the 10 percent level) than that of the best individual (65—calculated as the person with the highest average score across all the individual rounds) in each committee when playing alone. This provides further evidence that committees do more than just replicate the behavior of their best individual.

The significant decline in scores as participants move back to individual play is a striking feature of both our results and those of Blinder and Morgan (2000). By definition, this “residual” component of the committee improvement cannot be associated with learning about the game, because the information set of the players must be at least as great in round 13 as it was before. We argue therefore that at least some of this residual effect stems from the ability of committees to pool judgment, expertise, and skill. The distance C – D in figure 1 represents the difference between the average committee score and that of the median individual player in each committee. At 12.9 points, it is also significant at the 1 percent level ($t = 4.2$). This evidence also supports the view that there is something special about committees, beyond their ability to strip out the effects of bad players.

In order to try to abstract from learning effects in the initial rounds of the game, we can also compare committee scores with those achieved by the first, second, and third best players—as judged by their performance in rounds 13–16 of the game. Though committees do not outperform their best player (as judged by this criterion), scores are significantly higher for all other members (for example, for the second best player, $t = 2.91$, significant at the 1 percent level) when they set monetary policy together. This is a striking result because it suggests that even after players have had repeated opportunities to learn about the game, and from each other, most still perform better when they make collective decisions. Furthermore, given that it may be very difficult in practice to judge ex-ante who would be the best decision maker, this result shows that on balance it is better to make decisions by committee.

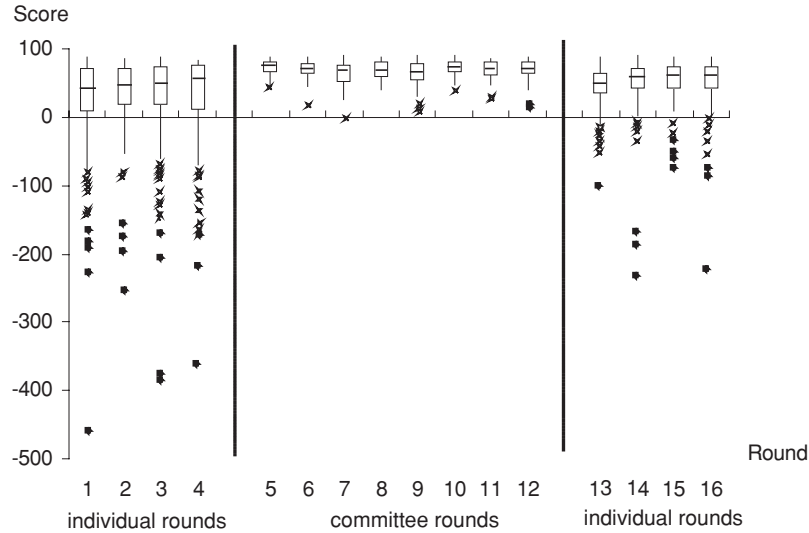
Figure 2. Distribution of Scores Across Players

Figure 2 shows a Tukey box-plot¹¹ of both the scores across the 170 individual players in rounds 1–4 and 13–16 and across the thirty-four committees in rounds 5–8. For the box-plots, the height of the box represents the interquartile range for each round, with the position of the median score shown by the horizontal line inside the box. The lines running vertically from the bottom and top edges of the boxes are “whiskers,” which extend to the furthest observation within one and a half interquartile ranges of the bottom and top of the box. Stars mark out “minor outliers” (outside the range of the whiskers), and dots mark “major outliers,” which lie greater than three interquartile ranges from the bottom of the box.

Several features are immediately striking. First, the range of scores is much wider for the individual than for the committee rounds, both in terms of the interquartile range of the distribution and the number of outliers. Individual scores are more often negative, and some particularly poor players achieve very low (negative) scores. For the committee games, only one outlier achieves a negative score (of -1.8 in round 7). Second, the individual scores

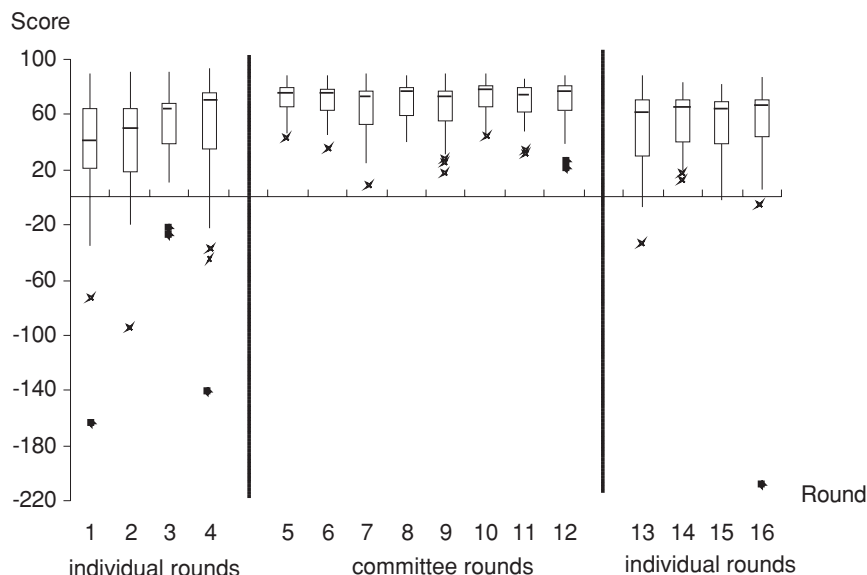
¹¹See Tukey (1977).

show a much larger negative skew: the median is much closer to the upper than lower quartile and there are long tails of negative scores.

Comparing the distribution of scores between the first and second individual stages may also give an insight into what players learn from being in a committee. The whole distribution is a lot narrower in stage 4, and although there is only a moderate shift up in the second and third quartiles, the first quartile is much higher, the very long tails in rounds 1–4 are absent, and the number of outliers falls somewhat. This is consistent with the hypothesis suggested above that the worst players learn a lot from being part of a committee.

We can also examine a breakdown of the performance of an “artificial committee” over time, using the median player in each individual round as a proxy for the committee (see figure 3). Again we see a much narrower distribution during the committee rounds—demonstrated by the smaller boxes (the interquartile range in the committee rounds is half the size on average) and shorter whiskers. And although the performance improvements are greatest for the

Figure 3. Distribution of Score for “Artificial Committee”



lower quartile, it is definitely possible to discern an upward shift in the positions of the boxes in the figure during rounds 5–12. The first, second, and third quartiles increase by twenty-eight, sixteen, and ten points, respectively.

3.2 *Does Discussion Help?*

The experiment also included two different ways of organizing committee decision making: one where participants were allowed to discuss their views and another where no verbal communication was allowed. Although the mean score of committees who were allowed to discuss their decisions (67) was significantly higher ($t = 7.9$, significant at the 1 percent level) than the mean individual score in rounds 13–16 of the experiment (51), perhaps the most surprising result was that the ability to discuss did not significantly improve committee performance.¹² The mean score of committees who did not discuss their decisions was 69, which was actually slightly (but not significantly, $t = 1.39$) higher than those committees who were able to discuss their decisions.

So our committees must have pooled information in other ways. The benefits of different forms of communication are likely to depend on the nature of the game, as well as the individuals taking part. There are many games—for example, snooker or chess—that may be easier to learn by watching, rather than discussion.

Our results are also supportive of those from the social psychology literature discussed above, which suggest that discussion may not always enhance group performance. And although it appears that for this experiment, and for this set of students, discussion did not provide more information than could be acquired by observing others' votes, we cannot tell for sure how much of this result is being driven by the particular specification of our experiment. Real-world policymaking is undoubtedly a much more complex affair, and the benefits of discussing decisions with fellow committee members are likely to be high.

¹²This contrasted with earlier trials on Bank of England staff. Observation of the LSE experiment in progress suggested that the quality of the discussion varied across committees—some would just talk about what interest rate to set, whereas others talked about what the structure of the model might be.

4. Policy Activism and Performance

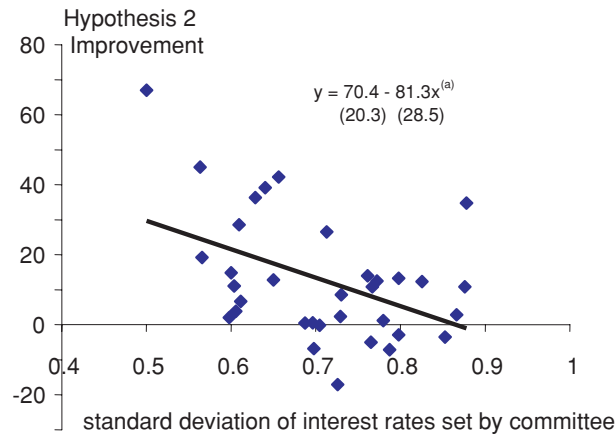
When the model described by equations (1) and (2) was simulated under the benchmark rule (3), interest rate movements were, on average, significantly less activist than those of the individuals ($t = 15.0$, significant at the 1 percent level), but not the committees ($t = -7.08$, significant at the 1 percent level), who played the game. If we calculate interest rate activism as the standard deviation of the interest rate during each ten-period round, 92 percent of individuals were more active than the benchmark rule, whereas only 9 percent of committees were. One interpretation might be that committees allow players to learn the appropriate amount of activism more quickly by pooling information.¹³ In other words, they can work out that aggressive movements in interest rates make the economy more difficult to control, and players understand this property of the model more quickly by either exchanging information or watching the votes of others.

Some supporting evidence is provided by figure 4, which shows a significant, negative relationship between hypothesis 2 improvement—that is, the portion of the committee improvement not explained by the move to majority voting—and policy activism (as measured by the standard deviation of committee interest rate decisions over time). However, figure 4 tells us nothing about causation.

A further test would be whether individuals are less active in rounds 13–16 than rounds 1–4. Although the standard deviation of rate changes falls slightly, the change is not significant, so another interpretation might be that committees naturally make more gradual decisions. This is not formally tested in our experiment, but Blinder and Morgan (2000) found that, on average, committees did not take longer to change their views than individuals. This was in direct contrast to what Alan Blinder expected in light of his experience sitting on the U.S. Federal Open Market Committee.

¹³Even if the causation were to run in the opposite direction—that is to say that bad players need to vary interest rates more because the economy is further away from target—it is still the case that good players can learn that this strategy is suboptimal over time.

Figure 4. Hypothesis 2 Improvement and Committee Activism



(a) standard errors in brackets

5. Conclusions and Suggestions for Further Work

In this paper, we have undertaken an experimental analysis of monetary policy decision making by individuals and committees. An experiment like this has obvious limitations: it can only be indicative of what might happen in the real world, where policymaking is a much more complex affair. Because of the need to design a relatively simple experiment, and the focus on testing the proposition that committees are better than individuals, our research has nothing to say on other aspects of optimal committee design such as committee size, composition, or how discussion should be structured.

Our experiment does suggest overwhelmingly that committees performed much better than the average of the individuals who composed them. There is also evidence to suggest that committees perform significantly better than all but their best member, and given that it might be difficult to discern ex-ante who is the best person, on balance it would seem advisable to make decisions by committee.

We argue that, while some of the improvement associated with group play reflects the averaging of errors across members, the ability of committees to allow the pooling of judgment and information (in

whatever form) also has a significant role to play in explaining why committees do better. Perhaps surprisingly, committees who were able to discuss their decisions did not perform better than those who could not. In our experiment, participants were able to glean the same amount of information about the game from observing each other's play as from discussion. These results certainly merit further investigation, perhaps using a more complicated or more lifelike setup than ours.

It is also possible to observe some evidence of learning within the experiment. The answers to the priors questionnaire suggest that participants learned a significant amount about certain aspects of the model during the game, although the experiment may not have been long enough for participants to learn much about the structural parameters. In addition, the distribution of scores across players narrowed during the course of the game—with the worst players learning most.

Appendix 1. Derivation of the Benchmark Rule

Assuming that players attempt to maximize their score (S_t) in each period of the game, the decision problem can be written as

$$\begin{aligned} \underset{r_t}{\text{Max}} \ E_{t-1}\{S_t\} \quad \text{s.t.} \quad (1) \ y_t &= 0.8y_{t-1} - 0.5r_t + \bar{g} + \eta_t \\ &\text{where } \eta_t \sim N(0, \sigma_\eta^2) \\ (2) \ \pi_t &= 0.7\pi_{t-1} + 0.3\pi_{t-2} + 0.2y_t + \nu_t \\ &\text{where } \nu_t \sim N(0, \sigma_\nu^2) \end{aligned}$$

$$\text{where} \quad (3) \ S_t = 100 - 40|y_t - y^*| - 40|\pi_t - \pi^*|$$

Equations (1) and (2) are written in deviation from equilibrium form, and r_t is the real interest rate.

Approximating (3) as a linear quadratic, we derive the benchmark rule by substituting in the constraints (1) and (2) and differentiating with respect to r_t , to give

$$r_t = 1.6y_{t-1} + 0.27\pi_{t-1} + 0.115\pi_{t-2} + 2\bar{g} \quad (4)$$

Obviously, the distribution of $\bar{g}$ is unknown to participants in the experiment, so (4) is the “certainty equivalence benchmark rule.” If our loss function were quadratic, the optimal policy rule under partial information would be the same as its full-information counterpart, according to Svensson and Woodford (2000).

We use this benchmark rule to conduct the simulations in section 3 and also to calibrate the responses to the “before” and “after” priors questionnaires.

Appendix 2. Priors Questionnaire

Players were asked to give numeric responses to the following questions. They could choose any value from $\alpha = 0$ to $\alpha = 1$, with intervals of 0.1, apart from question (2), where the options ranged from zero to ten periods. Although all players would have taken at least one university-level macroeconomics course, it is likely they may not have been familiar with the type of model used for the experiment, or the “correct” calibration of the parameter values in the questions below. Participants were told that if they did not understand the jargon in brackets, they should instead make a guess governed by how much they agreed or disagreed with the statement in the question.¹⁴

(1) To what extent should monetary policymakers respond cautiously to shocks (i.e., if their interest rate reaction function includes the expression $i_t = \alpha i_{t-1} + \dots$, what weight should they place on α)?

(2) After how many quarters is the maximum impact of monetary policy on inflation felt?

(3) What relative weight should monetary policymakers place on smoothing output compared with controlling inflation (i.e., if their reaction function includes the expression $i_t = \alpha(y_t - Y) + (1 - \alpha)(\pi_t - \pi^*) + \dots$, what weight should they place on α)?

¹⁴A complete copy of the questionnaire can be found in the Bank of England working paper version at www.bankofengland.co.uk.

(4) To what extent are shocks to output persistent (i.e., if the expression for output included the term $y_t = \alpha y_{t-1} + \dots$, what weight do you think α would take)?

(5) How sensitive is output to changes in interest rates (i.e., if the expression for output included the term $y_t = \alpha i_t + \dots$, what weight do you think α would take)?

(6) To what extent are shocks to inflation persistent (i.e., if the expression for inflation included the term $\pi_t = \alpha \pi_{t-1} + \dots$, what weight do you think α would take)?

(7) To what extent is inflation sensitive to deviations of output from trend in the short run (i.e., if the expression for inflation included the term $\pi_t = \alpha(y_{t-1} - Y) + \dots$, what weight do you think α would take)?

(8) To what extent is inflation sensitive to deviations of output from trend in the long run? Not at all sensitive (i.e., $\alpha = 0$) or highly sensitive (i.e., $\alpha = 1$)?

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Exchange Rate Volatility and the Credit Channel in Emerging Markets: A Vertical Perspective*

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Firms in emerging markets are exposed to severe financial frictions and credit constraints that are exacerbated by the sudden stop of capital inflows. Can monetary policy offset this external credit squeeze? We show that although this may be the case during moderate contractions (or in partial equilibrium), the expansionary effect of monetary policy vanishes during severe external crises. The exchange rate jumps to reduce the dollar value of domestic collateral until equilibrium in domestic financial markets is consistent with the external constraint. An expansionary monetary policy in this context raises the value of domestic collateral, but it exacerbates the exchange rate depreciation (beyond the standard interest parity effect) and has little effect on aggregate activity. However, there is a dynamic linkage between monetary policy and sudden stops. The anticipation of a dogged defense of the exchange rate worsens the consequences of sudden stops by distorting the private sector incentive to take precautions against these shocks. For similar general equilibrium reasons, dollarization

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of liabilities has limited impact during a sudden stop, but it has significant underinsurance consequences.

JEL Codes: E0, E4, E5, F0, F3, F4, G1

Emerging markets can suffer severe contractions when capital inflows suddenly reverse (“sudden stops”). What is the right monetary policy response to this event? Answering this important question requires understanding the mechanisms through which monetary policy can affect sudden stops and their consequences. The main contribution of this paper is to rethink the workings of the credit channel in emerging market economies and to propose an alternative view that shifts the focus from credit channel and aggregate demand to insurance considerations.

Much of the recent literature on emerging markets crises highlights the limited financial development of these economies and the severe credit squeeze experienced by local firms during crises. From this structure, two opposing arguments are commonly made regarding optimal monetary policy. Extrapolating from developed economy credit channel analysis, some advocate an expansionary monetary policy to offset the effect of the credit squeeze during downturns. Others advocate a contractionary monetary policy and dogged defense of the exchange rate during crises. Proponents of the latter view do not disagree on the centrality of the credit channel mechanism, but argue that a depreciating exchange rate will have dramatic effects in the credit channel mechanism, through a deterioration in borrowers’ balance sheets. The argument is that this effect is most likely to happen when the economy is dollarized and when inflation credibility is limited.¹

The starting point of our analysis is the observation from Caballero and Krishnamurthy (2001) that credit constraints in emerging markets may exist both at the country level as well as the firm level. A firm may have limited collateral and therefore be constrained in borrowing from either domestic or foreign lenders. Or the country as a whole may have limited international collateral, and therefore domestic firms are constrained in borrowing from foreign

¹For both sides of the argument see, e.g., Furman and Stiglitz (1998); Aghion, Bacchetta, and Banerjee (2000); Cespedes, Chang, and Velasco (2000); Gertler, Gilchrist, and Natalucci (2001); Christiano, Gust, and Roldos (2004); and Calvo and Reinhart (2003).

investors.² Extrapolation from developed economy analysis misses this distinction because in developed economies the credit constraint plausibly exists only at the firm level. The distinction is important for thinking through the effects of monetary policy because a sudden stop created by the reversal of capital inflows is predominantly about a binding international collateral constraint.

We argue that during mild crises, the distinction between domestic and international constraints is immaterial. Analysis of this case can be conducted in the standard credit channel framework. Much of the recent debate on monetary policy during crises implicitly assumes this to be the relevant scenario. As is stressed in the credit channel literature (e.g., Bernanke, Gertler, and Gilchrist 1999; Kiyotaki and Moore 1997), increasing the net worth of borrowers is the optimal response to a negative shock. Our analysis of this case reinforces the conclusions in the literature: there is some ambiguity regarding whether the optimal monetary policy is to raise or lower interest rates. It depends on the extent of liability dollarization and inflation credibility of the central bank.

The recipe of using monetary policy to increase the net worth of borrowers fails when the credit constraint exists at the country level. The reason is that monetary policy only regulates the borrowing of a domestic agent from a domestic lender. Thus, when the marginal lender is a foreign investor, monetary policy is ineffective because it does not alter the international collateral of the country.³ We argue that this is the relevant case for emerging markets' crises.

Although monetary expansions have little effect on output during a severe crisis, they have important general equilibrium effects on asset prices. In *partial* equilibrium, it is still true that a monetary expansion raises firms' borrowing capacity by increasing their domestic liquidity. But in general equilibrium, the expansion just translates

²The importance of international collateral constraints for emerging markets was first identified in the sovereign debt literature (see, e.g., Eaton and Gersovitz 1981 or Bulow and Rogoff 1989).

³Though in very different terms, Dornbusch (2001) also expresses his uneasiness about the assumption that emerging economies with access to monetary policy can use it to boost activity. For example, in criticizing the standard view, he writes: "The loss of the lender of last resort [argument] is intriguing. This argument is based on the assumption that the central bank—rather than the treasury or the world capital market—is the appropriate lender..."

into a decline in the relative price of domestic liquidity vis-à-vis international liquidity, not in an expansion in international liquidity (i.e., it does not loosen the international constraint). We show that the fall in the price of domestic liquidity implies a *temporary* exchange rate depreciation during the crisis. In fact, the depreciation of the exchange rate is beyond the depreciation due to the standard interest parity mechanism. The depreciation does weaken balance sheets, as some have argued in the context of the credit channel. But it is not firm-level balance sheets that are to blame. The exchange rate depreciation is just the general equilibrium counterpart of the tight international constraint.

Our conclusions regarding the effects of monetary policy are independent of the dollarization of balance sheets. Monetary policy is impotent during crises whether or not dollarized balance sheets undo the beneficial effects of lowering interest rates. The reason, again, is that domestic dollarization just tightens constraints at the firm level; it does not alter the aggregate international collateral of the country.⁴

The second main theme of the paper concerns the insurance effects of monetary policy. Despite the apparent impotence of monetary policy during sudden stops, expectations regarding a particular monetary rule have important effects on the private sector's ex-ante decisions. The anticipation of monetary policy actions during crises affects agents' expectations of the relative price of domestic to international liquidity, which in turn affects the private sector's insurance decisions against sudden stops. If the private sector expects a dogged defense of the exchange rate, then it also expects a smaller decline of the value of domestic to international liquidity. Conversely, if it expects an expansionary monetary policy, then it expects a sharper decline in the relative price of domestic liquidity. The incentive to hoard international over domestic liquidity is more pronounced in the latter than in the former case. Importantly, hoarding more international liquidity is desirable when there are negative pecuniary externalities in the use of international liquidity, a feature that is

⁴Dollarization of *external* liabilities has some additional external insurance implications that we discuss later in this paper and, more thoroughly, in Caballero and Krishnamurthy (2003). This is not an issue for the analysis of this paper since we allow for state-contingent debt.

inherent to economies with underdeveloped financial markets (see Caballero and Krishnamurthy 2001).

From this perspective, our argument connects with the literature on the virtues of flexible versus fixed exchange rate systems.⁵ In our model, a flexible exchange rate system coupled with a counter-cyclical monetary policy provides better *insurance* incentives to the private sector than a policy of defending the exchange rate during sudden stops.⁶ More generally, monetary policy and its constraints have (unintended) consequences for the private sector's insurance decisions with respect to sudden stops. Policies that exacerbate the (partial equilibrium) domestic credit squeeze during sudden stops, or constraints that limit the central bank's ability to relax domestic financial constraints, lead to ex-ante imprudent private sector actions.

In section 1 we present a model of market segmentation in which we draw a distinction between domestic and international liquidity. Section 2 describes the consequences of monetary policy in the segmented financial market. In particular, we show the oversensitivity of the exchange rate to monetary policy and the role of the exchange rate in endogenously aligning the extent of domestic credit squeeze with the limited international liquidity. In Section 3 we turn to the ex-ante consequences of monetary policy and its private insurance implications. We show that the strategy of tightening monetary policy during crises lowers the private return to taking preventive measures.

In section 4 we revisit the dollarization of liabilities argument, but now in the context of our dual-liquidity model. We show that the balance-sheet effect typically emphasized in this literature has no aggregate consequences in general equilibrium during the crisis. However, and following our argument above, there is a dynamic cost. Dollarization lowers the expected return on hoarding international liquidity and hence reduces the private sector incentives to take adequate precautions against sudden stops. In section 5 we highlight that not having the credibility or willingness to conduct a

⁵See, e.g., Edwards and Levy-Yeyati (2003) for evidence supporting the advantages of a flexible exchange rate in absorbing external shocks.

⁶Our mechanism is distinct (or in addition to) the standard free insurance argument in the moral hazard perspective of crises (see, e.g., Dooley 1999). We return to this comparison after developing our model.

countercyclical monetary policy means that an insurance mechanism against crises is lost. Thus, one should look for alternative measures that induce the private sector to carry more international liquidity into crisis states. Examples of these measures include taxation of capital inflows, international liquidity requirements, and large sterilizations of capital inflows. While enacting these measures may be costly, they should be seen as yet another cost of having lost the ability to use monetary policy in an environment of recurrent external crises.

Section 6 shows that our main conclusions are robust to the relaxation of some of the main stylized assumptions of our model, such as the presence of a “diagonal” supply of funds or an alternative model of money. Section 7 concludes and is followed by a technical appendix.

1. A Model of External Crises

We study a three-date economy ($t = 0, 1, 2$) with a single (tradeable) good. Date 0 is a fully flexible planning period. Agents make investment and financing decisions at this date. At date 1 an external crisis may occur. We assume that there are two states of the world at date 1, $\omega \in \{b, g\}$, which occur with probabilities $\{\pi, 1 - \pi\}$. The crisis can occur only if the b -state realizes. At date 2 agents repay all debts and consume.

1.1 Private Sector

There is a unit measure of domestic firms. Each has access to a production technology. Building a plant of size k at date 0 requires a firm to invest $c(k)$ —with $c(\cdot) \geq 0$, $c' > 0$ and $c'' > 0$. The output from production depends on the shocks at date 1. There is a maximum of Ak output goods at date 2, but if negative shocks hit firms, then only ak ($a < A$) output goods may be produced. A more detailed description is given below.

Domestic firms have no resources at date 0. They must import capital goods and borrow from foreigners to finance their investment. Each firm is run by a domestic entrepreneur/manager who has risk-neutral preferences over date 2 consumption of the single good. Thus,

financing and investment decisions are taken to maximize expected plant profits at date 2.

Domestic firms face significant financial constraints, and hence their financial net worth affects investment decisions. We assume that firms are endowed with w units of international collateral, in the form of receivables arriving at date 2. Our central assumption is that only these receivables have collateral value to foreigners (e.g., prime exports).

We disregard explicit equilibrium default and assume that all financing is done via fully collateralized debt contracts. Moreover, we assume that the debt contracts are made fully contingent on the state at date 1. We relax this in section 4 when discussing dollarized external debt, since the problem of dollar debt is essentially one of liabilities that are too inflexible.

At date 0, when firms sign debt contracts with foreigners, the contracted repayments of f^ω must not exceed w . Foreigners lend against this collateral at dates 0 and 1 at the rate i_0^* and i_1^* from period 0 to 1, and 1 to 2, respectively.

We also assume that domestic agents have some “domestic” assets that they can only trade to other domestic agents. First, we assume that the minimum date 2 output of ak can be pledged as collateral to other domestic agents. Second, we assume that agents are endowed with $\underline{M}$ units of domestic money (see below) that they can sell to other agents.

This modeling captures the distinction between domestic borrowing capacity and international borrowing capacity. We think of the domestic assets as “peso” assets, while the international collateral are “dollar” assets. Thus, changes in the exchange rate have an effect on the dollar value of domestic assets but not that of the international assets.

1.2 Credit Constraints During Crises

A crisis is a time when there is a shortage of resources available to finance investments. Formally, we model the external crisis by assuming that the demand for external resources rises in the b -state, while holding the international collateral fixed. An alternative (and more realistic) modeling of a shortage of external resources would be to assume that international collateral contracts during the crisis,

while demand for external resources remains fixed. The implications of these two modeling strategies are similar for our purposes, but our approach reduces the number of assumptions and equations required. In particular, we do not need to add an assumption of incomplete international insurance markets.⁷

In the g -state there are no shocks and investment is unaffected. Let us now turn to defining the shock in the b -state and explaining how it affects investments.

The plants of one-half of the firms receive a shock at date 1 that lowers output per plant from A to a . The shock only arrives in the b -state, but is idiosyncratic in that each firm receives the shock with probability $1/2$. The productivity decline can be offset by reinvesting θk ($\theta \leq 1$) goods, to give date 2 output of

$$\tilde{A}(\theta)k = (a + \theta\Delta)k \leq Ak \quad \text{where} \quad \Delta \equiv A - a,$$

so that $\tilde{A}(1) = A$. We assume that the return on reinvestment exceeds the international interest rate:

$$\Delta - 1 > i_1^*.$$

This means that firms will borrow as much as possible to finance reinvestment. A crisis occurs if firms are financially constrained at date 1, so that $\theta < 1$. We make parametric assumptions to ensure that this occurs in equilibrium only in the b -state (see the appendix).

A firm that receives a production shock is termed *distressed*. To cope with the shock, the firm first borrows directly from foreigners against its net international collateral of $w - f$. After this, it must turn for funds to the domestic firms that did not receive a shock (termed *intact*). As noted above, domestic agents accept the money held by distressed firms as payment. They also accept ak of the output from a firm's plants as collateral for any domestic debt. Thus the *domestic* (real) liquidity of firms entering date 1 (i.e., the date 2 goods that can be pledged to domestic agents) is

$$ak + \underline{M}/e_2 \tag{1}$$

⁷See Caballero and Krishnamurthy (2001) for the alternative approach. See Broner, Lorenzoni, and Schmukler (2003) for evidence that some of these international collateral shocks are due to shocks to specialist investors (in our framework, this is as if suddenly foreign lenders decide to reduce the share of w that is part of international collateral).

where e_2 is the date 2 nominal exchange rate (also the price level).

Intact firms have no output at date 1 either, so they must borrow from foreigners if they are to finance the distressed firms. At the interest rate of i_1^* , they can borrow up to $\frac{w-f}{1+i_1^*}$ from foreigners.

Since at date 1 firms, in aggregate, can borrow from foreigners up to

$$w^n \equiv \frac{w-f}{1+i_1^*}, \quad (2)$$

we refer to w^n as the *international* liquidity of the country during the crisis.

1.3 Central Bank

The central bank has $\underline{M}$ units of money outstanding at date 0. At date 1, it may choose to inject $(M - \underline{M})$ more money into firms. We assume that this money is distributed as “helicopter” drops to firms and redeemed at date 2 with taxes collected by the government.

We do not provide a detailed description of the government’s tax powers and its tax base. Instead we assume that the government collects T goods via nondistortionary taxation, and that these taxes do not come from firms. We think of these taxes as resources from a consumer sector that has a date 2 endowment of $y^c \geq T$. When it is effective, expansionary monetary policy transfers resources from consumers to the corporate sector.⁸

We assume that the central bank is credible in maintaining the date 2 price level at one, so that money injections do not lead to inflation. We relax this assumption in section 5 when we discuss inflation credibility. Thus, for now, the date 2 exchange rate is $e_2 = 1$.

Our model of the monetary transmission mechanism involves no monetary frictions, because our qualitative conclusions do not depend on the presence of these frictions and ignoring them simplifies the exposition. The domestic “peso” interest rate is zero in our environment and is unaffected by monetary policy. Therefore the usual channel (through the interest parity condition) by which monetary policy affects the exchange rate is absent in our model. Despite this, we show that in our model, monetary policy still affects the exchange

⁸See, e.g., Woodford (1990) and Holmstrom and Tirole (1998) for a similar assumption.

rate. For this reason, we refer to the exchange rate movements induced by monetary policy as “overresponsive.” Our results should be interpreted as *in addition* to those that arise from the movement in the peso rate and its effect through the interest parity condition. In section 6, we sketch a more standard model in which money provides a transaction service, thus confirming that our results are robust to this simplification.⁹

2. Monetary Policy, Exchange Rate Determination, and the Credit Channel

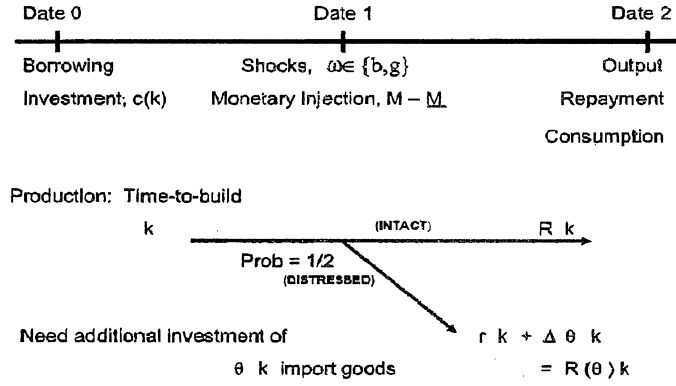
In this section we examine the equilibrium in the crisis state at date 1. Thus, for now, we take the date 0 investment decision of k and the financing decision of f as given. We study the case where credit constraints are binding in this crisis state, illustrating that the power of monetary policy to relax these constraints changes from mild (horizontal) to severe (vertical) crises.

All of the discussion in this section takes place after the bad state has realized at date 1 and the central bank has injected $(M - \underline{M})$ pesos in response. Figure 1 illustrates the timing of shocks and agents’ decisions.

2.1 Mild and Severe Crises (Date 1)

At date 1, distressed firms need to borrow in order to finance their production shock. Their total net worth, measured in dollars, is $\frac{ak+M}{e_1} + w^n$. Clearly, as M rises, so does net worth, at least for a given e_1 . The key question is whether e_1 depreciates with an expansionary monetary policy and by how much. We argue that the answer to this question depends critically on the severity of the crisis. If the latter is severe, a monetary expansion is ineffective in raising output and leads to an exchange rate overreaction. This happens despite full inflation target credibility ($e_2 = 1$), as it reflects the fall in the real value of domestic collateral (i.e., not a monetary phenomenon).

⁹The discussion in section 6 also clarifies exchange rate determination in our setup. When the peso interest rate is zero, e_1 , which we refer to as the date 1 exchange rate, can also be interpreted as the forward exchange rate standing at date 1 for exchange at date 2. In section 6 we show that when the peso interest rate is nonzero, this ambiguity disappears and e_1 is the date 1 exchange rate.

Figure 1. Timeline

Note: The figure presents the timing of investment, borrowing, and consumption decisions. Also pictured are the shocks affecting firms at date 1.

Let us consider equilibrium at date 1. Distressed firms can borrow w^n directly from foreign investors. Thereafter they must turn to other domestic agents to raise resources. Recall that intact firms can also borrow w^n from foreign investors. Moreover, since they are willing to accept domestic money as payment as well as purchase debt claims backed by ak of collateral, they can also finance the production shock of the distressed firms.

There are two regions of interest. If

$$\frac{1}{2}w^n > \frac{1}{2} \frac{ak + M}{1 + i_1^*}, \quad (3)$$

then intact firms have sufficient access to foreign funds to satisfy demand from all distressed firms. We refer to this case as the horizontal region, because the supply of funds from intact firms is elastic at the margin.

The other case is when

$$\frac{1}{2}w^n < \frac{1}{2} \frac{ak + M}{1 + i_1^*},$$

while $w^n < k/2$. That is, intact firms have insufficient resources to satisfy demand from distressed firms. We refer to this case as the vertical region, because the supply of funds is inelastic at the margin.

2.2 *Horizontal Region (Mild Crises): The Standard Credit Channel*

Intact firms have portfolios that include domestic loans, money, and international collateral. Indifference requires that i_1^* be the interest rate on the domestic loans as well. The exchange rate can also be determined from this indifference condition. Since the return on holding money must be equal to $1 + i_1^*$, the exchange rate is

$$e_1 = 1 + i_1^*. \quad (4)$$

This equation is the interest parity condition, given that $e_2 = 1$ and that money provides no special transaction service (so the peso interest rate is zero). Note that the exchange rate is unresponsive to monetary injections.

Total reinvestment is determined by the total resources of individual distressed firms,

$$\theta^H k = w^n + \frac{ak + M}{e_1} = \frac{w - f + ak + M}{1 + i_1^*} < 2w^n, \quad \theta^H < 1, \quad (5)$$

where the superscript H denotes the horizontal equilibrium. The first inequality shows that the economy has not used all of its international liquidity, while $\theta^H < 1$ indicates that the economy is in a crisis: distressed firms are not able to meet their production shock fully because of their binding financial constraints.

As we mentioned above, we refer to this as the horizontal region, because the price of loans is not affected by their quantity. In principle, a distressed firm could continue borrowing at the given interest rate i_1^* , as long as its own financial constraint is relaxed.

We can see the effects of monetary injections from the preceding expressions. They operate through a credit channel à la Bernanke and Gertler (1995). From (5) we note that injecting money increases the resources of the distressed firms and increases their investment. From (4) we note that this policy has no effect on exchange rates in the horizontal region, so the monetary expansion is a powerful mechanism to raise the dollar value of distressed firms' collateral. It is possible that reintroducing the standard interest parity effect (which we have suppressed) will work against this conclusion. But as we show next, in the vertical region, we reach the unambiguous conclusion that the exchange rate effect will neutralize monetary expansions.

2.3 The Vertical Region (Severe Crises): Overresponsive Exchange Rate

In this region, the international supply of funds faced by emerging market economies during external crises is vertical (i.e., price inelastic). Since all investment at date 1 is eventually financed by foreigners, the stock of international liquidity is what determines investment in this region:

$$\theta^V k = 2w^n \quad (6)$$

where the superscript V stands for vertical equilibrium. Note that domestic collateral does not appear at all in this expression.

Consider the exchange rate next. Since intact firms borrow up to their maximum capacity from foreigners and hold portfolios only composed of domestic money and loans against domestic collateral, it must be that the return on holding domestic money exceeds i_1^* . That is,

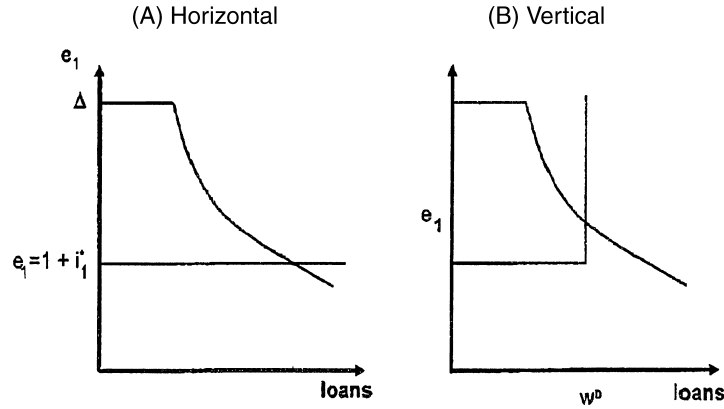
$$e_1 > 1 + i_1^*,$$

and the date 1 exchange rate is depreciated.

Indifference between holding money and domestic loans implies that the (dollar) interest rate on loans against domestic collateral also rises above i_1^* . In equilibrium, loans collateralized by w are made at rate i_1^* , while those collateralized by ak are made at a higher rate of i_1^d , where

$$i_1^d = e_1 - 1 > i_1^*.$$

Figure 2 represents the equilibrium determination of e_1 . The vertical axis is the price, e_1 , while the horizontal axis measures domestic reinvestment. For $e_1 = 1 + i_1^*$, intact firms elastically supply their international liquidity to distressed firms. However, at the point w^n , intact firms run out of international liquidity, and the supply curve turns vertical. On the other side of the domestic financial market, distressed firms demand as much international liquidity as they can obtain as long as the exchange rate is less than or equal to Δ . However, they are constrained in their domestic borrowing by their holdings of money and domestic collateral. The demand for international liquidity turns downward at some point, and effective demand is $\frac{M+ak}{e_1}$. Figure 2 represents two cases: one equilibrium in the horizontal region (panel A) and one in the vertical case (panel B).

Figure 2. Equilibrium Exchange Rate

Note: The figure illustrates the equilibrium determination of e_1 in both the horizontal and vertical regions of the supply of international liquidity.

This figure illustrates how e_1 rises above $1 + i_1^*$ in the vertical region when international liquidity is scarce. Note also that e_1 can never exceed Δ , the marginal product of reinvestment for distressed firms. However, there is a large interval for demand within which e_1 lies strictly between $1 + i_1^*$ and Δ . In this case,

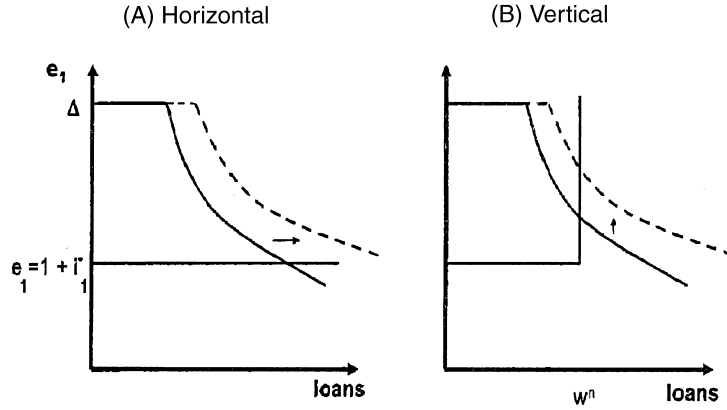
$$e_1 = \frac{M + ak}{w^n} > 1 + i_1^*. \quad (7)$$

We can see that monetary injections have a very different effect in the vertical than in the horizontal region. From (6), we note that injecting money has no effect on date 1 reinvestment. This is because the economy has a shortage of international liquidity, and reallocating domestic liquidity has no real (aggregate) effect.

As before, the net worth of distressed firms is

$$w^n + \frac{ak + M}{e_1}.$$

That is, the credit channel is still active, and the net worth of distressed firms is what determines reinvestment at date 1. But the exchange rate now is *overresponsive* to monetary injections. Referring back to equation (7), we see that monetary injections are offset one-for-one by a depreciation in the exchange rate. Essentially, the

Figure 3. Domestic Liquidity Expansion

Note: The figure illustrates the effect of an expansion in domestic liquidity on the exchange rate, e_1 , as well as the quantity of loans. The figure presents these effects in both the horizontal and vertical regions of the supply of international liquidity.

exchange rate depreciates to ensure that the dollar net worth remains at a level consistent with the availability of external funds. This role of the exchange rate, captured in (7), is what is behind the monetary ineffectiveness result.

2.4 Discussion: The Credit Channel in Emerging Markets

Panels A and B in figure 3 summarize the differences between the horizontal and vertical views. In their downward sloping segments, demands are equal to $(ak + M)/e_1$.

In the horizontal case, the increase in domestic net worth of distressed firms caused by expanding monetary policy raises date 1 investment, leaving e_1 unaffected. In the vertical region, the same increase has *no effect* on equilibrium investment and only raises e_1 .

We view the horizontal region as relevant for developed economies; w is plentiful, and there is no distinction between domestic and international liquidity. It may also apply for emerging economies experiencing moderate contractions. The vertical region, on the other hand, seems to be a better description of the environment faced by emerging markets during severe external crises.

There are two points to emphasize from this analysis. First, it may be that other mechanisms overturn the result that injecting money is expansionary in the horizontal region (for example, debts may be dollarized, as we discuss shortly). But in the vertical region, we reach the unambiguous conclusion that injecting money is fully offset by the depreciation in the exchange rate.

This leads to a second point. A researcher looking at the net worth of credit-constrained firms during a crisis will always conclude that the reason for monetary policy ineffectiveness is the deterioration in net worth triggered by the exchange rate depreciation—in particular, the dollarization of liabilities (more on this below). But this is *not* the right conclusion in the vertical region. The main credit constraint behind the crisis is an *aggregate* constraint, not a microeconomic one; the exchange rate depreciation is simply the equilibrium response to such restriction.

3. Underinsurance and Monetary Policy

Of course, all our claims so far are from the perspective of date 1 in the b -state, when w^n is already given. But what is the impact of anticipated monetary policy in the event of a crisis on date 0 decisions? We turn to this discussion next.

We show that a central bank that is expected to contract money in the event of a vertical crisis at date 1 actually causes the private sector to alter date 0 decisions so as to arrive at date 1 with a smaller w^n . By supporting the date 1 currency, the central bank causes the private sector to overborrow at date 0 and underinsure against the crisis. Conversely, expansionary monetary policy gives the private sector incentives to insure against the crisis and reduce date 0 borrowing.

3.1 Private Sector Date 0 Decisions and e_1^b

At date 0, the private sector decides how much to borrow from foreign investors and how much real investment to undertake. The borrowing contracts specify an amount loaned to a domestic firm and a repayment at date 2, f^ω , contingent on the date 1 (aggregate) state $\omega \in \{b, g\}$. Since the funds raised from this loan are used in date 0 investment, and since foreign investors are risk neutral, the date 0

budget constraint is

$$c(k) \leq \frac{1}{(1+i_0^*)(1+i_1^*)} \left((1-\pi)f^g + \pi f^b \right). \quad (8)$$

In the g -state at date 1, all firms make profits of

$$Ak + (w - f^g) + \underline{M}.$$

In the b -state, one-half of the firms are distressed and they make profits of

$$\left(\frac{ak + M}{e_1^b} + \frac{w - f^b}{1 + i_1^*} \right) \Delta,$$

while the other half are intact and make profits of

$$Ak + (w - f^b) \frac{e_1^b}{1 + i_1^*} + M.$$

Combining these expressions leads to the following problem for a firm at date 0:

PRIV:

$$\begin{aligned} \max_{k, f^g, f^b} \quad & (1-\pi)(Ak + w - f^g + \underline{M}) \\ & + \pi \frac{1}{2} \left(\left(A + a \frac{\Delta}{e_1^b} \right) k + (\Delta + e_1^b) \frac{w - f^b}{1 + i_1^*} + \left(1 + \frac{\Delta}{e_1^b} \right) M \right) \\ s.t. \quad & f^g, f^b \leq w \\ & c(k) \leq \frac{1}{(1+i_0^*)(1+i_1^*)} (\pi f^b + (1-\pi)f^g). \end{aligned}$$

Our technical assumptions (see the appendix) guarantee that $f^g = w$ and $f^b < w$. The former holds as long as increasing investment in k at date 0 is more profitable than investment in international markets. And $f^b < w$ as long as saving some resources to absorb the production shocks at date 1 is more valuable than using all of those resources toward investment at date 0. It is apparent from the private program that e_1^b is the *only* equilibrium price that influences the date 0 decision. Let us study this connection more closely.

Since $f^g = w$, we simply need to consider the trade-off between increasing f^b and reducing k . At date 0, building a marginally larger plant increases (expected) date 2 profits by

$$(1 - \pi)A + \pi \frac{1}{2} \left(A + a \frac{\Delta}{e_1^b} \right). \quad (9)$$

Building this larger plant requires the firm to raise an additional $c'(k)$ at date 0. The probability of a crisis is π , so in order to raise an additional $c'(k)$ at date 0, f^b must rise by

$$\frac{c'(k)(1 + i_0^*)(1 + i_1^*)}{\pi}.$$

The cost to the firm of raising f^b is that there are fewer resources to absorb the date 1 production shock. The fall in expected profits due to having fewer resources is

$$\pi \left(\frac{\Delta + e_1^b}{2(1 + i_1^*)} \right) \frac{c'(k)(1 + i_0^*)(1 + i_1^*)}{\pi},$$

which can be simplified to

$$c'(k)(1 + i_0^*) \left(\frac{\Delta + e_1^b}{2} \right). \quad (10)$$

The optimal choice of k by the private sector equalizes (9) and (10). We note two comparative statics and the conclusion that follows from them: (1) The benefit of building a larger plant size is decreasing in e_1^b ; (2) The marginal cost of investing is increasing in e_1^b . For both reasons, k is decreasing with respect to e_1^b .¹⁰

The investment/financing decision at date 0 is really to create a (real) “peso” plant at the cost of having fewer “dollar” assets in the b -state. The reason for this is that the output from k can only be used as domestic collateral and hence is affected by the value of the exchange rate. On the other hand, to build the plant, the firm has

¹⁰It is not e_1^b , per se, that matters in this trade-off, but $\frac{e_1^b}{e_2}$ (i.e., the expected future depreciation of the exchange rate) times one plus the i_1^p (the peso rate). Since we have set $e_2 = 1$ and $i_1^p = 0$, this point is obscured. We relax these assumptions later in the paper.

to raise f^b , meaning that there is less international liquidity in the b -state.

Intuitively, the reason why e_1^b matters is that it changes the terms of this investment/financing decision. A more depreciated e_1^b gives firms less incentive to increase k and more incentive to decrease f . This is the reason that k is decreasing with respect to e_1^b .

3.2 Consumption Maximizing e_1^b

We now ask what level of the exchange rate, e_1^b , is required in order for the date 0 decisions to maximize (expected) aggregate consumption. We show that for $e_1^b < \Delta$, the private sector generally overborrows and overinvests relative to this benchmark.

Consider first the date 0 decisions of the private sector, (k, f^g, f^b) , that maximize aggregate consumption. To arrive at this program, we simply substitute out the exchange rate of e_1^b from **PRIV**. If we substitute the market clearing condition for e_1^b from (7) into **PRIV** we find that

AGG:

$$\begin{aligned} \max_{k, f^g, f^b} \quad & (1 - \pi)(Ak + w - f^g) + \pi \left(\frac{w - f^b}{1 + i_1^*} \Delta + \frac{A + a}{2} k \right) \\ \text{s.t.} \quad & f^g, f^b \leq w \\ & c(k) \leq \frac{1}{(1 + i_0^*)(1 + i_1^*)} (\pi f^b + (1 - \pi) f^g). \end{aligned}$$

The trade-off between increasing k versus f^b is very different here than in **PRIV**. The benefit of increasing plant size is

$$(1 - \pi)A + \pi \frac{1}{2} (A + a).$$

For $e_1^b < \Delta$, this benefit is strictly lower than the private sector's computation. On the cost side, borrowing more to build this plant costs

$$c'(k)(1 + i_0^*)\Delta.$$

For $e_1^b < \Delta$, the cost lies strictly above the private sector's computation.

We conclude that, if $e_1^b < \Delta$, then the private sector's date 0 decisions involve overborrowing and overinvesting.

Intuitively, the cause of this date 0 overinvestment is that the private sector undervalues international liquidity in the b -state. At the aggregate level, resources in the b -state always generate a return of Δ from date 1 to date 2. On the other hand, firms see the cost of not having date 1 resources as proportional to e_1^b . As long as $e_1^b < \Delta$, the private sector's investment choices do not lead to maximization of date 2 expected aggregate consumption.

3.3 *The Effect of Anticipated Monetary Policy*

The previous result highlights a basic feature of the environment. Relative to the consumption-maximizing outcome, the private sector is always biased towards overborrowing and overinvesting. Thus we have a clear benchmark to evaluate policies that affect the exchange rate. In particular, the private sector's biases are exaggerated by a central bank that attempts to stabilize e_1^b . More generally, since the central bank can affect e_1^b with its choice of M , it can reduce this overborrowing problem through monetary policy. A straightforward argument establishes that if the private sector expects an increase in M if a crisis takes place at date 1, then k will be reduced at date 0 and expected date 2 consumption will rise.¹¹

4. Dollarization of Liabilities

The credit channel literature in emerging markets has highlighted the role played by dollarization of domestic liabilities in magnifying the contraction. Indeed, even in our framework, a depreciated exchange rate deteriorates balance sheets by lowering the value of domestic assets relative to liabilities. But our framework qualifies the main conclusion from the literature: while dollarization may have the balance-sheet effect highlighted in that literature during mild crises (horizontal region), it does *not* during severe crises (vertical region).

At the firm level, it will seem that the problem is one of dollarized liabilities: firms are credit constrained, and the depreciated exchange rate will worsen balance sheets. But the relevant constraint

¹¹The argument is by contradiction. Suppose that increasing M causes k to rise. Then from market clearing at date 1, it must also cause e_1^b to rise. However, since k is a decreasing function of e_1^b , k must fall. This is a contradiction. Therefore, k is decreasing in M , and aggregate consumption is increasing in M .

is a shortage of aggregate international liquidity, not a firm-level balance sheet problem. That is, in a vertical crisis, the country faces a macroeconomic constraint, not a microeconomic one.¹²

However, for the insurance reasons we discussed in the previous section, dollarization is not innocuous. It will generally lead to private sector underinsurance. Let us discuss these date 1 and 0 issues in turn.

4.1 *Dollarization During Severe Crises: A Fallacy of Aggregation*

In order to discuss domestic dollarization, we introduce a set of domestic consumers who lend to firms at date 0. For the purpose of this subsection, it is sufficient to start the analysis at date 1, given some preexisting debt. Suppose that firms arrive at the date 1 crisis state with some dollar-denominated debt, b^d , with domestic consumers. Thus, a distressed firm's investment is given as

$$\theta k = w^n + \frac{ak + M - e_1 b^d}{e_1}. \quad (11)$$

The literature emphasizes the fact that from (11) a depreciation reduces the dollar value of local assets ak and M , while it leaves unchanged the value of dollarized liabilities $e_1 b^d$. Moreover, since a monetary expansion depreciates the exchange rate, it may worsen balance sheets and, perversely, turn contractionary.

But, as we argue in section 2, this analysis does not consider that under severe crisis the main binding constraint is the lack of aggregate international liquidity. Thus, all these considerations of the effect of M on e_1 , and hence on the domestic credit squeeze, are important for distributional issues (from more to less dollarized firms and consumers) but not for the aggregate. The exchange rate reacts to monetary policy precisely to cause a credit squeeze that equilibrates investment demand with the limited external supply. At the aggregate level, distressed firms' investment is still

$$\theta k = 2w^n. \quad (12)$$

¹²There can be distributional effects—i.e., from those that don't receive M to those that do, or from those with more dollarized liabilities to those with fewer liabilities—but these are likely to be subordinate to the aggregate constraint.

Replacing (12) into (11) yields a new expression for the exchange rate as a function of M :

$$e_1^{b,d} = \frac{ak + M}{w^n + b^d}$$

where the $e_1^{b,d}$ stands for the equilibrium exchange rate with dollarized liabilities. This exchange rate expression needs to be contrasted with expression $e_1^{b,p}$ when domestic liabilities are denominated in pesos, b^p :

$$e_1^{b,p} = \frac{ak + M - b^p}{w^n}.$$

From these two expressions, we can consider the effect of monetary expansions on the exchange rate in economies with different degrees of dollarization:

$$0 < e_1^{b,d}(M)' = \frac{1}{w^n + b^d} < \frac{1}{w^n} = e_1^{b,p}(M)'. \quad (13)$$

The first implication of this expression is that expanding money always depreciates the exchange rate. A central bank that is concerned with the balance sheet position of firms (as in [11]) will be inclined to contract money and support the exchange rate. But since during a vertical crisis, the important constraint is that of (12), the central bank will be *protecting the wrong margin*.

The second implication of (13) is that during severe crises the exchange rate in a dollarized economy will be *less* responsive to a monetary expansion than that of the nondollarized economy. This is a sort of *market-based fear of floating*, as it results from equilibrium considerations for a given monetary policy, and not from any additional caution that the central bank may choose to adopt when liabilities are dollarized. The reason for this result is precisely that a depreciation is more contractionary in a dollarized economy and, therefore, can more easily generate the domestic credit squeeze needed to reduce investment demand to levels compatible with the limited availability of international liquidity.

4.2 Underinsurance Effect of Dollarization

There is a more subtle effect of dollarization in our framework. A central bank at date 1 that is concerned with preserving the balance

sheets of firms may, in the vertical region, choose to support the exchange rate. But boosting e_1^b causes firms to undervalue insuring against the b -state and set f^b higher. This follows from the underinsurance results of the previous section. In this sense, the negative effect of central bank support of the exchange rate is that external liabilities will become less contingent. In a model with richer options on the denomination of liabilities, this effect will lead to excessive dollarization of liabilities (see Caballero and Krishnamurthy 2003).¹³

That is, while we challenge the standard view that dollarization of liabilities is a key factor *during* severe crises, we argue that it is an important factor behind crises due to the negative effects it has on private agents' external *insurance* decisions.

4.3 Dollarization of External Liabilities

Before concluding this section, we should point out that there is a related but distinct issue—the dollarization of *external* liabilities. This refers to the fact that most emerging market external debt is not denominated in the issuing country's currency. If debt is denominated in a country's own currency, then the debt will be effectively contingent on aggregate outcomes. In practice, of course, unlike our theoretical assumptions, there is very little contingent debt issued by emerging markets. Thus, if developing economies could or would borrow abroad in their own currency, they would effectively obtain insurance from foreigners against events that depreciate their currency, such as the tightening of external financial constraints. The “original sin” literature in, e.g., Hausmann, Panizza, and Stein (2001) highlights reluctance on the supply side of that market, while Caballero and Krishnamurthy (2003) describe demand side problems behind the lack of contingency of external debt. In our current analysis, external dollarization simply means that there is less contingency in liabilities, so that f^b is higher and closer to f^g . As a result, the country will have less international liquidity in the crisis state. It is also worth noting that the problem of external liability dollarization is

¹³This implication is supported by Bleakley and Cowan (2002), who find that in economies with fixed exchange rates, the match between the denomination of liabilities and that of revenues is weaker than in economies with flexible exchange rates.

reinforced by the underinsurance effects of dollarization of domestic liabilities as discussed above.

Adding external liability dollarization to our problem is conceptually straightforward and makes the supply of international funds at date 1 effectively diagonal, but nothing substantive changes in our analysis. We discuss this extension in section 6.

5. Policy Constraints and Policy Options

In many instances, a central bank faces constraints that force it to support the exchange rate. Dollarization of domestic liabilities is one such instance (whether the market or the central bank determines this is not important for our argument here). Inflation credibility problems are another one. We begin this section with a discussion of inflation credibility, and then turn to alternative policy instruments when monetary policy is constrained to be procyclical or less countercyclical than desirable. There is an important difference in the costs of losing monetary policy during severe crises vis-à-vis doing so in mild ones. In the latter, as in the standard Mundell-Fleming context, the cost is that the central bank loses a countercyclical instrument to smooth fluctuations. In the former, on the other hand, the cost is mainly an “insurance” one. The central bank loses a relatively cheap instrument to induce the private sector to take adequate precautions against crises. In the standard framework, the natural response to the loss of monetary policy is to search for another countercyclical policy instrument (e.g., fiscal policy). In the vertical crisis environment, the natural response is to search for an alternative insurance mechanism. We discuss such alternatives in the second part of this section.

5.1 *Limited Inflation Credibility*

Limited inflation credibility has two effects in our environment. First, a central bank with a history of high inflation cannot afford the inflationary consequences of an exchange rate depreciation during a crisis. This is a common explanation for the apparent “fear of floating” of central banks in emerging economies (see, e.g., Calvo and Reinhart 2003). However, for the reasons we have outlined above, the anticipation of a contractionary monetary policy during a vertical

crisis will exacerbate the private sector's underinsurance problem and thereby worsen the crisis.

A second problem caused by limited inflation credibility is that a central bank that expands monetary policy in a crisis will have a limited effect on real balances. That is, in our environment, a central bank does better by increasing real money balances during a crisis. However, a central bank with limited inflation credibility will create inflation with little effect on real money balances and thereby on the expected return from hoarding international liquidity. In order to see this, note that in all our derivations up to now we have assumed there is long-run inflation (price level) credibility, so that e_2 remains at one regardless of the monetary policy adopted at date 1. But suppose that whenever there is a depreciation during a crisis, some of it is expected to pass through into e_2 .

To be concrete, suppose that e_2 (i.e., the date 2 price level) is an increasing function of $\frac{M}{\underline{M}}$, with $e_2(1) = 1$. In a vertical crisis, a distressed firm sells M units of money and ak of domestic collateral to raise funds for salvaging production. The intact firms lend w^n against these assets. To an intact firm, the money is worth $1/e_2$, proportionately less than the ak of domestic collateral. Thus, let us define the return on lending international liquidity as

$$i_1^d = \frac{M/e_2 + ak}{w^n} - 1.$$

The return to an intact firm of purchasing money with international liquidity is just $\frac{e_1}{e_2} - 1$. Since this return must equal the return to lending against the domestic collateral of ak , we find that the interest parity condition is:

$$\frac{e_1}{e_2} - 1 = i_1^d.$$

For the case where $e_2 = 1$, we previously showed that the date 0 level of investment, k , is a decreasing function of e_1 . We argued that supporting the currency at date 1 had the consequence of exacerbating the date 0 overinvestment problem of the private sector. Likewise, allowing e_1 to depreciate through an expansionary monetary policy reduced the overinvestment problem.

When e_2 is not fixed at one, a similar argument to that of section 3.1 establishes that k is a decreasing function of $\frac{e_1}{e_2}$. But since

$$\frac{e_1}{e_2} = \frac{M/e_2 + ak}{w^n},$$

the important term for policy purposes is the term involving real money balances (i.e., M/e_2). An expansionary monetary policy raises real balances only to

$$\frac{M}{e_2} < M.$$

Thus, a central bank with limited inflation credibility will have to resort to additional (and possibly very costly) mechanisms to induce adequate private sector insurance decisions. We turn to this discussion next.

5.2 Insurance Substitutes

The problem created by being unable to raise real balances is that the social-private spread, $\Delta - (1 + i_1^d)$, remains high. Thus, the return to hoarding international liquidity until date 1 remains undervalued, and private insurance decisions are distorted. As we argued in section 3, these considerations are unique to the vertical environment, because the insurance problem only arises if the aggregate international liquidity constraint binds.

There are two obvious ex-ante policy measures that can deal with the underinsurance problem: taxation of capital inflows during normal times (date 0) and international liquidity requirements at date 0. We now characterize the relationship between an ex-ante tax and i_1^d .

The first order condition in **AGG** is

$$c'(k^{AGG})(1 + i_0^*)\Delta = (1 - \pi)A + \pi\frac{1}{2}(A + a),$$

whereas for the private sector the condition sets (9) equal to (10) (with e_1 replaced by $1 + i_1^d$).

Aligning the date 0 private and consumption-maximizing incentives is a matter of choosing a tax/transfer policy. Suppose that the central bank levies a tax τ per unit of k , which is returned to firms

in a lump sum fashion. Then the first order condition for the private sector becomes

$$c'(k)(1 + i_0^*) \frac{\Delta + 1 + i_1^d}{2} = (1 - \pi)A + \pi \frac{1}{2} \left(A + a \frac{\Delta}{1 + i_1^d} \right) - \tau.$$

Choosing τ to align the private and central-bank incentives yields that for any equilibrium level of $i_{1,\tau}^d$, the optimal tax solves

$$\tau(i_{1,\tau}^d) = \frac{1}{2} \left(\frac{a\pi}{1 + i_{1,\tau}^d} + c'(k^{AGG})(1 + i_0^*) \right) (\Delta - (1 + i_{1,\tau}^d)).$$

The tax is increasing in $\Delta - (1 + i_{1,\tau}^d)$. Thus, economies where the central bank cannot follow countercyclical monetary policy, and therefore $\Delta - (1 + i_1^d)$ remains high, may need to rely on capital controls to correct the underinsurance problem. Note that the same result could be achieved via a contingent liquidity requirement. The tax solution gives the private sector incentives to choose the efficient k , thus resulting in the efficient $w - f^b$. Alternatively, the central bank could mandate directly that each firm preserve international liquidity for the date 1 crisis state, so that the efficient level of $w - f^b$ is realized.

In practice, taxes come with their own sets of distortions: dead-weight costs of taxation, costs of enforcement, evasion, etc. However, the important point to recognize is that, in the vertical environment, the cost of losing monetary policy is *not* being unable to manage aggregate demand or the extent of the domestic credit squeeze at date 1. Rather, the cost is the underinsurance by the private sector at date 0. In this sense, the cost of having to enforce capital controls may be seen as a direct cost of losing monetary policy.

6. Robustness: Peso Rates and Diagonal Supply

In this section, we show that our main conclusions are robust to modifications in two of the most stylized features of our model: the absence of a monetary friction and the absolute price inelasticity of the supply of funds during crises.

6.1 *Peso Rates and Interest Parity Condition*

Up to now we have simplified our analysis by removing all monetary frictions from the analysis, and therefore have set the domestic peso interest rate to zero. This simplifies exchange rate determination at date 1, but leaves us with an unusual model of money. Here we sketch a more standard model of money, in which money is special because it provides a transaction service.¹⁴ Our substantive results remain unchanged, although now we recover the standard exchange rate effect via the interest parity condition in addition to the excess sensitivity result we discussed in the previous sections.

Let us return to the full inflation credibility case ($e_2 = 1$) and let us focus on the vertical region and the bad aggregate state.

At the end of date 1, the government has liabilities of B bonds and M units of money per capita. Each bond is redeemed at date 2 for one unit of money, and the government is credible in ensuring that the price level at date 2 is one. At date 1, money is the only domestically liquid asset. Neither claims against ak (corporate bonds) nor the government bonds are liquid. Thus, at date 1, distressed firms sell M units of money to the intact firms in exchange for w^n units of international liquidity, which means that

$$e_1^b = \frac{M}{w^n}.$$

It is instructive to go through the steps that take us to this equation in order to disentangle the mechanisms through which money affects the exchange rate during severe crises in this economy.

We introduce a date 1^- in order to study peso interest rates and the effects of open market operations. At date 1^- the aggregate state of the world has been realized, but the identity of agents receiving the shock (distressed or intact) has not. Because of the latter, agents are all identical at 1^- .

At date 1^- , both the bond market and the money market are open to all agents. Entering date 1^- , the government has outstanding $\underline{B}$ bonds and $\underline{M}$ money. The government does an open market operation to purchase $(\underline{B} - B)$ bonds for $(M - \underline{M})$ money.

Let us consider the relative asset returns on bonds, money, and international liquidity at date 1^- .

¹⁴See Diamond and Rajan (2001, 2003) and Lorenzoni (2001) for alternative models of money in liquidity-based frameworks.

One bond yields one unit of money at date 2 and costs $1/(1+i_1^p)$ units of money at date 1^- , where i_1^p is the peso interest rate. One unit of money can be sold at date 1 to finance the liquidity shock if the agent is distressed. Money is sold for international liquidity at a price of $1/e_{1-}$. Each unit of international liquidity yields Δ at date 2. Thus the expected return on holding money from date 1^- to date 2 is $0.5(\Delta/e_{1-} + 1)$. As before, $\Delta \geq e_{1-}$, which means that the net return is positive. It immediately follows that indifference between holding money and bonds requires that

$$1 + i_1^p = \frac{1}{2} \left(\frac{\Delta}{e_{1-}} + 1 \right). \quad (14)$$

One unit of money also can be converted in the foreign exchange market into one unit of international liquidity, at the price of e_{1-} . One unit of international liquidity either can be used in production at date 1 or sold to a distressed firm at date 1. The expected benefit of the unit of international liquidity is $\frac{1}{2}(\Delta + e_{1-})$. Thus, the interest parity condition is

$$(1 + i_1^p)e_{1-} = \frac{1}{2}(\Delta + e_{1-}). \quad (15)$$

In the standard interest parity condition, the right-hand side of (15) is fixed, and a reduction in i_1^p must be offset by a depreciation in e_{1-} to keep the left-hand side constant as well. But the depreciation in our vertical environment raises the return to the dollar-lender in the right-hand side of (15), which means that the left-hand side must rise, and this is achieved by a further depreciation in e_{1-} . The latter is the excess sensitivity result we have discussed, and it results from the relaxation of the domestic credit squeeze caused by the monetary expansion.

Finally, combining (14) and (15), we can solve for the peso interest rate and the exchange rate as a function of monetary policy:

$$1 + i_1^p = \frac{1}{2} \left(\frac{\Delta}{M} w^n + 1 \right)$$

$$e_{1-} = \frac{M}{w^n}.$$

The latter is the expression we started this section with, and it summarizes the two channels through which money affects the exchange rate: interest parity and domestic credit squeeze.

Relative to our earlier model, it is the former (and standard) channel that is new. But neither channel can change the fact that international liquidity is fixed at date 1 and hence output cannot be affected by monetary policy:

$$C^V = \left(\frac{A+a}{2} \right) k + w^n \Delta + y^c.$$

And for date 0 decisions, it is only the *second* channel that matters, which is common to this and the model without monetary frictions.

6.2 “Diagonal” Supply

Let us now return to the model without monetary frictions, and continue to assume $e_2 = 1$. But rather than an inelastic supply of funds at date 1, let us consider instead a supply curve of the form

$$w^n s(e_1) \quad \text{where} \quad s(1) = 1, \quad s'(\cdot) > 0.$$

That is, the supply of international funds is “diagonal” as opposed to vertical.¹⁵

Equilibrium at date 1 is now

$$\frac{ak + M}{1 + i_1^d} = w^n s(e_1).$$

As in the pure vertical model, e_1 is increasing in M . However, consider the expression for total consumption at date 2. This is

$$C^V = \left(\frac{A+a}{2} \right) k + w^n s(e_1) \Delta + y^c,$$

¹⁵The diagonal supply also captures the idea that depreciating the exchange rate increases exports and, if the export sector is an important part of international collateral, thereby expands supply. Christiano, Gust, and Roldos (2004) offer a related perspective on diagonal supply. In their model, imperfect liquidity substitution stems from imperfect input-substitution and from the fact that different inputs are paid in different currencies. The “diagonal” aspect of their model arises from the (limited) possibility of substituting tradables and nontradables inputs.

which is now an increasing function of e_1 through the $s(\cdot)$ function. This implies that increasing M does have a contemporaneous effect on C^V .

Thus, from a date 1 perspective, the diagonal model has elements of both the horizontal model and the vertical model. As in the horizontal model, there is an aggregate demand channel/credit squeeze channel through which expanding money increases aggregate consumption. As in the vertical model, the money expansion depreciates the exchange rate beyond the standard interest parity effect.

Now, let us shift back to date 0. At this date, the firm contemplates borrowing some resources and increasing k , the size of the plant. As before, the shadow cost of the resources is increasing in e_1 . A higher expected e_1 induces a firm to save some dollar resources until date 1, at which point these resources can always be lent to return $e_1 - 1$. Moreover, as long as $e_1 < \Delta$, the private sector's decisions will not be consumption maximizing. Thus, as in the vertical model, there is an insurance channel for monetary policy in the diagonal model. Expanding M at date 1 depreciates e_1 . The anticipation of such depreciation makes the private sector reduce investment at date 0 and increase its insurance against the date 1 shock.

7. Final Remarks

The past decade has witnessed several episodes in which the sudden reversals of capital inflows triggers an emerging markets' crisis. These events have placed the risk of "sudden stops" as a central macroeconomic concern for emerging market economies. The main contribution of our paper is to illustrate how monetary policy affects financial constraints of firms during a sudden stop crisis and to highlight an *insurance* effect of monetary policy. Domestic monetary expansion relaxes individual financial constraints but is unable to relax the aggregate financial constraint faced by these economies during crises—it is the wrong kind of liquidity for such a purpose. However, while monetary policy is largely futile once the sudden stop has taken place, the anticipation of a particular reaction by the authority is important for private sector insurance decisions. A flexible exchange rate system, coupled with a countercyclical monetary policy, provides better *insurance* incentives to the private sector than a policy of defending the exchange rate during sudden stops.

Our insurance mechanism is distinct (or in addition to) the standard moral-hazard/free-insurance argument (see, e.g., Dooley 1999). In the latter argument, crises result from an implicit commitment by a local government or foreign institution to transfer resources to the imprudent borrower; a fixed exchange rate transfers reserves at submarket value (during crises) to the dollar-borrower or peso-investor. In our model, there is no direct transfer from the government. The monetary contraction implicit in the defense of the exchange rate lowers the domestic liquidity that borrowers can offer to dollar-lenders during the sudden stop, and in so doing, it lowers the effective return from *new* local dollar lending. By doing so, it affects the incentives to hoard international liquidity for crises.

More generally, monetary policy and its constraints have (unintended) consequences for the private sector's insurance decisions with respect to sudden stops. Policies that exacerbate the domestic credit squeeze during sudden stops, or constraints that limit the authorities' ability to relax domestic financial constraints, lead to socially imprudent private sector actions.

Our emphasis on dual liquidity and insurance has relevance for evaluation of other government policies. Proactively managing international reserves may result in benefits in our framework (see Caballero and Krishnamurthy 2004). Since international reserves are a form of international liquidity, and the private sector carries too little international liquidity into crises, the central bank has a role to play by carrying reserves in place of the private sector. Injecting reserves during a crisis relaxes the international financial constraint faced by the private sector and stabilizes the exchange rate. Relaxing the constraint is beneficial, but the effect on the private sector's own insurance incentives is not, for reasons akin to those we discussed in the context of monetary policy. Reserves management and monetary policy become complementary policy tools in this context. At the very least, the central bank ought to sterilize the forex intervention and possibly go beyond that in the monetary expansion in order to offset the perverse incentive effect of the reserves injection.

Of course, the insurance dimension we highlight in our analysis is not a substitute for conventional inflation credibility concerns. Quite the contrary, without inflation credibility, the central

bank will be unable to let the exchange rate float and expand monetary policy during sudden stops, and may well be forced into tightening monetary policy. The anticipation of this behavior is very costly in our setting, precisely because it exacerbates the private sector's underinsurance problem. In other words, our insurance consideration *raises* the value of achieving medium-term inflation credibility.

Our stylized model is subject to many caveats. One worth mentioning in concluding is the lack of true dynamics. In reality, crises build up, going first through a horizontal phase in which domestic financial conditions tighten and external borrowing becomes gradually more expensive, then falling into a sharp vertical sudden-stop phase. A central question for policymakers in this context is how to conduct monetary policy at the early stages of the crisis, when supply is still horizontal but there is a concern that events may lead to a binding international liquidity constraint. At this stage, tightening monetary policy will destroy financially constrained projects but save international liquidity for the vertical event. We conjecture that this trade-off can be analyzed in terms similar to those we have used throughout: if the commitment to an aggressive countercyclical monetary policy in case of a vertical event is credible, then there is little need to tighten during the horizontal phase. But if the commitment is not credible or feasible, then the appropriate response is to tighten during the early phase in order to protect international liquidity, very much as taxing capital flows at date 0 was advisable in our simplified model when there were constraints on monetary policy during crises. In fact, the costs in terms of the additional financial distress imposed on the domestic private sector are, to a large extent, comparable to the costs of the ex-ante measures we already discussed.

Finally, while our analysis has focused on emerging markets, the underlying structure may be a starting point for other applications. Our model illustrates how a bottleneck may segment financial markets and create liquidity premia on assets. It shows how monetary policy affects and is affected by these bottlenecks. There are many other scenarios, such as liquidity traps and post-bubble-collapses, where similar ingredients appear worthy of consideration. We are currently exploring these applications.

Appendix

Financing Assumptions

The financial frictions of the model are embodied in the following two assumptions:

Assumption 1 (*International Collateral*)

Foreigners lend to domestic firms only against the backing of w . Domestic agents lend against w , M , and ak .

Assumption 2 (*Domestic Collateral*)

A domestic lender can only be sure that a firm will produce ak units of goods at date 2. Any excess production based on physical reinvestment at date 1 is neither observable nor verifiable.

One last assumption is required to rule out date 0 insurance arrangements that transfer resources from distressed firms to intact firms.

Assumption 3 (*Nonobservability of Production Shock*)

The production shock at date 1 is idiosyncratic. The identity of firms receiving the shock is private information.

The mechanism design problem associated with these financing and informational constraints corresponds to the one in **AGG**. There is also a banking arrangement that, in principle, may get around the private information constraint, but this is very fragile.

Technical Assumptions

Consider next the technical assumptions on parameters that we have used. The program in **AGG** is

AGG:

$$\begin{aligned} \max_{k, f^g, f^b} \quad & (1 - \pi)(Ak + w - f^g) + \pi \left(\frac{w - f^b}{1 + i_1^*} \Delta + \frac{A + a}{2} k \right) \\ \text{s.t.} \quad & f^g, f^b \leq w \\ & c(k) \leq \frac{1}{(1 + i_0^*)(1 + i_1^*)} (\pi f^b + (1 - \pi) f^g). \end{aligned}$$

First, we require that $w = f^g$ in this program, or that the return to investing domestically exceeds that of investing abroad.

Assumption 4 (*High Investment Return*)

$$(1 - \pi)A + \pi \frac{A + a}{2} \geq c' \left(\frac{w}{(1 + i_0^*)(1 + i_1^*)} \right) (1 + i_0^*)(1 + i_1^*).$$

Second, we require that the solution features some insurance against the b -state, so that $f^b < w$.

Assumption 5 (*High Return to Insuring*)

$$c' \left(\frac{w}{(1 + i_0^*)(1 + i_1^*)} \right) (1 + i_0^*)\Delta \geq (1 - \pi)A + \pi \frac{A + a}{2}.$$

These last two assumptions can both be met by choosing a large enough value for Δ .

We require that equilibrium, with no central bank intervention, places us in the vertical region, or

$$1 + i_1^* < e_1 < \Delta.$$

The first-order condition for the program in **PRIV** is

$$c'(k)(1 + i_0^*) \frac{\Delta + e_1}{2} = (1 - \pi)A + \pi \frac{1}{2} \left(A + a \frac{\Delta}{e_1} \right).$$

Denote the solution to this equation as $k(e_1)$. Then the largest value of k is attained when $e_1 = 1 + i_1^*$, and the smallest value when $e_1 = \Delta$. Using this knowledge as well as the market clearing condition leads to:

Assumption 6 (*Equilibrium in Vertical Region*)

$$\begin{aligned} \frac{\pi(\underline{M} + ak(i_1^*))}{w - (1 + i_0^*)(1 + i_1^*)c(k(i_1^*))} &< \frac{\Delta}{1 + i_1^*} \\ \frac{\pi(\underline{M}ak(\Delta - 1))}{w - (1 + i_0^*)(1 + i_1^*)c(k(\Delta - 1))} &> 1 \end{aligned}$$

Finally, we have implicitly assumed that the maximum reinvestment constraint does not bind in the vertical equilibrium:

$$\frac{k}{2} > \frac{w - f^b}{1 + i_1^*} = \frac{w - c(k)(1 + i_0^*)(1 + i_1^*)}{\pi(1 + i_1^*)}$$

This can be rewritten as

$$e_1 = \frac{\pi(\underline{M} + ak)}{w - (1 + i_0^*)(1 + i_1^*)c(k)} > 2a.$$

This leads to:

Assumption 7 (*Reinvestment Constraint Does Not Bind in V*)

$$a < \frac{1 + i_1^*}{2}$$

Dollarization of Domestic Liabilities

We sketch an extension to the model of section 6 in order to address dollarization of liabilities. As mentioned in section 5.2, this is one of the primary reasons that policymakers give for being unable to lower interest rates during a crisis.

Suppose that firms have debts of D dollars that have to be settled at date 1^- . These debts are owed to domestic consumers, so that they do not affect the international liquidity of the country. Then the total peso net worth of agents at date 1^- , before any open market operations, is

$$NW = \frac{\underline{B}}{1 + i_1^p} + \underline{M} + \frac{ak}{1 + i_1^p} - De_{1^-} \quad (A1)$$

Now suppose that the government does an open market operation where it purchases bonds with money. Since this transaction is done at market prices, the net worth remains as in (A1).

The open market operation has two effects. First, it lowers i_1^p and thereby raises $\frac{ak}{1 + i_1^p}$. Second, it raises e_{1^-} and thereby increases the debt burden of domestic firms. For a large enough value of D , it is clear that the second effect can overwhelm the first, so that peso net worth *falls* rather than rises with the open market operation.

The interesting case for our model is when, after the open market operation, $NW < M$. That is, the peso net worth of the firms is less than the aggregate amount of money in the domestic economy. In this case, firms will not be able to sell their assets at date 1⁻ to acquire all of M . Thus, the market clearing condition at date 1 becomes

$$1 + i_1^d = \frac{NW}{w^n}.$$

In this case, expansionary open market operations lower i_1^d rather than raise it, but still have no effect on output. At date 1, since expansionary open market operations depreciate the exchange rate, the central bank will raise interest rates. The reaction in this case is ex-ante optimal, since raising interest rates causes i_1^d to rise and leads to better insurance decisions by the private sector. However, the fact that the central bank's time inconsistency problem is attenuated when domestic liabilities are dollarized does not mean that the underinsurance problem has been resolved. Quite the contrary, time inconsistency has been reduced because the country has *no* access to monetary policy.

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