Optimal Inflation Rates in a Non-linear New Keynesian Model: The Case of Japan and the United States*

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We investigate the optimal inflation rate using a New Keynesian model subject to non-linearity arising from downward nominal wage rigidity (DNWR) and prolonged spells of the zero lower bound of nominal interest rates (ZLB). We rigorously evaluate the model non-linearity and calibrate the model to the Japanese and U.S. economies. We find that the optimal inflation rate is close to 2 percent for both countries, though the main driver differs by country: ZLB for Japan, but DNWR for the United States. In addition, around 1 percentage point absolute deviation from the rate of close to 2 percent induces only a minor change in social welfare.

JEL Codes: E31, E43, E52.

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JANET YELLEN: *Mr. Chairman, will you define "price stability" for me?*

ALAN GREENSPAN: *Price stability is that state in which expected changes in the general price level do not effectively alter business or household decisions.*

JANET YELLEN: *Could you please put a number on that?*

— Transcript of Federal Open Market Committee Meeting, July 2–3, 1996

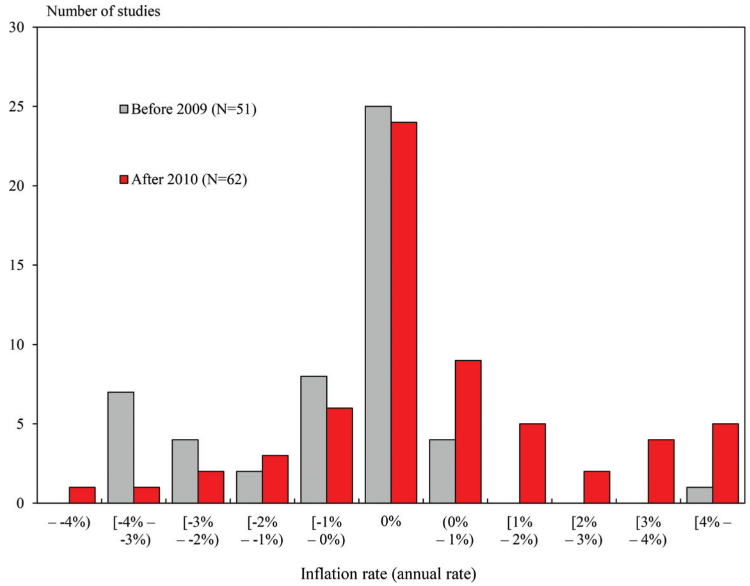
1. Introduction

Monetary economists have devoted themselves to the ever-growing debate on the optimal inflation rate. Even though the above statement by former Chair of the Board of Governors of the Federal Reserve System Alan Greenspan seems simple enough, assigning a precise number to the inflation rate that is consistent with the notion of price stability is a daunting task. This is because numerous frictions that generate monetary non-neutrality could affect social welfare and evaluating the precise effect of each channel requires a highly technical approach.

In fact, there is considerable difference among previous studies in the estimates of the optimal inflation rate. Figure 1 shows the distribution of the estimates for the U.S. economy, most of which were collected in the comprehensive survey conducted by Diercks (2019). Two basic facts about the agreements and disagreements over the optimal inflation rate are clear: they are centered around zero,[1] but there is a significant dispersion; there is a tendency for more studies made after the global financial crisis (GFC) to suggest positive optimal inflation rates. One reason for the variation is that each study focuses on different factors affecting the costs and benefits of inflation. In this regard, a comprehensive assessment of various factors is

[1] A survey by Schmitt-Grohé and Uribe (2010) pointed out that the observed regularity that many central banks in advanced economies have inflation target around 2 percent is hard to reconcile with the theoretical predictions in favor of inflation rates close to or below zero.

Figure 1. Optimal Inflation Rates in Previous Studies



Source: Diercks (2019) and others.
Note: The figure sums up the results of previous studies conducted between 1989 and 2019 regarding the optimal inflation rates for the U.S. economy. For those studies that give more than one estimate, their average estimate is shown. Square brackets, [ ], in the horizontal axis include threshold values, whereas round brackets, ( ), do not.

necessary for deriving realistic policy implications. Moreover, quantitative implication of each factor can be affected by the degree of precision of computation method given the highly non-linear nature of, for example, DNWR and the ZLB.

In this paper, we reinvestigate the optimal inflation rate in a non-linear New Keynesian model. Compared with previous studies, our analysis has the following features. First, in order to capture the trade-off regarding the level of the inflation rate in a balanced manner, our model embeds four major factors affecting the costs and benefits of inflation: (i) nominal price rigidity; (ii) money holdings; (iii) downward nominal wage rigidity (DNWR); and (iv) the zero lower bound of nominal interest rates (ZLB). Second, we derive a full non-linear solution using a global solution method. This methodology enables us to explicitly incorporate the non-linearity stemming from each element of the model, including the firms' price setting under non-zero inflation, DNWR, and the ZLB. Third, we calibrate our model to the Japanese and U.S. economies, reflecting the differences in the economic structure of the two countries. In particular, we employ a regime-switching shock to replicate the frequency and duration of ZLB episodes in each country. This will show whether the optimal inflation rate might vary across economies depending on their economic structure.

Our main findings are summarized as follows. First, our study confirms that positive steady-state inflation is welfare maximizing when we calibrate our model to the Japanese and U.S. economies. We find that the optimal inflation rate for both countries is close to an annual rate of 2 percent. The estimate is a little higher than most previous studies that dealt with DNWR and the ZLB, reflecting our rigorous approach to evaluate their non-linearity, as discussed in more detail shortly. Second, the main driver that supports the optimal inflation rate of close to 2 percent differs by country: the ZLB is the main driver for Japan, but for the United States it is DNWR. This country-specific aspect of the optimal inflation rate arises from our calibration strategy that aims to capture key characteristics of each economy, including the degree of DNWR and the experience of ZLB episodes. Third, around 1 percentage point absolute deviation from the rate of close to 2 percent induces only a minor change in social welfare. In the case of Japan, where the adverse effects of the ZLB are relatively large, incorporating a forward guidance measure through which private agents anticipate a prolonged zero interest rate policy once the ZLB binds reduces the lower end of the range that is acceptable in terms of welfare losses. Finally, we provide various sensitivity analyses. Among others, we examine the impact of changes in the frequency, duration, and severity of the ZLB, as ZLB parameterization is subject to a considerable margin of error.

This paper joins a wealth of literature on the optimal inflation rate. The early literature focused on the role of money holdings (e.g., Cooley and Hansen 1989; Schmitt-Grohé and Uribe 2004, SGU hereafter) and nominal price rigidity (e.g., King and Wolman 1999, SGU 2010), most of which found that an inflation rate below zero is welfare maximizing. On the other hand, recent studies have investigated the benefits of positive inflation with a particular focus on the ZLB (e.g., Coibion, Gorodnichenko, and Wieland 2012, CGW hereafter; Carreras et al. 2016, CCGW hereafter) and DNWR (e.g., Kim and Ruge-Murcia 2009, Carlsson and Westermark 2016).[2] While our study is closest to CGW and CCGW, our model extends that of CCGW by adding money holdings and DNWR, whereas it differs from that of CGW in that we consider a regime-switching shock that generates prolonged ZLB spells.

In terms of the difference of our quantitative results from these previous studies, our estimate of the optimal inflation rate for the United States is a little higher than many of previous studies that considered the ZLB. For example, CGW found that the optimal inflation rate is below 2 percent under various alternative settings. A key difference from their analysis is that we accommodate the prolonged ZLB episodes observed in the data by using a regime-switching shock, whereas they only use AR(1) shocks generating short-lived ZLB episodes. On the other hand, our estimate is lower than that of CCGW, who used a regime-switching shock and found that the optimal inflation rate can be higher than 4 percent. This is partly because we consider the cost of inflation associated with money holdings, which was abstracted by CCGW. In addition, our full non-linear solution captures the convex nature of the cost of inflation induced by nominal price rigidity, leading to a slightly lower optimal inflation rate. Regarding DNWR, our explicit specification of non-linearity of DNWR results in a higher optimal inflation rate compared with the previous studies that approximate DNWR by using a smooth asymmetric cost function (e.g., Kim and Ruge-Marcia 2009).[3]

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[2]Other studies explore the costs and benefits of inflation from a variety of perspectives, including measurement issues of the inflation rate (SGU 2012), trends in relative prices (Wolman 2011, Ikeda 2015), firms' entry and exit (Bilbiie, Fujiwara, and Ghironi (2014), and firms' productivity growth (Oikawa and Ueda 2018, Adam and Weber 2019).

[3]Several studies, including Kim and Ruge-Marcia (2009), considered the Ramsey policy and obtained relatively low optimal inflation rates, while we focus on a simple feedback rule for monetary policy in this paper. In this regard, Kim and Ruge-Marcia (2011) used a simple monetary policy rule and found that the optimal inflation rate is around 1 percent in an economy with DNWR, which is still lower than our estimate.

The literature above almost exclusively studies the U.S. economy. From the perspective of international comparison, Andrade et al. (2019) compared the optimal inflation rate in the United States and the euro area. This paper broadens the scope of the literature by providing a comparison with the Japanese economy.[4] The comparison of the Japanese and U.S. economies uncovers the consequences of prolonged ZLB spells on the optimal inflation rate, given that the Japanese economy has the longest experience of the ZLB in recent history, as well as those of different degree of DNWR in each country.

The remainder of this paper is organized as follows. Section 2 develops our model and presents our computation method. Section 3 describes our calibration. Section 4 provides the baseline results on the optimal inflation rate, and Section 5 investigates its determinants. Section 6 conducts various sensitivity analyses on parameter uncertainty and model specifications. Section 7 concludes.

2. Model

Our model is built upon a standard New Keynesian model. The economy consists of a representative household, monopolistically competitive firms, and a central bank. The household supplies labor service to the production sector, earns wages, consumes, and allocates wealth to nominal bonds and money. Firms produce differentiated goods and set prices under staggered contracts à la Calvo (1983). The central bank follows an interest rate feedback rule.

Our model embeds four major factors affecting the costs and benefits resulting from the level of the steady-state inflation rate: (i) nominal price rigidity; (ii) money holdings; (iii) DNWR; and

[4]With regard to Japan's case, Fuchi, Oda, and Ugai (2008) developed a New Keynesian model that includes the four factors affecting the trade-off in inflation, and found that the optimal inflation rate is between 0.5 and 2.0 percent for the Japanese economy. While they rely on a linearized model except for the ZLB, our study measures the non-linear dynamics stemming from other factors of the model, including firms' price setting under non-zero inflation and DNWR. In addition, whereas they only consider AR(1) shocks, we take into account a regime-switching shock to accommodate the prolonged ZLB experience of Japan. With these refinements, the optimal inflation rate in our model is slightly higher than that of Fuchi, Oda, and Ugai (2008).

(iv) the ZLB. Each element works on the optimal inflation rate in the following ways.

(i) Nominal Price Rigidity: Under staggered price setting, where only a proportion of firms can adjust their prices in response to current economic conditions, both inflation and deflation lead to the dispersion of relative prices among individual goods. The relative price dispersion makes individual goods demand un-uniform even when substitutability among goods is symmetric, and results in the misallocation of resources in the economy. Therefore, the presence of nominal price rigidity implies that zero inflation maximizes social welfare.

(ii) Money Holdings: Holding money brings about a variety of benefits, such as facilitating goods purchases. On the other hand, the opportunity to earn the nominal interest that would be paid on risk-free bonds is lost. In this regard, Friedman (1969) argued that the inflation rate should be negative so as to bring the nominal interest rate, the opportunity cost of holding money, down to zero.

(iii) DNWR: A variety of empirical evidence suggests that nominal wages are more rigid downwardly than upwardly. In the presence of DNWR, an adverse shock leads to misallocations in the labor market due to the lack of sufficient real wage adjustments. Tobin (1972) argued that positive inflation acts as the "grease of the wheels" in the labor market, i.e., it facilitates real wage adjustment in a downturn when nominal wages are downwardly rigid.

(iv) ZLB: The nominal interest rate is usually bounded at zero. The ZLB can be a constraint on the conduct of monetary policy that controls the nominal interest rate as a policy instrument. Summers (1991) argued that positive inflation provides the safety margin for cutting the nominal interest rate in a downturn. Blanchard, Dell'Ariccia, and Mauro (2010) reinforced this argument claiming that modern economies can hit the ZLB more often than was previously believed.

In the sections below, we describe our model settings.

2.1 Household

The representative household receives utility from consuming a composite good C_t and receives disutility from supplying labor service H_t. The expected lifetime utility is defined below:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \ln\left(C_{t+s}\right) - \frac{1}{1+\frac{1}{\eta}} \chi_{t+s} H_{t+s}^{1+\frac{1}{\eta}} \right\}, \tag{1}$$

where β is the subjective discount factor, η is the Frisch labor supply elasticity, and χ_t is exogenous labor disutility. The household has access to nominal bonds S_t, which carry the gross nominal interest rate $R_{n,t}$ in the next period and are subject to exogenous risk premium Q_t. We assume that nominal money holdings M_t facilitate goods purchases by reducing a transaction cost $s(V_t)$, which is a function of the consumption-based money velocity $V_t \equiv P_t C_t / M_t$, where P_t is the price index. The specification of the transaction cost function follows SGU (2004):

$$s\left(V_t\right) \equiv \delta_1 V_t + \frac{\delta_2}{V_t} - 2\sqrt{\delta_1 \delta_2}, \tag{2}$$

where $\delta_1,\ \delta_2 > 0$ are fixed parameters.[5] The household's budget constraint is given as follows:

$$\begin{aligned}(1+s(V_t))C_t + \frac{M_t}{P_t} + \frac{S_t}{P_t} \leq \frac{M_{t-1}}{P_t} + R_{n,t-1} Q_{t-1} \frac{S_{t-1}}{P_t} \\ + \frac{W_t}{P_t} H_t + \frac{T_t}{P_t} + \Phi_t,\end{aligned} \tag{3}$$

where W_t is nominal wage, T_t is lump-sum transfer from government, and Φ_t is firms' real profits.

The household chooses consumption C_t, labor supply H_t, nominal bond holdings S_t, and money holdings M_t, so as to maximize the expected utility (1) subject to the budget constraint (3).

[5] $s\left(V_t\right)$ is non-negative, and is increasing in V_t if and only if V_t is greater than the satiation point $\underline{V} = \sqrt{\delta_2/\delta_1} > 0$.

Consumption Euler Equation. The first-order conditions (FOCs) for C_t and S_t yield the consumption Euler equation:

$$\mathbb{E}_t\left[\beta\frac{\Xi_{t+1}}{\Xi_t}\frac{Q_tR_{n,t}}{\Pi_{t+1}}\right]=1, \tag{4}$$

$$\text{with}\quad \Xi_t=\frac{1}{C_t(1+s\left(V_t\right)+V_ts'\left(V_t\right))}, \tag{5}$$

where $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate, and Ξ_t denotes the Lagrange multiplier for the household's budget constraint.

Money Demand. From the FOC for M_t, the money demand function is given by

$$V_t^2s'\left(V_t\right)=1-\frac{1}{Q_tR_{n,t}}. \tag{6}$$

Equation (6) describes the trade-off regarding money holdings. The benefit is to reduce the transaction cost of goods purchases represented in the left-hand side, whereas the cost is to lose the opportunity to earn the nominal interest as described in the right-hand side.

Wage Determination. We assume DNWR. Following SGU (2016), we impose

$$W_t\geq\gamma W_{t-1}. \tag{7}$$

The parameter γ governs the degree of DNWR. The higher γ is, the more downwardly rigid nominal wages are. This setup nests the cases of absolute downward rigidity if $\gamma \geq 1$ and full wage flexibility if $\gamma = 0$. Along with the household's FOC for H_t, real wages are determined according to

$$\frac{W_t}{P_t}=\max\left\{\chi_t\frac{H_t^{1/\eta}}{\Xi_t},\gamma\frac{W_{t-1}}{\Pi_tP_{t-1}}\right\}. \tag{8}$$

The first element in the maximum operator represents the marginal rate of substitution of labor supply for consumption, while the second element represents the DNWR in real terms. Notice that real wages can decline up to the inflation rate even though nominal wages are downwardly rigid.

2.2 Firms

There is a continuum of monopolistically competitive firms indexed by $i \in [0,1]$. Each firm produces a differentiated good $Y_t(i)$ using labor input $H_t(i)$ with a linear production technology:

$$Y_t(i) = A_t Z_t H_t(i), \tag{9}$$

where productivity is common for each firm and consists of a stationary component Z_t and a non-stationary one that grows with a deterministic trend $g = A_t/A_{t-1}$. The FOC for cost minimization of labor inputs suggests that firms' real marginal cost MC_t is given by $MC_t = W_t/(P_t A_t Z_t)$.

The output Y_t is given by the CES aggregator of individual outputs $Y_t = (\int_0^1 Y_t(i)^{(\theta-1)/\theta} di)^{\theta/(\theta-1)}$, where θ is the elasticity of substitution across goods. Each firm faces the individual demand curve:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\theta} Y_t, \tag{10}$$

where the corresponding price index is given by $P_t = (\int_0^1 P_t(i)^{1-\theta} di)^{1/(1-\theta)}$.

Phillips Curve. Firms have monopolistic power over their products and are therefore price setters. We assume that they set their prices under a staggered contract as in Calvo (1983). In each period, a fraction $\lambda \in (0,1)$ of firms keeps their prices unchanged, while the remaining fraction $(1-\lambda)$ of firms resets their prices. The reset price B_t maximizes the expected real profits:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \lambda^s \underbrace{\left(\beta^s \frac{\Xi_{t+s}}{\Xi_t}\right)}_{\Lambda_{t,t+s}} \underbrace{\left(\frac{B_t}{P_{t+s}} Y_{t+s|t} - MC_{t+s} Y_{t+s|t}\right)}_{\Phi_{t+s|t}}, \tag{11}$$

where $\Phi_{t+s|t}$ is the real profit at time $t+s$ of the firms that reset their prices at time t, and $\Lambda_{t,t+s}$ is the stochastic discount factor between time t and $t+s$, subject to the goods demand:

$$Y_{t+s|t} = \left(\frac{B_t}{P_{t+s}}\right)^{-\theta} Y_{t+s}. \tag{12}$$

Notice that we drop the firm index i because the optimization problem here is identical across the firms that reset their prices at time t.

The FOC for the optimization problem above is written in a recursive manner:

$$\frac{B_t}{P_t} = \frac{\Omega_{1t}}{\Omega_{2t}}, \tag{13}$$

$$\text{where} \quad \Omega_{1t} = \frac{\theta}{\theta - 1} MC_t \Xi_t Y_t + \lambda\beta\mathbb{E}_t\left[\Pi_{t+1}^{\theta}\Omega_{1t+1}\right], \tag{14}$$

$$\Omega_{2t} = \Xi_t Y_t + \lambda\beta\mathbb{E}_t\left[\Pi_{t+1}^{\theta-1}\Omega_{2t+1}\right]. \tag{15}$$

The price index can be rearranged to the equation below:

$$1 = (1-\lambda)\left(\frac{B_t}{P_t}\right)^{1-\theta} + \lambda\Pi_t^{\theta-1}. \tag{16}$$

It is worth noting that the first-order approximation of the conditions above around the zero-inflation steady state leads to the well-known linearized form of the New Keynesian Phillips curve,

$$\pi_t = \beta\mathbb{E}_t\left[\pi_{t+1}\right] + \kappa\widehat{mc}_t, \tag{17}$$

where $\kappa \equiv (1-\lambda)(1-\beta\lambda)/\lambda$, $\pi_t \equiv \ln(\Pi_t)$, and $\widehat{x}$ denotes the log-deviation of variable X from the steady state. In what follows, on the other hand, we explicitly take into account the non-linearity arising from non-zero steady-state inflation. The setting allows us to investigate the welfare consequences of different levels of the steady-state inflation rate.

Aggregate Production and Price Dispersion. By integrating individual production function over firms, the aggregate production is given by

$$Y_t = \frac{A_t Z_t H_t}{D_t}, \tag{18}$$

where the relative price dispersion D_t is defined by $D_t \equiv \int_0^1 (P_t(i)/P_t)^{-\theta} di$. By using the definition of the price index, we can derive a recursive formula for D_t:

$$D_t = \lambda\Pi_t^{\theta} D_{t-1} + (1-\lambda)\left(\frac{B_t}{P_t}\right)^{-\theta}. \tag{19}$$

The goods market clearing condition is given below:

$$Y_t = (1 + s(V_t))\, C_t. \tag{20}$$

2.3 Central Bank

Monetary Policy Rule. In this paper, we assume the central bank follows an interest rate feedback rule when setting the policy rate. We focus on simple feedback rules since they have an advantage in implementation and public communication (see SGU 2007).[6] Although we might consider the Ramsey problem, i.e., the optimal commitment policy, as an alternative, it induces additional state variables in the model because the policymaker incorporates agents' expectations when formulating the future policy path. The computational burden to add state variables makes our numerical analysis nearly infeasible given the current state of our numerical method. In other words, we focus on the feasible monetary policy options that can be implemented in our non-linear model setup.

We consider two cases for the monetary policy rule. The first case is the so-called Taylor rule, in which the central bank sets the nominal interest rate $R_{n,t}$ responding to deviations in the inflation rate Π_t from its steady-state rate Π^* and the output gap Y_t/Y_t^f with interest rate smoothing.[7] Y_t^f denotes the output in the cashless economy under flexible prices and wages. We also consider the ZLB. Consequently, the monetary policy rule is given as follows:

$$R_{n,t}^d = \left(R_{n,t-1}^d\right)^{\rho_r} \left\{ R^*\Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^f}\right)^{\phi_y} \right\}^{1-\rho_r}, \tag{21}$$

$$R_{n,t} = \max\left\{R_{n,t}^d, 1\right\}, \tag{22}$$

[6] In fact, a number of previous studies found that the actual policy rates set by central banks can be approximated by simple rules (e.g., Taylor 1993, Coibion and Gorodnichenko 2011).

[7] The steady-state inflation rate Π^* refers to the level of the inflation rate in the deterministic steady state. Recent studies pointed out that the deterministic steady state does not necessarily coincide with the risky steady state in a non-linear and stochastic environment (e.g., Coeurdacier, Rey, and Winant 2011). We analyze the differences between the deterministic and risky steady states in our model in online Appendix C (at http://www.ijcb.org).

where ϕ_π and ϕ_y are the long-run responsiveness to inflation and that to the output gap, and $\rho_r \in (0,1)$ is the degree of interest rate smoothing. $R^* \equiv g/\beta$ is the natural rate of interest in the steady state. Notice that a higher Π^* ensures more room to cut the nominal interest rate upon an adverse shock, which is the so-called safety margin. The size of the safety margin also depends on the level of the steady-state value of R^*. The derivation of Y_t^f is provided in online Appendix A (available at http://www.ijcb.org).

Although the Taylor rule is used extensively in the literature on monetary policy analysis, major central banks in developed economies have conducted unconventional monetary policies to overcome the ZLB problem, especially after the GFC. As an example of such unconventional measures, we investigate the effects of forward guidance. Specifically, we consider the history-dependent rule that was proposed by Reifschneider and Williams (2000, the RW rule hereafter) as the second case. The rule takes the form below:

$$R_{n,t}^b = R^* \Pi^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^f} \right)^{\phi_y}, \tag{23}$$

$$R_{n,t} = \max \left\{ \frac{R_{n,t}^b}{\Gamma_t}, 1 \right\}, \tag{24}$$

$$\Gamma_t = \Gamma_{t-1} \left(\frac{R_{n,t-1}}{R_{n,t-1}^b} \right). \tag{25}$$

Under the RW rule, the central bank keeps track of the gap between the benchmark interest rate $R_{n,t}^b$ that responds to inflation and the output gap, and the actual interest rate $R_{n,t}$. These gaps are accumulated in the term Γ_t, and $R_{n,t}$ is kept lower than $R_{n,t}^b$ as long as the gap remains, i.e., $\Gamma_t > 1$. In other words, once the economy is constrained at the ZLB, the central bank will continue its low interest rate policy even if the economy begins to soar at future dates.

Money Supply. Money is supplied passively to fulfill the household's money demand. The government makes a lump-sum transfer to balance the consolidated government budget:

$$M_t - M_{t-1} = T_t. \tag{26}$$

2.4 Exogenous Processes

We consider three exogenous disturbances: productivity Z_t; labor disutility χ_t; and risk premium Q_t, each of which is often used in the business cycle literature (e.g., Smets and Wouters 2007).[8] Among others, fluctuations in the risk premium are the main drivers that bring the economy to the ZLB.[9] For descriptive purposes, we refer to exogenous variations in the risk premium as "ZLB shocks."

Following CCGW, we consider a regime-switching shock to the risk premium. CCGW argued that the regime-switching shock is key to replicating the long-lived ZLB episodes observed in the data, while a standard AR(1) shock generates only short-lived ones. We assume that the risk premium consists of a regime-switching component Q_t^{rs} and an AR(1) component Q_t^{ar}:

$$\ln(Q_t) = \ln(Q_t^{rs}) + \ln(Q_t^{ar}). \tag{27}$$

The regime-switching component Q_t^{rs} follows a two-state Markov chain:

$$\ln(Q_t^{rs}) = \begin{cases} -\frac{p_{21}}{p_{12}+p_{21}}\Delta \\ \frac{p_{12}}{p_{12}+p_{21}}\Delta \end{cases}, \tag{28}$$

where $p_{ij} \in (0,1)$ denotes the transition probabilities from Regime i to Regime j for $i, j = 1, 2$ with $\sum_{j=1}^{2} p_{ij} = 1$, and $\Delta > 0$ represents the magnitude of the regime-switching shock. Notice that the value in each regime is adjusted such that $\mathbb{E}[\ln(Q_t^{rs})] = 0$. Regime 2 is a recession regime when high risk premiums cause the household to lose the desire to consume in the current period.

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[8]Previous studies often introduce markup shocks to the Phillips curve, which generates monetary policy trade-off. In our specification, productivity shocks, fluctuations in the stochastic discount factor, and wage markup generated by labor disutility shocks and DNWR act as disturbances in the Phillips curve in terms of the output gap.

[9]A rise of risk premium decreases current consumption by raising the rate of return on nominal bonds held by the household relative to the nominal interest rate set by the central bank. CGW argued that the fluctuations in risk premium have similar effects to net-worth shocks in a model with financial frictions. Moreover, they can be interpreted as exogenous shocks to the aggregate demand of the economy in a parsimonious manner, given the fact that risk premium appears in the consumption Euler equation.

The laws of motion of Z_t, χ_t, and Q_t^{ar} are given by the following equations:

$$\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \epsilon_t^z, \quad \epsilon_t^z \sim i.i.d.N(0, \sigma_z^2), \tag{29}$$

$$\ln(\chi_t) = \rho_\chi \ln(\chi_{t-1}) + \epsilon_t^\chi, \quad \epsilon_t^\chi \sim i.i.d.N(0, \sigma_\chi^2), \tag{30}$$

$$\ln(Q_t^{ar}) = \rho_q \ln\left(Q_{t-1}^{ar}\right) + \epsilon_t^q, \quad \epsilon_t^q \sim i.i.d.N(0, \sigma_q^2), \tag{31}$$

where ρ_z, ρ_χ, $\rho_q \in (0,1)$ are the autoregressive coefficients, and ϵ_t^z, ϵ_t^χ, ϵ_t^q are i.i.d. exogenous innovations that are normally distributed with mean zero and variance σ_z^2, σ_χ^2, σ_q^2, respectively.

2.5 Equilibrium

An equilibrium consists of a set of prices $\{P_t, W_t, R_{n,t}\}_{t=0}^{\infty}$ and the allocations $\{Y_t, H_t, C_t, D_t, \; S_t, M_t, T_t, Y_t^f \; \}_{t=0}^{\infty}$, given exogenous variables $\{A_t, Z_t, \chi_t, Q_t\}_{t=0}^{\infty}$, such that the following conditions are satisfied for all t: (i) the household maximizes its utility; (ii) each firm maximizes its profits; (iii) the central bank sets the policy rate following the feedback rule; (iv) the consolidated government budget constraint holds; and (v) markets clear.

2.6 Social Welfare

We define social welfare as the unconditional expectation of the representative household's utility:

$$\mathbb{E}\left[\ln(C_t) - \frac{1}{1+\frac{1}{\eta}}\chi_t H_t^{1+\frac{1}{\eta}}\right]. \tag{32}$$

In what follows, we consider the cashless economy under flexible prices and wages as the benchmark. Then, we measure welfare losses as the deviations of the social welfare in the model economy from that in the benchmark economy. Formally, we define the consumption-equivalent welfare losses CE, the consumption changes that make the social welfare in the benchmark economy equal to that in the distorted economy, as

$$\mathbb{E}\left[\ln\left(C_t\right) - \frac{1}{1+\frac{1}{\eta}}\chi_t H_t^{1+\frac{1}{\eta}}\right] = \mathbb{E}\left[\ln\left((1+CE)C_t^f\right) - \frac{1}{1+\frac{1}{\eta}}\chi_t (H_t^f)^{1+\frac{1}{\eta}}\right], \tag{33}$$

where C_t^f and H_t^f denote, respectively, the consumption and labor input in the cashless economy under flexible prices and wages.

2.7 Numerical Method

One important issue in quantitative analysis is how we deal with the non-linearity of our model. The presence of DNWR and the ZLB introduces kinks into equilibrium conditions. Therefore, the perturbation method, which is used to solve a wide range of New Keynesian models, cannot be applied to our model. To address this issue, we numerically solve our model using the policy function iteration method of Coleman (1990). The method allows us to explicitly take into account the non-linearity. In addition, the method is applicable to the regime-switching environment. The details of the computation method are described in online Appendix B. Once we solve our model, we conduct stochastic simulations to evaluate welfare losses. At this stage, we approximate the unconditional expectation operator by taking the mean of the simulated series.

3. Calibration

3.1 Differences between Japan and the United States

The calibrated parameter values are illustrated in Table 1. The time frequency is quarterly. In our calibration, we take into account the differences in economic structure and experience of ZLB episodes between Japan and the United States as below. For reference, relevant data series are presented in online Appendix E.

Table 1. Calibrated Parameters

Symbol	Parameter	Japan	United States	Source
	Steady-State Values			
r^*	Natural Rate of Interest (Annual Rate)	0.95%	1.84%	Estimated by the method of Laubach and Williams (2003)
	Parameters for Utility Function			
β	Subjective Discount Factor	0.9975	0.9975	Externally fixed
η	Frisch Labor Supply Elasticity	0.82	1.00	JP: Kuroda and Yamamoto (2008)/US: CGW (2012)
	Parameters for Transaction Cost			
δ_1	Parameter in Transaction Cost	0.01110	0.01110	SGU (2004)
δ_2	Same as Above	0.07524	0.07524	SGU (2004)
	Parameters for Price and Wage Setting			
θ	Elasticity of Substitution across Individual Goods	7.00	7.00	CGW (2012)
λ	Degree of Price Stickiness	0.65	0.65	Nakamura and Steinsson (2008)
γ	Degree of Downward Nominal Wage Rigidity	0.9978	1.0045	Calibrated by the method of SGU (2016)

(continued)

Table 1. (Continued)

Symbol	Parameter	Japan	United States	Source
	Parameters for Monetary Policy Rule			
ϕ_π	Long-Run Responsiveness to Inflation	2.50	2.50	
ϕ_y	Long-Run Responsiveness to the Output Gap	0.25	0.25	JP: Sudo and Tanaka (2021)/US: Coibion and Gorodnichenko (2011)
ρ_r	Interest Rate Smoothing	0.90	0.90	
	Parameters for Exogenous Processes			
p_{12}	Transition Probability from Regime 1 to Regime 2	3.13%	1.14%	Frequency and duration of ZLB episodes
p_{21}	Transition Probability from Regime 2 to Regime 1	2.94%	3.45%	
Δ	Size of Regime-Switching Shock	0.0070	0.0135	Drop of output during ZLB episodes
ρ_z	Persistence of Productivity	0.90	0.90	Externally fixed
ρ_χ	Persistence of Labor Disutility	0.70	0.70	Externally fixed
ρ_q	Persistence of AR(1) Component of Risk Premium	0.85	0.85	Externally fixed
σ_z	S.D. of Innovations to Productivity	0.0015	0.0015	Externally fixed
σ_χ	S.D. of Innovations to Labor Disutility	0.0030	0.0030	Externally fixed
σ_q	S.D. of Innovations to AR(1) Component of Risk Premium	0.0025	0.0020	Externally fixed

3.1.1 Degree of DNWR: γ

For calibrating the degree of DNWR γ, we follow the method of SGU (2016). They set γ equal to the nominal wage growth during a severe recession period, such as the GFC, in which labor market was under strong downward pressure and therefore the lower bound of nominal wage growth was likely to bind. Specifically, the parameter is given by $\gamma = (W_{t_1}/W_{t_0-1})^{1/(t_1-t_0+1)}$, where t_0 and t_1 are the start and end periods of the severe recession episode and SGU (2016) chose a three-year window for the duration of episode.

For the U.S. calibration, we calculate the nominal wage growth rate during the three-year window around the GFC, i.e., 2008:Q1–2010:Q4. 2008:Q1 is the starting quarter for the recession according to the National Bureau of Economic Research (NBER) business cycle dating, and the window covers the periods when the unemployment rate peaked after the GFC. For Japan, we focus on two severe recession episodes: the recession starting from 2000:Q4, which followed Japan's banking crisis in the late 1990s, and the GFC starting from 2008:Q1. Both dates coincide with the business cycle peak of the previous expansion defined by the Cabinet Office of Japan. We calculate the three-year nominal wage growth rate for each recession episode, i.e., 2000:Q4–2002:Q3 and 2008:Q1–2010:Q4, and then take the mean of these two values. Note that Japan experienced negative nominal wage growth (of around 1 percent in annual rate) together with record high level of the unemployment rate during the two recession episodes.

Our measure of nominal wage is compensation per hour for both the United States and Japan. The calibrated γ takes into account a broad range of wage adjustment, including those using extra pay and benefits. Our wage measure is thus relevant for profit maximization of the firms that take into account a wide range of labor costs when setting prices and adjusting labor inputs. For the United States, we use the compensation per hour in the non-farm business sector. For Japan, we construct a series of total hours worked by multiplying the number of employees and the hours,worked per employee together, and then divide the total compensation of employees in the System of National Accounts (SNA) by the total hours,worked to obtain a series of compensation per hour. Data sources are provided in the notes for Table 2.

Table 2. Model Fits

Moment	Symbol	Variable	Japan		United States	
			Data	**Model**	**Data**	**Model**
Standard Deviation ×100	Y	Output	1.42	1.36	1.21	1.12
	C	Consumption	1.00	1.37	1.02	1.13
	H	Labor Input	0.82	1.38	1.77	1.14
	π	Inflation Rate	0.27	0.52	0.25	0.34
	π^w	Wage Inflation Rate	1.02	0.86	0.72	0.68
	R	Nominal Interest Rate	0.62	0.40	0.55	0.32
First-Order Auto-correlation	Y	Output	0.83	0.58	0.89	0.58
	C	Consumption	0.60	0.58	0.86	0.58
	H	Labor Input	0.62	0.54	0.95	0.56
	π	Inflation Rate	0.76	0.64	0.77	0.67
	π^w	Wage Inflation Rate	–0.22	0.19	0.08	0.19
	R	Nominal Interest Rate	0.95	0.92	0.94	0.93
Correlation with Output	Y	Output	1.00	1.00	1.00	1.00
	C	Consumption	0.82	1.00	0.92	1.00
	H	Labor Input	0.79	0.94	0.90	0.91
	π	Inflation Rate	0.48	0.55	0.15	0.69
	π^w	Wage Inflation Rate	0.20	0.66	0.05	0.68
	R	Nominal Interest Rate	0.28	0.08	0.49	0.06

Source: Data series in Japan: Cabinet Office, "System of National Accounts"; Ministry of Health, Labour and Welfare, "Monthly Labour Survey"; Ministry of Internal Affairs and Communications, "Labour Force Survey," "Consumer Price Index"; Bank of Japan, "Call Money Market Data." Data series in the United States: Bureau of Economic Analysis, "National Income and Product Accounts"; Bureau of Labor Statistics, "Current Employment Statistics"; Board of Governors of the Federal Reserve System, "Selected Interest Rates H.15"; Organization for Economic Co-operation and Development, "Main Economic Indicators." The series are retrieved from FRED provided by the Federal Reserve Bank of St. Louis.

Note:

1. The data moments are computed in the sample before the ZLB periods begin in each country. The sample period is 1985:Q1–1998:Q3 for Japan, and 1987:Q4–2008:Q4 for the United States.
2. The model moments are those in Regime 1 where a contractionary regime-switching shock is not present. For the simulation, the steady-state inflation rate is set to the mean inflation rate during 1985:Q1–2017:Q4 for Japan and 1987:Q4–2017:Q4 for the United States.
3. For the data series in Japan, the output is the GDP and the consumption is the private consumption in the System of National Accounts (SNA), deflated by the consumer price index (CPI, less fresh food). The labor input is the number of employees based on the Labour Force Survey, multiplied by hours worked per employee based on the Monthly Labour Survey. The inflation rate is the CPI (less fresh food). The series is adjusted for the introduction of the consumption tax and changes in the rates. The wage inflation rate is constructed from the compensation of employees in the SNA, divided by the labor input. The nominal interest rate is the uncollateralized overnight call rate after 1985:Q3, while the collateralized call rate is used before then due to the availability of the data.
4. For the data series in the United States, the output is the GDP and the consumption is the personal consumption expenditure (PCE) in the National Income and Product Accounts (NIPA), deflated by the PCE deflator (less food and energy). The labor input is total hours worked in the non-farm business sector. The inflation rate is the PCE deflator (less food and energy). The wage inflation rate is the compensation per hour in the non-farm business sector. The nominal interest rate is the effective federal funds rate.
5. The output, consumption, and labor input are on a per working-age person basis. These series are detrended using the Hodrick-Prescott filter.
6. The inflation rate and the wage inflation rate are on a quarter-on-quarter change.
7. The nominal interest rate is in quarterly rate.

Applying SGU (2016)'s method yields $\gamma = 0.9978$ for Japan, implying nominal wages decline up to around -0.9 percent in annual rate. In contrast, we obtain $\gamma = 1.0045$ for the United States, which means that the lower bound of nominal wage changes is around $+1.8$ percent.

The difference in the calibrated values aims to capture the empirical facts about the degree of DNWR for Japan and the United States: while Kuroda and Yamamoto (2005) reported that the DNWR that was measured using the total annual earnings of full-time employees in Japan disappeared after the late 1990s, studies on the U.S. economy such as Daly and Hobijn (2014) and Fallick, Lettau, and Wascher (2020) found that the nominal wages of individual workers were downwardly rigid even in the severe downturn after the GFC. Note, γ can take value greater than one in the aggregate, as we do for the U.S. economy, when some workers receive wage increases whereas other workers' wages are absolutely downward rigid.[10]

3.1.2 Steady-State Level of Natural Rate of Interest: R^*

We set the steady-state value of the natural rate of interest R^* equal to the time average of the estimates using the method of Laubach and Williams (2003), a seminal work of the estimation of the natural rate of interest. The sample period to calculate the average of R^* is 1985:Q1–2017:Q4 for Japan, and 1987:Q4–2017:Q4 for the United States. These periods correspond to the era of Great Moderation in which inflation rates and nominal interest rates became stable at low levels. By choosing these periods, we aim to replicate an environment that is seemingly calm, but potentially vulnerable to the ZLB constraint.

The calibrated R^* is 0.95 percent in annual rate in Japan whereas it is 1.84 percent in the United States. The lower value of R^* in Japan can be interpreted as an outcome of lower technological growth due to long-lasting stagnation in information technology (IT) and research and development (R&D) investment, and lower population growth in Japan (see, e.g., Fukao 2013 for the former hypothesis and Ikeda and Saito 2014 for the latter). This lower value of R^* implies

[10]SGU (2016) reported that the calibrated value exceeds one for many peripheral countries of Europe after the GFC.

that the Japanese economy is more often constrained at the ZLB given exogenous shocks.

Note that we interpret these values as the steady-state levels that are achieved when all the exogenous shocks are muted in the economy without nominal frictions. However, our quarter-by-quarter estimates of the natural rate of interest exhibit a mild downward trend,[11] as it is also reported in recent studies (see, e.g., Holston, Laubach, and Williams 2017). Hence the calibrated values of R^* in our study may understate the possibility of a permanent shift of R^* toward a lower level. To assess the impact of this possibility, we conduct a sensitivity analysis with respect to R^* in Section 6.

3.1.3 Frequency, Duration, and Size of ZLB Shocks: p_{12}, p_{21}, and Δ

The calibrated values of R^* are not powerful enough to generate the prolonged ZLB spells observed in Japan and the United States. We consider that the remaining gap between the model and data in terms of the frequency, duration, and severity of the ZLB episodes is driven by the exogenous ZLB shocks.

We calibrate the transition probabilities in the Markov chain of the regime-switching component of ZLB shocks, p_{12} and p_{21}, to match the frequency and duration of ZLB periods observed in the data. The ZLB periods in the data are defined as those periods when the short-term nominal interest rate stayed below 0.25 percent in annual rate. According to the criterion, the ZLB periods in Japan are 1998:Q4–2006:Q3 (32 quarters) and 2009:Q1–2015:Q4 (36 quarters), which we pick from our sample period for calibrating R^* (1985:Q1–2017:Q4, 132 quarters). In contrast, we observe only one ZLB episode for the United States, 2009:Q1–2015:Q4 (29 quarters), in our sample (1987:Q4–2017:Q4, 117 quarters). The unconditional probability of staying at the recession regime (Regime 2), $p_{12}/(p_{12} + p_{21})$, is

[11]For both Japan and the United States, the estimates of the natural rate of interest are around 3 percent in annual rate at the beginning of the sample period and are around 0 percent at the end of the sample. However, the estimates for Japan started to decline in the 1990s whereas those for the United States have gradually declined since around the 2000s, leading to a lower value for Japan on average.

matched with the frequency of ZLB episodes during the sample periods described above, which is 51.5 percent (= (32+36)/132) in Japan and 24.8 percent (= 29/117) in the United States. The expected duration of recession regime, $1/(1 - p_{21})$, is calibrated according to the (average) duration of ZLB episodes in the data, which is 34 quarters (= $(32 + 36)/2$) in Japan and 29 quarters in the United States.

Consequently, we obtain $p_{12} = 3.13$ percent and $p_{21} = 2.94$ percent for Japan, whereas we obtain $p_{12} = 1.14$ percent and $p_{21} = 3.45$ percent for the United States. The calibrated values suggest that the Japanese economy receives the ZLB shocks more often (higher p_{12}) and tends to remain in the recession regime for longer periods (lower p_{21}), reflecting the prolonged ZLB episodes in Japan.

The magnitude of the regime-switching shock Δ is calibrated to match the decline in the output gap during the ZLB periods in the data. To be precise, we match the declines in the output gap in Regime 2 of the model with those during two-year windows since the ZLB periods begin in the data.[12] The calibrated value is $\Delta = 0.0075$ for Japan and $\Delta = 0.0135$ for the United States. The smaller value for Japan is partly because the lower R^* in Japan causes ZLB episodes and resulting recessions even with relatively smaller shocks.

As for the implications of these differences between Japan and the United States, the lower degree of DNWR implies that the optimal inflation rate would be lower in Japan than in the United States, other things being equal. On the other hand, the lower steady-state value of the natural rate of interest and the frequent and long-lasting ZLB shocks would lead to a higher optimal inflation rate in Japan.

3.2 Other Parameter Values

For utility function, the subjective discount factor β is set to 0.9975. We set the Frisch labor supply elasticity η equal to 0.82 for Japan as in the empirical findings of Kuroda and Yamamoto (2008) and

[12]The choice of two-year windows broadly corresponds to the length of recession episodes around the beginning of the ZLB periods in the data. Regarding the data series, the output gap estimated by the Bank of Japan Research and Statistics Department is employed for Japan. For the United States, the output gap is the difference between the GDP in the National Income and Product Accounts (NIPA) and the potential GDP estimated by the Congressional Budget Office.

to 1.00 for the United States following CGW.[13] The parameters in the money demand function are taken from SGU (2004), that is, $\delta_1 = 0.01110$ and $\delta_2 = 0.07524$. The implied interest rate semi-elasticity of money demand in our model is around 1.5, which is consistent with the empirical estimate by Ireland (2009). We set the elasticity of substitution across individual goods θ equal to 7 following CGW. The degree of price stickiness λ is set to 0.65, based on the frequency of price changes reported by Nakamura and Steinsson (2008).[14] The value implies that the average duration of price changes is around three quarters. For monetary policy rules, the long-run responsiveness to inflation ϕ_π is 2.50 and that to the output gap ϕ_y is 0.25 with an interest rate smoothing parameter ρ_r equal to 0.90. These values are broadly consistent with the estimates obtained by Sudo and Tanaka (2021) for Japan, and Coibion and Gorodnichenko (2011) for the United States. Regarding exogenous processes, the parameter values for the persistence of productivity ρ_z, labor disutility ρ_χ, and the AR(1) component of risk premium ρ_q are set equal to 0.90, 0.70, and 0.85, while those for the standard deviation of innovations to productivity σ_z and labor disutility σ_χ are set to 0.0015 and 0.0030, respectively. We set the standard deviation of innovations to the AR(1) component of risk premium σ_q to 0.0025 for Japan and to 0.0020 for the United States to match the variations in output for each country.

3.3 Model Fits Regarding Non-ZLB Moments

Table 2 shows selected moments of the data and those of the simulated series of the calibrated model of the two countries when the ZLB is not binding. Specifically, the data moments show the business

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[13] Kuroda and Yamamoto (2008) reported that the Frisch labor supply elasticity fell in a range from 0.67 to 0.97 using Japanese micro data. We use the arithmetical mean of these two values.

[14] This value was reported for the U.S. economy. Although we could not find a comparable study for the Japanese economy, several studies indicate similar frequencies of price changes in the Japanese economy. For example, Kurachi, Hiraki, and Nishioka (2016) reported that the frequency of price changes is 6.5 percent per month in Japanese disaggregated price data after excluding temporary price changes by the running-mode filter of Kehoe and Midrigan (2015). The estimate is close to 6.9 percent per month reported by Kehoe and Midrigan (2015) for U.S. data.

cycle properties before the first ZLB episode unfolded in each country, while the model moments show those under Regime 1 described in Section 2. Though the model is not fully successful in matching all the business cycle moments in the data, partly due to the lack of a number of elements introduced in medium-scale dynamic stochastic general equilibrium (DSGE) models à la Smets and Wouters (2007), the model does capture the salient features of business cycles in both countries including (i) smaller standard deviation of the inflation rate and the wage inflation rate relative to that of output; (ii) moderate persistence of the inflation rate; and (iii) positive co-movements among variables. On the other hand, the persistence of output and inflation in the model is lower than that in the data. We introduce habit formation in consumption to add persistence to the model in Section 6 and assess the sensitivity of our quantitative results.

3.4 Model Fits Regarding Frequency and Duration of ZLB Episodes

Table 3 presents the frequency and average duration of ZLB episodes in the data and model as well as the corresponding moments reported by previous studies. The table shows that the model replicates the high frequency and long duration of ZLB episodes observed in the data. Especially, our regime-switching shock generates longer ZLB spells once the economy falls into Regime 2, the ZLB regime (see Table 3A).[15]

We can confirm this argument by comparing our result with those in the existing studies that only considered AR(1) shocks (see Table 3B). In those studies, including CGW, that are calibrated or estimated to the U.S. economy, the frequency of ZLB episodes

[15]Though the transition probabilities of a regime-switching shock are set to match these moments in the data, the frequency and duration of ZLB episodes in the simulated series of the model can deviate from those in the data for several reasons. First, in the simulation, short-lived ZLB episodes can be generated upon contractionary AR(1) shocks even in the low risk premium regime. Second, expansionary AR(1) shocks can bring the economy out of the ZLB in the high risk premium regime. In particular, in the case of Japan, since the steady-state interest rate is relatively low, short-lived ZLB episodes are often generated in the simulation, leading to shorter duration than in the data. Third, the interest rate smoothing term in the monetary policy rule renders the actual interest rate deviate from the state of the regime.

Table 3. Moments of ZLB Episodes

A. Moments of ZLB in Data and Model

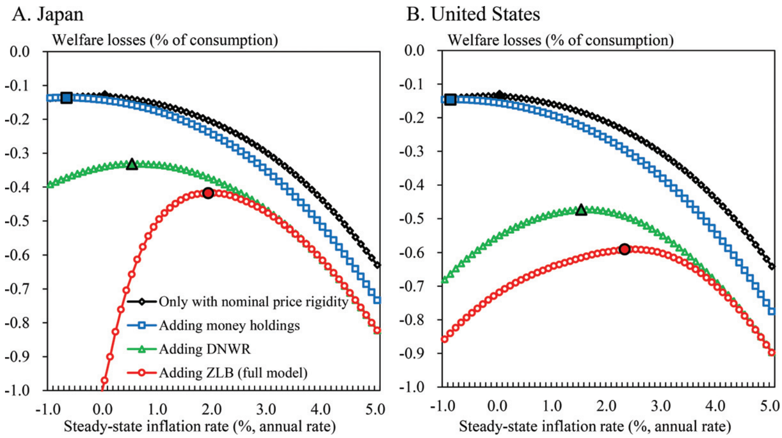
Moment	Japan			United States		
	Data	Model		Data	Model	
		Full Model	Without AR(1)		Full Model	Without AR(1)
Frequency of ZLB Episodes (%)	51.4	46.0	(49.0)	24.8	18.4	(19.6)
Average Duration of ZLB Episodes (Quarters)	34.0	26.8	(35.5)	29.0	29.3	(29.7)

B. Moments of ZLB in Previous Studies

	Frequency of ZLB Episodes (%)	Average Duration of ZLB Episodes (Quarters)	S.S. Inflation Rate (%, Annual Rate)	Optimal Inflation Rate
Billi (2011)	4.0	2.0	0.8	0.8
Carreras et al. (2016)	>20.0	>20.0	—	>4%
Coibion, Gorodnichenko, and Wieland (2012)	5.0	N.A.	3.5	<2%
Fernandez-Villaverde et al. (2015)	5.5	2.1	2.0	—
Gust et al. (2017)	4.0	3.5	2.5	—
Hirose and Sunakawa (2019)	12.1	4.3	2.4	—
Kiley and Roberts (2017), DSGE	7.8	6.5	2.0	>2%
Kiley and Roberts (2017), FRB/US	12.8	8.3	2.0	1–2%

Note:
1. For panel A, ZLB periods are defined as those in which the short-term nominal interest rate is below an annual rate of 0.25 percent during the sample of 1985:Q1–2017:Q4 (132 quarters) in Japan and 1987:Q4–2017:Q4 (117 quarters) for the United States. The details of the data series are described in the note for Table 2. According to the criterion, the periods 1998:Q4–2006:Q3 (32 quarters) and 2009:Q1–2017:Q4 (36 quarters) are classified as the ZLB periods in Japan, compared with 2009:Q1–2015:Q4 (29 quarters) in the United States. It follows that the frequency of ZLB episodes in the sample period is 51.5 percent (=(32+36)/132) in Japan, and 24.8 percent (=29/117) in the United States. The average duration of ZLB episodes in Japan is 34 quarters (=(32+36)/2).
2. For the model simulation in panel A, the steady-state inflation rate is set to the mean inflation rate during the sample period above, which is 0.38 percent in annual rate for Japan and 2.08 percent for the United States. The central bank is assumed to follow the Taylor rule.
3. The model moments that are reported in parentheses in panel A are those in the specification without AR(1) shocks. The average duration without parentheses excludes the ZLB episodes whose duration is less than eight quarters, to exclude short-lived ZLB episodes.
4. Unless otherwise noted, panel B reports the moments in the baseline specification of each paper. Billi (2011) reported the moments of ZLB episodes and the average inflation rate under the Ramsey policy. Carreras et al. (2016) reported that various frequency and duration—in particular, higher frequency and longer duration—can be generated by changing the parameter values for the transition probabilities of the regime-switching shock. Kiley and Roberts (2017) used the DSGE model and the FRB/US model, and the moments reported in the table are those when the steady-state real interest rate is 2 percent.

Figure 2. Welfare Losses under Taylor Rule



Note: Welfare losses are the deviations of social welfare from that in the cashless economy under flexible prices and wages. They are measured in terms of the percent of period consumption. The plot point with a filled marker indicates the optimal inflation rate in each specification (the same hereafter). The central bank is assumed to follow the Taylor rule.

typically ranges from 5 to 10 percent and the average duration is up to around five quarters. One notable exception is CCGW, who argued that the ZLB frequency and spell observed in the post-war period in the United States can be attained by introducing the regime-switching shock. Yet their study abstracts away from money holdings—which works out to reduce the optimal inflation rates—and DNWR—which prevents prices from falling, hence preventing real interest rates from shooting up in severe recessions. As a result, they justify a high optimal inflation rate above 4 percent for the United States.

4. Baseline Results

4.1 Welfare Losses under Taylor Rule

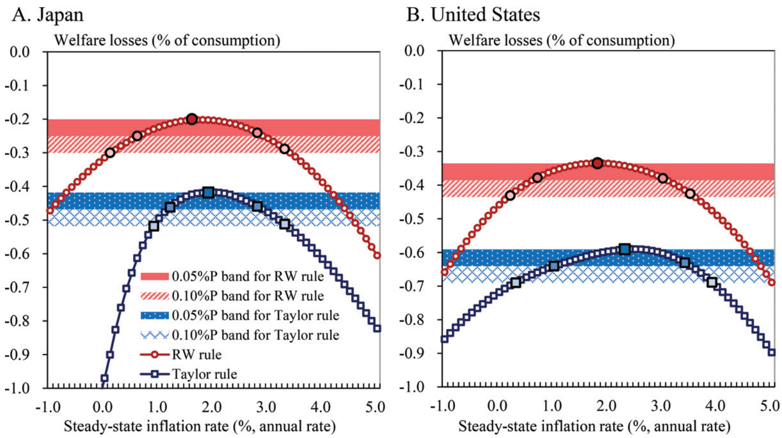
Figure 2 shows the welfare losses under different levels of the steady-state inflation rate Π^* when the central bank is assumed to follow the Taylor rule (21) and (22). In the figure, welfare losses are expressed as negative values, implying that an upper point delivers a smaller welfare loss. Each line corresponds to the specification that includes some or all of the four factors affecting the costs and benefits of inflation described in Section 2. We start by computing the welfare losses when only nominal price rigidity is present in the model. Then, we add money holdings, DNWR, and the ZLB sequentially, and examine their impact on the optimal inflation rate.[16]

Several points from the figure are noteworthy. First, in the specification that only includes nominal price rigidity as a source of welfare losses (the line with diamonds), 0 percent inflation in the steady state maximizes social welfare. This is consistent with the theoretical implication that both inflation and deflation generate welfare losses through the relative price dispersion. Second, when adding money holdings (the line with squares), the optimal inflation rate becomes negative because a lower nominal interest rate resulting from a lower Π^* reduces the opportunity costs of holding money. Third, incorporating DNWR (the line with triangles) and the ZLB (the line with circles) leads to a positive optimal inflation rate because positive inflation reduces the probability that these constraints bind. Fourth, when comparing Japan and the United States, the consequences of nominal price rigidity and money holdings are quite similar in the two countries. Regarding the benefits of inflation, however, the ZLB is the main driver justifying a positive optimal inflation rate in Japan, whereas DNWR plays a key role in the United States. These results are consistent with our calibration. In the full model that includes all four factors, the optimal inflation rate is an annual rate of 1.9 percent for Japan, while it is slightly higher for the United States at 2.3 percent.

The non-linearity of our model affects the level of the optimal inflation rate in a non-negligible manner. This is most notable for the effect of the ZLB for Japan. More frequent and longer ZLB incidences for Japan than for the United States induce significant

[16]We present the results when each element is added in the specific ordering. In this regard, it is true that in a non-linear environment the contribution of each element is not additively separable. However, we choose to consider this ordering because we find that, in some specifications, especially when the model does not embed DNWR but includes the ZLB, searching for equilibrium is not robustly attained for a wide range of Π^*. Related issues are discussed by Richter and Throckmorton (2015).

Figure 3. Welfare Losses under RW Rule



Note: The solid band and the band with pin-dots indicate the range in which the decline in social welfare from its maximum is within 0.05 percentage point, whereas the band with diagonal lines and the band with diagonal grids are those within 0.10 percentage point.

welfare losses when Π^* is set below the optimum: the curve for welfare loss for Japan is much steeper than the case for the United States for Π^* ranging between 0 to 1 percent, pointing to the importance of accounting for the non-linearity associated with the ZLB. Nonetheless, the curve becomes flat quickly when Π^* approaches 2 percent. This is due to the fact that the marginal benefit of having a larger safety margin—reduction in the ZLB probability and duration—diminishes with our calibration. We will discuss this point in Section 5 where we analyze the frequency and duration of ZLB under different Π^*.

4.2 Welfare Losses under RW Rule

In Figure 3, we show welfare losses under different levels of Π^* when the RW rule (23)–(25) is implemented (the line with circles). For ease of presentation, we only show the welfare losses when all four factors are introduced.

Under the RW rule, the welfare losses are smaller than those when the central bank follows the Taylor rule (the blue line with squares). (For figures in color, see the online version of the paper at http://www.ijcb.org.) This is because the RW rule mitigates the adverse effects of the ZLB by committing to a prolonged low interest rate policy when the economy is constrained at the ZLB. Since the benefits of holding the safety margin in the nominal interest rate provided for the ZLB are weakened under the RW rule, the optimal inflation rate is slightly reduced, compared with the case under the Taylor rule, to 1.6 percent in Japan and 1.8 percent in the United States. That being said, the estimates of the optimal inflation rate under the RW rule do not deviate too far from the conventional wisdom of 2 percent.

4.3 Welfare Consequences of Shifts in Steady-State Inflation Rate

Figure 3 also evaluates the changes in the welfare losses when Π^* deviates from the estimates in the previous analysis. The bands in the figure indicate the range of Π^* at which the decline in social welfare from its maximum remains within 0.05 and 0.10 percentage point in terms of the consumption-equivalent losses. For reference, 0.05 percent of consumption-equivalent losses amounts to roughly 20 to 30 U.S. dollars per working-age person in each year.[17]

From this figure, we can see that around 1 percentage point absolute deviation from the rate of close to 2 percent induces only a minor change in social welfare. In the case of Japan, the 0.05 percentage point band implies a range of Π^* from 1.2 to 2.8 percent under the Taylor rule. Moreover, implementing the RW rule substantially reduces the lower end of the range that is acceptable in terms of welfare losses because the RW rule is effective in mitigating the adverse effects of the ZLB even when Π^* is low. To be precise, the range is widened to include Π^* from 0.6 to 2.8 percent under the RW rule. Turning to the case of the United States, the 0.05 percentage point band forms a range of Π^* from 1.0 to 3.4 percent under the Taylor rule, and from 0.7 to 3.0 percent under the RW rule.

[17] These values are based on consumption per working-age person, which is calculated by dividing the consumption expenditure in the GDP statistics by the population aged 15 to 64 years old. As of 2017, 0.05 percent of consumption per working-age person is equivalent to 2,001 Japanese yen for Japan, and 32.4 U.S. dollars for the United States.

5. Determinants of Optimal Inflation Rates

In this section, we explore the determinants of the optimal inflation rates obtained in Section 4. Figure 4 displays the mean and standard deviation of key variables for Japan and the United States, respectively, under different levels of Π^* to see their effects on the dynamics of the model.

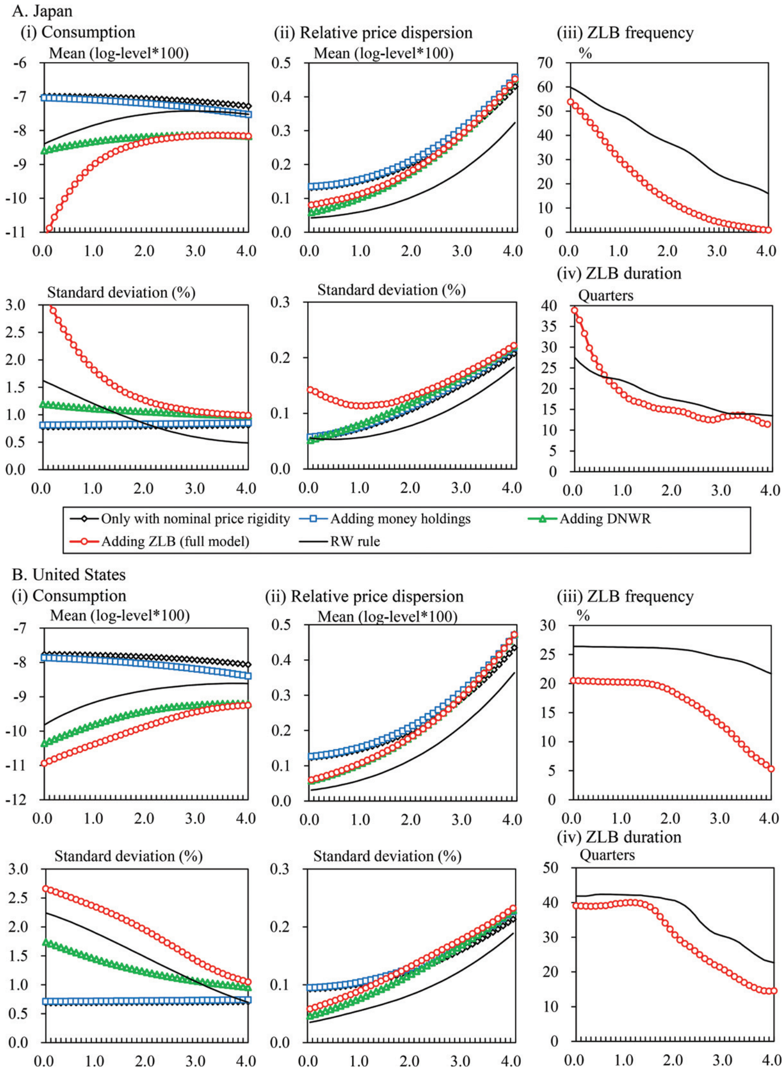
Several points are noteworthy in the figure. First, the specification only with nominal price rigidity (the lines with diamonds) and adding money holdings (the lines with squares) shows that the mean consumption shown in panel (i) is monotonically decreasing starting from a certain level of Π^* (from zero for only with the nominal price rigidity, and from a negative territory for adding money holdings). These are exactly stemming from a larger relative price dispersion (see panel (ii)) and a higher opportunity cost of money holdings under a higher Π^*.

Second, when adding DNWR (the lines with triangles), the mean consumption becomes increasing in Π^* at low inflation rates. This implies that the binding DNWR constraint at low inflation rates prevents wage adjustments upon contractionary shocks, leading to inefficiently low labor input, output, and consumption. This low average consumption is accompanied by high standard deviation since the level of economic activity diverges between binding and non-binding DNWR periods. Both of these mean and standard deviation effects increase welfare losses given concavity of utility function.

Third, when adding the ZLB (the lines with circles), the mean consumption becomes further lower and the standard deviation becomes higher under a low Π^*, due to the lack of offsetting monetary policy responses to exogenous shocks at the ZLB. Moreover, implementing the RW rule (the lines without markers) offsets the adverse effects of the ZLB as discussed in Section 4, though the implications of the ZLB are qualitatively unchanged.

It is worth mentioning that the costs associated with the ZLB are almost parallel to the frequency and duration of ZLB episodes for Japan, where DNWR affects social welfare to the lesser degree than in the United States (panels (iii) and (iv)). The level of consumption increases and its volatility declines as the frequency and duration of ZLB episodes decline in line with the increase in Π^* toward 2 percent from below. However, this benefit diminishes when Π^* surpasses

Figure 4. Time-Series Moments



Note: The moments are the unconditional mean of those in each regime. The upper and lower 2.5 percentile samples are removed to exclude the large variations when the regime is switched. The x-axis is the steady-state inflation rate. In (i) and (ii), the upper panels are the mean while the lower panels are the standard deviation of each variable. In (iv), the average duration excludes the ZLB episodes whose duration is less than eight quarters, to exclude short-lived ones.

2 percent, above which we do not observe a meaningful reduction in the ZLB frequency and duration. Moreover, the marginal cost of inflation in the form of price dispersion and the opportunity cost of money holdings begins to increase at that point, leading to the bliss point of Π^* around 2 percent.

6. Sensitivity Analysis

Though the baseline calibration reflects key moments of the data that potentially affect the optimal inflation rate, uncertainty regarding the parameter values and model specifications may have non-negligible impacts on the optimal inflation rate. In this section, we provide various sensitivity analyses on our baseline quantitative results presented in Section 4. We focus on (i) parameter uncertainty regarding the natural rate of interest, (ii) that regarding the ZLB shock, (iii) model specification on habit formation in consumption, (iv) that on money holdings, and (v) the piecewise-linear solution.

6.1 Parameter Uncertainty Regarding Natural Rate of Interest

Parameter uncertainty with respect to the ZLB, such as the level of the natural rate of interest and the specification of ZLB shocks, is considerably high because the data offers limited instances of ZLB episodes. In particular, as is recognized in the literature, the measurement of the natural rate of interest is subject to considerable uncertainty. Since the level of the natural rate of interest along with the inflation rate constitutes the distance of the nominal interest rate from the ZLB, it is one of crucial factors in determining the optimal inflation rate. While we use the time average of the estimates based on the Laubach and Williams (2003) model as the steady-state value of the natural rate of interest R^* in our baseline calibration, we consider alternative specifications in which the value deviates upwardly and downwardly by one standard deviation of the estimated series. Specifically, given that the mean of the estimated natural rate of interest is 0.95 percent with the standard deviation of 1.45 for Japan, we consider the natural rate of –0.50 and 2.40 percent for Japan. The corresponding alternatives for the United States are 0.75 and

2.93 percent, while the mean of the estimate is 1.84 percent and the standard deviation is 1.09.[18]

The upper panels of Figure 5 show the welfare losses when assuming the higher and lower R^*. Here we assume that the central bank follows the RW rule. In the figure, the higher (lower) R^*, the lower (higher) the optimal inflation rate. In principle, a lower R^* requires a higher Π^* to ensure the same size of the safety margin. Besides, the relationship is not necessarily one-to-one: consistent with Andrade et al. (2019), we find that the corresponding changes in the optimal inflation rate are smaller than the changes in R^*. For example, while the standard deviation of the estimated natural rate of interest is 1.45 in Japan, the resulting shift in the optimal inflation rate is around 0.8 percentage point on average in absolute terms. This is because a higher Π^* not only brings the benefit of a widening of the safety margin, but it also generates welfare losses through nominal price rigidity and money holdings.

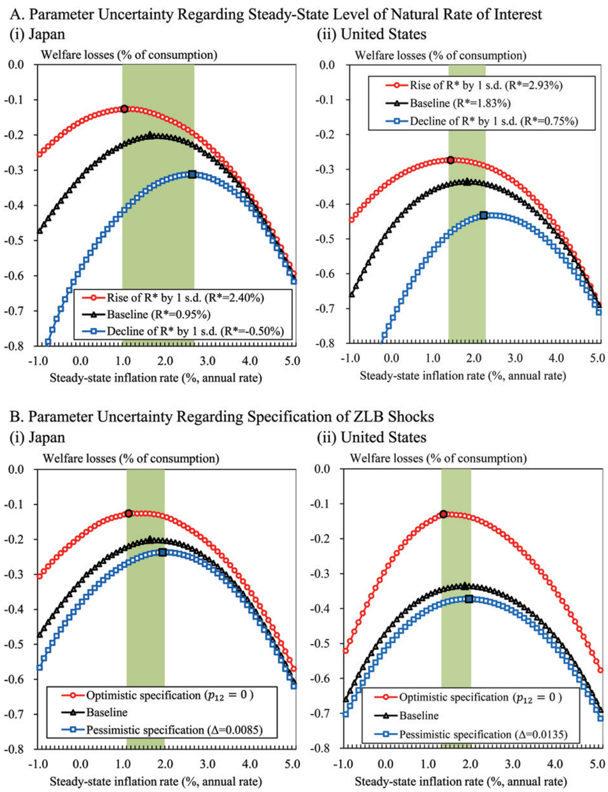
The uncertainty regarding R^* forms a considerable range of the optimal inflation rate for both countries. The implied range for Japan is 1.0 to 2.6 percent, a little wider than that for the United States, at 1.4 to 2.2 percent. The difference between Japan and the United States arises because the calibrated standard deviation of the estimated natural rate of interest is larger for Japan than for the United States, reflecting the larger time variations in the estimated series of the Laubach and Williams (2003) model. Moreover, since the Japanese economy is calibrated to be at the ZLB more frequently, the marginal effects of the changes in R^* are relatively larger.

6.2 Parameter Uncertainty Regarding ZLB Shock

The specification of ZLB shocks is another source of parameter uncertainty that potentially affects the optimal inflation rate. To evaluate the uncertainty, we formulate optimistic and pessimistic specifications based on the instances of ZLB periods in each country. In the optimistic specification, we remove the regime-switching

[18]The lower value of R^* for the United States is on the same level with the projections of the long-run policy rate by the Federal Open Market Committee (FOMC) participants. For example, the projections for the long-run federal funds rate of individual participants ranged from 2.0 to 3.3 percent in the December 2019 meeting, whereas the long-run inflation projection was 2.0 percent.

Figure 5. Sensitivity to Parameter Uncertainty



Note: In panel A, we consider the specifications where the steady-state value of the natural rate of interest (R*) deviates from the baseline value upward and downward by one standard deviation of the estimated series based on the Laubach and Williams (2003) model. In panel B, we assume that there are no regime-switching shocks for the optimistic specification. for the pessimistic specification, we calibrate the size of the regime-switching shock to replicate the largest decline in the output gap during ZLB periods in the data. The central bank is assumed to follow the RW rule.

shocks, i.e., $p_{12} = 0$. This specification means that ZLB shocks follow an AR(1) process, which is widely used in previous studies including CGW. On the other hand, for the pessimistic specification, we calibrate the size of the regime-switching shock to replicate the largest decline in the output gap during ZLB periods in the data. The data in Japan imply $\Delta = 0.0085$ compared with the baseline calibration of 0.0070. The calibration for the pessimistic case of the United States is $\Delta = 0.0143$ as opposed to 0.0135 in the baseline calibration.

The welfare losses under the optimistic and pessimistic specifications regarding ZLB shocks are given in the lower panels of Figure 5. Though somewhat smaller than in the case of the uncertainty regarding R^*, the specification of ZLB shocks forms the range of the optimal inflation rate, from 1.1 to 1.9 percent for Japan, and from 1.3 to 1.9 percent for the United States.

6.3 Habit Formation in Consumption

We extend the model to incorporate habit formation in consumption, which is often employed by previous studies including CGW. The analysis fundamentally assesses the sensitivity of our results with respect to the degree of inertia of the model. Following CGW, we consider the preference:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \ln \left(C_{t+s} - hgC_{t+s-1} \right) - \frac{1}{1+\frac{1}{\eta}} \chi_{t+s} H_{t+s}^{1+\frac{1}{\eta}} \right\}, \tag{34}$$

where h is the degree of habit formation. The Lagrange multiplier in the consumption Euler equation is modified to

$$\Xi_t = \left(\frac{1}{C_t - hgC_{t-1}} - \mathbb{E}_t \left[\frac{\beta hg}{C_{t+1} - hgC_t} \right] \right) \frac{1}{1 + s\left(V_t\right) + V_t s'\left(V_t\right)}. \tag{35}$$

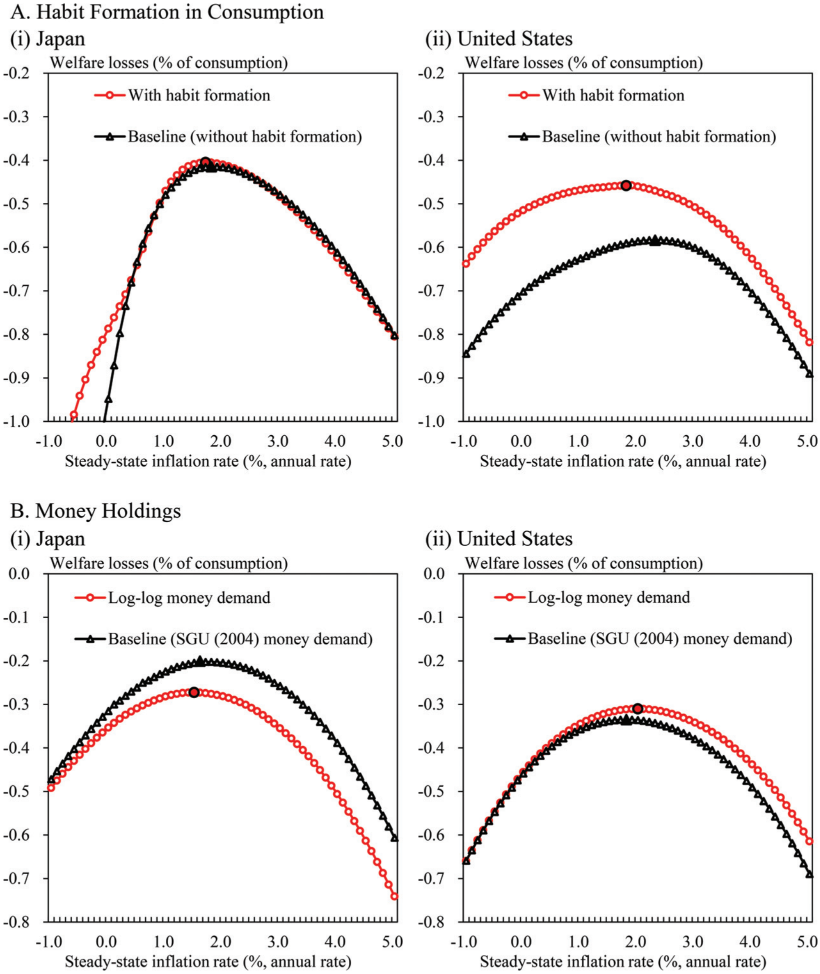
The other equilibrium conditions do not change from the baseline model conditional on the Lagrange multiplier. For monetary policy rule, we consider the case of the Taylor rule.[19] Regarding parameter

[19] With the habit formation in consumption, the output in the cashless economy under flexible prices and wages depends not only on the current exogenous variables but also on own lag. For the computational burden of adding another dimension of state space, we modify the monetary policy rule to respond to the deviations of output, instead of those of the output gap, from its steady-state value as below:

$$R_{n,t}^d = \left(R_{n,t-1}^d \right)^{\rho_r} \left\{ R^* \Pi^* \left(\frac{\Pi_t}{\Pi^*} \right)^{\phi_\pi} \left(\frac{Y_t}{Y^*} \right)^{\phi_y} \right\}^{1-\rho_r}, \tag{36}$$

where Y^* is output in the steady state. The monetary policy rule in the case without the habit formation is also modified accordingly.

Figure 6. Sensitivity to Model Specification



Note: In panel A, monetary policy rule is modified to respond to the deviations of output, instead of those of the output gap, from its steady-state value for both cases with and without habit formation. In panel B, the central bank is assumed to follow the RW rule.

values, the degree of habit formation in consumption h is set equal to 0.7 following CGW.

The upper panels of Figure 6 present the results. The optimal inflation rate in the model with the habit formation is quite close to the baseline specification, though the level of welfare losses is not directly comparable because the presence of habit formation changes the welfare function. The slight decline in the optimal inflation rate arises because the increased persistence due to the consumption habit makes the inflation rate less volatile and therefore leads to DNWR and the ZLB binding less frequently, which reduces the benefits of holding a positive inflation rate as a provision against recession.

6.4 Money Holdings

Money holdings, as measured by the money-to-income ratio, have risen in recent years under the ZLB in both Japan and the United States. Though there has been an active but unsettled debate on desirable specification of money demand function,[20] we employ the so-called log-log form as an alternative case. A potential advantage of the log-log form is that its strong concavity under low interest rates can accommodate high money holdings. Following Lucas (2000), we modify the transaction cost function (2) as below.[21]

$$s(V_t) = \frac{\delta_4}{1-\delta_4}\delta_3^{-\frac{1}{\delta_4}} V_t^{\frac{1-\delta_4}{\delta_4}}, \tag{37}$$

where $\delta_3,\ \delta_4 > 0$ are parameters that govern the overall level and curvature of transaction cost, respectively. The money demand function (6) takes the form

$$\ln(V_t) = \ln(\delta_3) + \delta_4 \ln(i_t), \tag{38}$$

[20] For instance, Ireland (2009) found that the semi-log form of money demand function performs better than the log-log form in the long-run U.S. data, whereas Watanabe and Yabu (2018) reported that the log-log form fits the recent data well. For Japan, Watanabe and Yabu (2019) argued that the heightened money holdings under the ZLB can be captured by the log-log form, whereas they also pointed out that empirical fit depends on whether or not structural breaks are allowed.

[21] While Lucas (2000) used the money-in-utility function to derive money demand function, we adopt a transaction cost to maintain the comparability with our baseline case. It is also notable that, though our baseline specification based on SGU (2004) can be written in the log-log form, $\ln(V_t) = -0.5\ln(\delta_1) + 0.5\ln(1 - 1/Q_tR_{n,t} + \delta_2)$, the alternative specification introduces additional flexibility (δ_4) to fit the curvature of money demand.

where $i_t \equiv \max\{1 - 1/Q_t R_{n,t}, \underline{i}\}$. Note that, though this is ad hoc, we suppose the presence of a lower bound on the rate of return $\underline{i} > 0$ to prevent the RHS of (38) from exploding.

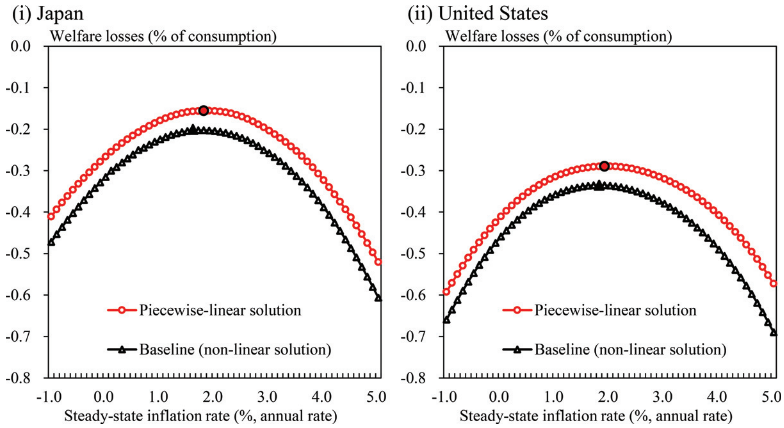
We calibrate δ_3 and δ_4 according to the empirical estimates of Watanabe and Yabu, who estimated money demand function in the recent U.S. and Japanese data. They obtained $\delta_3 = 1.561$ and $\delta_4 = 0.177$ for Japan in the sample of 1985–2017 (Watanabe and Yabu 2019), and $\delta_3 = 2.0893$ and $\delta_4 = 0.055$ for the United States in 1980–2013 (Watanabe and Yabu 2018). We set $\underline{i} = 0.01$ percent in annual rate and confirm that our result is not sensitive to the specific choice of $\underline{i}$.

The lower panels of Figure 6 compare the baseline and alternative specifications of money holdings. In the case of Japan, the alternative specification bears larger welfare losses of inflation than the baseline specification whereas it leads to slightly smaller welfare losses for the United States. The larger welfare losses in Japan arise from the high interest rate elasticity (higher δ_4) as well as the high overall level of money holdings (lower δ_3), consistent with the rise in money holdings under the ZLB observed in the data. However, the optimal inflations remain close to those in the baseline case: it is 1.5 percent for Japan and 2.0 percent for the United States in the alternative case. The result implies that the welfare losses arising from money holdings and their impacts on the optimal inflation rate are relatively small compared with those associated with DNWR and the ZLB.

6.5 Piecewise-Linear Solution

One of key features of this paper's analysis is that we keep track of full non-linearity of the model whereas most previous studies on this topic used the piecewise-linear solution (e.g., SGU 2010, CGW). To see the effects of the full non-linear solution, Figure 7 compares the welfare losses under the full non-linear and piecewise-linear solutions. We conduct a log-linear approximation around the deterministic steady state for each Π^*, and deal with DNWR and the ZLB in a piecewise fashion. The log-linearized equilibrium conditions are listed in online Appendix D. According to the figure, the welfare losses are smaller in the piecewise-linear solution than in the baseline full non-linear solution. In this regard, there are potentially two

Figure 7. Sensitivity to Computation Method



Note: The central bank is assumed to follow the RW rule.

effects of linearization. On the one hand, the linearization of the Euler equation eliminates precautionary saving, easing the contractionary effects induced by the future risks of DNWR and the ZLB. On the other hand, the linearization also removes the non-linearity of the Phillips curve. Since the Phillips curve becomes increasingly non-linear under a higher Π^* and generates larger price dispersion as discussed in Section 5, abstracting such non-linearity understates the welfare losses arising from nominal price rigidity. Indeed, the optimal inflation rate in the piecewise-linear solution is slightly higher than in the full non-linear solution. The result implies that the quantitative impacts of the two effects of linearization mentioned above are close with each other, but the latter effect is slightly dominant.

7. Concluding Remarks

In this paper, we investigate the optimal inflation rate in a non-linear New Keynesian model that embeds four major factors affecting the costs and benefits of inflation: nominal price rigidity, money holdings, downward nominal wage rigidity, and the zero lower bound. Although many previous studies have examined one or more of these four factors, we examine all four simultaneously and employ a computational methodology that is suitable for addressing the model non-linearity.

Most of the previous studies that focus on nominal price rigidity and money holdings find that the optimal inflation rate is close to or below 0 percent. On the other hand, when DNWR and the ZLB are taken into account in addition to these factors, a positive inflation rate in the steady state can maximize social welfare. In this regard, Janet Yellen, in her remarks during the July 1996 FOMC meeting, stressed that, among the four factors, DNWR and the ZLB are sources that support a positive steady-state inflation rate. Our quantitative result suggests that the optimal inflation rate is close to 2 percent for both Japan and the United States. Besides, the main driver that supports the optimal inflation rate of close to 2 percent differs by country: the ZLB is the main driver for Japan, but for the United States it is DNWR. At the same time, we find that around 1 percentage point absolute deviation from the rate of close to 2 percent induces only a minor change in social welfare. Effective forward guidance can reduce the lower end of the range that is acceptable in terms of welfare losses.

Though we incorporate major factors affecting the costs and benefits of inflation, and employ computation methodologies to address the non-linearity of the model, there may be other factors that potentially affect the optimal inflation rate. Our model restricts its focus to a closed economy and is therefore agnostic about the issues in an open economy; we also do not take into account the transmission mechanism of monetary policy through financial intermediaries especially when the nominal interest rate is in a negative territory; and finally, our analysis does not consider the full range of unconventional monetary policy measures, such as asset purchases and central bank lending. These issues are left for future research.

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