The Information Content and Statistical Properties of Diffusion Indexes*

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We study diffusion indexes constructed from qualitative surveys to provide real-time assessments of various aspects of economic activity. In particular, we highlight the role of diffusion indexes as estimates of change in a quasi-extensive margin, and characterize their distribution, focusing on the uncertainty implied by both sampling and the polarization of participants' responses. Because qualitative tendency surveys generally cover multiple questions around a topic, a key aspect of this uncertainty concerns the coincidence of responses, or the degree to which polarization co-moves, across individual questions. We illustrate these results using microdata on individual responses underlying different composite indexes published by the Michigan Survey of Consumers. We find a secular rise in consumer uncertainty starting around 2000, following a decade-long decline, and higher agreement among respondents in prior periods. In 2014, six years after the Great Recession, uncertainty arising from the polarization of responses in the Michigan Survey stood at its highest level, coinciding with the weakest recovery in U.S. postwar history. The formulas we derive allow for simple computations of approximate confidence intervals, thus affording a more complete real-time assessment of economic conditions using qualitative surveys.

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1. Introduction

Quantitative information regarding the state of U.S. economic activity is mostly compiled and published by various statistical agencies such as the Bureau of Labor Statistics (BLS), the Bureau of Economic Analysis, or the Federal Reserve Board. An important aspect of such data is that releases often involve a one-month lag and are further subject to revisions, typically at the three-month and one-year mark.[1] In part, as an attempt to provide information somewhat closer to real time, or to collect information simply not compiled by official statistical agencies, a growing number of institutions and government agencies produce diffusion indexes constructed from qualitative survey data. For example, the Michigan Survey of Consumers (MSC) provides several diffusion indexes of consumer sentiment based on monthly nationwide surveys; the Institute for Supply Management publishes different monthly diffusion indexes regarding various aspects of manufacturing, such as production, shipments, or new orders; and several regional Federal Reserve Banks including Atlanta, Dallas, Kansas City, New York, Philadelphia, and Richmond produce diffusion indexes meant to capture the direction of change, in real time, of different facets of economic activity, such as inventories, capital expenditures, or wages, at a more regional level. In the latter cases, the information is not otherwise compiled or readily available. Moreover, by sharing a common methodology, diffusion indexes allow for comparable benchmarks not just across U.S. regions but also across countries, as in the case of the manufacturing purchasing managers index (PMI) for France or Germany, for instance.

Diffusion indexes, throughout the paper, are defined in terms of the proportions of a set of disaggregated series or survey responses moving in different directions, thus defining a notion of optimism and pessimism, and provide a measure of the breadth of change in the corresponding aggregate series. This traditional interpretation is discussed in early work by Moore (1983), or the *Federal Reserve*

[1]See Croushore (2011) for a comprehensive survey of real-time data analysis and the caveats that arise from the distinction between real-time and revised data. See also Runkle (1998), Croushore and Stark (2001), Fernald and Wang (2005), and Croushore and Sill (2014).

Bulletin (Board of Governors of the Federal Reserve System 1991) among others, and is distinct from the interpretation of diffusion indexes constructed using principal components analysis in Stock and Watson (2002).

To this point, a large part of the literature on survey-based diffusion indexes has focused on their ability to forecast economic activity using various approaches, as surveyed, for example, in Nardo (2003).[2] Diffusion indexes have also been used to explore different properties of expectations, given assumptions on the underlying data, as surveyed in Pesaran and Weale (2006).[3] More recently, this second strand of research has been able to take advantage of the microdata on individual responses underlying diffusion indexes, when directly available, as in Bachmann and Elstner (2015) in the case of the IFO Business Climate Survey.[4] Our paper departs somewhat from this literature in that it studies the properties of diffusion indexes as estimates of the breadth of change in an aggregate series of interest. In particular, we highlight the role of the diffusion index as an estimate of change in a quasi-extensive margin, in a way closely related to the work of Gourio and Kashyap (2007). Moreover, we characterize the distribution of a general diffusion index, potentially averaging different individual indexes, focusing on the uncertainty implied by both sampling and the polarization of participants' responses.[5]

While the use of survey data has traditionally emphasized quantitative surveys—for example, with respect to properties of inflation expectations in the presence of nominal rigidities (Coibion and Gorodnichenko 2012; Coibion, Gorodnichenko, and Kumar 2015), or to measure disagreement among forecasters as an indication of uncertainty (Bomberger 1996; Boero, Smith, and Wallis 2008;

[2]See, for example, Kennedy (1994), Smith and McAleer (1995), and Bram and Ludvigson (1998).

[3]See also Ivaldi (1992), Jeong and Maddala (1996), and Claveria, Pons, and Suriñach (2006).

[4]A challenge remains, however, in that for many published diffusion indexes, individual survey responses are either not systematically recorded or not publicly available.

[5]Lahiri and Sheng (2010) also associate the notion of uncertainty to disagreement. Specifically, they decompose aggregate forecast uncertainty into two components: the first one captures disagreement among forecasters, and the second one, variability of future shocks.

D'Amico and Orphanides 2008; Rich and Tracy 2010; Sill 2012; Leduc and Sill 2013), the use of qualitative surveys is relatively more recent.[6] Specifically, we build on Bachmann, Elstner, and Sims (2013), who construct an empirical proxy for time-varying business-level uncertainty using qualitative survey data. These data capture notions of optimism and pessimism through the proportion of positive and negative responses to particular questions, and underlie a diffusion index reflective of business climate. We show by way of a central limit theorem (CLT) argument that all diffusion indexes, including composite indexes, are asymptotically normal in large samples, and that the uncertainty proxy in Bachmann, Elstner, and Sims (2013) is the variance of a particular, albeit widely used, individual diffusion index (scaled by the square root of the sample size). As such, this proxy can also be used to provide confidence intervals, taking into account sampling uncertainty, for the corresponding diffusion index.

In consumer and business tendency surveys, diffusion indexes are almost always published as composite indexes, combining multiple individual indexes corresponding to different questions around a given topic. Thus, we emphasize that a general notion of uncertainty based on multiple questions reflects not only the polarization of answers, or degree of disagreement, with respect to an individual survey question but also the extent to which agreement or disagreement coincides across individual questions. In particular, given individual response level data, we describe how the pairwise coincidence of answers across survey questions, or lack thereof, maps into the covariance between individual diffusion indexes making up the composite index. In deriving the distribution of composite indexes and their associated uncertainty, we show that given data on individual responses, one need only keep track of pairwise proportions of answer types across questions—for example, the proportion of responses indicating optimism to one question and pessimism to another.

We illustrate these results using microdata on individual responses used to construct a number of composite diffusion indexes around consumer sentiment published by the MSC. We find a

[6]See Barsky and Sims (2012) for an application using consumer sentiment indexes constructed by the MSC.

steadily rising trend in consumer uncertainty from 2000 to 2014 that contrasts with a gradual decline in uncertainty over the previous decade, and higher agreement in prior years. Uncertainty from the polarization of responses in the MSC reaches in 2014 its highest level since 1978, and only starts to decline six years after the Great Recession. We highlight that the continued increase in uncertainty until 2014 coincides with the weakest postwar expansion on record. The secular rise in uncertainty that we estimate among consumers is also evidenced by other uncertainty and polarization indicators, but to a lesser degree. In fact, the period starting around 2000 is somewhat unique. Most indicators of uncertainty tend to decline until 2005, increase after that year, and finally fall sharply after the Great Recession. In contrast, uncertainty around consumer sentiment tends to behave countercyclically.[7]

The rest of the paper is organized as follows. In section 2, we use employment growth data and the BLS employment diffusion index to highlight the role of diffusion indexes as estimates of extensive margin changes. In section 3, we provide a statistical foundation of individual diffusion indexes. In section 4 we extend these results to general composite diffusion indexes, and discuss several uses of these indexes in practice. In section 5, we illustrate our findings using the BLS employment diffusion index (as an example of an individual diffusion index) and several consumer sentiment indicators constructed by the MSC (as examples of composite diffusion indexes). Finally, section 6 concludes.

2. Diffusion Indexes: Measuring Changes in a Quasi-extensive Margin of Economic Activity

Diffusion indexes, such as the Index of Consumer Sentiment (ICS) produced by the MSC, receive widespread coverage in part because they have been empirically found, over time, to correlate well with economic activity (Bram and Ludvigson 1998). This section uses the diffusion index of employment, produced by the Bureau of Labor Statistics, to underscore two key points regarding diffusion indexes. First, scaled appropriately, diffusion indexes can be thought of as

[7] See Gourio (2014) for a setting with financial distress costs in which aggregate macroeconomic volatility arises endogenously and mostly in recessions.

capturing a quasi-extensive margin of change in economic activity or, in this case, employment. Second, this quasi-extensive margin accounts for a large portion of the variation in aggregate employment, although interestingly to different degrees in expansions and recessions.

Beginning in 1991, the BLS began publishing a diffusion index of employment in its monthly statistical release, covering roughly 264 sectors corresponding to the four-digit level of the North American Industrial Classification System (NAICS), separate from its release of measured employment. Overall employment growth, which reflects a weighted average of sectoral employment growth, gives us one measure of employment performance. Overall employment growth, however, does not provide a sense of how the change is shared across sectors; for instance, a given aggregate growth rate may be consistent with all sectors doing equally well or, instead, a relatively few sectors growing rapidly with most others simply muddling through. In contrast, the BLS employment diffusion index summarizes the direction of change in a set of disaggregated sectors over a given time period, thus providing a measure of the breadth of the change in employment. We now explore and describe the relationship between these two measures of economic performance.

The diffusion index is most conventionally reported as

$$\mu D + \kappa, \tag{1}$$

where D is the proportion of a set of disaggregated series that increased over a given time period less the proportion that decreased over the same period, $D = n^u/n - n^d/n$, where n is the total number of series, such as sectors, and n^u and n^d are the number of series that experienced an increase and decline in activity respectively; μ and κ are constants. In the case of the BLS employment diffusion index, $\mu = 1$ and $\kappa = 0$. Thus, if an individual series increases over the span of the diffusion index, it receives a value of 1; if it declines, it receives a value of -1; and if it is unchanged, it receives a value of zero. The diffusion index is then calculated by summing these values for each of the series and dividing the result by the number of series included in the diffusion index (in this case multiplied by 100). A value of the index above zero is then interpreted as an expansion in employment and vice versa for values less than zero; a value of 100 would be indicative of an expansion in all sectors.[8]

Consider employment in a given sector i and denote its monthly annualized growth rate by $\Delta x_{i,t} = 1200 \times \ln(x_{i,t}/x_{i,t-1})$. Because we are mainly concerned with issues related to the assessment of various economic conditions in real time, the key objective for many of the surveys considered below, our focus in this paper will be on monthly data. Let n represent the number of sectors covered by the employment index and denote overall employment growth at t by Δx_t. Then,

$$\Delta x_t = \underbrace{\frac{1}{n}\sum_{i=1}^{n_t^u} \Delta x_{i,t}^u}_{\Delta x_t^u} - \underbrace{\frac{1}{n}\sum_{i=1}^{n_t^d} \Delta x_{i,t}^d}_{\Delta x_t^d}, \tag{2}$$

where $\Delta x_{i,t}^u = \Delta x_{i,t}$ if $\Delta x_{i,t} \geq 0$ and zero otherwise, and $\Delta x_{i,t}^d = -\Delta x_{i,t}$ if $\Delta x_{i,t} < 0$ and zero otherwise.[9] Simply put, equation (2) distinguishes between those sectors that contribute positively to overall employment growth (up sectors), which sum to n_t^u and contribute Δx_t^u, and those that take away from overall growth (down sectors), which sum to n_t^d and contribute negatively Δx_t^d.

Let μ_t^u represent the cross-sectional average growth rate, at a point in time, of the sectors that add to overall growth in employment, $\mu_t^u = \frac{1}{n_t^u}\sum_{i=1}^{n_t^u} \Delta x_{i,t}^u$, and similarly let $\mu_t^d = \frac{1}{n_t^d}\sum_{i=1}^{n_t^d} \Delta x_{i,t}^d$ for those sectors where employment declined, with corresponding population conditional means, as of date t, $E_t(\Delta x_{i,t}|\Delta x_{i,t} \geq 0)$ and $E_t(\Delta x_{i,t}|\Delta x_{i,t} < 0)$, respectively. Then, equation (2) can alternatively be written as

[8] Note that the information conveyed by the index is invariant to an affine transformation. For example, the Federal Reserve Board defines the diffusion index of industrial production as the proportion of sectors where production increased plus half the sectors where production was unchanged, in which case $\mu = \kappa = 1/2$ in equation (1).

[9] We define overall growth in employment using uniform weights in this case. Foerster, Sarte, and Watson (2011) show, in the context of industrial production, that the choice of weights—either uniform, constant mean shares, or time-varying shares—is somewhat unimportant in aggregating growth rates across sectors.

$$\Delta x_t = \frac{n_t^u}{n}\mu_t^u - \frac{n_t^d}{n}\mu_t^d. \tag{3}$$

Define $\mu^u = T^{-1}\sum_{t=1}^T \mu_t^u$ and $\mu^d = T^{-1}\sum_{t=1}^T \mu_t^d$ as the time averages, or long-run cross-sectional averages, of sectors contributing to and subtracting from overall employment growth, respectively; we assume that the data is stationary with corresponding population means $E(\Delta x_{i,t}|\Delta x_{i,t} \geq 0)$ and $E(\Delta x_{i,t}|\Delta x_{i,t} < 0)$. We may then express positive contributions to Δx_t as

$$\begin{aligned}\Delta x_t^u = {} & \left(\frac{n_t^u}{n} - \varphi^u\right)\mu^u + \varphi^u\left(\mu_t^u - \mu^u\right) \\ & + \left(\frac{n_t^u}{n} - \varphi^u\right)\left(\mu_t^u - \mu^u\right) + \varphi^u\mu^u, \end{aligned} \tag{4}$$

where $\varphi^u = T^{-1}\sum_{t=1}^T n_t^u/n$ is the long-run average of proportion of sectors that raise overall growth.[10] In other words, at a point in time, a large increase in overall employment growth via Δx_t^u may arise because the proportion of expanding sectors is higher than usual given their average contribution, $\left(\frac{n_t^u}{n} - \varphi^u\right)\mu^u > 0$, akin to a rising extensive margin; the cross-sectional average of those positive contributions is higher than usual given the typical proportion of expanding sectors, $\varphi^u\left(\mu_t^u - \mu^u\right) > 0$, akin to a rising intensive margin; or both to the extent that both are true, $\left(\frac{n_t^u}{n} - \varphi^u\right)\left(\mu_t^u - \mu^u\right) > 0$. Similarly for the negative contributions, we have that

$$\Delta x_t^d = \left(\frac{n_t^d}{n} - \varphi^d\right)\mu^d + \varphi^d(\mu_t^d - \mu^d) + \left(\frac{n_t^d}{n} - \varphi^d\right)\left(\mu_t^d - \mu^d\right) + \varphi^d\mu^d, \tag{5}$$

where $\varphi^d = T^{-1}\sum_{t=1}^T n_t^d/n$. Then, it follows that overall employment growth may be approximated as

$$\Delta x_t \approx \underbrace{\varphi^u\left(\mu_t^u - \mu^u\right) - \varphi^d(\mu_t^d - \mu^d)}_{\substack{\text{Change in "how much"} \\ \text{or intensive margin}}} + \underbrace{\mu^u D_t}_{\substack{\text{Change in "how many"} \\ \text{or extensive margin}}}, \tag{6}$$

[10]Details of all derivations in the paper are given in online-only supplementary notes; see Pinto, Sharp, and Sarte (2015).

where $D_t = \left(\frac{n_t^u}{n} - \frac{n_t^d}{n}\right)$ is the difference in the proportions of increasing and decreasing series defined earlier.[11]

Put another way, by reinterpreting the scalar μ in (1), we may decompose employment growth as arising primarily from the change in an intensive margin—the difference between how fast "up" sectors grew and how badly "down" sectors declined—and the change in a quasi-extensive margin—the difference between the proportion of sectors that expanded versus those that declined or the breadth of the expansion, $\mu^u D_t$. The approximation in (6) comes about in part because the positive and negative cross-sectional growth rates μ^u and μ^d are not necessarily the same, and the fact that the difference in the way the extensive and intensive margins interact in (4) and (5) also matters in principle.[12] However, as we now illustrate, this difference in the interaction terms is largely immaterial for the behavior of overall employment growth, except in the last recession.

The top panel of figure 1 shows the decomposition of Δx_t^u (demeaned) in equation (4) for U.S. employment.[13] Examination of the figure indicates that variations in the proportion of expanding sectors, conditional on their average contribution, closely tracks the positive contributions to overall employment growth, Δx_t^u, while the interaction term between extensive and intensive margins is mostly unimportant. Figure 1 also illustrates, in the bottom panel, the decomposition of Δx_t^d in (5). As with the top panel, the variation in the proportion of declining sectors explains a large fraction of the negative contribution to overall employment growth, although the intensive margin now also plays an unambiguous role, especially at times when Δx_t^d spikes during recessions.

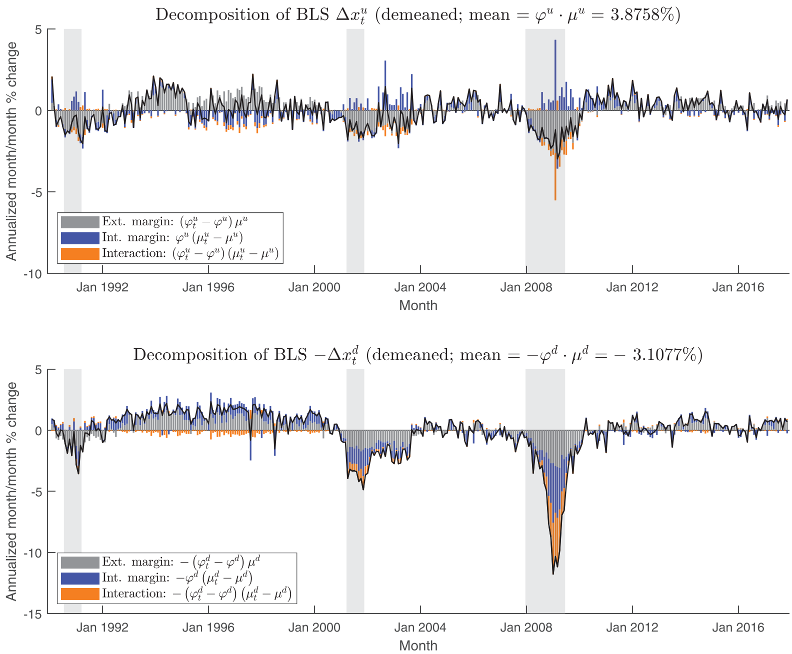
Figure 2 combines these two decompositions in addition to an interaction term, which, among other things, includes $\varepsilon = \mu^u - \mu^d$.

[11]The other components not included in equation (6) are some interaction terms and the time-invariant expression $(\varphi^u \mu^u - \varphi^d \mu^d)$.

[12]The derivation in (6) makes use of the fact that, without loss of generality, $\mu^u = \mu^d + \varepsilon$ for some $\varepsilon \lesseqgtr 0$. In the case of employment and over the period 1991–2017, μ^u and μ^d are close—7.16 percent versus 7.49 percent, respectively—so that either one may be used to scale the diffusion index to arrive at an approximate measure of the extensive margin.

[13]The color versions of the graphs can be found in the online version of the issue on the IJCB website (http://www.ijcb.org).

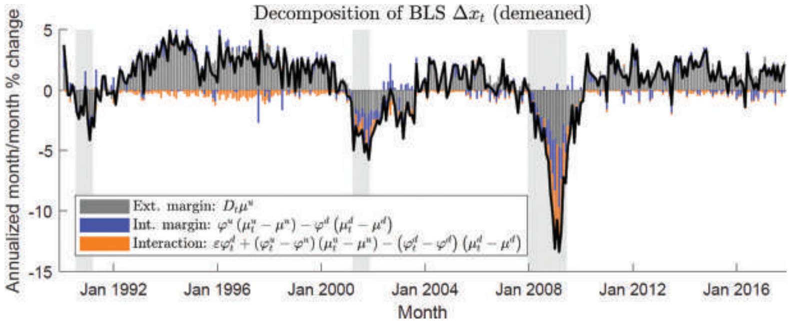
Figure 1. Decomposition of Positive and Negative Contributions to BLS Employment Growth



Notes: The top panel shows the behavior of the three components underlying the positive contributions to employment growth Δx_t^u according to the decomposition suggested in equation (4). The components include the extensive margin (in gray), the intensive margin (in blue), and the interaction term (in orange). The bold black describes the behavior of the series $(\Delta x_t^u - \varphi^u \mu^u)$, where $\varphi^u \mu^u$ is the last term on the right-hand side of (4). The panel on the bottom shows the same components but for the negative contributions to employment growth Δx_t^d, as described by the decomposition (5). The bold black line represents the series $-(\Delta x_t^d - \varphi^d \mu^d)$, where $\varphi^d \mu^d$ is the last term on the right-hand side of (5). Recession years are shaded in gray.

The figure shows that, to a large extent, variations in overall employment growth arise from changes in the quasi-extensive margin in (6). Put another way, movements in employment tend to reflect the breadth of change (scaled appropriately) rather than large variations in cross-sectional growth rates driven by particular sectors. Table 1 summarizes these results.

Figure 2. Decomposition of BLS Employment Growth



Notes: The figure displays the behavior of different components of Δx_t according to the decomposition suggested in equation (6). It illustrates the relative contribution of each component to the behavior of Δx_t. The bold black line shows the evolution of the series $\Delta x_t - (\varphi^u\mu^u - \varphi^d\mu^d)$. Recession years are shaded in gray.

Table 1. Decomposition of BLS Employment Growth Rates

Series	1990–2017
$\Delta x_t = \frac{1}{n}\sum \Delta x_{i,t}$	2.68
Components:	
$D_t\mu^u$	1.74
$\varphi^u(\mu_t^u - \mu^u) - \varphi^d(\mu_t^d - \mu^d)$	0.87
$(\varphi_t^u - \varphi^u)(\mu_t^u - \mu^u) - (\varphi_t^d - \varphi^d)(\mu_t^d - \mu^d) + \varepsilon\varphi_t^d$	0.66

Notes: Percentage points at annual rates; $\varepsilon = \mu^u - \mu^d$. Entries are the sample standard deviation of Δx_t and its components.

This observation is reminiscent of Gourio and Kashyap (2007) who show, using plant-level data from Chile and the United States, that "the number of establishments undergoing investment spikes (the "extensive margin") account for the bulk of variation in aggregate investment." In that sense, it is conceivable that variations in the breadth of change, D_t, captured in various surveys—say, regarding the business outlook of a sample of purchasing managers—would have proven a useful indicator of economic activity, even without more "concrete" data at hand. In the case of figure 2, notable exceptions concern pronounced downturns in employment that take place during recessions. In those periods, changes in the intensive margin defined in (6) contribute as much to the downturn as the extensive margin captured by the diffusion index, and thus so does the interaction of the two margins, and reveal a potentially interesting asymmetry across expansions and downturns.

It should be noted that the notion of a "quasi-extensive margin," in this context, abstracts from entry and exit since the number of sectors is held fixed in the BLS calculation of the employment diffusion index. In the many surveys that produce diffusion indexes, including those of the MSC that we study below, the sample size is also often targeted to a fixed level. In that sense, diffusion indexes are then specifically concerned with the distribution of change in an existing set of disaggregated series.

3. A Formal Description of Survey-Based Diffusion Indexes

While in the previous section we saw that the extensive margin captured by the BLS's employment diffusion index is an important component of overall employment growth, other types of diffusion indexes are generally constructed to capture changes in economic conditions in real time. A number of institutions and government agencies carry out qualitative surveys that are then similarly translated into a diffusion index. For instance, the MSC constructs widely used composite diffusion indexes that reflect consumer sentiment regarding various current and expected economic conditions, including household financial conditions, overall business conditions, and spending on durable goods, the latter being akin to a measure of investment. Similarly, several Federal Reserve Banks construct and publicly report diffusion indexes that are meant to track the breadth of economic performance in their respective regions in real time. Other diffusion indexes that receive widespread attention by markets and policymakers include the monthly index of manufacturing and service conditions released by the Institute for Supply Management (ISM). Similar to the employment diffusion index described in the previous section, these surveys most often rely on trichotomous classifications whereby participants are asked whether conditions are better, worse, or unchanged relative to the previous month.

In the rest of this section, we characterize the distribution of individual diffusion indexes, such as the BLS's employment diffusion index examined in section 2. The next section derives the distribution of composite diffusion indexes.

3.1 The Individual Diffusion Index and Its Cross-Sectional Distribution

Consider a sample of n survey participants drawn randomly from a population at a point in time. Given the focus on timeliness underlying diffusion indexes, the vast majority of indexes provide monthly information. Each participant answers a survey question relating to an economic series of interest, say household financial conditions or overall business conditions, according to a predefined set of qualitative responses. We denote the set of possible answer types by $\mathcal{A}$ and consider r possible qualitative responses, $\mathcal{A} = \{1, 2, \ldots, r\}$. Answer types from participants are indexed by $a \in \mathcal{A}$. In the most common example, there are three types of responses: "up" (u), "down" (d), and "same" (s), or alternatively "better off," "worse off," or "same." For example, a question in the Michigan Index of Consumer Sentiment asks: "Do you think that a year from now, you (and your family living there) will be better off, worse off, or just about the same as now?" In that case, we might write that $a \in \mathcal{A} = \{u, d, s\}$. A typical sample of responses might be summarized by the vector

$$(u, s, u, d, d, u, \ldots). \tag{7}$$

Let n^a denote the number of respondents associated with answer $a \in \mathcal{A}$, $\sum_{a=1}^{r} n^a = n$.[14] Answers of type a are assigned a value of $\omega^a \in \mathcal{R}$ in the diffusion index, where $\underline{\omega}^a$ and $\overline{\omega}^a$ denote lower and upper bounds, respectively, associated with possible values of ω^a. In the conventional example with three categories, $\omega^u = 1$, $\omega^s = 0$, and

[14] In principle, the number of responses of a particular type a (relative to the total number of responses) may be serially correlated over time. As will be made clear, however, our focus is on characterizing the cross-sectional distribution, and related uncertainty, that arises from the different survey responses at a point in time, so that we drop the time subscript. To the extent that the number of responses of each type are serially correlated, to reflect persistence in participants' views, so will be the measured uncertainty.

$\omega^d = -1$. An alternative formulation might have $\omega^u = 1$, $\omega^s = 1/2$, and $\omega^d = 0$, as with the Federal Reserve Board's diffusion index of industrial production.

The general diffusion index statistic is then given by[15]

$$\widehat{D} = \sum_{a=1}^{r} \omega^a \frac{n^a}{n}. \tag{8}$$

By construction, the resulting index will range from $\underline{\omega}^a$ to $\overline{\omega}^a$, with numbers above $(1/r)\sum_{a=1}^{r} \omega^a$ generally being interpreted as an expansion in the condition of interest, say household spending on durable goods. Observe that different combinations of answers can result in the same index. For example, in the case comprising three categories, $\omega^u = 1$, $\omega^s = 0$, and $\omega^d = -1$, and 100 firms being asked about overall business conditions, a value of $D = 0$ might emerge from having 50 firms responding "up" and 50 firms responding "down," or all firms responding "no change." While both cases may be interpreted as a reading of unchanged business conditions overall, we describe below the sense in which these two cases imply a different degree of confidence in the overall reading of "no change."

Let p^a denote the probability, in a given month, that a participant's answer is $a \in \mathcal{A} = \{1, 2, \ldots, r\}$, with $\sum_{a=1}^{r} p^a = 1$. The survey process, according to which n participants are drawn randomly from a population, has a natural interpretation in terms of n independent trials, each of which leads to a success for exactly one of r types of responses, with each answer type having a given fixed success probability, p^a.[16] The multinomial distribution then gives the probability, given n, of observing any particular combination of numbers of responses, $\{n^1, n^2, \ldots, n^r\}$, for the various categories $\{1, \ldots, r\}$. In particular, the probability mass function of this distribution is

[15] A constant is sometimes added to $\widehat{D}$, as in equation (1), to correct for sample design changes, but this modification is immaterial for questions related to uncertainty.

[16] The assumption of random sampling may not be entirely accurate in some contexts. However, as we discuss later, in many cases including those in which most of the sample is randomly drawn from phone book listings, as for the Michigan Survey of Consumers, this assumption represents a meaningful benchmark from which to build our results.

$$f(n^1, \ldots, n^r; n, p^1, \ldots, p^r) = \frac{n!}{\Pi_{a=1}^r n^a!} \Pi_{a=1}^r (p^a)^{n^a}, \tag{9}$$

where the expected number of answers of type a is $E(n^a) = np^a$, with associated variance $Var(n^a) = np^a(1 - p^a)$. The covariance between the numbers of answers of types $a \in \mathcal{A}$ and $a' \in \mathcal{A}$ is given by $Cov(n^a, n^{a'}) = -np^a p^{a'}$. Observe that $E(n^a)$ and $Var(n^a)$ in this case are, respectively, the mean and variance of a binomial distribution defined by the marginal distribution, for answers of type a, of the multinomial described in (9).

Let $\widehat{p}^a = n^a/n$ denote the proportion of answers of type $a \in \mathcal{A}$ observed in a given survey, which can interpreted as a Bernoulli average. Because the generic individual diffusion index, $\widehat{D}$, is a linear combination of sample Bernoulli means, $\sum_{a=1}^r \omega^a \widehat{p}^a$, that are asymptotically jointly normally distributed, the distribution of $\widehat{D}$ will also be asymptotically normal. Details of the derivations are provided in appendix section A.1. Thus, in large finite samples, the distribution of the diffusion index statistic is approximately normal and given by

$$\sqrt{n}\left(\widehat{D} - D\right) \overset{a}{\sim} \mathcal{N}\left(0, \sum_{a=1}^r (\omega^a)^2 p^a - D^2\right), \tag{10}$$

where $D = E(\widehat{D}) = \sum_{a=1}^r \omega^a p^a$.

Our approach complements that of Bachmann, Elstner, and Sims (2013) in a subtle but important way. In particular, the authors construct a measure of uncertainty that depends on the proportion of responses $\{u, d, s\}$, where it is implicitly assumed that ω^a is a random variable that takes the values $(\omega^u, \omega^d, \omega^s) = (1, -1, 0)$, with corresponding probabilities (p^u, p^d, p^s). In that case, the vector (p^u, p^d, p^s) fully describes the distribution of ω^a. Specifically, in the notation of Bachmann, Elstner, and Sims (2013), the expected value of ω^a is $Frac^+ - Frac^- = \sum_{a=1}^r \omega^a p^a$, and its standard deviation $\sqrt{Frac^+ + Frac^- - (Frac^+ - Frac^-)^2} = \sqrt{\sum_{a=1}^r (\omega^a)^2 p^a - \left(\sum_{a=1}^r (\omega^a) p^a\right)^2}$, where $Frac^+$ and $Frac^-$ correspond to p^u and p^d, respectively, in our framework. Put another way, their proxy for uncertainty, the standard deviation of ω^a, coincides with the asymptotic standard deviation of $\sqrt{n}\left(\widehat{D} - D\right)$ in our paper. Because our analysis starts with the diffusion index statistic $\widehat{D}$, it is more natural here to think of p^a as the random variable, with sample realization $\widehat{p}^{\,a}$, which then allows us to build on Bachmann, Elstner, and Sims (2013) as part of a full description of the asymptotic behavior of $\widehat{D}$ in (10). In addition, as we show below, this approach extends directly to the more general cases actually encountered in practice that involve composite diffusion indexes, such as those published by the MSC.[17]

Given equation (10), it immediately follows that confidence intervals for $\widehat{D}$ are symmetric and can be approximately constructed as $D \pm z\sqrt{\left(\sum_{a=1}^{r} (\omega^a)^2 p^a - D^2\right)/n}$, where z is the standard score corresponding to a critical value of interest. Confidence intervals, therefore, partly reflect sampling uncertainty, which decreases with the square root of the sample size, n, and the degree of polarization among survey respondents' answers, $\sum_{a=1}^{r} (\omega^a)^2 p^a - D^2$. To the degree that the weights, ω^a, or the number of responses, r, determine the scale of D, the standard deviation $\sqrt{\left(\sum_{a=1}^{r} (\omega^a)^2 p^a - D^2\right)}$ then preserves the same units. The following examples provide some intuition.

3.1.1 Practical Applications and Special Cases

In many common applications, such those regarding any question making up the MSC's Index of Consumer Sentiment, we have that $\mathcal{A} = \{u, d, s\}$, and $\omega^u = 1$, $\omega^s = 0$, $\omega^d = -1$. Then, $\widehat{D} = (\widehat{p}^u - \widehat{p}^d)$ and

$$\sqrt{n}\left(\widehat{D} - D\right) \overset{a}{\sim} \mathcal{N}\left(0, (1 - p^s) - D^2\right). \tag{11}$$

In this example, the variance of the diffusion index is also, in large samples, the population equivalent of the uncertainty proxy estimated in Bachmann, Elstner, and Sims (2013). We make two observations to provide intuition regarding the variance of $\widehat{D}$. First, it

[17]For example, this approach can in principle be used to analyze different functions of the probabilities p^a, including measures of concentration such as that captured by the Herfindahl index, $\sum_a (\widehat{p}^a)^2$, and whose distribution might be characterized following similar steps.

decreases as responses become less polarized, or alternatively split between extremes. In particular, the variance in Bachmann, Elstner, and Sims (2013) decreases with the magnitude of the diffusion index itself, $D^2 = (p^u - p^d)^2$; simply put, the only way for D^2 to be large, or more specifically to approach 1, is for all respondents to answer either "up" or "down," in which case answers coincide and there remains little room for uncertainty, $p^s = 0$ and $(1 - p^s) - D^2 = 0$. Alternatively, consider the case where "up" and "down" responses are evenly split, $p^u = p^d$, so that $D = 0$ irrespective of p^s. Then uncertainty decreases as p^s rises (and both p^u and p^d fall while D remains unchanged); in other words, we are relatively more confident of a reading of no change, $D = 0$, when everyone reports "no change," $p^s = 1$ and all answers coincide, than when half the respondents report "up" and half the respondents report "down," $p^s = 0$ (the largest even split). Second, since $\widehat{D}$ derives from a weighted sum of means, its variance simply decreases at rate n.

As another example, consider the Federal Reserve Board's diffusion index of industrial production, where $A = \{u, d, s\}$, $\omega^u = 1$, $\omega^s = 1/2$, $\omega^d = 0$, and $\widehat{D} = \widehat{p}^u + (1/2)\widehat{p}^s$. In that case,

$$\sqrt{n}\left(\widehat{D} - D\right) \overset{a}{\sim} \mathcal{N}\left(0, p^u + \frac{1}{4}p^s - D^2\right).$$

As before, the standard deviation of the index decreases as responses become less polarized; when either p^u or p^d is 1, $p^s = 0$ and $p^u + \frac{1}{4}p^s - D^2 = 0$.

At a more granular level, diffusion indexes may, in some cases, take into account different categories or groups of respondents. For example, participants may be reporting from different sectors, $j = 1, \ldots, J$,

$$\widehat{D} = \sum_{j=1}^{J}\sum_{a=1}^{r} \omega^a \frac{n_j^a}{n}, \tag{12}$$

where n_j^a denotes the number of responses of type a from group j with $\sum_{j=1}^{J} n_j^a = n^a$. This last expression may be equivalently written as

$$\widehat{D} = \sum_{j=1}^{J} \frac{n_j}{n} \underbrace{\sum_{a=1}^{r} \omega^a \frac{n_j^a}{n_j}}_{\widehat{D}_j},$$

where $\sum_{j=1}^{J} n_j = n$, so that the overall diffusion index may be interpreted as a weighted sum of group-specific diffusion indexes, $\widehat{D}_j$, with weights $\frac{n_j}{n}$, where n_j is the total number of responses obtained from group j. In principle, one may choose to scale those weights either up or down to emphasize particular target groups relative to the number of responses obtained in those groups, say by γ_j,[18]

$$\widehat{D} = \sum_{j=1}^{J} \gamma_j \frac{n_j}{n} \sum_{a=1}^{r} \omega^a \frac{n_j^a}{n_j}.$$

How then does this alternative weighting scheme affect the computation of confidence intervals? The answer depends not only on the alternative scaling, γ_j, but also on the more granular information captured in different groups by way of p_j^a, the probability of observing answer type "a" in group j in the overall survey, where $\sum_{a=1}^{r} \sum_{j=1}^{J} p_j^a = 1$. In particular, letting $\widehat{p}_j^a = n_j^a/n$ denote the proportion of answers of type $a \in \mathcal{A}$ observed in sector j in the overall sample, we have that

$$\sqrt{n}\left(\widehat{D} - D\right) \overset{a}{\sim} \mathcal{N}\left(0, \left(\sum_{a=1}^{r} \sum_{j=1}^{J} \left(\omega^a \gamma_j\right)^2 p_j^a\right) - D^2\right), \tag{13}$$

where $D = \sum_{a=1}^{r} \omega^a \sum_{j=1}^{J} \gamma_j p_j^a$.[19]

[18]For instance, the weights might ultimately reflect sectoral shares in value added or gross output. In practice, membership in the ISM Business Survey Committee is in fact initially based on each industry's contribution to gross domestic product (GDP).

[19]See appendix section A.2.

4. Composite Indexes and Their Distribution: Measuring Uncertainty Using Multiple Diffusion Indexes

In practice, very few, if any, diffusion indexes are reported as individual indexes. Rather, they are reported as composite indexes constructed as weighted averages of individual component indexes, described in the previous section, across different categories. For example, the headline Michigan Index of Consumer Sentiment combines information from individual indexes with respect to five categories: current and expected household financial conditions, current and expected overall business conditions, and spending on big-ticket items or durable goods. To the extent that each series is associated with some degree of uncertainty, arising from both sampling and fundamental polarization of survey answers, the uncertainty associated with the overall composite index will in turn depend on the extent to which this polarization co-moves across individual component indexes.

In characterizing the approximate distribution of diffusion indexes, one objective throughout the paper is to highlight the nature of the microdata needed to measure uncertainty in the survey responses, and thus how to organize and maintain incoming individual survey answers. As we will see, this involves keeping track of pairwise proportions of answer types across different questions. Intuitively, since composite indexes are weighted averages of individual indexes, each providing feedback on a particular question, the uncertainty surrounding the composite index will reflect not only the uncertainty associated with individual component indexes but also the covariances between them. For example, survey participants responding “up” to a particular category, such as their household financial condition, may be more or less likely to also respond “up” to another category, such as spending on durable goods, and this degree of co-movement may change as the state of the economy changes. Confidence intervals, therefore, must be adjusted accordingly. The next section formalizes these ideas.

4.1 Composite Diffusion Indexes

To describe composite diffusion indexes in a way that builds directly on the intuition presented thus far, we now consider a sample of n

survey participants responding to questions concerning $\overline{k}$ economic conditions or categories of interest. In practice, these might include financial conditions, employment conditions, etc. Answers from each participant corresponding to conditions of a particular component, k, are indexed by a_k, confined to a set $\mathcal{A}_k$, each comprising r possible types of responses, $\{1, 2, \ldots, r\}$. For example, $\mathcal{A}_1$ might describe whether conditions are "better off," "worse off," or "unchanged" in a given month, with the subscript "1" identifying the category "household financial conditions," and $a_1 \in \{u, d, s\}$, and similarly the subscript "2" in $\mathcal{A}_2$ might denote "overall business conditions," while the subscript "3" in $\mathcal{A}_3$ might denote "spending on big-ticket items."

A survey participant's answers across all components, $k = 1, \ldots, \overline{k}$, are collected in a k-tuple $\mathbf{a} = (a_1, \ldots, a_{\overline{k}})$ that lives in the set $\mathcal{A} = \Pi_{k=1}^{\overline{k}} \mathcal{A}_k$. In our simple example with three components, each comprising three possible responses, an example of $\mathbf{a}$ might be (u, u, u), indicating that conditions are "up," meaning improving, in household financial conditions, overall business conditions, and, say, spending on durable goods. In this case, $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3 = \{u, d, s\} \times \{u, d, s\} \times \{u, d, s\} = \{(u, u, s), (u, u, d), (u, u, s), (d, u, u), (d, d, u), (d, s, u), (s, u, u), (s, d, u), (s, s, u), \ldots\}$ has 27 elements. In general, $\mathcal{A}$ will have dimension $r^{\overline{k}}$.

It will be convenient below to distinguish between answers for a given component, k, and those for all other components, which we denote by $(a_k, \mathbf{a}_{-k})$, where $\mathbf{a}_{-k} \in \mathcal{A} \setminus \mathcal{A}_k$. We let $n^{\mathbf{a}}$ denote the number of respondents associated with answers $\mathbf{a} \in \mathcal{A}$, where $\sum_{\mathbf{a} \in \mathcal{A}} n^{\mathbf{a}} = n$. Observe that given our notation, $n^{\mathbf{a}}$ may also be expressed as $n^{(a_k, \mathbf{a}_{-k})}$. We let n_k^a denote the number of responses associated with answer $a_k \in \mathcal{A}_k$ for component k summed across all other components, $n_k^a = \sum_{\mathbf{a}_{-k} \in \mathcal{A} \setminus \mathcal{A}_k} n^{(a_k = a, \mathbf{a}_{-k})}$. For example, in the Michigan ICS, n_2^u might be the number of respondents reporting improving (u) overall business conditions (indexed by the subscript "2"), where $n_2^u = \sum_{\mathbf{a}_{-2} \in \mathcal{A} \setminus \mathcal{A}_2} n^{(u, \mathbf{a}_{-2})}$ includes all those reporting "up" on business conditions irrespective of their answers to other questions.

As before, answers of type $a_k \in \mathcal{A}_k$ for component k are assigned a value of $\omega^a \in \mathcal{R}$ independently of k. Then, an individual component diffusion index, D_k, associated with category or question k is given by

$$\widehat{D}_k = \sum_{a=1}^{r} \omega^a \sum_{\mathbf{a}_{-k} \in \mathcal{A} \backslash \mathcal{A}_k} \frac{n^{(a_k = a, \mathbf{a}_{-k})}}{n} = \sum_{a=1}^{r} \omega^a \frac{n_k^a}{n}, \tag{14}$$

and the composite index is averaged over all $\bar{k}$ components,

$$\widehat{D} = \sum_{k=1}^{\bar{k}} \delta_k \widehat{D}_k. \tag{15}$$

Note that the weights, δ_k, do not necessarily sum to 1, as in the Michigan ICS, which gives each of five questions an equal weight and uses this weight to normalize the overall index to a base year.

The approach followed earlier in section 3 can now be extended to characterize the distribution of the composite diffusion index $\widehat{D}$. Specifically, we show in appendix section A.3 that

$$\sqrt{n}\left(\widehat{D} - D\right) \overset{a}{\sim} \mathcal{N}\left(0, \sum_{k=1}^{\bar{k}} \delta_k^2 Var\left(\widehat{D}_k\right) + 2 \sum_{1 \le k < \ell \le \bar{k}} \delta_k \delta_\ell Cov\left(\widehat{D}_k, \widehat{D}_\ell\right)\right), \tag{16}$$

where $D = \sum_{k=1}^{\bar{k}} \delta_k \sum_{a=1}^{r} \omega^a p_k^a$,

$$Var\left(\widehat{D}_k\right) = \frac{1}{n}\left[\sum_{a=1}^{r} \left(\omega^a\right)^2 p_k^a - \left(D_k\right)^2\right], \tag{17}$$

$$Cov\left(\widehat{D}_k, \widehat{D}_\ell\right) = \frac{1}{n}\left[\sum_{(a,a') \in \mathcal{A}_k \times \mathcal{A}_\ell} \omega^a \omega^{a'} \times \left(p_{k\ell}^{aa'} - \sum_{(b,b') \in \mathcal{A}_k \times \mathcal{A}_\ell} p_{k\ell}^{ab} p_{k\ell}^{b'a'}\right)\right], \tag{18}$$

and

$$p_{k\ell}^{aa'} = \sum_{\mathbf{a}_{-\{k,\ell\}} \in \mathcal{A} \backslash \mathcal{A}_k \times \mathcal{A}_\ell} p^{(a_k = a, a_\ell = a', \mathbf{a}_{-\{k,\ell\}})}, \text{ for } k \neq \ell. \tag{19}$$

The probability $p_{k\ell}^{aa'}$ defined in (19) is the joint probability of observing $a_k = a$ and $a_\ell = a'$ for the components k and ℓ, and $p_k^a = \sum_{a' \in A_\ell} p_{k\ell}^{aa'}$ is the marginal probability with respect to component k.

5. Diffusion Indexes in Practice

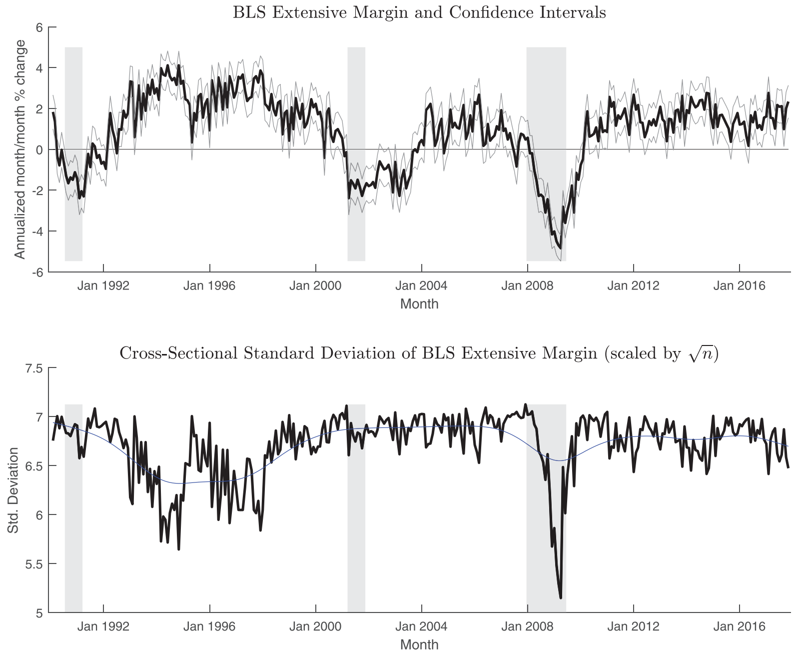
We now illustrate what the results derived in sections 3 and 4 imply in practice. First, we revisit the BLS employment diffusion index highlighted earlier as an example of an individual diffusion index. We then consider the historical behavior of consumer uncertainty, by way of composite diffusion indexes, using microdata on individual survey responses published by the MSC. In either case, measures of uncertainty associated with the index variance tend to fall the more a given direction of change is shared across sectors, or the more answers to given survey questions agree, and tend to rise as sectoral performance or survey responses become more polarized.

5.1 Revisiting the BLS Employment Diffusion Index

Consider once again the behavior of the BLS employment diffusion index in light of the results derived in the previous sections. The top panel of figure 3 reproduces the quasi-extensive margin contribution to employment growth depicted in figure 2, along with 95 percent confidence intervals. Since the distribution in (10) is normal, it is straightforward to construct confidence intervals for this measure, shown as the solid blue lines around μD in the top panel. Moreover, since the BLS considers 264 sectors, the sampling error associated with this quasi-extensive margin component is relatively small. The bottom panel of figure 3 shows the standard deviation of the BLS (scaled by $\sqrt{n}$) extensive margin component of employment growth in (10), along with its Hodrick-Prescott (HP) trend and with the recessions shaded in gray.[20] Observe that from 1990 to 2000, this measure of uncertainty, or polarization, tends to move opposite the index itself, as is typical of other conventional measures of uncertainty. As employment performance improves in the

[20]Since the data are monthly, we use a value of 14,400 for the HP smoothing parameter.

Figure 3. Uncertainty and the Breadth of BLS Employment Change



Notes: The top panel reproduces the extensive margin contribution ($D_t\mu^u$) to employment growth (Δx_t) from figure 2, along with 95 percent confidence intervals. The bottom panel shows the standard deviation of the BLS extensive margin component of employment growth (scaled by $\sqrt{n}$) in equation (10), along with its HP trend using a smoothing parameter of 14,400. Recession years are shaded in gray.

mid-1990s, the improvement also becomes more widespread across sectors and uncertainty falls; similarly, as overall employment starts declining toward the 2001 recession, it also becomes more polarized, with some sectors holding up while others lose employment.

In sharp contrast, however, the Great Recession stands out in that not only does BLS employment growth reach an all-time low during this period, but the breadth of the decline is also unprecedented, with the degree of polarization thus also reaching an all-time low. In that sense, the Great Recession was not only particularly severe along the intensive margin but also particularly widespread.

One key aspect of the notion of uncertainty emphasized here, as measured by the variance of diffusion indexes, is precisely that it is tied to the breadth of change; we become more or less confident in a given change as it becomes more or less widespread. Survey data, when they are purely qualitative, lack the intensive margin—for example, how strongly respondents might feel about overall business conditions. However, as we now show, we may nevertheless estimate a notion of uncertainty around sentiment based on how widely it is shared.

5.2 *The Michigan Surveys of Consumers*

The University of Michigan's Survey Research Center conducts consumer surveys and publishes three composite diffusion indexes: the Index of Current Conditions (ICC), the Index of Consumer Expectations (ICE), and the headline Index of Consumer Sentiment (ICS). The indexes are constructed from a number of questions to a sample of between 400 and 600 monthly interviews on average regarding household and overall economic conditions.[21]

Specifically, the composite indexes are derived from the following five questions:

- Q1: "We are interested in how people are getting along financially these days. Would you say that you (and your family living there) are better off or worse off financially than you were a year ago, or just about the same as now?"
- Q2: "Now looking ahead—do you think that a year from now you (and your family living there) will be better off financially, or worse off, or just about the same as now?"
- Q3: "Now turning to business conditions in the country as a whole—do you think that during the next twelve months we'll have good times financially, or bad times, or what?"

[21] Each month, the Survey of Consumer Attitudes chooses a representative sample of individuals from a list of national telephone users. The sample consists of two parts: a number of new contacts randomly selected for a specific month, which constitutes approximately two-thirds of the sample, and a sample of individuals that completed the survey six months previously. Thus, the majority of the sample is essentially entirely random, and the rest consists of a sample of respondents that participated six months ago. Moreover, note that the selection of the latter group is itself mostly a random subsample of previous respondents.

- Q4: “Looking ahead, which would you say is more likely—that in the country as a whole we’ll have continuous good times during the next five years or so, or that we will have periods of widespread unemployment or depression, or what?”
- Q5: “About the big things people buy for their homes—such as furniture, a refrigerator, stove, television, and things like that—generally speaking, do you think now is a good or bad time for people to buy major household items?”

A somewhat unique feature of the MSC composite indexes, relative to other published diffusion indexes, concerns the individual response level data underlying the index calculations, which are made readily available through the MSC’s online archives starting in 1978. In particular, from the archives, it is possible to obtain response-level data that allow keeping track, in each month, of a given respondent’s answers to each question making up the various MSC composite indexes. Given this level of detail, it is then possible to construct, in any given month, all pairwise responses, $n_{k\ell}^{aa'}$, and proportions, $\widehat{p}_{k\ell}^{aa'}$, where $k\ell$ denotes any pair of questions from the set of questions Q1 through Q5 listed above, and aa' any pair of answers—for example, “better off” and “good times.”

5.2.1 The Michigan Index of Current Conditions

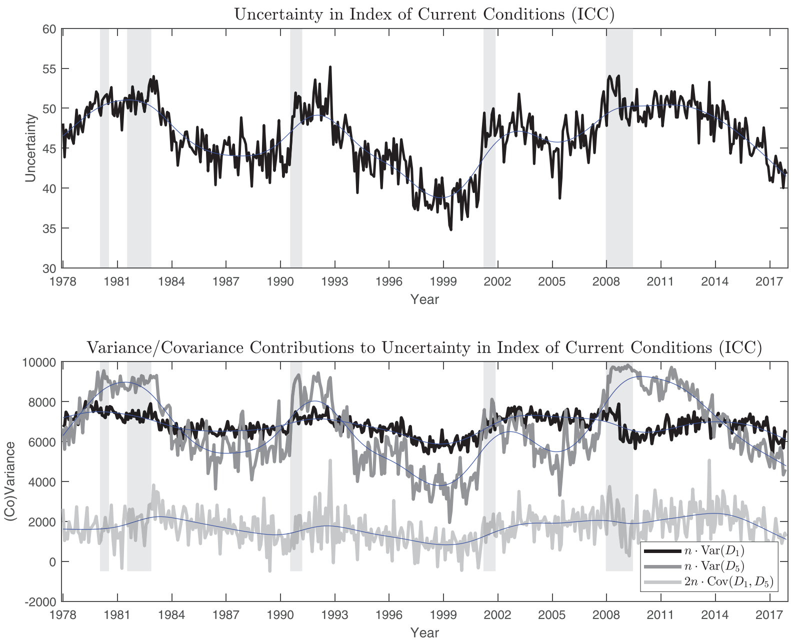
To calculate the Index of Current Conditions, the MSC first computes the diffusion indexes, or the proportion giving favorable replies less the proportion giving unfavorable replies (plus 100) for questions Q1 and Q5 listed above, denoted D_1 and D_5, respectively. Each individual index is rounded to the nearest whole number. The ICC is then calculated as

$$ICC = \frac{D_1 + D_5}{2.6424} + 2, \tag{20}$$

where the denominator in the above expression establishes a base period (1966), and the constant corrects for sample design changes in the 1950s.[22] Observe that D_5 summarizes a direction of change

[22]To the best of our knowledge, the micro- or response-level data are publicly available only since 1978.

Figure 4. Uncertainty in Consumers' Perception of Current Conditions



Notes: The top panel displays the behavior of the standard deviation of the ICC (scaled by $\sqrt{n}$), as given by equation (16), along with its HP trend. The bottom panel illustrates the decomposition of uncertainty in the ICC in terms of the variances of the individual diffusion indexes associated with questions Q1 and Q5 in the MSC, and the covariances between them. Note that the square of the series in the top panel is equal to the sum of the series in the bottom panel divided by 2.6424^2 (recall, from equation (20), that $ICC = (D_1 + D_5)/2.6424 + 2$). Recession years are shown in gray.

in households' current attitudes toward the purchase of big-ticket items or durable goods. As such, it is closely linked to households' attitudes toward investment. The individual index D_1 summarizes instead the direction of change in the state of households' finances, which likely has a direct bearing on their "ability and willingness to buy," which is used in part by the MSC as a working definition of consumer confidence.

The top panel of figure 4 shows the historical behavior of the standard deviation of the ICC (times $\sqrt{n}$), as given by equation (16), along with its HP trend and the recessions shaded in gray. Beginning in the early 1990s, the uncertainty reflecting the degree of polarization in responses underlying the ICC begins a decade-long decline to its lowest point in the sample, around 1999. This decade-long decline in uncertainty corresponds to one of the strongest expansions in postwar U.S. economic history, as measured in part by consumption growth, ending with the dot-com bust and the start of the 2001 recession. Prior to the 2001 recession, the degree of polarization in the ICC begins to rise as consumers increasingly disagree on the questions related to current conditions. As in Bachmann, Elstner, and Sims (2013) and Baker, Bloom, and Davis (2013), spikes in disagreement prior to the recessions of 1991, 2001, and 2007 are clearly visible. In principle, however, disagreement about the state of current conditions would not necessarily have to rise during recessions since it may be generally recognized, and agreed upon, by consumers that the state of the economy is poor or, more specifically in this case, that it is not a good time to purchase durable goods.

The recovery from the 2001 recession was somewhat subdued relative to other postwar U.S. recoveries, especially where employment is concerned, and the recovery from the 2007 recession is widely known to be the weakest on record from a number of standpoints, most notably per capita GDP growth. Interestingly, the top panel of figure 4 indicates that throughout the 2001 and 2007 recessions, and the recovery in between, uncertainty in consumers' views regarding the state of current conditions, at least as captured by the MSC, continued to rise steadily. Moreover, the degree of polarization in answers to questions regarding current conditions remained noticeably flat in the nearly six years that followed the Great Recession, although disagreement has recently started to fall somewhat.

Remarkably, the level of uncertainty in the ICC today, almost six years removed from the most recent recession, is comparable to that which emerged during the twin recessions of the 1980s and the period immediately surrounding the 1991 recession. The standard deviation of the ICC (normalized by $\sqrt{n}$) currently stands around 20 percent higher than in 1999, at which time it began to generally rise to the level we see today. In fact, the period since 2000 is notable relative to other postwar recoveries in the sample in that uncertainty unambiguously declined during the periods of pronounced economic expansion of the late 1980s and 1990s. These observations are suggestive, as argued by Bloom (2014) and others, that the state of economic activity is intrinsically linked to the degree of uncertainty households perceive, not only at business cycle frequencies but, as shown here, at longer frequencies as well. As with other measures of uncertainty in the literature, uncertainty in the ICC, in spite of currently being at a near all-time high six years into an expansion, tends to be countercyclical, falling in expansions and rising in and around recessions.

The bottom panel of figure 4 illustrates the decomposition of uncertainty in the ICC in terms of the variances of the individual diffusion indexes associated with questions Q1 and Q5 in the MSC, and the covariance between them (adding the series in the bottom panel, and dividing by 2.6424^2, gives the square of the series in the top panel). The main driver of uncertainty or variance in the ICC is the variance of the diffusion index associated with question Q5 in the MSC: the question related to consumers' perception of whether it is a good or bad time to purchase major household items or durable goods. The degree of polarization in answers to that question rose between 1999 and 2009, remained approximately constant until 2013, and has come down since then. Observe that since the end of the most recent recession until 2014, the covariance between the diffusion indexes D_1 and D_5 has risen slightly, a trend that began after the 2001 recession, indicating rising coincidence in answers to questions Q1 and Q5 over this time period. In other words, the extent to which consumers' answers to questions Q1 and Q5 have tended to locate in the extremes, good or bad rather than middle, became gradually more consistent across questions. As described in section 4, in the case of composite diffusion indexes, coincident polarization across questions increases uncertainty, as captured by the variance of the composite index. The covariance, however, started to decrease since 2014, and now all the components of the ICC uncertainty index point in the same declining direction.

5.2.2 *The Michigan Index of Consumer Expectations*

The Michigan Index of Consumer Expectations is based on diffusion indexes that summarize households' perceptions of times ahead, both for themselves and the country as a whole, over different time horizons, one and five years out. In particular, the ICE is given by

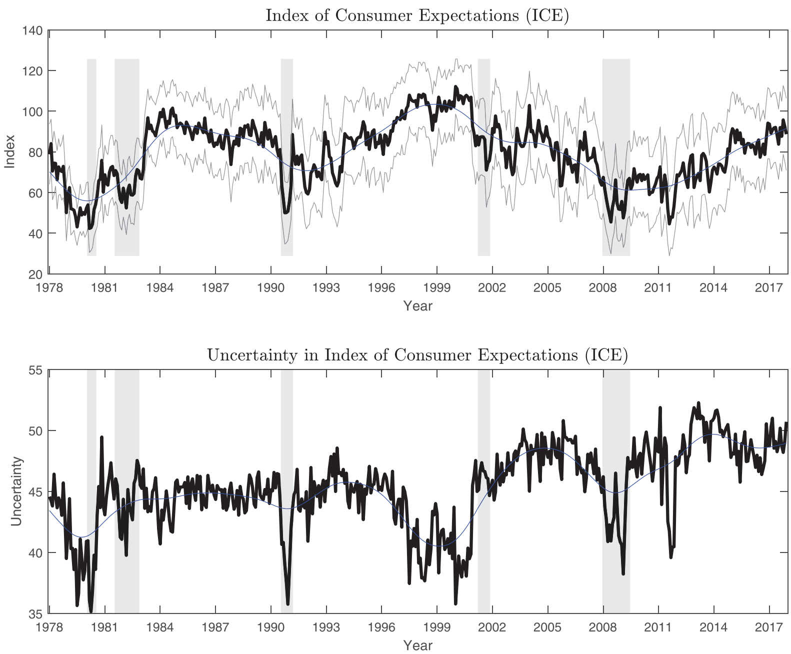
$$ICE = \frac{D_2 + D_3 + D_4}{4.1134} + 2,$$

where D_2, D_3, and D_4 are the diffusion indexes associated with questions Q2, Q3, and Q4 listed above, respectively. As with any survey, the questions are potentially subject to varying interpretations by consumers, and the level of polarization in answers might then also vary considerably over time. As we now illustrate, however, polarization in the ICE tends to be remarkably stable throughout the 1980s and into the 1990s, before starting a steady rise that is even more pronounced with respect to expectations than that shown with respect to consumers' uncertainty around current conditions.

The top panel of figure 5 shows the level of the ICE published by the MSC, and the bottom panel shows uncertainty in the ICE implied by the standard deviation in equation (16). Although more involved than in the case of the individual diffusion index, the distribution of composite indexes in (16) remains normal, so that constructing confidence intervals for the level of the ICE remains conceptually straightforward. These are shown as the solid blue lines around the MSC index in the top panel. However, as we address in more detail below, the calculations now entail the consideration of multiple pairwise objects, $\widehat{p}^{ab}_{k\ell}$, across answers and survey questions.

Observe first that the top and bottom panels of figure 5 paint distinctly different pictures: one provides a measure of the direction of change in households' perception of times ahead, while the other is indicative of the level of polarization in those perceptions. Starting in the early 1990s, the ICE begins to climb steadily, in the top panel, as the U.S. economy enters one of its longest expansion periods and households contemporaneously grow more optimistic about the future. At the same time, as with the ICC, consensus that the next one to five years are likely to be "good times" also grows among households, and the level of polarization in answers falls in the bottom panel. With the arrival of the 2001 recession, the ICE experiences a significant dip and levels off, while uncertainty among consumers about what to expect begins to increase dramatically. As the 2007 recession starts, the ICE falls further and remains at relatively subdued levels throughout the subsequent recovery period. At the same time, uncertainty about the next one to five years generally continues to increase until 2014. Polarization among consumers

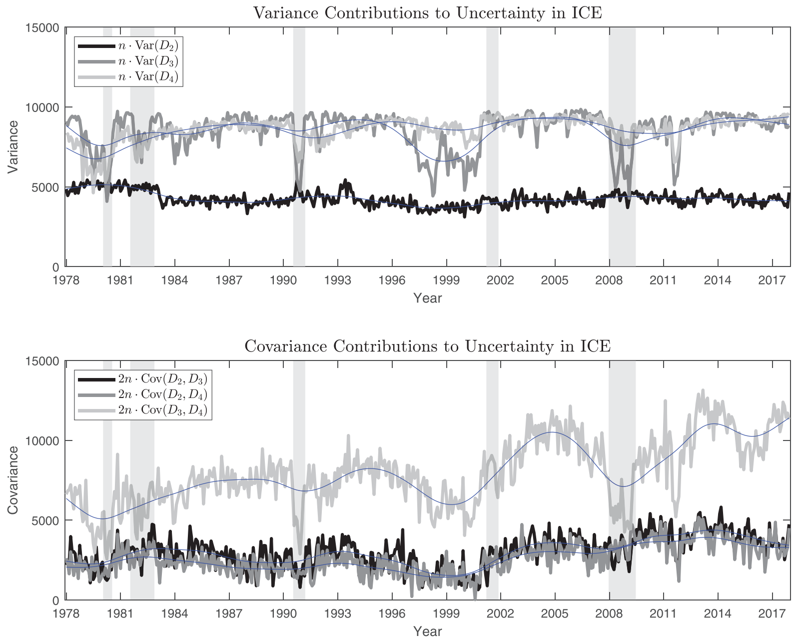
Figure 5. Direction of Change and Uncertainty in Consumers' Expectations



Notes: The top panel shows the behavior of the ICE series, along with 95 percent confidence intervals (solid blue line). The confidence intervals shown in the figure incorporate the serial correlation adjustment described in footnote 23. The bottom panel describes the behavior of uncertainty in the ICE implied by the standard deviation in equation (16), along with its HP trend (blue line). Recession years are shown in gray.

has slightly decreased since 2014 and has stabilized today at a level that is more than 20 percent above its 1999 level. Observe that, in contrast to the ICC, the 1991 and 2007 recessions are associated with pronounced downward spikes and a general pessimistic consensus about the future. In terms of trend, however, uncertainty about what lies ahead conveyed by consumers remains relatively stable throughout the 1980s and up to the mid-1990s. The period since 2000, in contrast, is one of striking rising uncertainty in consumers' perception of the future, indicating today a level of polarization in households' expectations unprecedented since 1978.

Figure 6. Decomposition of Uncertainty in Consumers' Expectations



Notes: The figures show the decomposition of uncertainty in the ICE (measured by the standard deviation in equation (16) and presented in figure 5) into its variance (equation (17)) and covariance components (equation (18)). The top panel illustrates the behavior over time of the variances (scaled by n) associated with the individual diffusion indexes, D_2, D_3, and D_5. The bottom panel shows the behavior of the pairwise covariances (scaled by $2n$). The figures also include the HP trends of each series. Recession years are shown in gray.

Figure 6 shows the decomposition of uncertainty in the Michigan ICE. The top panel of figure 6 illustrates the behavior over time of the variances (scaled by n) associated with the individual diffusion indexes, D_2, D_3, and D_5, while the bottom panel shows the behavior of the pairwise covariances. In the top panel, we see that the level of polarization in answers to question Q2, regarding personal finances one year ahead, is remarkably constant throughout the entire sample. Uncertainty regarding the national outlook, however, at both the one- and five horizons, sees more variations over time. In the bottom panel, all covariances experience a generally increasing tendency starting in 2000, but this tendency is especially pronounced where the co-movement between answers to questions Q3 and Q4 is concerned. Recall that questions Q3 and Q4 in the MSC differ mainly with respect to the time horizon over which households are asked about their expectations of the national outlook. Put another way, households' answers regarding the country as a whole grow more coincident between the one- and five-year horizons. This finding suggests that with the onset of the 2001 recession, households begin to perceive the state as more persistent, regardless of whether it is good or bad. This more coincident polarization of expectations across the one- and five-year horizons in turn is a key driver of the rising uncertainty in the ICE in the bottom panel of figure 6. In this case, the covariation in the degree of disagreement across different questions of a qualitative survey plays a substantive role in the determination of overall uncertainty conveyed by the survey.

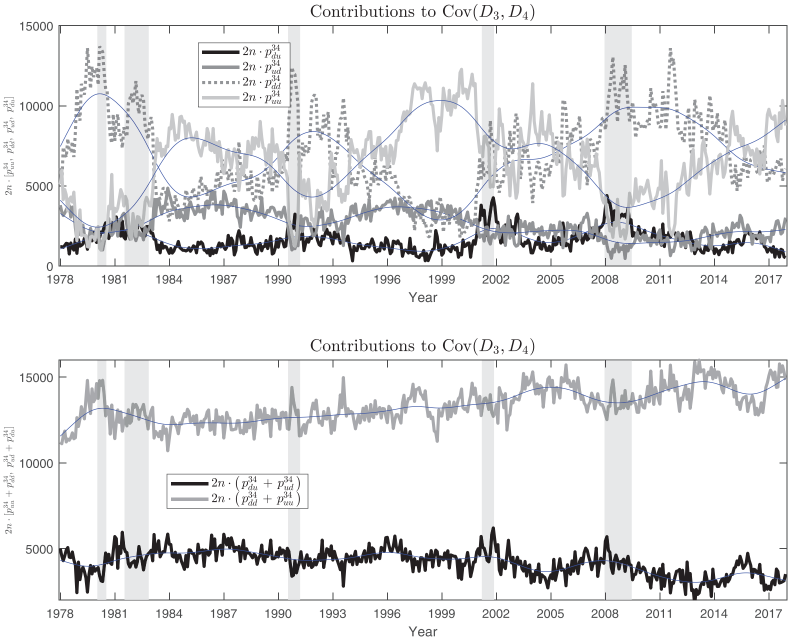
Figure 7 illustrates the more coincident polarization of responses with respect to the one- and five-year expectations of overall business conditions by plotting the various elements that make up the covariance between the diffusion indexes associated with questions Q3 and Q4, $Cov\left(\widehat{D}_3, \widehat{D}_4\right)$. In particular, in this case, the expression in (18) reduces to

$$Cov\left(D_3, D_4\right) = \left(p_{34}^{uu} + p_{34}^{dd}\right) - \left(p_{34}^{ud} + p_{34}^{du}\right) - D_3 \times D_4. \tag{21}$$

Intuitively, the more responses to questions Q3 and Q4 in the MSC coincide, or the larger p_{34}^{uu} and p_{34}^{dd}, the more D_3 and D_4 tend to co-move, while the reverse is true the more answers disagree across questions, or the larger p_{34}^{du} and p_{34}^{ud}. Furthermore, analogously to the squared term in the equation describing the variance of individual diffusion indexes (17), the more one-sided responses become in the same direction, or alternatively as both D_3 and D_4 approach either 1 or -1, the less room there is for the indexes to co-move.

In the case of questions Q3 and Q4 in the MSC, the top panel of figure 7 shows the behavior of the various proportions making up equation (21). First note that the terms capturing the coincidence in responses, p_{34}^{uu} and p_{34}^{dd}, tend to dominate relative to the terms reflecting disagreement, p_{34}^{ud} and p_{34}^{du}, and are noticeably procyclical and countercyclical, respectively. Second, in the bottom panel of figure 7,

Figure 7. Decomposition of Co-movement in Answers across the One- and Five-Year Horizons (D_3 and D_4)



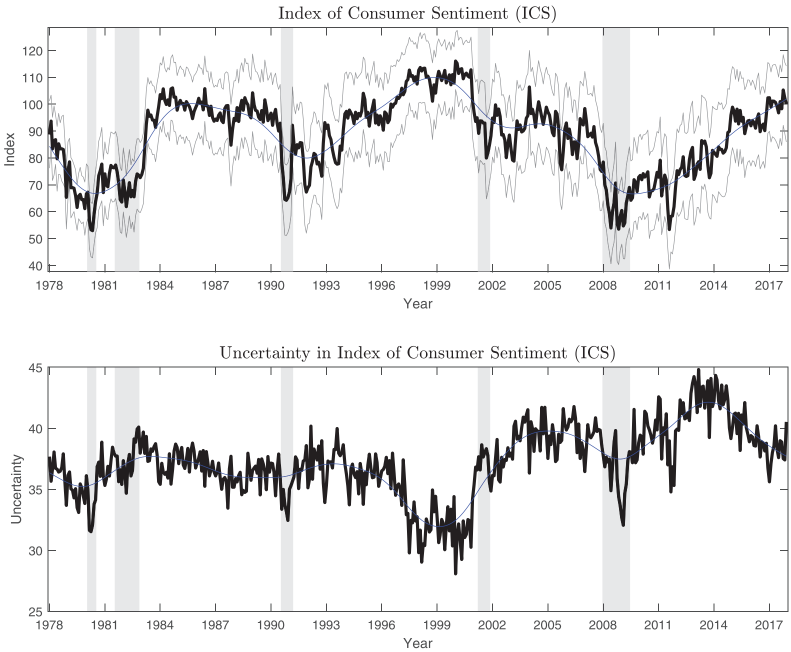
Notes: The figures show the behavior of the first two terms of $Cov\,(D_3, D_4)$ (equation (21)). The top panel shows the behavior of the individual proportions $p_{34}^{aa'}$. To make these terms comparable to the expressions described in figure 6, the proportions are scaled by $2n$. The bottom panel describes the behavior of the terms $(p_{34}^{uu} + p_{34}^{dd})$ and $(p_{34}^{ud} + p_{34}^{du})$ (also scaled by $2n$).

we see that the rise in co-movement between D_3 and D_4, depicted in the bottom panel of figure 6, indeed arises from increasing coincidence, and decreasing disagreement, among responses concerning overall business conditions at the one- and five-year horizons. In that sense, survey responses indicate that participants increasingly see the state one year ahead as persisting into a five-year horizon.

5.2.3 The Michigan Index of Consumer Sentiment

The headline Michigan Index of Consumer Sentiment summarizes the direction of change in consumer feedback by combining both

Figure 8. Direction of Change and Uncertainty in Consumer Sentiment



Notes: The top panel shows the behavior of the ICS series, along with its 95 percent confidence intervals (solid blue line). The confidence intervals incorporate the serial correlation adjustment described in footnote 23. The bottom panel describes the behavior of uncertainty in the ICS implied by the standard deviation in equation (16), along with its HP trend (blue line). Recession years are shown in gray.

their assessment of current conditions, which drives the ICC, and their expectations, which drive the ICE,

$$ICS = \frac{D_1 + D_2 + D_3 + D_4 + D_5}{6.7558} + 2.$$

Given the historical behavior of polarization in the ICC and ICE indexes, it is not surprising to observe, in the bottom panel of figure 8, an increase in uncertainty in the ICS index, as measured by the standard deviation of the ICS (times $\sqrt{n}$), starting in the year 2000, and again following the 2008 recession, to reach an unprecedented level over our sample period in the

Table 2. Consumer Surveys: 95 Percent Confidence Intervals

Series	1978–1999	2000–2017	1978–2017
Index of Current Conditions (ICC)	±4.19	±4.50	±4.33
Index of Consumer Expectations (ICE)	±4.02	±4.49	±4.23
Index of Consumer Sentiment (ICS)	±3.29	±3.70	±3.48

Notes: Entries are time averages of $\pm 1.96\sqrt{\left[\sum_{k=1}^{\bar{k}} \delta_k^2 Var\left(\widehat{D}_k\right) + 2\sum_{1\leq k<\ell\leq\bar{k}} \delta_k\delta_\ell Cov\left(\widehat{D}_k, \widehat{D}_\ell\right)\right]/n}$.

year 2014.[23] Uncertainty in the ICS was approximately 25 percent higher in 2014 than throughout the 1980s and 1990s. Observe, in particular, that as consumer sentiment, depicted in the top panel of figure 8, has steadily risen since the end of the last recession, the degree of polarization of responses reflected in the standard deviation of the ICS in the bottom panel has first increased and later declined. Specifically, the evolution of the variables describes two different subperiods after the recession. The first one goes from 2009 until 2014 and is described as a period in which consumers appeared to be more confident but also more polarized in their views according to the behavior based on the ICS index. In fact, overall polarization is driven mostly by the evolution of polarization in the ICE, which is increasing, since during this subperiod, polarization in ICC is relatively constant. From 2014 until now, consumers' confidence continues growing, but polarization declines. During this latter period, polarization in the ICE is approximately flat (lower than the 2014 peak, but still high and rising since 2016), polarization in ICC sharply declines, pushing polarization in the ICS index downward.

We end the discussion on the MSC indexes by highlighting the fact that, in characterizing the full distribution of general composite diffusion indexes, the formulas derived in (16) through (18) may be used to produce confidence intervals around any diffusion index estimate. In the case of the MSC, table 2

[23]The ICS index is given by $ICS = (ICC + ICE)/6.7558$. The evolution of polarization in the ICS index is driven mostly by polarization in the ICE index, rather than by polarization in the ICC. The correlation between polarization in the ICE and the ICS indexes is 0.87. In contrast, the correlation between the ICC and ICE indexes is low, approximately 0.20.

summarizes the margins of error associated with 95 percent confidence intervals for its different indexes averaged over different sample periods.[24]

As shown in the top panels of figures 5 and 8, with around 400 respondents, these margins are relatively tight compared with the span of the indexes, although variations within those margins are not infrequently taken up or discussed as meaningful changes in direction.[25] The Institute for Supply Management, and the Federal Reserve Bank of Philadelphia's Business Outlook Survey, do not

[24]In the context of asymptotic results deriving from a statistic (i.e., the diffusion index) based on draws from a multinomial distribution, and where these draws are then repeated in subsequent periods from potentially different multinomial distributions (i.e., indexed by different probabilities), we know of no general approach that also considers time dependence across periods. As noted by Zwiers and von Storch (1995), with time dependence, information in each observation is not totally separate from the information in other observations, so that the effective sample size is smaller than the actual sample size. Specifically, in the case of binomial draws with successes and failures, the effective sample size is given by $n_t^* = an_t$, where $a = 1/\left[1 + 2\sum_{\tau=1}^{n-1}(1 - \tau/n)\,\rho(\tau)\right]$, and $\rho(\tau)$ is the correlation in successes between period t and $t + \tau$. If this series then follows an AR(1) process with autocorrelation coefficient ρ, such that $\rho(\tau) = \rho^{|\tau|}$, $a = 1/\left[1 + 2\sum_{\tau=1}^{n-1}(1 - \tau/n)\,\rho^{\tau})\right]$. For large n_t, therefore, the effective sample size can then be approximated by $n_t^* \approx n_t[(1 - \rho)/(1 + \rho)]$. In the case of the MSC, there are more than two answer types so that we take ρ to be the average of the autocorrelation coefficients calculated for each answer type. An alternative would be to take the max of these autocorrelation coefficients. However, the autocorrelation coefficients across answer types in our example are relatively close so that findings aren't materially different in the latter case. These calculations are reflected in the confidence intervals shown in the top panels of figures 5 and 8. Observe that since the measures of polarization or uncertainty that we show at the bottom panels of figures 5 and 8 (as well as the top panel in figure 4) are independent of the sample size (they represent an estimate of the asymptotic standard deviation of D_t, where $n_t^* \to \infty$ when $n_t \to \infty$), the correction modeled as in Zwiers and von Storch (1995) does not modify the behavior of the latter series.

[25]Specialized media follows closely changes in consumer sentiment. In the wake of the government shutdown of 2013, for example, the *Wall Street Journal* reported that "U.S. consumers turned less optimistic about the economy in early October, according to data released Friday. The Thomson-Reuters/University of Michigan preliminary October sentiment index slipped to 75.2 from an end-September level of 77.5, according to an economist who has seen the numbers" (Madigan 2013). In this case, the index decreased only 2.3 points in anticipation of the government shutdown, which statistically did not indicate a change in consumer sentiment.

publicly report margins of errors, and indeed few if any of the most widely published diffusion indexes do.

5.2.4 Other Indexes of Uncertainty

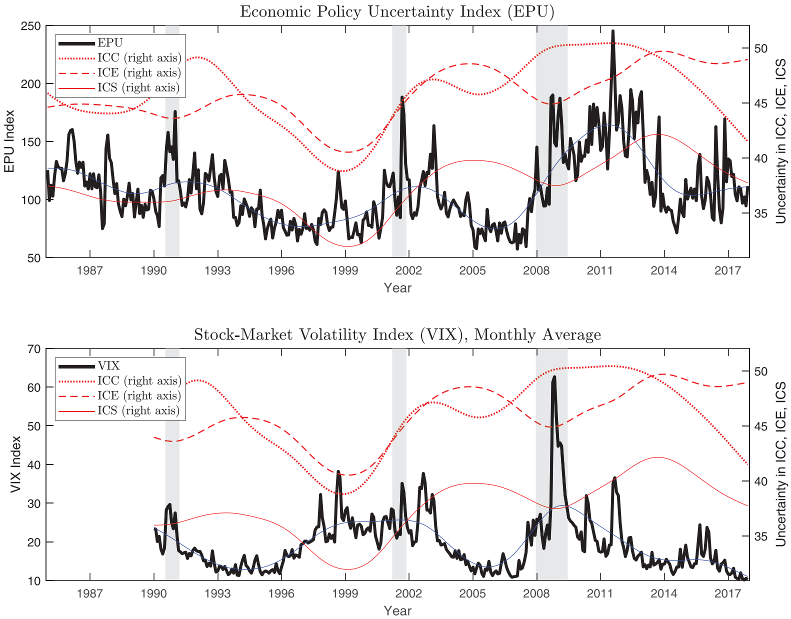
To further explore the notion of polarization in survey responses developed in previous sections, we next compare their behavior with that of a number of existing measures of uncertainty, financial volatility, and political polarization. The latter indexes capture different aspects of uncertainty and thus do not necessarily coincide.

Several indicators are generally used to measure economic uncertainty. The Chicago Board Options Exchange Volatility Index (VIX), for instance, reflects the stock market's expectation of volatility over the next 30 days, and is constructed using a series of options included in the S&P 500 index. Alternatively, the Economic Policy Uncertainty (EPU) index developed by Baker, Bloom, and Davis (2013) intends to convey uncertainty about economic policy. It consists of three components, the first of which is based on media coverage of economic policy uncertainty, the second of which tracks federal tax provisions that are set to expire in the next few years, and the third of which captures disagreement between economic forecasters. Both the VIX and the EPU, along with the (smoothed) polarization indicators associated with the ICC, ICE, and ICS, are shown in figure 9. The figure reveals that the evolution of polarization in the ICC and in the EPU are remarkably similar, with both indicators reaching their peaks well after the end of the Great Recession in 2012.[26] A key difference, however, between ICC polarization and the EPU concerns the behavior of these measures since 2014. Specifically, beginning in 2014, the EPU index has been increasing while ICC uncertainty has sharply declined. This recent divergence underscores the notion that there can be different facets to uncertainty which can move in different directions over a given time period.

When we consider the entire period since 1978, our indicators do not seem to move closely with stock market volatility described

[26]The correlation between polarization in the ICC and the EPU is 0.5.

Figure 9. EPU and VIX Indicators



Notes: The top panel shows the evolution of the Economic Policy Uncertainty index developed by Baker, Bloom, and Davis (2013). The bottom panel shows the evolution of the Chicago Board Options Exchange Market Volatility Index. The figures also include the HP-filtered polarization indicators associated with the ICC, ICE, and ICS for comparison (in red; scale on the right axis). Recession years are shown in gray.

by the VIX, as shown in the bottom panel of figure 9.[27] That said, since 2003, polarization in the ICC and the VIX have been moving in the same direction.

We also compare the behavior of our indicators with other more recent measures of uncertainty developed in the literature. We focus here on different measures suggested by Jurado, Ludvigson, and Ng (2015) (JLN), and by Rossi, Sekhposyan, and Soupre (2016) (RSS). The JLN index is essentially an h-step-ahead forecast error variance derived from a factor model that includes a large set of

[27] Over the sample period, the correlations between the VIX and polarization in the ICC and the ICE are either small or negative, 0.08 and −0.45, respectively.

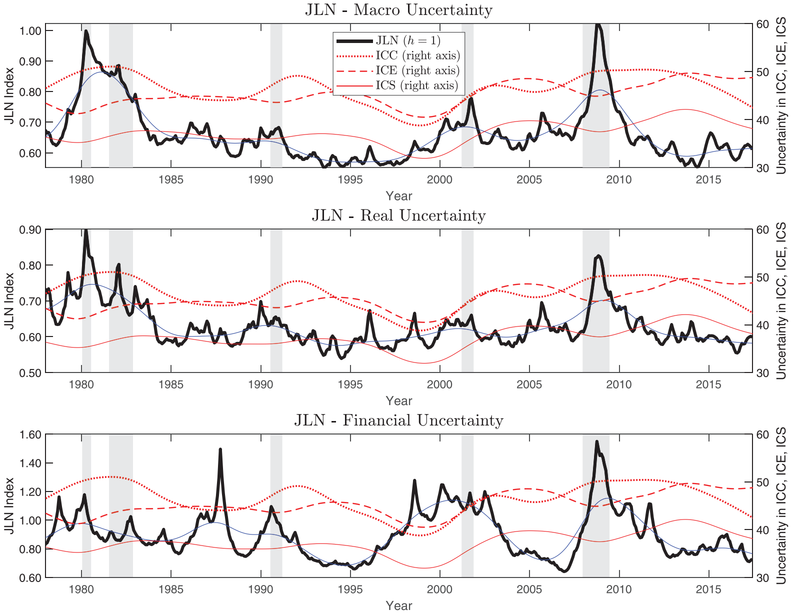
variables. The authors estimate a number of indexes that capture different types of uncertainty, including macroeconomic, real, and financial uncertainty, using alternative time horizons—specifically, $h = 1, 3, 12$ months ahead. The evolution of the three uncertainty measures, i.e., macro, real, and financial uncertainty, is presented in figure 10, along with our measures of polarization. We only show in the graph the series for $h = 1$. It appears from figure 10 that the JLN macro and real uncertainty measures and polarization in the ICC index vary in a similar way.[28]

The RSS index uses data from the Survey of Professional Forecasters (SPF) and is constructed from the density of forecasts. Consistent with the idea that there are different aspects to uncertainty, Rossi, Sekhposyan, and Soupre (2016) decompose their uncertainty index into a number of components. The first decomposition breaks down overall uncertainty into aggregate uncertainty and disagreement. A second decomposition differentiates between risk and Knightian uncertainty. Their last decomposition separates ex ante from ex post uncertainty. Figure 11 shows the different decompositions of uncertainty and compares the RSS indexes with our polarization measures. Overall, movements in the RSS indexes and our polarization indexes are only weakly related. Among all indexes, the highest correlation emerges between polarization in the ICC index and the risk component of RSS uncertainty.

Finally, we compare our indexes of polarization with the Partisan Conflict Index (PCI) published by the Federal Reserve Bank of Philadelphia. This index, constructed using text analytic techniques on media publications, attempts to capture partisan disagreement among lawmakers at a given point in time. When compared with our indexes of polarization, we find that movements in the PCI seem to be more closely related to movements in polarization in the ICE (and in the ICS), as shown in figure 12. In particular, both indicators increased between the last recession and 2014. After reaching a sample maximum in 2014, polarization in the ICE began to decline.

[28]The correlations between JLN macro uncertainty and polarization in the ICC, and between JLN real uncertainty and polarization in the ICC, are above 0.40 for $h = 1$ and $h = 3$. The correlations between all JLN indicators and polarization in the ICE are in fact negative (in absolute value, the highest correlations are between JLN financial uncertainty and polarization in the ICE).

Figure 10. JLN Uncertainty Indicators

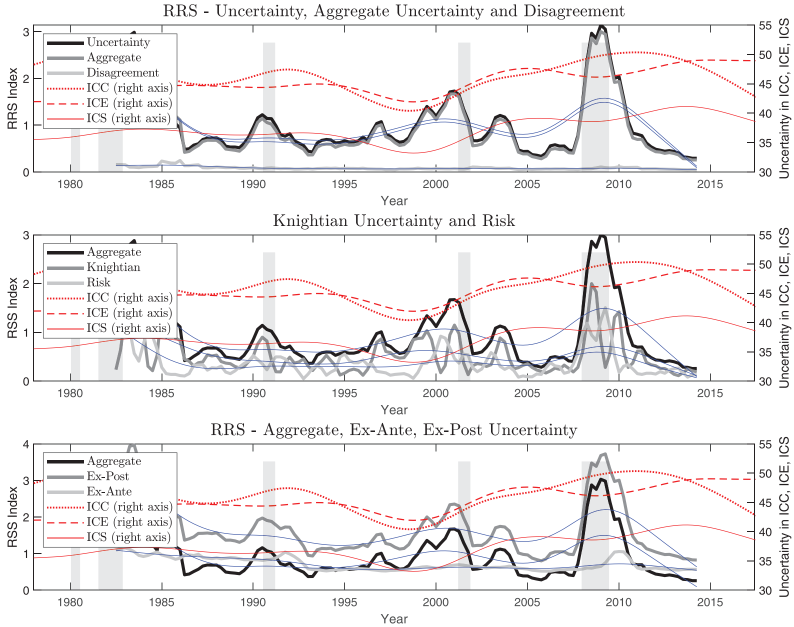


Notes: The figures show the behavior of three indicators developed by Jurado, Ludvigson, and Ng (2015). These indicators capture different types of uncertainty: macroeconomic (top), real (middle), and financial (bottom) uncertainty. The figures display the behavior of the series for $h = 1$ month ahead. Each figure shows the evolution of the respective indicator, along with its HP-filtered trend, and the HP-filtered measures of polarization in the ICC, ICE, and ICS developed in the paper (in red; scale on the right axis). Recession years are shown in gray.

In contrast, after being stable over the period 2014–15, the PCI continued rising and since 2016, both indicators have been increasing. The PCI and polarization in the ICC series, however, do not show a high degree of co-movement.

The comparisons above make clear that there are different aspects of uncertainty, and researchers have used different approaches to quantify them. Jurado, Ludvigon, and Ng (2015) and Rossi, Sekhposyan, and Soupre (2016) estimate uncertainty in different ways. While the JLN uncertainty indicators are based on forecast errors of a large number of economic variables, the RSS indexes reflect uncertainty from subjective predictions about the evolution of

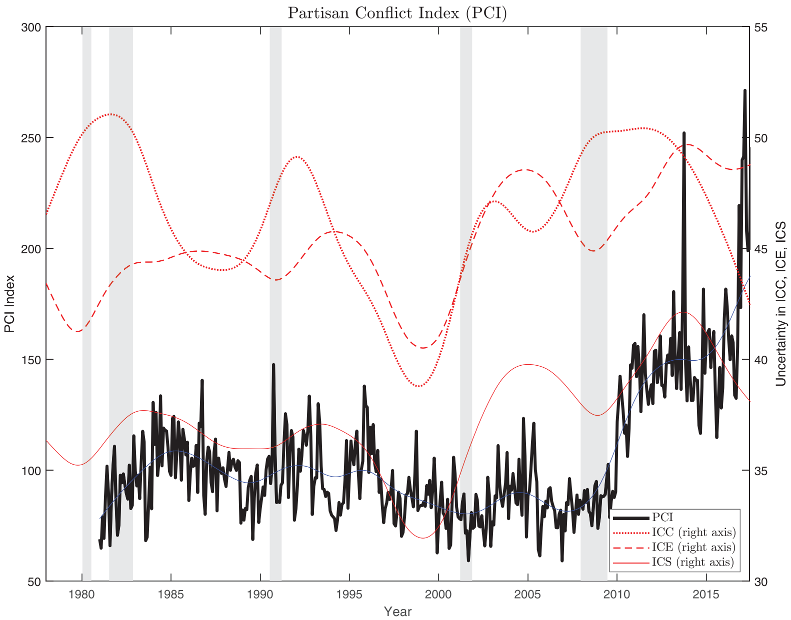
Figure 11. RSS Uncertainty Indicators



Notes: The figure shows the behavior of different indicators of uncertainty developed by Rossi, Sekhposyan, and Soupre (2016) (RSS). Aggregate uncertainty, as captured by the RRS index, is decomposed into different components: (i) aggregate uncertainty and disagreement, (ii) risk and Knightian uncertainty, and (iii) ex ante and ex post uncertainty. The evolution of these three components (and their respective HP-filtered trends), along with the evolution of our (HP-filtered) measures of polarization in the ICC, ICE, and ICS (in red, scale on the right axis), are shown in the top, middle, and bottom panels. Recession years are shown in gray.

certain variables. These proxies of uncertainty, as a result, can move in different directions over a given period of time. We contribute additional measures of uncertainty that focus on polarization in survey responses regarding current economic conditions (ICC), and expectations with respect to the future (ICE), as well as polarization in the overall ICS. We note that polarization in the ICC and the ICE can also move in different directions. Here, therefore, disagreement in responses to questions about current conditions and expectations of future economic conditions can differ materially. Moreover,

Figure 12. Partisan Conflict Index (PCI)



Notes: The figure compares the evolution of the Partisan Conflict Index (PCI), published by the Federal Reserve Bank of Philadelphia, with our (HP-filtered) measures of polarization in the ICC, ICE, and ICS (in red, scale on the right axis). Recession years are shown in gray.

polarization in the ICC and the ICE can themselves co-move with different measures of uncertainty computed in previous work. While polarization in the ICC tends to move more closely with the JLN index, polarization in the ICE tends to be more closely aligned with the PCI.

6. Concluding Remarks

In this paper, we thoroughly examine the properties of diffusion indexes defined as estimates of the breadth of change in an aggregate series of interest. The analysis highlights, in the first place, the relevance of diffusion indexes in capturing changes in the extensive margin. We show, for instance, that the BLS's employment diffusion index explains a large part of overall employment growth.

Next, we characterize the distribution of general composite diffusion indexes, defined as the weighted sum of individual indexes based on responses to different individual survey questions. We show that diffusion indexes are asymptotically normal, and that the uncertainty proxy in Bachmann, Elstner, and Sims (2013) is the variance of a particular, albeit widely used, individual diffusion index (scaled by the square root of the sample size). This proxy, which takes into account the uncertainty implied by both sampling and the polarization or disagreement of participants' responses, can be used to construct confidence intervals for the diffusion index. Our approach reveals that a general notion of uncertainty based on composite indexes reflects both the degree of disagreement with respect to an individual survey question and the degree of disagreement across individual questions. In particular, we show that only pairwise proportions of answer types across questions are relevant to derive the variance of composite indexes.

Finally, we use microdata published by the MSC to illustrate our results. We find that consumer uncertainty calculated using our approach and measured by polarization of the ICS index is relatively constant from 1978 until approximately 1993 and declines thereafter until the end of the 1990s. It steadily increased until 2014, when it reached the highest level since 1978, and declines thereafter. A salient aspect of the behavior of this measure is that uncertainty continued increasing and remained relatively high even after six years after the Great Recession.

The analysis and results of this paper have direct and practical applications in terms of efforts carried by various institutions, including Federal Reserve Banks, the ISM, the BLS, and the MSC, that report different types of diffusion indexes based on survey work. For example, the general practice has thus far consisted of reporting point estimates without much emphasis on confidence intervals around those estimates. The formulas we derive can hopefully serve as a first step toward providing a more complete picture of the information conveyed by diffusion indexes.

Appendix

A.1 Distribution of Individual Diffusion Indexes, $\widehat{D} = \sum_{a=1}^{r} \omega^a \widehat{p}^a$

We can write $\widehat{p}^a$ as

$$\widehat{p}^a = \frac{1}{n} \sum_{i=1}^{n} x_i^a, \tag{A.1}$$

where x_i^a is an indicator variable that takes on the value 1 when survey participant i answers a and is zero otherwise. Since $\widehat{p}^a$ then has the interpretation of a sample Bernoulli mean, the multivariate central limit theorem, combined with the fact that sums of Bernoulli random variables are binomial, immediately gives

$$\sqrt{n} \begin{pmatrix} \widehat{p}^1 - p^1 \\ \widehat{p}^2 - p^2 \\ \cdots \\ \widehat{p}^{r-1} - p^{r-1} \end{pmatrix} \overset{a}{\sim} \mathcal{N} \left(\begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} p^1(1-p^1) & -p^1 p^2 & \cdots & -p^1 p^{r-1} \\ -p^2 p^1 & p^2(1-p^2) & \cdots & -p^2 p^{r-1} \\ \cdots & \cdots & \cdots & \cdots \\ -p^{r-1} p^1 & -p^{r-1} p^2 & \cdots & p^{r-1}(1-p^{r-1}) \end{pmatrix} \right), \tag{A.2}$$

as shown in Wasserman (2004) or Agresti (2012).[29] Because the generic individual diffusion index $\widehat{D}$ is a linear combination of sample Bernoulli means, $\sum_{a=1}^{r} \omega^a \widehat{p}^a$, that are asymptotically jointly normally distributed according to (A.2), the distribution of $\widehat{D}$ will also be asymptotically normal. Moreover, we have that

$$\begin{aligned} D &= E(\widehat{D}) \\ &= \sum_{a=1}^{r} \omega^a E(\widehat{p}^a) = \sum_{a=1}^{r} \omega^a p^a. \end{aligned}$$

[29] Note that $\widehat{p}^r$ is simply defined as a residual, $\widehat{p}^r = 1 - \sum_{a=1}^{r-1} \widehat{p}^a$, and thus need not be included in expression (A.2).

Finally,

$$
\begin{aligned}
&Var\left(\widehat{D}\right)\\
&= Var\left(\sum_{a=1}^{r}\omega^a\widehat{p}^a\right),\\
&= \sum_{a=1}^{r}(\omega^a)^2 Var\left(\widehat{p}^a\right) + \sum_{a\neq a'}\omega^a\omega^{a'}Cov(\widehat{p}^a,\widehat{p}^{a'}),\\
&= \sum_{a=1}^{r}(\omega^a)^2\frac{p^a(1-p^a)}{n} - 2\sum_{1\leq a<a'\leq r}\omega^a\omega^{a'}\frac{p^a p^{a'}}{n},\\
&= \frac{1}{n}\left\{\sum_{a=1}^{r}(\omega^a)^2 p^a - \sum_{a=1}^{r}(\omega^a)^2(p^a)^2 - 2\sum_{1\leq a<a'\leq r}\omega^a\omega^{a'}p^a p^{a'}\right\},
\end{aligned}
$$

or

$$
Var\left(\widehat{D}\right) = \frac{1}{n}\left\{\sum_{a=1}^{r}(\omega^a)^2 p^a - D^2\right\}, \tag{A.3}
$$

where the last line follows from the multinomial theorem. The variance in equation (10) is precisely (A.3).

A.2 Distribution of Weighted Individual Diffusion Indexes, $\widehat{D} = \sum_{j=1}^{J}\gamma_j\sum_{a=1}^{r}\omega^a\widehat{p}_j^a$

Let

$$
\widehat{p}_j^a = \frac{1}{n}\sum_{i=1}^{n}x_i^a(j),
$$

where $x_i^a(j)$ is an indicator variable that takes on the value 1 when survey participant i answers a and belongs to group or sector j, and is zero otherwise. The multivariate central limit theorem gives

$$\sqrt{n}\begin{pmatrix} \widehat{p}_1^1 - p_1^1 \\ \widehat{p}_1^2 - p_1^2 \\ \cdots \\ \widehat{p}_J^{r-1} - p_J^{r-1} \end{pmatrix} \overset{a}{\sim} \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} p_1^1(1-p_1^1) & -p_1^1 p_1^2 & \cdots & -p_1^1 p_J^{r-1} \\ -p_1^2 p_1^1 & p_1^2(1-p_1^2) & \cdots & -p_1^2 p_J^{r-1} \\ \cdots & \cdots & \cdots & \cdots \\ -p_J^{r-1} p_1^1 & -p_J^{r-1} p_1^2 & \cdots & p_J^{r-1}(1-p_J^{r-1}) \end{pmatrix}\right).$$

Thus, in large finite samples, the distribution of the diffusion index statistic, $\widehat{D} = \sum_{j=1}^{J} \gamma_j \sum_{a=1}^{r} \omega^a \widehat{p}_j^a$, is approximately normal with mean $D = \sum_{j=1}^{J} \gamma_j \sum_{a=1}^{r} \omega^a E\left(\widehat{p}_j^a\right) = \sum_{j=1}^{J} \gamma_j \sum_{a=1}^{r} \omega^a p_j^a$ and variance

$$\begin{aligned}
Var(\widehat{D}) &= Var\left(\sum_{(a,j)} (\omega^a \gamma_j)\, \widehat{p}_j^a\right) \\
&= \sum_{(a,j)} (\omega^a \gamma_j)^2\, Var\left(\widehat{p}_j^a\right) + \sum_{(a,j)\neq(a'j')} \left(\omega^a \omega^{a'} \gamma_j \gamma_{j'}\right) \\
&\quad \times cov\left(\widehat{p}_j^a, \widehat{p}_{j'}^{a'}\right) \\
&= \sum_{(a,j)} (\omega^a \gamma_j)^2\, \frac{p_j^a(1-p_j^a)}{n} - \sum_{(a,j)\neq(a'j')} \left(\omega^a \omega^{a'} \gamma_j \gamma_{j'}\right) \frac{p_j^a p_{j'}^{a'}}{n} \\
&= \frac{1}{n}\left\{\sum_{(a,j)} (\omega^a \gamma_j)^2\, p_j^a - \left(\sum_{(a,j)} (\omega^a \gamma_j)^2 \left(p_j^a\right)^2\right.\right. \\
&\quad \left.\left. + \sum_{(a,j)\neq(a'j')} \left(\omega^a \omega^{a'} \gamma_j \gamma_{j'}\right) p_j^a p_{j'}^{a'}\right)\right\} \\
&= \frac{1}{n}\left\{\left(\sum_{(a,j)} (\omega^a \gamma_j)^2\, p_j^a\right) - D^2\right\}.
\end{aligned}$$

A.3 Distribution of Composite Individual Indexes, $\widehat{D} = \sum_{k=1}^{\overline{k}} \delta_k \widehat{D}_k$*, where* $\widehat{D}_k = \sum_{a=1}^{r} \omega^a \frac{n_k^a}{n}$

Let the probability of drawing answers $\mathbf{a} = (a_1, \ldots a_{\overline{k}}) \in \mathcal{A}$ be denoted by $p^{\mathbf{a}}$, with $\sum_{\mathbf{a} \in \mathcal{A}} p^{\mathbf{a}} = 1$, where $p^{\mathbf{a}}$ can also be expressed as $p^{(a_k, \mathbf{a}_{-k})}$. Thus, we denote the marginal probability of drawing a given response $a \in \mathcal{A}_k$ for component k as $p_k^a = \sum_{\mathbf{a}_{-k} \in \mathcal{A} \backslash \mathcal{A}_k} p^{(a_k = a, \mathbf{a}_{-k})}$. Because the composite index (15) averages across different component indexes, say D_k and D_ℓ, we will need to take into account the pairwise covariance between components. We now describe how this process in turn relies on keeping track of all pairwise joint probabilities between any two components of the composite diffusion index.

Denote the joint probability of observing $a_k = a$ and $a_\ell = a'$ for the components k and ℓ by $p_{k\ell}^{aa'}$. This probability is given by

$$p_{k\ell}^{aa'} = \sum_{\mathbf{a}_{-\{k,\ell\}} \in \mathcal{A} \backslash \mathcal{A}_k \times \mathcal{A}_\ell} p^{(a_k = a, a_\ell = a', \mathbf{a}_{-\{k,\ell\}})}, \ k \neq \ell, \tag{A.4}$$

where the notation $(a_k = a, a_\ell = a', \mathbf{a}_{-\{k,\ell\}})$ distinguishes between answers for component k, component ℓ, and all other components, $-\{k, \ell\}$. The marginal with respect to component k satisfies $p_k^a = \sum_{a' \in A_\ell} p_{k\ell}^{aa'}$. We let $\mathbf{p}_{k\ell}$ denote the vector comprising all pairwise joint probabilities, $p_{k\ell}^{aa'}$ for given components k and ℓ, where the dimension of $\mathbf{p}_{k\ell}$ is r^2. Thus, for the example with three components and three possible responses, the element p_{13}^{ud} in $\mathbf{p}_{13}$ gives the joint probability of observing "up" along dimension "1," or improving household financial conditions, and "down" along dimension "3," or deteriorating spending on durable goods.

Denote the number of survey participants answering $a_k = a$ and $a_\ell = a'$ for the components k and ℓ by $n_{k\ell}^{aa'}$, where similarly to equation (A.4),

$$n_{aa'}^{k\ell} = \sum_{\mathbf{a}_{-\{k,\ell\}} \in \mathcal{A} \backslash \mathcal{A}_k \times \mathcal{A}_\ell} n^{(a_k = a, a_\ell = a', \mathbf{a}_{-\{k,\ell\}})}, \ k \neq \ell.$$

As before, the number of participants answering a given response a for component k satisfies $n_k^a = \sum_{a' \in A_\ell} n_{k\ell}^{aa'}$.

Let $\widehat{p}_k^a = n_k^a/n$ in the component index (14) and $\widehat{p}_{k\ell}^{aa'} = n_{k\ell}^{aa'}/n$. Observe that

$$\widehat{D}_k = \sum_{a=1}^{r} \omega^a \widehat{p}_k^a = \sum_{a=1}^{r} \omega^a \sum_{a' \in \mathcal{A}_\ell} \widehat{p}_{k\ell}^{aa'} \tag{A.5}$$

for any component ℓ, where the components other than ℓ have already been integrated out in (A.4). Equation (A.5) effectively allows us to write each component index, D_k, in the overall index, $\widehat{D}$, in terms of joint pairwise probabilities with any other component index, D_ℓ, and, therefore, capture the pairwise co-movement across these indexes.

As in section 2, each element $\widehat{p}_{k\ell}^{aa'}$, which we collect in the vector $\widehat{\mathbf{p}}_{k\ell}$, may be interpreted as the sample mean of Bernoulli random variables. Then, it follows that $\widehat{\mathbf{p}}_{k\ell}$ is approximately normally distributed in large samples, i.e.,

$$\sqrt{n}(\widehat{\mathbf{p}}_{k\ell} - \mathbf{p}_{k\ell}) \overset{a}{\sim} \mathcal{N}(0, \Sigma_{\mathbf{p}_{k\ell}}). \tag{A.6}$$

A typical element of $\widehat{\mathbf{p}}_{k\ell}$, say $\widehat{p}_{k\ell}^{ab}$, is such that $E(\widehat{p}_{k\ell}^{ab}) = p_{k\ell}^{ab}$, $Var(\widehat{p}_{k\ell}^{ab}) = n^{-1}(1 - p_{k\ell}^{ab})p_{k\ell}^{ab}$, and its covariance with any other element $\widehat{p}_{k\ell}^{b'a'}$ is $Cov\left(\widehat{p}_{k\ell}^{ab}, \widehat{p}_{k\ell}^{b'a'}\right) = -n^{-1}p_{k\ell}^{ab}p_{k\ell}^{b'a'}$. Observe that the elements of $\mathbf{p}_{k\ell}$ sum to 1, in that $\mathbf{p}_{k\ell}$ can be used to define a marginal multinomial distribution obtained by integrating out all categories other than k and ℓ from the underlying primitive multinomial distribution over all categories (i.e., marginal distributions constructed from integrating out dimensions of a multinomial distribution remain multinomial).

The information contained in (A.6), constructed for all pairs of questions k and ℓ in the survey, is all that is needed to construct sampling uncertainty around a composite index that combines different individual indexes. Moreover, we know that

$$\begin{aligned} D &= E(\widehat{D}) \\ &= \sum_{k=1}^{\bar{k}} \delta_k \sum_{a=1}^{r} \omega^a E\left(\widehat{p}_k^a\right) = \sum_{k=1}^{\bar{k}} \delta_k \sum_{a=1}^{r} \omega^a p_k^a, \end{aligned}$$

and

$$\begin{aligned} Var(\widehat{D}) &= Var\left(\sum_{k=1}^{\overline{k}} \delta_k \widehat{D}_k\right) \\ &= \sum_{k=1}^{\overline{k}} \delta_k^2 Var\left(\widehat{D}_k\right) + \sum_{k\neq\ell} \delta_k \delta_\ell Cov\left(\widehat{D}_k, \widehat{D}_\ell\right) \\ &= \sum_{k=1}^{\overline{k}} \delta_k^2 Var\left(\widehat{D}_k\right) + 2 \sum_{1\leq k<\ell\leq\overline{k}} \delta_k \delta_\ell Cov\left(\widehat{D}_k, \widehat{D}_\ell\right). \quad \text{(A.7)} \end{aligned}$$

Hence, it follows from the CLT that

$$\begin{aligned} &\sqrt{n}\left(\widehat{D} - D\right) \\ &\quad \overset{a}{\sim} \mathcal{N}\left(0, \sum_{k=1}^{\overline{k}} \delta_k^2 Var\left(\widehat{D}_k\right) + 2 \sum_{1\leq k<\ell\leq\overline{k}} \delta_k \delta_\ell Cov\left(\widehat{D}_k, \widehat{D}_\ell\right)\right). \end{aligned} \quad \text{(A.8)}$$

In equation (A.7), analogously to the expression in (A.3), we have that for each individual diffusion index D_k,

$$Var\left(\widehat{D}_k\right) = \frac{1}{n}\left\{\left(\sum_{a=1}^{r} (\omega^a)^2 p_k^a\right) - (D_k)^2\right\},$$

where $p_k^a = \sum_{a'\in A_\ell} p_{k\ell}^{aa'} = \sum_{\mathbf{a}_{-\{k,\ell\}}\in\mathcal{A}\setminus\mathcal{A}_k\times\mathcal{A}_\ell} p^{(a_k=a, a_\ell=a', \mathbf{a}_{-\{k,\ell\}})}$, $k \neq \ell$, using the notation introduced in the text. In addition,

$$\begin{aligned} Cov\left(\widehat{D}_k, \widehat{D}_\ell\right) &= Cov\left(\sum_{a=1}^{r} \omega^a \widehat{p}_k^a, \sum_{a'=1}^{r} \omega^{a'} \widehat{p}_\ell^{a'}\right) \\ &= Cov\left(\sum_{a=1}^{r} \omega^a \sum_{b\in\mathcal{A}_\ell} \widehat{p}_{k\ell}^{ab}, \sum_{a'=1}^{r} \omega^{a'} \sum_{b'\in\mathcal{A}_k} \widehat{p}_{k\ell}^{b'a'}\right) \end{aligned}$$

$$= \sum_{(a,a')\in\mathcal{A}_k\times\mathcal{A}_\ell} \omega^a\omega^{a'} Cov\left(\sum_{b\in\mathcal{A}_\ell}\widehat{p}_{k\ell}^{ab}, \sum_{b'\in\mathcal{A}_k}\widehat{p}_{k\ell}^{b'a'}\right)$$
$$= \sum_{(a,a')\in\mathcal{A}_k\times\mathcal{A}_\ell} \omega^a\omega^{a'} \sum_{(b',b)\in\mathcal{A}_k\times\mathcal{A}_\ell} Cov\left(\widehat{p}_{k\ell}^{ab}, \widehat{p}_{k\ell}^{b'a'}\right). \tag{A.9}$$

Using $Cov\left(\widehat{p}_{k\ell}^{ab}, \widehat{p}_{k\ell}^{b'a'}\right) = -n^{-1}p_{k\ell}^{ab}p_{k\ell}^{b'a'}$ and the definition of $p_{k\ell}^{aa'}$ in (A.4), we can rewrite equation (A.9) as

$$Cov\left(\widehat{D}_k, \widehat{D}_\ell\right)$$
$$= \frac{1}{n}\left[\sum_{(a,a')\in\mathcal{A}_k\times\mathcal{A}_\ell} \omega^a\omega^{a'}\left(p_{k\ell}^{aa'} - \sum_{(b,b')\in\mathcal{A}_k\times\mathcal{A}_\ell} p_{k\ell}^{ab}p_{k\ell}^{b'a'}\right)\right].$$

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