Are QE and Conventional Monetary Policy Substitutable?*

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Yes! We study the substitutability between conventional monetary policy based on the adjustment of a short-term policy interest rate with quantitative easing (QE). We do so in a four-equation New Keynesian model featuring financial frictions that allows QE to be economically relevant. We analytically derive how much QE versus conventional policy is necessary to implement an inflation target. Quantitatively, the observed expansion of the Federal Reserve's balance sheet over the zero lower bound (ZLB) period provides stimulus equivalent to cutting the policy rate to 2 percentage points below zero. This is in line with the decline in the empirical shadow federal funds rate series. Moreover, we show that the amount of QE required to achieve price stability depends on the expected duration of the ZLB.

JEL Codes: E32, E52, E58.

1. Introduction

Prior to the financial crisis and ensuing Great Recession of 2007–09, the Federal Reserve (Fed) in the United States and other central banks around the world implemented monetary policy via the adjustment of short-term interest rates. In response to the crisis, central banks pushed short-term policy rates to the zero lower bound (ZLB) or, in some cases, slightly below zero. Lacking the ability to pursue conventional easing policies by pushing short-term rates even lower, central banks instead resorted to a sequence of unconventional
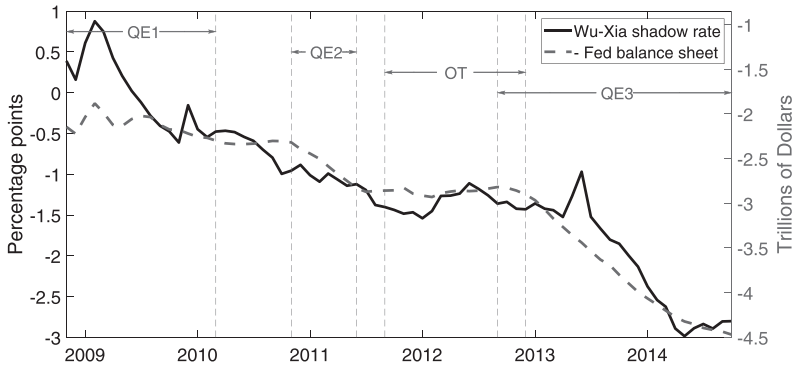
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policy interventions. The most prominent unconventional intervention has been large-scale asset purchases, more commonly known as quantitative easing (QE).[1] In the United States, for example, the Fed bought longer-maturity Treasury and residential mortgaged-backed securities and ended up with an unprecedentedly large balance sheet of 4.5 trillion dollars.

In spite of its expansive use and the likelihood that QE operations will be deployed again to fight future economic downturns, economists' understanding of the magnitudes and mechanisms by which QE affects the economy remains somewhat limited. In particular, there is no consensus on how much QE is equivalent to a conventional policy rate cut. Our paper contributes to this important question. Not only do we provide an affirmative answer to the question posed in the title, we also calculate a direct quantitative mapping between QE and conventional policy.

The starting point of our analysis is the shadow federal funds rate. The shadow rate is an older concept originally introduced by Black (1995) to circumvent issues arising in term structure models from the ZLB on the short end of the yield curve. It has more recently been used by a number of researchers as a summary statistic for the overall stance of monetary policy during periods in which policy rates are at their lower bound. A shadow rate series uses information from longer-term interest rates to infer a hypothetical short-term interest rate were there no ZLB. Wu and Xia (2016), for example, compute a shadow rate series for the United States and find that it reaches a nadir of approximately 3 percentage points below zero at the end of the Fed's QE operations. This is suggestive that unconventional operations have provided a significant amount of economic stimulus and that perhaps the ZLB has not been much of a constraint on policy. Indeed, as we document in figure 1, visually there is a tight connection between the size of the Fed's balance sheet and the shadow rate series. Without more structure, however,

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[1]Another widely used unconventional tool has been forward guidance, which involves the central bank communicating its expected path of short-term policy rates after a ZLB episode has ended. We focus on QE in this paper. For excellent reviews and discussions of forward guidance, see Campbell et al. (2012) or Del Negro, Giannoni, and Patterson (2015).

Figure 1. Shadow Rate and the Fed's Balance Sheet

Data Sources: Wu-Xia/Federal Reserve Bank of Atlanta and Federal Reserve Economic Data.
Notes: We reproduce Wu and Zhang's (2017) figure 1 between November 2008 and October 2014. Solid line: the Wu-Xia shadow rate; dashed line: the negative of the Fed's balance sheet. Left scale: shadow rate in annualized percentage points; right scale: negative Fed's balance sheet in trillions of dollars. QE1: the first round of QE from November 2008 to March 2010; QE2: the second round of QE from November 2010 to June 2011. OT: Operation Twist from September 2011 to December 2012. QE3: the third round of QE from September 2012 to October 2014.

it is impossible to move beyond interpreting the tight temporal connection between the two series as more than coincidental. Further, since the term structure models upon which the construction of the shadow rate is based are mute about structural economic mechanisms, it is not possible to draw a tight, quantitative link between QE purchases and movements in the shadow rate.

Our paper seeks to fill this void. We do so using the four-equation linearized New Keynesian model of Sims and Wu (2019b). The model features an IS curve summarizing aggregate demand and a Phillips curve describing aggregate supply, along with policy rules for the short-term interest rate as well as the size of the central bank's long bond portfolio. The underlying environment features two types of households, short- and long-term debt, and financial intermediaries subject to a leverage constraint. Bond market segmentation in conjunction with a leverage constraint on intermediaries allow the central bank's long bond portfolio to be economically relevant. Linearization about the non-stochastic steady state gives rise to the

four key equations. They look similar to their counterparts in the textbook three-equation model (e.g., Galí 2008), except the IS and Phillips curve contain additional terms related to credit market disturbances and the central bank's long bond portfolio. Though substantially simpler and more tractable, the four-equation model is based on similar building blocks to more complicated quantitative dynamic stochastic general equilibrium (DSGE) models like those of Gertler and Karadi (2011, 2013); Carlstrom, Fuerst, and Paustian (2017); and Sims and Wu (2019a).

We focus on a framework in which conventional policy entails adjusting the short-term interest rate to implement a strict inflation target. Inflation targeting is the explicit mandate for many of the world's central banks, and since 2012 the Fed in the United States has adopted an official target of 2 percent. With no QE, implementing the inflation target absent a ZLB constraint in our model requires adjusting the interest rate one-for-one with fluctuations in the natural rate of interest and moving the policy rate to counterbalance credit market disturbances. An inability to adjust the policy rate because of a binding ZLB causes the central bank to miss on its inflation target and results in substantial fluctuations in the output gap in response to both natural rate and credit shocks.

We then derive an expression for the central bank's QE holdings so as to implement its inflation target when the policy rate is constrained by the ZLB. In our model, QE should move in the opposite direction of how the policy rate ordinarily would in response to shocks. From the perspective of implementing an inflation target, QE is perfectly substitutable with conventional interest rate policy in a ZLB environment. The implications of the two policies for the behavior of the output gap are different, however. Nevertheless, the output gap reacts significantly less to both natural rate and credit shocks with our endogenous QE rule compared with a policy of doing nothing at the ZLB.

Relating back to the empirical shadow federal funds rate, we derive an analytical substitution factor between QE and conventional monetary policy in the model. Calibrated to U.S. data, we find that a doubling of the central bank's long bond portfolio is approximately equivalent to a cut in the policy rate of 3 percentage points at an annualized rate. Feeding the observed time series of the Fed's balance sheet into our analytical substitution expression

results in an implied shadow rate series that aligns closely with Wu
and Xia's (2016) empirical series. In particular, our model predicts
that QE1 through QE3 provided stimulus equivalent to cutting the
policy rate to roughly 2 percentage points below zero, a number that
is consistent with Wu and Xia (2016).

    The remainder of the paper proceeds as follows. Section 2 reviews
the current monetary policy framework. This includes a discussion of
conventional policy rate adjustment relative to unconventional tools
like QE, a review of some of the recent literature, and a description
of the empirical shadow federal funds rate and its close connection to
the size of the Fed's balance sheet. Section 3 describes the model, and
section 4 discusses both conventional monetary policy and QE at the
ZLB. Section 5 discusses the analytical conversion between conven-
tional policy rate movements and QE and quantitatively documents
how the shadow rate implied by our substitution factor using the
Fed's balance sheet closely aligns with empirical shadow rate series.
Section 6 concludes.

2.   Review of the Monetary Policy Framework

In this section, we provide a brief intuitive review of the past and
current monetary policy framework employed by the Federal Reserve
and other leading central banks. We compare and contrast the frame-
work prior to the financial crisis with one based on QE policies
deployed to circumvent the constraints on conventional policy posed
by the ZLB. Next, we review some of the empirical literature on the
effects of large-scale asset purchases. We then tie these frameworks
into the empirical shadow rate literature typified by Wu and Xia
(2016).

2.1   Conventional Monetary Policy versus QE

Although most macro models only feature one interest rate, in real-
ity there are myriad interest rates facing consumers and firms. The
interest rates relevant for the most cyclically sensitive components of
expenditure are long term and account for default risk. Prior to the
crisis, in contrast, central banks implemented policy largely through
the adjustment of short-term, risk-free rates.

Riskless, short-term rates are related to economically relevant longer-term rates through the simple decomposition expressed as follows:

$$Long\ rate = expectation + risk\ premium. \qquad (1)$$

Long-term rates can be broken into two components. The expectations component is based on the expected sequence of short-term policy rates. The risk premium component accounts for duration and default risk associated with longer-term, risky debt. Conventional monetary policy works through the expectations component of (1)—adjusting short-term rates in the present affects long rates through the expected path of policy rates, and in turn affects spending categories that are especially sensitive to long-term rates (e.g., consumer durables and residential investment).

Unconventional policies were deployed to circumvent constraints on conventional policy posed by the ZLB in the wake of the financial crisis. Loosely speaking, unconventional policies seek to influence economically relevant long-term rates independently of adjusting current short-term rates. Because such policies seek to influence the long end of a yield curve without adjusting the "short" end, Eberly, Stock, and Wright (2019) refer to them as "slope policies." Like conventional interest rate policy, forward guidance seeks to influence relevant rates through the expectations component of (1), albeit by changing expectations of future policy rates rather than current rates.

Quantitative easing seeks to influence economically relevant rates instead through the risk premium channel. From the perspective of conventional macroeconomic theory with unconstrained agents and frictionless markets, it is not clear why central bank purchases of long bonds might be beneficial. It is against this background that Ben Bernanke famously said, "The problem with QE is that it works in practice but not in theory."[2]

A number of authors have advanced different theories for how QE policies might work to lower long-term rates. Vayanos and Vila (2009) develop a preferred-habitat theory of the term structure in which central bank purchases or sales of bonds can affect supply and demand in particular segments of the bond market. Ray (2019)

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[2]See the transcript associated with Ahamed and Bernanke (2014).

incorporates this preferred-habitat environment into an otherwise standard New Keynesian model. The framework upon which our model is based relies upon financial market segmentation with constrained intermediaries (see, e.g., Gertler and Karadi 2011, 2013; Carlstrom, Fuerst, and Paustian 2017; and Sims and Wu 2019a). In this type of framework, bond purchases by a central bank can ease constraints facing intermediaries, resulting in an expansion of credit supply and lower credit spreads.[3]

There is by now an expansive empirical literature on the effects of QE. Much of this literature has converged to the conclusion that QE has been effective. Gagnon et al. (2011) find that QE purchases were successful in driving down long-term interest rates primarily through lower risk and term premiums. Hamilton and Wu (2012), Bauer and Rudebusch (2014), and Greenwood and Vayanos (2014) study the empirical effects of QE on the term structure. Gagnon et al. (2011), Krishnamurthy and Vissing-Jorgensen (2011), and D'Amico and King (2013) use an event-study methodology to quantify the effects of QE in the United States on a variety of different interest rates. Eberly, Stock, and Wright (2019) employ a reduced-form statistical model to conclude that so-called slope policies provided substantial stimulus to the U.S. economy during the ZLB period. Among these, Gagnon et al. (2011), Hamilton and Wu (2012), and Greenwood and Vayanos (2014) highlight the risk premium channel as described in (1). Different from the rest of the literature, Greenlaw et al. (2018) offer a more skeptical view on the efficacy of QE.

2.2   Shadow Rate

It has become increasingly popular to summarize the overall stance of monetary policy at the ZLB with the so-called shadow rate. We focus on the Wu and Xia (2016) shadow rate, which has been widely used by researchers, policymakers, and media. For empirical studies, see Nikolsko-Rzhevskyy, Papell, and Prodan (2014), Aizenman,

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[3]An alternative theory of the transmission of QE rests on a so-called signaling hypothesis—expansive QE operations can serve as a credible signal of lower future policy rates, thereby affecting long rates through the expectations channel. See, e.g., Bauer and Rudebusch (2014), Bhattarai, Eggertsson, and Gafarov (2015), or some of the associated discussion in Krishnamurthy and Vissing-Jorgensen (2011).

Chinn, and Ito (2016), and Aastveit, Natvik, and Sola (2017), and Basu and Bundick (2017). For policy analyses, see Altig (2014) and Hakkio and Kahn (2014). For media discussions, see articles in *Bloomberg Businessweek* (Coy 2014), the *Wall Street Journal* (Lahart 2014), *Forbes* (Wakatabe 2015), *Bloomberg News* (Miller 2016), and *Business Insider* (2016).

Initially, the shadow rate was a concept introduced by Black (1995) into the term structure literature to circumvent issues arising from the ZLB. In particular, let

$$r_t = \max\{0, s_t\}, \tag{2}$$

where r_t is the short-term policy rate and s_t is the shadow rate. Although the policy rate is subject to a zero lower bound, the shadow rate is not. When above zero, both of them are the same, whereas when r_t is at its ZLB, the shadow rate still displays variation and contains economically meaningful information.

Empirically, the shadow rate is extracted from the term structure of interest rates. At the ZLB, the short end of the yield curve is at or very close to zero. However, medium- and long-term interest rates still contain economically relevant information. The shadow rate inferred from longer-term interest rates represents the hypothetical short end of the yield curve if the ZLB were not a constraint. Kim and Singleton (2012) and Ichiue and Ueno (2013) apply such a model to Japan, whereas Lombardi and Zhu (2014), Christensen and Rudebusch (2015), Bauer and Rudebusch (2016), and Wu and Xia (2016) focus on the United States. Kortela (2016), Lemke and Vladu (2016), and Wu and Xia (2018) extend the model to Europe.

2.3   QE and the Shadow Rate

Figure 1 plots the empirical shadow rate series as the solid black line from November of 2008 through October of 2014. This corresponds to the time frame over which the Fed was actively engaged in large-scale asset purchases.

After an initial upward blip at the very beginning of the sample period, the shadow rate series exhibits a sustained downward trajectory, ultimately falling to about 3 percentage points below zero. Several authors, most notably Bullard (2012), Wu and Xia (2016), Mouabbi and Sahuc (2017), and Wu and Zhang (2017, 2019),

have interpreted the large and persistent decline in the shadow rate as evidence of the efficacy of unconventional monetary policies deployed in the wake of the Great Recession. This work aligns with a growing literature arguing that unconventional policies have served as a good substitute for conventional monetary policy and that the ZLB on policy rates has ultimately not been much of a hindrance to effective stabilization policy. See, for example, Swanson and Williams (2014), Debortoli, Galí, and Gambetti (2016), Swanson (2018a, 2018b), Garín, Lester, and Sims (2019), and Sims and Wu (2019b).

The construction of a shadow rate series is based on empirical term structure models that do not have an explicit mapping back into structural economic models or particular unconventional tools. Nevertheless, a number of the papers cited above have associated the Fed's expansive QE operations with the observed empirical behavior of the shadow rate. The dashed line in figure 1 plots the negative of the Fed's balance sheet from the end of 2008 through 2014. The balance sheet is measured in trillions of dollars and is on the right scale. The two series are obviously highly correlated. From QE1 through QE3, the Fed expanded its balance sheet from under 2 trillion dollars to more than 4.5 trillion dollars. Over the same time period, the shadow rate goes from slightly positive to about 3 percentage points below zero. This figure is suggestive, though of course not dispositive, that QE operations contributed significantly to the monetary easing as captured by the shadow rate.

Next, we formalize the relationship between the shadow rate and QE. In particular, we use the four-equation New Keynesian model of Sims and Wu (2019b) to theoretically derive a conversion factor between QE and conventional monetary policy. We show in section 5 that a conventionally calibrated version of our model is quantitatively consistent with the expansion in the Fed's balance sheet explaining much of the downward drift in the empirical shadow rate.

3.   Model

Our analysis is based on the New Keynesian model developed in Sims and Wu (2019b). The model features short- and long-term bonds as well as financial intermediaries standing between borrowers and savers. Bond market segmentation combined with intermediaries

being subject to a risk-weighted leverage constraint allows QE operations to have real economic effects. The model captures features of more involved DSGE models of intermediation (e.g., Gertler and Karadi 2011, 2013; Carlstrom, Fuerst, and Paustian 2017; Sims and Wu 2019b) while retaining the elegance and tractability of the textbook three-equation model.

The model reduces to four linearized equations.[4] For details, see Sims and Wu (2019b). It consists of an IS curve,

$$x_t = E_t x_{t+1} - \frac{1-z}{\sigma} \left(r_t - E_t \pi_{t+1} - r_t^f \right)$$
$$- z \left[\bar{b}^{FI}(E_t \theta_{t+1} - \theta_t) + \bar{b}^{cb}(E_t qe_{t+1} - qe_t) \right], \qquad (3)$$

and a Phillips curve,

$$\pi_t = \gamma \zeta x_t - \frac{\gamma \sigma z}{1-z} \left[\bar{b}^{FI} \theta_t + \bar{b}^{cb} qe_t \right] + \beta E_t \pi_{t+1}, \qquad (4)$$

together with two policy rules, one characterizing the behavior of the short-term interest rate, r_t, and one the central bank's long bond portfolio, which we denote qe_t.

Lowercase variables with a t subscript denote log-deviations about the nonstochastic steady state. π_t is inflation and $x_t = y_t - y_t^f$ denotes the output gap, where y_t^f is the equilibrium level of output consistent with price flexibility and no credit shocks. Similarly, r_t^f denotes the natural rate of interest—i.e., the real interest rate consistent with output equaling potential. θ_t captures credit conditions in the financial market; positive values correspond to more favorable conditions. We refer to it as a credit shock. qe_t denotes the real market value of the central bank's long-term bond portfolio.

Letters without t subscripts are parameters or steady-state values. σ, β, and γ are standard parameters—σ measures the inverse intertemporal elasticity of substitution, β is a subjective discount factor, and γ is the elasticity of inflation with respect to real marginal cost. ζ is the elasticity of real marginal cost with respect to

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[4]To reduce the model to four equations, we make a number of simplifying assumptions. These assumptions are not crucial for the qualitative properties of the model.

the output gap.[5] $\bar{b}^{FI}$ and $\bar{b}^{CB}$ are parameters measuring the steady-state long-term bond holdings of financial intermediaries and the central bank, respectively, relative to total outstanding long-term bonds. These coefficients sum to one, i.e., $\bar{b}^{FI} + \bar{b}^{CB} = 1$.

In Sims and Wu's (2019b) model, there are two types of households, and the parameter $z \in [0,1)$ represents the share of non-standard households (who are relatively impatient) in the total population. The model collapses to the standard three-equation New Keynesian model when $z = 0$. In this case, credit shocks θ_t and the central bank's long bond portfolio qe_t are irrelevant for the equilibrium dynamics of output and inflation.

We assume that the credit shock obeys an exogenous autoregressive process with one lag, AR(1),

$$\theta_t = \rho_\theta \theta_{t-1} + s_\theta \varepsilon_{\theta,t}. \tag{5}$$

The natural rate of interest is driven by fundamental shocks to preferences and technology, but can be thought of as exogenous with respect to output and inflation. We therefore model it as an exogenous AR(1) process,

$$r_t^f = \rho_f r_{t-1}^f + s_f \varepsilon_{f,t}. \tag{6}$$

The autoregressive parameters ρ_θ and ρ_f lie strictly between zero and one; the shocks are drawn from standard normal distributions, and are scaled by s_θ and s_f.

To close the model, it is necessary to specify policy rules for the short-term interest rate and the central bank's long bond portfolio. We turn to such specifications, as well as the potential substitutability between the two kinds of policy instruments, next in section 4 and section 5.

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[5] In particular, $\gamma = \frac{(1-\phi)(1-\phi\beta)}{\phi}$, where $\phi \in [0,1)$ measures the probability of nonprice adjustment. This is exactly the same expression as in the three-equation model. The elasticity of real marginal cost with respect to the output gap is $\zeta = \frac{\chi(1-z)+\sigma}{1-z}$, where χ is the inverse Frisch labor supply elasticity. When $z = 0$, this would also be the same as in the three-equation model.

4.   Policy Rules

For our analysis, we focus on a central bank that adopts a strict infla-
tion target. Inflation targeting is the working framework for many
central banks. Among advanced economies, leading examples include
New Zealand, Canada, and the United Kingdom. While the Federal
Reserve in the United States officially has a dual mandate of price
stability and maximum employment, since 2012 it has adopted an
explicit inflation target of 2 percent. In addition to being a realistic
description of central bank policies in actual economies, inflation tar-
geting permits a clean analytical expression for the substitutability
between conventional interest rate policy and QE.

4.1   Conventional Monetary Policy

As a starting point, suppose that the central bank is free to adjust
the short-term interest rate but does not engage in QE operations,
i.e., $qe_t = 0$. This provides a good characterization of central bank
policies in advanced economies prior to the financial crisis. The cen-
tral bank endogenously adjusts r_t so as to implement $\pi_t = 0$. Doing
so requires the following path of the policy rate:

$$r_t = r_t^f + \frac{\sigma z \bar{b}^{FI}(1 - \rho_\theta)\chi}{(1 - z)\zeta}\theta_t, \tag{7}$$

where $\chi \geq 0$ is the inverse Frisch labor supply elasticity.

To implement the inflation target, the policy rate must respond
one-for-one to movements in the natural rate of interest. This is the
same as in the standard three-equation New Keynesian model. How-
ever, in the four-equation model, the policy rate must also react to
the credit shock in order to fully stabilize inflation. The required
policy rate reaction to credit shocks is positive—that is, a tighten-
ing of credit conditions (i.e., a decrease in θ_t) should be met by a
decrease in the policy rate to stabilize inflation. In the special case
in which $z = 0$, the model collapses to the textbook model and the
reaction to the credit shock is zero.

With the policy rule described in (7), the output gap follows

$$x_t = \frac{\sigma z \bar{b}^{FI}}{(1 - z)\zeta}\theta_t, \tag{8}$$

which depends on the credit shock, but not on the natural rate shock. The so-called divine coincidence (Blanchard and Galí 2007) holds conditional on natural rate shocks, wherein stabilizing inflation about target automatically closes the output gap. But the divine coincidence does not hold conditional on credit market disturbances. As discussed in Sims and Wu (2019b), it is therefore not possible to simultaneously stabilize both inflation and the output gap with only one policy instrument. See derivations in appendix A.

4.2   The ZLB

Now let us suppose that the nominal interest rate is stuck at zero for a deterministic number of periods, H. This is the policy experiment considered in Carlstrom, Fuerst, and Paustian (2014) to approximate the effects of a binding ZLB. There is no uncertainty over the duration of the interest rate peg, H. In this experiment, the policy rate is held fixed for the current and subsequent $H-1$ periods, after which time it reverts to the rule necessary to implement a strict inflation target as described above, (7). Formally,

$$
r_{t+j} = \begin{cases} 0 & \text{if } j < H \\ r_{t+j}^f + \frac{\sigma z \bar{b}^{FI}(1-\rho_\theta)\chi}{(1-z)\zeta}\theta_{t+j} & \text{if } j \geq H. \end{cases} \tag{9}
$$

Starting in period $t+H$, the central bank reverts to implementing an inflation target, where $\pi_{t+H+j} = 0$ and $x_{t+H+j} = \frac{\sigma z \bar{b}^{FI}}{(1-z)\zeta}\theta_{t+H+j}$ for $j \geq 0$. Assuming there is no possibility of using QE (i.e., $qe_{t+j} = 0$ $\forall j$), we can use these terminal conditions to then solve backwards for the paths of inflation and the output gap.

   To illustrate the consequences of a binding ZLB, we parameterize and solve the model. The parameterization is described in table 1. The discount factor takes on a standard value of $\beta = 0.99$. The share of impatient households is set as in Sims and Wu (2019b), at $z = 1/3$. The elasticity of intertemporal substitution is unity. The inverse Frisch elasticity is set to $\chi = 1$. The parameterization of $\bar{b}^{FI}$ and $\bar{b}^{cb}$ follows Sims and Wu (2019b). The parameters γ and ζ imply a slope of the Phillips curve of 0.22, which is fairly standard.

   Figure 2 plots impulse responses to a 1 percentage point negative shock to the natural rate of interest. Solid lines are responses when

Table 1. Parameter Values of Linearized Model

Parameter	Value	Description
β	0.99	Discount Factor
z	0.33	Consumption Share of Impatient Households
σ	1	Inverse Elasticity of Substitution
$\bar{b}^{FI}$	0.70	Weight on Leverage in IS/PC Curves
$\bar{b}^{cb}$	0.30	Weight on QE in IS/PC Curves
γ	0.086	Elasticity of Inflation w.r.t. Marginal Cost
ζ	2.5	Elasticity of Gap w.r.t. Marginal Cost
χ	1	Inverse Frisch Elasticity
ρ_f	0.9	AR Natural Rate
ρ_θ	0.9	AR Leverage

Note: This table lists the values of calibrated parameters of the linearized four-equation model.
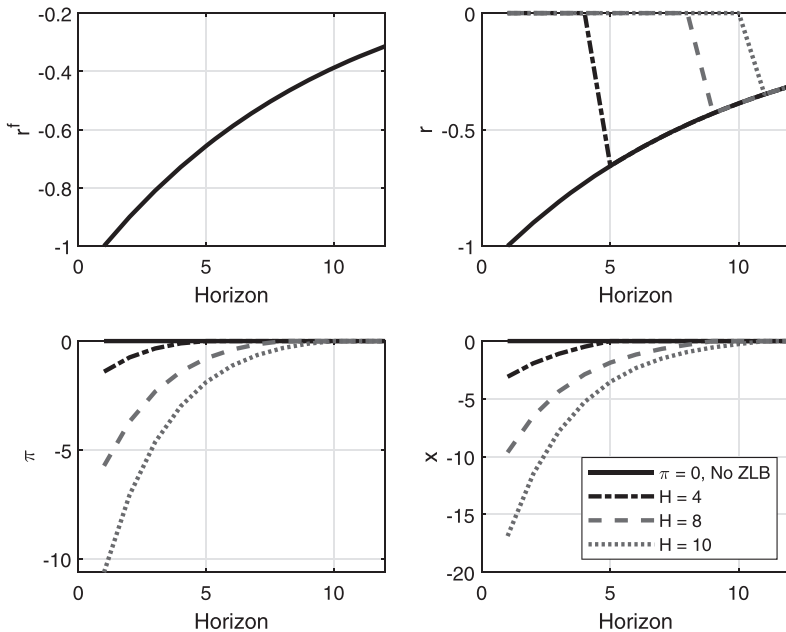
there is no constraint on the policy rate and the central bank implements the inflation target. Alternative line styles show responses when the policy rate is constrained for H periods. We consider peg lengths of $H = 4$ (dash-dotted), $H = 8$ (dashed), and $H = 10$ (dotted).

Absent a ZLB constraint, the central bank would lower the policy rate one-for-one with the natural rate, resulting in no movements in either inflation or the output gap. At the ZLB, the inability to lower the interest rate means that monetary policy is too tight for H periods, resulting in both inflation and the output gap falling significantly. The longer is the expected duration of the ZLB, the more output and inflation decline in response to the shock. After H periods, the policy rate declines to match the natural rate and the output gap and inflation return to zero.

Figure 3 shows impulse responses to a contractionary credit shock of 20 percent.[6] The figure is constructed similarly to figure 2.

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[6]As discussed in Sims and Wu (2019b), θ_t has the interpretation as the log-deviation of a risk-weighted leverage requirement on intermediaries. In their baseline calibration, the steady-state leverage requirement is set to 5. Hence, a 20 percent decline in θ_t is equivalent to the mandatory leverage ratio falling from 5 to 4.

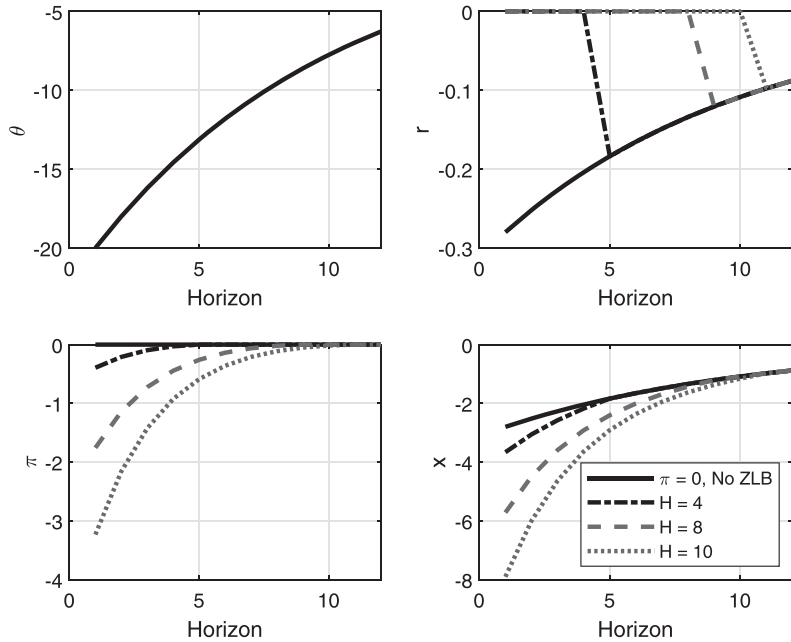## Figure 2. Impulse Responses to Natural Rate Shock at ZLB

Notes: This figure plots impulse responses to a negative shock to the natural rate under a strict inflation target (solid line) and when the policy rate is constrained for H periods. After H periods policy reverts to implementing the strict inflation target via adjustment of the short-term interest rate. Dash-dotted lines correspond to $H = 4$, dashed lines to $H = 8$, and dotted lines to $H = 10$.

Absent any constraints, the central bank would lower the policy rate. This would stabilize inflation about target, but the output gap would still decline somewhat. When the policy rate is constrained, in contrast, the inability to lower the policy rate results in inflation declining and the output gap falling by substantially more. Once again, these effects are exacerbated the larger is H.

Figure 2 and figure 3 make the simple and well-known point that an inability to adjust policy rates has adverse consequences—inflation does not hit target and the output gap declines more in response to both types of shocks. These effects are larger the longer is the duration of the ZLB constraint. In other words, the ZLB is

Figure 3. Impulse Responses to Credit Shock at ZLB

Notes: This figure plots impulse responses to a negative credit shock under a strict inflation target (solid line) and when the policy rate is constrained for H periods. After H periods policy reverts to implementing the strict inflation target via adjustment of the short-term interest rate. Dash-dotted lines correspond to $H = 4$, dashed lines to $H = 8$, and dotted lines to $H = 10$.

quite costly when the central bank has no additional tools at its disposal and simply has to wait until the ZLB lifts.

4.3   Endogenous QE

In reality, central banks did not just sit idly by when policy rates hit the ZLB during the recent financial crisis. And in our four-equation model, they need not—it is possible for QE to adjust so as to hit the same target for inflation when the policy rate is constrained.

When the policy rate is constrained for H periods as in (9), the strict inflation target can be implemented via the following QE rule.

Details are in appendix B.

$$
qe_{t+j} = \begin{cases} -\frac{\bar{b}^{FI}}{\bar{b}^{cb}}(1 - \rho_\theta^{H-j})\theta_{t+j} - \frac{(1-z)\zeta}{\sigma z \bar{b}^{cb}\chi}\frac{1-\rho_f^{H-j}}{1-\rho_f}r_{t+j}^f & \text{if } j < H \\ 0 & \text{if } j \geq H \end{cases}
$$
(10)

According to (10), QE reacts to both natural rate and credit shocks during the periods in which the policy rate is pegged, and returns to steady state thereafter. The required reaction to credit shocks is opposite the sign of the shock (i.e., when θ_t goes down, qe_t must increase). The necessary reaction to the natural rate shock is also negative. Rather naturally, QE moves in the opposite direction as the policy rate would absent a ZLB constraint in response to both shocks.

With QE policy so implemented, inflation will remain at target but the dynamics of the output gap will depend on the path of qe_t. In particular, the path of x_t can be solved for from (4):
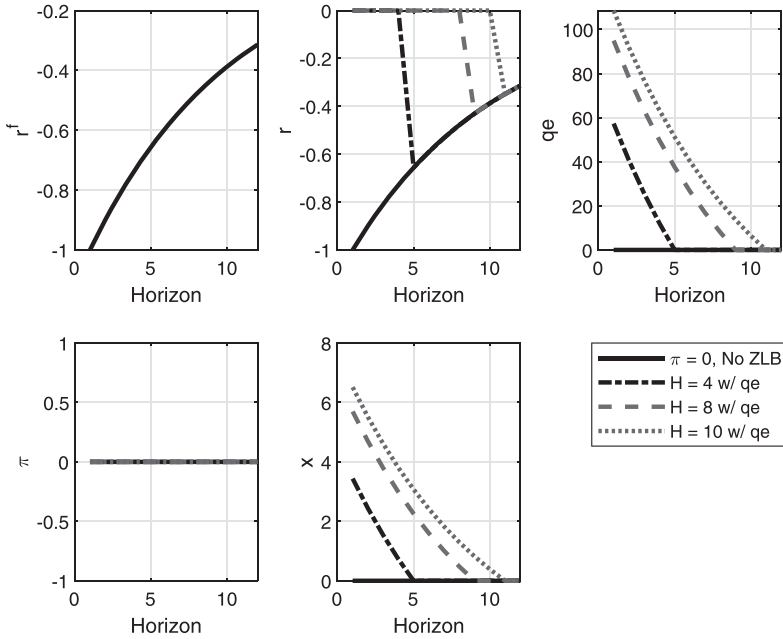
$$
x_t = \frac{\sigma z}{(1-z)\zeta}\left[\bar{b}^{FI}\theta_t + \bar{b}^{cb}qe_t\right].
$$
(11)

From (11), $qe_t > 0$ will be expansionary for output. This means that x_t ought to fall less (in comparison with the ZLB with no unconventional policy reaction) in response to both contractionary natural rate and credit shocks when QE is deployed so as to stabilize inflation.

Figure 4 plots impulse responses to contractionary natural rate shocks. Solid lines show the base case of no ZLB and no QE, with the policy rate adjusting to implement a zero inflation rate. Alternative line styles plot responses when the ZLB binds but endogenous QE is implemented as described in (10). We again do so for three different durations of the ZLB: $H = 4$ (dash-dotted), $H = 8$ (dashed), and $H = 10$ (dotted).

As shown in figure 2, inflation and output both decline in response to a negative natural rate shock at the ZLB. Figure 4 shows that, to offset the decline in inflation, the central bank can increase its long bond holdings. The amount by which it must increase its bond holdings depends on the duration of the peg, a point to which we return below. Engaging in bond purchases keeps inflation at target. Consistent with (11), output declines by less and indeed rises

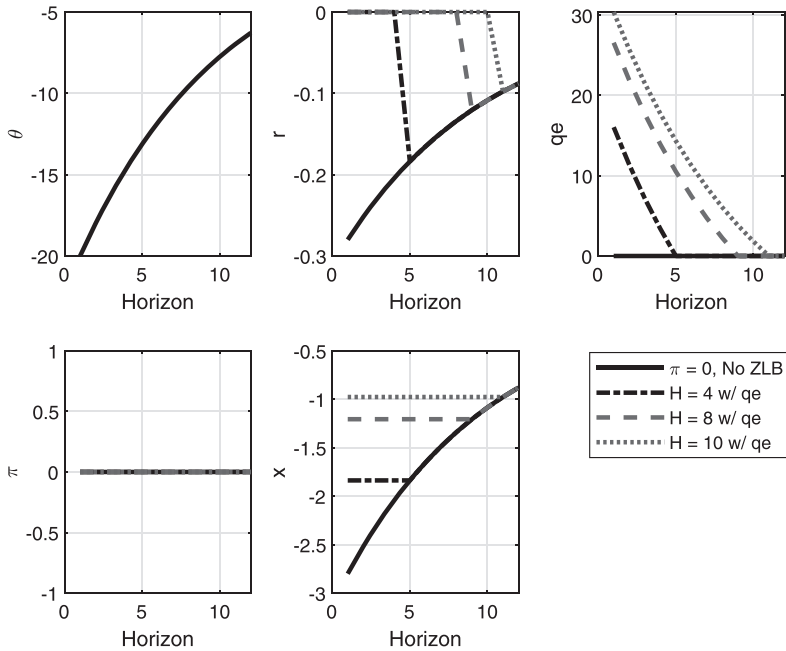## Figure 4. Impulse Responses to Natural Rate Shock at ZLB with Endogenous QE

Notes: This figure plots impulse responses to a negative shock to the natural rate under a strict inflation target (solid line) and when the policy rate is constrained but endogenous QE is undertaken via (10). Dash-dotted lines correspond to $H = 4$, dashed lines to $H = 8$, and dotted lines to $H = 10$. After H periods policy reverts to implementing the strict inflation target via adjustment of the short-term interest rate with no QE.

(instead of falling) when positive bond purchases are undertaken so as to stabilize inflation at target.

Figure 5 plots responses to contractionary credit shocks. It is structured similarly to figure 4. At the ZLB with no QE, the output gap and inflation both decline in response to the shock. In contrast, figure 5 illustrates that when the central bank engages in endogenous QE, inflation remains at target and the output gap declines significantly less during the ZLB period in comparison to what happens when the central bank does nothing. Interestingly, the output

Figure 5. Impulse Responses to Credit Shock at ZLB with Endogenous QE

Notes: This figure plots impulse responses to a negative credit shock under a strict inflation target (solid line) and when the policy rate is constrained but endogenous QE is undertaken via (10). Alternative line styles show responses for different constraint lengths: dash-dotted lines correspond to $H = 4$, dashed lines to $H = 8$, and dotted lines to $H = 10$. After H periods policy reverts to implementing the strict inflation target via adjustment of the short-term interest rate with no QE.

gap responds less negatively to the credit shock at the ZLB with endogenous QE than it does absent a ZLB with conventional policy.

One will also note that the path of the output gap during the ZLB conditional on a credit shock is constant with endogenous QE. Combining (10) with (11), one can show that the path of the gap during the ZLB period is given by

$$E_t x_{t+j} = \frac{\sigma z \bar{b}^{FI}}{(1 - z)\zeta} \rho_\theta^{H-j} E_t \theta_{t+j} \text{ for } j < H. \tag{12}$$

Since $E_t\theta_{t+j} = \rho_\theta^j \theta_t$, this expression reduces to

$$E_t x_{t+j} = \frac{\sigma z \bar{b}^{FI}}{(1-z)\zeta} \rho_\theta^H \theta_t, \tag{13}$$

which does not vary with j. Further, the bigger is H, the more qe_t reacts to the credit shock, and hence the output gap response is less negative during the period of the peg. When $H \to \infty$, (13) becomes

$$E_t x_{t+j} = 0. \tag{14}$$

In other words, if the ZLB persists forever, then stabilizing inflation conditional on credit shocks implies completely stabilizing the output gap. This may seem non-intuitive but is consistent with the results in Sims and Wu (2019b) that QE policy can completely neutralize the consequences of credit market shocks with no movement in the short-term policy rate. The reason why the gap is not completely stabilized for finite peg values here is that the central bank in these experiments reverts to using interest rate policy to stabilize inflation after the peg. This implies fluctuations in future output gaps that in turn affect the current output gap. When $H \to \infty$, future gaps are constant because interest rate policy is never resumed, and so the current output gap does not move.

The results in this section demonstrate that a central bank can significantly mitigate the costs of the ZLB by engaging in long bond purchases in our model. Inflation remains at target in response to both natural rate and credit shocks, and the response of the output gap is smaller compared with the case of engaging in no unconventional policy action. The response of the output gap is also smaller compared with normal times when QE is deployed in response to the credit shock. How much bond buying the central bank must do, and how this relates to the expected duration of the ZLB and the actual practice of the Federal Reserve in the wake of the financial crisis, are issues to which we turn next.

5.   Substitutability between QE and the Shadow Rate

In this section, we expound upon the substitutability of QE with conventional monetary policy. In particular, we show how to map QE

purchases into a shadow rate measure and compare this conversion with the observed patterns in U.S. data.

5.1   Theory

How to convert between QE purchases and the shadow rate—i.e., how much bond purchases are equivalent to a reduction in the policy rate of a given amount—remains a key question of interest for central banks. We address this question utilizing (7) and (10).

Suppose the economy is only subject to a natural rate shock. Then the ratio of the required QE purchase to stabilize prices relative to the necessary policy rate reaction is

$$\frac{qe_t}{r_t} = -\frac{(1-z)\zeta}{\sigma z \bar{b}^{cb} \chi} \frac{1-\rho_f^H}{1-\rho_f}. \tag{15}$$

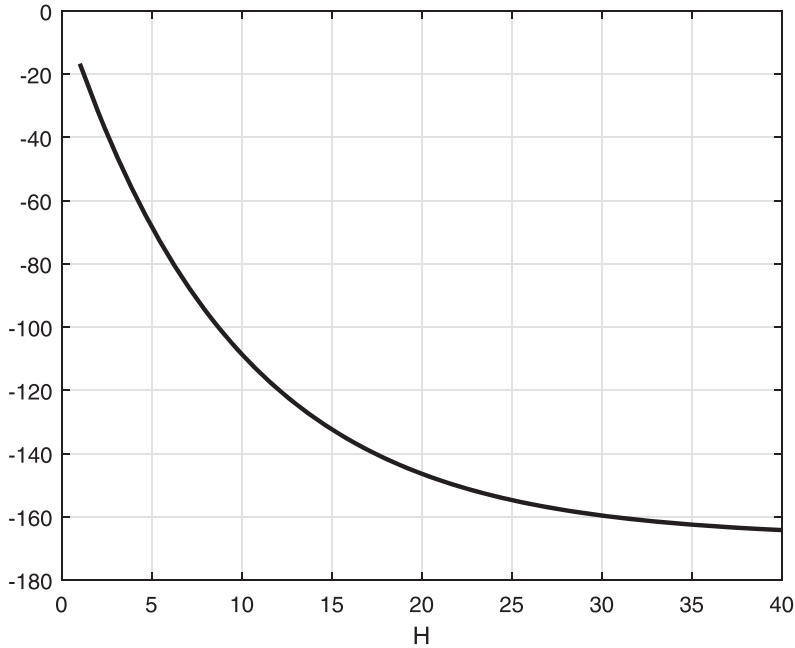Alternatively, suppose there is only the credit shock. Then the same ratio is

$$\frac{qe_t}{r_t} = -\frac{(1-z)\zeta}{\sigma z \bar{b}^{cb} \chi} \frac{1-\rho_\theta^H}{1-\rho_\theta}. \tag{16}$$

In practice, given their latent nature, identifying structural shocks remains a challenge. With the simple and plausible assumption that $\rho_f = \rho_\theta = \rho$, (15) and (16) collapse to the same expression:

$$\frac{qe_t}{r_t} = -\frac{(1-z)\zeta}{\sigma z \bar{b}^{cb} \chi} \frac{1-\rho^H}{1-\rho}. \tag{17}$$

Equation (17) provides an exact mapping between the requisite policy rate movement and QE purchase necessary to stabilize inflation about target. This ratio is nonpositive for all parameter values, meaning that QE purchases must move opposite of the policy rate.

Our model makes a novel yet intuitive prediction about the amount of QE required to stabilize prices as a function of the duration of an interest rate peg. To our knowledge, we are the first to note this interesting relationship. Figure 6 plots the required QE response to stabilize inflation relative to the necessary policy rate response (17) as a function of H. The QE response always has an opposite

Figure 6. Substitution Factor as a Function of H

Notes: This figure plots the substitution factor between QE and conventional policy, (17), as a function of the duration of the ZLB, H. Parameter values are set as described in table 1.

sign of the policy rate response and is decreasing in the duration of the ZLB, H. Rather naturally, for a longer expected duration of a ZLB episode, QE must move by *more* in order to attain price stability. The units are as follows. When $H = 8$, a 4 percentage point cut in the annualized policy rate is roughly equivalent to increasing the size of the balance sheet by about 160 percent.[7]

---

[7]The unit of time in our model is one quarter, so a 4 percentage point cut in the annualized policy rate is a 1 percent cut at a quarterly frequency. When $H = 8$, for our parameterization we have $\frac{qe_t}{r_t} = -94.92$. Thus, cutting r_t by 0.01 is equivalent to raising by qe_t by 0.9492. $\exp(0.9492) = 2.58$, so this corresponds to about a 160 percent increase in the size of the balance sheet.

5.2   QE and the Empirical Shadow Rate

In this subsection, we apply the conversion factor from above and use actual data on the expansion in the Fed's balance sheet to quantify its effect on the empirical shadow rate. Using the relationship in (17), we can estimate the model implied shadow rate as follows:

$$\widehat{s}_t = -\frac{qe_t}{\frac{(1-z)\zeta}{\sigma z \overline{b}^{cb} \chi} \frac{1-\rho^H}{1-\rho}}. \tag{18}$$

We use the baseline parameterization described in table 1. We assume an expected duration of the ZLB of $H = 8$, or two years. While the ZLB episode in the United States in fact lasted seven years, what is relevant is how long agents expect the ZLB to last ex ante. A two-year duration is roughly consistent with the estimated durations in Bauer and Rudebusch (2016) and Wu and Xia (2016). Using this parameterization, we then take actual data on the size of the Fed's balance sheet over the course of its QE operations to measure qe_t in (18) at each month from November of 2008 to October of 2014, which in turn allows us to calculate an implied time series of the hypothetical shadow rate series, $\widehat{s}_t$.[8]
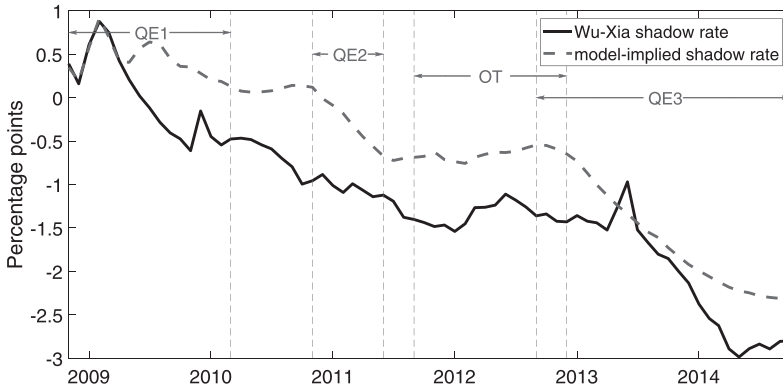
5.2.1   Results

We plot the relationship between the actual shadow rate and the shadow rate implied by the level of the Fed's balance sheet in figure 7. The solid line is the Wu and Xia (2016) shadow rate, and the dashed line plots the implied shadow rate calculated with the Fed's balance sheet using (18). The two lines appear very similar, implying that the theoretical relationship developed in subsection 5.1 works well in practice.

Figure 7 appears similar to figure 1. However, the crucial difference lies in how they are produced. This highlights the main

---

[8]For each period from November 2008 to October 2014, we measure $qe_t = \ln BS_t - \ln$ BS2007m12, where BS refers to total assets held by the Federal Reserve and we take December of 2007 as the reference point. We then multiply this by 400 (to translate into annualized units from our model), divide by the conversion factor, and add 4, the latter of which is necessary because $\widehat{s}_t$ is an absolute deviation relative to steady state, and in our model the steady-state policy rate is 4 percentage points annualized.

**Figure 7. Wu and Xia (2016) Shadow Rate and
Model-Implied Shadow Rate**

Data Sources: Wu-Xia/Federal Reserve Bank of Atlanta and Federal Reserve
Economic Data.
Notes: Solid line: the Wu-Xia shadow rate; dashed line: implied shadow rate
calculated with the Fed's balance sheet using (18). Scale: interest rates in per-
centage points. QE1: the first round of QE from November 2008 to March 2010;
QE2: the second round of QE from November 2010 to June 2011. OT: Operation
Twist from September 2011 to December 2012. QE3: the third round of QE from
September 2012 to October 2014.

contribution of our paper. In figure 1, the two lines use different
scales (the Wu and Xia 2016 shadow rate on the left, and the actual
Fed's balance sheet on the right). In figure 7, we are plotting the
implied shadow rate from (18) using data on the size of the Fed's
balance sheet, and both series are on the same scale.

Qualitatively, movements in the model-implied shadow rate
series closely track those in the empirical series. The model-implied
shadow rate series mostly lies above the actual Wu and Xia (2016)
shadow rate. This suggests, quite naturally, that our model—which
focuses solely on quantitative easing—does not capture all of the
observed movements in the shadow rate that might arise from other
types of unconventional policies (such as forward guidance). More-
over, there are other channels by which QE might provide stimulus
to the economy that are not captured in our model—for example,
the scarcity of safe assets (see Krishnamurthy and Vissing-Jorgensen
2011) or signaling the future path of the policy rate (see Bauer and
Rudebusch 2014). After a minor upward blip, the model predicts

a modest reduction of the shadow rate into negative territory during the QE1 period. The model predicts the largest decline in the shadow rate during QE3. These declines correspond with significant movements in the estimated Wu and Xia (2016) shadow rate. During the so-called Operation Twist episode, in which the Fed bought long-maturity securities financed through the sale of short-maturity bonds so as to maintain the size of its balance sheet, our model predicts little change in the shadow rate. This is precisely what one observes in the empirical Wu and Xia (2016) series. In May 2013, then Federal Reserve Board Chair Bernanke communicated a plan for winding down QE, an event which led to the so-called taper tantrum in financial markets. The taper tantrum coincides with a large temporary upward spike in the empirical shadow rate, but does not appear in the model implied shadow rate because this communication did not materialize in practice.

6.   Conclusion

We use the four-equation linear New Keynesian model of Sims and Wu (2019b) to assess the substitutability between conventional monetary policy and QE at the ZLB. When short-term interest rates are fixed, QE can be utilized to achieve price stability, albeit with different implications for the output gap compared with conventional policy. Moreover, we show that the amount of QE required to implement an inflation target depends on the expected duration of the ZLB.

We use the model to derive an analytical substitution factor between conventional monetary policy and QE. We find that a doubling of the central bank's balance sheet provides stimulus roughly equivalent to a 3 percentage point cut in the policy interest rate. Taking the observed time series of the Fed's balance sheet over its QE operations as given, we use our substitution factor to assess how much of the decline in the Wu and Xia (2016) shadow rate series can be accounted for by QE. We find that QE1 through QE3 is equivalent to moving the policy rate a little more than 2 percentage points into negative territory. This lines up very closely with the empirical results in Wu and Xia (2016).

The results of our paper have a number of potentially important implications for the conduct of monetary policy going forward.

First, our finding that QE can serve as an effective substitute for conventional interest rate policy suggests that ZLB is not as costly as once thought. Therefore, implementing policy to reduce the likelihood of the ZLB binding again in the future—such as raising the inflation target (Ball 2014)—may not be desirable. Second, our conclusion that QE can serve as an effective substitute for conventional policy hinges on the ability of long-term interest rates to fall. This suggests that balance sheet normalization (i.e., quantitative tightening) after periods of substantial QE is likely desirable so as to provide more space for QE to be effective in subsequent episodes. Third, our results have implications for the desirability of negative interest rate policy. Studied in more depth in Sims and Wu (2019a), we urge caution in deploying negative short-term policy rates in the current environment. While doing so may lower long-term rates, negative short-term rates would likely leave less scope for QE to be effective by restricting the amount by which longer-term rates could decline. Finally, our analysis calls for heightened attention to monetary-fiscal interactions. In our model, increased issuance of long-term debt by the Treasury could undermine bond-purchasing programs by the central bank. This suggests that greater cooperation between fiscal and monetary authorities, particularly during strained times, is likely warranted.

Appendix A. No QE

Inflation targeting and no QE implies $\pi_t = 0$ and $qe_t = 0$. Plug these into the Phillips curve. We get (8). The IS equation becomes

$$r_t - r_t^f = \frac{\sigma}{1-z}(E_t x_{t+1} - x_t) - \frac{\sigma z \bar{b}^{FI}}{1-z}(E_t \theta_{t+1} - \theta_t). \qquad \text{(A.1)}$$

Using (8), we can write this as

$$r_t - r_t^f = \frac{\sigma}{1-z}\frac{\sigma z \bar{b}^{FI}}{(1-z)\zeta}(E_t \theta_{t+1} - \theta_t) - \frac{\sigma z \bar{b}^{FI}}{1-z}(E_t \theta_{t+1} - \theta_t). \text{ (A.2)}$$

Since $E_t \theta_{t+1} = \rho_\theta \theta_t$, we can simplify this further to

$$r_t - r_t^f = \frac{\sigma z \bar{b}^{FI}(\rho_\theta - 1)}{1-z}\left[\frac{\sigma}{(1-z)\zeta} - 1\right]\theta_t. \qquad \text{(A.3)}$$

Note $\zeta = \frac{\chi(1-z)+\sigma}{1-z}$, therefore $\frac{\sigma}{(1-z)\zeta} - 1 = -\frac{\chi}{\zeta}$. Hence (A.3) gives us (7). To derive (8), simply impose $\pi_t = qe_t = 0$ in (4) and rearrange so as to isolate x_t on the left-hand side.

Appendix B. Interest Rate Pegs

The short rate is constrained for H periods, after which time we revert to the conventional monetary policy described in (7). We derive the QE policy during the peg that generates the same zero inflation response. To do so, we solve backwards from $t + H - 1$ (the last period the interest rate is pegged) to t.

B.1 Period $t + H - 1$

In period $t + H$, the interest rate peg ends, and we have r_{t+H} obeying (7), which means x_{t+H} obeys (8), $\pi_{t+H} = 0$, and $qe_{t+H} = 0$. In the last period of the peg, $t + H - 1$, $r_{t+H-1} = 0$, hence the IS curve is

$$
\begin{aligned}
x_{t+H-1} = {} & \frac{\sigma z \bar{b}^{FI}}{(1-z)\zeta} E_{t+H-1}\theta_{t+H} + \frac{1-z}{\sigma} r^f_{t+H-1} \\
& - z\bar{b}^{FI}(E_{t+H-1}\theta_{t+H} - \theta_{t+H-1}) + z\bar{b}^{cb} qe_{t+H-1}. \quad \text{(B.1)}
\end{aligned}
$$

From the Phillips curve, x_{t+H-1} satisfies

$$
x_{t+H-1} = \frac{\sigma z}{(1-z)\zeta} \left[\bar{b}^{FI}\theta_{t+H-1} + \bar{b}^{cb} qe_{t+H-1} \right]. \quad \text{(B.2)}
$$

Combine (B.2) with (B.1) to eliminate x_{t+H-1}:

$$
\begin{aligned}
& \frac{\sigma z}{(1-z)\zeta} \left[\bar{b}^{FI}\theta_{t+H-1} + \bar{b}^{cb} qe_{t+H-1} \right] \\
& = \frac{\sigma z \bar{b}^{FI}}{(1-z)\zeta} E_{t+H-1}\theta_{t+H} + \frac{1-z}{\sigma} r^f_{t+H-1} \\
& \quad - z\bar{b}^{FI}(E_{t+H-1}\theta_{t+H} - \theta_{t+H-1}) + z\bar{b}^{cb} qe_{t+H-1}. \quad \text{(B.3)}
\end{aligned}
$$

Eliminate fractions:

$$
\bar{b}^{FI}\theta_{t+H-1} + \bar{b}^{cb}qe_{t+H-1} = \bar{b}^{FI}E_{t+H-1}\theta_{t+H} + \frac{(1-z)^2\zeta}{\sigma^2 z}r^f_{t+H-1}
$$
$$
- \frac{(1-z)\zeta\bar{b}^{FI}}{\sigma}(E_{t+H-1}\theta_{t+H} - \theta_{t+H-1}) + \frac{(1-z)\zeta\bar{b}^{cb}}{\sigma}qe_{t+H-1}. \tag{B.4}
$$

Write $E_{t+H-1}\theta_{t+H} = \rho_\theta\theta_{t+H-1}$ and simplify:

$$
\bar{b}^{cb}\left[1 - \frac{(1-z)\zeta}{\sigma}\right]qe_{t+H-1} = \frac{(1-z)^2\zeta}{\sigma^2 z}r^f_{t+H-1}
$$
$$
+ (\rho_\theta - 1)\bar{b}^{FI}\left[1 - \frac{(1-z)\zeta}{\sigma}\right]\theta_{t+H-1}, \tag{B.5}
$$

or

$$
\bar{b}^{cb}\left[\frac{\sigma - (1-z)\zeta}{\sigma}\right]qe_{t+H-1} = \frac{(1-z)^2\zeta}{\sigma^2 z}r^f_{t+H-1}
$$
$$
+ (\rho_\theta - 1)\bar{b}^{FI}\left[\frac{\sigma - (1-z)\zeta}{\sigma}\right]\theta_{t+H-1}. \tag{B.6}
$$

Simplifying further, we obtain

$$
qe_{t+H-1} = (\rho_\theta - 1)\frac{\bar{b}^{FI}}{\bar{b}^{cb}}\theta_{t+H-1} + \frac{(1-z)^2\zeta}{\sigma z\bar{b}^{cb}(\sigma - (1-z)\zeta)}r^f_{t+H-1}. \tag{B.7}
$$

B.2 Period $t + H - 2$

Next, we go back to $t + H - 2$, taking (B.7) as given. Writing out the IS curve, we have

$$
x_{t+H-2} = E_{t+H-2}x_{t+H-1} + \frac{1-z}{\sigma}r^f_{t+H-2}
$$
$$
- z\bar{b}^{FI}(E_{t+H-2}\theta_{t+H-1} - \theta_{t+H-2}) - z\bar{b}^{cb}(E_{t+H-2}qe_{t+H-1} - qe_{t+H-2}). \tag{B.8}
$$

Plug in for x_{t+H-1} in terms of qe_{t+H-1} from (B.2):

$$x_{t+H-2} = \frac{\sigma z}{(1-z)\zeta} \left[\bar{b}^{FI} E_{t+H-2}\theta_{t+H-1} + \bar{b}^{cb} E_{t+H-2}qe_{t+H-1} \right]$$

$$+ \frac{1-z}{\sigma} r^f_{t+H-2} - z\bar{b}^{FI} (E_{t+H-2}\theta_{t+H-1} - \theta_{t+H-2})$$

$$- z\bar{b}^{cb}(E_{t+H-2}qe_{t+H-1} - qe_{t+H-2}). \tag{B.9}$$

From the Phillips curve, x_{t+H-2} also satisfies

$$x_{t+H-2} = \frac{\sigma z}{(1-z)\zeta} \left[\bar{b}^{FI}\theta_{t+H-2} + \bar{b}^{cb}qe_{t+H-2} \right]. \tag{B.10}$$

Plug this into (B.9):

$$\frac{\sigma z}{(1-z)\zeta} \left[\bar{b}^{FI}\theta_{t+H-2} + \bar{b}^{cb}qe_{t+H-2} \right]$$

$$= \frac{\sigma z}{(1-z)\zeta} \left[\bar{b}^{FI} E_{t+H-2}\theta_{t+H-1} + \bar{b}^{cb} E_{t+H-2}qe_{t+H-1} \right]$$

$$+ \frac{1-z}{\sigma} r^f_{t+H-2} - z\bar{b}^{FI}(E_{t+H-2}\theta_{t+H-1} - \theta_{t+H-2})$$

$$- z\bar{b}^{cb}(E_{t+H-2}qe_{t+H-1} - qe_{t+H-2}). \tag{B.11}$$

Eliminate the fraction on the left-hand side:

$$\bar{b}^{FI}\theta_{t+H-2} + \bar{b}^{cb}qe_{t+H-2}$$

$$= \bar{b}^{FI} E_{t+H-2}\theta_{t+H-1} + \bar{b}^{cb} E_{t+H-2}qe_{t+H-1} + \frac{(1-z)^2\zeta}{\sigma^2 z} r^f_{t+H-2}$$

$$- \frac{(1-z)\zeta\bar{b}^{FI}}{\sigma}(E_{t+H-2}\theta_{t+H-1} - \theta_{t+H-2})$$

$$- \frac{(1-z)\zeta\bar{b}^{cb}}{\sigma}(E_{t+H-2}qe_{t+H-1} - qe_{t+H-2}). \tag{B.12}$$

Write $E_{t+H-2}\theta_{t+H-1} = \rho_\theta\theta_{t+H-2}$ and rearrange terms:

$$\bar{b}^{cb}\left[1 - \frac{(1-z)\zeta}{\sigma}\right] qe_{t+H-2} = \bar{b}^{cb}\left[1 - \frac{(1-z)\zeta}{\sigma}\right] E_{t+H-2}qe_{t+H-1}$$

$$+ \frac{(1-z)^2\zeta}{\sigma^2 z} r^f_{t+H-2} + (\rho_\theta - 1)\bar{b}^{FI}\left[1 - \frac{(1-z)\zeta}{\sigma}\right]\theta_{t+H-2}. \tag{B.13}$$

This implies

$$qe_{t+H-2} = E_{t+H-2}qe_{t+H-1} + \frac{(1-z)^2\zeta}{\sigma z \bar{b}^{cb}(\sigma - (1-z)\zeta)}r^f_{t+H-2}$$

$$+ (\rho_\theta - 1)\frac{\bar{b}^{FI}}{\bar{b}^{cb}}\theta_{t+H-2}. \tag{B.14}$$

Next, plug in for qe_{t+H-1} from (B.7):

$$qe_{t+H-2} = (\rho_\theta - 1)\frac{\bar{b}^{FI}}{\bar{b}^{cb}}E_{t+H-2}\theta_{t+H-1} + (\rho_\theta - 1)\frac{\bar{b}^{FI}}{\bar{b}^{cb}}\theta_{t+H-2}$$

$$+ \frac{(1-z)^2\zeta}{\sigma z \bar{b}^{cb}(\sigma - (1-z)\zeta)}E_{t+H-2}r^f_{t+H-1}$$

$$+ \frac{(1-z)^2\zeta}{\sigma z \bar{b}^{cb}(\sigma - (1-z)\zeta)}r^f_{t+H-2}. \tag{B.15}$$

Noting that $E_{t+H-2}\theta_{t+H-1} = \rho_\theta\theta_{t+H-2}$ and $E_{t+H-2}r^f_{t+H-1} = \rho_f r^f_{t+H-2}$, we have

$$qe_{t+H-2} = (1+\rho_\theta)(\rho_\theta-1)\frac{\bar{b}^{FI}}{\bar{b}^{cb}}\theta_{t+H-2} + \frac{(1-z)^2\zeta(1+\rho_f)}{\sigma z \bar{b}^{cb}(\sigma - (1-z)\zeta)}r^f_{t+H-2}. \tag{B.16}$$

B.3 Period $t + H - 3$

Next, we go back to $t + H - 3$. Writing out the IS curve, we have

$$x_{t+H-3} = E_{t+H-3}x_{t+H-2} + \frac{1-z}{\sigma}r^f_{t+H-3}$$

$$- z\bar{b}^{FI}(E_{t+H-3}\theta_{t+H-2} - \theta_{t+H-3})$$

$$- z\bar{b}^{cb}(E_{t+H-3}qe_{t+H-2} - qe_{t+H-3}). \tag{B.17}$$

Plug in for x_{t+H-2} in terms of qe_{t+H-2} from (B.10):

$$x_{t+H-3} = \frac{\sigma z}{(1-z)\zeta}\left[\bar{b}^{FI}E_{t+H-3}\theta_{t+H-2} + \bar{b}^{cb}E_{t+H-3}qe_{t+H-2}\right]$$

$$+ \frac{1-z}{\sigma}r^f_{t+H-3} - z\bar{b}^{FI}(E_{t+H-3}\theta_{t+H-2} - \theta_{t+H-3})$$

$$- z\bar{b}^{cb}(E_{t+H-3}qe_{t+H-2} - qe_{t+H-3}). \tag{B.18}$$

From the Phillips curve, x_{t+H-3} also satisfies

$$x_{t+H-3} = \frac{\sigma z}{(1-z)\zeta} \left[\bar{b}^{FI}\theta_{t+H-3} + \bar{b}^{cb}qe_{t+H-3} \right]. \tag{B.19}$$

Plug this into (B.18):

$$\frac{\sigma z}{(1-z)\zeta} \left[\bar{b}^{FI}\theta_{t+H-3} + \bar{b}^{cb}qe_{t+H-3} \right]$$

$$= \frac{\sigma z}{(1-z)\zeta} \left[\bar{b}^{FI}E_{t+H-3}\theta_{t+H-2} + \bar{b}^{cb}E_{t+H-3}qe_{t+H-2} \right]$$

$$+ \frac{1-z}{\sigma}r^f_{t+H-3} - z\bar{b}^{FI}(E_{t+H-3}\theta_{t+H-2} - \theta_{t+H-3})$$

$$- z\bar{b}^{cb}(E_{t+H-3}qe_{t+H-2} - qe_{t+H-3}). \tag{B.20}$$

Eliminate the fraction on the left-hand side:

$$\bar{b}^{FI}\theta_{t+H-3} + \bar{b}^{cb}qe_{t+H-3}$$

$$= \bar{b}^{FI}E_{t+H-3}\theta_{t+H-2} + \bar{b}^{cb}E_{t+H-3}qe_{t+H-2}$$

$$+ \frac{(1-z)^2\zeta}{\sigma^2 z}r^f_{t+H-3} - \frac{(1-z)\zeta\bar{b}^{FI}}{\sigma}(E_{t+H-3}\theta_{t+H-2} - \theta_{t+H-3})$$

$$- \frac{(1-z)\zeta\bar{b}^{cb}}{\sigma}(E_{t+H-3}qe_{t+H-2} - qe_{t+H-3}). \tag{B.21}$$

Write $E_{t+H-3}\theta_{t+H-2} = \rho_\theta\theta_{t+H-3}$ and rearrange terms:

$$\bar{b}^{cb}\left[1 - \frac{(1-z)\zeta}{\sigma}\right]qe_{t+H-3} = \bar{b}^{cb}\left[1 - \frac{(1-z)\zeta}{\sigma}\right]E_{t+H-3}qe_{t+H-2}$$

$$+ \frac{(1-z)^2\zeta}{\sigma^2 z}r^f_{t+H-3} + (\rho_\theta - 1)\bar{b}^{FI}\left[1 - \frac{(1-z)\zeta}{\sigma}\right]\theta_{t+H-3}. \tag{B.22}$$

This implies

$$qe_{t+H-3} = E_{t+H-3}qe_{t+H-2} + \frac{(1-z)^2\zeta}{\sigma z\bar{b}^{cb}(\sigma - (1-z)\zeta)}r^f_{t+H-3}$$

$$+ (\rho_\theta - 1)\frac{\bar{b}^{FI}}{\bar{b}^{cb}}\theta_{t+H-3}. \tag{B.23}$$

Next, plug in for qe_{t+H-2} from (B.16):

$$qe_{t+H-3} = (\rho_\theta+1)(\rho_\theta-1)\frac{\bar{b}^{FI}}{\bar{b}^{cb}}E_{t+H-3}\theta_{t+H-2}+(\rho_\theta - 1)\frac{\bar{b}^{FI}}{\bar{b}^{cb}}\theta_{t+H-3}$$

$$+ \frac{(1-z)^2\zeta(1+\rho_f)}{\sigma z \bar{b}^{cb}(\sigma - (1-z)\zeta)}E_{t+H-3}r^f_{t+H-2}$$

$$+ \frac{(1-z)^2\zeta}{\sigma z \bar{b}^{cb}(\sigma - (1-z)\zeta)}r^f_{t+H-3}. \tag{B.24}$$

Noting that $E_{t+H-3}\theta_{t+H-2} = \rho_\theta\theta_{t+H-3}$ and $E_{t+H-3}r^f_{t+H-2} = \rho_f r^f_{t+H-3}$, we have

$$qe_{t+H-3} = (1 + \rho_\theta + \rho_\theta^2)(\rho_\theta - 1)\frac{\bar{b}^{FI}}{\bar{b}^{cb}}\theta_{t+H-3}$$

$$+ \frac{(1-z)^2\zeta(1+\rho_f+\rho_f^2)}{\sigma z \bar{b}^{cb}(\sigma - (1-z)\zeta)}r^f_{t+H-3}. \tag{B.25}$$

B.4 Period $t + j$

Comparing (B.7), (B.16), and (B.25), we can generalize to period $t + j$

$$qe_{t+j} = \frac{\bar{b}^{FI}}{\bar{b}^{cb}}(\rho_\theta - 1)\sum_{i=0}^{H-j-1}\rho_\theta^i\theta_{t+j}+\frac{(1-z)^2\zeta}{\sigma z \bar{b}^{cb}(\sigma - (1-z)\zeta)}\sum_{i=0}^{H-j-1}\rho_f^i r^f_{t+j}$$

$$= -\frac{\bar{b}^{FI}}{\bar{b}^{cb}}(1 - \rho_\theta^{H-j})\theta_{t+j} + \frac{(1-z)^2\zeta}{\sigma z \bar{b}^{cb}(\sigma - (1-z)\zeta)}\frac{1 - \rho_f^{H-j}}{1 - \rho_f}r^f_{t+j}. \tag{B.26}$$

Using $\sigma - (1-z)\zeta = -\chi(1-z)$ yields (10).

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