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International Journal of Central Banking

Volume 15, Number 3 September 2019

Optimal Negative Interest Rate under Uncertainty 1
Kuk Mo Jung

Exchange Rate Pass-Through: What Has Changed Since the Crisis? 27
Martina Jašová, Richhild Moessner, and Előd Takáts

What Drives the Strength of Monetary Policy Transmission? 59
Jakub Matějů

Banks' Wealth, Banks' Creation of Money, and Central Banking 89
Tianxi Wang

Macroeconomic Effects of Banking-Sector Losses across Structural Models 137
Luca Guerrieri, Matteo Iacoviello, Francisco Covas, John C. Driscoll, Mohammad Jahan-Parvar, Michael Kiley, Albert Queralto, and Jae Sim

Deleveraging and Consumer Credit Supply in the Wake of the 2008–09 Financial Crisis 205
Reint Gropp, John Krainer, and Elizabeth Laderman

What Drives Bank-Intermediated Trade Finance? Evidence from Cross-Country Analysis 253
José María Serena Garralda and Garima Vasishtha

Comparing Fiscal Consolidation Multipliers across Models in Europe 285
Juha Kilponen, Massimiliano Pisani, Sebastian Schmidt, Vesna Corbo, Tibor Hledik, Josef Hollmayr, Samuel Hurtado, Paulo Júlio, Dmitry Kulikov, Matthieu Lemoine, Matija Lozej, Henrik Lundvall, José R. Maria, Brian Micallef, Dimitris Papageorgiou, Jakub Rysanek, Dimitrios Sideris, Carlos Thomas, and Gregory de Walque

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Optimal Negative Interest Rate under Uncertainty*

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I employ a simple overlapping-generations model of money and nominal bonds with Epstein-Zin preferences and study the optimal negative interest rate. A subzero lower bound can arise in the model due to the illiquidity of money as a savings instrument. This model of negative interest rates differentiates from conventional ones based on exogenous money holding costs in that the subzero lower bound as well as the optimal negative rate turn out to crucially depend upon agents' preferences for the timing of uncertainty resolution. Both the lower bound and the optimal interest rate for aggregate consumption can fall into a negative territory only if agents prefer late resolution of uncertainty. In the latter case, the lower bound and the optimal rate both decrease even further when aggregate output uncertainty rises. However, the optimal interest rate turns out to be non-negative and to have a positive relationship with the degree of aggregate uncertainty if agents prefer early resolution of uncertainty.

JEL Codes: E43, E50, E52, G12.

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1. Introduction

Since 2011 the world monetary system has experienced something that only a few had imagined in a hypothetical world, namely negative interest rates. As of May 2019, five central banks are now in the negative-rate club, including the European Central Bank, the Bank of Japan, the National Bank of Denmark, the Swiss National Bank, and Sweden's Riksbank. Radical as it sounds, this recent phenomenon has spurred public debates, topics of which can be basically categorized into two questions: (i) Can (substantially) negative rates be implemented? (ii) If yes, is it desirable to pursue such policy?

The former question largely revolves around how to substantially get rid of incentives to hoard cash under negative interest rates. Many possible mechanisms were already suggested. Examples include abolishing cash, levying a tax on paper currency, imposing a fee for converting cash deposits to electronic bank reserves at the central bank, etc.[1] Whether these suggested methods could really clear the path for negative interest rates remains to be seen.

Despite its importance, this paper will be agnostic about the former question. Instead, it aims to shed new light on the latter one. In particular, I am interested in how low (negative) rates should go once they are possible. In short, this paper doesn't aim to fully microfound the feasibility of negative rates. Instead, it aims to find the optimal negative interest rate, if any, in a hypothetical world where negative rates are already feasible. Two pivotal issues remain for this task. The first one is how to introduce a subzero lower bound for interest rates, and the second issue is about appropriately incorporating various costs and benefits associated with negative interest rates.

The first issue should be taken seriously because the Freidman rule (FR), i.e., a zero nominal interest rate, becomes the lower bound for interest rate policy in a standard monetary model. One conventional way to bypass this problem is to introduce some exogenous fixed costs associated with holding cash. Yet, this approach is problematic since it would usually imply both the negative lower bound and the optimal rate being effectively equivalent to the FR minus

[1] Interested readers may refer to Rogoff (2016) for an excellent review of these suggested mechanisms.

the exogenous holding cost. Therefore, I adopt another approach where money is used for exchange only, while agents use bonds to save (but not to exchange). The lower bound can be negative in this case to the extent that agents' intertemporal desire to save through bonds outweighs costs from their negative returns. While it is admittedly true that this way of introducing the negative lower bound is still a reduced form, the optimal result in my model turns out to be richer than and produce more far-reaching policy implications than the conventional approach. Detailed and intuitive explanation will follow shortly.

The second issue is even more important. The literature has already pointed out major pros and cons of negative interest rates. Possible benefits discussed so far cover various factors: more effective inflation expectations control, less distortion from intertemporal price fluctuations—especially given the long-term declining trend in real interest rates, and stimulation effects on lending and investment, etc. On the contrary, serious concerns have been raised as well: downward pressure on banks' net interest margin, threats of currency wars, and negative signaling effects on the effectiveness of monetary policy, etc. (See Agarwal and Kimball 2015, Goodfriend 2016, and references therein for detailed review.) Though certainly important, incorporating all these factors into a unified framework is not the purpose of this study. Instead, I only focus on one novel channel through which the optimality of negative rates can be analyzed in a tractable manner. This newly offered channel is "aggregate uncertainty," defined as second-moment changes in the distribution of aggregate output. Despite its narrow focus, many interesting and novel policy implications also emerge, as will be shown subsequently.

The model constructed in this paper is, therefore, meant to rigorously reflect upon the aforementioned ways of dealing with the two issues in a unified manner. It builds upon a standard two-period overlapping-generations (OLG) model with endowments. The young receive a fixed amount of goods, while the old face uncertain amounts of the same good. I also introduce decentralized trading in which agents acquire a separate second good through the use of a medium of exchange (MOE). Two financial assets are introduced: *fiat* money and nominal bonds. As mentioned earlier, I assume that money only serves an MOE role, while bonds only function as a savings instrument. Lastly, the Epstein-Zin (EZ) preferences are adopted to materialize the idea of uncertainty-driven optimal negative rates. A key and well-known innovation of the EZ preferences is that they allow agents to separate their degree of aversion to cross-sectional and intertemporal risks; see Epstein and Zin (1989), Bansal and Yaron (2004), and Jung (2017) for details.

The model delivers very interesting equilibrium allocations and welfare implications. First, the lower bound for interest rates is no longer zero. Intuition is not hard. Agents can still demand bonds as a savings instrument in light of negative interest rates, because intergenerational transfers of the endowment good are only possible through bonds. It is still true that interest rates should be bounded from below; otherwise, agents are better off simply with autarky under too-negative rates. The extent to which the lower bound can fall, therefore, depends upon the degree to which the intertemporal inequality in endowments prevails. In other words, the lower bound can go deep into a negative territory further when the average level of old-age endowments relatively decreases.

One novel prediction of the current model is that the lower bound also depends on the second moment of random old-age endowments, i.e., aggregate uncertainty, and an agent's preference for the timing of uncertainty resolution, i.e., the relative degree to which agents dislike intertemporal risks compared with the cross-sectional ones. As opposed to the standard constant relative risk aversion (CRRA) utility case, the uncertainty *directly* affects an agent's utility under EZ preferences because of its negative effect on the certainty equivalent value of old-age consumption. Accordingly, the uncertainty undoubtedly affects an agent's willingness to save using bonds in this framework. The point is that such incentives act differently depending on an agent's attitude towards cross-sectional and intertemporal risks.

Intuitively, when an agent's aversion to intertemporal risks is relatively greater, they dislike intertemporal inequality in consumption more than the cross-sectional variation in old-age consumption. Under this case, a higher uncertainty means more intertemporal consumption inequality. Thus, it surely increases an agent's willingness to purchase bonds. As a result, the lower bound must also fall in response to a higher uncertainty. The exact opposite logic applies to the case where an agent's aversion to intertemporal risks is relatively smaller. In this opposite case, agents become relatively more averse to cross-sectional variations in old-age consumption, and therefore a higher uncertainty actually induces them to transfer consumption towards the young through less bond savings. This means the lower bound must increase in response to a rise in uncertainty, as opposed to the former case.

Main results of the paper, i.e., what the optimal negative rate is and how it is affected by uncertainty, also turn out to be interesting. A pure-utility-based welfare as a function of the nominal interest rate exhibits a hump-shaped pattern with the FR being a maximizer. This property is the same regardless of the agent's relative intertemporal risk aversion. The idea is that only the Friedman rule guarantees no distortion in the price level of the intergenerational transfer instrument, which is also common in other major OLG monetary models.

However, welfare measured in terms of aggregate consumption shows novel and richer patterns. The maximum aggregate consumption in this endowment economy is achieved only when the consumption of the three different goods, i.e., the (numeraire) good consumed in both periods and the special good, are equally distributed. Yet, changes in the rate of return on bonds certainly affect that distribution. Again, the point is that such interest rate effects crucially differ depending on the agent's relative aversion to intertemporal risks. For instance, when agents are indifferent to cross-sectional and intertemporal risks, i.e., agents have the CRRA utility, uncertainty has no effects on agents' consumption decision because the certainty equivalent and the average value of old-age consumption are always equal to each other. Thus, nominal interest rates distort the aggregate consumption distribution in the standard way, i.e., a higher interest rate leads to more old-age consumption. To avoid such a distortion, a zero nominal interest rate must be taken, i.e., the FR becomes the optimal rule to achieve the maximum aggregate consumption.

When agents have different attitudes towards cross-sectional and intertemporal risks, the FR breaks up. In light of a greater aversion to intertemporal risks, the aggregate consumption portfolio becomes more biased towards old-age consumption even with a zero nominal interest rate, for the reasons mentioned earlier. Thus, a subzero level of nominal interest rate is required to mitigate such negative distribution effects, meaning that the optimal interest rate should be negative in this case. Moreover, it naturally follows that the negative optimal interest rate would fall even further as uncertainty rises in this case. Nevertheless, these properties get totally reversed when agents dislike cross-sectional risks more. The aggregate consumption distribution this time is biased towards young-age consumption even under the FR. Therefore, a positive level of interest rate becomes an optimal policy to pursue. Finally, the (positive) optimal interest rate in this case should increase in uncertainty so as to further transfer consumption towards the old.

2. Related Literature

Full-fledged academic literature on negative interest rates has yet to be developed. Nevertheless, there are some excellent review papers on the feasibility, desirability, and implementation of negative interest rates. Interested readers should refer to Agarwal and Kimball (2015), Goodfriend (2016), and Williams (2016). The main theme of this paper is the optimal level of negative interest rate, and papers on this particular issue are still scarce to the best knowledge of the author. In what follows, I, therefore, briefly introduce existing studies that are related to the current paper in terms of methodology and contribution only.

Goodfriend (2000) was the first one to theoretically explore the possibility of a negative nominal interest rate policy. He largely focused on how to enable central banks to actually target negative interest rates. Suggested options are a carry tax on money, open market operations in long bonds, and monetary transfers. Unlike his emphasis on technical implementation issues, the current paper explores what might be the optimal negative interest rate once negative interest rates are implementable. Agarwal and Kimball (2015) also suggest a scheme for a changeable exchange rate between currency and reserves for negative interest rates to be implementable. The idea is that central banks can lower the rate at which reserves can be converted to cash so that the negative interest rate on reserves becomes arbitrage free. Haldane (2015) and Kocherlakota (2016) argue that the best way to make negative interest rates practically feasible is to abolish cash and move completely to electronic cash with any yield. Again, none of them aims to look for the optimal negative interest rate based on a general equilibrium monetary framework.

Brunnermeier and Koby (2016) is probably the study most related to the current paper in terms of finding the optimal interest rate subject to the negative lower bound. They explicitly take into account banking sectors to find what they call the “reversal interest rate,” the rate at which accommodative monetary policy reverses its effect and becomes contractionary for lending. According to them, that rate critically depends upon various microstructures in banking sectors, and could well be negative. The current paper differs in that the effects of negative interest rates on aggregate variables and welfare are analyzed through an agent’s general preferences for the timing of uncertainty resolution rather than banking intermediation channels.

Lastly, this paper is related to those using OLG monetary models. Schreft and Smith (2002) and Bhattacharya, Haslag, and Martin (2005) introduce financial intermediation and limited communication into the OLG framework, and show the suboptimality of the Friedman rule. In contrast to this line of research, the suboptimality arises in the current model from interactions between uncertainty and an agent’s relative aversion to intertemporal risks. The current model follows most closely Jung (2018) in terms of methodology, where decentralized trading is explicitly incorporated into the OLG framework. Yet, it is a pure currency model, while the current model extends it to include nominal bonds.

3. The Model

The current model is a discrete-time and two-period overlapping-generations model with no time discounting and no population growth. The economy consists of one main island at the center and a unit measure of periphery islands around it. Each period, a unit measure of households is born in the main island, and lives only for two periods. When households are born, they get endowed with fixed units, i.e., x, of *numeraire* goods. By assumption, these goods are perishable such that carrying them across periods and outside the main island is not possible. When households move into the second period of their life, they also receive an identical endowment, ε units of *numeraire* goods, but this time it is random and follows a uniform distribution, $\mathcal{U}(y-b, y+b)$, where $y \geq b$ and $x > y$.

A key feature of the model is that the household is divided into two independent individuals, a *worker* and a *shopper*. The worker needs to consume *numeraire* goods in both periods, while a shopper only needs to consume in the second period of life. Furthermore, the shopper must consume something different from the *numeraire* good. We call it "special goods." The problem is households are never endowed with special goods, which only "sellers" living on periphery islands can produce. To be more precise, we assume that one seller is born every period on each periphery island, and lives only one period. Each seller is born with a homogeneous technology to produce the special good with a linear cost of labor disutility. However, they only get the utility from consuming *numeraire* goods. This framework basically gives rise to a trading motive between old shoppers and sellers each period.

Two key trading characteristics are worth noting. First, trades between sellers and old shoppers must take place in a bilateral fashion due to spatial separation among islands. Second, any kind of credit arrangement and/or barters between sellers and old shoppers are also ruled out due to anonymity and limited commitment within a bilateral meeting along with the assumption that *numeraire* goods are perishable across periods and islands. In consequence, the medium of exchange (MOE) is required for this mutually beneficial trade to take place.

To that end, we introduce two potential candidates: an intrinsically useless object called "(fiat) money" and a one-period nominal government bond. Money in this economy is issued by the government and assumed to be in fixed supply. Therefore, we denote M as the total money supply every period. We rule out lump-sum money transfers by the government so as to introduce a nominal interest rate as the only available government policy instrument.[2] We denote φ_t as the real price of money in terms of *numeraire* goods at period t. Apart from the money, there exist one-period pure discount nominal bonds. They take the form of a book entry such as the U.S. Treasury bonds. The real price of one unit of the nominal bond at period t is denoted by ψ_t. This means that a unit of money at period $t+1$ can be redeemed from one unit of nominal bonds sold at the price level of ψ_t. Importantly, we assume that the nominal bond

[2]The constancy of money supply is chosen purely for the sake of simplicity. One could easily introduce a constant (gross) money growth rate, say μ, as an additional policy instrument. However, this would not change welfare results qualitatively, but only generate level effects.

price level at period t, i.e., ψ_t/φ_t, is a policy variable set by the government. Furthermore, we assume that the government is always on the balanced budget. That is the total real value of the bonds the government must redeem each period, i.e., $\varphi_t A_{t-1}$, and it ought to equal the total real value of the nominal bonds issued each period, i.e., $\psi_t A_t$, where A_t denotes the total amount of nominal bonds held by agents in period t.

Regarding bilateral trading frictions, we adopt a simple mechanism.[3] First, a perfect match between each old shopper and each seller is assumed. That is, every seller and old shopper gets to match and consume every period. Second, we adopt a *take-it-or-leave-it* offer by (old) shoppers to sellers as a pricing protocol within the pairwise trade. As in Jung (2018), bargaining solutions are trivial. Old shoppers always hand over all of their real balances to young sellers who produce exactly the same amount of special goods as the real money balances they receive. Lastly and most importantly, we assume that nominal bonds are perfectly illiquid, meaning that sellers never accept nominal bonds as a payment method within a pairwise trade. This means each old shopper's real balances consist of only money.

Old workers' real balances, on the other hand, can potentially become a portfolio of money and bonds. Technically speaking, money can potentially serve both as an MOE and as a savings instrument. Under positive nominal interest rates, it is obvious that workers will never use money as a savings instrument due to a higher rate of return on nominal bonds. The problem here is what happens in light of negative nominal interest rates. In such a case, nominal bonds whose return rate is lower than that of money will never be valued in equilibrium. Thus, this economy would return to a pure currency economy as in Jung (2018). This is an undesirable feature of the model since the very purpose of this paper is to search for optimal negative interests under an economy with both money and bonds being valued. For this reason, we take a shortcut, as we did with respect to the illiquidity of nominal bonds. Simply, we assume

[3]See search-based monetary theory literature, e.g., Lagos and Wright (2005) and Geromichalos and Jung (2018) for a detailed introduction to bilateral trading frictions.

that money can never be used as a savings instrument under any circumstances.[4]

Finally, a household born in period t has EZ-type preferences, $U(c_t, s_{t+1}, c_{t+1})$, given by the following form:

$$U(c_t, s_{t+1}, c_{t+1}) = \left[c_t^{1-\rho} + s_{t+1}^{1-\rho} + [R_t(c_{t+1})]^{1-\rho}\right]^{\frac{1}{1-\rho}}$$

$$\text{where } R_t(c_{t+1}) = \left(E_t\left[c_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}}, \rho > 0.$$

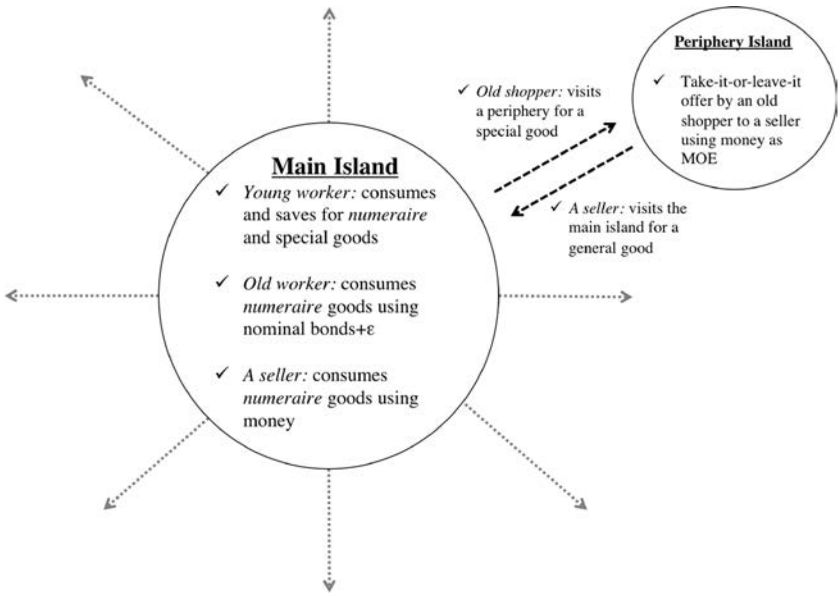
c_t and c_{t+1} denote the amount of *numeraire* goods consumed by the worker in period t and $t+1$, while s_{t+1} denotes the amount of special goods consumed by the shopper in period $t+1$. Figure 1 provides a graphical illustration of the timing of key events. Finally, one can refer to Jung (2018) for an intuitive interpretation of EZ preferences.

4. Constrained Efficiency

We first study efficient allocations by solving a social planner problem. In doing so, we restrict our attention to constrained efficiency. The planner is prevented from achieving the first best because, like private agents, she is assumed to be unable to provide full insurance for old-age consumption. The rationale goes as follows. Suppose she was allowed to achieve complete risk sharing between old agents; she would no longer face EZ preferences due to no uncertainty on old-age consumption, i.e., the certainty equivalent value of old-age consumption would always be maximized to its expected value. Hence, a fully efficient allocation can be achieved. It is, however, important to note that private agents in competitive equilibrium and the planner would face different objective functions, i.e., a constant elasticity of substitution (CES) aggregate of c_y, s_o, and $R(c_o)$ for the former and a CES

[4]It is important to note that the perfect illiquidity of money as a savings instrument is assumed for simplicity. For instance, one could instead introduce a partial illiquidity of money as a savings instrument, say by assuming that only a fixed portion θ of agents can use money to save every period (think of $1-\theta$ as a portion of the population that has no bank accounts or cash storage technology). This relaxation wouldn't change the results qualitatively. So, technically speaking, one would only need some degree of illiquidity of money as a savings instrument to get my results. In this sense, the restriction on the money as a store of value may not be as ad hoc as it first seems.

Figure 1. Timing of Key Events



aggregate of c_y, s_o, and c_o for the latter. Thus, this (fully efficient) allocation would not be a fair benchmark to compare with the competitive equilibrium allocation. Furthermore, if agents are not able to set up an insurance arrangement that allows them to share their endowment risk in old age, it is not clear why the planner should be able to do so.

Second, we only focus on stationary allocations. That is, the social planner only gets to choose stationary allocations for *numeraire* goods consumed by young workers, c_y^*; special goods consumed by old shoppers, s^*; *numeraire* goods consumed by old workers, c_o^*; and *numeraire* goods consumed by sellers, n^*. Then, the planner's solution solves for the following problem:

$$\max_{c_y^*, c_o^*} \left\{ \left[(c_y^*)^{1-\rho} + (s^*)^{1-\rho} + [R(c_o^*)]^{1-\rho} \right]^{\frac{1}{1-\rho}} + [n^* - s^*] \right\}, \qquad (1)$$

$$\text{s.t.} \quad c_y^* + c_o^* + n^* = x + y,$$

$$\text{and} \quad s^* = n^*,$$

where $R(c_o^*) = \left(E\left[(c_o^*)^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}, \rho > 0$.

The first aggregate (resource) constraint implies that total *numeraire* goods consumed by households and sellers must be the same as total endowments of the *numeraire* good in each period. The second aggregate (resource) constraint simply implies that the planner also faces the bilateral trading friction on each island as private agents in competitive equilibrium. Specifically, it tells that the amount of total special goods consumed by (old) shoppers is equal to the total special goods produced by sellers each period. Note that the amount of total special goods produced by sellers is equal to the total *numeraire* goods consumed by sellers due to the *take-it-or-leave-it* offer, which also explains the second linear part in the objective function. The following lemma summarizes the socially optimal stationary allocations of consumptions by households and sellers.

LEMMA 1. *The constrained efficient stationary allocations can be expressed as* $c_y^* = x - T^c - T^s$; $c_o^* = T^c + y$; $s^* = n^* = T^s$, *where* T^c *and* T^s *must meet the following two conditions:*

$$(i)\; Q(T^c, T^s) = 1 \quad \forall t,$$

$$(ii)\; T^s = [x - T^c - T^s]^{\gamma/\rho} \left[E\left[(T^c + \varepsilon)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \right]^{(\rho-\gamma)/\rho} \qquad \forall t,$$

$$\textit{where} \quad Q(T^c, T^s) = \left[\frac{T^c + \varepsilon}{x - T^c - T^s} \right]^{-\rho} \left[\frac{T^c + \varepsilon}{E\left[(T^c + \varepsilon)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}} \right]^{\rho-\gamma},$$

$$\textit{and} \quad E\left[(T^c + \varepsilon)^{1-\gamma}\right] = \frac{(T^c + y + b)^{(2-\gamma)} - (T^c + y - b)^{(2-\gamma)}}{2b(2-\gamma)}.$$

Proof. See the appendix in Jung (2018).

Intuition for these results is exactly the same as the one in Jung (2018).

5. Competitive Equilibrium

While the constrained efficient outcome in this economy is identical to Jung (2018), a competitive equilibrium in this economy is different from it due to the introduction of nominal bonds. A key difference is that households need to decide how much real money balances to acquire for special good consumption, m_t, and how much nominal bonds are used to purchase for *numeriare* good consumption, a_t.[5] Then, a young household's choice problem can be given by

$$\max_{m_t, a_t} \left[(c_t)^{1-\rho} + (s_{t+1})^{1-\rho} + R_t(c_{t+1})^{1-\rho} \right]^{\frac{1}{1-\rho}}, \tag{2}$$

$$\text{s.t.} \quad c_t + \varphi_t m_t + \psi_t a_t = x, \quad c_{t+1} = \varphi_{t+1} a_t + \varepsilon_{t+1}, \quad \text{and}$$
$$s_{t+1} = \varphi_{t+1} m_t,$$

where $R_t(c_{t+1}) = \left(E_t \left[c_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}$, $\rho > 0$, and ε_{t+1} follows a uniform distribution of $\mathcal{U}(y-b, y+b)$.

Intuitively, households face uncertainty with regard to old-age endowment. Thus, nominal bonds serve as a savings instrument for households to consume *numeraire* goods in the second period of their life. Again, nominal bonds here are assumed to be perfectly illiquid in a pairwise trade between old shoppers and sellers. Consequently, they serve only as a store of value.

Intuitive explanation for the three constraints in problem (2) can be provided as well. The first one refers to a budget constraint for young households. The second one simply says that *numeraire* goods consumption in old age must be financed by nominal bond savings from the previous period and the current endowment. The third constraint simply follows from two crucial assumptions: perfectly illiquid nominal bonds and the *take-it-or-leave-it* offer.

Using some properties of the EZ preferences, the following lemma summarizes individual optimal choice by the young household.

LEMMA 2. *Given aggregate real prices $\{\varphi_t, \varphi_{t+1}, \psi_t, \psi_{t+1}\}$ and old-age endowment shocks ε_{t+1}, the young household's optimal portfolio choice of $\{m_t, a_t\}$ must satisfy the following conditions:*

$$(i)\ \frac{\psi_t}{\varphi_{t+1}} = Q_{t,t+1}(m_t, a_t, \varepsilon_{t+1}) \quad \forall t,$$

[5] As in Jung (2018), we assume the endowment shocks are realized after shoppers leave the main island in order for a household to choose a portfolio ex ante.

$$(ii)\ \varphi_{t+1}m_t = [x - \varphi_t m_t - \psi_t a_t]^{\gamma/\rho} \times \left[E_t\left[(\varphi_{t+1}a_t + \varepsilon_{t+1})^{1-\gamma}\right]^{\frac{1}{1-\gamma}}\right]^{(\rho-\gamma)/\rho} \quad \forall t,$$

$$\textit{where} \quad Q_{t,t+1}(m_t, a_t, \varepsilon_{t+1}) = \left[\frac{\varphi_{t+1}a_t + \varepsilon_{t+1}}{x - \varphi_t m_t - \psi_t a_t}\right]^{-\rho} \times \left[\frac{\varphi_{t+1}a_t + \varepsilon_{t+1}}{E_t\left[(\varphi_{t+1}a_t + \varepsilon_{t+1})^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right]^{\rho-\gamma},$$

$$\textit{and} \quad E_t\left[(\varphi_{t+1}a_t + \varepsilon_{t+1})^{1-\gamma}\right] = \frac{(\varphi_{t+1}a_t + y + b)^{(2-\gamma)} - (\varphi_{t+1}a_t + y - b)^{(2-\gamma)}}{2b(2-\gamma)}.$$

Proof. The proof is the same as the one for lemma 4 of Jung (2018). One just needs to replace m_t^c, m_t^s, $h(m_t^s)$, and $h(m_t^c)$ in Jung (2018) with a_t, m_t, $\varphi_{t+1}m_t$, and $\varphi_{t+1}a_t$, respectively.

Interpretation of lemma 2 follows similarly from Jung (2018). The first condition refers to an intertemporal optimality between c_t and c_{t+1}, while the second one represents an intratemporal optimality between s_{t+1} and c_{t+1}. Note that agents' preferences for the timing of the uncertainty resolution, i.e., whether $\rho > \gamma$ or $\rho < \gamma$, along with the level of future endowment uncertainty, b, critically affect both optimal conditions.

Now we can define competitive equilibrium. As in Jung (2018), we restrict attention to the symmetric, monetary, and stationary equilibrium.

DEFINITION 1. *A competitive, symmetric, monetary, and stationary equilibrium is a list* $\{Z, W, n, s, c_y, c_o\}$*, where* $Z \equiv Z_t \equiv Z_{t+1} \equiv \varphi_t m_t \equiv \varphi_{t+1}m_{t+1} \equiv \bar{\varphi}M$, $\forall t$*, where* $\varphi_t = \bar{\varphi}, \forall t$. $W \equiv W_t \equiv \psi_t a_t = \psi_{t+1}a_{t+1}$*, where the last equality follows from the government budget constraint, i.e.,* $\varphi_t a_{t-1} = \psi_t a_t$, $\forall t$ *and an aggregate resource constraint,* $a_t = A_t\ \forall t$. *Lastly,* $\{c_y, c_o, s, n\} = \{x - Z - W, W + y, Z, Z\}$. *The equilibrium real money and bond balances* $\{Z, W\}$ *satisfy lemma 2 given that* $\varphi_t/\varphi_{t+1} = 1$, $\varepsilon_{t+1} = E[\varepsilon] = y$*, and lastly a (gross) nominal interest rate set by the government, i.e.,* $i = \varphi_t/\psi_t$, $\forall t$.

This definition results in a system of two log-linearized equations that $\{Z, W\}$ must satisfy as below:

$$-\ln i - \rho \ln (x - Z - W) + \gamma \ln (W + y) = (\gamma - \rho) \ln (R(W + y)), \tag{3}$$

$$\ln (Z) = (\gamma/\rho) \ln (x - Z - W) + \{(\rho - \gamma)/\rho\} \ln (R(W + y)), \tag{4}$$

where $\ln (R(W + y))$

$$= \frac{1}{1-\gamma} \left\{ \ln \left[\frac{(W + y + b)^{2-\gamma}}{2b(2-\gamma)} - \frac{(W + y - b)^{2-\gamma}}{2b(2-\gamma)} \right] \right\}.$$

Now, we conduct comparative static analyses based on the three cases regarding agents' preferences for the timing of the uncertainty resolution. Most importantly, we study the stationary welfare in two versions as in Jung (2018). The first one is in terms of pure utility, i.e., the CES aggregate of c_y, s, and $R(c_o)$ in the unique stationary monetary equilibrium. The second welfare measure shows aggregate consumption equivalents, i.e., the CES aggregate of c_y, s, and c_o in the unique stationary monetary equilibrium. We use $1 - \rho$ as a CES aggregate parameter for both cases. For notational convenience, we denote the former (latter) as W^1 (W^2). The following proposition discusses characteristics of equilibrium when agents are indifferent to the timing of uncertainty resolution.

PROPOSITION 1. *Consider the case where $\rho = \gamma$ and/or $b = 0$. Let Z_{EZ} and W_{EZ} denote real money and bond balances, respectively, in stationary equilibrium. $\exists!\, Z_{EZ}$ and $\exists!\, W_{EZ}$, only if $i \geq 2y^{\gamma}/x$. Then, the following holds true in the unique stationary monetary equilibrium:*

(i) $\partial Z_{EZ}/\partial i < 0$ and $\partial W_{EZ}/i > 0$.

(ii) $W^1 = W^2$ and the Friedman rule achieves the constrained efficiency both in terms of pure utility and aggregate consumption.

Proof. See the proof for proposition 3.

The first case admits intuitive welfare implications. First, i must be bounded from below to guarantee a unique equilibrium where bonds coexist with money. Intuition is straightforward. A too-low rate of return on bonds makes households lose incentives to carry bonds over periods. That lower bound is positively (negatively) related to the old-age (young-age) endowment, as the formula, i.e., $2y^\gamma/x$, in the proposition indicates.

Next, the *Friedman rule* achieves the constrained efficiency in both pure-utility and aggregate consumption terms. This can be easily verified from the two equations in lemma 2 and equations (3) and (4). Since uncertainty does not affect households' preferences, all that matters is the relative returns on bonds. A positive rate of return on bond holdings would induce households to bias their portfolio towards bonds, i.e., part (i) of the proposition. In turn, households would spend on *numeraire* goods more than the constrained efficient amount. The exact opposite analysis could apply in the case of negative returns on bonds. A zero nominal interest rate is the only way to achieve the social optimum in this case.

Next, we consider the second case, $\rho > \gamma$, which brings about a much richer set of comparative static analyses on stationary allocations. The next proposition summarizes the results.

PROPOSITION 2. *Consider the second case, where* $\rho > \gamma$. *Let* $\tilde{Z}$ *equal* Z *such that* $\ln Z = (\gamma/\rho)\ln(x - Z) + (\rho - \gamma)/\rho \ln([R(y)])$. $\exists!\, Z_{EZ}$ *and* $\exists!\, W_{EZ}$*, only if*

$$i \geq \underline{i} \equiv \frac{y^\gamma}{(x - \tilde{Z})[R(y)]^{\gamma-\rho}}.$$

The following holds true in the unique stationary monetary equilibrium:

- *(i)* $\partial \underline{i}/\partial b < 0$.
- *(ii)* $\partial Z_{EZ}/\partial b < 0$ *and* $\partial W_{EZ}/\partial b > 0$.
- *(iii)* $\partial Z_{EZ}/\partial i < 0$ *and* $\partial W_{EZ}/\partial i > 0$.
- *(iv) The Friedman rule achieves the same* W^1 *that the planner does.*
- *(v) The Friedman rule usually does not maximize* W^2. *Define the optimal (gross) nominal interest rate that maximizes the*

W^2 as i^*. If $\underline{i} \geq 1$, then $i^* = \underline{i}$. Otherwise, $\underline{i} \leq i^* < 1$ and $\partial i^*/\partial b < 0$.

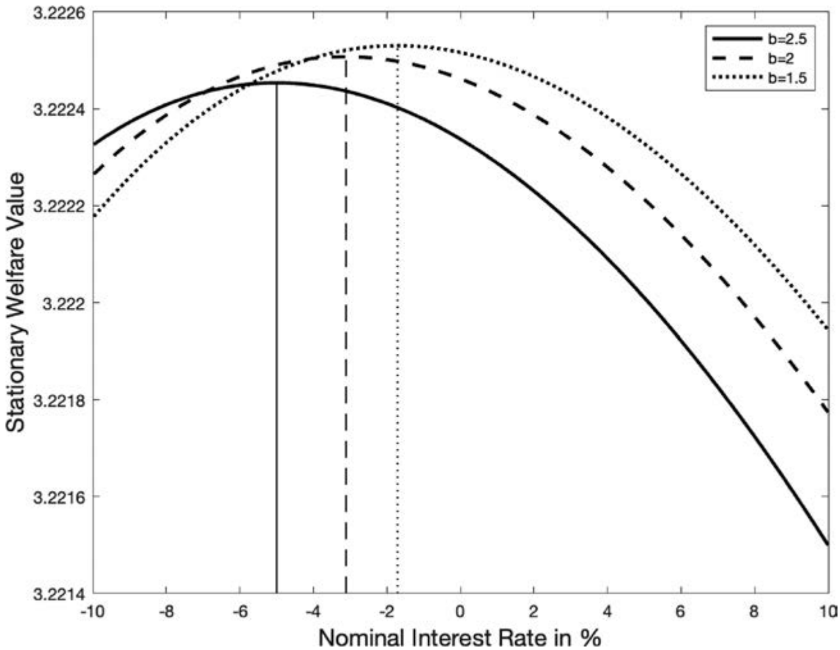
Proof. See the proof for proposition 3.

Unlike the case in proposition 1, the lower bound for nominal interest rates is now negatively affected by uncertainty, i.e., b. Intuition follows from households' preferences for the timing of the uncertainty resolution. Households in this case have a greater relative dislike for intertemporal inequality. Since a higher degree of uncertainty means more intertemporal inequality, households' incentives to save are strengthened. This eventually would reduce the lower bound.

Because households dislike intertemporal inequality in terms of general good consumption to a greater extent, they accumulate more bond holdings in response to a higher b, i.e., $\partial W_{EZ}/\partial b > 0$. This in turn means that young workers underconsume general goods in equilibrium. Given that households equalize the marginal utility from consuming general goods at a young age and special goods in old age, cash holdings for special goods must fall too when b goes up, i.e., $\partial Z_{EZ}/\partial b < 0$. As before, changes in nominal interest rates render qualitatively the same substitution effects. A higher rate of return on bond holdings would induce households to bias their portfolio towards bonds, i.e., $\partial Z_{EZ}/\partial i < 0$, and $\partial W_{EZ}/\partial i > 0$.

Welfare implications are also richer. In terms of pure utility, the Friedman rule is the unique optimal policy, i.e., part (iv) in proposition 2. Again, this is hardly surprising since the intergenerational transfer instrument price is never distorted only under the Friedman rule. What is interesting is that the optimal inflation rate for aggregate consumption is usually not the *Friedman rule* and negative. This follows from two important facts: (i) an equal division of c_y, c_o, and s achieves the maximum aggregate consumption, and (ii) uncertainty distorts that distribution. Intuitively, a higher degree of uncertainty would bias the aggregate consumption basket towards old-age consumption, i.e., $\partial Z_{EZ}/\partial b < 0$ and $\partial W_{EZ}/\partial b > 0$. To mitigate this effect, a subzero level of i is required, subject to the subzero lower bound (because a lower rate of return on bonds always leads to undersavings and, thus, lower old-age consumptions). Lastly, the higher aggregate output uncertainty gets, the bigger bond holdings

Figure 2. Numerical Examples for W^2 with $\rho > \gamma$



become. Thus, an even lower negative interest rate is required to bring down the overbond holdings, i.e., $\partial i^*/\partial b < 0$.

In what follows, a simple numerical example is illustrated. Figure 2 shows how W^2 (aggregate consumption level) responds to changes in nominal interest rates, i, under the case where $\rho > \gamma$. For various reasons of tractability, the model is not rigorously calibrated; see Jung (2018) for a detailed explanation on why two-period OLG models are problematic for calibration. In order to justify the parameter values as much as possible, nevertheless, the following measures are taken. First, the intertemporal risk-aversion parameter, ρ, is chosen to equal 2, i.e., intertemporal elasticity of substitution approximately equals 0.5. This is within reach of the usual values in the macro-finance literature that heavily rely on the EZ preferences for quantitative work. Note that the γ value used here (1.5) is somewhat lower than the usual in the literature. However, that γ value, along with $x = 10$ and $y = 4$, is chosen to make sure that the lower bound interest rate ($\underline{i}$) is sufficiently negative. Please check the formula in proposition 2. Finally, I also illustrate W^2 under three different values for b, i.e., 1.5, 2, and 2.5, so as to analyze the effects of uncertainty simultaneously. This example clearly shows that the lower bound is negative, i.e., –10 percent, and the optimal negative interest rate decreases in aggregate output uncertainty, consistent with predictions in proposition 2.

Using the intuition so far, it follows easily that the third case, $\rho < \gamma$, brings about opposite comparative static analyses on stationary allocations in general. First, proposition 3 summarizes such results.

PROPOSITION 3. *Consider the third case, where* $\rho < \gamma$. $\exists!\, Z_{EZ}$ *and* $\exists!\, W_{EZ}$, *only if* $i \geq \underline{i}$. *The following holds true in the unique stationary monetary equilibrium:*

(i) $\partial \underline{i}/\partial b > 0$.

(ii) $\partial Z_{EZ}/\partial b > 0$ *and* $\partial W_{EZ}/\partial b < 0$.

(iii) $\partial Z_{EZ}/\partial i < 0$ *and* $\partial W_{EZ}/\partial i > 0$.

(iv) The Friedman rule achieves the same W^1 *that the planner does.*

(v) The Friedman rule does not maximize W^2 *unless uncertainty disappears. Under* $b > 0$, $i^* > 1$ *and* $\partial i^*/\partial b > 0$.

Proof. See the appendix.

Uncertainty effects on the stationary allocation are exactly opposite to the second case. Now, households dislike cross-sectional variation in old-age consumption relatively more. Therefore, their incentives to hold nominal bonds get weaker. As a consequence, a higher degree of uncertainty effectively pushes up the lower bound for nominal interest rates, i.e., $\partial \underline{i}/\partial b > 0$. Again, since households are very averse to cross-section variation in the old-age general good consumption, they accumulate *fewer* bonds in response to a higher b, i.e., $\partial W_{EZ}/\partial b < 0$. This in turn means that young workers overconsume general goods in equilibrium. Given that households equalize the marginal utility from consuming general goods at a young age and special goods in old age, cash holdings for special goods must increase as well when b goes up, i.e., $\partial Z_{EZ}/\partial b > 0$. Interest rate effects on the stationary allocation are same as before, i.e., $\partial Z_{EZ}/\partial i < 0$, and $\partial W_{EZ}/\partial i > 0$, for obvious reasons.

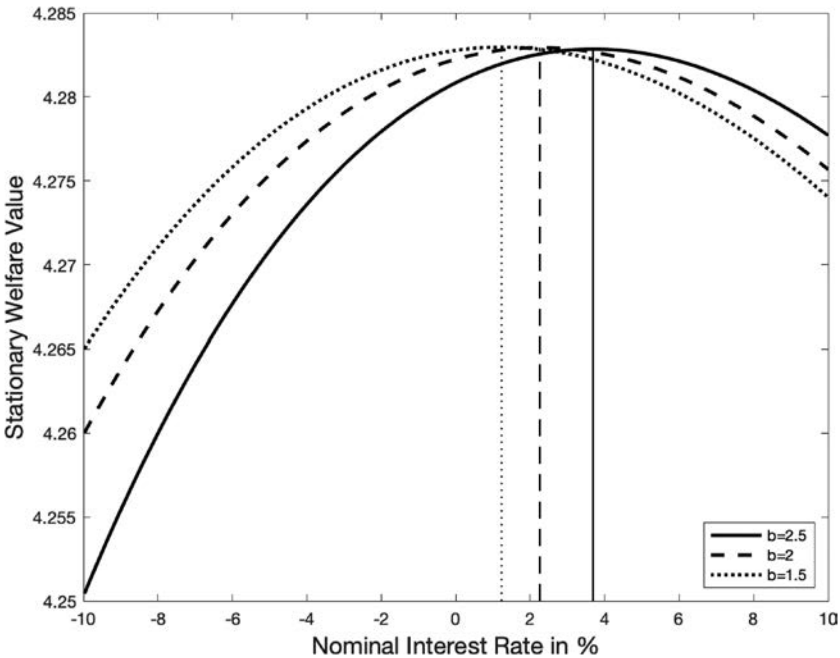
Welfare effects are also reversed except for the fact that the Friedman rule still achieves the constrained efficiency in terms of pure-utility-based welfare. In particular, the optimal inflation rate for aggregate consumption is positive this time. Unlike the second case, a higher degree of uncertainty would bias the aggregate consumption basket towards young-age general good consumption, i.e., $\partial Z_{EZ}/\partial b > 0$ and $\partial W_{EZ}/\partial b < 0$. To mitigate this effect, a positive level of i is required because a higher rate of return on bonds always brings about oversavings and, thus, greater old-age general good consumptions. Lastly, the higher aggregate output uncertainty gets, the smaller bond holdings become. Thus, an even more positive interest rate is required to boost the underbond holdings, i.e., $\partial i^*/\partial b > 0$.

Similar to the second case, a numerical example is shown in figure 3. All the parameter values except for $\gamma = 1.1$ and $\rho = 0.7$ are the same as in the second case. Consistent with predictions from proposition 3, the optimal interest rate happens to be positive this time, and the latter has a strictly positive relationship with the degree of uncertainty, as opposed to the second case.

6. Conclusion

To sum up, the current model delivers important policy implications. Negative interest rates can be beneficial only if an economic agent's aversion to intertemporal inequality in consumption is relatively greater. Under such a case, monetary policy authorities should target a negative interest rate that moves in the direction opposite to that of aggregate output uncertainty. It also leaves a completely reversed set of policy recommendations under the case where agents mind cross-sectional consumption inequality relatively more. In that case, a strictly positive interest rate should be targeted and positively tied to the degree of aggregate output uncertainty. Limitations of the current model for policy recommendations certainly exist, e.g.,

Figure 3. Numerical Examples for W^2 with $\rho < \gamma$



too-low transactional frequency inherent in two-period OLG models, etc. However, the current model can be used as a benchmark upon which more realistic and complex features of the economy can build.

For instance, much debate on negative interest rates recently revolves around their potential adverse effects on banking sectors; see Brunnermeier and Koby (2016). Many policymakers are concerned about financial instability that negative interest rates might cause through squeezing banks' net interest margin, which is completely absent in the current model. Embedding financial intermediaries into the current OLG structure following Schreft and Smith (2002) and/or Duffie, Gârleanu, and Pedersen (2005), therefore, might be a useful future research avenue. One could also extend the current model into a two-country environment to analyze the effects of negative interest rates on capital flows, currency markets, international trade, etc. I leave all these fruitful exercises to future research.

Appendix

Proof for Proposition 3

Proofs for proposition 1 and 2 are simply subcases of what follows. One just needs to impose $\rho = \gamma$ and $\rho < \gamma$, respectively. First, $Z \equiv Z(W)$ from equation (3). Then, it can be shown that $\partial Z/\partial W < 0$. Proof for the latter follows from the fact that

$$-\ln i - \rho \ln (x - Z - W) + \gamma \ln \left[\frac{W + y}{R(W) + y}\right] = -\rho \ln R(W + y),$$

which is from equation (3). Also note that $Z(x) = 0$ and $Z(0) = \tilde{Z}$, where

$$\tilde{Z} = \left\{Z : \ln Z = \frac{\gamma}{\rho} \ln (x - Z) + \frac{\rho - \gamma}{\rho} \ln[R(y)]\right\}.$$

Also note that $\tilde{Z} < x$ since $\ln x = \gamma/\rho(-\infty) + \text{constant}$.

Next, one can also derive $W \equiv W(Z)$ from equation (4). $\partial W/\partial Z < 0$. Proof is given through applying the implicit function theorem to equation (4).

$$\partial W/\partial Z = -\frac{\rho/(x - Z - W)}{\frac{\rho}{x-Z-W} + \gamma\left(\frac{1}{W+y} - \frac{\partial\gamma/\partial W}{R(W+y)}\right) + \rho\frac{\partial\gamma/\partial W}{R(W+y)}} < 0,$$

which again follows from $1/W + y > (\partial\gamma/\partial W)/R(W + y)$ due to the concavity of the R function. Also, $W(0) = \tilde{W}$, where

$$\begin{aligned}\tilde{W} = \big\{W : &-\ln i - \rho \ln (x - W) + \gamma \ln (W + y) \\ &= (\gamma - \rho) \ln[R(W + y)]\big\}.\end{aligned}$$

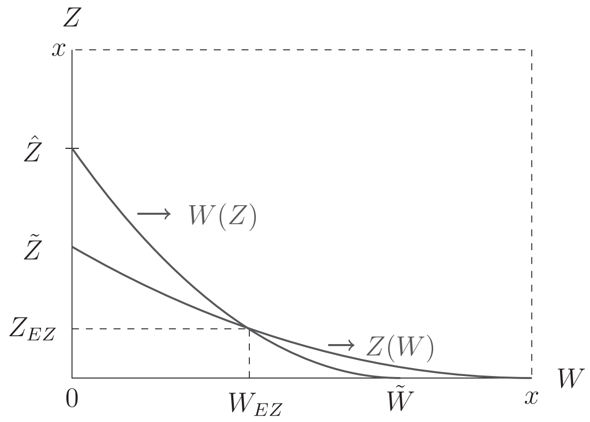
It's easy to show that $\tilde{W} < x$ since if $\tilde{W} = x$, then

$$-\ln i - \rho \ln (x - x) + \gamma \ln (W + y) > (\gamma - \rho) \ln[R(W + y)].$$

Thus, $\tilde{W}$ must be below x to lower the left-hand side of the above equation. Lastly, $\hat{Z}$ such that $W(\hat{Z}) = 0$ must satisfy the following:

$$\hat{Z} = \{-\ln i - \rho \ln (x - Z) + \gamma \ln y = (\gamma - \rho) \ln[R(y)]\}.$$

Figure 4. A Unique Stationary Equilibrium



So to ensure $\exists!(Z_{EZ}, W_{EZ})$, one must make sure $\hat{Z} \geq \tilde{Z}$, which is equivalent to

$$-\rho \ln (x - \tilde{Z}) + \gamma \ln y - (\gamma - \rho) \ln[R(y)] < \ln i.$$

This means $\exists!$ minimum $\underline{i}$ so that $\exists!(Z_{EZ}, W_{EZ})$, and

$$\underline{i} = \left\{ \underline{i} : \ln i = -\rho \ln (x - \tilde{Z}) + \gamma \ln y - (\gamma - \rho) \ln[R(y)] \right\}.$$

This proves for the $\underline{i}$ for all three cases, i.e., $\rho = \gamma$, $\rho < \gamma$, and $\rho < \gamma$.

One can finally visualize all of these in the diagram shown in figure 4. Given figure 4, one can easily check the effects of changes in i on Z_{EZ} and W_{EZ}. $i \uparrow \rightarrow W(Z)$ shifts out for all γ and ρ values. Hence, $\partial Z/\partial i < 0$ and $\partial W/\partial i > 0$ for all three cases. The effects of changes in b differ depending on the relative size of γ and ρ. When $\gamma > \rho$, $b \uparrow \rightarrow R(\cdot) \downarrow$. Thus, $Z(W)\,(W(Z))$ shifts up (down). This also proves why $i^* > 1$ and $\partial i^*/\partial b > 0$ under $\gamma > \rho$. On the contrary, when $\gamma < \rho$, $b \uparrow \rightarrow R(\cdot) \downarrow$. Thus, $Z(W)\,(W(Z))$ shifts down (up). This also proves why $i^* < 1$ and $\partial i^*/\partial b < 0$ under $\gamma > \rho$. Q.E.D.

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Exchange Rate Pass-Through: What Has Changed Since the Crisis?*

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We study how exchange rate pass-through to CPI inflation has changed since the global financial crisis. We have three main findings. First, exchange rate pass-through in emerging economies decreased after the financial crisis, while exchange rate pass-through in advanced economies has remained relatively low and stable over time. Second, we show that the declining pass-through in emerging markets is related to declining inflation. Third, we show that it is important to control for non-linearities when estimating exchange rate pass-through. These results hold for both short-run and long-run pass-through and remain robust to extensive changes in the specifications.

JEL Codes: E31, E58, F31.

1. Introduction

Exchange rate pass-through is again at the center of economic policy and central bank thinking (Forbes 2014, 2015). We have to understand how the observed large exchange rate movements translate to

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consumer price inflation, especially as inflation remains well below central bank targets in many advanced economies. From the perspective of some emerging market economies (EMEs), another question arises on how large exchange rate movements affect inflation, especially when it is already above target. In addition, as Plantin and Shin (2016) find, exchange rate pass-through can affect the financial risk-taking channel of monetary policy.

In this paper we aim to provide an overall picture of how exchange rate pass-through has evolved for both advanced and emerging market economies. We find that exchange rate pass-through in emerging economies on average decreased after the financial crisis, and that this decline in pass-through is linked to declining inflation. By contrast, in advanced economies, where inflation has tended to be consistently low, exchange rate pass-through has also remained low. Yet, in spite of the recent decline in emerging economies, pass-through estimates are still lower in advanced economies than in emerging economies. The results are consistent with the implications of the menu cost theory of price setting: when inflation is higher, exchange rate changes are passed through more quickly and to a larger extent because firms have to adjust prices frequently anyway (see further Taylor 2000 for a sticky price setup).

We also confirm that the results hold robustly. The pattern of declining pass-through in EMEs and low pass-through in advanced economies holds similarly for contemporaneous (quarterly), yearly, and long-run pass-through estimates. This pattern also does not depend on the length of rolling window estimates: three-, four-, five-, six-, and eight-year rolling windows all show the same pattern. The results are also not dependent on the econometric methodology: while our main methodology uses an Arellano and Bover (1995) and Blundell and Bond (1998) type of system GMM panel estimates, the pattern remains under difference GMM and within-group estimators. While we control for time fixed effects to ensure that common global shocks do not affect the estimates, the results also hold when dropping these fixed effects and explicitly controlling for the global business cycle or oil prices.

We also find that controlling for non-linear effects of exchange rate movements can be crucial when estimating exchange rate pass-through: as one would expect based on the menu cost theory, larger exchange rate movements have a stronger chance to overcome the menu cost of price changes and thereby are more likely to be passed through to consumer prices. Hence, naive linear estimates of pass-through would show an increase in emerging markets after the taper tantrum when exchange rate volatility increased sharply. However, we show that this increase disappears when one properly controls for non-linearities.

The contribution of our paper to the literature is threefold. First, we document the overall pattern of more than twenty years of exchange rate pass-through developments for a large group of economies. We report that the pass-through has been low and stable in advanced economies, and higher but declining in emerging economies. The advanced economy results extend the link found earlier, for instance, by Engel (2002) and Devereux and Yetman (2008), between low pass-through and low inflation in advanced economies in the post-crisis data set. As for the EMEs, our results on declining pass-through extend the earlier finding in Mihaljek and Klau (2008), Aleem and Lahiani (2014), and Lopez-Villavicencio and Mignon (2016) to a more recent period and/or to a larger set of economies. Our finding of a recent decline in linear pass-through slightly contrasts with De Gregorio (2016), who finds that pass-through for large depreciations in the 2008–15 period was lower than in the 1970s but comparable to the 1990s.[1] These results might be reconciled by the fact that we consider linear pass-through when controlling for non-linearities, while De Gregorio (2016) considers the full effects of large depreciations.

Second, we provide solid empirical evidence for a causal link between lower inflation and lower pass-through in emerging market data, as was proposed in Calvo and Reinhart (2002) and Choudhri and Hakura (2006). Our results can also be seen as extending the analysis of the low inflation/low pass-through link from advanced economies in the 1990s of Takhtamanova (2010) to emerging markets in the 2000s.

[1] Importantly, we do not exclude the possibility that the link between lower pass-through and lower inflation works through more credible monetary policy, as Bailliu and Fujii (2004) and Gagnon and Ihrig (2004) argued for advanced economies.

Third, we provide evidence that larger exchange rate movements lead to disproportionally larger price changes. Therefore, it is useful to control for non-linearities when estimating pass-through, especially when exchange rate volatility is changing in the sample period. One crucial example is the period after the taper tantrum when exchange rate volatility increased—and the inclusion of such periods in a naïve linear setup can misleadingly suggest an increase in pass-through. This result confirms the findings in Bussière (2013), Cheikh and Rault (2015), and Alvarez, Lippi, and Passadore (2017) of the relevance of non-linearity and provides additional support to control for non-linearities to exclude the possibility that linear pass-through estimates pick up changes in exchange rate volatility. This result is also consistent with evidence in Campa and Goldberg (2005) and Kohlscheen (2010) that pass-through to consumer prices and import prices, respectively, is higher for countries with greater nominal exchange rate volatilities.

Furthermore, the results also have policy relevance when thinking about changing global conditions for monetary and economic policy setting. The average low pass-through levels today imply that central banks in general should have less "fear of floating," at least from an inflation perspective. Yet, the lower pass-through in emerging markets also implies that the exchange rate channel of monetary policy might be less effective to affect inflation than before the financial crisis. Finally, the results further reinforce the importance of price stability by showing that lower inflation also reduces pass-through. In fact, there might be a positive feedback loop: lower pass-through could in turn further contribute to price stability.

However, the results should be read with appropriate caveats. Importantly, our results apply only for groups of countries, and not for individual economies. Hence, our results do not offer direct implications for individual countries. Furthermore, our setup is necessarily limited to macroeconomic factors and only captures time-invariant microeconomic factors, such as pricing power, through country fixed effects. Finally, our approach does not distinguish between exogenous and endogenous exchange rate shocks—and this distinction might matter, as Shambaugh (2008) and Forbes, Hjortsoe, and Nenova (2015) show. However, we mitigate this problem by consistently controlling for global shocks through time fixed effects.

The remainder of the paper is organized as follows. The second section outlines the methodology, and the third section introduces the data. The fourth section discusses the results, and the fifth section presents robustness checks. Finally, the sixth section concludes.

2. Estimation Methodology

We estimate exchange rate pass-through based on the following hybrid New Keynesian Phillips-curve framework, using dynamic panel regression with system GMM,

$$\pi_{it} = \alpha_i + \beta_t + \rho E_t \pi_{it+1} + \delta \pi_{it-1} - \sum_{j=0}^{3} \gamma_j \Delta NEER_{it-j} - \sum_{k=0}^{3} \mu_k \Delta NEER^2_{it-k} - \sum_{l=0}^{3} \nu_l \Delta NEER^3_{it-l} + \phi y_{it} + \varepsilon_{it}. \quad (1)$$

Here, π_{it} denotes log differences in quarterly seasonally adjusted consumer price indexes (CPI) in country i in quarter t; y_{it} is the domestic output gap in country i in quarter t; $\Delta NEER_{it}$ is the (change in the log of) the nominal effective exchange rate; α_i are country fixed effects; and β_t are time (quarter) fixed effects.

We choose a hybrid New Keynesian Phillips-curve framework, which also includes a forward-looking inflation expectations term, $E_t \pi_{it+1}$, in order to be consistent with the New Keynesian Phillips-curve framework which has commonly been used in macroeconomic and monetary policy analysis for capturing inflation dynamics (Clarida, Galí, and Gertler 1999; Smets 2003; Woodford 2003; Levin and Moessner 2005).

In our benchmark specification, we exclude inflation expectations by setting $\rho = 0$ in equation (1), in order for our results to be more easily comparable with both (i) the earlier empirical cross-country exchange rate pass-through literature and (ii) the pass-through literature focusing on import prices, as neither literature has generally included inflation or import price expectations. We also estimate pass-through using the hybrid New Keynesian Phillips-curve framework of equation (1) including the forward-looking inflation expectations term, $E_t\pi_{it+1}$, in order to be consistent with the hybrid New Keynesian Phillips-curve framework commonly used in monetary policy analysis for analyzing inflation dynamics. Later we check and find that our main results are robust to the inclusion of the forward-looking inflation expectations according to the hybrid New Keynesian framework (see tables 1 and 2).

We are not assuming that the inflation expectations formation process within the hybrid New Keynesian Phillips-curve framework is the same for advanced and emerging economies. Since we estimate the specifications separately for advanced and for emerging economies, we allow for the coefficients on lagged inflation and on inflation expectations to differ between advanced and emerging economies.

Most of the cross-country empirical exchange rate pass-through literature has not used a New Keynesian Phillips-curve framework including inflation expectations, but has been based on a backward-looking generic specification of Goldberg and Knetter (1997) (e.g., Mihaljek and Klau 2008; Lopez-Villavicencio and Mignon 2016). A recent paper has used a New Keynesian Phillips-curve framework for cross-country panel estimation of exchange rate pass-through in advanced economies (Takhtamanova 2010). Takhtamanova (2010) states about her paper that "much of the existing research focuses on the relationship between movements in nominal exchange rates and import prices. A smaller but equally important strand of the literature concentrates on the macroeconomic exchange rate pass-through to aggregate price indices (Bachetta and van Wincoop 2003, Campa and Goldberg 2006, Gagnon and Ihrig 2004). This paper also focuses on the relationship between aggregate prices and inflation, but departs from existing studies by using a Phillips curve framework to analyze exchange rate pass-through." To the best of our knowledge, our paper is the first to provide empirical evidence for a causal link between lower CPI inflation and lower exchange rate pass-through for a large separate emerging country sample using a New Keynesian Phillips-curve framework, when including the period after the global financial crisis.

To capture any non-linearities in the exchange rate pass-through, we extend the specification to include quadratic and cubic changes in exchange rates. The exchange rate terms are presented with a negative sign, given that in the original series local exchange rate depreciation is reflected as a decrease in the nominal effective exchange rate (NEER). The model works with contemporaneous exchange rate change and three additional lags to capture exchange rate pass-through over the period of one year. Furthermore, the specification also satisfies the optimal lag structure based on Akaike and Bayesian information criteria. We present estimates for advanced and emerging economies separately. We also include country fixed effects to control for unobserved country heterogeneity. Moreover, we include time fixed effects to control for global factors driving inflation.

Our estimation assumes a non-linear structure, since the underlying pass-through process may be non-linear (Bussière 2013; Cheikh and Rault 2015). Such non-linearity might arise due to menu costs, i.e., due to the presence of non-negligible costs of adjusting prices. Firms might prefer to avoid these menu costs when exchange rate moves are small, but they could be forced to adjust prices for larger exchange rate movements (Forbes 2014). Alternatively, firms might absorb small changes in input prices but not large ones. Non-linearities might also be explained by imperfect competition which would lead to observationally similar results.

Our estimation period is 1994:Q1–2017:Q4. To estimate equation (1), we use generalized method of moments (GMM) following Arellano and Bover (1995) and Blundell and Bond (1998). This method has been widely used to deal with panel data with endogenous explanatory variables, and in our case it is able to control for common shocks that affect both inflation and exchange rates (Shambaugh 2008; Aron and Muellbauer 2014; Forbes 2015). The benchmark model uses system GMM technique with two to eight lags of log NEER changes, the output gap, and lags of log CPI changes as GMM instruments for levels and first-differences equations. We also repeat the estimates with difference GMM and within-group estimators for robustness.

Based on equation (1), we estimate linear contemporaneous, yearly, and long-run exchange rate pass-through. Contemporaneous linear exchange rate pass-through is defined as the coefficient on the contemporaneous log change in the NEER in equation (1), i.e., γ_0. Yearly linear pass-through is the sum of the coefficients on log changes in the NEER over four quarters, i.e., $\gamma_0+\gamma_1+\gamma_2+\gamma_3$. Linear long-run pass-through is defined as yearly pass-through divided by one minus the coefficient on lagged inflation, i.e., $(\gamma_0 + \gamma_1 + \gamma_2 + \gamma_3)/(1 - \delta)$.

3. Data

We analyze quarterly time-series data for twenty-two emerging[2] and eleven advanced[3] economies over the period 1994:Q1–2017:Q4.

We focus on exchange rate pass-through (ERPT) to consumer price inflation. To do so, we use log differences in quarterly seasonally adjusted consumer price indexes (CPI) as our dependent variable.

We use several explanatory variables. The exchange rate series are chosen as the Bank for International Settlements (BIS) NEER broad indexes available from 1994, with 2010 as the indexes' base year. In the regression analysis we use log differences in the average quarterly NEER indexes. In our definition, an increase in the NEER implies an appreciation of the local exchange rate. Later, we also use log differences in average quarterly bilateral U.S. dollar exchange rates.

We also control for the business cycle by including measures of the output gap. The underlying real GDP series are taken from national sources. The output gap is calculated by employing the standard univariate Hodrick-Prescott filtering method with the smoothing parameter λ set to 1,600 for all available quarterly GDP data. For the analysis, we use the data starting in 1994:Q1 or later depending on their availability.[4]

In addition, we use control variables for some global factors, namely oil prices and the global output gap. For oil prices we use average quarterly West Texas Intermediate (WTI) crude oil spot prices in U.S. dollars transformed into quarterly log changes. The global output gap is calculated according to the same methodology

[2] Argentina, Brazil, Chile, China, Colombia, the Czech Republic, Hong Kong SAR, Hungary, India, Indonesia, Israel, Korea, Mexico, Malaysia, Peru, the Philippines, Poland, Russia, Singapore, South Africa, Thailand, and Turkey.

[3] Australia, Canada, Denmark, the euro area, Japan, New Zealand, Norway, Sweden, Switzerland, the United Kingdom, and the United States.

[4] Data are available since 1995:Q1 for Hungary, Israel, and Poland; since 1996:Q1 for Chile and the Czech Republic; since 1996:Q2 for India; and since 1998:Q1 for the Philippines.

as the domestic output gap, and is computed from national data as a weighted average.

In some specifications, we also include inflation expectations to evaluate the pass-through according to a New Keynesian Phillips-curve setup. The end-year inflation expectations are taken from Consensus Economics. We estimate the expectation series with a quarterly frequency by subtracting realized quarterly inflation from the forecasts (Q2 and Q3), using end-year figures (Q4), or linearly interpolating end-year's estimates (Q1).

Appendix 1 provides a detailed description of the data, including additional information on data availability.

4. Results

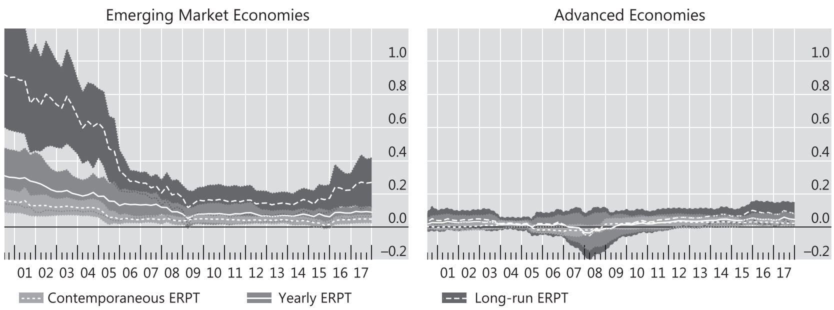
4.1 Evolution of Pass-Through

As a first step, we run the benchmark regression, equation (1) excluding inflation expectations (setting $\rho = 0$), on six-year windows to assess the evolution of pass-through over time (figure 1). We run the regression separately for emerging markets (left-hand panel) and advanced economies (right-hand panel).

For emerging markets (left-hand panel), all three pass-through measures (contemporaneous, yearly, and long run) decline strongly from the pre-crisis levels after the financial crisis. A similar declining pattern also holds when choosing different rolling windows (see figure 4 in appendix 2). Interestingly, there is some small-scale pickup in pass-through in the last two years. Looking into this result, it turns out that the pickup is almost entirely driven by a single country, Argentina, and it is almost eliminated when Argentina is excluded from the estimation (see figure 7 in appendix 2). This might be supporting our main hypothesis, i.e., the causal link between inflation and pass-through, as Argentina experienced a high rate of inflation in this period.

For advanced economies (right-hand panel, figure 1), all three pass-through measures (contemporaneous, yearly, and long run) remain relatively stable, at low levels, throughout our estimation period. This results again holds for different rolling windows (see figure 5 in appendix 2).

Figure 1. Exchange Rate Pass-Through (baseline specification, six-year rolling windows)



4.2 Post-Crisis Decline in Pass-Through in EMEs

In order to evaluate whether the decline in pass-through in emerging economies after the financial crisis was indeed significant, we add to the benchmark equation a dummy variable for the post-crisis period. The dummy variable D_t takes the value of one in the post-crisis period (2009:Q3–2017:Q4) and zero in the pre-crisis period (1994:Q1–2008:Q2)—while we omit the volatile crisis years.[5] In sum, we estimate the following equation:

$$\begin{aligned}\pi_{it} = {} & \alpha_i + \beta_t + \rho E_t\pi_{it+1} + \delta\pi_{it-1} - \sum_{j=0}^{3}\gamma_j \Delta NEER_{it-j} \\ & - \sum_{k=0}^{3}\mu_k \Delta NEER^2_{it-k} - \sum_{l=0}^{3}\nu_l \Delta NEER^3_{it-1} + \varphi y_{it} \\ & + \rho_D D_t E_t\pi_{it+1} + \delta_D D_t\pi_{it-1} - \sum_{j=0}^{3}\gamma_{jD} D_t \Delta NEER_{it-j} \\ & - \sum_{k=0}^{3}\mu_{kD} D_t \Delta NEER^2_{it-k} - \sum_{l=0}^{3}\nu_{lD} D_t \Delta NEER^3_{it-1} \\ & + \varphi_D D_t y_{it} + \varepsilon_{it}. \qquad (2)\end{aligned}$$

[5] In the presented specifications we omit the crisis period (2008:Q3–2009:Q2). However, the results are robust to including the crisis years, i.e., when D_t is set to one for the period 2008:Q3–2017:Q4 and zero otherwise. These results are available upon request from the authors.

In our benchmark specification, exactly as in our benchmark specification of equation (1), we exclude inflation expectations by setting $\rho = \rho_D = 0$ in equation (2) (see columns 1–3 and 5–7 of table 1). For robustness, we also estimate equation (2) including the inflation expectation terms (see columns 4 and 8 of table 1). Table 1 shows that the decrease in linear coefficients of the pass-through in emerging markets after the crisis is statistically significant at the 5 percent level for all three pass-through measures (contemporaneous, yearly, and long run); see the coefficient estimates of the post-crisis interaction dummy in column 1 of table 1. By contrast, this pass-through appears to increase slightly, and mostly only at the 10 percent significance level, in advanced economies in the post-crisis period.

For all three pass-through horizons, these results are consistent with the results reported in figure 1. Table 1 shows that, pre-crisis, an exchange rate appreciation of 10 percent in EMEs was associated with an average decrease in consumer prices of around 2.3 percent within the same year; post-crisis, a 10 percent appreciation was associated with a lower decrease in consumer prices of 0.9 percent. The estimates of table 1 also demonstrate that the conclusion is robust to different control variables for global factors, namely to using changes in oil prices or the global output gap instead of time fixed effects; see columns 2 and 3. Table 1 also shows that the results are robust to including inflation expectations to evaluate the pass-through according to the New Keynesian Phillips-curve setup of equation (1) (see column 4 of table 1).

For advanced economies, some results seem to suggest an increase in pass-through especially when measured over the one-year or long-run horizons (see columns 5–8). While all pre-crisis pass-through estimates do not appear to be significantly different from zero, we report some positive and statistically significant post-crisis pass-through estimates. Yet, one should be careful when interpreting this: the increase in advanced economies is not robust (as, for instance, the EME post-crisis decline is). Furthermore, the magnitude of the increase is also smaller.

4.3 Pass-Through and Inflation

The large and significant decline in emerging market pass-through requires explanation: what has changed that could account for it?

Table 1. How Did the ERPT Change in the Post-Crisis Period? (pass-through coefficients, based on equation (2))

Explanatory Variables	Emerging Market Economies				Advanced Economies			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Exchange Rate Pass-Through (Pre-crisis)								
Contemporaneous ERPT	0.120***	0.116***	0.117***	0.104***	0.00470	−0.000989	−0.00380	0.00594
	(0.0341)	(0.0315)	(0.0312)	(0.0331)	(0.00595)	(0.00534)	(0.00763)	(0.00570)
Yearly ERPT	0.231***	0.222***	0.223***	0.230***	0.00592	−0.00646	−0.0127	0.00413
	(0.0785)	(0.0741)	(0.0740)	(0.0737)	(0.00736)	(0.00893)	(0.0137)	(0.0123)
Long-Run ERPT	0.712***	0.717***	0.719***	0.585***	0.00878	−0.00921	−0.0184	0.0627
	(0.150)	(0.140)	(0.141)	(0.136)	(0.0115)	(0.0121)	(0.0185)	(0.0192)
Post-Crisis Interaction Dummy								
D_t*Contemporaneous ERPT	−0.0764**	−0.0753**	−0.0810***	−0.0675*	0.0182*	0.0214**	0.00691	0.0185*
	(0.0328)	(0.0294)	(0.0284)	(0.0328)	(0.00893)	(0.00845)	(0.00837)	(0.00973)
D_t*Yearly ERPT	−0.146**	−0.150**	−0.155**	−0.148**	0.0417*	0.0573**	0.0408	0.0453*
	(0.0670)	(0.0634)	(0.0627)	(0.0640)	(0.0194)	(0.0209)	(0.0228)	(0.0233)
D_t*Long-Run ERPT	−0.143**	−0.142**	−0.146**	−0.128**	0.0483*	0.0630**	0.0432	0.0498
	(0.0685)	(0.0601)	(0.0591)	(0.0580)	(0.0253)	(0.0246)	(0.0250)	(0.0284)
Exchange Rate Pass-Through (Post-crisis)								
Contemporaneous ERPT + D_t*Contemporaneous ERPT	0.0434***	0.0404***	0.0359**	0.0362***	0.0229**	0.0204**	0.00311	0.0245**
	(0.0126)	(0.0110)	(0.0128)	(0.0111)	(0.00839)	(0.00667)	(0.00970)	(0.00796)
Yearly ERPT + D_t*Yearly ERPT	0.0856***	0.0713***	0.0676***	0.0825***	0.0476***	0.0509***	0.0281	0.0494***
	(0.0171)	(0.0183)	(0.0201)	(0.0170)	(0.0131)	(0.0132)	(0.0170)	(0.0137)
Long-Run ERPT + D_t*Long-Run ERPT	0.247***	0.193**	0.180**	0.152***	0.0886***	0.0833***	0.0446	0.0869***
	(0.0797)	(0.0689)	(0.0719)	(0.0274)	(0.0251)	(0.0181)	(0.0248)	(0.0231)

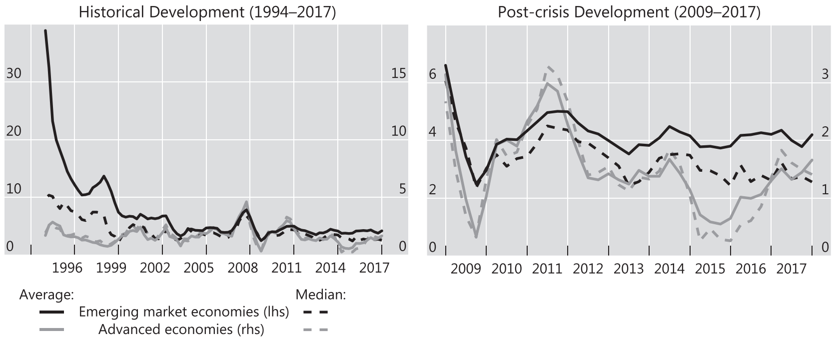
(*continued*)

Table 1. (Continued)

Explanatory Variables	Emerging Market Economies (1)	(2)	(3)	(4)	Advanced Economies (5)	(6)	(7)	(8)
Lagged Dependent Variable	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Inflation Expectations	No	No	No	Yes	No	No	No	Yes
Control Variables for Local Factors[a]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Control Variables for Global Factors	Time Fixed Effects	ΔOil Prices_t	Global Output Gap_t	Time Fixed Effects	Time Fixed Effects	ΔOil Prices_t	Global Output Gap_t	Time Fixed Effects
Country Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,889	1,889	1,889	1,889	957	957	957	907
Number of Countries	22	22	22	22	11	11	11	11
Sargan Test[b]	0.972	1	1	0.965	0.0752	0.695	0.604	0.0439
Hansen Test[b]	1	1	1	1	1	1	1	1
Serial Correlation Test[c]	0.435	0.477	0.488	0.353	0.0319	0.0811	0.196	0.0273

Notes: System GMM estimation using Arellano and Bover (1995) and Blundell and Bond (1998) dynamic panel estimator. Robust standard errors are in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. Full results are reported in online appendix table A1.
[a]Control variables for local factors include domestic output gap in all specifications.
[b]Reports p-values for the null hypothesis that the instruments used are not correlated with the residuals.
[c]Reports p-values for the null hypothesis that the errors in the first-difference regression exhibit no second-order serial correlation.

Figure 2. Inflation Dynamics (consumer prices, year-on-year changes, in percent)



Sources: National data, authors' calculations.

Our hypothesis is that the level of inflation affects the level of pass-through. Such a hypothesis is consistent with the menu cost theory of price setting: when inflation is higher, exchange rate changes are passed through more quickly and to a larger extent because firms have to adjust prices frequently anyway.

Indeed, inflation has declined substantially in emerging markets in the years preceding the financial crisis, i.e., around the time when estimated pass-through fell too (figure 2). While EME inflation was generally high in the 1990s, it fell rapidly afterwards (solid black line, left-hand panel). However, in spite of the fall, EME inflation levels tended to remain higher than advanced economy levels even after the financial crisis (right-hand panel).

Having seen that average inflation fell around the time when pass-through fell, we move to formally test whether lower inflation can indeed explain the decline in pass-through in EMEs. To do so, we formally estimate equation (3),[6]

[6] The four-lag structure ensures that we do not interact contemporaneous inflation and exchange rate terms. However, this is not critical for our results; the results remain robust under fewer lags.

$$\pi_{it} = \alpha_i + \beta_t + \rho E_t \pi_{\mathrm{i}t+1} + \delta \pi_{it-1} - \sum_{j=0}^{3} \gamma_j \Delta NEER_{it-j} - \sum_{k=0}^{3} \mu_k \Delta NEER^2_{it-k} - \sum_{l=0}^{3} \nu_l \Delta NEER^3_{it-1} + \varphi y_{it} - \sum_{j=0}^{3} \gamma_{j\pi} \pi_{it-4} \Delta NEER_{it-j} - \sum_{k=0}^{3} \mu_{k\pi} \pi_{it-4} \Delta NEER^2_{it-k} - \sum_{l=0}^{3} \nu_{l\pi} \pi_{it-4} \Delta NEER^3_{it-1} + \varepsilon_{it}. \quad (3)$$

In our benchmark specification, exactly as in our benchmark specification of equation (1), we exclude inflation expectations by setting $\rho = 0$ in equation (3) (see columns 1–3 and 5–7 of table 2). For robustness, we also estimate equation (3) including the inflation expectation terms (see columns 4 and 8 of table 2).

The results, shown in detail in table 2, suggest that lower inflation can indeed explain lower pass-through at least at the yearly or long-run horizons. This can be seen, as the coefficient on the interaction term of linear exchange rate changes with lagged inflation is positive and significant for EMEs at these horizons.

The results provide evidence that lower inflation can induce firms to decide to adjust prices more slowly in response to exchange rate changes, consistent with the existence of menu costs. These results are robust to using changes in oil prices or the global output gap as controls for global factors instead of using time fixed effects (see columns 2 and 3). The results are also robust to including inflation expectations within the New Keynesian framework, with the interaction term for both yearly and long-run pass-through again remaining significant (see column 4 of table 2).[7]

[7]The fact that our EME pass-through estimates show a slight increase in the last two years when we include Argentina in the sample, but not when we exclude it from the sample, lends further support to this link, as Argentina experienced a relatively high rate of inflation in this period.

Table 2. Lower Inflation/Lower Pass-Through in Emerging Markets (pass-through coefficients, based on equation (3))

	Dependent Variable: Inflation$_t$							
	Emerging Market Economies				Advanced Economies			
Explanatory Variables	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
	Exchange Rate Pass-Through							
Contemporaneous ERPT	0.0655***	0.0581***	0.0524***	0.0607***	0.00476	0.00569	0.000809	0.00580
	(0.0179)	(0.0179)	(0.0185)	(0.0144)	(0.00510)	(0.00405)	(0.00677)	(0.00527)
Yearly ERPT	0.126***	0.109***	0.107***	0.125***	0.0187*	0.0166**	0.00277	0.0220*
	(0.0318)	(0.0265)	(0.0287)	(0.0289)	(0.00911)	(0.00710)	(0.0111)	(0.0101)
Long-Run ERPT	0.271***	0.246***	0.237***	0.291***	0.0292*	0.0249*	0.00402	0.0339*
	(0.0705)	(0.0707)	(0.0734)	(0.0446)	(0.0150)	(0.0120)	(0.0163)	(0.0170)
	Inflation Interaction							
Inflation$_{t-4}$*Contemporaneous ERPT	0.921	0.936	0.973	0.775	1.582	0.979	−0.278	1.668
	(0.614)	(0.602)	(0.611)	(0.576)	(1.285)	(0.868)	(1.287)	(1.261)
Inflation$_{t-4}$*Yearly ERPT	1.741*	1.775*	1.830*	1.712*	1.026	0.643	−0.200	0.972
	(0.921)	(0.938)	(0.943)	(0.845)	(1.333)	(1.034)	(1.070)	(1.401)
Inflation$_{t-4}$*Long-Run ERPT	3.737*	4.014*	4.059*	3.163*	1.601	0.966	−0.291	1.500
	(1.254)	(1.312)	(1.288)	(1.178)	(1.935)	(1.464)	(1.577)	(2.047)

(continued)

Table 2. (Continued)

Explanatory Variables	Dependent Variable: $Inflation_t$							
	Emerging Market Economies				**Advanced Economies**			
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Lagged Dependent Variable	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Inflation Expectations	No	No	No	Yes	No	No	No	Yes
Control Variables for Local Factors[a]	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Control Variables for Global Factors	Time Fixed Effects	ΔOil $Prices_t$	Global Output Gap_t	Time Fixed Effects	Time Fixed Effects	ΔOil $Prices_t$	Global Output Gap_t	Time Fixed Effects
Country Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,977	1,977	1,977	1,977	1,001	1,001	1,001	951
Number of Countries	22	22	22	22	11	11	11	11
Sargan Test[b]	0.860	0.997	0.990	0.941	0.211	0.922	0.608	0.204
Hansen Test[b]	1	1	1	1	1	1	1	1
Serial Correlation Test[c]	0.514	0.520	0.587	0.401	0.0847	0.0843	0.683	0.0872

Notes: System GMM estimation using Arellano and Bover (1995) and Blundell and Bond (1998) dynamic panel estimator. Robust standard errors are in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. Full results are reported in online appendix table A2.
[a]Control variables for local factors include domestic output gap in all specifications.
[b]Reports p-values for the null hypothesis that the instruments used are not correlated with the residuals.
[c]Reports p-values for the null hypothesis that the errors in the first-difference regression exhibit no second-order serial correlation.

The estimated impact of lower inflation on lowering pass-through is also economically significant. The results imply that 1 percentage point lower inflation lowers the long-term average pass-through exchange rate move by around 0.3–0.4 percentage point. This is a sizable impact, as the average pass-through of such a 10 percent exchange rate move is around 2.7 percentage points.

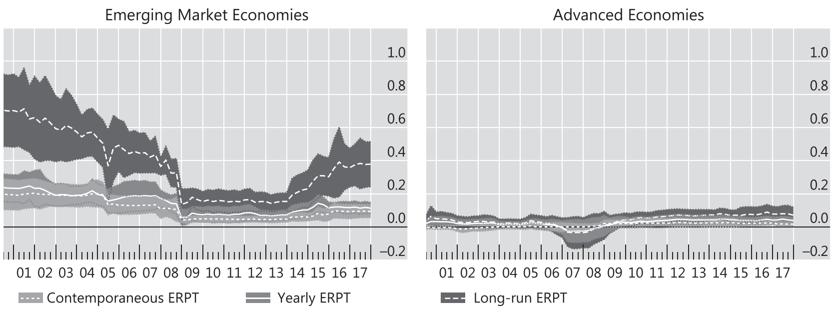
Our paper is not the first to provide empirical evidence for a causal link between lower inflation and lower exchange rate pass-through in emerging economies. Earlier evidence on this has been found for consumer prices by Choudhri and Hakura (2006) for a combined sample of emerging and developing countries, and by Devereux and Yetman (2008) for a combined sample of advanced, emerging, and developing countries. But to our knowledge our paper is the first to provide cross-country empirical evidence for a causal link between lower CPI inflation and lower exchange rate pass-through for a large separate emerging country sample when including the period after the global financial crisis using a New Keynesian Phillips-curve framework. Such evidence has also been found in Lopez-Villavicencio and Mignon (2016) for fifteen emerging economies, but not using a New Keynesian framework as in our paper.

4.4 Omitting Non-linearity

For comparison, we also present pass-through estimates when omitting the non-linear terms in the benchmark specification of equation (1) excluding inflation expectations (setting $\rho = 0$). The aim of the exercise is to demonstrate that omitting these non-linear terms can cause pass-through estimates to pick up the impact of exchange rate volatility. This exercise is very relevant, as non-linear terms are often neglected in the literature.

The basic pattern in linear pass-through over time is broadly similar for both emerging markets and advanced economies (figure 3). Pass-through is declining in emerging markets (left-hand panel), while it remains low and stable in advanced economies (right-hand panel). However, the linear pass-through estimates show a steady increase after mid-2013, i.e., following tapering of asset purchases by the Federal Reserve and the increase of exchange rate volatility in emerging markets. These larger exchange rate movements are expected to pass through more strongly to consumer prices

Figure 3. Pass-Through Omitting Non-linear Terms (baseline specification without non-linear terms, six-year rolling windows)



than smaller movements, because they are more likely to overcome the menu costs associated with price changes. Consequently, simple linear pass-through estimates, which ignore these non-linearities, would suggest some increase in pass-through in EMEs after tapering by the Federal Reserve, while such an increase is not visible to this extent in the specification that takes non-linearities into account (figure 1).

This underlines the importance of controlling for non-linearities. Furthermore, when estimating the benchmark specification of equation (1) excluding inflation expectations (setting $\rho = 0$), the coefficients on some of the non-linear terms are significant for EMEs (see online appendix table A1, available at www.ijcb.org).

Our paper is not the first to supply cross-country evidence that larger exchange rate movements lead to disproportionally larger price changes. Cross-country evidence on this has previously been found for import prices in G-7 economies by Bussière (2013) and for CPI inflation by Alvarez, Lippi, and Passadore (2017) for a sample of advanced and emerging economies (neither of these papers uses a New Keynesian Phillips-curve framework). To our knowledge our paper is the first to find evidence of non-linearity in exchange rate pass-through to consumer prices using a New Keynesian framework for a large emerging country sample and including the post-crisis period. Kohlscheen (2010) finds that pass-through to consumer prices for EMEs is higher for countries with greater nominal exchange rate volatilities, using bivariate VARs, and Campa and Goldberg (2005) find that pass-through to import prices is higher for countries with greater nominal exchange rate volatilities.

5. Robustness

Next we extend our analysis to check the robustness of the main findings.

First, we change the size of the rolling window from six to eight, five, four, and three years in the main specification of equation (1) excluding inflation expectations (setting $\rho = 0$) and report the results in figures 4 and 5 in appendix 2. We find that for all horizons (contemporaneous, yearly, and long run) the pattern for EMEs of lower linear pass-through post-crisis is robust to the length of the estimation window, for all the window sizes considered. Similarly, the pattern that the pass-through has been relatively stable in advanced economies is preserved for different rolling window sizes.

Second, we present the results for the main specification of equation (3) excluding inflation expectations (setting $\rho = 0$) when using log changes in bilateral exchange rates against the U.S. dollar, instead of in NEERs (table 3 and appendix figure 6). The reason is, as Gopinath (2016) found, that the pass-through might work through the invoicing currency, typically the U.S. dollar (USD), and not through the effective exchange rates. We find that the patterns of the pass-through estimates are roughly similar whether we use changes in the nominal exchange rate or the U.S. dollar bilateral exchange rate, though some of our results are actually stronger when using U.S. dollar bilateral exchange rates. For EMEs, the inflation interaction terms appear somewhat larger and more significant than in case of the NEERs (see table 3 and online appendix table A3). In particular, when using the U.S. dollar bilateral exchange rates, the inflation interaction term also becomes significant for contemporaneous pass-through. Moreover, the coefficients on the inflation interaction terms are slightly larger for yearly and long-run pass-through, and more significant, namely at the 5 percent level, than when using NEERs.

Third, our results also remain robust to changes in the empirical estimation techniques. While our benchmark specification was

Table 3. Lower Inflation/Lower Pass-Through with USD Bilateral Exchange Rates (pass-through coefficients, based on equation (3) excluding inflation expectations)

	Dependent Variable: Inflation$_t$			
	Emerging Market Economies		**Advanced Economies**	
	NEER	**Bilateral USD Exchange Rate**	**NEER**	**Bilateral USD Exchange Rate**
Explanatory Variables	**(1)**	**(2)**	**(3)**	**(4)**
Exchange Rate Pass-Through				
Contemporaneous ERPT	0.0655***	0.0410**	0.00476	0.00657
	(0.0179)	(0.0190)	(0.00510)	(0.00489)
Yearly ERPT	0.126***	0.0922***	0.0187*	0.0206**
	(0.0318)	(0.0209)	(0.00911)	(0.00653)
Long-Run ERPT	0.271***	0.184***	0.0292*	0.0329**
	(0.0705)	(0.0508)	(0.0150)	(0.0112)
Inflation Interaction				
Inflation$_{t-4}$*Contemporaneous ERPT	0.921	1.201*	1.582	0.694
	(0.614)	(0.661)	(1.285)	(0.625)
Inflation$_{t-4}$*Yearly ERPT	1.741*	2.061**	1.026	0.644
	(0.921)	(0.870)	(1.333)	(1.122)
Inflation$_{t-4}$*Long-Run ERPT	3.737*	4.123**	1.601	1.030
	(1.254)	(1.077)	(1.935)	(1.677)
Lagged Dependent Variable	Yes	Yes	Yes	Yes
Inflation Expectations	No	No	No	No
Control Variables for Local Factors	Domestic Output Gap	Domestic Output Gap	Domestic Output Gap	Domestic Output Gap
Time Fixed Effect	Yes	Yes	Yes	Yes
Country Fixed Effects	Yes	Yes	Yes	Yes
Observations	1,977	1,977	1,001	910
Number of Countries	22	22	11	10
Sargan Test[a]	0.860	0.636	0.211	0.182
Hansen Test[a]	1	1	1	1
Serial Correlation Test[b]	0.514	0.528	0.0847	0.0721

Notes: System GMM estimation using Arellano and Bover (1995) and Blundell and Bond (1998) dynamic panel estimator. Robust standard errors are in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. Full results are reported in online appendix table A3.

[a]Reports p-values for the null hypothesis that the instruments used are not correlated with the residuals.

[b]Reports p-values for the null hypothesis that the errors in the first-difference regression exhibit no second-order serial correlation.

system GMM, the results remain materially unchanged when using difference GMM and within-group estimators (table 4). This suggests that the methodological choice is not critical for our results. In all three cases we also test for different lag structures of instrumental variables to confirm that the results do not depend on the instrument lag choice (table A5 in the online appendix). Furthermore, the basic pattern—the post-crisis decrease for EMEs and relative stability of pass-through for advanced economies—remains unchanged for all three pass-through horizons.

Furthermore, we also modify the main specification to allow for asymmetry in pass-through for exchange rate depreciations and appreciations. However, we do not find evidence for consistent asymmetries when estimating the exchange rate pass-through separately for depreciations and appreciations.

5.1 Caveats

However, the results should be read with appropriate caveats. Importantly, they apply only for groups of countries, and not for individual economies. In particular, while we see a large drop in pass-through for emerging markets as a group after the financial crisis, some emerging economies could still have experienced stable or even increasing pass-through. Similarly, in spite of the stable average results, different pass-through trends might prevail in some advanced economies.

Furthermore, our setup is limited to macroeconomic factors, while microeconomic factors, such as price competition or pricing to market, might also play a role in determining pass-through (Campa and Goldberg 2005). For instance, the more oligopolistic/less price taking behavior is, the weaker pass-through from input prices (which might be affected by exchange rate movements) to final prices is. However, these concerns are mitigated by the fact that our setup captures the time-invariant microeconomic factors by the country fixed effects. Further mitigation, as Forbes (2015) argues, is that these structural microeconomic differences might matter less than thought earlier: recent pass-through estimates for the United Kingdom do not show much difference between goods with differing import content, or between economic sectors with different tradability or degree of international competition.

Table 4. Lower Inflation/Lower Pass-Through: Different Methodologies (pass-through coefficients, based on equation (3) excluding inflation expectations)

	Dependent Variable: Inflation$_t$					
	Emerging Market Economies			**Advanced Economies**		
	System GMM	**Difference GMM**	**Within-Group Estimator**	**System GMM**	**Difference GMM**	**Within-Group Estimator**
Explanatory Variables	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Exchange Rate Pass-Through						
Contemporaneous ERPT	0.0655***	0.0567***	0.0569***	0.00476	0.00340	0.00340
	(0.0179)	(0.0170)	(0.0168)	(0.00510)	(0.00482)	(0.00480)
Yearly ERPT	0.126***	0.112***	0.112***	0.0187*	0.0283***	0.0283***
	(0.0318)	(0.0267)	(0.0265)	(0.00911)	(0.00530)	(0.00528)
Long-Run ERPT	0.271***	0.208***	0.210***	0.0292*	0.0340***	0.0340***
	(0.0705)	(0.0537)	(0.0529)	(0.0150)	(0.00708)	(0.00705)
Inflation Interaction						
Inflation$_{t-4}$ *Contemporaneous ERPT	0.921	0.921	0.915	1.582	2.305*	2.305*
	(0.614)	(0.605)	(0.598)	(1.285)	(1.121)	(1.116)
Inflation$_{t-4}$*Yearly ERPT	1.741*	1.861**	1.845**	1.026	2.216*	2.216*
	(0.921)	(0.891)	(0.878)	(1.333)	(1.121)	(1.216)
Inflation$_{t-4}$*Long-Run ERPT	3.737*	3.465**	3.449**	1.601	2.656*	2.656*
	(1.254)	(1.071)	(1.057)	(1.935)	(1.380)	(1.374)

(continued)

Table 4. (Continued)

	Dependent Variable: $Inflation_t$					
	Emerging Market Economies			Advanced Economies		
	System GMM	Difference GMM	Within-Group Estimator	System GMM	Difference GMM	Within-Group Estimator
Explanatory Variables	(1)	(2)	(3)	(4)	(5)	(6)
Lagged Dependent Variable	Yes	Yes	Yes	Yes	Yes	Yes
Inflation Expectations	No	No	No	No	No	No
Control Variables for Local Factors	Domestic Output Gap	Domestic Output Gap	Domestic Output Gap	Domestic Output Gap	Domestic Output Gap	Domestic Output Gap
Time Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Country Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1,977	1,955	1,977	1,001	990	1,001
Number of Countries	22	22	22	11	11	11
Sargan Test[a]	0.860	0.567		0.211	0.010	
Hansen Test[a]	1	1		1	1	
Serial Correlation Test[b]	0.514	0.551		0.0847	0.317	
Within R^2			0.813			0.465

Notes: Robust standard errors are in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1.
[a]Reports p-values for the null hypothesis that the instruments used are not correlated with the residuals.
[b]Reports p-values for the null hypothesis that the errors in the first-difference regression exhibit no second-order serial correlation.

Another caveat arises due to new limitations in monetary policy, namely reaching the zero lower bound and in some cases outright negative interest rates. To the degree that this constrains monetary policy, pass-through could be affected. Having said that, this constraint is unlikely to affect our main conclusion: First, there is no such constraint in emerging markets where we see a larger decline in pass-through. Second, pass-through estimates seem to remain low in advanced economies even under low interest rates. Yet, this issue highlights that one should not be complacent about low pass-through in advanced economies: the slight and not very robust increase in pass-through in some of our estimates shown for advanced economies could warrant further investigation and research in light of these policy constraints.

Finally, our approach does not distinguish between exogenous and endogenous exchange rate shocks—and this distinction might matter, as Shambaugh (2008) and Forbes, Hjortsoe, and Nenova (2015) show. However, we consciously control for global factors, either through time fixed effects or explicitly, in order to consistently exclude global shocks.[8] On the one hand, this exclusion of global factors is reassuring: the results are not contaminated by shifting global shocks. On the other hand, this also implies that the inclusion of global shocks could add further dynamics in principle—though our tests suggest that removing the time-fixed effects does not materially affect the main results.[9]

6. Conclusions

We studied how exchange rate pass-through has changed since the global financial crisis. We found that exchange rate pass-through to CPI inflation in emerging economies decreased in the wake of the financial crisis, and that this decline in pass-through in emerging economies is linked to declining inflation. By contrast, exchange rate pass-through in advanced economies has remained relatively stable

[8]The dynamic panel estimates in the baseline specification include time fixed effects, and thereby the pass-through estimates are derived only from the cross-sectional variation.

[9]We explored this point above by running our specification without time fixed effects for robustness (see columns 2, 3, 6, and 7 of tables 1 and 2 for robustness).

over time, at a lower level than in emerging economies. These results hold for both short-run and long-run pass-through. The results are found to be robust to a range of controls and specifications.

The results have policy relevance, particularly when assessing broad changes in how exchange rate changes are transmitted to consumer prices in the global economy. Providing such a global context might help thinking about monetary policy in many countries, even if the pass-through estimates are not directly applicable to any individual country. In this regard, the generally low pass-through levels today imply that central banks in general should “fear” less the “floating” of their exchange rates, at least from an inflation perspective. Yet, the lower pass-through in emerging markets also implies that the exchange rate channel of monetary policy might be less effective to affect inflation than before the financial crisis.

Finally, the results further confirm the importance of price stability by showing that lower inflation, among its other benefits, also reduces exchange rate pass-through to consumer prices.

Appendix 1. Data Sources

Table 5. Data Sources

Variable	Description	Source
	Inflation	
Consumer Price Index	Quarter-on-quarter log changes, seasonally adjusted.	Datastream National Data BIS
	Exchange Rates	
Nominal Effective Exchange Rate	Nominal effective exchange rate indexes are calculated as geometric weighted averages of bilateral exchange rates. Broad indexes comprise sixty-one economies, with data from 1994. Quarterly averages, quarter-on-quarter log changes.	BIS
Bilateral USD Exchange Rates	Bilateral U.S. dollar exchange rate against local currency. Quarterly averages, quarter-on-quarter log changes.	National Data BIS
	Control Variables for Local Factors	
Domestic Output Gap	Standard Hodrick-Prescott filter applied on quarterly real GDP series. GDP in levels; domestic currency units.	National Data BIS Authors' calculations
Inflation Expectations	Quarter-on-quarter inflation expectations. Data are derived from yearly Consensus surveys' inflation expectations by assuming constant inflation over the coming quarters within the year.	Consensus Economics Datastream National Data BIS Authors' caculations
	Control Variables for Global Factors	
Oil Prices	West Texas Intermediate (WTI) crude oil spot price. Quarterly averages, quarter-on-quarter log changes.	Bloomberg
Global Output Gap	Standard Hodrick-Prescott filter applied on quarterly real GDP series.	National Data BIS Authors' calculations

Appendix 2. Robustness Checks

Figure 4. Pass-Through to Emerging Market Economies (baseline specification with different rolling window sizes)

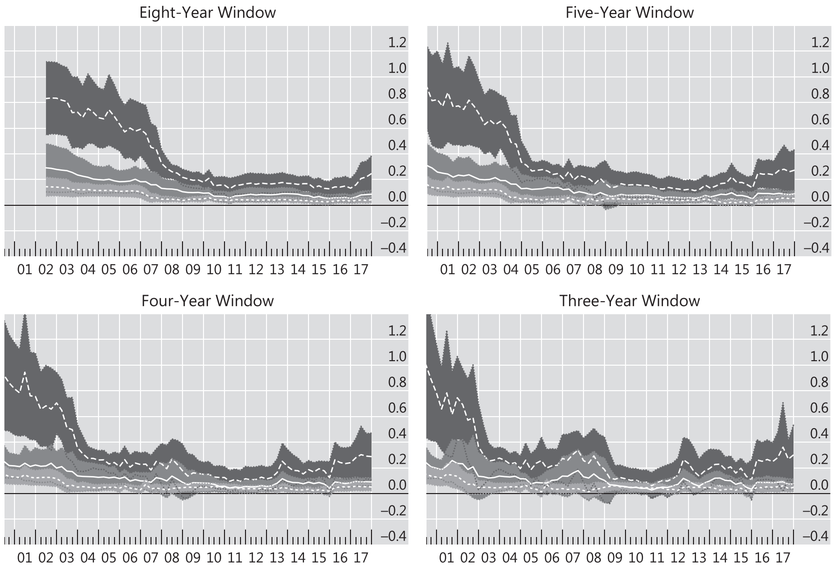


Figure 5. Pass-Through to Advanced Economies (baseline specification with different rolling window sizes)

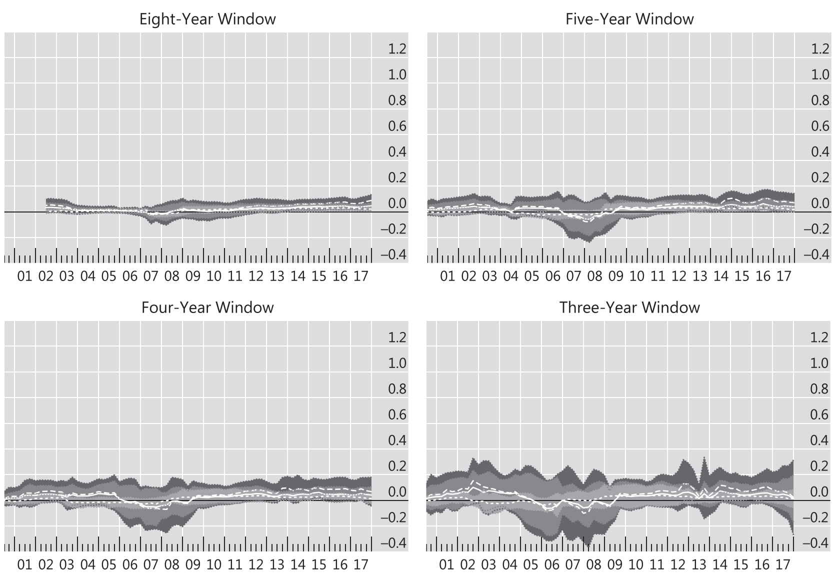


Figure 6. Pass-Through Using Bilateral USD Exchange Rate (six-year rolling windows, based on equation (1) without inflation expectations)

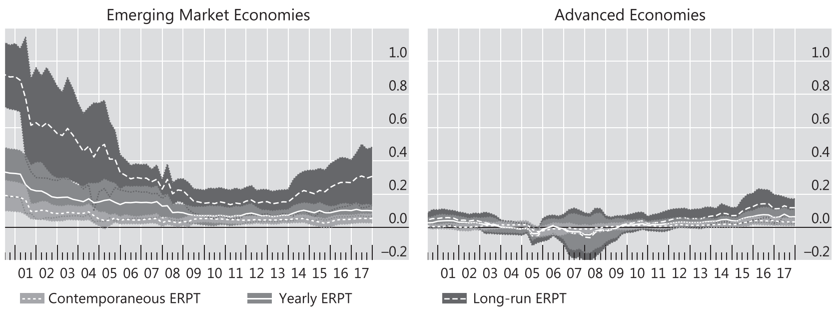
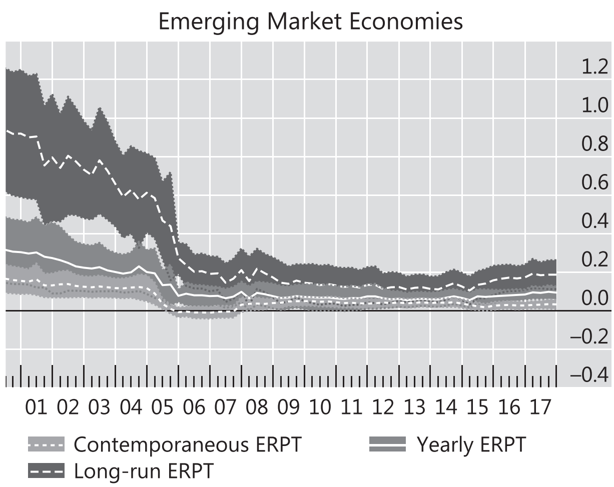


Figure 7. Pass-Through to Emerging Market Economies, excluding Argentina (baseline specification, six-year rolling windows)



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Online Appendix to Exchange Rate Pass-Through: What Has Changed Since the Crisis?

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Table A1. How Did the ERPT Change in the Post-Crisis Period? (full results for table 1 in main text)

	Dependent Variable: $Inflation_t$							
	Emerging Market Economies				Advanced Economies			
Explanatory Variables	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
$Inflation_{t-1}$	0.675***	0.691***	0.690***	0.606***	0.326***	0.299***	0.313***	0.342***
	(0.0458)	(0.0456)	(0.0449)	(0.0458)	(0.0999)	(0.0817)	(0.0891)	(0.0906)
$\Delta NEER_t$ = Contemporaneous ERPT	0.120***	0.116***	0.117***	0.0104***	0.00470	−0.000989	−0.00380	0.00594
	(0.0341)	(0.0315)	(0.0312)	(0.0331)	(0.00595)	(0.00534)	(0.00763)	(0.00570)
$\Delta NEER_{t-1}$	0.0725**	0.0716**	0.0711**	0.0763**	0.00286	0.00228	0.000844	7.94e-05
	(0.0295)	(0.0320)	(0.0320)	(0.0293)	(0.0100)	(0.00897)	(0.00776)	(0.0104)
$\Delta NEER_{t-2}$	0.0117	0.00728	0.00736	0.0161	0.000848	0.000326	0.000548	0.000225
	(0.0226)	(0.0163)	(0.0165)	(0.0237)	(0.00805)	(0.00731)	(0.00722)	(0.0108)
$\Delta NEER_{t-3}$	0.0275**	0.0271**	0.0273**	0.0340**	−0.00249	−0.00808	−0.0103	−0.00212
	(0.0118)	(0.0109)	(0.0105)	(0.0132)	(0.0106)	(0.00738)	(0.00745)	(0.0111)
$\Delta NEER^2_t$	0.190*	0.188	0.198*	0.204**	0.0386	−0.132	−0.124	0.0481
	(0.105)	(0.114)	(0.112)	(0.0861)	(0.165)	(0.134)	(0.177)	(0.173)
$\Delta NEER^2_{t-1}$	−0.166	−0.151	−0.150	−0.155	−0.0542	−0.0125	0.0214	−0.0618
	(0.148)	(0.161)	(0.161)	(0.140)	(0.412)	(0.308)	(0.288)	(0.394)
$\Delta NEER^2_{t-2}$	0.130	0.138	0.136	0.157*	0.268	0.187	0.163	0.269
	(0.0818)	(0.0815)	(0.0822)	(0.0790)	(0.223)	(0.162)	(0.209)	(0.245)
$\Delta NEER^2_{t-3}$	−0.0572	−0.0558	−0.0574	−0.0101	0.0650	0.0811	0.0110	0.0994
	(0.0829)	(0.0796)	(0.0782)	(0.0885)	(0.223)	(0.215)	(0.228)	(0.242)
$\Delta NEER^3_t$	0.392***	0.401***	0.410***	0.416***	−1.507	0.464	0.533	−1.634
	(0.120)	(0.109)	(0.108)	(0.125)	(0.900)	(1.317)	(1.703)	(0.936)
$\Delta NEER^3_{t-1}$	−0.346	−0.329	−0.329	−0.327	2.798	3.432	3.197	3.098
	(0.214)	(0.234)	(0.234)	(0.207)	(2.633)	(2.143)	(2.173)	(2.890)
$\Delta NEER^3_{t-2}$	0.0677	0.0889	0.0859	0.119	0.0145	0.984	−0.168	0.313
	(0.138)	(0.142)	(0.139)	(0.146)	(1.079)	(1.384)	(1.647)	(1.127)
$\Delta NEER^3_{t-3}$	−0.0789	−0.0744	−0.0755	−0.0382	0.935	2.009	1.648	0.755
	(0.0884)	(0.0849)	(0.0838)	(0.0848)	(1.464)	(1.187)	(1.231)	(1.725)
Output Gap_t	−0.0118	0.0292	0.00954	0.0199	0.0296*	0.0538***	0.0456**	0.0269
	(0.0403)	(0.0384)	(0.0429)	(0.0241)	(0.0150)	(0.00756)	(0.0166)	(0.0153)
D_t*$Inflation_{t-1}$	−0.0213	−0.0605	−0.0655	−0.150***	0.137	0.0901	0.0563	0.0891
	(0.0767)	(0.0596)	(0.0604)	(0.0421)	(0.124)	(0.0767)	(0.0859)	(0.121)
D_t*$\Delta NEER_t$ = D_t*Contemporaneous ERPT	−0.0764**	−0.0753**	−0.0810***	−0.0675*	0.0182*	0.0214**	0.00691	0.0185*
	(0.0328)	(0.0294)	(0.0284)	(0.0328)	(0.00893)	(0.00845)	(0.00837)	(0.00973)

(continued)

Table A1. (Continued)

Explanatory Variables	Dependent Variable: $Inflation_t$							
	Emerging Market Economies				Advanced Economies			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
D_t*$\Delta NEER_{t-1}$	−0.0423*	−0.0464*	−0.0468*	−0.0457*	0.0113	0.0138	0.0119	0.0158
	(0.0245)	(0.0269)	(0.0270)	(0.0243)	(0.0136)	(0.0107)	(0.00840)	(0.0144)
D_t*$\Delta NEER_{t-2}$	−0.0131	−0.00609	−0.00752	−0.00937	0.000589	0.00398	−0.00245	0.000285
	(0.0245)	(0.0181)	(0.0179)	(0.0236)	(0.00837)	(0.0117)	(0.0123)	(0.0102)
D_t*$\Delta NEER_{t-3}$	−0.0141	−0.0225*	−0.0198*	−0.0250	0.0116	0.0181*	0.0245**	0.017
	(0.0120)	(0.0114)	(0.0106)	(0.0152)	(0.0112)	(0.00859)	(0.00883)	(0.0121)
D_t*$\Delta NEER^2_t$	−0.0937	−0.150	−0.151	−0.221	0.158	0.358**	0.719***	0.136
	(0.277)	(0.270)	(0.263)	(0.243)	(0.191)	(0.124)	(0.159)	(0.187)
D_t*$\Delta NEER^2_{t-1}$	−0.0348	0.00924	0.0124	−0.102	−0.0919	−0.0914	−0.298	−0.0930
	(0.184)	(0.184)	(0.175)	(0.165)	(0.380)	(0.338)	(0.295)	(0.369)
D_t*$\Delta NEER^2_{t-2}$	0.101	0.0680	0.0775	0.0571	−0.335	−0.221	−0.211	−0.342
	(0.129)	(0.125)	(0.124)	(0.118)	(0.310)	(0.214)	(0.274)	(0.326)
D_t*$\Delta NEER^2_{t-3}$	−0.0877	−0.101	−0.135	−0.0481	−0.00153	−0.0135	0.0481	−0.0265
	(0.0987)	(0.0895)	(0.0938)	(0.0914)	(0.205)	(0.233)	(0.228)	(0.219)
D_t*$\Delta NEER^3_t$	1.425*	1.291*	1.294*	0.953	1.309	−1.004	2.696	1.217
	(0.790)	(0.749)	(0.734)	(0.743)	(1.481)	(1.764)	(2.006)	(1.682)
D_t*$\Delta NEER^3_{t-1}$	0.259	0.434*	0.501**	0.172	−4.788	−6.405*	−6.721*	−5.219
	(0.269)	(0.227)	(0.206)	(0.310)	(3.925)	(3.451)	(3.609)	(4.162)
D_t*$\Delta NEER^3_{t-2}$	0.0956	0.0668	0.107	−0.0226	−0.273	−1.129	0.481	−0.236
	(0.337)	(0.305)	(0.296)	(0.330)	(1.732)	(2.003)	(2.434)	(1.691)
D_t*$\Delta NEER^3_{t-3}$	−0.403*	−0.355	−0.462**	−0.197	−0.799	−1.922	−1.609	−0.624
	(0.213)	(0.222)	(0.210)	(0.146)	(1.413)	(1.112)	(1.253)	(1.663)
D_t*Output Gap_t	0.0417	−0.0183	−0.0239	0.0173	−0.0126	−0.0511	−0.0933**	−0.00577
	(0.0483)	(0.0450)	(0.0448)	(0.0343)	(0.0422)	(0.0344)	(0.0390)	(0.0411)
ΔOil $Prices_t$		0.00701***				0.0117***		
		(0.00221)				(0.00127)		
Global Output Gap_t			0.100**				0.0442*	
			(0.0380)				(0.0224)	
Inflation $Expectations^{t+1}_t$				0.151***				−0.0542
				(0.0407)				(0.133)
D_t*Inflation $Expectations^{t+1}_t$				0.163*				0.148
				(0.0803)				(0.0992)
Constant	0.0112**	0.00344***	0.00353***	−0.00186	0.00588***	0.00268***	0.00280***	0.00391**
	(0.00527)	(0.000698)	(0.000644)	(0.00621)	(0.000958)	(0.000487)	(0.000505)	(0.00143)

(continued)

Table A1. (Continued)

Explanatory Variables	Dependent Variable: Inflation$_t$							
	Emerging Market Economies				Advanced Economies			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Yearly ERPT	0.231***	0.222***	0.0223***	0.230***	0.00592	−0.00646	−0.0127	0.00413
	(0.0785)	(0.0741)	(0.0740)	(0.0737)	(0.00736)	(0.00893)	(0.0137)	(0.0123)
Long-Run ERPT	0.712***	0.717***	0.719***	0.585***	0.00878	−0.00921	−0.0184	0.00627
	(0.150)	(0.140)	(0.141)	(0.136)	(0.0115)	(0.0121)	(0.0185)	(0.0192)
D_t*Yearly ERPT	−0.146**	−0.150**	−0.155**	−0.148**	0.0417*	0.0573**	0.0408	0.0453*
	(0.0670)	(0.0634)	(0.0627)	(0.0640)	(0.0194)	(0.0209)	(0.0228)	(0.0233)
D_t*Long-Run ERPT	−0.143**	−0.142**	−0.146**	−0.128**	0.0483*	0.0630**	0.0432	0.0498
	(0.0685)	(0.0601)	(0.0591)	(0.0580)	(0.0253)	(0.0246)	(0.0250)	(0.0284)
Contemporaneous ERPT + D_t*Contemporaneous ERPT	0.0434***	0.0404***	0.0359**	0.0362***	0.0229**	0.0204**	0.00311	0.0245**
	(0.0126)	(0.0110)	(0.0128)	(0.0111)	(0.00839)	(0.00667)	(0.00970)	(0.00796)
Yearly ERPT + D_t*Yearly ERPT	0.0856***	0.0713***	0.0676***	0.0825***	0.0476***	0.0509***	0.0281	0.0494***
	(0.0171)	(0.0183)	(0.0201)	(0.0170)	(0.0131)	(0.0132)	(0.0170)	(0.0137)
Long-Run ERPT + D_t*Long-Run ERPT	0.247***	0.193**	0.180**	0.152***	0.0886***	0.0833***	0.0446	0.0869***
	(0.0797)	(0.0689)	(0.0719)	(0.0274)	(0.0251)	(0.0181)	(0.0248)	(0.0231)
Observations	1,889	1,889	1,889	1,889	957	957	957	907
Number of Countries	22	22	22	22	11	11	11	11
Country Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time Fixed Effects	Yes	No	No	No	Yes	No	No	No
Sargan Test[a]	0.972	1	1	0.965	0.0752	0.695	0.604	0.0439
Hansen Test[a]	1	1	1	1	1	1	1	1
Serial Correlation Test[b]	0.435	0.477	0.488	0.353	0.0319	0.0811	0.196	0.0273

Notes: System GMM estimation using Arellano and Bover (1995) and Blundell and Bond (1998) dynamic panel estimator. Robust standard errors are in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1.
[a]Reports p-values for the null hypothesis that the instruments used are not correlated with the residuals.
[b]Reports p-values for the null hypothesis that the errors in the first-difference regression exhibit no second-order serial correlation.

Table A2. Can Lower Inflation Explain Lower Pass-Through in EMEs? (full results for table 2 in main text)

	Dependent Variable: Inflation_t							
	Emerging Market Economies				Advanced Economies			
Explanatory Variables	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**	**(7)**	**(8)**
Inflation_{t-1}	0.534***	0.558***	0.549***	0.459***	0.359***	0.334***	0.310***	0.352***
	(0.0997)	(0.0971)	(0.0967)	(0.0825)	(0.0751)	(0.0687)	(0.0701)	(0.0619)
ΔNEER_t = Contemporaneous ERPT	0.0655***	0.0581***	0.0524***	0.0607***	0.00476	0.00569	0.000809	0.00580
	(0.0179)	(0.0179)	(0.0185)	(0.0144)	(0.00510)	(0.00405)	(0.00677)	(0.00527)
ΔNEER_{t-1}	0.0299**	0.0266**	0.0277**	0.0247**	0.00753	0.00564	0.00252	0.00735
	(0.125)	(0.0110)	(0.0118)	(0.0108)	(0.00611)	(0.00533)	(0.00587)	(0.00597)
ΔNEER_{t-2}	−0.00242	−0.00432	−0.00354	0.00430	0.00637	0.0060**	0.000956	0.00782
	(0.0140)	(0.0112)	(0.0115)	(0.0161)	(0.00438)	(0.00242)	(0.00541)	(0.00566)
ΔNEER_{t-3}	0.0331**	0.0283**	0.0303**	0.0355**	2.82e-05	−0.000377	−0.00152	0.001000
	(0.0144)	(0.0130)	(0.0127)	(0.0130)	(0.00498)	(0.00377)	(0.00479)	(0.00485)
ΔNEER^2_t	0.0369	0.0302	0.0449	0.0842	0.0453	0.0757	0.272*	0.0440
	(0.0769)	(0.0759)	(0.0748)	(0.0913)	(0.0524)	(0.0580)	(0.123)	(0.0580)
$\Delta\text{NEER}^2_{t-1}$	−0.145	−0.115	−0.116	−0.170	0.0287	0.0467	0.0867	0.0256
	(0.135)	(0.135)	(0.134)	(0.121)	(0.103)	(0.0680)	(0.0637)	(0.0946)
$\Delta\text{NEER}^2_{t-2}$	0.0881	0.0943	0.0834	0.121	0.0405	0.0647	−0.0806	0.0365*
	(0.123)	(0.116)	(0.123)	(0.112)	(0.0234)	(0.0386)	(0.0556)	(0.0178)
$\Delta\text{NEER}^2_{t-3}$	−0.0960	−0.101	−0.107	−0.0271	0.0452	0.0124	−0.0400	0.0530
	(0.0847)	(0.0867)	(0.0859)	(0.0872)	(0.0525)	(0.0334)	(0.0452)	(0.0552)
ΔNEER^3_t	0.314**	0.327***	0.352***	0.360**	−0.0394	0.0780	0.606	−0.114
	(0.124)	(0.110)	(0.114)	(0.155)	(0.276)	(0.234)	(0.381)	(0.341)
$\Delta\text{NEER}^3_{t-1}$	−0.184	−0.149	−0.151	−0.202	0.101	0.0954	0.356	0.0715
	(0.165)	(0.165)	(0.164)	(0.148)	(0.281)	(0.252)	(0.339)	(0.206)
$\Delta\text{NEER}^3_{t-2}$	0.0681	0.0834	0.0684	0.130	0.479	0.358	0.334	0.487
	(0.114)	(0.107)	(0.111)	(0.111)	(0.286)	(0.221)	(0.275)	(0.317)
$\Delta\text{NEER}^3_{t-3}$	−0.137	−0.136	−0.147	−0.0487	0.491*	0.630**	0.582**	0.483
	(0.0882)	(0.0856)	(0.0871)	(0.0886)	(0.271)	(0.216)	(0.231)	(0.288)
Output Gap_t	−0.00691	0.0343	0.00424	0.0214	0.0349*	0.0463***	0.0119	0.0340*
	(0.0300)	(0.0317)	(0.0367)	(0.0174)	(0.0162)	(0.0129)	(0.0190)	(0.0159)
$\text{Inflation}_{t-4}{*}\Delta\text{NEER}_t$ = Inflation_{t-4}*Contemp. ERPT	0.921	0.936	0.973	0.775	1.582	0.979	−0.278	1.668
	(0.614)	(0.602)	(0.611)	(0.576)	(1.285)	(0.868)	(1.287)	(1.261)
$\text{Inflation}_{t-4}{*}\Delta\text{NEER}_{t-1}$	0.609**	0.623**	0.632**	0.761***	0.346	0.572	0.0597	0.433
	(0.238)	(0.258)	(0.254)	(0.205)	(0.539)	(0.703)	(0.741)	(0.503)

(continued)

Table A2. (Continued)

	Dependent Variable: $Inflation_t$							
	Emerging Market Economies				Advanced Economies			
Explanatory Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$Inflation_{t-4}*\Delta NEER_{t-2}$	0.324	0.328	0.340*	0.278	−1.390**	−0.889*	−0.214	−1.569**
	(0.193)	(0.194)	(0.197)	(0.180)	(0.530)	(0.443)	(0.756)	(0.496)
$Inflation_{t-4}*\Delta NEER_{t-3}$	−0.112	−0.112	−0.116	−0.102	0.488	−0.0187	0.232	0.441
	(0.0895)	(0.0899)	(0.0911)	(0.0797)	(0.507)	(0.403)	(0.455)	(0.677)
$\Delta Oil\ Prices_t$		0.0119***				0.0122***		
		(0.00196)				(0.00144)		
$Global\ Output\ Gap_t$			0.137***				0.0742***	
			(0.0434)				(0.0188)	
$Inflation\ Expectations_t^{t+1}$				0.168***				0.0420
				(0.0467)				(0.106)
Constant	0.00858	0.0044***	0.0046***	0.00434	−0.0081**	0.0028***	0.0030***	0.00139
	(0.00718)	(0.00093)	(0.00092)	(0.00319)	(0.00277)	(0.00055)	(0.00057)	(0.00143)
Yearly ERPT	0.126***	0.109***	0.107***	0.125***	0.0187*	0.0166**	0.00277	0.0220*
	(0.0318)	(0.0265)	(0.0287)	(0.0289)	(0.00911)	(0.00710)	(0.0111)	(0.0101)
Long-Run ERPT	0.271***	0.246***	0.237***	0.231***	0.0292*	0.0249*	0.00402	0.0339*
	(0.0705)	(0.0707)	(0.0734)	(0.0446)	(0.0150)	(0.0120)	(0.0163)	(0.0170)
$Inflation_{t-4}$*Yearly ERPT	1.741*	1.775*	1.830*	1.712*	1.026	0.643	−0.200	0.972
	(0.921)	(0.938)	(0.943)	(0.845)	(1.333)	(1.034)	(1.070)	(1.401)
$Inflation_{t-4}$*Long-Run ERPT	3.737*	4.014*	4.059*	3.163*	1.601	0.966	−0.291	1.500
	(1.254)	(1.312)	(1.288)	(1.178)	(1.935)	(1.464)	(1.577)	(2.047)
Observations	1,977	1,977	1,977	1,977	1,001	1,001	1,001	951
Number of Countries	22	22	22	22	11	11	11	11
Country Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time Fixed Effects	Yes	No	No	No	Yes	No	No	No
Sargan Test[a]	0.860	0.997	0.990	0.941	0.211	0.922	0.608	0.204
Hansen Test[a]	1	1	1	1	1	1	1	1
Serial Correlation Test[b]	0.514	0.520	0.587	0.401	0.0847	0.0843	0.683	0.0872

Notes: System GMM estimation using Arellano and Bover (1995) and Blundell and Bond (1998) dynamic panel estimator. Robust standard errors are in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
[a]Reports p-values for the null hypothesis that the instruments used are not correlated with the residuals.
[b]Reports p-values for the null hypothesis that the errors in the first-difference regression exhibit no second-order serial correlation.

Table A3. Lower Inflation/Lower Pass-Through: NEER vs. USD Exchange Rates (full results for table 3 in main text)

	Dependent Variable: $Inflation_t$			
	Emerging Market Economies		Advanced Economies	
	NEER	Bilateral USD Exchange Rate	NEER	Bilateral USD Exchange Rate
Explanatory Variables	(1)	(2)	(3)	(4)
$Inflation_{t-1}$	0.534***	0.500***	0.359***	0.375***
	(0.0997)	(0.0904)	(0.0751)	(0.0828)
ΔExchange $Rate_t$ = Contemporaneous ERPT	0.0655***	0.0410**	0.00476	0.00657
	(0.0179)	(0.0190)	(0.00510)	(0.00489)
ΔExchange $Rate_{t-1}$	0.0299**	0.0335**	0.00753	0.00798
	(0.0125)	(0.0140)	(0.00611)	(0.00506)
ΔExchange $Rate_{t-2}$	−0.00242	−0.0219	0.00637	0.00347
	(0.0140)	(0.0193)	(0.00438)	(0.00536)
ΔExchange $Rate_{t-3}$	0.0331**	0.0395**	2.82e-05	0.00254
	(0.0144)	(0.0149)	(0.00498)	(0.00535)
ΔExchange $Rate^2_t$	0.0369	0.201**	0.0453	0.0422
	(0.0769)	(0.0794)	(0.0524)	(0.0698)
ΔExchange $Rate^2_{t-1}$	−0.145	0.121	0.0287	−0.0204
	(0.135)	(0.0726)	(0.103)	(0.128)
ΔExchange $Rate^2_{t-2}$	0.0881	−0.0994	0.0405	−0.00978
	(0.123)	(0.143)	(0.0234)	(0.0518)
ΔExchange $Rate^2_{t-3}$	−0.0960	0.0683	0.0452	−0.0419
	(0.0847)	(0.0632)	(0.0525)	(0.0368)
ΔExchange $Rate^3_t$	0.314**	0.0107	−0.0394	−0.132
	(0.124)	(0.121)	(0.276)	(0.243)
ΔExchange $Rate^3_{t-1}$	−0.184	−0.0736	0.101	0.107
	(0.165)	(0.0732)	(0.281)	(0.512)
ΔExchange $Rate^3_{t-2}$	0.0681	0.151	0.479	0.104
	(0.114)	(0.110)	(0.286)	(0.206)
ΔExchange $Rate^3_{t-3}$	−0.137	−0.193***	0.491*	0.331
	(0.0882)	(0.0593)	(0.271)	(0.238)
Output Gap_t	−0.00691	−0.00630	0.0349*	0.0353*
	(0.0300)	(0.0259)	(0.0162)	(0.0167)
$Inflation_{t-4}$*ΔExchange $Rate_t$ = $Inflation_{t-4}$*Contemporaneous ERPT	0.921	1.201*	1.582	0.694
	(0.614)	(0.661)	(1.285)	(0.625)
$Inflation_{t-4}$*ΔExchange $Rate_{t-1}$	0.609**	0.501**	0.346	0.0335
	(0.238)	(0.180)	(0.539)	(0.673)
$Inflation_{t-4}$*ΔExchange $Rate_{t-2}$	0.324	0.439***	−1.390**	−0.528*
	(0.193)	(0.154)	(0.530)	(0.267)
$Inflation_{t-4}$*ΔExchange $Rate_{t-3}$	−0.112	−0.0803	0.488	0.444
	(0.0895)	(0.0826)	(0.507)	(0.431)
Constant	0.00858	0.0134*	−0.0081**	0.000605
	(0.00718)	(0.00666)	(0.00277)	(0.000799)
Yearly ERPT	0.126***	0.0922***	0.0187*	0.0206**
	(0.0318)	(0.0209)	(0.00911)	(0.00653)
Long-Run ERPT	0.271***	0.184***	0.0292*	0.0329**
	(0.0705)	(0.0508)	(0.0150)	(0.0112)
$Inflation_{t-4}$*Yearly ERPT	1.741*	2.061**	1.026	0.644
	(0.921)	(0.870)	(1.333)	(1.122)
$Inflation_{t-4}$*Long-Run ERPT	3.737*	4.123**	1.601	1.030
	(1.254)	(1.077)	(1.935)	(1.677)

(*continued*)

Table A3. (Continued)

	Dependent Variable: Inflation$_t$			
	Emerging Market Economies		**Advanced Economies**	
	NEER	**Bilateral USD Exchange Rate**	**NEER**	**Bilateral USD Exchange Rate**
Explanatory Variables	**(1)**	**(2)**	**(3)**	**(4)**
Observations	1,977	1,977	1,001	910
Number of Countries	22	22	11	10
Country Fixed Effect	Yes	Yes	Yes	Yes
Time Fixed Effects	Yes	Yes	Yes	Yes
Sargan Test[a]	0.860	0.636	0.211	0.182
Hansen Test[a]	1	1	1	1
Serial Correlation Test[b]	0.514	0.528	0.0847	0.0721

Notes: System GMM estimation using Arellano and Bover (1995) and Blundell and Bond (1998) dynamic panel estimator. Robust standard errors are in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.

[a]Reports p-values for the null hypothesis that the instruments used are not correlated with the residuals.

[b]Reports p-values for the null hypothesis that the errors in the first-difference regression exhibit no second-order serial correlation.

Table A4. Lower Inflation/Lower Pass-Through: Different Methodologies (full results for table 4 in main text)

	Dependent Variable: $Inflation_t$					
	Emerging Market Economies			Advanced Economies		
	System GMM	Difference GMM	Within-Group Estimator	System GMM	Difference GMM	Within-Group Estimator
Explanatory Variables	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
$Inflation_{t-1}$	0.534***	0.463***	0.465***	0.359***	0.166***	0.166***
	(0.0997)	(0.102)	(0.102)	(0.0751)	(0.0464)	(0.0462)
$\Delta NEER_t$=	0.0655***	0.0567***	0.0569***	0.00476	0.00340	0.00340
Contemporaneous ERPT	(0.0179)	(0.0170)	(0.0168)	(0.00510)	(0.00482)	(0.00480)
$\Delta NEER_{t-1}$	0.0299**	0.0242**	0.0245**	0.00753	0.00930	0.00930
	(0.0125)	(0.0107)	(0.0106)	(0.00611)	(0.00585)	(0.00583)
$\Delta NEER_{t-2}$	−0.00242	−0.00185	−0.00175	0.00637	0.0108**	0.0108**
	(0.0140)	(0.0135)	(0.0135)	(0.00438)	(0.0448)	(0.00446)
$\Delta NEER_{t-3}$	0.0331**	0.0326**	0.0326**	2.82e-05	0.00485	0.00485
	(0.0144)	(0.0135)	(0.0135)	(0.00498)	(0.00313)	(0.00312)
$\Delta NEER^2_t$	0.0369	0.0238	0.0257	0.0453	−0.00103	−0.00103
	(0.0769)	(0.0745)	(0.0764)	(0.0524)	(0.0723)	(0.0720)
$\Delta NEER^2_{t-1}$	−0.145	−0.160	−0.158	0.0287	−0.0131	−0.0131
	(0.135)	(0.126)	(0.128)	(0.103)	(0.0639)	(0.0636)
$\Delta NEER^2_{t-2}$	0.0881	0.0963	0.0970	0.0405	−0.00703	−0.00703
	(0.123)	(0.119)	(0.118)	(0.0234)	(0.0290)	(0.0289)
$\Delta NEER^2_{t-3}$	−0.0960	−0.0767	−0.0750	0.0452	0.00108	0.00108
	(0.0847)	(0.0815)	(0.0805)	(0.0525)	(0.0353)	(0.0351)
$\Delta NEER^3_t$	0.314**	0.312**	0.314**	−0.0394	−0.499*	−0.499
	(0.124)	(0.130)	(0.133)	(0.276)	(0.277)	(0.276)
$\Delta NEER^3_{t-1}$	−0.184	−0.167	−0.167	0.101	−0.338	−0.338
	(0.165)	(0.152)	(0.154)	(0.281)	(0.250)	(0.249)
$\Delta NEER^3_{t-2}$	0.0681	0.0963	0.0967	0.479	0.0913	0.0913
	(0.114)	(0.107)	(0.107)	(0.286)	(0.290)	(0.289)

(*continued*)

Table A4. (Continued)

	Dependent Variable: $Inflation_t$					
	Emerging Market Economies			Advanced Economies		
	System GMM	**Difference GMM**	**Within-Group Estimator**	**System GMM**	**Difference GMM**	**Within-Group Estimator**
Explanatory Variables	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
$\Delta NEER^3_{t-3}$	−0.137	−0.116	−0.114	0.491*	0.216	0.216
	(0.0882)	(0.0809)	(0.0797)	(0.271)	(0.260)	(0.259)
Output Gap_t	−0.00691	−0.0115	−0.0111	0.0349*	0.0429*	0.0429*
	(0.0300)	(0.0298)	(0.0300)	(0.0162)	(0.0210)	(0.0209)
$Inflation_{t-4}$*$\Delta NEER_t$ =	0.921	0.921	0.915	1.582	2.305*	2.305*
$Inflation_{t-4}$*Contemp. ERPT	(0.614)	(0.605)	(0.598)	(1.285)	(1.121)	(1.116)
$Inflation_{t-4}$*$\Delta NEER_{t-1}$	0.609**	0.718***	0.711***	0.346	0.997	0.997
	(0.238)	(0.220)	(0.217)	(0.539)	(0.597)	(0.594)
$Inflation_{t-4}$*$\Delta NEER_{t-2}$	0.324	0.333*	0.330*	−1.390**	−1.212	−1.212
	(0.193)	(0.183)	(0.181)	(0.530)	(0.769)	(0.766)
$Inflation_{t-4}$*$\Delta NEER_{t-3}$	−0.112	−0.111	−0.111	0.488	0.125	0.125
	(0.0895)	(0.0874)	(0.0865)	(0.507)	(0.533)	(0.531)
Constant	0.00858		0.0155***	−0.00809**		0.00785***
	(0.00718)		(0.00499)	(0.00277)		(0.00108)
Yearly ERPT	0.126***	0.112***	0.112***	0.0187*	0.0283***	0.0283***
	(0.0318)	(0.0267)	(0.0265)	(0.00911)	(0.00530)	(0.00528)
Long-Run ERPT	0.271***	0.208***	0.210***	0.0292*	0.0340***	0.0340***
	(0.0705)	(0.0537)	(0.0529)	(0.0150)	(0.00708)	(0.00705)
$Inflation_{t-4}$*Yearly ERPT	1.741*	1.861**	1.845**	1.026	2.216*	2.216*
	(0.921)	(0.891)	(0.878)	(1.333)	(1.221)	(1.216)
$Inflation_{t-4}$*Long-Run ERPT	3.737*	3.465**	3.449**	1.601	2.656*	2.656*
	(1.254)	(1.071)	(1.057)	(1.935)	(1.380)	(1.374)

(*continued*)

Table A4. (Continued)

	Dependent Variable: Inflation_t					
	Emerging Market Economies			**Advanced Economies**		
	System GMM	**Difference GMM**	**Within-Group Estimator**	**System GMM**	**Difference GMM**	**Within-Group Estimator**
Explanatory Variables	**(1)**	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Observations	1,977	1,955	1,977	1,001	990	1,001
Number of Countries	22	22	22	11	11	11
Country Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Time Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes
Sargan Test[a]	0.860	0.567		0.211	0.010	
Hansen Test[a]	1	1		1	1	
Serial Correlation Test[b]	0.514	0.551		0.0847	0.317	
Within R^2			0.813			0.465

Notes: Robust standard errors are in parentheses. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$.
[a]Reports p-values for the null hypothesis that the instruments used are not correlated with the residuals.
[b]Reports p-values for the null hypothesis that the errors in the first-difference regression exhibit no second-order serial correlation.

Table A5. Lower Inflation/Lower Pass-Through: Different Structure of GMM Instruments (system GMM, different lag structure of GMM-type instruments)

	Dependent Variable: Inflation_t			
	Emerging Market Economies		Advanced Economies	
	GMM Instruments: 2–8 Lags	GMM Instruments: 2–7 Lags	GMM Instruments: 2–6 Lags	GMM Instruments: 2–5 Lags
Explanatory Variables	(1)	(2)	(3)	(4)
Inflation_{t-1}	0.534***	0.534***	0.529***	0.526***
	(0.0997)	(0.0997)	(0.100)	(0.103)
ΔNEER_t = Contemporaneous ERPT	0.0655***	0.0655***	0.0678***	0.0692***
	(0.0179)	(0.0180)	(0.0175)	(0.0180)
ΔNEER_{t-1}	0.0299**	0.0299**	0.0295**	0.0281**
	(0.0125)	(0.0125)	(0.0127)	(0.0128)
ΔNEER_{t-2}	−0.00242	−0.00240	−0.00190	−0.00194
	(0.0140)	(0.0140)	(0.0139)	(0.0145)
ΔNEER_{t-3}	0.0331**	0.0331**	0.0332**	0.0332**
	(0.0144)	(0.0144)	(0.0146)	(0.0144)
ΔNEER^2_t	0.0369	0.0368	0.0409	0.0410
	(0.0769)	(0.0770)	(0.0822)	(0.0832)
$\Delta\text{NEER}^2_{t-1}$	−0.145	−0.145	−0.150	−0.158
	(0.135)	(0.135)	(0.139)	(0.140)
$\Delta\text{NEER}^2_{t-2}$	0.0881	0.0882	0.0891	0.0895
	(0.123)	(0.123)	(0.124)	(0.126)
$\Delta\text{NEER}^2_{t-3}$	−0.0960	−0.0960	−0.104	−0.106
	(0.0847)	(0.0847)	(0.0861)	(0.0894)
ΔNEER^3_t	0.314**	0.314**	0.319**	0.319**
	(0.124)	(0.124)	(0.130)	(0.130)
$\Delta\text{NEER}^3_{t-1}$	−0.184	−0.185	−0.189	−0.198
	(0.165)	(0.165)	(0.169)	(0.172)
$\Delta\text{NEER}^3_{t-2}$	0.0681	0.0681	0.0689	0.0692
	(0.114)	(0.114)	(0.115)	(0.118)
$\Delta\text{NEER}^3_{t-3}$	−0.137	−0.137	−0.147	−0.149
	(0.0882)	(0.0882)	(0.0879)	(0.0914)
Output Gap_t	−0.00691	−0.00690	−0.00759	−0.00903
	(0.0300)	(0.0301)	(0.0304)	(0.0316)
$\text{Inflation}_{t-4}*\Delta\text{NEER}_t$ = Inflation_{t-4}*Contemp. ERPT	0.921	0.921	0.910	0.901
	(0.614)	(0.614)	(0.615)	(0.624)
$\text{Inflation}_{t-4}*\Delta\text{NEER}_{t-1}$	0.609**	0.609**	0.623**	0.635**
	(0.238)	(0.238)	(0.240)	(0.240)
$\text{Inflation}_{t-4}*\Delta\text{NEER}_{t-2}$	0.324	0.323	0.326	0.328
	(0.193)	(0.193)	(0.193)	(0.196)
$\text{Inflation}_{t-4}*\Delta\text{NEER}_{t-3}$	−0.112	−0.112	−0.111	−0.110
	(0.0895)	(0.0895)	(0.0896)	(0.0908)
Constant	0.00858	0.0625	0.0119**	0.0118**
	(0.00718)	(0.00364)	(0.00481)	(0.00482)
Yearly ERPT	0.126***	0.126***	0.128***	0.129***
	(0.0318)	(0.0318)	(0.0319)	(0.0335)
Long-Run ERPT	0.271***	0.271***	0.273***	0.271***
	(0.0705)	(0.0706)	(0.0692)	(0.0724)

(*continued*)

Table A5. (Continued)

	Dependent Variable: Inflation$_t$			
	Emerging Market Economies		Advanced Economies	
	GMM Instruments: 2–8 Lags	GMM Instruments: 2–7 Lags	GMM Instruments: 2–6 Lags	GMM Instruments: 2–5 Lags
Explanatory Variables	**(1)**	**(2)**	**(3)**	**(4)**
Inflation$_{t-4}$*Yearly ERPT	1.741*	1.741*	1.749*	1.754*
	(0.921)	(0.921)	(0.920)	(0.931)
Inflation$_{t-4}$*Long-Run ERPT	3.737*	3.737*	3.714*	3.703*
	(1.254)	(1.254)	(1.237)	(1.239)
Observations	1,977	1,977	1,977	1,977
Number of Countries	22	22	22	22
Country Fixed Effect	Yes	Yes	Yes	Yes
Time Fixed Effects	Yes	Yes	Yes	Yes
Sargan Test[a]	0.860	0.853	0.0262	7.19e-08
Hansen Test[a]	1	1	1	1
Serial Correlation Test[b]	0.514	0.515	0.518	0.519

Notes: System GMM estimation using Arellano and Bover (1995) and Blundell and Bond (1998) dynamic panel estimator. Robust standard errors are in parentheses. ***p < 0.01, **p < 0.05, *p < 0.1.
[a]Reports p-values for the null hypothesis that the instruments used are not correlated with the residuals.
[b]Reports p-values for the null hypothesis that the errors in the first-difference regression exhibit no second-order serial correlation.

What Drives the Strength of Monetary Policy Transmission?*

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This paper analyzes the cross-country and time variation in monetary policy transmission from short-term interest rates to price level. Using Bayesian TVP-VAR models where the structural monetary policy shocks are identified using zero and sign restrictions, the results suggest that monetary policy transmission has become stronger over time and sacrifice ratios have decreased. Exploring the cross-country and time variation in monetary policy responses using panel regressions, I show that stronger monetary policy transmission and lower sacrifice ratios were associated with an inflation-targeting regime. In periods of banking crises, the transmission was weaker and output costs were higher.

JEL Codes: E52, C54.

1. Introduction

The knowledge of the functioning of transmission mechanism from the short-term policy interest rate to price level and output is crucial for the conduct of monetary policy. Central bankers need to know how their decisions about policy interest rates affect the economy. How large is the expected impact of an interest rate cut on output and price level? What is the time profile of the response of targeted variables? Obviously, these questions have been frequently addressed in the academic literature. For the quantitative summary

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of studies estimating the monetary transmission in various countries, over different time periods and using different modeling strategies, see Havránek and Rusnák (2012). The current consensus is that the response of price level to a monetary policy shock reaches its peak in four to eight quarters. Some studies (Canova, Gambetti, and Pappa 2007; Koop, Leon-Gonzalez, and Strachan 2009) show that monetary policy has changed over time in some countries. Other studies illustrate the cross-country heterogeneity in monetary transmission (Elbourne and de Haan 2006; Jarocinski 2010). A natural policymakers' question would be: what are the drivers of the strength and speed of monetary transmission?

This paper aims to explain the cross-country and time variation in the strength of monetary transmission focusing on the possible role of monetary regime and the characteristics of the economy and financial sector. I start by estimating time-varying impulse responses to monetary policy shocks on a sample of thirty-three OECD and EU countries, keeping the model specification fixed across countries. Having obtained the panel of impulse responses to monetary policy shocks (monetary policy shock is identified by a mixture of short-term and sign restrictions in the framework of Bayesian time-varying parameter vector autoregression, or TVP-VAR; see the methodological part for details), I illustrate the time evolution of impulse responses over the last four decades. Specifically, the results show that the lags of price-level responses to monetary policy shocks have shortened during the observed period, which may reflect the changes in monetary policy strategies and smoother functioning of financial markets. Further, using panel fixed-effects regressions, I show the differences in monetary transmission for different monetary policy regimes: when a country adopted an inflation-targeting monetary policy regime, the transmission from monetary policy interest rates to prices became significantly stronger (by 0.1–0.2 percentage point in a response to a 1 pp shock to policy interest rate). On the other hand, inflation targeters as a whole have a marginally weaker transmission than euro-area countries. Further, a part of the time variation in a fixed-effects model is explained by variables linked to the functioning of the financial sector: while higher leverage in the economy (with higher domestic private credit) is related to stronger transmission, the occurrence of a banking crisis decreases the magnitude of the response of prices to a monetary policy shock.

In a banking crisis, the response of prices to a 1 pp monetary policy shock is lower by 0.05–0.1 pp. The between-effects regressions show that euro-area member countries and countries more open to international trade have stronger transmission.

Finally, I examine the "sacrifice ratios" of monetary policy in the spirit of Jarocinski (2010), measuring the ratio of output losses to price-level decrease as a result of non-systematic monetary policy tightening. The results suggest that the sacrifice ratios have decreased during the last decades from their peak in the 1970s (which coincides with the U.S. disinflation under Volcker chairmanship of the Federal Reserve). A monetary policy regime switch to an inflation-targeting regime was typically followed by lower output costs of disinflation. In the periods of stress in the banking sector the sacrifice ratios were higher.

Still, the standard shortcomings of small-size VAR analysis apply. Due to the limited number of variables in the VAR system, the observed responses may be influenced by shocks to omitted or unobserved variables. The analysis relies on the identification strategy based on a mixture of short-term (zero) and sign restrictions, using the established understanding of the nature of monetary policy shock. Further, one has to be cautious about interpreting the obtained relationships between the transmission strength and the explanatory variables as causalities. The fixed-effects model mitigates the identification problem to some extent by avoiding the time-invariant endogeneity, but other types of endogeneity may be still present.

Following the Introduction, section 2 describes the empirical methodology in more detail, section 3 presents the estimated time-varying impulse responses illustrating the time-evolution and cross-country heterogeneity of monetary transmission, and section 4 examines the role of monetary policy regime, financial-sector characteristics, and other economic factors in explaining the cross-country and time variation in the strength of monetary transmission. Finally, section 5 concludes.

2. Relation to Existing Literature

Early attempts to explain cross-country heterogeneity in monetary policy transmission have often focused on the differences in transmission among the members of the European Economic and Monetary Union, both before (Ehrmann 2000; Mojon and Peersman 2001) and after euro adoption (Angeloni and Ehrmann 2003; Ciccarelli and Rebucci 2006) with mixed conclusions regarding prior euro adoption heterogeneity of transmission and its homogenization in euro-area countries after the adoption of the single currency. Weak conclusions are largely caused by wide confidence intervals of the estimates. The heterogeneity of transmission in euro-area countries has been explored by Ciccarelli, Maddaloni, and Peydró (2013), showing the importance of the credit channel during the crisis. Another cross-country study, by Aysun, Brady, and Honig (2013), has focused on the impact of financial structure on transmission, similarly showing that in economies with higher financial frictions, the credit channel is stronger, as the theory of financial accelerator (Carlstrom and Fuerst 1997; Bernanke, Gertler, and Gilchrist 1999) would predict. The role of financial system and banks in transmission has been examined by Ehrmann et al. (2001) using bank-level data. A paper by Mishra, Montiel, and Spilimbergo (2012) focuses on the role of financial-sector development in transmission differences among low-income countries, stressing the role of the degree of development of financial markets and the exchange rate arrangements. Berben et al. (2004) have examined the heterogeneity in transmission through the lens of forecasting and policy-evaluation models of national central banks of the euro area, concluding that the differences stem from the heterogeneity of economic conditions rather than from the differences in modeling strategies.

A work close to the analysis conducted here is that of Georgiadis (2012), who attributes the cross-country differences in transmission estimated through VAR impulse responses to differences in the financial sector, labor market, and industrial mix. However, his work is restricted to a cross-country analysis only and so it cannot control for the country fixed effects. There might be an omitted-variable bias present in such estimations. For better identification of the effects, this paper explores also the time variation in monetary policy transmission. A Bayesian methodology for multi-country estimation of time-varying parameter VARs was proposed by Canova and Ciccarelli (2009). Due to better control over the estimation process, we stick to a set of single-country TVP-VAR models using the methodology of Primiceri (2005), which allows for time-varying (stochastic) volatility.[1] The sample used in this paper is considerably broader than in any previous work, using the data for thirty-three now-developed countries, with the time span from 1970 to 2010 where the data permit. This to some extent restricts the set of possible explanatory variables of the strength of monetary transmission; notably, many financial and labor market indicators started to be collected in a broader set of economies only after 2000. The analysis presented here sticks to variables available in a longer time series for most countries.

3. Empirical Methodology

To be able to analyze both cross-country and time variation in the strength of monetary transmission, I need to estimate the impulse responses of monetary policy shocks in the respective countries and periods. For this purpose, I make use of the time-varying parameter vector autoregressive model (TVP-VAR) with stochastic volatility as proposed by Primiceri (2005). I include the standard set of variables: output, price level, nominal effective exchange rate, interest rate (as endogenous variables), and oil prices (as an exogenous variable[2]). The reduced-form TVP-VAR with stochastic volatility is estimated using Bayesian Markov chain Monte Carlo (MC^2) technique (the Gibbs sampler) based on the procedure developed by Primiceri (2005). The structural monetary policy shocks are then identified using a theory-based mix of short-term and sign restrictions. After obtaining impulse responses to monetary policy shocks, I examine the determinants of cross-country and time variation using panel regressions.

[1]Because I estimate the TVP-VARs separately for each country, the international spillovers of monetary policy are not analyzed.

[2]Although the original contribution by Sims (1980) defines VAR with endogenous variables only, the later empirical applications have frequently used the restrictions that some variables are exogenous to the system (the VAR system with exogenous variables has been called VARX). See Lütkepohl (2007) for details.

The process can be summarized as follows:

(i) Assemble data set of thirty-three OECD+EU countries (where the data coverage permits), including the endogenous and exogenous variables of the VAR and possible determinants of the strength of monetary transmission.

(ii) Estimate Bayesian TVP-VAR with stochastic volatility, identify structural shocks using short-term and sign restrictions, and compute impulse response functions (IRFs).

(iii) Plot the impulse response functions, exploring both the time and cross-country patterns in monetary transmission, including the “sacrifice ratios.”

(iv) Test whether monetary policy regime and financial-sector characteristics matter for the strength of monetary transmission using panel regressions.

3.1 Data

For the estimation of TVP-VARs, I use a quarterly data set consisting of the seasonally adjusted log-GDP index in constant prices, log-CPI, and money market interest rate—a market-based proxy for the monetary policy target rate and a logarithm of nominal effective exchange rate. Further, I include the logarithm of the oil price index as an exogenous variable.

For the examination of the possible covariates of the strength of monetary transmission and the sacrifice ratios, I use a set of variables consisting of the characteristics of the financial sector, the monetary policy regime, and other economic characteristics such as trade openness. The majority of the data have been downloaded through Thomson Datastream and come from national sources. Data on the exchange rate arrangements and monetary policy regimes were taken from the updated data set of Reinhart and Rogoff (2004). The data on the occurrence of banking crises are taken from Babecký et al. (2012), where an aggregation of crises recorded by academic papers is complemented by a survey among central bank experts in respective countries.

3.2 TVP-VAR Estimation and Identification

3.2.1 Reduced-Form TVP-VAR

I estimate the reduced-form time-varying parameter vector autoregressive model with stochastic volatility, similarly to Primiceri (2005) and Koop, Leon-Gonzalez, and Strachan (2009). In models with stochastic volatility, not only the coefficients but also the covariance matrix of residuals are allowed to change over time. Therefore, these models are able to distinguish between the structural changes in the parameters of monetary policy—the "good/bad policy" story (Cogley and Sargent 2002; Lubik and Schorfheide 2004; Boivin and Giannoni 2006)—and changes in the magnitude of shocks—the "good/bad luck" story (Sims and Zha 2006). We estimate the reduced-form system of equations

$$y_t = Z_t\beta_t + u_t,$$

where y_t is a vector of endogenous variables (consisting of the logs of output and price level, the interest rate, and the log of the nominal effective exchange rate), Z_t is a vector of explanatory variables consisting of the lags of endogenous variables, an exogenous variable (the log of the commodity price index), and the intercept. I use one lag of endogenous variables in the TVP-VAR system to reduce the number of coefficients to be estimated.[3] The coefficients are time varying, and I assume they follow a random-walk process

$$\beta_t = \beta_{t-1} + \nu_t,$$

where ν_t is iid $\mathcal{N}(0, Q)$. The error term u_t has a potentially time-varying distribution $\mathcal{N}(0, \Omega_t)$. Similarly to Primiceri (2005), I use a triangular reduction of Ω_t such that

$$A_t\Omega_t A_t' = \Sigma_t\Sigma_t',$$

[3]Although Bayesian estimation principally avoids the dimensionality problem, the set of parameters to be estimated consists of 4×6 time-varying coefficients in VAR equations (including coefficients on the exogenous variables and the intercept), plus the $4 \times 4 + 4$ elements of time-varying variance-covariance matrices and a number of time-invariant parameters. To maintain a reasonable degree of estimation efficiency, I use one lag of the endogenous variables in the VAR system, which is sufficient for capturing the impulse response dynamics.

where A_t is a lower triangular matrix consisting of elements $\alpha_{ij,t}$ for $i > j$, ones on the diagonal $(i = j)$, and zeros elsewhere $(i < j)$. Σ_t is a diagonal matrix with $\sigma_{ii,t}$ elements. It follows that

$$\Omega_t = A_t^{-1}\Sigma_t\Sigma_t'(A_t^{-1})'$$

and consequently

$$y_t = Z_t\beta_t + A_t^{-1}\Sigma_t\epsilon_t,$$

where ϵ_t is iid $\mathcal{N}(0, 1)$. The elements of A_t and Σ_t matrices follow

$$\alpha_{ij,t} = \alpha_{ij,t-1} + \xi_t$$
$$\log\sigma_{ii,t} = \log\sigma_{ii,t-1} + \eta_t,$$

where ξ_t is iid $\mathcal{N}(0, S)$ and η_t is iid $\mathcal{N}(0, W)$. The matrix describing the whole covariance structure of the model,

$$V = \begin{pmatrix} I_n & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{pmatrix},$$

is block diagonal, i.e., the shocks in each random-walk equation governing the time variation of each model parameter are assumed to be independent across model parameters.

For the technical details of Bayesian MC^2 estimation (Gibbs sampler), see Primiceri (2005) or Koop and Korobilis (2009). The priors on the parameters of the model, as well as the hyperparameters (see appendix 1 for details) are held constant across the cross-section of countries for which the TVP-VARs are estimated, and I take them from Primiceri (2005).

3.2.2 Sign Restrictions

Subsequently to estimating the reduced-form TVP-VAR, I identify structural shocks using a mixture of short-term and sign restrictions (Fry and Pagan 2011). Sign-restrictions identification has been frequently used for the analysis of monetary policy (Canova and Nicolo 2002; Uhlig 2005; Rafiq and Mallick 2008; Scholl and Uhlig

2008; Jarocinski 2010). Due to computing-time reasons, this paper focuses only on the identification of monetary policy shock. I restrict the monetary policy shock to have the following pattern:

$$\begin{bmatrix} \text{Response to MP Shock} & \text{Log-GDP} & \text{Log-CPI} & \text{IR} & \text{ER} \\ \text{On Impact} & 0 & 0 & + & + \\ \text{In the Short Run (1–4 quarters)} & ? & ? & + & + \\ \text{On the MP Horizon (4–6 quarters)} & - & - & ? & ? \\ & & & & \end{bmatrix},$$

i.e., I search for such structural transformation of (transitory) reduced-form shocks where output and price level react to a monetary policy shock only with a lag, and on the monetary policy horizon (four to six quarters) the response of prices and output is negative. Interest rate and exchange rate can react on impact, while the direction of their responses is positive in the short run (up to four quarters) and unrestricted further on. The set of restrictions may appear exhaustive, but as Matthias (2007) shows on data simulated from a structural model, a large number of sign restrictions is needed to correctly identify shocks.[4] The sign pattern used here is consistent with the theoretical consensus of New Keynesian general equilibrium models (Smets and Wouters 2003; Christiano, Eichenbaum, and Evans 2005; Galí and Gertler 2007).[5] The structural shocks are identified according to Fry and Pagan (2011). First, the candidate structural shocks are obtained from orthogonal (Givens) rotations of a set of uncorrelated shocks which come from the recursive (Choleski) identification. Specifically, in this case the candidate structural shocks are created by multiplying a set of uncorrelated shocks by a Givens matrix $Q(\theta)$, which takes the following form:

[4]Canova and Paustian (2011) even show that a large number of restrictions can make up for modest model misspecifications. Another reason for using the mixture of sign and short-run restrictions is that using the sign restrictions alone leads to a bias in estimates of the impulse responses, as the posterior distribution of estimated IRFs is effectively truncated by the sign restriction. The size of the truncation bias was estimated by Liu and Theodoridis (2012), who show that it increases with IRF horizons and VAR lags and amounts to up to 0.5 pp.

[5]However, a number of assumptions such as habit formation, capital installment costs, persistent autoregressive shocks, etc. need to be made to generate hump-shaped responses in those models.

$$Q(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta & \cos\theta \end{pmatrix},$$

where the parameters θ are drawn from $U[0, \pi]$. That way, I obtain candidate structural (uncorrelated) shocks. Among the candidate shocks, only those which satisfy the sign restrictions are picked. This identification strategy does not deliver exact model identification, because I generally find multiple candidate sets of structural shocks which satisfy the restrictions. We follow the consensus (Fry and Pagan 2011) to report the median impulse response of the ones which satisfy sign restrictions. As I identify only one structural shock, the choice of median model is straightforward.

The shocks are normalized to a 1 pp increase in interest rate on impact.

3.3 Panel of Impulse Responses

As a result, I obtain a panel of impulse responses of price level to (transitory) monetary policy shocks. I examine the cross-country and time variation first graphically (showing, for example, that the lag of monetary transmission has gradually decreased over the last decades) and then test for the possible correlates of stronger monetary transmission using panel regressions. As a measure of the transmission strength (the explained variable), I use the value of impulse response function at the conventional monetary policy horizons ($h = 4, 6, 8$ quarters). Different horizons generally yield similar conclusions, so I show only the results for the cumulative response from the impact to $h = 6$ quarters after the initial shock. Using panel regressions, I proceed by testing the relationships between the strength of the impulse responses and various characteristics of the economy and financial sector, such as monetary policy regime, the stock market capitalization, trade openness, or the occurrence of banking crises. The standard errors are heteroskedasticity robust and clustered at the country level.

Using panel fixed effects and between estimators, I am able to distinguish which of the factors capture the cross-country variation and which are better at capturing the time variation in the strength of monetary transmission. For example, the results suggest that although euro-area countries generally feature stronger monetary transmission of monetary policy shocks than inflation targeters, the fixed-effects estimator (explaining the time variation) shows that when a country adopted inflation targeting, its transmission on average strengthened. Moreover, the results suggest that in the periods of elevated financial stress (captured by the occurrence of banking crises), monetary policy transmission is relatively weaker. On the other hand, a part of the cross-country variation can be explained by the degree of openness: more open economies have on average stronger transmission, probably because of the exchange rate channel. The transmission is also stronger with increasing financial leverage.

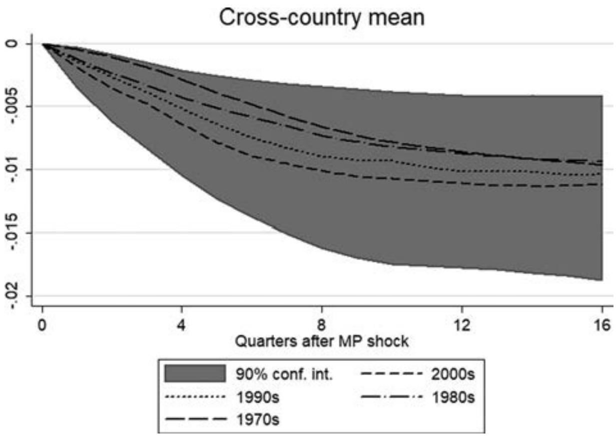
Finally, I compute and analyze the behavior of the monetary policy sacrifice ratios in the spirit of Jarocinski (2010). The sacrifice ratios measure the output cost of disinflation, i.e., how much output needs to be sacrificed to deliver lower inflation after a monetary tightening, and are defined as $\frac{IRF^{GDP}(h)}{IRF^{P}(h)}$ on the horizon h. According to the results, sacrifice ratios have decreased over time, reaching their maximum values during the 1980s. The figures below also show that an inflation-targeting regime of monetary policy is related to lower output costs of disinflation. Moreover, the sacrifice ratios increase in the periods of financial stress in the banking sector.

In this section, the impulse responses from the TVP-VAR on the sample of thirty-three countries are presented, illustrating both cross-country and time variation in the impulse responses to a monetary policy shock of a magnitude of 1 pp. In the main text I focus on the impulse response functions of CPI. Responses of other variables are in the appendix.

Figure 1 shows the mean impulse response (IRF) of CPI to a monetary policy (MP) shock averaged across countries (data permitting) as it evolved during the last four decades. Note that the panel of impulse responses is unbalanced because of the short time series for some countries, affecting the first two decades of the sample.[6]

[6]Confidence intervals for the estimates are not explicitly presented, as the uncertainty consists of both model uncertainty (generally more impulse responses satisfy the restrictions) and parameter uncertainty. However, the parameter

Figure 1. Evolution of IRFs of CPI to an MP Shock over Decades



Notes: Response of log-CPI to a 1 pp monetary policy interest rate shock, estimated within a VAR model and identified using sign restrictions. Confidence bands are derived from the cross-country sample distribution, and do not explicitly reflect parameter or model uncertainty.

Although the changes are relatively small to be statistically significant, the figure suggests that monetary transmission has strengthened gradually over the last forty years. At the same time, the hump shape of the IRF of log-CPI to a monetary policy restriction has become more pronounced, with the peak response now occurring earlier after the initial shocks. This illustrates that the lags of the response of prices to monetary policy shocks have become shorter.[7]

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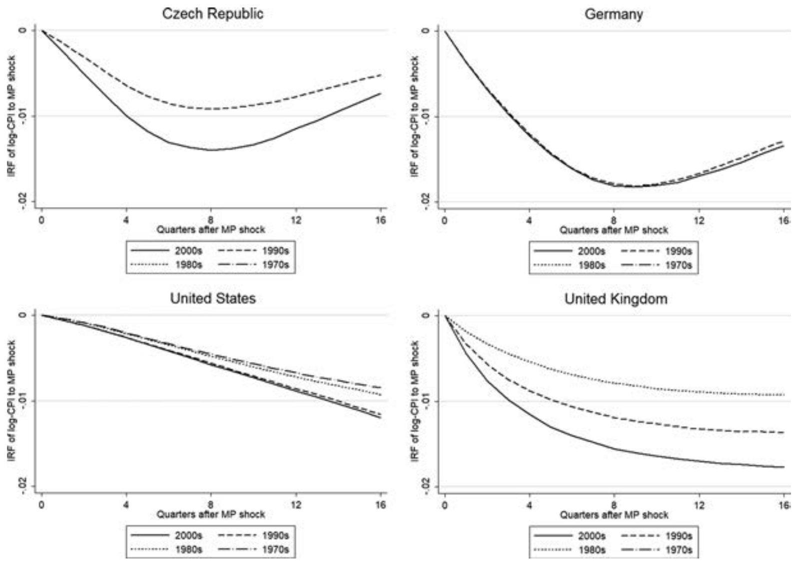
uncertainty around mean impulse responses is fairly stable across countries, periods, and horizons and amounts to about 0.5–0.8 pp.

[7]Note that all referred shocks represent deviations from a systematic monetary policy rule rather than actual actions of monetary policy. In other words, the presented IRFs illustrate the responses to non-systematic shocks. These could represent either a situation where the policy rule would imply no reaction, but the monetary policy reacted, or an opposite situation where the monetary policy should have reacted according to the rule, but it did not, or the reaction was weaker than reaction implied by the policy rule (or stronger, or even in the opposite direction). The (time-varying) monetary policy rule is estimated as a part of the TVP-VAR model, constituting one of the equations of the VAR system. The model allows for immediate reaction of the monetary policy rate to developments in all endogenous variables and the exogenous variable, and as such it can be considered a generalization of the simple empirical Taylor (1993) rule.

Figure 2. Evolution of IRFs of CPI to an MP Shock in Different Countries

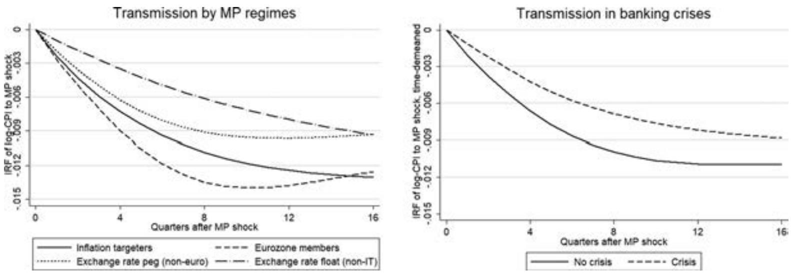


Notes: Response of log-CPI to a 1 pp monetary policy interest rate shock, estimated within a VAR model and identified using sign restrictions.

Figure 2 shows the evolution of impulse responses in several selected countries. The Czech Republic is chosen to represent (historically) developing inflation-targeting countries, the United Kingdom represents a developed inflation-targeting economy, Germany represents a major euro-area economy, and the United States represents the world's leading economy. The development of monetary transmission in these four countries represents the overall trends in the last decades. The transmission from short-term interest rates to prices became stronger, and the duration between the initial shock and the peak response in CPI (where there is one) became shorter.

Figure 3 (left panel) illustrates the different strength and speed of monetary transmission under different monetary policy regimes. The figure presents a typical (mean) impulse responses of price level to 1 pp monetary policy restriction for countries operating under inflation targeting, members of the euro area, non-euro pegged exchange rate regimes, and non-inflation-targeting floaters. As the monetary

Figure 3. IRFs of CPI to an MP Shock under Different MP Regimes and In/Out of Banking Crises



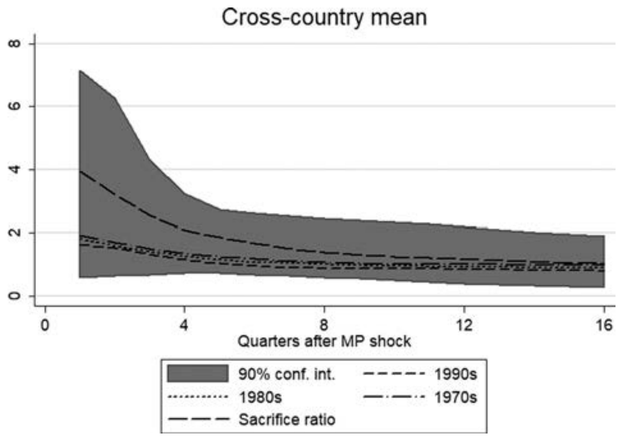
Notes: Response of log-CPI to a 1 pp monetary policy interest rate shock, estimated within a VAR model and identified using sign restrictions.

policy transmission has changed over time (as illustrated in figure 1), and, at the same time, countries were adopting inflation targeting and entering the euro area, spurious relationships are likely to arise. We control for the time trends here by subtracting time effects (cross-country mean for each time period) from the values of IRFs in respective periods. Still, the differences come from both the cross-country variation and the time variation, as there have been changes of monetary policy regimes during the sample period.

Interestingly, transmission of (non-systematic) monetary policy is the strongest in euro-area countries, even after controlling for the time effects of generally stronger transmission in more recent time periods. This finding may not be that surprising given that euro-area members face common monetary policy which may not be fully suited for the needs of individual economies. Therefore the deviations from country-specific, idiosyncratic monetary rules (represented by the VAR equation with interest rate as the explained variable) are generally larger and more frequent. As Matthias (2007) and Castelnuovo (2012) show, when the variance of shocks is large, their identification using sign restrictions is better. Another explanation may be that euro-area countries are among the ones with the most developed financial sectors in the sample, which may be related to a better functioning of transmission.

Figure 3 (right panel) illustrates the mean IRFs for countries going through a banking crisis. Again controlling for the time effects,

Figure 4. Evolution of the Monetary Policy Sacrifice Ratios over Decades

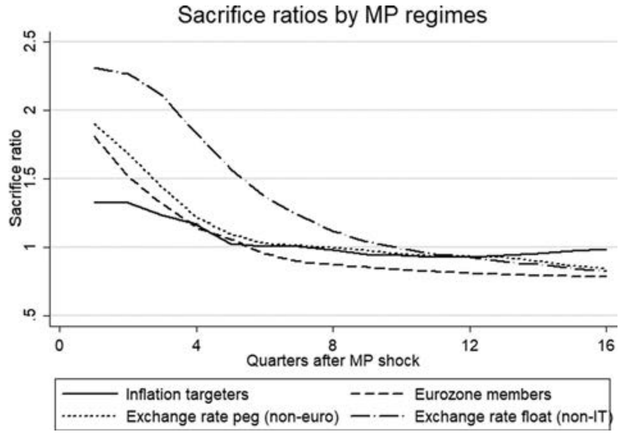


Notes: Sacrifice ratio is defined as a share of IRFs of output and CPI in percentage points to a 1 pp monetary policy interest rate shock, estimated within a VAR model and identified using sign restrictions. Confidence bands are derived from the cross-country sample distribution, and do not explicitly reflect parameter or model uncertainty.

the figure shows that during banking crises the transmission is marginally weaker. It is important to note that the presented typical responses for different regimes do mix two sources of variation coming from different dimensions: cross-country variation and time variation. For example, transmission can become weaker when a country enters a banking crisis, but it may be stronger in countries which generally experience crises more frequently. The presented figures are, pooling the two dimensions together. As the identification of pure cross-country effects is a rather difficult task because of possibly many omitted variables (unobserved country-specific endogeneity), more reliable relationships can be obtained from the time variation in each. To distinguish between the two dimensions of variation, I run both between-effects and fixed-effects panel estimators in the following section.

Figure 4 illustrates the evolution of the sacrifice ratios of monetary policy over the last decades. As defined earlier, the sacrifice ratios show how much output (in %) on average has to be sacrificed

Figure 5. Sacrifice Ratios under Different MP Regimes



Notes: Sacrifice ratio is defined as a share of IRFs of output and CPI in percentage points to a 1 pp monetary policy interest rate shock, estimated within a VAR model and identified using sign restrictions.

to induce a 1 percent decrease in price level at a desired horizon, given the estimated impulse response functions of (real) log-GDP and log-CPI to a monetary policy shock. In other words, the sacrifice ratios measure the output costs of disinflation. Although the changes are again too small to be significant, the results illustrate that the monetary policy sacrifice ratios might have decreased since the 1970s. Finally, figure 5 shows that the sacrifice ratios are marginally lower for inflation targeters at the short horizons. Possible determinants of lower sacrifice ratios are also analyzed in the next section.

It is important to relate these findings to the literature on the slope of the Phillips curve. Several influential studies have suggested that the Phillips curve has flattened (Simon, Matheson, and Sandri 2013; Blanchard, Cerutti, and Summers 2015), partly relating this phenomenon to better anchoring of short-term inflation expectations. This may suggest that monetary policy needed to do more to affect inflation and that sacrifice ratios have increased.[8]

[8]There are several methodological questions regarding the findings of declining Phillips-curve slope, often related to the single-equation estimation and the

Taking a closer look, this does not directly contradict findings of this paper.[9] First recall that sacrifice ratios here are defined as the ratio of the impulse responses of output and inflation. The finding of decreasing sacrifice ratios rather indicates a growing importance of other channels of monetary policy (than the standard interest rate channel), those affecting inflation directly with a relatively weaker effect on economic activity. The exchange rate channel is one of the candidates, as are other asset price channels (such as housing, directly reflected in CPI measures of many countries). Another one may be the expectation channel, which affects inflation directly through managing expectations without the need to force changes in output.

To sum up, the above-presented findings indicate gradual, structural improvements in the strength and efficiency in monetary policy transmission, which can be related to changes in monetary policy frameworks. This process may be disrupted or delayed by banking crises.

4. Results: Factors Associated with the Characteristics of Monetary Transmission

Finally, I examine the cross-country and within-country variation in using panel regressions. While the above-presented IRF plots mix these two dimensions of variation together, in this section I show that when controlling for country fixed effects, the results can be somewhat different.

In the fixed-effects regressions the following equation is estimated:

$$IRF^P(h)_{i,t} = \alpha_i^{FE} + \beta^{FE} X_{i,t} + \varepsilon_{i,t}^{FE}, \tag{1}$$

possible effects of omitted variables. As Jarocinski and Bobeica (2017) suggest, the “missing inflation” puzzles attributed to flattening Phillips curves disappear once using a VAR with full endogenous relationships. Further, other studies have suggested that Phillips curves may not have flattened if using different measures of slack.

[9] Additionally, the effects presented in this paper are conditional on nonsystematic monetary policy shock, which may be substantially different from unconditional relationships such as the Phillips-curve slope.

where $X_{i,t}$ captures possible explanatory variables for the strength of monetary policy transmission. The fixed-effects regression measures the correlation of within-country variation in explanatory variables with the cumulative response of price level to a monetary policy shock. I focus on the typical monetary policy horizon and present the results for $h = 6$.

The between-effects regression explores the cross-country variation, which is filtered out in the above fixed-effects regression. The between-effects equation is the following:

$$IRF^P(h)_i = \alpha^{BE} + \beta^{BE} X_i + \varepsilon_i^{BE}, \tag{2}$$

where both the left- and the right-hand-side variables are within-country averages. The between-effects regression may be largely influenced by the problem of omitted variables, as there are possibly many country-specific factors not explicitly included in the regression. I believe that there is more information value in the fixed-effects model which filters out this time-invariant endogeneity, while the coefficients of the between-effects model can be interpreted rather as correlations.

It is important to note that the dependent variable is estimated and comes with a potentially large measurement error into the regression. This does not bias the estimates (as opposed to the estimated explanatory variable) but may pose inference problems, as the errors might be heteroskedastic. To account for this, Lewis and Linzer (2005) for most applications suggest using robust standard errors, e.g., the standard Hubert-White. Additionally, I use robust standard errors clustered at the country level, as the errors might well be correlated within country.

The results presented in table 1 suggest that when a country adopted an inflation-targeting regime of monetary policy, the transmission of monetary policy to price level became stronger by 0.1–0.2 pp in a response to a 1 pp shock to monetary policy rate. For comparison and the evaluation of the economic significance of this effect, as figure 1 shows, the mean cumulative response at the monetary policy horizon has amounted to 1 pp. The relationship between inflation targeting and stronger transmission can be attributed to an increased transparency, more careful communication, and increased credibility of monetary policy, typical features of an

Table 1. Correlates of the Cumulative Response of Inflation to an MP Shock after Six Quarters

	FE	BE	FE Full	BE Full
InflTargeters	−0.176***		−0.137***	−0.0172
	(−4.52)		(−3.81)	(−0.06)
DomPrivCredit	−0.00151***		−0.00132***	−0.000994
	(−4.12)		(−2.84)	(−0.41)
BankCrisis	0.0725***		0.0759***	0.862
	(3.69)		(3.58)	(0.99)
Eurozone		−0.844***	−0.0152	−0.568
		(−3.25)	(−0.47)	(−1.41)
Openness		−0.00353*	−0.00139	−0.00208
		(−1.75)	(−1.27)	(−0.83)
MktCapitalization			6.07e-16	1.18e-14
			(0.07)	(0.21)
GovtDebt			0.000507	0.00248
			(0.69)	(0.71)
Constant	−0.579***	−0.364*	−0.564***	−0.666*
	(−17.05)	(−2.02)	(−6.34)	(−1.78)
Observations	3,256	3,284	2,631	2,631
Adjusted R^2	0.274	0.262	0.194	0.055

Notes: t-statistics are in parentheses. The dependent variable is the cumulative response of inflation in percentage points to a 1 pp MP shock. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

inflation-targeting regime. Transparency has been found crucial for monetary transmission also in the empirical study by Neuenkirch (2011), while theoretical work of Amato, Morris, and Shin (2002) emphasized the role of public information for coordination. For a survey on the evolution and the role of central bank communication, see Blinder et al. (2008).

Further, the stress in the banking sector seems to disrupt the functioning of monetary policy transmission. The occurrence of banking crises shows a significant relationship to the weaker effect of monetary policy interest rate shocks on prices. In the periods of banking crises, the response of price level to a 1 pp monetary policy shock has been lower by about 0.05–0.1 pp on average. This is likely because of disrupted pass-through from the policy rates to client interest rates on loans, caused by elevated risk premiums on both interbank market and client loans. The disruptions to the monetary policy transmission mechanism related to financial stress (Adrian and Shin 2009) seem to overweight the potential amplification effects of the financial accelerator, which would predict stronger transmission when frictions in financial markets are more substantial (which is likely during banking crises). On the other hand, a higher ratio of domestic private credit to GDP is related to stronger monetary policy transmission, as more leveraged economies respond more strongly to changes in interest rates through credit and balance sheet channels, while also the wealth effects of interest rate changes are larger. This finding is in line with the suggestions of the financial accelerator literature (Carlstrom and Fuerst 1997; Bernanke, Gertler, and Gilchrist 1999) and the related empirical studies (Ciccarelli, Maddaloni, and Peydró 2013).[10]

The fixed-effects regression using the full set of explanatory variables (third column in table 1) shows that other variables are of lower relevance.

The cross-country variation suggests that more open economies (with a higher share of the volume of trade to GDP) seem to feature stronger monetary policy transmission, which is likely an effect of a stronger exchange rate channel. Euro-area members have also seen a stronger transmission, which may be an effect of country-specific unobserved factors, as euro-area countries are among the most developed financial sectors in the sample.

Table 2 shows the correlates of monetary policy sacrifice ratios with the explanatory variables. Notably, inflation targeting is shown to be related to more favorable sacrifice ratios, i.e., lower output losses are needed for disinflation. Again, this can be attributed to higher credibility and transparency related to inflation targeting, although a similarly favorable (albeit slightly weaker) result applies to euro-area members. In addition to a generally weaker monetary policy transmission in banking crises, the analysis shows that banking stress (both the crisis occurrence and the increased

[10]It is important to note that the presented effects might come from two sources, namely change in monetary policy strategy or a structural change in the economy/financial sector, affecting the transmission mechanism. Ciccarelli and Rebucci (2006) explore this distinction while analyzing changes in transmission after the establishment of EMU. However, this distinction would require a different modeling strategy than I use here.

Table 2. Correlates of the MP Sacrifice Ratios at the Horizon of Six Quarters

	FE	FE Full	BE Full
InflTargeters	−0.108***	−0.0862***	−0.863
	(−4.95)	(−4.57)	(−0.96)
Eurozone	−0.0507***	−0.0301	−1.307
	(−3.75)	(−1.62)	(−1.11)
BankCrisis	0.0865	0.0521**	0.221
	(1.46)	(2.31)	(0.09)
MktCapitalization		−2.41e-16	−8.17e-14
		(−0.08)	(−0.49)
DomPrivCredit		0.0000692	0.0000552
		(0.27)	(0.01)
GovtDebt		0.00217	0.00314
		(1.32)	(0.31)
Openness		−0.000140	−0.00817
		(−0.33)	(−1.11)
Constant	1.452***	1.319***	2.469**
	(147.19)	(11.98)	(2.26)
Observations	3,311	2,631	2,631
Adjusted R^2	0.085	0.152	−0.117

Notes: t-statistics are in parentheses. *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$.

non-performing loans ratio) is related also to higher output costs of monetary-policy-induced disinflation. As these effects are symmetric, this result also offers an optimistic interpretation: during banking crises, monetary policy can help to recover the real economy using monetary stimulus without increasing inflation much.

5. Concluding Remarks

This paper has mapped the monetary policy transmission across time and space. Taking an aggregate view on the transmission from monetary policy interest rates to price level, it has documented the role of financial-sector characteristics and the monetary policy regime on the strength and efficiency of monetary policy transmission. The aggregate approach allows for the quantification of the effects.

I have constructed a panel of time-varying impulse response functions for thirty-three countries, with the time span ranging from 1970 to 2010 where the data permitted. Estimating Bayesian TVP-VAR models with the same specification for all countries and identifying the monetary policy interest rate shock using a mixture of short-term and sign restrictions, I have obtained the time-varying impulse responses of price level and GDP to a monetary policy shock for each country in the sample. The time-varying model suggests that the transmission of monetary policy has strengthened over time and the lags of transmission became shorter, while the monetary policy sacrifice ratios have gradually decreased.

Further, I have analyzed the possible determinants of the strength of monetary policy transmission. Exploiting both the cross-country and the within-country variance using panel regressions, the role of various financial and institutional characteristics of economies for the strength of monetary policy transmission has been examined. The results suggest that inflation targeting is related to stronger transmission and more favorable sacrifice ratios of monetary policy. When a country adopted inflation targeting, the response of prices to a 1 pp policy interest rate shock became on average stronger by 0.1–0.2 pp. On the other hand, stress in the banking sector is related to disrupted monetary policy transmission and higher sacrifice ratios. In a banking crisis, the response of CPI to a 1 pp shock was lower by about 0.05–0.1 pp. Further, a higher ratio of domestic private credit to GDP is related to stronger transmission, as a higher leverage ratio implies that interest rate changes have a larger impact through the credit and balance sheet channels. Finally, more open economies feature stronger responses to monetary policy shocks, likely due to the exchange rate channel.

Finally, note that the observed relationships cannot be interpreted as causalities, as various forms of endogeneity may still be present even when time and country fixed effects are controlled for. Further work addressing the endogeneity issues, as well as exploring the quantitative implications of financial and institutional factors for transmission in a more structural framework, could be valuable topics for future research.

Appendix 1. Bayesian TVP-VAR Estimation: Technical Annex

We use one lag in the VAR system, to keep the space of estimated (time-varying) coefficients reasonably sized. The number of Gibbs sampler iterations varies across countries. Those with longer time series typically need more draws for reaching convergence, as there are longer paths of time-varying parameters (or, equivalently, the respective errors) to be estimated. Generally we have used 10,000 burn-in draws and 2,000–10,000 effective draws depending on whether a reasonable degree of convergence was reached. Convergence was diagnosed using autocorrelation functions of the Markov chain and Raftery and Lewis (1992) convergence diagnostics. The priors on the parameters of the model and the hyperparameters are set according to Primiceri (2005). Specifically,

$$\beta_0 \sim \mathcal{N}(\hat{\beta}_{OLS}, 4.\text{var}[\hat{\beta}_{OLS}])$$
$$A_0 \sim \mathcal{N}(\hat{A}_{OLS}, 4.\text{var}[\hat{A}_{OLS}])$$
$$\log \sigma_0 \sim \mathcal{N}(\log \hat{\sigma}_{OLS}, 4I_n),$$

i.e., the prior distributions are normal for matrices Z and A and log-normal for vector σ, where the means are the OLS estimates of Z, A, and σ from a time-invariant VAR. Prior variances on Z and A are set at four times the variances from a time-invariant VAR, while the prior variance on $\log \sigma$ is four times the identity matrix. The priors on the hyperparameters are set as follows:

$$Q \sim \mathcal{IW}(k_Q^2.\tau.\text{var}[\hat{Z}_{OLS}], \tau)$$
$$W \sim \mathcal{IG}(k_W^2.(1 + \dim(W)).I_n, (1 + \dim(W)))$$
$$S_\tau \sim \mathcal{IW}(k_S^2.(1 + \dim(S_\tau)).\text{var}[\hat{A}_{\tau,OLS}], (1 + \dim(S_\tau))),$$

where τ is the size of training sample, and S_τ and $\hat{A}_{\tau,OLS}$ are the corresponding parts of the respective matrices. We use the whole time series as the training sample. The prior hyperparameter mean factors are set to $k_Q = 0.01$, so that we attribute 1 percent of the uncertainty around the time-invariant OLS estimate to time variation. Further $k_W = 0.01$ and $k_S = 0.1$, we again follow Primiceri (2005) here.

Appendix 2. Additional Results

Figure 6 illustrates the impulse response functions of remaining variables to a monetary policy shock, including their estimated evolution over time and the confidence bands. Figure 7 shows three-dimensional plots of the impulse response functions of log-CPI to a 1 pp monetary policy shock in selected countries.

Figure 6. IRFs of Log-Output, Interest Rate (pp), and Log-Effective Exchange Rate to a Monetary Policy Shock (normalized to 1 pp to interest rate on impact)

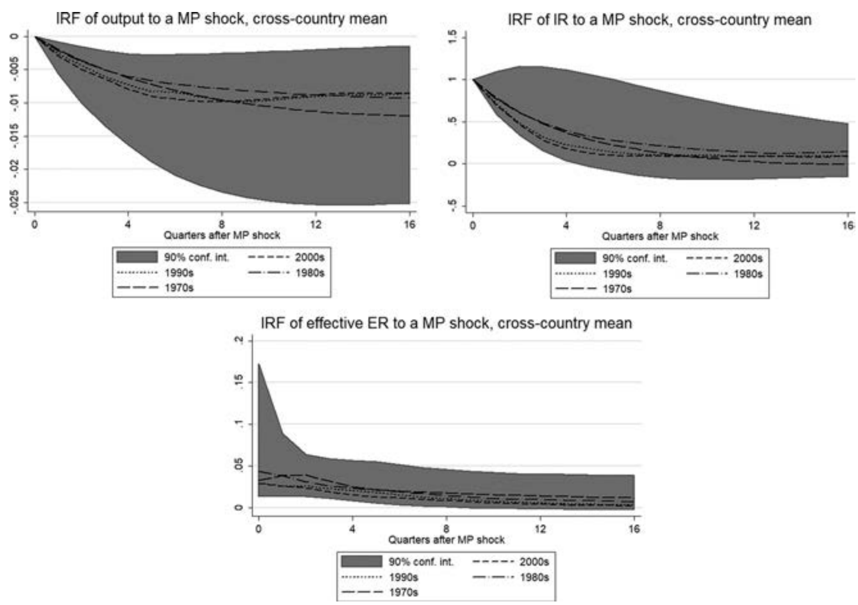
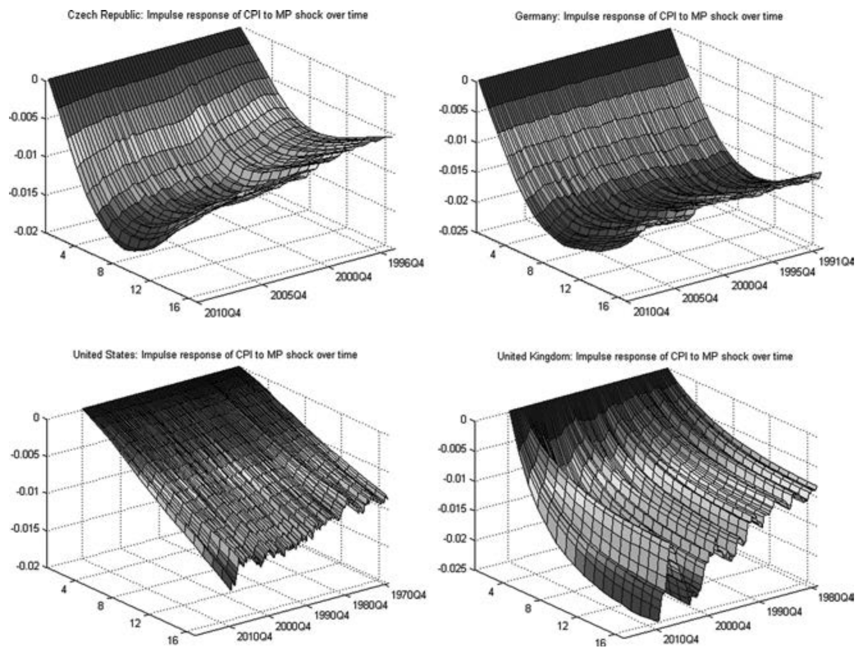


Figure 7. Evolution of IRFs of CPI to an MP Shock in Selected Countries



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Banks' Wealth, Banks' Creation of Money, and Central Banking*

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Banks are special in that their liabilities are widely accepted as a means of payment, thereby often needed by real sectors to obtain resources. This paper studies this interaction between the banking sector and real sectors on competitive markets and the policy response of the central bank to market inefficiency, which is determined by the aggregate wealth of banks. In the circumstance of a credit crunch, the central bank improves efficiency by allowing banks to borrow its fiat money at zero interest up to a limit. This policy bears the flavor of quantitative easing policies (QE). It produces real effects in the absence of surprises and nominal rigidity. The mechanism in which it works depends on a difference in nature between bank-created money and fiat money. Furthermore, this policy, while expanding the money supply, induces deflation under the positive productivity shock. Lastly, this paper explains when interest rate policy and capital adequacy regulation are among the optimal policies within a unified model.

JEL Codes: E5, E65, G21.

"The community cannot get rid of its currency supply... The 'hot potato' analogy truly applies. For bank-created money, however, there is an economic mechanism of extinction as well as creation, contraction as well as expansion."

James Tobin (1963)

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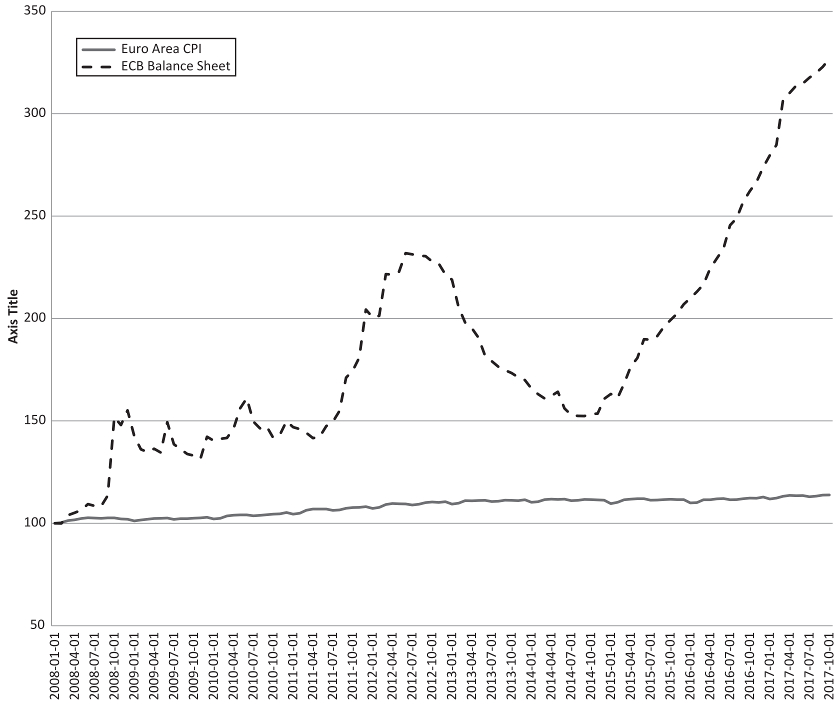
1. Introduction

Commercial banks are special in that their liability, especially that in the form of demand deposit, is *widely* accepted as a means of payment,[1] whereas rarely so are the liabilities of non-bank firms or households. Due to this difference, real-sector firms often need to borrow a bank's liability (usually called *money* in everyday language) as a means of paying for resources that they want. Banks' decisions on the price and quantity of money in lending and the competition between them, therefore, have a profound impact on the economic activity of real sectors, which, inversely, affects the decisions of and competition between banks. Furthermore, often, the central bank's policy produces effects by affecting banks' lending decisions and is based on its ability to create an alternative means of payment, namely fiat money. While money creation by banks is well known and widely introduced in macroeconomic textbooks, these interactions between real sectors, the banking sector, and the central bank in relation to means of payment have not been studied much yet,[2] a relationship which this paper makes an attempt to understand. Specifically, it presents a general equilibrium analysis of money creation by banks and of how certain policies of the central bank improve efficiency over the market allocations. This analysis

[1]Often lay people perceive that depositing cash is to let the bank store the money deposited. However, this perception is wrong. Depositing is an exchange of cash to the bank's liability, which is what the depositor owns with the bank account and, when he makes a purchase with the account, is what he uses for the payment.

[2]The interaction between the banking sector and real sectors is studied in the literature on financial intermediation, where money creation by banks is not a concern; see Gorton and Winton (2003) for a survey. A strand of literature uses search-matching frameworks—see, e.g., Cavalcanti, Erosa, and Temzalides (1999) and Williamson (1999)—to examine how and when certain privately issued claims circulate as a means of payment, but has difficulty accommodating banks' decision on the price and scale of lending and competition between them. Lastly, New Keynesian literature and the literature that uses frameworks of cash in advance (CIA) both study monetary policy. However, the former abstracts banks completely, while the latter, when concerned with money creation by banks, lets the scale of banks' lending be pinned down either by a binding reserve constraint (see, e.g., Goodfriend and McCallum 2007) or by an exogenous rule of holding excessive reserves (see, e.g., Chen 2018 and Mishkin 2016), which leaves little role for banks' own decisions.

Figure 1. The Balance Sheet of the European Central Bank (ECB) and the Consumer Price Index in the Euro Area over Jan. 1, 2008–Oct. 10, 2017 (with Jan. 1, 2008 = 100)



Source of the Data: www.https://fred.stlouisfed.org/.
Note: The three hikes in the ECB's balance sheet during the period are associated with only slight price increases, if anything at all.

results in new insights on the mechanism in which the quantitative easing policy (QE) works, based on which this paper offers an explanation for the observation that QE, while causing enormous monetary expansion, is associated with low inflation or even deflation pressure in certain economies. One of them is the euro area, as is illustrated in figure 1.

The model economy of the paper is populated by workers, entrepreneurs, and banks. Workers can produce the consumption good, corn, in autarky, or work for entrepreneurs. The specialty of banks is modeled with the assumption that workers accept banks' promises to pay, but not entrepreneurs', as a means of wage payment. At date 0, therefore, entrepreneurs first borrow banks' promises to pay—more specifically, notes that read like "X bank promises to pay the bearer 10 kilograms of corn tomorrow"—and then use these notes to hire workers. As a result, entrepreneurs owe a debt to the lender banks and banks owe a debt to the workers. At date 1, entrepreneurs produce corn and use it to settle their debts to the banks. Banks then use this repayment and their own corn, which is stored over time and represents their wealth, to redeem their notes from the workers by fulfilling the promises written on the notes.

In this economy, the real resources are workers' labor and entrepreneurs' capital, and efficiency is measured with the number of workers that entrepreneurs hire. Banks matter, however, because entrepreneurs need to borrow banks' liabilities to hire workers. How much banks lend in terms of real value determines how many workers entrepreneurs hire and hence economic efficiency. The aggregate lending of banks is in turn determined by competition between them. What banks supply is a means of payment, which is a homogenous good. They thus engage in Bertrand competition. Moreover, what a bank lends is its liability. Hence the scale of its lending is anchored to its wealth by a borrowing constraint.[3] Put together, banks are engaged in Bertrand competition with a limited capacity à la Kreps and Scheinkman (1983). If the borrowing constraint is binding, banks' aggregate wealth determines the quantity of money supplied to entrepreneurs, and hence efficiency. If this wealth is below a threshold, the money supply is inadequate and so is the number of workers that entrepreneurs hire.

This problem of meager bank wealth depressing economic activity has been diagnosed in many studies such as Gertler and Kiyotaki (2010) and Holmstrom and Tirole (1997). However, these studies are not concerned with means of payment. Hence they do not consider the possibility of a remedy with the central bank (CB) issuing fiat money. By contrast, in this paper the CB can offer a remedy by allowing banks to borrow its fiat money at zero interest up to a

[3]This borrowing constraint could be due to a moral hazard friction à la Gertler and Kiyotaki (2010) or Holmstrom and Tirole (1997), or due to the risk-shifting problem à la Jensen and Meckling (1976).

bound. We show that although the economy lasts for only two periods, in one equilibrium the fiat money circulates, flowing through banks to entrepreneurs, who thereby hire more workers. That is, the policy eases the constraints imposed upon their economic activity by an inadequate supply of bank-created money. Hence, the policy is called *quantitative easing policy*, or QE.

QE enables banks to raise lending capacity without violating the borrowing constraint, while backed by the same amount of wealth. It does so because fiat money is different in nature from bank-created money. A central bank never commits to redeem its fiat money at a specified value, whereas bank-created money is the bank's liability, which it commits to redeem at a pre-specified value and thus bears a real obligation of repayment.[4] It is to this difference in nature that Tobin (1963) alludes above. As fiat money is not redeemable, its value freely adjusts with the state of the economy. In particular, its value falls in the event of the negative productivity shock, which means *inflation*. Inflation reduces the real value of banks' liability to the CB and thus reduces their borrowing constraint, giving them room to increase lending.[5]

While QE induces inflation in the event of the negative shock, it induces deflation in the event of the positive shock. This result might partly explain the aforementioned observation that in some economies QE causes a great monetary expansion on the one hand and is accompanied by or followed by lingering low inflation or even deflation pressure on the other hand, which is a puzzle if considered from the point of view of quantitative theory of money. A key difference in this paper is that the money created with QE is utilized to expand real economic activity, resulting in a rise in the output of goods, which keeps the price down. Indeed, all the QE-created money

[4]In the model economy, this fact is straightforward because banks use corn, the real good, to redeem their liability. Even if they redeem it with fiat money, which is typically the case in modern times, they still need to spend real resources to obtain it. Hence bank-created money still bears real obligations of repayment. For more discussion, see the remark in section 5.

[5]The point that monetary policy can help banks by affecting the real value of their liabilities is also considered by Diamond and Rajan (2006). The way in which fiat money circulates in the model economy is also in Allen and Gale (1998). However, these papers are not concerned with banks lending their liabilities to real sectors to be used as means of payment, nor with the inefficiency related to banks' wealth, nor with the quantitative easing policy.

is utilized this way if the scale of QE is below a threshold. If it is beyond it, the excess is not put into circulation, we also find. This finding partly explains another related observation, that in some economies a substantial fraction of the monetary bases created with QE is not lent out but stays in banks' reserve accounts.

This paper is in line with nascent literature that examines banks' specialty of their liability being accepted as a means of payment. Most closely related is Donaldson, Piacentino, and Thakor (2018). Both their paper and this one consider how banks' issuance of means of payment affects the real economy as well as policy implications. The two papers, however, have different focuses. Their paper explains how this specialty of banks is derived from their superior technology of warehousing, while this paper emphasizes the importance of banks' wealth for economic efficiency. Also the policy implications are different. Their paper shows that, contrary to the received wisdom, a higher central bank rate could raise bank lending, while this paper considers QE. Jakab and Kumhof (2015) describe in detail how banks create money with double bookkeeping. They focus on the quantitative implications of this facet of banking in a full-fledged dynamic stochastic framework, but are not concerned with economic efficiency and policy responses, on which this paper focuses.

Since the recent crisis, many studies have examined QE; see Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), among others. Their diagnosis on the source of the problem is shared by this paper: banks' wealth is too low, causing inadequate lending.[6] However, those studies model banks not as issuers of means of payment but as intermediaries of trading real goods. As a result, in those studies what the government issues must be backed by tax incomes and is essentially the sovereign debt, whereas what the central bank issues in this paper is purely nominal. Furthermore, the mechanism by which QE works is different. In those studies, it works by transferring wealth to banks, whereas in this paper it works by reducing the real value of banks' liability via inflation.

[6]This is also true in a study by Brunnermeier and Sannikov (2013), where money plays the role of a saving instrument, as in Samuelson (1958), rather than the role of means of payment.

Other cases of inefficiency in connection with private issuance of means of payment are considered by Hart and Zingales (2015), Monnet and Sanches (2015), and Stein (2012). In Hart and Zingales (2015) and Stein (2012) inefficiency is driven by fire-sale externalities. In Monnet and Sanches (2015) inefficiency arises because banks offer a return rate on the liability side that is lower than the inverse of time discount. Besides this difference in the source of inefficiency, none of those studies considers the circumstance of banks under-lending or the policy of QE, as this paper does.

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 analyzes a benchmark case where banks face no borrowing constraint and are thus subject to unfettered forces of competition. This analysis not only examines the bank competition in the purest form but also bears relevance to the historical banking. Section 4 introduces the borrowing constraint of banks. Section 5 studies QE, and section 6 studies interest rate policy and capital adequacy regulation. Section 7 concludes. All proofs are relegated to the appendix.

2. The Model

The economy has one storable good, corn, used as the numeraire, and lasts for two days. Contracting and production occur at $t = 0$, yielding and consumption at $t = 1$. There are N banks, N^2 entrepreneurs, and N^3 workers, where N is a large number (later, in section 5, the central bank will be introduced).[7] Thus, banks are in perfect competition and each serves a large number of entrepreneurs; and there are more workers than entrepreneurs can hire. All agents are risk neutral and protected by limited liability.

Workers either produce w kilograms (kg) of corn in autarky or are hired by entrepreneurs, who each have h units of human or physical capital. If an entrepreneur hires L workers at $t = 0$, then his project yields at $t = 1$

$$y = \widetilde{A} h^{1-\alpha} L^{\alpha},$$

[7] A setting with the same feature is to be found in Wang (2015), where a continuum of $[0, 1]$ and $[0, 1] \times [0, 1]$ instead of N and N^2 is used. These two representations are equivalent.

where $0 < \alpha < 1$. Without losing any generality, normalize $h = 1$. Productivity, $\widetilde{A}$, is subject to a common shock. At $t = 0$, it is common knowledge that the good state with $\widetilde{A} = \overline{A}$ occurs with probability $q > 0$ and the bad state with $\widetilde{A} = \underline{A}$ occurs with probability $1 - q > 0$. Let $A_e \equiv q\overline{A} + (1 - q)\underline{A}$ denote the mean. Assume the following:

$$0 < \underline{A} < A_e \alpha. \tag{1}$$

As there are more workers than can be hired by entrepreneurs, in equilibrium workers are indifferent in working for the latter or in autarky. Therefore they earn a real wage of w. This wage is independent of economic activity of the other sectors, which gives a convenience for exposition.

Banks each have G units of corn, where a unit is defined as N kg and used wherever banks are concerned. Banks supply no real resources for corn production. What makes banks relevant is due to the following assumption.

ASSUMPTION 1. *Workers do not accept entrepreneurs' promise to pay but banks' as a means of wage payment.*

This assumption captures the specialty of banks explicated in the Introduction. According to Kiyotaki and Moore (2001), this difference between banks and entrepreneurs arises because the former has stronger commitment power than the latter.

Due to this assumption, entrepreneurs cannot hire workers at $t = 0$ with a promise to pay them later at $t = 1$. To hire workers, entrepreneurs need to borrow banks' promise to pay them at $t = 1$. We assume that banks have no difficulty enforcing repayment from entrepreneurs and hence this borrowing is feasible. To fix the idea, suppose that banks' promises to pay are printed on notes. That is, a note issued by a bank reads that this bank promises to pay the bearer of this note with a certain quantity of corn at $t = 1$.[8] This quantity is the note's *face value* or *par value*.

[8] In the model economy banks promise to pay the real good in redeeming their liability, whereas in modern times, they typically redeem it with fiat money, the central bank's issues. Other instances of claims on real goods used as a means of

ASSUMPTION 2. *The face values of banks' notes cannot be contingent on the realization of* $\widetilde{A}$.

This assumption captures the observation that in real life a private security that serves as a means of payment—such as demand deposit, promissory notes, cheques, or trade credit—commonly bears fixed claims and is of debt, and is rarely a contingent claim.[9]

Besides the friction of payment, entrepreneurs lack commitment power in another dimension.

ASSUMPTION 3. *Entrepreneurs are unable to make commitment on the scale of their projects in terms of the number of workers they will hire.*

This friction is *real* in the sense that it is unrelated to means of payment. In the absence of the friction, a bank would be willing to lower the interest rate, denoted by r, to a borrower entrepreneur who commits to a smaller scale, because thereby his project delivers a higher average return rate. This, however, would give entrepreneurs an incentive to borrow from multiple banks, each in a small amount and thus at a favorable rate. The presence of the real friction, therefore, is justified if entrepreneurs cannot be prevented from doing so. The importance of its presence is that it engenders a wedge between the first- and second-best allocations, as will be shown.

Due to the friction, a bank posts a single interest rate r for loans of any size E rather than a menu of $r(E)$. A loan contract is represented by a profile of (E, r): at $t = 0$ the entrepreneur borrows the bank's notes of overall face value E, and at $t = 1$ he is obliged to pay $E(1 + r)$ kg corn back to the bank.

The timing of events is as follows. At $t = 0$, each bank posts the aggregate face value of notes that it will issue, D, and the interest rate that it will charge, r. Observing all these offers, each entrepreneur then chooses one bank to go to and asks to borrow face value

payment incude Hart and Zingales (2015) and Williamson (1999). This abstraction is harmless in the baseline model where inflation is not addressed. Where it is, as in section 5 below, the abstraction helps clarify the mechanism by which QE works.

[9]For the reason why that is the case, see Dang, Gorton, and Holmstrom (2012) and Gorton and Pennacchi (1990).

E of the bank's notes. If one bank's notes are over-demanded, only a fraction of the entrepreneurs have their demand met. Entrepreneurs then use borrowed notes to hire workers and start the production. Banks store their G units of corn.

At $t = 1$, entrepreneurs produce corn. They either repay $E(1+r)$ kg corn to the lender banks or default. In that case they give all their output of corn to the banks. After receiving repayments from the entrepreneurs, the sum total denoted by $\widetilde{Y}$, banks redeem notes from workers. If $\widetilde{Y}+G \geq D$ to a bank, its notes are redeemed at par. If $\widetilde{Y} + G < D$, the bank defaults and the notes are redeemed at a fraction $(\widetilde{Y} + G)/D$ of their par values. Finally, the agents consume the corn in their possession.

In anticipation of the possibility of default, at $t = 0$ a bank's notes are discounted with a factor of

$$\delta = \mathrm{E}_{\widetilde{A}}\left[\min\left(1, \frac{\widetilde{Y} + G}{D}\right)\right]. \tag{2}$$

Moving on to the equilibrium analysis, we figure out two benchmark allocations.

2.1 The First-Best and Second-Best Allocations

Efficiency concerns the number of workers allocated to entrepreneurs. Define the first-best allocation as the number of workers that maximizes the social surplus of projects, which is $A_e L^\alpha - wL$ due to universal risk neutrality and the opportunity cost of labor being w. The first-best allocation is thus

$$L^{FB} = \left(\frac{A_e \alpha}{w}\right)^{\frac{1}{1-\alpha}}.$$

The second-best allocation is defined as the number of workers that entrepreneurs would hire in the competitive equilibrium if the friction of payment (in assumption 1) were absent but the real friction (in assumption 3) remained—that is, if entrepreneurs could hire workers with their own promise to pay, but their wage offer could not be conditional on the scale of their projects. The equilibrium allocation is as follows.

LEMMA 1. *The second-best number of workers that entrepreneurs hire is*

$$L^{SB} = \left(\frac{q\overline{A}\alpha + (1-q)\underline{A}}{w} \right)^{\frac{1}{1-\alpha}}.$$

Obviously, $L^{SB} > L^{FB}$. That is, the real friction induces the entrepreneurs to hire too many workers.[10] This fact engenders a circumstance of banks over-lending, among remedies to which are interest rate policy and capital adequacy regulation, as will be shown in sections 4 and 6.

Below we first analyze the baseline model in which bank issuance is subject to no borrowing constraints nor any other restrictions. This analysis serves two purposes: (i) it studies the competitive equilibrium of money creation by banks in the purest form; (ii) it bears relevance to early periods of banking history.

3. The Least-Fettered Issuance

In the least-fettered issuance, banks *can* finance an unlimited quantity of assets by issuing promises to pay. The *equilibrium* quantity is of course limited, as examined below. We first consider the demand side of the market for banks' notes, then the supply side, and, finally, the meeting of the two.

3.1 The Demand Side of the Market for Notes

Consider a representative entrepreneur who borrows from a bank offering (D, r). If he borrows notes of a face value E, then they are worth δE, where the discount factor δ, as will be shown, is a function of (D, r). With these notes, he hires workers in a number of

$$L = \frac{\delta E}{w}, \tag{3}$$

because they earn real wage w. At $t = 1$, the entrepreneur either repays $E(1 + r)$ of corn to the bank or defaults. Thus, his decision problem is

[10] For a more general analysis of this type of inefficiency, see Wang (2010).

$$\max_{E} \mathrm{E}_{\widetilde{A}}[\max(\widetilde{A}L^{\alpha} - E(1+r), 0)], s.t.(3).$$

LEMMA 2. *For any (w, δ, r), the solution to the above problem satisfies $\underline{A}L^{\alpha} < E(1+r)$. That is, entrepreneurs all default in the bad state.*

This lemma is driven by the assumption that $\underline{A} < A_e\alpha$, which says that the negative shock is severe enough to knock entrepreneurs into default.

At the optimum, the demand of the representative entrepreneur for the notes is

$$E(\delta, r) = \left(\frac{\overline{A}\alpha}{1+r}\right)^{\frac{1}{1-\alpha}} \left(\frac{\delta}{w}\right)^{\frac{\alpha}{1-\alpha}} \tag{4}$$

and the number of workers that he hires and his profit are, respectively,

$$L(R) = \left(\frac{\overline{A}\alpha}{wR}\right)^{\frac{1}{1-\alpha}} \tag{5}$$

$$V(R) = q(1-\alpha)\left(\frac{\overline{A}^{\frac{1}{\alpha}}\alpha}{wR}\right)^{\frac{\alpha}{1-\alpha}}, \tag{6}$$

where

$$R \equiv \frac{1+r}{\delta}. \tag{7}$$

Note that, so defined, R is the *actual* interest rate and measures the cost of borrowing: to obtain a means of payment that is worth 1, the entrepreneur borrows notes of face value $1/\delta$, thus acquiring a debt of $(1+r)/\delta$. Naturally, his scale of hiring and his profit, both of which are real variables, are inversely related to the cost of borrowing, namely, R.

Recall that the efficiency concerns only the number of the workers hired by entrepreneurs, which depends only on the actual interest rate. The efficiency of market equilibrium is thus determined solely by the actual interest rate in equilibrium. Define R^{FB} (R^{SB}) as the value of the actual interest rate at which entrepreneurs hire the first-best (second-best) number of workers, that is, $L(R) = L^{FB}$ (L^{SB}). With (5),

$$R^{FB} = \frac{\overline{A}}{A_e}$$

$$R^{SB} = \frac{\overline{A}\alpha}{q\overline{A}\alpha + (1-q)\underline{A}}.$$

After banks all have posted (D, r), each entrepreneur decides which bank to go to. In the equilibriums of this subgame, an entrepreneur gets the same expected profit, $\widehat{V}$, from any bank that attracts a number of entrepreneurs.[11] As entrepreneurs' profit depends only on the actual interest rate, define $\widehat{R}$ by $V(\widehat{R}) = \widehat{V}$. Then, $\widehat{R}$ is the actual interest rate that prevails on the notes market, conditional on banks' choices of (D, r). Given that there is a large number of banks, any single bank is too small to affect $\widehat{R}$ with its choice of (D, r) and takes it as given when making that choice.

3.2 The Supply Side of the Market for Notes

Consider a representative bank. To attract entrepreneurs, the bank's choice of (D, r) satisfies $V(R) \geq \widehat{V}$ or, equivalently, $(1 + r)/\delta \leq \widehat{R}$. While the interest rate r is directly chosen by the bank, the discount factor of its notes δ is determined by (D, r). It depends on whether the bank ever defaults or not, that is, whether $D > \widetilde{Y} + G$ in some state at $t = 1$. In the good state, $\widetilde{Y} = \overline{Y} = D(1 + r)$ because the entrepreneurs do not default. As a result, the bank does not default. In the bad state, by lemma 2, all the entrepreneurs default and pass the output of their projects on to the bank, of which the aggregate value is given below.

[11] By offering (D, r), which determines δ, each bank chooses an actual interest rate $R = (1 + r)/\delta$. No entrepreneur goes to a bank offering $V(R) < \widehat{V}$ when he can get $\widehat{V}$ elsewhere. On the other hand, if a bank offers $V(R) > \widehat{V}$, it induces over-demand for its notes (which is never optimal), so an entrepreneur coming to it is served with such a probability l that $l \cdot V(R) = \widehat{V}$.

LEMMA 3. *The aggregate value of the bank's loans in the bad state is*

$$\underline{Y} = \frac{\underline{A}}{\overline{A}\alpha} \times D(1+r).^{12} \tag{8}$$

In the bad state, the bank does not default if and only if $D \leq G + \underline{Y}$, which, with $\underline{Y}$ given by (8), is equivalent to

$$D \cdot \left(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha}\right) \leq G. \tag{9}$$

On the left-hand side is the loss made by lending in the bad state, $D - \underline{Y}$. Thus, the inequality says that banks will stay solvent in the bad state if and only if their lending scale, D, is not too large relative to their wealth, G, so that the loss from loans can be absorbed by the wealth.

Substitute the value of $\widetilde{Y}$ given above into (2), and the discount factor of the bank's notes is determined by its choice of (D, r) via

$$\delta(D,r) = \left\{ \begin{array}{c} 1, \text{ if (9) is satisfied} \\ q \times 1 + (1-q) \times (\frac{G}{D} + \frac{\underline{A}(1+r)}{\overline{A}\alpha}), \text{ otherwise} \end{array} \right\}. \tag{10}$$

Now consider the representative bank's decision problem at $t = 0$. Taking into account the possibility of default, its economic profit with the choice of (D, r), denoted by $\Pi(D, r)$, is $E_{\widetilde{Y}} \max(G + \widetilde{Y} - D, 0) - G$.

LEMMA 4.

$$\Pi(D,r) = D\left[\frac{1+r}{R^{SB}} - \delta\right]. \tag{11}$$

Therefore, the profit margin of lending—that is, the profit of lending out a note of face value 1—is $(1+r)/R^{SB} - \delta$. Intuitively, the present value of this note, which is part of the bank's liability, is δ, while the bank's revenue from such lending is $E_{\widetilde{Y}}\left(\widetilde{Y}/D\right) = (1+r)/R^{SB}$.

[12] $\underline{Y} < \overline{Y}$ because $\underline{A} < A_e\alpha$ by assumption and $A_e\alpha < \overline{A}\alpha$.

The representative bank chooses (D, r) to maximize $\Pi(D, r)$ subject to the constraint that it can attract entrepreneurs, that is,

$$\frac{1+r}{\delta(D,r)} \leq \widehat{R}. \tag{12}$$

Given the scale of lending D, the bank wants to charge an interest rate as high as possible so long as it can attract entrepreneurs. Therefore, at the optimum, the constraint (12) is binding.[13] Hence, the bank's optimal choice of r as a function of its choice of D, denoted by $r(D)$, is determined by

$$\frac{1+r}{\delta(D,r)} = \widehat{R},$$

and banks' problem becomes

$$\max_D D\delta(D, r(D)) \times \left[\frac{\widehat{R}}{R^{SB}} - 1\right].$$

As $\delta > q$ always (because the bank always redeems its notes at par in the good state), the following proposition is self-evident.

PROPOSITION 1. *The solution to and the value of the representative bank's problem are*

(i) if $\widehat{R} > R^{SB}$, then $D = \infty$ and $\Pi = \infty$;

(ii) if $\widehat{R} = R^{SB}$, then $\Pi = 0$, and the bank is indifferent to any value of D, with $r = r(D)$;

(iii) if $\widehat{R} < R^{SB}$, then lending makes a loss, and thus $D = 0$ and $\Pi = 0$.

As was shown, the profit margin of lending equals $\delta\left(\widehat{R}/R^{SB} - 1\right)$ and is positive if and only if $\widehat{R} > R^{SB}$. Moreover, if the profit margin of lending is positive, banks obtain $\Pi = \infty$. That is because,

[13] Mathematically, given D, both $\frac{1+r}{R^{SB}} - \delta(D,r)$ and $\frac{1+r}{\delta(D,r)}$ increase with r.

despite their limited stocks of corn, they all have an unlimited lending capacity (i.e., $D = \infty$), due to two reasons: (i) what banks lend out is their promise to pay, essentially word of mouth, which they can infinitely supply; (ii) banks are subject to no constraints on the quantity of supply in the baseline model.

3.3 The Equilibrium: The Second-Best Allocation Attained

The prevailing actual interest rate, $\widehat{R}$, which plays the role of price, clears the market for notes in equilibrium. We define the symmetric equilibrium below and discuss other equilibriums later.

DEFINITION 1. *A profile* $(\{D, r, \delta, E\}; \widehat{R})$ *forms an equilibrium if*

(i) given $\widehat{R}$*, banks' choice of* (D, r) *is optimal and thus given in proposition 1. This choice determines* $\delta = \delta(D, r)$ *through (10);*

(ii) given that all banks offer the same (D, r, δ)*, entrepreneurs go to each bank with the same probability and, by the law of large numbers, each bank receives* N *of them. Their demand for notes,* E*, is optimal, that is,* $E = E(\delta, r)$ *given by (4);*

(iii) the market clears: $D = E$*.*[14]

In any equilibrium, banks neither obtain an infinitely large profit nor abstain from lending, which, by proposition 1, is the case if and only if

$$\widehat{R} = R^{SB}.$$

Therefore, the real allocation in equilibrium is unique and conforms with the second-best allocation, the one that would arise if the friction of payment were absent and entrepreneurs could hire workers with their own promises to pay. Intuitively, what banks supply is a means of payment, a homogeneous good. Furthermore, in this case of unfettered issuance, they all have unlimited capacity. Therefore,

[14] Note that D is in the unit of N kg, while entrepreneurs' demand is denoted with the unit of kg.

they are in Bertrand competition, which annihilates their profit margin. As a result, entrepreneurs overcome the friction of payment at no costs and the real allocation is the one that would arise if the friction were absent, that is, the second-best allocation.

On the nominal side, however, there is indeterminacy. At $\widehat{R} = R^{SB}$, by proposition 1, the profit margin of lending is 0, and individual banks are indifferent to any quantity of issues, although in aggregation their issues exactly suffice for entrepreneurs to hire L^{SB} workers. This indeterminacy leads to a continuum of equilibriums besides the symmetric one defined above. In the symmetric equilibrium, all banks issue the same quantity of notes and their notes are discounted at the same factor; therefore, one bank's notes are perfect substitutes for another's. In asymmetric equilibriums, however, some banks issue more than others despite ex ante being identical, and see their notes more heavily discounted.

PROPOSITION 2.

(i) *In any equilibrium, independent of banks' wealth G, $\widehat{R} = R^{SB}$, the profit margin of bank lending is 0, and the second-best allocation is attained.*

(ii) *In any equilibrium, a fraction of banks default at $t = 1$ upon the realization of $\widetilde{A} = \underline{A}$ if and only if banks' wealth is below a threshold G^*, where*

$$G^* = [(q\overline{A}\alpha + (1-q)\underline{A}) - \underline{A}] \left(\frac{q\overline{A}\alpha + (1-q)\underline{A}}{w} \right)^{\frac{\alpha}{1-\alpha}}.$$

An intuition for result (i) is given above. As for (ii), note that, given that banks are indifferent to any quantity of issues, the quantity of money circulated—that is, the aggregate bank lending—is determined by the demand side, namely entrepreneurs. Thus, the aggregate bank issues are fixed at a quantity exactly sufficient for entrepreneurs to hire the second-best number of workers. With this scale of lending, if banks' wealth, G, is sufficiently low—namely $G < G^*$—then it is insufficient to absorb the loss incurred in the bad state and bank default occurs accordingly.

The baseline mode examined above, besides providing a simple framework to consider money creation by banks on competitive markets, also bears empirical relevance to the early periods of the banking history during which a large number of banks issued their own notes and discounted others', such as the Free Banking Era in the United States (1838–63) and a period between 1750 and 1844 in England.[15] Specifically, the baseline model derives a relationship between the loan interest rate r, the discount factor δ, the leverage ratio $\Lambda := D/G$, and the risks of default $1-q$ of individual banks. First, $\frac{1+r}{\delta} = \widehat{R}$, a constant across banks, by the binding constraint (12). Therefore, the paper predicts that *across banks, the gross interest rate that a bank charges for loans (i.e.,* $1+r$*) is positively related to and solely determined by the discount factor of its notes* δ, *independent of the bank's other attributes.* Second, regarding the discount factor, from (10) and binding (12) it follows that

$$\delta = \left\{ \begin{array}{c} 1 \text{ if } \Lambda \leq (1 - \frac{A}{\overline{A}\alpha}\widehat{R})^{-1} \\ \frac{q+(1-q)\Lambda^{-1}}{1-(1-q)\frac{A}{\overline{A}\alpha}\widehat{R}} \text{ if } \Lambda \geq (1 - \frac{A}{\overline{A}\alpha}\widehat{R})^{-1} \end{array} \right\}. \tag{13}$$

Therefore, *the discount factor of a bank's notes* δ *is inversely related to the bank's default risk* $1-q$[16] *and its leverage* Λ *(which is endogenous).* The first part of this prediction is empirically confirmed by Gorton (1999), who studies the pricing of bank notes during the U.S. Free Banking Era.[17]

The applicability of the baseline model to modern banking is more limited. A fundamental assumption of the model, namely that banks face no constraints in making loans, is probably far from reality nowadays. In particular, due to this assumption, the baseline

[15] For the case of the United States, according to Gorton (1999), thousands of different banks' notes were in circulation, while for the case of England, Cameron et al. (1967) report that in year 1810 there were 783 note-issuing county banks in England.

[16] In the lower branch of (13) $\partial\delta/\partial q$ share the sign of $1 - \frac{A}{\overline{A}\alpha}\widehat{R} - \Lambda^{-1}$, which is positive because in that branch $\Lambda \geq (1 - \frac{A}{\overline{A}\alpha}\widehat{R})^{-1}$, that is, $\Lambda^{-1} \leq 1 - \frac{A}{\overline{A}\alpha}\widehat{R}$.

[17] Specifically, he found that possessing traits of low default risks, such as being a member of the Suffolk System or under a state-sponsored insurance, is negatively correlated with the *implied volatility* of the bank, of which the discount factor—or price, in his paper—is a decreasing function. Therefore, the low-risk traits are associated with a higher value of δ.

model predicts that the aggregate quantity of bank credit is independent of banks' wealth, G, whereas the aftermath of the recent financial crisis witnesses that the banking sector reduces credit issuance after suffering a severe loss. To give an account for this observation in particular and to examine the modern banking in general, we shall introduce a borrowing constraint to banks, as below.

4. Banks' Wealth Matters in the Presence of a Borrowing Constraint

In this section, we assume banks are subject to a borrowing constraint, that their equity value should never fall below $(1-\mu)G$.[18] The equity value is higher in the good state than it is in the bad state. Thus, we only need to consider the constraint in the bad state, which is $G + \underline{Y} - D \geq (1-\mu)G$. With $\underline{Y} = \underline{A}(1+r)/(\overline{A}\alpha) \times D$ by (8) and rearrangement, it becomes

$$D \cdot \left[1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha}\right] \leq \mu G. \tag{14}$$

The introduction of the constraint has two immediate implications. The first is that, because banks always maintain a positive equity value, they never default. Therefore, their notes are not discounted, that is, $\delta = 1$ and hence $1 + r = \widehat{R}$ hereafter. As a result, the borrowing constraint (14) becomes

$$D \cdot \left(1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}\right) \leq \mu G. \tag{15}$$

The other implication is that, unlike in the preceding case of least-fettered issuance, banks now are in Bertrand competition with *limited* capacities à la Kreps and Scheinkman (1983). With the borrowing constraint, the market equilibrium is as follows.

[18] This borrowing constraint can be due to several reasons. One reason is that, in modern times, bank default becomes very costly and banks' managers want to maintain solvency in any contingency, which is a special case of the above constraint with $\mu = 1$. Another reason, as in Gertler and Kiyotaki (2010) and Holmstrom and Tirole (1997), is that banks have a moral hazard issue: at $t = 1$, the owner of a bank—the banker—can abscond with a fraction of $1-\mu$ of its stored wealth to a remote island. Hence, banks' equity value should never fall below $(1-\mu)G$ in any contingency.

PROPOSITION 3.

(i) *If* $G \geq \frac{1}{\mu} G^*$*, then in all equilibriums the real allocation is the same as in the case of least-fettered issuance:* $\widehat{R} = R^{SB}$*, the profit margin of bank lending is 0, and* $L = L^{SB}$*. If* $G < \frac{1}{\mu} G^*$*, there is a unique equilibrium in which the profit margin of bank lending is positive; and* $\widehat{R} > R^{SB}$ *and* $\widehat{R}$ *is determined by* G *through*

$$G = \frac{1}{\mu} \left(\frac{\overline{A}\alpha}{w^\alpha} \right)^{\frac{1}{1-\alpha}} \frac{1 - \frac{\underline{A}}{\overline{A}\alpha} \widehat{R}}{\widehat{R}^{\frac{1}{1-\alpha}}} \equiv G(\widehat{R}). \tag{16}$$

(ii) *If* G *decreases from* $\frac{1}{\mu} G^*$ *to 0, the equilibrium interest rate,* $\widehat{R}$*, increases from* R^{SB} *to* $\overline{A}\alpha / \underline{A}$ *and the number of workers hired by entrepreneurs decreases from* L^{SB} *to* $L^{SB} \times \left[\frac{\underline{A}}{q\overline{A}\alpha + (1-q)\underline{A}} \right]^{\frac{1}{1-\alpha}}$.

The borrowing constraint limits banks' capacity of issuance to an endogenous proportion of their wealth. If their wealth is high enough—that is, beyond $\frac{1}{\mu} G^*$—then banks still possess a capacity large enough to annihilate the profit margin of issuance, which leads to the second-best allocation, as was in the case of least-fettered issuance. Hence arises result (i).

If $G < \frac{1}{\mu} G^*$, banks' wealth does not suffice to back an issuance of that size. As a result, issuance bears a positive profit margin. This positive profit margin drives all banks to issue as much as possible, until the borrowing constraint, (15), is binding. This clears the indeterminacy in the quantity of individual banks' issues and gives rise to a unique equilibrium. It also shows that, now, the quantity of money circulated is determined by the supply side—namely, the banking sector—rather than by the demand side, as was in the case of least-fettered issuance. Therefore, the lower the banks' wealth, the lower the total value of money that they supply and, as a result, the higher the interest rate of borrowing ($\widehat{R}$) and the fewer the workers that entrepreneurs hire.

The last comparative static, however, does not mean the aggregate output always decreases with G for $G < \frac{1}{\mu} G^*$ because of the wedge between the first-best and second-best allocations, namely $L^{FB} < L^{SB}$. Define

$$G^{FB} \equiv G(R^{FB}),$$

where function $G(\widehat{R})$ is defined in equation (16); with $R^{FB} = \overline{A}/A_e$,

$$G^{FB} = \frac{1}{\mu}(A_e\alpha - \underline{A})(A_e\alpha/w)^{\frac{\alpha}{1-\alpha}}.$$

Then G^{FB} is the level of banks' wealth at which the actual interest rate exactly takes the first-best value and therefore the aggregate output is maximized. $G^{FB} < \frac{1}{\mu}G^*$.[19] By proposition 3, then, there are two types of efficiency:

(i) $G < G^{FB}$. In this case, $\widehat{R} > R^{FB}$ and $L < L^{FB}$. That is, relative to the first-best allocation, banks under-lend, whereby the cost of bank credit is too high and the sector that depends on it—namely entrepreneurs—obtains inadequate resources, namely labor.

(ii) $G > G^{FB}$. In this case, $\widehat{R} < R^{FB}$ and $L > L^{FB}$. That is, relative to the first-best allocation, banks over-lend, whereby the cost of bank credit is too low and the sector that depends on it obtains excessive resources.

The existence of the second type of inefficiency is due to assumption 3, which drives a wedge between the first-best and second-best allocations. In the absence of this wedge, R^{FB} would be equal to R^{SB}, hence $G(R^{FB})$ to $\frac{1}{\mu}G^*$, and the case of bank over-lending would not exist. This case of over-lending provides the paper with room to accommodate interest rate policy and capital adequacy regulation, as will be shown in section 6.

The first type of inefficiency is caused by inadequate creation of money by banks. Can the central bank enlarge the money supply through the banking system? It can, as we show in the next section, with a policy which, because it eases the constraints imposed by a dearth of money, is called the quantitative easing policy, or QE.

[19]That is because $R^{FB} > R^{SB}$, $\frac{1}{\mu}G^* = G(R^{SB})$ and function $G(R)$ is decreasing.

5. QE Reduces the Borrowing Constraint of Banks

The central bank (CB hereafter) in this economy is modeled as the unique entity that is able to costlessly produce another means of payment which has no intrinsic values; to fix the idea, let it be shells. Shells are different in nature to bank notes. The CB does not promise to pay a holder of shells with any corn, the real good. Shells, therefore, are purely nominal. By contrast, a bank commits to redeem its notes with the promised amounts of corn. Hence banks' notes are not nominal.

Although shells are fiat money and the model economy lasts for only two dates, shells can circulate in it, whereby the CB can conduct meaningful monetary policy, such as the following QE. At $t = 0$ the CB announces a facility whereby each of the N banks can borrow up to S units of shells (again, one unit is defined as N kg) at a small interest rate $\epsilon > 0$. A borrower bank is obliged to repay its debt to the CB at $t = 1$ either with shells or with corn, with a fixed rate of exchanging corn for shells—for example, 1 kg corn for 1 kg shells. This exchange rate is arbitrary and is made to confer a nominal value on shells, as in real life the Bank of England can print an arbitrary number, like £10, on a piece of paper, and then this piece of paper has a nominal value of £10 and can be used to buy real goods of this value (e.g., two packs of cherries at M&S). As such, we call 1 kg corn the *par value* of 1 kg shells, etc., although shells are nominal. The banks that have used the CB's facility thus have two types of money to lend to entrepreneurs. One is their own notes; the other is shells. We assume that no entrepreneurs borrow both types of money, in order to avoid the unnecessary complication of trying to determine which debt, of the two, is senior in the case of default.

The contract of borrowing a bank's notes is still represented by (E, r), whereby the entrepreneur borrows the bank's notes of face value E at $t = 0$ and then owes it a debt of $E(1+r)$, which he repays at $t = 1$ with corn.

The contract of borrowing shells is similarly represented by (E_s, r_s), whereby the entrepreneur borrows E_s kg shells at $t = 0$ and then owes the bank a debt of $E_s(1 + r_s)$, which he repays at $t = 1$ with either corn or shells, with 1 kg corn equivalent to 1 kg shells.

The timing of events at $t = 0$ is as follows. First, the CB chooses S and publicly announces the policy. Then, banks decide the quantity of shells to borrow from the CB, $Q \leq S$, and post offers of $(D, r; Q, r_s)$. Based on these offers, entrepreneurs decide which bank to go to and how much to borrow. Lastly, they use borrowed means of payment (bank notes or shells) to hire workers and start the production.

The timing of events at $t = 1$ is as follows. First, entrepreneurs produce corn. Second, the market for shells opens, on which the shell-borrowing entrepreneurs use corn to buy shells from workers. Let $\overline{p_1}$ ($\underline{p_1}$) denote the shell price in the good (bad) state. Third, entrepreneurs settle their debts to the lender banks, using corn and/or shells. Fourth, the market for shells *may* open the second time, on which banks having excessive shells sell to banks in shortage. Lastly, banks redeem their notes from workers and settle their debts to the CB. At this stage, the CB may end up holding a certain quantity of corn, which it will transfer to the agents of the economy before the consumption starts.

Note that there is no surprise or nominal rigidity associated with QE. All the private-sector agents (i.e., banks, entrepreneurs, and workers) made decisions after having observed the CB's move and they can freely adjust their decisions with it. Still, QE produces real effects in this economy in one equilibrium if $G < \frac{1}{\mu} G^*$.[20] For any $G < G^{FB}$, moreover, there exists a unique value of S at which this equilibrium attains the first-best allocation. This is the equilibrium that we discuss below.

In this equilibrium, at $t = 0$, workers believe that they will be able to use shells to buy corn at $t = 1$ and thus they accept a wage payment with shells; and at $t = 1$, indeed shells receive a positive real value, intuitively because they can be used to settle certain debts

[20] As shells are fiat money, there is another equilibrium in which workers disbelieve that shells will have real value at $t = 1$ and thus do not accept to be paid with shells at $t = 0$, which are thus not circulated. This type of equilibrium exists for fiat money in general. By contrast, it never exists for banks' notes: banks each have G units of corn, therefore their notes always have a positive real value at $t = 1$; indeed, if a bank issues $D \leq G$, workers know, without any equilibrium calculation, that its notes are worth par. This difference results from the difference in nature between shells and banks' notes stated at the beginning of this section.

that otherwise have to be settled with corn. To solve the equilibrium, we first examine what happens at $t = 1$ and then what happens at $t = 0$. Consider, first, the case in which S is below a threshold (which is given in proposition 4 below) and therefore $r_s > \epsilon$ in equilibrium. In this case, all the banks borrow to the full capacity of QE, namely, $Q = S$, and lend all the shells out to entrepreneurs. Therefore, at the beginning of $t = 1$, there are NS units of shells in workers' hands and the aggregate debt of the shell-borrowing entrepreneurs is $NS \times (1 + r_s)$ units.

In the good state, as no entrepreneurs default, the shell price $\overline{p_1} = 1$. On the one hand, shells can never be worth more than their nominal values: if the entrepreneurs use 1 kg corn only to buy less than 1 kg shells, they will not buy them but instead use 1 kg corn to settle a debt of 1 kg shells to the banks. On the other hand, in the good state, it cannot be that $\overline{p_1} < 1$ either. Otherwise, the shell-borrowing entrepreneurs would want only shells, not corn, to repay all their debts. Their aggregate demand for shells would thus be $NS(1 + r_s)$, as they do not default. This demand, with $r_s > 0$, is bigger than NS, the supply of shells, leaving the market uncleared.

As $\overline{p_1} = 1$, the entrepreneurs are indifferent to repaying their banks with shells or with corn in the good state. As a result, some banks may end up with more than $S(1+\epsilon)$ units of shells, some less. If so, the former wants to sell the excess to the latter and the second shell market opens. On this market, the equilibrium shell price, p_B, equals 1 also, following the same argument as above. On the one hand, it is impossible that $p_B > 1$. On the other hand, if $p_B < 1$, banks all want to use only shells, not corn, to settle their debts to the CB, and their aggregate demand would be $NS(1 + \epsilon)$, greater than the total supply, NS, thus leaving the market uncleared. This argument holds true, and hence $p_B = 1$ is the only market clearing price, so long as $\epsilon > 0$, however small. To simplify exposition, in what follows, we consider the limit case where $\epsilon \rightarrow 0$.[21]

[21]Note, however, that if $\epsilon = 0$, then there is indeterminancy: any price $p_B \in [0, 1]$ can clear this second shell market. If $\epsilon = 0$, the aggregate excess exactly equals the aggregate shortage. Then, the supply side is willing to sell all the excessive shells at any $p_B \geq 0$, while the demand side is willing to buy exactly the same amount at any $p_B \leq 1$.

In the bad state, the output is low. Hence $\underline{p_1} < 1$, as will be verified later. Thus, the shell-borrowing entrepreneurs use only shells, no corn, to repay the banks. Furthermore, in this state entrepreneurs all default according to lemma 2. This means all their outputs are used to settle their debts to the banks. With these two facts put together, all the shell-borrowing entrepreneurs' output is used to buy shells. Denote by Y_s the total output of one bank's shell-borrowing entrepreneurs. Then the shell market clearing commands that $\underline{p_1} \times NS = NY_s$, or

$$\underline{p_1} = Y_s/S. \tag{17}$$

With a calculation similar to that leading to equation (8), we find $\underline{p_1} = \underline{A}(1 + r_s)/(\overline{A}\alpha)$, which, as is shown below in proposition 4, is smaller than 1 indeed, as was intuitively argued. Moreover, from $\underline{p_1} = Y_s/S$ it follows that the shell-borrowing entrepreneurs of one bank altogether buy $Y_s/\underline{p_1} = S$ units of shells. Hence, each bank ends up with having S units of shell equally and the second shell market does not open in the bad state.

The value of 1 kg shells at $t = 0$, p_0, is the mean of their values at $t = 1$:

$$p_0 = q \times 1 + (1 - q) \times \frac{\underline{A}(1 + r_s)}{\overline{A}\alpha}. \tag{18}$$

Workers thus accept a wage payment with w/p_0 kg shells at $t = 0$ in the equilibrium.

The effect of this policy is presented in the following proposition.

PROPOSITION 4. *Suppose $G < \frac{1}{\mu}G^*$.*

(i) If QE's scale $S < \underline{S}(G) \equiv \frac{\overline{A}\alpha}{q(\overline{A}\alpha - \underline{A})}(\frac{1}{\mu}G^* - G)$, then there is a unique equilibrium in which all the S units of shells are in circulation. In this equilibrium, QE produces real effects: the actual interest rate, $\widehat{R}$, is determined by S through

$$S = \left(\frac{\underline{A}}{w^\alpha}\right)^{\frac{1}{1-\alpha}} \cdot \frac{1 - (1 - q)\theta}{q} \left(\frac{1}{\theta^{\frac{1}{1-\alpha}}} - \frac{\mu G}{1 - \theta}\right) \equiv F(\theta; G),$$

$$\text{with } \theta \equiv \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}. \tag{19}$$

Moreover, the interest rate of lending shells is $r_s = \frac{[q\overline{A}\alpha+(1-q)\underline{A}]\widehat{R}-\overline{A}\alpha}{\overline{A}\alpha-(1-q)\underline{A}\widehat{R}}$. *It satisfies* $r_s > 0$ *and* $\underline{A}(1+r_s)/(\overline{A}\alpha) < 1$, *as was said.*

(ii) If S *increases,* $\widehat{R}$ *and* r_s *decrease and* L *increases. At* $S = \underline{S}(G), \widehat{R} = R^{SB}$, $r_s = 0$ *and* $L = L^{SB}$.

QE works by increasing the real value of money that banks lend out at $t = 0$. That explains why, compared with the situation of its absence (namely with $S = 0$), the cost of borrowing money ($\widehat{R}$) is reduced, whereby entrepreneurs hire more workers. And the larger the scale of QE (i.e., S), the greater are these effects (so long as S is below $\underline{S}(G)$).

The question is: how is QE able to make banks enlarge their lending scale, backed by the same amount of wealth, without breaking the borrowing constraint? To answer this question, consider a representative bank's balance sheet at $t = 1$, as shown in table 1.

Consider the bad state, in which the borrowing constraint is binding. The constraint commands that $G + \underline{Y} + Y_s - D - \underline{p_1}S \geq (1 - \mu)G$. This inequality, with $\underline{p_1} = Y_s/S$ by (17), is equivalent to $G + \underline{Y} - D \geq (1 - \mu)G$, which leads to inequality (14), the same borrowing constraint in the absence of QE. Therefore, the injection of the fiat money with QE does not reduce the capacity of private issuance at all. This is due to $\underline{p_1} = Y_s/S < p_0$,[22] which means that in the bad state the shell price goes down, so that the real value of a bank's liability to the CB, $\underline{p_1}S$, falls enough to be covered by the value of the assets, Y_s. The decrease in the real value of shells means inflation. Put differently, QE enables banks to lend more without breaking the borrowing constraint because it induces inflation in the bad state, which lowers the real value of banks' liability to the CB. The inflation arises because shells are fiat money. The CB never commits to redeem shells with a real good. Their value, therefore, freely adjusts with the state of the economy. By contrast, banks commit to redeem their notes (i.e., their promise to pay) at

[22] $\underline{p_1} < p_0 \Leftrightarrow \underline{p_1} < q \times 1 + (1 - q) \times \underline{p_1} \Leftrightarrow \underline{p_1} < 1$, which holds true by the proposition.

Table 1. The Balance Sheet (in real value) of the Representative Bank with QE

Assets	Liabilities
Corn Stored (G)	Equity
Loans in Notes (Y)	Liability to the Note Holders (D)
Loans in Shells (Y_s)	Liability to the CB ($p_1 S$)

a pre-specified, non-contingent, value.[23] This difference in nature between banks' notes and shells, therefore, drives the functioning of QE.

Remark: An abstraction of the model helps elucidate this mechanism of QE. That is the denomination of banks' liabilities with corn, the real good. In reality, however, typically they are denominated with the fiat money currency of the economy. Incorporating this feature would not necessarily invalidate the working of the mechanism. First, the aforementioned difference in nature between bank-created money and currency is still there. Although banks now use the currency to redeem their liability, to obtain the currency they need to expend real resources. Therefore, their liability still bears real obligations of repayment. In contrast, fiat money is not redeemable and bears no such obligations. Second, it is true that if banks' liability is denominated with the currency, its real value falls under the negative shocks, which improves banks' conditions. However, a credit crunch still happens. So long as they face a binding borrowing constraint, they will still reduce lending if their wealth is profusely eaten off, as is evident in the aftermath of the 2008–09 crisis. Third, in this circumstance, if QE is able to raise banks' lending capacities, it has to ease their borrowing constraint. As it does not give real resources to banks, to do so it has to further reduce the real value of banks'

[23]In the model economy, the value of these issues at $t = 1$ could be made contingent on the state in two ways. One is to let the specified value be contingent on the state, which is disallowed by assumption 2. The other is default, which is disallowed because of the borrowing constraint, (14). While default is disallowed altogether because of the particular form that the constraint takes, in general so long as any borrowing constraint restricts banks' capacity, there is only a limited extent to which the value of their liabilities can adjust to the real economic conditions.

liability, and this can only be done via inflation. Fourth, this reduction can be done by the CB with QE, but not by banks themselves, exactly because of the above said difference in nature between the two types of money.

At the core is the point that this difference gives the CB leeway to act that banks do not have. This point can be seen even more clearly in a modified version of the setting where the CB implements QE by directly lending shells to entrepreneurs (rather than through banks) at interest rate r_s, which would result in the same prices of shells and the same allocation as in the original setting. Obviously, in this setting the CB can lend out the currency, whereas banks cannot lend more of their liabilities, because shells are not redeemable but banks' liabilities are. Moreover, if banks' liabilities were now denominated with the currency, this policy would still work. Actually it would work better because it would ease banks' borrowing constraint by inducing inflation, which enables them to enlarge the lending capacity.

Returning to the model, while QE induces inflation in the bad state, it causes deflation in the good state: $\overline{p_1} = 1 > p_0$. This result might partly explain the lingering deflation pressure or the low inflation observed during or after the implementation of QE in several economies, such as the United Kingdom and the euro zone. This phenomenon would be a puzzle if considered from the point of view of the quantitative theory of money, given that QE enormously expands the monetary base.

Thus far we have considered the case in which $S < \underline{S}(G)$ and therefore $r_s > 0$. Now we consider what happens if QE is in a bigger scale. At $S = \underline{S}(G)$, $\widehat{R} = R^{SB}$ and $r_s = 0$. By proposition 1(iii), $\widehat{R}$ can never fall below R^{SB}, otherwise banks would stop lending altogether, not a case in equilibrium. Therefore, if the scale of QE is larger than $\underline{S}(G)$, then $\widehat{R}$ stays at R^{SB}, r_s stays at 0, and the excessive shells beyond the threshold are not put into circulation; indeed, with $r_s = 0$, banks are indifferent regarding the quantity of shells to borrow and lend. This result partly offers an explanation for the phenomenon that, in certain economies such as the United States and the euro zone, a substantial fraction of monetary bases created with QE is not lent out but stays in banks' reserve accounts.

Now consider the optimal scale of QE. The first-best value of the interest rate is $\widehat{R} = R^{FB}$. In the circumstance where $G \geq G^{FB}$ and hence $\widehat{R} \leq R^{FB}$ at $S = 0$, any $S > 0$ only makes $\widehat{R}$ still smaller and efficiency even lower. Therefore, the optimal $S = 0$, namely, the CB should not implement QE at all in the circumstance of banks over-lending. However, in the circumstance where $G < G^{FB}$ and hence $\widehat{R} > R^{FB}$ at $S = 0$, there is a unique value of S at which QE drags down $\widehat{R}$ to R^{FB}. This is summarized below.

PROPOSITION 5.

(i) If $G \geq G^{FB}$, the optimal scale of QE is $S = 0$, that is, no QE should be implemented.

(ii) If $G < G^{FB}$, the optimal $S = F(\frac{A}{\bar{A}\alpha} \cdot R^{FB}; G) > 0$, where function $F(\cdot)$ is given by (19). At this scale QE attains the first-best allocation (i.e., $\widehat{R} = R^{FB}$). Moreover, with optimal QE, banks earn profit from lending shells (i.e., $r_s > 0$), but their overall profit is reduced compared with the case without QE if $G \geq \underline{G}$ for some $\underline{G} < G^{FB}$.

According to the last result, although with QE banks receive free funding from the CB and lend it at a positive interest rate, surprisingly they lose from the policy, which thus does not subsidize them. That is because to banks, besides this positive effect on the scale, QE induces a negative, general equilibrium effect on the profit margin by enlarging the lending capacities of all banks. As a result, banks obtain a reduced profit from lending their notes. As $\partial F/\partial G < 0$, if banks' wealth is above a threshold (i.e., $\underline{G}$), then the scale of optimal QE is small enough. As a result, the positive effect from the enlargement of the lending scale is small and dominated by the negative effect from the reduction of the profit margin. However, even in this case of banks all losing from the free funding of the CB, given $r_s > 0$, individual banks still strictly prefer taking it rather than abstaining from it—thus QE can be conducted on a voluntary basis. The reason is that a single bank takes the profit margin of lending as given and neglects the effect on it of enlarging its own capacity.

In the circumstance where $G > G^{FB}$ and banks over-lend, we have seen that QE does not help. What the CB can do is discussed in the next section.

6. Interest Rate Policy and Capital Adequacy Regulation to Curb Bank Lending

If $G > G^{FB}$, then banks over-lend, making the actual interest rate too low, and entrepreneurs hire too many workers relative to the first-best allocation. In this circumstance, it is usually expected that the CB can help by setting a high policy interest rate. This idea is explored in this section, where we show that interest rate policy produces real effects and is able to attain the first-best allocation if and only if there is a nominal rigidity. In its absence, we show, capital adequacy regulation is always a remedy.

To explain the meaning of nominal rigidity in this paper, we define nominal wage as the total face value of banks' notes that workers receive as the wage payment. In the absence of any intervention from the CB, the nominal wage is w because banks do not default (due to the borrowing constraint (14)). The nominal rigidity in this paper is defined as the friction that keeps the nominal wage of workers staying at w and unchanged with the CB's policy.

6.1 Interest Rate Policy Works If and Only If the Nominal Rigidity Is Present

In the two-date economy of this paper, the CB sets the policy rate r_p by offering to workers a savings account which takes in the deposit of banks' notes at $t = 0$ and pays out with shells at $t = 1$. Specifically, if the CB receives a deposit of some banks' notes of overall face value F kg corn at $t = 0$, it issues to the depositor $F(1 + r_p)$ kg shells at $t = 1$. Moreover, by taking in the notes, the CB becomes a creditor to the issuer banks and charges these banks an interest rate of $1 + r_p + \varepsilon$ for some $\varepsilon > 0$. Thus it obliges these banks altogether to pay back $F(1 + r_p + \varepsilon)$ at $t = 1$, either with corn or with shells, counting 1 kg corn equivalent to 1 kg shells. There is one equilibrium in which this policy is meaningful.[24] In this equilibrium *if* the aggregate face value of notes deposited with the CB at $t = 0$ is F and consequently $F(1+r_p)$ kg shells are created at $t = 1$, then these

[24] As shells are fiat money, there is an equilibrium in which no one believes that shells have real value at $t = 1$ and thus no one deposits any bank notes with the CB at $t = 0$, which makes the policy rate meaningless.

shells are priced at par (i.e., 1 kg shells worth 1 kg corn) at $t = 1$. On the one hand, they can never be priced above par. On the other hand, shells are not priced below par either. Otherwise, the banks indebted to the CB would want to use only shells to settle all their debts. Thus, their aggregate demand for shells would be $F(1+r_p+\varepsilon)$ kg, but only $F(1+r_p)$ kg shells are created, insufficient to meet the demand. The deposit of bank notes with the CB and the creation of shells, however, will not actually happen in equilibrium. Banks will pay their note holders the same interest rate of r_p (or even a little more) to stop them from depositing the notes with the CB and avoid the payment of the even higher interest rate of $r_p + \varepsilon$. As a result of the policy rate set at r_p, therefore, a bank's note of face value 1 issued at $t = 0$ is worth $1 + r_p$ at $t = 1$, and banks that issue notes of aggregate face value D at $t = 0$ are in a liability of $D(1 + r_p)$ at $t = 1$.

Having explained how the interest rate policy is conducted in this economy, we state its effects in the following proposition.

PROPOSITION 6. *Suppose $G > G^{FB}$.*

(i) If workers' nominal wage adjusts with the policy rate, the interest rate policy produces no real effects but deflates the nominal wage to $w/(1 + r_p)$.

(ii) If workers' nominal wage stays at w invariant to the CB's policy rate, the equilibrium interest rate of bank lending, $\widehat{R}$, increases with the policy rate r_p. At a unique value of r_p, $\widehat{R} = R^{FB}$ and the first-best allocation is attained. With the optimal interest rate policy, banks obtain zero profit if $G \geq G_s$, where $G_s \equiv \frac{1}{\mu}[(q\overline{A}\alpha + (1-q)\underline{A}) - \underline{A}](A_e\alpha/w)^{\frac{\alpha}{1-\alpha}}$ and satisfies $G^{FB} < G_s < \frac{1}{\mu}G^*$.

Intuitively, in both the regime of flexible nominal wage and that of sticky nominal wage, a higher policy rate, by increasing the cost of deposit, induces the bank to set a higher lending rate in the hope of shifting the increased cost of deposit to entrepreneurs: r increases with r_p in both regimes, as shown in the proof. As a result, in both regimes, a higher policy rate reduces the quantity of bank credit that entrepreneurs borrow. In the regime of flexible wage, this reduction produces no effect on the number of workers they employ because it is exactly offset by the decrease in the nominal wage that workers accept. In the regime of sticky wage, by contrast, the nominal wage stays the same and this reduction in bank credit forces entrepreneurs to hire fewer workers. With a proper policy rate, the number of workers they hire is brought down to the first-best level, that is, the first-best allocation is attained. The decrease in entrepreneurs' demand for bank credit subjects banks to stronger competition. As a result, their profit margin of lending is nullified if their lending capacity is not too small, that is, if their wealth is not too low—i.e., no less than G_s.

The interest rate policy, therefore, works to improve efficiency over the market equilibrium if and only if the nominal rigidity is present. However, regulation that sets a lower bound on banks' capital adequacy ratio always works to curb the over-lending by banks (if the CB has the authority for such regulation), as is shown below.

6.2 Capital Adequacy Regulation Always Works

The aggregate face value of money that is needed for entrepreneurs to hire the first-best number of workers is $wL^{FB} := D^{FB}$. Suppose the CB imposes the constraint that banks cannot issue more than D^{FB}/G times their wealth, G. Then, this constraint is binding in the competitive equilibrium if $G > G^{FB}$ because, in its absence, banks issue more than D^{FB} (as $\widehat{R} < R^{FB}$) by proposition 3, and thus the constraint is violated. The constraint, therefore, is binding, so banks issue D^{FB} and the first-best allocation is attained. One way to implement this constraint is to impose at $t = 0$ a lower bound on banks' capital adequacy ratio, which is defined as the equity-to-asset ratio in market value, that is, $(G+Y-D)/(G+Y)$, where Y is the value of banks' loans at $t = 0$, that is, $Y = q \times \overline{Y} + (1-q) \times \underline{Y}$. Define

$$c^{FB} := \frac{A_e \alpha G + (1-q)\underline{A}(1-\alpha)D^{FB}}{A_e \alpha G + [q\overline{A}\alpha + (1-q)\underline{A}]D^{FB}},$$

which is the capital adequacy ratio if banks lend the first-best quantity of money (as is shown in the proof of the following proposition).

PROPOSITION 7. *Suppose $G > G^{FB}$. If the CB imposes regulation that restricts banks' capital adequacy ratio from being smaller than c^{FB}, then banks issue the first-best quantity of money and the first-best allocation is attained.*

The intuition for why the regulation works is simple: the money that banks create is their liability. Therefore, the scale of its issuance is subject to the capital adequacy regulation. If the regulation is tight, then banks are restricted from over-lending.

7. Conclusion

Banks are special because their liability, especially that in the form of demand deposit, is widely accepted as a means of payment. Often, real sectors need to borrow banks' liability (so-called *money* or *credit*) to obtain resources that they want. This interaction between the real sectors and the banking sector on competitive markets, as well as the central bank's remedy to market inefficiency, is the focus of this paper. It underlines the importance of banks' aggregate wealth. Depending on the size of this wealth, there are two types of inefficiency.

If banks' wealth is below a threshold, then they issue too little money, with symptoms that the interest rate of bank credit is too high and the sectors that depend on it obtain inadequate resources. In this circumstance, the central bank can improve efficiency by lending to all banks its fiat money at zero interest rate. This policy, which bears the flavor of QE, works because of a difference in nature between bank-created money and fiat money. The latter is not redeemable, whereas the former bears real obligations of redemption. Furthermore, the policy induces deflation under the positive productivity shock. Lastly, while the policy gives to banks free funding, which they lend out at a positive interest rate, it does not subsidize them unless their wealth is very low.

If banks' wealth is above the threshold, on the other hand, banks lend out too much money, with symptoms that the interest rate of bank credit is too low and the resources are skewed to the sectors that highly depend on it. To curb over-lending by banks, the central bank may set a high policy rate, which, however, works if and only if the nominal wage cannot freely adjust with the policy. By contrast, the regulation that sets a tight capital adequacy ratio always works because the money that banks create is their liability, and its quantity is thus subject to the regulation.

Appendix: The Proofs

Proof of Lemma 1

In equilibrium, only one promised wage, denoted by F, prevails on the market, as will be shown. Competitive equilibrium is thus defined as a profile of (F, L), such that

(i) given that F prevails on the market, the optimal demand for labor of each entrepreneur is L;

(ii) given that each entrepreneur demands L workers, F clears the labor market.

The two conditions are elaborated as follows.

For (i), given F, a representative entrepreneur's decision problem on labor demand is

$$\max_L q(\overline{A}L^\alpha - FL) + (1-q)\max(\underline{A}L^\alpha - FL, 0),$$

where the "max" term appears because the entrepreneur might default in the bad state. That is indeed the case at the optimum. Otherwise, the entrepreneur's problem is

$$\max_L q(\overline{A}L^\alpha - FL) + (1-q)(\underline{A}L^\alpha - FL).$$

The solution is $L = (\frac{A_e\alpha}{F})^{\frac{1}{1-\alpha}}$. Then in the bad state his output is $\underline{A}((\frac{A_e\alpha}{F})^{\frac{\alpha}{1-\alpha}}$, which is smaller than $F \cdot (\frac{A_e\alpha}{F})^{\frac{1}{1-\alpha}}$, the wage obligation, because $\underline{A} < A_e\alpha$ as assumed in (1). Hence, he defaults in the bad state, contradictory to what was supposed.

Defaulting in the bad state, entrepreneurs choose L to maximize the profit in the good state, $\overline{A}L^\alpha - FL$. Therefore, given F, the labor demand is

$$L = \left(\frac{\overline{A}\alpha}{F}\right)^{\frac{1}{1-\alpha}}. \tag{20}$$

For (ii), as there are a lot more workers than entrepreneurs can hire, the labor market is cleared by an expected wage income of w, the output of workers in autarky. In the good state, the workers hired get the promised wage, F. In the bad state, entrepreneurs default and all the output goes to the workers, of which each obtains $\frac{\underline{A}L^\alpha}{L} = \underline{A}L^{\alpha-1}$. The labor market clears if

$$qF + (1-q)\underline{A}L^{\alpha-1} = w. \tag{21}$$

Equations (20) and (21) together pin down L^{SB} as given in the lemma.

Now, show that only one F prevails on the market. If an entrepreneur posts F, then by (20) he hires $L = (\frac{\overline{A}\alpha}{F})^{\frac{1}{1-\alpha}}$ workers, whose wage income is F in the good state and $\underline{A}L^{\alpha-1} = \frac{\underline{A}}{\overline{A}\alpha}F$ in the bad state. Both increase with F. Therefore, workers go only to entrepreneurs who post the highest F, and in competitive equilibrium, only one F prevails.

Q.E.D.

Proof of Lemma 2

Suppose, otherwise, an entrepreneur does not default in the bad state. Then, his problem is

$$\max_E q(\overline{A}L^\alpha - E(1+r)) + (1-q)(\underline{A}L^\alpha - E(1+r)) \text{ s.t. } (3).$$

From the constraint, $E = wL/\delta$. Substitute it into the objective and let $\gamma \equiv w(1+r)/\delta$. Then, the problem becomes

$$\max_L A_e L^\alpha - \gamma L.$$

The solution is $L = (A_e\alpha/\gamma)^{\frac{1}{1-\alpha}}$. At this scale, the entrepreneur will default in the bad state: $\underline{A}L^\alpha < E(1+r)|_{E=wL/\delta;\gamma\equiv w(1+r)/\delta} \Leftrightarrow \underline{A}L^\alpha < \gamma L \Leftrightarrow \underline{A}L^{\alpha-1} < \gamma|_{L=(A_e\alpha/\gamma)^{\frac{1}{1-\alpha}}} \Leftrightarrow \underline{A} < A_e\alpha$, which is assumed in (1)—hence a contraction to what was supposed.

Q.E.D.

Proof of Lemma 3

As the entrepreneurs all default in the bad state and each hands over his whole output, $\underline{y}$, to the bank, the value of the bank's loans in the bad state, $\underline{Y}$, equals $\underline{y}$ times the number of entrepreneurs that the bank finances, D/E. With $\underline{y} = \underline{A}L^{\alpha}$, and E and L given by (4) and (5),

$$\frac{\underline{y}}{E} = \frac{\underline{A}(1+r)}{\overline{A}\alpha}. \tag{22}$$

Then, $\underline{Y} = \underline{y} \cdot D/E = D \cdot \underline{y}/E = \underline{A}(1+r)/\overline{A}\alpha \cdot D$, that is, (8).
Q.E.D.

Proof of Lemma 4

$\Pi(D,r) = E_{\widetilde{Y}} \max(G + \widetilde{Y} - D, 0) - G = E_{\widetilde{Y}} \max(\widetilde{Y} - D, -G) = E_{\widetilde{Y}}\left[\widetilde{Y} + \max(-D, -G - \widetilde{Y})\right] = E_{\widetilde{Y}}\left[\widetilde{Y} - \min(D, G + \widetilde{Y})\right] = E_{\widetilde{Y}}\left[\widetilde{Y} - D \times \min(1, \frac{G+\widetilde{Y}}{D})\right] = D \times E_{\widetilde{Y}}\left[\frac{\widetilde{Y}}{D} - \min(1, \frac{G+\widetilde{Y}}{D})\right]$, which, with $\delta = \min(1, \frac{G+\widetilde{Y}}{D})$, equals $D \times E_{\widetilde{Y}}\left[\frac{\widetilde{Y}}{D} - \delta\right] = D \times \left[q\frac{\overline{Y}}{D} + (1-q)\frac{\underline{Y}}{D} - \delta\right]$, which, as $\overline{Y} = D(1+r)$ and $\underline{Y} = D \times \frac{\underline{A}}{\overline{A}\alpha}(1+r)$, equals $D \times \left[q(1+r) + (1-q)\frac{\underline{A}}{\overline{A}\alpha}(1+r) - \delta\right] = D \times \left[\frac{1+r}{R^{SB}} - \delta\right]$.
Q.E.D.

Proof of Proposition 2

Proof of proposition 2(i) has been shown in the main text. For proposition 2(ii), prove the "if" part by reduction to absurdity. Suppose that in one equilibrium, no banks default; namely, (9) is honored for all banks. Then, for all banks $\delta = 1$ and $1 + r = \widehat{R}|_{\text{result }(i)} = R^{SB}$. By (9), each bank issues

$$D \leq G/\left(1 - \frac{\underline{A}}{\overline{A}\alpha}R^{SB}\right) = \frac{G[q\overline{A}\alpha + (1-q)\underline{A}]}{q(\overline{A}\alpha - \underline{A})}.$$

With $\delta = 1$ and $1+r = R^{SB}$, by (4) the demand by entrepreneurs is $E = (q\overline{A}\alpha + (1-q)\underline{A})^{\frac{1}{1-\alpha}} w^{\frac{-\alpha}{1-\alpha}}$. If $G < G^*$, then $D < E$, that is, the supply is below the demand—thus not in equilibrium. To prove the "only if" part of proposition 2(ii), it suffices to show that if $G \geq G^*$, no banks default in the symmetric equilibrium, which is constructed as follows. Banks all choose $r = R^{SB} - 1$ and D to satisfy entrepreneurs' demand for notes $E = (q\overline{A}\alpha + (1-q)\underline{A})^{\frac{1}{1-\alpha}} w^{\frac{-\alpha}{1-\alpha}}$. With this value of (D, r), it is straightforward to check that if $G \geq G^*$, the no-default condition, (9), is honored and hence no banks default in the bad state. Q.E.D.

Proof of Proposition 3

Note that, as there is no default, $\delta = 1$ and $1 + r = R$. In any equilibrium, the actual interest rate that all banks offer equals the market clearing rate, that is, $R = \widehat{R}$. As $1 + r = \widehat{R}$ and $\delta = 1$, by (4), the demand by entrepreneurs is $E = [\overline{A}\alpha/(\widehat{R}w^{\alpha})]^{\frac{1}{1-\alpha}} := E(\widehat{R})$, which decreases with $\widehat{R}$. Due to the borrowing constraint, (14), and with $R = \widehat{R}$, the quantity of notes supply satisfies

$$D \leq \mu G \left(1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}\right)^{-1} := \overline{D}\left(\widehat{R}; G\right).$$

Moreover, if $\widehat{R} > R^{SB}$, the profit margin of lending is positive, banks want to lend as much as they can, and therefore the constraint is binding. It follows that the supply function is

$$D\left(\widehat{R}; G\right) = \left\{ \begin{array}{c} 0 \text{ if } \widehat{R} < R^{SB} \\ [0, \overline{D}\left(\widehat{R}; G\right)] \text{ if } \widehat{R} = R^{SB} \\ \overline{D}\left(\widehat{R}; G\right) \text{ if } \widehat{R} > R^{SB} \end{array} \right\}.$$

The equilibrium value of $\widehat{R}$ is determined by $E(\widehat{R}) = D\left(\widehat{R}; G\right)$.

For proposition 3(i), if $G \geq \frac{1}{\mu}G^*$, then $D\left(\widehat{R}; G\right) > E(\widehat{R})$ for $\widehat{R} > R^{SB}$ and $D\left(\widehat{R}; G\right) < E(\widehat{R})$ for $\widehat{R} < R^{SB}$. Therefore, in equilibrium $\widehat{R} = R^{SB}$. Hence $L = L^{SB}$—the second-best allocation is attained—and the profit margin of lending is 0, both of which hold in the case of least-fettered issuance.

If $G < \frac{1}{\mu}G^*$, $D\left(\widehat{R};G\right) < E(\widehat{R})$ for $\widehat{R} \leq R^{SB}$. Therefore, in equilibrium $\widehat{R} > R^{SB}$ and is determined by $E(\widehat{R}) = \overline{D}\left(\widehat{R};G\right)$ or, equivalently, equation (16), replicated below:

$$G = G(\widehat{R}) \equiv \frac{1}{\mu}\left(\frac{\overline{A}\alpha}{w^\alpha}\right)^{\frac{1}{1-\alpha}} \cdot \frac{1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}}{\widehat{R}^{\frac{1}{1-\alpha}}}. \tag{23}$$

It is straightforward to show for this function $G(\cdot)$ that $G' < 0$, $G(R^{SB}) = \frac{1}{\mu}G^*$ and $G(\overline{A}\alpha/\underline{A}) = 0$.

Furthermore, as all the banks issue $\overline{D}$, there is no indeterminacy in the scale of issuance by individual banks and the equilibrium uniquely exists.

To prove proposition 3(ii), let $\widehat{R}(G)$ be the inverse function of $G(\widehat{R})$. Then, in the equilibrium $\widehat{R} = \widehat{R}(G)$ if $G < \frac{1}{\mu}G^*$. As $G(\widehat{R})$ is decreasing, so is $\widehat{R}(G)$. Moreover, $\widehat{R}(\frac{1}{\mu}G^*) = R^{SB}$ and $\widehat{R}(0) = \overline{A}\alpha/\underline{A}$ because $G(R^{SB}) = \frac{1}{\mu}G^*$ and $G(\overline{A}\alpha/\underline{A}) = 0$. With $R = \widehat{R}$, the number of workers hired in equilibrium, by (5), is $L = (\overline{A}\alpha/w\widehat{R})^{\frac{1}{1-\alpha}}$, which decreases with $\widehat{R}$. Thus, L increases with G. Moreover, $L = L^{SB}$ at $\widehat{R} = R^{SB}$ which holds if $G = \frac{1}{\mu}G^*$. At the other end, if $G = 0$, then $\widehat{R} = \overline{A}\alpha/\underline{A}$. As L is in proportion to $(1/\widehat{R})^{\frac{1}{1-\alpha}}$, we have $L = L^{SB} \times (R^{SB}/\widehat{R})^{\frac{1}{1-\alpha}} = L^{SB} \times [\frac{R^{SB}}{\overline{A}\alpha/\underline{A}}]^{\frac{1}{1-\alpha}} = L^{SB} \times [\frac{\underline{A}}{q\overline{A}\alpha+(1-q)\underline{A}}]^{\frac{1}{1-\alpha}}$.

Q.E.D.

Proof of Proposition 4

For proposition 4(i), to characterize the equilibrium, observe that if QE's scale S is small enough—the threshold for which will be found later—then $\widehat{R} > R^{SB}$ still holds and $r_s > 0$. That is, lending bears a positive profit margin. It has two implications: (a) both notes and shells are lent out; and (b) they are lent in the maximum quantity, that is, banks' borrowing constraint (15) is binding and all the S units of shells are in circulation.

From implication (a) it follows that the actual interest rates of borrowing the two types of money are equalized, and equal to $\widehat{R}$, the market rate. Following the discussion in subsection 3.1, the actual interest rate of borrowing shells is $(1+r_s)/p_0$, while the actual interest rate of borrowing banks' notes is $1+r$ (since $\delta = 1$ as banks do not default). Therefore,

$$\frac{1+r_s}{p_0} = 1+r = \widehat{R}. \tag{24}$$

From implication (b), the quantity of notes issued is $D = \mu G/(1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R})$. Given that they are not discounted (i.e., $\delta = 1$) and the ex ante value of shells per unit is p_0, the aggregate value of means of payment supplied is $p_0 S + D$, which, when the market clears, equals the wage payment that entrepreneurs demand to hire workers:

$$wL = p_0 S + \frac{\mu G}{1 - \underline{A}\widehat{R}/(\overline{A}\alpha)}. \tag{25}$$

By (5), the number of workers they hire is

$$L = \left(\frac{\overline{A}\alpha}{w}\right)^{\frac{1}{1-\alpha}} \widehat{R}^{\frac{-1}{1-\alpha}}. \tag{26}$$

These four equations (note that (24) has two) together with equation (18) (which settles p_0), as shown below, determine a unique profile of $(p_0, r_s, r, \widehat{R}, L)$ in equilibrium—thus, the equilibrium in which shells circulate exists uniquely. Moving on to show that, we first derive equations (19). By (24), $1 + r_s = \widehat{R}p_0$. Substituting it into (18) and rearranging, we have

$$p_0 = \frac{q}{1-(1-q)\underline{A}/(\overline{A}\alpha)\cdot\widehat{R}}. \tag{27}$$

Substitute it and (26) into (25), rearrange, let $\theta \equiv \underline{A}/(\overline{A}\alpha)\cdot\widehat{R}$, and we come to (19):

$$S = \left(\frac{\underline{A}}{w^{\alpha}}\right)^{\frac{1}{1-\alpha}} \frac{1-(1-q)\theta}{q}\left(\frac{1}{\theta^{\frac{1}{1-\alpha}}} - \frac{\mu G}{1-\theta}\right) \equiv F(\theta). \tag{28}$$

It is straightforward to verify that (a) $F'(\theta) < 0$; (b) equation $F(\theta) = 0$ is equivalent to (16). Thus, at $S = 0$, $\widehat{R} = \widehat{R}(G)$, where $\widehat{R}(G)$ is the interest rate determined by (16) in proposition 3, namely the actual interest rate without QE; and (c) at $\widehat{R} = R^{SB}$, $F(\frac{\underline{A}}{\overline{A}\alpha} R^{SB}) = \underline{S}(G)$. It follows that for any $S < \underline{S}(G)$, equation (28) determines a unique $\widehat{R} \in (R^{SB}, \widehat{R}(G)]$; hence $\underline{S}(G)$ is the threshold that was to be found at the beginning of the proof. After $\widehat{R}$ is found, it uniquely determines r, L, and p_0, respectively, through equations (24), (26), and (27). It also uniquely determines r_s by putting equations (24) and (27) together, which leads to

$$r_s = \frac{[q\overline{A}\alpha + (1-q)\underline{A}]\widehat{R} - \overline{A}\alpha}{\overline{A}\alpha - (1-q)\underline{A}\widehat{R}}. \tag{29}$$

Therefore, any $S \in [0, \underline{S}(G))$ pins down a unique equilibrium profile of $(p_0, r_s, r, \widehat{R}, L)$.

By (29), $r_s > 0 \Leftrightarrow \widehat{R} > R^{SB} \Leftrightarrow S < \underline{S}(G)$. Moreover, $\underline{A}(1+r_s)/(\overline{A}\alpha) < 1 \Leftrightarrow 1+r_s < \overline{A}\alpha/\underline{A}|_{(29)} \Leftrightarrow q\widehat{R}/[1-(1-q)\underline{A}/(\overline{A}\alpha)\cdot \widehat{R}] < \overline{A}\alpha/\underline{A} \Leftrightarrow \widehat{R} < \overline{A}\alpha/\underline{A}|_{\widehat{R}<\widehat{R}(G)} \Leftarrow \widehat{R}(G) < \overline{A}\alpha/\underline{A}$, which is affirmed by proposition 3(ii).

For proposition 4(ii), as $F'(\theta) < 0$, we have θ, and thus $\widehat{R}$, decrease with S. By property (c) of function $F(\cdot)$ above, $\widehat{R} = R^{SB}$ at $S = \underline{S}(G)$. By (29), r_s increases with $\widehat{R}$ and $r_s = 0$ at $\widehat{R} = R^{SB}$. Therefore, r_s decreases with S and equals 0 at $S = \underline{S}(G)$.

Q.E.D.

Proof of Proposition 5

Result (i) is proved in the main text. As for (ii), as $G < G^{FB}$, $\widehat{R}(G) > R^{FB} > R^{SB}$. Note that $\widehat{R} = \widehat{R}(G)$ at $S = 0$ and $\widehat{R} = R^{SB}$ at $S = \underline{S}(G)$. Therefore, there is a unique S between 0 and $\underline{S}(G)$ at which $\widehat{R} = R^{FB}$ and this S equals $F(\frac{\underline{A}}{\overline{A}\alpha} \cdot R^{FB})$ by (19). As for the profit of banks, in the unique equilibrium, each bank serves N entrepreneurs and the profit obtained from one of them, $\widehat{\pi}$, is the difference of the social value of his project minus his profit from it; that is, $\widehat{\pi} = A_e L^\alpha - wL - V$. With L and V given by (5) and (6) and $R = \widehat{R}$,

$$\widehat{\pi} = \left(\frac{\overline{A}\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} (q\overline{A}\alpha + (1-q)\underline{A}) \cdot \widehat{R}^{\frac{-1}{1-\alpha}} (\widehat{R} - R^{SB}).$$

Note that $\widehat{\pi}$ increases with $\widehat{R}$ for $\widehat{R} \in [R^{SB}, \frac{1}{\alpha}R^{SB}]$. Define $\underline{G} \equiv G(\frac{1}{\alpha}R^{SB})$. Then, if $G \geq \underline{G}$, we have $\widehat{R} \leq \widehat{R}(G) \leq \widehat{R}(\underline{G}) = \frac{1}{\alpha}R^{SB}$ for any $S \geq 0$. Therefore, QE of any $S > 0$—in particular, the optimal one—lowers $\widehat{\pi}$ and hence reduces banks' profit by decreasing $\widehat{R}$.

Q.E.D.

Proof of Proposition 6

(i) The equilibrium is found in a way parallel to that of proposition 3: first find the demand for notes given $\widehat{R}$, then the supply, and finally the equilibrium value of $\widehat{R}$. Given $\widehat{R}$, entrepreneurs hire $L = (\frac{\overline{A}\alpha}{w})^{\frac{1}{1-\alpha}}\widehat{R}^{\frac{-1}{1-\alpha}}$ workers by (5). With flexible nominal wage, workers accept a nominal wage of $w/(1 + r_p)$. Therefore, the demand for notes is

$$E^*\left(\widehat{R}\right) = w\left(\frac{\overline{A}\alpha}{w}\right)^{\frac{1}{1-\alpha}} \frac{\widehat{R}^{\frac{-1}{1-\alpha}}}{1 + r_p}.$$

Now consider the supply side. By borrowing face value E, an entrepreneur hires $E(1+r_p)/w$ workers, which means that $\delta = 1 + r_p$ in equation (3). Hence if a bank charges interest r, the actual rate of its loans is $R = (1 + r)/(1 + r_p)$. At the optimum, all banks offer $R = \widehat{R}$, the market clearing rate. It follows that

$$1 + r = \widehat{R}(1 + r_p).$$

That is, banks mark up the interest rate on loans to pass the cost of deposit due to the interest rate policy on to entrepreneurs. If a bank issues notes of aggregate face value D, then at $t = 1$, with the interest rate policy, its liability to the note holders is $D(1+r_p)$. The equity value in the bad state is thus $G+\underline{Y}-D(1+r_p)$, which, by the borrowing constraint, cannot be smaller than $(1-\mu)G$. With $\underline{Y} = \frac{\underline{A}(1+r)}{\overline{A}\alpha}D$, this constraint is equivalent to

$$D(1 + r_p) \leq \mu G + \frac{\underline{A}(1 + r)}{\overline{A}\alpha}D, \tag{30}$$

which, with $1+r=\widehat{R}(1+r_p)$, is equivalent to

$$D \leq \frac{\mu G}{1+r_p}\left(1-\frac{A}{\overline{A}\alpha}\widehat{R}\right)^{-1} := \overline{D}^{*}\left(\widehat{R};G\right).$$

According to proposition 1, if $\widehat{R}=R^{SB}$, the profit margin of lending is zero and banks are indifferent in any D, while if $\widehat{R}>R^{SB}$, the profit margin is positive and banks want to lend as much as they can, that is, the above borrowing constraint is binding. Hence, the supply of notes is

$$D^{*}\left(\widehat{R};G\right)=\left\{\begin{array}{c} 0 \text{ if } \widehat{R}<R^{SB} \\ [0,\overline{D}\left(\widehat{R};G\right)] \text{ if } \widehat{R}=R^{SB} \\ \overline{D}\left(\widehat{R};G\right) \text{ if } \widehat{R}>R^{SB} \end{array}\right\}.$$

Observe that in equation $E^{*}\left(\widehat{R}\right)=\overline{D}^{*}\left(\widehat{R};G\right)$, the factor $1+r_p$ is canceled out, and the equation is equivalent to $E\left(\widehat{R}\right)=\overline{D}\left(\widehat{R};G\right)$, where $E\left(\cdot\right)$ and $\overline{D}\left(\cdot\right)$ are the demand and maximum supply functions of notes in proposition 3, namely in the absence of the policy. It follows that $E^{*}\left(\widehat{R}\right)=D^{*}\left(\widehat{R};G\right)$ if and only if $E\left(\widehat{R}\right)=D\left(\widehat{R};G\right)$, where $D\left(\cdot\right)$ is the supply functions of notes in proposition 3. That is, the equilibrium $\widehat{R}$ in the presence of the interest rate policy is the same as that in its absence. Therefore, the interest rate policy produces no real effects.

(ii) In the circumstance where the nominal wage of workers stays at w invariant to the policy rate r_p, if an entrepreneur borrows notes of face value E, he still hires E/w (rather than $E(1+r_p)/w$) workers. This implies (a) that the aggregate demand for notes is wL (rather than $wL/(1+r_p)$), which is equalized to the aggregate supply D in equilibrium; and (b) that the discount factor in (3) is $\delta=1$, whereby the actual interest rate to entrepreneurs is $R=1+r$ (rather than $R=(1+r)/(1+r_p)$). Given $\widehat{R}$, $L=(\frac{\overline{A}\alpha}{w})^{\frac{1}{1-\alpha}}\widehat{R}^{\frac{-1}{1-\alpha}}$ by (5). It follows from implication (a) that $D=w(\frac{\overline{A}\alpha}{w})^{\frac{1}{1-\alpha}}\widehat{R}^{\frac{-1}{1-\alpha}}$.

At the optimum all banks choose $R = \widehat{R}$, which together with implication (b) means $1 + r = \widehat{R}$. Substitute these into (30) and banks' borrowing constraint becomes

$$w\left(\frac{\overline{A}\alpha}{w\widehat{R}}\right)^{\frac{1}{1-\alpha}}\left[(1+r_p) - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}\right) \leq \mu G. \tag{31}$$

This borrowing constraint is binding if the profit margin of lending is positive, as before. Given that the workers' real wage is now $w(1 + r_p)$, the profit to a bank from lending to one entrepreneur becomes $A_e L^\alpha - w(1 + r_p)L - V$. With L and V as functions of $R = \widehat{R}$ given by (5) and (6), the profit margin with sticky wage is $[\overline{A}\alpha/(w^\alpha \widehat{R})]^{\frac{1}{1-\alpha}}[\widehat{R}/R^{SB} - (1 + r_p)]$. This profit margin never goes below 0. Therefore,

$$1 + r_p \leq \widehat{R}/R^{SB}, \tag{32}$$

which implies that $\widehat{R} \geq R^{SB}$.

The equilibrium actual interest rate $\widehat{R}$ is pinned down by conditions (31) and (32), and the fact that one of them must be binding: if (32) is not binding—that is, if the profit margin of issuance is positive—then banks keep issuing notes until the no-default constraint, (31), is binding. Note that (31) is equivalent to

$$1 + r_p \leq \Phi(\widehat{R}), \tag{33}$$

where

$$\Phi(R) \equiv \frac{\mu G w^{\frac{\alpha}{1-\alpha}}}{(\overline{A}\alpha)^{\frac{1}{1-\alpha}}} R^{\frac{1}{1-\alpha}} + \frac{\underline{A}}{\overline{A}\alpha} R, \tag{34}$$

and obviously $\Phi'(\cdot) > 0$, and the inverse function, $\Phi^{-1}(\cdot)$, exists. Then in equilibrium, inequalities (33) and (32) hold, and one of them must be binding. Therefore,

$$1 + r_p = \min\left(\widehat{R}/R^{SB}, \Phi(\widehat{R})\right) := H(\widehat{R}). \tag{35}$$

We only need to consider function $H(\cdot)$ over $\widehat{R} \geq R^{SB}$.

CLAIM P6. *Over* $\widehat{R} \geq R^{SB}$, *if* $G \geq \frac{1}{\mu}G^*$, $H(\widehat{R}) = \widehat{R}/R^{SB}$; *and if* $G < \frac{1}{\mu}G^*$, *then*

$$H(\widehat{R}) = \left\{ \begin{array}{c} \Phi(\widehat{R}) \text{ if } \widehat{R} \leq R^* \\ \widehat{R}/R^{SB} \text{ if } \widehat{R} \geq R^* \end{array} \right\},$$

where

$$R^* = \frac{\overline{A}\alpha}{w}\left[\frac{q(\overline{A}\alpha - \underline{A})}{\mu G}\right]^{\frac{1-\alpha}{\alpha}}. \tag{36}$$

Proof. Let $\chi(R) \equiv \Phi(R) - R/R^{SB}$ for $R \geq 0$. Then $\chi(R) = 0$ has two roots: 0 and R^*. Then (a) $R^* \geq R^{SB}$ if and only if $G \leq \frac{1}{\mu}G^*$; (b) $\chi < 0$ for $R \in (0, R^*)$ and $\chi > 0$ for $R > R^*$, because $\chi'(0) < 0$. These two results lead to the claim. Q.E.D.

With help of the claim, we solve $\widehat{R}$ as a function of r_p from (35) as follows. If $G \geq \frac{1}{\mu}G^*$, $\widehat{R} = R^{SB}(1 + r_p)$. If $G < \frac{1}{\mu}G^*$, then

$$\widehat{R} = \left\{ \begin{array}{c} \Phi^{-1}(1 + r_p) \text{ if } r_p \leq r_p^* \\ R^{SB}(1 + r_p) \text{ if } r_p \geq r_p^* \end{array} \right\},$$

where $r_p^* \equiv R^*/R^{SB} - 1$, which, as $\chi(R^*) = 0$, also equals $\Phi(R^*) - 1$; hence $r_p^* \geq 0 \Leftrightarrow R^* \geq R^{SB} \Leftrightarrow G \leq \frac{1}{\mu}G^*$.

In both cases, $\widehat{R}$ increases with the policy rate r_p to infinity. And at $r_p = 0$, namely in the absence of the CB's intervention, $\widehat{R} < R^{FB}$ in this circumstance of banks over-lending. Therefore, there exists a unique policy rate at which $\widehat{R} = R^{FB}$ and the first-best allocation is attained.

Banks obtain zero profit if the non-negative profit constraint, (32), is binding, namely $H(\widehat{R}) = \widehat{R}/R^{SB}$, which, by claim P6, is the case if $\widehat{R} \geq R^*$. With the optimal policy rate, $\widehat{R} = R^{FB}$, and hence banks obtain zero profit if $R^{FB} \geq R^*$, which is equivalent to $G \geq G_s := \frac{1}{\mu}[(q\overline{A}\alpha + (1-q)\underline{A}) - \underline{A}](A_e\alpha/w)^{\frac{\alpha}{1-\alpha}}$. It is straightforward to check that $G^{FB} < G_s < \frac{1}{\mu}G^*$. Q.E.D.

Proof of Proposition 7

According to the discussion of subsection 3.2, $\overline{Y} = D(1 + r)$ and $\underline{Y} = \underline{A}/(\overline{A}\alpha) \times D(1+r)$. Therefore, $Y = [q + (1-q)\underline{A}/(\overline{A}\alpha)] \times D(1+r)$.

Moreover, at the optimum, banks all offer $1 + r = \widehat{R}$. It follows that if banks issue D, the capital adequacy ratio is

$$c = \frac{G + \{[q + (1-q)\underline{A}/(\overline{A}\alpha)]\widehat{R} - 1\}D}{G + [q + (1-q)\underline{A}/(\overline{A}\alpha)]\widehat{R}D}.$$

If banks' issuance is at the first quantity and the first-best allocation is attained, then $D = D^{FB}$ and $R = R^{FB}$ in the above formula and the capital adequacy ratio equals

$$c^{FB} := \frac{A_e \alpha G + (1-q)\underline{A}(1-\alpha)D^{FB}}{A_e \alpha G + [q\overline{A}\alpha + (1-q)\underline{A}]D^{FB}}.$$

The regulation restricts $c \geq c^{FB}$. To prove the proposition, it suffices to show that this restriction is equivalent to $D \leq D^{FB}$, the constraint which, as we saw in the main text, will lead to the first-best allocation. This equivalence follows from two observations: (i) $c = c^{FB}$ if $D = D^{FB}$ and (ii) c decreases with D because $c = 1 - \frac{D}{G + [q + (1-q)\underline{A}/(\overline{A}\alpha)]\widehat{R}D} := c(D, \widehat{R})$ and moreover, $\frac{\partial c}{\partial D} < 0$ and $\frac{\partial c}{\partial \widehat{R}} > 0$ while $\widehat{R}$ decreases with D.

Q.E.D.

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Banks' Wealth, Banks' Creation of Money, and Central Banking*

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Banks are special in that their liabilities are widely accepted as a means of payment, thereby often needed by real sectors to obtain resources. This paper studies this interaction between the banking sector and real sectors on competitive markets and the policy response of the central bank to market inefficiency, which is determined by the aggregate wealth of banks. In the circumstance of a credit crunch, the central bank improves efficiency by allowing banks to borrow its fiat money at zero interest up to a limit. This policy bears the flavor of quantitative easing policies (QE). It produces real effects in the absence of surprises and nominal rigidity. The mechanism in which it works depends on a difference in nature between bank-created money and fiat money. Furthermore, this policy, while expanding the money supply, induces deflation under the positive productivity shock. Lastly, this paper explains when interest rate policy and capital adequacy regulation are among the optimal policies within a unified model.

JEL Codes: E5, E65, G21.

"The community cannot get rid of its currency supply... The 'hot potato' analogy truly applies. For bank-created money, however, there is an economic mechanism of extinction as well as creation, contraction as well as expansion."

James Tobin (1963)

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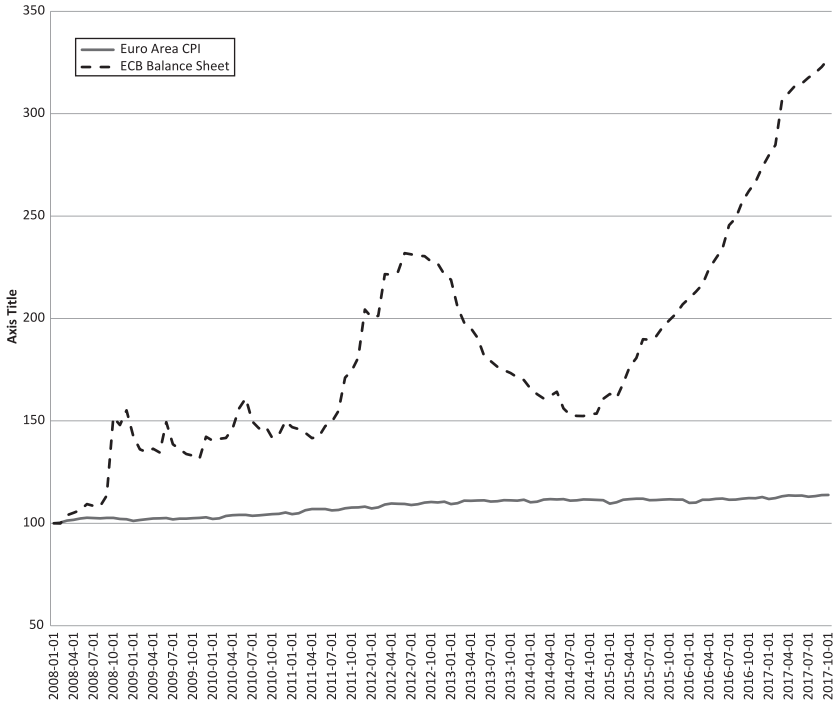
1. Introduction

Commercial banks are special in that their liability, especially that in the form of demand deposit, is *widely* accepted as a means of payment,[1] whereas rarely so are the liabilities of non-bank firms or households. Due to this difference, real-sector firms often need to borrow a bank's liability (usually called *money* in everyday language) as a means of paying for resources that they want. Banks' decisions on the price and quantity of money in lending and the competition between them, therefore, have a profound impact on the economic activity of real sectors, which, inversely, affects the decisions of and competition between banks. Furthermore, often, the central bank's policy produces effects by affecting banks' lending decisions and is based on its ability to create an alternative means of payment, namely fiat money. While money creation by banks is well known and widely introduced in macroeconomic textbooks, these interactions between real sectors, the banking sector, and the central bank in relation to means of payment have not been studied much yet,[2] a relationship which this paper makes an attempt to understand. Specifically, it presents a general equilibrium analysis of money creation by banks and of how certain policies of the central bank improve efficiency over the market allocations. This analysis

[1] Often lay people perceive that depositing cash is to let the bank store the money deposited. However, this perception is wrong. Depositing is an exchange of cash to the bank's liability, which is what the depositor owns with the bank account and, when he makes a purchase with the account, is what he uses for the payment.

[2] The interaction between the banking sector and real sectors is studied in the literature on financial intermediation, where money creation by banks is not a concern; see Gorton and Winton (2003) for a survey. A strand of literature uses search-matching frameworks—see, e.g., Cavalcanti, Erosa, and Temzalides (1999) and Williamson (1999)—to examine how and when certain privately issued claims circulate as a means of payment, but has difficulty accommodating banks' decision on the price and scale of lending and competition between them. Lastly, New Keynesian literature and the literature that uses frameworks of cash in advance (CIA) both study monetary policy. However, the former abstracts banks completely, while the latter, when concerned with money creation by banks, lets the scale of banks' lending be pinned down either by a binding reserve constraint (see, e.g., Goodfriend and McCallum 2007) or by an exogenous rule of holding excessive reserves (see, e.g., Chen 2018 and Mishkin 2016), which leaves little role for banks' own decisions.

Figure 1. The Balance Sheet of the European Central Bank (ECB) and the Consumer Price Index in the Euro Area over Jan. 1, 2008–Oct. 10, 2017 (with Jan. 1, 2008 = 100)



Source of the Data: www.https://fred.stlouisfed.org/.
Note: The three hikes in the ECB's balance sheet during the period are associated with only slight price increases, if anything at all.

results in new insights on the mechanism in which the quantitative easing policy (QE) works, based on which this paper offers an explanation for the observation that QE, while causing enormous monetary expansion, is associated with low inflation or even deflation pressure in certain economies. One of them is the euro area, as is illustrated in figure 1.

The model economy of the paper is populated by workers, entrepreneurs, and banks. Workers can produce the consumption good, corn, in autarky, or work for entrepreneurs. The specialty of banks is modeled with the assumption that workers accept banks' promises to pay, but not entrepreneurs', as a means of wage payment. At date 0, therefore, entrepreneurs first borrow banks' promises to pay—more specifically, notes that read like "X bank promises to pay the bearer 10 kilograms of corn tomorrow"—and then use these notes to hire workers. As a result, entrepreneurs owe a debt to the lender banks and banks owe a debt to the workers. At date 1, entrepreneurs produce corn and use it to settle their debts to the banks. Banks then use this repayment and their own corn, which is stored over time and represents their wealth, to redeem their notes from the workers by fulfilling the promises written on the notes.

In this economy, the real resources are workers' labor and entrepreneurs' capital, and efficiency is measured with the number of workers that entrepreneurs hire. Banks matter, however, because entrepreneurs need to borrow banks' liabilities to hire workers. How much banks lend in terms of real value determines how many workers entrepreneurs hire and hence economic efficiency. The aggregate lending of banks is in turn determined by competition between them. What banks supply is a means of payment, which is a homogenous good. They thus engage in Bertrand competition. Moreover, what a bank lends is its liability. Hence the scale of its lending is anchored to its wealth by a borrowing constraint.[3] Put together, banks are engaged in Bertrand competition with a limited capacity à la Kreps and Scheinkman (1983). If the borrowing constraint is binding, banks' aggregate wealth determines the quantity of money supplied to entrepreneurs, and hence efficiency. If this wealth is below a threshold, the money supply is inadequate and so is the number of workers that entrepreneurs hire.

This problem of meager bank wealth depressing economic activity has been diagnosed in many studies such as Gertler and Kiyotaki (2010) and Holmstrom and Tirole (1997). However, these studies are not concerned with means of payment. Hence they do not consider the possibility of a remedy with the central bank (CB) issuing fiat money. By contrast, in this paper the CB can offer a remedy by allowing banks to borrow its fiat money at zero interest up to a

[3] This borrowing constraint could be due to a moral hazard friction à la Gertler and Kiyotaki (2010) or Holmstrom and Tirole (1997), or due to the risk-shifting problem à la Jensen and Meckling (1976).

bound. We show that although the economy lasts for only two periods, in one equilibrium the fiat money circulates, flowing through banks to entrepreneurs, who thereby hire more workers. That is, the policy eases the constraints imposed upon their economic activity by an inadequate supply of bank-created money. Hence, the policy is called *quantitative easing policy*, or QE.

QE enables banks to raise lending capacity without violating the borrowing constraint, while backed by the same amount of wealth. It does so because fiat money is different in nature from bank-created money. A central bank never commits to redeem its fiat money at a specified value, whereas bank-created money is the bank's liability, which it commits to redeem at a pre-specified value and thus bears a real obligation of repayment.[4] It is to this difference in nature that Tobin (1963) alludes above. As fiat money is not redeemable, its value freely adjusts with the state of the economy. In particular, its value falls in the event of the negative productivity shock, which means *inflation*. Inflation reduces the real value of banks' liability to the CB and thus reduces their borrowing constraint, giving them room to increase lending.[5]

While QE induces inflation in the event of the negative shock, it induces deflation in the event of the positive shock. This result might partly explain the aforementioned observation that in some economies QE causes a great monetary expansion on the one hand and is accompanied by or followed by lingering low inflation or even deflation pressure on the other hand, which is a puzzle if considered from the point of view of quantitative theory of money. A key difference in this paper is that the money created with QE is utilized to expand real economic activity, resulting in a rise in the output of goods, which keeps the price down. Indeed, all the QE-created money

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[4]In the model economy, this fact is straightforward because banks use corn, the real good, to redeem their liability. Even if they redeem it with fiat money, which is typically the case in modern times, they still need to spend real resources to obtain it. Hence bank-created money still bears real obligations of repayment. For more discussion, see the remark in section 5.

[5]The point that monetary policy can help banks by affecting the real value of their liabilities is also considered by Diamond and Rajan (2006). The way in which fiat money circulates in the model economy is also in Allen and Gale (1998). However, these papers are not concerned with banks lending their liabilities to real sectors to be used as means of payment, nor with the inefficiency related to banks' wealth, nor with the quantitative easing policy.

is utilized this way if the scale of QE is below a threshold. If it is beyond it, the excess is not put into circulation, we also find. This finding partly explains another related observation, that in some economies a substantial fraction of the monetary bases created with QE is not lent out but stays in banks' reserve accounts.

This paper is in line with nascent literature that examines banks' specialty of their liability being accepted as a means of payment. Most closely related is Donaldson, Piacentino, and Thakor (2018). Both their paper and this one consider how banks' issuance of means of payment affects the real economy as well as policy implications. The two papers, however, have different focuses. Their paper explains how this specialty of banks is derived from their superior technology of warehousing, while this paper emphasizes the importance of banks' wealth for economic efficiency. Also the policy implications are different. Their paper shows that, contrary to the received wisdom, a higher central bank rate could raise bank lending, while this paper considers QE. Jakab and Kumhof (2015) describe in detail how banks create money with double bookkeeping. They focus on the quantitative implications of this facet of banking in a full-fledged dynamic stochastic framework, but are not concerned with economic efficiency and policy responses, on which this paper focuses.

Since the recent crisis, many studies have examined QE; see Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), among others. Their diagnosis on the source of the problem is shared by this paper: banks' wealth is too low, causing inadequate lending.[6] However, those studies model banks not as issuers of means of payment but as intermediaries of trading real goods. As a result, in those studies what the government issues must be backed by tax incomes and is essentially the sovereign debt, whereas what the central bank issues in this paper is purely nominal. Furthermore, the mechanism by which QE works is different. In those studies, it works by transferring wealth to banks, whereas in this paper it works by reducing the real value of banks' liability via inflation.

[6]This is also true in a study by Brunnermeier and Sannikov (2013), where money plays the role of a saving instrument, as in Samuelson (1958), rather than the role of means of payment.

Other cases of inefficiency in connection with private issuance of means of payment are considered by Hart and Zingales (2015), Monnet and Sanches (2015), and Stein (2012). In Hart and Zingales (2015) and Stein (2012) inefficiency is driven by fire-sale externalities. In Monnet and Sanches (2015) inefficiency arises because banks offer a return rate on the liability side that is lower than the inverse of time discount. Besides this difference in the source of inefficiency, none of those studies considers the circumstance of banks under-lending or the policy of QE, as this paper does.

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 analyzes a benchmark case where banks face no borrowing constraint and are thus subject to unfettered forces of competition. This analysis not only examines the bank competition in the purest form but also bears relevance to the historical banking. Section 4 introduces the borrowing constraint of banks. Section 5 studies QE, and section 6 studies interest rate policy and capital adequacy regulation. Section 7 concludes. All proofs are relegated to the appendix.

2. The Model

The economy has one storable good, corn, used as the numeraire, and lasts for two days. Contracting and production occur at $t = 0$, yielding and consumption at $t = 1$. There are N banks, N^2 entrepreneurs, and N^3 workers, where N is a large number (later, in section 5, the central bank will be introduced).[7] Thus, banks are in perfect competition and each serves a large number of entrepreneurs; and there are more workers than entrepreneurs can hire. All agents are risk neutral and protected by limited liability.

Workers either produce w kilograms (kg) of corn in autarky or are hired by entrepreneurs, who each have h units of human or physical capital. If an entrepreneur hires L workers at $t = 0$, then his project yields at $t = 1$

$$y = \widetilde{A}h^{1-\alpha}L^{\alpha},$$

[7] A setting with the same feature is to be found in Wang (2015), where a continuum of $[0, 1]$ and $[0, 1] \times [0, 1]$ instead of N and N^2 is used. These two representations are equivalent.

where $0 < \alpha < 1$. Without losing any generality, normalize $h = 1$. Productivity, $\widetilde{A}$, is subject to a common shock. At $t = 0$, it is common knowledge that the good state with $\widetilde{A} = \overline{A}$ occurs with probability $q > 0$ and the bad state with $\widetilde{A} = \underline{A}$ occurs with probability $1 - q > 0$. Let $A_e \equiv q\overline{A} + (1 - q)\underline{A}$ denote the mean. Assume the following:

$$0 < \underline{A} < A_e\alpha. \tag{1}$$

As there are more workers than can be hired by entrepreneurs, in equilibrium workers are indifferent in working for the latter or in autarky. Therefore they earn a real wage of w. This wage is independent of economic activity of the other sectors, which gives a convenience for exposition.

Banks each have G units of corn, where a unit is defined as N kg and used wherever banks are concerned. Banks supply no real resources for corn production. What makes banks relevant is due to the following assumption.

ASSUMPTION 1. *Workers do not accept entrepreneurs' promise to pay but banks' as a means of wage payment.*

This assumption captures the specialty of banks explicated in the Introduction. According to Kiyotaki and Moore (2001), this difference between banks and entrepreneurs arises because the former has stronger commitment power than the latter.

Due to this assumption, entrepreneurs cannot hire workers at $t = 0$ with a promise to pay them later at $t = 1$. To hire workers, entrepreneurs need to borrow banks' promise to pay them at $t = 1$. We assume that banks have no difficulty enforcing repayment from entrepreneurs and hence this borrowing is feasible. To fix the idea, suppose that banks' promises to pay are printed on notes. That is, a note issued by a bank reads that this bank promises to pay the bearer of this note with a certain quantity of corn at $t = 1$.[8] This quantity is the note's *face value* or *par value*.

[8]In the model economy banks promise to pay the real good in redeeming their liability, whereas in modern times, they typically redeem it with fiat money, the central bank's issues. Other instances of claims on real goods used as a means of

ASSUMPTION 2. *The face values of banks' notes cannot be contingent on the realization of* $\widetilde{A}$.

This assumption captures the observation that in real life a private security that serves as a means of payment—such as demand deposit, promissory notes, cheques, or trade credit—commonly bears fixed claims and is of debt, and is rarely a contingent claim.[9]

Besides the friction of payment, entrepreneurs lack commitment power in another dimension.

ASSUMPTION 3. *Entrepreneurs are unable to make commitment on the scale of their projects in terms of the number of workers they will hire.*

This friction is *real* in the sense that it is unrelated to means of payment. In the absence of the friction, a bank would be willing to lower the interest rate, denoted by r, to a borrower entrepreneur who commits to a smaller scale, because thereby his project delivers a higher average return rate. This, however, would give entrepreneurs an incentive to borrow from multiple banks, each in a small amount and thus at a favorable rate. The presence of the real friction, therefore, is justified if entrepreneurs cannot be prevented from doing so. The importance of its presence is that it engenders a wedge between the first- and second-best allocations, as will be shown.

Due to the friction, a bank posts a single interest rate r for loans of any size E rather than a menu of $r(E)$. A loan contract is represented by a profile of (E, r): at $t = 0$ the entrepreneur borrows the bank's notes of overall face value E, and at $t = 1$ he is obliged to pay $E(1 + r)$ kg corn back to the bank.

The timing of events is as follows. At $t = 0$, each bank posts the aggregate face value of notes that it will issue, D, and the interest rate that it will charge, r. Observing all these offers, each entrepreneur then chooses one bank to go to and asks to borrow face value

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payment incude Hart and Zingales (2015) and Williamson (1999). This abstraction is harmless in the baseline model where inflation is not addressed. Where it is, as in section 5 below, the abstraction helps clarify the mechanism by which QE works.

[9]For the reason why that is the case, see Dang, Gorton, and Holmstrom (2012) and Gorton and Pennacchi (1990).

E of the bank's notes. If one bank's notes are over-demanded, only a fraction of the entrepreneurs have their demand met. Entrepreneurs then use borrowed notes to hire workers and start the production. Banks store their G units of corn.

At $t = 1$, entrepreneurs produce corn. They either repay $E(1+r)$ kg corn to the lender banks or default. In that case they give all their output of corn to the banks. After receiving repayments from the entrepreneurs, the sum total denoted by $\widetilde{Y}$, banks redeem notes from workers. If $\widetilde{Y}+G \geq D$ to a bank, its notes are redeemed at par. If $\widetilde{Y}+G < D$, the bank defaults and the notes are redeemed at a fraction $(\widetilde{Y}+G)/D$ of their par values. Finally, the agents consume the corn in their possession.

In anticipation of the possibility of default, at $t = 0$ a bank's notes are discounted with a factor of

$$\delta = \mathrm{E}_{\widetilde{A}}\left[\min\left(1, \frac{\widetilde{Y}+G}{D}\right)\right]. \tag{2}$$

Moving on to the equilibrium analysis, we figure out two benchmark allocations.

2.1 The First-Best and Second-Best Allocations

Efficiency concerns the number of workers allocated to entrepreneurs. Define the first-best allocation as the number of workers that maximizes the social surplus of projects, which is $A_e L^{\alpha} - wL$ due to universal risk neutrality and the opportunity cost of labor being w. The first-best allocation is thus

$$L^{FB} = \left(\frac{A_e \alpha}{w}\right)^{\frac{1}{1-\alpha}}.$$

The second-best allocation is defined as the number of workers that entrepreneurs would hire in the competitive equilibrium if the friction of payment (in assumption 1) were absent but the real friction (in assumption 3) remained—that is, if entrepreneurs could hire workers with their own promise to pay, but their wage offer could not be conditional on the scale of their projects. The equilibrium allocation is as follows.

LEMMA 1. *The second-best number of workers that entrepreneurs hire is*

$$L^{SB} = \left(\frac{q\overline{A}\alpha + (1-q)\underline{A}}{w} \right)^{\frac{1}{1-\alpha}}.$$

Obviously, $L^{SB} > L^{FB}$. That is, the real friction induces the entrepreneurs to hire too many workers.[10] This fact engenders a circumstance of banks over-lending, among remedies to which are interest rate policy and capital adequacy regulation, as will be shown in sections 4 and 6.

Below we first analyze the baseline model in which bank issuance is subject to no borrowing constraints nor any other restrictions. This analysis serves two purposes: (i) it studies the competitive equilibrium of money creation by banks in the purest form; (ii) it bears relevance to early periods of banking history.

3. The Least-Fettered Issuance

In the least-fettered issuance, banks *can* finance an unlimited quantity of assets by issuing promises to pay. The *equilibrium* quantity is of course limited, as examined below. We first consider the demand side of the market for banks' notes, then the supply side, and, finally, the meeting of the two.

3.1 The Demand Side of the Market for Notes

Consider a representative entrepreneur who borrows from a bank offering (D, r). If he borrows notes of a face value E, then they are worth δE, where the discount factor δ, as will be shown, is a function of (D, r). With these notes, he hires workers in a number of

$$L = \frac{\delta E}{w}, \tag{3}$$

because they earn real wage w. At $t = 1$, the entrepreneur either repays $E(1 + r)$ of corn to the bank or defaults. Thus, his decision problem is

[10] For a more general analysis of this type of inefficiency, see Wang (2010).

$$\max_{E} \mathrm{E}_{\widetilde{A}}[\max(\widetilde{A}L^{\alpha} - E(1+r), 0)], s.t.(3).$$

LEMMA 2. *For any (w, δ, r), the solution to the above problem satisfies $\underline{A}L^{\alpha} < E(1+r)$. That is, entrepreneurs all default in the bad state.*

This lemma is driven by the assumption that $\underline{A} < A_e\alpha$, which says that the negative shock is severe enough to knock entrepreneurs into default.

At the optimum, the demand of the representative entrepreneur for the notes is

$$E(\delta, r) = \left(\frac{\overline{A}\alpha}{1+r}\right)^{\frac{1}{1-\alpha}} \left(\frac{\delta}{w}\right)^{\frac{\alpha}{1-\alpha}} \tag{4}$$

and the number of workers that he hires and his profit are, respectively,

$$L(R) = \left(\frac{\overline{A}\alpha}{wR}\right)^{\frac{1}{1-\alpha}} \tag{5}$$

$$V(R) = q(1-\alpha)\left(\frac{\overline{A}^{\frac{1}{\alpha}}\alpha}{wR}\right)^{\frac{\alpha}{1-\alpha}}, \tag{6}$$

where

$$R \equiv \frac{1+r}{\delta}. \tag{7}$$

Note that, so defined, R is the *actual* interest rate and measures the cost of borrowing: to obtain a means of payment that is worth 1, the entrepreneur borrows notes of face value $1/\delta$, thus acquiring a debt of $(1+r)/\delta$. Naturally, his scale of hiring and his profit, both of which are real variables, are inversely related to the cost of borrowing, namely, R.

Recall that the efficiency concerns only the number of the workers hired by entrepreneurs, which depends only on the actual interest rate. The efficiency of market equilibrium is thus determined solely by the actual interest rate in equilibrium. Define R^{FB} (R^{SB}) as the value of the actual interest rate at which entrepreneurs hire the first-best (second-best) number of workers, that is, $L(R) = L^{FB}$ (L^{SB}). With (5),

$$R^{FB} = \frac{\overline{A}}{A_e}$$

$$R^{SB} = \frac{\overline{A}\alpha}{q\overline{A}\alpha + (1-q)\underline{A}}.$$

After banks all have posted (D, r), each entrepreneur decides which bank to go to. In the equilibriums of this subgame, an entrepreneur gets the same expected profit, $\widehat{V}$, from any bank that attracts a number of entrepreneurs.[11] As entrepreneurs' profit depends only on the actual interest rate, define $\widehat{R}$ by $V(\widehat{R}) = \widehat{V}$. Then, $\widehat{R}$ is the actual interest rate that prevails on the notes market, conditional on banks' choices of (D, r). Given that there is a large number of banks, any single bank is too small to affect $\widehat{R}$ with its choice of (D, r) and takes it as given when making that choice.

3.2 The Supply Side of the Market for Notes

Consider a representative bank. To attract entrepreneurs, the bank's choice of (D, r) satisfies $V(R) \geq \widehat{V}$ or, equivalently, $(1 + r)/\delta \leq \widehat{R}$. While the interest rate r is directly chosen by the bank, the discount factor of its notes δ is determined by (D, r). It depends on whether the bank ever defaults or not, that is, whether $D > \widetilde{Y} + G$ in some state at $t = 1$. In the good state, $\widetilde{Y} = \overline{Y} = D(1 + r)$ because the entrepreneurs do not default. As a result, the bank does not default. In the bad state, by lemma 2, all the entrepreneurs default and pass the output of their projects on to the bank, of which the aggregate value is given below.

[11]By offering (D, r), which determines δ, each bank chooses an actual interest rate $R = (1 + r)/\delta$. No entrepreneur goes to a bank offering $V(R) < \widehat{V}$ when he can get $\widehat{V}$ elsewhere. On the other hand, if a bank offers $V(R) > \widehat{V}$, it induces over-demand for its notes (which is never optimal), so an entrepreneur coming to it is served with such a probability l that $l \cdot V(R) = \widehat{V}$.

LEMMA 3. *The aggregate value of the bank's loans in the bad state is*

$$\underline{Y} = \frac{\underline{A}}{\overline{A}\alpha} \times D(1+r).^{12} \tag{8}$$

In the bad state, the bank does not default if and only if $D \leq G + \underline{Y}$, which, with $\underline{Y}$ given by (8), is equivalent to

$$D \cdot \left(1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha}\right) \leq G. \tag{9}$$

On the left-hand side is the loss made by lending in the bad state, $D - \underline{Y}$. Thus, the inequality says that banks will stay solvent in the bad state if and only if their lending scale, D, is not too large relative to their wealth, G, so that the loss from loans can be absorbed by the wealth.

Substitute the value of $\widetilde{Y}$ given above into (2), and the discount factor of the bank's notes is determined by its choice of (D, r) via

$$\delta(D,r) = \left\{ \begin{array}{c} 1, \text{ if (9) is satisfied} \\ q \times 1 + (1-q) \times (\frac{G}{D} + \frac{\underline{A}(1+r)}{\overline{A}\alpha}), \text{ otherwise} \end{array} \right\}. \tag{10}$$

Now consider the representative bank's decision problem at $t = 0$. Taking into account the possibility of default, its economic profit with the choice of (D, r), denoted by $\Pi(D, r)$, is $E_{\widetilde{Y}} \max(G + \widetilde{Y} - D, 0) - G$.

LEMMA 4.

$$\Pi(D,r) = D\left[\frac{1+r}{R^{SB}} - \delta\right]. \tag{11}$$

Therefore, the profit margin of lending—that is, the profit of lending out a note of face value 1—is $(1+r)/R^{SB} - \delta$. Intuitively, the present value of this note, which is part of the bank's liability, is δ, while the bank's revenue from such lending is $E_{\widetilde{Y}}\left(\widetilde{Y}/D\right) = (1+r)/R^{SB}$.

[12] $\underline{Y} < \overline{Y}$ because $\underline{A} < A_e\alpha$ by assumption and $A_e\alpha < \overline{A}\alpha$.

The representative bank chooses (D, r) to maximize $\Pi(D, r)$ subject to the constraint that it can attract entrepreneurs, that is,

$$\frac{1+r}{\delta(D,r)} \leq \widehat{R}. \tag{12}$$

Given the scale of lending D, the bank wants to charge an interest rate as high as possible so long as it can attract entrepreneurs. Therefore, at the optimum, the constraint (12) is binding.[13] Hence, the bank's optimal choice of r as a function of its choice of D, denoted by $r(D)$, is determined by

$$\frac{1+r}{\delta(D,r)} = \widehat{R},$$

and banks' problem becomes

$$\max_D D\delta(D, r(D)) \times \left[\frac{\widehat{R}}{R^{SB}} - 1\right].$$

As $\delta > q$ always (because the bank always redeems its notes at par in the good state), the following proposition is self-evident.

PROPOSITION 1. *The solution to and the value of the representative bank's problem are*

(i) if $\widehat{R} > R^{SB}$, then $D = \infty$ and $\Pi = \infty$;

(ii) if $\widehat{R} = R^{SB}$, then $\Pi = 0$, and the bank is indifferent to any value of D, with $r = r(D)$;

(iii) if $\widehat{R} < R^{SB}$, then lending makes a loss, and thus $D = 0$ and $\Pi = 0$.

As was shown, the profit margin of lending equals $\delta\left(\widehat{R}/R^{SB} - 1\right)$ and is positive if and only if $\widehat{R} > R^{SB}$. Moreover, if the profit margin of lending is positive, banks obtain $\Pi = \infty$. That is because,

[13] Mathematically, given D, both $\frac{1+r}{R^{SB}} - \delta(D,r)$ and $\frac{1+r}{\delta(D,r)}$ increase with r.

despite their limited stocks of corn, they all have an unlimited lending capacity (i.e., $D = \infty$), due to two reasons: (i) what banks lend out is their promise to pay, essentially word of mouth, which they can infinitely supply; (ii) banks are subject to no constraints on the quantity of supply in the baseline model.

3.3 The Equilibrium: The Second-Best Allocation Attained

The prevailing actual interest rate, $\widehat{R}$, which plays the role of price, clears the market for notes in equilibrium. We define the symmetric equilibrium below and discuss other equilibriums later.

DEFINITION 1. *A profile* $(\{D, r, \delta, E\}; \widehat{R})$ *forms an equilibrium if*

(i) given $\widehat{R}$*, banks' choice of* (D, r) *is optimal and thus given in proposition 1. This choice determines* $\delta = \delta(D, r)$ *through (10);*

(ii) given that all banks offer the same (D, r, δ)*, entrepreneurs go to each bank with the same probability and, by the law of large numbers, each bank receives* N *of them. Their demand for notes,* E*, is optimal, that is,* $E = E(\delta, r)$ *given by (4);*

(iii) the market clears: $D = E$*.*[14]

In any equilibrium, banks neither obtain an infinitely large profit nor abstain from lending, which, by proposition 1, is the case if and only if

$$\widehat{R} = R^{SB}.$$

Therefore, the real allocation in equilibrium is unique and conforms with the second-best allocation, the one that would arise if the friction of payment were absent and entrepreneurs could hire workers with their own promises to pay. Intuitively, what banks supply is a means of payment, a homogeneous good. Furthermore, in this case of unfettered issuance, they all have unlimited capacity. Therefore,

[14] Note that D is in the unit of N kg, while entrepreneurs' demand is denoted with the unit of kg.

they are in Bertrand competition, which annihilates their profit margin. As a result, entrepreneurs overcome the friction of payment at no costs and the real allocation is the one that would arise if the friction were absent, that is, the second-best allocation.

On the nominal side, however, there is indeterminacy. At $\widehat{R} = R^{SB}$, by proposition 1, the profit margin of lending is 0, and individual banks are indifferent to any quantity of issues, although in aggregation their issues exactly suffice for entrepreneurs to hire L^{SB} workers. This indeterminacy leads to a continuum of equilibriums besides the symmetric one defined above. In the symmetric equilibrium, all banks issue the same quantity of notes and their notes are discounted at the same factor; therefore, one bank's notes are perfect substitutes for another's. In asymmetric equilibriums, however, some banks issue more than others despite ex ante being identical, and see their notes more heavily discounted.

Proposition 2.

(i) *In any equilibrium, independent of banks' wealth G, $\widehat{R} = R^{SB}$, the profit margin of bank lending is 0, and the second-best allocation is attained.*

(ii) *In any equilibrium, a fraction of banks default at $t = 1$ upon the realization of $\widetilde{A} = \underline{A}$ if and only if banks' wealth is below a threshold G^*, where*

$$G^* = [(q\overline{A}\alpha + (1-q)\underline{A}) - \underline{A}] \left(\frac{q\overline{A}\alpha + (1-q)\underline{A}}{w} \right)^{\frac{\alpha}{1-\alpha}}.$$

An intuition for result (i) is given above. As for (ii), note that, given that banks are indifferent to any quantity of issues, the quantity of money circulated—that is, the aggregate bank lending—is determined by the demand side, namely entrepreneurs. Thus, the aggregate bank issues are fixed at a quantity exactly sufficient for entrepreneurs to hire the second-best number of workers. With this scale of lending, if banks' wealth, G, is sufficiently low—namely $G < G^*$—then it is insufficient to absorb the loss incurred in the bad state and bank default occurs accordingly.

The baseline mode examined above, besides providing a simple framework to consider money creation by banks on competitive markets, also bears empirical relevance to the early periods of the banking history during which a large number of banks issued their own notes and discounted others', such as the Free Banking Era in the United States (1838–63) and a period between 1750 and 1844 in England.[15] Specifically, the baseline model derives a relationship between the loan interest rate r, the discount factor δ, the leverage ratio $\Lambda := D/G$, and the risks of default $1 - q$ of individual banks. First, $\frac{1+r}{\delta} = \widehat{R}$, a constant across banks, by the binding constraint (12). Therefore, the paper predicts that *across banks, the gross interest rate that a bank charges for loans (i.e.,* $1+r$*) is positively related to and solely determined by the discount factor of its notes* δ*, independent of the bank's other attributes.* Second, regarding the discount factor, from (10) and binding (12) it follows that

$$\delta = \left\{ \begin{array}{c} 1 \text{ if } \Lambda \leq (1 - \frac{A}{\overline{A}\alpha}\widehat{R})^{-1} \\ \frac{q+(1-q)\Lambda^{-1}}{1-(1-q)\frac{A}{\overline{A}\alpha}\widehat{R}} \text{ if } \Lambda \geq (1 - \frac{A}{\overline{A}\alpha}\widehat{R})^{-1} \end{array} \right\}. \tag{13}$$

Therefore, *the discount factor of a bank's notes* δ *is inversely related to the bank's default risk* $1-q$[16] *and its leverage* Λ *(which is endogenous).* The first part of this prediction is empirically confirmed by Gorton (1999), who studies the pricing of bank notes during the U.S. Free Banking Era.[17]

The applicability of the baseline model to modern banking is more limited. A fundamental assumption of the model, namely that banks face no constraints in making loans, is probably far from reality nowadays. In particular, due to this assumption, the baseline

[15] For the case of the United States, according to Gorton (1999), thousands of different banks' notes were in circulation, while for the case of England, Cameron et al. (1967) report that in year 1810 there were 783 note-issuing county banks in England.

[16] In the lower branch of (13) $\partial\delta/\partial q$ share the sign of $1 - \frac{A}{\overline{A}\alpha}\widehat{R} - \Lambda^{-1}$, which is positive because in that branch $\Lambda \geq (1 - \frac{A}{\overline{A}\alpha}\widehat{R})^{-1}$, that is, $\Lambda^{-1} \leq 1 - \frac{A}{\overline{A}\alpha}\widehat{R}$.

[17] Specifically, he found that possessing traits of low default risks, such as being a member of the Suffolk System or under a state-sponsored insurance, is negatively correlated with the *implied volatility* of the bank, of which the discount factor—or price, in his paper—is a decreasing function. Therefore, the low-risk traits are associated with a higher value of δ.

model predicts that the aggregate quantity of bank credit is independent of banks' wealth, G, whereas the aftermath of the recent financial crisis witnesses that the banking sector reduces credit issuance after suffering a severe loss. To give an account for this observation in particular and to examine the modern banking in general, we shall introduce a borrowing constraint to banks, as below.

4. Banks' Wealth Matters in the Presence of a Borrowing Constraint

In this section, we assume banks are subject to a borrowing constraint, that their equity value should never fall below $(1-\mu)G$.[18] The equity value is higher in the good state than it is in the bad state. Thus, we only need to consider the constraint in the bad state, which is $G + \underline{Y} - D \geq (1-\mu)G$. With $\underline{Y} = \underline{A}(1+r)/(\overline{A}\alpha) \times D$ by (8) and rearrangement, it becomes

$$D \cdot \left[1 - \frac{\underline{A}(1+r)}{\overline{A}\alpha}\right] \leq \mu G. \tag{14}$$

The introduction of the constraint has two immediate implications. The first is that, because banks always maintain a positive equity value, they never default. Therefore, their notes are not discounted, that is, $\delta = 1$ and hence $1 + r = \widehat{R}$ hereafter. As a result, the borrowing constraint (14) becomes

$$D \cdot \left(1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}\right) \leq \mu G. \tag{15}$$

The other implication is that, unlike in the preceding case of least-fettered issuance, banks now are in Bertrand competition with *limited* capacities à la Kreps and Scheinkman (1983). With the borrowing constraint, the market equilibrium is as follows.

[18] This borrowing constraint can be due to several reasons. One reason is that, in modern times, bank default becomes very costly and banks' managers want to maintain solvency in any contingency, which is a special case of the above constraint with $\mu = 1$. Another reason, as in Gertler and Kiyotaki (2010) and Holmstrom and Tirole (1997), is that banks have a moral hazard issue: at $t = 1$, the owner of a bank—the banker—can abscond with a fraction of $1-\mu$ of its stored wealth to a remote island. Hence, banks' equity value should never fall below $(1-\mu)G$ in any contingency.

PROPOSITION 3.

(i) If $G \geq \frac{1}{\mu}G^*$, then in all equilibriums the real allocation is the same as in the case of least-fettered issuance: $\widehat{R} = R^{SB}$, the profit margin of bank lending is 0, and $L = L^{SB}$. If $G < \frac{1}{\mu}G^*$, there is a unique equilibrium in which the profit margin of bank lending is positive; and $\widehat{R} > R^{SB}$ and $\widehat{R}$ is determined by G through

$$G = \frac{1}{\mu}\left(\frac{\overline{A}\alpha}{w^\alpha}\right)^{\frac{1}{1-\alpha}} \frac{1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}}{\widehat{R}^{\frac{1}{1-\alpha}}} \equiv G(\widehat{R}). \tag{16}$$

(ii) If G decreases from $\frac{1}{\mu}G^*$ to 0, the equilibrium interest rate, $\widehat{R}$, increases from R^{SB} to $\overline{A}\alpha/\underline{A}$ and the number of workers hired by entrepreneurs decreases from L^{SB} to $L^{SB} \times \left[\frac{\underline{A}}{q\overline{A}\alpha+(1-q)\underline{A}}\right]^{\frac{1}{1-\alpha}}$.

The borrowing constraint limits banks' capacity of issuance to an endogenous proportion of their wealth. If their wealth is high enough—that is, beyond $\frac{1}{\mu}G^*$—then banks still possess a capacity large enough to annihilate the profit margin of issuance, which leads to the second-best allocation, as was in the case of least-fettered issuance. Hence arises result (i).

If $G < \frac{1}{\mu}G^*$, banks' wealth does not suffice to back an issuance of that size. As a result, issuance bears a positive profit margin. This positive profit margin drives all banks to issue as much as possible, until the borrowing constraint, (15), is binding. This clears the indeterminacy in the quantity of individual banks' issues and gives rise to a unique equilibrium. It also shows that, now, the quantity of money circulated is determined by the supply side—namely, the banking sector—rather than by the demand side, as was in the case of least-fettered issuance. Therefore, the lower the banks' wealth, the lower the total value of money that they supply and, as a result, the higher the interest rate of borrowing ($\widehat{R}$) and the fewer the workers that entrepreneurs hire.

The last comparative static, however, does not mean the aggregate output always decreases with G for $G < \frac{1}{\mu}G^*$ because of the wedge between the first-best and second-best allocations, namely $L^{FB} < L^{SB}$. Define

$$G^{FB} \equiv G(R^{FB}),$$

where function $G(\widehat{R})$ is defined in equation (16); with $R^{FB} = \overline{A}/A_e$,

$$G^{FB} = \frac{1}{\mu}(A_e\alpha - \underline{A})(A_e\alpha/w)^{\frac{\alpha}{1-\alpha}}.$$

Then G^{FB} is the level of banks' wealth at which the actual interest rate exactly takes the first-best value and therefore the aggregate output is maximized. $G^{FB} < \frac{1}{\mu}G^*$.[19] By proposition 3, then, there are two types of efficiency:

(i) $G < G^{FB}$. In this case, $\widehat{R} > R^{FB}$ and $L < L^{FB}$. That is, relative to the first-best allocation, banks under-lend, whereby the cost of bank credit is too high and the sector that depends on it—namely entrepreneurs—obtains inadequate resources, namely labor.

(ii) $G > G^{FB}$. In this case, $\widehat{R} < R^{FB}$ and $L > L^{FB}$. That is, relative to the first-best allocation, banks over-lend, whereby the cost of bank credit is too low and the sector that depends on it obtains excessive resources.

The existence of the second type of inefficiency is due to assumption 3, which drives a wedge between the first-best and second-best allocations. In the absence of this wedge, R^{FB} would be equal to R^{SB}, hence $G(R^{FB})$ to $\frac{1}{\mu}G^*$, and the case of bank over-lending would not exist. This case of over-lending provides the paper with room to accommodate interest rate policy and capital adequacy regulation, as will be shown in section 6.

The first type of inefficiency is caused by inadequate creation of money by banks. Can the central bank enlarge the money supply through the banking system? It can, as we show in the next section, with a policy which, because it eases the constraints imposed by a dearth of money, is called the quantitative easing policy, or QE.

[19] That is because $R^{FB} > R^{SB}$, $\frac{1}{\mu}G^* = G(R^{SB})$ and function $G(R)$ is decreasing.

5. QE Reduces the Borrowing Constraint of Banks

The central bank (CB hereafter) in this economy is modeled as the unique entity that is able to costlessly produce another means of payment which has no intrinsic values; to fix the idea, let it be shells. Shells are different in nature to bank notes. The CB does not promise to pay a holder of shells with any corn, the real good. Shells, therefore, are purely nominal. By contrast, a bank commits to redeem its notes with the promised amounts of corn. Hence banks' notes are not nominal.

Although shells are fiat money and the model economy lasts for only two dates, shells can circulate in it, whereby the CB can conduct meaningful monetary policy, such as the following QE. At $t = 0$ the CB announces a facility whereby each of the N banks can borrow up to S units of shells (again, one unit is defined as N kg) at a small interest rate $\epsilon > 0$. A borrower bank is obliged to repay its debt to the CB at $t = 1$ either with shells or with corn, with a fixed rate of exchanging corn for shells—for example, 1 kg corn for 1 kg shells. This exchange rate is arbitrary and is made to confer a nominal value on shells, as in real life the Bank of England can print an arbitrary number, like £10, on a piece of paper, and then this piece of paper has a nominal value of £10 and can be used to buy real goods of this value (e.g., two packs of cherries at M&S). As such, we call 1 kg corn the *par value* of 1 kg shells, etc., although shells are nominal. The banks that have used the CB's facility thus have two types of money to lend to entrepreneurs. One is their own notes; the other is shells. We assume that no entrepreneurs borrow both types of money, in order to avoid the unnecessary complication of trying to determine which debt, of the two, is senior in the case of default.

The contract of borrowing a bank's notes is still represented by (E, r), whereby the entrepreneur borrows the bank's notes of face value E at $t = 0$ and then owes it a debt of $E(1+r)$, which he repays at $t = 1$ with corn.

The contract of borrowing shells is similarly represented by (E_s, r_s), whereby the entrepreneur borrows E_s kg shells at $t = 0$ and then owes the bank a debt of $E_s(1 + r_s)$, which he repays at $t = 1$ with either corn or shells, with 1 kg corn equivalent to 1 kg shells.

The timing of events at $t = 0$ is as follows. First, the CB chooses S and publicly announces the policy. Then, banks decide the quantity of shells to borrow from the CB, $Q \leq S$, and post offers of $(D, r; Q, r_s)$. Based on these offers, entrepreneurs decide which bank to go to and how much to borrow. Lastly, they use borrowed means of payment (bank notes or shells) to hire workers and start the production.

The timing of events at $t = 1$ is as follows. First, entrepreneurs produce corn. Second, the market for shells opens, on which the shell-borrowing entrepreneurs use corn to buy shells from workers. Let $\overline{p_1}$ ($\underline{p_1}$) denote the shell price in the good (bad) state. Third, entrepreneurs settle their debts to the lender banks, using corn and/or shells. Fourth, the market for shells *may* open the second time, on which banks having excessive shells sell to banks in shortage. Lastly, banks redeem their notes from workers and settle their debts to the CB. At this stage, the CB may end up holding a certain quantity of corn, which it will transfer to the agents of the economy before the consumption starts.

Note that there is no surprise or nominal rigidity associated with QE. All the private-sector agents (i.e., banks, entrepreneurs, and workers) made decisions after having observed the CB's move and they can freely adjust their decisions with it. Still, QE produces real effects in this economy in one equilibrium if $G < \frac{1}{\mu}G^*$.[20] For any $G < G^{FB}$, moreover, there exists a unique value of S at which this equilibrium attains the first-best allocation. This is the equilibrium that we discuss below.

In this equilibrium, at $t = 0$, workers believe that they will be able to use shells to buy corn at $t = 1$ and thus they accept a wage payment with shells; and at $t = 1$, indeed shells receive a positive real value, intuitively because they can be used to settle certain debts

[20]As shells are fiat money, there is another equilibrium in which workers disbelieve that shells will have real value at $t = 1$ and thus do not accept to be paid with shells at $t = 0$, which are thus not circulated. This type of equilibrium exists for fiat money in general. By contrast, it never exists for banks' notes: banks each have G units of corn, therefore their notes always have a positive real value at $t = 1$; indeed, if a bank issues $D \leq G$, workers know, without any equilibrium calculation, that its notes are worth par. This difference results from the difference in nature between shells and banks' notes stated at the beginning of this section.

that otherwise have to be settled with corn. To solve the equilibrium, we first examine what happens at $t = 1$ and then what happens at $t = 0$. Consider, first, the case in which S is below a threshold (which is given in proposition 4 below) and therefore $r_s > \epsilon$ in equilibrium. In this case, all the banks borrow to the full capacity of QE, namely, $Q = S$, and lend all the shells out to entrepreneurs. Therefore, at the beginning of $t = 1$, there are NS units of shells in workers' hands and the aggregate debt of the shell-borrowing entrepreneurs is $NS \times (1 + r_s)$ units.

In the good state, as no entrepreneurs default, the shell price $\overline{p_1} = 1$. On the one hand, shells can never be worth more than their nominal values: if the entrepreneurs use 1 kg corn only to buy less than 1 kg shells, they will not buy them but instead use 1 kg corn to settle a debt of 1 kg shells to the banks. On the other hand, in the good state, it cannot be that $\overline{p_1} < 1$ either. Otherwise, the shell-borrowing entrepreneurs would want only shells, not corn, to repay all their debts. Their aggregate demand for shells would thus be $NS(1 + r_s)$, as they do not default. This demand, with $r_s > 0$, is bigger than NS, the supply of shells, leaving the market uncleared.

As $\overline{p_1} = 1$, the entrepreneurs are indifferent to repaying their banks with shells or with corn in the good state. As a result, some banks may end up with more than $S(1+\epsilon)$ units of shells, some less. If so, the former wants to sell the excess to the latter and the second shell market opens. On this market, the equilibrium shell price, p_B, equals 1 also, following the same argument as above. On the one hand, it is impossible that $p_B > 1$. On the other hand, if $p_B < 1$, banks all want to use only shells, not corn, to settle their debts to the CB, and their aggregate demand would be $NS(1 + \epsilon)$, greater than the total supply, NS, thus leaving the market uncleared. This argument holds true, and hence $p_B = 1$ is the only market clearing price, so long as $\epsilon > 0$, however small. To simplify exposition, in what follows, we consider the limit case where $\epsilon \to 0$.[21]

[21]Note, however, that if $\epsilon = 0$, then there is indeterminancy: any price $p_B \in [0, 1]$ can clear this second shell market. If $\epsilon = 0$, the aggregate excess exactly equals the aggregate shortage. Then, the supply side is willing to sell all the excessive shells at any $p_B \geq 0$, while the demand side is willing to buy exactly the same amount at any $p_B \leq 1$.

In the bad state, the output is low. Hence $\underline{p_1} < 1$, as will be verified later. Thus, the shell-borrowing entrepreneurs use only shells, no corn, to repay the banks. Furthermore, in this state entrepreneurs all default according to lemma 2. This means all their outputs are used to settle their debts to the banks. With these two facts put together, all the shell-borrowing entrepreneurs' output is used to buy shells. Denote by Y_s the total output of one bank's shell-borrowing entrepreneurs. Then the shell market clearing commands that $\underline{p_1} \times NS = NY_s$, or

$$\underline{p_1} = Y_s/S. \tag{17}$$

With a calculation similar to that leading to equation (8), we find $\underline{p_1} = \underline{A}(1 + r_s)/(\overline{A}\alpha)$, which, as is shown below in proposition 4, is smaller than 1 indeed, as was intuitively argued. Moreover, from $\underline{p_1} = Y_s/S$ it follows that the shell-borrowing entrepreneurs of one bank altogether buy $Y_s/\underline{p_1} = S$ units of shells. Hence, each bank ends up with having S units of shell equally and the second shell market does not open in the bad state.

The value of 1 kg shells at $t = 0$, p_0, is the mean of their values at $t = 1$:

$$p_0 = q \times 1 + (1 - q) \times \frac{\underline{A}(1 + r_s)}{\overline{A}\alpha}. \tag{18}$$

Workers thus accept a wage payment with w/p_0 kg shells at $t = 0$ in the equilibrium.

The effect of this policy is presented in the following proposition.

PROPOSITION 4. *Suppose $G < \frac{1}{\mu}G^*$.*

(i) If QE's scale $S < \underline{S}(G) \equiv \frac{\overline{A}\alpha}{q(\overline{A}\alpha - \underline{A})}(\frac{1}{\mu}G^* - G)$, then there is a unique equilibrium in which all the S units of shells are in circulation. In this equilibrium, QE produces real effects: the actual interest rate, $\widehat{R}$, is determined by S through

$$S = \left(\frac{\underline{A}}{w^\alpha}\right)^{\frac{1}{1-\alpha}} \cdot \frac{1 - (1 - q)\theta}{q} \left(\frac{1}{\theta^{\frac{1}{1-\alpha}}} - \frac{\mu G}{1 - \theta}\right) \equiv F(\theta; G),$$

$$\text{with } \theta \equiv \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}. \tag{19}$$

Moreover, the interest rate of lending shells is $r_s = \frac{[q\overline{A}\alpha+(1-q)\underline{A}]\widehat{R}-\overline{A}\alpha}{\overline{A}\alpha-(1-q)\underline{A}\widehat{R}}$. *It satisfies* $r_s > 0$ *and* $\underline{A}(1+r_s)/(\overline{A}\alpha) < 1$, *as was said.*

(ii) If S *increases,* $\widehat{R}$ *and* r_s *decrease and* L *increases. At* $S = \underline{S}(G), \widehat{R} = R^{SB}$, $r_s = 0$ *and* $L = L^{SB}$.

QE works by increasing the real value of money that banks lend out at $t = 0$. That explains why, compared with the situation of its absence (namely with $S = 0$), the cost of borrowing money ($\widehat{R}$) is reduced, whereby entrepreneurs hire more workers. And the larger the scale of QE (i.e., S), the greater are these effects (so long as S is below $\underline{S}(G)$).

The question is: how is QE able to make banks enlarge their lending scale, backed by the same amount of wealth, without breaking the borrowing constraint? To answer this question, consider a representative bank's balance sheet at $t = 1$, as shown in table 1.

Consider the bad state, in which the borrowing constraint is binding. The constraint commands that $G + \underline{Y} + Y_s - D - \underline{p_1}S \geq (1-\mu)G$. This inequality, with $\underline{p_1} = Y_s/S$ by (17), is equivalent to $G + \underline{Y} - D \geq (1-\mu)G$, which leads to inequality (14), the same borrowing constraint in the absence of QE. Therefore, the injection of the fiat money with QE does not reduce the capacity of private issuance at all. This is due to $\underline{p_1} = Y_s/S < p_0$,[22] which means that in the bad state the shell price goes down, so that the real value of a bank's liability to the CB, $\underline{p_1}S$, falls enough to be covered by the value of the assets, Y_s. The decrease in the real value of shells means inflation. Put differently, QE enables banks to lend more without breaking the borrowing constraint because it induces inflation in the bad state, which lowers the real value of banks' liability to the CB. The inflation arises because shells are fiat money. The CB never commits to redeem shells with a real good. Their value, therefore, freely adjusts with the state of the economy. By contrast, banks commit to redeem their notes (i.e., their promise to pay) at

[22] $\underline{p_1} < p_0 \Leftrightarrow \underline{p_1} < q \times 1 + (1-q) \times \underline{p_1} \Leftrightarrow \underline{p_1} < 1$, which holds true by the proposition.

Table 1. The Balance Sheet (in real value) of the Representative Bank with QE

Assets	Liabilities
Corn Stored (G) Loans in Notes (Y) Loans in Shells (Y_s)	Equity Liability to the Note Holders (D) Liability to the CB (p_1S)

a pre-specified, non-contingent, value.[23] This difference in nature between banks' notes and shells, therefore, drives the functioning of QE.

Remark: An abstraction of the model helps elucidate this mechanism of QE. That is the denomination of banks' liabilities with corn, the real good. In reality, however, typically they are denominated with the fiat money currency of the economy. Incorporating this feature would not necessarily invalidate the working of the mechanism. First, the aforementioned difference in nature between bank-created money and currency is still there. Although banks now use the currency to redeem their liability, to obtain the currency they need to expend real resources. Therefore, their liability still bears real obligations of repayment. In contrast, fiat money is not redeemable and bears no such obligations. Second, it is true that if banks' liability is denominated with the currency, its real value falls under the negative shocks, which improves banks' conditions. However, a credit crunch still happens. So long as they face a binding borrowing constraint, they will still reduce lending if their wealth is profusely eaten off, as is evident in the aftermath of the 2008–09 crisis. Third, in this circumstance, if QE is able to raise banks' lending capacities, it has to ease their borrowing constraint. As it does not give real resources to banks, to do so it has to further reduce the real value of banks'

[23]In the model economy, the value of these issues at $t = 1$ could be made contingent on the state in two ways. One is to let the specified value be contingent on the state, which is disallowed by assumption 2. The other is default, which is disallowed because of the borrowing constraint, (14). While default is disallowed altogether because of the particular form that the constraint takes, in general so long as any borrowing constraint restricts banks' capacity, there is only a limited extent to which the value of their liabilities can adjust to the real economic conditions.

liability, and this can only be done via inflation. Fourth, this reduction can be done by the CB with QE, but not by banks themselves, exactly because of the above said difference in nature between the two types of money.

At the core is the point that this difference gives the CB leeway to act that banks do not have. This point can be seen even more clearly in a modified version of the setting where the CB implements QE by directly lending shells to entrepreneurs (rather than through banks) at interest rate r_s, which would result in the same prices of shells and the same allocation as in the original setting. Obviously, in this setting the CB can lend out the currency, whereas banks cannot lend more of their liabilities, because shells are not redeemable but banks' liabilities are. Moreover, if banks' liabilities were now denominated with the currency, this policy would still work. Actually it would work better because it would ease banks' borrowing constraint by inducing inflation, which enables them to enlarge the lending capacity.

Returning to the model, while QE induces inflation in the bad state, it causes deflation in the good state: $\overline{p_1} = 1 > p_0$. This result might partly explain the lingering deflation pressure or the low inflation observed during or after the implementation of QE in several economies, such as the United Kingdom and the euro zone. This phenomenon would be a puzzle if considered from the point of view of the quantitative theory of money, given that QE enormously expands the monetary base.

Thus far we have considered the case in which $S < \underline{S}(G)$ and therefore $r_s > 0$. Now we consider what happens if QE is in a bigger scale. At $S = \underline{S}(G)$, $\widehat{R} = R^{SB}$ and $r_s = 0$. By proposition 1(iii), $\widehat{R}$ can never fall below R^{SB}, otherwise banks would stop lending altogether, not a case in equilibrium. Therefore, if the scale of QE is larger than $\underline{S}(G)$, then $\widehat{R}$ stays at R^{SB}, r_s stays at 0, and the excessive shells beyond the threshold are not put into circulation; indeed, with $r_s = 0$, banks are indifferent regarding the quantity of shells to borrow and lend. This result partly offers an explanation for the phenomenon that, in certain economies such as the United States and the euro zone, a substantial fraction of monetary bases created with QE is not lent out but stays in banks' reserve accounts.

Now consider the optimal scale of QE. The first-best value of the interest rate is $\widehat{R} = R^{FB}$. In the circumstance where $G \geq G^{FB}$ and hence $\widehat{R} \leq R^{FB}$ at $S = 0$, any $S > 0$ only makes $\widehat{R}$ still smaller and efficiency even lower. Therefore, the optimal $S = 0$, namely, the CB should not implement QE at all in the circumstance of banks over-lending. However, in the circumstance where $G < G^{FB}$ and hence $\widehat{R} > R^{FB}$ at $S = 0$, there is a unique value of S at which QE drags down $\widehat{R}$ to R^{FB}. This is summarized below.

Proposition 5.

(i) If $G \geq G^{FB}$, the optimal scale of QE is $S = 0$, that is, no QE should be implemented.

(ii) If $G < G^{FB}$, the optimal $S = F(\frac{A}{\bar{A}\alpha} \cdot R^{FB}; G) > 0$, where function $F(\cdot)$ is given by (19). At this scale QE attains the first-best allocation (i.e., $\widehat{R} = R^{FB}$). Moreover, with optimal QE, banks earn profit from lending shells (i.e., $r_s > 0$), but their overall profit is reduced compared with the case without QE if $G \geq \underline{G}$ for some $\underline{G} < G^{FB}$.

According to the last result, although with QE banks receive free funding from the CB and lend it at a positive interest rate, surprisingly they lose from the policy, which thus does not subsidize them. That is because to banks, besides this positive effect on the scale, QE induces a negative, general equilibrium effect on the profit margin by enlarging the lending capacities of all banks. As a result, banks obtain a reduced profit from lending their notes. As $\partial F/\partial G < 0$, if banks' wealth is above a threshold (i.e., $\underline{G}$), then the scale of optimal QE is small enough. As a result, the positive effect from the enlargement of the lending scale is small and dominated by the negative effect from the reduction of the profit margin. However, even in this case of banks all losing from the free funding of the CB, given $r_s > 0$, individual banks still strictly prefer taking it rather than abstaining from it—thus QE can be conducted on a voluntary basis. The reason is that a single bank takes the profit margin of lending as given and neglects the effect on it of enlarging its own capacity.

In the circumstance where $G > G^{FB}$ and banks over-lend, we have seen that QE does not help. What the CB can do is discussed in the next section.

6. Interest Rate Policy and Capital Adequacy Regulation to Curb Bank Lending

If $G > G^{FB}$, then banks over-lend, making the actual interest rate too low, and entrepreneurs hire too many workers relative to the first-best allocation. In this circumstance, it is usually expected that the CB can help by setting a high policy interest rate. This idea is explored in this section, where we show that interest rate policy produces real effects and is able to attain the first-best allocation if and only if there is a nominal rigidity. In its absence, we show, capital adequacy regulation is always a remedy.

To explain the meaning of nominal rigidity in this paper, we define nominal wage as the total face value of banks' notes that workers receive as the wage payment. In the absence of any intervention from the CB, the nominal wage is w because banks do not default (due to the borrowing constraint (14)). The nominal rigidity in this paper is defined as the friction that keeps the nominal wage of workers staying at w and unchanged with the CB's policy.

6.1 Interest Rate Policy Works If and Only If the Nominal Rigidity Is Present

In the two-date economy of this paper, the CB sets the policy rate r_p by offering to workers a savings account which takes in the deposit of banks' notes at $t = 0$ and pays out with shells at $t = 1$. Specifically, if the CB receives a deposit of some banks' notes of overall face value F kg corn at $t = 0$, it issues to the depositor $F(1 + r_p)$ kg shells at $t = 1$. Moreover, by taking in the notes, the CB becomes a creditor to the issuer banks and charges these banks an interest rate of $1 + r_p + \varepsilon$ for some $\varepsilon > 0$. Thus it obliges these banks altogether to pay back $F(1 + r_p + \varepsilon)$ at $t = 1$, either with corn or with shells, counting 1 kg corn equivalent to 1 kg shells. There is one equilibrium in which this policy is meaningful.[24] In this equilibrium *if* the aggregate face value of notes deposited with the CB at $t = 0$ is F and consequently $F(1+r_p)$ kg shells are created at $t = 1$, then these

[24] As shells are fiat money, there is an equilibrium in which no one believes that shells have real value at $t = 1$ and thus no one deposits any bank notes with the CB at $t = 0$, which makes the policy rate meaningless.

shells are priced at par (i.e., 1 kg shells worth 1 kg corn) at $t = 1$. On the one hand, they can never be priced above par. On the other hand, shells are not priced below par either. Otherwise, the banks indebted to the CB would want to use only shells to settle all their debts. Thus, their aggregate demand for shells would be $F(1+r_p+\varepsilon)$ kg, but only $F(1+r_p)$ kg shells are created, insufficient to meet the demand. The deposit of bank notes with the CB and the creation of shells, however, will not actually happen in equilibrium. Banks will pay their note holders the same interest rate of r_p (or even a little more) to stop them from depositing the notes with the CB and avoid the payment of the even higher interest rate of $r_p + \varepsilon$. As a result of the policy rate set at r_p, therefore, a bank's note of face value 1 issued at $t = 0$ is worth $1 + r_p$ at $t = 1$, and banks that issue notes of aggregate face value D at $t = 0$ are in a liability of $D(1 + r_p)$ at $t = 1$.

Having explained how the interest rate policy is conducted in this economy, we state its effects in the following proposition.

PROPOSITION 6. *Suppose* $G > G^{FB}$.

(i) *If workers' nominal wage adjusts with the policy rate, the interest rate policy produces no real effects but deflates the nominal wage to* $w/(1 + r_p)$.

(ii) *If workers' nominal wage stays at* w *invariant to the CB's policy rate, the equilibrium interest rate of bank lending,* $\widehat{R}$, *increases with the policy rate* r_p. *At a unique value of* r_p, $\widehat{R} = R^{FB}$ *and the first-best allocation is attained. With the optimal interest rate policy, banks obtain zero profit if* $G \geq G_s$, *where* $G_s \equiv \frac{1}{\mu}[(q\overline{A}\alpha + (1-q)\underline{A}) - \underline{A}](A_e\alpha/w)^{\frac{\alpha}{1-\alpha}}$ *and satisfies* $G^{FB} < G_s < \frac{1}{\mu}G^*$.

Intuitively, in both the regime of flexible nominal wage and that of sticky nominal wage, a higher policy rate, by increasing the cost of deposit, induces the bank to set a higher lending rate in the hope of shifting the increased cost of deposit to entrepreneurs: r increases with r_p in both regimes, as shown in the proof. As a result, in both regimes, a higher policy rate reduces the quantity of bank credit that entrepreneurs borrow. In the regime of flexible wage, this reduction produces no effect on the number of workers they employ because it is exactly offset by the decrease in the nominal wage that workers accept. In the regime of sticky wage, by contrast, the nominal wage stays the same and this reduction in bank credit forces entrepreneurs to hire fewer workers. With a proper policy rate, the number of workers they hire is brought down to the first-best level, that is, the first-best allocation is attained. The decrease in entrepreneurs' demand for bank credit subjects banks to stronger competition. As a result, their profit margin of lending is nullified if their lending capacity is not too small, that is, if their wealth is not too low—i.e., no less than G_s.

The interest rate policy, therefore, works to improve efficiency over the market equilibrium if and only if the nominal rigidity is present. However, regulation that sets a lower bound on banks' capital adequacy ratio always works to curb the over-lending by banks (if the CB has the authority for such regulation), as is shown below.

6.2 Capital Adequacy Regulation Always Works

The aggregate face value of money that is needed for entrepreneurs to hire the first-best number of workers is $wL^{FB} := D^{FB}$. Suppose the CB imposes the constraint that banks cannot issue more than D^{FB}/G times their wealth, G. Then, this constraint is binding in the competitive equilibrium if $G > G^{FB}$ because, in its absence, banks issue more than D^{FB} (as $\widehat{R} < R^{FB}$) by proposition 3, and thus the constraint is violated. The constraint, therefore, is binding, so banks issue D^{FB} and the first-best allocation is attained. One way to implement this constraint is to impose at $t = 0$ a lower bound on banks' capital adequacy ratio, which is defined as the equity-to-asset ratio in market value, that is, $(G+Y-D)/(G+Y)$, where Y is the value of banks' loans at $t = 0$, that is, $Y = q \times \overline{Y} + (1-q) \times \underline{Y}$. Define

$$c^{FB} := \frac{A_e \alpha G + (1-q)\underline{A}(1-\alpha)D^{FB}}{A_e \alpha G + [q\overline{A}\alpha + (1-q)\underline{A}]D^{FB}},$$

which is the capital adequacy ratio if banks lend the first-best quantity of money (as is shown in the proof of the following proposition).

PROPOSITION 7. *Suppose $G > G^{FB}$. If the CB imposes regulation that restricts banks' capital adequacy ratio from being smaller than c^{FB}, then banks issue the first-best quantity of money and the first-best allocation is attained.*

The intuition for why the regulation works is simple: the money that banks create is their liability. Therefore, the scale of its issuance is subject to the capital adequacy regulation. If the regulation is tight, then banks are restricted from over-lending.

7. Conclusion

Banks are special because their liability, especially that in the form of demand deposit, is widely accepted as a means of payment. Often, real sectors need to borrow banks' liability (so-called *money* or *credit*) to obtain resources that they want. This interaction between the real sectors and the banking sector on competitive markets, as well as the central bank's remedy to market inefficiency, is the focus of this paper. It underlines the importance of banks' aggregate wealth. Depending on the size of this wealth, there are two types of inefficiency.

If banks' wealth is below a threshold, then they issue too little money, with symptoms that the interest rate of bank credit is too high and the sectors that depend on it obtain inadequate resources. In this circumstance, the central bank can improve efficiency by lending to all banks its fiat money at zero interest rate. This policy, which bears the flavor of QE, works because of a difference in nature between bank-created money and fiat money. The latter is not redeemable, whereas the former bears real obligations of redemption. Furthermore, the policy induces deflation under the positive productivity shock. Lastly, while the policy gives to banks free funding, which they lend out at a positive interest rate, it does not subsidize them unless their wealth is very low.

If banks' wealth is above the threshold, on the other hand, banks lend out too much money, with symptoms that the interest rate of bank credit is too low and the resources are skewed to the sectors that highly depend on it. To curb over-lending by banks, the central bank may set a high policy rate, which, however, works if and only if the nominal wage cannot freely adjust with the policy. By contrast, the regulation that sets a tight capital adequacy ratio always works because the money that banks create is their liability, and its quantity is thus subject to the regulation.

Appendix: The Proofs

Proof of Lemma 1

In equilibrium, only one promised wage, denoted by F, prevails on the market, as will be shown. Competitive equilibrium is thus defined as a profile of (F, L), such that

(i) given that F prevails on the market, the optimal demand for labor of each entrepreneur is L;

(ii) given that each entrepreneur demands L workers, F clears the labor market.

The two conditions are elaborated as follows.

For (i), given F, a representative entrepreneur's decision problem on labor demand is

$$\max_L q(\overline{A}L^\alpha - FL) + (1-q)\max(\underline{A}L^\alpha - FL, 0),$$

where the "max" term appears because the entrepreneur might default in the bad state. That is indeed the case at the optimum. Otherwise, the entrepreneur's problem is

$$\max_L q(\overline{A}L^\alpha - FL) + (1-q)(\underline{A}L^\alpha - FL).$$

The solution is $L = (\frac{A_e\alpha}{F})^{\frac{1}{1-\alpha}}$. Then in the bad state his output is $\underline{A}((\frac{A_e\alpha}{F})^{\frac{\alpha}{1-\alpha}}$, which is smaller than $F \cdot (\frac{A_e\alpha}{F})^{\frac{1}{1-\alpha}}$, the wage obligation, because $\underline{A} < A_e\alpha$ as assumed in (1). Hence, he defaults in the bad state, contradictory to what was supposed.

Defaulting in the bad state, entrepreneurs choose L to maximize the profit in the good state, $\overline{A}L^\alpha - FL$. Therefore, given F, the labor demand is

$$L = \left(\frac{\overline{A}\alpha}{F}\right)^{\frac{1}{1-\alpha}}. \tag{20}$$

For (ii), as there are a lot more workers than entrepreneurs can hire, the labor market is cleared by an expected wage income of w, the output of workers in autarky. In the good state, the workers hired get the promised wage, F. In the bad state, entrepreneurs default and all the output goes to the workers, of which each obtains $\frac{\underline{A}L^\alpha}{L} = \underline{A}L^{\alpha-1}$. The labor market clears if

$$qF + (1-q)\underline{A}L^{\alpha-1} = w. \tag{21}$$

Equations (20) and (21) together pin down L^{SB} as given in the lemma.

Now, show that only one F prevails on the market. If an entrepreneur posts F, then by (20) he hires $L = (\frac{\overline{A}\alpha}{F})^{\frac{1}{1-\alpha}}$ workers, whose wage income is F in the good state and $\underline{A}L^{\alpha-1} = \frac{\underline{A}}{\overline{A}\alpha}F$ in the bad state. Both increase with F. Therefore, workers go only to entrepreneurs who post the highest F, and in competitive equilibrium, only one F prevails.

Q.E.D.

Proof of Lemma 2

Suppose, otherwise, an entrepreneur does not default in the bad state. Then, his problem is

$$\max_E q(\overline{A}L^\alpha - E(1+r)) + (1-q)(\underline{A}L^\alpha - E(1+r)) \text{ s.t. (3).}$$

From the constraint, $E = wL/\delta$. Substitute it into the objective and let $\gamma \equiv w(1+r)/\delta$. Then, the problem becomes

$$\max_L A_e L^\alpha - \gamma L.$$

The solution is $L = (A_e\alpha/\gamma)^{\frac{1}{1-\alpha}}$. At this scale, the entrepreneur will default in the bad state: $\underline{A}L^\alpha < E(1+r)|_{E=wL/\delta;\gamma\equiv w(1+r)/\delta} \Leftrightarrow \underline{A}L^\alpha < \gamma L \Leftrightarrow \underline{A}L^{\alpha-1} < \gamma|_{L=(A_e\alpha/\gamma)^{\frac{1}{1-\alpha}}} \Leftrightarrow \underline{A} < A_e\alpha$, which is assumed in (1)—hence a contraction to what was supposed.

Q.E.D.

Proof of Lemma 3

As the entrepreneurs all default in the bad state and each hands over his whole output, $\underline{y}$, to the bank, the value of the bank's loans in the bad state, $\underline{Y}$, equals $\underline{y}$ times the number of entrepreneurs that the bank finances, D/E. With $\underline{y} = \underline{A}L^{\alpha}$, and E and L given by (4) and (5),

$$\frac{\underline{y}}{E} = \frac{\underline{A}(1+r)}{\overline{A}\alpha}. \tag{22}$$

Then, $\underline{Y} = \underline{y} \cdot D/E = D \cdot \underline{y}/E = \underline{A}(1+r)/\overline{A}\alpha \cdot D$, that is, (8).
Q.E.D.

Proof of Lemma 4

$\Pi(D,r) = E_{\widetilde{Y}} \max(G + \widetilde{Y} - D, 0) - G = E_{\widetilde{Y}} \max(\widetilde{Y} - D, -G) = E_{\widetilde{Y}} \left[\widetilde{Y} + \max(-D, -G - \widetilde{Y})\right] = E_{\widetilde{Y}} \left[\widetilde{Y} - \min(D, G + \widetilde{Y})\right] = E_{\widetilde{Y}} \left[\widetilde{Y} - D \times \min(1, \frac{G+\widetilde{Y}}{D})\right] = D \times E_{\widetilde{Y}} \left[\frac{\widetilde{Y}}{D} - \min(1, \frac{G+\widetilde{Y}}{D})\right]$, which, with $\delta = \min(1, \frac{G+\widetilde{Y}}{D})$, equals $D \times E_{\widetilde{Y}} \left[\frac{\widetilde{Y}}{D} - \delta\right] = D \times \left[q\frac{\overline{Y}}{D} + (1-q)\frac{\underline{Y}}{D} - \delta\right]$, which, as $\overline{Y} = D(1+r)$ and $\underline{Y} = D \times \frac{\underline{A}}{\overline{A}\alpha}(1+r)$, equals $D \times \left[q(1+r) + (1-q)\frac{\underline{A}}{\overline{A}\alpha}(1+r) - \delta\right] = D \times \left[\frac{1+r}{R^{SB}} - \delta\right]$.
Q.E.D.

Proof of Proposition 2

Proof of proposition 2(i) has been shown in the main text. For proposition 2(ii), prove the "if" part by reduction to absurdity. Suppose that in one equilibrium, no banks default; namely, (9) is honored for all banks. Then, for all banks $\delta = 1$ and $1 + r = \widehat{R}|_{\text{result }(i)} = R^{SB}$. By (9), each bank issues

$$D \leq G/\left(1 - \frac{\underline{A}}{\overline{A}\alpha}R^{SB}\right) = \frac{G[q\overline{A}\alpha + (1-q)\underline{A}]}{q(\overline{A}\alpha - \underline{A})}.$$

With $\delta = 1$ and $1+r = R^{SB}$, by (4) the demand by entrepreneurs is $E = (q\overline{A}\alpha + (1-q)\underline{A})^{\frac{1}{1-\alpha}} w^{\frac{-\alpha}{1-\alpha}}$. If $G < G^*$, then $D < E$, that is, the supply is below the demand—thus not in equilibrium. To prove the "only if" part of proposition 2(ii), it suffices to show that if $G \geq G^*$, no banks default in the symmetric equilibrium, which is constructed as follows. Banks all choose $r = R^{SB} - 1$ and D to satisfy entrepreneurs' demand for notes $E = (q\overline{A}\alpha + (1-q)\underline{A})^{\frac{1}{1-\alpha}} w^{\frac{-\alpha}{1-\alpha}}$. With this value of (D, r), it is straightforward to check that if $G \geq G^*$, the no-default condition, (9), is honored and hence no banks default in the bad state. Q.E.D.

Proof of Proposition 3

Note that, as there is no default, $\delta = 1$ and $1 + r = R$. In any equilibrium, the actual interest rate that all banks offer equals the market clearing rate, that is, $R = \widehat{R}$. As $1 + r = \widehat{R}$ and $\delta = 1$, by (4), the demand by entrepreneurs is $E = [\overline{A}\alpha/(\widehat{R}w^{\alpha})]^{\frac{1}{1-\alpha}} := E(\widehat{R})$, which decreases with $\widehat{R}$. Due to the borrowing constraint, (14), and with $R = \widehat{R}$, the quantity of notes supply satisfies

$$D \leq \mu G \left(1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}\right)^{-1} := \overline{D}\left(\widehat{R}; G\right).$$

Moreover, if $\widehat{R} > R^{SB}$, the profit margin of lending is positive, banks want to lend as much as they can, and therefore the constraint is binding. It follows that the supply function is

$$D\left(\widehat{R}; G\right) = \left\{ \begin{array}{c} 0 \text{ if } \widehat{R} < R^{SB} \\ [0, \overline{D}\left(\widehat{R}; G\right)] \text{ if } \widehat{R} = R^{SB} \\ \overline{D}\left(\widehat{R}; G\right) \text{ if } \widehat{R} > R^{SB} \end{array} \right\}.$$

The equilibrium value of $\widehat{R}$ is determined by $E(\widehat{R}) = D\left(\widehat{R}; G\right)$.

For proposition 3(i), if $G \geq \frac{1}{\mu}G^*$, then $D\left(\widehat{R}; G\right) > E(\widehat{R})$ for $\widehat{R} > R^{SB}$ and $D\left(\widehat{R}; G\right) < E(\widehat{R})$ for $\widehat{R} < R^{SB}$. Therefore, in equilibrium $\widehat{R} = R^{SB}$. Hence $L = L^{SB}$—the second-best allocation is attained—and the profit margin of lending is 0, both of which hold in the case of least-fettered issuance.

If $G < \frac{1}{\mu}G^*$, $D\left(\widehat{R};G\right) < E(\widehat{R})$ for $\widehat{R} \leq R^{SB}$. Therefore, in equilibrium $\widehat{R} > R^{SB}$ and is determined by $E(\widehat{R}) = \overline{D}\left(\widehat{R};G\right)$ or, equivalently, equation (16), replicated below:

$$G = G(\widehat{R}) \equiv \frac{1}{\mu}\left(\frac{\overline{A}\alpha}{w^{\alpha}}\right)^{\frac{1}{1-\alpha}} \cdot \frac{1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}}{\widehat{R}^{\frac{1}{1-\alpha}}}. \tag{23}$$

It is straightforward to show for this function $G(\cdot)$ that $G' < 0$, $G(R^{SB}) = \frac{1}{\mu}G^*$ and $G(\overline{A}\alpha/\underline{A}) = 0$.

Furthermore, as all the banks issue $\overline{D}$, there is no indeterminacy in the scale of issuance by individual banks and the equilibrium uniquely exists.

To prove proposition 3(ii), let $\widehat{R}(G)$ be the inverse function of $G(\widehat{R})$. Then, in the equilibrium $\widehat{R} = \widehat{R}(G)$ if $G < \frac{1}{\mu}G^*$. As $G(\widehat{R})$ is decreasing, so is $\widehat{R}(G)$. Moreover, $\widehat{R}(\frac{1}{\mu}G^*) = R^{SB}$ and $\widehat{R}(0) = \overline{A}\alpha/\underline{A}$ because $G(R^{SB}) = \frac{1}{\mu}G^*$ and $G(\overline{A}\alpha/\underline{A}) = 0$. With $R = \widehat{R}$, the number of workers hired in equilibrium, by (5), is $L = (\overline{A}\alpha/w\widehat{R})^{\frac{1}{1-\alpha}}$, which decreases with $\widehat{R}$. Thus, L increases with G. Moreover, $L = L^{SB}$ at $\widehat{R} = R^{SB}$ which holds if $G = \frac{1}{\mu}G^*$. At the other end, if $G = 0$, then $\widehat{R} = \overline{A}\alpha/\underline{A}$. As L is in proportion to $(1/\widehat{R})^{\frac{1}{1-\alpha}}$, we have $L = L^{SB} \times (R^{SB}/\widehat{R})^{\frac{1}{1-\alpha}} = L^{SB} \times [\frac{R^{SB}}{\overline{A}\alpha/\underline{A}}]^{\frac{1}{1-\alpha}} = L^{SB} \times [\frac{\underline{A}}{q\overline{A}\alpha+(1-q)\underline{A}}]^{\frac{1}{1-\alpha}}$.

Q.E.D.

Proof of Proposition 4

For proposition 4(i), to characterize the equilibrium, observe that if QE's scale S is small enough—the threshold for which will be found later—then $\widehat{R} > R^{SB}$ still holds and $r_s > 0$. That is, lending bears a positive profit margin. It has two implications: (a) both notes and shells are lent out; and (b) they are lent in the maximum quantity, that is, banks' borrowing constraint (15) is binding and all the S units of shells are in circulation.

From implication (a) it follows that the actual interest rates of borrowing the two types of money are equalized, and equal to $\widehat{R}$, the market rate. Following the discussion in subsection 3.1, the actual interest rate of borrowing shells is $(1+r_s)/p_0$, while the actual interest rate of borrowing banks' notes is $1 + r$ (since $\delta = 1$ as banks do not default). Therefore,

$$\frac{1+r_s}{p_0} = 1 + r = \widehat{R}. \tag{24}$$

From implication (b), the quantity of notes issued is $D = \mu G/(1 - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R})$. Given that they are not discounted (i.e., $\delta = 1$) and the ex ante value of shells per unit is p_0, the aggregate value of means of payment supplied is $p_0 S + D$, which, when the market clears, equals the wage payment that entrepreneurs demand to hire workers:

$$wL = p_0 S + \frac{\mu G}{1 - \underline{A}\widehat{R}/(\overline{A}\alpha)}. \tag{25}$$

By (5), the number of workers they hire is

$$L = \left(\frac{\overline{A}\alpha}{w}\right)^{\frac{1}{1-\alpha}} \widehat{R}^{\frac{-1}{1-\alpha}}. \tag{26}$$

These four equations (note that (24) has two) together with equation (18) (which settles p_0), as shown below, determine a unique profile of $(p_0, r_s, r, \widehat{R}, L)$ in equilibrium—thus, the equilibrium in which shells circulate exists uniquely. Moving on to show that, we first derive equations (19). By (24), $1 + r_s = \widehat{R}p_0$. Substituting it into (18) and rearranging, we have

$$p_0 = \frac{q}{1 - (1-q)\underline{A}/(\overline{A}\alpha) \cdot \widehat{R}}. \tag{27}$$

Substitute it and (26) into (25), rearrange, let $\theta \equiv \underline{A}/(\overline{A}\alpha) \cdot \widehat{R}$, and we come to (19):

$$S = \left(\frac{\underline{A}}{w^{\alpha}}\right)^{\frac{1}{1-\alpha}} \frac{1-(1-q)\theta}{q} \left(\frac{1}{\theta^{\frac{1}{1-\alpha}}} - \frac{\mu G}{1-\theta}\right) \equiv F(\theta). \tag{28}$$

It is straightforward to verify that (a) $F'(\theta) < 0$; (b) equation $F(\theta) = 0$ is equivalent to (16). Thus, at $S = 0$, $\widehat{R} = \widehat{R}(G)$, where $\widehat{R}(G)$ is the interest rate determined by (16) in proposition 3, namely the actual interest rate without QE; and (c) at $\widehat{R} = R^{SB}$, $F(\frac{\underline{A}}{\overline{A}\alpha} R^{SB}) = \underline{S}(G)$. It follows that for any $S < \underline{S}(G)$, equation (28) determines a unique $\widehat{R} \in (R^{SB}, \widehat{R}(G)]$; hence $\underline{S}(G)$ is the threshold that was to be found at the beginning of the proof. After $\widehat{R}$ is found, it uniquely determines r, L, and p_0, respectively, through equations (24), (26), and (27). It also uniquely determines r_s by putting equations (24) and (27) together, which leads to

$$r_s = \frac{[q\overline{A}\alpha + (1-q)\underline{A}]\widehat{R} - \overline{A}\alpha}{\overline{A}\alpha - (1-q)\underline{A}\widehat{R}}. \tag{29}$$

Therefore, any $S \in [0, \underline{S}(G))$ pins down a unique equilibrium profile of $(p_0, r_s, r, \widehat{R}, L)$.

By (29), $r_s > 0 \Leftrightarrow \widehat{R} > R^{SB} \Leftrightarrow S < \underline{S}(G)$. Moreover, $\underline{A}(1+r_s)/(\overline{A}\alpha) < 1 \Leftrightarrow 1+r_s < \overline{A}\alpha/\underline{A}|_{(29)} \Leftrightarrow q\widehat{R}/[1-(1-q)\underline{A}/(\overline{A}\alpha)\cdot \widehat{R}] < \overline{A}\alpha/\underline{A} \Leftrightarrow \widehat{R} < \overline{A}\alpha/\underline{A}|_{\widehat{R}<\widehat{R}(G)} \Leftarrow \widehat{R}(G) < \overline{A}\alpha/\underline{A}$, which is affirmed by proposition 3(ii).

For proposition 4(ii), as $F'(\theta) < 0$, we have θ, and thus $\widehat{R}$, decrease with S. By property (c) of function $F(\cdot)$ above, $\widehat{R} = R^{SB}$ at $S = \underline{S}(G)$. By (29), r_s increases with $\widehat{R}$ and $r_s = 0$ at $\widehat{R} = R^{SB}$. Therefore, r_s decreases with S and equals 0 at $S = \underline{S}(G)$.

Q.E.D.

Proof of Proposition 5

Result (i) is proved in the main text. As for (ii), as $G < G^{FB}$, $\widehat{R}(G) > R^{FB} > R^{SB}$. Note that $\widehat{R} = \widehat{R}(G)$ at $S = 0$ and $\widehat{R} = R^{SB}$ at $S = \underline{S}(G)$. Therefore, there is a unique S between 0 and $\underline{S}(G)$ at which $\widehat{R} = R^{FB}$ and this S equals $F(\frac{\underline{A}}{\overline{A}\alpha} \cdot R^{FB})$ by (19). As for the profit of banks, in the unique equilibrium, each bank serves N entrepreneurs and the profit obtained from one of them, $\widehat{\pi}$, is the difference of the social value of his project minus his profit from it; that is, $\widehat{\pi} = A_e L^\alpha - wL - V$. With L and V given by (5) and (6) and $R = \widehat{R}$,

$$\widehat{\pi} = \left(\frac{\overline{A}\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} (q\overline{A}\alpha + (1-q)\underline{A}) \cdot \widehat{R}^{\frac{-1}{1-\alpha}} (\widehat{R} - R^{SB}).$$

Note that $\widehat{\pi}$ increases with $\widehat{R}$ for $\widehat{R} \in [R^{SB}, \frac{1}{\alpha}R^{SB}]$. Define $\underline{G} \equiv G(\frac{1}{\alpha}R^{SB})$. Then, if $G \geq \underline{G}$, we have $\widehat{R} \leq \widehat{R}(G) \leq \widehat{R}(\underline{G}) = \frac{1}{\alpha}R^{SB}$ for any $S \geq 0$. Therefore, QE of any $S > 0$—in particular, the optimal one—lowers $\widehat{\pi}$ and hence reduces banks' profit by decreasing $\widehat{R}$.

Q.E.D.

Proof of Proposition 6

(i) The equilibrium is found in a way parallel to that of proposition 3: first find the demand for notes given $\widehat{R}$, then the supply, and finally the equilibrium value of $\widehat{R}$. Given $\widehat{R}$, entrepreneurs hire $L = (\frac{\overline{A}\alpha}{w})^{\frac{1}{1-\alpha}}\widehat{R}^{\frac{-1}{1-\alpha}}$ workers by (5). With flexible nominal wage, workers accept a nominal wage of $w/(1+r_p)$. Therefore, the demand for notes is

$$E^*\left(\widehat{R}\right) = w\left(\frac{\overline{A}\alpha}{w}\right)^{\frac{1}{1-\alpha}} \frac{\widehat{R}^{\frac{-1}{1-\alpha}}}{1+r_p}.$$

Now consider the supply side. By borrowing face value E, an entrepreneur hires $E(1+r_p)/w$ workers, which means that $\delta = 1 + r_p$ in equation (3). Hence if a bank charges interest r, the actual rate of its loans is $R = (1+r)/(1+r_p)$. At the optimum, all banks offer $R = \widehat{R}$, the market clearing rate. It follows that

$$1 + r = \widehat{R}(1 + r_p).$$

That is, banks mark up the interest rate on loans to pass the cost of deposit due to the interest rate policy on to entrepreneurs. If a bank issues notes of aggregate face value D, then at $t = 1$, with the interest rate policy, its liability to the note holders is $D(1+r_p)$. The equity value in the bad state is thus $G+\underline{Y}-D(1+r_p)$, which, by the borrowing constraint, cannot be smaller than $(1-\mu)G$. With $\underline{Y} = \frac{\underline{A}(1+r)}{\overline{A}\alpha}D$, this constraint is equivalent to

$$D(1 + r_p) \leq \mu G + \frac{\underline{A}(1+r)}{\overline{A}\alpha}D, \tag{30}$$

which, with $1+r=\widehat{R}(1+r_p)$, is equivalent to

$$D \leq \frac{\mu G}{1+r_p}\left(1-\frac{A}{\overline{A}\alpha}\widehat{R}\right)^{-1} := \overline{D}^*\left(\widehat{R};G\right).$$

According to proposition 1, if $\widehat{R} = R^{SB}$, the profit margin of lending is zero and banks are indifferent in any D, while if $\widehat{R} > R^{SB}$, the profit margin is positive and banks want to lend as much as they can, that is, the above borrowing constraint is binding. Hence, the supply of notes is

$$D^*\left(\widehat{R};G\right) = \left\{\begin{array}{c} 0 \text{ if } \widehat{R} < R^{SB} \\ [0, \overline{D}\left(\widehat{R};G\right)] \text{ if } \widehat{R} = R^{SB} \\ \overline{D}\left(\widehat{R};G\right) \text{ if } \widehat{R} > R^{SB} \end{array}\right\}.$$

Observe that in equation $E^*\left(\widehat{R}\right) = \overline{D}^*\left(\widehat{R};G\right)$, the factor $1+r_p$ is canceled out, and the equation is equivalent to $E\left(\widehat{R}\right) = \overline{D}\left(\widehat{R};G\right)$, where $E(\cdot)$ and $\overline{D}(\cdot)$ are the demand and maximum supply functions of notes in proposition 3, namely in the absence of the policy. It follows that $E^*\left(\widehat{R}\right) = D^*\left(\widehat{R};G\right)$ if and only if $E\left(\widehat{R}\right) = D\left(\widehat{R};G\right)$, where $D(\cdot)$ is the supply functions of notes in proposition 3. That is, the equilibrium $\widehat{R}$ in the presence of the interest rate policy is the same as that in its absence. Therefore, the interest rate policy produces no real effects.

(ii) In the circumstance where the nominal wage of workers stays at w invariant to the policy rate r_p, if an entrepreneur borrows notes of face value E, he still hires E/w (rather than $E(1+r_p)/w$) workers. This implies (a) that the aggregate demand for notes is wL (rather than $wL/(1+r_p)$), which is equalized to the aggregate supply D in equilibrium; and (b) that the discount factor in (3) is $\delta = 1$, whereby the actual interest rate to entrepreneurs is $R = 1+r$ (rather than $R = (1+r)/(1+r_p)$). Given $\widehat{R}$, $L = (\frac{\overline{A}\alpha}{w})^{\frac{1}{1-\alpha}}\widehat{R}^{\frac{-1}{1-\alpha}}$ by (5). It follows from implication (a) that $D = w(\frac{\overline{A}\alpha}{w})^{\frac{1}{1-\alpha}}\widehat{R}^{\frac{-1}{1-\alpha}}$.

At the optimum all banks choose $R = \widehat{R}$, which together with implication (b) means $1 + r = \widehat{R}$. Substitute these into (30) and banks' borrowing constraint becomes

$$w\left(\frac{\overline{A}\alpha}{w\widehat{R}}\right)^{\frac{1}{1-\alpha}}\left[(1+r_p) - \frac{\underline{A}}{\overline{A}\alpha}\widehat{R}\right) \leq \mu G. \tag{31}$$

This borrowing constraint is binding if the profit margin of lending is positive, as before. Given that the workers' real wage is now $w(1 + r_p)$, the profit to a bank from lending to one entrepreneur becomes $A_e L^\alpha - w(1 + r_p)L - V$. With L and V as functions of $R = \widehat{R}$ given by (5) and (6), the profit margin with sticky wage is $[\overline{A}\alpha/(w^\alpha \widehat{R})]^{\frac{1}{1-\alpha}}[\widehat{R}/R^{SB} - (1 + r_p)]$. This profit margin never goes below 0. Therefore,

$$1 + r_p \leq \widehat{R}/R^{SB}, \tag{32}$$

which implies that $\widehat{R} \geq R^{SB}$.

The equilibrium actual interest rate $\widehat{R}$ is pinned down by conditions (31) and (32), and the fact that one of them must be binding: if (32) is not binding—that is, if the profit margin of issuance is positive—then banks keep issuing notes until the no-default constraint, (31), is binding. Note that (31) is equivalent to

$$1 + r_p \leq \Phi(\widehat{R}), \tag{33}$$

where

$$\Phi(R) \equiv \frac{\mu G w^{\frac{\alpha}{1-\alpha}}}{(\overline{A}\alpha)^{\frac{1}{1-\alpha}}} R^{\frac{1}{1-\alpha}} + \frac{\underline{A}}{\overline{A}\alpha}R, \tag{34}$$

and obviously $\Phi'(\cdot) > 0$, and the inverse function, $\Phi^{-1}(\cdot)$, exists. Then in equilibrium, inequalities (33) and (32) hold, and one of them must be binding. Therefore,

$$1 + r_p = \min\left(\widehat{R}/R^{SB}, \Phi(\widehat{R})\right) := H(\widehat{R}). \tag{35}$$

We only need to consider function $H(\cdot)$ over $\widehat{R} \geq R^{SB}$.

CLAIM P6. *Over* $\widehat{R} \geq R^{SB}$, *if* $G \geq \frac{1}{\mu}G^*$, $H(\widehat{R}) = \widehat{R}/R^{SB}$; *and if* $G < \frac{1}{\mu}G^*$, *then*

$$H(\widehat{R}) = \left\{ \begin{array}{c} \Phi(\widehat{R}) \text{ if } \widehat{R} \leq R^* \\ \widehat{R}/R^{SB} \text{ if } \widehat{R} \geq R^* \end{array} \right\},$$

where

$$R^* = \frac{\overline{A}\alpha}{w}\left[\frac{q(\overline{A}\alpha - \underline{A})}{\mu G}\right]^{\frac{1-\alpha}{\alpha}}. \tag{36}$$

Proof. Let $\chi(R) \equiv \Phi(R) - R/R^{SB}$ for $R \geq 0$. Then $\chi(R) = 0$ has two roots: 0 and R^*. Then (a) $R^* \geq R^{SB}$ if and only if $G \leq \frac{1}{\mu}G^*$; (b) $\chi < 0$ for $R \in (0, R^*)$ and $\chi > 0$ for $R > R^*$, because $\chi'(0) < 0$. These two results lead to the claim. Q.E.D.

With help of the claim, we solve $\widehat{R}$ as a function of r_p from (35) as follows. If $G \geq \frac{1}{\mu}G^*$, $\widehat{R} = R^{SB}(1 + r_p)$. If $G < \frac{1}{\mu}G^*$, then

$$\widehat{R} = \left\{ \begin{array}{c} \Phi^{-1}(1 + r_p) \text{ if } r_p \leq r_p^* \\ R^{SB}(1 + r_p) \text{ if } r_p \geq r_p^* \end{array} \right\},$$

where $r_p^* \equiv R^*/R^{SB} - 1$, which, as $\chi(R^*) = 0$, also equals $\Phi(R^*) - 1$; hence $r_p^* \geq 0 \Leftrightarrow R^* \geq R^{SB} \Leftrightarrow G \leq \frac{1}{\mu}G^*$.

In both cases, $\widehat{R}$ increases with the policy rate r_p to infinity. And at $r_p = 0$, namely in the absence of the CB's intervention, $\widehat{R} < R^{FB}$ in this circumstance of banks over-lending. Therefore, there exists a unique policy rate at which $\widehat{R} = R^{FB}$ and the first-best allocation is attained.

Banks obtain zero profit if the non-negative profit constraint, (32), is binding, namely $H(\widehat{R}) = \widehat{R}/R^{SB}$, which, by claim P6, is the case if $\widehat{R} \geq R^*$. With the optimal policy rate, $\widehat{R} = R^{FB}$, and hence banks obtain zero profit if $R^{FB} \geq R^*$, which is equivalent to $G \geq G_s := \frac{1}{\mu}[(q\overline{A}\alpha + (1-q)\underline{A}) - \underline{A}](A_e\alpha/w)^{\frac{\alpha}{1-\alpha}}$. It is straightforward to check that $G^{FB} < G_s < \frac{1}{\mu}G^*$. Q.E.D.

Proof of Proposition 7

According to the discussion of subsection 3.2, $\overline{Y} = D(1 + r)$ and $\underline{Y} = \underline{A}/(\overline{A}\alpha) \times D(1+r)$. Therefore, $Y = [q + (1-q)\underline{A}/(\overline{A}\alpha)] \times D(1+r)$.

Moreover, at the optimum, banks all offer $1 + r = \widehat{R}$. It follows that if banks issue D, the capital adequacy ratio is

$$c = \frac{G + \{[q + (1-q)\underline{A}/(\overline{A}\alpha)]\widehat{R} - 1\}D}{G + [q + (1-q)\underline{A}/(\overline{A}\alpha)]\widehat{R}D}.$$

If banks' issuance is at the first quantity and the first-best allocation is attained, then $D = D^{FB}$ and $R = R^{FB}$ in the above formula and the capital adequacy ratio equals

$$c^{FB} := \frac{A_e \alpha G + (1-q)\underline{A}(1-\alpha)D^{FB}}{A_e \alpha G + [q\overline{A}\alpha + (1-q)\underline{A}]D^{FB}}.$$

The regulation restricts $c \geq c^{FB}$. To prove the proposition, it suffices to show that this restriction is equivalent to $D \leq D^{FB}$, the constraint which, as we saw in the main text, will lead to the first-best allocation. This equivalence follows from two observations: (i) $c = c^{FB}$ if $D = D^{FB}$ and (ii) c decreases with D because $c = 1 - \frac{D}{G+[q+(1-q)\underline{A}/(\overline{A}\alpha)]\widehat{R}D} := c(D, \widehat{R})$ and moreover, $\frac{\partial c}{\partial D} < 0$ and $\frac{\partial c}{\partial \widehat{R}} > 0$ while $\widehat{R}$ decreases with D.

Q.E.D.

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Macroeconomic Effects of Banking-Sector Losses across Structural Models*

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The macroeconomic effects of capital shortfalls in the financial intermediation sector are compared across five dynamic equilibrium models for policy analysis. Although all the models considered share antecedents and a methodological core, each model emphasizes different transmission channels. This approach delivers model-based confidence intervals for the real and financial effects of shocks originating in the financial sector. The width of 90 percent confidence interval for the GDP response to a banking-sector shock produced by a VAR is comparable to the range of outcomes featured in our model-comparison exercise.

JEL Codes: E32, E44, E47.

1. Introduction

The financial crisis has proved a catalyst for academic research to incorporate financial frictions and an explicit role for an intermediation sector in a general equilibrium framework. In addition, the crisis has reignited the interest in the causes and consequences of shocks affecting the balance sheet of banks, as shown for instance by the increased reliance on regulatory stress tests as an instrument of macroprudential policy.

*We thank Joe LaBriola for excellent research assistance. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System. Corresponding author (Iacoviello) e-mail: matteo.iacoviello@frb.gov.

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In this paper, we argue that this research can offer insights into the propagation of capital shortfalls in the intermediation sector to the rest of the economy. Some of this research has been mirrored and expanded at the Federal Reserve Board by different groups of economists. This paper includes models developed by five of these groups. Our original contribution lies in the meta-analysis of results from the different models rather than in the formulation of the models themselves.[1] Although all the models presented share common antecedents and a common methodological core, they have evolved in complementary directions. Accordingly, comparisons of simulation results from these models, with an eye to identifying the structural features chiefly responsible for quantitative differences, can provide a useful assessment of the spillover effects of shortfalls in capital to the rest of the macroeconomy. Moreover, to the extent that quantitative models are needed for policy analysis, and to the extent that different models give starkly different quantitative predictions, it is useful to investigate the origins of these differences.

Each of the models presented emphasizes different aspects of the nexus between a financial sector and the rest of the economy.

(i) The model by *Iacoviello* allows two financial frictions to coexist in that both bankers and entrepreneurs are constrained in how much they can borrow from patient savers. A key feature of the model is that entrepreneurs own commercial real estate, which enters the production function for final goods and which can be posted as collateral against loans.

(ii) The model by *Covas and Driscoll* also features credit constraints on bankers and entrepreneurs. In addition, a corporate sector is included so that the banking sector need not fund the entire economy. A key distinction of their approach is that the model is solved with global non-linear methods, rather than by a linear approximation that imposes that all credit constraints are always binding.

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[1] Each of the five models in this paper is described more fully in related work cited below.

(iii) The model by *Kiley and Sim* is set up to study the interaction between financial frictions and monetary policy. In all the models covered here, financial intermediaries have access to debt markets and retained earnings. In addition, a key feature of the model developed by Kiley and Sim is that financial intermediaries can access external equity markets to finance their investments, which allows them an explicit treatment of dilution costs related to the expansion of external equity.

(iv) The interaction between inside and outside equity is also at the center of *Queralto*'s model. An agency problem justifies the constraints on borrowing faced by the financial sector in his model. The agency problem is devised in such a way that financial intermediaries face a tradeoff between short-term debt and outside equity. In turn, this endogenous tradeoff is affected differently by different sources of fluctuations.

(v) Finally, the model developed by *Guerrieri and Jahan-Parvar* is geared to the analysis of monetary policy and takes into account the zero lower bound on nominal interest rates. A salient characteristic of the model is the interaction between two groups of firms. One group of firms can only raise external funds through financial intermediaries, while the other group of firms has direct access to financing from households.

To facilitate comparisons across models, each of the self-contained model sections to follow considers one particular form of capital shortfall, namely a transfer of funds from the banking sector to the household sector. This transfer takes place in a lump-sum fashion and does not distort at the margin the actions of the household sector. Accordingly, it could also be thought of as a shock that simply destroys some assets on the balance sheet of the banking sector. While each model has features that can be used to analyze a plethora of distinct financial shocks, the baseline transfer shock has the virtue of being easily implemented and comparable across all models. In addition, the baseline transfer shock is a initially a "pure" financial shock in that it does not imply, per se, the depletion of real resources. In that respect, it is fair to characterize the macro repercussions as "spillover effects" from the financial sector to the rest of the macroeconomy.

Each model section presents results for the evolution of key macro variables, such as aggregate output, consumption, and investment. It also reports some key financial variables, such as bank capital and spreads between interest rates on deposits and on loans.

Rather than coordinating on the same structural parameters across different models, each model adopts a parameterization best suited to its specific features. Sensitivity analysis with respect to key parameters focuses on plausible changes in calibration that can result in large differences in macro outcomes. From this sensitivity analysis, we learn, for instance, that the choice of labor supply elasticity, while having very little bearing on financial outcomes, can exert an outsize influence on the macro spillover effects of financial shocks. More broadly, we highlight that features unrelated to the modeling of financial frictions can be just as important in determining the macroeconomic impact of financial shocks as specific aspects of the financial frictions.

In addition to the effects of the baseline transfer shock, each model section presents the effects of a distinct financial shock that leads to a shortfall in capital for the banking sector, e.g., a housing shock or a change in capital requirements. These additional shocks are calibrated to produce a capital shortfall that is comparable to that of the transfer shock. Because each distinct shock considered has different propagation channels, this exercise provides additional insights on the mechanisms by which financial shocks affect the macroeconomy.

Our model comparisons can deliver "model-based confidence intervals" relative to the effects of financial shocks. The results are informative about the importance of different modeling approaches in influencing the quantitative implications of standardized shocks. Moreover, the sensitivity analysis regarding parameter choices is meant to produce envelope results relative to the possible spillover effects of capital shortfalls. By harmonizing the calibration of the different models, we confirmed that the differences in results highlighted are extant economic differences, rather than differences merely driven by plausible alternative calibrations. Finally, the comparison of shocks other than the baseline transfer shocks across models reinforces the intuition that the underlying causes of a capital shortfall in the financial sector are important in predicting the subsequent spillover effects to the rest of the economy.[2]

The rest of the paper proceeds as follows. Section 2 describes the calibration of the baseline transfer shock. Each of the sections from 3 to 7 describes results from individual models. Section 8 provides a horizontal comparison of the effects of the baseline transfer shocks across models. Section 9 concludes. An online appendix, available at http://www.ijcb.org, provides additional details on each of the models.

2. Calibration of the Baseline Transfer Shock

In order to provide informative comparisons across the linear and non-linear models considered, the calibration of the size for the baseline transfer shock is chosen to be large but empirically realistic. We consider a transfer shock in line with the results from the stress tests for the U.S. banking sector mandated by the Financial Reform Act. These stress tests, whose main goal is to assess the solvency of the banking system in the face of rapidly deteriorating macroeconomic conditions, provide useful information regarding the magnitude of empirically relevant capital shortfalls. We use the results for the Comprehensive Capital Analysis and Review (CCAR) of 2013. According to these results, under a severely adverse scenario for the U.S. economy, total projected losses of the eighteen bank holding companies included in the stress test amounted to a cumulative total of $462 billion for the nine quarters from 2012:Q4 through 2014:Q4. For context, these losses are conditional on a scenario designed to be comparable to the Great Recession.[3]

These losses amount to about 3 percent of 2012 GDP. Only the top eighteen banks by assets were included in the stress-test exercise. To calibrate the baseline transfer shock to capture plausible losses for the entire banking system and not just the largest banks,

[2]As pointed out in Wieland et al. (2012), model comparison exercises have helped produce influential insights, such as the robustness of the Taylor rule across many models, but are infrequent and costly, because they require the input of many teams of researchers.

[3]Cumulative losses are disclosed in a press release issued by the Federal Reserve, available at http://www.federalreserve.gov/newsevents/press/bcreg/dfast_2013_results_20130314.pdf.

we scale up the magnitude of the transfer to reflect that the CCAR banks account for about 60 percent of banking assets (the sum of assets of depository institutions and bank holding companies in the flow of funds). Furthermore, a second rescaling is applied to reflect that traditional banks account for about two-thirds of the asset of the banking sector, defined as traditional banking institutions in addition to bank-like institutions.[4]

Accordingly, the *baseline transfer shock* entails a reduction in assets equal to 7.5 percent of GDP (=3%/0.6/0.66) cumulatively over the first nine quarters following the transfer. The shock is phased in using an autoregressive process of order 1 with a persistence equal to 0.9. The desired cumulative transfer over nine quarters is used to pin down the initial innovation to the shock process (roughly 1.2 percent of GDP). Given these choices, after ten years, the total cumulative transfer amounts to about 12 percent of GDP.

3. Matteo Iacoviello: An Estimated Model of Banks with Financing Frictions

3.1 Model Description

The economy in Iacoviello (2015) features four agents: patient households (savers), impatient households (borrowers), bankers, and entrepreneurs. In the following, we present key elements of Iacoviello's model abstracting from a variety of frictions—such as habits, adjustment costs, and variable capital utilization—that bolster the empirical realism of the model. The full model description (including the calibrated parameters for the exercises

[4]The share of assets of traditional banking institutions is derived from the following flow of funds series: 1 – ((FL704090005.Q + FL734090005.Q) / ((FL413065005.Q + FL674090005.Q + FL614090005.Q + FL664090005.Q + FL504090005.Q) + (FL704090005.Q + FL734090005.Q))), that is: 1 – ((Total Financial Assets of Private Depository Institutions + Total Financial Assets of Holding companies)/((Agency- and GSE-backed mortgage pools; total mortgages; asset + Issuers of asset-backed securities; total financial assets + Finance companies; total financial assets + Security brokers and dealers; total financial assets + Funding corporations; total financial assets) + (Total Financial Assets of Private Depository Institutions + Total Financial Assets of Holding companies))).

below) can be found in the online appendix accompanying this paper.

Each agent has a unit mass.[5] Households work, consume and buy real estate, and make one-period deposits into a bank. The household sector in the aggregate is the net saver. Entrepreneurs accumulate real estate, hire households, and borrow from banks. In between the households and the entrepreneurs, bankers intermediate funds. The nature of the banking activity implies that bankers are borrowers when it comes to their relationship with households, and are lenders when it comes to their relationship with the credit-dependent sector—entrepreneurs—of the economy. Iacoviello designs preferences in a way that two frictions coexist and interact in the model's equilibrium: first, bankers are credit constrained in how much they can borrow from the patient savers; second, entrepreneurs are credit constrained in how much they can borrow from bankers.

Entrepreneurs own housing $H_{E,t}$, priced at q_t, which, combined with household labor, is used by final good firms to produce the final output Y_t. They are subject to a borrowing constraint of the form

$$L_{E,t} \leq m_H E_t \left(\frac{q_{t+1}}{R_{E,t+1}} H_{E,t} \right) - m_N W_{H,t} N_{H,t}. \tag{1}$$

Here, $L_{E,t}$ are loans that banks extend to entrepreneurs (yielding a gross return $R_{E,t}$). The borrowing constraint states that entrepreneurs cannot borrow more than a fraction m_H of the expected value of their housing stock, discounted by the interest rate. The constraint also stipulates that a fraction m_N of the wage bill $W_{H,t}N_{H,t}$ must be paid in advance. Entrepreneurs discount the future more heavily than households and bankers: this assumption guarantees that the borrowing constraint will bind in a neighborhood of the steady state. Denoting with $\lambda_{E,t}$ the Lagrange multiplier on the borrowing constraint, and with $u_{CE,t}$

[5]Except for the introduction of the banking sector, the model structure closely follows a flexible price version of the basic model in Iacoviello (2005), where credit-constrained entrepreneurs borrow from households directly. Here, banks intermediate between households and entrepreneurs.

the entrepreneur's marginal utility of consumption, the first-order condition for loans is

$$(1 - \lambda_{E,t})\, u_{CE,t} = \beta_E E_t(R_{E,t+1} u_{CE,t+1}). \tag{2}$$

This first-order condition shows that the credit constraint introduces a wedge in the intertemporal optimization condition of the entrepreneur. Additionally, when this first-order condition is combined with the entrepreneur's factor demands for N_H and H_E, the borrowing constraint acts as a tax not just on the demand for credit but also on the demand for the factors of production.

The other key agents in the model are the bankers, who solve the following problem:

$$\max \sum_{t=0}^{\infty} \beta_B^t \log C_{B,t},$$

where $\beta_B < \beta_H$, and where β_H is the household's discount factor, subject to

$$C_{B,t} + R_{H,t-1} D_{t-1} + L_{E,t} = D_t + R_{E,t} L_{E,t-1} - \varepsilon_t, \tag{3}$$

where D_t denotes household deposits (yielding $R_{H,t}$), $L_{E,t}$ are loans to entrepreneurs, $C_{B,t}$ is bankers' consumption, and ε_t are loan losses suffered by bankers in the conduct of their business. This formulation is analogous to a formulation where bankers maximize a convex function of dividends (discounted at rate β_B), once C_B is reinterpreted as the residual income of the bankers, after depositors have been repaid and loans have been issued. Iacoviello assumes that bankers are constrained in their ability to issue liabilities by the amount of equity capital in their portfolio. This constraint can be motivated by regulatory concerns or by standard moral hazard problems. Letting $K_{B,t} = L_{E,t} - \varepsilon_t - D_t$ denote bank capital at the end of the period (after loan losses caused by transfer shocks have been realized), a capital requirement can be reinterpreted as a standard borrowing constraint, such as

$$D_t \leq \gamma_E \left(L_{E,t} - \varepsilon_t \right). \tag{4}$$

Above, the left-hand side denotes banks' liabilities D_t, while the right-hand side denotes which fraction of each of the banks' assets can be used as collateral.

Let $m_{B,t} \equiv \beta_B E_t \left(\frac{C_{B,t}}{C_{B,t+1}} \right)$ denote the bankers' stochastic discount factor, and let $\lambda_{B,t}$ denote the multiplier on the bankers' capital requirement. The optimality conditions for deposits and loans are, respectively,

$$1 - \lambda_{B,t} = E_t \left(m_{B,t} R_{H,t} \right), \tag{5}$$

$$1 - \gamma_E \lambda_{B,t} = E_t \left(m_{B,t} R_{E,t+1} \right). \tag{6}$$

The interpretation of the two first-order conditions is straightforward. Consider the ways that bankers can increase their consumption by one extra unit today:

(i) Bankers can borrow from households, increasing deposits D_t by one unit today: in doing so, the banker reduces its equity by one unit, thus tightening the capital requirement one-for-one and reducing the utility value of an extra deposit by $\lambda_{B,t}$. Overall, today's payoff from the deposit is $1-\lambda_{B,t}$. The next-period cost is given by the stochastic discount factor times the interest rate R_H.

(ii) Bankers can consume more today by reducing loans by one unit. When lending less, bankers face a tighter capital requirement, since the reduction in loans mechanically translates into a reduction in equity. The utility cost of tightening the borrowing constraint through lower loans is equal to $\gamma_E \lambda_{B,t}$. Intuitively, the higher the value of loans as collateral for the bank activity (the higher γ_E is), the larger is the utility cost of not making loans. Overall, today's cost of making a loan is $1-\gamma_E \lambda_{B,t}$. The next-period benefit is given by the stochastic discount factor times the interest rate R_E.

For bankers to be indifferent between collecting deposits (borrowing) and making loans (saving), the returns across assets must be equalized. Given that R_H is determined from the household problem, bankers will be borrowing constrained, and λ_B will be positive, so long as $m_{B,t}$ is sufficiently lower than the inverse of R_H. In turn, if λ_B is positive, the required return on loans R_E will be higher, the lower γ_E is. Intuitively, the lower γ_E is, the lower is the liquidity value of loans in relaxing the bankers' borrowing constraint, and the higher is the compensation required by bankers to be indifferent between lending and borrowing. Moreover, loans will pay a return that is (near the steady state) higher than the cost of deposits, since, so long as γ_E is lower than one, loans are less liquid than the deposits.

The bankers' capital requirement on the one hand, and the entrepreneurs' credit constraint on the other, create a wedge between steady-state output in absence of financial frictions and output when financial frictions are present. The capital requirement on banks limits the amount of savings that banks can transform into loans. Likewise, the credit constraint on entrepreneurs limits the amount of loans that can be invested for production. Both forces lower steady-state output.

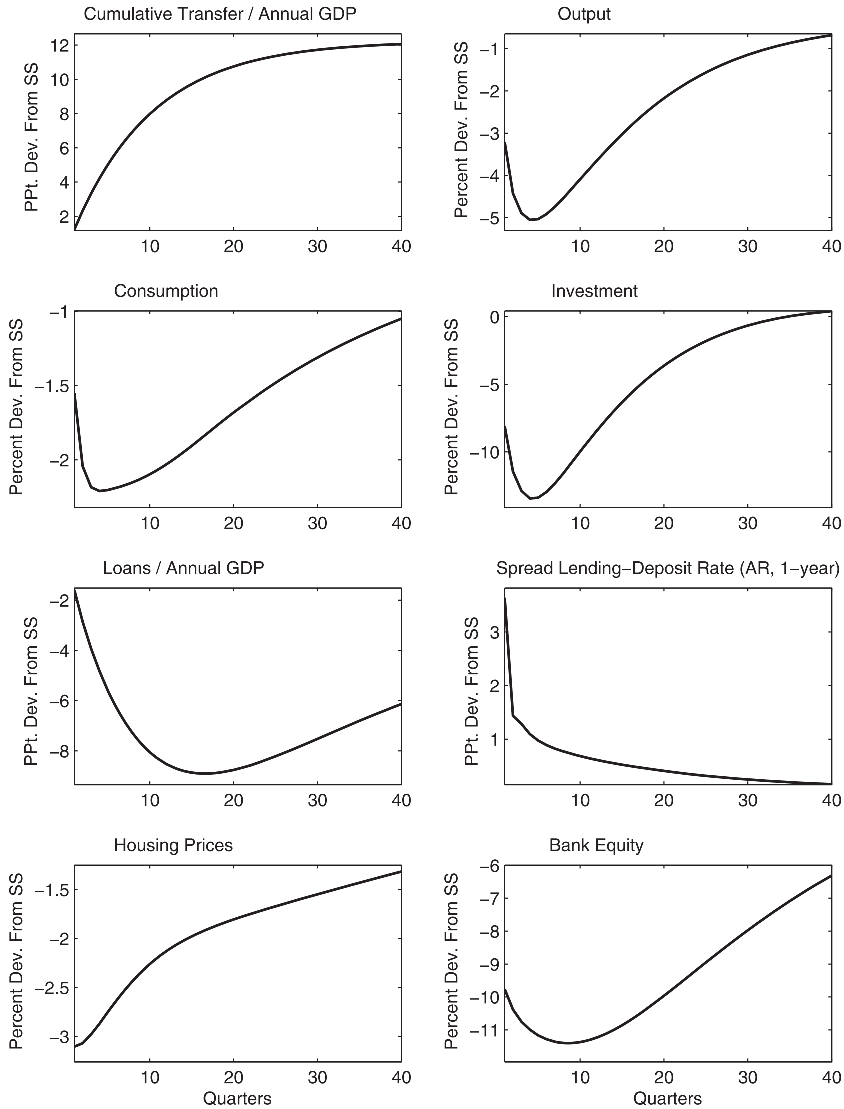
3.2 Transfer Shock

Analogous forces are also at work for shocks that move the economy away from the steady state, to the extent that these shocks tighten or loosen the severity of the borrowing constraints. To illustrate their importance, consider the dynamic effects of a transfer shock ε_t. An interpretation of this shock is that it captures losses for the banking system caused by a wave of defaults. Figure 1 plots a dynamic simulation for the model economy. The stochastic process for ε_t follows:

$$\varepsilon_t = 0.9\varepsilon_{t-1} + \iota_t. \tag{7}$$

The transfer shock is calibrated as already discussed in section 2. The shock impairs the bankers' balance sheet, by reducing the value of bank assets (total loans minus loan losses) relative to the liabilities (household deposits): at that point, in absence of any further adjustment to either loans or deposits, bankers would have a capital asset ratio that is below target. Bankers could restore their capital-to-asset ratio either deleveraging (reducing deposits from households) or reducing consumption in order to restore the equity cushion. If reducing consumption is costly, bankers reduce loans and give rise to a vicious, dynamic cycle of reductions in

Figure 1. Transfer Shock in Iacoviello's Model



both loans and deposits, which propagates the credit crunch. In particular, the decline in loans to the credit-dependent sector of the economy (entrepreneurs) acts as a drag on consumption and productive investment. It drags investment down because credit-constrained entrepreneurs reduce their real estate holdings and labor demand as credit supply is reduced. And it drags consumption down because the decline in labor demand and the reduction in entrepreneurial investment induce a decline in total output. All told, GDP declines almost 5 percent after about one year.[6]

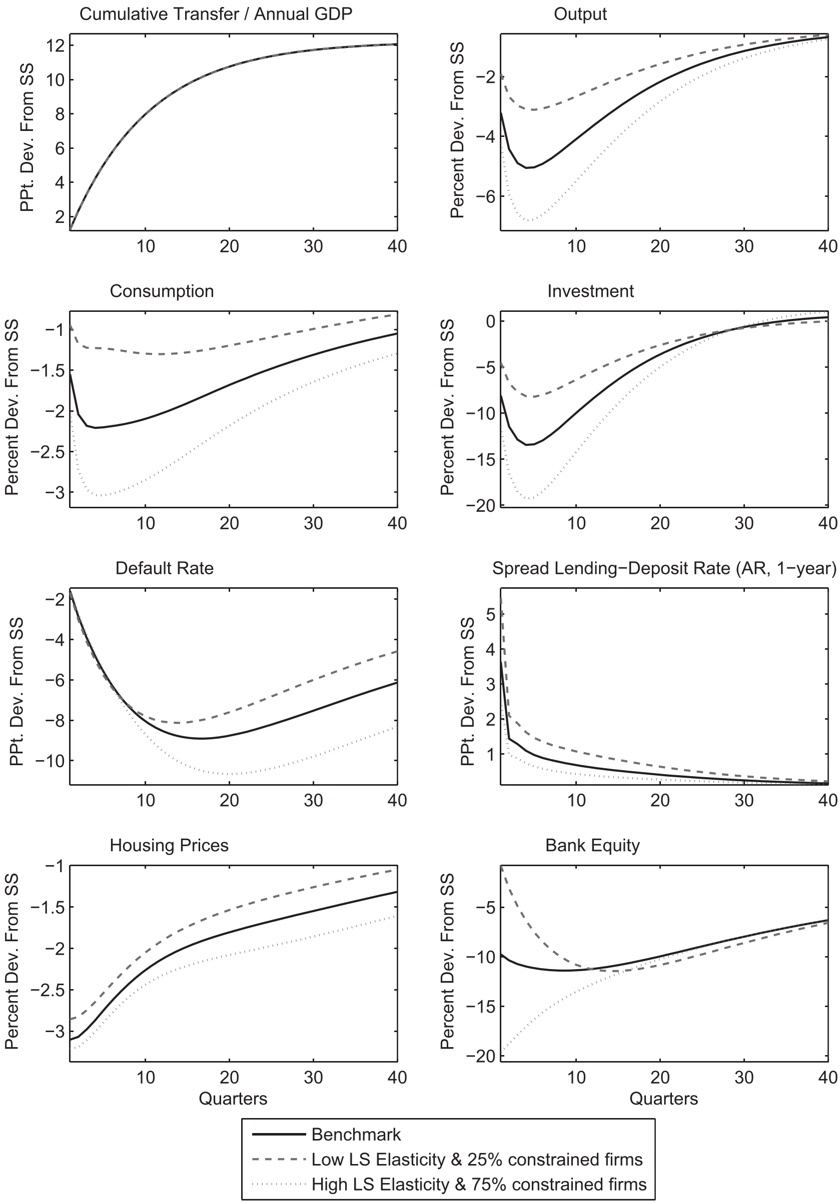
3.3 Robustness Analysis

Figure 2 presents robustness analysis around the baseline parameterization. In the benchmark case, labor supply elasticity is 2, and the capital share of credit-constrained entrepreneurs is about one-half. A higher labor supply elasticity and capital share of constrained entrepreneurs both work to reinforce, as one would expect, the effects of a shock to bank capital. A lower labor supply elasticity (slightly less than 1) and a 25 percent share of credit-constrained entrepreneurs both work to reduce the magnitude of the decline in output from 5 to 3 percent. Conversely, a higher labor supply elasticity (around 5) and a 75 percent share of credit-constrained entrepreneurs concomitantly boost the decline in economic activity from 5 to 7 percent.

Figure 3 considers the effects of another shock that endogenously leads to a reduction in bank capital, namely a decline in housing prices. Through a decline in lending activity, consumption, and investment, the shock to housing prices leads to a reduction in bank capital, even in the absence of direct shocks to bank capital (such as those taking place with the transfer shock). When the housing price shock is sized to reduce bank equity by 10 percent (namely, the same percent decline in bank equity following the transfer shock), aggregate output falls by approximately 4 percent, slightly less than in the case of the transfer shock.

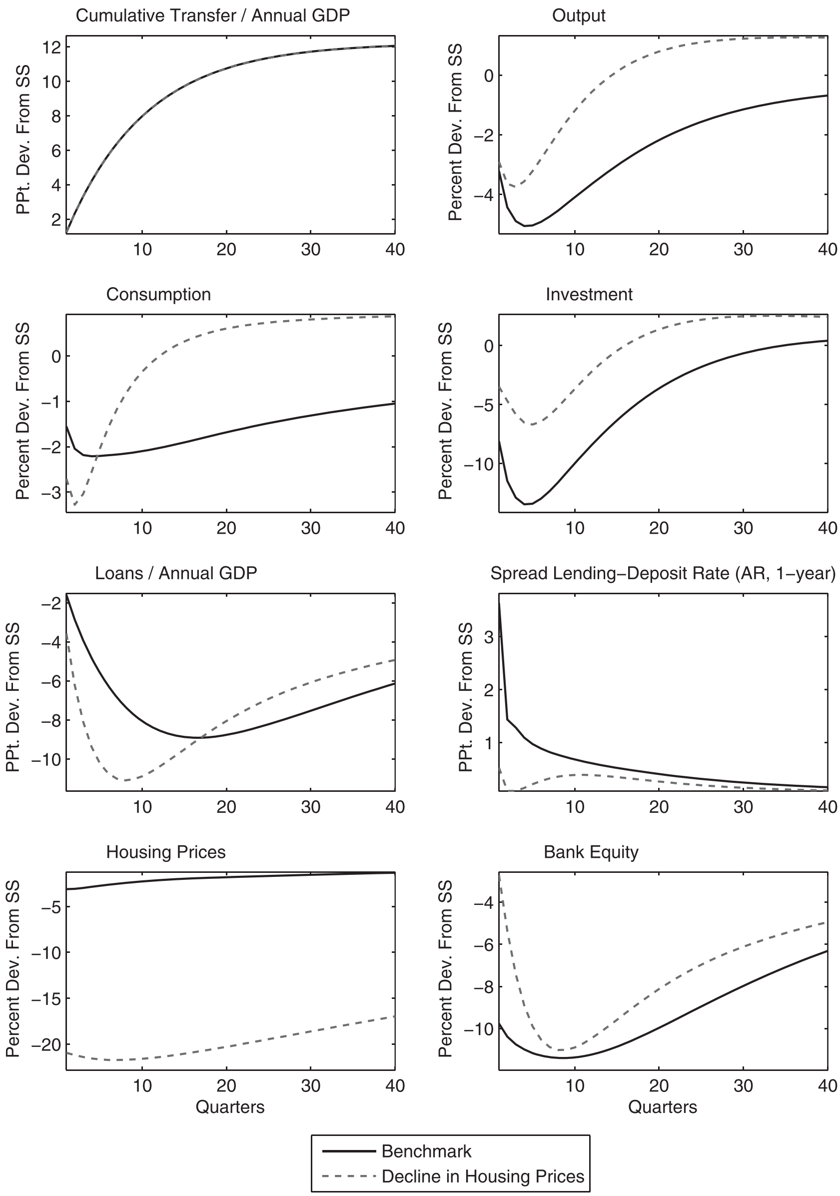
[6]An additional force that reduces output in the wake of a transfer shock is a negative wealth effect on labor supply for the households who receive funds from the bank. This effect contributes to less than one-quarter of the decline in output.

Figure 2. Transfer Shock in Iacoviello's Model: Robustness Analysis



Notes: In the benchmark case, the labor supply elasticity is around 2, and the capital share of credit-constrained entrepreneurs is about one-half. A higher labor supply elasticity and a higher capital share of constrained entrepreneurs both work to reinforce the effects of a shock to bank capital. The lower labor supply elasticity considered is slightly less than unity. The higher labor supply elasticity considered is nearly 5.

Figure 3. Transfer Shock vis-à-vis a Housing Price Shock in Iacoviello's Model



Notes: The figure considers the effects of a shock to housing prices that leads to a decline in bank equity comparable to the decline induced by the transfer shock.

4. Francisco Covas and John Driscoll: A Non-linear Model of Borrowing Constraints

4.1 Model Description

The model of this section is also described in Covas and Driscoll (2013). That paper evaluates the aggregate effects of imposing a liquidity coverage ratio requirement in addition to a risk-based capital requirement on the banking sector. Covas and Driscoll sketch key features of their model below.[7] The model is based on that of Aiyagari (1994), in which a continuum of heterogeneous workers are subject to idiosyncratic labor income risk under the presence of a borrowing constraint. In addition, the model adds heterogeneous entrepreneurs who face investment risk under the presence of a borrowing constraint and heterogeneous bankers which are subject to profitability risk and a capital requirement.[8] The model with workers and entrepreneurs is very similar to the model specifications used by Angeletos (2007) and Covas (2006). The banker's problem is similar to the partial equilibrium setup analyzed by De Nicolò, Gamba, and Lucchetta (2013). The key frictions in the banking sector are the capital requirement and the inability of bankers to issue outside equity, that is, all the increase in equity occurs via retained earnings. The combination of these two frictions and the fact that entrepreneurs are assumed to be bank dependent creates a setting in which the Modigliani-Miller theorem does not apply.[9] As a result, an exogenous shock to bankers' equity leads to adjustments in the supply of credit by banks and loan spreads, with corresponding real effects.

[7]The full model description (including the calibrated parameters for the exercises below) can be found in the online appendix accompanying this paper.

[8]To better preserve comparability with the other models, for the simulations below the liquidity requirement is not included.

[9]The assumption of bank dependence for the entrepreneurial sector is in accordance with the literature on the credit channel of monetary policy, which also assumes that some firms, particularly smaller ones, do not have the same amount of access to other forms of finance.

Workers supply one exogenous unit of labor to the entrepreneurs and a corporate sector. They are subject to labor productivity shocks that affect their earnings. They choose consumption, deposits, and asset holdings to maximize utility subject to a borrowing constraint. Entrepreneurs can invest in an individual-specific risky technology and in riskless securities. They supply one exogenous unit of labor to their entrepreneurial businesses and also to the corporate sector. Entrepreneurs choose consumption, investment, and loans (from the banking sector) to maximize lifetime utility subject to a borrowing constraint. The reliance on bank loans as a form of finance and the presence of a borrowing constraint violate the Modigliani-Miller theorem for the entrepreneurial sector, in which changes in the quantity and price of bank loans force entrepreneurs to chance the consumption and investment choices.

Bankers hold loans and riskless securities; the latter, which are assumed to be in positive net supply, may also be used to fund loans, and therefore net securities holdings may be negative. Loans mature at a constant rate and have a constant servicing cost; to capture the illiquidity of loans relative to securities, banks pay (asymmetric) adjustment costs to changing the quantity of loans outstanding. In addition, loans and other banking activities generate non-interest income which is a concave function of the size of the loan portfolio and is subject to idiosyncratic profitability shocks. Loans are funded through deposits and equity. Banks face a risk-based capital constraint, in which the amount of equity must be at least equal to a risk-weighted sum of loans and securities (the latter of which has a zero risk weight). Bankers maximize utility subject to the above constraints. In equilibrium, banks will choose to hold a (precautionary) buffer of equity capital above the requirement; however, the capital constraint may still bind for some banks. As mentioned earlier, bankers are not allowed to issue outside equity, and the increase in capital has to be done via retained earnings.

The model is completed by a corporate sector, which produces output with capital supplied by workers and labor supplied by both workers and entrepreneurs. This sector is included so that the banking sector need not fund the entire economy.

Table 1. Selected Moments of Covas and Driscoll's Model

Moments	Data	Model
Tier 1 Capital Ratio	10.0	9.7
Share of Constrained Banks	0.1	0.3
Leverage Ratio	7.0	6.3
Adjusted Return on Assets, % (AR)	2.9	3.4
Cross-Sectional Volatility of Adjusted Return on Assets	1.3	1.4
Safe Assets Held by Banks, %	33.1	34.4
Ratio of Interest Income to Non-interest Income	1.3	0.3
Share of Non-interest Expenses	3.0	8.5
Return on Securities, % (AR)	0.5	0.5
Loan Rate, % (AR)	4.0	4.1
Consumption to Output	0.7	0.7
Banking Assets to Output	0.9	1.2
Safe-to-Total Assets	0.3	0.3
Memo: Deposit Rate, % (AR)	0.1	0.1

Notes: Moments are based on sample averages using quarterly observations between 1997:Q1 and 2012:Q3, with the exception of the percentage of safe assets held by banks, which is only available starting in 2001:Q1, and averages for the ratio of interest income to non-interest income and banking assets to output are calculated only for the period after the fourth quarter of 2008 when investment banks became bank holding companies. The adjusted return on assets is defined as net income excluding income taxes and salaries and employee benefits. The percentage of safe assets held by banks includes all assets with a zero risk weight plus assets with a 20 percent risk weight. The sample includes all bank holding companies and commercial banks that are not part of a BHC, or that are part of a BHC which does not file the Y-9C report. The share of constrained banks is estimated using banks' responses in the Senior Loan Officer Opinion Survey and reported by Bassett and Covas (2013). The safe-asset share is obtained from Gorton, Lewellen, and Metrick (2012). All interest rates reported are annual.

In steady-state equilibrium, the loan, security, and deposit markets clear, factor prices equal marginal products, and distributions of agents' characteristics are invariant. The model is calibrated so that parameters from the bankers' problem match certain moments from bank holding company Call Report data as summarized in table 1. A summary of the calibration of the model is provided in the online appendix. The model is solved numerically by iterating the policy function over time, as in Coleman (1990).

Table 2. Details of the Transfer Shock in Covas and Driscoll's Model

Sector	Year 1	Year 2
Workers	0.9	0.6
Entrepreneurs	0.2	0.2
Bankers	−37.3	−24.5

Note: Entries in the table denote the size of transfer in each year as a percent of the steady-state level of wealth of each sector.

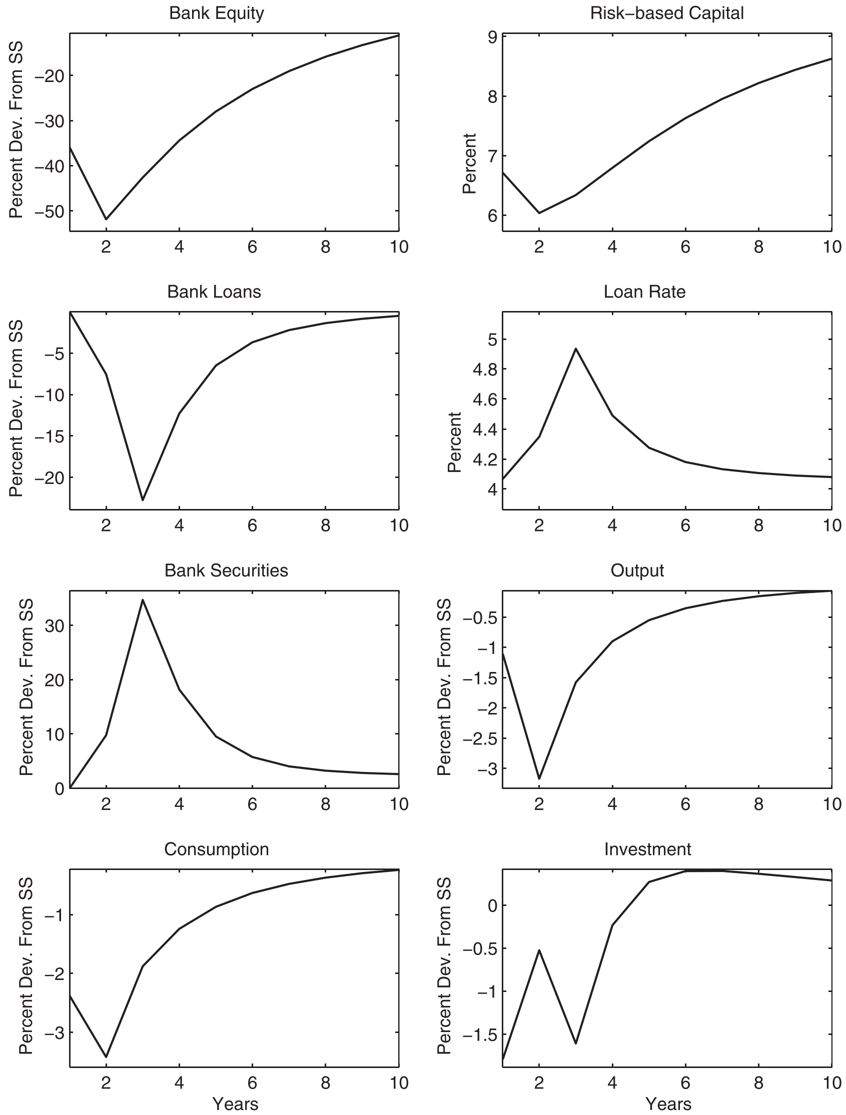
The steady-state solution also solves for the loan rate, the return on securities, and the capital-labor ratio of the corporate sector using a quasi-Newton method. Finally, the simulation results presented below are based on transition dynamics which simulate the evolution of the density function for each sector using the optimal policy functions and the time path for the loan rate, the return on securities, and the capital-labor ratio of the corporate sector.

4.2 Transfer Shock

The baseline simulation reports the effects of a transfer of wealth from bankers to entrepreneurs and workers equivalent to 7.5 percent of steady-state output, in line with the calibration of the shock discussed in section 2. Furthermore, we assume that 60 percent of the transfer occurs in the first year, and 40 percent in the second year (hewing closely to the quarterly autoregressive progress with a coefficient of 0.9 as for the other models in this paper). The transfer of wealth between the three sectors is assumed to be unexpected in both years. Table 2 gives the size of the transfer in each year relative to the level of steady-state wealth of each sector.

The large reduction in bankers' wealth drives down bank equity by about 35 percent in the first year and 50 percent in the second year, as seen in figure 4. This generates a reduction in the average tier 1 ratio of 300 basis points in the first year and 70 basis

Figure 4. Transfer Shock in the Covas/Driscoll Model



points in the second year. Despite the larger decrease in wealth in the second year, the decrease in the tier 1 ratio is lower in the second year because the large majority of banks have a binding tier 1 capital ratio which cannot go below 6 percent. In order to meet the capital requirement, banks slash consumption (i.e., dividends) by 40 percent in the first year and 60 percent in the second year, reduce loan outstandings by about 8 percent in the first year and 23 percent in the second year, and increase holdings of securities by 10 and 35 percent in years 1 and 2, respectively.[10] The abrupt reduction in loans hinges partially on the assumption that bankers do not have access to outside equity and in our model all equity capital accumulation is done via retained earnings. The magnitude of the transfer shock would likely be dampened if banks had access to outside equity or started the exercise with a larger capital buffer. The reduction in the supply of loans by banks causes the loan rate to increase by about 30 basis points in the first year and 65 basis points in the second year, and, similarly, the rate on securities to fall by 40 and 110 basis points, respectively. The change in these two interest rates combined implies that the loan spread would increase by 70 basis points in the first year and 170 basis points in the second year.

The transfer shock initially benefits the entrepreneurs, with both wealth and consumption increasing by small amounts for the first two years. However, the increase in the loan rate reduces investment by entrepreneurs and causes their wealth and consumption to fall in subsequent years. As a result, entrepreneurs' capital and holdings of securities fall, as do their labor demand and output. Investment is initially negative, before rising as the economy returns to its steady state.

Throughout the transition period, workers are better off, as they receive the benefit of increased wealth without incurring the direct cost of higher loan rates since they do not borrow from banks. In response to an increase in wealth, workers increase consumption and savings. Some of the increase in savings is done through the accumulation of capital that is rented to the corporate sector, whose output rises as a result.

[10]Based on Call Report data, total loans at commercial banks declined by 10 and 6 percent in 2009 and 2010, respectively.

In the aggregate, consumption and output both fall by about 3 percent in the second year of the transfer shock, and investment declines by about 1 percent. The decline in investment is less pronounced relative to the decline in output because of the large boom in investment in the corporate sector.[11]

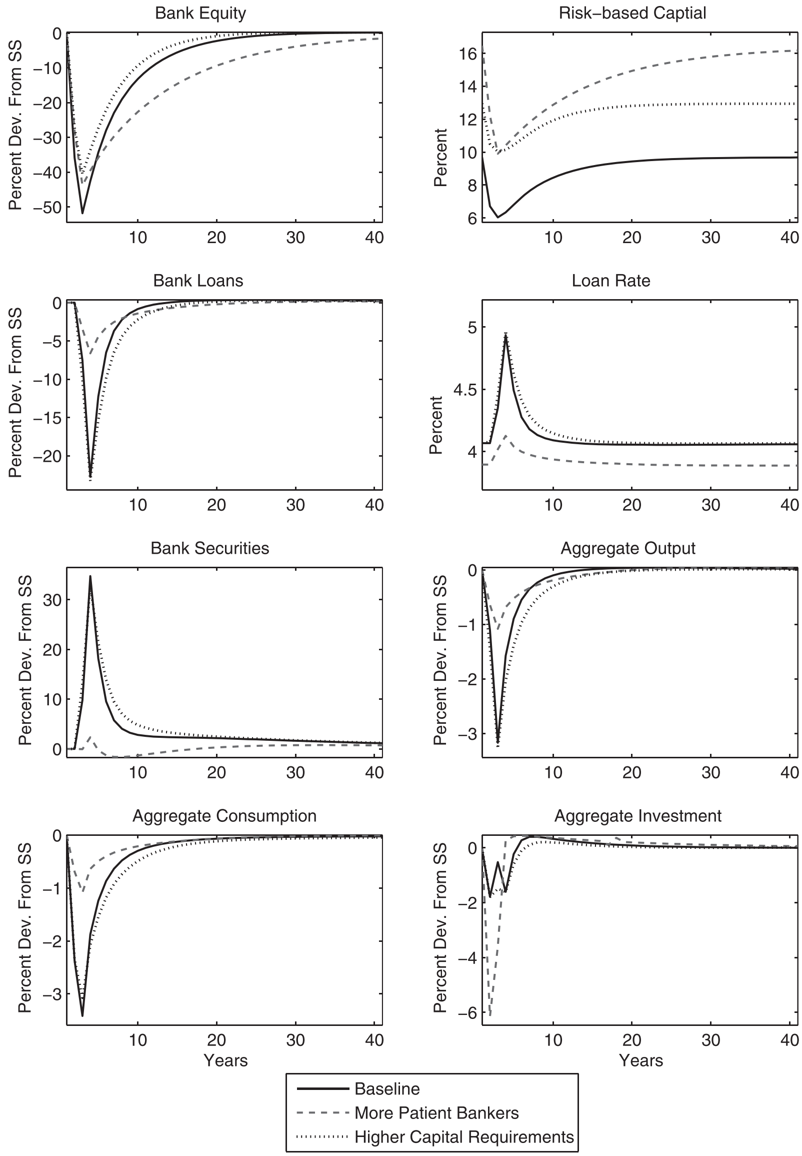
4.3 Sensitivity Analysis

A key feature of the model is the capital requirement for bankers. As shown in table 1, the capital requirement constraint binds for about one-third of banks in the steady state. The capital constraint is the key friction in the banking sector, and for that reason we conduct two types of sensitivity analysis. In the first exercise, we reduce the fraction of capital-constrained bankers to about half of the steady-state share. We do so by increasing the discount factor of bankers, which increases the size of the capital buffer above the minimum capital requirement. In the second exercise, we also increase the amount of equity held by bankers. However, we do so by raising the capital requirement, and so the capital buffer above the minimum remains relatively unchanged. We show in figure 5 that these two experiments generate different sets of aggregate responses, and we conclude that the key driver of bankers' responses following the transfer shock is the share of capital-constrained banks.

Reducing the share of capital-constrained banks reduces the effects of the transfer shock—output and consumption now both decline by about 0.6 percent in the first year and 1 percent in the second year. Since bankers have larger capital buffers when the transfer shock occurs, the responses of bank loans and the corresponding interest rate are considerably less pronounced in this case. In particular, the transfer shock now increases the loan rate only by 20 basis points, and the return on securities declines by 10 basis

[11]This result is a bit counterintuitive. The reason is that the transfer shock is very large and bankers cannot absorb more deposits because the capital constraint binds for almost all banks. In equilibrium, workers invest even more in the corporate sector. In the next section, we reduce the share of constrained banks and get the standard result that the response of investment is larger (more negative) than the response of output.

Figure 5. Transfer Shock in the Covas/Driscoll Model: Robustness Analysis



Notes: Under the baseline calibration the fraction of capital-constrained bankers is 0.3 in the steady state. Under "More Patient Bankers," an increase in the discount factor of bankers reduces the fraction of capital-constrained bankers to 0.15 by increasing the capital buffer above the statutory minimum. Under "Higher Capital Requirements," the increase in the discount factor is accompanied by an increase in the capital requirement, sized so that the buffer over the statutory minimum is unchanged relative to the baseline calibration.

points. As a result, the spillover effects of the shock in the banking sector to the entrepreneurial and worker sectors are considerably smaller.

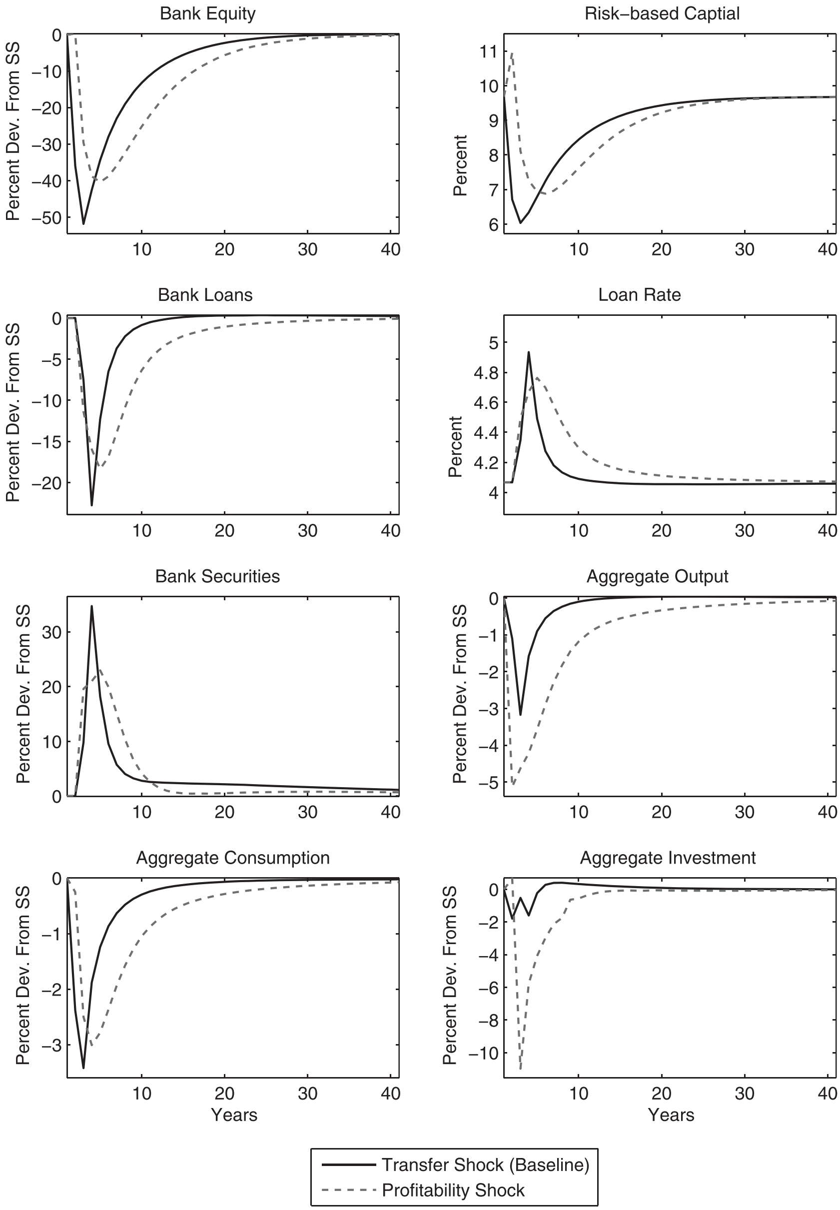
Finally, increasing the size of the capital requirement and requiring bankers to hold more equity prior to the transfer shock generates very similar responses in aggregate output and consumption relative to the baseline case. This suggests that the key mechanism in this model is driven by likelihood of banks to be capital constrained and not the level of equity held by banks. An important assumption in the model is that banks are not allowed to violate the capital requirement of 10 percent. Taken together, these two experiments suggest that allowing banks to go below the capital requirement at the same time the transfer shock occurs would yield sizable welfare gains relative to the case in which capital requirements are left unchanged.

4.4 Responses to an Alternative Shock Affecting the Balance Sheet of Banks

A final alternative looks at the effect of another shock: a reduction in bank revenues. In particular, we model the decrease in bank revenues by assuming a persistence shock to the non-interest component of bank revenues. Bankers are assumed to have perfect foresight of the shock. The shock is calibrated so that the change in wealth of bankers is roughly the same as the change of wealth induced by the transfer shock in the baseline calibration.

As seen in figure 6, the effect of the revenue shock reduces aggregate output by about 5 percent in the first year and 4.5 percent in the second year, which is considerably more than the response found above for the case of the transfer shock. This is not surprising since in this exercise bankers' wealth is no longer transferred to the entrepreneurial and workers' sectors. As a result, the reduction in output driven by the entrepreneurial sector is not partially offset by the increase in output in the corporate sector. Finally, consumption falls by substantially less—bottoming out at about a 3 percent reduction since bankers have perfect foresight of the shock and are able to smooth consumption more effectively.

Figure 6. Transfer Shock vs. Bank Revenue Shock in the Covas/Driscoll Model



Notes: Under the "Profitability Shock," bank revenues decline following a persistent shock to the non-interest revenues. The transfer shock is sized so that the change in wealth of bankers is comparable to the change of wealth induced by the transfer shock.

5. Michael Kiley and Jae Sim: Intermediary Leverage, Macroeconomic Dynamics, and Macroprudential Policy

5.1 Model Description

Kiley and Sim (2013, KS below) study the nexus between macroprudential policy and monetary policy. To that end, Kiley and Sim develop a macroeconomic model in which the financial intermediaries mix debt and equity capital to finance their investments subject to financial frictions that make intermediary choice of capital structure deviate from the Miller-Modigliani theorem within an otherwise standard dynamic general equilibrium model of the type used in monetary policy analysis such as that found in Smets and Wouters (2007). Thus, the capital structure of intermediaries in KS is optimized to balance the benefits of leverage and the costs of bankruptcy under the costly recapitalization option rather than being imposed by a regulatory fiat, a feature that helps explain the role of an unregulated financial sector in the propagation of macroeconomic shocks. The description below sketches the main details of the model and its calibration.[12]

The model economy consists of (i) a representative household, (ii) a representative firm producing intermediate goods, (iii) a continuum of monopolistically competitive retailers, (iv) a representative firm producing investment goods, and (v) a continuum of financial intermediaries. A key assumption that makes the model's asset pricing implication in sharp contrast with that of frictionless neoclassical models is that the representative household lacks the knowledge needed to manage financial investments, and thus turns to the financial intermediaries that have special knowledge in selecting and managing financial projects but face financial friction in funding their operations. This delegation of investment function from a financially unconstrained agent to a constrained agent with limits of arbitrage makes the model's propagation mechanism of financial disturbances drastically different from that of

[12]A detailed description can be found in the online appendix.

frictionless business cycle models through the dynamics of pecuniary externality.[13]

The important role of liquidity condition of financial intermediaries in asset price dynamics can be seen in the following asset pricing equation of KS:

$$1 = \mathbb{E}_t \left\{ M^B_{t,t+1} \cdot \frac{1}{m_t} \left[\frac{\mathcal{R}^A_{t+1}}{\Pi_{t+1}} - (1 - m_t) \frac{R^B_{t+1}}{\Pi_{t+1}} \right] \right\}, \tag{8}$$

where $M^B_{t,t+1}$ is the intermediary pricing kernel, m_t is the capital ratio optimally chosen by the intermediaries, and $\mathcal{R}^A_{t+1}/\Pi_{t+1}$ and R^B_{t+1}/Π_{t+1} are intermediaries' real return on assets and borrowing rates. Equation (8) summarizes all the important deviations of the model from standard asset pricing models: (i) (8) is a levered asset pricing formula, and the *net* asset returns is scaled up by a factor $1/m_t$; (ii) the intermediary pricing kernel is a filtered version of the household's stochastic discounting factor, where the filter is due to the liquidity condition of the intermediaries measured by the ratio of shadow value of internal funds today versus tomorrow, i.e., $M^B_{t,t+1} = \mathbb{E}_{t+1}[\lambda_{t+1}|\Omega_{t+1}]/\mathbb{E}_t[\lambda_t|\Omega_t] \cdot M_{t,t+1}$, where $\mathbb{E}_t[\lambda_t|\Omega_t]$ measures the ex ante shadow value of internal funds based on all the available macroeconomic information (Ω_t); (iii) the return on asset deviates from the frictionless counterpart because, first, raising outside capital is costly due to dilution effects[14] and thus lowers the effective return on equity, and second, the limited liability of financial intermediaries create a strictly positive value of default option, which then interacts with risk-taking of intermediaries.[15]

[13]A similar assumption also plays an important role in the majority of the recently developed macroeconomic models featuring intermediary funding constraints such as Brunnermeier and Sannikov (2014), Gertler and Karadi (2011), Gertler and Kiyotaki (2010), and He and Krishnamurthy (2012).

[14]Dilution costs arise when firms announce new offerings of seasoned equities and the announcement leads to a drop in the market value of existing shares. The dominant interpretation of the phenomenon in the literature is provided by Myers and Majluf (1984), who show that asymmetric information in capital markets may lead uninformed investors to discount the value of new shares to avoid *lemons*, which then causes the market value of existing shares to drop by arbitrage.

[15]In contrast to the majority of this literature, defaults of financial institutions are equilibrium outcomes. In this aspect, the model is akin to Brunnermeier and Sannikov (2014).

5.2 Calibration

The calibration of parameters regarding preferences and technology reflects conventional values. The constant relative risk aversion, habit formation, and the elasticity of labor supply are set equal to 3, 0.8., and 3, respectively, to be consistent with the micro-level evidence. The capital share of production function is set equal to 0.4. The quadratic adjustment cost of investment is chosen as 2. KS does not posit a utilization cost of capital and takes a constant depreciation rate of 0.025. The quadratic cost of price adjustment is set equal to 120. This choice is equivalent to a quarter fraction of firms resetting prices at any point in time given the steady-state markup of 1.11. Inflation indexation and wage rigidity are not considered for the transparency of the results. The monetary policy reaction function parameters are chosen as 1.5, 0.125, and 0.8 for inflation gap, production-based output gap, and monetary policy inertia, respectively (see the online appendix for details).

There are parameters associated with the long-run capital structure, dilution cost of equity issuance, corporate income tax shield, bankruptcy cost of failed institution, and idiosyncratic volatility. The dilution cost is set to 0.15 in the steady state, which is in the middle of the range reported in corporate finance literature. The tax differential between corporate and personal income tax rates is set to 0.20. Given all other parameters, the idiosyncratic volatility is chosen to match the 0.40 capital ratio, which facilitates the comparison with other papers in this literature. The bankruptcy cost is then specified as 3 percent of the size of the balance sheet to match the steady-state, short-term funding spreads.[16]

5.3 Impact of Balance Sheet Shock: Baseline Results and Robustness

To illustrate the importance of the intermediary liquidity position on macroeconomic outcomes, we consider a financial shock that transfers a certain amount of resources from financial intermediaries to

[16] All parameter values are broadly consistent with the original choices made in KS.

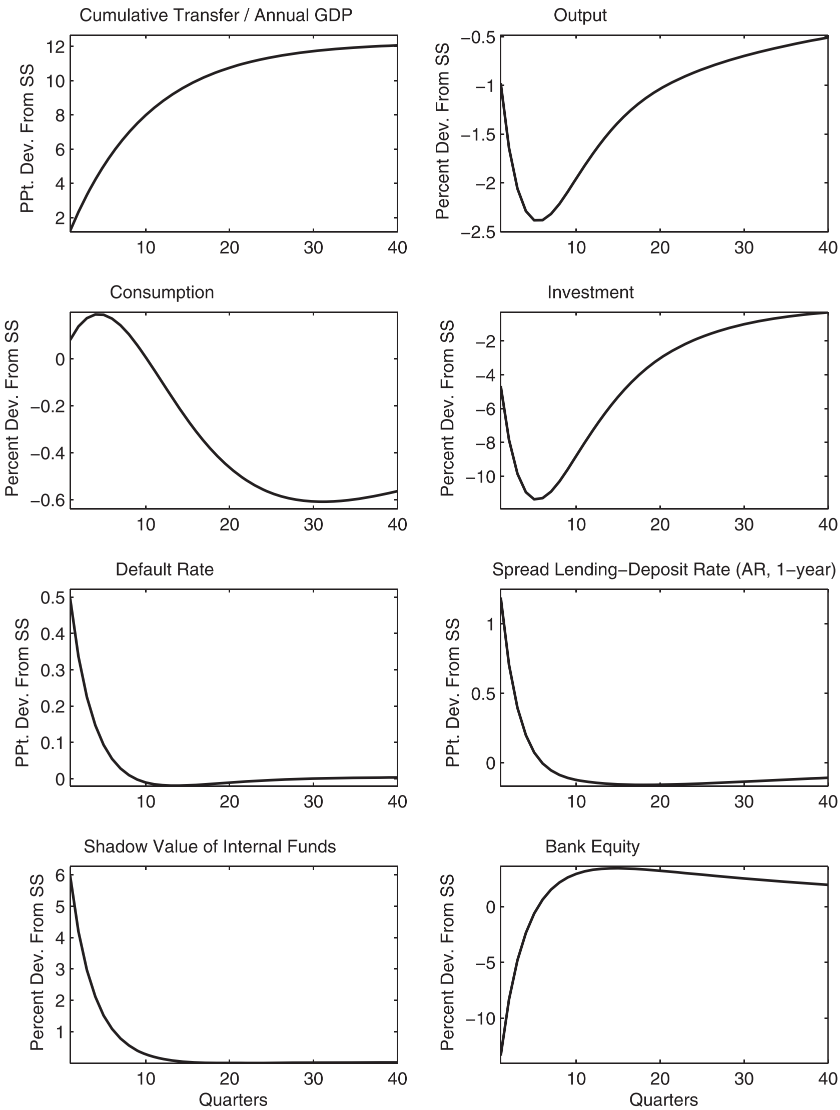
the representative household in a lump-sum fashion. This stylized shock helps highlight the role of financial market friction in the model since it does not directly affect the marginal productivity of physical capital in the economy and thus would have no impact on the allocation of real resources in a frictionless economy because, first, the investment decisions of the financial intermediaries are not affected by their liquidity condition, and second, the loss in the wealth of households due to the decline in the value of equities of financial institutions is exactly offset by the positive wealth transfer to households. The size and persistence of the shock follow the calibration choices discussed in section 2.

Figure 7 shows the impact of the shock on the real economy and financial markets. By construction, the shock does not have any impact if the financial friction in the model is taken out. However, as shown in the figure, the shock leads to a massive contraction in the real economy: maximum contraction on output, consumption, and investment amount to 2.5 percent, 0.6 percent, and 11 percent, respectively.

The reason for this strong reaction of the real economy can be found in the response of financial markets also shown in figure 7. On the impact, the default rate of intermediaries shoots up 0.5 percentage point. This is due to both the direct hit to the internal funding condition by the transfer shock and the indirect result of the endogenous decline in the asset prices. While the financial intermediaries try to raise outside capital as shown by the stiff increase in equity issuance, as much as 20 percent relative to its normal level, doing so in the KS model is costly due to a dilution cost. Finally, the increased cost of capital is passed through to the lending spreads, resulting in a large reduction in overall credit and a sizable contraction in economic activity.

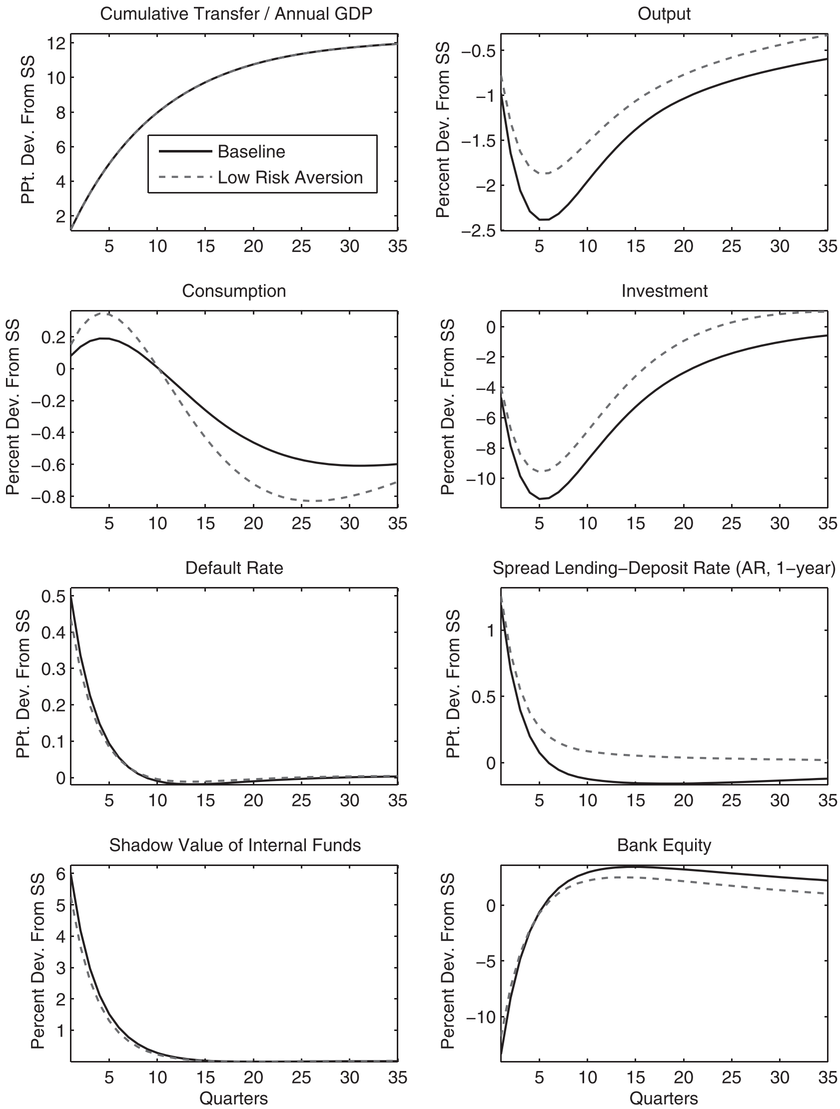
The results shown in figure 7 are sensitive to calibration choices. Among others, the relative risk aversion turns out to be very important in assessing the overall impact of the balance sheet transfer shock, as shown in figure 8. On impact, household consumption increases moderately, as a decline in household wealth, stemming from the reduced value of intermediary shares, is not perfectly offset by the transfer shock under the financial friction. This initial increase in consumption plays an important role in determining the overall size of the impact, as consumption accounts for about 80 percent of

Figure 7. Transfer Shock in the Kiley/Sim Model



total spending in the model. Having a lower degree of relative risk aversion makes the initial hump of household consumption bigger, reducing the size of overall impact on the economy. For instance, setting the parameter equal to 1 (log utility) reduces the maximum

Figure 8. Transfer Shock in the Kiley/Sim Model: Sensitivity Analysis



Notes: Under the baseline calibration, the coefficient of relative risk aversion is set to 3. Under "Low Risk Aversion," the coefficient of relative risk aversion is set to 1.

impact on the output to 2 percent, about 50 bps lower than what is shown in the figure.[17]

5.4 Alternative Financial Shock: Dilution Cost Shock

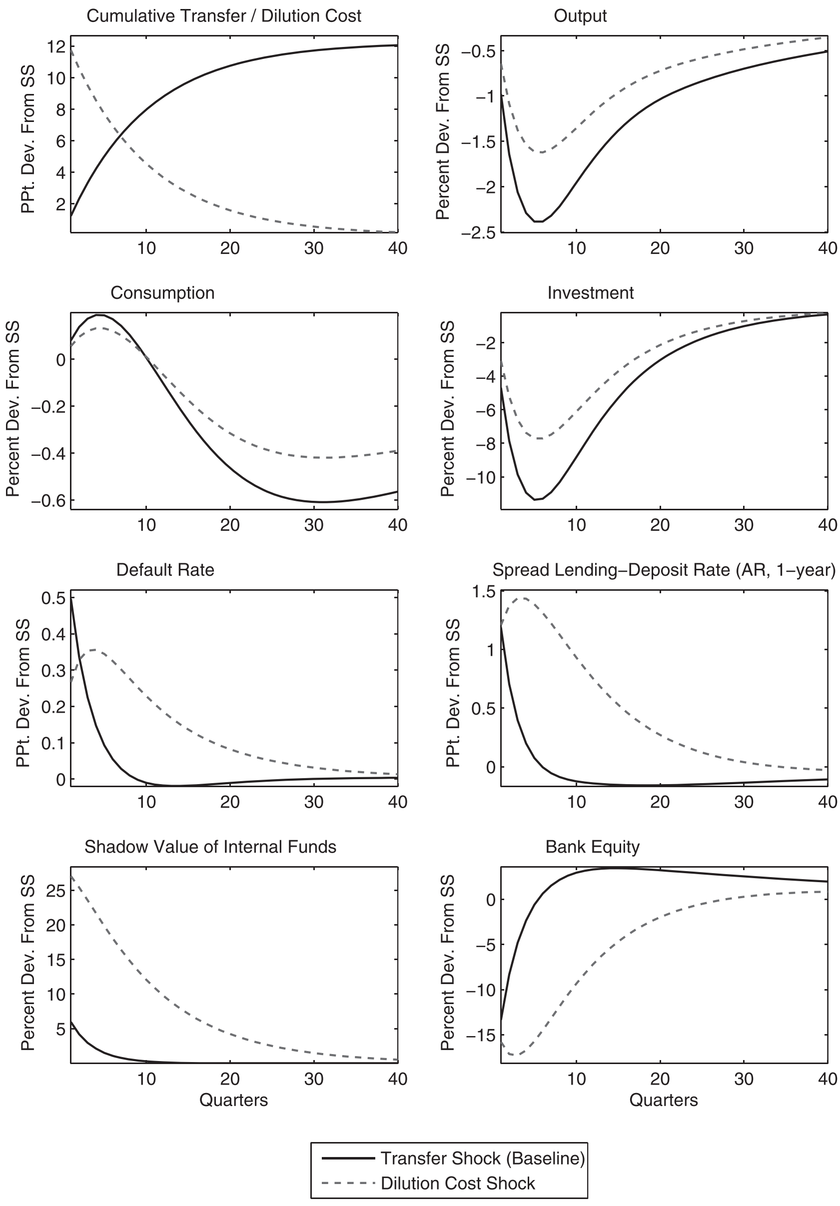
KS uses the balance sheet shock only as an illustration device. A financial shock that plays a more important role is a shock to the cost of raising outside equity—what we call a dilution cost shock. This shock has more desirable features in generating an economic crisis induced by stressed financial system. Financial stresses are usually associated with greater uncertainty, which can aggravate the asymmetric information in financial markets and lead to a greater lemon premium that elevates the cost of equity capital for financial intermediaries.

Figure 9 reports the impact of a dilution shock on the real economy and financial markets when calibrated to match the initial capital shortfall induced by the transfer shock. For ease of comparison, the persistence of the shock is set the same as in the transfer shock. As shown in the figure, the shock elevates dilution costs by a little less than 5 percentage points. The contour of the dynamic responses of real variables is broadly similar to those in the case of transfer shock.

While the peak impacts on output, consumption, and investment are about half the size of the peak impacts of the transfer shock, the shock and the propagation mechanisms appear empirically relevant. In contrast to the case of the transfer shock, equity issuance shows a hump-shaped response. Facing a greater cost of raising outside equity, the intermediaries can only gradually recapitalize in response to the shortfall in capital, which is, unlike in the case of transfer shock, entirely due to the endogenous fall in asset prices resulting from preemptive downsizing of intermediary balance sheets. As a

[17] The degree of nominal rigidity, and hence the flatness of the Phillips curve, is also important. For instance, halving the price adjustment cost to let the impact of the shock be absorbed by greater adjustment in prices reduces the maximum response of output by 30 bps. However, even with completely frictionless price setting, the maximum impact is reduced only by 60 bps. Finally, the size of investment adjustment friction also matters. While a greater adjustment friction in this sector increases the asset price volatility in general, it leads to a smoother response in aggregate investment and output. As a result, for instance, doubling the size of this friction can reduce the maximum impact on the output by 60 bps.

Figure 9. Effects of a Dilution Cost Shock vs. a Transfer Shock in the Kiley/Sim Model



Notes: The dilution shock raises the dilution costs by a little less than 5 percentage points. The shock is sized to match the initial capital shortfall induced by the baseline transfer shock.

consequence, the capital shortfall persists, and the resulting defaults and elevated funding costs persist as well, prolonging the downturn in a way consistent with recent experience.

6. Albert Queralto: Banks and Outside Equity

6.1 Model Description

The model of this section builds on recent papers that introduce financial intermediation in a business cycle framework—for example, Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). These papers extend the basic financial accelerator mechanism developed by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) to financial intermediaries (banks) in order to capture a disruption of intermediation. In this class of models, banks borrow short-term non-contingent debt from depositors and use these funds (together with their own internal funds) to make loans to non-financial firms. As in the earlier literature on the financial accelerator, financial market frictions are endogenized by introducing an agency problem that potentially constrains the ability of banks to obtain funds from depositors. When the constraint binds, the balance sheets limit the ability of banks to obtain deposits. In this instance, the constraint effectively introduces a wedge between loan and deposit rates, which rises as the balance sheets of banks deteriorate. This raises the cost of credit that non-financial borrowers face. In this way, when banks are highly leveraged, adverse returns to their balance sheet may lead to sharp increases in credit spreads and declines in investment and economic activity.[18]

Key to motivating a crisis within these frameworks is the heavy reliance of banks on short-term debt. This feature makes these institutions highly exposed to the risk of adverse returns to their balance sheet in way that is consistent with recent experience. Within these frameworks and most others in this class, however, by assumption the only way banks can obtain external funds is by issuing short-term debt. Thus, in their present form, these models are not equipped to

[18]The full model description can be found in the online appendix accompanying this paper.

address how the financial system found itself so vulnerable in the first place.

In the model analyzed here, banks are allowed to issue outside equity as well as short-term debt.[19] This feature makes bank risk exposure an endogenous choice, as outside equity allows banks to share risk with equity holders. The goal is to have a model that can not only capture a crisis when financial institutions are highly vulnerable to risk, but also account for why these institutions adopt such a risky balance sheet structure in the first place. Accordingly, the model extends the agency problem between banks and savers to allow intermediaries a meaningful tradeoff between short-term debt and equity. Ultimately, a bank's decision over its balance sheet will depend on its perceptions of risk. Thus, the model allows a quantitative analysis of the interplay between risk perceptions by banks, the liability structure that they choose, and the vulnerability of the economy to a crisis.

The production side of the model is analogous to a standard frictionless real business cycle (RBC) economy. The production function, capital accumulation, and resource constraint are as follows:

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha}, \tag{9}$$

$$K_{t+1} = \psi_{t+1} \left[(1-\delta) K_t + I_t \right], \tag{10}$$

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}} \right) \right] I_t, \tag{11}$$

where Y_t, K_t, L_t, I_t, and C_t denote output, physical capital, labor, investment, and consumption, respectively. In (10), ψ_{t+1} is a capital quality shock, which serves as a trigger of movements in the quality of banks' assets. It can be thought of as capturing a form of economic obsolescence.[20]

[19]The structure of the model closely follows Gertler, Kiyotaki, and Queralto (2012). See their paper for a complete description.

[20]See the appendix of Gertler, Kiyotaki, and Queralto (2012) for a microfoundation of the capital quality shock along these lines. This appendix is available as supplementary material at Elsevier's website: http://www.journals.elsevier.com/journal-of-monetary-economics/.

The preference structure follows Miao and Wang (2010), in turn based on Greenwood, Hercowitz, and Huffman (1988) (GHH hereafter). The preference specification allows for (internal) habit formation and, as in GHH, abstracts from wealth effects on labor supply. The household's problem is as follows:

$$\max \qquad \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\gamma} \left(C_\tau - hC_{\tau-1} - \frac{\chi}{1+\varphi} L_\tau^{1+\varphi} \right)^{1-\gamma},$$

subject to

$$\begin{aligned} C_t + D_{h,t} + q_t e_t = W_t L_t + \Pi_t - T_t + R_t D_{h,t-1} \\ + \left[Z_t + (1-\delta) q_t \right] \psi_t e_{t-1}. \end{aligned} \qquad (12)$$

Note that the household has access to non-contingent riskless short-term debt (deposits), denoted $D_{h,t}$ and paying gross interest R_t, as well as bank (outside) equity, e_t.[21] The price of a unit of outside equity is q_t, and Z_t denotes the flow returns at t generated by one unit of the bank's assets. The units of outside equity are normalized so that each unit is a claim to the future returns of one unit of the asset held by the bank.

Each bank raises funds by issuing deposits d_t and outside equity to purchase producers' equity, s_t, at price Q_t:

$$Q_t s_t = n_t + q_t e_t + d_t. \qquad (13)$$

The evolution of a bank's net worth (or inside equity), n_t, is as follows:

$$n_t = \left[Z_t + (1-\delta) Q_t \right] \psi_t s_{t-1} - \left[Z_t + (1-\delta) q_t \right] \psi_t e_{t-1} - R_t d_{t-1} - \xi_t. \qquad (14)$$

Above, ξ_t is a capital transfer which subtracts from the bank's resources at the beginning of the period. It is assumed to be taken from the bankers and given to the households, and therefore only has effects insofar as net worth constrains banks' ability to obtain

[21] Outside equity refers to equity issued by banks and held by households, while inside equity (or net worth) refers to the accumulated retained earnings of a banker who manages an intermediary.

funds. Accordingly, in the RBC version of the model, the effects of the transfer are nil, as it is just a redistribution of wealth within the representative household.

From equation (14), note that the use of outside equity reduces the impact of return fluctuations on net worth. When e is higher, movements in returns to the bank's assets are passed on to outside equity holders (households) to a greater extent, thus acting as a hedge. By contrast, deposit financing is risky for the bank, since its cost is non-contingent. In our quantitative analysis, we interpret outside equity as capturing securities that allow banks to share risk with the security holders broadly. In particular, we assume that outside equity in the model corresponds to common equity, while inside equity, n, corresponds to the sum of preferred equity and subordinate debt. We calibrate the parameters of the model so that the ratio of outside to inside equity in the steady state equals two-thirds, which roughly matches the U.S. banking sector prior to the crisis.

The value of the bank at the end of period t is

$$V_t = V(s_t, x_t, n_t) = \mathbb{E}_t \sum_{\tau=t+1}^{\infty} (1-\sigma)\sigma^{\tau-t}\Lambda_{t,\tau}n_\tau, \tag{15}$$

where $x_t = \frac{q_t e_t}{Q_t s_t}$, and σ is the banker's survival probability. After obtaining funds, the banker may default on debt and divert a fraction $\Theta(x_t)$ of assets. The incentive constraint for the bank not to steal is

$$V(s_t, x_t, n_t) \geq \Theta(x_t) Q_t s_t. \tag{16}$$

The divertable fraction is a convex function of x_t:

$$\Theta(x_t) = \theta\left(1 + \epsilon x_t + \frac{\kappa}{2}x_t^2\right). \tag{17}$$

We assume that the amount divertable is increasing in the degree of outside equity x_t, and therefore the constraint of the bank is tighter the larger is x_t.[22] This represents a cost of outside equity which the bank trades off against its hedging benefit.

[22]The idea is that short-term deposits give banks less discretion over payouts than equity, and therefore offer more discipline over bank managers than does outside equity. This idea is due to Calomiris and Kahn (1991).

Table 3. Calibrated Parameters in Queralto's Model

γ	2	Risk Aversion
β	0.99	Discount Factor
α	0.33	Capital Share
δ	0.025	Depreciation Rate
χ	0.25	Utility Weight of Labor
φ	1/3	Inverse Frisch Elasticity of Labor Supply
If''/f'	1	Inverse Elasticity of Investment to the Price of Capital
h	0.75	Habit Parameter
σ	0.9685	Survival Rate of Bankers
ξ	0.0289	Transfer to Entering Bankers
Asset Diversion Parameters		
θ	0.264	
ε	−1.21	
κ	13.41	

6.2 Calibration and Model Solution

Table 3 reports the baseline parameter values. The preference and technology parameters are set to reasonably conventional values. The banking-sector parameters are chosen to match salient features of the U.S. financial intermediation sector.

In the model, bank balance sheet structure depends on risk perceptions. It is thus important to take risk into account in the computation of the model. Similar to Coeurdacier, Rey, and Winant (2011), a "risk-adjusted" steady state is constructed, where given agents' perceptions of second moments, variables remain unchanged if the realization of the (mean-zero) exogenous disturbance is zero. The risk-adjusted steady state differs from the non-stochastic state only by terms that are second order. These second-order terms, which depend on variances and covariances of the endogenous variables, pin down banks' balance sheet. Model dynamics are then analyzed by computing a first-order log-linear approximation around the risk-adjusted steady state.

To calculate the relevant second moments, we use an iterative procedure. We first log-linearize the model around the non-stochastic steady state. We then use the second moments calculated from this exercise to compute the risk-adjusted steady state. We repeat the exercise, this time calculating the moments from the risk-adjusted steady state. We keep iterating until the moments generated by the first-order approximation around the risk-adjusted steady state are consistent with the moments used to construct it.[23]

6.3 Transfer Shock

Figure 10 plots a dynamic simulation of the model economy following a transfer shock. Here the amount of exogenous volatility is calibrated as the average between a low-risk economy (meant to reproduce the Great Moderation period) and a high-risk economy (which captures the period of volatility in the two decades prior to the Great Moderation). The idea is that the aftermath of the Global Financial Crisis is characterized by heightened uncertainty relative to the Great Moderation, but risk is still not as high as in the high-volatility period of the 1960s and 1970s. The size and persistence of the transfer shock follow the calibration choices discussed in section 2.

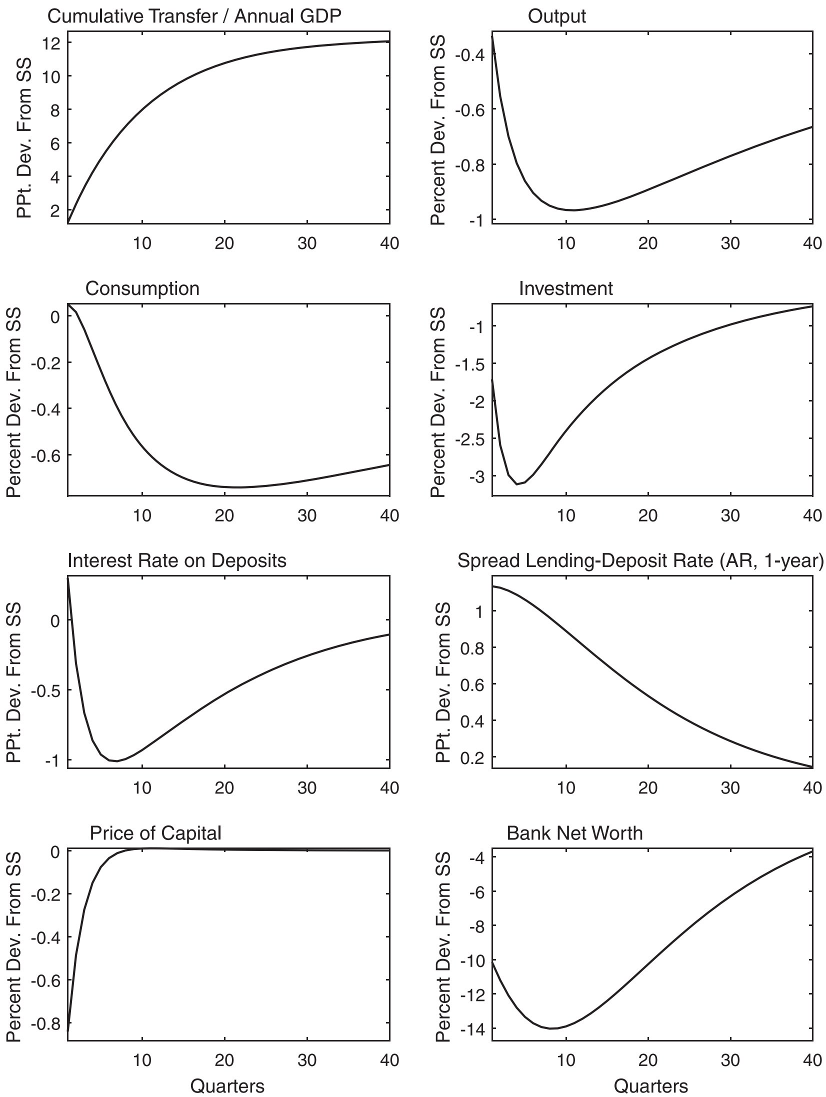
The loss in capital in the intermediation sector worsens the agency problem between banks and their creditors, leading the credit spread to rise by more than 100 basis points. With their balance sheets impaired, banks' ability to lend is diminished, and aggregate investment and asset prices drop as a result. Along the way, the financial accelerator mechanism operates, as drops in bank net worth and in the asset price Q_t reinforce each other. All told, output falls by about 1 percentage point, and investment drops by more than 3 percentage points.

6.4 Robustness Analysis

Several authors have suggested that the low volatility during the Great Moderation period may have induced a sense of complacency about risk in financial markets, which ultimately contributed

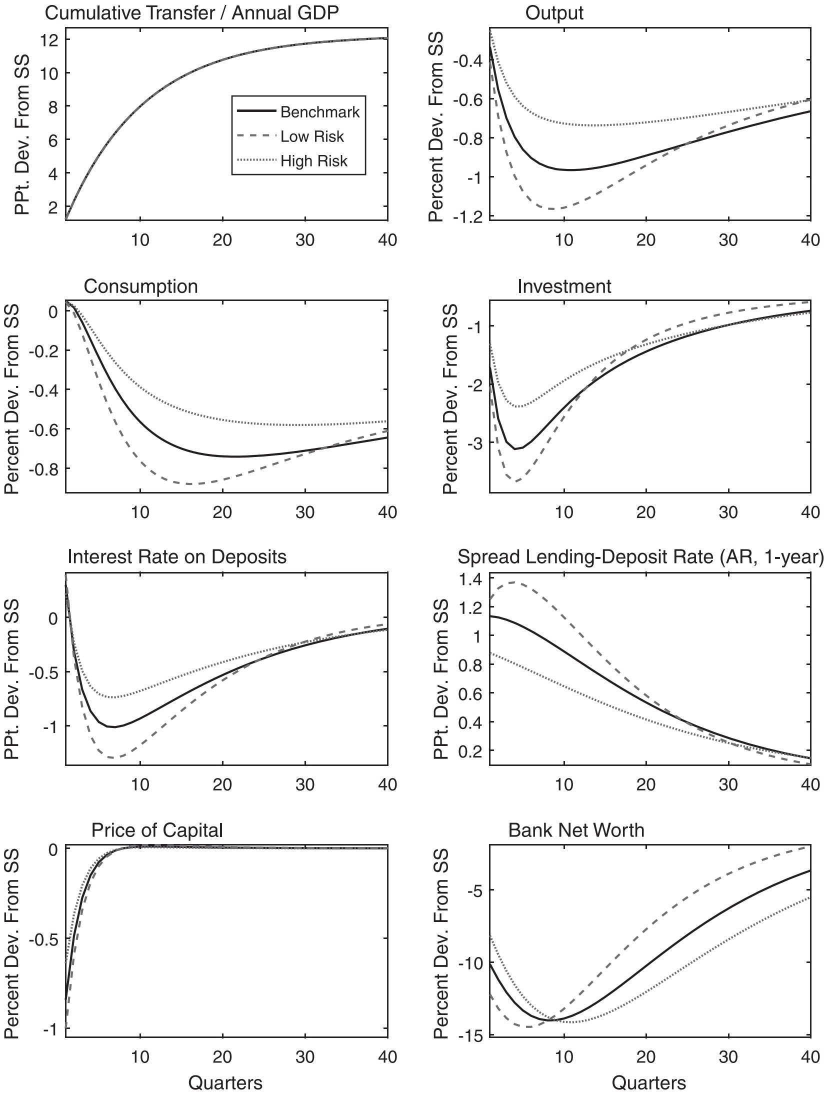
[23]See the appendix of Gertler, Kiyotaki, and Queralto (2012) for details.

Figure 10. Transfer Shock in Queralto's Model



to the vulnerability of the system once the crisis hit. To illustrate this possibility, figure 11 performs robustness analysis by modifying the level of exogenous risk. The low-risk economy features standard deviations of the shock processes so that the standard deviation

Figure 11. Transfer Shock in Queralto's Model: Sensitivity Analysis



Notes: "Low Risk" refers to an economy in which the standard deviation of the exogenous capital quality shock is 0.69 percent. In the "High Risk" economy, the standard deviation of the capital quality shock equals 2.07 percent. For ease of comparison, "Benchmark" reports again the results for the benchmark calibration of the Queralto model that sets the standard deviation of the capital quality shock at an intermediate value.

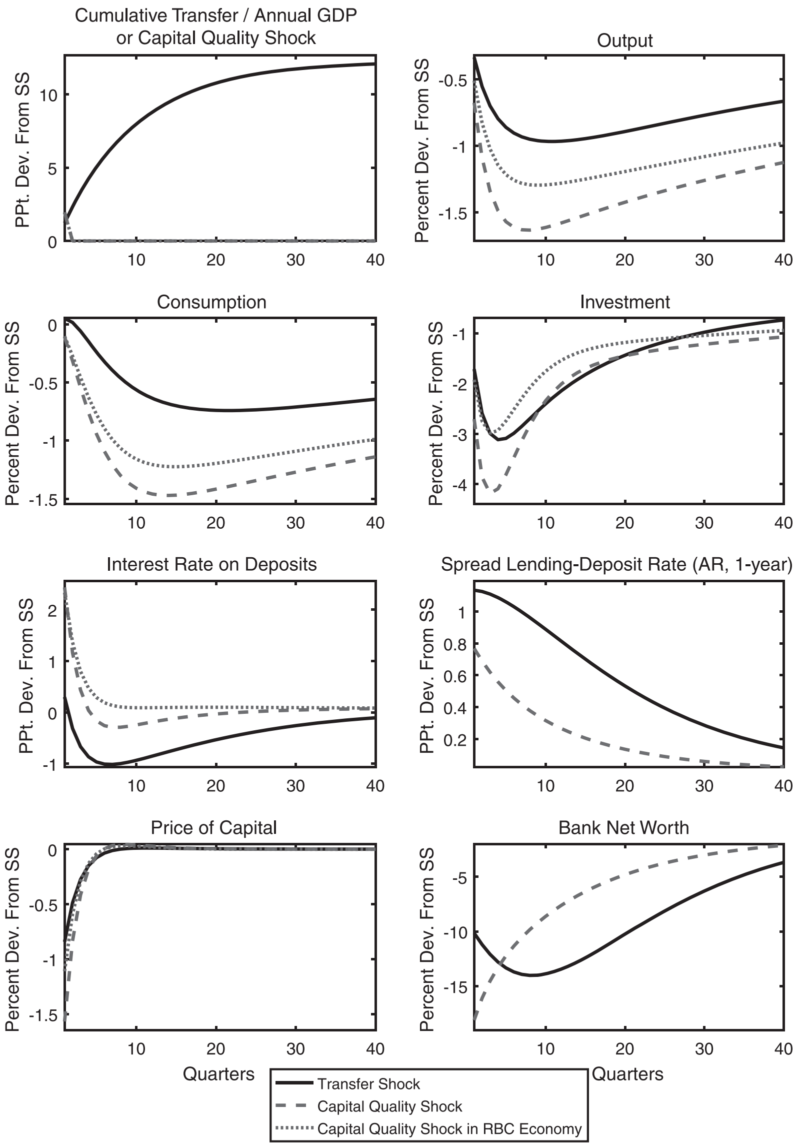
of annual output growth corresponds roughly to that in the Great Moderation period, while the high-risk economy features a level of risk corresponding to the period of volatility in the two decades prior.

When risk is high, the effects of the shock are weaker, and with low risk the effects are stronger. The reason is straightforward: the anticipation of high risk induces banks to substitute outside equity for short-term debt, as higher risk increases the hedging value of outside equity. When the shock hits, outside equity acts as a buffer in two ways. First, it moderates the drop in inside equity induced by the decline in asset values. Second, as the effects of the shock unfold after the initiating disturbance, banks are able to relax their borrowing constraint a bit by substituting short-term debt for outside equity (recall that short-term debt permits creditors greater discipline over bankers).

The differences in exogenous risk lead to quantitatively significant differences in the effects of the transfer shock. When risk is high, the peak decline in investment moderates from over 3 percent in the baseline case to a little above $2^1/_4$ percent, and the peak output loss is less than $^3/_4$ percent, compared with about 1 percent in the baseline case. Conversely, when exogenous risk is low—leading banks to adopt a more risky balance sheet structure—investment drops $3^3/_4$ percent at its trough. The drop in output reaches nearly $1^1/_4$ percent.

Figure 12 compares the effects of the transfer shock (in the low-risk economy) with those of a decline in capital quality, where the magnitude of the latter is calibrated to induce the same average decline in bank net worth as the transfer shock over the five years following the shock. The effects of the capital quality shock are considerably larger, leading output to drop more than 2 percent at the trough. The reason is that the decline in capital quality effectively leads to a reduction in the amount of physical capital in the economy, and therefore has adverse effects even in an economy with no financial frictions, as indicated by the dotted line. On the other hand, the degree of financial-sector spillovers (i.e., the contraction over and above what would happen in a frictionless economy) is comparable across the two shocks (recall that the transfer shock is only a redistribution of resources within the representative household, and therefore has no effects in a frictionless economy).

Figure 12. Transfer Shock vs. Capital Quality Shock in Queralto's Model



Notes: "Capital Quality Shock" denotes the responses to a capital quality shock for the benchmark calibration of the Queralto model. The size of the shock was chosen to match the evolution of bank net worth, on average, over the first nine quarters with the effects of the transfer shock—reported again for ease of reference. The figure also shows a capital quality shock in a frictionless RBC economy.

7. Luca Guerrieri and Mohammad Jahan-Parvar: Capital Shortfalls in a Two-Sector Production Economy

7.1 Model Description

Guerrieri and Jahan-Parvar consider the effects of sectoral and aggregate financial shocks in a two-sector model. Firms in one sector have access to equity markets, while firms in the other sector can only finance capital purchases through credit extended by financial intermediaries (hereafter, banks, for short). The interactions of these two types of firms can buffet the macro effects of shocks that reduce the equity position of banks. The demand for capital by equity-financed firms acts to curb equilibrium movements in the price of capital which otherwise amplifies the macro response to variation in credit from the banking sector. However, aggregate valuation shocks that affect both equity markets and banks continue to have sizable macro repercussions. Apart from sensitivity analysis relative to the size of the credit-dependent sector, the results highlight the implication of the zero lower bound on policy interest rates for the transmission of the baseline transfer shock.

The model is an extension of Gertler and Karadi (2011), hereafter abbreviated as GK. The extension is that not all firms are dependent on bank credit. Firms in the equity-financed sector are able to write a financing contract directly with households. A special case of the model with all firms financed by household equity reproduces the one-sector model considered by Boldrin, Christiano, and Fisher (2001). In the model, final goods are a Cobb-Douglas composite of goods produced by firms that are credit dependent and by firms that are equity financed. A retail sector purchases the intermediate goods and repackages them for consumers in a way that supports the inclusion of nominal rigidities. Monetary policy follows an interest rate reaction function that responds to current inflation and allows for interest rate smoothing. Production subsidies, in the absence of financial frictions, reproduce the efficient allocations in steady state. The description that follows highlights the credit-related friction but leaves the full description of the model for the online appendix.

The key financial friction for bank-dependent firms follows Gertler and Karadi (2011). Banks lend funds obtained from households to non-financial firms. Let $N_t(j)$ be the amount of wealth—or net worth—that a banker j has at the end of period t.

$$Q_t S_t^b(j) = N_t(j) + D_t(j) \tag{18}$$

Deposits $D_t(j)$ pay a return $(1 + R_t)$ at time $t + 1$. Thus $D_t(j)$ may be thought of as the debt of bank j, and $N_t(j)$ as its capital. Credit extended to firms $S_t^b(j)$ earns the stochastic return $(1 + R_{t+1}^{bs})$ at time $t + 1$. Over time, the capital of banks evolves as the difference between earnings on assets and interest payments on deposits:

$$N_{t+1}(j) = (1 + R_{t+1}^{bs})Q_t S_t(j) - (1 + R_t)D_t(j). \tag{19}$$

Because banks may be financially constrained, they have an incentive to retain earnings, but bank capital does not expand indefinitely, because bankers cease operations with iid probability $1 - \theta$ each period. Upon exiting, a banker becomes a worker and all retained earnings are transferred back to his original household. Each period, a fraction $1 - \theta$ of all workers is selected to join the existing bankers and receives a startup transfer, so that the fraction of household members acting as workers and bankers is constant over time.

The objective of bank j is to maximize expected terminal wealth, given by

$$\max_{S_{t+i}^b(j)} V_t(j) = E_t \sum_{i=0}^{\infty} (1 - \theta)\, \theta^i \psi_{t,t+1+i} \big[\big(R_{t+1+i}^{bs} - R_{t+i}\big) Q_{t+i} S_{t+i}^b(j) + (1 + R_{t+i}) N_{t+i}(j) \big], \tag{20}$$

where $\psi_{t,t+1+i}$ is the stochastic discount factor of households.

An agency problem limits the ability of banks to attract deposits. At the beginning of each period, a banker can choose to transfer a fraction λ of assets (in period t those assets equal $Q_t S_t(j)$) back to his household. If the banker makes the transfer, depositors will force the bank into bankruptcy and recover the remaining fraction $1 - \lambda$ of assets. Thus, households are willing to make deposits only if the incentive-compatibility constraint is satisfied:

$$V_t(j) \geq \lambda Q_t S_t(j). \tag{21}$$

When solving the model with a standard first-order perturbation solution, we assume that this constraint binds always with equality.

The setup of GK is nested and is reproduced when the share of equity-financed firms in production is zero. The model departs from GK along a few dimensions. Notably, unlike in GK, capacity utilization is constant; monetary policy responds only to inflation and to a lag of the monetary policy rate, and does not attempt to stabilize output around its steady-state value; the Frisch elasticity of labor supply is set to one, at the upper range of micro estimates, but well below the elasticity in GK. In the two-sector model, the equity- and credit-dependent firms produce intermediate goods that are necessary to produce an undifferentiated final good using a Cobb-Douglas production function. The sectoral shares are set to 0.5. A retail sector produces differentiated goods that are subject to nominal rigidities.[24]

7.2 Baseline Shock and Comparisons with One-Sector Model

Figure 13 shows the effects of the baseline transfer shock from banks to households in our two-sector model. The shock is calibrated as discussed in section 2. The macro effects of the shock are modest. The drop in aggregate output grows in magnitude over two years to a peak of 0.45 percent of its steady-state value. The modest size of the spillover effects of the shock is related to the fact that the reduction in the demand for capital by credit-dependent firms is compensated by an increase in demand from the equity-financed firms.

As shown in figure 14, the macro spillover effects of the baseline transfer shock are greatly amplified in a one-sector model in which all firms are credit dependent. The main reason for this amplification is that lack of access to alternative funding leads to a large reaction in the equilibrium price of capital. In turn, in the one-sector model, the drop in the price of capital boosts the magnitude of the drop in bank capital and leads to a further curtailing of credit supply. By contrast, in a two-sector model, the price of capital barely responds to a transfer shock. Higher demand for capital from equity-financed firms acts to reduce downward pressures on the equilibrium price

[24]Notice that when either $\lambda = 0$ or when all firms are equity financed, a monetary policy rule that stabilizes inflation would reproduce the allocations chosen by the benevolent planner.

Figure 13. Transfer Shock in Guerrieri/Jahan-Parvar Model

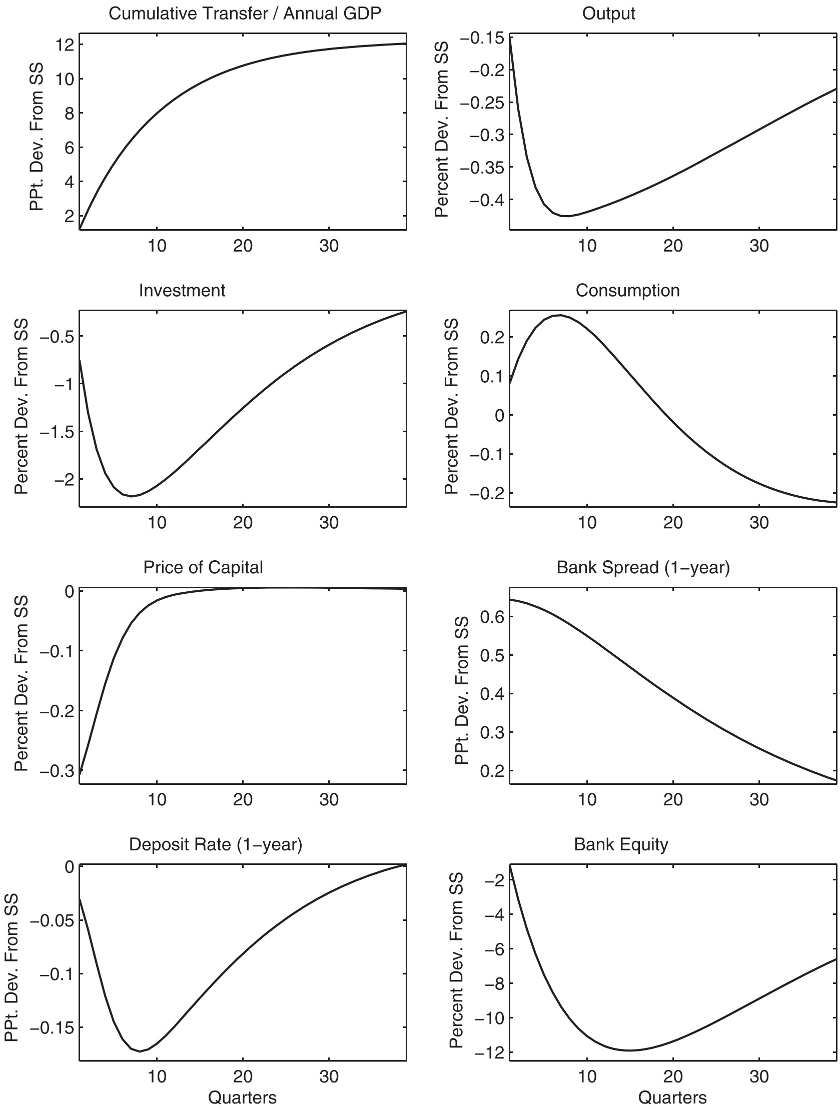
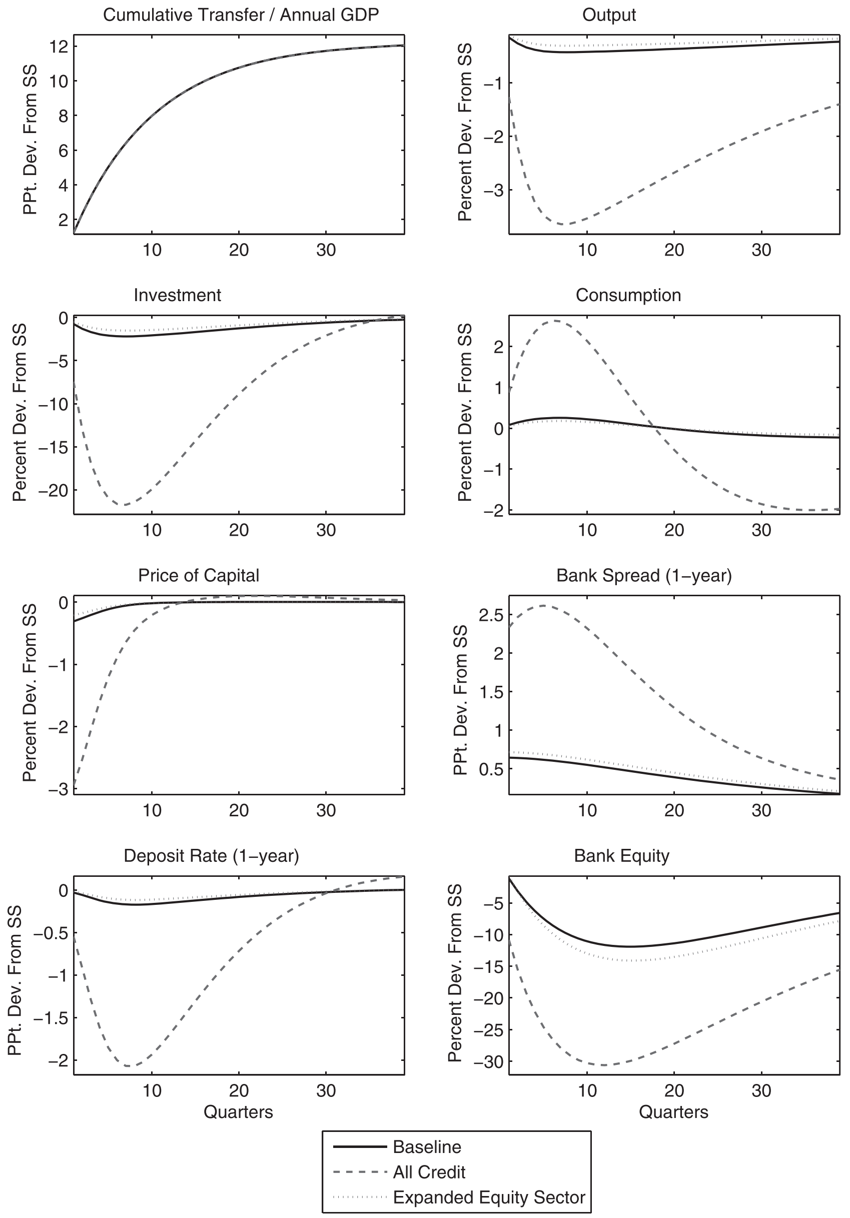


Figure 14. Transfer Shock in the Guerrieri/Jahan-Parvar Model: Sensitivity Analysis



Notes: The line denoted "All Credit" refers to a one-sector model in which all firms are credit dependent. Under "Expanded Equity Sector," firms with access to equity financing account for 75 percent of aggregate output.

of capital. This stability in the price of capital has one principal implication—it reduces the endogenous response of bank equity to the exogenous transfer shock.

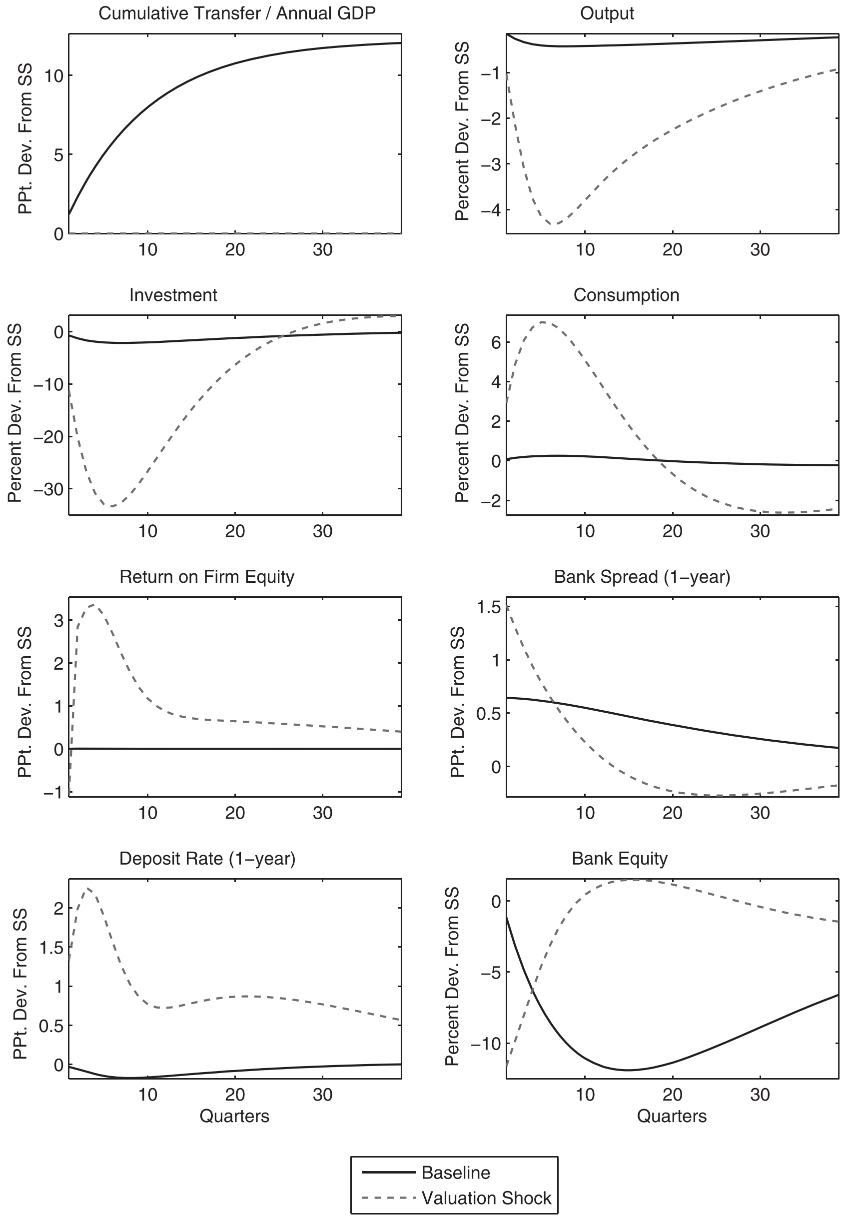
7.3 Sensitivity Analysis: The Response to an Economy-Wide Shock

The benchmark model introduced in section 7.2 focuses on a sector-specific shock whose aggregate effects are buffeted by the reaction of the other sector. This section considers the implications of an economy-wide valuation shock, following Albuquerque, Eichenbaum, and Rebelo (2012). Accordingly, the discount factor of households drops and the risk-free interest rate increases. The size of the shocks is set to match the average *endogenous* shortfall in bank capital from the baseline transfer shock (taking into account the equilibrium response of capital prices) over the first nine quarters.

Figure 15 reports the comparison of the impact of a valuation shock with that of a transfer shock in the baseline two-sector model. In this case, the macro effects of a financial shock are much closer to those that obtain in the special case of one-sector model in which all firms are credit dependent. Similar to GK and in contrast to the benchmark model, an economy-wide shock can cause a significant drop in output and investment. Since households own all the assets in the economy, a shock that lowers the discount factor implies less appetite for risk and a reduction in the funds available to both equity-financed firms and banks. The aggregate nature of the valuation shock dampens the role played by the equity-financed firms in counterbalancing the shocks to banks in the benchmark model. Accordingly, the equilibrium loan rate rises to compensate the shortage of available funds, resulting in a large drop in investment. Similar to GK, the macro spillovers of the financial shocks are amplified by a fall in the price of capital.

The benchmark model implies that the presence of additional financial assets issued by firms that are capable of direct intermediation with the households can mitigate the impact of a financial shock to banks. However, shocks that affect both equity- and credit-financed firms still lead to sizable macro spillover effects comparable to those that obtain in a one-sector model with all firms

Figure 15. Transfer Shock vs. Valuation Shock in Guerrieri/Jahan-Parvar Model



Notes: The line denoted "Baseline" shows the effects of the transfer shock under the baseline calibration. The line denoted "Valuation Shock" shows the effects of a shock that leads to a revaluation of bank equity that matches the drop in bank equity from the baseline transfer shock, on average, over the first nine quarters.

credit dependent. The analysis also highlights that shocks that have comparable impacts on the equity position of banks can have dramatically different macro spillover effects.

7.4 The Response to the Baseline Transfer Shock at the Zero Lower Bound

We revisit this amplification for the baseline transfer shock in our two-sector model against a deep recession that brings the economy to the zero lower bound. In the model, the deposit contract between banks and households is tantamount to an indexed bond with maturity equal to one quarter. In normal times, the real return on deposits hews closely to the nominal deposit rate and to the monetary policy rate. However, at the zero lower bound there can be a decoupling between the real return on deposits and nominal short-term interest rate.

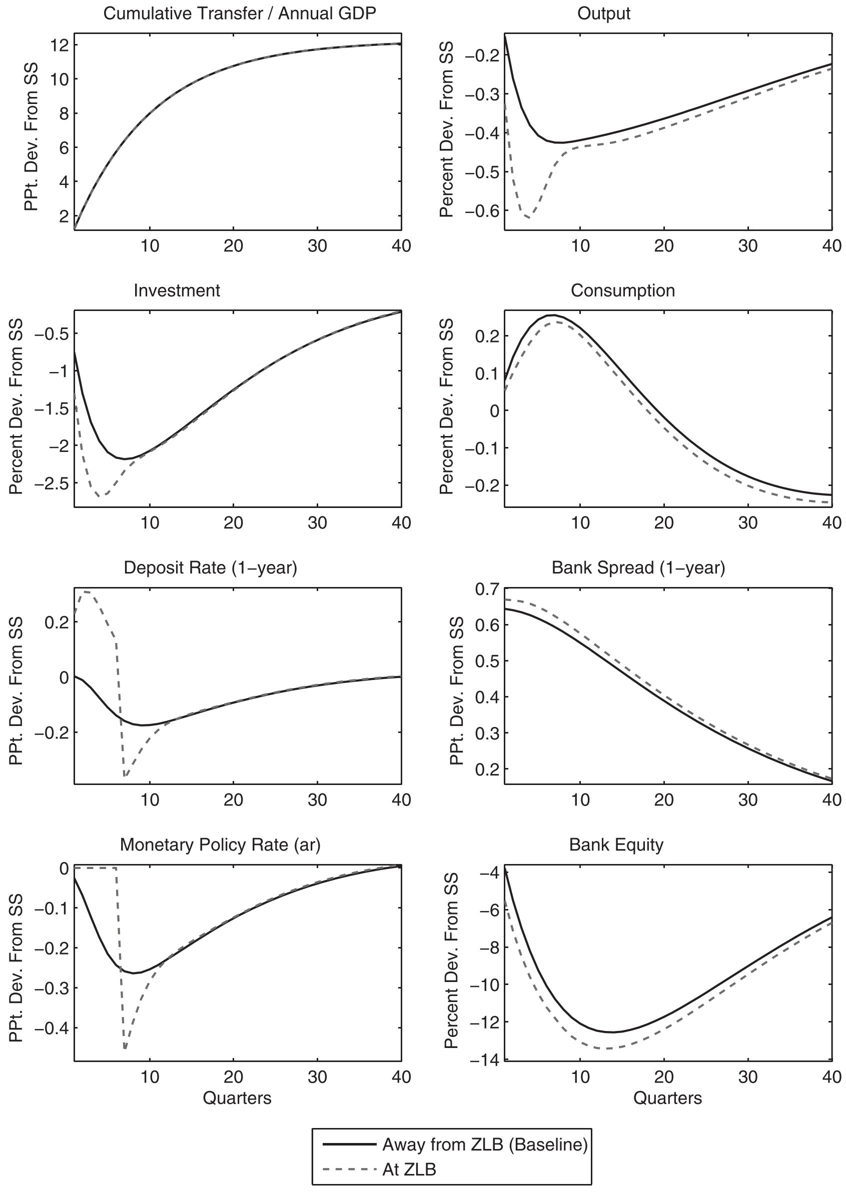
The stylized shock that leads the economy to the zero lower bound is a shock to preferences. The utility function of the representative household is modified as follows:

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i} - \gamma C_{t+i-1} + \epsilon_{ct}) - \frac{\chi}{1+\varepsilon} L_{t+i}^{1+\varepsilon} \right], \qquad (22)$$

where, again, C_t denotes consumption of final goods, and L_t denotes hours worked. The term ϵ_{ct} is a shock to consumption preferences. The shock itself is assumed to follow an autoregressive process of order 1, with a persistence coefficient equal to 0.7. The shock is sized so that households expect the policy interest rate to remain at the zero lower bound for six quarters in the absence of additional shocks. For the purpose of this section, the model is solved using a piecewise linear solution technique as developed by Guerrieri and Iacoviello (2015). As shown in Bodenstein, Erceg, and Guerrieri (2009), the particular mix of shocks that leads to recession and the attainment of the zero lower bound have a role in determining the marginal effects of additional shocks that is well summarized by the expected duration of the zero lower bound.

Figure 16 shows the effect of the transfer shock from banks to households under two configurations. In one case the transfer shock

Figure 16. Transfer Shock at the Zero Lower Bound in Guerrieri/Jahan-Parvar Model



Notes: The line denoted "Away from ZLB (Baseline)" shows the effects of the transfer shock under the baseline calibration, away from the zero lower bound on nominal interest rates. The line denoted "At ZLB" shows the effects of the transfer shock at the zero lower bound on nominal interest rates.

occurs against the background of a deep recession and the responses are shown in deviation from the outlook for the economy that agents expected prior to the realization of the transfer shock. In the other case, the responses are shown in deviation from their steady-state values (interpreted as "normal times" given the linear approximation used to solve the model).

Since banks cannot attract deposits at negative nominal rates, in the face of deflationary shocks, such as the transfer shock considered, the real return on deposits *rises* instead of falling. The unexpected rise in real deposit rate, equal in size but opposite in sign to the response of inflation in deviation from baseline, amplifies the drop in bank equity relative to normal times. In turn, the further drop in bank equity amplifies the rationing of credit and the contraction of investment and output relative to normal times, away from the zero lower bound.

It is well understood that the amplification of the responses of the economy to contractionary shocks in a liquidity trap is driven by the evolution of inflation expectations. In the model, the deflationary effects of the shock are kept to a relatively modest size—inflation drops $\frac{1}{4}$ percentage point at its nadir—principally by monetary policy. The policy rule is anticipated to respond aggressively to stabilize inflation away from the zero lower bound. The credible response of monetary policy away from the zero lower bound provides forward guidance. By contrast, with a less aggressive monetary policy rule, inflation is more volatile and the zero lower bound would amplify the effects of contractionary shocks in a more pronounced fashion. Similarly, the expected duration of the zero lower bound is a key determinant of the non-linear amplification effects at the zero lower bound. For an extended discussion of these issues see Bodenstein, Erceg, and Guerrieri (2009) and Bodenstein, Guerrieri, and Gust (2013).

8. Horizontal Comparison of Results

Table 4 summarizes the choices available to financial intermediaries that are salient in the reaction to a capital shortfall. The

Table 4. Model Characteristics

	Iacoviello	Covas/Driscoll	Kiley/Sim	Queralto	Guerrieri/ Jahan-Parvar
Choices Available to Banks:					
Issue New Equity	No	No	Yes	Yes	No
Reduce Dividend Payments	Yes	Yes	Yes	No	No
Increase Operating Efficiency	No	No	No	No	No
Raise Interest Spread	Yes	Yes	Yes	Yes	Yes
Increase Non-interest Income	No	No	No	No	No
Services Offered by Banks:					
Liquidity Provision	Yes	Yes	Yes	Yes	Yes
Liquidity Transformation	No	No	No	No	No
Other Features of the Model:					
Multiple Sources of Funding*	Yes	Yes	No	No	Yes
Nominal Rigidities	No	No	Yes	No	Yes
Solution Method	1st Order	Non-linear	1st–2nd Order	1st Order	Piecewise Linear

*"Multiple Sources of Funding" refers to the presence of sources of funding other than bank credit.

summary hews closely to the action set available to banks in reaction to changes in capital requirements, as summarized in an interim report of the BIS Macroeconomic Assessment Group (BCBS 2010). In addition to issuing new equity and to increasing retained earnings, the BIS report highlights that banks may in fact attempt to increase risk-weighted assets by shifting balance sheet composition towards less risky assets in ways not captured by any of the models presented here. Another feature not captured by any of the models presented is the possibility that banks could speed up the recapitalization process by increasing fees or, more generally, other sources of non-interest income. The table highlights that the models presented do in fact expand a core framework in complementary directions. Nonetheless, one source of homogeneity across models is that the financial sector is engaged in liquidity provision, and not in liquidity transformation, which could contribute to understating the macroeconomic repercussions of financial shocks.

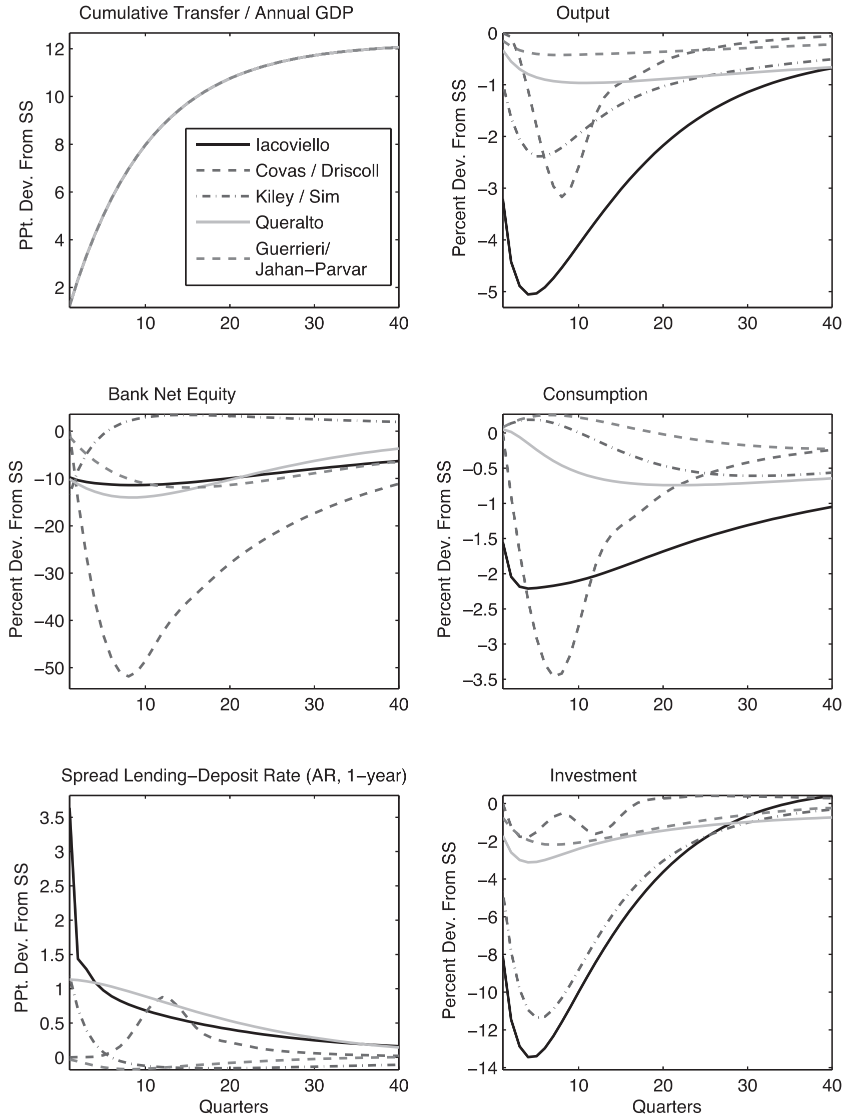
Figure 17 provides a horizontal comparison of the effects of the baseline transfer shock across models. The responses shown are in deviation from each model's steady state.[25] For completeness, table 5 reports key steady-state values for each of the models.

As shown in the top-left panel of the figure, the size of the transfer shock is standardized.[26] Despite the standardization of the cumulative transfer, the hit to bank equity across models differs greatly.

[25]This approach hews closely to actual practice in several model comparison exercises in the literature: see, e.g., the work in Wieland et al. (2012) and references therein, which mostly compares model dynamics in deviation from each model's respective steady state. Steady-state comparisons could be useful to study, but, in keeping with common practice, we do not attempt to do so here. Our approach follows the common practice at many central banks that typically separates the construction of the baseline outlook from the construction of scenarios around the baseline outlook itself. Often, a common baseline is constructed using a combination of large-scale models, nowcasting, and judgmental projections. However, the scenarios are routinely constructed using a diverse set of models, where each model response is constructed in deviations from each model's baseline, and only later added to the common baseline.

[26]Notice that to facilitate the comparison of the results of the model of Covas and Driscoll, calibrated at a yearly frequency, we have interpolated the model's responses to quarterly frequency using splines.

Figure 17. Horizontal Model Comparison



Notes: The figure shows responses to the baseline transfer shock (whose calibration is described in section 2) across all the models of the sections from 3 to 7 using the baseline calibration of each model. For ease of comparison, the responses for the yearly model of Covas and Driscoll were interpolated to quarterly frequency using splines.

Table 5. Steady-State Values for Selected Model Variables

Variable	Iacoviello	Covas/Driscoll	Kiley/Sim	Queralto	Guerrieri/ Jahan-Parvar
Ratio of Consumption to Output	0.75	0.7	0.81	0.78	0.58
Ratio of Capital to Annual Output	1.81	2.3	1.93	2.18	2.23
Ratio of Investment to Output	0.25	0.3	0.19	0.22	0.22
Ratio of Government Spending to Output	—	—	—	—	0.20
Risk-Free Rate, Annual	3.0%	3.4%	6.2%	4.0%	4.0%
Net Interest Margin, Annual	2.0%	3.6%	2.3%	1.1%	1.6%

Note: The net interest margin is defined as net interest income divided by interest-earning assets.

In the general equilibrium approach common to all the models, the exogenous shortfall has drastically varied implications for bank net equity. Apart from additional model-specific mechanisms, bank net equity does not simply reflect the size of the exogenous transfer shock because the general equilibrium nature of the models imply important movements in asset prices, which feed back into the determination of the hit to bank net equity. At one end of the extreme, the multi-sector model of Guerrieri and Jahan-Parvar implies only a modest drop in bank net equity which mounts as the transfer shock builds in size. Demand from firms not reliant on bank credit keeps asset prices afloat.[27] At the other end of the spectrum, in the model by Covas and Driscoll, the anticipated drop in credit resulting from the mounting transfer shock leads to a sizable fall in bank equity since their non-linear modeling approach does not assume capital constraints that bind all the time. Accordingly, banks can lower their equilibrium capital ratios offset the effect of the capital shortfall shock.

Across all models, the drop in net equity leads to a contraction in the supply of credit and an increase in the spread between interest rates on lending and on deposits. Despite differences in magnitudes, the persistence of the movements is elevated across all models and reflects the persistence of the drop in net equity. In this respect, the model of Kiley and Sim is an outlier in our group. In that model, access to outside equity allows for a quicker recapitalization of the financial sector that reduces the persistence of the drop in net equity and of the change in spreads between lending and deposit rates. While firms in the model of Queralto also have access to outside equity that could potentially curb the persistence of the response of bank equity in a similar fashion, in that model outside equity is intertwined with the specification of the principal-agent problem at the core of the model in such a way that financial intermediaries prefer to avoid recapitalizing more quickly and rely more prominently on the accumulation of internal equity through retained earnings. Because

[27] As shown above, even in the model of Guerrieri and Jahan-Parvar, an economy-wide shock would lead to large reductions in asset prices more closely comparable to those obtained in the other models presented.

these modeling differences ride through general equilibrium channels, a limited information approach to the estimation of the cost of issuance of outside equity would be ill suited to discriminating between these different results.

Notably, some of the differences in the response of net equity in the model by Covas and Driscoll are made more apparent by a different calibration approach that focuses on matching details of the commercial banking sector, rather than a stylized overall financial sector in the other models, and results in a magnification in the drop of bank capital in percent terms.

The disparities in the behavior of bank equity account to a large extent for the different spillover effects to the rest of the macroeconomy, as made apparent by the right column of figure 17. Focusing again on outliers, the drop in investment ranges between about 2 and 14 percent. A variety of modeling choices accounts for these disparities. The responses in the model of Guerrieri and Jahan-Parvar and in the model of Covas and Driscoll are compressed due to the interaction among sectors—the sector-specific transfer shock is compensated by increased lending from complementary sources. Such mechanism, by contrast, is muted in Iacoviello's model and Kiley and Sim's model. In Iacoviello's model, even if 50 percent of capital is produced by unconstrained firms, the complementarities across constrained and unconstrained firms are such that unconstrained firms cannot undo the drop in labor and capital demand that follow a credit crunch. Similar mechanisms also apply to Kiley and Sim's model.

The consumption side reflects an even broader range of outcomes. In some models, the baseline financial shock *boosts* aggregate consumption—the transfer shock considered is a windfall for the household sector. In other models, such as that of Covas and Driscoll and that of Iacoviello, the windfall is offset by the fact that the banking sector cuts dividends sharply in order to boost the recapitalization process by retaining earnings. The models of Queralto and of Guerrieri and Jahan-Parvar do not embed this mechanism, as dividends are not explicitly modeled.

Finally, all models predict a contraction in output, but the magnitudes differ greatly, ranging from a 5 percent contraction of the model of Iacoviello to a contraction below 0.5 percent in the model by Guerrieri and Jahan-Parvar. Apart from the interaction across sectors, sensitivity analysis to parametric assumptions brings out the importance of the interaction between financial frictions and the labor market to gauge the effects on aggregate output. With capital predetermined in all models and with the transfer shock not able to affect real assets on impact, the immediate fall in output has to ride through a contraction in hours worked. In this respect, apart from extant differences in modeling approaches, different calibration choices regarding the Frisch elasticity of labor supply across the models play an important role in determining disparities in results.

8.1 Harmonized Calibration

Table 6 summarizes key parameters across models, including the Frisch elasticity of labor supply. To gauge the importance of differences related to alternative calibration choices, we also considered a harmonized calibration, reported in the last column of the table. As some of the parameters govern features not included across all of the models considered, for ease of comparison, where necessary, we shut down some of the missing features, as in the case of adjustment costs for loans or deposits, or for consumption habits. For the other parameters we settled on representative estimates from the literature.

Figure 18 compares the effects of the baseline transfer shock across models under the harmonized calibration of table 6.[28] The harmonization of the parameter values slightly compresses the range of model responses, but, even with a common calibration, there remain substantial differences across models. In sum, the figure reinforces the headline finding of our analysis that economic modeling choices (more so than different parameter choices) can dramatically affect the results across models to the same size shock.

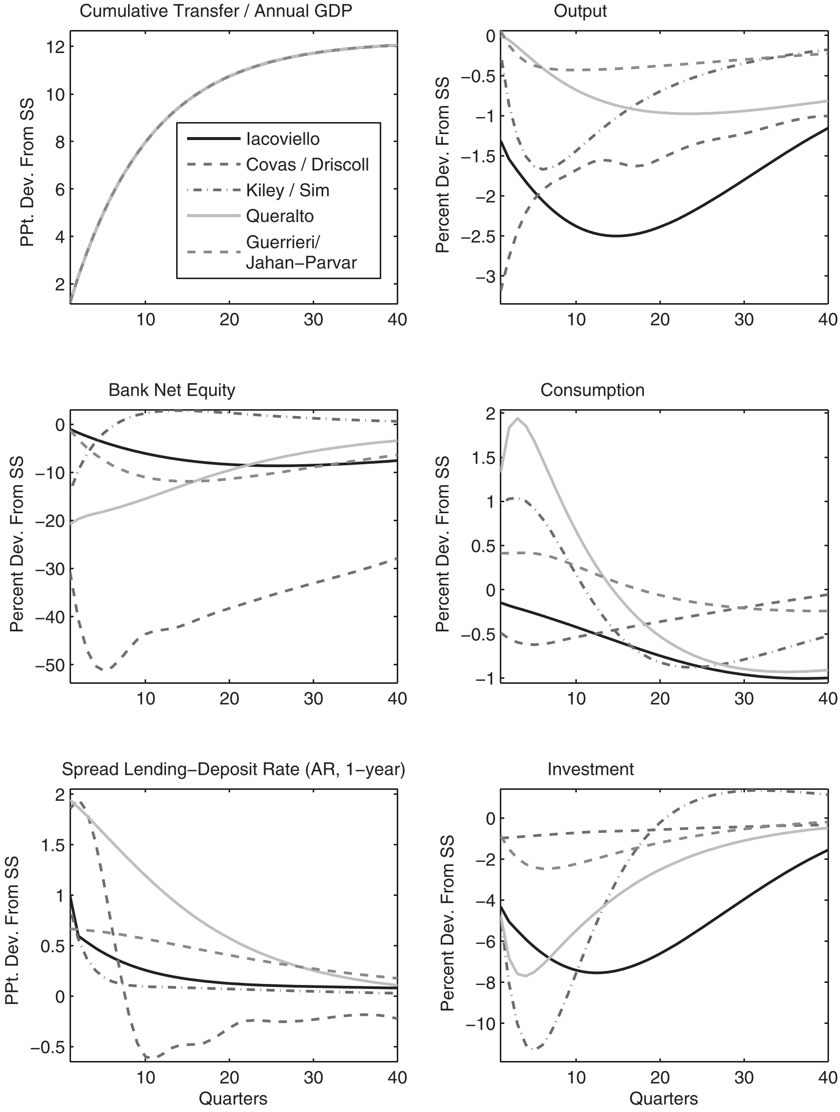
[28]Parameters not included in table 6 are unchanged relative to the calibration tables in the online appendix.

Table 6. Harmonized Calibration

Parameter	Iacoviello	Covas/ Driscoll	Kiley/Sim	Queralto	Guerrieri/ Jahan-Parvar	Harmonized Value
Capital Share	0.35	0.36	0.4	0.33	0.33	0.33
Capital Depreciation Rate	0.035	0.021	0.025	0.025	0.025	0.025
Average Discount Factor	0.975	0.988	0.985	0.99	0.99	0.99
Are There Adjustment Costs for Loans or Deposits?	Yes	Yes	No	No	No	No
Consumption Habits	0.46	0	0.37	0.75	0.82	0
Risk Aversion for Households	1	2	1.57	2	1	1
Frisch Labor Supply Elasticity	2	0	1.05	3	1	1

Note: Columns 2–6 report the parameters used in each model. Column 7 reports the parameters chosen for a harmonized calibration.

Figure 18. Horizontal Model Comparison with Harmonized Calibration



Notes: The figure shows responses to the baseline transfer shock (whose calibration is described in section 2) across all the models of the sections from 3 to 7 using the harmonized calibration described in section 8. For ease of comparison, the responses for the yearly model of Covas and Driscoll were interpolated to quarterly frequency using splines.

8.2 VAR Estimates

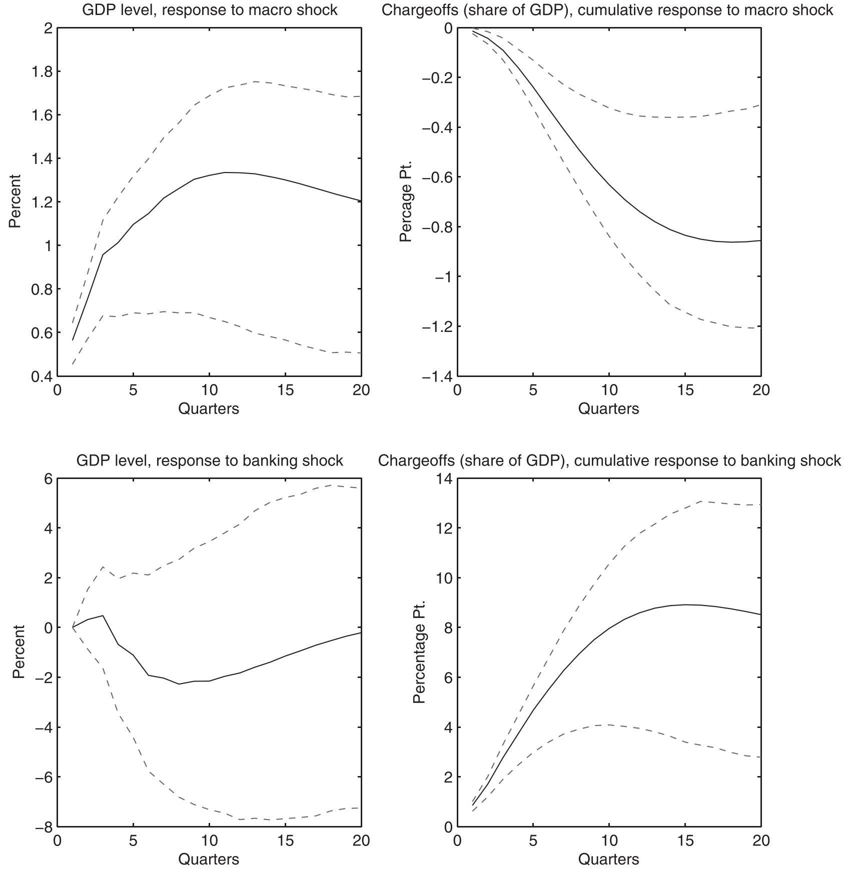
In order to check if any of the models presented are outliers relative to simple empirical evidence, we considered a variety of vector autoregressions. Capital shortfalls can stem from sources ranging from changes in the valuation of available-for-sale assets on the portfolio of banks to reductions in income. The simple empirical evidence presented below focuses on increases in charge-offs on loans and leases.

We run a bivariate VAR of U.S. real GDP growth and charge-off rates on loans and leases for the period 1985:Q1–2016:Q3 using four lags.[29] Using a simple recursive identification scheme, we identify two shocks: a macro shock, an innovation to GDP that contemporaneously affects loan charge-offs; and a loan charge-off shock (a banking shock), which does not affect GDP contemporaneously. We then rescale the loan charge-off shock so that, when expressed as a fraction of GDP, total loan charge-offs rise after nine quarters so as to imply a shortfall sized at 7.5 percent of GDP, just like in our model comparison exercise. The VAR results are shown in figure 19. The shock to loan charge-offs, shown in the bottom row, produces a mean contraction for GDP in the neighborhood of 3 percent after two years. The shaded areas in the figure represent 90 percent bootstrap confidence intervals.

Figure 20 overlays the 90 percent confidence interval for the GDP response from the VAR with the model responses under the original calibrations (top panel of the figure) and under the harmonized calibrations (bottom panel of the figure). From this comparison we conclude that the range of outcomes consistent with sampling uncertainty from the empirical VAR is similar to the range of outcomes from our models. Moreover, this range of uncertainty is also consistent with the outcomes from simple empirical frameworks presented, for instance, in BCBS (2010).

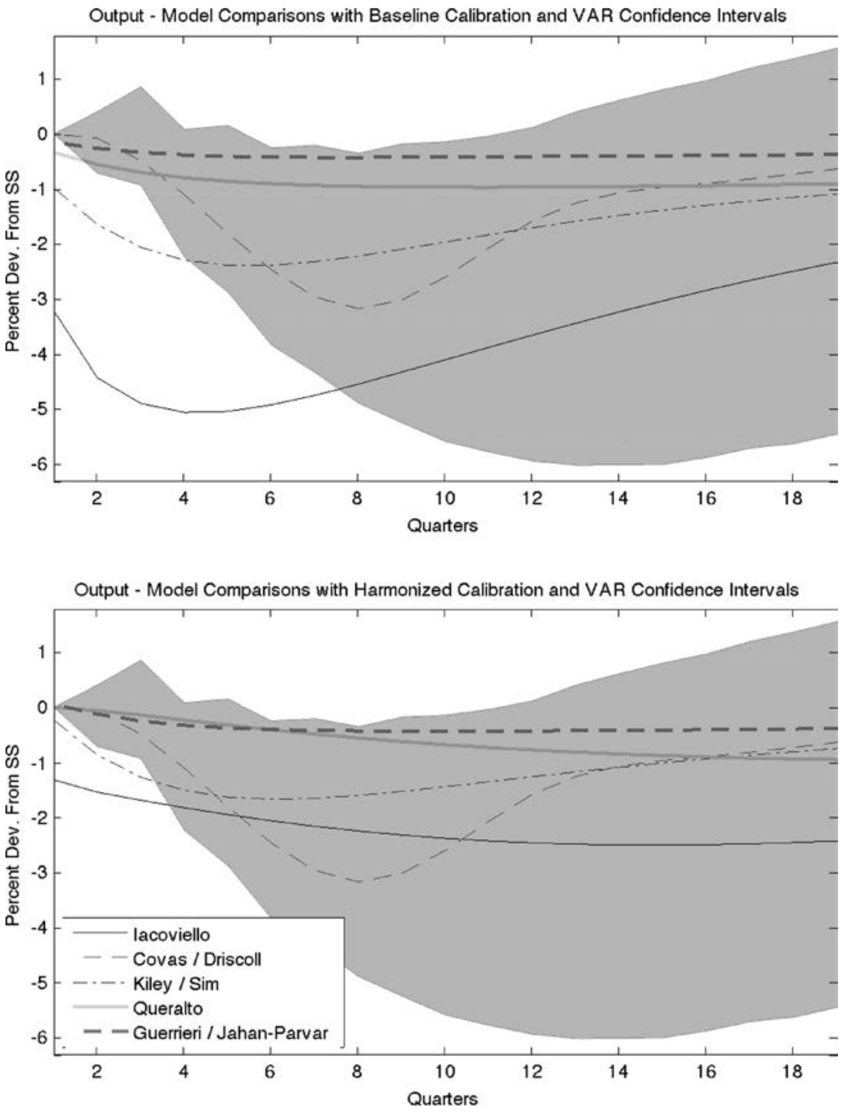
[29] Charge-offs are expressed as a share of GDP by multiplying loan charge-off rates at all insured commercial banks (Haver mnemonic: DY@USECON) by loans and leases in bank credit (FABWA@USECON) and dividing by nominal GDP (GDP@USECON).

Figure 19. Responses Estimated Using a VAR



Notes: The solid lines show point estimates for the response to recursively identified shocks from a bivariate VAR of order 4, which includes GDP growth and charge-off rates for all commercial banks. The dashed lines denote 90 percent bootstrap confidence intervals.

Figure 20. Comparing Model Uncertainty and VAR Sampling Uncertainty



Notes: The shaded area represents a 90 percent bootstrap confidence interval for the response of U.S. GDP to a banking shock identified from a VAR. The top panel compares this measure of VAR sampling uncertainty with the output responses from the models under the original calibrations. The bottom panel shows analogous model responses under a harmonized calibration. See table 5 for the calibration details.

9. Conclusions

Despite a common core, models that emphasize a complementary set of linkages between the financial and the real sectors produce a wide array of predictions for the macroeconomic effects of a shortfall in capital. All the models presented imply that the baseline shock that produces a capital shortfall similar in size to that gauged under the U.S. stress-test program would lead to a contraction in output. However, the size of this contraction varies greatly across models.

We draw the following conclusions:

- General equilibrium channels can exert a large influence on the spillover effects of capital shortfalls through the response of asset prices such as the price of capital and interest rate spreads.
- The interaction between alternative sectors that can provide financing is an important determinant of the availability of credit and of the size of the macroeconomic consequences of shortfalls in capital. In turn, important implications of this interaction ride through asset prices.
- The modeling of alternative sources of financing can lead to large differences in results. The interaction among alternative financing sources can generate subtle differences across models. For instance, recapitalization associated with outside equity can be influenced by readily measurable costs, such as costs of issuance, as well as by more subtle structural features of the economy, such as the effect of outside equity on incentives of banks.
- If the financial shock does not imply the destruction of physical resources, as for our baseline transfer shock, the macroeconomic spillover has to work through a contraction in hours worked on impact. Accordingly, refinements of the linkages between financial frictions and frictions in the labor market would bolster our understanding of the macro effects of financial shocks.
- Finally, sensitivity analysis shows that the sources of shocks to financial positions can have a large influence on their macroeconomic effects.

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Online Appendix to Macroeconomic Effects of Banking-Sector Losses across Structural Models

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Contents

1 Matteo Iacoviello: An Estimated Model of Banks with Financing Frictions 4

1.1 The Dynamic Model 4

1.1.1 Household Savers 4

1.1.2 Household Borrowers 5

1.1.3 Bankers 6

1.1.4 Entrepreneurs 8

1.1.5 Equilibrium 10

1.1.6 Shocks 10

1.2 Calibration 11

2 Francisco Covas and John Driscoll: A Non-linear Model of Borrowing Constraints 13

2.1 Introduction 13

2.2 The Model 13

2.2.1 Workers 13

2.2.2 Entrepreneurs 14

2.2.3 Bankers 17

2.2.4 Corporate Sector 21

2.2.5 Equilibrium 21

(continued)

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2.3 Calibration 23
2.3.1 Workers' and Entrepreneurs' Problems 23
2.3.2 Bankers' Problem 24
2.4 Solution Techniques 26
2.4.1 Numerical Solution 26
2.4.2 Transitional Dynamics 27
2.4.3 Markov Chains 27
3 Michael Kiley and Jae Sim: Intermediary Leverage, Macroeconomic Dynamics, and Macroprudential Policy 28
3.1 Model without Pigovian Tax 28
3.1.1 The Financial Intermediary Sector 28
3.1.2 Debt Contract 29
3.1.3 Intermediary Equity Finance 30
3.1.4 Value Maximization 31
3.1.5 Production and Investment 33
3.1.6 Households 34
3.1.7 Nominal Rigidity and Monetary Policy 35
3.1.8 Fiscal Policy 35
3.2 Pigovian Tax 35
3.3 Calibration/Estimation of Key Parameters 36
4 Albert Queralto: Banks and Outside Equity 40
4.1 Model Setup 40
4.1.1 Households 40
4.1.2 Non-financial Firms 40
4.1.3 Goods Producers 41
4.1.4 Capital Producers 41
4.1.5 Banks 41
4.2 Equilibrium Conditions 43
4.3 Calibration 45
5 Luca Guerrieri and Mohammad Jahan-Parvar: Capital Shortfalls in a Two-Sector Production Economy 46
5.1 Asset Pricing in a Basic RBC Model 47
5.1.1 Production 47
5.1.2 Households 51

(continued)

5.1.3 Capital-Producing Firms 51
5.1.4 Necessary Conditions for an Equilibrium 52
5.2 Introducing Financial Constraints Following Gertler and Karadi (2011) 54
5.2.1 Households 54
5.2.2 Banks 55
5.3 Introducing Transfer Shocks between Banks and Households 62
5.3.1 Banks 63
5.4 Introducing Heterogenous Firms 69
5.4.1 Households 69
5.4.2 Output-Producing Firms 71
5.4.3 Capital-Producing Firms 74
5.4.4 Banks 75
5.5 Introducing Nominal Rigidities 76
5.6 Calibration 83

List of Tables

Table 1.1 Calibrated Parameters for the Extended Model 11
Table 1.2 Estimated Structural Parameters and Shock Processes 12
Table 2.1 Parameter Values under Baseline Calibration 15
Table 2.2 Selected Moments 26
Table 3.1 Baseline Calibration 38
Table 3.2 Posterior Moments of Key Parameters 38
Table 3.3 Variance Decomposition for Observable Variables 39
Table 4.1 Calibration 46
Table 5.1 Calibration 83

1. Matteo Iacoviello: An Estimated Model of Banks with Financing Frictions

This appendix contains the complete set of equations for the model described in section 3 of the paper "Macroeconomic Effects of Banking-Sector Losses across Structural Models." The material borrows heavily on the technical appendix of the paper "Financial Business Cycles," described in Iacoviello (2015).

1.1 The Dynamic Model

1.1.1 Household Savers

Savers (denoted with subscript $_H$) choose consumption C, housing H, and hours N to solve

$$\max \sum_{t=0}^{\infty} \beta_H^t \left(A_{p,t} \left(1-\eta\right) \log \left(C_{H,t} - \eta C_{H,t-1}\right) + j A_{j,t} A_{p,t} \log H_{H,t} + \tau \log \left(1 - N_{H,t}\right)\right)$$

subject to

$$C_{H,t} + \frac{K_{H,t}}{A_{K,t}} + D_t + q_t \left(H_{H,t} - H_{H,t-1}\right) + ac_{KH,t} + ac_{DH,t} = \left(R_{M,t} z_{KH,t} + \frac{1-\delta_{KH,t}}{A_{K,t}}\right) K_{H,t-1} + R_{H,t-1} D_{t-1} + W_{H,t} N_{H,t}, \quad (1.1)$$

where the adjustment costs take the following form,

$$ac_{KH,t} = \frac{\phi_{KH}}{2} \frac{\left(K_{H,t} - K_{H,t-1}\right)^2}{K_H}$$

$$ac_{DH,t} = \frac{\phi_{DH}}{2} \frac{\left(D_t - D_{t-1}\right)^2}{D},$$

and the depreciation function is

$$\delta_{KH,t} = \delta_{KH} + b_{KH} \left(0.5 \zeta_H' z_{KH,t}^2 + \left(1 - \zeta_H'\right) z_{KH,t} + \left(0.5 \zeta_H' - 1\right)\right),$$

where $\zeta'_H = \frac{\zeta_H}{1-\zeta_H}$ is a parameter measuring the curvature of the utilization rate function. $\zeta_H = 0$ implies $\zeta'_H = 0$; ζ_H approaching 1 implies ζ'_H approaches infinity and $\delta_{KH,t}$ stays constant. $b_{KH} = \frac{1}{\beta_H} + 1 - \delta_{KH}$ and implies a unitary steady-state utilization rate. ac measures a quadratic adjustment cost for changing the quantity i between time t and time $t+1$. The adjustment cost is external. Habits are external too.

The household problem yields, denoting with $u_{CH,t} = \frac{A_{p,t}}{C_{H,t}-\eta C_{H,t-1}}$ and $u_{HH,t} = \frac{jA_{j,t}A_{p,t}}{H_{H,t}}$ the marginal utilities of consumption and housing,

$$u_{CH,t}\left(1+\frac{\partial ac_{DH,t}}{\partial D_t}\right) = \beta_H R_{H,t} u_{CH,t+1} \tag{1.2}$$

$$W_{H,t} u_{CH,t} = \frac{\tau_H}{1-N_{H,t}} \tag{1.3}$$

$$\begin{aligned}&\frac{1}{A_{K,t}} u_{CH,t}\left(1+\frac{\partial ac_{KH,t}}{\partial K_{H,t}}\right)\\ &\quad = \beta_H\left(R_{M,t+1} z_{KH,t+1} + \frac{1-\delta_{KH,t+1}}{A_{K,t+1}}\right) u_{CH,t+1}\end{aligned} \tag{1.4}$$

$$q_t u_{CH,t} = u_{HH,t} + \beta_H q_{t+1} u_{CH,t+1} \tag{1.5}$$

$$R_{M,t} = \delta'\left(z_{KH,t}\right), \tag{1.6}$$

where $A_{K,t}$ is an investment shock, $A_{p,t}$ is a consumption preference shock, and $A_{j,t}$ is a housing demand shock.

1.1.2 Household Borrowers

They solve

$$\begin{aligned}\max \sum_{t=0}^{\infty} \beta_S^t \,&(A_{p,t}(1-\eta)\log(C_{S,t}-\eta C_{S,t-1})\\ &+ jA_{j,t}A_{p,t}\log H_{S,t} + \tau \log(1-N_{S,t}))\end{aligned}$$

subject to

$$\begin{aligned}&C_{S,t} + q_t\left(H_{S,t}-H_{S,t-1}\right) + R_{S,t-1}L_{S,t-1} - \varepsilon_{H,t} + ac_{SS,t}\\ &\quad = L_{S,t} + W_{S,t}N_{S,t}\end{aligned} \tag{1.7}$$

and to

$$L_{S,t} \leq \rho_S L_{S,t-1} + (1-\rho_S)\, m_S A_{MH,t} \frac{q_{t+1}}{R_{S,t}} H_{S,t} - \varepsilon_{H,t}, \tag{1.8}$$

where $\varepsilon_{H,t}$ is the borrower repayment shock; $A_{M,t}$ is a loan-to-value ratio shock. The adjustment cost is

$$ac_{SS,t} = \frac{\phi_{SS}}{2} \frac{(L_{S,t} - L_{S,t-1})^2}{L_S}.$$

The first-order conditions are, denoting with $u_{CS,t} = \frac{A_{p,t}}{C_{S,t}}$ and $u_{HS,t} = \frac{jA_{j,t}A_{p,t}}{H_{S,t}}$ the marginal utilities of consumption and housing, and with $\lambda_{S,t} u_{CS,t}$ the (normalized) multiplier on the borrowing constraint,

$$\left(1 - \frac{\partial ac_{SS,t}}{\partial L_{S,t}} - \lambda_{S,t}\right) u_{CS,t} = \beta_S \left(R_{S,t} - \rho_S \lambda_{S,t+1}\right) u_{CS,t+1} \tag{1.9}$$

$$W_{S,t} u_{CS,t} = \frac{\tau_S}{1 - N_{S,t}} \tag{1.10}$$

$$\left(q_t - \lambda_{S,t}(1-\rho_S)\, m_S A_{MH,t} \frac{q_{t+1}}{R_{S,t}}\right) u_{CS,t} = u_{HS,t} + \beta_S q_{t+1} u_{CS,t+1}. \tag{1.11}$$

1.1.3 Bankers

Bankers solve

$$\max \sum_{t=0}^{\infty} \beta_B^t \log (C_{B,t} - \eta C_{B,t-1})$$

subject to

$$C_{B,t} + R_{H,t-1} D_{t-1} + L_{E,t} + L_{S,t} + ac_{DB,t} + ac_{EB,t} + ac_{SB,t}$$
$$= D_t + R_{E,t} L_{E,t-1} + R_{S,t} L_{S,t-1} - \varepsilon_{E,t} - \varepsilon_{S,t}, \tag{1.12}$$

where $\varepsilon_{E,t}$ is the entrepreneur repayment shock. The adjustment costs are

$$ac_{DB,t} = \frac{\phi_{DB}}{2}\frac{(D_t - D_{t-1})^2}{D}$$
$$ac_{EB,t} = \frac{\phi_{EB}}{2}\frac{(L_{E,t} - L_{E,t-1})^2}{L_E}$$
$$ac_{SB,t} = \frac{\phi_{SB}}{2}\frac{(L_{S,t} - L_{S,t-1})^2}{L_S}.$$

Denote $\varepsilon_t = \varepsilon_{E,t}+\varepsilon_{S,t}$. Let $L_t = L_{E,t}+L_{S,t}$. The banker's constraint is a capital adequacy constraint of the form

$$\underbrace{(L_t - D_t - \varepsilon_t)}_{\text{bank equity}} \geq \rho_D (L_{t-1} - D_{t-1} - \varepsilon_{t-1}) + (1-\gamma)(1-\rho_D)\underbrace{(L_t - \varepsilon_t)}_{\text{bank assets}}$$

stating that bank equity (after losses) must exceed a fraction of bank assets, allowing for a partial adjustment in bank capital given by ρ_D. Such constraint can be rewritten as a leverage constraint of the form

$$D_t \leq \rho_D (D_{t-1} - (L_{E,t-1} + L_{S,t-1} - (\varepsilon_{E,t-1} + \varepsilon_{S,t-1}))) + (1-(1-\gamma)(1-\rho_D))(L_{Et} + L_{S,t} - (\varepsilon_{E,t} + \varepsilon_{S,t})). \quad (1.13)$$

The first-order conditions to the banker's problem imply, choosing D, L_E, L_S and letting $\lambda_{B,t}u_{CB,t}$ be the normalized multiplier on the borrowing constraint,

$$\left(1 - \lambda_{B,t} - \frac{\partial ac_{DB,t}}{\partial D_t}\right) u_{CB,t} = \beta_B (R_{H,t} - \rho_D \lambda_{B,t+1}) u_{CB,t+1} \quad (1.14)$$

$$\left(1 - (\gamma_E (1-\rho_D) + \rho_D)\lambda_{B,t} + \frac{\partial ac_{EB,t}}{\partial L_{E,t}}\right) u_{CB,t} = \beta_B (R_{E,t+1} - \rho_D \lambda_{B,t+1}) u_{CB,t+1} \quad (1.15)$$

$$\left(1 - (\gamma_S (1-\rho_D) + \rho_D)\lambda_{B,t} + \frac{\partial ac_{SB,t}}{\partial L_{S,t}}\right) u_{CB,t} = \beta_B (R_{S,t} - \rho_D \lambda_{B,t+1}) u_{CB,t+1}. \quad (1.16)$$

1.1.4 Entrepreneurs

Entrepreneurs obtain loans and produce goods (including capital). Entrepreneurs hire workers and demand capital supplied by the household sector,

$$\max \sum_{t=0}^{\infty} \beta_E^t \log \left(C_{E,t} - \eta C_{E,t-1}\right),$$

subject to

$$\begin{aligned} C_{E,t} &+ \frac{K_{E,t}}{A_{K,t}} + q_t H_{E,t} + R_{E,t} L_{E,t-1} + W_{H,t} N_{H,t} + W_{S,t} N_{S,t} \\ &+ R_{M,t} z_{KH,t} K_{H,t-1} \\ &= Y_t + \frac{1-\delta_{KE,t}}{A_{K,t}} K_{E,t-1} + q_t H_{E,t-1} + L_{E,t} + \varepsilon_{E,t} \\ &+ ac_{KE,t} + ac_{EE,t} \end{aligned} \tag{1.17}$$

and to

$$\begin{aligned} Y_t = A_{Z,t} \left(z_{KH,t} K_{H,t-1}\right)^{\alpha\mu} \left(z_{KE,t} K_{E,t-1}\right)^{\alpha(1-\mu)} \\ \times H_{E,t-1}^{\nu} N_{H,t}^{(1-\alpha-\nu)(1-\sigma)} N_{S,t}^{(1-\alpha-\nu)\sigma}, \end{aligned} \tag{1.18}$$

where $A_{Z,t}$ is a shock to total factor productivity. The adjustment costs are

$$\begin{aligned} ac_{KE,t} &= \frac{\phi_{KE}}{2} \frac{\left(K_{E,t} - K_{E,t-1}\right)^2}{K_E} \\ ac_{EE,t} &= \frac{\phi_{EE}}{2} \frac{\left(L_{E,t} - L_{E,t-1}\right)^2}{L_E}. \end{aligned}$$

Note that symmetrically to the household problem entrepreneurs are subject to an investment shock, can adjust the capital utilization rate, and pay a quadratic capital adjustment cost. The depreciation rate is governed by

$$\delta_{KE,t} = \delta_{KE} + b_{KE} \left(0.5\zeta_E' z_{KE,t}^2 + \left(1-\zeta_E'\right) z_{KE,t} + \left(0.5\zeta_E' - 1\right)\right),$$

where setting $b_{KE} = \frac{1}{\beta_E} + 1 - \delta_{KE}$ implies a unitary steady-state utilization rate.

Entrepreneurs are subject to a borrowing/pay-in-advance constraint that acts as a wedge on the capital and labor demand. The constraint is

$$L_{E,t} = \rho_E L_{E,t-1} + (1-\rho_E) A_{ME,t} \times \left(m_H \frac{q_{t+1}}{R_{E,t+1}} H_{E,t} + m_K K_{E,t} - m_N (W_{H,t} N_{H,t} + W_{S,t} N_{S,t}) \right). \quad (1.19)$$

Letting $u_{CE,t}$ be the marginal utility of consumption and $\lambda_{E,t} u_{CE,t}$ the normalized borrowing constraint, the first-order conditions for L_E, K_E, and H_E are

$$\left(1 - \lambda_{E,t} + \frac{\partial ac_{LE,t}}{\partial L_{E,t}}\right) u_{CE,t} = \beta_E (R_{E,t+1} - \rho_E \lambda_{E,t+1}) u_{CE,t+1} \quad (1.20)$$

$$\left(1 + \frac{\partial ac_{KE,t}}{\partial K_{E,t}} - \lambda_{E,t} (1-\rho_E) m_K A_{ME,t}\right) u_{CE,t} = \beta_E (1 - \delta_{KE,t+1} + R_{K,t+1} z_{KE,t+1}) u_{CE,t+1} \quad (1.21)$$

$$\left(q_t - \lambda_{E,t} (1-\rho_E) m_H A_{ME,t} \frac{q_{t+1}}{R_{E,t+1}}\right) u_{CE,t} = \beta_E q_{t+1} (1 + R_{V,t+1}) u_{CE,t+1}. \quad (1.22)$$

Additionally, these conditions can be combined with those of the "production arm" of the firm, giving

$$\alpha \mu Y_t = R_{K,t} z_{KE,t} K_{E,t-1} \quad (1.23)$$

$$\alpha (1-\mu) Y_t = R_{M,t} z_{KH,t} K_{H,t-1} \quad (1.24)$$

$$\nu Y_t = R_{V,t} q_t H_{E,t-1} \quad (1.25)$$

$$(1-\alpha-\nu)(1-\sigma) Y_t = W_{H,t} N_{H,t} (1 + m_N A_{ME,t} \lambda_{E,t}) \quad (1.26)$$

$$(1-\alpha-\nu) \sigma Y_t = W_{S,t} N_{S,t} (1 + m_N A_{ME,t} \lambda_{E,t}) \quad (1.27)$$

$$R_{K,t} = \delta' (z_{KE,t}). \quad (1.28)$$

1.1.5 Equilibrium

Market clearing is implied by Walras's law by aggregating all the budget constraints. For housing, we have the following market clearing condition:

$$H_{H,t} + H_{S,t} + H_{E,t} = 1. \tag{1.29}$$

The model dynamics (except for the stochastic properties of the exogenous shocks, described separately below) are fully described by equations (1.1) to (1.29). These equations—together with the definition of the depreciation rate functions and the adjustment cost functions given above—represent a dynamic system in the following twenty-nine endogenous variables:

- Fourteen quantities: Y, H_E, H_H, H_S, K_E, K_H, N_H, N_S, C_B, C_E, C_H, C_S, z_{KH}, z_{KE}.
- Three loans and deposits: L_E, L_S, D.
- Three prices: q, W_H, W_S.
- Six interest rates: R_K, R_M, R_V, R_E, R_S, R_H.
- Three Lagrange multipliers: λ_E, λ_S, λ_B.

1.1.6 Shocks

The shocks obey the following stochastic processes:

$$\varepsilon_{E,t} = \rho_{be}\varepsilon_{E,t-1} + u_{E,t}, \quad u_E \sim N(0, \sigma_{be})$$

$$\varepsilon_{H,t} = \rho_{bh}\varepsilon_{H,t-1} + \upsilon_{H,t}, \quad u_H \sim N(0, \sigma_{bh})$$

$$\log A_{j,t} = \rho_j \log A_{j,t-1} + \upsilon_{j,t}, \quad u_j \sim N(0, \sigma_j)$$

$$\log A_{K,t} = \rho_K \log A_{K,t-1} + \upsilon_{K,t}, \quad u_K \sim N(0, \sigma_k)$$

$$\log A_{ME,t} = \rho_{me} \log A_{ME,t-1} + \upsilon_{ME,t}, \quad u_{ME} \sim N(0, \sigma_{me})$$

$$\log A_{MH,t} = \rho_{mh} \log A_{MH,t-1} + \upsilon_{MH,t}, \quad u_{MH} \sim N(0, \sigma_{mh})$$

$$\log A_{p,t} = \rho_p \log A_{p,t-1} + \upsilon_{p,t}, \quad u_p \sim N(0, \sigma_p)$$

$$\log A_{Z,t} = \rho_z \log A_{Z,t-1} + \upsilon_{z,t}, \quad u_z \sim N(0, \sigma_z).$$

1.2 Calibration

Table 1.1 Calibrated Parameters for the Extended Model

Calibrated Parameter		**Value**
Household-Saver (HS) Discount Factor	β_H	0.9925
Household-Borrower (HB) Discount Factor	β_S	0.94
Banker Discount Factor	β_B	0.945
Entrepreneur (E) Discount Factor	β_E	0.94
Total Capital Share in Production	α	0.35
Loan-to-Value Ratio on Housing, HB	m_S	0.9
Loan-to-Value Ratio on Housing, E	m_H	0.9
Loan-to-Value Ratio on Capital, E	m_K	0.9
Wage Bill Paid in Advance	m_N	1
Liabilities-to-Assets Ratio for Banker	γ_E, γ_S	0.9
Housing Preference Share	j	0.075
Capital Depreciation Rates	δ_{KE}, δ_{KH}	0.035
Labor Supply Parameter	τ	2

Table 1.2 Estimated Structural Parameters and Shock Processes

Estimated Parameter		Value
A. Estimated Structural Parameters		
Habit in Consumption	η	0.46
D Adj. Cost, Banks	ϕ_{DB}	0.14
D Adj. Cost, Household Saver (HS)	ϕ_{DH}	0.10
K Adj. Cost, Entrepreneurs (E)	ϕ_{KE}	0.59
K Adj. Cost, Household Saver (HS)	ϕ_{KH}	1.73
Loan to E Adj. Cost, Banks	ϕ_{EB}	0.07
Loan to E Adj. Cost, E	ϕ_{EE}	0.06
Loan to HB Adj. Cost, Banks	ϕ_{SB}	0.47
Loan to HB Adj. Cost, HH Borrower HB	ϕ_{SS}	0.37
Capital Share of E	μ	0.46
Housing Share of E	ν	0.04
Inertia in Capital Adequacy Constraint	ρ_D	0.24
Inertia in E Borrowing Constraint	ρ_E	0.65
Inertia in HB Borrowing Constraint	ρ_S	0.70
Wage Share HB	σ	0.33
Curvature for Utilization Function E	ζ_E	0.42
Curvature for Utilization Function HS	ζ_H	0.38
B. Estimated Shock Processes		
Autocorrelation E Default Shock	ρ_{be}	0.932
Autocorrelation HB Default Shock	ρ_{bh}	0.969
Autocorrelation Housing Demand Shcok	ρ_j	0.992
Autocorrelation Investment Shock	ρ_k	0.916
Autocorrelation LTV Shock, E	ρ_{me}	0.839
Autocorrelation LTV Shock, HB	ρ_{mh}	0.873
Autocorrelation Preference Shock	ρ_p	0.994
Autocorrelation Technology Shock	ρ_z	0.988
St. Dev., Default Shock, E	σ_{be}	0.0011
St. Dev., Default Shock, HB	σ_{bh}	0.0013
St. Dev., Housing Demand Shock	σ_j	0.0346
St. Dev., Investment Shock	σ_k	0.0081
St. Dev., LTV Shock, E	σ_{me}	0.0204
St. Dev., LTV Shock, HB	σ_{mh}	0.0115
St. Dev., Preference Shock	σ_p	0.0205
St. Dev., Technology Shock	σ_z	0.0070

2. Francisco Covas and John Driscoll: A Non-linear Model of Borrowing Constraints

2.1 Introduction

In this appendix, we describe the setup of the model by Covas and Driscoll included in "Macroeconomic Effects of Banking-Sector Losses across Structural Models." We construct a general equilibrium model augmenting that of Aiyagari (1994) by having three types of agents that face uninsurable risks: workers, entrepreneurs, and bankers. Workers supply labor to entrepreneurs and face labor productivity shocks which dictate their earning potential. Entrepreneurs can invest in their own technology and face investment risk shocks which determine their potential profitability. Bankers play the role of financial intermediaries in this economy by accepting deposits from workers and making loans to entrepreneurs. In addition, bankers can also invest in riskless securities. Bankers are subject to revenue shocks that determine their potential profitability. An important feature of the banker's problem is the presence of occasionally binding capital and liquidity constraints.

2.2 The Model

The model includes three groups of agents: workers, entrepreneurs, and bankers. We describe the economic problems faced by each group of agents below.

2.2.1 Workers

As in Aiyagari (1994) workers are heterogeneous with respect to wealth holdings and earnings ability. Since there are idiosyncratic shocks, the variables of the model will differ across workers. To simplify notation, we do not index the variables to indicate this cross-sectional variation. Let c_t^w denote the worker's consumption in period t, d_t^w denote the deposit holdings, and a_t^w denote the worker's asset holdings in the same period, and ϵ_t is a labor-efficiency process which follows a first-order Markov process. Workers choose consumption to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta_w^t u(c_t^w, d_{t+1}^w),$$

subject to the following budget constraint:

$$c_t^w + d_{t+1}^w + a_{t+1}^w = w\,\epsilon_t + R^D d_t^w + R\,a_t^w,$$

where $0 < \beta_w < 1$ is the worker's discount factor, w is the worker's wage rate, R^D is the gross rate on deposits, and R is gross return on capital. We assume workers are subject to an ad hoc borrowing constraint; that is, $a_{t+1}^w \geq \underline{a}$, where $\underline{a} \leq 0$. The wage rate and the return on capital are determined in general equilibrium such that labor and corporate capital markets clear in the steady state. Note that we have introduced a demand for deposits by assuming that their holdings bring utility to the worker. However, the deposit rate is assumed to be exogenous since, as described later, bankers take as given the stock of deposits supplied by the workers.

Let $v^w(\epsilon, x_w)$ be the optimal value function for a worker with earnings ability ϵ and cash on hand x_w.[1] The worker's optimization problem can be specified in terms of the following dynamic programming problem:

$$\begin{aligned} v^w(\epsilon, x_w) = \max_{c_w, d_w', a_w'} \; & u(c_w, d_w') + \beta_w E[v(\epsilon', x_w')|\epsilon], \qquad (2.1)\\ \text{s.t.} \quad & c_w + d_w' + a_w' = x_w,\\ & x_w' = w\,\epsilon' + R^D d_w' + R\,a_w',\\ & a_w' \geq \underline{a}. \end{aligned}$$

The full list of parameters of the worker's problem is shown at the top of table 2.1.

2.2.2 Entrepreneurs

Entrepreneurs are also heterogeneous with respect to wealth holdings and productivity of the individual-specific technology that they

[1]Because the worker's problem is recursive, the subscript t is omitted in the current period, and a prime denotes the value of the variables one period ahead.

Table 2.1 Parameter Values under Baseline Calibration

Parameter	Description	Value
	Workers' Parameters	
β_w	Discount Factor	0.96
γ_w	Coefficient of Relative Risk Aversion	2.0
ω	Weight on Consumption	0.97
ρ_ϵ	Persistence of Earnings Risk	0.80
σ_ϵ	Unconditional s.d. of Earnings Risk	0.16
$\underline{a}$	Borrowing Constraint	0.0
η_w	Mass of Workers	0.666
	Entrepreneurs' Parameters	
β_e	Discount Factor	0.95
γ_e	Coefficient of Relative Risk Aversion	2.0
ρ_z	Persistence of Productivity Risk	0.70
σ_z	Unconditional s.d. of Productivity Risk	0.22
κ	Borrowing Constraint	0.50
α	Capital Share	0.45
ν	Labor Share	0.35
δ	Depreciation Rate	0.08
η_e	Mass of Entrepreneurs	0.333
	Bankers' Parameters	
β_b	Discount Factor	0.95
γ_b	Coefficient of Relative Risk Aversion	1.0
$\underline{\chi}$	Capital Requirements	0.06
$\bar{\delta}$	Loan Maturity	0.24
α_b	Curvature of Loan Revenues	0.75
ρ_θ	Persistence of Shock to Loan Revenues	0.70
σ_θ	Unconditional s.d. of Shock to Loan Revenues	0.09
ρ_d	Persistence of Shock to Deposits	0.80
σ_d	Unconditional s.d. of Shock to Deposits	0.15
ϕ_b	Intermediation Cost	0.15
ν^-	Adjustment Cost for Decreasing Loans	0.04
ν^+	Adjustment Cost for Increasing Loans	0.02
	Corporate Sector's Parameters	
α_c	Capital Share	0.36
δ_c	Depreciation Rate	0.08

operate. Entrepreneurs choose consumption to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta_e^t u(c_t^e),$$

where $0 < \beta_e < 1$ is the entrepreneur's discount factor. Each period, the entrepreneur can invest in an individual-specific technology (risky investment) or invest its savings in securities. The risky technology available to the entrepreneur is represented by

$$y_t = z_t f(k_t, l_t),$$

where z_t denotes productivity, k_t is the capital stock in the risky investment, and l_t is labor. This investment is risky because the stock of capital is chosen before productivity is observed. The labor input is chosen after observing productivity. The idiosyncratic productivity process follows a first-order Markov process. As is standard, capital depreciates at a fixed rate δ.

In addition, the entrepreneur is allowed to borrow to finance consumption and the risky investment. Let b_{t+1}^e denote the amount borrowed by the entrepreneur and R^L denote the gross rate on bank loans. The loan rate is determined in general equilibrium. Borrowing is constrained, for reasons of moral hazard and adverse selection that are not explicitly modeled, to be no more than a fraction of entrepreneurial capital:

$$b_{t+1}^e \geq -\kappa\, k_{t+1},$$

where κ represents the fraction of capital that can be pledged at the bank as collateral. Entrepreneurs that are not borrowing to finance investment can save through a riskless security, denoted by s^e with a gross return R^S which will also be determined in general equilibrium.

Under this set of assumptions, the entrepreneur's budget constraint is as follows:

$$\begin{aligned} c_t^e + k_{t+1} + b_{t+1}^e + s_{t+1}^e &= x_t^e, \\ x_{t+1}^e &= z_{t+1} f(k_{t+1}, l_{t+1}) + (1 - l_{t+1}) w \\ &\quad + (1-\delta) k_{t+1} + R^L b_{t+1}^e + R^S\, s_{t+1}^e, \end{aligned}$$

where x_t^e denotes the entrepreneur's period-t wealth. It should be noted that the entrepreneur can also supply labor to the corporate sector or other entrepreneurial businesses.

Let $v^e(z, x_e)$ be the optimal value function for an entrepreneur with productivity z and wealth x_e.[2] The entrepreneur's optimization problem can be specified in terms of the following dynamic programming problem:

$$v^e(z, x_e) = \max_{c_e, k', b'_e, s'_e} u(c_e) + \beta_e E[v(z', x'_e)|z], \tag{2.2}$$

$$\text{s.t.} \quad c_e + k' + s'_e + b'_e = x_e,$$

$$x'_e = \pi(z', k'; w) + (1 - \delta)k' + R^L b'_e + R^S s'_e,$$

$$0 \geq b'_e \geq -\kappa\, k',$$

$$s'_e \geq 0,$$

$$k' \geq 0,$$

where $\pi(z', k'; w)$ represents the operating profits of the entrepreneur's and incorporates the static optimization labor choice. From the properties of the utility and production functions of the entrepreneur, the optimal levels of consumption and the risky investment are always strictly positive. The constraints that may be binding are the choices of bank loans, b'_e, and security holdings, s'_e. The full list of parameters of the entrepreneur's problem is shown in the middle panel of table 2.1.

2.2.3 Bankers

Bankers are heterogeneous with respect to wealth holdings, loan balances, deposit balances, and productivity. Bankers choose consumption to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta_b^t u(c_t^b),$$

where $0 < \beta_b < 1$ is the banker's discount factor.

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[2]Because the entrepreneur's problem is recursive, the subscript t is omitted in the current period, and we let the prime denote the value of the variables one period ahead.

Bankers hold two types of assets—risky loans (b) and riskless securities (s)—and fund those assets with deposits (d) and equity (e). Loans can also be funded by short-selling securities—implying s can be negative.

Each period, the banker chooses the amount of loans it makes to the entrepreneurs, denoted by b_{t+1}. Loans, which are assumed to mature at a rate $\bar{\delta}$, yield both interest and non-interest income (the latter arises, for example, from fees, which in practice are a substantial part of bank income). Banks may differ in their ability to extract net revenue from loans due to (unmodeled) differences in their ability to screen applicants or monitor borrowers, or in market power. For analytical convenience, we represent net revenue in period t from the existing stock of loans b_t as

$$y_t^b = (R^L - \phi_b)b_t + \theta_t g(b_t),$$

where θ_t denotes the idiosyncratic productivity of the bank, the function $g(b_t)$ exhibits decreasing returns to scale, and ϕ_b is the cost of operating the loan technology.

The banks also face adjustment costs in changing the quantity of loans, which allows us to capture the relative illiquidity of such assets. The adjustment costs are parametrized by

$$\Psi(b_{t+1}, \bar{\delta}b_t) \equiv \frac{\nu_t}{2}\left(\frac{b_{t+1} - \bar{\delta}b_t}{b_t}\right)^2 b_t,$$

where

$$\nu_t \equiv \nu^+ 1_{\{b_{t+1} \geq \bar{\delta}b_t\}} + \nu^- 1_{\{b_{t+1} < \bar{\delta}b_t\}}.$$

In our calibration, we will assume that the cost of adjusting the stock of loans downwards is much greater than the cost of adjusting it upwards—reflecting the idea that calling in or selling loans is more costly than originating loans.

Gross returns from the bank's securities holdings is given by

$$y_t^s = R^S s_t,$$

which may be negative if the bank is short-selling securities. The banker's budget constraint is written as follows:

$$c_t^b + b_{t+1} + s_{t+1} + d_{t+1} = x_t^b - \Psi(b_{t+1}, \bar{\delta}b_t),$$

$$x_{t+1}^b = (R^L - \phi_b)b_{t+1} + \theta_{t+1}g(b_{t+1}) + R^S s_{t+1} + R^D d_{t+1},$$

where x_t^b denotes the banker's period-t wealth and d_{t+1} the stock of deposits. The bank borrows through deposits that it receives from the workers, but it can also borrow by selling securities to other bankers or entrepreneurs. For simplicity, we assume the share of deposits received by each bank is exogenous and follows a four-state first-order Markov chain (see section 2.4.3 of this appendix for further details). However, borrowing from entrepreneurs and other bankers is endogenous and is constrained by capital requirements. Letting e_{t+1} denote banks' equity, the capital requirement may be written as

$$e_{t+1} \geq \chi b_{t+1},$$

which is equivalent to a risk-based capital requirement, giving a zero risk weight to securities. The capital requirement may in turn be rewritten in terms of securities holdings as follows (since $e_{t+1} = x_t^b - \Psi(b_{t+1}, \bar{\delta} b_t) - c_t^b$):

$$s_{t+1} \geq (\chi - 1) b_{t+1} - d_{t+1}.$$

We also impose a liquidity requirement, in which we assume that cash on hand—which consists of the return on existing securities holdings, $R^S s_{t+1}$m, and the net revenue from paydowns on existing loans, $\bar{\delta} b_{t+1}$—must be sufficient to satisfy demand for deposit withdrawals under a liquidity stress scenario and interest payments on deposits. This can be represented as

$$R^S s_{t+1} + \bar{\delta} b_{t+1} \geq (d_{\{s-1,1\}^+} - R^D d_{t+1}), \tag{2.3}$$

where $d_{\{s-1,1\}^+}$ represents a decline in the stock of deposits (note that $d < 0$). Since d_t follows a Markov chain, if in period t the bank is in state s, then deposit withdrawals correspond to state $\{s-1,1\}^+$. The stringency of the liquidity requirement is given by the assumption about the relative size of the bad deposits realization.[3] It will be calibrated through an assumption of how quickly deposits would run off in a crisis situation.

[3]When not in a crisis, the deposits runoff will be smaller, and the constraint will not bind.

Let $v^b(\theta, x^b, b, d')$ be the optimal value function for a banker with wealth x^b, loans b, deposits d', and productivity θ. The banker's optimization problem can be specified in terms of the following dynamic programming problem:

$$v^b(\theta, x_b, b, d') = \max_{c_b, b', s'} u(c^b) + \beta_b E[v^b(x'_b, b', d'', \theta')|\theta, d'], \tag{2.4}$$

$$\text{s.t.} \quad c_b + b' + s' + d' = x_b - \Psi(b', \bar{\delta}b),$$

$$x'_b = (R^L - \phi_b)b' + \theta' g(b') + R^S s' + R^D d',$$

$$e' \geq \chi b',$$

$$R^S s' + \bar{\delta}b' \geq (d_{\{s-1,1\}^+} - R^D d').$$

Banker's Capital Constraint. The balance sheet constraint of the banker is given by

$$b' + s' = x_b - c_b - \Phi(b', \bar{\delta}b) - d',$$

where the left-hand side of this expression is the banker's assets, $b' + s'$, and the right-hand side is the banker's equity, $e_b \equiv x_b - c_b - \Phi(b', \bar{\delta}b)$, and debt, $-d'$. The capital constraint can be written as

$$e_b \geq \chi b'$$

$$b' + s' + d' \geq \chi b'$$

$$d' \geq (\chi - 1)b' - s'.$$

Banker's First-Order Conditions. The first-order conditions for b' and s' are as follows:

$$\left[1 + \frac{\partial \Phi(b', b)}{\partial b'}\right] u_c(c) = \beta_b E\left[\frac{\partial v_b}{\partial x_b}\frac{\partial x_b}{\partial b'} + \frac{\partial v_b}{\partial b'}\middle|\theta, d'\right] + (1 - \chi)\lambda + \bar{\delta}\mu$$

$$u_c(c) = \beta_b E\left[\frac{\partial v_b}{\partial x_b}\frac{\partial x_b}{\partial s'}\middle|\theta, d'\right] + \lambda + \mu R_S,$$

where λ is the Lagrange multiplier associated with the capital constraint and μ is the Lagrange multiplier associated with the liquidity constraint. Note that the envelope conditions are

$$\frac{\partial v_b}{\partial x_b} = u_c(c)$$
$$\frac{\partial v_b}{\partial b} = -u_c(c)\frac{\partial \Phi}{\partial b}.$$

Using the envelope condition on the set of first-order conditions, one obtains

$$\left[1 + \frac{\partial \Phi(b', b)}{\partial b'}\right] u_c(c)$$
$$= \beta_b E\left[\left(\theta' g_b(b') + R_L - \phi_b - \frac{\partial \Phi(b'', b')}{\partial b'}\right) u_c(c') \middle| \theta, d'\right]$$
$$+ (1-\chi)\lambda + \bar{\delta}\mu$$
$$u_c(c) = \beta_b E\left[R_S u_c(c') \middle| \theta, d'\right] + \lambda + \mu R_S.$$

2.2.4 Corporate Sector

In this economy there is also a corporate sector that uses a constant-returns-to-scale Cobb-Douglas production function, which uses the capital and labor or workers and entrepreneurs as inputs. The aggregate technology is represented by

$$Y_t = F(K_t, L_t),$$

and aggregate capital, K_t, is assumed to depreciate at rate δ.

2.2.5 Equilibrium

Definition 1 summarizes the steady-state equilibrium in this economy.

Definition 1. *The steady-state equilibrium in this economy is a value function for the worker, $v^w(\epsilon, x^w)$, for the entrepreneur $v^e(z, x^e)$, and for the banker, $v^b(\theta, x_b, b, d')$; the worker's policy functions $\{c^w(\epsilon, x^w), d^w(\epsilon, x^w), a^w(\epsilon, x^w)\}$; the entrepreneur's policy functions $\{c^e(z, x_e), k(z, x_e), l(z, x_e), b^e(z, x_e), a^e(z, x_e)\}$; the banker's policy functions $\{c^b(x_b, b, \theta, d')$, $b^b(x_b, b, \theta, d')$, $s(x_b, b, \theta, d')$, $d(x_b, b, \theta, d')\}$; a constant cross-sectional distribution of worker's* *characteristics, $\Gamma_w(\epsilon, x^w)$ with mass η_w; a constant cross-sectional distribution of entrepreneur's characteristics, $\Gamma_e(z, x^e)$ with mass η_e; a constant cross-sectional distribution of banker's characteristics, $\Gamma_b(x_b, b, \theta, d')$, with mass $(1 - \eta_w - \eta_e)$; and prices (R^D, R^L, R^S, R, w), such that*

(i) *Given R^D, R, and w, the worker's policy functions solve the worker's decision problem (2.1).*

(ii) *Given R, R^L, and w, the entrepreneur's policy functions solve the entrepreneur's decision problem (2.2).*

(iii) *Given R^D, R^L, and R^S, the banker's policy functions solve the banker's decision problem (2.4).*

(iv) *The loan, securities, and deposit markets clear:*

$$\eta_e \int b^e \, d\Gamma_e + (1 - \eta_w - \eta_e) \int b^b \, d\Gamma_b = 0, \qquad \textit{(Loan market)}$$

$$\bar{S} = \eta_e \int s^e \, d\Gamma_e + (1 - \eta_w - \eta_e) \int s^b \, d\Gamma_b, \qquad \textit{(Securities market)}$$

$$\eta_w \int d^w \, d\Gamma_w + (1 - \eta_w - \eta_e) \int d^b \, d\Gamma_b = 0. \qquad \textit{(Deposit market)}$$

(v) *Corporate-sector capital and labor are given by*

$$K = \eta_w \int a^w \, d\Gamma_w$$

$$L = (\eta_w + \eta_e) - \eta_e \int l \, d\Gamma_e.$$

(vi) *Given K and L, the factor prices are equal to factor marginal productivities:*

$$R = 1 + F_K(K, L) - \delta,$$

$$w = F_L(K, L).$$

(vii) *Given the policy functions of workers, entrepreneurs, and bankers, the probability measures of workers, Γ_w, entrepreneurs, Γ_e, and bankers, Γ_b, are invariant.*

2.3 Calibration

The properties of the model can be evaluated only numerically. We assign functional forms and parameters values to obtain the solution of the model and conduct comparative statics exercises. We choose one period in the model to represent one year.

2.3.1 Workers' and Entrepreneurs' Problems

The parameters of the workers' and entrepreneurs' problems are fairly standard, with the exception of the discount factor of entrepreneurs, which is chosen to match the loan rate. The period utility of the workers is assumed to have the following form:

$$u(c_e, d'_w) = \omega\left(\frac{c_w^{1-\gamma_w}}{1-\gamma_w}\right) + (1-\omega)\ln(d'_w),$$

where ω is the relative weight on the marginal utility of consumption and deposits and γ_w is the risk-aversion parameter. We set γ_w to 2, a number often used in representative-agent macroeconomic models. We set ω equal to 0.97 to match the ratio of banking assets relative to output, since this parameter controls the stock of deposits in our economy. The discount factor of workers is set at 0.96, which is standard.

We adopt a constant relative risk-aversion (CRRA) specification for the utility function of entrepreneurs:

$$u(c_e) = \frac{c_e^{1-\gamma_e}}{1-\gamma_e}.$$

We set γ_e to 2, close to that of Quadrini (2000). The idiosyncratic earnings process of workers is first-order Markov with the serial correlation parameter, ρ_ϵ, set to 0.80 and the unconditional standard deviation, σ_ϵ, set to 0.16. Although we lack direct information to calibrate the stochastic process for entrepreneurs, we make the reasonable assumption that the process should be persistent and consistent with the evidence provided by Hamilton (2000) and Moskowitz and Vissing-Jørgensen (2002) that the idiosyncratic risk facing entrepreneurs is larger than the idiosyncratic risk facing workers. Hence, we set the serial correlation of entrepreneurs to 0.70 and the unconditional standard deviation to 0.22.

As is standard in the business cycle literature, we choose a depreciation rate δ of 8 percent for the entrepreneurial as well as the corporate sector. The degree of decreasing returns to scale for entrepreneurs is equal to 0.80—slightly less than Cagetti and De Nardi (2006)—with capital and labor shares of 0.45 and 0.35, respectively. As in Aiyagari (1994), we assume workers are not allowed to have negative assets, and let the maximum leverage ratio of entrepreneurs be at about 50 percent, which corresponds to κ set to 0.5.[4]

The discount factor of entrepreneurs is chosen to match the average loan rate between 1997 and 2012. Based on bank holding company and Call Report data, the weighted average real interest rate charged on loans of all types was 4.6 percent. By setting β_e to 0.95, we obtain approximately this calibration.

2.3.2 Bankers' Problem

We divide the set of parameters of the bankers' problem into two parts: (i) parameters set externally, and (ii) parameters set internally. The parameters set externally are taken directly from outside sources. These include the loan maturity, $\bar{\delta}$, and the capital constraint parameter, χ. In addition, we assume the banker has log utility to minimize the amount of precautionary savings induced by the occasionally binding capital constraint. The remaining nine parameters of the banker's problem are determined so that a set of nine moments in the model are close to a set of nine moments available in the bank holding company and commercial bank Call Reports. The lower panel in table 2.1 reports the parameter values assumed in the parametrization of the banker's problem.

We now describe the parameters set externally. For the capital constraint we assume that the minimum capital requirement in the

[4]Leverage is defined as debt to assets, that is, $-b/k$. At the constraint $b = -\kappa k$, the maximum leverage in the model is equal to $\kappa = 0.50$.

model is equal to 6 percent, which corresponds to the minimum tier 1 ratio a bank must maintain to be considered well capitalized. Thus, χ equals 0.06. The loan maturity parameter, $\bar{\delta}$, is set to 0.24 so that the average maturity of loans is 4.2 years based on the maturity buckets available on banks' Call Reports.

The parameters set internally—namely the banker's discount factor, the intermediation cost, the parameters of the banker's loan technology, the persistence and standard deviation of the shock to deposits, and the adjustment cost parameters—are chosen to match a set of nine moments calculated from regulatory reports. The moments selected are (i) tier 1 capital ratio, (ii) the fraction of capital-constrained banks, (iii) leverage ratio, (iv) adjusted return on assets, (v) the cross-sectional volatility of adjusted return on assets, (vi) the share of assets with a zero or 20 percent Basel I risk weight, (vii) the share of interest income relative to total revenues, (viii) the share of non-interest expenses, and (ix) the return on securities. The upper panel of table 2.2 presents a comparison between the data and the model for this selected set of moments. Given the relatively large number of parameters and that we are solving the model using non-linear methods, it is difficult to match closely the moments of the model with those in the data.

As discussed above, the supplies of certain types of safe assets such as U.S. Treasury securities, Agency debt, and municipal bonds are not directly modeled in our framework. We capture the supply of these assets using the parameter $\overline{S}$. We calibrate this parameter using the estimates of the share of safe assets provided by Gorton, Lewellen, and Metrick (2012). Specifically, that paper estimates that during the post-war period the safe-asset share has fluctuated between 30 and 35 percent. In the model we define the safe-asset share as follows. The numerator includes bank deposits, the exogenous amount of safe assets, $\overline{S}$, and the amount of borrowing by banks in the securities market. The denominator includes all assets in the economy for each of the three types of agents: workers' deposits and corporate-sector assets; entrepreneurs' capital and securities; and bankers' loans and securities. By setting $\overline{S}$ to 9, we obtain a safe-asset share of 33 percent in our calibrated model. The solution of the model is obtained via computational methods and additional details are provided in section 2.4 below.

Table 2.2 Selected Moments

Moment	Data	Model
Tier 1 Capital Ratio	10.0	9.7
Share of Constrained Banks	0.1	0.3
Leverage Ratio	7.0	6.3
Adjusted Return on Assets	2.9	3.4
Cross-Sectional Volatility of Adjusted Return on Assets	1.3	1.4
% Safe Assets Held by Banks	33.1	34.4
Share of Interest Income in Revenues	1.3	0.3
Share of Non-interest Expenses	3.0	8.5
Return on Securities	0.5	0.5
Loan Rate	4.0	4.1
Consumption to Output	0.7	0.7
Banking Assets to Output	0.9	1.2
Safe-to-Total Assets	0.3	0.3
Memo: Deposit Rate	0.1	0.1
% Labor in Entrepreneurial Sector	—	37.6
% Labor in Corporate Sector	—	62.4
% Output of Entrepreneurial Sector	—	48.6
% Output of Corporate Sector	—	44.0
% Output of Banking Sector	—	7.5

Notes: Moments are based on sample averages using quarterly observations between 1997:Q1 and 2012:Q3, with the exception of the percentage of safe assets held by banks, which is only available starting in 2001:Q1, and averages for share of interest income in revenues and banking assets to output are calculated only for the period after the fourth quarter of 2008 when investment banks became bank holding companies. The adjusted return on assets is defined as net income excluding income taxes and salaries and employee benefits. The percentage of safe assets held by banks includes all assets with a zero and with a 20 percent risk weight. The deposit rate is a parameter. The sample includes all banking holding companies and commercial banks that are not part of a BHC, or that are part of a BHC which does not file the Y-9C report. The share of constrained banks is based on banks' responses in the Senior Loan Officer Opinion Survey. The safe-asset share is obtained from Gorton, Lewellen, and Metrick (2012).

2.4 Solution Techniques

2.4.1 Numerical Solution

The numerical algorithm solves the banker's problem by solving for a fixed point in the consumption function by time iteration as in

Coleman (1990). The policy function $c_b(\theta, x_b, b, d')$ is approximated using piecewise bilinear interpolation of the state variables x_b and b. The variables x_b and b are discretized in a non-uniformly spaced grid points with 100 nodes each. More grid points are allocated to lower levels of each state variable. The two stochastic processes, θ and d', are discretized into five and four states, respectively, using the method proposed by Tauchen (1986). The policy functions of consumption for workers and entrepreneurs are also solved by time iteration. Because the state space is smaller, the variables x_w and x_e are discretized in a non-uniformly spaced grid with 900 nodes. The invariant distributions of bankers, workers, and entrepreneurs are derived by computing the inverse decision rules on a finer grid than the one used to compute the optimal decision rules. Finally, the equilibrium prices are determined using a standard quasi-newton method.

2.4.2 Transitional Dynamics

The transition to the new stationary equilibrium is calculated assuming the new steady state is reached after sixty periods ($T = 60$). We take as inputs the steady-state distribution of agents in period $t = 1$ (prior to the change in policy); guesses for the path of R^L, R^S, and K/L between $t = 1$ and $t = T$; and the optimal decision functions at the new steady state. Using those guesses we solve the problem of each agent backwards in time, for $t = T - 1, \ldots, 1$. With the time-series sequence of decision rules for each agent, we simulate the dynamics of the distribution for workers, entrepreneurs, and bankers and check if the loan market, the deposit market, and goods market clear. If these markets are not in equilibrium, we update the path of R^L, R^S, and K/L using a simple linear updating rule. Finally, after convergence of the algorithm, we compare the simulated distribution at $T = 60$ with the steady-state distribution of each agent type obtained after the change in the policy parameters.

2.4.3 Markov Chains

Both the revenue and deposit shocks of the banker follow a first-order Markov process with five and four states, respectively. The Markov-chain process for the revenue process is as follows:

$$\bar{\theta} = [0.69; 0.83; 1.0; 1.21; 1.46]$$

$$\Pi(\theta', \theta) = \begin{bmatrix} 0.42 & 0.55 & 0.03 & 0.00 & 0.00 \\ 0.05 & 0.62 & 0.33 & 0.00 & 0.00 \\ 0.00 & 0.15 & 0.70 & 0.15 & 0.00 \\ 0.00 & 0.00 & 0.33 & 0.62 & 0.05 \\ 0.00 & 0.00 & 0.03 & 0.55 & 0.42 \\ \end{bmatrix}.$$

As for the deposit shock process, we assume

$$\bar{d} = [0.47; 0.78; 1.28; 2.12]$$

$$\Pi(d'|d) = \begin{bmatrix} 0.75 & 0.25 & 0.00 & 0.00 \\ 0.02 & 0.89 & 0.09 & 0.00 \\ 0.00 & 0.09 & 0.89 & 0.02 \\ 0.00 & 0.00 & 0.25 & 0.75 \\ \end{bmatrix}.$$

3. Michael Kiley and Jae Sim: Intermediary Leverage, Macroeconomic Dynamics, and Macroprudential Policy

This appendix provides the description of the structure of the model and the estimation/calibration strategy used in Kiley and Sim (2015). Since the focus of the analysis is on the financial intermediary, the description is more in detail for the sector. However, the description of the other sectors will be very brief.

3.1 Model without Pigovian Tax

The model economy consists of (i) a representative household, (ii) a representative firm producing intermediate goods, (iii) a continuum of monopolistically competitive retailers, (iv) a representative firm producing investment goods, and (v) a continuum of financial intermediaries.

3.1.1 The Financial Intermediary Sector

Financial intermediaries fund investment projects by issuing debt and equity securities. Debt is tax advantaged and is subject to default, while equity issuance is associated with a sizable issuance cost. We adopt the following timing convention: a time period is split into two subperiods where lending and borrowing (e.g., asset and liability) decisions have to be made in the first half of period t; idiosyncratic shocks to the returns of the projects are realized in the second half of period t, at which point lending and borrowing decisions cannot be reversed (until period $t+1$).

3.1.2 Debt Contract

We denote the return on intermediary project by $1 + r^F_{t+1} = \epsilon_{t+1}(1 + r^A_{t+1})$, where r^A_{t+1} is the aggregate component and ϵ_{t+1} is the idiosyncratic component. The latter follows a time-varying log-normal distribution: $\log \epsilon_t \sim N(-0.5\sigma_t^2, \sigma_t^2)$. The time-varying volatility follows:

$$\log \sigma_t = (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log \sigma_{t-1} + \sigma_\sigma v_{\sigma t}, \quad v_{\sigma t} \sim N(0, 1). \quad (3.1)$$

We let $F_t(\cdot) = F(\cdot|\sigma_t)$ denote the cumulative distribution function of ϵ given the realization of σ_t. We also denote the fraction of balance sheet asset funded through equity by m_t. $1 - m_t$ then represents the fraction funded by debts. For each unit of debt financing, the financial intermediary owes $1 + (1 - \tau_c)r^B_{t+1}$, where r^B_{t+1} is the borrowing rate and τ_c is a flat-rate corporate income tax rate. The intermediary is insolvent when the realized return is below its debt obligation:

$$\epsilon_{t+1}(1 + r^A_{t+1}) \leq [1 + (1 - \tau_c)r^B_{t+1}](1 - m_t).$$

We define the default threshold shock as

$$\epsilon^D_{t+1} \equiv \frac{1 + (1 - \tau_c)r^B_{t+1}}{1 + r^A_{t+1}}(1 - m_t). \quad (3.2)$$

Using the default threshold, investors' participation constraint can be expressed as

$$1 - m_t \leq \mathbb{E}_t \Bigg\{ M_{t,t+1} \Bigg[(1 - \eta) \int_0^{\epsilon^D_{t+1}} \epsilon(1 + r^A_{t+1}) dF_{t+1}(\epsilon) + \int_{\epsilon^D_{t+1}}^{\infty} (1 - m_t)(1 + r^B_{t+1}) dF_{t+1} \Bigg] \Bigg\}, \quad (3.3)$$

where the default recovery is discounted by a factor $1 - \eta$ owing to bankruptcy costs and $M_{t,t+1}$ is the stochastic discount factor of the representative household.

3.1.3 Intermediary Equity Finance

We denote the dividend payouts of the intermediary by D_t. When D_t is negative, it should be interpreted as equity issuance. We express equity-related cash flow $\bar{\varphi}(D_t)$ as

$$\bar{\varphi}(D_t) = \begin{cases} D_t & \text{if } D_t \geq 0 \\ -(1-\varphi)D_t & \text{if } D_t < 0. \end{cases} \tag{3.4}$$

Note that $-(1-\varphi_t)D_t < -D_t$ when D_t is negative. This implies that the actual cash flow from the equity issuance of $-D_t$ is strictly less than $-D_t$ owing to equity dilution cost $\varphi \in (0,1)$. The dilution cost is a transfer from old shareholders to new shareholders. In general equilibrium, both are an identical entity. As a result, investors, as a whole, do not gain from this dilution cost. In the extreme of $\varphi = 1$, this would be equivalent to the assumption that the intermediary cannot issue equities. We denote the number of claims that the intermediary purchases by S_t and its unit price by Q_t. The flow of funds constraint for the intermediary is

$$\begin{aligned} Q_t S_t = {} & \max\{0, \epsilon_t(1+r_t^A) - [1+(1-\tau_c)r_t^B](1-m_{t-1})\}Q_{t-1}S_{t-1} \\ & + (1-m_t)Q_t S_t - \bar{\varphi}(D_t). \end{aligned} \tag{3.5}$$

We define an equity-financing trigger ϵ_t^E as the level of idiosyncratic shock below which financial intermediary must raise external funds. The shock threshold can be found by setting $\bar{\varphi}(D_t) = 0$ and solves (3.5) for ϵ_t, guessing that at this level of shock, the intermediary does not default, i.e., $\epsilon_t^E > \epsilon_t^D$:

$$\begin{aligned} \epsilon_t^E &= (1-m_{t-1})\frac{1+(1-\tau_c)r_t^B}{1+r_t^A} + \frac{m_t Q_t}{(1+r_t^A)Q_{t-1}S_{t-1}} \\ &= \epsilon_t^D + \frac{m_t Q_t}{(1+r_t^A)Q_{t-1}S_{t-1}}. \end{aligned} \tag{3.6}$$

(3.6) shows that $\epsilon_t^E > \epsilon_t^D$ indeed.

3.1.4 Value Maximization

The intermediary problem is presented in two stages. We denote the ex ante value of the intermediary by J_t prior to the realization of the idiosyncratic shock. We denote the ex post value $V_t(N_t)$ after the realization. Before the realization of the idiosyncratic shock, the intermediary solves

$$J_t = \max_{Q_t S_t, m_t, \epsilon^B_{t+1}} \left\{ \mathbb{E}^{\epsilon}_t[D_t] + \mathbb{E}_t \left[M_{t,t+1} \mathbb{E}^{\epsilon}_{t+1}[V_{t+1}(N_{t+1})] \right] \right\}$$
$$\text{s.t. (3.3) and (3.5),} \tag{3.7}$$

where the expectation operator $\mathbb{E}^{\epsilon}_t[\cdot]$ is defined with respect to ϵ. After the realization of the idiosyncratic shock, the intermediary solves

$$V_t(N_t) = \max_{D_t} \left\{ D_t + \mathbb{E}_t \left[M_{t,t+1} J_{t+1} \right] \right\} \quad \text{s.t. (3.5).} \tag{3.8}$$

We denote the shadow value of the flow of funds constraint (3.5) by λ_t. The first-order condition for (3.8) is

$$\lambda_t = \begin{cases} 1 & \text{if } D_t \geq 0 \\ 1/(1-\varphi) & \text{if } D_t < 0. \end{cases} \tag{3.9}$$

What matters for the investment problem is not λ_t, but its expected value $\mathbb{E}^{\epsilon}_t[\lambda_t]$. Using (3.6) and (3.9), one can evaluate the expected value as

$$\mathbb{E}^{\epsilon}_t[\lambda_t] = 1 - F_t(\epsilon^E_t) + \frac{F_t(\epsilon^E_t)}{1-\varphi} = 1 + \mu F_t(\epsilon^E_t) > 1, \quad \mu \equiv \frac{\varphi}{1-\varphi}. \tag{3.10}$$

We define standardized default and equity issuance thresholds as $s^D_{t+1} \equiv \sigma^{-1}_{t+1}(\log \epsilon^D_{t+1} + 0.5\sigma^2_{t+1})$ and $s^E_{t+1} \equiv \sigma^{-1}_{t+1}(\log \epsilon^E_{t+1} + 0.5\sigma^2_{t+1})$, respectively. The appendix of Kiley and Sim (2015) derives the first-order conditions of problem (3.7) as

$$Q_t S_t : 1 = \mathbb{E}_t \left\{ M^B_{t,t+1} \frac{1}{m_t} \left[1 + \tilde{r}^A_{t+1} - (1-m_t)[1 + (1-\tau_c) r^B_{t+1}] \right] \right\}, \tag{3.11}$$

$$m_t : \mathbb{E}_t^{\epsilon}[\lambda_t] = \theta_t \Bigg\{ 1 - \mathbb{E}_t \Bigg[M_{t,t+1} \Bigg((1-\eta) r_t^m \Phi(s_{t+1}^D) - \frac{\tau_c - r_t^m}{1-\tau_c} [1 - \Phi(s_{t+1}^D)] \Bigg) \Bigg] \Bigg\}, \tag{3.12}$$

$$\begin{aligned} \epsilon_{t+1}^D : 0 = {} & \mathbb{E}_t \left[M_{t,t+1} \left(\frac{\Phi(s_{t+1}^D)}{1-\varphi_{t+1}} - [1 + \mu_{t+1} \Phi(s_{t+1}^E)] \right) (1 + r_{t+1}^A) \right] \\ & + \theta_t \mathbb{E}_t \Bigg\{ M_{t,t+1} \Bigg[(1-\eta) \frac{\phi(s_{t+1}^D - \sigma_{t+1})}{\sigma_{t+1} \epsilon_{t+1}^D} \\ & + \frac{1}{1-\tau_c} \left(1 - \Phi(s_{t+1}^D) - \frac{\phi(s_{t+1}^D)}{\sigma_{t+1}} \right) \Bigg] (1 + r_{t+1}^A) \Bigg\} \\ & + \theta \, (1 - m_t) \, \mathbb{E}_t \left[M_{t,t+1} \frac{\phi(s_{t+1}^D)}{\sigma_{t+1} \epsilon_{t+1}^D} \frac{\tau_c}{1-\tau_c} \right], \end{aligned} \tag{3.13}$$

where θ_t is the shadow value of the constraint (3.3), the intermediary asset pricing kernel is given by

$$M_{t,t+1}^B \equiv M_{t,t+1} \frac{\mathbb{E}_{t+1}^{\epsilon}[\lambda_{t+1}]}{\mathbb{E}_t^{\epsilon}[\lambda_t]} = M_{t,t+1} \frac{1 + \mu \Phi(s_{t+1}^E)}{1 + \mu \Phi(s_t^E)}, \tag{3.14}$$

and the modified asset return $1 + \tilde{r}_{t+1}^A$ is defined as

$$1 + \tilde{r}_{t+1}^A \equiv \left[\frac{1 + \mu_{t+1} \Phi(s_{t+1}^E - \sigma_{t+1})}{1 + \mu \Phi(s_{t+1}^E)} + \frac{\epsilon_{t+1}^D \Phi(s_{t+1}^D) - \Phi(s_{t+1}^D - \sigma_t)}{(1-\varphi)[1 + \mu_{t+1} \Phi(s_{t+1}^E)]} \right] (1 + r_{t+1}^A). \tag{3.15}$$

The appendix of Kiley and Sim (2015) further shows that the analytical solution for (3.15) is given by

$$1 + \tilde{r}_{t+1}^A \equiv \left[\frac{1 + \mu_{t+1} \Phi(s_{t+1}^E - \sigma_{t+1})}{1 + \mu \Phi(s_{t+1}^E)} + \frac{\epsilon_{t+1}^D \Phi(s_{t+1}^D) - \Phi(s_{t+1}^D - \sigma_t)}{(1-\varphi)[1 + \mu_{t+1} \Phi(s_{t+1}^E)]} \right] (1 + r_{t+1}^A). \tag{3.16}$$

3.1.5 Production and Investment

There is a competitive industry that produces intermediate goods using a constant-returns-to-scale technology; without loss of generality, we assume the existence of a representative firm. The firm combines capital (K) and labor (H) to produce the intermediate goods using a Cobb-Douglas production function,

$$Y_t^M = a_t H_t^{\alpha} K_t^{1-\alpha}, \tag{3.17}$$

where the technology shock follows a Markov process,

$$\log a_t = \rho_a \log a_{t-1} + \sigma_a v_{at}, \quad v_{at} \sim N(0,1). \tag{3.18}$$

The intermediate goods producer issues state-contingent claims S_t to a financial intermediary and uses the proceeds to finance capital purchases, $Q_t K_{t+1}$. No arbitrage implies that the price of the state-contingent claim must be equal to Q_t such that $Q_t S_t = Q_t K_{t+1}$. The firm's static profit per unit of capital is determined by the capital share of revenue, i.e., $r_t^K = (1-\alpha) P_t^M Y_t / K_t$, where P_t^M is the relative price of the intermediate goods. The aggregate return on asset is given by

$$1 + r_t^A = \frac{(1-\tau_c)(1-\alpha)P_t^M Y_t / K_t + [1-(1-\tau_c)\delta] Q_t}{Q_{t-1}}. \tag{3.19}$$

To endogenize the price of capital, we introduce a competitive investment-goods industry, which produces investment goods by combining and consumption goods and undepreciated capital using a quadratic adjustment cost of investment, $\chi_t/2(I_t/I_{t-1} - 1)^2 I_{t-1}$, where χ_t follows a Markov process,

$$\log \chi_t = (1-\rho_\chi)\log \bar{\chi} + \rho_\chi \log \chi_{t-1} + \sigma_\chi v_{\chi t}, \quad v_{\chi t} \sim N(0,1). \tag{3.20}$$

The optimization condition of the investment-goods firm leads to a well-known investment Euler equation.

3.1.6 Households

The preferences of the representative household is specified as

$$\sum_{s=0}^{\infty} \beta^s \left[\frac{1}{1-\gamma} [(C_{t+s} - hC_{t+s-1})^{1-\gamma} - 1] - \frac{1}{1+\nu} H_{t+s}^{1+\nu} \right], \qquad (3.21)$$

where C_t is consumption, H_t is hours worked, β is the time discount factor, γ governs the curvature in the utility function, h is the external habit, and ν is the inverse of the Frisch elasticity of labor supply. The household problem is to optimize over the choices of intermediary bond holdings, intermediary equity holdings, risk-free nominal bond holdings, and labor hours. Of these we skip the static optimizing condition for hours.

The household invests in a perfectly diversified portfolio of intermediary debts, $B_t = \int [1 - m_{t-1}(i)] Q_{t-1} S_{t-1} di$. The optimization condition for bond investment leads to the participation constraint (3.3).

The appendix of Kiley and Sim (2015) shows that the optimization condition of equity investment in intermediary shares satisfies

$$1 = \mathbb{E}_t \left[M_{t,t+1} \frac{\mathbb{E}_{t+1}^{\epsilon}[\max\{D_{t+1}, 0\} + (1 - \varphi_{t+1}) \min\{D_{t+1}, 0\}] + P_{t+1}^S}{P_t^S} \right], \qquad (3.22)$$

where P_t^S is the ex-dividend price of an intermediary share. In our symmetric equilibrium, $P_t^S(i) = P_t^S$ for all $i \in [0, 1]$ because $\mathbb{E}_t[M_{t,t+1} \cdot J_{t+1}]$ does not depend on intermediary-specific variables.[5]

Finally, the household's optimizing condition for risk-free bond holding leads to the well-known consumption Euler equation:

$$1 = \mathbb{E}_t \left[M_{t,t+1} R_t \Xi_t \right]. \qquad (3.23)$$

We assume that the "risk premium" follows a Markov process,

$$\log \Xi_t = \rho_{\Xi} \log \Xi_{t-1} + \sigma_{\Xi} v_{\Xi t}, \quad v_{\Xi t} \sim N(0, 1). \qquad (3.24)$$

[5]In general equilibrium, the existing shareholders and the investors in the new shares are the same entity, the representative household. Hence, costly equity financing does not create a wealth effect for the household, but affects the aggregate allocation through the marginal efficiency conditions of the intermediaries.

3.1.7 Nominal Rigidity and Monetary Policy

To generate nominal rigidity, we assume that the retailers face a quadratic cost in adjusting their prices $P_t(i)$ given by $\chi^p/2\left(P_t(i)/P_{t-1}(i) - (\bar{\Pi}^{1-\kappa}\Pi_{t-1}^{\kappa})\right)^2 Y_t$, where Y_t is the CES aggregate of the differentiated products with an elasticity of substitution ε_t, which follows a Markov process,

$$\log \varepsilon_t = (1-\rho_\varepsilon)\log \bar{\varepsilon} + \rho_\varepsilon \log \varepsilon_{t-1} + \sigma_\varepsilon v_{\varepsilon t}, \quad v_{\varepsilon t} \sim N(0,1). \quad (3.25)$$

κ is the inflation indexation parameter. The optimal pricing decision leads to a well-known Phillips curve, which is both backward and forward looking.

Monetary policy is specified by the following Taylor rule:

$$R_t = R_{t-1}^{\rho_R}\left[\frac{\bar{\Pi}}{\beta}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{r_\Pi}\left(\frac{Y_t - Y^*}{Y^*}\right)^{r_{y^*}}\left(\frac{Y_t}{Y_{t-1}}\right)^{r_{\Delta y}}\right]^{1-\rho_R}\exp(e_t^R), \quad (3.26)$$

where e_t^R is iid monetary policy shock.

3.1.8 Fiscal Policy

The fiscal policy is simply dictated by the period-by-period balanced budget constraint. The revenues for government come from two sources: corporate income tax of the financial intermediaries and lump-sum tax on households. The proceeds from the corporate income tax are assumed to be transferred back to the financial intermediaries in a lump-sum fashion. We also assume that the distortionary subsidies on product prices and wages are funded by the lump-sum tax on the households. In addition, fluctuations in government purchases are a source of autonomous demand shocks, as in Smets and Wouters (2007).

3.2 Pigovian Tax

When the Pigovian tax is introduced, the flow of funds constraint facing the intermediaries becomes

$$Q_tS_t = \max\{0, \epsilon_t(1 + r_t^A) - [1 + (1 - \tau_c)r_t^B](1 - m_{t-1})\}Q_{t-1}S_{t-1} + (1 - \tau_t^m)(1 - m_t)Q_tS_t - \bar{\varphi}(D_t), \tag{3.27}$$

where T_t is the lump-sum transfer of the proceeds from the leverage taxation. In equilibrium $\tau_t^m(1 - m_t)Q_tS_t = T_t$, though T_t is taken as given by the intermediaries. The default threshold is now given by

$$\epsilon_{t+1} \leq \epsilon_{t+1}^D \equiv (1 - m_t)\left[\frac{1 + (1 - \tau_c)r_{t+1}^B}{1 + r_{t+1}^A}\right]. \tag{3.28}$$

Following the same steps, one can derive the following efficiency conditions:

$$Q_tS_t : 1 = \mathbb{E}_t\left[M_{t,t+1}^B \frac{1}{m_t + \tau_t^m(1 - m_t)} \times \left[1 + \tilde{r}_{t+1}^A - (1 - m_t)[1 + (1 - \tau_c)r_{t+1}^B]\right]\right] \tag{3.29}$$

$$m_t : 1 - \tau_t^m)\mathbb{E}_t^\epsilon[\lambda_t] = \theta_t\left\{1 + \frac{\tau_c}{1 - \tau_c}\mathbb{E}_t\left[M_{t,t+1}[1 - \Phi(s_{t+1}^D)]\right]\right\} \tag{3.30}$$

$$\begin{aligned}\epsilon_{t+1}^D : 0 = {} & \mathbb{E}_t\left\{M_{t,t+1}\left[\frac{\Phi(s_{t+1}^D)}{1 - \varphi_{t+1}} - [1 + \mu_{t+1}\Phi(s_{t+1}^E)]\right](1 + r_{t+1}^A)\right\} \\ & + \theta_t\mathbb{E}_t\left\{M_{t,t+1}\left[(1 - \eta)\frac{\phi(s_{t+1}^D - \sigma_{t+1})}{\sigma_{t+1}\epsilon_{t+1}^D}\right.\right. \\ & \left.\left. + \frac{1}{1 - \tau_c}\left(1 - \Phi(s_{t+1}^D) - \frac{\phi(s_{t+1}^D)}{\sigma_{t+1}}\right)\right](1 + r_{t+1}^A)\right\} \\ & + \theta_t(1 - m_t)\mathbb{E}_t\left\{M_{t,t+1}\frac{\tau_c}{1 - \tau_c}\frac{\phi(s_{t+1}^D)}{\sigma_{t+1}\epsilon_{t+1}^D}\right\}.\end{aligned} \tag{3.31}$$

3.3 Calibration/Estimation of Key Parameters

Our approach involves calibration of certain parameters and estimation of others—we assign parameters to each category based on the degree to which observed fluctuations in the data are likely to be informative about parameter values. Our estimation is informed by eight macroeconomic time series. The first six are among those in Smets and Wouters (2007), given below.

$$\text{Change in output per capita} = \hat{y}_t - \hat{y}_{t-1}$$
$$\text{Change in consumption per capita} = \hat{c}_t - \hat{c}_{t-1}$$
$$\text{Change in investment per capita} = \hat{\imath}_t - \hat{\imath}_{t-1}$$
$$\text{Change in nominal wage per capita} = \hat{w}_t - \hat{w}_{t-1}$$
$$\text{Change in hours worked per capita} = \hat{l}_t - \hat{l}_{t-1}$$
$$\text{GDP price inflation} = \hat{\pi}_t$$
$$\text{Nominal federal funds rate} = \hat{r}_t$$

In each case, lowercase letters refer to the natural logarithm of a variable, and we remove the mean from the series prior to estimation.

The last two time series used in estimation are data on long-run expected inflation from the Survey of Professional Forecasters and the excess bond premium from Gilchrist and Zakrajsek (2012), which we link to the model by

$$\text{Expected inflation} = \frac{1}{40}\sum_{j=1}^{40} E_t[\hat{\pi}_{t+j}]$$
$$\text{Excess bond premium} = \frac{1}{20}\sum_{j=1}^{40} E_t[\hat{R}^L_{t+j} - \hat{R}_{t+j}].$$

Table 3.1 summarizes the calibrated parameters. Tables 3.2 and 3.3 report the key estimated parameters and the variance decomposition implied by the estimation results. Our estimation sample spans the periods from 1965 to 2008.

Table 3.1 Baseline Calibration

Description	Calibration
Preferences and Production	
Time Discounting Factor	$\beta = 0.985$
Value-Added Share of Labor	$\alpha = 0.600$
Depreciation Rate	$\delta = 0.025$
Financial Frictions	
Liquidation Cost	$\eta = 0.050$
Corporate Income Tax	$\tau_c = 0.200$
Long-Run Level of Uncertainty	$\bar{\sigma} = 0.030$

Table 3.2 Posterior Moments of Key Parameters

Parameter	Mean	[0.05, 0.95]
Preferences		
γ	1.57	[1.41, 1.72]
h	0.37	[0.30, 0.44]
ν	0.95	[0.63, 1.27]
Financial Friction		
$\bar{\varphi}$	0.24	[0.20, 0.28]
$\bar{\chi}$	4.44	[3.76, 5.13]
Nominal Rigidities		
$\bar{\varepsilon}$	51.69	[41.14, 59.06]
κ	0.07	[0.01, 0.12]
Monetary Policy		
ρ_R	0.72	[0.68, 0.75]
r_{y*}	0.02	[–0.01, 0.06]
$r_{\Delta y}$	0.53	[0.41, 0.64]
r_{Π}	0.72	[0.59, 0.84]

Table 3.3 Variance Decomposition for Observable Variables

	Fin. Vol.	**Nat. Rate of Int.**	**Tobin's Q**	**Technology**	**Markup**	**Nominal Int. Rate**	**Infl. Target**	**Auto. Dem.**
Δy	2.2	15.5	17.2	3.3	27.0	7.5	0.4	27.0
Δc	10.6	38.0	5.3	5.8	13.3	18.3	0.6	8.0
Δi	11.5	4.0	50.6	1.6	29.1	1.9	1.3	0.1
Δh	1.2	11.8	13.3	27.7	20.2	5.7	0.2	20.0
EBP	41.0	51.7	0.0	0.1	2.2	4.3	0.6	0.0
R	0.1	0.1	0.1	0.0	0.0	0.0	99.7	0.0
Π	0.0	0.0	0.0	0.0	0.1	0.0	99.9	0.0
$E\Pi^{40}$	0.0	0.0	0.0	0.0	0.0	0.0	100.0	0.0

4. Albert Queralto: Banks and Outside Equity

This appendix provides details on the model by Gertler, Kiyotaki, and Queralto (2012) included in "Macroeconomic Effects of Banking-Sector Losses across Structural Models." Section 4.1 describes the agents' optimization problems. Section 4.2 contains the model's full set of equilibrium conditions. Section 4.3 describes the calibration of the model parameters.

4.1 Model Setup

4.1.1 Households

The household chooses consumption, labor supply, riskless debt, and outside equity $(C_t, L_t, D_{h,t}, \overline{e}_t)$ to maximize

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\gamma} \left(C_\tau - hC_{\tau-1} - \frac{\chi}{1+\varphi} L_\tau^{1+\varphi} \right)^{1-\gamma} \tag{4.1}$$

subject to

$$\begin{aligned} C_t + D_{h,t} + q_t \overline{e}_t = {} & W_t L_t + \Pi_t - T_t + R_t D_{h,t-1} \\ & + \left[Z_t + (1-\delta) q_t \right] \psi_t \overline{e}_{t-1}. \end{aligned} \tag{4.2}$$

Here q_t is the price of a unit of outside equity, normalized so that each equity is a claim to the future returns of one unit of the asset that the bank holds. Z_t is the flow returns generated by one unit of the bank's asset, δ is the depreciation rate of capital, and ψ_t is the capital quality shock. Thus, the total payoff at t for a share of outside equity acquired at $t-1$ is $[Z_t + (1-\delta)q_t]\psi_t$.

W_t is the wage rate, T_t is lump-sum taxes, and Π_t is net profit from both banks and non-financial firms.

4.1.2 Non-financial Firms

There are two types of non-financial firms: goods producers and capital producers.

4.1.3 Goods Producers

Competitive goods producers use capital K_t and labor L_t as inputs to produce final goods. They operate a production function given by

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}. \tag{4.3}$$

Good producers purchase capital one period in advance. To finance their capital purchases, they issue state-contingent securities to banks, at price Q_t (the price of a unit of physical capital). Then, given capital, in period t firms choose labor to satisfy

$$W_t = (1-\alpha)\frac{Y_t}{L_t}. \tag{4.4}$$

Gross profits per unit of capital, Z_t, are then

$$Z_t \equiv \frac{Y_t - W_t L_t}{K_t} = \alpha \left(\frac{L_t}{K_t}\right)^{1-\alpha}. \tag{4.5}$$

Since there are no financial frictions between firms and banks, through perfect competition the (gross) return on goods firms' securities is $\psi_t\left[Z_t + (1-\delta)Q_t\right]$, and these firms earn zero residual profits state by state.

4.1.4 Capital Producers

Capital producers make new capital goods using final output as input, and are subject to adjustment costs given by $f(I_t/I_{t-1})I_t$, with $f(1) = f'(1) = 0$ and $f''(I_t/I_{t-1}) > 0$. A capital producer chooses I_t to solve

$$\max \quad \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_\tau I_\tau - \left[1 + f\left(\frac{I_\tau}{I_{\tau-1}}\right)\right] I_\tau \right\}. \tag{4.6}$$

Above, $\Lambda_{t,\tau}$ is the household's discount factor between periods t and τ.

4.1.5 Banks

Each bank raises funds by issuing deposits d_t and outside equity to purchase producers' equity, s_t:

$$Q_t s_t = n_t + q_t e_t + d_t. \tag{4.7}$$

The evolution of a bank's net worth (or inside equity), n_t, is

$$n_t = [Z_t + (1-\delta)Q_t]\,\psi_t s_{t-1} - [Z_t + (1-\delta)q_t]\psi_t e_{t-1} - R_t d_{t-1} - \epsilon_t n_{t-1}. \tag{4.8}$$

Above, $\epsilon_t n_{t-1}$ is a capital transfer which subtracts from the bank's resources at the beginning of the period. We assume that the transfer is equal to fraction ϵ_t of previous-period inside equity n_{t-1}, where ϵ_t is an exogenous stochastic process.

The value of the bank at the end of period t is

$$V_t = V(s_t, x_t, n_t) = \mathbb{E}_t \sum_{\tau=t+1}^{\infty} (1-\sigma)\sigma^{\tau-t}\Lambda_{t,\tau} n_\tau, \tag{4.9}$$

where $x_t \equiv \frac{q_t e_t}{Q_t s_t}$, and σ is the banker's survival probability. After obtaining funds, the banker may default on its debt and divert a fraction $\Theta(x_t)$ of assets. The incentive constraint for the bank not to steal is

$$V(s_t, x_t, n_t) \geq \Theta(x_t) Q_t s_t. \tag{4.10}$$

The divertable fraction is

$$\Theta(x_t) = \theta\left(1 + \epsilon x_t + \frac{\kappa}{2}x_t^2\right). \tag{4.11}$$

The bank's problem is to choose assets and outside equity, (s_t, x_t), to maximize (4.9) subject to (4.7), (4.8), and (4.10). To solve the problem, we first conjecture that the bank's value function takes the following form:

$$V_t(s_t, x_t, n_t) = (\mu_{s,t} + x_t \mu_{e,t}) Q_t s_t + \nu_t n_t, \tag{4.12}$$

where $\mu_{s,t}$, $\mu_{e,t}$ and ν_t are coefficients to be determined, which do not depend on the bank's individual state. The Lagrangian for the bank's problem, $\mathcal{L}_t$, is then

$$\mathcal{L}_t = [(\mu_{s,t} + x_t\mu_{e,t})\,Q_t s_t + \nu_t n_t]\,(1+\lambda_t) - \lambda_t\theta\left(1 + \epsilon x_t + \frac{\kappa}{2}x_t^2\right), \tag{4.13}$$

where λ_t is the multiplier on (4.10).

As shown in the working paper version of Gertler, Kiyotaki, and Queralto (2012), the bank's optimality conditions are as follows:

$$Q_t s_t = \phi_t n_t \tag{4.14}$$

$$\phi_t = \frac{\nu_t}{\Theta(x_t) - (\mu_{s,t} + x_t \mu_{e,t})} \tag{4.15}$$

$$\begin{aligned} x_t &= -\frac{\mu_{s,t}}{\mu_{e,t}} + \left[\left(\frac{\mu_{s,t}}{\mu_{e,t}}\right)^2 + \frac{2}{\kappa}\left(1 - \epsilon\frac{\mu_{s,t}}{\mu_{e,t}}\right)\right]^{1/2} \\ &\equiv x\left(\frac{\mu_{s,t}}{\mu_{e,t}}\right), \quad \text{where } \ x' > 0 \ \text{ given } \ \kappa > \frac{1}{2}\epsilon^2 \end{aligned} \tag{4.16}$$

with

$$\nu_t = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{t+1} - \epsilon_{t+1}\right)\right] \tag{4.17}$$

$$\mu_{s,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{k,t+1} - R_{t+1}\right)\right] \tag{4.18}$$

$$\mu_{e,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{t+1} - R_{e,t}\right)\right] \tag{4.19}$$

$$\Omega_{t+1} = 1 - \sigma + \sigma\left[\nu_{t+1} + \phi_{t+1}\left(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1}\right)\right]. \tag{4.20}$$

Note that the marginal value of inside equity, ν_t, includes the term $-\epsilon_{t+1}$, capturing the inside equity transfer in period $t+1$. Above, we have defined the rates of return to non-financial firms' securities and to banks' outside equity, $R_{k,t}$ and $R_{e,t}$, respectively, as

$$R_{k,t} \equiv \psi_t \frac{Z_t + (1-\delta)Q_t}{Q_{t-1}} \tag{4.21}$$

$$R_{e,t} \equiv \psi_t \frac{Z_t + (1-\delta)q_t}{q_{t-1}}. \tag{4.22}$$

4.2 Equilibrium Conditions

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{4.23}$$

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - \mathbb{E}_t\left[\Lambda_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right)\right] \tag{4.24}$$

$$Q_t S_t = \phi_t N_t \tag{4.25}$$

$$S_t = I_t + (1-\delta)K_t \tag{4.26}$$

$$K_{t+1} = \psi_{t+1} S_t \tag{4.27}$$

$$N_t = \sigma\left\{\left[R_{k,t} - R_t - (R_{e,t} - R_t)x_{t-1}\right] Q_{t-1}S_{t-1} + R_t N_{t-1}\right\} + (1-\sigma)\xi Q_{t-1}S_{t-1} - \epsilon_t N_{t-1} \tag{4.28}$$

$$1 = \mathbb{E}_t\left(\Lambda_{t+1} R_{t+1}\right) \tag{4.29}$$

$$0 = \mathbb{E}_t\left[\Lambda_{t+1}\left(R_{e,t+1} - R_{t+1}\right)\right] \tag{4.30}$$

$$\Lambda_t = \beta \frac{u_{c,t}}{u_{c,t-1}} \tag{4.31}$$

$$\phi_t = \frac{\nu_t}{\theta\left(1 + \epsilon x_t + \frac{\kappa}{2}x_t^2\right) - (\mu_{s,t} + x_t \mu_{e,t})} \tag{4.32}$$

$$\nu_t = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{t+1} - \epsilon_{t+1}\right)\right] \tag{4.33}$$

$$\mu_{s,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{k,t+1} - R_{t+1}\right)\right] \tag{4.34}$$

$$\mu_{e,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{t+1} - R_{e,t}\right)\right] \tag{4.35}$$

$$x_t = -\frac{\mu_{s,t}}{\mu_{e,t}} + \left[\left(\frac{\mu_{s,t}}{\mu_{e,t}}\right)^2 + \frac{2}{\kappa}\left(1 - \epsilon\frac{\mu_{s,t}}{\mu_{e,t}}\right)\right]^{1/2} \tag{4.36}$$

$$\Omega_{t+1} = 1 - \sigma + \sigma\left[\nu_{t+1} + \phi_{t+1}\left(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1}\right)\right] \tag{4.37}$$

$$R_{k,t} = \psi_t \frac{\alpha\left(\frac{L_t}{K_t}\right)^{1-\alpha} + (1-\delta)Q_t}{Q_{t-1}} \tag{4.38}$$

$$R_{e,t} = \psi_t \frac{\alpha\left(\frac{L_t}{K_t}\right)^{1-\alpha} + (1-\delta)q_t}{q_{t-1}} \tag{4.39}$$

$$(1-\alpha)\frac{Y_t}{L_t}u_{C,t} = \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi}\right)^{-\gamma}\chi L_t^{\varphi} \tag{4.40}$$

$$u_{C,t} = \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \right)^{-\gamma} - \beta h \mathbb{E}_t \left(C_{t+1} - hC_t - \frac{\chi}{1+\varphi} L_{t+1}^{1+\varphi} \right)^{-\gamma} \tag{4.41}$$

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \tag{4.42}$$

The twenty equilibrium conditions (4.23)–(4.42) determine the twenty endogenous variables Y_t, C_t, I_t, Q_t, q_t, ϕ_t, N_t, S_t, K_{t+1}, $R_{k,t}$, $R_{e,t}$, R_{t+1}, x_t, Λ_t, $u_{C,t}$, ν_t, $\mu_{s,t}$, $\mu_{e,t}$, Ω_t, L_t. The exogenous variables are the capital quality shock, ψ_t, and the bank capital transfer, ϵ_t.

4.3 Calibration

Table 4.1 contains the values assigned to the model's parameters. We choose conventional values for the standard preference and technology parameters: $\gamma, \beta, \alpha, \delta, \chi, \varphi, h$, and the elasticity of investment to Q.

There are five parameters specific to our model: σ, ξ, θ, ϵ, and κ. We set the survival rate of bankers, σ, to 0.9685, implying that bankers survive for eight years on average. We set the remaining four parameters to hit four targets. The first three targets involve characteristics of the low-risk economy, which is meant to capture the "Great Moderation" period. In particular, we target an aggregate leverage ratio (assets to the sum of inside and outside equity) of 4, an average credit spread of 100 basis points annually, and a ratio of outside to inside equity of two-thirds. The final target is having the aggregate leverage ratio fall by a third as the economy moves from low to high risk. The choice of an aggregate leverage of 4 represents a first-pass attempt to average across sectors with vastly different financial structures, from housing finance (featuring very large leverage ratios) to other sectors of the economy where leverage is clearly lower. The target for the spread is based on a rough average of the following spreads over the Great Moderation period: mortgage rates relative to government bonds rates, BAA corporate rates versus government bond rates, and commercial paper rates versus T-bill rates. The target of outside to inside equity approximates the ratio of common equity to the sum of preferred equity and subordinate debt in the banking sector prior to the crisis. Finally, the

Table 4.1 Calibration

γ	2	Risk Aversion
β	0.99	Discount Factor
α	0.33	Capital Share
δ	0.025	Depreciation Rate
χ	0.25	Utility Weight of Labor
φ	1/3	Inverse Frisch Elasticity of Labor Supply
If''/f	1	Inverse Elasticity of Investment to the Price of Capital
h	0.75	Habit Parameter
σ	0.9685	Survival Rate of Bankers
ξ	0.0289	Transfer to Entering Bankers
θ	0.264	Parameter in Asset Diversion Function (1)
ϵ	−1.21	Parameter in Asset Diversion Function (2)
κ	13.41	Parameter in Asset Diversion Function (3)

drop in the aggregate leverage ratio of a third as the economy moves from low to high risk is a rough estimate of what would occur if the financial system undid the buildup of leverage over the last decade.

5. Luca Guerrieri and Mohammad Jahan-Parvar: Capital Shortfalls in a Two-Sector Production Economy

In this appendix we describe the setup of the model by Guerrieri and Jahan-Parvar included in "Macroeconomic Effects of Banking-Sector Losses across Structural Models."

We build the model in layers. We start with a frictionless real business cycle (RBC) model, decentralized in a way that firms operate for only two periods. In the first period they plan and raise equity from households to buy capital and produce the following period. The next layer puts financial intermediaries between households and firms introducing the same principal-agent problem considered by Gertler and Karadi (2011). Building up, we show how to introduce a transfer shock from banks to households. Expanding the one-sector model, we consider an environment in which a fraction of firms can access equity markets directly, without having to reach them through banks. Finally, we layer on nominal rigidities and monetary policy.

5.1 Asset Pricing in a Basic RBC Model

5.1.1 Production

The production technology of the representative firm is

$$Y_t = A_t {K_t}^{\alpha} L_t^{1-\alpha}. \tag{5.1}$$

Firms operate for only one period, but some of the planning for production is done one period in advance. To operate capital in period $t+1$, a firm must purchase it in period t. To do so, the firm issues shares in period t. There are as many shares S_t as units of capital purchased. By arbitrage, the current value of capital equals the value of shares. Thus,

$$Q_t K_{t+1} = Q_t S_t. \tag{5.2}$$

Let π_{t+1} denote the revenue of firms in period $t+1$ net of expenses. Revenues include proceeds from the sale of output as well as from the sale of the undepreciated fraction of capital. Expenses include obligations connected with the servicing of shares and with the compensation for labor services. Thus,

$$\pi_{t+1} = Y_{t+1} + Q_{t+1}(1-\delta)K_{t+1} - W_{t+1}L_{t+1} - (1 + R^s_{t+1})Q_t S_t. \tag{5.3}$$

At time t the problem of firms is to choose S_t and K_{t+1} to maximize the expected profits in period $t+1$, knowing that the firms will be able to choose the optimal quantity of labor in that period. The firm takes Q_t, Q_{t+1}, R^s_{t+1}, and W_{t+1} as given. This maximization problem can be expressed as

$$\max_{S_t, K_{t+1}} E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \max_{L_{t+1}} \pi_{t+1} \tag{5.4}$$

subject to constraints of the production technology $Y_t = A_t {K_t}^{\alpha} L_t^{1-\alpha}$ and financing $Q_t K_{t+1} = Q_t S_t$. The solution of $\max_{L_{t+1}} \pi_{t+1}$ implies that

$$W_{t+1} = (1-\alpha)\,\frac{Y_{t+1}}{L_{t+1}} \tag{5.5}$$

$$L_{t+1} = (1-\alpha)\,\frac{Y_{t+1}}{W_{t+1}} \tag{5.6}$$

under all states of nature. Accordingly, $\max_{S_t,K_{t+1}} E_t \max_{L_{t+1}} \pi_{t+1}$ collapses to

$$\begin{aligned}
\max_{S_t K_{t+1}, L_{t+1}} \; & E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}[A_{t+1}{K_{t+1}}^{\alpha}L_{t+1}^{1-\alpha} \\
& + Q_{t+1}(1-\delta)K_{t+1} - W_{t+1}L_{t+1} - (1+R^s_{t+1})Q_tS_{t.} + \qquad (5.7)\\
& \lambda_{lt+1t}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\,\frac{A_{t+1}{K_{t+1}}^{\alpha}L_{t+1}^{1-\alpha}}{W_{t+1}} - L_{t+1}\right] \\
& + \lambda_{st}\left(Q_tS_{t.} - Q_tK_{t+1}\right). \qquad (5.8)
\end{aligned}$$

Notice that there is no expectation operator on the Lagrangian multipliers because those constraints hold under every state of nature.

The problem implies the following conditions:

$$\frac{\partial}{S_t} = -E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^s_{t+1})Q_t + \lambda_{st}Q_t = 0 \tag{5.9}$$

$$\begin{aligned}
\frac{\partial}{\partial K_{t+1}} &= E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)\right] \\
&\quad + \lambda_{lt+1}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\,\frac{\alpha\frac{Y_{t+1}}{K_{t+1}}}{W_{t+1}}\right] - \lambda_{st}Q_t \qquad (5.10)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial L_{t+1}} &= \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\,\frac{Y_{t+1}}{L_{t+1}} - W_{t+1}\right] + \\
&\quad \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\lambda_{lt+1}\left[(1-\alpha)^2\,\frac{Y_{t+1}}{L_{t+1}W_{t+1}} - 1\right]. \qquad (5.11)
\end{aligned}$$

Working on $\frac{\partial}{S_t}$,

$$\lambda_{st} = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^s_{t+1}). \tag{5.12}$$

From $\frac{\partial}{\partial K_{t+1}}$,

$$\lambda_{st} Q_t = E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta) \right] + \lambda_{lt+1} \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[(1-\alpha) \frac{\alpha \frac{Y_{t+1}}{K_{t+1}}}{W_{t+1}} \right] \tag{5.13}$$

$$E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} (1 + R^s_{t+1}) Q_t = E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[\alpha \frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta) \right] + \lambda_{lt+1} \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[(1-\alpha) \frac{\alpha \frac{Y_{t+1}}{K_{t+1}}}{W_{t+1}} \right] \tag{5.14}$$

$$E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} (1 + R^s_{t+1}) = E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[\frac{1}{Q_t} \alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta) \frac{Q_{t+1}}{Q_t} \right] + E_t \lambda_{lt+1} \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[(1-\alpha) \frac{\alpha \frac{Y_{t+1}}{K_{t+1}}}{W_{t+1}} \right]. \tag{5.15}$$

Next work on $\frac{\partial}{\partial L_{t+1}}$. Again, since $(1-\alpha) \frac{Y_{t+1}}{L_{t+1}} = W_{t+1}$,

$$\frac{\partial}{\partial L_{t+1}} = \lambda_{lt+1} \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[(1-\alpha)^2 \frac{Y_{t+1}}{L_{t+1} W_{t+1}} - 1 \right] = 0. \tag{5.16}$$

Substituting $(1-\alpha) \frac{Y_{t+1}}{L_{t+1}} = W_{t+1}$ again in the equation above, one can see that

$$\frac{\partial}{\partial L_{t+1}} = \lambda_{lt+1} \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} [(1-\alpha) - 1] = 0 \tag{5.17}$$

$$\lambda_{lt+1} = 0. \tag{5.18}$$

Then, combining the implications of $\frac{\partial}{\partial L_{t+1}} = 0$ and $\frac{\partial}{\partial K_{t+1}} = 0$ yields

$$E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} (1 + R^s_{t+1}) = E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \left[\frac{1}{Q_t} \alpha \frac{Y_{t+1}}{K_{t+1}} + (1-\delta) \frac{Q_{t+1}}{Q_t} \right]. \tag{5.19}$$

We can also think of this equation as determining the demand for capital K_{t+1} (or loans S_t). Remembering that K_{t+1} is in the information set at time t, and rearranging,

$$K_{t+1}E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^s_{t+1}) = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha Y_{t+1}+(1-\delta)\frac{Q_{t+1}}{Q_t}K_{t+1}\right] \tag{5.20}$$

$$K_{t+1}E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1+R^s_{t+1})-(1-\delta)\frac{Q_{t+1}}{Q_t}\right] = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha Y_{t+1}\right] \tag{5.21}$$

$$K_{t+1} = \frac{E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha Y_{t+1}\right]}{E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1+R^s_{t+1})-(1-\delta)\frac{Q_{t+1}}{Q_t}\right]}. \tag{5.22}$$

Notice that firms will make zero profits under all states of nature (and that's why we can drop the expectation operator). Thus,

$$0 = Y_{t+1}+Q_{t+1}(1-\delta)K_{t+1}-W_{t+1}L_{t+1}-(1+R^s_{t+1})Q_tS_t \tag{5.23}$$

$$(1+R^s_{t+1})Q_tS_{t.} = Y_{t+1}+Q_{t+1}(1-\delta)K_{t+1}-W_{t+1}L_{t+1} \tag{5.24}$$

$$(1+R^s_{t+1}) = \frac{Y_{t+1}+Q_{t+1}(1-\delta)K_{t+1}-W_{t+1}L_{t+1}}{Q_tS_t} \tag{5.25}$$

$$(1+R^s_{t+1}) = \frac{Y_{t+1}+Q_{t+1}(1-\delta)K_{t+1}-W_{t+1}L_{t+1}}{Q_tK_{t+1}} \tag{5.26}$$

$$(1+R^s_{t+1}) = \frac{Y_{t+1}+Q_{t+1}(1-\delta)K_{t+1}-W_{t+1}\left(1-\alpha\right)\frac{Y_{t+1}}{W_{t+1}}}{Q_tK_{t+1}} \tag{5.27}$$

$$(1+R^s_{t+1}) = \frac{1}{Q_t}\frac{\alpha Y_{t+1}}{K_{t+1}}+\frac{(1-\delta)}{Q_t}Q_{t+1}. \tag{5.28}$$

This condition will also imply $E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^s_{t+1}) = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha\frac{Y_{t+1}}{K_{t+1}}+(1-\delta)\frac{Q_{t+1}}{Q_t}\right]$ derived above (if profits are always zero, it does not matter how you discount them). To interpret the zero-profit condition, notice that if Q_t is the price of capital normalized by the price of consumption, then $\frac{1}{Q_t}$ must be the capital obtained by giving up one unit of consumption. That quantity of capital $\frac{1}{Q_t}$ obtains a rental rate $\frac{\alpha Y_{t+1}}{K_{t+1}}$. After production takes place, the underpreciated portion can be resold at price Q_{t+1}, so the same quantity of capital $\frac{1}{Q_t}$ obtains additionally capital gains equal to $(1-\delta)Q_{t+1}$. Also note that because the condition above holds under every state of nature, it can be written as

$$(1+R_t^s) = \frac{1}{Q_{t-1}}\frac{\alpha Y_t}{K_t} + \frac{(1-\delta)}{Q_{t-1}}Q_t. \tag{5.29}$$

Firms sell their output to households, to the government, and to investment-goods producers. Consequently, the resource constraint can be expressed as

$$Y_t = C_t + I_t^g + G_t. \tag{5.30}$$

5.1.2 Households

A representative household maximizes utility given by

$$\max_{C_{t+i},L_{t+i},S_t,B_t} E_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon} L_{t+i}^{1+\varepsilon}\right]. \tag{5.31}$$

In the absence of financial frictions, households buy shares of firms directly. Then, the budget constraint of households takes the following form:

$$C_t = W_t L_t - T_t - Q_t S_t + (1+R_t^s)Q_{t-1}S_{t-1} - B_t + (1+R_{t-1})B_{t-1}. \tag{5.32}$$

There is a riskless government bond B_t. In period t households purchase B_t of the riskless bond and earn $(1+R_{t-1})B_{t-1}$ from previous purchases. Households take R_t^s, R_t, W_t, and T_t as given.

5.1.3 Capital-Producing Firms

The evolution of capital takes the form

$$K_{t+1} = I_t^n + (1-\delta)\, K_t. \tag{5.33}$$

Net investment is simply given by

$$I_t^n = K_{t+1} - (1-\delta)\, K_t. \tag{5.34}$$

The production technology for investment involves a quadratic adjustment for current production relative to past production, thus the supply of investment goods is given by

$$I_t^n = \left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g. \tag{5.35}$$

Capital-producing firms solve the problem

$$\max_{I_{t+i}^g} E_t \sum_{i=0}^{\infty} \psi_{t,t+i} \left[Q_{t+i}\left[1 - \frac{\phi}{2}\left(\frac{I_{t+i}^g}{I_{t+i-1}^g} - 1\right)^2\right] I_{t+i}^g - I_{t+i.}^g\right]. \tag{5.36}$$

In the maximization, Q_t is taken as given and $\psi_{t,t+i}$ is the stochastic discount factor of households who own the capital-producing firms (defined below).

5.1.4 Necessary Conditions for an Equilibrium

From the side of firms,

$$K_{t+1} = S_{t.} \tag{5.37}$$

$$Y_t = A_t {K_t}^{\alpha} L_t^{1-\alpha}. \tag{5.38}$$

From the solution of $\max_{L_{t+1}} \pi_{t+1}$,

$$L_t = (1-\alpha)\, \frac{Y_t}{W_t}. \tag{5.39}$$

From the zero-profit condition for firms,

$$(1 + R_t^s) = \frac{1}{Q_{t-1}} \frac{\alpha Y_t}{K_t} + \frac{(1-\delta)}{Q_{t-1}} Q_t. \tag{5.40}$$

From the problem for households,

$$\max_{C_{t+i},L_{t+i},S_{t+i},B_{t+i}} U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon} L_{t+i}^{1+\varepsilon} \right]$$
$$+ \beta^i \lambda_{ct+i} \left(-C_{t+i} + W_{t+i}L_{t+i} - T_{t+i}\right.$$
$$- Q_{t+i}S_{t+i} + (1+R^s_{t+i})Q_{t-1+i}S_{t-1+i} - B_{t+i}$$
$$\left. + (1+R_{t-1+i})B_{t-1+i}\right) \tag{5.41}$$

$$\frac{\partial U_t}{\partial C_t} = \frac{1}{C_t - \gamma C_{t-1}} - \lambda_{ct} - E_t\beta \frac{\gamma}{C_{t+1} - \gamma C_t} = 0 \tag{5.42}$$

$$\frac{\partial U_t}{\partial L_t} = -\chi L_t^{\varepsilon} + \lambda_{ct} W_t = 0 \tag{5.43}$$

$$\frac{\partial U_t}{\partial S_t} = -\lambda_{ct} Q_t + E_t \beta \lambda_{ct+1} Q_t (1 + R^s_{t+1}) = 0 \tag{5.44}$$

$$\lambda_{ct} = E_t \beta \lambda_{ct+1} (1 + R^s_{t+1}) \tag{5.45}$$

$$\frac{\partial U_t}{\partial B_t} = -\lambda_{ct} + E_t \beta \lambda_{ct+1} (1 + R_t) = 0 \tag{5.46}$$

$$\lambda_{ct} = E_t \beta \lambda_{ct+1} (1 + R_t) \tag{5.47}$$

$$E_t \frac{\lambda_{ct+1}}{\lambda_{ct}} = \frac{1}{\beta(1+R_t)}. \tag{5.48}$$

Define the stochastic discount factor $\psi_{t,t+i}$ as $E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}} = \frac{1}{1+R_t}$.

The evolution of capital takes the form

$$K_{t+1} = I_t^n + (1-\delta)\, K_t. \tag{5.49}$$

From the maximization problem for capital-producing firms,

$$\max_{I^g_{t+i}} E_t \sum_{i=0}^{\infty} \psi_{t,t+i} \left[Q_{t+i} \left[1 - \frac{\phi}{2} \left(\frac{I^g_{t+i}}{I^g_{t+i-1}} - 1 \right)^2 \right] I^g_{t+i} - I^g_{t+i.} \right] \tag{5.50}$$

$$\frac{\partial}{\partial I_t^g} = Q_t\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] - Q_t\phi\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)\frac{I_t^g}{I_{t-1}^g} - 1$$
$$+ \psi_{t,t+1}Q_{t+1}\phi\left(\frac{I_{t+1}^g}{I_t^g} - 1\right)\frac{I_{t+1}^g}{I_t^{g2}}I_{t+1}^g. \tag{5.51}$$

And from the resource constraint,

$$Y_t = C_t + I_t^g + G_t. \tag{5.52}$$

Finally, G_t is set as a fixed share of Y_t and the government's budget is balanced every period.

5.2 Introducing Financial Constraints Following Gertler and Karadi (2011)

The problem of the firms is unchanged, but they are prevented from issuing shares to households directly. Instead, they need to use financial intermediaries, which are dubbed "banks" and are described below.

5.2.1 Households

The representative household has a continuum of members. A fraction $1 - f$ of members in this continuum supplies labor to firms and returns the wage earned to the household. A fraction f of members in the continuum works as bankers. The consumption of workers and bankers within the household is equalized. As before, the utility function is

$$U_t = E_t\sum_{i=0}^{\infty}\beta^i\left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon}L_{t+i}^{1+\varepsilon}\right]. \tag{5.53}$$

However, in this case, the budget constraint takes the form

$$C_t = W_tL_t + \Pi_t - T_t - D_t + (1 + R_{t-1})D_{t-1}. \tag{5.54}$$

The term D_t represents the amount of deposits with banks (not owned by the household).

Because banks may be financially constrained, they have an incentive to retain earnings. To avoid making the financial constraint irrelevant with iid probability $1-\theta$, a banker exits next period. Upon exiting, bankers transfer retained earnings back to the households and become workers. Each period $(1-\theta)\,f$ workers are selected to become bankers. These new bankers receive a startup transfer from the family. By construction, the fraction of household members in each group is constant over time. Π_t is net funds transferred to the household from its banker members; that is, funds transferred from existing bankers minus the funds transferred to new bankers.

5.2.2 Banks

Banks lend funds obtained from households to non-financial firms. Let $N_t(j)$ be the amount of wealth—or net worth—that a banker j has at the end of period t.

$$Q_t S_t(j) = N_t(j) + D_t(j) \tag{5.55}$$

As noted earlier, deposits $D_t(j)$ pay the non-state-contingent return $(1+R_t)$ at time $t+1$. Thus $D_t(j)$ may be thought of as the debt of bank j, and $N_t(j)$ as its capital. As seen above, the shares $S_t(j)$ earn the stochastic return $(1+R^s_{t+1})$ at time $t+1$.

Over time, the banker's equity capital evolves as the difference between earnings on assets and interest payments on liabilities:

$$N_{t+1}(j) = (1+R^s_{t+1})Q_t S_t(j) - (1+R_t)D_t(j) \tag{5.56}$$

$$D_t(j) = Q_t S_t(j) - N_t(j) \tag{5.57}$$

$$N_{t+1}(j) = (1+R^s_{t+1})Q_t S_t(j) - (1+R_t)\left(Q_t S_t(j) - N_t(j)\right) \tag{5.58}$$

$$N_{t+1}(j) = \left[(1+R^s_{t+1}) - (1+R_t)\right] Q_t S_t(j) + (1+R_t)N_t(j) \tag{5.59}$$

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right) Q_t S_t(j) + (1+R_t)N_t(j). \tag{5.60}$$

Let $\psi_{t,t+j} = \beta^j \frac{\lambda_{ct+j}}{\lambda_{ct}}$ be the stochastic discount factor between periods t and $t+i$. The banker's objective is to maximize expected terminal wealth, given by

$$\max_{s_{t+i}(j)} V_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i} \left[\left(R^s_{t+1+i} - R_{t+i} \right) Q_{t+i} S_{t+i}(j) + (1+R_{t+i}) N_{t+i}(j) \right]. \tag{5.61}$$

Notice that there is an asymmetry between period 0 and all subsequent periods. If a bank has to quit in period 0, it does not conduct any operations and revenues are 0. Since the banker will not fund assets with a discounted return less than the discounted cost of borrowing, for the bank to operate in period $t + i$, it must be that $E_t\psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right) \geq 0$, i.e., there are expected positive excess returns from holding stocks even after discounting and adjusting for risk through $\psi_{t,t+1+i}$. In the absence of financial frictions, when $E_t\psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right)$ is positive, the bank will want to expand its balance sheet by attracting additional deposits from households.

To limit the ability of banks to attract deposits indefinitely, consider the following agency problem. At the beginning of each period, a banker can choose to transfer a fraction λ of assets (in period t those assets equal $Q_tS_t(j)$) back to his household. If the banker makes the transfer, depositors will force the bank into bankruptcy and recover the remaining fraction $1 - \lambda$ of assets. Thus, households are willing to make deposits only if the incentive-compatibility constraint is satisfied:

$$V_t(j) \geq \lambda Q_t S_t(j). \tag{5.62}$$

This constraint says that the expected terminal wealth for period t needs to be at least as large as the fraction of assets that can be diverted in that period. The left-hand side is the cost of diverting assets; the right-hand side is the benefit. When the constraint binds, it affects the ability to raise deposits and will imply expected positive excess returns in equilibrium. Next we show that the ability of the banks to attract deposits is related to their net worth. For this purpose, it is useful to separate the recursive form of net worth into a component that depends on total assets $v_t(j)$ and a component that depends on net worth $\eta_t(j)$. The form we are after is the following:

$$V_t(j) = v_t Q_t S_t(j) + \eta_t N_t(j) \tag{5.63}$$

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) + \psi_{t,t+1}\theta\frac{Q_{t+i}S_{t+i}(j)}{Q_tS_t(j)}v_{t+1}(j) \tag{5.64}$$

$$\eta_t(j) = E_t\,(1-\theta) + \psi_{t,t+1}\theta\frac{N_{t+1}(j)}{N_t(j)}\eta_{t+1}(j). \tag{5.65}$$

Notice that

$$\begin{aligned} V_t(j) &= E_t\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right)Q_{t+i}S_{t+i}(j) \\ &\quad + E_t\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t,t+1+i}(1+R_{t+i})N_{t+i}(j). \end{aligned} \tag{5.66}$$

Define

$$v_t(j) = E_t\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right)\frac{Q_{t+i}S_{t+i}(j)}{Q_tS_t(j)} \tag{5.67}$$

$$\eta_t(j) = E_t\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t,t+1+i}(1+R_{t+i})\frac{N_{t+i}(j)}{N_t(j)}. \tag{5.68}$$

Then

$$V_t(j) = v_t(j)Q_tS_t(j) + \eta_t(j)N_t(j). \tag{5.69}$$

Next write $v_t(j)$ and $\eta_t(j)$ recursively. Start by pulling out the first term in each summation,

$$\begin{aligned} v_t(j) &= E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right)\frac{Q_tS_t(j)}{Q_tS_t(j)} \\ &\quad + \sum_{i=1}^{\infty}(1-\theta)\,\theta^i\psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right)\frac{Q_{t+i}S_{t+i}(j)}{Q_tS_t(j)} \end{aligned} \tag{5.70}$$

$$\eta_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t)\frac{N_t(j)}{N_t(j)} + \sum_{i=1}^{\infty}(1-\theta)\,\theta^i\psi_{t,t+1+i}(1+R_{t+i})\frac{N_{t+i}(j)}{N_t(j)}. \tag{5.71}$$

Now transform the summations so that they start from 0:

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) + \theta\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t,t+2+i}\left(R^s_{t+2+i} - R_{t+1+i}\right)\frac{Q_{t+1+i}S_{t+1+i}(j)}{Q_tS_t(j)} \tag{5.72}$$

$$\eta_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) + \theta\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t,t+2+i}(1+R_{t+1+i})\frac{N_{t+1+i}(j)}{N_t(j)}. \tag{5.73}$$

Express $\psi_{t,t+2+i}$ as a function of $\psi_{t+1,t+2+i}$. Remember that $\psi_{t,t+j} = \beta^j\frac{\lambda_{ct+j}}{\lambda_{ct}}$. Thus, $\psi_{t+1,t+2+i} = \beta^{1+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct+1}}$ and $\psi_{t,t+2+i} = \beta^{2+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct}}$. Notice that

$$\psi_{t,t+2+i} = \beta\beta^{1+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct}}\frac{\lambda_{ct+1}}{\lambda_{ct+1}} \tag{5.74}$$

$$= \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\beta^{1+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct+1}} \tag{5.75}$$

$$= \psi_{t,t+1}\psi_{t+1,t+2+i}. \tag{5.76}$$

Substituting $\psi_{t,t+2+i} = \psi_{t,t+1}\psi_{t+1,t+2+i}$ into the last equations for $v_t(j)$ and for $\eta_t(j)$, one can see that

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) + \theta\psi_{t,t+1}\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t+1,t+2+i}\left(R^s_{t+2+i} - R_{t+1+i}\right) \times \frac{Q_{t+1+i}S_{t+1+i}(j)}{Q_tS_t(j)} \tag{5.77}$$

$$\eta_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) + \theta\psi_{t,t+1}\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t+1,t+2+i}(1+R_{t+1+i})\frac{N_{t+1+i}(j)}{N_t(j)}. \tag{5.78}$$

But the above equations can also be written as

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t+1,t+2+i} \times \left(R^s_{t+2+i}-R_{t+1+i}\right)\frac{Q_{t+1+i}S_{t+1+i}(j)}{Q_{t+1}S_{t+1}(j)}$$

$$\eta_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) + \theta\psi_{t,t+1}\frac{N_{t+1}(j)}{N_t(j)}\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t+1,t+2+i}(1+R_{t+1+i}) \times \frac{N_{t+1+i}(j)}{N_{t+1}(j)},$$

which yields

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}v_{t+1}(j) \tag{5.79}$$

$$\eta_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) + \theta\psi_{t,t+1}\frac{N_{t+1}(j)}{N_t(j)}\eta_{t+1}(j), \tag{5.80}$$

but remember that from the households' problem $E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}} = \frac{1}{(1+R_t)}$

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}v_{t+1}(j) \tag{5.81}$$

$$\eta_t(j) = E_t\,(1-\theta) + \theta\psi_{t,t+1}\frac{N_{t+1}(j)}{N_t(j)}\eta_{t+1}(j). \tag{5.82}$$

Q.E.D.

Since all banks have access to the same investment opportunities, $\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}$ will be equalized across all j and similarly for $\frac{N_{t+1+i}(j)}{N_{t+1}(j)}$. Consequently, we can drop the dependence on j and simply carry around υ_t and η_t. Notice that υ_t and η_t have an interesting interpretation: υ_t is the expected discounted marginal gain of expanding assets Q_tS_t by one unit holding net worth constant; η_t is the expected discounted value of having another unit of net worth $N_t(j)$ holding Q_tS_t constant. Notice that υ_t is zero in a frictionless market without the agency problem.

Substituting

$$V_t(j) = \upsilon_t Q_t S_t(j) + \eta_t N_t(j) \tag{5.83}$$

into the incentive-compatibility constraint

$$V_t(j) \geq \lambda Q_t S_t(j), \tag{5.84}$$

one obtains that

$$\upsilon_t Q_t S_t(j) + \eta_t N_t(j) \geq \lambda Q_t S_t(j). \tag{5.85}$$

When this constraint binds,

$$Q_t S_t(j) = \frac{\eta_t}{(\lambda - \upsilon_t)} N_t(j). \tag{5.86}$$

Therefore, $\frac{\eta_t}{(\lambda-\upsilon_t)}$ is the ratio of assets to equity. This constraint limits the leverage ratio of the intermediary to the point where the banker's incentive to cheat is exactly balanced by the costs. Holding $N_t(j)$ constant, expanding $S_t(j)$ raises the banker's incentive to divert funds. To prove this, I need to show that $\frac{\partial V_t(j)}{\partial S_t(j)} < \frac{\partial \lambda Q_t S_t(j)}{\partial S_t(j)} = \lambda Q_t$. From

$$\upsilon_t Q_t S_t(j) + \eta_t N_t(j) \geq \lambda Q_t S_t(j), \tag{5.87}$$

given that $\eta_t N_t(j) > 0$, it must be that the constraint binds if $\upsilon_t < \lambda$. Additionally, we know that if the constraint binds, $\upsilon_t > 0$. Hence, for the constraint to bind, it must be that $\lambda > 0$.

Using $Q_tS_t(j) = \frac{\eta_t}{(\lambda-\upsilon_t)}N_t(j)$ and the evolution of net worth derived above,

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right) Q_t S_t(j) + (1 + R_t) N_t(j) \tag{5.88}$$

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} N_t(j) + (1 + R_t) N_t(j)$$
$$= \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] N_t(j). \tag{5.89}$$

It also follows that $\frac{N_{t+1}(j)}{N_t(j)}$, conditional on surviving, as used above, is given by

$$\frac{N_{t+1}(j)}{N_t(j)} = \left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1 + R_t). \tag{5.90}$$

In turn, $\frac{Q_{t+1} S_{t+1}(j)}{Q_t S_t(j)}$ is given by

$$\frac{Q_{t+1} S_{t+1}(j)}{Q_t S_t(j)} = \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}} \frac{N_{t+1}(j)}{N_t(j)}$$
$$= \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}} \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right]. \tag{5.91}$$

Consequently, v_t and η_t are equalized across all j and evolve according to

$$v_t = E_t\,(1 - \theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right)$$
$$+ \theta \psi_{t,t+1} \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}} \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] v_{t+1}(j) \tag{5.92}$$

$$\eta_t = E_t\,(1 - \theta) + \theta \psi_{t,t+1} \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] \eta_{t+1}(j). \tag{5.93}$$

Since $\frac{\eta_{t+1}}{(\lambda - v_{t+1})}$ is independent of j, one can aggregate across banks to obtain

$$\int_j Q_t S_t(j) dj = \int_j \frac{\eta_t}{(\lambda - v_t)} N_t(j) dj \tag{5.94}$$

$$Q_t S_t = \frac{\eta_t}{(\lambda - v_t)} N_t. \tag{5.95}$$

Finally, recognize that there is a distinction between the net worth of continuing and new bankers. Aggregate net worth is the sum the two types: Bankers that survive from period $t-1$ to period t will have aggregate net worth equal to

$$\theta\left[(R_t^s - R_{t-1})\frac{\eta_{t-1}}{(\lambda - v_{t-1})} + (1+R_{t-1})\right]N_{t-1}. \tag{5.96}$$

Assume that new bankers receive as endowment a fixed fraction of the current value of the assets intermediated by exiting bankers in the previous period, amounting to $(1- \theta)\,Q_t S_{t-1}$. Assume that the household transfers the fraction $\frac{\omega}{(1- \theta)}$ of that amount to new bankers. Thus, in the aggregate,

$$N_t^n = \frac{\omega}{(1- \theta)}(1- \theta)\,Q_t S_{t-1} = \omega Q_t S_{t-1}. \tag{5.97}$$

Then, current net worth is the sum of net worth carried from the previous period by surviving firms $\theta\big[(R_t^s - R_{t-1})\frac{\eta_{t-1}}{(\lambda - v_{t-1})} + (1+R_{t-1})\big]N_{t-1}$, plus the net worth of new entrants, $\omega Q_t S_{t-1}$, i.e.,

$$N_t = \theta\left[(R_t^s - R_{t-1})\frac{\eta_{t-1}}{(\lambda - v_{t-1})} + (1+R_{t-1})\right]N_{t-1} + \omega Q_t S_{t-1}. \tag{5.98}$$

5.3 Introducing Transfer Shocks between Banks and Households

Change the problem of the households to be

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon} L_{t+i}^{1+\varepsilon}\right]. \tag{5.99}$$

However, in this case, the budget constraint takes the form

$$C_t = W_t L_t + \Pi_t - T_t + \tau_t N_t - D_t + (1+R_{t-1})D_{t-1}. \tag{5.100}$$

Notice that BT_t is a transfer shock from banks back to households in a lump-sum fashion.

5.3.1 Banks

Banks lend funds obtained from households to non-financial firms. Let $N_t(j)$ be the amount of wealth—or net worth—that a banker j has at the end of period t.

$$Q_t S_t(j) = N_t(j)\,(1-\tau_t) + D_t(j) \tag{5.101}$$

As noted earlier, deposits $D_t(j)$ pay the non-state-contingent return $(1 + R_t)$ at time $t + 1$. Thus $D_t(j)$ may be thought of as the debt of bank j, and $N_t(j)$ as its capital. As seen above, the shares $S_t(j)$ earn the stochastic return $(1 + R^s_{t+1})$ at time $t + 1$.

Over time, the banker's equity capital evolves as the difference between earnings on assets and interest payments on liabilities:

$$N_{t+1}(j) = (1 + R^s_{t+1})Q_t S_t(j) - (1 + R_t)D_t(j) \tag{5.102}$$

$$D_t(j) = Q_t S_t(j) - N_t(j)\,(1-\tau_t) \tag{5.103}$$

$$N_{t+1}(j) = (1 + R^s_{t+1})Q_t S_t(j) - (1 + R_t)\,(Q_t S_t(j) - N_t(j)\,(1-\tau_t)) \tag{5.104}$$

$$N_{t+1}(j) = \left[(1 + R^s_{t+1}) - (1 + R_t)\right] Q_t S_t(j) + (1 + R_t)N_t(j)\,(1-\tau_t) \tag{5.105}$$

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right) Q_t S_t(j) + (1 + R_t)N_t(j)\,(1-\tau_t)\,. \tag{5.106}$$

Let $\psi_{t,t+j} = \beta^j \frac{\lambda_{ct+j}}{\lambda_{ct}}$ be the stochastic discount factor between periods t and $t+i$. The banker's objective is to maximize expected terminal wealth, given by

$$\max_{s_{t+i}(j)} V_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i}[\left(R^s_{t+1+i} - R_{t+i}\right) Q_{t+i} S_{t+i}(j) + (1 + R_{t+i})N_{t+i}(j)\,(1-\tau_{t+i})]. \tag{5.107}$$

Since the banker will not fund assets with a discounted return less than the discounted cost of borrowing, for the bank to operate in period $t + i$, it must be that $E_t\psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right) \geq 0$, i.e., there are expected positive excess returns from holding stocks even after discounting and adjusting for risk through $\psi_{t,t+1+i}$. In the absence of financial frictions, when $E_t\psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right)$ is positive, the bank will want to expand its balance sheet by attracting additional deposits from households.

To limit the ability of banks to attract deposits indefinitely, consider the following agency problem. At the beginning of each period, a banker can choose to transfer a fraction λ of assets (in period t those assets equal $Q_t S_t(j)$) back to his household. If the banker makes the transfer, depositors will force the bank into bankruptcy and recover the remaining fraction $1 - \lambda$ of assets. Thus, households are willing to make deposits only if the incentive-compatibility constraint is satisfied:

$$V_t(j) \geq \lambda Q_t S_t(j). \tag{5.108}$$

This constraint says that the expected terminal wealth for period t needs to be at least as large as the fraction of assets that can be diverted in that period. The left-hand side is the cost of diverting assets; the right-hand side is the benefit. When the constraint binds, it affects the ability to raise deposits and will imply expected positive excess returns in equilibrium. Next we show that the ability of the banks to attract deposits is related to their net worth. For this purpose, it is useful to separate the recursive form of net worth into a component that depends on total assets $v_t(j)$ and a component that depends on net worth $\eta_t(j)$.

Notice that

$$\begin{aligned} V_t(j) = {} & E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i}\right) Q_{t+i} S_{t+i}(j) \\ & + E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i} (1 + R_{t+i}) N_{t+i}(j) \left(1 - \tau_{t+i}\right). \end{aligned} \tag{5.109}$$

Define

$$v_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i}\right) \frac{Q_{t+i} S_{t+i}(j)}{Q_t S_t(j)} \tag{5.110}$$

$$\eta_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i}(1+R_{t+i}) \frac{N_{t+i}(j)\,(1-\tau_{t+i})}{N_t(j)\,(1-\tau_t)}. \tag{5.111}$$

Then

$$V_t(j) = v_t(j) Q_t S_t(j) + \eta_t(j) N_t(j)\,(1-\tau_t)\,. \tag{5.112}$$

Next write $v_t(j)$ and $\eta_t(j)$ recursively. Start by pulling out the first term in each summation,

$$\begin{aligned} v_t(j) = {} & E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) \frac{Q_t S_t(j)}{Q_t S_t(j)} \\ & + \sum_{i=1}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right) \frac{Q_{t+i} S_{t+i}(j)}{Q_t S_t(j)} \end{aligned} \tag{5.113}$$

$$\begin{aligned} \eta_t(j) = {} & E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) \frac{N_t(j)\,(1-\tau_t)}{N_t(j)\,(1-\tau_t)} \\ & + \sum_{i=1}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i}(1+R_{t+i}) \frac{N_{t+i}(j)\,(1-\tau_{t+i})}{N_t(j)\,(1-\tau_t)}. \end{aligned} \tag{5.114}$$

Now transform the summations so that they start from 0:

$$\begin{aligned} v_t(j) = {} & E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) \\ & + \theta \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+2+i}\left(R^s_{t+2+i} - R_{t+1+i}\right) \frac{Q_{t+1+i} S_{t+1+i}(j)}{Q_t S_t(j)} \end{aligned} \tag{5.115}$$

$$\begin{aligned} \eta_t(j) = {} & E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) \\ & + \theta \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+2+i}(1+R_{t+1+i}) \frac{N_{t+1+i}(j)\,(1-\tau_{t+i})}{N_t(j)\,(1-\tau_t)}. \end{aligned} \tag{5.116}$$

Express $\psi_{t,t+2+i}$ as a function of $\psi_{t+1,t+2+i}$. Remember that $\psi_{t,t+j} = \beta^j \frac{\lambda_{ct+j}}{\lambda_{ct}}$. Thus, $\psi_{t+1,t+2+i} = \beta^{1+i} \frac{\lambda_{ct+2+i}}{\lambda_{ct+1}}$ and $\psi_{t,t+2+i} = \beta^{2+i} \frac{\lambda_{ct+2+i}}{\lambda_{ct}}$. Notice that

$$\begin{aligned}\psi_{t,t+2+i} &= \beta\beta^{1+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct}}\frac{\lambda_{ct+1}}{\lambda_{ct+1}} \\ &= \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\beta^{1+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct+1}} \\ &= \psi_{t,t+1}\psi_{t+1,t+2+i}. \end{aligned} \tag{5.117}$$

Substituting $\psi_{t,t+2+i} = \psi_{t,t+1}\psi_{t+1,t+2+i}$ into the last equations for $v_t(j)$ and for $\eta_t(j)$, one can see that

$$\begin{aligned} v_t(j) = {} & E_t\left(1-\theta\right)\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) \\ & + \theta\psi_{t,t+1}\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^i\psi_{t+1,t+2+i}\left(R^s_{t+2+i}-R_{t+1+i}\right) \\ & \times \frac{Q_{t+1+i}S_{t+1+i}(j)}{Q_tS_t(j)} \end{aligned} \tag{5.118}$$

$$\begin{aligned} \eta_t(j) = {} & E_t\left(1-\theta\right)\psi_{t,t+1}(1+R_t) \\ & + \theta\psi_{t,t+1}\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^i\psi_{t+1,t+2+i}(1+R_{t+1+i})\frac{N_{t+1+i}(j)}{N_t(j)} \\ & \times \frac{(1-\tau_{t+i})}{(1-\tau_t)}. \end{aligned} \tag{5.119}$$

But the above equations can also be written as

$$\begin{aligned} v_t(j) = {} & E_t\left(1-\theta\right)\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) \\ & + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^i\psi_{t+1,t+2+i} \\ & \times\left(R^s_{t+2+i}-R_{t+1+i}\right)\frac{Q_{t+1+i}S_{t+1+i}(j)}{Q_{t+1}S_{t+1}(j)} \\ \eta_t(j) = {} & E_t\left(1-\theta\right)\psi_{t,t+1}(1+R_t) \\ & + \theta\psi_{t,t+1}\frac{N_{t+1}(j)}{N_t(j)}\frac{(1-\tau_{t+1})}{(1-\tau_t)}\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^i\psi_{t+1,t+2+i} \\ & \times(1+R_{t+1+i})\frac{N_{t+1+i}(j)}{N_{t+1}(j)}\frac{(1-\tau_{t+1+i})}{(1-\tau_{t+1})}, \end{aligned}$$

which yields

$$v_t(j) = E_t (1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_t S_t(j)} v_{t+1}(j) \tag{5.120}$$

$$\eta_t(j) = E_t (1-\theta)\,\psi_{t,t+1}(1+R_t) + \theta\psi_{t,t+1} \times \frac{N_{t+1}(j)}{N_t(j)}\frac{(1-\tau_{t+1})}{(1-\tau_t)}\eta_{t+1}(j), \tag{5.121}$$

but remember that from the households' problem $E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}} = \frac{1}{(1+R_t)}$,

$$v_t(j) = E_t (1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_t S_t(j)} v_{t+1}(j) \tag{5.122}$$

$$\eta_t(j) = E_t (1-\theta) + \theta\psi_{t,t+1}\frac{N_{t+1}(j)}{N_t(j)}\frac{(1-\tau_{t+1})}{(1-\tau_t)}\eta_{t+1}(j). \tag{5.123}$$

Since all banks have access to the same investment opportunities, $\frac{Q_{t+1}S_{t+1}(j)}{Q_t S_t(j)}$ will be equalized across all j and similarly for $\frac{N_{t+1+i}(j)}{N_{t+1}(j)}$. Consequently, we can drop the dependence on j and simply carry around v_t and η_t.

Substituting

$$V_t(j) = v_t Q_t S_t(j) + \eta_t N_t(j)\,(1-\tau_t) \tag{5.124}$$

into the incentive-compatibility constraint

$$V_t(j) \geq \lambda Q_t S_t(j), \tag{5.125}$$

one obtains that

$$v_t Q_t S_t(j) + \eta_t N_t(j)\,(1-\tau_t) \geq \lambda Q_t S_t(j). \tag{5.126}$$

When this constraint binds,

$$Q_t S_t(j) = \frac{\eta_t}{(\lambda - v_t)} N_t(j)\,(1-\tau_t)\,. \tag{5.127}$$

Therefore, $\frac{\eta_t}{(\lambda - v_t)}$ is the ratio of assets to equity. This constraint limits the leverage ratio of the intermediary to the point where the banker's incentive to cheat is exactly balanced by the costs. Next derive $\frac{N_{t+1}(j)}{N_t(j)}$ and $\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}$.

$$N_{t+1}(j) = \left[(R^s_{t+1} - R_t)\frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] N_t(j)(1 - \tau_t)$$

$$\frac{N_{t+1}(j)}{N_t(j)} = \left[(R^s_{t+1} - R_t)\frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right](1 - \tau_t)$$

Taking the lead of $Q_tS_t(j) = \frac{\eta_t}{(\lambda - v_t)} N_t(j)\,(1 - \tau_t)$ and dividing it by $Q_tS_t(j)$, one can see that

$$\begin{aligned}\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)} &= \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})} N_{t+1}(j)\,(1 - \tau_{t+1})}{\frac{\eta_t}{(\lambda - v_t)} N_t(j)\,(1 - \tau_t)} \\ &= \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}}\left[(R^s_{t+1} - R_t)\frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] \\ &\quad \times (1 - \tau_{t+1})\,.\end{aligned}$$

Accordingly,

$$\begin{aligned}v_t(j) = E_t\,&(1 - \theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) \\ &+ \theta\psi_{t,t+1}\frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}}\left[(R^s_{t+1} - R_t)\frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] \\ &\times (1 - \tau_{t+1})\,v_{t+1}(j)\end{aligned} \tag{5.128}$$

$$\begin{aligned}\eta_t(j) = E_t\,&(1 - \theta) + \theta\psi_{t,t+1}\left[(R^s_{t+1} - R_t)\frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] \\ &\times (1 - \tau_{t+1})\,\eta_{t+1}(j).\end{aligned} \tag{5.129}$$

Since $\frac{\eta_{t+1}}{(\lambda - v_{t+1})}$ is independent of j, one can aggregate across banks to obtain

$$\int_j Q_tS_t(j)dj = \int_j \frac{\eta_t}{(\lambda - v_t)} N_t(j)(1 - \tau_t)dj \tag{5.130}$$

$$Q_tS_t = \frac{\eta_t}{(\lambda - v_t)} N_t(1 - \tau_t). \tag{5.131}$$

Finally, recognize that there is a distinction between the net worth of continuing and new bankers. Aggregate net worth is the sum the two types: Bankers that survive from period $t-1$ to period t will have aggregate net worth equal to

$$\theta\left[\left(R_t^s - R_{t-1}\right)\frac{\eta_{t-1}}{\left(\lambda - \upsilon_{t-1}\right)} + \left(1 + R_{t-1}\right)\right] N_{t-1}\left(1 - \tau_{t-1}\right). \quad (5.132)$$

Assume that new bankers receive as endowment a fixed fraction of the current value of the assets intermediated by exiting bankers in the previous period, amounting to $(1-\ \theta)\, Q_t S_{t-1}$. Assume that the household transfers the fraction $\frac{\omega}{(1-\ \theta)}$ of that amount to new bankers. Thus, in the aggregate,

$$N_t^n = \frac{\omega}{(1-\ \theta)}\left(1-\ \theta\right) Q_t S_{t-1} = \omega Q_t S_{t-1}. \quad (5.133)$$

Then, current net worth is the sum of net worth carried from the previous period by surviving firms $\theta\big[\left(R_t^s - R_{t-1}\right)\frac{\eta_{t-1}}{(\lambda-\upsilon_{t-1})} + (1+R_{t-1})\big]$ $N_{t-1}\left(1-\tau_t\right)$, plus the net worth of new entrants, $\omega Q_t S_{t-1}\left(1-\tau_t\right)$, i.e.,

$$N_t = \theta\left[\left(R_t^s - R_{t-1}\right)\frac{\eta_{t-1}}{\left(\lambda - \upsilon_{t-1}\right)} + \left(1 + R_{t-1}\right)\right] \times N_{t-1}\left(1 - \tau_{t-1}\right) + \omega Q_t S_{t-1}. \quad (5.134)$$

5.4 Introducing Heterogenous Firms

Now suppose that a fraction of firms can access equity markets directly, without having to reach them through banks. Call the type of such firms u. The other firms have to rely on banks to fund their capital purchases. Call the type of such firms b. The cost structure of the two types of firms will be different, and their products will have different prices in equilibrium. Both types of firms will coexist in equilibrium because the final consumption and investment goods are assumed to be a composite of both types of intermediate goods (possibly in different proportions).

5.4.1 Households

As before, the representative household has a continuum of members. A fraction $1-f$ of members in this continuum supplies labor to firms and returns the wage earned to the household. A fraction f of members in the continuum works as bankers. The consumption of workers and bankers within the household is equalized. As before, the utility function is

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon} L_{t+i}^{1+\varepsilon} \right]. \quad (5.135)$$

However, in this case, the budget constraint takes the form

$$C_t = W_t L_t + \Pi_t - T_t + Q_t S_t^u - (1 + R_t^{su}) Q_{t-1} S_{t-1}^u + D_t - (1 + R_{t-1}) D_{t-1}. \quad (5.136)$$

The term D_t represents the amount of deposits with banks (not owned by the household). R_{t-1} is non-state contingent. When the price of consumption is chosen to be the numeraire, the interest rate on deposits is “risk free” (under other normalization of prices deposits would not insure against the risk of changes in the price of consumption). The term S_t^u represents the shares issued by final product firms that have direct access to equity markets. Shares acquired the previous period pay the risky rate R_t^{su}.

The division between bankers and workers within the representative family remains unchanged relative to the setup considered before.

Households allocate consumption between two goods produced by firms of type u and by firms of type b. The production of final goods takes place through perfectly competitive firms. Their production technology is

$$Y_t = (Y_t^u)^{\alpha} \left(Y_t^b\right)^{1-\alpha}. \quad (5.137)$$

Each period they minimize the cost of production subject to meeting demand:

$$\min_{Y_t^u, Y_t^b, P_t^F} P_t^u Y_t^u + P_t^b Y_t^b + P_t^F \left(Y_t - (Y_t^u)^{\alpha^F} \left(Y_t^b\right)^{1-\alpha^F} \right). \quad (5.138)$$

We are using the prices of final goods to be the numeraire units, hence the Lagrange multiplier on the technology of production P_t^F is set to 1.

First-order conditions:

$$P_t^u + P_t^F \left(-\alpha^F \left(Y_t^u\right)^{\alpha^F - 1} \left(Y_t^b\right)^{1-\alpha^F} \right) = 0$$

$$P_t^u = P_t^F \alpha^F \frac{Y_t}{Y_t^u}$$

$$Y_t^u = \alpha^F Y_t \frac{P_t^F}{P_t^u}.$$

But $P_t^F = 1$:

$$Y_t^u = \alpha^F Y_t \frac{1}{P_t^u}.$$

5.4.2 Output-Producing Firms

There are two kinds of firms: firms that have direct access to equity markets and firms that have to use banks for their financing requirements. Both have production technologies

$$Y_t^j = A_t K_t^{j\,\alpha} L_t^{j 1-\alpha}, \tag{5.139}$$

where j is either u for the firms that have access to equity markets or b for the firms that have to use banks. Firms operate for only one period, but some of the planning for production is done one period in advance. To operate capital in period $t+1$, a firm must purchase it in period t. To do so, the firm issues shares in period t. There are as many shares S_t^j as units of capital purchased. By arbitrage, the current value of capital equals the value of shares. Thus,

$$Q_t K_{t+1}^j = Q_t S_{t.}^j. \tag{5.140}$$

Let π_{t+1} denote the revenue of firms in period $t+1$ net of expenses. Revenues include proceeds from the sale of output as well as from the sale of the undepreciated fraction of capital. Expenses include obligations connected with the servicing of shares and with the compensation for labor services. Thus,

$$\pi_{t+1}^j = P_{t+1}^j Y_{t+1}^j + Q_{t+1}(1-\delta) K_{t+1}^j - W_{t+1} L_{t+1}^j - (1 + R_{t+1}^{js}) Q_t S_{t.}^j \tag{5.141}$$

At time t the problem of firms is to choose S_t^j and K_{t+1}^j to maximize the expected profits in period $t+1$, knowing that the firms will be able to choose the optimal quantity of labor in that period. The firm takes Q_t, Q_{t+1}, R_{t+1}^{js}, and W_{t+1} as given. This maximization problem can be expressed as

$$\max_{S_t, K_{t+1}^j} E_t \max_{L_{t+1}^j} \pi_{t+1}^j. \tag{5.142}$$

Notice that the equalization of Q_t and W_{t+1} across types of firms arises because of the absence of sector-specific frictions in physical markets for labor and capital.

At time t the problem of firms is to choose S_t and K_{t+1} to maximize the expected profits in period $t+1$, knowing that the firms will be able to choose the optimal quantity of labor in that period. The firm takes Q_t, Q_{t+1}, R_{t+1}^{s}, and W_{t+1} as given. This maximization problem can be expressed as

$$\max_{S_t^j, K_{t+1}^j} E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \max_{L_{t+1}^j} \pi_{t+1}^j \tag{5.143}$$

subject to constraints of the production technology $Y_t^j = A_t K_t^{j\,\alpha} L_t^{j\,1-\alpha}$ and financing $Q_t K_{t+1}^j = Q_t S_t^j$. The solution of $\max_{L_{t+1}^j} \pi_{t+1}^j$ implies that

$$W_{t+1} = (1-\alpha)\,\frac{P_{t+1}^j Y_{t+1}^j}{L_{t+1}^j} \tag{5.144}$$

$$L_{t+1}^j = (1-\alpha)\,\frac{P_{t+1}^j Y_{t+1}^j}{W_{t+1}} \tag{5.145}$$

under all states of nature. Accordingly, $\max_{S_t, K_{t+1}} E_t \max_{L_{t+1}} \pi_{t+1}$ collapses to

$$\max_{S_t K_{t+1}, L_{t+1}} E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \Big[P_{t+1}^j A_{t+1} K_{t+1}{}^{\alpha} L_{t+1}^{1-\alpha} + Q_{t+1}(1-\delta) K_{t+1}^j - W_{t+1} L_{t+1}^j - (1 + R_{t+1}^{sj}) Q_t S_{t.} \Big]$$

$$+ \lambda^j_{lt+1t}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\frac{P^j_{t+1}A_{t+1}K^j_{t+1}{}^{\alpha}L^{j1-\alpha}_{t+1}}{W_{t+1}} - L_{t+1}\right]$$
$$+ \lambda^j_{st}\left(Q_t S_{t.} - Q_t K^j_{t+1}\right). \tag{5.146}$$

Notice that there is no expectation operator on the Lagrangian multipliers because those constraints hold under every state of nature.

The problem implies the following conditions:

$$\frac{\partial}{S_t} = -E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^{sj}_{t+1})Q_t + \lambda_{st}Q_t = 0 \tag{5.147}$$

$$\frac{\partial}{\partial K_{t+1}} = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\alpha\frac{P^j_{t+1}Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)\right]$$
$$+ \lambda^j_{lt+1}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\frac{\alpha P^j_{t+1}\frac{Y_{t+1}}{K_{t+1}}}{W_{t+1}}\right] - \lambda^j_{st}Q_t \tag{5.148}$$

$$\frac{\partial}{\partial L_{t+1}} = \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\frac{P^j_{t+1}Y_{t+1}}{L_{t+1}} - W_{t+1}\right]$$
$$+ \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\lambda_{lt+1}\left[(1-\alpha)^2\frac{P^j_{t+1}Y_{t+1}}{L_{t+1}W_{t+1}} - 1\right]. \tag{5.149}$$

Working on $\frac{\partial}{S_t}$,

$$\lambda^j_{st} = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^{js}_{t+1}). \tag{5.150}$$

From $\frac{\partial}{\partial K_{t+1}}$,

$$E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^{js}_{t+1}) = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha\frac{P^j_{t+1}Y^j_{t+1}}{K^j_{t+1}} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right]$$
$$+ E_t\lambda_{lt+1}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\frac{\alpha\frac{P^j_{t+1}Y^j_{t+1}}{K^j_{t+1}}}{W_{t+1}}\right].$$

Next work on $\frac{\partial}{\partial L_{t+1}}$. Again, since $(1-\alpha)\frac{P^j_{t+1}Y^j_{t+1}}{L^j_{t+1}} = W_{t+1}$,

$$\frac{\partial}{\partial L_{t+1}} = \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}[0] + \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\lambda_{lt+1}[(1-\alpha)-1] = 0 \qquad (5.151)$$

$$\lambda_{lt+1} = 0. \qquad (5.152)$$

Then, combining the implications of $\frac{\partial}{\partial L_{t+1}} = 0$ and $\frac{\partial}{\partial K_{t+1}} = 0$ yields

$$E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^{js}_{t+1}) = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha\frac{P^j_{t+1}Y^j_{t+1}}{K^j_{t+1}} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right]. \qquad (5.153)$$

Notice that firms will make zero profits under all states of nature (and that's why we can drop the expectation operator). Thus,

$$0 = P^j_{t+1}Y^j_{t+1} + Q_{t+1}(1-\delta)K^j_{t+1} - W_{t+1}L^j_{t+1} - (1+R^{sj}_{t+1})Q_tS^j_{t.} \qquad (5.154)$$

$$(1+R^{sj}_{t+1}) = \frac{1}{Q_t}\frac{\alpha P^j_{t+1}Y^j_{t+1}}{K^j_{t+1}} + \frac{(1-\delta)}{Q_t}Q_{t+1}. \qquad (5.155)$$

This condition will also imply $E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^{sj}_{t+1}) = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha\frac{P^j_{t+1}Y^j_{t+1}}{K^j_{t+1}} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right]$ derived above (if profits are always zero, it does not matter how you discount them).

5.4.3 Capital-Producing Firms

The evolution of capital takes the form

$$K_{t+1} = I^n_t + (1-\delta)K_t. \qquad (5.156)$$

Net investment is simply given by

$$I^n_t = K_{t+1} - (1-\delta)K_t. \qquad (5.157)$$

The production technology for investment involves a quadratic adjustment for current production relative to past production, thus the supply of investment goods is given by

$$I_t^n = \left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I^g_{t.} \tag{5.158}$$

Capital-producing firms solve the problem

$$\max_{I^g_{t+i}} E_t \sum_{i=0}^{\infty} \psi_{t,t+i} \left[Q_{t+i}\left[1 - \frac{\phi}{2}\left(\frac{I^g_{t+i}}{I^g_{t+i-1}} - 1\right)^2\right] I^g_{t+i} - P^i_t I^g_{t+i.}\right]. \tag{5.159}$$

In the maximization, Q_t is taken as given and $\psi_{t,t+i}$ is the stochastic discount factor of households who own the capital-producing firms (defined below).

5.4.4 Banks

Banks lend funds obtained from households to non-financial firms. Let $N_t(j)$ be the amount of wealth—or net worth—that a banker j has at the end of period t.

$$Q_t S_t^b(j) = N_t(j) + D_t(j) \tag{5.160}$$

As noted earlier, deposits $D_t(j)$ pay the non-state-contingent return $(1 + R_t)$ at time $t + 1$. Thus $D_t(j)$ may be thought of as the debt of bank j, and $N_t(j)$ as its capital. As seen above, the shares $S_t^b(j)$ earn the stochastic return $(1 + R^{bs}_{t+1})$ at time $t + 1$.

Over time, the banker's equity capital evolves as the difference between earnings on assets and interest payments on liabilities:

$$N_{t+1}(j) = (1 + R^{bs}_{t+1})Q_t S_t(j) - (1 + R_t)D_t(j) \tag{5.161}$$

$$D_t(j) = Q_t S_t(j) - N_t(j) \tag{5.162}$$

$$N_{t+1}(j) = (1 + R^{bs}_{t+1})Q_t S_t(j) - (1 + R_t)\left(Q_t S_t(j) - N_t(j)\right) \tag{5.163}$$

$$N_{t+1}(j) = \left[(1 + R^{bs}_{t+1}) - (1 + R_t)\right] Q_t S_t^b(j) + (1 + R_t)N_t(j) \tag{5.164}$$

$$N_{t+1}(j) = \left(R^{bs}_{t+1} - R_t\right) Q_t S_t^b(j) + (1 + R_t)N_t(j). \tag{5.165}$$

Let $\psi_{t,t+i} = \beta^i \frac{\lambda_{ct+i}}{\lambda_{ct}}$ be the stochastic discount factor between periods t and $t + i$. The banker's objective is to maximize expected terminal wealth, given by

$$\max_{S^b_{t+i}(j)} V_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i}[\left(R^{bs}_{t+1+i} - R_{t+i}\right) Q_{t+i} S^b_{t+i}(j) \\ + (1+R_{t+i}) N_{t+i}(j)]. \tag{5.166}$$

Since the banker will not fund assets with a discounted return less than the discounted cost of borrowing, for the bank to operate in period $t+i$, it must be that $E_t \psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right) \geq 0$, i.e., there are expected positive excess returns from holding stocks even after discounting and adjusting for risk through $\psi_{t,t+1+i}$. In the absence of financial frictions, when $E_t \psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right)$ is positive, the bank will want to expand its balance sheet by attracting additional deposits from households.

To limit the ability of banks to attract deposits indefinitely, now impose the external requirement λ_t:

$$N_t(j) \geq \lambda_t Q_t S^b_t(j). \tag{5.167}$$

Log-linearizing,

$$\lambda_t = \frac{N_t}{Q_t S^b_t}. \tag{5.168}$$

As before, then

$$N_t = \theta \left[\left(R^{bs}_t - R_{t-1}\right) \frac{1}{\lambda_{t-1}} + (1+R_{t-1}) \right] N_{t-1} + \omega Q_t S^b_{t-1}. \tag{5.169}$$

5.5 Introducing Nominal Rigidities

Modify the problem of households to be

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon} L^{1+\varepsilon}_{t+i} \right]. \tag{5.170}$$

However, in this case, the budget constraint takes the form

$$P_t C_t = P_t W_t L_t + P_t \Pi_t - P_t T_t + P_t Q_t S^u_t - (1+R^{su}_t) P_t Q_{t-1} S^u_{t-1} \\ + P_t D_t - (1+R_{t-1}) P_t D_{t-1}. \tag{5.171}$$

Note that, despite writing the budget constraint in nominal terms, we are guaranteeing a real return R_t. In this respect, deposits are akin to indexed bonds.

Consider the first order-condition with respect to deposit holdings:

$$\lambda_{ct}^N P_t - E_t \beta \lambda_{ct+1}^N (1 + R_{t+1}) P_{t+1} = 0$$
$$\lambda_{ct}^N P_t = E_t \beta \lambda_{ct+1}^N (1 + R_{t+1}) P_{t+1}$$
$$E_t \beta \frac{\lambda_{ct+1}^N}{\lambda_{ct}^N} \frac{P_{t+1}}{P_t} (1 + R_{t+1}) = 1$$
$$E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} (1 + R_{t+1}) = 1.$$

Proceed as before, but modify the problem of producing firms.

The production technology of the representative firm is $Y_t = (Y_t^u)^\alpha \left(Y_t^b\right)^{1-\alpha}$. Firms operate for only one period, but some of the planning for production is done one period in advance. To operate capital in period $t+1$, a firm must purchase it in period t. To do so, the firm issues shares in period t. There are as many shares S_t as units of capital purchased. By arbitrage, the current value of capital equals the value of shares. Thus,

$$P_t Q_t K_{t+1} = P_t Q_t S_{t.} \tag{5.172}$$

Let π_{t+1} denote the revenue of firms in period $t+1$ net of expenses. Revenues include proceeds from the sale of output as well as from the sale of the undepreciated fraction of capital. Expenses include obligations connected with the servicing of shares and with the compensation for labor services. Thus,

$$\begin{aligned}\pi_{t+1} = {} & \sigma_{t+1} Y_{t+1} + P_{t+1} Q_{t+1} (1 - \delta) K_{t+1} - P_{t+1} W_{t+1} L_{t+1} \\ & - (1 + r_{t+1}^s) P_t Q_t S_{t.}\end{aligned} \tag{5.173}$$

At time t the problem of firms is to choose S_t and K_{t+1} to maximize the expected profits in period $t+1$, knowing that the firms will be able to choose the optimal quantity of labor in that period. The firm takes Q_t, Q_{t+1}, R_{t+1}^s, and W_{t+1} as given. This maximization problem can be expressed as

$$\max_{S_t, K_{t+1}} E_t m_{t+1/t} \max_{L_{t+1}} \pi_{t+1} \tag{5.174}$$

subject to constraints of the production technology $Y_t = A_t {K_t}^{\alpha} L_t^{1-\alpha}$ and financing $Q_t P_t K_{t+1} = Q_t P_t S_t$. The solution of $\max_{L_{t+1}} \pi_{t+1}$ implies that

$$P_{t+1} W_{t+1} = (1-\alpha) \frac{\sigma_{t+1} Y_{t+1}}{L_{t+1}} \tag{5.175}$$

$$L_{t+1} = (1-\alpha) \frac{Y_{t+1}}{W_{t+1}} \frac{\sigma_{t+1}}{P_{t+1}} \tag{5.176}$$

under all states of nature. Accordingly, $\max_{S_t, K_{t+1}} E_t \max_{L_{t+1}} \pi_{t+1}$ collapses to

$$\begin{aligned} &\max_{S_t K_{t+1}, L_{t+1}} E_t m_{t+1/t} \left[\sigma_{t+1} A_{t+1} {K_{t+1}}^{\alpha} L_{t+1}^{1-\alpha} + P_{t+1} Q_{t+1} (1-\delta) K_{t+1} \right. \\ &\quad \left. - P_{t+1} W_{t+1} L_{t+1} - (1 + r_{t+1}^s) P_t Q_t S_{t.} \right] \\ &\quad + \lambda_{lt+1t} m_{t+1/t} \left[(1-\alpha) \frac{A_{t+1} {K_{t+1}}^{\alpha} L_{t+1}^{1-\alpha}}{W_{t+1}} \frac{\sigma_{t+1}}{P_{t+1}} - L_{t+1}\right] \\ &\quad + \lambda_{st} \left(Q_t S_{t.} - Q_t K_{t+1}\right). \end{aligned} \tag{5.177}$$

Subject to the modifications above, the derivations follow closely what we had in the absence of nominal rigidities. The conditions for an equilibrium from the side of producing firms are as follows.

From the zero-profit condition,

$$\begin{aligned} 0 &= \sigma_{t+1} Y_{t+1} + P_{t+1} Q_{t+1} (1-\delta) K_{t+1} - P_{t+1} W_{t+1} L_{t+1} \\ &\quad - (1 + r_{t+1}^s) P_t Q_t S_{t.} \end{aligned} \tag{5.178}$$

$$\begin{aligned} (1 + r_{t+1}^s) P_t Q_t S_{t.} &= \sigma_{t+1} Y_{t+1} + Q_{t+1} P_{t+1} (1-\delta) K_{t+1} \\ &\quad - P_{t+1} W_{t+1} L_{t+1} \end{aligned} \tag{5.179}$$

$$(1 + r_{t+1}^s) = \frac{\sigma_{t+1} Y_{t+1} + Q_{t+1} P_{t+1} (1-\delta) K_{t+1} - P_{t+1} W_{t+1} L_{t+1}}{P_t Q_t S_{t.}} \tag{5.180}$$

$$(1 + r_{t+1}^s) = \frac{\sigma_{t+1} Y_{t+1} + Q_{t+1} P_{t+1} (1-\delta) K_{t+1} - P_{t+1} W_{t+1} L_{t+1}}{P_t Q_t K_{t+1.}} \tag{5.181}$$

$$(1+r^s_{t+1}) = \frac{\begin{array}{c}\sigma_{t+1}Y_{t+1} + Q_{t+1}P_{t+1}(1-\delta)K_{t+1} \\ -P_{t+1}W_{t+1}\left(1-\alpha\right)\frac{\sigma_{t+1}Y_{t+1}}{P_{t+1}W_{t+1}}\end{array}}{P_tQ_tK_{t+1.}} \tag{5.182}$$

$$(1+r^s_{t+1}) = \frac{1}{Q_t}\frac{\alpha\sigma_{t+1}Y_{t+1}}{P_tK_{t+1}} + \frac{(1-\delta)}{P_tQ_t}P_{t+1}Q_{t+1} \tag{5.183}$$

$$(1+r^s_{t+1}) = \frac{1}{Q_t}\frac{\alpha\sigma_{t+1}Y_{t+1}}{P_{t+1}K_{t+1}}\frac{P_{t+1}}{P_t} + \frac{(1-\delta)}{Q_t}\frac{P_{t+1}}{P_t}Q_{t+1} \tag{5.184}$$

$$(1+r^s_{t+1}) = \frac{1}{Q_t}\frac{\alpha\sigma_{t+1}Y_{t+1}}{P_{t+1}K_{t+1}}\frac{P_{t+1}}{P_t} + \frac{(1-\delta)}{Q_t}\frac{P_{t+1}}{P_t}Q_{t+1} \tag{5.185}$$

$$\frac{(1+r^s_{t+1})}{\frac{P_{t+1}}{P_t}} = \frac{1}{Q_t}\frac{\alpha\sigma_{t+1}Y_{t+1}}{P_{t+1}K_{t+1}} + \frac{(1-\delta)}{Q_t}Q_{t+1}. \tag{5.186}$$

Define

$$(1+R^s_t) = \frac{(1+r^s_t)}{\frac{P_t}{P_{t-1}}}.$$

Accordingly,

$$(1+R^s_{t+1}) = \frac{1}{Q_t}\frac{\alpha\sigma_{t+1}Y_{t+1}}{P_{t+1}K_{t+1}} + \frac{(1-\delta)}{Q_t}Q_{t+1}, \tag{5.187}$$

and from above,

$$L_{t+1} = (1-\alpha)\frac{Y_{t+1}}{W_{t+1}}\frac{\sigma_{t+1}}{P_{t+1}}. \tag{5.188}$$

The problem of the final firms is

$$\max_{P_{t+i(}(f)} E_t\sum_{i=0}^{\infty}\psi_{t,t+i}\left\{(1+\tau_p)\,P_{t+i}\,(f) - \sigma_{t+i}\right\}\left(1-\phi_{P,t+i}\,(f)\right)Y_{t+i} \times \left(\frac{P_{t+i}\,(f)}{P_{t+i}}\right)^{-\frac{1+\theta_p}{\theta_p}},$$

where

$$\phi_{P,t} = \frac{\phi_p}{2}\left(\frac{P_t\,(f)}{\pi P_{t-1}\,(f)} - 1\right)^2.$$

The first-order conditions are

$$E_t\begin{bmatrix} (1+\tau_p)\left(1-\phi_{P,t}\left(f\right)\right)Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \\ -\frac{1+\theta_p}{\theta_p}\left\{(1+\tau_p)\,P_t\left(f\right)-\sigma_t\right\}\left(1-\phi_{P,t}\left(f\right)\right)Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}-1}\frac{1}{P_t} \\ -\left\{(1+\tau_p)\,P_t\left(f\right)-\sigma_t\right\}Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}}\frac{\partial\phi_{P,t}(f)}{\partial P_t(f)} \\ -\psi_{t,t+1}\left\{(1+\tau_p)\,P_{t+1}\left(f\right)-\sigma_{t+1}\right\}Y_{t+1}\left(\frac{P_{t+1}(f)}{P_{t+1}}\right)^{-\frac{1+\theta_p}{\theta_p}}\frac{\partial\phi_{P,t+1}(f)}{\partial P_t(f)} \end{bmatrix}=0$$

$$E_t\begin{bmatrix} (1+\tau_p)\left(1-\phi_{P,t}\left(f\right)\right)Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \\ -\frac{1+\theta_p}{\theta_p}\left\{(1+\tau_p)-\frac{\sigma_t}{P_t(f)}\right\}\left(1-\phi_{P,t}\left(f\right)\right)Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}-1}\frac{P_t(f)}{P_t} \\ -\left\{(1+\tau_p)\,P_t\left(f\right)-\sigma_t\right\}Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}}\frac{\partial\phi_{P,t}(f)}{\partial P_t(f)} \\ -\psi_{t,t+1}\left\{(1+\tau_p)\,P_{t+1}\left(f\right)-\sigma_{t+1}\right\}Y_{t+1}\left(\frac{P_{t+1}(f)}{P_{t+1}}\right)^{-\frac{1+\theta_p}{\theta_p}}\frac{\partial\phi_{P,t+1}(f)}{\partial P_t(f)} \end{bmatrix}=0$$

$$E_t\begin{bmatrix} (1+\tau_p)\left(1-\phi_{P,t}\left(f\right)\right)Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \\ +\left\{-\frac{1+\theta_p}{\theta_p}\left(1+\tau_p\right)+\frac{1+\theta_p}{\theta_p}\frac{\sigma_t}{P_t(f)}\right\}\left(1-\phi_{P,t}\left(f\right)\right)Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \\ -\left\{(1+\tau_p)\,P_t\left(f\right)-\sigma_t\right\}Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}}\frac{\partial\phi_{P,t}(f)}{\partial P_t(f)} \\ -\psi_{t,t+1}\left\{(1+\tau_p)\,P_{t+1}\left(f\right)-\sigma_{t+1}\right\}Y_{t+1}\left(\frac{P_{t+1}(f)}{P_{t+1}}\right)^{-\frac{1+\theta_p}{\theta_p}}\frac{\partial\phi_{P,t+1}(f)}{\partial P_t(f)} \end{bmatrix}=0$$

$$E_t\begin{bmatrix} +\left\{\left(1-\frac{1+\theta_p}{\theta_p}\right)\left(1+\tau_p\right)+\frac{1+\theta_p}{\theta_p}\frac{\sigma_t}{P_t(f)}\right\}\left(1-\phi_{P,t}\left(f\right)\right) \\ Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}}-\left\{(1+\tau_p)\,P_t\left(f\right)-\sigma_t\right\}Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}}\frac{\partial\phi_{P,t}(f)}{\partial P_t(f)} \\ -\psi_{t,t+1}\left\{(1+\tau_p)\,P_{t+1}\left(f\right)-\sigma_{t+1}\right\}Y_{t+1}\left(\frac{P_{t+1}(f)}{P_{t+1}}\right)^{-\frac{1+\theta_p}{\theta_p}}\frac{\partial\phi_{P,t+1}(f)}{\partial P_t(f)} \end{bmatrix}=0$$

$$E_t\begin{bmatrix} \left[-\frac{1}{\theta_p}\left(1+\tau_p\right)+\frac{1+\theta_p}{\theta_p}\frac{\sigma_t}{P_t(f)}\right]\left(1-\phi_{P,t}\left(f\right)\right)Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \\ -\left\{(1+\tau_p)\,P_t\left(f\right)-\sigma_t\right\}Y_t\left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}}\frac{\partial\phi_{P,t}(f)}{\partial P_t(f)} \\ -\psi_{t,t+1}\left\{(1+\tau_p)\,P_{t+1}\left(f\right)-\sigma_{t+1}\right\}Y_{t+1}\left(\frac{P_{t+1}(f)}{P_{t+1}}\right)^{-\frac{1+\theta_p}{\theta_p}}\frac{\partial\phi_{P,t+1}(f)}{\partial P_t(f)} \end{bmatrix}=0.$$

Due to symmetry,

$$E_t \begin{bmatrix} \left[-\frac{1}{\theta_p}(1+\tau_p)+\frac{1+\theta_p}{\theta_p}\frac{\sigma_t}{P_t}\right](1-\phi_{P,t})Y_t \\ -\left\{(1+\tau_p)-\frac{\sigma_t}{P_t}\right\}Y_t P_t \frac{\partial \phi_{P,t}}{\partial P_t} \\ -\psi_{t,t+1}\left\{(1+\tau_p)-\frac{\sigma_{t+1}}{P_{t+1}}\right\}Y_{t+1}P_{t+1}\frac{\partial \phi_{P,t+1}(f)}{\partial P_t(f)} \end{bmatrix} = 0$$

with the adjustment costs

$$\phi_{P,t} = \frac{\phi_p}{2}\left(\frac{P_t(f)}{\pi P_{t-1}(f)} - 1\right)^2$$
$$\frac{\partial \phi_{P,t}}{\partial P_t} = \phi_p\left(\frac{P_t(f)}{\pi P_{t-1}(f)} - 1\right)\frac{1}{\pi P_{t-1}(f)}$$
$$\frac{\partial \phi_{P,t}}{\partial P_{t-1}(f)} = -\phi_p\left(\frac{P_t(f)}{\pi P_{t-1}(f)} - 1\right)\frac{P_t(f)}{\pi P_{t-1}(f)}\frac{1}{P_{t-1}(f)}$$

or

$$\phi_{P,t} = \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2$$
$$\frac{\partial \phi_{P,t}}{\partial P_t}P_t = \phi_p\left(\frac{\pi_t}{\pi} - 1\right)\frac{\pi_t}{\pi}$$
$$\frac{\partial \phi_{P,t}}{\partial P_{t-1}}P_t = -\phi_p\left(\frac{\pi_t}{\pi} - 1\right)\frac{\pi_t}{\pi}\pi_t.$$

As a small detour, let's map the parameter ϕ_p into the parameterization of sticky price contracts following the Calvo scheme.

Let $\hat{\pi}_t = \pi_t - \pi$. Let $\hat{\sigma}_t = \frac{\frac{\sigma_t}{P_t} - \frac{\sigma}{P}}{\frac{\sigma}{P}}$. But notice that with $P = 1$, in our model $\sigma = 1$ (since we impose $\tau_p = \theta_p$), so $\hat{\sigma}_t = \frac{\frac{\sigma_t}{P_t} - \frac{\sigma}{P}}{\frac{\sigma}{P}} = \frac{\sigma_t}{P_t} - \frac{\sigma}{P} = \frac{\sigma_t}{P_t} - 1$. Standard results are that, under Calvo contracts, the first-order approximation of the firms' pricing equation yields

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \kappa_p\hat{\sigma}_t,$$

where $\kappa_p = \frac{(1-\beta\xi)(1-\xi)}{\xi}$, and where $1 - \xi$ is the probability that a firm will be allowed to reoptimize its price. Now, consider the pricing condition for Rotemberg contracts:

$$E_t \begin{bmatrix} \left[-\frac{1}{\theta_p}(1+\tau_p) + \frac{1+\theta_p}{\theta_p}\frac{\sigma_t}{P_t}\right](1-\phi_{P,t})Y_t \\ -\left\{(1+\tau_p) - \frac{\sigma_t}{P_t}\right\} Y_t \phi_p \left(\frac{\pi_t}{\pi} - 1\right)\frac{\pi_t}{\pi} \\ \psi_{t,t+1}\left\{(1+\tau_p) - \frac{\sigma_{t+1}}{P_{t+1}}\right\} Y_{t+1}\phi_p\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{\pi_{t+1}}{\pi}\pi_{t+1} \end{bmatrix} = 0,$$

with $P = 1$ in steady state. Using the first-order Taylor series expansion around the steady-states π and σ, we find

$$0 = \frac{1+\theta_p}{\theta_p} Y\hat{\sigma}_t - [(1+\tau_p) - \sigma] Y\phi_p\hat{\pi}_t + \beta[(1+\tau_p) - \sigma] Y\phi_p\hat{\pi}_{t+1}.$$

But remembering $\tau_p = \theta_p$,

$$0 = \frac{1+\theta_p}{\theta_p} Y\hat{\sigma}_t - [(1+\theta_p) - \sigma] Y\phi_p\hat{\pi}_t + \beta[(1+\theta_p) - \sigma] Y\phi_p\hat{\pi}_{t+1}.$$

Remembering that $\sigma = 1$ in steady state,

$$\theta_p Y\phi_p\hat{\pi}_t = \beta\theta_p Y\phi_p\hat{\pi}_{t+1} + \frac{1+\theta_p}{\theta_p} Y\hat{\sigma}_t$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{1+\theta_p}{\theta_p^2\phi_p}\hat{\sigma}_t.$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{(\varepsilon-1)\varepsilon}{\phi_p}\hat{\sigma}_t.$$

Matching the coefficients on marginal costs from Calvo and Rotemberg contracts, we obtain $\frac{1+\theta_p}{\theta_p^2\phi_p} = \kappa_p$ or

$$\phi_p = \frac{1+\theta_p}{\theta_p^2\kappa_p}.$$

Finally, monetary policy is set according to a interest rate reaction function of the following form:

$$R_t = \phi_R(R_{t-1} - \bar{R}) + (1-\phi_R)(\pi_t - \bar{pi}).$$

Table 5.1 Calibration

Parameter	Description	Sector
$\alpha = 0.33$	Share of Capital in Production	Production
$\rho = 0.95$	Autoregressive Coefficient of the Productivity Growth Process	
$\delta = 0.025$	Capital Depreciation Rate	Capital-Producing Firms
$\phi = 1.5$	Investment Adjustment Coefficient	Capital-Producing Firms
$\beta = 0.99$	Household Subjective Discount Factor	Households
$\gamma = 0.82$	Habit Persistence Parameter	
$\epsilon = 1.00$	Inverse Frisch Elasticity of Labor Supply	
$\theta = 0.97$	Expected Number of Periods as Banker = 30	Banks
$\alpha_{fp} = 0.60$	Share of Bank-Financed Firms	
$\xi_p = 0.88$	Coefficient of Average Contract Duration	
$\theta_p = 0.1$	Steady-State Markup	Nominal Rigidities
$\xi_p = 0.88$	Calvo Probability of Not-Adjusting Price	
$\phi_R = 0.7$	Interest Rate Smoothing	Monetary Policy Rule
$\phi_\pi = 3$	Weight on Inflation	

5.6 Calibration

The share of output devoted to government spending is 20 percent. The fraction of time spent working is 0.5 in steady state. Following Gertler and Karadi (2011), the parameter θ is set to deliver an expected duration of a banker's assignment of thirty-five quarters. The steady-state loan-to-equity ratio is set to 4 and the steady-state spread is 0.5 percent, or 2 percent when annualized. These latter two steady-state choices are achieved by setting λ to 0.60 and ω to 0.0011. The persistence of the transfer shock to households is 0.9. All the other calibrated parameters are shown in table 5.1.

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Deleveraging and Consumer Credit Supply in the Wake of the 2008–09 Financial Crisis*

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We explore the sources of the decline in household non-mortgage debt following the collapse of the housing market in 2006. First, we use data from the Federal Reserve Board's Senior Loan Officer Opinion Survey to document that, post-2006, banks tightened consumer lending standards more in counties that experienced a more pronounced house price decline (the pre-2006 "boom" counties). We then use the idea that renters did not experience an adverse wealth or collateral shock when the housing market collapsed to identify a general consumer credit supply shock. Our evidence suggests that a tightening of the supply of non-mortgage credit that was independent of the direct effects of lower housing collateral values played an important role in households' non-mortgage debt reduction. Renters decreased their non-mortgage debt more in boom counties than in non-boom counties, but homeowners did not. We argue that this wedge between renters and homeowners can only have arisen from a general tightening of banks' consumer lending stance. Using an IV approach, we trace this effect back to a reduction in bank capital of banks in boom counties.

JEL Codes: E21, G21.

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1. Introduction

Between 2008 and 2012, total household debt fell by about 6 percent, and debt-to-income by about 10 percent.[1] These household balance sheet adjustments are thought to have weighed on aggregate consumption as the U.S. economy struggled to emerge from the downturn. Mian, Rao, and Sufi (2012) (MRS hereafter) use regional variation to assemble evidence on the links between declines in housing wealth, deleveraging, and changes in consumption expenditures after the 2008–09 financial crisis. MRS argue that the combination of a large accumulation of household debt in counties with high house price appreciation before the mortgage crisis (here, "boom" counties) and the subsequent sharp decline in house prices in roughly the same counties resulted in household deleveraging and a concomitant reduction of household consumption expenditure. In support of their story, MRS show that household consumption expenditures declined more in boom counties than in non-boom counties.

One question that remains in this line of research is how the household deleveraging was accomplished. MRS mention two possible, not necessarily mutually exclusive, mechanisms of deleveraging. One possible mechanism for consumer deleveraging stems from a demand-side story. In a simple model of household consumption planning, homeowners would optimally choose to reduce their lifetime consumption, and thereby reduce their household debt, upon perceiving a negative and permanent shock to their housing wealth.[2] The second mechanism for household deleveraging focuses on credit supply. MRS's story here is that homeowners are forced to delever because banks are less willing to lend to them, refinance their mortgages, or roll over existing debt because the value of their collateral has declined.[3] Our story is different. We show that credit supply was

[1]Federal Reserve Flow of Funds and National Income and Product Accounts, 2008:Q3–2012:Q1.

[2]One alternative demand story is that homeowners with limited self-control may increase home equity borrowing when house prices climb in order to finance greater current consumption (Laibson 1997) and then cut back on current consumption and borrowing, perhaps out of remorse or excess prudence, when house prices fall.

[3]Mortgage refinancing usually decreases mortgage debt. But it can also be accompanied by an increase in total household debt through, for example, a more than offsetting increase in credit card spending.

tightened even for households without housing collateral. Hence, we document an overall credit supply effect as an important factor in the deleveraging process of households in the wake of the crisis.[4]

To set the stage for our analysis, figure 1 presents the cumulative tightening from 2008 through 2012 of lending standards in boom versus non-boom counties for consumer installment loans and credit card loans from the Federal Reserve Board's Senior Loan Officer Opinion Survey (SLOOS).[5] For both loan categories, banks tightened lending standards markedly more in boom counties than in non-boom counties.[6] This evidence supports the notion that non-mortgage credit extension to consumers tightened significantly

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[4]One other source of deleveraging came through the mortgage foreclosure process. More than 4 million foreclosures took place in the housing crisis, with the houses backing the defaulted mortgages remaining vacant or sold for considerably lower amounts. This process had a large impact on bank profits, but economic growth in the recovery was more closely linked to the decisions of non-defaulting households regarding desired spending and borrowing and the willingness of banks to lend to this population.

[5]Even in the credit card market, there is scope for geographic variation in credit standards. Stango (2002) finds empirical evidence of switching costs for consumers in the credit card market. In addition, only the top 50 of the 250 largest credit card issuers operate nationally. (Stango 2002, p. 481.)

[6]Figure 1 was prepared as follows: Every quarter, the SLOOS asks a panel of large and medium-sized banks how their credit standards and terms have changed over the past three months. Banks are asked to rate the direction and extent of any change by picking one of five qualitative answers. For example, banks are asked to pick one of the following answers characterizing the change in their willingness to lend since the preceding survey: much less willing, somewhat less willing, no change, somewhat more willing, or much more willing. For one consumer installment loan question (asking about the bank's change in its willingness to make such loans) and one credit card loan question (asking about the bank's change in lending standards on such loans), we assigned numerical values to each of the five choices and constructed, for each county, an index equal to the sum of the SLOOS banks' responses, with each bank's response weighted by the proportion of the total deposits at SLOOS banks' branches in the county that is held by that bank. Our values range from -2, for the most easing of credit conditions, to 2, for the most tightening of credit conditions. For each county, we then calculated running totals of these weighted sums over time to construct the cumulative change in lending conditions since the beginning of 2008. So, although in any single period, the weighted sum of responses can range between -2 and 2, if banks are tightening or easing over consecutive time periods, the value of our cumulative weighted sum can move outside of this range. The graphs in figure 1 depict the mean values of these running totals for boom and non-boom counties.

We note the sharp increase in the cumulative tightening of consumer installment loan credit standards in the fourth quarter of 2011 in non-boom counties.

more in boom counties than in non-boom counties in the aftermath of the 2008–09 financial crisis. However, a tightening of credit standards by itself does not tell us whether those standards were tightened mostly for homeowners due to declines in the value of their housing collateral.

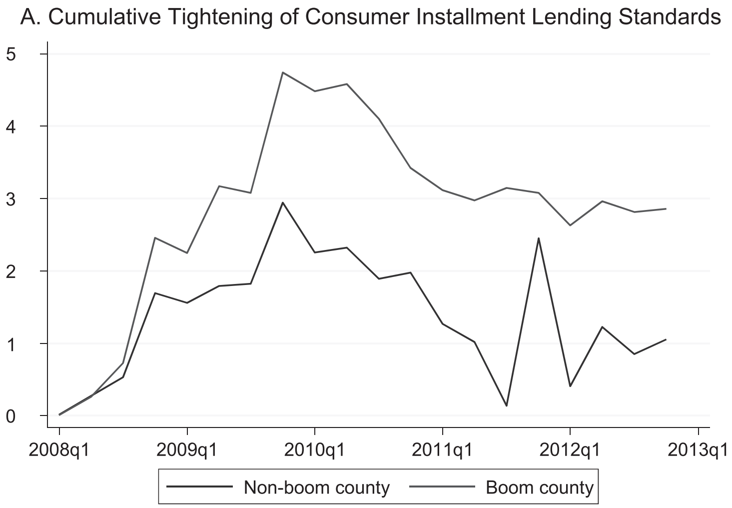
In this paper we attempt to identify a role for a general tightening of credit supply in explaining the decline in household debt. Using individual credit file data from 1999 through 2008, we estimate a model of the level of debt in which we control for many factors likely to affect the demand for such debt. We confirm the reasonableness of the empirical results we obtain from estimating that model. We then estimate a model of the probability of a consumer living in a boom county, using the same controls as in our debt-level regressions. Following this, we construct a sample of matched consumers, with each consumer in a boom county matched to the consumer in the non-boom county that has the closest *predicted probability* of living in the boom county as the consumer in the non-boom county. Within slices of the distribution of predicted debt for 2008:Q3, we then compare the change in non-mortgage debt between 2008 and 2011 for the non-boom sample with the change in non-mortgage debt for the boom sample.[7] Finally, using the matched sample, we

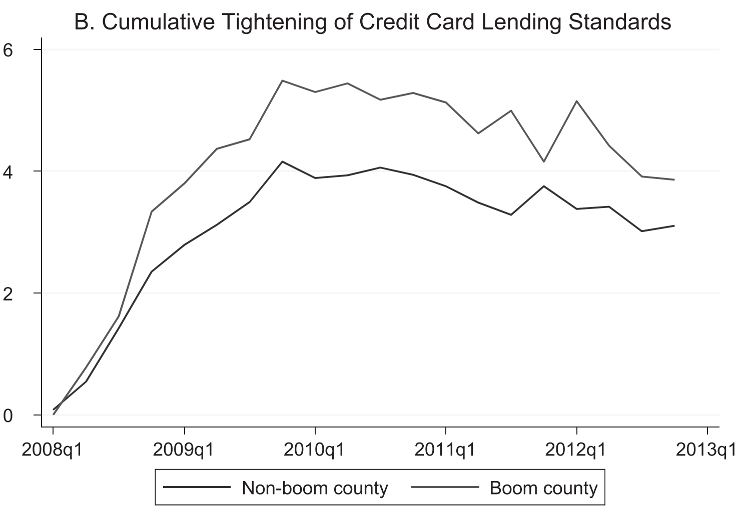
The underlying data indicate fairly widespread tightening across the relevant banks. Therefore, we simply accept the sharp increase as a feature of the data.

We also note that each bank's response to the survey likely is weighted by the distribution of its own deposits across boom versus non-boom counties. In principle, this could temper any correlation between boom counties and changes in credit standards as we measure them. For example, a bank may have a large share of the total deposits in a boom county, but the share of its own deposits in boom counties overall may be small. If such were the case, even though the bank likely would respond on the basis of its being, in general, a non-boom county lender, its response would be heavily weighted in our index for the particular boom county. We observe, however, a strong correlation between county types and changes in credit standards, in the expected direction. A contributing factor may be that, in practice, banks that have large shares of the total deposits in boom counties tend to also have large shares of their own deposits in boom counties.

[7]Since house price changes during the crisis were highly correlated with pre-crisis house price appreciation (that is, whether or not a county was a boom county (figure 7)), we are, in essence, assessing the effect of house price changes between 2008 and 2011 on the change in debt between 2008 and 2011. We follow MRS in differentiating counties according to their degree of pre-crisis house price appreciation (high versus low) instead of according to their crisis-era house price changes.

Figure 1. Cumulative Tightening of Lending Standards in Boom and Non-boom Counties



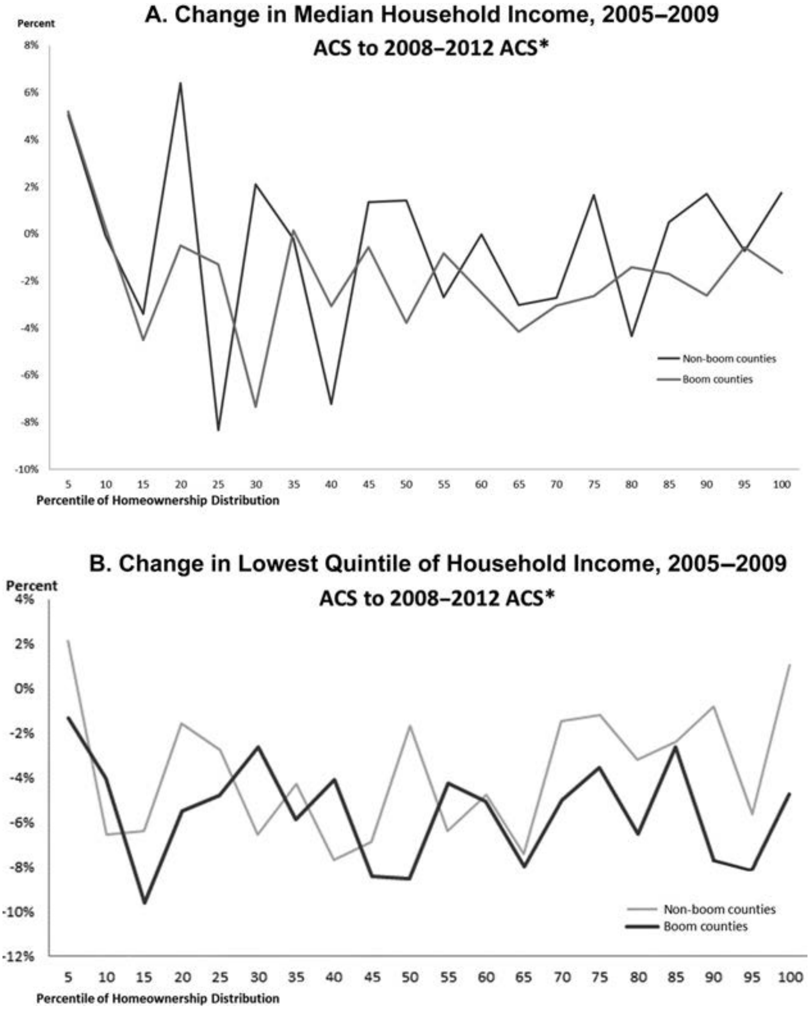


Source: Senior Loan Officer Opinion Survey and authors' calculations (see text).

estimate regressions of the change in non-mortgage debt, controlling for boom county and interacting boom county with the homeowner status of the consumer.

We rely on the following idea to identify general credit supply effects. We make use of the fact that renters experienced neither an adverse housing wealth shock in boom counties nor a drop in collateral values. Hence, if the difference in debt reduction between renters in boom counties and renters in non-boom counties is greater than or equal to the difference between homeowners in boom counties and homeowners in non-boom counties, this would indicate the presence of general credit supply effects that are independent of house value effects. If, on the other hand, the difference between renters in non-boom counties and renters in boom counties is smaller than the difference between the debt reduction of homeowners in boom counties versus that of homeowners in non-boom counties, we cannot identify supply effects. Hence, our identification of credit supply effects relies on a double difference-in-differences term. The first difference is the one between boom and non-boom counties and the second is the difference between homeowners and renters. One possible challenge to our empirical approach is that the economic downturn affected renters more in boom than in non-boom counties. For example, the shock to consumer fundamentals (consumer expectations, incomes, unemployment) may have been more severe for renter households than for homeowner households. We check for this possibility and find that boom versus non-boom county changes in median income over this period across U.S. counties appear to be uncorrelated with the percentile of the homeownership rate (see figure 2A). In addition, examining changes in the 20th percentile of income, we find that counties at the *higher* end of the homeownership rate distribution show a more consistent pattern of deeper declines in income in boom than in non-boom counties than do counties at the lower end of the homeownership rate distribution. We also use Panel Study of Income Dynamics (PSID) data to show that renter income growth over our period did not vary significantly with pre-recession house price growth across different markets (see figure 3). We take this finding as supportive of our overall approach of using renters as a control group for studying the impact of the housing bust on credit supply.

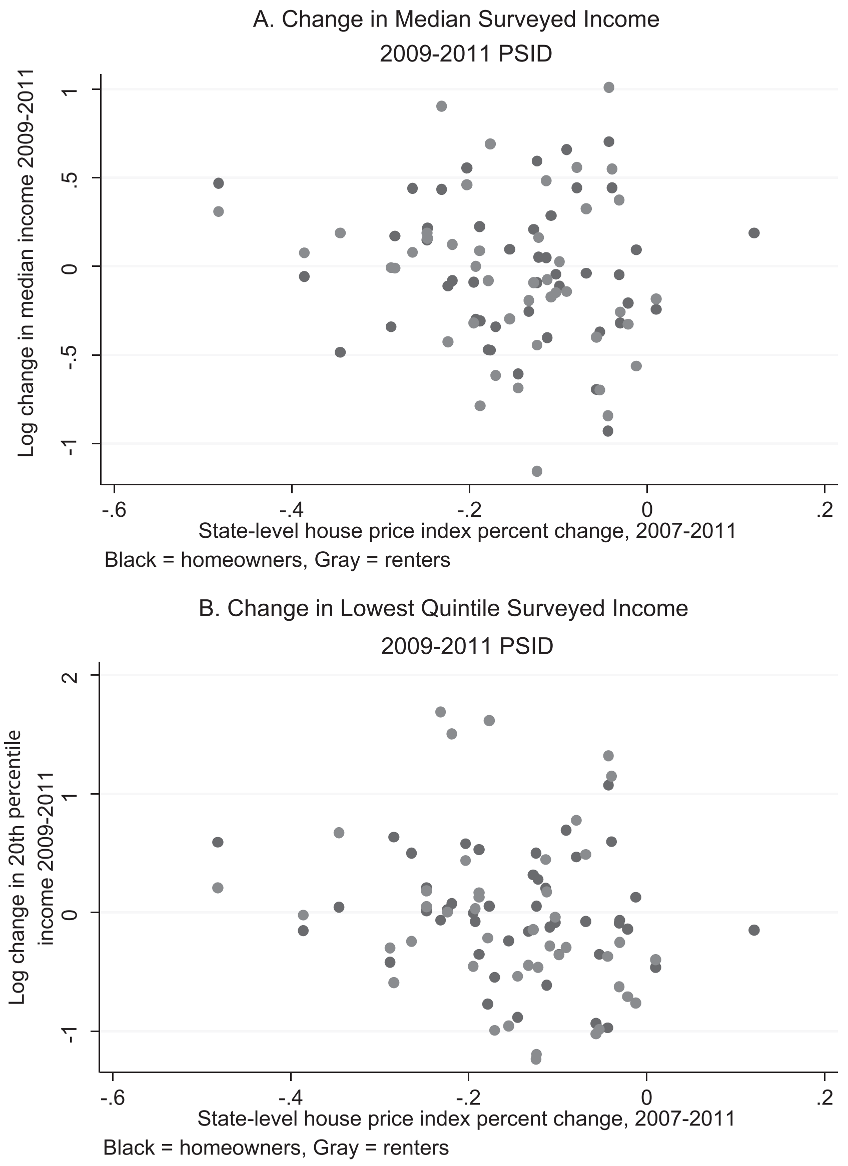
Figure 2. Changes in Median and 20th Percentile of Household Income



Source: American Community Survey and Federal Reserve Bank of San Francisco calculations.

Our results indicate the presence of general credit supply effects. Controlling for demand for debt using our methodology, we find stronger deleveraging in boom counties after the crisis, consistent with MRS. However, this difference appears only for renters. We

Figure 3. Changes in Median and 20th Percentile Surveyed Income



argue that, as renters were not hit by an adverse wealth shock, this finding must be due to differences in the availability of credit in boom versus non-boom counties. Finally, we trace the effect back to weaker bank balance sheets in boom counties compared with non-boom counties, using an instrumental-variables (IV) approach, and map out the transmission channel from bank balance sheets to household consumer borrowing.

This paper is an empirical microeconomic investigation. However, in the background, on the demand side, we have in mind a general consumption-smoothing framework. A sharp unanticipated drop in house prices may cause leveraged households to want to reduce debt by way of a standard wealth effect. On the supply side, the drop in collateral values may reduce the provision of credit to households, which may result in a larger reduction in debt than the wealth effect alone would have generated. Decreases in the provision of credit due to the collateral channel are consistent with arguments in Eggertsson and Krugman (2012) and Midrigan and Philippon (2011). Of interest to us are various other sources of reduced credit provision during this period. For example, see Damar, Gropp, and Mordell (2012) for an empirical investigation using Canadian data of the effect of bank financial distress on household consumption. In addition, see Dynan and Edelberg (2013) for a comprehensive list of potential supply and demand factors that may affect household leverage, and Bhutta (2013) for evidence of decreases in the supply of mortgage credit since the housing bust. Brown, Stein, and Zafar (2015) also use the Federal Reserve Bank of New York's (FRBNY's) Consumer Credit Panel to report interesting differences in the way different demographic groups (e.g., older and prime homeowners) substituted between home equity and credit card debt over the sample period.

We complement the large literature that links household debt, household wealth, and consumption with a focus on the Great Recession. For example, Carroll (2013), Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2011), Hall (2011), and Midrigan and Philippon (2011) all point to a high level of household debt as an important precursor of the Great Recession. In these expositions, a negative shock to homeowner collateral values causes lenders to restrict credit to homeowners. We show, however, that the effect coming from bank balance sheets and the consequent general restriction in credit supply for all borrowers may have been at least as important as the channel these papers point to. Additional empirical evidence linking high levels of household debt to economic downturns in a macroeconomic context can be found in Glick and Lansing (2009, 2010), Jordà, Schularick, and Taylor (2012), and Mian and Sufi (2010).

The paper is organized as follows. In section 2 we describe the data. In section 3, we present the results of our pre-crisis debt-level regressions. In section 4, we present the results of our post-crisis debt-change regressions. Section 5 concludes.

2. Data

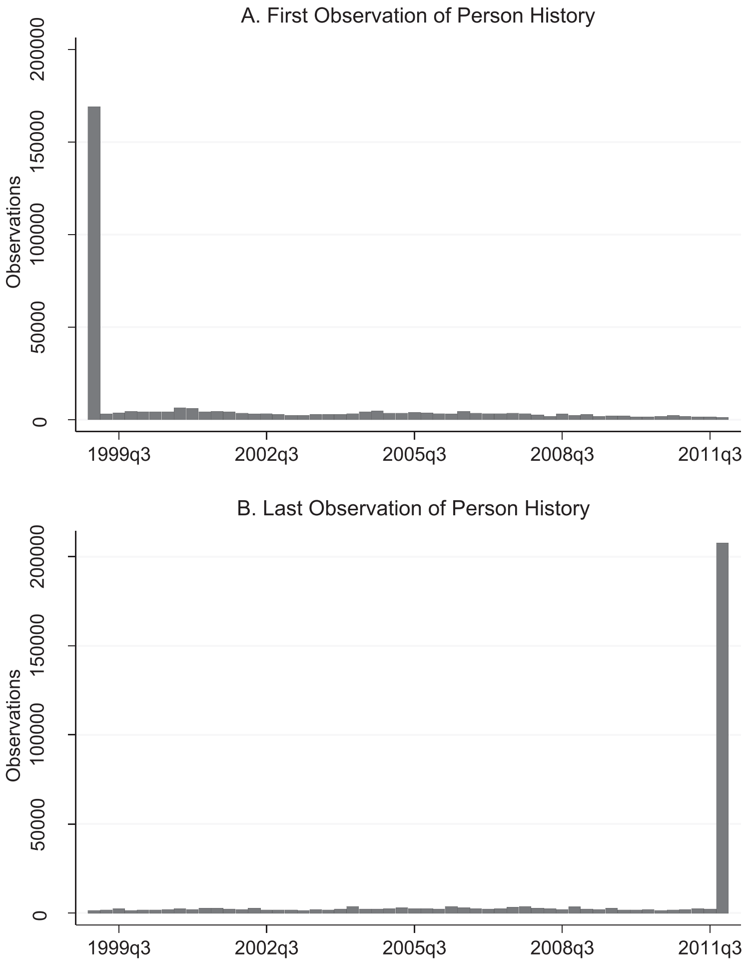
The data come from Equifax, a large credit-reporting agency. The data span 1999:Q1 to 2011:Q4 and contain a large amount of information on consumer liabilities—mortgage, home equity, auto, credit card, etc., and some borrower characteristics such as age, risk score, and delinquency status on their liabilities.[8] All analysis is based on data from the Federal Reserve Bank of New York's Consumer Credit Panel, which is a 5 percent random sample of consumers with credit histories that is nationally representative in a given quarter and also designed to reproduce the transitions of young and old into and out of the credit pool.[9] To make the data set more manageable, we used a 10 percent random sample of the Consumer Credit Panel, implying a .5 percent random sample of the U.S. population with credit histories. Consumers are located by the Zip code of their home mailing address. After identifying the county of each residence and merging with house price indexes available from CoreLogic, our sample consists of over 900,000 individuals living in more than 1,100 counties across the country.

The distribution of entry into and exit from our sample may be seen in figure 4. On average, about 3,000 new credit histories appear in our sample each quarter, offset somewhat by about 2,300

[8] The risk score is the Equifax Risk Score.

[9] See Lee and Van der Klaauw (2010) for a description of the sampling methodology used in the construction of the Consumer Credit Panel.

Figure 4. Distribution of Entry into and Exit from Sample



Source: FRBNY Consumer Credit Panel/Equifax.

history terminations each quarter, so that our sample is slowly growing over time.[10] The commencement of a borrower history, however,

[10]Termination of a borrower record could take place for a variety of reasons, including death or instances where a trade line (credit type) has no recorded activity for a length of time greater than Equifax's predefined limits. Also, the

does appear closely related to age. The new borrowers entering the sample had a median age of about twenty-eight, a number which trended downward over the sample period. Newly appearing borrowers had median risk scores of about 660, well below the overall sample median of 712.

Only one-third of our sample of consumer histories spans the full 1999:Q1–2011:Q4 period. While this is still a substantial amount of data, in this analysis we use the entire, unbalanced panel to allow some of the compositional changes that we experienced over the 2000s to enter into the analysis. As alluded to above, many of the new entrants to the panel were young households with low risk scores. These household borrowers were particularly susceptible to the economic volatility that occurred at the end of our sample, and we will want their credit experiences present in our data. In many ways, this group bore the brunt of the shock that hit the U.S. housing market starting in 2006. By contrast, a more seasoned homeowner that bought in 1999 and stayed in the house most likely experienced net house price appreciation over the entire period.[11] Indeed, average risk scores for borrowers present for the entire sample period actually increased over the thirteen-year period, whereas average risk scores for the population at large fell quite notably.

For a first glimpse at the loan balance data, we compare the Flow of Funds data with the aggregated totals in our sample. While the match is not perfect, the correlation of our total mortgage series (figure 5) and total non-mortgage debt series (figure 6) with the Flow of Funds counterpart is quite high. Throughout, we define non-mortgage debt as the sum of auto, credit card, and other non-mortgage consumer loan balances outstanding, excluding student debt.[12] This particular concept of non-mortgage debt does not match

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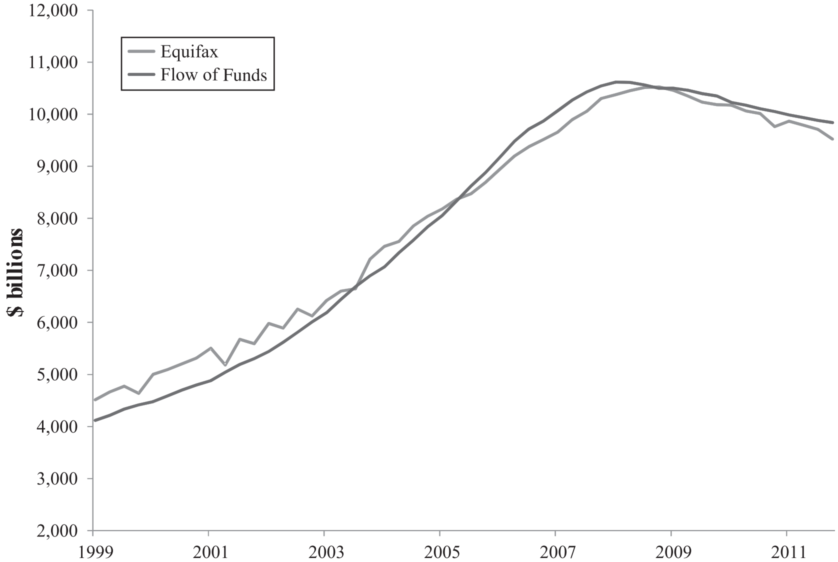
Consumer Credit Panel draws from borrowers with specific sequences of digits in their social security numbers. If an individual changes their social security number, they could drop out of the sample.

[11] Of course, the same homeowner may still have changed leverage over the period and become underwater relative to their mortgage debt despite the overall price appreciation.

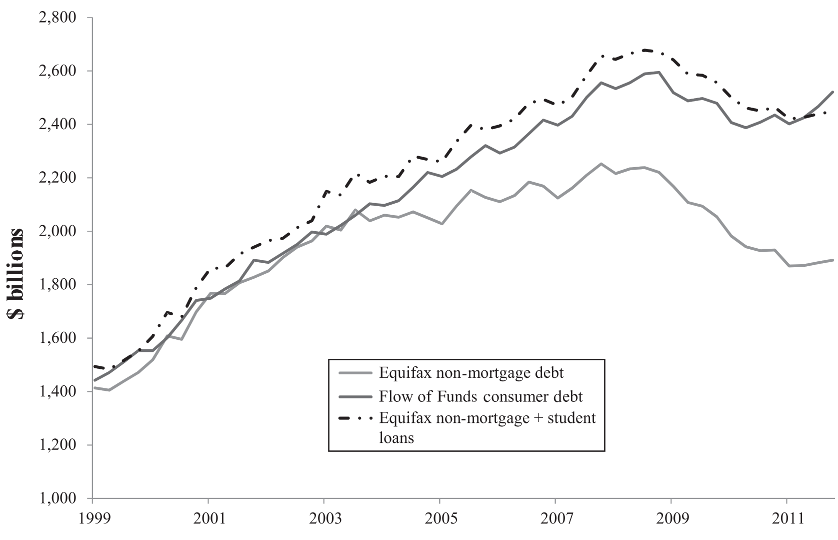
[12] Our non-mortgage consumer debt excludes any consumer debt secured by a house, so it excludes, for example, home equity debt. In the appendix, we discuss results obtained when we add home equity debt to total non-mortgage debt.

Figure 5. Total Mortgage Debt (includes home equity)



Source: Federal Reserve Board, FRBNY Consumer Credit Panel/Equifax.

Figure 6. Non-mortgage Debt



Source: Federal Reserve Board, FRBNY Consumer Credit Panel/Equifax.

the dynamics of the Flow of Funds, because we do not include student loans in our measure. This choice was made because of the uncertainties of measuring student loan debt as well as our desire to focus on a non-mortgage debt series that is plausibly linked to debt put in place to finance consumption.

As seen in figures 5 and 6, through the course of our sample period, total mortgage debt more than doubled, peaking in 2008 and then falling by about 10 percent. Non-mortgage debt declined somewhat more, as balances fell 15 percent from the peak.

Following MRS, our debt-level regressions use the estimates from Saiz (2010) of the elasticity of local housing supply with respect to price as a way of controlling for exogenous features of the land that might lead to differential house price levels and, hence, differences in debt. For example, MRS present empirical evidence that, in counties with highly inelastic housing supply and rapidly increasing house prices during the boom, homeowners were especially likely to increase their debt. We manually link the 806 MSA-level elasticity estimates in Saiz (2010) to the counties in our data set. About 10 percent of our observations are from locales not covered by Saiz (2010). Inspection reveals that the vast majority of these match failures are in less-populated areas. To conserve data, we assigned an imputed elasticity equal to the sample maximum to these observations and included a missing elasticity dummy variable in all our debt-level regression specifications.

At the individual borrower level, our debt-level regressions include the number of inquiries made to Equifax over the preceding four quarters regarding the consumer's credit record as well as the borrower's age and risk score. The inquiries are usually made as a result of the consumer seeking more credit and therefore are a useful gauge of overall credit demand.

We also use information from the U.S. Census Bureau's American Community Survey (ACS), based on pooled data from 2006 through 2010. These demographic data are at the census tract level. All of the variables here are meant to proxy for income, wealth, family attributes, and the many other factors that would be expected to influence an individual's demand for credit independent of their influence through changes in credit record inquiries. Finally, we use the unemployment rate, from the Bureau of Labor Statistics, which is monthly and is at the county level. Many of our demographic variables are the same as those in Cohen-Cole (2011) and Musto and Souleles (2006).

Table 1 gives some summary statistics on the demographic variables we use in the analysis.

3. Pre-crisis Debt-Level Regressions

The first step in establishing a benchmark for the demand for credit takes the form of debt-level regressions as given by

$$D_i = \alpha + f(X_i) + \Gamma X + \varepsilon, \tag{1}$$

where D_i is an individual debt category for individual i, X_i consists of borrower i's age and risk score and the number of inquiries requesting borrower i's credit report in the previous twelve months, and X is a vector of control variables at the census tract or county level.[13] We estimate the above equation as a pooled regression on a subsample of our data using observations from 1999:Q1 to 2008:Q2, the quarter at which total household debt peaked in our overall sample. All debt categories are in logs. The results from the OLS specifications may be found in table 2.

The number of credit report inquiries comes in strongly positive in all categories, as expected. Interestingly, we do not find a particularly strong role for the supply elasticity in explaining cross-sectional differences in consumer debt. Only for home equity does the elasticity coefficient estimate have both the expected sign and statistical and economic significance that would be fully consistent with MRS. Since we are looking at the individual borrower level and focusing on debt levels—not debt-to-income ratios—this result is not necessarily a contradiction of the results in the MRS paper. In addition, we include more control variables than MRS. This is an important result for our study in that the supply elasticity is one of only two variables in our set of controls that vary meaningfully

[13] The regression in equation (1) also contains a complete set of time dummies to capture macroeconomic fluctuations over the period. The individual's age and risk score enter the equation as piecewise linear splines. Explanatory variables also include squared terms of all the other non-categorical variables. We do not report the coefficients on the time dummies, on the age or risk score splines, or on the squared terms.

Table 1. Summary Statistics

	Mean	Std. Dev.	p25	p50	p75
Age	48.1	17.84	34	46	60
Credit Inquiries (Trailing Twelve Month)	2.598	6.722	1	3	4
Risk Score	688.5	107.4	610	710	781
County Unemployment	6.107	2.61	4.3	5.4	7.4
Housing Supply Elasticity	1.729	0.95	0.998	1.645	2.175
High Small Business Percent	23.614	10.867	15.878	24.924	29.427
Median Age	37.96	6.673	33	38	42
Median Homeownship Costs (Monthly)	1,391	628.1	925	1,267	1,710
Median Rent (Monthly)	836.5	368.5	568	749	1,010
Median Tract Income (Annual)	60,048	27,504	41,103	54,561	73,150
Median Year Moved into Housing Unit	2001	3.423	1999	2001	2003
Percent Black	12.72	21.6	0.856	3.608	12.882
Percent College Education	29.88	18.63	15.37	25.549	41.146
Percent Food Stamps	9.546	9.231	3.005	6.671	13.048
Percent High School Only	27.92	10.64	20.525	28.436	35.524
Percent Hispanic	8.816	13.08	1.248	3.802	10.367
Percent Homeowner	66.39	21.45	52.99	70.729	83.521
Percent in Labor Force	66.14	8.403	61.52	66.887	71.604
Percent Male	0.487	0.037	0.466	0.487	0.508
Percent Married	49.48	12.93	41.453	50.95	59.097
Percent One Vehicle	95.2	10.06	95.643	98.273	99.426
Percent Some High School	14.15	11.29	6.084	11.087	18.904
Percent Working	78.48	7.787	74.452	79.756	83.893
Total Tract Population	4,990	2,038	3,618	4,799	6,115
Observations	16,024,144				

Source: FRBNY Consumer Credit Panel/Equifax, Saiz (2010), and the 2010 American Community Survey.
Notes: The table shows the summary statistics (means and percentiles) of the explanatory variables used in the debt-level and debt-change regressions. The sample period is from 1999:Q1 to 2011:Q4. Age, credit inquiries, and risk score are observed at the consumer level. The housing supply elasticity is taken from MSA-level estimates in Saiz (2010) and mapped to the county level by the authors. All other variables are observed at the county or census tract level.

Table 2. Debt-Level Regressions

	Total Debt (1)	Mortgage (2)	Home Equity (3)	Non-mortgage (4)	Auto (5)	Credit Card (6)
Housing Supply Elasticity	0.021***	0.052***	−0.014***	0.008***	0.001	−0.008***
	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.002)
Missing Elasticity Dummy	−0.008	−0.014*	−0.145***	−0.031***	−0.125***	−0.031***
	(0.005)	(0.006)	(0.005)	(0.005)	(0.006)	(0.005)
Credit Inquiries (Trailing Twelve Month)	0.335***	0.184***	0.100***	0.321***	0.346***	0.204***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.011)
County Unemployment	0.010***	−0.017***	0.048***	0.027***	0.034***	0.033***
	(0.003)	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)
Median Tract Income	−4.089***	−15.373***	−2.114***	−0.854***	0.265	−1.371***
	(0.195)	(0.237)	(0.166)	(0.191)	(0.215)	(0.191)
Percent College Education	−0.003***	0.002*	0.003***	0.000	−0.010***	0.007***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)
Percent Black	−0.007***	−0.005***	−0.001***	−0.003***	0.002***	−0.004***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Percent Hispanic	0.002***	0.003***	−0.009***	0.006***	0.015***	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
High Small Business Percent	−0.174***	−0.247***	−0.151***	−0.072***	−0.336***	0.077***
	(0.015)	(0.019)	(0.013)	(0.015)	(0.017)	(0.015)
Percent One Vehicle	−0.008*	−0.051***	0.016***	−0.005	0.002	−0.024***
	(0.004)	(0.005)	(0.003)	(0.004)	(0.004)	(0.004)
Percent Married	0.004***	−0.008***	−0.003***	0.007***	0.010***	0.001
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Percent Food Stamps	−0.012***	0.001	−0.002***	−0.019***	−0.018***	−0.018***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.001)
Percent Working	−0.018***	−0.024***	0.014***	−0.013***	−0.008**	−0.023***
	(0.002)	(0.003)	(0.002)	(0.002)	(0.003)	(0.002)
Percent Homeowner	0.012***	0.023***	0.011***	0.001**	−0.000	−0.002***
	(0.000)	(0.001)	(0.000)	(0.000)	(0.001)	(0.000)

(*continued*)

Table 2. (Continued)

	Total Debt (1)	**Mortgage (2)**	**Home Equity (3)**	**Non-mortgage (4)**	**Auto (5)**	**Credit Card (6)**
Median Age	-0.002^{***}	-0.012^{***}	-0.002^{***}	0.004^{***}	-0.006^{***}	0.011^{***}
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Total Tract Population	-0.676^{***}	-1.331^{***}	0.086	−0.073	-0.884^{***}	0.227^{**}
(in logs)	(0.080)	(0.097)	(0.068)	(0.079)	(0.088)	(0.079)
Median Rent (in logs)	0.195	3.169^{***}	1.063^{***}	0.308^{**}	0.948^{***}	1.001^{***}
	(0.108)	(0.132)	(0.093)	(0.107)	(0.120)	(0.107)
Median Homeowner Cost	-0.210^{*}	1.219^{***}	-0.463^{***}	0.308^{**}	1.342^{***}	0.958^{***}
(in logs)	(0.101)	(0.123)	(0.086)	(0.099)	(0.112)	(0.099)
Percent Some High School	-0.019^{***}	-0.013^{***}	−0.0000	-0.021^{***}	-0.019^{***}	-0.021^{***}
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Percent High School Only	-0.008^{***}	-0.013^{***}	0.003^{***}	-0.005^{***}	-0.002^{***}	-0.006^{***}
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Constant	15.993^{***}	58.421^{***}	0.698	-2.533^{*}	-20.309^{***}	1.011
	(1.155)	(1.403)	(0.987)	(1.135)	(1.277)	(1.136)
Observations	6,351,515	6,351,515	6,351,515	6,351,515	6,351,515	6,351,515
R-squared Adjusted	0.211	0.172	0.056	0.161	0.103	0.122
Log Likelihood	−1.722e+07	−1.845e+07	−1.622e+07	−1.710e+07	−1.785e+07	−1.711e+07

Source: FRBNY Consumer Credit Panel/Equifax.
Notes: This table depicts the debt-level regressions using data from 2004:Q2 to 2008:Q2. These specifications will be used to generate predicted debt levels for consumers living in boom and non-boom counties in 2008:Q3. Debt levels enter specification in logs. Explanatory variables also include squared terms of all the non-categorical variables, as well as splines for age and risk score as described in the text.

across counties. As we will see, there are not large differences in our distribution of predicted debt levels across counties when we sort by house price appreciation during the housing boom. This finding is consistent with our finding of a muted role for cross-county variation in supply elasticity.

The other variable in our set of controls that varies meaningfully across counties is the current unemployment rate. As with all the other controls, the presence of credit report inquiries in the regression complicates interpretation of the coefficients on the unemployment rate. However, we do note that the unemployment coefficient for all but the mortgage category is positive. This finding is consistent with the results in Hurst and Stafford (2004), who document household consumption smoothing in the face of income shocks by way of drawing on credit lines—particularly home equity credit.

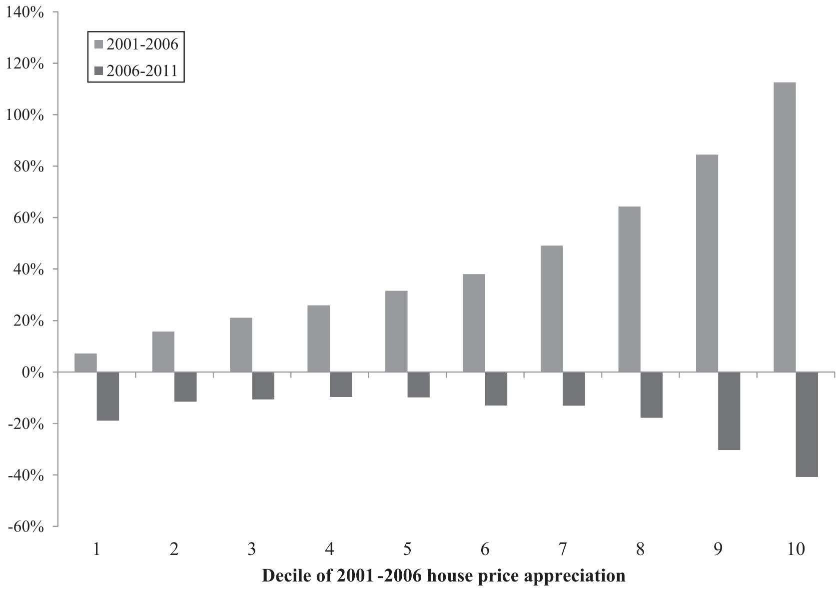
4. Post-crisis Debt-Change Analysis

4.1 Empirical Setup

The ultimate objective of this paper is to ascertain whether a contributing factor to households in counties with particularly strong pre-crisis house price appreciation reducing their debt was because they were unable to obtain the desired amount of credit. Following MRS, we break our sample up into regional groupings according to county house price appreciation during the 2001–06 period (figure 7). We form a group of low-appreciation counties that were in the bottom two deciles of boom-period house price appreciation ("non-boom" counties). We also form a group of high-appreciation counties from the top two deciles of this same distribution ("boom" counties).

Disentangling the demand for credit from the supply of credit is no easy task. Ideally, we would be able to identify consumers with identical demand for credit but living in counties with different exposures to house price shocks. Further, despite differing house price shocks, we would like all other economic conditions in these different counties to evolve in exactly the same way. In this idealized setting, with underlying credit demand controlled for, differences in household debt changes across the counties would be interpreted as

Figure 7. County-Level Deciles of House Price Appreciation Rates



Source: CoreLogic.

differences in credit availability. Using matching techniques and taking advantage of one group of households not directly affected by an adverse housing shock, renters, we feel we come quite close to such an idealized setting.

We proceed as follows. First, the data set does not contain a variable identifying consumers as "homeowners" or "renters" directly. Instead, we classify consumers with a mortgage outstanding as "homeowners" and those that do not have mortgages as "renters."[14] This classifies those consumers that paid off their mortgages fully as

[14]Thus, a "renter" is a long-term renter, and similarly for homeowners. When working with a renter- and homeowner-only sample, this means that we will be dropping consumers who switch tenure choice during our window of analysis. The assumption also implies that our total sample size (column 1 of table 3) will be greater than the sum of sample sizes of the renter- and homeowner-only samples (columns 2 and 3 of table 3).

renters, causing some measurement error. However, with a fully paid-off mortgage, it does seem reasonable to classify these consumers as not being subject to a severe housing shock, as while the value of their home may have declined recently, they are likely to have owned their home for a long time. In any case, the data suggest that the values of the homes of these consumers were above the purchase levels even after the collapse of the housing market in 2009–10.

Next, we perform a propensity-score matching exercise that pairs consumers that are similar in terms of their probability of living in a boom county but are different according to whether they actually live in a boom county or not. We perform the matching exercise separately for homeowners and for renters. We then take a difference-in-differences approach among the matched consumers within their predicted 2008:Q2 total debt-level category, to investigate whether we observe differences in deleveraging following the post-2008 boom. Finally, we estimate a set of regressions of changes in debt at the individual consumer level that allows us to control not just for proxies for initial-period consumer-specific credit demand but also for differences in subsequent changes in the economic environment that might have led to changes in demand as the housing bust and recession set in in mid-2008.

Before presenting the results, we note that our identification strategy requires that, conditional on our matching routine, renters in boom counties differ from their counterparts in non-boom counties only by their county of residence. One concern might be that once the recession was under way, the effects of the economic slowdown were stronger in boom counties, and renters there were disproportionately affected compared with renters in non-boom counties. In the robustness section we show that there is no evidence for such differential exposure to the recession. Among other tests, we use data from the PSID to show that renters did not suffer significantly larger income declines in boom markets than in non-boom markets; nor did they suffer significantly higher incidences of unemployment.[15] Compared with homeowners, renters did indeed suffer larger income declines.

[15]The PSID does not allow us to identify individuals at the same county level we use in our analysis, so we base our tests on whether renter income growth in the states containing boom counties was different than growth in states containing non-boom counties.

But these growth rates do not vary across markets or with measures of pre-recession house price growth, leaving our identification strategy intact.

The matching routine is based on a probit model of the assignment of consumers into boom versus non-boom counties, estimated separately for homeowners and renters.[16] Our predictors in the probit model are the same variables as are in the debt-level regressions, with the exception that we omit the squared terms for the demographic variables and we replace the spline terms for age and risk score with the level of these variables. The fitted probabilities of living in a boom county are estimated over all consumers present in the data in 2008:Q2. Homeowners (renters) in the boom counties are matched to homeowners (renters) in the non-boom counties by virtue of the similarity of their fitted probabilities of living in the boom county. We match with replacement.

Using this matched sample, we then take slices of the predicted total debt distribution in our base year (2008:Q2) to focus on how non-mortgage debt changed amongst the consumers as a function of their overall level of indebtedness. In this spirit we focus on consumers with low debt (less than 20th percentile of total predicted debt in 2008:Q2), medium debt (40th–60th percentile of total predicted debt in 2008:Q2), and high debt ranges (greater than 80th percentile). The predictions of total debt in 2008:Q3 are based on the estimates of the debt-level regressions in column 1 of table 2.

Finally, we then compare the change in non-mortgage debt from 2008:Q3 to 2011:Q4 for residents of boom counties with the same difference for their matches in non-boom counties, again comparing renters who, even in boom counties, were not subject to an adverse housing wealth shock with homeowners who were.

4.2 Matching Results

The results from the probit estimation may be found in table 3. The results suggest that consumers living in boom counties have a higher rate of credit inquiries, on average. Homeowners in boom counties also have higher risk scores than those in non-boom counties. As of

[16]We omit borrowers living in counties other than the bottom two and top two deciles from all of the empirical analyses from here on in the paper.

Table 3. Probability of Living in High-Appreciation County

	All Consumers	Homeowners	Renters
	b/se	b/se	b/se
Age	0.001***	0.006***	0.001*
	(0.000)	(0.001)	(0.000)
Credit Inquiries (Trailing Twelve Month)	0.031***	0.071***	0.026***
	(0.002)	(0.007)	(0.003)
Risk Score	0.077	0.914***	0.006
	(0.054)	(0.164)	(0.069)
County Unemployment	−0.151***	−0.216***	−0.143***
	(0.003)	(0.008)	(0.004)
High Small Business Percent	0.856***	0.364***	0.941***
	(0.044)	(0.104)	(0.057)
Median Age	0.046***	0.060***	0.041***
	(0.001)	(0.003)	(0.001)
Median Homeowner Cost (in logs)	1.244***	1.981***	1.102***
	(0.022)	(0.070)	(0.028)
Median Rent (in logs)	1.775***	1.409***	1.871***
	(0.019)	(0.044)	(0.026)
Median Tract Income	1.240***	1.237***	1.168***
	(0.036)	(0.101)	(0.045)
Median Year Moved into Housing Unit	−0.001	−0.019***	0.004
	(0.002)	(0.005)	(0.003)
Percent Black	−0.002***	−0.004***	−0.001***
	(0.000)	(0.001)	(0.000)
Percent College Education	−0.049***	−0.057***	−0.045***
	(0.001)	(0.002)	(0.001)
Percent Food Stamps	−0.040***	−0.033***	−0.041***
	(0.001)	(0.003)	(0.001)
Percent High School Only	−0.015***	−0.022***	−0.014***
	(0.001)	(0.003)	(0.001)
Percent Hispanic	0.050***	0.065***	0.046***
	(0.001)	(0.002)	(0.001)
Percent Homeowner	−0.019***	−0.022***	−0.017***
	(0.001)	(0.002)	(0.001)
Percent in Labor Force	−0.026***	−0.024***	−0.026***
	(0.001)	(0.003)	(0.001)
Percent Male	−1.562***	−2.264***	−1.468***
	(0.140)	(0.399)	(0.175)

(*continued*)

Table 3. (Continued)

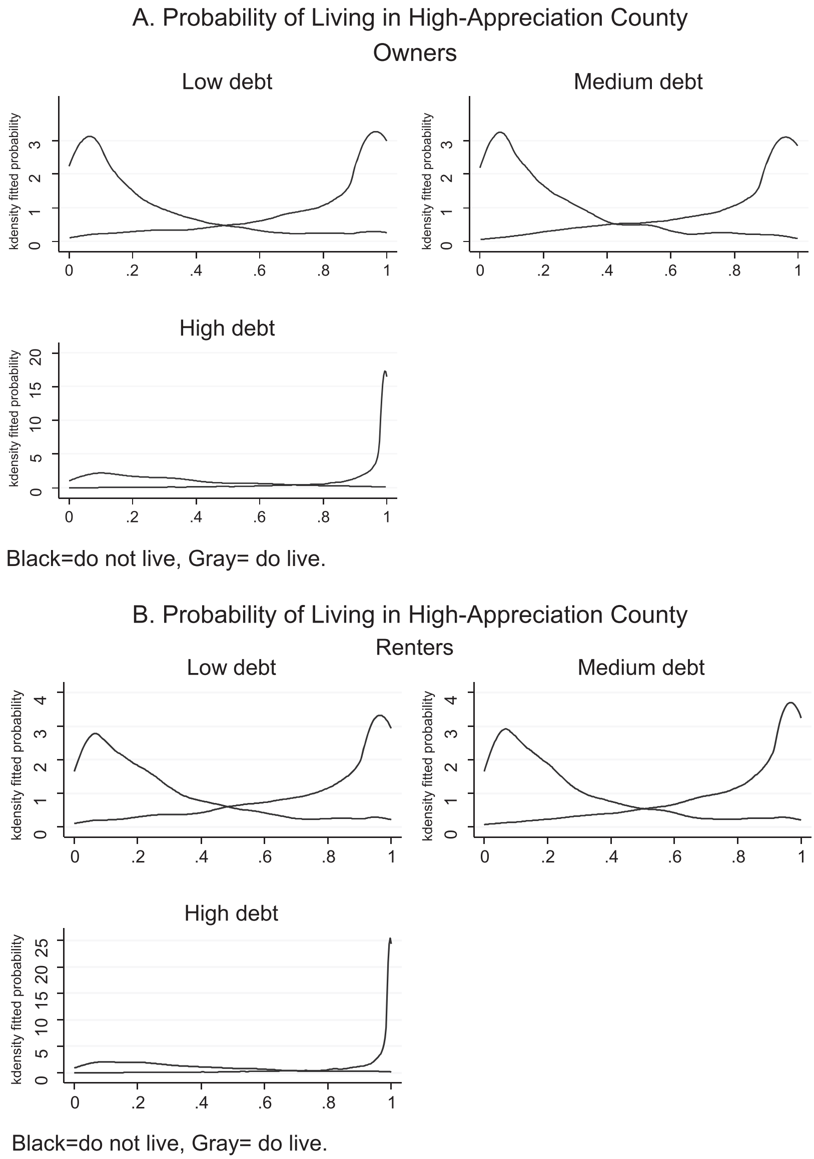
	All Consumers	Homeowners	Renters
	b/se	b/se	b/se
Percent Married	−0.028***	−0.041***	−0.024***
	(0.001)	(0.002)	(0.001)
Percent One Vehicle	−0.058***	−0.050***	−0.061***
	(0.001)	(0.004)	(0.001)
Percent Some High School	0.008***	0.004	0.010***
	(0.001)	(0.003)	(0.001)
Percent Working	−0.027***	−0.036***	−0.025***
	(0.001)	(0.004)	(0.002)
Total Tract Population (in logs)	0.386***	0.366***	0.402***
	(0.013)	(0.034)	(0.017)
Constant	−22.303***	11.205	−30.062***
	(4.172)	(10.815)	(5.299)
Observations	143,523	22,998	87,801
Log Likelihood	−42,789.858	−6,764.014	−26,074.814

Notes: This table presents the results from the probit model that forms the basis for the propensity-score matching routine. The dependent variable is the binary variable taking the value of one if a consumer lives in a high-appreciation boom county in 2008:Q2, and zero if the consumer lives in a low-appreciation non-boom county. Among the covariates, inquiries, age, and risk score are observed at the individual level. All other controls are observed at the county or census tract level. The "All Consumers" column reflects all records of consumers living in the boom and non-boom counties in 2008:Q2. The "Homeowner" ("Renter") column reflects all consumers who have had a mortgage (no mortgage) continuously for three years leading up to 2008:Q2. ***, **, and * denote significance at the 1 percent, 5 percent, and 10 percent level, respectively.

2008, the geographic controls identify the boom counties as having census tracts with somewhat better economic conditions as given by a higher incidence of small business activity, lower unemployment, and lower shares of households living on food stamps.

In figure 8, we plot the kernel density estimates of the fitted probabilities of living in a boom county, conditional on actual county of residence and conditional on our estimate of predicted total debt. We focus on low-debt consumers (in the 20th percentile of the predicted debt distribution), consumers with predicted total debt in a middle range (40th–60th percentile), and consumers with high levels

Figure 8. Probability of Living in High-Appreciation County: Owners and Renters



Source: FRBNY Consumer Credit Panel/Equifax.

of predicted debt (above the 80th percentile of predicted total debt). We generally have ample common support for the two distributions of fitted probabilities, meaning that for most of the boom county consumers in these particular slices of the predicted total debt distribution, there exists a counterpart actually living in a non-boom county with a similar propensity to live in a boom county. This observation is equally valid for the sample of homeowners (figure 8A) as it is for the renter sample (figure 8B). For consumers with high predicted levels of total debt, however, we see that there is less common support in the distribution of fitted probabilities of living in the boom counties. As can readily be seen in both the homeowner and renter figures, there are relatively few individuals with high predicted debt who live in a non-boom county but have attributes that give them a high probability of living in a boom county. We have 1,284 renters with high predicted debt in non-boom counties matched to the renters with high predicted debt in boom counties. We have 192 homeowners with high predicted debt in non-boom counties matched to the homeowners with high predicted debt in boom counties.

In tables 4–6, we summarize the demographic variables in our matched sample. As we would expect, once we have matched on the consumer attributes from the probit regression (see tables 4–6), the average characteristics of consumers in the remaining sample look quite similar. This basic conclusion holds when we restrict our matching according to whether we observe a mortgage on the consumers' balance sheets (homeowners in table 4 and renters in table 5). The main exception to this result, however, is the percent Hispanic variable. Even after controlling for observables, our matched sample consists of a higher percentage of borrowers in the boom counties that live in census tracts with relatively high Hispanic representation.[17]

With matches in hand we can then test for differences in debt changes across counties and across different types of borrowers (homeowners and renters). Before we do this, however, we note in figure 9 that, unconditionally and in aggregate, we do see a noticeable difference in declines in non-mortgage debt across non-boom

[17]This is not surprising, given the prevalence of boom counties in California and other parts of the southwestern United States.

Table 4. Characteristics of Matched Sample of Consumers: All Borrowers

	Non-boom Counties		Boom Counties	
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**
Age	49.32	18.3	48.41	18.05
Credit Inquiries (Trailing Twelve Month)	1.639	2.691	1.645	2.492
Risk Score	690.2	114.1	687.4	111.6
County Unemployment Rate	5.672	1.462	5.649	1.67
High Small Business Percent	22.2	14.5	23.6	10.6
Median Age	38.57	6.491	37.45	7.401
Median Ownership Costs	7.073	0.338	7.323	0.423
Median Rent	6.595	0.354	6.872	0.358
Median Tract Income	10.87	0.454	10.95	0.435
Median Year Moved into Housing Unit	2001	3.468	2001	3.384
Percent Black	15.52	24.95	13.52	22.02
Percent College Education	29.6	18.62	28.78	18.08
Percent Food Stamps	9.707	9.908	8.744	8.721
Percent High School Only	28.25	10.55	26.17	9.727
Percent Hispanic	6.088	9.865	14.32	15.81
Percent Homeowner	68.71	21.64	62.11	23.19
Percent in Labor Force	65.52	8.629	65.44	8.725
Percent Male	48.5	4.17	48.7	3.75
Percent Married	49.11	14.93	47.93	12.17
Percent One Vehicle	96.5	5.759	93.45	12.25
Percent Some High School	12.93	10.14	16.93	13.37
Percent Working	78.14	8.369	76.59	7.708
Total Population	8.385	0.422	8.467	0.411
Observations	13,517		96,639	

Source: FRBNY Consumer Credit Panel/Equifax.
Notes: This table presents the summary statistics for the matched sample of boom and non-boom county consumers. All consumers in the boom counties are matched to corresponding non-boom county consumers. The matching is done through a propensity-score matching routine. The matching is done with replacement. Non-matched consumers are dropped from the sample for purposes of calculating the summary tables.

Table 5. Characteristics of Matched Sample of Consumers: Homeowners

	Non-boom Counties		Boom Counties	
	Mean	**Std. Dev.**	**Mean**	**Std. Dev.**
Age	51.05	11.51	51.87	11.77
Credit Inquiries (Trailing Twelve Month)	1.34	2.013	1.372	1.878
Risk Score	749.9	84.69	758.3	80.8
County Unemployment Rate	5.485	1.386	5.527	1.715
High Small Business Percent	23.5	15.5	24.6	10.2
Median Age	39.8	5.933	39.01	6.814
Median Ownership Costs	7.198	0.314	7.443	0.381
Median Rent	6.684	0.367	6.968	0.373
Median Tract Income	11.06	0.417	11.15	0.406
Median Year Moved in	2000	3.272	2000	3.513
Percent Black	9.234	18.11	8.463	15.47
Percent Food Stamps	6.588	7.208	5.796	6.118
Percent High School Only	25.84	10.56	23.89	9.873
Percent Hispanic	5.078	8.082	11.86	13.43
Percent Homeowner	75.48	18.49	71.84	18.88
Percent in Labor Force	66.75	7.851	66.21	7.907
Percent Male	48.8	3.79	48.9	3.25
Percent Married	54.63	13.23	53.18	10.76
Percent One Vehicle	97.96	3.552	96.89	6.615
Percent Some High School	9.753	8.384	12.6	11.16
Percent Working	79.92	6.522	78.13	6.575
Percent Colege Education	35.02	18.42	34.41	18.93
Total Population	4,962	1,949	5,336	2,307
Observations	2,143		14,809	

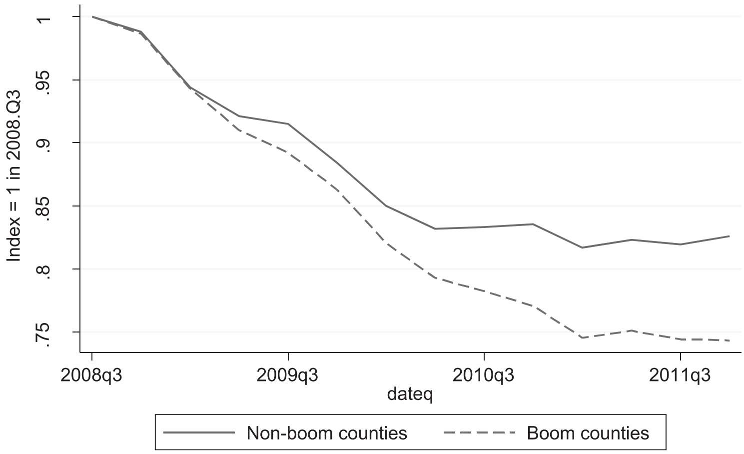
Source: FRBNY Consumer Credit Panel/Equifax.
Notes: This table presents the summary statistics for the matched sample of boom and non-boom county consumers. All consumers in the boom counties are matched to corresponding non-boom county consumers. The matching is done through a propensity-score matching routine. The matching is done with replacement. Non-matched consumers are dropped from the sample for purposes of calculating the summary tables. Homeowners are identified as consumers with positive mortgage balances over the preceding twelve quarters.

Table 6. Characteristics of Matched Sample of Consumers: Renters

	Non-boom Counties		Boom Counties	
	Mean	Std. Dev.	Mean	Std. Dev.
Age	52.05	19.47	50.64	18.9
Credit Inquiries (Trailing Twelve Month)	1.553	2.685	1.514	2.442
Risk Score	679.9	115.2	678.1	112.5
County Unemployment Rate	5.729	1.47	5.676	1.633
High Small Business Percent	22	14.2	23.5	10.6
Median Age	38.34	6.622	37.27	7.533
Median Ownership Costs	7.026	0.339	7.292	0.433
Median Rent	6.558	0.339	6.849	0.354
Median Tract Income	10.81	0.448	10.9	0.431
Median Year Moved in	2001	3.533	2001	3.364
Percent Black	17.76	26.94	14.88	23.55
Percent College Education	27.72	18.5	27.57	17.66
Percent Food Stamps	10.95	10.68	9.425	9.127
Percent High School Only	29.1	10.46	26.67	9.649
Percent Hispanic	6.357	10.49	14.76	16.14
Percent Homeowner	66.6	21.8	59.77	23.48
Percent in Labor Force	64.79	8.805	65.12	8.96
Percent Male	48.4	4.31	48.6	3.86
Percent Married	47.1	14.84	46.58	12.15
Percent One Vehicle	96.02	6.201	92.64	13.02
Percent Some High School	14.2	10.58	17.88	13.56
Percent Working	77.36	8.9	76.19	7.952
Total Population	8.358	0.423	8.452	0.407
Observations	8,231		59,120	

Source: FRBNY Consumer Credit Panel/Equifax.
Notes: This table presents the summary statistics for the matched sample of boom and non-boom county consumers. All consumers in the boom counties are matched to corresponding non-boom county consumers. The matching is done through a propensity-score matching routine. The matching is done with replacement. Non-matched consumers are dropped from the sample for purposes of calculating the summary tables. Renters are identified as consumers with zero mortgage balances over the preceding twelve quarters.

Figure 9. Non-mortgage Debt in Matched Sample



Source: FRBNY Consumer Credit Panel/Equifax.

and boom counties. Consumers in the boom counties reduced non-mortgage debt at a faster pace early on in the recession, and continued to reduce debt for a longer period than their matched counterparts in the non-boom counties.

The results of difference-in-difference tests are found in table 7, where we report the difference in changes in debt levels between the matched individuals. Negative numbers mean that individuals living in boom counties reduced their debt levels more than did their matched counterparts in non-boom counties. With the exception of low-debt homeowners, all statistically significant difference-in-difference estimates are negative, and all of these indicate economically important effects. In particular, for consumers with high levels of predicted total debt (top two deciles of the predicted debt distribution), we see more evidence of greater non-mortgage debt reductions in boom counties than in non-boom counties for renters than for homeowners.

4.3 Post-crisis Regressions on the Matched Sample

The results in table 7 suggest the possibility that an inward shift in credit supply played an important role in the deleveraging of

Table 7. Difference-in-Difference Results for Non-mortgage Debt

Percentile of Total Predicted Debt 2008:Q2	20th Percentile	40th–60th Percentile	80th Percentile
All Consumers	.088	0.029	$-.190^{***}$
	(.061)	(.060)	(.021)
Homeowners	$.265^{*}$	$-.735^{***}$	$-.536^{***}$
	(.137)	(.107)	(.040)
Renters	−0.010	$-.144^{*}$	$-.814^{***}$
	(.075)	(.077)	(.027)

Source: FRBNY Consumer Credit Panel/Equifax.
Notes: This table presents the results of differences-in-means t-tests of changes in non-mortgage debt from 2008:Q3 to 2011:Q4 among consumers living in boom counties vs. consumers living in non-boom counties. All results are based on a sample of matched consumers as described in the propensity-score matching method. With consumers matched by propensity scores, we also filter by restricting the sample to consist of matched consumers belonging to specific quantiles of the distribution of predicted total debt in 2008:Q2. A negative difference in mean debt change signifies that debt declined more in the boom counties than in the non-boom counties. ***, **, and * denote significance at the 1 percent, 5 percent, and 10 percent level, respectively.

households in boom counties. However, even though we control for debt demand and household- and tract-level characteristics, we would like to rule out that *changes* in these characteristics between 2008 and 2011 drive the differences in deleveraging of renters between boom and non-boom counties. Specifically, it is possible that renters fared worse from 2008 to 2011 in boom counties than in non-boom counties and that this underlies our differences in deleveraging. In order to address this point, we estimate variants of the following regression:

$$\Delta D_{i,2011-2008} = \alpha + \rho_1 BC + \rho_2 BC * Renter_i + \rho_3 BC * risk_i + \Pi X_i + \Theta X + \xi \Delta ue + \varepsilon, \quad (2)$$

where $\Delta D_{i,2011-2008}$ represents, for individual i, the difference between their 2008:Q3 non-mortgage debt level and their 2011:Q4 non-mortgage debt level.[18] The coefficients ρ_1, ρ_2, and ρ_3 on the indicator variable BC ("boom county") are of most interest. The first is intended to measure the degree to which households in boom counties reduce debt more than those in non-boom counties, controlling for changes in the non-house-price elements of the demand for debt. The second allows us to see how that differential depends on whether the borrower is a renter. The third allows us to see how that differential depends on the risk score of the borrower. We also include a triple interaction of boom county, renter status, and risk score to further explore the patterns in consumer debt reduction.

In all of the changes in debt regressions to follow, we work with the matched sample of borrowers from the propensity-score matching analysis above. That is, while we have both homeowners and renters from the two groups of counties, a non-boom homeowner only appears if it is matched to a homeowner in a boom county, and similarly for renters. The controls in X_i include, analogous to equation (1), the individual's age and risk score, and the number of credit report inquiries on the individual over the previous twelve months, all as of 2008:Q3. In addition, X_i includes the change in the individual's risk score between 2008 and 2011, whether the individual is a homeowner, and the individual's actual level of total debt, as of 2008:Q3. The controls in X also include the census tract and county level control variables as they appear in equation (1), with the exception of the unemployment rate, which, instead, enters equation (2) as a change between 2008 and 2011.

Table 8 presents the results for non-mortgage debt-change regressions. The coefficients on each of the individual level controls are highly statistically significant. As expected, consumers with a low risk score (<650) in 2008 tended to reduce non-mortgage debt more than those with a higher risk score. However, those consumers with risk scores that improved (i.e., a positive change in risk score) reduced debt more than consumers whose risk score worsened.[19]

[18] Our regression sample consists of pairs of individuals, matched on their probability of living in a boom county, within predicted 2008:Q2 debt ranges, as described in the text above.

[19] In part this may just be due to reverse causality, as those who reduced debt more may have been rewarded with a higher risk score.

Table 8. Change in Non-mortgage Debt Regressions

	(1)	(2)	(3)	(4)	(5)
Boom County	−.076**	−0.050	0.081	0.093	0.107
	(0.039)	(0.049)	(0.065)	(0.063)	(0.067)
Borrower Age in 2008:Q3		−0.018***	−0.018***	−0.018***	−0.018***
		(0.001)	(0.001)	(0.001)	(0.001)
Borrower Credit Inquiries in 2008:Q3		0.071***	0.070***	0.070***	0.070***
		(0.007)	(0.007)	(0.007)	(0.007)
Borrower Total Debt in 2008:Q3		−0.340***	−0.340***	−0.340***	−0.340***
		(0.005)	(0.005)	(0.005)	(0.005)
Change in Risk Score (2008:Q3–2011:Q4)		−0.007***	−0.007***	−0.007***	−0.007***
		(0.000)	(0.000)	(0.000)	(0.000)
Low Risk Score (<650) in 2008:Q3		−1.250***	−1.249***	−1.167***	−0.954***
		(0.031)	(0.031)	(0.071)	(0.235)
Renter (2008:Q3–2011:Q4)		−1.819***	−1.673***	−1.694***	−1.652***
		(0.052)	(0.057)	(0.059)	(0.067)
Boom County × Renter			−0.170**	−0.146*	−0.167*
			(0.064)	(0.068)	(0.078)
Boom County × Low Risk Score				−0.094	−0.186
				(0.074)	(0.252)
Renter × Low Risk					−0.245
					(0.253)
Boom County × Renter × Low Risk					0.109
					(0.274)
Constant	−0.562***	14.892	14.639	14.575	14.562
	(0.036)	(10.663)	(10.673)	(10.682)	(10.678)
Observations	76,437	76,437	76,437	76,437	76,437
R-squared Adjusted	.000	0.162	0.162	0.162	0.162

Notes: This table presents the results from the regressions of changes in non-mortgage debt on the boom county dummy and control variables. The change in non-mortgage debt is computed over 2008:Q3–2011:Q4. Among the covariates, age, credit inquiries, risk score, and total debt are calculated for each consumer for the base year 2008:Q3. We also compute a change in risk score over the 2008:Q3–2011:Q4 period. All regressions include tract- and county-level controls, and clustered standard errors at the county level. ***, **, and * denote significance at the 1 percent, 5 percent, and 10 percent level, respectively.

Individuals with more inquiries tended to reduce debt less, as expected. More debt in 2008 was correlated with greater reductions in debt. All other factors held constant, renters reduced debt more than homeowners. Again, this result points to the role of restricted credit supply during the 2008:Q3–2011:Q4 period. Controlling for the demographics we have available to us, we would naturally expect renters to reduce debt by less because their demand would not have been affected by the negative wealth shock to housing.

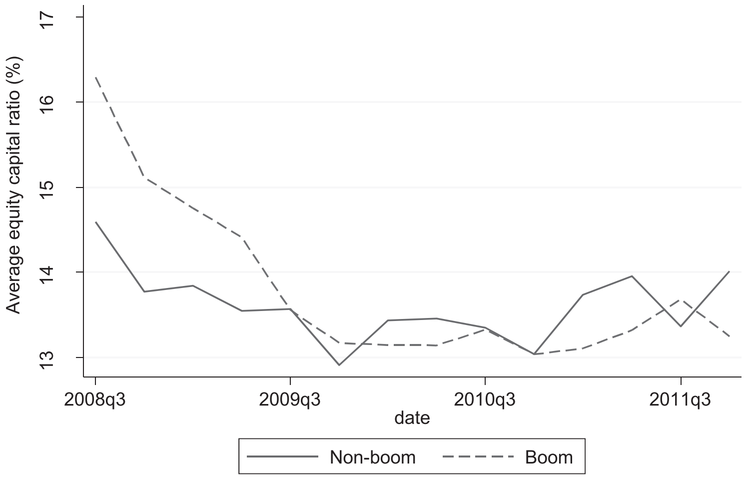
The coefficients on the interaction between boom county and renter in table 8, in columns 3–5, are negative and statistically significant. Living in a boom county strengthens the positive association between renters and reduction in non-mortgage debt. This contrasts with the lack of statistical significance of the effect of boom county on homeowners (the coefficient on boom county by itself) in the same specifications.

These results about the relationship between boom county and debt reduction point towards a more subtle interpretation of the determinants of household deleveraging during the aftermath of the housing boom than is typically reported. We generally corroborate the results in MRS with the positive association between boom county and deleveraging and also the finding that borrowers entering our analysis period with relatively large overhangs of total debt tended to delever more. However, the important point here is that the effect of residence in a boom county on any non-mortgage debt reduction tends to be larger for renters than for equivalent homeowners. This is indicated by the negative coefficient on the interaction between renter and boom county. This suggests that a general negative credit supply shock contributed to debt reduction in the post-2008 period. In addition, the lack of statistical significance of the coefficient on the three-way interaction between boom county, renter, and low risk score indicates that the cutback in supply was not confined to the borrowers with the highest levels of credit risk.

4.4 IV Results

The findings above naturally suggest a story where a bank lending channel restricted consumers' ability to borrow in the aftermath of the housing boom. To explore this idea more fully, we compare measures of bank conditions across the boom and non-boom counties during the 2008:Q3–2011:Q4 period. We associate banks with local markets on the basis of their headquarters of operation as reported in the Federal Reserve's Call Reports. When we restrict our analysis to include only those commercial banks with less than $1 billion in assets in 2008, we can be reasonably confident that our banks are

Figure 10. Average Equity Capital Ratios by County



small enough to be lending primarily in the counties where they are headquartered.[20]

As a first pass, in figure 10 we plot the average equity capital ratio of our sample of banks in both the boom and non-boom counties. As the figure indicates, local banks in the boom counties suffered losses during the recession and experienced a decline in their equity capital ratios. Since new lending must be supported by equity capital, the boom county banks were progressively more constrained in terms of capital as the analysis period rolled forward from 2008:Q3 and into 2011. The figure also makes clear that if we find any evidence of bank effects on consumer deleveraging, then the channel will be through a change in bank conditions. Indeed, the figure shows that average equity capital ratios in the boom counties declined to become more similar to the equity capital ratios observed in the non-boom counties. Thus, our sample of banks clearly indicates that boom county banks suffered a shock, but there is much less evidence of the non-boom county banks experiencing a similar shock.

Proceeding more formally, we estimate an IV model, in which we use boom county residence as an instrument for changes in bank

[20]The $1 billion asset threshold is a commonly used definition of a "community bank" in the U.S. banking sector.

capital of local banks. The model permits us to map out the transmission channel from banks' balance sheets to deleveraging. The approach should map out the part of extra deleveraging in boom counties due to differential changes in bank capital. Hence, a significant coefficient in the IV model shows the relative differences in deleveraging between homeowners and renters in boom versus non-boom counties limited to differences in credit supply of banks. As we are interested in the relative effect for homeowners and renters, we estimate the model separately for the two groups.

The results are presented in table 9. Columns 1 and 3 show that our instrument is strong: we obtain negative coefficients on the boom county indicator and large negative t-statistics for both the homeowner and renter samples. Economically, being headquartered in a boom county is highly correlated with a strong decline in bank capital in the 2008 to 2011 period. In the second stage (columns 2 and 4 of table 9), we find that changes in bank capital and changes in debt exhibit a negative and insignificant correlation for homeowners and a positive and significant correlation for renters. This evidence suggests that credit supply was important in the deleveraging of renters and not in the deleveraging of homeowners. This confirms our earlier findings that a significant part of the deleveraging process of households in the wake of the real estate boom before the crisis resulted from more restrictive lending of banks that were trying to rebuild capital after sustaining large losses.

5. Robustness

We conduct several robustness checks on our results. First, we investigate whether our results may be due to renters' demand for debt being more negatively affected by residence in a boom county than homeowners' demand for debt. Recall that it is not a problem if renters overall (regardless of being in a boom or non-boom county) were more affected by the economic downturn that followed the crisis than homeowners. What is important for our identification strategy to hold is that renters in boom counties were not more affected than renters in non-boom counties. Hence, we examine this question in detail here. Start with the observation that, according to the debt-level regression results in table 2, income is correlated with auto debt (positively) and credit card debt (negatively). No doubt some

Table 9. Local Bank Conditions and Changes in Non-mortgage Debt

	Homeowners Only		Renters Only	
	First Stage	Second Stage	First Stage	Second Stage
Dependent Variable	Change in Bank Equity	Change in Non-mortgage Debt	Change in Bank Equity	Change in Non-mortgage Debt
Boom County	−0.014***		−0.016***	
	(0.002)		(0.001)	
Predicted Change in Bank Equity Capital		−5.462		5.871*
		(5.543)		(2.935)
Constant		1.610		25.117
		(24.098)		(12.985)
Borrower Age in 2008:Q3		−0.011***		−0.019***
		(0.002)		(0.001)
Borrower Credit Inquiries in 2008:Q3		−0.015		0.085***
		(0.014)		(0.006)
Borrower Total Debt in 2008:Q3		−0.141***		−0.344***
		(0.028)		(0.004)
Change in Risk Score (2008:Q3–2011:Q4)		−0.005***		−0.007***
		(0.000)		(0.000)
Low Risk Score (<650) in 2008:Q3		−1.021***		−1.280***
		(0.083)		(0.034)
Demographic Controls	Yes	Yes	Yes	Yes
Observations	16,121	16,121	60,263	60,263
R-squared	0.024	0.016	0.026	0.175

Notes: The table reports the results of an instrumental-variables regression of the change in non-mortgage debt (2008:Q3–2011:Q4) on a local banking conditions variable and controls. The first stage is a regression of change in bank equity on the boom county indicator variable and demographic controls. The second stage is a regression of change in non-mortgage debt on the instrumented change in equity and the control variables. The control variables include county- and tract-level demographic variables used in table 8 and throughout the paper. The banking conditions variable is the average change in equity capital ratio (2008:Q3–2011:Q4) for banks headquartered in the consumer's county of residence. The average equity capital ratio is instrumented using the boom county indicator. A positive coefficient on the predicted change in equity capital variable indicates that growth in non-morntgage debt is positively associated with growth in local bank equity ratios. ***, **, and * denote significance at the 1 percent, 5 percent, and 10 percent level, respectively.

of this correlation is coming from the demand side and some from the supply side. In any case, if there were an association between boom versus non-boom county residence and changes in income, and if such an association showed a differential boom effect for renters versus homeowners, then our results might be due to differential demand changes, not the presence of credit supply shocks.

As a first piece of evidence consider figure 2. Figure 2 gives no evidence of differential boom effects that would confound the interpretation of our results. The figure uses data from the ACS vintages that correspond most closely with 2008, the beginning of the window over which we measure debt changes, and 2011, the end of the debt-change window. In figure 2A, we plot changes in median household income at the county level. In figure 2B, we plot changes in the 20th percentile of household income. On the horizontal axis of the two panels we plot the percentiles of the county homeownership rate, increasing from left to right. In figure 2A, there is no association between homeownership rate and any differences between changes in income in boom versus non-boom counties. Changes in incomes for boom counties that are systematically lower than changes in incomes for non-boom counties (the gray line below the black line) *along with* a shrinkage in this differential from left to right would support a view of differential demand changes that would be detrimental to our argument. Figure 2A shows no such pattern. Neither does figure 2B. In fact, figure 2B shows the opposite pattern. For households at the 20th percentile of income, there appears to be a more consistent pattern of deeper declines in income in boom than in non-boom counties on the right side of the graph than on the left, but the right side of the graph represents the *high*-homeownership counties, not the low-homeownership counties.

We also conducted a similar exercise to the one depicted in figures 2A and 2B using data from the Panel Study of Income Dynamics (PSID). The PSID allows us to look at the actual income distributions for both renters and homeowners, instead of using the local homeownership rates as proxies for renting versus owning. One drawback to the PSID, however, is that it only identifies the state of residence of the survey respondents. Thus, we cannot look at how income changed for renters and owners in our specific boom and non-boom *counties*, but are forced to analyze this at the state level. We use observations on total family income from the family files in 2009 and 2011 waves, which corresponds most closely to the sample period in our study. To parallel the analysis with the ACS data above, in each wave we calculate the median and the 20th percentile of the income distribution within each state, by renter and homeowner. Our change in income measure is the (log) difference of incomes at each of these percentiles. We then correlate these income changes with the four-year past house price appreciation in the state. We plot these relationships for the change in median income (figure 3A) and the change in income at the 20th percentile (figure 3B). In the figures, the dots depict the change in income and four-year change in house prices for each state, with black representing the observations for homeowners and gray representing the observations for renters. As we saw with the ACS data, at the aggregated market level, changes in income do not vary systematically with house price changes for renter households.

The analysis reported in figures 3A and 3B is conducted at the state level. Differences in average renter and homeowner income growth do not vary with the house price appreciation in their states of residence. This level of aggregation parallels the analysis with the ACS data in figures 2A and 2B. We can also use the PSID data to track income growth at the family unit. We restrict our sample to include observations on total family income only for individuals who identified themselves as head of household and reported no change in family composition between the 2009 and 2011 PSID waves.

In table 10 we report regressions of income growth on renter status and our proxies for exposure to the housing market wealth shock. The dependent variable is the log change in total income. All regressions include a standard set of controls observed in 2009 (log income, age, and years of education). A household is considered a renter if it rents in both the 2009 and 2011 waves. We use two measures of exposure to the housing market shock. First, we use the five-year state-level house price appreciation between 2001 and 2006, similar to the measure that we use to define the boom and non-boom counties in the main part of our analysis. We also include a "boom state" indicator variable, equal to one for states with boom counties from our earlier analysis and zero for states with non-boom counties.[21] The regressions are weighted using the PSID cross-sectional sampling weights.

In general, the results in table 10 are consistent with our earlier results. Family income growth is negatively related to renter status, suggesting that renters are more vulnerable than homeowners to income shocks over the business cycle. However, the coefficient

[21] We are forced to drop New York and North Carolina as states that have both boom and non-boom counties.

Table 10. PSID Family Income Growth Regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	b/se	b/se	b/se	b/se	b/se	b/se	b/se
House Price Change, 2001–06	0.138**			0.159***		0.188***	
	(0.045)			(0.045)		(0.052)	
Boom State		0.092**			0.103**		0.130***
		(0.033)			(0.033)		(0.037)
Renter			−0.277***	−0.283***	−0.291***	−0.238***	−0.248***
			(0.051)	(0.051)	(0.072)	(0.059)	(0.073)
Renter × House Price Change						−0.090	
						(0.079)	
Renter × Boom State							−0.087
							(0.062)
Age, 2009	−0.148	−0.136	−0.574***	−0.583***	−0.574***	−0.586***	−0.581***
	(0.119)	(0.144)	(0.135)	(0.135)	(0.173)	(0.135)	(0.172)
Years Education, 2009	0.047***	0.047***	0.044***	0.044***	0.044***	0.044***	0.044***
	(0.009)	(0.011)	(0.008)	(0.008)	(0.010)	(0.008)	(0.010)
Total Income (log), 2009	−0.273***	−0.294***	−0.320***	−0.324***	−0.347***	−0.324***	−0.347***
	(0.046)	(0.062)	(0.050)	(0.050)	(0.068)	(0.049)	(0.068)
Constant	2.415***	2.638***	3.682***	3.668***	3.932***	3.652***	3.928***
	(0.416)	(0.568)	(0.577)	(0.565)	(0.790)	(0.561)	(0.788)
Observations	5,319	3,278	5,319	5,319	3,278	5,319	3,278
Log Likelihood	−4,953.620	−3,131.567	−4,878.349	−4,861.509	−3,075.109	−4,860.326	−3,073.365

Notes: This table shows regressions of 2009–11 total family income growth on renter status and housing market exposure as reported in the 2009 and 2011 PSID. All regressions are weighted using the PSID cross-sectional sampling weights in 2011. The sample is restricted to observations from the PSID individual file for individuals self-identified as head of household who reported no change in family composition over 2009–11. A family is designated as a "renter" if it was a renter in both 2009 and 2011. Housing market exposure is measured as house price appreciation between 2001 and 2006, and also with the boom state indicator variable equal to one if the state includes a boom county and zero if it includes a non-boom county. Observations are dropped from the sample if a household lives in a state with both boom and non-boom counties.

estimates on the interactions of renter status with past house price appreciation and boom state are insignificant. We do not find evidence that renter family income growth was systematically lower in states corresponding to our boom markets. Thus, weak income growth among the renter population in boom markets does not appear to explain why renter households in these markets had significantly weaker growth in credit during the economic recovery.

Perhaps renters' optimism about the future declined more than did homeowners'. Here, we do not have evidence on boom versus non-boom, but we can proxy for renter versus owner. We investigate the Michigan Index of Consumer Expectations and find no strong evidence that this was true. We assume that income is positively correlated with homeownership. We find that the change in the Index between 2008:Q3 and 2011:Q4 was −3.2 for the lowest income quartile, −2.5 for the next highest quartile, −2.3 for the next highest quartile, and −3.8 for the highest income quartile. Optimism about the future declined the most for the group most likely to be homeowners. We assume that demand for debt is positively associated with optimism. Therefore, this result does not support the view that renters' demand declined more than homeowners' demand.

There are several other assumptions in our research design that need closer examination. First, our method of identifying homeowners and renters as having (or not having) a mortgage balance is somewhat restrictive. In table 11, we loosen this assumption. We define homeowners in the usual way, as consumers who had a mortgage balance throughout the 2008:Q3–2011:Q4 period. Renters, by contrast, include everyone else in the matched sample. This definition of renters encompasses the old definition and admits in other consumers who may have transitioned between homeownership and renter status. The results in table 11 generally confirm our earlier debt-change regression results in table 8. The more expansive definition still shows that renters reduced debt significantly more than homeowners, and particularly so in the boom counties. Indeed, the boom county × renter interaction term in columns 3–5 is always negative and significant at the 1 percent confidence level.

For another robustness check we run parallel debt-change regressions using all consumers living in the boom and non-boom counties, and not just the consumers that have been matched. While we believe that using some kind of matching approach is desirable to try

Table 11. Change in Non-mortgage Debt Regressions: Alternative Definition of Renters

	(1)	(2)	(3)	(4)	(5)
Boom County	−0.092*	−0.065	0.087	0.096	0.115
	(0.037)	(0.047)	(0.062)	(0.060)	(0.065)
Borrower Age in 2008:Q3		−0.018***	−0.018***	−0.018***	−0.018***
		(0.001)	(0.001)	(0.001)	(0.001)
Borrower Credit Inquiries in 2008:Q3		0.064***	0.064***	0.064***	0.064***
		(0.007)	(0.007)	(0.007)	(0.007)
Borrower Total Debt in 2008:Q3		−0.331***	−0.331***	−0.331***	−0.331***
		(0.004)	(0.004)	(0.004)	(0.004)
Change in Risk Score (2008:Q3–2011:Q4)		−0.007***	−0.007***	−0.007***	−0.007***
		(0.000)	(0.000)	(0.000)	(0.000)
Low Risk Score (<650) in 2008:Q3		−1.289***	−1.289***	−1.228***	−0.927***
		(0.031)	(0.031)	(0.071)	(0.233)
Renter (2008:Q3–2011:Q4)		−1.669***	−1.503***	−1.517***	−1.460***
		(0.043)	(0.055)	(0.058)	(0.061)
Boom County × Renter			−0.194**	−0.177**	−0.204**
			(0.062)	(0.065)	(0.071)
Boom County × Low Risk Score				−0.069	−0.187
				(0.076)	(0.251)
Renter × Low Risk					−0.343
					(0.238)
Boom County × Renter × Low Risk					0.138
					(0.259)
Constant	−0.567***	15.422	15.140	15.088	15.084
	(0.035)	(10.736)	(10.746)	(10.756)	(10.751)
Observations	83,071	83,071	83,071	83,071	83,071
R-squared Adjusted	0.000	0.157	0.157	0.157	0.157

Notes: This table presents the results from the regressions of changes in non-mortgage debt on the boom county dummy and control variables. In this table the definition of renter is expanded to include all consumers who do not have a mortgage balance for the entire 2008:Q3–2011:Q4 sample period. Partial spells with mortgages are classified as renters. The change in non-mortgage debt is computed over 2008:Q3–2011:Q4. Among the covariates, age, credit inquiries, risk score, and total debt are calculated for each consumer for the base year 2008:Q3. We also compute a change in risk score over the 2008:Q3–2011:Q4 period. All regressions include tract- and county-level controls, and clustered standard errors at the county level. ***, **, and * denote significance at the 1 percent, 5 percent, and 10 percent level, respectively.

to put consumers on an even footing as of 2008:Q3, this robustness check helps to alleviate concerns that the matching somehow selects renters in the non-boom counties who were particularly sensitive to changes in economic conditions. The results from this exercise are in table 12. Again, we see that renters are still more likely to have reduced their non-mortgage debt during the 2008:Q3–2011:Q4 period, and particularly so in the boom counties.

Table 12. Change in Non-mortgage Debt Regressions: Non-matched Sample

	(1)	(2)	(3)	(4)	(5)
Boom County	−0.142***	−0.066	0.122*	0.149**	0.144*
	(0.024)	(0.039)	(0.055)	(0.055)	(0.058)
Borrower Age in 2008:Q3		−0.020***	−0.020***	−0.020***	−0.020***
		(0.001)	(0.001)	(0.001)	(0.001)
Borrower Credit Inquiries in 2008:Q3		0.066***	0.066***	0.065***	0.065***
		(0.006)	(0.006)	(0.006)	(0.006)
Borrower Total Debt in 2008:Q3		−0.354***	−0.354***	−0.354***	−0.354***
		(0.005)	(0.005)	(0.005)	(0.005)
Change in Risk Score (2008:Q3–2011:Q4)		−0.007***	−0.007***	−0.007***	−0.007***
		(0.000)	(0.000)	(0.000)	(0.000)
Low Risk Score (<650) in 2008:Q3		−1.176***	−1.175***	−1.052***	−1.032***
		(0.032)	(0.032)	(0.047)	(0.120)
Renter (2008:Q3–2011:Q4)		−1.810***	−1.651***	−1.681***	−1.677***
		(0.050)	(0.047)	(0.048)	(0.052)
Boom County × Renter			−0.241***	−0.192***	−0.186**
			(0.055)	(0.057)	(0.062)
Boom County × Low Risk Score				−0.181***	−0.135
				(0.052)	(0.152)
Renter × Low Risk					−0.024
					(0.123)
Boom County × Renter × Low Risk					−0.050
					(0.156)
Constant	−0.461***	7.967	7.508	7.455	7.471
	(0.020)	(8.606)	(8.627)	(8.677)	(8.690)
Observations	107,686	107,686	107,686	107,686	107,686
R-squared Adjusted	0.000	0.163	0.164	0.164	0.164

Notes: This table presents the results from the regressions of changes in non-mortgage debt on the boom county dummy and control variables. In this table the definition of renter is expanded to include all consumers who do not have a mortgage balance for the entire 2008:Q3–2011:Q4 sample period. Partial spells with mortgages are classified as renters. The change in non-mortgage debt is computed over 2008:Q3–2011:Q4. Among the covariates, age, credit inquiries, risk score, and total debt are calculated for each consumer for the base year 2008:Q3. We also compute a change in risk score over the 2008:Q– 2011:Q4 period. All regressions include tract- and county-level controls, and clustered standard errors at the county level. ***, **, and * denote significance at the 1 percent, 5 percent, and 10 percent level, respectively.

6. Conclusion

Previous empirical research has suggested that the tremendous blow to the housing market contributed to the sharp drop in aggregate consumer debt. We investigate whether such a link may be at least partially due to cutbacks in the supply of credit. First using the Federal Reserve Board's Senior Loan Officer Opinion Survey, we show that banks did indeed tighten credit standards in boom counties more than in non-boom counties. In order to further explore the role of credit supply conditions, we examine individual credit file data. Here, we find evidence suggesting that credit supply effects independent of collateral effects were important. In our estimations, we find that, for non-mortgage debt, homeowners did not reduce debt more in counties with large house price declines than in counties with small house price declines. In contrast, renters did reduce debt more in the boom counties. Using an IV estimation, we map out the transmission channel from banks with weak bank balance sheets to a higher degree of deleveraging after the crisis. Our evidence supports the view that a general tightening of credit supply that was independent of the effects of the drop in value of housing collateral contributed to cutbacks in consumer borrowing.

Appendix

As mentioned in the main text, we also conduct additional estimations of the non-mortgage debt change regressions. Table 13 shows that our main results hold up when we add home equity debt to non-mortgage debt. When we excluded home equity debt, in table 8, we saw that renters saw a negative effect on debt change from living in a boom county, but that homeowners saw no effect from living in a boom county. In most specifications in table 13, boom county has no effect on the debt change of either homeowners or renters, either by itself or in interaction with any other variable. The important point here is that homeowners did not undergo a greater reduction in debt in boom counties than in non-boom counties wherein that differential was stronger than for renters. This too suggests that a general negative credit supply shock contributed to debt reduction in the post-2008 period.

Table 13. Change in Non-mortgage + Home Equity Debt Regressions

	(1)	(2)	(3)	(4)	(5)
Boom County	−0.023	−0.189	0.132	0.136	0.188
	(0.048)	(0.106)	(0.096)	(0.093)	(0.102)
Borrower Age in 2008:Q3		−0.021***	−0.021***	−0.021***	−0.021***
		(0.001)	(0.001)	(0.001)	(0.001)
Borrower Credit Inquiries in 2008:Q3		0.067***	0.067***	0.067***	0.067***
		(0.008)	(0.008)	(0.008)	(0.008)
Borrower Total Debt in 2008:Q3		−0.437***	−0.415***	−0.415***	−0.415***
		(0.012)	(0.007)	(0.007)	(0.007)
Change in Risk Score (2008:Q3–2011:Q4)		−0.007***	−0.007***	−0.007***	−0.007***
		(0.000)	(0.000)	(0.000)	(0.000)
Low Risk Score (<650) in 2008:Q3		−1.341***	−1.340***	−1.317***	−1.002***
		(0.032)	(0.032)	(0.087)	(0.289)
Renter (2008:Q3–2011:Q4)		−2.017***	−1.885***	−1.891***	−1.831***
		(0.060)	(0.084)	(0.089)	(0.099)
Boom County × Renter			−0.153	−0.146	−0.220*
			(0.088)	(0.096)	(0.109)
Boom County × Low Risk Score				−0.027	−0.430
				(0.091)	(0.314)
Renter × Low Risk					−0.360
					(0.304)
Boom County × Renter × Low Risk					0.458
					(0.333)
Constant	−0.817***	11.137	10.898	10.880	10.799
	(0.045)	(13.509)	(13.513)	(13.517)	(13.535)
Observations	76,437	76,437	76,437	76,437	76,437
R-squared Adjusted	−0.000	0.140	0.140	0.140	0.140

Notes: This table presents the results from the regressions of changes in non-mortgage debt plus home equity debt on the boom county dummy and control variables. The change in non-mortgage debt is computed over 2008:Q3–2011:Q4. Among the covariates, age, credit inquiries, risk score, and total debt are calculated for each consumer for the base year 2008:Q3. We also compute a change in risk score over the 2008:Q– 2011:Q4 period. All regressions include tract- and county-level controls, and clustered standard errors at the county level. ***, **, and * denote significance at the 1 percent, 5 percent, and 10 percent level, respectively.

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What Drives Bank-Intermediated Trade Finance? Evidence from Cross-Country Analysis*

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Several important policy questions raised by the drop in trade finance during the global financial crisis remain unsettled due to the lack of hard data on trade finance. This paper provides fresh empirical evidence on the determinants of bank-intermediated trade finance using a novel panel data set. Results indicate that trade finance is driven by demand-side factors, such as a country's trade flows growth, and global import growth. In addition, trade finance is dependent on funding availability for domestic banks, as well as global financial conditions and dollar funding costs. These results are robust to different model specifications.

JEL Codes: F14, F19.

1. Introduction

The market for global trade finance was generally regarded as well functioning and liquid until the global financial crisis of 2008–09, and thus did not attract much attention from policymakers and scholars.

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Since the crisis, however, trade finance has experienced episodes of stress, particularly after the collapse of Lehman Brothers in September 2008 and again in late 2011, when several European banks were under funding pressures.[1] In this context, understanding the drivers of trade finance becomes important for two main reasons. First, international trade is heavily dependent on trade finance since it involves certain forms of commercial risks that are elevated relative to domestic trade, such as payment risk and transportation risk, in addition to exchange rate risk, which is unique to this line of activity. These risks are generally assumed by banks since importers and exporters are often unwilling to bear them. Indeed, estimates for the share of global trade relying on trade finance instruments range from 30 to 40 percent (Committee on the Global Financial System 2014). This paper focuses on trade finance intermediated by banks.[2]

Second, research has shown that shocks to banks in general and the supply of trade finance, in particular, affect exports and imports and have contributed to the drop in trade in recent years (Ahn 2013; Amiti and Weinstein 2011; Del Prete and Federico 2014; Niepmann and Schmidt-Eisenlohr 2017; Paravisini et al. 2011). The importance of trade finance in supporting the functioning of global trade is also underscored by the fact that many multilateral and national institutions expanded their trade finance programs to facilitate exports and imports in some emerging market and advanced economies following the call from leaders of the Group of Twenty (G-20) countries in the aftermath of the global financial crisis.[3]

[1] Broadly speaking, the term "trade finance" refers to payment arrangements between buyers and sellers. The focus of this paper is on the international dimension of trade finance, i.e., financing for cross-border transactions.

[2] Trade finance includes both bank-intermediated trade finance, in which banks facilitate transactions between buyers and sellers, and non-bank trade finance, in which buyers and sellers extend credit to each other; this paper focuses on the first category only. Estimates for the share of global trade finance relying on different financing options, including open accounts, interfirm trade credit, and bank-intermediated trade finance, are much higher, in the range of 80 to 90 percent (Auboin 2009). However, global estimates should be treated with caution, as gauging the overall size of the bank-intermediated trade finance market requires extrapolation from partial data, which makes these estimates imprecise.

[3] For a detailed discussion, see Asmundson et al. (2011) and CGFS (2014). The G-20 communiqués, including the communiqué from the April 2009 summit, can be found at http://www.g20.utoronto.ca/2009/2009communique 0402.html.

Empirical work on the underlying causes as well as the impact of the recent dislocations in trade finance has grown rapidly after the global financial crisis. However, the evidence so far is largely based on surveys or country-specific analyses using firm-level data since cross-country analyses are absent due to the paucity of hard data. As a consequence, many of the important policy questions raised by the drop in trade finance during the global financial crisis remain largely unanswered: Did declines in cross-border bank-intermediated trade financing transmit financial shocks across borders? Or did they simply reflect the lesser need for trade financing due, for instance, to weaker growth in trading partners, or subdued domestic economic growth? In other words, did supply or demand drive bank-intermediated trade finance during the global financial crisis?

To shed some light on these issues, this paper presents fresh evidence on the key determinants of bank-intermediated trade finance using a unique, newly constructed panel data set on trade finance. As such, this paper is the first attempt at understanding the determinants of bank-intermediated trade finance for a set of countries and thus makes an important contribution to the empirical literature in this field. Our results indicate that both global and country-specific factors, such as growth in trade flows and the funding availability of domestic banks, are important determinants of trade finance. The results are robust to alternative model specifications as well as changes in the cross-sectional dimension of the data set. In particular, the main results hold when we use instrumental variables to control for the potential endogeneity of trade flows growth. Results are also robust to random-effects estimation, which we undertake in order to address any potential small sample bias.

Overall, our findings suggest that the short-term, self-liquidating nature of trade finance could generate some scope for negative externalities for the global economy, especially if the banking sector is subject to global shocks. These externalities can be amplified if a large number of banks simultaneously run down their liquidity pool embodied in their trade finance portfolios. We elaborate on the implications of the results in sections 5 and 6.

The remainder of the paper is structured as follows. Section 2 summarizes the related literature. Section 3 provides a brief description of trade finance instruments and summarizes the evolution of trade finance during the global financial crisis. Section 4 describes the data, the empirical methodology, and the choice of explanatory variables. Section 5 discusses the results and section 6 concludes.

2. Related Literature

Our work makes two key contributions to the empirical literature on trade finance. First, it provides fresh evidence on the determinants of bank-intermediated trade finance—an area that has been hitherto unexplored due to the lack of hard data. Previous empirical work has generally focused on firm-level data in a country-specific setting to analyze firms' choice with regard to different payment contracts. For example, Ahn (2013) finds evidence of a substantial impact of bank liquidity shocks on the supply of letters of credit import transactions in Colombia during the 2008–09 crisis. In a similar vein, Antras and Foley (2011) use detailed transaction-level data for a U.S.-based exporter to study how the choice between cash-in-advance and open account is affected by the characteristics of the country in which the importer is located. In a more recent paper, Niepmann and Schmidt-Eisenlohr (2013), using U.S. banking data, find that the volume of banks' trade finance claims differs substantially across destination countries, with claims being hump shaped in country credit risk and increasing with the time to import of a destination market. The authors also find that trade finance claims vary systematically with global conditions, expanding when aggregate risk is higher and funding is cheaper.

Rather than taking these firm-specific approaches and focusing on certain instruments, such as letters of credit, our study takes a more comprehensive view to examine the key determinants of bank-intermediated trade finance as a whole and for a set of countries.[4] To the best of our knowledge, this is the first study to use such an approach, one that allows us to investigate the role of country-of-origin variables (frequently called "home variables") in driving

[4]See tables 8 and 9 in the data appendix for a description of the country-specific data on trade finance used in this paper. We refer the reader to CGFS (2014) for an in-depth description of the data, as well as a comparison with other sources of information.

trade finance. The results, therefore, complement previous empirical work that has focused primarily on either bank-level information or country-of-destination information. Further, it is informative to analyze trade finance as a whole since it encompasses a wide range of instruments, and market intelligence suggests that firms can switch relatively easily from one instrument to another, making the distinction between instruments blurred at times.

Second, our work is also related to the strand of empirical literature focusing on the relationship between financial conditions, trade credit, and trade at the firm and sectoral levels. This literature has aimed at understanding and measuring the impact of disruptions in trade finance on the so-called Great Trade Collapse. At the firm level, Behrens, Corcos, and Mion (2013), Bricongne et al. (2012), and Coulibaly, Sapriza, and Zlate (2011) all find that financial constraints explain part of the decline in production and exports during the trade collapse. Using sector-level data, Chor and Manova (2012) examine how the sector composition of exports to the United States varies across countries depending on the cost of finance in those source countries. The authors find that tight financial conditions (i.e., higher interbank interest rates) led exports to fall more during the 2008–09 crisis in sectors with high external finance dependence or low asset tangibility. Further, they demonstrate that countries with tight financial conditions exported less to the United States than countries where financial conditions were less tight. Our paper contributes to this literature by showing that, controlling for trade flows, trade finance depends on global financial conditions and funding availability for domestic banks, and accordingly can be impaired by financial shocks.

3. Trade Finance: Instruments and Dynamics during the Global Financial Crisis

The term “trade finance” is generally used for financial instruments that are specifically linked to underlying international trade transactions (exports or imports). Banks and other institutions typically provide trade finance for two purposes. First, it serves as a source of working capital for individual traders and international companies in need of liquid assets.[5] Second, trade finance provides credit insurance against the risks involved in international trade, such as currency or price fluctuations, or political risk. While we acknowledge that some trade finance instruments may be long term in nature, in this paper we focus only on short-term bank-intermediated trade finance, because it funds a much larger volume of trade and is also closely linked with overall bank funding conditions.

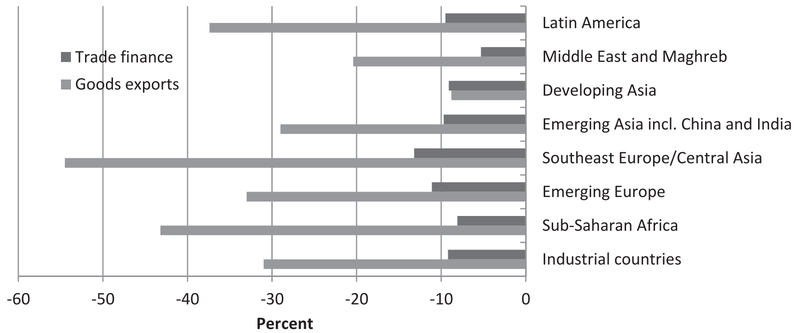
Trade finance comprises a wide range of products used to reduce risks related to international payments between importers and exporters. One of the most common and standardized forms of bank-intermediated trade finance is a letter of credit (or L/C). L/Cs reduce payment risk by providing a framework under which a bank makes (or guarantees) the payment to an exporter on behalf of an importer once goods have been shipped or delivered.[6] Banks may also help meet working capital needs by providing trade finance loans to exporters or importers, i.e., short-term loans used to buy the inputs necessary to produce goods ordered by foreign customers. In this case, the loan documentation is linked either to an L/C or to other forms of documentation related to the underlying trade transaction. Working capital is more important for financing export shipments than for domestic shipments because of the longer lag between production and payment for exports (Amiti and Weinstein 2011).

With respect to the recent developments in the market for trade finance, figure 1 shows the drop in trade finance and trade at the peak of the global financial crisis—between October 2008 and January 2009. The fall in trade finance was about one-third of the contraction in global merchandise trade, with the largest declines witnessed in Emerging Europe and Central Asia. In the aftermath of the crisis, the International Monetary Fund (IMF) together with the Bankers' Association for Finance and Trade–International Financial Services Association (BAFT-IFSA) undertook a survey on pricing,

[5] A working capital loan not specifically tied to trade is typically not included in this definition.

[6] For the most part, L/Cs represent off-balance-sheet commitments, though they may, at times, be associated with an extension of credit. This can occur, for example, if an import L/C is structured to allow the importer a period of time (known as "usance") before repaying the bank for the payment it made on the importer's behalf.

Figure 1. Changes in Merchandise Exports and Trade Finance between October 2008 and January 2009 (percentage change)



Source: Asmundson et al. (2011).

volumes, and drivers in trade finance markets in March 2009.[7] This was followed by several additional survey rounds. The results of these surveys showed that changes in trade finance conditions were particularly pronounced among large banks that suffered most from the crisis and were thus in greater need to deleverage quickly. Further, the surveys showed that banks also increased the cost to borrowers. Regarding the underlying causes for the decline in trade finance, the surveyed banks identified the fall in demand for trade as the major reason for the decline in trade finance, and attributed about 30 percent of the fall to reduced credit availability at either their own institutions or counterparty banks.[8] While these surveys provide valuable insights into the developments in the market for trade finance, quantitative estimates derived from them should be treated with caution, as survey respondents usually provide only directional indications instead of details for their firm which can then be aggregated (CGFS 2014). In contrast, by using objective data, this paper

[7]The 2009 survey, by the International Monetary Fund and the Bankers' Association for Finance and Trade, is titled "IMF-BAFT Trade Finance Survey: A Survey Among Banks Assessing the Current Trade Finance Environment."

[8]Asmundson et al. (2011) provides a summary of the first four IMF surveys.

provides a more nuanced perspective on the underlying determinants of trade finance.

4. Data and Methodology

4.1 Data

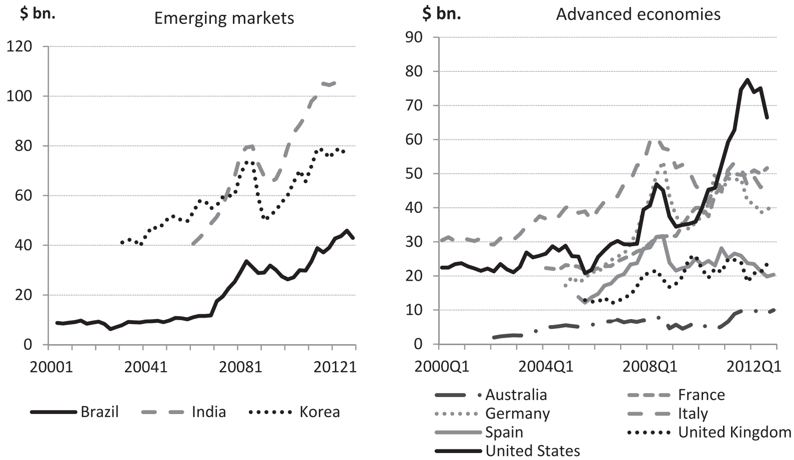
The data on trade finance used in this paper were put together by members of the Study Group on Trade Finance under the auspices of the Committee on the Global Financial System (CGFS).[9] The sample includes the following ten countries: Australia, Brazil, France, Germany, India, Italy, Korea, Spain, the United Kingdom, and the United States. The sample spans the time period 2001:Q1 to 2012:Q4, although the trade finance data are not available for the entire time period for some countries, resulting in an unbalanced panel.

A few recent trends in the data are worth highlighting. Globally bank-intermediated trade finance has increased substantially in dollar terms over the past decade, particularly since the end of 2006. The pace, however, has diverged significantly across countries in recent years (figure 2). The growth in trade finance is particularly notable in some of the emerging market countries in our sample, which in turn corroborates anecdotal evidence that local banks in these countries are playing a greater role in the provision of trade finance (CGFS 2014).

Table 8 in the data appendix provides a detailed description of the country-specific data on trade finance. Data coverage in terms of trade finance instruments differs substantially across countries. While countries like Brazil, India, Italy, and Korea have detailed data covering a significant share of overall trade finance activities, others have statistics capturing only specific components of their trade finance markets, such as export-related trade finance or letters of credit (L/Cs). For most countries, the available data capture only on-balance-sheet lending activities (i.e., L/Cs are excluded, except when they are tied to or become funded loans) by resident banks, and focus on lending to domestic borrowers. Further, it is important

[9]The final report of the Study Group (CGFS 2014) can be found at http://www.bis.org/publ/cgfs50.htm.

Figure 2. Trends in Trade Finance (in billions of U.S. dollars; quarterly data)



Sources: CGFS and national authorities.

to note that bank-intermediated trade finance is a part of international credit. The latter comprises banks' cross-border lending and their claims on non-banks in foreign currency, regardless of whether the foreign-currency lending is extended inside or outside the country (Avdjiev, McCauley, and McGuire 2012).

To provide a comparison of the data across countries, table 9 in the data appendix briefly summarizes the main features of the data in terms of reporting population, cross-border reach, main counterparties, and instruments. In eight out of the ten jurisdictions in our sample, trade finance data reflect lending booked by resident banks in a given home country vis-à-vis residents of that home country. These seven jurisdictions are Australia, Brazil, France, India, Italy, Korea, and Spain. Overall, these data are consistent with the spirit of the Bank for International Settlements (BIS) Locational Banking Statistics (LBS) data.[10]

In contrast, in the case of Germany and the United States, some aspects of the data are more similar to the BIS Consolidated Banking

[10] In the case of Spain and Australia, the data also include claims by foreign branches and subsidiaries of banks headquartered in these countries.

Statistics (CBS): trade finance includes claims by domestic banking institutions worldwide, with a focus on non-resident borrowers.[11] Further, in these two countries trade finance by foreign branches and subsidiaries of national banks is also included.

The data should be well suited to analyze the role of home factors in driving trade finance as long as a strong decoupling between residence and nationality does not exist. Such a decoupling occurs when a jurisdiction hosts many foreign banks, whose activities are likely not affected by domestic factors, such as sovereign creditworthiness, trade flows, or other country-specific macroeconomic fundamentals. For example, this decoupling could pose a challenge when analyzing countries that are "financial hubs," acting as hosts of branches and subsidiaries of foreign banks. While banks in these jurisdictions book their trade finance activities locally, their lending likely has little to do with host-country factors such as trade flows or sovereign risk (see, for example, Fender and McGuire 2010). Among the countries in our sample, only the United Kingdom exhibits features of a financial hub. This becomes clear when one examines the lending patterns by resident banks using the newly published BIS LBS data, which show that the proportion of local claims (i.e., lending to residents) to total claims (on all borrowers) is very high in all jurisdictions except for the United Kingdom.[12]

In light of the heterogeneous nature of trade finance across countries described above, each national data source should be viewed as providing a partial window into aspects of the bank-intermediated trade finance activities conducted in that country (CGFS 2014). We include country-specific fixed effects in our estimations to account

[11]It is important to note that for the United States there are some key differences between the trade finance data used in the paper and that in the BIS CBS, although the source data for both come from the FFIEC Country Exposure Lending Survey. Our data set comprises trade finance lending by U.S. banks, which includes U.S. holding companies owned by foreign banks, but excludes U.S. branches of foreign banks. In the BIS CBS, however, only U.S. holding companies owned by banks headquartered in a non-reporting country are included in the sample, while other entities are removed to avoid double counting.

[12]Specifically, resident banks' local claims relative to total claims are as follows: Australia: 84 percent; Brazil: 96 percent; France: 70 percent; Germany: 72 percent; India: 97 percent; Korea: 89 percent; Spain: 83 percent; and the United Kingdom: 58 percent. For Italy and the United States, this ratio cannot be computed due to lack of data.

for the differences in data measurement across countries. In addition, we also examine the sensitivity of our main results to the exclusion of the key countries for which the trade finance data are more dissimilar (i.e., Germany, United Kingdom, and United States).

4.2 Empirical Framework

Our research question is closely related to the growing literature on the determinants of international credit. Recent papers in this strand of literature, such as Avdjiev, Kuti, and Takáts (2012), Avdjiev, McCauley, and McGuire (2012), Bruno and Shin (2015), Hermann and Mihaljek (2010), McCauley, McGuire, and Sushko (2015), and Takáts (2010), suggest that such flows are driven by both global and local (i.e., country-specific) factors. Since bank-intermediated trade finance is mostly dollar denominated, it is a subset of total international credit. Thus, our empirical specification and choice of explanatory variables are also guided by the above literature. Specifically, following Avdjiev, Kuti, and Takáts (2012) and Bruno and Shin (2015), our benchmark specification in its general form is given by

$$\Delta TF_{it} = \alpha + \Sigma_k \beta_k X_{k,i,t} + \Sigma_l \beta_l Y_{l,t} + \alpha_i + \varphi_y + \gamma_q + \varepsilon_{i,t}, \qquad (1)$$

where ΔTF_{it} denotes the growth in the outstanding volume of bank-intermediated trade finance for country $i, X_{k,i,t}$ are k country-specific exogenous variables, and $Y_{l,t}$ denotes l global variables. The benchmark measure of the dependent variable is the quarter-over-quarter (qoq) rate of growth of trade finance, calculated as the difference between $\log(\mathit{tradefinance})_t$ and $\log(\mathit{tradefinance})_{t-1}$.[13] We include country-specific fixed effects, α_i, in equation (1) to account for the differences in the way in which trade finance is measured across countries, as well as for any additional country-level effects not captured by our control variables. However, admittedly, not every potential source of heterogeneity can be controlled for. We also acknowledge the possible existence of year-specific and

[13]An alternative would be to define trade finance relative to trade flows. Instead, we include trade flows as an explanatory variable in the benchmark estimations.

quarter-specific factors, which are captured by the time dummies φ_y and γ_q, respectively.[14] Table 10 in the data appendix provides further details on all the variables included in the analysis.

In what follows, we describe our selection of the global and country-specific explanatory variables in detail. With regard to global factors, Bruno and Shin (2015) argue that global financial conditions are the key drivers of cross-border bank flows—and, accordingly, we expect them to be drivers of bank-intermediated trade finance. We include three different measures as proxies for global financial conditions. First, we use the VIX index of implied volatility of S&P 500 equity index options—which is the most widely used measure of global financial conditions in the literature. Second, we use a synthetic indicator of financial stress, namely, the financial conditions index (FCI), which is based on the methodology of Guichard, Haugh, and Turner (2009). The FCI is derived from real short-term interest rates, real long-term interest rates, the real effective exchange rate, bond spreads, stock market capitalization, and credit standards in the euro area, Japan, the United Kingdom, and the United States.[15] As such, it is a more comprehensive measure of global financial conditions than the widely used VIX index.

Finally, since trade finance is predominantly denominated in U.S. dollars—even more so than global trade—the ability of banks to provide trade finance can be disrupted if banks' dollar funding lines are curtailed (CGFS 2014).[16] Indeed, this seems to have been the case in some instances in 2008–09 and in 2011–12.[17] We account for dollar funding pressures in our framework by including the three-month U.S. LIBOR-OIS spread as an explanatory variable. We expect this variable to have a negative impact on bank-intermediated trade finance. Trade finance may also be affected by other global

[14]A potential caveat with respect to these time dummies is that they may not fully capture the seasonal patterns in the data which might differ across countries from the Northern and Southern Hemisphere included in our sample.

[15]For details on the construction of this variable, see Guichard, Haugh, and Turner (2009).

[16]More than 80 percent of L/Cs are settled in U.S. dollars.

[17]For instance, reduced dollar funding in the aftermath of the Lehman failure was one of the main reasons for the Brazilian and Korean central banks to provide both direct and indirect support to trade finance markets (CGFS 2014).

variables such as world demand, which we proxy here by global imports growth.[18]

Trade finance is also likely to depend on country-specific macroeconomic fundamentals or "pull" factors. We include nominal GDP growth since faster-growing economies are likely to have greater demand for credit (Bruno and Shin 2015).[19] Following recent literature on the links between private and sovereign debt, we include the S&P rating as a measure of sovereign creditworthiness. Sovereign defaults are frequently accompanied by domestic banking crises, usually due to the fact that the government postpones the default decision and strains the banking system in order to service the debt, until it is no longer feasible (Arteta and Hale 2008). This would make domestic liquidity more scarce, which in turn would put upward pressure on the cost of trade finance, since banks set rates that account for the higher probabilities of defaults by importers and exporters.

Trade finance may also be facilitated by country-specific financial factors such as the leverage, equity, and funding costs of local banks, although the lack of good-quality data acts as a constraint in testing these hypotheses. As a proxy for local banks' soundness, again following Bruno and Shin (2015), we use the banks' capital-to-assets ratio. We expect this measure to be positively correlated with bank-intermediated trade finance growth. Following the recent literature, we also include the five-year credit default swap (CDS) spreads for each banking sector as a measure of banks' riskiness and short-term funding costs in wholesale markets (for example, Chui et al. 2010). We construct this measure as a simple average of the CDS spreads for the main banks in each country (see table 11 in the data appendix). Finally, we also include the (country-specific) growth in trade flows (defined as the sum of exports and imports), which is expected to be an important determinant of trade finance. Table 1 provides the summary statistics for the main variables.

Estimating equation (1) poses some challenges. First, there is potential endogeneity arising from the inclusion of growth in trade flows as an explanatory variable. Second, global factors are likely

[18]Another alternative is to use global real GDP growth, but we consider global imports growth to be a better proxy for global demand conditions.

[19]There is also evidence that foreign bank lending to emerging markets is procyclical (see Jeanneau and Micu 2002).

Table 1. Summary Statistics

	Mean	Standard Deviation	Min.	Max.
Trade Finance Growth (Percentage Points)	0.02	0.10	−0.57	0.39
Trade Flows Growth (Percentage Points)	0.02	0.09	−0.53	−0.27
Bank Capital to Total Assets (Percentage Points)	6.98	2.14	3.7	12.1
CDS Spreads (Percentage Points)	1.15	1.14	0.05	7.14
S&P Rating*	16.87	4.15	7	20
GDP Growth (Percentage Points)	1.42	1.47	−3.7	7.93
World Imports Growth (Percentage Points)	0.87	3.74	−16.71	6.57
VIX (Level)	21.94	8.49	11.03	58.32
Financial Conditions Index (Level)	0.09	0.48	−0.53	1.96
LIBOR-OIS Spread (Percentage Points)	0.28	0.36	0.07	2.13

Note: Table 10 provides a detailed description of the variables.
*S&P ratings are expressed as numerical values using a linear mapping. AAA corresponds to 20, while D corresponds to 0. The threshold between investment grade and junk is 12.

to be important determinants of trade finance, which in turn can be a source of large cross-sectional correlation. Indeed, inference can be misleading if the standard errors are not robust to such cross-sectional correlation. Taking these issues into account, we estimate the benchmark model based on the fixed-effects estimation with Driscoll-Kraay standard errors, which renders errors robust to cross-sectional correlation (Driscoll and Kraay 1998; Hoechle 2007). As mentioned above, we also include the time (i.e., year) dummies in order to control for any additional time-specific sources of cross-sectional correlation. However, this estimation procedure does not eliminate the biases stemming from the potentially endogenous variable, namely, trade flows growth.

Therefore, as a robustness check, we also estimate equation (1) using instrumental variables (IVs) to account for the potential endogeneity of trade flows growth. The IVs should be such that they do not directly determine TF_{it} but are correlated with the variable being instrumented. We consider two potential instruments capturing a country's external demand. To construct these variables, we first identify the top ten trading partners for each country in our sample and compute the share of exports to each of these trading partners in total exports of the particular country. Next, using these trade shares as weights, we compute two weighted aggregate measures and test their validity as instruments. The first variable is the trade-weighted measure of real GDP growth for each country's top ten trading partners. The second instrumental variable is a similar trade-weighted measure of growth in exports to the main trading partners for each country. Both variables are potentially valid instruments, as we expect them to be highly correlated with trade flows growth but uncorrelated with the error term.[20] We then estimate equation (1) using the within-2SLS estimator.

An additional issue with regard to estimating equation (1) is the correlation between the global variables, which turns out to be quite strong (table 2). Thus, we include these variables on an individual basis in the regression analysis. We also perform panel unit-root tests (Fisher-type tests) to check for non-stationarity in certain variables (trade finance growth, GDP growth, and banks' capital-to-assets ratio) and do not find any evidence of unit roots.

A final issue relates to the size of the panel. Our data set contains ten countries and an average of thirty quarters.[21] In this setup, the

[20]We also consider three competitiveness indicators from the OECD as potential instruments: relative unit labor costs, relative consumer prices, and relative price of exported goods. However, these indicators did not pass the test of validity of instruments.

[21]Incidentally, the small number of countries covered could potentially cast doubts about the generality of the results. However, the jurisdictions covered account for a remarkable fraction of the overall lending. International claims booked in the jurisdictions covered account for 48 percent of total international claims; the corresponding number is 43 percent for total claims ("total claims" include international plus local claims by resident banks, where local claims are defined as lending by resident banks to resident borrowers in domestic currency). The bulk of the international lending not covered in our sample corresponds to just three jurisdictions (Japan, Netherlands, and Switzerland). Data on trade finance are not available for these countries (CGFS 2014).

Table 2. Matrix of Correlations between Global Variables

	Financial Conditions Index (FCI)	VIX	World Imports Growth (QoQ)	LIBOR-OIS Spread
Financial Conditions Index (FCI)	1			
VIX	0.93	1		
World Imports Growth (QoQ)	−0.65	−0.57	1	
LIBOR-OIS Spread	0.80	0.76	−0.62	1

Note: The correlations between the financial variables are generally lower once we exclude the peak crisis periods, i.e., 2008:Q4 and 2009:Q1.

fixed-effects estimator can be unstable and non-robust to potentially anomalous samples. As a robustness check, we also reestimate the regressions using the random-effects estimator.

5. Results

5.1 Benchmark Specification

Table 3 shows the benchmark regressions with the quarter-over-quarter growth in outstanding trade finance volumes as the dependent variable. As discussed, all regressions are estimated using fixed-effects estimation with Driscoll-Kraay standard errors. Column 1 includes world imports growth as an explanatory variable, while columns 2–4 include our three measures of global financial conditions individually. Results show that growth in bank-intermediated trade finance is positively associated with trade flows growth (i.e., demand), as expected.[22] The CDS spread also has the expected (negative) sign and is statistically significant, indicating that an increase in the riskiness of banks and their short-term funding costs curtails

[22]We also included other country-specific macroeconomic fundamentals, such as inflation, budget balance/GDP, and external debt/GDP in the benchmark specification, but the coefficients on these variables were generally not significant. Hence, we do not include them in the benchmark regressions reported here.

Table 3. Determinants of Growth in Trade Finance: Fixed Effects Estimations with Driscoll-Kraay Standard Errors

	(1)	(2)	(3)	(4)
Trade Flows Growth	0.123*	0.201***	0.195***	0.229***
	[0.070]	[0.056]	[0.071]	[0.049]
Bank Capital to Total Assets	0.007	0.006	0.007	0.006
	[0.010]	[0.010]	[0.009]	[0.010]
CDS Spreads	−0.011**	−0.013**	−0.012**	−0.016**
	[0.005]	[0.006]	[0.005]	[0.007]
S&P Rating	0.001	−0.001	−0.000	−0.003
	[0.006]	[0.007]	[0.007]	[0.008]
GDP Growth	−0.007	−0.006	−0.007	−0.005
	[0.004]	[0.005]	[0.005]	[0.006]
World Imports Growth	0.009***			
	[0.001]			
VIX		−0.004***		
		[0.001]		
Financial Conditions Index			−0.068***	
			[0.022]	
LIBOR-OIS Spread				−0.076***
				[0.037]
Observations	294	294	294	294
Within R-squared	0.30	0.27	0.26	0.25

Notes: Driscoll-Kraay standard errors are in brackets. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. The dependent variable is the quarter-over-quarter (qoq) growth in the outstanding volume of trade finance. All explanatory variables in growth rates are also in terms of qoq growth. All specifications include year and quarter dummies and country-specific fixed effects and are estimated using the fixed-effects estimation with Driscoll-Kraay standard errors.

trade finance growth. An increase in CDS spreads of 100 basis points is associated with a reduction in trade finance of roughly 1.3 percentage points on a quarter-over-quarter basis.[23] The capital-to-assets ratio of banks, however, does not have a statistically significant impact on trade finance. Unsurprisingly, growth in world imports

[23]This is the average impact across the four specifications in table 3.

(i.e., demand) is associated with stronger trade finance growth, and the effect is highly statistically significant.

International financial strains are found to impair trade finance, as exemplified by the negative and highly significant relationship between the VIX, the financial conditions index (FCI), and the LIBOR-OIS spread (columns 2–4). Specifically, an increase of 100 basis points in both the FCI and the LIBOR-OIS spread translates into a drop in trade finance of roughly 7 percentage points. Higher dollar funding costs tend to restrict trade finance probably because of the significant role played by the dollar in global trade. These results are also in line with recent literature, including Bruno and Shin (2015), Rey (2016), and Takáts (2010), which finds that the VIX can explain a substantial part of the variation in cross-border bank lending. Overall, these sizable impacts of the financial variables illustrate well the importance of global financial conditions in determining trade finance flows. Our findings suggest that the short-term nature of trade finance may result in some negative externalities for the global economy, particularly if the banking sector is subject to global shocks.

All in all, the results suggest that trade finance growth depends on global financial conditions, global imports growth, as well as country-specific trade flows growth and funding availability for domestic banks.

5.2 Endogeneity of Trade Flows: Instrumental-Variables Estimation

We reestimate the regressions in table 3 using instrumental variables (IVs). As described in section 4.2, to instrument trade flows growth, we use the two valid instruments identified: (i) a weighted measure of the trade flows growth for each country's main trading partners and (ii) a weighted measure of real GDP growth for each country's main trading partners. The results are reported in table 4.

The Sargan-Hansen test of over-identification confirms that the instruments are valid. The IV estimation results are broadly in line with the benchmark results. Trade flows growth has a positive impact on the growth of bank-intermediated trade finance. This effect is not statistically significant in the first column, perhaps due to the high correlation between the instruments and world

Table 4. Determinants of Growth in Trade Finance: Instrumental-Variables Approach

	(1)	(2)	(3)	(4)
Trade Flows Growth	0.168	0.268**	0.262**	0.307***
	[0.109]	[0.105]	[0.107]	[0.099]
Bank Capital to Total Assets	0.007	0.007	0.007	0.006
	[0.011]	[0.011]	[0.011]	[0.011]
CDS Spreads	−0.011	−0.012	−0.012	−0.015*
	[0.009]	[0.009]	[0.009]	[0.009]
S&P Rating	0.001	−0.001	0.000	−0.003
	[0.010]	[0.011]	[0.011]	[0.011]
GDP Growth	−0.008	−0.007	−0.008	−0.007
	[0.006]	[0.006]	[0.006]	[0.006]
World Imports Growth	0.009***			
	[0.002			
VIX		−0.003***		
		[0.001]		
Financial Conditions Index			−0.061***	
			[0.021]	
LIBOR-OIS Spread				−0.067***
				[0.029]
Observations	294	294	294	294
R-squared	0.27	0.24	0.24	0.24
Sargan-Hansen Statistic	0.03	0.08	0.01	0.08

Notes: Robust standard errors are in brackets. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. All specifications include year and quarter dummies and country-specific fixed effects. The dependent variable is the quarter-over-quarter (qoq) growth in the outstanding volume of trade finance. All explanatory variables in growth rates are also in terms of qoq growth. The variable instrumented is "trade flows growth." We use two measures as instruments: the trade-weighted real GDP growth of the top ten trading partners for each country and the weighted trade flows growth of the top ten trading partners for each country. We assess the validity of instruments using the Sargan-Hansen test.

import growth. Higher world import growth is positively associated with trade finance growth. Global financial conditions have a negative and statistically significant impact on trade finance growth. The coefficient for CDS spreads is only significant in one specification (column 4). A potential explanation for this could be that the impact of the country-specific variables is not properly identified by the instrumental-variables approach given the cross-sectional correlation that could arise from the presence of the global financial variables.

5.3 Random-Effects Estimator

When the cross-sectional dimension of the data set is small, as in our case, the fixed-effects estimator can be unstable and exhibit lack of robustness to potentially anomalous samples. The variance of the estimator can be large in such cases (see, for example, Clark and Linzer 2015). At the same time, random-effects estimation also has its own disadvantages, particularly the possibility of biases in the estimation due to unobserved, time-invariant heterogeneity. In our case, this problem is somewhat mitigated by the fact that in some country-specific covariates a substantial fraction of the variation is within groups.[24] Nevertheless, in order to test the sensitivity of our results to the fixed-effects estimator, we reestimate the model using the random-effects estimator. The results are shown in table 5. They are broadly consistent with the benchmark model, thus underscoring the role of both demand and supply factors as determinants of trade finance lending.

5.4 Sensitivity Analysis: Excluding Germany, United Kingdom, and United States

As discussed in section 4.1, the trade finance data being used are not entirely homogeneous across jurisdictions. Although in most jurisdictions the trade finance data cover lending by resident banks vis-à-vis domestic residents (i.e., locally booked), there are three jurisdictions where this is not the case, although for different reasons: Germany, the United States, and the United Kingdom. In the first two cases, trade finance is generally compiled on a consolidated basis, thus tracking lending by domestically owned banks vis-à-vis non-resident borrowers. In the United Kingdom, these data are compiled

[24]For instance, 90 percent of the variance in the growth in trade flows occurs within groups. This fraction is over two-thirds in the case of banks' CDS. However, the corresponding fraction for sovereign ratings and banks' capital-to-assets ratio is much lower, at 20 percent and 30 percent, respectively.

Table 5. Determinants of Growth in Trade Finance: Random-Effects Estimations

	(1)	(2)	(3)	(4)
Trade Flows Growth	0.114	0.194**	0.188**	0.223***
	[0.077]	[0.076]	[0.077]	[0.075]
Bank Capital to Total Assets	0.001	0.001	0.001	0.002
	[0.003]	[0.003]	[0.003]	[0.003]
CDS Spreads	−0.010	−0.011	−0.011	−0.013*
	[0.007]	[0.007]	[0.007]	[0.007]
S&P Rating	−0.003	−0.003	−0.003	−0.003
	[0.002]	[0.002]	[0.002]	[0.002]
GDP Growth	−0.002	−0.001	−0.002	−0.001
	[0.005]	[0.005]	[0.005]	[0.005]
World Imports Growth	0.009***			
	[0.002]			
VIX		−0.004***		
		[0.001]		
Financial Conditions Index			−0.062***	
			[0.018]	
LIBOR-OIS Spread				−0.071***
				[0.025]
Observations	294	294	294	294
R-squared	0.299	0.263	0.262	0.251

Notes: Robust standard errors are in brackets. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. All specifications include year and quarter dummies. The dependent variable is the quarter-over-quarter (qoq) growth in the outstanding volume of trade finance. All explanatory variables in growth rates are also in terms of qoq growth. All regressions are estimated using the random-effects estimator.

on a residence basis, but there is a large presence of foreign banks with outward-oriented activities. Thus, there is potential decoupling between residence and nationality, and resident banks' activity could be independent of U.K.-specific factors.[25]

[25]We would like to thank an anonymous referee for pointing this out. As an illustration, consider a U.K. branch of, say, a German bank which is highly active in the trade finance sector. This branch would book its claims in the United Kingdom. It is reasonable to expect that its ability to extend trade finance will be

To examine the sensitivity of our results to this heterogeneity in trade finance data, we estimate the regressions by excluding Germany, the United Kingdom, and the United States, one at a time. The results are shown in table 6 (panels A, B, and C) and are broadly unchanged.[26]

5.5 Additional Analysis: Advanced Economies versus EMEs

Next, we investigate whether emerging market economies (EMEs) and advanced economies are affected differently by some global variables.[27] To do so, we create a dummy variable called "EMEs" which takes a value of 1 if a country is classified as an emerging market economy and 0 otherwise. We then interact this variable with each of the four global variables and include the resulting (four) explanatory variables in the regression framework. The coefficient on each variable indicates whether global variables have any differential impact on emerging economies, compared with the benchmark effect.

The results are reported in table 7 and are very similar to the benchmark specification (table 3). Global factors are important in explaining bank-intermediated trade finance in both EMEs and advanced economies; they do not have any additional impact on EMEs. Note that the variables capturing global financial conditions are not significant for EMEs, but this is most likely a consequence of the small sample size.

6. Conclusion

Understanding the drivers of trade finance is important from an academic as well as policymakers' standpoint since 30 to 40 percent of global trade relies on some version of trade finance. The sharp drop in trade finance during the global financial crisis has raised some important policy questions. However, empirical work on the determinants of trade finance has been very limited due to the lack of

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largely unrelated to liquidity conditions in the United Kingdom, such as banks' capital-to-asset ratios.

[26]The results remain broadly similar even when all three countries are excluded. These results are not reported for the sake of brevity.

[27] Advanced economies include Australia, France, Germany, Italy, Spain, the United Kingdom, and the United States. EMEs include Brazil, India, and Korea.

Table 6. Sensitivity Analysis: Excluding Germany, United Kingdom, and United States

	A. Excluding Germany				B. Excluding United Kingdom				C. Excluding United States			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Trade Flows Growth	0.101	0.186**	0.185**	0.214***	0.095	0.162**	0.154**	0.188***	0.110	0.179**	0.175**	0.206***
	[0.073]	[0.062]	[0.078]	[0.062]	[0.072]	[0.055]	[0.068]	[0.057]	[0.067]	[0.057]	[0.073]	[0.057]
Bank Capital to Total Assets	0.008	0.007	0.008	0.007	0.006	0.005	0.006	0.004	0.005	0.004	0.005	0.004
	[0.009]	[0.009]	[0.009]	[0.009]	[0.010]	[0.010]	[0.010]	[0.010]	[0.010]	[0.010]	[0.010]	[0.010]
CDS Spreads	−0.012**	−0.014**	−0.014**	−0.017**	−0.008*	−0.008	−0.007	−0.011	−0.013**	−0.014**	−0.014**	−0.018**
	[0.005]	[0.005]	[0.005]	[0.007]	[0.004]	[0.005]	[0.004]	[0.007]	[0.004]	[0.005]	[0.005]	[0.008]
S&P Rating	−0.000	−0.002	−0.002	−0.004	0.004	0.003	0.004	0.001	−0.000	−0.002	−0.001	−0.004
	[0.006]	[0.007]	[0.007]	[0.007]	[0.006]	[0.006]	[0.006]	[0.007]	[0.006]	[0.007]	[0.007]	[0.008]
GDP Growth	−0.007	−0.006	−0.006	−0.004	−0.006	−0.006	−0.006	−0.005	−0.006	−0.005	−0.006	−0.004
	[0.006]	[0.006]	[0.005]	[0.007]	[0.004]	[0.004]	[0.004]	[0.005]	[0.004]	[0.005]	[0.004]	[0.005]
World Imports Growth	0.009***				0.010***				0.009***			
	[0.001]				[0.002]				[0.001]			
VIX		−0.004***				−0.005***				−0.004***		
		[0.001]				[0.001]				[0.001]		
Financial Conditions Index			0.061**				−0.080***				−0.069***	
			[0.022]				[0.023]				[0.021]	
LIBOR-OIS Spread				−0.071*				−0.097**				−0.078**
				[0.035]				[0.038]				[0.033]
Observations	263	263	263	263	269	269	269	269	259	259	259	259
Within R-squared	0.30	0.26	0.26	0.25	0.31	0.28	0.27	0.26	0.30	0.27	0.27	0.27

Notes: Robust standard errors are in brackets. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. The dependent variable is the quarter-over-quarter (qoq) growth in the outstanding volume of trade finance. All explanatory variables in growth rates are also in terms of qoq growth. All specifications include year and quarter dummies and country-specific fixed effects. In panels A, B, and C, the model is estimated removing Germany, the United Kindom, and the United States, respectively.

Table 7. Additional Analysis: Effect of Global Variables on Advanced Economies vs. EMEs

	(1)	(2)	(3)	(4)
Trade Flows Growth	0.126*	0.204***	0.199***	0.227***
	[0.067]	[0.057]	[0.072]	[0.061]
Bank Capital to Total Assets	0.006	0.005	0.006	0.003
	[0.010]	[0.010]	[0.010]	[0.010]
CDS Spreads	−0.018**	−0.017**	−0.015**	−0.023**
	[0.007]	[0.006]	[0.007]	[0.008]
S&P Rating	−0.007	−0.007	−0.004	−0.012
	[0.009]	[0.010]	[0.011]	[0.009]
GDP Growth	−0.004	−0.005	−0.006	−0.004
	[0.004]	[0.005]	[0.005]	[0.005]
World Imports Growth	0.010***			
	[0.001]			
EMEs*World Imports Growth†	−0.005*			
	[0.002]			
VIX		−0.004***		
		[0.001]		
EMEs*VIX		0.001		
		[0.001]		
Financial Conditions Index			−0.068***	
			[0.021]	
EMEs*Financial Conditions Index			0.012	
			[0.021]	
LIBOR-OIS Spread				−0.078***
				[0.034]
EMEs*LIBOR-OIS Spread				0.044
				[0.027]
Observations	294	294	294	294
Within R-squared	0.31	0.27	0.26	0.26

Notes: Driscoll-Kraay standard errors are in brackets. ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. The dependent variable in regression (1)–(4) is the qoq growth in the outstanding volume of trade finance. All explanatory variables in growth rates are also in terms of qoq growth. All specifications include year and quarter dummies and country-specific fixed effects.
†This variable is constructed as the interaction of the "EME" dummy with the qoq growth in world imports. The variables EMEs*VIX, EMEs*Financial Conditions Index, and EMEs*LIBOR-OIS Spread are constructed in a similar fashion.

availability of data, with previous studies having focused on developments in specific countries. Our paper addresses this gap by analyzing the main determinants of bank-intermediated trade finance using a newly constructed data set in a panel estimation framework.

Results indicate that bank-intermediated trade finance is impaired by global financial strains, while it depends positively on global imports growth. Country-specific variables, namely, growth in trade flows, and the funding availability of domestic banks—as measured by the banks' CDS spreads—are also important determinants of trade finance. These results are robust to different model specifications. We believe that our results mark an improvement over existing research, which has focused either on firm-specific data or on specific instruments, while our data set provides a more comprehensive coverage of trade finance within each country.

We acknowledge that there are other potentially relevant drivers of bank-intermediated trade finance that have not been included in our analysis. These include measures of contractual enforcement, bank lending restrictions, and foreign exchange restrictions, as well as additional country-specific measures of banking system soundness. However, including these indicators in an econometric framework is challenging given the lack of good-quality data. Further, policy responses to mitigate the impact of global financial conditions probably played an important role in determining trade finance developments. Given the small sample size and the significant heterogeneity in policy responses across countries, we leave these questions to be addressed in future research.

Data Appendix

Table 8. Trade Finance Data

Country	Data Description	Source
Australia	Stock of banks' contingent liabilities arising from trade-related obligations (e.g., documentary L/Cs issued, acceptances on trade bills or shipping guarantees issued).	Reserve Bank of Australia
Brazil	Stock and flows of resident banks' trade finance vis-à-vis residents for exports and imports.	Central Bank of Brazil
France	Stock of trade finance, including both buyer and supplier credit lines.	Bank of France
Germany	Estimations for the volume of short-term trade finance to emerging and developing countries, covering maturities of twelve months or less.	Deutsche Bundesbank (Not Public)
India	Stock of short-term loans and advances of pre- and post-shipment bank-intermediated export credit, as well as stock and flows of import credit extended by banks with maturities of less than three years.	Reserve Bank of India
Italy	Stock of loans and guarantees for import and export purposes by domestic banks.	Italian Credit Register (Not Public)
Korea	Stock of documentary bills, domestic import usance bills, and pre-shipment finance.	Bank of Korea, Financial Supervisory Service
Spain	Stock of commercial credit to non-residents and documentary credit to residents and non-residents, granted by banks operating in Spain.	Bank of Spain (Not Public)
United Kingdom	Estimates derived from reported amounts for "lending under Export Credit Guarantee Department bank guarantee" and "holdings of non-resident bills," which may be discounted for trade finance purposes.	Bank of England (Not Public)
United States	Stock of bank-intermediated, short-term trade finance (including funded loans and unfunded off-balance-sheet commitments and guarantees) vis-à-vis foreign residents on an ultimate risk basis.	FFIEC Country Exposure Lending Survey*

Source: CGFS (2014).
*FFIEC stands for the Federal Financial Institutions Examination Council.

Table 9. Trade Finance Data: Coverage, Counterparties, and Instruments

	Institutions Covered			**Counterparties**		**Instruments**	
	Resident Banks	**Cross-Border Branches**[1]	**Cross-Border Subsidiaries**[1]	**Residents**	**Non-residents**	**Loans**	**L/C and Guarantees**
Australia	X	X	X	X			
Brazil	X			X		X	
France	X			X			P
Germany	X	P	P		P[2]	X	P
India	S			X		X	
Italy	X + OFIs*			X		X	X
Korea	X			X		X	
Spain	X	X		X		P	X
United Kingdom	X				X	P	P
United States	X	X	X		X	X	X

Sources: CGFS (2014), country-specific sources, and authors' classification.
Note: X = dimension is captured; P = dimension is partially captured.
*OFIs = other financial institutions.
[1]Foreign operations of home-headquartered banks.
[2]Only to residents of emerging markets and developing countries.

Table 10. Description of Variables

Variable	Source	Description
Country-Specific Variables		
Nominal GDP Growth	National Statistical Agencies via Haver	Quarter-over-quarter (annualized rate) growth in percentage points; seasonally adjusted.
S&P Rating	Standard & Poor's	We transform ratings into numerical values, using a linear mapping. AAA corresponds to 20, while D corresponds to 0. The threshold between investment grade and junk is 12.
Banks' Capital-to-Assets Ratio	WDI (Annual Data—Interpolated)	In percentage points.
Trade Flows Growth (Sum of Exports and Imports)	CGFS (2014)	Qoq growth in percentage points, calculated as log-differences.
CDS Spreads	Datastream	Five-year CDS spreads, in percentage points, measured as the average of the CDS spreads for the main banks in each country (as listed in table 9).
Global Variables		
VIX Index	Haver Analytics	In level.
Financial Conditions Index (FCI)	Banco de España, National Statistical Agencies	Constructed folllowing Guichard, Haugh, and Turner (2009). The index covers four economic areas: the United States, the United Kingdom, Japan, and the euro area. In each country/region, the index aggregates information on credit conditions, bond spreads, real and short-term interest rates, and real effective exchange rates.
Imports Growth	World Bank, World Development Indicators	Qoq growth in percentage points, calculated as log-differences.
LIBOR-OIS Spread	Bloomberg	The spread between the three-month LIBOR and the three-month USD overnight indexed swap quarterly average, expressed in percentage points.

Table 11. List of Banks Included in CDS Spreads (Five Years) Measure

Country	Banks
Australia	National Australia Bank
Brazil	Banco do Brasil, Bradesco, Votorantim
France	BNP Paribas, Credit Agricole, Société Générale
Germany	Deutsche Bank AG, Commerzbank AG
India	ICICI Bank, State Bank of India
Italy	Intesa Sanpaolo, Unicredito Italiano
Korea	Hana Bank, Woori Bank, Kookmin Bank
Spain	Banco Santander, BBVA
United Kingdom	Barclays Bank, Lloyds Bank, RBS Group PLC, HSBC Bank PLC
United States	Bank of America, Citigroup Inc., Goldman Sachs, JP Morgan Chase, Morgan Stanley, Wells Fargo

Source: Datastream.
Note: For Brazil, we use the sovereign CDS spreads for the period prior to 2011 as a proxy since the information on banks' CDS spreads is only available from 2011.

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Comparing Fiscal Consolidation Multipliers across Models in Europe*

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This paper employs fifteen dynamic macroeconomic models maintained within the European System of Central Banks to assess the macroeconomic effects of a temporary fiscal tightening when the zero lower bound (ZLB) on monetary policy holds for two years. The main results are as follows. First, the ZLB does not greatly affect short-run multipliers in the case of a temporary fiscal tightening implemented in isolation by a generic euro-area (EA) country. Second, the ZLB unfolds quite sizable effects on the size of multipliers if the same fiscal tightening measure is simultaneously implemented in the whole EA. Third, public consumption multipliers are typically larger in absolute value than short-run tax (on labor income, capital income, and consumption) multipliers. Fourth, recessionary effects of the initial fiscal tightening are lower if distortionary taxes are reduced in the medium and long run.

JEL Codes: E12, E13, E17, E62, E63.

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1. Introduction

The Great Recession has triggered a new wave of research on evaluating fiscal multipliers when monetary stabilization policy is constrained by the zero lower bound (ZLB) on the monetary policy rate. In Europe, the Great Recession merged into the European sovereign debt crisis and the subsequent period of fiscal consolidations. In the case of the euro area (EA), consolidation was country specific and took place against the background of a union-wide monetary policy. This adds two intertwined issues to the academic and policy debate on fiscal multipliers: (i) the implementation, country specific or cross-country simultaneous, of fiscal measures in a monetary union and (ii) the interaction between fiscal measures and the union-wide ZLB holdings.

Our paper addresses these issues from a quantitative (positive) perspective. We employ fifteen structural, calibrated or estimated, macroeconomic models maintained within the European System of Central Banks (ESCB) to evaluate country-specific fiscal multipliers in correspondence of the ZLB on the union-wide monetary policy rate and of alternative fiscal consolidation plans.

Since our suite of models contains (i) models representing individual EA countries of different size, (ii) models of countries not belonging to the EA, and (iii) a model of the EA as a whole, we can assess how the fiscal multipliers of measures unilaterally enacted by an EA country or simultaneously implemented by all EA countries are affected by the union-wide monetary policy stance, in particular when the ZLB holds. The simulation of country-specific quantitative models allows us to reduce model uncertainty and increase the robustness of the results along key features of EA countries, such as size, degree of openness, and nominal and financial frictions.

In each of the simulated scenarios, we consider the effects of a standardized discretionary change in a single fiscal policy instrument on domestic real GDP at the country level or, because we simulate the model featuring the EA as a single entity, at the union-wide level. We compare scenarios in which the union-wide policy rate is determined by a Taylor rule with scenarios in which the policy rate is constrained by the ZLB during the first two years of the fiscal tightening.[1] The change in the policy instrument amounts to 1 percent of baseline GDP and represents a tightening of the fiscal stance. Specifically, we consider a reduction in (unproductive) government consumption and increases in tax rates on households' labor income, capital income, and consumption. To assess the impact of the design of the fiscal plan on the short-run fiscal multipliers, we make alternative assumptions on the taxes that are reduced in the long run in correspondence of the fiscal room created by the initial fiscal tightening (in all simulations it is assumed that the debt-to-GDP ratio, after decreasing, gradually returns to the initial pre-shock level). The reduction is fully anticipated by households and firms and, thus, factored into their (short- and long-run) optimal decisions.

Our first main result, *common to all models of individual EA countries*, is that imposing the ZLB to bind for two years does not greatly affect short-run multipliers in the case of a temporary fiscal tightening implemented in isolation by a generic EA country. The reason is that the monetary policy rate stays essentially at its baseline level even when the monetary authority is free to adjust it, reflecting the limited impact of a country-specific fiscal shock on the EA economy. In contrast, and this is our second main result, the ZLB unfolds quite sizable effects on the size of multipliers if the same fiscal (tightening) measure is simultaneously implemented in the whole EA. In particular, short-run government consumption multipliers become larger than one. The same holds true for non-EA countries in which monetary policy is determined domestically. The third result is that government consumption multipliers are typically larger in absolute value than short-run tax (on labor income, capital income, and consumption) multipliers. In the short run, tax multipliers are in general negative and smaller than one in absolute value. This result is quite robust with respect to the considered country, the considered fiscal instrument, and the duration of the fiscal shock. The fourth result is that the short-run multipliers tend to be more favorable if in the long run the distortionary taxes are reduced to exploit the fiscal room created by the initial tightening, since households anticipate long-run effects at the outset of the simulations.

[1] The common nominal interest rate is set in response to union-wide inflation and economic activity, to which each EA country contributes according to its share of the union-wide GDP.

Finally, long-run multipliers are in general negative when the budgetary room materializing after the fiscal tightening is used to reduce lump-sum taxes. Instead, long-run multipliers are typically positive if the households' labor income tax rate is reduced in the medium to long term.

Our paper is most closely related to a small set of studies that examine the robustness of fiscal multiplier estimates among structural models. Cwik and Wieland (2011) use five macroeconomic models to estimate multipliers associated with the European Economic Recovery Plan and related national fiscal policy measures in the EA. They focus on the announced government purchases component of the plan for 2009 and 2010. In the majority of models, private consumption and investment are crowded out by the rise in government spending unless the ZLB is anticipated to be binding for at least two years. Unlike our paper, they do not consider tax policies. Coenen at al. (2012) employ seven dynamic stochastic general equilibrium models maintained by policymaking institutions to assess the GDP effects of expansionary fiscal shocks. They find that fiscal stimulus is most effective if it is temporary and accompanied by an accommodative monetary policy stance. Unlike our paper, they do not focus on the role of a monetary union, and they analyze fiscal stimulus programs instead of fiscal consolidation plans. Erceg and Lindé (2013) and Forni, Gerali, and Pisani (2010) evaluate fiscal consolidation in a monetary union, and Farhi and Werning (2016) study fiscal multipliers in a currency union with a liquidity trap. Different from them, we make a comprehensive cross-country assessment of fiscal multipliers in the EA, and find a set of rather robust results. The novelty of our study is that while previous studies investigate multipliers associated with expansionary fiscal shocks, we consider fiscal retrenchments. The sign of the fiscal shocks matters in particular in those situations where the economy is at the ZLB. Otherwise, the fiscal multipliers studied in this paper are not sign dependent.

More broadly, our paper is related to a large and growing set of studies that examine the size of fiscal multipliers within one or two macroeconomic models. Prominent recent examples include Christiano, Eichenbaum, and Rebelo (2011), Cogan et al. (2010), Eggertsson (2011), and Woodford (2011). In particular, our results are qualitatively similar to those in Drautzburg and Uhlig (2015). Different from them, our aim is to find robust cross-country results on the size of short-run fiscal multipliers in a monetary union, and in particular how they are affected by the common monetary policy stance and by the design of the fiscal consolidation.

The remainder of the paper is structured as follows. Section 2 summarizes the models used in the simulation exercises. Section 3 describes the simulations and presents the results. Section 4 summarizes the results of the sensitivity analysis. Finally, section 5 concludes.

2. Model Setup

We use fifteen quarterly models from national central banks (NCBs) and the European Central Bank (ECB) in the simulation exercises. Fourteen out of fifteen are New Keynesian dynamic general equilibrium models. Ten are calibrated and five are estimated. A complete list of the models is presented in table 9 in the appendix.

The majority of models from NCBs of EA countries are based on multi-country setups, namely those of Belgium, Estonia, France, Germany, Italy, Malta, Slovenia, and Spain. These models exhibit a "home" country, the rest of the EA (possibly subdivided), and in some cases the rest of the world. In these models the EA monetary policy responds to economic fluctuations in the home country only proportionally to its weight in the monetary union.

A second set of models comprises small open economy setups, with an exogenous rest of the EA and/or rest of the world: the Czech Republic, Finland, Greece, Netherlands, Portugal, and Sweden. If the corresponding country is in the EA, monetary policy is assumed to be exogenous. If not, the monetary policy is set according to a standard Taylor rule.

Finally, the ECB's New Area-Wide Model (NAWM) has also been used. It is a two-country model of the EA and the United States. Monetary policy in both model blocks is characterized by standard nominal interest rate rules.

Responses to fiscal shocks can be influenced by the fiscal instrument that, through the fiscal rule, endogenously adjusts to stabilize public debt. In the vast majority of the models, this fiscal instrument reacts to deviations of the government debt-to-GDP ratio from the target, but in a few cases it reacts also to deviations of the public deficit or public consumption from the target. Typically, either the labor income tax or lump-sum transfers are used as the fiscal instrument. In some of the simulations, the choice of the fiscal rule has been left to the discretion of each country's modelers. However, whenever the fiscal rule becomes critical for the results, we harmonized the instrument that is specified by the rule across models.

In general, the models share the pros and cons of structural models that have recently been discussed in the literature; see, e.g., Blanchard (2016). The models are theoretically motivated and well suited to this type of policy analysis, in which the stock-flow consistency and aggregate resource constraint are key features of the analysis. At the same time, there are elements of interaction that models may not fully capture, such as the impact of sovereign risk on sovereign funding conditions and spillovers of sovereign risk to banking and to financial-sector lending.

2.1 Steady-State Values and Calibration

Key parameters and their calibration are listed in tables 10–12 in the appendix. The models differ in various aspects.

In terms of steady-state values, the models differ significantly as regards the imports-to-GDP ratio, which to some extent measures the degree of openness of the economy. The lowest import penetration is found for Greece and the largest is found for Estonia. The models also differ substantially in terms of how public expenditures are financed. As an example, in the German model the labor income tax revenues amount to 35 percent of GDP, while in Spain they account for only 7 percent of GDP. The steady-state values of the debt-to-GDP ratio vary from 0 percent to 120 percent. The models also vary in the degree of home bias in government consumption. Most of the models assume full home bias, as is typical in this type of setup, and only a few feature somewhat lower home bias of around 90 percent. Finally, the share of liquidity-constrained consumers, i.e., households that have at most limited ability to smooth consumption over time, varies between 0 to 40 percent.

Regarding the calibration of some key parameters, household preferences, investment (or capital) adjustment costs, price and wage stickiness, and the proportion of firms (workers) that index their price (wage) to inflation are quite different among models:[2] the Frisch elasticity of labor supply varies from 0.50 to 11, wage indexation from 0 to 0.90, and investment adjustment costs from 0.20 to about 14.

All these differences can play an important role in explaining differences in fiscal multipliers across the models.

3. Simulation Experiments and Results

In each of the simulation scenarios reported below, we consider the short-run and—if applicable—the long-run effects of a discretionary change in a single fiscal policy instrument on real GDP. The change in the policy instrument amounts to 1 percent of baseline (pre-shock) GDP and represents a tightening of the fiscal stance. The tightening can be temporary (lasting for two years) or permanent. Specifically, we consider a reduction in government consumption and increases in the tax rate on households' labor income, capital income, and consumption. Fiscal items (including social security contributions) other than the ones subject to discretionary change are held constant.[3] In the medium to long run (after the initial two years), either lump-sum or labor income taxes are allowed to adjust according to the country-specific fiscal rules to stabilize the public debt-to-GDP or deficit-to-GDP ratio at their target values. In the case of permanent fiscal shocks, the multipliers can be quite sensitive to the fiscal instrument that stabilizes the debt or the deficit. Therefore, we conduct these simulations twice with each model, in one case imposing a lump-sum tax rule and in the other a households' labor income tax rule.[4]

Monetary policy is harmonized across models, assuming that the short-term nominal interest rate is determined by the Taylor rule used in Gomes, Jacquinot, and Pisani (2012), where the policy rate

[2]Nominal rigidities are characterized by a Calvo parameter or, if the value is larger than one, by a Rotemberg adjustment cost parameter.

[3]In the case of the Swedish model, the fiscal rule is implicit: lump-sum transfers make sure that government expenditures and tax revenues are equal in every period. For the simulations carried out in this model, lump-sum transfers are thus allowed to adjust also in the short run.

[4]The specification of the country-specific fiscal rule has only very modest effects on multipliers if the fiscal shock is transitory.

responds to EA-wide inflation and output growth.[5] We also assess fiscal multipliers when the ZLB holds, assuming that the Taylor rule is deactivated and the short-term nominal interest rate is held constant at its baseline level during the initial two years.

All simulations are run under perfect foresight. Therefore, policies are fully anticipated by households and firms.

Broadly speaking, the term "fiscal multiplier" describes the effects of changes in fiscal instruments on real GDP. Typically, it is defined as the ratio of the change in real GDP to the change in the fiscal balance. In this paper, we compare the effects on real GDP of different fiscal instruments. We therefore normalize the fiscal impulses in the experiments so that the size of the discretionary shock in each case represents a decrease in public consumption or an increase in revenues equal to 1 percent of baseline, pre-consolidation GDP for two years or on a permanent basis. The first-year multiplier is calculated by averaging the change of output from quarter 1 to quarter 4,

$$\textit{First-year multiplier} = \frac{\sum_{i=1}^{4} \frac{\Delta y_{t+i}}{4}}{\sum_{i=1}^{4} \frac{\Delta f_{t+i}}{4}},$$

where Δy_{t+i} and Δf_{t+i}, respectively, refer to changes in output and the fiscal instrument, i.e., government expenditures or tax revenues in quarter i relative to their corresponding before-shock values. The second-year multiplier is calculated similarly, by averaging the effects from quarter 5 to quarter 8. The long-run multiplier is

$$\textit{Long-run multiplier} = \frac{\Delta y_T}{\Delta f_T},$$

where Δy_T and Δf_T measure the permanent, long-run steady-state changes in output and fiscal instrument, respectively.

In what follows we first report GDP multipliers for transitory changes in each fiscal instrument implemented unilaterally by a single country. Subsequently we present multipliers associated with

[5]The rule is specified as $R_t^4 = \phi_R(R_{t-1}^4) + (1 - \phi_R)[\bar{R}^4 + \phi_{\prod}(\prod_t^4 - \bar{\prod}^4)] + \phi_Y\left(\frac{y_t}{y_{t-1}} - 1\right)$, where R_t is the quarterly gross interest rate ($\bar{R}$ is the steady-state value), $\prod$ is the quarterly inflation rate ($\bar{\prod}$ is the steady-state value), and y is the output level. $0 < \phi_R < 1, \phi_{\prod}, \phi_Y > 0$ are parameters.

Table 1. Short-Run Fiscal Multipliers: Temporary (Two-Year) Reduction in Government Consumption

	No ZLB		Two-Year ZLB	
	Year 1	**Year 2**	**Year 1**	**Year 2**
Belgium	−0.93	−0.90	−0.97	−0.95
Czech Republic	−0.54	−0.54	−1.79	−1.57
Estonia	−0.83	−0.66	−0.98	−0.77
Euro Area	−0.98	−0.91	−1.39	−1.30
Finland*	−0.78	−0.76	−0.78	−0.76
France	−0.92	−0.71	−1.05	−0.87
Germany	−0.52	−0.48	−0.72	−0.68
Greece*	−0.90	−0.73	−0.90	−0.73
Italy	−0.79	−0.67	−0.86	−0.73
Malta	−0.73	−0.49	−0.73	−0.49
Netherlands*	−0.74	−0.72	−0.74	−0.72
Portugal*	−0.76	−0.23	−0.76	−0.23
Portugal* (ff)	−0.85	−0.37	−0.85	−0.37
Slovenia	−0.66	−0.48	−0.68	−0.50
Spain	−0.50	−0.29	−0.50	−0.29
Sweden	−0.60	−0.63	−1.63	−2.07

*In these countries, monetary policy is exogenous. Portugal (ff) indicates the presence of financial frictions following Bernanke, Gertler, and Gilchrist (1999).

permanent changes in each fiscal instrument. In both cases, two years after the initial shock the country-specific fiscal rule starts to operate, gradually bringing the debt-to-GDP ratio or deficit-to-GDP ratio back to its target level (the initial pre-shock level).

3.1 Temporary Fiscal Shocks

3.1.1 Government Consumption

Table 1 shows the government consumption multipliers, i.e., the response of GDP to a temporary (two-year) decrease in government consumption.

In the first case, denoted "No ZLB," the ZLB is not imposed as a constraint and the nominal interest rate adjusts according to the Taylor rule (see footnote 5). In the second case, denoted "Two-Year

ZLB," the nominal interest rate is kept constant during the first two years of the simulation and follows the Taylor rule thereafter. Similarly, all the other fiscal items are held constant at their corresponding baseline levels, including the fiscal instrument during the first two years as the fiscal rule kicks in thereafter.

When the ZLB is not binding, all multipliers are below one in absolute terms. In the majority of the models the first-year multipliers are between 0.7 and 0.9, but in some cases they are lower. They are close to 0.5 in Germany, Spain, the Czech Republic, and Sweden.

The intuition for multipliers being lower than one in absolute value is based on the crowding-in effect on private-sector spending, which partially compensates for the reduction in public consumption. In the majority of models, private-sector consumption and investment (not reported) increase. The crowding-in effect is not very large in the case of EA countries. The country-specific real interest rate does not greatly decrease and, thus, does not contribute significantly to the crowding in of private demand. This is for two reasons. First, the monetary policy rate, set at the union-wide level, is not greatly reduced after a country-specific shock, because the latter has a small effect on EA inflation and economic activity. Second, the response of the country-specific inflation rate is rather contained, because prices are sticky in the short run and the lower aggregate demand is also matched by lower imports (trade channel). Finally, the positive wealth effect is relatively small, because the fiscal retrenchment is temporary and, thus, the reduction in the present value of future tax payments required to balance the government's budget is contained.

The multipliers being lower than one is a result robust also to the introduction of financial frictions. In the case of Portugal, the multipliers increase around 10 percent in the first year when the model includes financial frictions. Along with lower aggregate demand, the price of capital decreases, as well as net worth. The entrepreneurial sector becomes more leveraged and is forced to face a higher external finance premium, which dampens investment. The presence of financial frictions also creates some persistence effects, as it takes time to rebuild lost net worth.

Overall, second-year multipliers are only to some extent lower in absolute terms than the first-year multipliers. Adjustment costs on investment, habit persistence in consumption, and nominal wage and price rigidities make the positive response of private spending gradual.

Interestingly, this is a crucial result of the paper, for EA countries multipliers are lower than one also under the ZLB (the only exception is France, which exhibits a multiplier slightly larger than one). They are either unchanged or only slightly larger than in the case of the nominal interest rate set according to the Taylor rule. The intuition is that EA countries are de facto at the ZLB also when the area-wide Taylor rule holds. Specifically, in the case of EA country-specific fiscal retrenchment, responses of economic activity and inflation in the rest of the EA are muted and, thus, in the case of the active Taylor rule the EA-wide policy rate does not greatly change. Moreover, the ZLB lasts for a relatively small (but plausible) number of periods (eight quarters) and, thus, is not able to largely amplify the cross-country spillovers of the fiscal shock. Overall, the responses of the region-specific real interest rates (in the considered country and in the rest of the EA) are muted and similar in both scenarios.

To the opposite, the ZLB makes the difference in the case of the *area-wide* (simultaneous across EA countries) decrease in public consumption. In this case, obtained by simulating the NAWM, the EA policy rate is reduced in response to the decrease in EA inflation, which is larger than in the case of country-specific fiscal shocks. The interest rate response favors the crowding in of private spending. When the ZLB holds, the constant nominal interest rate and the decrease in EA inflation lead to a rather strong increase in the EA real interest rate that depresses private spending. Consistent with this intuition, multipliers become significantly larger when the ZLB binds in the case of the Czech Republic and Sweden, which have their own monetary policy, reaching values that are clearly larger than one.

3.1.2 Taxes

Households' Labor Income Tax Rate. Table 2 reports the short-run GDP multipliers in the case of a transitory (two-year) increase in the households' labor income tax rate.[6]

[6]For some simulations Czech and Dutch results are not available.

Table 2. Short-Run Fiscal Multipliers: Temporary (Two-Year) Increase in Households' Labor Income Tax Rate

	No ZLB		Two-Year ZLB	
	Year 1	Year 2	Year 1	Year 2
Belgium	−0.04	−0.10	−0.03	−0.10
Czech Republic	−0.36	−0.40	−0.38	−0.28
Estonia	−0.21	−0.43	0.04	−0.22
Euro Area	−0.11	−0.19	−0.04	−0.12
Finland*	−0.10	−0.13	−0.10	−0.13
France	−0.13	−0.30	−0.09	−0.25
Germany	−0.10	−0.09	−0.15	−0.14
Greece*	−0.50	−0.77	−0.50	−0.77
Italy	−0.06	−0.13	−0.05	−0.12
Malta	−0.09	−0.20	−0.09	−0.20
Netherlands*	−0.11	−0.15	−0.11	−0.15
Portugal*	−0.51	−0.91	−0.51	−0.91
Portugal* (ff)	−0.49	−0.86	−0.49	−0.86
Slovenia	−0.10	−0.19	−0.10	−0.19
Spain	−0.13	−0.11	−0.13	−0.11
Sweden	−0.27	−0.31	0.56	0.88

*In these countries, monetary policy is exogenous. Portugal (ff) indicates the presence of financial frictions following Bernanke, Gertler, and Gilchrist (1999).

Multipliers are lower than one in absolute value and smaller than those associated with the reduction in government consumption. They are generally around 0.1 in the first year and between 0.2 and 0.4 in the second year.

The labor tax multiplier is relatively small because it operates mainly through its effects on wealth (the permanent income is reduced, inducing an increase in labor effort) and incentives to substitute leisure for labor effort. As in the case of public consumption, the wealth effect is rather small because the fiscal measure is transitory. The role of income is emphasized also by the larger multipliers in some of those models that feature strong non-Ricardian features and thus relatively large consumption responses to current income.

This is the case for the models that include a relatively high percentage of liquidity-constrained consumers (Greece, Portugal, and the Czech Republic).[7]

Unlike in the scenario of public consumption reduction, a labor tax hike leads to an increase in the multiplier for the majority of countries when moving from the first to the second year. Similar to that scenario, this reflects the presence of nominal and real frictions, which leads to a gradual response of private demand to the labor income tax hike.

As in the case of government consumption, multipliers associated with country-specific tax increases are not greatly affected by the ZLB. For the majority of countries, the multipliers are slightly smaller when the nominal interest rate is held constant for two years. The reason is the rather contained increase in inflation (associated with negative supply side effects of higher labor taxes) and the fixed policy rate assumption holding for a relatively small number of periods. They result in a slight decrease in the real interest rate, partially limiting the decrease in aggregate demand. To the opposite, and similarly to the case of public consumption, the ZLB does affect multipliers relatively more when the fiscal retrenchment is implemented in the whole monetary union (see the NAWM-based results) and in countries that do not belong to a monetary union and have their own monetary policy, nominal exchange rate, and, thus, inflation, widely responding to the fiscal shock.[8]

Capital Income Tax Rate. Table 3 shows the short-run output multipliers of a transitory (two-year) increase in capital income taxation. The multipliers are generally rather small, below 0.3 in absolute terms. There is no strong incentive to reduce investment

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[7] To some extent, the size of the labor tax multiplier is also related to the share of labor income tax revenues to GDP and to the degree of wage indexation. For countries with a large labor income tax base, the multiplier tends to be smaller in absolute terms. This is explained by the fact that, e.g., labor supply reacts to a change in the labor income *tax rate*, whereby a change in the tax rate needs to be smaller for those countries with a large labor income tax base to achieve a 1 percent increase in the ratio of labor income tax revenues to GDP.

[8] Multipliers can greatly change because of the relatively large change in nominal exchange rate and inflation when the ZLB holds. See, for example, the results of the Swedish model, the discussion in Laséen and Svensson (2011), and Carlstrom, Fuerst, and Paustian (2012). The results from the ZLB experiments with the Swedish model should therefore be interpreted with some caution.

Table 3. Short-Run Fiscal Multipliers: Temporary (Two-Year) Increase in Capital Tax Rate

	No ZLB		Two-Year ZLB	
	Year 1	**Year 2**	**Year 1**	**Year 2**
Belgium	−0.06	−0.08	−0.06	−0.08
Estonia	−0.10	−0.11	−0.10	−0.12
Euro Area	−0.12	−0.10	−0.19	−0.17
Finland*	−0.10	−0.12	−0.10	−0.12
France	−0.07	−0.08	−0.09	−0.10
Germany	−0.05	−0.08	−0.11	−0.14
Greece*	−0.65	−1.06	−0.65	−1.06
Italy	−0.08	−0.11	−0.09	−0.12
Malta	−0.02	−0.04	−0.02	−0.04
Portugal*	−0.10	−0.01	−0.10	−0.01
Portugal* (ff)	−0.19	−0.15	−0.19	−0.15
Slovenia	−0.11	−0.11	−0.12	−0.12
Spain	−0.09	−0.07	−0.09	−0.07
Sweden	−0.33	−0.50	−2.18	−3.14

*In these countries, monetary policy is exogenous. Portugal (ff) indicates the presence of financial frictions following Bernanke, Gertler, and Gilchrist (1999).

since the increase in the capital income tax is transitory. Multipliers are also relatively low in the Portuguese case, in which credit market frictions work to propagate and amplify the negative impact on GDP (the higher capital income tax negatively affects entrepreneurial returns and, thus, increases leverage and the cost of external finance, which reduces investment).

There are some exceptions. Multipliers are rather large in the case of Sweden and Greece. In the Greek model, the large multiplier is driven by the sizable reduction in the utilization rate of capital and the price of capital that induce a strong negative response of output to the tax shock.

The short-run response of investment to an increase in the capital income tax is rather gradual, because of the short-run adjustment costs of investment.

Multipliers increase slightly under the two-year ZLB scenario. As in the previous simulations, the decrease in union-wide inflation and economic activity due to the temporary drop in the country-specific demand is rather muted for countries belonging to the EA. Under standard monetary policy, the policy rate does not greatly change and the country-specific real interest rate hardly moves. Similarly, the slowdown in country-specific inflation, and hence the increase in the country-specific real interest rate, is small under the ZLB. As a result, the ZLB does not significantly amplify the negative macroeconomic effects of the capital income tax increase. In the Portuguese case, the presence of credit market frictions has a slight amplification effect on the multipliers, also creating some persistence effects. The increase in multipliers is much larger in the case of the EA-wide shock (see the NAWM results) and in the Swedish case, where the role of exchange rate (that appreciates) in shaping inflation (which widely decreases) and, thus, the real interest rate, is more relevant.[9]

Consumption Tax Rate. Table 4 reports the short-run output multipliers associated with a transitory (two-year) increase in consumption taxation. In the absence of the ZLB, all multipliers are below one in absolute value. The largest multiplier is equal to 0.7 and the smallest is equal to 0.1.

The differences reflect the calibration of the intertemporal elasticity of substitution and consumption habit persistence. Higher intertemporal elasticity of substitution and lower habit persistence make current consumption more responsive to changes in consumer prices, which are directly affected by the transitory increase in consumption taxes. Habit persistence also tends to increase the multiplier in the second year relative to the first year, because households favor a gradual response of private consumption.

The ZLB does not change the overall picture significantly. The only exceptions are, again, the EA as a whole and Sweden, where the monetary policy rate strongly reacts to the changing inflation conditions.

3.2 Permanent Fiscal Shocks

In the previous section, we have considered transitory fiscal shocks. We now turn to permanent fiscal shocks, which allow us to assess

[9] Concerning the effects for Sweden, the results under the ZLB should again be interpreted with some caution.

Table 4. Short-Run Fiscal Multipliers: Temporary (Two-Year) Increase in Consumption Tax Rate

	No ZLB		Two-Year ZLB	
	Year 1	Year 2	Year 1	Year 2
Belgium	−0.19	−0.43	−0.20	−0.43
Czech Republic	−0.19	−0.09	−0.15	−0.03
Estonia	−0.25	−0.08	−0.25	−0.08
Euro Area	−0.48	−0.62	−0.78	−0.92
Finland*	−0.72	−0.70	−0.72	−0.70
France	−0.14	−0.23	−0.18	−0.29
Germany	−0.17	−0.22	−0.17	−0.17
Greece*	−0.48	−0.56	−0.48	−0.56
Italy	−0.29	−0.36	−0.35	−0.41
Malta	−0.15	−0.18	−0.15	−0.18
Portugal*	−0.49	−0.38	−0.49	−0.38
Portugal* (ff)	−0.52	−0.43	−0.52	−0.43
Slovenia	−0.24	−0.25	−0.24	−0.25
Spain	−0.14	−0.19	−0.14	−0.19
Sweden	−0.17	−0.21	−1.05	−1.45

*In these countries, monetary policy is exogenous. Portugal (ff) indicates the presence of financial frictions following Bernanke, Gertler, and Gilchrist (1999).

both the short- and long-run effects of discretionary changes in fiscal instruments. A permanent fiscal shock can be interpreted as "fiscal reform," or an "announced and fully credible fiscal plan," which permanently alters the fiscal structure of the economy. For instance, the combination of permanent reduction in government consumption and permanent reduction in labor income taxes reduces the size of the public sector and tax burden of the economy permanently. Similarly, a permanent change in one type of tax financed by an opposite change in another type of tax represents a permanent change in the tax structure of the economy. As in previous simulations, the fiscal rule is deactivated in the first two years, and thereafter it becomes active again to stabilize the public debt and/or the deficit at their target values, which remain unchanged (thus, we do not consider the case of a permanent reduction in public debt and/or public deficit).

Table 5. Short- and Long-Run Fiscal Multipliers: Permanent Reduction in Government Consumption

Fiscal Rule:	Lump-Sum Tax			Households' Labor Income Tax		
	Year 1	Year 2	Long Run	Year 1	Year 2	Long Run
Belgium	−0.95	−0.90	−0.63	−0.93	−0.83	0.70
Czech Republic	−0.25	−0.21	−0.43	—	—	—
Estonia	−0.65	−0.61	−0.68	−0.32	−0.22	0.84
Euro Area	−0.83	−0.62	−0.61	−0.46	−0.29	0.34
Finland*	−0.40	−0.31	−0.63	−0.33	−0.25	0.91
France	−0.97	−0.76	−0.82	−0.82	−0.48	1.28
Germany	−0.62	−0.40	−0.24	−0.61	−0.51	0.06
Greece*	−0.87	−0.74	−1.05	−0.83	−0.81	0.53
Italy	−0.68	−0.52	−0.58	−0.51	−0.19	0.54
Malta	−0.68	−0.37	−0.51	−0.62	−0.21	0.30
Portugal*	−0.58	−0.35	−0.67	−0.62	−0.05	1.64
Portugal* (ff)	−0.67	−0.44	−0.66	−0.72	−0.20	1.55
Slovenia	−0.66	−0.41	−0.38	−0.56	−0.15	0.82
Spain	−0.57	−0.35	−0.39	−0.48	−0.38	0.31
Sweden	−0.48	−0.44	−0.60	—	—	—

*In these countries, monetary policy is exogenous. Portugal (ff) indicates the presence of financial frictions following Bernanke, Gertler, and Gilchrist (1999).

Since the long-run response of output critically depends on the fiscal instrument that is determined by the fiscal rule, we compare two cases. In the first case, the fiscal rule is specified in terms of the lump-sum taxes (benchmark assumption). In the second, arguably more plausible, case the fiscal rule is instead specified in terms of the (distortionary) households' labor income tax.

3.2.1 Government Consumption, Lump-Sum Tax Rule

The first three columns of table 5 contain the short- and long-run output multipliers for a permanent reduction in government consumption when lump-sum taxes endogenously adjust according to the fiscal rule.

The estimated short-run multipliers are smaller than one in absolute value, ranging from 0.25 to 0.97 in the first year. The multipliers are generally smaller than their counterparts in the case of a transitory reduction in public consumption (see table 1), because of the large positive wealth effect on households and firms. The permanent reduction in public consumption makes more resources available for private spending on a permanent basis; this induces a larger crowding-in effect on private consumption and investment. As in the case of transitory shocks, the multipliers are smaller in the second year than in the first year, because nominal and real rigidities lead to a gradual adjustment of private demand for consumption and investment.

The long-run multipliers are negative across all models and, with the exception of the Greek model, remain smaller than one in absolute value. In the long run, a decrease in government consumption translates into lower lump-sum taxes for households. Since lump-sum taxes or transfers do not alter labor supply of Ricardian households or affect relative prices in the long run, lower aggregate demand due to lower public expenditure leads to a negative GDP effect.

3.2.2 Government Consumption, Labor Tax Rule

The last three columns of table 5 contain the short- and long-run multipliers of a permanent reduction in government consumption when the households' labor income tax rate endogenously adjusts according to the fiscal rule. Short-run multipliers are generally smaller than in the case of the lump-sum tax rule. Lower future labor income taxes induce households to gradually substitute labor for leisure. The increase in labor makes capital more productive, inducing firms to increase demand for investment. There is also a positive wealth effect, which induces households to increase their demand of consumption goods.[10]

[10] In the German and Spanish models, the second-year multipliers are larger when labor taxes adjust. In the case of Germany, the labor supply elasticity is calibrated to a very large number (see tables 10 and 11 in the appendix), and thus households widely shift labor effort to the long run, when labor taxes are lower. In the case of Spain, the labor market is modeled following the search-matching literature. Also in this case households postpone their labor effort.

In contrast to the previous results, long-run multipliers are now positive and, in some cases, larger than one. The largest multiplier is equal to 1.6 (Portuguese model), the smallest to 0.1 (German model). Typically, multipliers turn positive after three to six years. The permanent reduction in the labor tax rate leads to an outward shift of labor supply, providing incentives to increase employment. Higher employment in turn makes capital more productive. Since capital is rather elastic in the long run, there is a relatively large (supply-side) effect on production and economic activity. The long-run multipliers tend to be smaller and, thus, economic benefits of the reform are smaller for those countries that have a higher import penetration, i.e., higher import-to-GDP ratio.

3.2.3 Distortionary Taxes, Lump-Sum Tax Rule

Table 6 contains the multipliers for a permanent increase in distortionary tax revenues when the fiscal rule is specified in terms of lump-sum taxes.

We first consider the permanent increase in labor income taxes. Short-run multipliers are negative and generally lower than one in absolute value, ranging between 0.0 and 0.8 in the first year and between 0.1 and 1.0 in the second year. Short-run multipliers are in general larger than in the case of a transitory fiscal shock (see table 2). Long-run multipliers are negative as well and, in seven out of fifteen cases, larger than one in absolute value. As labor income taxation is distortionary, its increase induces households to reduce labor in favor of leisure. Moreover, the rather large negative wealth effect, due to the fact that the measure is permanent, induces households to reduce aggregate demand.

The estimates of short-run multipliers associated with capital income taxation vary quite a lot across models. In absolute values, the range goes from 0.0 for the German model in the first year to 2.5 in case of the Greek model in the second year. Long-run multipliers are unequivocally negative and much larger in absolute value than the multipliers associated with labor taxation. Long-run multipliers are larger than three in France, Greece, Slovenia, and Spain, and are equal to or larger than two in the EA, Belgium, Finland, Italy, and Portugal. In the long run, the physical capital fully adjusts to the new tax level, inducing a strong decline in labor and, thus, economic

Table 6. Short- and Long-Run Fiscal Multipliers: Permanent Increase in Tax Rate—Lump-Sum Taxes Adjust

Tax Rate:	Labor Income Tax			Capital Income Tax			Consumption Tax		
	Year 1	Year 2	Long Run	Year 1	Year 2	Long Run	Year 1	Year 2	Long Run
Belgium	−0.02	−0.18	−1.03	−0.29	−0.58	−2.11	−0.24	−0.49	−0.57
Czech Republic	−0.20	−0.32	−0.11	—	—	—	−0.03	−0.07	−0.03
Estonia	−0.56	−0.65	−0.60	−0.92	−0.76	−1.25	0.00	0.01	−0.16
Euro Area	−0.52	−0.66	−0.87	−1.69	−2.21	−2.56	−0.40	−0.45	−0.51
Finland*	−0.79	−0.64	−1.48	−0.12	−0.99	−1.97	−0.47	−0.10	−0.74
France	−0.28	−0.63	−1.24	−0.36	−0.61	−3.27	−0.18	−0.36	−0.61
Germany	−0.19	−0.15	−0.29	−0.02	−0.11	−0.79	−0.04	−0.06	−0.13
Greece*	−0.57	−0.82	−1.41	−1.18	−2.46	−3.77	−0.39	−0.58	−0.96
Italy	−0.19	−0.38	−0.91	−0.21	−0.57	−2.50	−0.08	−0.15	−0.36
Malta	−0.14	−0.33	−0.72	−0.06	−0.16	−1.67	−0.09	−0.17	−0.31
Portugal*	−0.47	−1.04	−1.27	−0.34	−0.45	−2.01	−0.29	−0.52	−0.66
Portugal* (ff)	−0.45	−0.98	−1.27	−0.54	−0.53	−2.00	−0.28	−0.50	−0.66
Slovenia	−0.26	−0.55	−1.42	−0.48	−0.77	−3.26	−0.13	−0.23	−0.54
Spain	−0.12	−0.11	−0.53	−0.26	−0.45	−3.25	−0.16	−0.18	0.00
Sweden	−0.35	−0.50	−0.68	−0.43	−0.80	−1.81	−0.15	−0.21	−0.28

*In these countries, monetary policy is exogenous. Lump-sum transfers adjust in the long run. Portugal (ff) indicates the presence of financial frictions following Bernanke, Gertler, and Gilchrist (1999).

activity. Both short-run and long-run multipliers tend to be larger (in absolute terms) for those countries in which the ratio of private investment to GDP is larger and where the initial capital tax revenues are lower. In the Portuguese model, financial frictions amplify the negative short-run impact on GDP, as a deterioration of entrepreneurs' net worth, due to higher capital income taxes, increases leverage and the cost of external funds.

Finally, the short-run multipliers associated with the consumption tax hike are between 0.0 and 0.5 in absolute value, while long-run multipliers are between 0 and 1. Long-run multipliers are larger than the short-run counterparts because habit formation in consumption leads to a gradual response of consumption to the increase in taxation. For the same reason, the multiplier is usually larger in the second year than in the first year.

3.2.4 Distortionary Taxes, Labor Tax Rule

Finally, we assess the value of multipliers when the fiscal room created by the permanent increase in capital income or consumption taxation is used to permanently reduce households' labor income taxes (instead of lump-sum taxes).

Table 7 reports the results. Short-run multipliers associated with a permanent increase in capital income taxes are somewhat smaller in absolute value than in the case of fiscal rules specified in terms of lump-sum taxes. In the case of Finland, the multiplier even becomes positive in the first year. The reason is that anticipation of the permanent reduction in labor taxation provides an incentive to gradually increase labor supply. This partially counterbalances the incentive to reduce investment associated with a higher taxation of capital.

Long-run multipliers are negative and, again, much larger than one in absolute value in most cases, given that investment is very elastic in the long run. Thus, the expansionary effects of lower labor taxation compensate only partially for the strong recessionary effect of permanently higher capital income taxes.

Short-run multipliers associated with a permanent increase in consumption taxes are lower when the fiscal rule is specified in terms of the labor income tax instead of the lump-sum tax. In some cases they become positive (Estonia, Italy, Slovenia) due to the quick

Table 7. Short- and Long-Run Fiscal Multipliers: Permanent Increase in Tax Rate—Households' Labor Income Tax Rate Adjusts

Tax Rate:	Capital Income Tax			Consumption Tax		
	Year 1	Year 2	Long Run	Year 1	Year 2	Long Run
Belgium	−0.29	−0.44	−1.04	−0.18	−0.35	0.53
Czech Republic	—	—	—	—	—	—
Estonia	−0.48	−0.64	−0.16	0.27	0.38	1.73
Euro Area	−1.23	−1.82	−1.17	−0.09	−0.17	0.33
Finland*	0.13	−0.91	−1.52	−0.37	−0.28	1.07
France	−0.22	−0.41	−2.43	−0.05	−0.11	1.31
Germany	−0.14	−0.15	−0.98	−0.17	−0.20	1.41
Greece*	−1.17	−2.51	−2.69	−0.35	−0.56	0.55
Italy	−0.08	−0.30	−1.92	0.10	0.20	0.66
Malta	−0.02	−0.08	−1.26	−0.02	0.01	0.47
Portugal*	−0.34	−0.17	−0.79	−0.30	−0.36	0.58
Portugal* (ff)	−0.57	−0.30	−0.82	−0.31	−0.37	0.53
Slovenia	−0.39	−0.52	−2.36	−0.02	0.07	0.59
Spain	−0.29	−0.48	−2.79	−0.18	−0.21	0.74

*In these countries, monetary policy is exogenous. Labor income tax rate adjusts in the long run. Portugal (ff) indicates the presence of financial frictions following Bernanke, Gertler, and Gilchrist (1999).

positive response of labor and the gradual decrease in consumption associated with habit. In the case of Estonia, the rather large multiplier is explained by strong competitiveness gains due to reduced labor costs, combined with the fact that trade effects in the Estonian model have a relatively large weight in the overall dynamics.

In contrast to the capital-income-tax-based scenario, the long-run consumption-tax-based multipliers are positive. Lower labor taxes favor the increase in employment, counterbalancing the negative effects due to an increase in consumption taxes. As such, economic activity increases in the long run. Higher short-run consumption tax multipliers tend to be associated with models that exhibit a larger share of liquidity-constrained consumers. The latter have at most a limited ability to smooth consumption over time, and thus are more affected by the negative income effect associated with the increase in consumption taxes. Thus, their reaction to the consumption tax hike is large.

4. Sensitivity Analysis

Results presented so far are based on the benchmark calibrations of the models (tables 10–12 in the appendix). In this section, we analyze the sensitivity of the results with respect to the following changes in the models' calibration:

- 30 percentage point increase in the share of liquidity-constrained households
- 10 percent reduction in the degree of price stickiness
- 10 percent reduction in the degree of wage stickiness
- 50 percent reduction in households' risk aversion
- 50 percent increase in investment adjustment costs

The sensitivity analysis is conducted with and without the ZLB. It focuses on two scenarios: (i) a permanent reduction in government consumption and (ii) a permanent increase in labor income taxes. All other fiscal items are held constant. After two years, lump-sum taxes are allowed to adjust according to the fiscal rules. Only a subset of models was used (EA, Finland, Italy, Malta, Portugal, and Slovenia).

Table 8 reports the average short-run and long-run multipliers across models. Short-run multipliers become larger in absolute terms when there are more liquidity-constrained households, because they are less able to smooth consumption than unconstrained (Ricardian) households. This effect becomes exacerbated when the ZLB is binding.

Absent the ZLB, the short-run government consumption multipliers are typically smaller when prices are less sticky. Firms adjust goods prices faster, leading to a quicker accommodating monetary policy response. Results are similar with regard to wage stickiness. Absent the ZLB, the short-run government consumption multipliers are typically smaller when wages are more flexible.

A lower degree of risk aversion translates into a higher interest rate elasticity of aggregate demand so that the accommodating

Table 8. Fiscal Multipliers: Sensitivity with Respect to Key Parameters

	Baseline	**More Liquidity-Constrained Households**	**Less Price Stickiness**	**Less Wage Stickiness**	**Lower Risk Aversion**	**Higher Investment Adjustment Costs**
A. Average Multipliers across Models in the First Two Years						
Government Consumption	−0.53	−0.70	−0.50	−0.51	−0.49	−0.54
Gov. Consumption + Two-Year ZLB	−0.59	−0.81	−0.59	−0.57	−0.56	−0.60
Labor Income Tax	−0.50	−0.56	−0.50	−0.53	−0.51	−0.51
Labor Income Tax + Two-Year ZLB	−0.55	−0.66	−0.58	−0.59	−0.58	−0.56
B. Average Long-Run Multipliers across Models						
Government Consumption	−0.56	−0.63	−0.56	−0.56	−0.47	−0.56
Labor Income Tax	−1.11	−1.07	−1.12	−1.12	−1.21	−1.12

Note: The sensitivity analysis is based on the results from the NAWM, three multi-country models, and two small open economy models with fixed interest rate.

monetary policy response has a stronger effect, thereby lowering the absolute value of short-run spending multipliers. It should also be noted that the fiscal multipliers are sensitive to the degree of financial frictions, as shown in tables 1–7 for the Portuguese model. The presence of financial frictions increases in particular the government consumption and the capital income tax multipliers. For other taxes, however, these frictions seem less relevant.

At the same time, the sensitivity of the multipliers with respect to investment adjustment costs differs across models, thus precluding the derivation of any straightforward conclusion.

5. Conclusions

We have provided estimates of the size and sign of fiscal multipliers—both in the short run and in the long run—for European countries based on simulations of structural models used at the NCBs and the ECB. The heterogeneity of the models with regard to the specific model features and the calibration provided a useful environment to study the driving factors of fiscal multipliers. Cross-country differences in fiscal multipliers can be traced back to country-specific features, such as the share of liquidity-constrained consumers, financial frictions, and different degrees of price and wage rigidities.

At the same time, while acknowledging the importance of these country differences, some of the findings are fairly robust across the variety of models.

Our first robust result is that under standard monetary policy the short-run multipliers are smaller than one in absolute terms in the vast majority of models, irrespective of the fiscal instrument, the considered country, or the nature of the fiscal shock. Temporary reductions in government consumption are typically associated with larger short-run GDP effects than temporary increases in the tax rate on households' labor income, capital income, and consumption.

The second robust finding is that a two-year-long ZLB episode has relatively small effects on the multipliers in the case of a temporary measure enacted by a single EA country. Cross-country spillovers are rather weak, and the response of EA inflation to the country-specific fiscal shocks is rather muted. In contrast, when the same fiscal measure is simultaneously implemented by many EA members, the ZLB has a relatively strong impact on short-run government consumption multipliers, which can become larger than one. The same holds true for non-EA countries that exhibit a country-specific monetary policy rule.

Third, if fiscal measures are implemented permanently, short-run government consumption and consumption tax multipliers are smaller in absolute value than in the case of a temporary implementation. Long-run multipliers are in general negative when the budgetary room materializing after the fiscal tightening is used to adjust lump-sum taxes. Instead, long-run multipliers are typically positive if the households' labor income tax rate is reduced in the medium to long term. Since households anticipate these long-run GDP effects at the outset of the simulations, short-run multipliers are more favorable when the budgetary room that materializes after the fiscal tightening is used to reduce distortionary taxes.

Finally, expenditure-based fiscal adjustments typically have larger negative short-run effects than tax-based adjustments. However, in the long run, tax-based fiscal adjustments lower the long-run output potential of the economy, while expenditure-based fiscal adjustments can result in positive long-run output effects.

The suite-of-models approach followed in this paper makes results robust to model uncertainty. However, there are some important dimensions that are missing from the models. For instance, limitations of the exercise can be associated with (i) the role of fiscal consolidation (in a ZLB situation) in reducing spillovers of sovereign risk to private-sector credit (à la Corsetti, Kuester, and Maier 2011); (ii) the role of accompanying government support for banks, à la Kollman et al. (2013), and more generally the way in which the banking sector is modeled (or not) in the simulated models; and (iii) the likely mismeasurement of fiscal policy changes, as addressed by Guajardo, Leigh, and Pescatori (2011), whereby changes in fiscal policy aiming at deficit reduction should be separated from those responding to prospective economic conditions. As model development within the central banks progresses and models become richer in the above-mentioned dimensions, the general approach provided in this paper could be repeated in order to gain more understanding of the relevance of these missing elements for evaluating fiscal multipliers.

Appendix

Table 9. Simulated Models

Country	Model	Reference
Belgium	BE-3C	Jeanfils, Wouters, and de Walque (2012)
Czech Republic	g3	Ambrisko et al. (2012)
Estonia	EP DSGE	Gelain and Kulikov (2009)
Euro Area	NAWM	Coenen, McAdam, and Straub (2008)
Finland	Aino	Kilponen, Kinnunen, and Ripatti (2006)
France	EAGLE	Jacquinot and Lemoine (2013)
Germany	GEAR	Gadatsch, Hauzenberger, and Stähler (2015)
Greece	BoGGEM	Papageorgiou (2014)
Italy	IDEA-BI-EAGLE	Forni, Gerali, and Pisani (2010)
Malta	EAGLE	Micallef (2013)
Netherlands	DELFI	De Nederlandsche Bank (2011)
Portugal	PESSOA	Almeida et al. (2013)
Slovenia	EAGLE	Gomes, Jacquinot, and Pisani (2010)
Spain	FiMod	Stähler and Thomas (2012)
Sweden	Ramses II	Adolfson et al. (2013)

Table 10. Elements of Calibration

	Belg.	Cz. Rep.	Estonia	EA	Finland
Name of the Model	BE-3C	g3	EP DSGE	NAWM	Aino
Model Calibrated/Estimated	Est.	Est.	Est.	Cal.	Cal.
Open Economy Features					
Number of Countries	3	2	2	2	1
Number of Countries in Monetary Union	2	0	1	1	1
RoW/RoEA Exogenous	No	Yes	Yes	No	Yes
Tradable/Non-tradable Goods	Both	Tr. Only	Both	Tr.	Tr. Only
Steady-State Values					
Private Consumption-to-GDP Ratio	0.60	0.59	0.60	0.60	0.62
Private Investment-to-GDP Ratio	0.15	0.12	0.25	0.22	0.19
Imports-to-GDP Ratio	0.74	0.29	0.90	0.18	0.38
Public Consumption-to-GDP Ratio	0.14	0.22	0.25	0.16	0.17
Public Investment-to-GDP Ratio	0.00	0.06	0.00	0.03	0.02
Public-Sector Interest Payment-to-GDP Ratio	0.03	0.01	0.00	0.05	0.03
Labor Income Tax Revenues-to-GDP Ratio	0.31	0.11	0.11	0.07	0.14
Capital Income Tax Revenues-to-GDP Ratio	0.04	0.01	0.03	0.03	0.03
Consumption Tax Revenues-to-GDP Ratio	0.10	0.12	0.11	0.11	0.14
Value of the Public Debt-to-Annualized GDP	0.60	0.45	—	0.90	0.60
Value of the Net Foreign Asset-to-Yearly GDP	0.00	0.00	—	0.00	0.00
Annualized Nominal Interest Rate	0.05	0.03	0.05	0.05	0.05
Annualized Inflation	0.02	0.02	0.00	0.02	0.02

(*continued*)

Table 10. (Continued)

	Belg.	Cz. Rep.	Estonia	EA	Finland
Calibration					
Share of Liquidity-Constrained Households	0.00	0.40	0.00	0.25	0.00
Coefficient of Risk Aversion	2.12	N/A	1.61	1.00	3.00
Frisch Elasticity of Labor Supply	2.08	2.84	1.78	2.00	> 2
Habit	0.65	0.75	0.65	0.60	0.00
Adjustment Costs on Investment	13.66	0.20	6.42	3.00	1.40
Price Stickiness	0.71	0.50	0.69	0.90	0.85
Price Indexation	0.59		0.27	0.70	1.00
Wage Stickiness	0.78	0.80	0.55	0.75	0.85
Wage Indexation	0.90		0.37	0.75	1.00
Own Taylor Rule (Cal. as ECBWP1195)	No	Yes	No	Yes	No
Fiscal Rule React on Deviation of Pub. Debt	Yes	Yes	Yes	No	Yes
Fiscal Rule React on Deviation of Pub. Deficit	No	No	No	Yes	No
Fiscal Rule React on Deviation of Gov. Cons.	No	Yes	No	No	No

Table 11. Elements of Calibration

	France	**Germany**	**Greece**	**Italy**	**Malta**
Name of the Model	EAGLE	GEAR	BoGGEM	EAGLE	EAGLE
Model Calibrated/Estimated	Cal.	Cal.	Cal.	Cal.	Cal.
Open Economy Features					
Number of Countries	5	3	1	3	4
Number of Countries in Monetary Union	3	2	1	2	2
RoW/RoEA Exogenous	No	No	Yes	No	No
Tradable/Non-tradable Goods	Both		Tr. Only	Both	Both
Steady-State Values					
Private Consumption-to-GDP Ratio	0.57	0.62	0.63	0.59	0.63
Private Investment-to-GDP Ratio	0.19	0.23	0.26	0.18	0.20
Imports-to-GDP Ratio	0.27	0.22	0.17	0.25	0.50
Public Consumption-to-GDP Ratio	0.23	0.12	0.18	0.20	0.20
Public Investment-to-GDP Ratio	0.00	0.03	0.03	0.02	0.00
Public-Sector Interest Payment-to-GDP Ratio	0.03	0.01	0.05	0.04	0.03
Labor Income Tax Revenues-to-GDP Ratio	0.24	0.35	0.23	0.21	0.15
Capital Income Tax Revenues-to-GDP Ratio	0.05	0.02	0.09	0.13	0.03
Consumption Tax Revenues-to-GDP Ratio	0.10	0.09	0.11	0.10	0.11
Value of the Public Debt-to-Annualized GDP	0.62	0.60	1.20	1.19	0.60
Value of the Net Foreign Asset-to-Yearly GDP	0.04	0.00	0.00	0.00	0.04
Annualized Nominal Interest Rate	0.05	0.02	0.04	0.03	0.05
Annualized Inflation	0.02	1.80	0.00	0.00	0.02

(continued)

Table 11. (Continued)

	France	Germany	Greece	Italy	Malta
Calibration					
Share of Liquidity-Constrained Households	0.25	0.40	0.40	0.00	0.25
Coefficient of Risk Aversion	1.00	1.40	1.00	1.00	1.00
Frisch Elasticity of Labor Supply	2.00	11.00	1.00	0.50	2.00
Habit	0.90	0.60	0.65	0.60	0.70
Adjustment Costs on Investment	6.00	6.90	10.00	6.00	4.00
Price Stickiness	0.75	0.90	0.71	0.75–0.8	0.75
Price Indexation	0.75	0.45	0.27	0.50	0.50
Wage Stickiness	0.92	200.00	*	0.75–0.8	0.75
Wage Indexation	0.50	0.75		0.50	0.75
Own Taylor Rule (Cal. as ECBWP1195)	No	No	No	No	No
Fiscal Rule React on Deviation of Pub. Debt	Yes	Yes	Yes	Yes	Yes
Fiscal Rule React on Deviation of Pub. Deficit	No	No	No	Yes	No
Fiscal Rule React on Deviation of Gov. Cons.	No	No	No	No	No

*The model features real wage rigidity following Blanchard and Galí (2007).

Table 12. Elements of Calibration

	Nether.*	**Portugal**	**Slovenia**	**Spain**	**Sweden**
Name of the Model	DELFI	PESSOA	EAGLE	FiMod	Ramses II
Model Calibrated/Estimated	Est.	Cal.	Cal.	Cal.	Est.
Open Economy Features					
Number of Countries	1	1	4	2	2
Number of Countries in Monetary Union	1	1	2	2	0
RoW/RoEA Exogenous	Yes	Yes	No	No	Yes
Tradable/Non-tradable Goods		Tr. Only	Both	Tr. Only	Tr. Only
Steady-State Values					
Private Consumption-to-GDP Ratio		0.60	0.55	0.57	0.63
Private Investment-to-GDP Ratio		0.21	0.27	0.21	0.17
Imports-to-GDP Ratio		0.33	0.61	0.27	0.44
Public Consumption-to-GDP Ratio		0.23	0.19	0.18	0.30
Public Investment-to-GDP Ratio				0.04	0.00
Public-Sector Interest Payment-to-GDP Ratio		0.02	0.03	0.02	0.00
Labor Income Tax Revenues-to-GDP Ratio		0.11	0.23	0.07	0.29
Capital Income Tax Revenues-to-GDP Ratio		0.03	0.01	0.02	
Consumption Tax Revenues-to-GDP Ratio		0.43	0.09	0.04	0.16
Value of the Public Debt-to-Annualized GDP		0.53	0.60	0.48	0.00
Value of the Net Foreign Asset-to-Yearly GDP		−0.23	−0.09	0.00	0.00
Annualized Nominal Interest Rate		0.05	0.05	0.04	0.04
Annualized Inflation		0.02	0.02	0.00	0.02

(*continued*)

Table 12. (Continued)

	Nether.*	Portugal	Slovenia	Spain	Sweden
Calibration					
Share of Liquidity-Constrained Households		0.40	0.25	0.40	0.00
Coefficient of Risk Aversion		5.00	1.00	2.00	1.00
Frisch Elasticity of Labor Supply		0.85	2.00	Match. funct.	2.98
Habit		0.90	0.80	0.85	0.66
Adjustment Costs on Investment		10.00	5.00	2.50	2.35
Price Stickiness		100.00	0.75	0.75	0.88
Price Indexation		0.00	0.50	0.00	0.16
Wage Stickiness		100.00	0.81	0.75/0.7	0.75
Wage Indexation		0.00	0.75	0.50	0.34
Own Taylor Rule (Cal. as ECBWP1195)	No	No	No	No	Yes
Fiscal Rule React on Deviation of Pub. Debt		Yes	Yes	Yes	No
Fiscal Rule React on Deviation of Pub. Deficit		No	No	No	No
Fiscal Rule React on Deviation of Gov. Cons.		No	No	No	Yes

*Netherland's model is not a DSGE model, hence some of the data is not available.

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