# Risk Shifting with Fuzzy Capital Constraints*

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We construct a model where risk shifting can be moderated by capital requirements. Imperfect information about the level of capital per unit of risk, however, introduces uncertainty about the risk exposure of intermediaries. Over-estimation of the capital held by financial intermediaries, or the extent of regulatory arbitrage, may induce households to wrongly infer from higher asset prices that the fundamentals of risky assets have improved. This mechanism can notably explain the low risk premia paid by U.S. financial intermediaries between 2000 and 2007 in spite of their increased exposure to risk through higher leverage. Moreover, the lower the level of the risk-free interest rate, the more risk is under-estimated.

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## 1. Introduction

The goal of this paper is to explain why, in the run-up to the sub-prime crisis, U.S. financial intermediaries were able to pay

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non-increasing risk premia while their leverage increased.[1] To do so, we introduce the capital of financial intermediaries into the risk-shifting model developed by Allen and Gale (2000). In their model, households can invest in risky assets only indirectly, by lending to financial intermediaries. Households require a risk premium on this loan because they anticipate that financial intermediaries will default in the bad state of the world. However, intermediaries that have limited liability will take too much risk. A bubble results, as the price of the risky asset will be higher than in the case in which households, which do not have limited liability, can directly invest in the risky asset.

In our model, the amount of risk taken by households through lending to intermediaries—i.e., the amount and the interest rate at which they lend—will crucially depend on the level of capital held by intermediaries. Households, which cannot observe the degree of risk of the risky asset ex ante, try to infer it from the price of the risky asset and from their assumption of the level of capital held by financial intermediaries.

Our contribution is twofold. First, we show that the patterns of risk premia and leverage ratios observed in the United States between 2000 and 2007 can be understood only if investors under-estimated the intermediaries' incentives to take risks. This is likely when capital constraints are fuzzy, meaning that households can form wrong beliefs on the level of banks' capital. We show how investors may wrongly infer from rising asset prices that the aggregate risk is decreasing, and thus charge a low risk premium on their loans to intermediaries. This will be the case if investors under-estimate the degree of regulatory arbitrage, which allows intermediaries to minimize the capital they pledge on risky assets. As argued by Acharya and Schnabl (2009) and Rochet (2008), one of the reasons why the risk-weighted regulatory capital ratio may be opaque is that intermediaries use off-balance-sheet conduits to "play" the level of capital. Uncertainty about the level of capital of banks then implies uncertainty about the risk characteristics of their assets. We show that investors under-estimate the risk of some assets (and thus charge low risk premia) if they over-estimate the level of capital of

[1] See Acharya and Schnabl (2009), Blanchard (2008), Brender and Pisani (2010), Brunnermeier (2009), and Greenlaw et al. (2008).

intermediaries. This model can therefore show why risk premia did not increase before the crisis, which is one of the most puzzling stylized facts of this period. In addition, the mechanism we formalize might be relevant for other periods of major financial innovation or deregulation, when risk-weighted capital positions are likely to be opaque.

Second, the model points to a risk-taking channel for the impact of low real interest rates: the misperception of risk is greater at lower levels of interest rates. This is because the impact of the interest rate on asset prices is larger when the leverage of financial intermediaries is high. This implies that changes in the level of capital have a larger impact on the price of risky assets and, in turn, on the perception of risk by investors at lower levels of interest rates.[2] We discuss in this paper how this channel may translate into a risk-taking channel of monetary policy.

## *1.1 Related Literature*

This article focuses on the link between the leverage of financial intermediaries, asset prices, and interest rates. It draws on the results of Adrian and Shin (2010) and Geanakoplos (2009), who have highlighted the impact of financial intermediaries' leverage on asset prices. It also provides a theoretical underpinning for the empirical results of Adrian and Shin (2010), Altunbas, Gambacorta, and Marques-Ibanez (2010), Ciccarelli, Maddaloni, and Peydró (2010), Ioanidou, Ongena, and Peydró (2008), Maddaloni, Peydró, and Scopel (2008), and Shin (2010), who showed that accommodative monetary policies leading to low real interest rates are associated with increased risk taking by banks. We hence provide a theory for what Adrian and Shin (2010) and Borio and Zhu (2009) call the "risk-taking" channel of monetary policy.

Among the literature on risk shifting, our paper relates first to the contribution of Allen and Gale (2000), where they showed how limited liability on the part of debt issuers leads to over-investment

[2]In fact, the model we use is real, and the interest rate is also real, rather than nominal. We assume that monetary policy can affect, possibly only temporarily, the level of the real interest rate on the storage asset. Section 4 discusses the impact of monetary policy and the savings glut on the level of real and nominal interest rates in the run-up to the crisis.

in risky assets. Barlevy (2008) proved that risk shifting also implies bubbles within more general frameworks of financial intermediation (i.e., when the formation of financial contracts is endogenous). He also generalized risk shifting to a continuous-time dynamic framework. Challe and Ragot (2011) expand the risk-shifting model to the case in which the supply of loans is endogenous.[3]

Finally, two recent papers developed models on similar issues. Dell'Ariccia, Laeven, and Marquez (2010) developed another model of the risk-taking channel of monetary policy framed in a moral hazard setup for banks' capital. Challe, Mojon, and Ragot (2012) show that the proportion of banks that prefer a risky investment portfolio over a diversified, less risky, one decreases with the level of interest rates.

The paper proceeds as follows. Section 2 documents the stylized facts about the sub-prime crisis. Section 3 presents the model. Section 4 solves the model and shows in turn the implications of different assumptions on households' beliefs on risk-weighted capital. Section 5 discusses robustness of the main conclusions of the paper for alternative specifications of the model. Section 6 reports alternative explanations of low risk premia in the run-up to the crisis. Section 7 concludes.

## 2. Stylized Facts on the Period Preceding the Sub-Prime Crisis

We underline three major stylized facts from the period that preceded the sub-prime crisis: the U.S. banking sector increased its exposure to credit risk and liquidity risk; the perceived riskiness of U.S. financial intermediaries did not increase; and the effective level of banks' capital was difficult to assess during the period.

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[3]It is also important to underline the difference between the risk-shifting literature and the literature on endogenous credit constraints. The latter analyzes how asymmetric information introduces external finance premia and collateral constraints. This literature effectively accounts for the financial accelerator, either in the boom phase, when the rising price of collateral relaxes credit constraints (Kiyotaki and Moore 1997), or in the bust phase, when the collapse in asset prices tightens the credit constraints considerably (Holmstrom and Tirole 1997). However, these models face some difficulties in explaining why there are equilibria with too much credit and over-investment in the risky asset.

### *2.1 Risk Taking in the U.S. Banking Sector*

There is now a consensus view that U.S. financial intermediaries increased their risk exposure during the decade leading up to the crisis. This took the form of an expansion of balance sheets and increased leverage on the part of U.S. investment banks. For instance, the Security and Exchange Commission (SEC) reports that, between 2003 and 2007, the mean leverage ratio (defined as the ratio between overall debt and bank's equity) of the five major investment banks[4] jumped from 22 to 30. Among these five investment banks, only one survived the crisis as a stand-alone institution.

This expansion in the size of banks' balance sheets was accompanied by an increase in "off-balance-sheet leverage," as documented in Acharya and Schnabl (2009). This allowed financial intermediaries to generate higher profits without additional capital, in spite of increased potential future losses: in ex ante terms, the unit of risk borne by each dollar of the U.S. banking system's equity increased markedly.

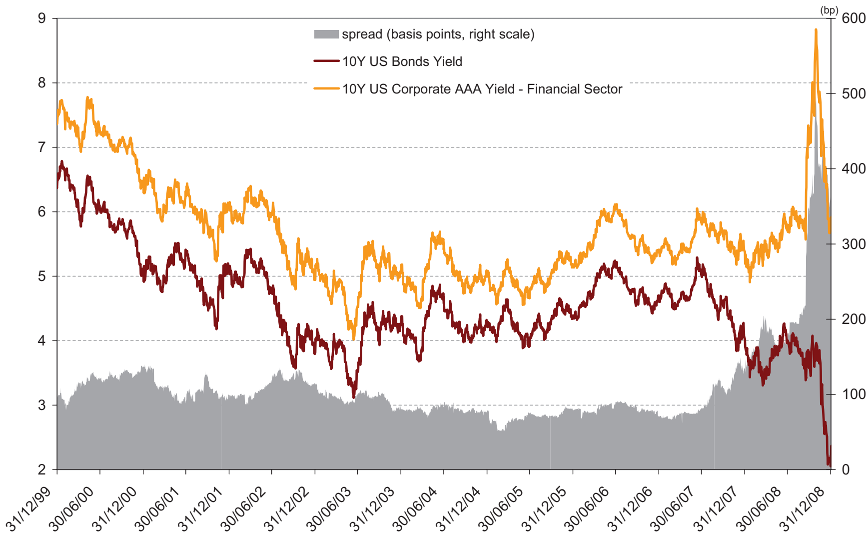
### *2.2 The Perceived Riskiness of Financial Intermediaries Was Stable*

U.S. banks, however, benefited from very low risk premia paid on their debt. The spread between bonds of U.S. financial companies and government bonds (figure 1) shows that the premia paid on banks' default risk did not increase from 2000 to mid-2007. The price of credit risk for banks even declined somewhat between 2002 and 2007.

The change in banks' expected default frequencies (EDFs) is another indicator of the ease with which banks accessed market funding between 2002 and 2007. Banks' EDFs decreased worldwide between 2002 and 2007 (see figure 2), suggesting that market investors either assigned lower probabilities to defaults in the banking sector or required lower risk premia to invest in banks' debt instruments. The same observation that credit risk for banks

[4]Lehman Brothers, Bear Stern, Merrill Lynch, Goldman Sachs, and Morgan Stanley.

**Figure 1. Spread between Ten-Year U.S. T-Bonds and Ten-Year Bonds of U.S. AAA Financial Companies**



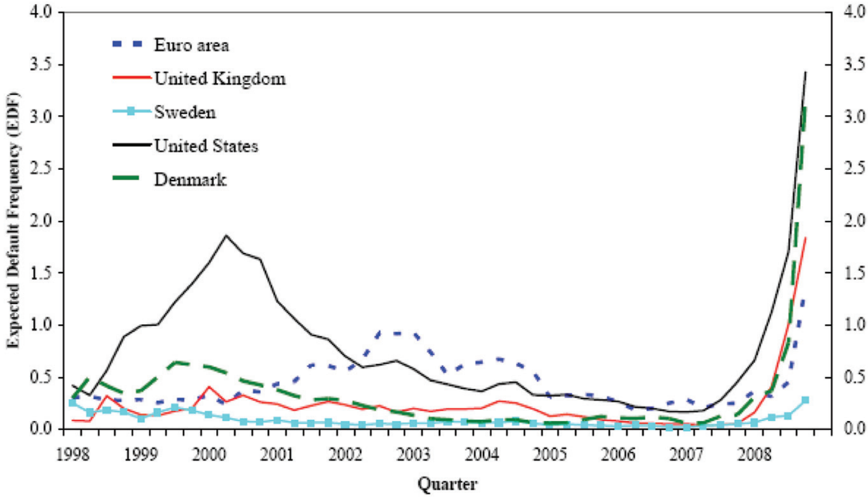

**Source:** Bloomberg.

was perceived to be negligible can be made by comparing CDS contracts on U.S. banks intermediaries and on issuers in others economic sectors.[5]

To sum up, we observe that, during the years leading up to the crisis, the U.S. banking sector experienced very favorable funding conditions and faced very low risk premia on its debt, while at the same time increasing its leverage (for investment banks) or, more generally, its exposure to credit risk and liquidity risk via off-balance-sheet vehicles. We also note that the increase in banks' assets was concentrated among assets that require very low capital funding. These products, considered to be quite safe by regulatory standards, were however at the root of significant losses for banks after the beginning of the financial crisis.

[5]This is also particularly striking when comparing banks and non-financial corporate credit spreads in the euro area between 2000 and 2007 (Gilchrist and Mojon 2014).

**Figure 2. Expected Default Frequency of Banks, Over a One-Year Horizon (averages by country and groups of countries)**




**Source:** Moody's KMV, from Altunbas, Gambacorta, and Marques-Ibanez (2010).

### *2.3 Changes in Capital Requirements*

Several factors explain why capital requirements and capital norms were weak during this period. Most of all, one of the very purposes of Basel II was to authorize banks to reduce their capital base through the use of internal risk models. Blundell-Wignall and Atkinson (2008) and Rochet (2008) highlight the difficulty for outsiders to obtain extensive information on the level of risk borne by financial intermediaries. Such complexity must have led financial intermediaries to minimize risk per unit of risk in line with the vested interest of the industry. Finally, the accounting rules concerning the consolidation of off-balance-sheet entities were singled out by the Financial Stability Forum (2008) for creating "a belief that risk did not lie with arrangers and led market participants to underestimate firms' risk exposures."[6]

[6]This issue is actually on the agenda of the G20, and similar concerns about off-balance-sheet vehicles have been brought up by academics (see Acharya and Schnabl 2009), official regulators, and central bankers (see, for instance, speeches of Christian Noyer and Ben Bernanke in 2008).

## 3. The Model

There are two dates $t = 1, 2$. The economy comprises four types of agents: households, financial intermediaries, entrepreneurs, and initial sellers. We first describe the assets available to these agents and then their investment decision.

### *3.1 Assets*

Agents make their investment choices at date 1 and get asset returns at date 2. Four financial assets are available in the economy:

(i) A storage asset $F$, which has a constant return $\tau$. This asset is available in infinite supply.

(ii) A safe asset whose supply $X_S$ is variable, and whose return is $r_S$. The safe asset will be issued by entrepreneurs who have access to an iso-elastic production function $f(.) = X^{1-\eta}/(1-\eta), \eta < 1$.

(iii) A risky asset in fixed supply $X_R$, whose return is $R^*$. $R^*$ equals $R$ with probability $\pi$ and 0 with probability $(1-\pi)$, which is the level of "economic risk" in the model. The price of the risky asset in period 1 is denoted as $P$. The assumption of fixed supply simplifies the model and is considered to be the benchmark case. It is relaxed in section 5.1, as a robustness check.

Moreover, we make the following technical assumption, which relates the concavity of the production function $\eta$ and the extent of economic risk $1-\pi$:

$$\eta > \frac{1-\pi}{\pi}. \tag{1}$$

This assumption, which ensures the uniqueness of the equilibrium, is satisfied for reasonable values of the parameters.[7]

[7]In equilibrium we will obtain $f'(X_S) = r_S$ or $X_S = r_S^{-\frac{1}{\eta}}$. If $\eta$ is too small, the volume (and the share) of safe assets held by financial intermediaries is quite sensitive to interest rates. Both the riskiness and the ex ante return of the entire

(iv) Debt $B$ issued by financial intermediaries and acquired by households.

Financial assets in this economy can be interpreted in the following way:

- The storage asset may, for example, include deposit facilities at the central bank or cash. Indeed, it allows agents to invest without limit at a low and constant rate. In what follows, we will use the return on the storage asset as a proxy for the interest rate set by the monetary policy authority.
- The safe asset represents investment-grade bonds. It can be interpreted as a loan to the "real" sector in order to finance investment or production.
- Finally, the risky asset encompasses all types of investments whose expected returns are higher than the return on the safe asset. It can be either real estate mortgages, junk bonds, or stocks.

## *3.2 Agents*

### *3.2.1 Financial Intermediaries*

There is a unit mass of financial intermediaries (which we also designate as "banks") that are risk neutral and receive an endowment $W^f$ at the beginning of date 1. Agents maximize their consumption over the two periods with a discount factor $\beta$, such that

$$\beta < 1/\tau. \tag{2}$$

This assumption implies that the intermediaries are comparatively impatient that they want to borrow in period 1. In addition, they enjoy a private benefit $U$ from being intermediaries. This benefit guarantees that these agents agree to operate as intermediaries rather than consuming all their endowment in period 1. They thus seek to maximize $c_1^f + \beta E\left[c_2^f\right] + U$, where $c_1^f$ and $c_2^f$ are the period 1

portfolio could decrease when $r_S$ decreases. It implies that the ex ante return on the whole portfolio could increase when $r_S$ decreases, which would generate multiple equilibria. (1) is a sufficient condition relating the curvature $\eta$ and the risk $\pi$ to ensure that it is not the case.

and period 2 consumption levels. Financial markets open in period 1 after goods markets. This implies that financial intermediaries can bring to financial markets as equity only $K$ of their wealth that is not consumed in period 1:

$$c_1^f \leq W^f - K. \tag{3}$$

Financial intermediaries can invest in all existing assets. They do not invest in the storage asset, because they have access to the safe asset, which yields a higher return. Thus, their balance sheet is composed of a risky asset, $PX_R$, and a safe asset, $X_S$, on the asset side, whereas their liabilities are either equity, $K$, or debt, $B$. The amount $K$ stands for the fraction of resources invested by the intermediaries themselves in their business. The resource constraint of financial intermediaries is

$$PX_R + X_S = B + K. \tag{4}$$

We assume that financial intermediaries are subject to a norm of "risk coverage" or "risk-weighted capital requirements" by either financial regulation or market discipline.[8] They have to invest from their endowment at least $\Delta$ per unit of risky asset:

$$K \geq \Delta PX_R. \tag{5}$$

Following Allen and Gale (2000), we assume that financial intermediaries raise some funds using debt contracts, and that households, who lend to them, are not able to fully observe the investment decisions of financial intermediaries. Hence they will demand the same interest rate $r$ irrespective of the size of the loan they grant to the financial intermediary.

Financial intermediaries can default on their debt. Default occurs when the intermediary's wealth is negative in period 2. Their second-period consumption $c_2^f$ is thus

$$c_2^f \leq \max\{R^* X_R + r_S X_S - rB, 0\}. \tag{6}$$

[8] Indeed, the level of capital requirements need not exclusively be the one set by regulators. It can also be the market norm on the acceptable level of capital for a given level of risk taking.

They therefore choose their debt level $B$, the equity $K$, the composition of their portfolio $(X_S, X_R)$, and their consumption profile $c_1^f$ and $c_2^f$ to solve the following program:

$$\max_{K,B,X_R,X_S,c_1^f,c_2^f} c_1^f + \beta E\left[c_2^f\right] + U$$

subject to the conditions (4)–(6).

#### *3.2.2 Households*

There is a unit mass of households who are risk neutral and who are uniformly distributed on the $[0, 1]$ interval. Each household $j$ receives an endowment $W_j^H$ at the beginning of date 1. To simplify the algebra, and without any loss of generality, households maximize their date 2 consumption only.

As in Allen and Gale (2000), we introduce the following form of market segmentation. Households cannot invest directly in the risky asset or in the safe asset, and they can only either invest in the storage asset or lend to financial intermediaries an amount $B$ at the interest rate $r$. This assumption captures the advanced skills and accumulated rents (asset-management abilities, private information, and so on) needed to trade sophisticated financial products.

Households do not know the level of risk in the economy, summarized by $\pi$. Moreover, they cannot observe the composition of the liabilities of financial intermediaries, $K$ and $B$. This assumption reflects the inability of each household to observe the extent of the total indebtedness of financial intermediaries. As argued above, the rationale for this assumption is the complexity of their liability structure due to off-balance-sheet liabilities. Technically, households know their portfolio, but they do not observe the portfolio of other households and we do not restrict our analysis to symmetric equilibria.

The key assumption of our model is that households do not know the true value of capital requirement $\Delta$, but their belief $\Delta^H$ may differ from $\Delta$. This assumption is meant to reflect that households, investors, or even rating agencies have a hard time assessing the degree of risk effectively borne by financial intermediaries. This opaqueness can be due to regulatory arbitrage or financial innovations. In brief, the information set of households comprises prices and both risky and safe assets held by financial intermediaries ($X_R$ and $X_S$). However, households cannot observe the composition of banks' liabilities ($K$ and $B$).

Each household $j$ chooses the composition of its financial portfolio in order to maximize its consumption:

$$\max_{F_j, B_j^H, c_j^H} \beta E\left[c_j^H\right] \tag{7}$$

$$F_j + B_j^H \leq W_j^H \text{ (at date 1)} \tag{8}$$

$$c_j^H \leq \rho B_j^H + \tau F_j \text{ (at date 2)}, \tag{9}$$

where $E[\cdot]$ is the expectation operator and $\beta$ is the discount factor. The expectations are formed on $R^*$ for a given value $\Delta^H$. In the budget constraints, $F_j$ is the amount invested in the storage asset, and $B_j^H$ is the amount lent to intermediaries. We further denote $B^H = \int_j B_j^H dj$, $F = \int_j F_j dj$, $W^H = \int_j W_j^H dj$, and $c = \int_j c_j^H dj$ the aggregate amount of loans to intermediaries, the aggregate investment in the storage asset, the aggregate households endowment, and the aggregate households consumption, respectively. The stochastic interest rate $\rho$ that all households receive ex post on their loans to financial intermediaries is uncertain and depends on the probability of default of intermediaries. Intermediaries default in the bad state of the world, in which the return on the risky asset is 0, because their debt burden will be greater than their portfolio invested in the safe asset. In the event of default, households receive the residual value of the portfolio of financial intermediaries $r_S X_S$, so that $\rho$ in case of default is $r_S X_S / B$. When intermediaries do not default, households get the return $r$.

$$\rho = \begin{cases} r \text{ if no default, with probability } \pi \\ \frac{r_S X_S}{B} \text{ if default, with probability } (1-\pi) \end{cases}$$

#### 3.2.3 Entrepreneurs and Initial Sellers

There is a unit mass of entrepreneurs who maximize period 2 consumption, denoted as $c^e$. They have no wealth and they need to borrow in period 1 to produce in period 2. Their production function is $f(X) = X^{1-\eta}/(1-\eta)$. They borrow an amount denoted as

$X_S^e$ from financial intermediaries at a rate $r_S$, to maximize their period 2 consumption $c^e = f\left(X_S^e\right) - r_S X_S^e$. This maximization over $X_S^e$ yields the simple relationship

$$r_s = f'\left(X_S^e\right). \tag{10}$$

Initial sellers are agents who sell risky assets to intermediaries at period 1, consume, and leave the economy. These agents are only introduced as a simple way of creating a supply of the risky asset, and thus an observable asset price, at the beginning of period 1. Initial sellers have no choices to make and simply consume in period 1 the amount obtained from the sale of the risky asset:

$$c^i = PX_R.$$

In the benchmark model, the quantity of the risky asset is fixed and equal to 1. We relax this assumption below.

#### *3.2.4 Equilibrium*

For given parameters, and a given value of households' belief $\Delta^H$, an equilibrium of this economy is a set of prices $r, r_S, P$ and quantities $F_j, B_j^H, c_j^H, K, B, c_1^f, c_2^f, X_S, X_R, X_S^e$, and a risk assessment by households $\pi^H$, such that (i) quantities solve the program of all agents at given prices and given households' belief $\Delta^H$, and (ii) markets clear $X_R = 1$, $X_S = X_S^e$ and $B = B^H$.

### *3.3 Pareto-Efficient Equilibria*

We first derive the set of Pareto-efficient allocations. In order to do this, we maximize a general welfare function, which weights the utility of the four types of agents. This can be written, with obvious notations for the Pareto weights, as follows:

$$W = \omega^H \beta E\left[c^H\right] + \omega^f \left(c_1^f + \beta E\left[c_2^f\right]\right) + \omega^i c^i + \omega^e \beta c^e \tag{11}$$
$$\text{with } \omega^H, \omega^f, \omega^i, \omega^e > 0 \text{ and } \omega^H + \omega^f + \omega^i + \omega^e = 1.$$

Expectations in the objective function are only taken with respect to the economic risk, $R^* = R$ with probability $\pi$ and $R^* = 0$ with probability $1 - \pi$. The feasibility constraints are

$$W^f + W^H = c_1^f + F + X_S + c^i \tag{12}$$

$$\tau F + f(X_S) + RX_R = c^H + c_2^f + c^e. \tag{13}$$

As $\tau < 1/\beta$, forming the Lagrangian for the maximization of (11) subject to the constraints (12) and (13), one can check that the solution has the following properties:

$$F = 0 \text{ and } f'(X_S) = \frac{1}{\beta}.$$

The allocation of the central planner can be achieved in the decentralized economy if we remove any market segmentation and allow for lump-sum transfers. In this case, households can directly lend to entrepreneurs and buy the safe asset. In this equilibrium, the interest rate on the safe asset is $1/\beta$ and the price of the risky asset is equal to its fundamental value $P^* = \beta\pi R$.

## 4. Model Solution

### *4.1 Asset Prices and Households' Beliefs*

In this section we derive the price of the risky asset by solving the program of financial intermediaries, and provide the intuitions for the main results of the paper.

We solve the program of financial intermediaries under two assumptions. The first is that the capital requirement constraint is always binding, hence $K = \Delta P X_R$. This case is, of course, the one of interest for this model. The capital norm is binding if financial intermediaries are sufficiently impatient, i.e.,

$$\pi r < 1/\beta. \tag{14}$$

This inequality stipulates that the expected cost of the debt $\pi r$ (because the debt is repaid only outside the bad state, which occurs with probability $\pi$) must not be too high. If the expected cost of the debt is too high, intermediaries would want to invest all of their wealth to decrease their expected debt burden, and the capital norm constraint would therefore not bind. As $r$ is determined in equilibrium, we show below that the condition (14) is fulfilled for a wide range of parameter values.

The solution of the program of intermediaries yields the equilibrium price of the risky asset:

$$P = \frac{\beta \pi R}{\Delta + \beta \pi r \left(1 - \Delta\right)}. \tag{15}$$

This asset price equilibrium is the main equation of the model. First note that when there is no capital requirement ($\Delta = 0$), the price is simply $P = R/r$, which is the case studied by Allen and Gale (2000).[9] As intermediaries default in the bad state, their demand for the risky asset is always higher than under the first-best equilibrium. Indeed, since $\pi r < 1/\beta$, one finds $P > P^*$. Asset prices are thus too high. Second, when capital requirements increase, the price of the risky asset decreases. Taking $r$ as given, increasing $\Delta$ implies a cost in the form of additional foregone consumption in period 1, an effect that dominates the reduction in size of the loan that needs to be repaid with probability $\pi$.

Thus, in partial equilibrium, the price of the risky asset can increase for two reasons: either because $\pi$ increases, which means that the expected return of the risky asset is higher, i.e., "fundamentals are better," or because $\Delta$ decreases (the amount of ex ante risk shifting increases).

Maximization with respect to the demand for the safe asset $X_S$ implies that the funding cost of financial intermediaries is equal to the return on the safe asset, as in Allen and Gale (2000): $r = r_s$. This is necessary and sufficient in order to avoid infinite riskless profit opportunities on the part of financial intermediaries, while guaranteeing a positive demand in equilibrium.

The demand for the safe asset implies

$$f'\left(X_S\right) = r_S = r \Longrightarrow X_S = \left[f'(r)\right]^{-1}. \tag{16}$$

[9] In their model, Allen and Gale show how incomplete debt contracts limit debtors' losses in the bad state of the world (losses fall on lenders). In other words, debt contracts act as call options for borrowers. This implies that borrowers only focus on the good state of the world when deciding the composition of their portfolio: the share of the portfolio at risk is higher and the price of risky assets is inflated above its level in a world without segmentation or incomplete contracts.

The previous equalities are valid irrespective of the beliefs on the part of households about the economic environment. These beliefs will, however, determine the interest rate charged by households.

The basic assumption of the model is that households infer the probability of default from their observation of the risky asset price and their belief on risk-weighted capital $\Delta^H$. The price of the asset is given by (15) for the true value of $\Delta$ and $\pi$, because it results from a no-arbitrage condition for intermediaries, who know the real value $\pi$ and the real $\Delta$. Households deduce a value of $\pi^H$ that is consistent with price $P$ and their belief $\Delta^H$. It therefore follows that

$$P = \frac{\beta \pi R}{\Delta + r\beta\pi\left(1-\Delta\right)} = \frac{\beta \pi^H R}{\Delta^H + r\beta\pi^H\left(1-\Delta^H\right)}. \tag{17}$$

We deduce the following inference for households:

$$\pi^H = \pi \frac{\Delta^H}{\Delta + r\beta\pi\left(\Delta^H-\Delta\right)}. \tag{18}$$

Due to the condition $r\beta\pi < 1$, if $\Delta^H > \Delta$, then $\pi^H > \pi$, and if $\Delta^H < \Delta$, then $\pi^H < \pi$. In addition, if $\Delta^H = \Delta$, then $\pi^H = \pi$. In other words, if households over-estimate risk-weighted capital, they under-estimate the risk, and if they under-estimate risk-weighted capital, they over-estimate the risk. Moreover, when households have correct beliefs concerning risk-weighted capital (i.e., $\Delta^H \equiv \Delta$ irrespective of the value of $\Delta$), they correctly infer the right level of aggregate risk.

We now solve the model for the various cases concerning the relationship between $\Delta^H$ and $\Delta$.

### *4.2 Symmetric Information Over $\Delta$*

If both households and financial intermediaries have correct beliefs on $\Delta$, households can deduce the level of aggregate risk $\pi$, as shown in the discussion of equation (18). In addition, knowing $\Delta$, they deduce $K = \Delta P X_R$. They can also infer the amount of aggregate debt $B$ from the budget constraint of financial intermediaries.

With the expression for $X_S$ given by (16) and $r = r_S$, the no-arbitrage condition for household can be written as

$$\pi r + (1-\pi)\frac{rf'^{-1}(r)}{B} = \tau. \tag{19}$$

This condition states that average return for each unit invested in financial intermediaries (taking into consideration the possibility of default) should be equal to the return on the storage technology. We introduce the main conclusions of the paper as propositions. All proofs are presented in the appendix.

PROPOSITION 1. *If* $\Delta^H \equiv \Delta$, *households' expectations of the aggregate risk are correct* ($\pi^H = \pi$) *and*

$$\frac{\partial B}{\partial \Delta} < 0 \text{ and } \frac{\partial (r-\tau)}{\partial \Delta} < 0.$$

Proposition 1 states that decreasing capital per unit of the risky asset increases both the volume of debt of intermediaries and the credit risk premium, $r - \tau$. This result is due to two effects. First, the overall general equilibrium effect of a decrease in $\Delta$ is an increase in the intermediaries' debt level, as financial intermediaries have a greater incentive to increase their exposure to risk by issuing debt. Second, when $\Delta$ decreases, households understand that the residual value of the assets they receive in the event of default decreases. They hence request a larger default risk premium $r - \tau$ to compensate for the increased cost of default. This version of the model is therefore *not* consistent with the stylized facts of the sub-prime cycle. As shown in figure 1, banks have been able to borrow at lower risk premia during the five years leading up to the crisis, in spite of increasing leverage and decreasing risk-weighted capital with respect to, for instance, U.S. housing loans.

To summarize, the change in $\Delta$ can account for an increase in the debt level of banks, but it cannot explain the path of the risk premia between 2000 and 2007. We therefore assert that risk shifting, per se, is not sufficient to replicate the stylized fact of the *sub-prime crisis*. Before the crisis, banks and financial intermediaries benefited in fact from extremely favorable funding conditions, which would not be the case if changes to risk-weighted capital were fully understood by investors.

The next sections will discuss the more likely cases where capital constraints are fuzzy. In view of the complexity of financial intermediaries' balance sheets, and off-balance-sheets transactions, this case is more likely to reflect the real world. Households may either over-estimate or under-estimate $\Delta$.

### *4.3 Over-Estimation of* $\Delta$

We now assume that households believe that the level of risk-weighted capital is higher than that actually faced by financial intermediaries: $\Delta^H > \Delta$. In this case, the amount of capital pledged by financial intermediaries is lower than that expected by households, and households over-estimate the probability of success of the risky asset $\pi^H > \pi$, due to the relationship (18).

Households form their inference about the residual value of their portfolio, $\frac{rX_S}{B^H}$, in the following way. First, from the observation of the amount of risky assets in the economy $X_R$, and from their belief about $\Delta^H$, households infer that the level of the capital of financial intermediaries is

$$K^H = X_R \Delta^H P.$$

Second, from the budget constraint of financial intermediaries, households form the following expectation about the amount of debt:

$$B^H = X_S + PX_R \left(1 - \Delta^H\right). \tag{20}$$

Third, the no-arbitrage condition for households must now be written according to their expectations:

$$\pi^H r + \left(1 - \pi^H\right) \frac{rX_S}{B^H} = \tau. \tag{21}$$

Using equation (18) in order to substitute for $\pi^H$, the expressions for $X_S$ given by (16), the value of $B$ implied by the balance sheet constraint of the intermediary, and the fact that $r_S = r$, we obtain an equation for the equilibrium interest rate $r$ which depends only on known parameters and functional forms.

In order to obtain analytical insight, we assume that households' belief about capital requirement is not too far away from the true one, i.e., we assume that $\varepsilon \equiv \Delta^H - \Delta$ is small. In this case, we can perform first-order approximations.

PROPOSITION 2. *If* $\varepsilon \equiv \Delta^H - \Delta$ *is small, we have*

$$\frac{\partial \pi^H}{\partial \varepsilon} > 0, \frac{\partial (r - \tau)}{\partial \varepsilon} < 0 \textit{ and } \frac{\partial B}{\partial \varepsilon} > 0.$$

Proposition 2 summarizes the effect of an increase in households' estimation of risk-weighted capital $\Delta^H - \Delta$ (or a decrease in risk-weighted capital $\Delta$ keeping belief $\Delta^H$ constant). Households become more optimistic about the risk of the asset $\pi^H$. They hence charge a lower risk premium, which allows financial intermediaries to borrow more. This proposition illustrates how unexpected regulatory arbitrage might explain why, before the crisis, banks increased the risk they took without being sanctioned by higher risk premia.

PROPOSITION 3. *If* $\varepsilon \equiv \Delta^H - \Delta$ *is small and positive, we have*

$$\frac{\partial \pi^H}{\partial \tau} < 0, \frac{\partial (r - \tau)}{\partial \tau} > 0.$$

When households over-estimate risk-weighted capital, a decrease in the risk-free rate $\tau$ exacerbates their optimistic bias about the risky asset. The reason stems from equation (17). When the level of the risk-free rate $\tau$ decreases, the lending rate to financial intermediaries $r$ also decreases under general conditions. The asset price $P$ increases and households assign part of this increase to a decrease in the riskiness of the asset, leading to an increase in borrowing by financial intermediaries. The model can therefore explain one of the channels through which monetary policy might affect risk taking by financial intermediaries.[10]

The predictions of the model for risk perception are actually consistent with the empirical results produced by Altunbas, Gambacorta, and Marques-Ibanez (2010). They found that the expected

[10]Borio and Zhu (2009) coined the term "the risk taking channel of monetary policy" that they define as "the impact of changes in policy rates on either risk perceptions or risk-tolerance and hence on the degree of risk in the portfolios, on the pricing of assets, and on the price and non-price terms of the extension of funding."

default frequencies, and other market-based measures of bank's risks as perceived by financial market participants, react positively to changes in interest rates: a lower interest rate leads investors to perceive banks as less risky. Turning to banks' risk taking, which may be interpreted as banks exploiting their ability to borrow cheaply from financial markets, a number of recent studies—including Ciccarelli, Maddaloni, and Peydró (2010), Ioannidou, Ongena, and Peydró (2008), and Jimenez et al. (2007)—show that credit standards are correlated to the level of interest rates. Lower interest rates therefore imply lower credit standards, including for customers who are perceived as presenting a higher credit risk.

It is important to stress, however, that in our model the impact of the level of interest rates on risk perception and risk taking does not depend on the source of variation in interest rates. The interest rate in the model is real and can, therefore, be influenced by several factors. During the decade leading up to the crisis, several explanations were put forth in order to explain the low level of nominal and real interest rates. According to Taylor and Williams (2009), U.S. monetary policy was overly accommodative. Bernanke (2010), however, stressed instead that China's excess savings have played a major role in keeping the long end of the U.S. yield curve at comparatively low levels. Either of these factors may in turn have been amplified by the phenomenon of "search for yield," as emphasized by Rajan (2005). We do not take a position on these alternative possible drivers of the level of interest rates, and only stress that the endogenous mechanism described in our model would hold for either of them.

What the model highlights, however, is that the search for yield and risk taking can in part result from the wrong inference of risks from asset prices. This is because interest rates are central in the valuation of assets and the inference on risk incentives. It points to the interdependence of interest rates, asset prices, and capital-based prudential policies in a world where risk incentives and exposure cannot be assessed with certainty.

### *4.4 Under-Estimation of* $\Delta$

The previous section focused on the case in which households overestimate capital and wrongly infer the level of collapse risk. We show that this is consistent with the stylized facts on the pre-crisis period. The symmetric case is, however, also interesting.

The case of excess caution, where the risk-weighted capital of banks is believed to be too low, may help understand other periods of history. In a recent paper, Malmendier and Nagel (2011) show that households who experienced the Great Depression are less likely to invest in stock markets or participate in financial markets. Our model is able to rationalize this behavior by an under-estimation of the constraints imposed on banks after the Glass-Steagall Act. Anticipating that both the banking system and risky assets are more risky than they really are, households ask for higher returns or guarantees to compensate for the perceived risk. Empirical support for the view that investors' appetite for risk varies over time can also be found in Gilchrist and Zakrajsek (2012), who show large and persistent swings in the price of risk, defined as the part of bond risk premia that are not explained by "fundamentals" on the risk of default, where the latter is derived from the Merton's valuation of firms' stocks as an option to default. The price of credit risk was consistently negative from 2003 to 2007. It has also remained positive for several periods, such as around 2000, around 2008, and throughout the 1980s (see figure 1 in Gilchrist and Zakrajsek 2012). Again, the periods of over-estimation of risk may be due to an under-estimation of the capital constraints imposed on financial intermediaries.

## 5. Alternative Specifications of the Model

### *5.1 Elastic Supply of the Risky Asset*

In the baseline model, we assume that the supply of the risky asset was fixed, i.e., $X_R = 1$. This section shows that the results are robust even if this supply may respond to prices, provided this response is not too large.

Let us now assume that instead of being sold in period 1 by initial sellers, the risky asset is produced by a new class of entrepreneurs. There is a unit mass of such entrepreneurs. They have access to a risky technology and consume in period 2. The risky technology yields $g(Y) = \lambda Y^{1-\theta}/(1-\theta)$, with a probability $\pi$ in period 2, or fails to produce anything with probability $1-\pi$. We assume that risks are perfectly correlated among these entrepreneurs.

They sell the risky asset to financial intermediaries in period 1, at a unit price $P$. One unit of risky asset costs $P$ in period 1 and pays off $R$ with a probability $\pi$ and 0 with a probability $1-\pi$ in period 2. Entrepreneurs choose how many units $X_R$ of risky asset to sell. And they have no alternative to investing it. Their objective function is to maximize their period 2 consumption, denoted as $c^R$, with $c^R = \pi\left[g\left(PX_R\right) - RX_R\right]$. It yields

$$X_R = \left(\frac{\lambda}{R}\right)^{\theta} P^{\frac{1-\theta}{\theta}}.$$

Our baseline model is a special case where $\theta = 1$, and $\lambda = R$. It can be shown that our results on the effects of wrong beliefs (proposition 2) and the change in the risk-free rate $\tau$ (proposition 3) are still valid when $\theta$ is not too low. For low values of $\theta, P$ cannot deviate enough from its fundamental value for obvious reasons, so wrong beliefs concerning $\Delta$ have negligible effects.

This extension is important because financial innovation is likely to be stimulated when investors are optimists, or when either interest rates or risk premia are low. In the case of the U.S. sub-prime crisis, the supply of several forms of risky assets increased. More houses and condos were constructed, especially in states where housing prices increased the fastest (Florida, Nevada, etc.). The "originate-to-distribute" business model exposed mortgages to U.S. households that had decreasing creditworthiness. And more mortgage-backed securities and collateralized debt obligations that packaged these mortgages were sold to investors. The under-estimation of credit risk, as shown in figure 1, carried on for as long as the price of risky assets increased, i.e., the supply could not catch up with demand.

## *5.2 Risk about the Effectiveness of Capital Requirements*

Our baseline model assumes that households may have wrong beliefs about the effectiveness of capital requirements, i.e., how much loss would be absorbed by holders of banks' stocks in the bad state of the world. Relaxing this assumption, while feasible, would greatly increase the complexity of the model. It is indeed possible to introduce an additional shock on the level of capital requirement. This shock is known by banks but unknown to the households. In this framework households form on average correct expectations about the average level of capital requirements. For an unexpected negative shock to this level, we would find the same results as in the current model where $\Delta^H > \Delta$. We chose a simpler structure to derive theoretical results in a transparent way.

### *5.3 Uncertain Return*

In the baseline model, we assumed that the return in case of success $R$ was known but that the probability distribution of risk $\pi$ was unknown. An alternative modeling strategy would be to consider $\pi$ as known but that the return $R = R(e)$ is uncertain and affected by the private actions of financial intermediaries. In this case, the main result would be preserved: if households know capital requirements $\Delta$, they can infer from asset prices the real return $R(e)$ and thus the private action $e$. Changes in capital requirements may drive changes in private actions, but these changes would be anticipated and thus reflected in risk premia. Alternatively, when capital requirements are unknown, changes in capital requirements, and thus in equilibrium prices, will be partly understood to be a higher return and would thus bias the estimate of credit risk. Although it may be hard to distinguish between uncertain return and uncertain probabilities for specific assets, our modeling choice is motivated by the direct evidence of a sharp change in the expected probability of default during the crisis, as mentioned in section 2. Our model is thus designed to explain this bias in expectations of default.

### *5.4 Return and Risk*

In our model, a change in the probability of default $\pi$ affects both the mean and the variance of the return on the risky asset. As agents are risk neutral, the effect on the variance does not affect prices, but it would still be useful to express the model in order to analyze the effect of a change in the mean return keeping its variance constant. It is possible to do so by introducing an additional risk. Let us assume that the risky asset is equal to 0 with a probability $1 - \pi$ and equal to a stochastic variable $\tilde{R}$ with a probability $\pi$. $\tilde{R}$ has a mean $R$ and is uniformly distributed in the support $[R - \delta; R + \delta]$. If the support is small enough, default will occur only when $R$ is equal to 0. It is then possible to jointly choose $\pi$ and $\delta$ to study the effect of a change in mean which keeps the variance constant.

## 6. Alternative Explanations of Low Risk Premia

### *6.1 Expectations of Bailouts*

The model focuses on uncertainty of capital requirements to explain why risk premia were low before the crisis. An alternative explanation is that investors expected to be collectively bailed out by governments and central banks. Farhi and Tirole (2012) propose a model in which financial institutions coordinate their exposure to risks in order to increase systemic risk, and therefore the likelihood that public authorities will bail them out. Their model explores the issue of risk shifting from investors to taxpayers. We focus instead on the shifting of risk from banks to bondholders.

It is clear that the concepts of "too big to fail" or "too interconnected to fail" are likely to have influenced investors. The expectations of bailouts may have played a role in the evolution of risk premia. In particular, the big change in expected default frequency after the collapse of Lehman in September 2008 may have been due partly to a revision in the perceived probability of a public bailout and partly to a reassessment of the underlying risk. To the best of our knowledge, there is no evidence that allows us to rule out that these phenomena may have influenced risk premia.

It should also be stressed, however, that the political process that leads to bailing out financial institutions is uncertain. Financial institutions usually pay a credit risk premia with respect to the Treasury. These premia vary over time for a number of reasons, including the collective moral hazard hypothesis of Farhi and Tirole (2012) and the one we propose in this paper.

### *6.2 General Under-Estimation of Risk*

Another explanation of low risk premia would be that all agents, including financial intermediaries, under-estimated the risk of default on housing assets. This is, for instance, the view held by Schleifer (2011). He stresses that the ex post losses of the financial industry were so large as to discard the "expectation of bailouts" hypothesis as well as hypotheses, like the one we present in this paper, which highlight asymmetry of information between the financial industry and non-financial agents. Again, a global underestimation of risk cannot be rejected, and it may be hard to claim that banks correctly anticipated the real risk of all assets. However, some evidence, such as legal actions against financial intermediaries, suggests that financial intermediaries and households did not all have the same information. In 2007 Chuck Prince, then chairman of Citigroup, declared: "When the music stops, in terms of liquidity, things will be complicated. But, as long as the music is playing, you've got to get up and dance. We are still dancing." (*Financial Times* 2007)

This is highly suggestive that banks knew they were taking risks.[11]

## 7. Concluding Remarks: Can the Model Explain the Buildup of Financial Fragility?

In this paper we show that the combination of risk shifting and fuzzy capital requirements may explain one of the sub-prime crisis puzzles, i.e., that financial intermediaries were able to increase their exposure to risk without having to pay higher risk premia on their debt.

In an opaque banking system where regulatory constraints are difficult to observe, an increase in asset prices can be interpreted as a lower aggregate risk in the economy while, in fact, asset prices are driven by greater risk taking on the part of financial intermediaries. We also showed that this model gives rise to a risk-taking channel of low interest rates, which reduces the perceived risk of some agents and increase the exposure to risk of others.

Our result resonates with the popular notion that financial markets participants can draw incorrect inferences about risks. In particular, when the effectiveness of capital requirements is not observable by agents, the signal extracted from market prices is contaminated by noise coming from excessive risk-taking behavior. In our model,

[11] Another paper elaborating on the information asymmetry between financial intermediaries and households is Shleifer and Vishny (2010). In this paper, the expectations of households are taken as given. Instead, we endogenize them through an inflation-extraction problem.

market forces, by themselves, do not result in an optimal allocation of capital, because risk incentives are not correctly understood.

We see two obvious extensions to our model. First, it is possible to endogenize the expectations of households within a dynamic setting in which households learn about the relevant parameters. The results of our model would still hold if the priors of the households were far enough from the true parameters. The resulting dynamics of their learning process should generate useful patterns. Second, it would be interesting to study the political economy aspects associated with the assessment of risk within such an economy. Sellers of the assets have an incentive to under-estimate the degree of risk, or to generate complexity in order to increase the cost of signal extraction. This should be anticipated by households, who would then look for other sources of information. It is in this context, for example, that we interpret the current discussion about rating agencies to be part of the debate on the management of risk expectations in economies where intermediaries play an important role.

## Appendix

### *Proof of Proposition 1*

Since $\pi \in ]0,1[$ and $X_S < B$, the no-arbitrage condition (19) implies $r > \tau$. Equality (19) can be written as

$$B(r) = \frac{(1-\pi)\, r X_S}{\tau - \pi r}. \tag{22}$$

We can substitute $K$, $X_S$, and $P$ with their equilibrium values given by equations (15) and (16) to obtain an expression $B\,(r)$:

$$B\,(r) = \frac{(1-\Delta)\, R}{\frac{\Delta}{\beta\pi} + r\,(1-\Delta)} X_R + f'^{-1}\,(r)\,. \tag{23}$$

Let us define: $\Theta \equiv \frac{\Delta}{1-\Delta}$. Then from (16), (23), and (19), we find that the real interest rate $r$ satisfies the equality

$$\tau = \pi r + (1-\pi)\, \frac{(\Theta + \beta\pi r)\, r^{1-1/\eta}}{\beta\pi R X_R + \Theta r^{-1/\eta} + \beta\pi r^{1-1/\eta}}.$$

This last equality implicitly defines the interest rate by equality $M(r) = \Theta$, where

$$M(r) \equiv \beta\pi \left( \frac{(\tau - \pi r)\, r^{1/\eta} \mu X_R}{r - \tau} - r \right).$$

In the equilibrium under consideration, $\pi r < \tau < 1$ and $r > \tau$. As a consequence, we can check that a sufficient condition for $M'(r) < 0$ is $\eta > \frac{1-\pi}{\pi}$, which is (1). In this case, the equality $M(r) = \Theta$ implies that $r$ is decreasing with $\Delta$.

From equality (19), one finds $B(r) = \frac{1-\pi}{\tau - \pi r} r^{1-\frac{1}{\eta}}$. After some algebra, we find that $B(r)$ increases with $r$ when (1) is fulfilled. As a consequence, $\frac{\partial B}{\partial r} > 0$, $\frac{\partial (r-\tau)}{\partial \Delta} < 0$, and $\frac{\partial B}{\partial \Delta} < 0$.

*Proof of Proposition 2*

Denote $\varepsilon = \Delta^H - \Delta$. From equations (16), (21), (17), (18), and (20) we find that the real interest rate satisfies $G(r, \tau, \Delta, \varepsilon) = 0$, where

$$G(r, \tau, \Delta, \varepsilon) \equiv \beta\pi \left[ \frac{\left( \tau - \pi r \frac{\Delta + \varepsilon}{\Delta + r\beta\pi\varepsilon} \right) R r^{\frac{1}{\eta}}}{r - \tau} - r \left( 1 - \frac{\varepsilon}{1 - \Delta} \right)^{-1} \right] - \frac{\Delta}{1 - \Delta - \varepsilon}. \tag{24}$$

As $\tau, \Delta, \varepsilon$ are given parameters, the equality $G(r, \tau, \Delta, \varepsilon) = 0$ defines the equilibrium interest rate as a function of the parameters of the model. Studying the derivative of the function $G$, we find the following signs for the derivatives (with obvious notations):

$$G\left( \underset{-}{r}, \underset{+}{\tau}, \underset{-}{\Delta}, \underset{-}{\varepsilon} \right) = 0.$$

As a consequence and by the implicit function theorem, we find that $r$ has the following variations: $r = r\left( \underset{+}{\tau}, \underset{-}{\Delta}, \underset{-}{\varepsilon} \right)$. This proves $\frac{\partial (r-\tau)}{\partial \varepsilon} < 0$.

Next, the anticipated probability can be written as $\pi^H = \pi(\varepsilon + \Delta) / (\Delta + r\beta\pi\varepsilon)$ from (18). As a consequence, one finds the following variations: $\pi^H = \pi^H \left( \underset{-}{r}, \underset{+}{\varepsilon} \right)$. This proves $\frac{\partial \pi^H}{\partial \varepsilon} > 0$ (as $r$ decreases when $\varepsilon$ increases).

Finally, the budget constraint of financial intermediaries, together with the price (17), gives the debt level $B$:

$$B = r^{-\frac{1}{\eta}} + \frac{\beta \pi R}{\frac{\Delta}{1-\Delta} + r\beta\pi}.$$

One can easily deduce the variation $B = B\left(\underset{-}{r}\right)$ and $r = r\left(\underset{+}{\tau}, \underset{-}{\Delta}, \underset{-}{\varepsilon}\right)$. We have thus the variations $B = B\left(\underset{-}{\tau}, \underset{+}{\varepsilon}\right)$.

*Proof of Proposition 3*

From the proof of proposition 2, we obtain $r = r\left(\underset{+}{\tau}, \underset{-}{\Delta}, \underset{-}{\varepsilon}\right)$ and $\pi^H = \pi^H \left( \underset{-}{r}, \underset{+}{\varepsilon} \right)$. As a consequence, we find $\frac{\partial \pi^H}{\partial \tau} < 0$. The proof of the inequality $\frac{\partial (r-\tau)}{\partial \tau} > 0$ requires more algebra. This inequality is first proven for $\varepsilon = 0$. Then, a continuity argument is invoked. From the definition (24) and the equality $G(r, \tau, \Delta, \varepsilon) = 0$, we find

$$r - \tau = r \frac{1 - \pi \frac{\Delta + \varepsilon}{\Delta + r\beta\pi\varepsilon}}{\frac{1}{\mu_1 r^{\frac{1}{\eta}}} \left[ \frac{\Delta}{1-\Delta} \frac{1}{\beta\pi} + r \right] \frac{1}{1 - \frac{\varepsilon}{1-\Delta}} + 1}.$$

Taking the derivative with respect to $\tau$ and setting $\varepsilon = 0$, we easily find $\frac{\partial (r-\tau)}{\partial \tau} > 0$. By continuity, this inequality is fulfilled when $\varepsilon$ is small.

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