Introducing Funding Liquidity Risk in a Macro Stress-Testing Framework*

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The main contribution of this paper is to introduce a funding liquidity component à la Morris and Shin (2009) in a stress-testing framework. As a result, funding liquidity risk arises as an endogenous outcome of the interactions between market liquidity and solvency risks, and banks' liquidity profiles. We perform a calibration exercise that highlights the vulnerability of leveraged institutions to the combination of low cash holdings and the prevalence of short-term debt, a key feature of the 2008 credit crisis. We also analyze the trade-offs between higher capital ratios, more liquid assets, and/or less short-term liabilities in reducing systemic risk.

JEL Codes: G01, G21, G28, C72, E58.

"Over the years [...], funding liquidity came to replace asset liquidity. The idea was that, so long as bank capital sufficiency was assured, which adherence to Basel II was supposed to achieve, banks could always rely on access to these large, efficient, wholesale markets. [...] The Basel Committee on Banking Supervision (BCBS) had attempted, in the mid 1980s, to put together an Accord on banking liquidity as a supplement to the

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Capital Accord of 1988. But when that initiative failed, [...] no individual country regulator felt able to halt, let alone to reverse, the developing trend away from asset liquidity. When short-term wholesale markets did collapse after August 9, 2007, banks were left with little internal asset liquidity with which to ride out the storm." (Goodhart 2010)

1. Introduction

The collective reactions by market participants during the recent financial crisis led to mutually reinforcing solvency and liquidity problems at banks around the globe. As funding liquidity evaporated, many strongly capitalized financial institutions in Europe and the United States had to write off or sell at a loss illiquid assets, creating uncertainty about their solvency among market participants. Many institutions avoided bankruptcy thanks to massive public intervention. The extent of the support required to alleviate the crisis made clear that an approach to regulation with more emphasis on funding liquidity risk and the risk of collapse of the entire financial system was needed. This so-called macroprudential approach to regulation requires tractable models for measuring those risks, with a view to elaborating appropriate policy recommendations.

Models of the financial system and its complex interactions are at a relatively early stage of development but have become a major research priority at institutions responsible for financial stability. In particular, central banks in many jurisdictions have been developing macro stress-test models that provide a rigorous and consistent quantitative framework for risk assessment and help to sharpen the analysis of banking system vulnerabilities. These models examine the impact of key risks on a bank-by-bank as well as system-wide basis, and can be an aid in the assessment of the impact of potential policy measures.[1]

Prior to the crisis, stress-testing models were mainly focusing on solvency risk at individual institutions and spillover effects between banks due to interbank exposures. In recent years, there has been an increasing interest in incorporating liquidity risk in these models.

[1] See Foglia (2009) for a review of stress-testing models.

Van den End (2008) relates market and funding liquidity risks to banks' behavioral and reputational effects using Monte Carlo simulations. Wong and Hui (2009) propose an econometric model linking banks' deposit outflows to solvency concerns triggered by asset fire sales. Aikman et al. (2009) provide a tractable stress-testing approach to modeling funding liquidity risks, solvency risk, and externalities through spillover effects due to interbank exposures. In their framework, rules of thumb are used to impose funding constraints once banks' balance sheets deteriorate beyond certain exogenous thresholds.

In this paper, we propose a framework similar in its broad structure to Aikman et al. (2009) but provide analytical underpinnings for the link between solvency risk, market liquidity risk, and funding liquidity risk, rather than relying on exogenous thresholds. We build on recent advances in the global game theory to introduce an endogenous funding liquidity component à la Morris and Shin (2009) (MS hereafter) in which rollover risk occurs through a coordination problem among short-term creditors. This approach is consistent with the events of the recent financial crisis: banks' creditors refused to roll over their short-term claims if they had serious concerns about their future solvency. To our best knowledge, we are the first to introduce such rigorous microeconomic foundations in macro stress-testing models.

To illustrate the functioning of the framework and show its ability to replicate some stylized facts, we perform two simulation exercises. We first estimate the different types of risk in the Canadian banking system at the height of the sub-prime crisis and reproduce its robustness. As a second exercise, we consider a hypothetical banking system in which leverage levels and liquidity profiles are similar to those of banks that needed to be rescued during the recent financial crisis and find that funding liquidity and systemic risks can be significant. We also find that the combined impact of a common macro shock, a broader set of linkages among banks, and funding liquidity risk increases significantly the likelihood of network spillover effects of leveraged banks.

We then suggest how the model can be used as a policy analysis tool by quantifying the trade-offs between higher capital ratios, more liquid assets, and/or less short-term liabilities in reducing risks in the banking system. Our results suggest that liquidity requirements can be effective in forestalling contagious failures, which is supportive of the new Basel III liquidity requirements. However, we find that a regulatory framework that properly controls for systemic risk should consider both minimum capital and liquidity requirements in a holistic way.

The rest of the paper is organized as follows. We summarize the related literature in section 2. We present our framework in section 3, with an emphasis on how funding liquidity risk is introduced. We illustrate the use of the model in section 4, through two simulation exercises and a quantitative analysis of the trade-offs between liquidity and capital in reducing systemic risk. Finally, we conclude and highlight future extensions of the framework in section 5.

2. Related Literature

Our paper is related to three lines of literature. It first uses recent developments in the theoretical literature on deposit-based financial fragility and bank run. This literature, which was initiated by the seminal work of Diamond and Dybvig (1983), emphasizes the role of coordination failure between depositors in banking panics. They may run when they fear that the bank becomes insolvent because other depositors suddenly withdraw their funds. In this literature, however, a bank run is not a unique equilibrium. The theory of global games first introduced in Carlsson and van Damme (1993), developed in Morris and Shin (2001), and applied in Rochet and Vives (2004), shows that such equilibrium becomes unique when a small amount of uncertainty is introduced about the long-term return of banks' assets. In this work, banks' illiquidity occurs as a function of current insolvency risk. Our paper adapts the model developed in Morris and Shin (2009) where coordination failure occurs due to *future* rather than current fundamental uncertainty. Other recent theoretical studies of the interactions between different risks include Acharya, Gale, and Yorulmazer (2011), which show that high rollover frequency can lead to diminishing debt capacity of risky assets, and Brunnermeier and Pedersen (2009), in which market liquidity risk and traders' funding liquidity mutually reinforce, giving rise to a liquidity spiral.

Second, our paper is related to the literature on financial contagion in interbank markets. For example, Flannery (1996) and Rochet and Tirole (1996) emphasize the potential benefits from peer monitoring when banks are interconnected. Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000) show that an interbank market where all banks are interconnected may serve as a means for banks to mutually insure against idiosyncratic bank liquidity shocks. In parallel to this theoretical literature, there are numerous empirical studies applying network techniques developed in mathematics to assess interbank spillover effects in national banking systems.[2] For example, using the clearing algorithm introduced in Eisenberg and Noe (2001), Elsinger, Lehar, and Summer (2006) and Alessandri et al. (2009) find that those effects are low-probability/high-impact events.

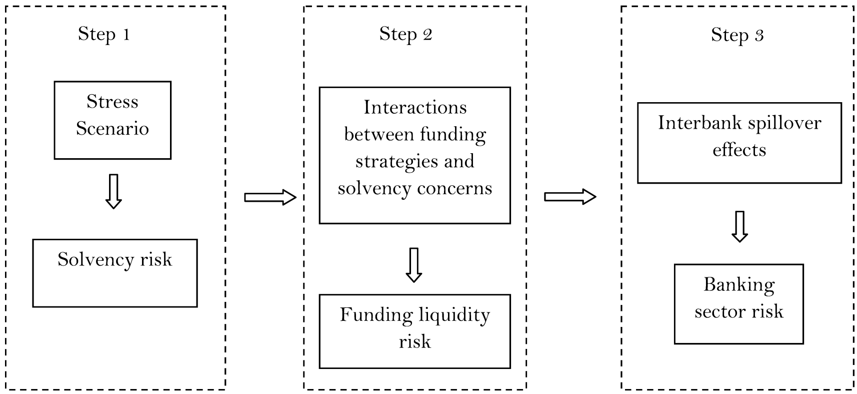
Finally, our paper is related to the empirical literature on the assessment of systemic risk in financial systems. One strand of this literature uses banks' balance sheets to assess systemic risk. For example, Gauthier, Lehar, and Souissi (2012) use a network of Canadian financial institutions and compare alternative mechanisms for allocating the overall risk of a banking system to its member banks. Another strand of the literature uses market data to statistically estimate reduced-form models of value at risk and co-movements (such as Adrian and Brunnermeier 2011, Gravelle and Li 2011, Huang, Zhou, and Zhu 2012, and Acharya et al. 2013). Although market data are more timely than balance sheet data, they do not necessarily incorporate appropriately ex ante externalities due to contagion and spillover effects.

3. A Systemic Risk Assessment Framework

Our framework is derived from a class of macro stress-testing models used by central banks and international institutions to identify vulnerabilities in national banking sectors. Based on a set of banks' balance sheet data, it allows for solvency risk, reflecting potential losses associated with banks' assets (such as credit risk), and for funding liquidity risk and network interactions between banks. The basic structure of the framework and the mapping from macroeconomic shocks to banking-sector risk are illustrated in figure 1.

[2]See Upper (2011) for a summary of the empirical literature on contagion through interbank markets.

Figure 1. Basic Structure of the Model



The framework can be divided into three sequential steps. First, banks are subject to common adverse macroeconomic shocks that engender asset losses over a one-year horizon.[3] In the second step, funding liquidity risk is introduced. As initial losses reduce bank capital, they raise concerns about banks' future solvency. This induces short-term lenders to refrain from rolling over their claims, thus generating an increase in funding risk. In the third step, failure or distress at one bank—due to solvency risk or funding problems—can spill over to other banks through mutual interbank exposures. A defaulting bank (or a bank with a serious capital shortfall) may not be able to fulfill its obligations in the interbank market, imposing counterparty credit losses on other banks and potentially leading to their default. We next describe each step in more detail.

3.1 Solvency Risk

We first simulate asset losses at individual banks over a one-year horizon under an adverse but plausible macroeconomic scenario. In this paper, asset losses come from exposures to non-bank borrowers only. They are due to the decline in the credit quality of banks' loans,

[3]An obvious criticism of this approach is that the last crisis was triggered by a financial shock, losses on sub-prime loans, which was amplified into a banking crisis and eventually a recession (and not the other way around). However, the framework can accommodate any type of initial shock as long as it is mapped into an impact on banks' capital.

as expected defaults climb with the deterioration of macroeconomic conditions. Losses reflect both systematic and idiosyncratic factors.

To capture systematic factors in different business sectors, we use an updated version of the empirical macroeconomic credit risk model developed in Misina, Tessier, and Dey (2006) as part of the 2007 International Monetary Fund (IMF) FSAP update for Canada.[4] This model builds on early work by Boss (2002) and Sorge and Virolainen (2006) which assesses credit risk in loan portfolios of banks in Austria and Finland, respectively. It relates default rates to changes in the overall performance of the economy.[5] Macroeconomic indicators used as control variables capture systematic factors affecting the probability of default of business-sector banks' loans simultaneously. They include Canadian GDP growth, the unemployment rate, the interest rate (medium-term business-loan rate), and the ratio of credit to GDP. We also use the model described in Djoudad (2011) to estimate default rates on household-sector loans.[6] These models are used to infer expected default rates consistent with the paths of stressed macroeconomic variables. Distributions of default rates are then obtained by adding random draws based on the correlation structure of historical default rates across sectors.

Idiosyncratic risk factors are incorporated using an extended CreditRisk+ model as in Elsinger, Lehar, and Summer (2006) and Gauthier, Lehar, and Souissi (2012). It generates, for a given default rate, a distribution of expected loan losses that reflects the size distribution of banks' loans. This distribution is used as an input in the following step.

[4]The IMF helps national authorities assessing the robustness of their financial sector with its Financial Sector Assessment Program, known by its acronym, "FSAP." The FSAP is a tool for conducting systematic assessments of a country's financial sector and benchmarking regulatory efforts against international best practice. It includes a macro stress-testing exercise tailored to the country being assessed.

[5]Sectors of economic activity are selected based on the classification employed by banks in regulatory reporting and include the following eight sectors: accommodation (or services), agriculture, construction, manufacturing, wholesale, uninsured residential mortgages, other consumer loans, and loans to small businesses.

[6]Djoudad (2011) constructs a formal analytic framework to simulate the impact of various economic shocks on the household debt-service ratio, using data from the Canadian Financial Monitor (CFM) survey. The impact of these shocks on individual households depends on their socioeconomic characteristics.

3.2 Interactions between Solvency and Funding Liquidity Risks

We then adapt the MS model to measure the interactions between funding liquidity and solvency concerns. To do so, the one-year assessment horizon of credit losses is divided into three periods: period 0, at the beginning of the year, where only the distribution of credit losses at the end of the year is known; the interim period, after six months, at which time some loan book losses are realized, potentially leading to interim illiquidity or insolvency; and period 2, the end of the year, at which time total credit losses are observed and liquid banks could be insolvent.[7]

We consider the following balance sheet of a bank in period 2:

Assets	Liabilities
M	S
$Y - p_1 - p_2$	L
	$E - p_1 - p_2$,

where M is the amount of liquid assets on the institution's balance sheet in period 0; p_1 and p_2 are the credit losses on the risky (and potentially illiquid) assets Y in the semi-annual period 1 and 2, respectively; S and L are the short-term and long-term liabilities, respectively; and $E - p_1 - p_2$ is the remaining capital after the total credit write-downs. We assume that the only available capital to absorb losses is tier 1 capital. The total cash available to the bank at the interim date is

$$C^* = M + \psi(Y - p_1), \tag{1}$$

where $0 \leq \psi < 1$ can be seen as either the fire-sale price of the illiquid assets or the collateral value of the assets (one minus the haircut) in stressed funding markets.[8]

[7] In this paper, we set the interim period at six months. Further details on how the model is mapped to the real world are provided in section 4.1, and a sensitivity analysis of our results to changes in the interim date parameter is performed in section 4.5.

[8] ψ is akin to exogenous market liquidity risk. See Cifuentes, Ferrucci, and Shin (2005), Nier et al. (2007), or Gauthier, Lehar, and Souissi (2012) for a modeling of endogenous market liquidity risk in a similar framework.

The bank fails from a run if the proportion of short-term debt holders not rolling over is more than $\lambda = \frac{C^*}{S}$, i.e., if the total cash available on the balance sheet is not enough to cover for the creditors not rolling over $(\lambda * S)$.

Following MS, solvency risk is defined as the likelihood of default conditional on there being no run on the bank, and liquidity risk as the probability of a run. To characterize the ex ante liquidity and solvency risks, we need to solve the problem backwards and first characterize those risks in the interim period, once p_1 is observed.

3.2.1 Interim Solvency Risk

An institution is insolvent in period 2 if

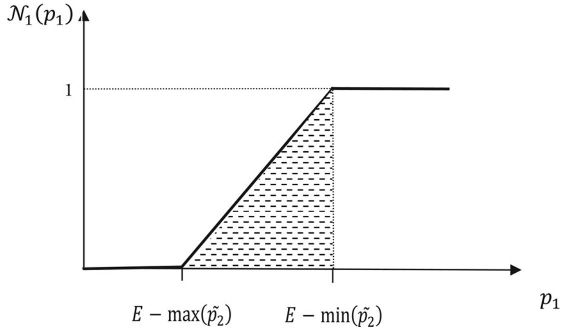
$$E - p_1 - p_2 < 0. \tag{2}$$

The probability of this happening, conditional on having observed losses of p_1 after the first six months, and conditional on not having a run by short-term creditors, is

$$\begin{aligned} \mathcal{N}_1(p_1) &= P(\tilde{p}_2 > E - p_1) \\ &= \begin{cases} 1, & \text{if } p_1 > E - \min(\tilde{p}_2) \\ 1 - F_2(E - p_1), & \text{if } E - \max(\tilde{p}_2) < p_1 \leq E - \min(\tilde{p}_2) \\ 0, & \text{if } p_1 \leq E - \max(\tilde{p}_2), \end{cases} \end{aligned} \tag{3}$$

where $\tilde{p}_2$ is the ex ante credit loss in period 2, in contrast to the realized value p_2, and follows the cumulative uniform distribution $F_2(\cdot)$ on the support $[\min(\tilde{p}_2), \max(\tilde{p}_2)]$. When $p_1 > E - \min(\tilde{p}_2)$, the realized credit losses are such that even the smallest possible loss in period 2 would wipe out the entire bank's capital. In the remaining, we refer to $p_1 = E - \min(\tilde{p}_2)$ as the *"insolvency point."* If, on the other hand, the bank's capital can cover for the largest possible loss in period 2, given p_1, then the bank will be solvent for sure at the end of the year. In between these two extremes, the likelihood of the bank being insolvent in period 2 is $1 - F_2(E - p_1)$. Interim solvency risk is illustrated by the area with dashed horizontal lines in figure 2.

Figure 2. Interim Solvency Risk



3.2.2 Interim Liquidity Risk and the Rollover Decision of Short-Term Creditors

In the interim period, short-term creditors must decide whether or not to roll over their funds after observing the credit losses in the first period. Let r_s be the notional return of short-term debt from period 1 to period 2. The total interim return of rolling over, conditional on there not being a run, is therefore

$$r_s(1 - \mathcal{N}_1(p_1)) = \begin{cases} 0, & \text{if } p_1 > E - \min(\tilde{p}_2) \\ r_s F_2(E - p_1), & \text{if } E - \max(\tilde{p}_2) < p_1 \leq E - \min(\tilde{p}_2) \\ r_s, & \text{if } p_1 \leq E - \max(\tilde{p}_2), \end{cases} \tag{4}$$

There is a coordination problem among short-term debt holders in the interim period, and a key variable in their decision to roll over or not will be their beliefs about the proportion of other short-term creditors expected to roll over. It is easy to verify that the global game version of our model satisfies the necessary conditions for the existence of a unique equilibrium. This equilibrium is the same as the one obtained under the assumption that each short-term creditor believes that the proportion of short-term creditors not rolling over is uniformly distributed on the interval [0,1].[9] A successful run will not

[9]Note that the uniformity assumption for $F_2(\cdot)$ is a sufficient but not necessary condition for the assumption made on the short-term creditors' beliefs to hold.

occur if this proportion is smaller than the liquidity ratio λ, and each short-term creditor will assume this to happen with probability λ.[10]

Given those beliefs, the expected return to rolling over is $\lambda r_s F_2(E - p_1)$. Assume now that short-term creditors have an alternative investment opportunity in which they can earn gross return r^*. Since both the likelihood of not having a successful run, (λ), and the probability of being solvent, $F_2(E - p_1)$, are decreasing with p_1, a successful run will occur for values of p_1 above a *"run point,"* p_1^*, at which the expected return to rolling over is just equal to the creditor's opportunity cost, r^*:

$$\lambda(p_1^*) r_s F_2(E - p_1^*) = r^*. \tag{5}$$

Since both $\lambda(\cdot)$ and $F_2(\cdot)$ are decreasing functions of p_1, it is easy to verify from equation (6) that the *"run point"* is an increasing function of bank's capital (E), liquid asset holdings (M), and the return on short-term debt (r_s), and a decreasing function of the amount of short-term funding (S) and the opportunity cost of short-term creditors (r^*). Define $\mu \equiv r^*/r_s$.[11] The run point can be expressed as

$$p_1^* = E - F_2^{-1}(\mu/\lambda) = E - F_2^{-1}\left(\frac{r^* * S}{r_s[M + \psi(Y - p_1^*)]}\right) \tag{6}$$

and interim liquidity risk is equal to[12]

$$\mathcal{L}_1(p_1) = \begin{cases} 0, & \text{if } p_1 < p_1^* \\ F_2(E - p_1), & \text{if } p_1^* \leq p_1 \leq E - \min(\tilde{p}_2) \\ 0, & \text{if } p_1 > E \min(\tilde{p}_2). \end{cases} \tag{7}$$

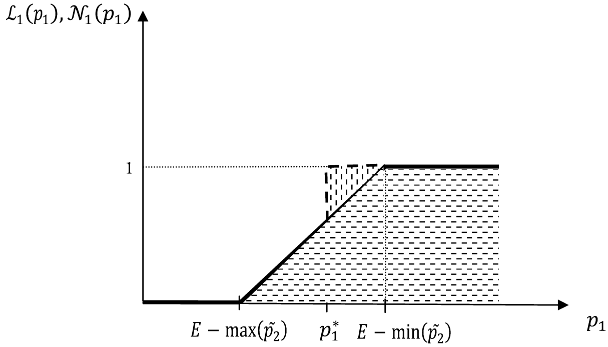
See MS for a sketch of how results from the global games literature provide a foundation for the assumption made about the short-term creditors' beliefs, and Morris and Shin (2003) for a fully rational foundation.

[10] Note that for $\lambda \geq 1$, there will always be enough liquidity to cover for short-term funding withdrawal, and a successful run will not occur with probability 1. We concentrate in this section on cases where a run is possible and relegate singular cases to footnotes.

[11] The run point is a function of the exogenous ratio μ. Therefore, it is not affected by changes in interest rates that would keep this ratio fixed.

[12] If $\lambda > 1$, or $p_1^* > E - \min(\tilde{p}_2)$, the liquidity risk is 0 and hence $\mathcal{C}_1(p_1) = \mathcal{N}_1(p_1)$.

Figure 3. Interim Liquidity Risk



For values of p_1 smaller than the *"run point,"* the expected return to rolling over is higher than the opportunity cost and liquidity risk is 0. When $p_1 > E - \min(\tilde{p}_2)$, i.e., the observed credit losses in the interim period are sufficiently large, the institution will be bankrupt for sure even without any run by the creditors. Hence, solvency risk is 1 and, by definition, liquidity risk is zero.[13] For values of p_1 sufficiently close to the *"insolvency point,"* the bank defaults because of a run by its short-term creditors, whereas it would have been solvent in the absence of a run. Liquidity risk is illustrated by the area with dashed vertical lines in figure 3.

The characterization of interim liquidity risk in (8) ignores the case where $\mu/\lambda > 1$, which implies maximal liquidity risk or $L_1(p_1) = 1$. Indeed in that case, the opportunity cost of the creditors is higher than the expected return to rolling over irrespective of the losses incurred in period 1 or the level of capital. This case is more likely when the reliance on short-term funding is high, liquid asset holdings are low, or the fire-sale price of illiquid assets (ψ) is low.

Summing the solvency and liquidity risk gives the interim (total) credit risk:

$$\mathcal{C}_1(p_1) = \begin{cases} 0, & \text{if } p_1 \leq E - \max(\tilde{p}_2) \\ 1 - F_2(E - p_1), & \text{if } E - \max(\tilde{p}_2) < p_1 < p_1^* \\ 1, & \text{if } p_1 \geq p_1^*. \end{cases} \tag{8}$$

[13] Note that it is also possible that $p_1^* > E - \min(\tilde{p}_2)$, in which case $\mathcal{L}_1(p_1) = 0$ for the same reason.

3.2.3 Ex Ante Risks

Given that the distribution of credit losses in period 1, $f_1(\cdot)$, is known at period 0, the ex ante solvency risk (i.e., the expectation at $t = 0$ of being insolvent at $t = 1$) is

$$\mathcal{N}_0 = \int \mathcal{N}_1(p_1) f_1(p_1) dp_1, \tag{9}$$

that is,

$$\mathcal{N}_0 = \int_{E-\max(\tilde{p}_2)}^{E-\min(\tilde{p}_2)} (1 - F_2(E - p_1)) f_1(p_1) dp_1 + \int_{E-\min(\tilde{p}_2)}^{\max(\tilde{p}_1)} f_1(p_1) dp_1. \tag{10}$$

Similarly, the ex ante liquidity risk is

$$\mathcal{L}_0 = \int \mathcal{L}_1(p_1) f_1(p_1) dp_1 = \int_{p_1^*}^{E-\min(\tilde{p}_2)} F_2(E - p_1) f_1(p_1) dp_1. \tag{11}$$

For given $F_2(\cdot)$, $f_1(\cdot)$, and E, the ex ante liquidity risk $\mathcal{L}_0$ is a decreasing function of the run point p_1^* and is maximized when $\mu/\lambda > 1$.[14]

The ex ante total credit risk $\mathcal{C}_0 = 0$ is simply the sum of $\mathcal{N}_0$ and $\mathcal{L}_0$:

$$\mathcal{C}_0 = \int_{E-\max(\tilde{p}_2)}^{p_1^*} (1 - F_2(E - p_1)) f_1(p_1) dp_1 + \int_{p_1^*}^{\max(\tilde{p}_1)} f_1(p_1) dp_1. \tag{12}$$

3.3 Network of Exposures between Banks

Our framework is designed to include network externalities caused by counterparties' default (recall the right-hand box before the calculation of aggregate losses in figure 1). Defaulting banks, either because of insolvency or illiquidity, are unable to fully honor their interbank liabilities, potentially causing, in turn, the default of other

[14]If $\lambda > 1$, or $p_1^* > E - \min(\tilde{p}_2)$, the interim liquidity risk is always 0 and hence $\mathcal{L}_0 = 0$ and $\mathcal{C}_0 = \mathcal{N}_0$; if $\mu/\lambda > 1$, we have $\mathcal{L}_0 = 1$ and $\mathcal{C}_0 = 1$.

banks.[15] We model these network externalities explicitly by clearing the interbank market and identifying banks that are in spillover default.[16] The clearing is done by extending the model of Eisenberg and Noe (2001) to include bankruptcy costs and uncertainty as in Elsinger, Lehar, and Summer (2006).[17]

To model the network of interbank obligations, we distinguish between claims outside the banking system and those to counterparties inside the banking system. Consider a set $\Lambda = \{1, \ldots, N\}$ of banks. Each bank $i \in \Lambda$ has a claim on specific assets A_i outside of the banking system, which we can interpret as the bank's portfolio of non-bank loans and securities. Each bank is partially funded by issuing senior debt or deposits, D_i, to outside investors. Bank i's obligations against other banks $j \in \Lambda$ are characterized by the nominal liabilities x_{ij}.

The total value of a bank after credit losses is the value of its net assets minus the outside liabilities, $A_i - p_{1i} - p_{2i} - D_i$, plus the value of all net payments to and from counterparties in the banking system. We denote by $d \in \mathbb{R}_+^N$ the vector of total obligations of banks toward the rest of the system, i.e., $d_i = \sum_{j \in \Lambda} x_{ij}$. We define a new matrix $\prod \in [0,1]^{N \times N}$ which is derived by normalizing x_{ij} by total obligations.

$$\pi_{ij} = \begin{cases} \frac{x_{ij}}{d_i}, & \text{if } d_i > 0 \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

The net value of bank i, assuming all interbank claims are paid in full, is

$$A_i - p_{1i} - p_{2i} + \sum_{j=1}^{N} \pi_{ji} d_j - D_i - d_i. \tag{14}$$

where the summation term represents the total interbank assets of bank i. We define a *clearing payment vector* X^* which has to respect

[15]The capital threshold for banks to default on their counterparty exposures is set at zero in this paper but can be set at a higher level to reflect liquidity hoarding in stressed periods.

[16]Jorion and Zhang (2009) provide empirical evidence of contagion from counterparty risk for a sample of U.S. banks and corporations.

[17]See Eisenberg and Noe (2001) for a proof of the existence and uniqueness of the clearing payment vector.

limited liability of banks and proportional sharing in case of default. It denotes the total payments made by the banks under the clearing mechanism. Each component of X^* is defined by

$$x_i^* = \min\left[d_i, \max\left(A_i - p_{1i} - p_{2i} - \phi_i + \sum_{j=1}^{N} x_j^* \pi_{ji} - D_i, 0\right)\right], \tag{15}$$

where the proportional bankruptcy cost is similarly applied to all types of bankruptcies and is defined as

$$\phi_i = \begin{cases} \psi_i y_i, & \text{if } p_{1i} + p_{2i} > E_1 \text{ or } \mathcal{L}_1(p_{1i}) > 0 \text{ or } x_i^* < d_i \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

4. Simulations

In this section, we show how this framework can be used to assess risks in the banking sector and address questions related to prudential policy. We first calibrate the model and then present the results from three applications.

4.1 Calibration of the Framework

As mentioned above, our framework requires the design of a severe but plausible scenario.[18] Following Gauthier, Lehar, and Souissi (2012), we use a scenario with a recession that is about 20 percent larger than the one experienced in 2009. We simulate sectoral distributions of 10,000 default rates for 2009:Q2 under this scenario. Descriptive statistics of these distributions as well as historic peaks over the 1988–2012 period are presented in table 1. Consistent with the severity of the macro scenario, mean default rates are much higher than historic peaks. Default rates in the tail of the distributions are still higher.

[18] As a means to interpret the stress scenario considered here, Canadian banks recorded about \$22 billion of trading book losses from the onset of the sub-prime crisis to the third quarter of 2009. This is comparable with the range of credit losses assumed in table 2 below.

Table 1. Summary Statistics of Simulated Default Rate Distributions for 2009:Q2

	Minimum	Average	Maximum	Historic Peaks
Accommodation	3.0	11.7	21.0	7.6
Agriculture	1.0	1.7	2.0	0.8
Construction	2.0	6.4	10.0	3.3
Manufacturing	5.0	12.2	20.0	8.3
Retail	0.0	4.3	8.0	5.3
Wholesale	2.0	7.0	12.0	4.6
Mortgage	0.0	0.6	1.0	0.6

Source: Gauthier, Lehar, and Souissi (2012).
Notes: Columns 2–4 show the minimum, maximum, and average default rates generated for each sector. Column 5 gives the historic peak over the 1988–2012 period.

For given default rates, a portfolio with a higher number of large exposures will experience higher expected losses. In order to capture idiosyncratic risk factors arising from the size distribution of banks' exposures, we estimate banks' loan portfolio compositions by sector using the Bank of Canada Banking and Financial Statistics, and private data on banks' largest exposures towards non-banks obtained from the Office of the Superintendent of Financial Institutions (OSFI). For each bank, we draw 100 independent loan sizes for each of the 10,000 sectoral default rates simulated previously, yielding a total of one million sectoral loan loss scenarios. The individual distributions of total expected losses are derived by adding expected sectoral loan losses.[19]

Table 2 presents the main features of the aggregate loss distribution in 2008:Q4 (i.e., the sum of p_1 across banks). It illustrates the importance of considering both sources of uncertainty. When considering systematic factors only, expected losses to the six major banks average \$22.8 billion, or 23.8 percent of total tier 1 capital, with a standard deviation of \$4.2 billion. Taking both systematic and idiosyncratic factors into account, the expected losses are approximately the same (\$23.2 billion on average) and, not surprisingly, are larger

[19]These losses are derived assuming 50 percent loss given default.

Table 2. Aggregate Losses Conditional on Extreme Stress Scenario

	Systematic Factors		Systematic and Idiosyncratic Factors	
	$Billion	% of Tier 1 Capital	$Billion	% of Tier 1 Capital
Mean	−22.8	23.8	−23.2	24.3
Standard Deviation	3.9	4.2	4.8	4.9
Quantiles				
99%	−13.6	14.2	−12.8	13.4
10%	−27.9	29.2	−29.4	30.7
1%	−31.8	33.3	−34.3	35.9

Source: Gauthier, Lehar, and Souissi (2012).
Notes: The table provides descriptive statistics of aggregate losses (sum of p_1 across banks) considering systematic factors only (columns 2 and 3) and both systematic and idiosyncratic factors (columns 4 and 5).

in the tail of the distribution (the 99 percent Value-at-Risk (VaR) is \$34.3 billion as compared with \$31.8 billion in the first distribution).

We assume that losses in the second half of the simulation horizon (p_2) follow a uniform distribution on the support $[\min(\tilde{p}_1), \max(\tilde{p}_1)]$. This reflects heightened uncertainty about the quality of banks' assets as the crisis unfolds, and conveniently ensures that the short-term creditors deciding to run or not will have the uniform beliefs hypothesized above. Our measure of capital is tier 1 and is calibrated as of 2008:Q2. It averages 10 percent across banks. Bankruptcy costs are set at 10 percent of the failed banks' assets.[20]

We also calibrate the key determinants of funding liquidity risk (M, S, r_s, r^*, and ψ) at the interim date (2008:Q4), using public information on banks' balance sheets and other available market data. We define M as the sum of banks' holdings of cash and

[20]Bris, Welch, and Zhu (2006) find that bankruptcy costs are considerable for a wide sample of corporate bankruptcies in the United States, sometimes amounting to up to 20 percent of assets. Meanwhile, for the banking sector, James (1991) estimates direct bankruptcy costs to be equivalent to approximately 10 percent of the failed bank's assets.

government securities. Data come from banks' monthly balance sheets reported to the OSFI. On average, liquid holdings of the Big Six Canadian banks were around 13 percent of total assets and as much as 23 percent for one bank.

To estimate the amount of funding that comes due at the interim date (S), we consider both short- and long-term liabilities regardless of their original contract maturity.[21] The only source of data that covers all types of funding liabilities is banks' annual reports. Assuming that banks constantly manage short- or long-term debt to keep their mix constant, we use the information reported by banks at the end of 2007. It covers funding liabilities with a maturity of less than three months—such as bankers' acceptances (BAs), repurchase agreements (repos), and personal and non-personal on-demand and notice deposits—and other fixed-term liabilities maturing in less than twelve months. For liabilities maturing in less than three months, we assume that the amount to roll over in 2008:Q4 is the total reported. For the longer-term liabilities, we assume that half of the reported amount matures in 2008:Q4.

We present descriptive statistics in table 3. Under the assumptions outlined above, total funding of the Big Six Canadian banks maturing at the interim date was significant, representing close to 47 percent of total liabilities or 64 percent of total banks' funding on average.[22] Funding liabilities with a maturity of less than three months is the most important source of major Canadian banks' funding (around 43 percent of banks' total funding on average). For illustrative purposes, we include personal deposits and repos as potential sources of run even though they may be considered as stable sources of funding. This could be seen as a worst-case scenario.[23] We relax

[21]We assume for simplicity that M, Y, S, L, and E stay constant over the simulation horizon.

[22]It remains quite large even when personal source of funding is excluded (more than 30 percent of total liabilities or 42 percent of total funding on average).

[23]Despite the availability of government deposit insurance in Canada (up to $100,000), one can argue that retail depositors might run due to concerns about the immediate availability of their funds in the event of a bank default. As for Canadian repo markets, which ceased to function at one point during the crisis except for overnight maturities, we could foresee their total disruption should central banks decide not to provide extraordinary liquidity facilities.

Table 3. Summary Statistics on Liabilities Maturing at the Interim Date for the Big Six Canadian Banks

	As Percentage of Total Funding	As Percentage of Total Liabilities		
	(Average)	Minimum	Average	Maximum
A. Funding Liabilities with a Maturity Less than Three Months				
Repurchase Agreements	8.0	1.8	5.9	8.5
Bankers' Acceptance	3.7	2.0	2.7	3.6
On-Demand and Notice Deposits				
Personal	15.8	8.4	11.5	12.7
Wholesale	15.3	9.6	11.1	14.4
Total	42.8	24.0	31.2	37.3
B. Fixed-Term Funding Liabilities Maturing in Less than Twelve Months				
Personal Deposits	5.6	2.5	4.1	5.7
Wholesale Deposits	15.1	7.9	11.1	13.2
C. Total Funding Assumed to Roll Over at the Interim Period				
Total	63.5	37.9	46.4	50.4
Excluding Personal	42.1	25.3	30.8	35.9

Notes: Column 2 reports the average amounts across banks as a percentage of total funding. Columns 3–5 report minimum, average, and maximum amounts of funding as a percentage of total liabilities. Panel A gives the amount of funding with a maturity less than three months. Panels B and C provide the fixed-term funding maturing and total funding, respectively.

this assumption in the last simulation exercise, where we consider different levels of stable short-term funding.

For each bank, the cost of short-term funding, r_s, is calibrated as the average cost of the short-term liabilities described above weighted by their respective size. The calibrated values range between 2.7 and 3.3 percent.[24] The opportunity cost (r^*) is calculated as the average six-month Government of Canada Treasury-bill rates in 2008:Q4. The calibrated value is 1.57 percent.

The fire-sale price of illiquid assets, ψ, is derived as

$$\psi = \sum_i \psi_i * y_i, \tag{17}$$

where ψ_i is the fire-sale price of illiquid asset y_i. Three types of illiquid assets are considered: securities, loans, and others (including derivatives). Individual banks' security holdings come from 2008 banks' annual reports, and fire-sale prices are obtained from table 1 in Committee on the Global Financial System (2010). Loans and derivatives are assumed totally illiquid (zero fire-sale price).[25] The calibrated ψ is 25 percent for the average bank.

As for the calibration of the network, we use actual exposures among the Big Six Canadian banks as of 2008:Q2. This includes an extended set of exposures that go beyond those covered in the related literature, which are generally limited to traditional lending (Upper 2011). In addition to traditional lending (unsecured interbank loans and deposits), our set includes interbank exposures arising from cross-shareholdings, and exchange-traded and over-the-counter (OTC) derivatives.[26] Data on deposits and unsecured loans

[24]The cost of short-term deposits is calculated from the banks' financial statements. The BA rates are based on data from the trading room of the Bank of Canada, which are collected from market dealers. The repo rates are calculated as the Canadian Overnight Repo Rate Average (CORRA) from the Bank of Canada database.

[25] Different assumptions on banks' liquid holdings are considered below.

[26]Derivatives exposures are netted, but loans and other non-derivatives exposures are gross values. Zero-risk exposures such as repo-style transactions are excluded. Owing to data limitations, the data set does not cover the exposures from intraday payment and settlement, from bank holdings of banks' preferred shares (and other forms of capital), and from holdings of debt instruments issued by banks like debentures and subordinated debt.

come from the banks' monthly balance sheet reports to OSFI. These monthly reports reflect the aggregate asset and liability exposures of a bank for deposits, and only aggregate asset exposures for unsecured loans. Data on exposures related to derivatives come from a survey initiated by OSFI at the end of 2007. In that survey, banks are asked to report their 100 largest mark-to-market counterparty exposures that were larger than $25 million. These exposures were related to both OTC and exchange-traded derivatives. They are reported after netting and before collateral and guarantees.[27] The reported data are used to construct a matrix of Big Six banks' bilateral exposures. Data on cross-shareholdings exposures were collected from Bank of Canada's quarterly securities returns.[28] We present descriptive statistics in table 4.

A full description of linkages between Canadian banks requires a complete matrix of the bilateral exposures. Such a complete matrix was available only for exposures related to derivatives. Unavailable bilateral exposures were estimated under the assumption that banks spread their lending and borrowing as widely as possible across all other banks using an entropy-maximization algorithm. A difficulty with this solution is that it rules out the possibility of relationship banking, i.e., a bank preferring some counterparties to others (as reflected in the structure of banks' exposures related to derivatives). Banks' bilateral exposures were also estimated under the assumption that concentrations of exposures between banks are broadly consistent with their asset sizes. As shown in table 4, banks' bilateral exposures are comparable under these two assumptions. Indeed, they are consistent with the concentration of bilateral exposures related to derivatives.

[27]The derivatives exposures reported may be biased upward, since they were reported before collateral and guarantees. In particular, anecdotal evidence suggests that the major Canadian banks often rely on high-quality collateral to mitigate their exposures to OTC derivatives.

[28]These returns provide for each bank aggregate holdings of all domestic financial institutions' shares. Due to data limitations, cross-shareholdings among the Big Six banks were estimated by (i) distributing the aggregate holdings of a given bank according to the ratio of its assets to total assets of domestic financial institutions, and (ii) excluding shares that were held for trading (assuming that they are perfectly hedged).

Table 4. Summary Statistics on Exposures between Canadian Banks

	Aggregate Exposure ($Billion)	As Percentage of Tier 1 Capital		
		Minimum	Average	Maximum
A. Aggregate Exposures between Canadian Banks				
Traditional Lending Lending	12.7	5.25	16.3	38.6
Derivatives Exposures	5.4	0.0	5.9	21.1
Cross-Shareholdings	3.5	0.3	4.1	8.8
Total Exposures	21.6	5.5	26.4	51.2
B. Banks' Bilateral Exposures as a Percentage of Tier 1 Capital				
Assumption:				
Entropy Maximization		0.6	4.4	15.6
Relationship Banking		0.5	4.4	16.2

Source: Gauthier, Lehar, and Souissi (2012).
Notes: Panel A gives the aggregate size of interbank exposures related to traditional lending, derivatives, and cross-shareholdings (reported in $billion and as a percentage of banks' tier 1 capital). Panel B gives banks' bilateral exposures as a percentage of tier 1 capital under two assumptions: entropy maximization and relationship banking.

The aggregate size of interbank exposures was approximately $21.6 billion for the six major Canadian banks. As summarized in table 4, total exposures between banks accounted for around 25 percent of bank capital on average. The available data suggest that exposures related to traditional lending (deposits and unsecured loans) were the largest ones compared with mark-to-market derivatives and cross-shareholdings exposures. Indeed, in May 2008, exposures related to traditional lending represented around $12.7 billion on aggregate and 16.3 percent of banks' tier 1 capital on average. Together, mark-to-market derivatives and cross-shareholdings represented 10 percent of banks' tier 1 capital on average.

Table 5. Bank Defaults in the 2008:Q2 Calibrated Case

Bank	Solvency PD ($\mathcal{N}_0$) (%)	Liquidity PD ($\mathcal{L}_0$) (%)	Contagious PD (%)	Total PD(%)
1	0	0	0	0
2	0.0023	0	0	0.0023
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	0.0010	0	0	0.0010

Notes: Column 2 shows the individual probabilities of default due to solvency risk. Column 3 shows the PDs due to liquidity risk, and column 4 the PDs due to interbank contagion. Total PD is the sum of columns 2, 3, and 4.

4.2 Assessment of Risks in the Canadian Banking System at the Height of the Sub-Prime Crisis (2008:Q2–2009:Q2)

As a first exercise, we estimate the different types of risk in the Canadian banking system at the height of the sub-prime crisis. We focus on the six major Canadian banks which represent more than 90 percent of total banking assets. The simulation horizon is the 2008:Q2–2009:Q2 period. Using the notation of section 3, 2008:Q2 is period 0, 2008:Q4 is the interim period where creditors decide whether or not to roll over their claims, and 2009:Q2 is period 2.

We decompose the bank's total risk into solvency, funding liquidity, and network spillover risks (table 5). Consistent with what was observed during the last crisis, our framework suggests that the Canadian banking system was very stable, with total probabilities of default (PDs) below 0.003 percent for all banks. Canadian banks were strongly capitalized and liquid at the starting point of the simulation. Consequently, solvency risk is very low for all banks. Moreover, the run point, p_1^*, above which creditors would run, is higher than the maximum possible realization of p_1 under the stress scenario, and liquidity risk is zero for all banks. Allen, Hortacsu, and Kastl (2011), who study the Canadian banks' bidding behavior at the liquidity auctions of the Bank of Canada during the financial crisis, reach a similar conclusion. Participating Canadian banks did not seem to put persistently high value on liquidity before mid-September 2008. These results suggest that the funding liquidity risk was generally low for Canadian banks prior to Lehman's collapse in September 2008.

4.3 Assessment of Risks in a Hypothetical Highly Leveraged and Illiquid Banking System

Canadian banks stood out for their robustness during the 2007–9 crisis because of their low leverage and reliance on wholesale short-term funding markets relative to their peers in Europe and the United States. As a second application of the model, we illustrate how the Canadian banking system could have been affected had Canadian banks been more highly leveraged and vulnerable to a liquidity shortfall.

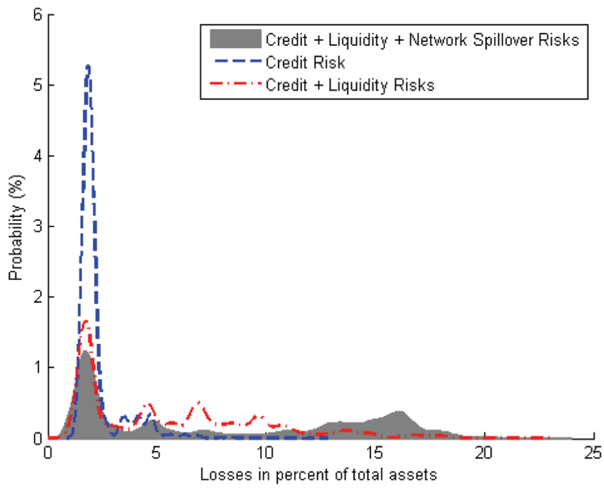
We create a hypothetical banking system made of six major banks whose main balance sheet parameters (capital level, reliance on short-term funding, and liquid asset holdings) are calibrated to levels closer to those observed in 2007 for banks that were subsequently rescued during the financial crisis. We set all banks' capital at 6 percent of risk-weighted assets, short-term liabilities at 50 percent of total liabilities, and liquid asset holdings at 10 percent of total assets.[29] We keep all other parameters at the values set in the previous exercise.[30]

Figure 4 shows the impact of the various risks considered in the framework on the distribution of aggregated losses as a percentage of total assets for this hypothetical banking system. When only the direct impact of credit risk is considered (represented by the dashed line), maximum system-wide losses are 3 percent of total assets and average losses amount to less than 2 percent of total assets. The tail is, however, significantly affected by adding funding liquidity risk to credit risk (the dashed-dotted line). The negative tail outcomes of

[29] Six percent of tier 1 capital corresponds to a leverage ratio—a measure of total assets to shareholder equity—of approximately 33, which compares with 35 for the average European bank in 2008. Fifty percent of short-term funding is much higher than what Canadian banks had at the onset of the crisis (around 30 percent) but lower than other banks such as Northern Rock (around 65 percent).

[30] Our calibration of funding costs reflects the lower short-term funding and higher capital of Canadian banks, which bias upward our measure of liquidity risk. However, we also ignore the negative impact that higher funding costs would have on capital through negative earnings. We implicitly assume that these two effects cancel out.

Figure 4. Loss Distributions of a Hypothetical Banking System for Various Groups of Risks (as a percentage of total assets)

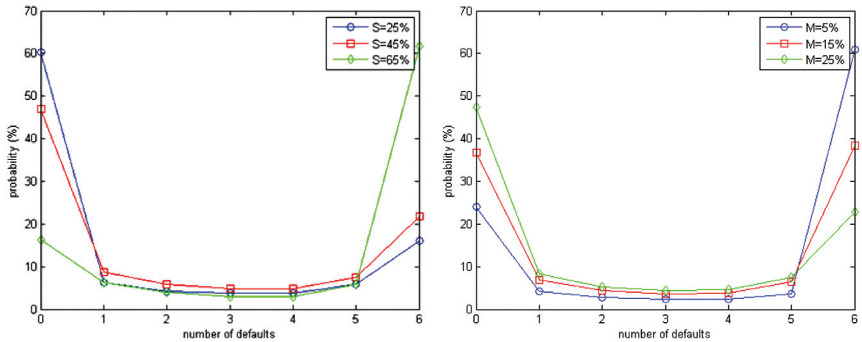


potential runs by short-term creditors are much more adverse and more likely to occur. Indeed, when factoring in liquidity risk, the likelihood of the banking system suffering losses larger than 10 percent of its total assets increases. This result demonstrates that a failure to account for liquidity risk could significantly underestimate the extent of systemic risk in an undercapitalized banking system that relies extensively on the short-term funding market.

The inclusion of interbank network spillover risks on the system-wide loss distribution also leads to multiple peaks (the shaded area). One is associated with the average direct outcome of credit losses, while the peak in the right-hand tail captures the combined impact of network spillover risks and liquidity runs.[31] The effects of network spillover risks can significantly amplify the impact of initial shocks, especially when banks are highly leveraged and dependent

[31] Alessandri et al. (2009) also obtain a system loss distribution with multiple peaks that takes into account the effects of network spillover risks and asset price feedback. When the effects of network spillover risks and liquidity risk are considered, some banks may fail. The multiple modes are driven mainly by calibrated bankruptcy costs.

Figure 5. Distribution of Bank Defaults for a Level of Capital of 6 Percent



Notes: The left panel shows distributions of multiple bank defaults for different levels of short-term financing in banks' liabilities. These distributions integrate over banks' short-term liability levels, which are assumed to follow a uniform distribution on [0.25 0.75]. The right panel plots distributions of multiple defaults for different levels of liquid asset holdings and obtained by integrating over liquid asset holdings, which are assumed to follow a uniform distribution on [0.05 0.25].

on short-term funding. This illustrates the importance of obtaining timely information on exposures among banks and suggests that current initiatives under the Basel III framework to promote greater use of central counterparties could be useful in mitigating this risk.

A different way to look at the results is through the probability of multiple bank failure. In this paper, there are two dimensions of liquidity: the reliance on short-term liability and the amount of liquid asset holdings. To show the effects of liquidity on systemic risk, as measured with the probability of multiple bank defaults, figure 5 plots the distributions of the number of defaults for a capital level of 6 percent, and for different levels of short-term liabilities (the left panel) as well as holding of liquid assets (the right panel). In the left (right) panel, the distributions are calculated by integrating over short-term liabilities which are assumed to be uniformly distributed on [0.25 0.75] (liquid asset holdings are assumed to follow a uniform distribution on [0.05 0.25]).

The left panel of figure 5 suggests that the level of short-term financing in banks' liabilities is a key driver of systemic risk. As it goes up, the probability of the whole banking system being wiped out increases significantly, while the probability of no default is reduced correspondingly. For example, increasing short-term liability from 25 percent to 65 percent raises the probability of all banks being simultaneously in default, conditional on our stress scenario, from about 15 percent to more than 60 percent, and reduces the probability of no default from about 60 percent to less than 20 percent. The right panel of figure 5 shows a similar pattern of default distribution for liquid asset holdings. The changes in the distribution of bank defaults are larger when the liquid asset holding goes up from 5 to 15 percent than from 15 to 25 percent. For example, going from 5 to 15 percent of liquid holdings decreases the probability of observing at least one bankruptcy by about 15 percent, whereas going from 15 to 25 percent decreases it by only 10 percent. Interestingly, the likelihood of observing a single or two to four simultaneous defaults seems not significantly affected by either the level of short-term liabilities or the level of liquid asset holdings.

4.4 Trade-Offs between Capital and Liquidity

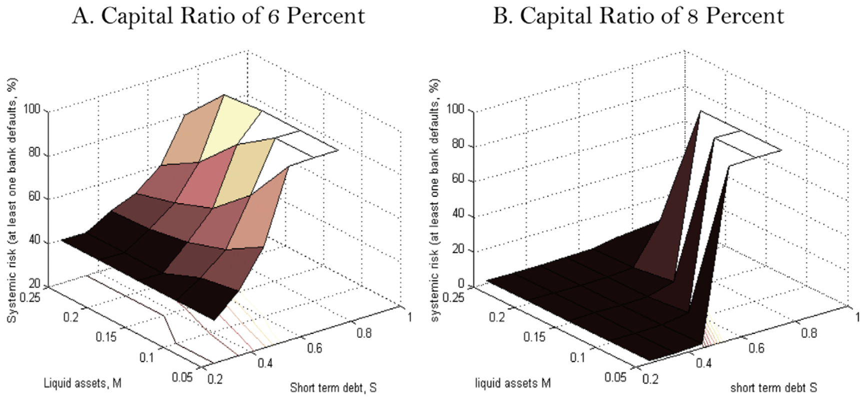
Our framework can also contribute to the current work to reform liquidity regulation by measuring the trade-offs between higher levels of capital and a more-secure funding structure in reducing systemic risk. To illustrate this, we set the parameters for capital and funding liquidity at the same values for all banks in our hypothetical system. We then let short-term liabilities (S) vary uniformly between 25 percent and 75 percent of total liabilities and allow holdings of liquid assets (M) to vary between 5 percent and 25 percent of total assets, for two different levels of bank capitalization.[32]

Figure 6 plots systemic risk—measured as the probability of having at least one bank default—as a function of M and S for capital ratio levels of 6 percent (panel A) and 8 percent (panel B).[33] As expected, systemic risk generally decreases as the capital ratio

[32] In recent years, Canadian banks, on average, have relied on unsecured short-term funding (liquid asset holdings) at levels close to the lower (upper) bound of our simulated values.

[33] Given their large size, we consider the failure of any of the Big Six banks to be a systemic event.

Figure 6. Systemic Risk for Different Capital Ratios

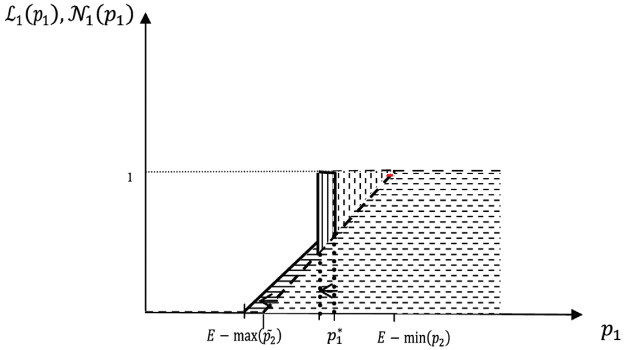


increases from 6 percent to 8 percent. For a given capital ratio, systemic risk rises as holdings of liquid assets decrease and short-term liabilities increase.

The distributions in figure 6 (panels A and B) show the relationship between systemic risk and the two dimensions of liquidity. In particular, the positive relationship between systemic risk and reliance on short-term funding is much steeper when banks have fewer liquid asset holdings, for both levels of capital: An illiquid bank is more sensitive to disruptions in short-term funding markets. Similarly, the negative relationship between systemic risk and holdings of liquid assets is more significant when banks have a greater reliance on short-term funding. Our framework allows us to assess the degree to which an increase in liquid asset holdings would offset the negative effect on systemic risk arising from increases in short-term liabilities. By demonstrating that both sufficient liquid asset holdings and the structure of funding are relevant for the containment of liquidity risk, these results provide support to the new Basel III liquidity standards.

Finally, our results suggest that a regulatory framework that takes systemic risk into account should consider capital, holdings of liquid assets, and short-term liabilities in a comprehensive manner. For example, we find that an increase in the capital ratio from 6 to 8 percent would completely eliminate systemic risk for short-term funding below 40 percent, and consequently make more liquidity.

Figure 7. Impact of Postponing the Rollover Game on Risks in the Model



This result is highly sensitive to the assumed capital threshold below which funding problems and network effects occur (zero in the current simulation). Assuming that these effects would be triggered much earlier (for capital close to the minimum requirements, for example), much more capital and liquidity would be required to eliminate systemic risk.

4.5 Sensitivity Analysis

In this section, we perform a robustness analysis to assess the sensitivity of our model to changes in the calibration of the key parameters. We first analyze the impact on banks' riskiness of a change in the interim date at which financiers can decide not to roll over their claims. We then consider international linkages of Canadian banks.

4.5.1 A Change in the Interim Date

A key parameter in the mapping of our model to the real world is the choice of the interim date, which can affect both solvency and liquidity risks. Assume, for example, that we are interested in the likelihood of a run nine months after the initial shock instead of six months as assumed in the main calibration exercise (areas with dashed lines represent risks in the base case). Impacts on solvency and liquidity risks are shown in figure 7. The amount of short-term funding to be rolled over (S) increases by the additional amount of long-term funding that would mature at the new interim date. As the run point is a decreasing function of S, liquidity risk increases (as represented by the area with solid vertical lines). Postponing the interim date will also increase the realized bank losses at the interim date, and therefore increase the likelihood that losses in period 2 would wipe out the available capital. This therefore leads to greater solvency risk (as represented by the area with solid horizontal lines). Of course, the impact on liquidity and solvency risks depends on the importance of changes in the size of interim losses relative to the pre-stress amount of capital: The larger is initial capital, the lower would be the change in risks.

To get a sense of the quantitative impact on our risk measures, we derive the results shown in table 5 (for the calibrated case of Canadian banks, section 4.2) and in figure 4 (for a hypothetical banking system, section 4.3), assuming that short-term creditors play the rollover game nine months after the beginning of the stress, and we report the results of the simulations in tables 6 and 7.

As mentioned above, postponing the interim date increases the likelihood of future insolvency of banks. In fact, the 99 percent Value-at-Risk of the interim loss distribution increases from 36 percent of tier 1 capital in the original simulation to 59 percent for the average bank. However, this has a very marginal impact on bank solvency PDs, which increase by only 0.003 percent on average. This is mainly due to the large pre-stress amounts of capital held by banks which represent three times the amount of expected losses in this simulation exercise. Note that banks 2 and 6, which have less capital than other banks (the 99 percent Value-at-Risk in the original simulation represents 44 percent of their tier 1 capital compared with an average of 36 percent for the remaining four banks), would be the most affected.

Postponing the interim date also increases the rollover risk through the amount of maturing liabilities, which increases from 46.4 percent of total liabilities to 54 percent on average. This does not trigger additional liquidity risk for most banks, however, since their capital at the interim date remains very strong. Again, the strong capitalization of banks plays a key role in determining their exposure to funding liquidity risk (even for the least capitalized bank, liquidity risk remains very low, at less than 0.01 percent).

Table 6. Bank Defaults in the Calibrated Case

Bank	Solvency PD (%)	Liquidity PD (%)	Contagious PD (%)	Total PD (%)
A. Interim Date = 2008:Q4				
1	0.000004	0	0	0.000004
2	0.002261	0	0.000006	0.002267
3	0	0	0.000001	0.000001
4	0.000016	0	0.000071	0.000087
5	0	0	0	0
6	0.000994	0	0.000049	0.001043
B. Interim Date = 2009:Q1				
1	0.000006	0	0	0.000006
2	0.015941	0.009187	0.000004	0.025132
3	0	0	0.000003	0.000003
4	0.000035	0	0.000351	0.000386
5	0	0	0	0
6	0.005370	0	0.000175	0.005545

Notes: Panel A displays banks' PDs in the original simulation. In panel B, the interim date is set at 2009:Q1 and both losses and short-term funding are adjusted accordingly. For both panels, column 2 shows the individual probabilities of default due to solvency risk. Column 3 shows the PDs due to liquidity risk, and column 4 the PDs due to interbank contagion. Total PD is the sum of columns 2, 3, and 4.

Finally, note that individual contagion PDs increase marginally following the small changes in solvency and liquidity risk. Bank 2, which is relatively more exposed to solvency and liquidity risks, triggers the network effects, which explains why its contagion PD remains constant while that of other banks increases marginally.

The bottom line is that total PD goes up marginally as the interim date is pushed forward. Since the increase in vulnerability is very small, the main result that such a calibration produces a robust financial system is maintained.

The conclusion from the simulation exercise for the hypothetical banking system remains also roughly unchanged. As the losses in the first period increase, the likelihood of banks being insolvent increases as well. This increases both solvency risk and the risk of a run. Table 7 provides details on the changes in PDs. The bottom

Table 7. Bank Defaults for the Hypothetical Banking System

Bank	Solvency PD (%)	Liquidity PD (%)	Contagious PD (%)	Total PD (%)
A. Interim Date = 2008:Q4				
1	17.8799	17.9103	7.2533	43.0435
2	18.3807	18.4312	9.7187	46.5306
3	4.776	4.8184	37.587	47.1814
4	8.6526	8.2549	31.5969	48.5044
5	3.2561	2.456	36.6908	42.4029
6	9.2986	9.1756	25.3441	43.8183
B. Interim Date = 2009:Q1 (S = 60%)				
1	20.1988	21.2707	5.0942	46.5637
2	20.3408	25.2094	8.6289	54.1791
3	6.849	11.1403	38.0953	56.0846
4	10.936	16.5608	30.1616	57.6584
5	4.6803	9.2969	37.5296	51.5068
6	12.0575	13.5722	26.8871	52.5168

Notes: Panel A displays banks' PDs in the original simulation (whose results were illustrated in figure 4 of the paper). In panel B, the interim date is set at 2009:Q1 and both losses and short-term funding are adjusted accordingly. For both panels, column 2 shows the individual probabilities of default due to solvency risk. Column 3 shows the PDs due to liquidity risk, and column 4 the PDs due to interbank contagion. Total PD is the sum of columns 2, 3, and 4.

line remains that the overall results of the paper are not dramatically sensitive to the change in the interim date.

4.5.2 International Linkages

In the paper so far, Canadian banks are subject only to a shock to their domestic loan portfolios. The shock could also have a different origin, such as a failure of a bank outside Canada. The possibility of a shock from outside the banking system is certainly an important risk factor for financial stability in any country and especially in Canada.

As mentioned above, our framework can easily accommodate any type of initial shock as long as the impact of that shock can be

Table 8. Bank Defaults in the 2008:Q2 Calibrated Case

Bank	Solvency PD (%)	Liquidity PD (%)	Contagious PD (%)	Total PD (%)
1	0.02	0.00	0.00	0.02
2	0.15	0.01	0.03	0.19
3	0.34	0.09	0.17	0.60
4	0.11	0.03	0.12	0.26
5	0.01	0.00	0.06	0.07
6	1.04	0.00	0.13	1.17

Notes: Banks are assumed to incur additional losses from their exposures to foreign banks: 25 percent of these exposures are lost. Column 2 shows the individual probabilities of default due to solvency risk. Column 3 shows the PDs due to liquidity risk, and column 4 the PDs due to interbank contagion. Total PD is the sum of columns 2, 3, and 4.

translated into an impact on banks' capital. As an additional robustness check, we expose banks to an additional shock of foreign bank defaults. We collect data on Canadian banks' deposits with foreign banks and assume that 25 percent of them are completely lost. This assumption is relatively harsh and contrasts with our approach in the main part of our analysis, where we model loan losses as probabilistic and assume a loss given default of 50 percent. Also, it is very unlikely that Canadian banks have such a high exposure to a country, and that enough foreign banks default to cause a 25 percent loss in foreign interbank deposits. The results of this simulation are reported in table 8. In this scenario, the probability of default due to insolvency increases to 0.28 percent on average, with banks 3 and 6 being the most affected due to their relatively large exposures to foreign banks. This explains why these banks also have the largest probabilities of default due to illiquidity.

5. Conclusion and Future Work

We propose a framework for assessing systemic risk. Our main contribution is to build on recent theoretical literature to integrate funding liquidity risk as an endogenous outcome of the interaction between market liquidity risk, solvency risk, and the structure of banks' balance sheets.

We find that the failure to account for network effects and liquidity risk would significantly underestimate the extent of systemic risk in the financial system. Our calibration exercise highlights the vulnerability of leveraged institutions to the combination of low cash holdings and the prevalence of short-term debt funding, a key feature of the recent global financial crisis.

Moreover, we measure the extent to which higher capital, more liquid assets, and/or less short-term liabilities induce lower systemic risk in the financial system and find support for the broad approach contained in Basel III's new liquidity standards. A regulatory framework that properly controls for systemic risk should consider capital, liquid asset holdings, and short-term liability in a holistic way. Treating any of them in isolation would produce a misleading assessment of systemic risk and hence impair the effectiveness of the regulatory policies.

In its current form, our framework could be used to address various policy questions, such as the impact of central bank interventions on systemic risk during periods of financial stress. Central bank liquidity facilities could reduce the discount on illiquid assets, which would in turn reduce funding liquidity and systemic risks. Other policy topics include the measurement of the rate of exchange between regular and contingent capital, and whether bank size is an ideal determinant of a capital surcharge for systemically important financial institutions (see Gauthier et al. 2012).

The framework can also be extended in different directions. Work is under way to introduce the potential for the contagion among banks in short-term funding markets owing to negative information about other financial institutions. Another challenging extension of the model would be to include any negative feedback that could occur between heightened risks to the banking system and the real economy.

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