Countering Appreciation Pressure with Unconventional Monetary Policy: The Role of Financial Frictions*

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In a simple two-country framework with imperfect financial intermediation we analyze and compare the effectiveness of two unconventional monetary policy measures: foreign exchange interventions and credit easing. Central bank interventions only have real effects when banks are financially constrained. For a country facing excess demand for its bonds, we study three external sources of appreciation pressure: increased financial frictions in the international credit market, an increase in capital inflows, and increased financial frictions in the foreign investment market. Only in the first two cases, foreign exchange interventions can reverse the appreciation and the resulting misallocation of capital. Under certain conditions, credit easing is a substitute for foreign exchange interventions.


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1. Introduction

In response to the global financial crisis of 2007–09, the world’s leading central banks lowered their policy rates to very low levels. However, the liquidity they provided did not reach the private sector because financial intermediation was severely disrupted. At the same time, economies like Switzerland, Denmark, and Israel were facing intense appreciation pressure on their currencies, which meant a further tightening of monetary conditions. With low short-term interest rates proving to be of limited effectiveness and policy rates close to the effective lower bound, their central banks started engaging in unconventional monetary policy.

For a central bank wishing to dampen appreciation pressure on its currency by using unconventional policy tools, a number of questions arise before it can take action: What is the source of the appreciation? What tools would be effective at reducing the appreciation pressure in the first place? And if there are several tools, which one would be most suitable? In this paper, we propose a simple framework to address these questions. We extend a simplified, real version of the closed-economy financial frictions model of Gertler and Karadi (2013) by adding an open-economy dimension and by incorporating the idea of Gabaix and Maggiori (2015) that financial frictions lead to deviations from interest parity. We focus on two of the most prominent unconventional measures: foreign exchange (FX) interventions and credit easing (CE), i.e., a special form of quantitative easing (QE).

We examine under what conditions a central bank can use these two tools to reduce financial frictions and, in particular, respond to appreciation pressure.

There already exists substantial research on unconventional monetary policy tools. On the theoretical side, both in the literature on asset purchase programs and in the literature on FX interventions, there are models that introduce financial frictions as limited commitment of financial intermediaries. In the past few years, financial

\[1\] Credit easing (central bank purchases of bonds issued by private sector borrowers) is a special form of QE if it involves increasing the monetary base. However, the intentions of credit-easing programs differ from those of QE programs. The goal of credit easing is to change the asset composition on the central bank’s balance sheet in order to reduce specific interest rates or restore the functioning of specific markets (see Bernanke 2009).
Frictions and imperfect financial intermediation have become an important modeling tool, since they are considered to be crucial for the strong spread of the 2007–09 financial crisis. They provide a plausible explanation for credit spreads and deviations from interest parity and generate a portfolio balance channel through which QE/CE and FX interventions are effective. The key feature for the portfolio balance channel to work is imperfect substitutability between the assets the central bank purchases and those that it uses to finance these purchases. In QE/CE models, different types of domestic assets are modeled as imperfect substitutes, while in FX interventions models, domestic and foreign assets are imperfectly substitutable. Despite the similarities in the literature on QE/CE and FX interventions, what is missing to our knowledge is a framework in which the two tools are directly compared. Our paper presents a first contribution to fill this gap.

From an empirical point of view, there are many arguments in favor of studying the effectiveness of foreign exchange interventions and credit easing in a unified framework. Crisis times tend to be reflected in both the foreign exchange market and credit markets. During periods of high uncertainty, credit spreads tend to widen (see, for example, Kwon 2020). At the same time, an increase in uncertainty tends to trigger exchange rate movements. Safe-haven currencies like the U.S. dollar (USD) and the Swiss franc, for example, typically appreciate and see an increase in their safety premia, indicating an increase in the deviation from uncovered interest parity (see Maggiori 2013 for the USD and Leutert 2018 for the Swiss franc). As we show in Appendix A, at least in periods of high uncertainty, changes in credit spreads in the United States and in Switzerland are significantly correlated with changes in the exchange rate, suggesting that there is a co-movement between credit spreads and deviations from uncovered interest parity. This potentially tight link between the domestic credit market and the foreign exchange market has important implications for the transmission channels of unconventional monetary policies. On the one hand, there is a large literature documenting that QE/CE leads to a depreciation of the domestic currency. For the case of the European Central

For an overview of the literature, see for example Dedola et al. (2021). While most contributions assess the exchange rate effects of QE in general, Saadi Sedik,
Bank (ECB) and Federal Reserve QE and credit-easing programs, Dedola et al. (2021) find that adjustments in deviations from covered and uncovered interest parity (UIP) add to explaining this depreciation. On the other hand, empirical evidence suggests that foreign exchange interventions, in turn, have an impact on domestic credit market conditions. Fuhrer, Nitschka, and Wunderli (2021) look at the case of Switzerland where, since the global financial crisis, large-scale foreign exchange interventions have led to a strong increase in central bank reserves in the financial system. They provide evidence that this increase in reserves has lowered banks’ lending spreads. In this paper, we provide a theoretical framework that can explain both the potential co-movement between credit spreads and deviations from interest parity and the potential spillovers of unconventional monetary policies.

Banks are at the core of our model. An agency problem between borrowers and lenders generates the portfolio balance channel: Limited commitment of banks leads to an endogenous credit constraint and results in limits to arbitrage in both the domestic investment markets and the international credit market, which is reflected in excess returns on the corresponding assets. Compared with a frictionless equilibrium, capital costs in the investment markets are higher and there is a deviation from interest parity in the international credit market.

For a country facing an excess demand for its bonds, we identify three external sources of appreciation pressure related to financial frictions. The first is financial frictions in the international credit market: Banks are less able to bear exchange rate risk and to absorb the excess supply of foreign bonds resulting from trade and financial imbalances. Less intermediation in the international credit market leads to a deviation from interest parity and causes a home real appreciation. The second source of external appreciation pressure is capital inflows. If banks are credit constrained in the international credit market, they lead to an increase in the deviation from interest parity and therefore an appreciation. This is because such inflows absorb a large part of the banks’ limited intermediation capacity. The third source of external appreciation pressure is financial

Jacome H., and Ziegenbein (2018) focus on credit easing and provide evidence that it leads to a depreciation of the domestic currency.
frictions in the foreign investment market. When foreign banks are less able to intermediate funds, investment in the foreign country decreases. The higher relative level of future home output induces a permanent home appreciation but does not lead to a deviation from interest parity.

We show that within our model the home central bank can use unconventional monetary policy to reduce the appreciation pressure in the first two cases only. Both financial frictions in the international credit market and capital inflows lead to a temporary appreciation because of an increase in the deviation from interest parity, i.e., the excess return on foreign bonds. By acquiring foreign bonds and issuing domestic bonds in return, the central bank can increase overall financial intermediation and thereby help to reduce excess returns and thus bring the home country’s economy closer to the frictionless state. In contrast, since financial frictions in the foreign investment market lead to a permanent home appreciation, unconventional policy, if effective at all, can lower the appreciation today only at the price of a future appreciation.

Finally, we show that credit easing can achieve the same goal as foreign exchange interventions if banks are not only credit constrained in the international credit market but also in the domestic investment market. Interventions in one market make banks shift their assets to the other market, reducing the excess returns in both markets. In this case, central bank intervention should target the market that exhibits the highest excess returns. This ensures that the balance sheet extension of the central bank needed to reach its goal is minimized.

Most of the literature on asset purchase programs emerged after the financial crisis of 2007–09. Overall, there is broad empirical evidence that QE programs were successful at flattening the yield curve, while the precise mechanism through which they work remains unclear. Theory suggests two main channels: the signaling channel (the intervention is a signal about the future stance of monetary policy; see, e.g., Eggertsson and Woodford 2003) and the portfolio balance channel. The portfolio balance channel was first described

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3 Although the central bank extends the quantity of reserves under QE, only a few studies discuss the liquidity channel through which QE may work (see, e.g., Christensen and Krogstrup 2019).
by Tobin (1958, 1969). Since then, various theoretical foundations for imperfect asset substitutability have emerged. Gertler and Karadi (2013) follow Gertler and Karadi (2011) and Gertler and Kiyotaki (2011) by looking at QE as a form of financial intermediation, performed by the central bank. The central bank acquires assets by issuing interest-bearing short-term debt. The assets that the central bank purchases and those that it issues are imperfectly substitutable because of limits to arbitrage in private financial intermediation caused by financial frictions. Such limits to arbitrage lead to extraordinary returns on assets, and thus generate a role for central bank intermediation to drive these returns down. As the central bank can intermediate both long-term government bonds and short-term private securities, the model presents a unified approach to analyze a variety of programs used in practice. While most of the theoretical literature models asset purchase programs in closed economies, Dedola, Karadi, and Lombardo (2013) provide a two-country model to analyze the international dimension of unconventional policies in economies with financial frictions, but do not study exchange rates.

The literature on FX interventions grew rapidly after the end of the Bretton Woods system. It finds that non-sterilized interventions do have an effect on the exchange rate because they change the monetary base. The effectiveness of sterilized interventions is less clear on both theoretical and empirical grounds. Similar to the theory on QE, there are two main channels through which FX interventions may affect the exchange rate: the signaling channel and the portfolio balance channel. Early theoretical foundations for the portfolio balance channel were provided by Kouri (1976), Henderson and Rogoff (1982), and Branson and Henderson (1985). More recent advances are Kumhof (2010) and Gabaix and Maggiori (2015). The latter’s model of exchange rate determination is a modern version of the traditional portfolio balance models. It illustrates how gross capital

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4 Andrés, López-Salido, and Nelson (2004) introduce an adjustment to household preferences leading to imperfect asset substitutability between holdings of long-term bonds and money in a New Keynesian model. Similarly, using a partial equilibrium approach, Vayanos and Vila (2009) propose a model where investors have heterogeneous preferences for assets of different maturities (“preferred-habitat” motive).

5 For a literature review see, e.g., Sarno and Taylor (2001), Neely (2005), or Menkhoff (2010).
flows matter for the determination of exchange rates and what the effects of foreign exchange interventions are. Similar to Gertler and Karadi (2013), they use limited commitment of financial intermediaries to introduce financial frictions and endogenize a deviation from interest parity, reflecting a currency risk premium. Cavallino (2019) derives the optimal foreign exchange intervention policy in a New Keynesian small open economy version of the Gabaix and Maggiori (2015) model. He finds that in response to a capital inflow shock, the optimal foreign exchange intervention leans against the wind to stabilize the path of the exchange rate. In addition, using Swiss data, Cavallino (2019) provides some empirical evidence suggesting that capital inflows lead to an appreciation of the Swiss franc. While our model contains the key mechanisms of Gabaix and Maggiori (2015) and Cavallino (2019) and hence their main results regarding the exchange rate determination, capital flows, and foreign exchange interventions also hold in our setup, our framework incorporates capital and the domestic credit markets as additional elements. This allows us to study the spillovers between the foreign exchange market and domestic credit markets and include credit easing as an additional policy option.

The key mechanisms that are at work in our model are also contained in the ECB’s New Area-Wide Model II (NAWM II; see Coenen et al. 2018). In particular, large-scale asset purchases in this open-economy dynamic stochastic general equilibrium (DSGE) framework exert their influence through both a credit and an exchange rate channel. The key contributions of our paper relative to the ECB model are to provide a detailed discussion of the role of financial frictions in the exchange rate determination and the preconditions for asset purchases to be effective, and to contrast credit easing to FX interventions. In addition, we shed light on the potential importance of gross capital flows and gross foreign asset positions in the determination of the exchange rate.

The main building blocks of our model are also similar to Nuguer (2018). Proposing a two-country model with globally acting banks, she studies the international transmission of a financial crisis through the international interbank market and looks at the welfare effects of unconventional credit policies that help to mitigate the effects of a financial disruption. In contrast, in our paper, we mainly focus on the role of financial frictions in the exchange rate determination and
compare the effectiveness of foreign exchange interventions to the effectiveness of credit easing at reducing spreads in financial markets, abstracting from a DSGE framework and a welfare analysis. Yet, relative to Nuguer (2018), our model has some insightful additional dimensions. First, in her model, domestic and foreign banks trade a global bond and the global interbank market is assumed to be frictionless. In our model, we abstract from such a bond but assume that there is not only an agency problem between households and banks, but also between domestic banks and foreign banks. As a result, in our model, there is a straightforward link between deviations from interest parity and any excess supply of foreign bonds in the spirit of Gabaix and Maggiori (2015). Second, as opposed to Nuguer, we allow the friction parameters to differ across asset classes (as in Gertler and Karadi 2013), thereby capturing the fact that not all markets need simultaneously be subject to distortions. As a result, in our model, the effectiveness of different unconventional monetary policies depends strongly on the types of frictions present. Finally, in contrast to Nuguer, we allow households to hold not only domestic bonds but also foreign bonds. Thereby, our model highlights the potentially important role of gross foreign asset positions in the distortion of financial markets and, in particular, the exchange rate.

Finally, even though we use quite a different modeling approach, our model has some similar channels as Adrian et al. (2020). They have developed a New Keynesian model to assess the effects of FX interventions and capital controls. They find these two tools to be useful in improving the trade-offs of conventional monetary policy or when policy rates hit the effective lower bound. Our model can replicate the main features and predictions of their model. Yet,

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6Nuguer (2018) assumes that foreign banks can only divert assets funded by households, but not assets funded by domestic banks. This seems unrealistic. As pointed out by Kollmann (2016), the interbank market was severely impaired during the global financial crisis.

7For example, Adrian et al. (2020) assume the UIP risk premium to rise when net foreign liabilities increase—this is a key result in our model—or corporate spreads in emerging economies to rise as their currencies depreciate. In our model, for a country facing an excess supply of its bonds, increases in corporate spreads go hand in hand with a currency depreciation when there is an increase in risk perception regarding the investment market.
our setup has some additional insightful features. For instance, our model suggests that changes in the UIP risk premium may affect corporate spreads also in advanced economies (not only in emerging economies), namely through the portfolio decision made by the banking sector.\footnote{In Adrian et al. (2020), corporate spreads in emerging economies are assumed to increase as their currency depreciates. The authors motivate this feature with a balance sheet channel resulting from large foreign-currency debt.} As a result, central banks in these economies have another potential tool at their disposal when wishing to counter appreciation pressure by reducing interest rate spreads, namely credit easing. Furthermore, as opposed to Adrian et al. (2020), our model emphasizes the potential importance of gross capital flows and gross foreign asset positions in deviations from UIP and, therefore, the level of the exchange rate.

2. **Model**

Our general equilibrium model combines the main elements of Gertler and Karadi (2013) and Gabaix and Maggiori (2015). As a starting point, we take a simplified version of the closed-economy model by Gertler and Karadi (2013), breaking it down to two periods ($t = 0, 1$) and assuming a deterministic and real model environment. We extend this setup to a two-country model with one homogeneous traded good, and one non-traded good in each country. The prices of the non-traded goods act as numéros. There are households, firms, and banks. Later, we also introduce a central bank.

By choosing a real setup, we abstract from exchange rate movements stemming from monetary phenomena and nominal frictions. This allows us to study the credit channels in isolation from any other influences. Even though highly reduced in some respects, our model captures the fundamental structure of the domestic and international bond markets and thus contains the main economic intuitions.

2.1 **Households**

In each of the two countries, there is a continuum of identical households having unit mass. The representative household in the home
country works, consumes, and saves. It is endowed with traded goods in the first period and non-traded goods in both periods. It provides labor in two ways: it runs a bank and it works for the non-financial firm (in the second period only). The supply of labor to the non-financial firm \( L \) is inelastic. The household saves in the first period by transferring some exogenous amount \( N_0 \) of its tradable goods endowment as seed capital to its bank and by buying domestic bonds issued by a bank other than the one it owns. The household consumes the consumption basket \( C_t \), which is a composite of non-traded goods consumption \( C_{NT,t} \) and tradable goods consumption \( C_{T,t} \). The consumption index is of Cobb-Douglas form:

\[
C_t = \left( C_{NT,t} \chi C_{T,t} \right)^{\frac{1}{1+\chi}},
\]

where \( \chi \) is a preference parameter. We consider a log utility function. The household’s optimization problem is

\[
\max_{B_0, (C_{NT,t}, C_{T,t})_{t=0,1}} \ln C_0 + \beta \ln C_1 \quad \text{subject to (1) and}
\]

\[
P_0 C_0 + B_0 = p_0 Y_{T,0} - p_0 N_0 + Y_{NT,0}, \tag{2}
\]

\[
P_1 C_1 = R_1 B_0 + w_1 L + p_1 N_1 + Y_{NT,1}, \tag{3}
\]

where \( P_t \) is the price index, \( Y_{NT,t} \) and \( Y_{T,0} \) are the endowments of the non-traded and the traded good, and \( B_0 \) are bond holdings. Note that while these bonds represent a claim on traded goods, they are expressed in terms of the numéraire, i.e., the domestic non-traded good. \( p_t N_t \) are transfers to and from the household’s bank and taken as given by the household. \( p_t \) is the price of the traded good, \( w_1 \) is the wage, and \( R_1 \) is the gross return on bond holdings, all measured in terms of the numéraire. The price index is defined as the minimum cost of obtaining one unit of the consumption basket. Thus, given the optimal choice of \( C_{NT,t} \) and \( C_{T,t} \), total consumption expenditure is

\[
P_t C_t = C_{NT,t} + p_t C_{T,t}. \tag{4}
\]

Solving the household’s intertemporal problem yields the standard Euler condition

\[
1 = R_1 \Lambda_{0,1}, \tag{5}
\]
where $\Lambda_{0,1} = \beta \frac{P_0 C_0}{P_1 C_1}$ is the household’s intertemporal marginal rate of substitution.

Solving the intratemporal problem yields the demand functions for tradables and non-tradables:

$$C_{NT} = \frac{\chi}{1 + \chi} \left( \frac{1}{P} \right)^{-1} C,$$

$$C_T = \frac{1}{1 + \chi} \left( \frac{P}{P} \right)^{-1} C.$$  

(6)  

(7)

The foreign household is modeled in an equivalent way. It is the owner of a foreign bank and faces an identical maximization problem as the home household. Foreign country-variables will be denoted with an asterisk (*). The foreign household receives the same amount of endowment and has the same intratemporal preferences as the home household. Yet, the households in the two countries differ in one respect. To induce a trade imbalance, and hence an excess supply of the foreign country’s bonds, we assume that $\beta^* < \beta$, i.e., that the foreign household has a relatively higher discount rate. As a consequence, in an otherwise symmetric setup, in equilibrium the home country runs a trade surplus in the first period and, accordingly, there is an excess supply of the foreign country’s bonds.

2.2 Non-financial Firms

The traded good is produced by perfectly competitive firms in the second period. The representative firm in the home country operates according to the following constant-returns-to-scale technology:

$$Y_{T,1} = K_1^\alpha L_1^{1-\alpha}.$$  

(8)

Labor $L_1$ and capital $K_1$ are internationally immobile. In the first period, the firm can transform traded goods into capital and then use it for production in the second period. One unit of output invested raises capital by one unit. This process is reversible, so that a unit of capital, after having been used to produce output, can be retransformed into the tradable consumption good. The firm obtains the necessary funds for this investment by issuing investment securities $S_{p,0}$ at price $q_0$. One security finances one unit of capital, so we have $S_{p,0} = I_0 = K_1$ and $q_0 = p_0$. Given our assumption about the
capital transformation process, the price of capital is always equal to the price of output. From now on we will use \( p_t \) as the price per investment security and refer to \( K_1 \) as the total supply of investment securities.

The firm’s first-order conditions are

\[
Z_1 = \alpha \left( \frac{L_1}{K_1} \right)^{1-\alpha} p_1, 
\]
(9)

\[
w_1 = (1 - \alpha) \left( \frac{K_1}{L_1} \right)^\alpha p_1, 
\]
(10)

where \( Z_1 \) is the cost of capital to the firm, or the profit flow from a security financing one unit of capital to the holder of this security, measured in terms of the domestic numéraire. In period 1, after production, the firm is left with \((1 - \delta)K_1\) units of capital, which represent the outstanding claims of the security holders. Therefore, the rate of return on one home investment security is

\[
R_{k,1} = \frac{Z_1 + (1 - \delta)p_1}{p_0}. 
\]
(11)

Foreign firms are modeled in an equivalent way, i.e., they face the same production technology as the home firms. By assumption, the law of one price holds for the traded good, so \( p_0 = e_0 p_0^* \) and \( p_1 = e_1 p_1^* \). \( e_t \) is the real exchange rate, defined as the price of the foreign numéraire (i.e., the foreign non-traded good) in terms of home numéraire (i.e., the home non-traded good).

2.3 Banks

Banks are at the core of our model. We assume financial markets are segmented, implying that non-financial agents cannot lend funds directly to each other. Banks act as the financial intermediaries in two types of financial markets: the investment market in each country and the international credit market. In the former, banks intermediate funds between households and firms and in the latter between the agents of the two countries by financing trade imbalances (later we will also consider imbalances from financial flows). Due to an agency problem between creditors and banks, however, this financial intermediation is imperfect.
The imperfect nature of financial intermediation will become visible in the form of spreads or excess returns in the two financial markets. In our—for the sake of simplicity—deterministic setup, these spreads reflect arbitrage opportunities. In practice, though, such excess returns might rather be related to risk. For instance, excess returns in the international credit market, i.e., deviations from uncovered interest parity, are the rule rather than the exception and there is broad agreement in the literature that these deviations do not necessarily imply arbitrage opportunities, but rather reflect a fair compensation for holding currency risk (for a review of the literature, see Engel 2014).

In our model, “banks” are meant to capture the large players in the global financial market like investment banks, currency hedge funds, active investment managers, and pension funds. As stressed in Gabaix and Maggiori (2015), these institutions have in common that they often bear the ultimate risk (which arises because household medium-term currency demand is unbalanced). In this sense, the spreads in our model can be interpreted as a compensation that such institutions demand for holding currency risk. In particular, we follow Gabaix and Maggiori (2015) and refer to the deviation from interest parity as a risk premium, even though in our model it does not stem from uncertainty and is therefore not a risk premium in the traditional sense.

Yet, “true” limits to arbitrage that can cause deviations from covered interest parity are not unrealistic neither, even in the huge foreign exchange market where vast amounts of capital are around. Even the biggest players can face financial constraints, depending on their risk-bearing capacities and their existing balance sheet risks. Moreover, as noted by Shleifer and Vishny (1997), textbook-style arbitrage, i.e., involving neither capital nor risk, hardly exists in practice. Thus, in the context of the international credit market, the spreads in our model can be interpreted in either way: deviations from the covered or the uncovered interest parity.

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9The failure of covered interest parity is, for example, documented and discussed by Borio et al. (2016) and Du, Tepper, and Verdelhan (2018); deviations from covered interest parity were particularly large during the global financial crisis.
Figure 1 provides an overview of the first-period (period-0) flows in the banking sector. In each country, there is a continuum of identical banks that have unit mass. The representative home and foreign banks are modeled in a similar way. Differences between the two result from their interaction in the international credit market. This interaction in turn is a result of the trade flows (and later also portfolio flows) between the two countries. Since the domestic country runs a trade surplus in the first period, there is an excess supply of foreign bonds, i.e., claims on traded goods issued in the foreign country and denominated in the foreign numéraire. More precisely, a trade surplus in the home country and, hence, a trade deficit in the foreign country imply that foreign households consume relatively more, bringing less of their endowment of traded goods to the foreign bank to save \( D_0 > D_0^* \). As a consequence, the foreign bank has a relatively smaller amount of traded goods available to invest in local firms. To increase these investments it issues foreign bonds to the home bank and receives traded goods in exchange.

Let us now take a more detailed look at the home bank. Its balance sheet is

\[
p_0 S_{p,0} + e_0 A_{p,0} = D_0 + p_0 N_0. \tag{12}
\]

In addition to obtaining funds from households by issuing domestic bonds \( D_0 \), the bank receives seed capital from its owner, \( N_0 > 0 \). This seed capital represents the bank’s net worth in the first period.
The bank uses these funds, first, to invest in domestic firms by buying home investment securities to the amount of $p_0 S_{p,0}$ in the domestic investment market. Second, it purchases foreign bonds, issued by the foreign bank, to the amount of $e_0 A_{p,0}$ in the international credit market. All variables on the home bank’s balance sheet are measured in terms of the domestic numéraire, except for $A_{p,0}$, which is measured in terms of the foreign numéraire.

The interaction between the bank and the firm, and hence the transfer of funds from banks to firms, is frictionless. However, this is not the case for the interaction between the bank and its creditors, where financial frictions lead to imperfect financial intermediation. These frictions are the key feature of our model. To generate them endogenously, we introduce a limited commitment problem. Following Gertler and Karadi (2013), we assume that in period 0, after taking positions, the bank can choose to divert a certain fraction of its assets and transfer the proceeds to its owner. In particular, the bank can divert a fraction $\theta$ of its holdings of investment securities $p_0 S_{p,0}$ and a fraction $\Delta$ of its foreign bond holdings $e_0 A_{p,0}$.

The creditors can react by forcing the bank into bankruptcy and claiming the remaining fraction of assets. Like Gertler and Karadi (2013), we allow the friction parameters in the two markets to differ ($\Delta \leq \theta$). For instance, when $\Delta > \theta$ the bank can more easily divert foreign bonds than investment securities, and vice versa. The intuition behind this is that the performance of some assets in the bank’s portfolio may be less transparent for creditors and therefore an easier target for diversion.

Creditors anticipate the possibility of diversion and limit the amount of funds that they lend to the bank. Hence, the bank can only issue domestic bonds as long as it has no incentive to misbehave, i.e., as long as its discounted profit $V_0$, which is given up under diversion, is greater than or equal to the gain from diversion. Thus, the bank’s incentive constraint (IC) is

$$IC: \quad V_0 \geq \theta p_0 S_{p,0} + \Delta e_0 A_{p,0},$$

(13)

where

$$V_0 = \Lambda_{0,1} \left( R_{k,1} p_0 S_{p,0} + R_{1}^* e_1 A_{p,0} - R_1 D_0 \right).$$

(14)

Note that this setup of the banking sector is valid only for a non-negative excess supply of foreign bonds.
$R_{k,1}$, $R_1^*$, and $R_1$ are the gross returns on home investment securities, foreign bonds, and domestic bonds, respectively. Using balance sheet equation (12), we can rewrite the discounted profit in terms of excess returns ($R_{k,1} - R_1$) and $(R_1^* e_1 - R_1)$:

$$V_0 = \Lambda_{0,1} \left( (R_{k,1} - R_1) p_0 S_{p,0} + \left( R_1^* \frac{e_1}{e_0} - R_1 \right) e_0 A_{p,0} + R_1 p_0 N_0 \right).$$

(15)

The optimization problem of the bank is

$$\max_{S_{p,0}, A_{p,0}} V_0 \quad \text{subject to (13) and (15)}$$

and the first-order conditions are ($\lambda$ is the Lagrange multiplier attached to the incentive constraint):

$$\Lambda_{0,1} (R_{k,1} - R_1) = \frac{\lambda}{1 + \lambda} \theta,$$

(16)

$$\Lambda_{0,1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right) = \frac{\lambda}{1 + \lambda} \Delta.$$  

(17)

If the financial friction parameters ($\theta, \Delta$) are small, we are in a frictionless environment. The incentive constraint is not binding and $\lambda = 0$ because the divertable part of the bank’s assets is lower than the equity capital it would lose in case of misbehavior. Banks acquire assets up to the point where no arbitrage possibilities are left and excess returns are zero. Firms can borrow at the home interest rate, $R_{k,1} = R_1$, and the interest parity holds, $R_1 = R_1^* \frac{e_1}{e_0}$.

If $\theta$ and/or $\Delta$ are above a certain threshold, however, the incentive constraint is binding and $\lambda > 0$. We are in a model environment of financial frictions that become visible in the form of positive excess returns in the respective market(s). Compared with the frictionless equilibrium, there is less financial intermediation. The bank would like to borrow more funds and invest them in the respective market(s) to earn the excess returns, but creditors are unwilling to provide them. These limits to arbitrage lead to higher returns on securities in the home investment market, $R_{k,1} > R_1$, and to a deviation from interest parity in the international credit market,
As explained above, the latter can be interpreted as a premium that the home bank requires in order to be willing to absorb the imbalance in the international credit market and not divert any of its assets. Moreover, the size of this spread reflects the risk of shifting funds from the home to the foreign country. In this sense, the exchange rate change between periods 0 and 1 incorporates a risk premium on foreign bonds (or, equivalently, a safety premium on domestic bonds).

Note that Equations (16) and (17) imply the following no-arbitrage relation:

\[ (R_{k,1} - R_1) = \frac{\theta}{\Delta} \left( R_1^* \frac{e_1}{e_0} - R_1 \right). \]  

(18)

The households’ willingness to lend, and hence the size of the bank’s portfolio, depends not only on the fractions that the bank can divert but also on the size of the bank’s equity capital. The limited commitment of the bank generates an endogenous capital constraint (CC) (for the derivation, see Appendix B):

\[
\text{CC} = \begin{cases} 
\frac{\Delta \Lambda_{0,1} R_1}{\Delta - \Lambda_{0,1} (R_1^* \frac{e_1}{e_0} - R_1)} p_0 N_0 \geq \theta p_0 S_{p,0} + \Delta e_0 A_{p,0} & \text{if } \theta \geq 0, \Delta > 0 \\
\frac{\theta \Lambda_{0,1} R_1}{\theta \Lambda_{0,1} (R_{k,1} - R_1)} p_0 N_0 \geq \theta p_0 S_{p,0} + \Delta e_0 A_{p,0} & \text{if } \theta > 0, \Delta \geq 0 \\
\text{no CC} & \text{if } \theta = 0, \Delta = 0.
\end{cases}
\]

(19)

The left-hand side gives the bank’s net worth multiplied by some weight. The right-hand side depicts a measure of the bank’s portfolio. The weights \(\theta\) and \(\Delta\) represent the weaker (stronger) limits to arbitrage in the investment market if \(\theta < \Delta\) (\(\theta > \Delta\)), which allows the bank to acquire a relatively higher (lower) share of investment securities compared with foreign bonds in its portfolio. The higher \(N_0\), the more assets the bank can buy. Capital constraint (19) reveals that a high \(\Delta\) stands for a low ability of the bank to intermediate international funds, which, viewed from the aggregate perspective, implies a disruption in the international credit market. Likewise, a high \(\theta\) means that the bank is limited in its capacity to intermediate investment funds, reflecting a disruption in the home investment market. In this respect, we can also interpret the divertable fractions \(\theta\) and \(\Delta\) as a measure of the risk-bearing capacity of the banks. The
higher the fractions are, the lower is the banks’ risk-bearing capacity and the lower the funds that they can intermediate in the respective markets.

The setup of the foreign bank is similar to the domestic bank with some differences in the balance sheet, reflecting the interaction of the two in the international credit market. Thus, the balance sheet of the foreign bank is

\[ p_0^* S_{p,0}^* = D_0^* + A_{p,0} + p_0^* N_0^*. \]  

(20)

It can divert a fraction \( \theta^* \) of its holdings of investment securities \( p_0^* S_{p,0} \). Accordingly, the foreign bank’s optimization problem is

\[
\max_{S_{p,0}^*} V_0^* \quad \text{subject to} \quad V_0^* \geq \theta^* p_0^* S_{p,0}^* \quad \text{and} \quad V_0^* = \Lambda_{0,1}^* \left( (R_{k,1}^* - R_1^*) p_0^* S_{p,0}^* + R_1^* p_0^* N_0^* \right),
\]

(21)

(22)

which yields the following first-order condition:

\[
\Lambda_{0,1}^* (R_{k,1}^* - R_1^*) = \frac{\lambda^*}{1 + \lambda^*} \theta^*.
\]

(23)

The restriction on the foreign bank’s portfolio, i.e., the foreign endogenous capital constraint, is

\[
\text{CC}^* = \begin{cases} 
\frac{1}{R_{k,1}^* - R_1^*} p_0^* N_0^* \geq p_0^* S_{p,0}^* & \text{if } \theta^* > 0 \\
\text{no CC}^* & \text{if } \theta^* = 0.
\end{cases}
\]

(24)

2.4 Market Clearing

To close the model, we require the markets for assets, labor, and goods to clear. Therefore, the home and foreign capital markets as well as the markets for home and foreign bonds are characterized by

\[
S_{p,0} = K_1; \quad S_{p,0}^* = K_1^* \quad \text{and} \quad B_0 = D_0; \quad B_0^* = D_0^*.
\]

(25)

(26)

Remember that \( K_1 \) and \( K_1^* \) are the total supplies of domestic and foreign investment securities. In the labor market, labor demand in
each country needs to equal the inelastic labor supply: $L_1 = L$ and $L^*_1 = L^*$. In the goods markets, market clearing for traded goods requires that

$$Y_{T,0} + Y^*_{T,0} = C_{T,0} + C^*_{T,0} + K_1 + K^*_1 \quad \text{and} \quad (27)$$

$$Y_{T,1} + Y^*_{T,1} + (1 - \delta)K_1 + (1 - \delta)K^*_1 = C_{T,1} + C^*_{T,1}. \quad (28)$$

Finally, for simplicity we assume that the endowment of non-traded goods is constant across countries and time: $Y_{NT,t} = Y^*_{NT,t} = \chi$. Hence, it must hold that $C_{NT,t} = C^*_{NT,t} = \chi$.

Note that combining the budget constraints of the home households and the home banks in period 0 and using the market clearing condition for domestic bonds, domestic investment securities, and non-traded goods yields the market clearing equation of the international credit market:

$$e_{0} A_{p,0} = p_{0} (Y_{T,0} - K_1 - C_{T,0}). \quad (29)$$

The right-hand side is equal to $p_{0}$ times net exports of the home country in the first period, $NX_0$. Equilibrium in the international credit market requires that the excess supply of foreign bonds in the amount of $p_{0}NX_0$ is fully absorbed by home banks and therefore equal to their demand for foreign bonds $e_{0} A_{p,0}$.

A summary of the equilibrium conditions is provided in Appendix B.

### 3. The Role of Financial Frictions

Even though this is a parsimonious model, a closed-form solution only exists for the frictionless case. With non-zero financial frictions, it is not possible to solve the model analytically. In this section, we use graphical and numerical illustrations to visualize the implications of the different frictions and to obtain an intuition about the mechanisms at work.

Before starting, note that putting the market clearing condition for the non-tradable good into the demand equation (6) yields the general result that total consumption expenditure is constant over time ($P_{0}C_{0} = P_{1}C_{1}$). From this, it follows that the rate of return on
domestic bonds must always satisfy $R_1 = 1/\beta$ (see Equation (5)). Likewise, the rate of return on foreign bonds is $R^*_1 = 1/\beta^*$. It follows that any movements in excess returns must come from a change in $R_{k,1}, R^*_{k,1}$, or $e_1/e_0$, respectively.

3.1 Effect of Financial Frictions in the International Credit Market

Starting from the frictionless case, consider first the effect of financial frictions in the international credit market, captured by an increase in $\Delta$, with $\theta$ and $\theta^*$ set to zero. When $\Delta$ is sufficiently large for the domestic capital constraint to become binding ($\lambda > 0$), home banks invest less funds in foreign bonds, which, as explained earlier, results in a deviation from interest parity: $R^*_1 e_1/e_0 - R_1 > 0$ (see Equation (17)). Furthermore, the level of the home country’s net exports is limited as follows (plug (29) into (19)):\(^{11}\)

$$\frac{1}{\Delta - \frac{1}{R_1} \left( R^*_1 e_1/e_0 - R_1 \right)} p_0 N_0 \geq p_0 N X_0. \tag{30}$$

Graphically, the global equilibrium can be illustrated by the Metzler diagram in Figure 2. It depicts how first-period savings ($S_0 = Y_{T,0} - C_{T,0}$ and $S^*_0 = Y^*_{T,0} - C^*_{T,0}$) and investment ($I_0 = K_1$ and $I^*_0 = K^*_1$) schedules\(^{12}\) change with real interest rates\(^{13}\) ($p_1 R_1$ and $p^*_1 R^*_1 = e_1/e_0 p_1 R^*_1$). Starting from the frictionless equilibrium, the consequences of financial frictions in the international credit market are twofold. First, due to the limits to arbitrage, the equilibrium real rate of return is higher in the foreign country than in the home country ($R^*_1 e_1/e_0 p_1 > R_1$). Second, there is a slight leftward shift of the home country’s savings curve as home households increase their first-period consumption given that they can expect a positive return on the home banks’ equity capital they hold: the excess return on foreign bonds is positive and the home banks will make positive profits. Overall, compared with the frictionless case, a larger fraction

\(^{11}\) As $\lambda > 0$, inequality (30) will hold with equality.

\(^{12}\) For a formal definition of the savings ($SS, SS^*$) and investment ($KK, KK^*$) schedules, see Appendix E.

\(^{13}\) We use the term real to mean in terms of the traded good.
of the home country’s first-period endowment is either consumed in period 0 or invested domestically, making its net exports shrink. Frictions in the international credit market thus make intertemporal trade more costly: they act like a tax on capital flows and make the current account shrink. The international mobility of funds is limited, leading to a misallocation of capital: the majority of capital is invested in the home country, whereas in the frictionless case, identical production technologies and equal labor forces imply that both countries invest the same amount.

In this scenario, the home country, i.e., the country facing excess demand for its bonds, experiences an appreciation in period 0, while at the same time the foreign country, i.e., the country facing an excess supply of bonds, experiences a depreciation. There are two mechanisms driving these exchange rate movements. First and foremost, the frictions in the international credit market cause a deviation from interest parity (i.e., a safety premium on domestic bonds), which incorporates a home appreciation in the first and a depreciation in the second period. Moreover, there is an increase in the home country’s relative lifetime resources coming from the change in

Note: The solid lines represent the frictionless equilibrium, the dashed lines the equilibrium with frictions.
the allocation of capital. This implies that, relative to the frictionless case, the home country’s second-period output increases while the foreign country’s second-period output decreases. This change in fundamentals induces further appreciation pressure in the first period (and offsets part of the depreciation in the second period).

Figure 3 provides a numerical illustration of these results. With \( \theta \) and \( \theta^* \) set to zero and using the parameterization in Table 1 to calibrate the remaining parameters, it shows the evolution of the model’s equilibrium as \( \Delta \) increases. For small values, the home banks’ incentive constraint is not binding. However, as soon as \( \Delta \) is above a certain threshold, the equilibrium adjusts as described in the graphical analysis above. Any further increase in \( \Delta \) amplifies these effects, i.e., it leads to further increases in the deviation from interest parity and an additional home appreciation (depreciation) in the first (second) period.

3.2 Effect of Financial Frictions in the Foreign Investment Market

Next, consider the effect of financial frictions in the foreign investment market, captured by an increase in \( \theta^* \), with \( \theta \) and \( \Delta \) set to zero. When \( \theta^* \) is sufficiently large for the foreign banks’ capital constraint to become binding (\( \lambda^* > 0 \)), foreign banks are unable to exploit all arbitrage opportunities. Excess returns in the foreign investment market thus become positive: \( R^*_{k,1} - R^*_1 > 0 \) (see Equation (23)). The level of foreign capital is limited by (plug Equation (25) into CC* (24))

\[
\theta^* - \frac{1}{R^*_1} \left( R^*_{k,1} - R^*_1 \right) p^*_0 N^*_0 \geq p^*_0 K^*_1. \tag{31}
\]

In contrast, excess returns in both the home investment market and the international credit market remain zero (see Equations (16) and (17)).

\[\text{As } \lambda^* > 0, \text{ inequality (31) will hold with equality.}\]
Figure 3. Effect of an Increase in International Credit Market Frictions $\Delta$ ($\Delta$ on x-axis)

Note: Evolution of the model’s equilibrium as the friction parameter in the international credit market increases, starting from a frictionless point. $\theta = 0$, $\theta^* = 0$, $\Delta \geq 0$. 
Table 1. Parameterization

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
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<td>$\beta$</td>
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<td>$\delta$</td>
<td>0.33</td>
</tr>
<tr>
<td>$\beta^*$</td>
<td>0.5</td>
<td>$Y_{T,0}, Y_{T,*}$</td>
<td>1</td>
</tr>
<tr>
<td>$\frac{1}{1+\chi}$</td>
<td>0.5</td>
<td>$\alpha$</td>
<td>0.33</td>
</tr>
<tr>
<td>$L, L^*$</td>
<td>1</td>
<td>$N_0, N_0^*$</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Figure 4. Financial Frictions in the Foreign Investment Market: $\theta^* > 0$

In a Metzler diagram, financial frictions in the foreign investment market shift the foreign investment curve to the left (see Figure 4).\(^{15}\) Intuitively, for a given real rate of return $\frac{e_1}{e_0} \frac{p_2}{p_1} R_1^*$, investment in the foreign country declines compared with the frictionless case, because foreign banks are now less able to intermediate funds in this market. Costs of capital in the foreign market thus increase. In order to

\(^{15}\)As opposed to the case of an increase in $\Delta$, an increase in $\theta$ or $\theta^*$ does not lead to a shift in the savings curve of the respective country, as the higher return on the households’ holdings of equity capital (in this case due to the increase in the spreads $R_{k,1} - R_1$ and $R_{k,1}^* - R_1^*$, respectively) is just nullified by a decrease in their second-period labor income due to the lower level of capital. For the formal proofs, see Appendix E.
maintain the global equilibrium, the equilibrium real rate of return has to decrease.

The frictions in the foreign investment market affect savings and investment in both countries. The home country lowers its savings and invests a larger share of its remaining savings domestically, which causes its net exports to decrease. The foreign country also lowers its savings, but since it lowers investment even more, its net imports are reduced. Overall, the foreign investment market frictions lead to a decrease in global savings and, consequently, global investment, implying a lower level of global output in the second period. Furthermore, there is a misallocation of capital in the first period. Investment is not equal in the two countries as it would be in the frictionless case; instead, a majority is invested in the home country. From this change in the investment allocation it follows that, compared with the frictionless case, the second-period output is now higher in the home country but lower in the foreign country, implying a change in the two countries’ fundamentals. The relative increase in the home country’s lifetime resources induces a home appreciation (foreign depreciation) in both periods.\cite{16}

The charts in Figure 5 provide a numerical illustration of the effects that have just been described. Setting $\theta$ and $\Delta$ equal to zero, it shows how the model’s equilibrium evolves as $\theta^*$ increases. For small values, the foreign banks’ incentive constraint is not binding yet. However, as soon as $\theta^*$ is above a certain threshold, the equilibrium adjusts as described in the graphical analysis above. Any further increase in $\theta^*$ has the same effects, i.e., it leads to another

\footnote{The home country’s lifetime resources are apparent in the home households’ intertemporal budget constraint:}

\[
C_{NT,0} + p_0C_{T,0} + \frac{1}{R_1} (C_{NT,1} + p_1C_{T,1}) = Y_{NT,0} + p_0Y_{T,0} + \frac{1}{R_1} (Y_{NT,1} + w_1L + p_1N_1) - p_0N_0.
\]

As the endowment of non-traded goods and period-0 traded goods is given and the law of one price holds, any relative changes in lifetime resources between the two countries must either come from relative changes in labor income $w_1L = (1 - \alpha)p_1Y_{T,1}$ or changes in the relative payoffs to equity capital $p_1N_1 = (R_{k,1} - R_1)p_0S_{p,0} + \left(R_1^* \frac{e_1}{e_0} - R_1\right) e_0A_{p,0} + R_1p_0N_0$.\cite{16}
Figure 5. Effect of an Increase in Foreign Financial Frictions $\theta^*$ ($\theta^*$ on x-axis)

Note: Evolution of the model’s equilibrium as the friction parameter in the foreign investment market increases, starting from a frictionless point. $\theta = 0$, $\theta^* \geq 0$, $\Delta = 0$. Given that the setup of the banking sector is only valid with $e_0 A_{p,0} \geq 0$, the plots only cover a limited range of possible values for $\theta^*$. 
increase in excess returns in the foreign investment market and an additional home appreciation.

The case of financial frictions in the home investment market, captured by an increase in $\theta$, is symmetric to the one just described. When $\theta$ is sufficiently large for the home incentive constraint to become binding, the home investment curve shifts to the left, resulting in a home depreciation (foreign appreciation) in both periods. A detailed analysis is provided in Appendix B.

3.3 General Case

After studying the effects of financial frictions for different markets one at a time, we now turn to a description of the general model in which all friction parameters are positive and banks in both countries face binding incentive constraints. In this case, excess returns are positive in all three markets, i.e., there is a deviation from interest parity and home as well as foreign firms face capital costs above the frictionless level. The banks’ capital constraints (see Equations (19) and (24)), combined with market clearing in both the investment markets and in the international credit market (Equations (25) and (29)) describe the restrictions on capital and net exports:

\[
\frac{1}{\Delta - \frac{1}{R_1} \left( R_{1e}^{*} - R_1 \right)} p_0 N_0 \geq \frac{\theta}{\Delta} p_0 K_1 + p_0 N X_0, \tag{32}
\]

\[
\frac{1}{\theta^* - \frac{1}{R_1} \left( R_{k1}^{*} - R_1^{*} \right)} p_0^{*} N_0^{*} \geq p_0^{*} K_1^{*}. \tag{33}
\]

In line with our earlier reasoning, Equation (32) shows that, when limits to arbitrage are higher in the home investment market than in the international credit market (i.e., when $\theta > \Delta$), intermediating capital in the home country is more constraining than intermediating net exports.

In general, further increases in any of the friction parameters make the banks shift funds away from the respective market. Overall, the mechanisms are the same as those described in Sections 3.1 and 3.2, even though graphically (regarding the shifts in the savings

\[\text{As by assumption } \lambda > 0 \text{ and } \lambda^* > 0, \text{ both inequalities will hold with equality.}\]
and investment curves of the Metzler diagrams) there can be small differences due to the non-zero Lagrange multipliers $\lambda$ and $\lambda^*$ and excess returns. Accordingly, a further increase in $\Delta$ leads to a shift of funds away from foreign assets and herewith home net exports and foreign capital towards home assets and home capital. Likewise, a further increase in $\theta^*$ leads to a decrease in the total amount of global investment ($K_1 + K^*_1$) and a shift of funds away from foreign investment securities and thus foreign capital towards home investment securities and thus home capital. A numerical illustration of the corresponding effects is provided in Figures B.3 to B.5 in Appendix B.

Obviously, higher frictions in one of the markets lead to a direct increase in excess returns in this specific market. In contrast, the spreads in the other markets are only affected marginally. Higher frictions in the foreign investment market, for instance, make the home banks’ incentive constraint slightly less binding because the foreign banks issue fewer foreign bonds and the supply of foreign assets thus decreases. Home banks thus now need to absorb fewer of these and therefore can invest more of their constrained funds in home investment securities. So, overall, the constraint on home banks is slightly eased and, accordingly, excess returns in both the home investment market and the international credit market decrease.

4. Impact of International Portfolio Flows

So far, we have assumed that households are only able to trade bonds denominated in the numéraire of their own country, and hence that the imbalance in the demand for domestic and foreign bonds that is absorbed by the home banks results from trade flows only. Now, we introduce financial flows other than those resulting from trade imbalances and refer to them as international portfolio flows. In particular, we allow households in both countries to hold bonds issued by the other country’s banks. Home households are assumed to have an inelastic demand for foreign bonds $f$ that is funded by an offsetting position $e_0 f$ in domestic bonds. Equivalently, foreign households have an exogenous inelastic demand for domestic bonds $f^*$ that is funded by an offsetting position $f^*/e_0$ in foreign bonds. For simplicity, we set all these portfolio flows to be exogenous, which helps to
avoid mixing up different transmission channels. Hence, one could think of \( f \) and \( f^* \) as the result of simple noise or liquidity trading or as “deliberate” holdings of foreign bonds motivated, for example, by practical reasons in daily business or foreign direct investment.

With portfolio flows, market clearing conditions (26) for home and foreign bonds change to

\[
B_0 + f^* = D_0 + e_0 f; \quad B_0^* + f = D_0^* + f^*/e_0. \tag{34}
\]

Accordingly, the richer set of financial flows also alters equilibrium condition (29) in the international credit market. Home banks now need to absorb the imbalance in the demand for foreign bonds stemming from both trade and portfolio flows:

\[
e_0 A_{p,0} = p_0 N X_0 + f^* - e_0 f, \tag{35}
\]

where \( p_0 N X_0 + f^* \) reflects the total supply of foreign bonds and \( e_0 f \) is the domestic households’ demand for foreign bonds. All else equal, the larger the inflows \( f^* \), the larger the international funds that the home banks need to intermediate. Hence, as will become evident in the rest of the section, it is not necessarily net exports \( (p_0 N X_0) \) and thus net foreign assets that matter in the determination of the exchange rate, but rather the excess supply of foreign bonds that need to be absorbed by the private financial sector \( (p_0 N X_0 + f^* - e_0 f) \). In practice, the two are likely to have the same sign for most countries, given that the excess supply of foreign bonds depends positively on net foreign assets, but they can also have opposite signs. An extreme example is certainly the United States. While the U.S. net exports and current account are persistently negative, the demand for U.S. dollar assets by the rest of the world (in the model captured by \( f^* \)) is huge, among other things due to the U.S. dollar’s role as a vehicle and reserve currency. Hence, even though

\[\text{In practice, a large part of the demand for foreign bonds is likely to be endogenous and depend on present and expected future fundamentals, such as interest rate differentials. For this endogenous part, it would be more realistic to have } f = f(R_1, R_1^*, e_0, e_1, \ldots) \text{ and } f^* = f^*(R_1, R_1^*, e_0, e_1, \ldots). \text{ For instance, as suggested in Gabaix and Maggiori (2015), a straightforward way to model a popular trading strategy, carry trade, would be to set } f = a + b(R_1 - R_1^*) \text{ and } f^* = c + d(R_1 - R_1^*) \text{ for some constants } a, b, c, \text{ and } d. \text{ However, given that our main goal is to improve understanding of the effects of international financial flows, but not the origin of these, we abstract from such dependencies.}\]
U.S. net foreign assets are negative, the excess demand for U.S. dollars (and hence the excess supply of foreign currency that need to be absorbed by the private financial sector) is likely to be positive.\footnote{The U.S. dollar’s safe-haven property of appreciating during periods of high uncertainty is thus in line with the predictions of our model.}

When the international credit market frictions parameter $\Delta$ is equal to zero, home banks are able to absorb any imbalance in the international credit market. That is, interest parity holds and gross capital flows have no effect on any variable other than $A_{p,0}$. Consider the example of foreign households suddenly wanting to hold a certain amount of domestic bonds $f^*$. When banks are not constrained in the international credit market and the returns on domestic and foreign bonds are equalized, home banks are willing to issue any additional amount of domestic bonds and, in return, increase their holdings of foreign bonds correspondingly. Hence, home banks increase their holdings of foreign bonds $e_0A_{p,0}$ one for one with the inflow of capital $f^*$. It is a trade that concerns only foreign households and home banks and does not affect the rest of the economy. This irrelevance of gross capital flows is a common feature of the traditional international economics literature inspired by Dornbusch (1976) and Obstfeld and Rogoff (1995), where interest parity (or more specifically, uncovered interest parity) is often either directly assumed to hold or imposed in the process of first-order linearization.

Gross capital flows start to matter once the international credit market friction parameter $\Delta$ is positive and the home banks’ incentive constraint is, or starts to be, binding. When banks are credit constrained in the international credit market, gross capital flows have an impact on the tightness of the capital constraint they are facing.\footnote{The inequalities in (36) will hold with equality in this case given that banks are balance sheet constrained.}

$$CC = \begin{cases} \frac{\Delta}{\pi_1(R_{k,1} - R_{1})} p_0 N_0 \geq \theta p_0 K_1 \\ + \Delta(p_0 N X_0 + f^* - e_0 f) & \text{if } \theta \geq 0, \Delta > 0 \\ \frac{\theta}{\pi_1(R_{k,1} - R_{1})} p_0 N_0 \geq \theta p_0 K_1 \\ + \Delta(p_0 N X_0 + f^* - e_0 f) & \text{if } \theta > 0, \Delta \geq 0 \\ \text{no CC} & \text{if } \theta = 0, \Delta = 0. \end{cases}$$
Note that the critical value of $\Delta$ at which the constraint becomes binding is endogenous and depends negatively (positively) on capital inflows $f^*$ (outflows $e_0f$). For instance, if $f^*$ is very high, i.e., the excess supply of foreign bonds that the home banks need to absorb is large, a relatively low $\Delta$ suffices to make the banks’ incentive constraint binding.

An increase in capital inflows to the home country or a decline in capital outflows make the home banks’ capital constraint more binding, as they need a larger part of their risk-bearing capacity to intermediate these flows. Consider again the example of foreign households suddenly wanting to hold a certain amount $f^*$ of home country bonds. If home banks are constrained in the international credit market, we are at a point at which creditors are not willing to provide them with more funds, since the banks would invest these (at least partially) in foreign bonds, which in turn would result in higher proceeds under diversion and hence a higher incentive to misbehave. Capital inflows from foreign households, however, represent an exogenous increase in the funds available to home banks, but as a consequence there is also a higher amount of foreign bonds that they need to absorb in order to maintain equilibrium in the international credit market (see Equation (35)). Due to the binding balance sheet constraint, home banks are only able to intermediate these exogenous capital inflows if they can simultaneously relax their capital constraint in some other way. This can happen through two channels. The first is an adjustment on the creditor side. Home households will find it optimal to increase their period-0 consumption and decrease their savings, i.e., provide home banks with less funds. Together with the concurrent decrease in net exports (and hence in the demand for domestic bonds), this leads to a relaxation of the capital constraint. The second potential channel is an adjustment in the home banks’ portfolio towards home investment securities. The relative importance of these two channels can vary. The first channel always plays an important role, and if home banks are not constrained in the home investment market (i.e., $\theta = 0$), the second channel also becomes significant. If, however, there are frictions in both the international credit market and the home investment market, substituting foreign bonds (and thereby the intermediation of net exports) for additional home investment securities will not necessarily relax the balance sheet constraint. The higher the investment
market frictions, the less such a substitution yields the necessary loosening of the constraint, and hence the stronger the first channel, i.e., the more households will increase their period-0 consumption and reduce the amount of funds they provide the banks with.

In the case where only the international credit market friction parameter is positive, an increase in capital inflows has similar effects to an increase in Δ. While an increase in Δ represents a direct reduction in the banks’ ability to intermediate international funds, higher capital inflows imply that a larger part of the banks’ risk-bearing capacity is absorbed by these exogenous flows, representing an indirect reduction in their ability to intermediate international funds. Graphically, an increase in capital inflows implies a widening of the spread between the two dashed vertical lines in Figure 2, corresponding to a larger deviation from interest parity and a home appreciation (foreign depreciation) in period 0. Accordingly, the results in the numerical example correspond for the most part to those of an increase in Δ, as can be seen in Figure D.1 in Appendix D. Higher portfolio inflows increase the misallocation of capital that is potentially already present due to frictions in the international credit market. The higher the portfolio inflows that the home banks need to intermediate, the larger the amount of funds that they find optimal to invest in the home country. The resulting increase in the home country’s second-period output relative to the foreign country raises the home country’s relative lifetime resources, leading to further appreciation pressure in period 0.

Finally, Figure 6 provides a numerical illustration of the case in which both home and foreign investment markets exhibit limits to arbitrage. Note that since we set Δ = θ, excess returns in the international credit and the home investment market are equally large. Again, exogenous capital inflows trigger an increase in the excess return in the international credit market (i.e., an increase in the safety premium on domestic bonds) and thus an appreciation in period 0. Note that the levels of capital $K_1$ and $K_1^*$ change only marginally. As the home banks are equally constrained in the home investment market and the international credit market, there is no portfolio adjustment. While net exports decline strongly, variations in the level of investment are of second order only. Hence, relative second-period output in the two countries changes but marginally, if at all. However, compared with foreign households, home households
Figure 6. Effect of an Increase in Capital Inflows $f^*$ ($f^*$ on x-axis)

Note: Evolution of the model’s equilibrium as capital inflows increase, starting from a point where there are limits to arbitrage in all financial markets. $\theta = 1/3, \theta^* = 1/3, \Delta = 1/3$. 
will have a relatively higher payoff from their equity capital. The implicit decrease in the banks’ ability to intermediate funds in the international credit market and in the investment market drives up excess returns in both of these markets (see Equation (18)). Hence, in this case too, there is an increase in the home country’s relative lifetime resources, which induces a further appreciation in the first period.

Regarding the foreign country, i.e., the country facing an excess supply of its bonds, note that there an exogenous increase in portfolio inflows (captured by an increase in $e_0 f$) would also generate an appreciation, at least as long as the home banks’ capital constraint is binding. Yet, the underlying reason differs across the two countries. In the case of the home country, the appreciation pressure resulting from higher capital inflows is explained by a tightening of the home banks’ capital constraint and, therefore, a widening of the interest parity spread. The foreign appreciation in the case of higher capital inflows to the foreign country, on the other hand, stems from a relaxation of the home banks’ capital constraint. The excess supply of foreign bonds declines, making the interest parity spread narrow. In other words, higher capital inflows to the foreign country bring the economy closer to its frictionless state and the exchange rate closer to its frictionless value, i.e., the value explained by economic fundamentals.

5. Central Bank Asset Purchases

The previous sections have shed light on how frictions in financial markets—as they are experienced during a financial crisis—and global imbalances in gross flows can lead to excess returns on domestic and foreign bonds and distortions in real activity, namely in the allocation of capital and in international trade. We now analyze a central bank’s policy options to counteract such distortions. In this section, we show how large-scale asset purchases by the home central bank can be used to lower excess returns in general. In the next section, we then look at three specific cases of external appreciation pressure to the home country and identify the possible policy responses of the home central bank.

Our model allows us to study and compare two policies that were repeatedly put into practice in the course of the 2007–09
financial crisis: credit easing and foreign exchange (FX) interventions. By applying these policies, the home central bank itself plays the role of an intermediary, reducing the (excess) supply of investment securities and foreign bonds that need to be absorbed by the private intermediaries, and thus relaxes the banks’ capital constraint. In our model, the two policy options are implemented as follows. The central bank intervenes in the domestic investment market by purchasing domestic investment securities \( p_0 S_{CB,0} \) and in the international credit market by purchasing foreign bonds \( e_0 A_{CB,0} \), issuing domestic bonds \( D_{CB,0} \) to finance these transactions.\(^{21}\) Following the baseline scenario of Gertler and Karadi (2013), we assume that the central bank issues these domestic bonds directly to households.\(^{22}\) The central bank’s profits in period 1 are transferred to the home households.

In contrast to private intermediaries, the central bank has the crucial advantage that it is not balance sheet constrained because it is not facing a limited commitment problem. We furthermore make the simplifying assumption that both types of interventions, i.e., FX interventions and credit easing, are costless to the central bank. This implies that it is as efficient as private intermediaries at intermediating funds. This assumption simplifies the analysis but is not critical for our results.\(^{23}\)

As a result of the interventions, the central bank’s balance sheet,

\[
p_0 S_{CB,0} + e_0 A_{CB,0} = D_{CB,0},
\]

\(^{21}\)Theoretically, we could also look at a policy where the home central bank directly acquires foreign investment securities instead of foreign bonds. However, this seems little realistic, as the foreign investment market tends to be the responsibility of the foreign authorities.

\(^{22}\)As Gertler and Karadi (2013) point out, \( D_{CB,0} \) can also be interpreted as reserves held by banks on account at the central bank.

\(^{23}\)It would be straightforward to introduce relative efficiency costs, as in Gertler and Karadi (2013), to capture the fact that a central bank is less efficient at intermediating funds than ordinary banks. For welfare considerations (from which we abstract in this paper), central bank interventions would then only be desirable when private intermediation is significantly constrained and even then only if efficiency costs are not too large. The latter, however, is a reasonable assumption, so that in the end these costs would not change the qualitative implications of the model.
expands. Accordingly, the market clearing condition (26) for home bonds extends to \( B_0 + f^* = D_{p,0} + D_{CB,0} + e_0 f \).

In the home investment market, the market clearing condition (25) changes to

\[
S_{p,0} + S_{CB,0} = K_1, \tag{38}
\]

which reflects the fact that capital is now partly intermediated by the central bank.

Finally, the consolidation of the home households’ first-period budget constraint and the balance sheet equations of the home banks and the central bank yields the new market clearing condition of the international credit market:

\[
e_0 A_{p,0} + e_0 A_{CB,0} = p_0 N x_0 + f^* - e_0 f. \tag{39}
\]

The excess supply of foreign bonds, determined by the home country’s trade surplus as well as the portfolio flows of home and foreign households, is now jointly absorbed by the home banks and central bank.

As the home central bank only intervenes in the domestic investment market and the international credit market, only the home banks’ incentive constraint is relevant for determining whether the interventions are effective. When the constraint is not binding, returns in the home investment market and the international credit market are determined by frictionless arbitrage and interventions by the central bank are neutral. Its purchases of either home investment securities or foreign bonds simply replace part of the private intermediation, but have no effect on returns and the exchange rate. However, central bank interventions are non-neutral in markets where the financial friction parameters are high enough to generate limits to arbitrage. When banks are balance sheet constrained in one or both markets, central bank purchases of the respective assets do not just replace private intermediation one for one, but rather expand the total demand for the respective asset type, which in turn drives down the excess return(s).

The precise effect on different excess returns depends on whether the balance sheet constraint is binding in just one or in both markets. In the former case, central bank interventions obviously only
have an effect on excess returns in the market that exhibits limits to arbitrage while the other market is unaffected. In the latter case, purchases of either asset affect excess returns in both markets. This spillover effect results from the no-arbitrage relation (18). For a better intuition, consider the example where the central bank intervenes by purchasing foreign bonds. According to the reasoning above, this reduces excess returns in the international credit market. Home banks then shift part of their funds towards the home investment market where excess returns are still high and therefore more attractive. This in turn also reduces the excess return on investment securities. The banks’ portfolio adjustment ends once excess returns in the two markets, adjusted by the weight $\frac{\theta}{\Delta}$, are equalized. Note that, no matter which of the two assets the central bank purchases, its interventions change excess returns most (in absolute terms) in the market where banks are most constrained.

The additional intermediation by the central bank allows for higher levels of home capital and net exports to be intermediated, as can be observed in the new capital constraint

$$CC = \begin{cases} \frac{\Delta}{\theta} \left( R_1 \frac{1}{1+\theta} - R_1 \right) p_0 N_0 \geq \theta p_0 (K_1 - S_{CB,0}) \\ + \Delta (p_0 N X_0 + f^* - e_0 f - e_0 A_{CB,0}) & \text{if } \theta \geq 0, \Delta > 0 \\ \frac{\theta}{\theta} \left( R_1 \frac{1}{1+\theta} - R_1 \right) p_0 N_0 \geq \theta p_0 (K_1 - S_{CB,0}) \\ + \Delta (p_0 N X_0 + f^* - e_0 f - e_0 A_{CB,0}) & \text{if } \theta > 0, \Delta \geq 0 \\ \text{no CC} & \text{if } \theta = 0, \Delta = 0. \end{cases}$$

(40)

Central bank asset purchases $p_0 S_{CB,0}$ and $e_0 A_{CB,0}$ reduce the amount of funds that need to be intermediated by home banks, which relaxes their capital constraint and brings the intermediated quantities closer to their frictionless level. In the limit, intermediation by the home central bank can make the excess returns in the home investment market and the international credit market disappear completely. As long as $\theta^*$ is small enough for the foreign banks’

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24 The inequalities in (40) hold with equality given that we are considering the case where banks are constrained.
capital constraints not to be binding, the economy would then be back in the frictionless state.²⁵

Furthermore, Equation (40) reveals that in terms of the amount of intervention needed to reach a given reduction of excess returns, it matters which asset the central bank buys. If both markets are affected by limits to arbitrage, buying a certain amount of investment securities \( p_0 S_{CB,0} \) relaxes the constraint to the same extent, and therefore has exactly the same effect, as buying foreign assets \( e_0 A_{CB,0} \) to the amount of \( \frac{\theta}{\Delta} p_0 S_{CB,0} \). Intuitively, a central bank intervention involving the issuance of a given amount of domestic bonds frees up a higher amount of bank capital if purchases are made in the market that faces higher limits to arbitrage. This implies that when \( \Delta > \theta \), i.e., when the international credit market exhibits higher excess returns than the home investment market, FX interventions have a stronger effect than credit easing and are preferable to the latter in order to avoid an unnecessary expansion of the central bank’s balance sheet.²⁶ Likewise, credit easing is the preferred instrument when \( \theta > \Delta \).

An overview of the model’s formal equilibrium conditions with portfolio flows and central bank interventions is provided in Appendix D. Here, Figures 7 and 8 provide numerical illustrations of the effects of credit easing and FX interventions, respectively, when banks are equally constrained in both markets (\( \theta = \Delta \)) and \( \theta^* = 0 \).²⁷ As just described, in this case both credit easing and FX interventions reduce excess returns and raise the level of intermediated funds in both markets. However, to make excess returns in both markets disappear completely and return the economy to the frictionless equilibrium, it may not be enough to intervene in just one market. The reason is that the amount of assets outstanding in

²⁵Remember that, realistically, we would have to introduce efficiency costs to central bank intermediation, which in turn could make such extreme interventions less desirable.

²⁶The larger a central bank’s balance sheet, the larger the size of potential losses due to valuation changes on its assets (e.g., because of exchange rate fluctuations). Substantial reductions in a central bank’s net worth may be considered problematic in the public and—in the worst case—lead to a loss in confidence. However, as pointed out by Jordan (2011), low or even temporarily negative levels of equity do not constrain a central bank’s capacity to act.

²⁷Numerical illustrations for when \( \Delta = \theta = \theta^* > 0 \) are provided in Figures D.2 and D.3 in Appendix D.
Figure 7. Effect of an Increase in $S_{CB,0}$ ($p_0S_{CB,0}$ on x-axis)

Note: Evolution of the model’s equilibrium as central bank intermediation in the domestic investment market increases, starting from a point where there are limits to arbitrage in the domestic investment market and the international credit market. $\theta = 1/3$, $\theta^* = 0$, $\Delta = 1/3$. The red line represents the value in the frictionless equilibrium.
Figure 8. Effect of an Increase in $A_{CB,0}$ ($e_0A_{CB,0}$ on x-axis)

Note: Evolution of the model’s equilibrium as central bank intermediation in the international credit market increases, starting from a point where there are limits to arbitrage in the domestic investment market and the international credit market. $\theta = 1/3$, $\theta^* = 0$, $\Delta = 1/3$. The red line represents the value in the frictionless equilibrium. The dashed part of the lines captures the range of FX interventions where the latter would require the home banks to go short in foreign bonds in order to fulfill the central bank’s demand for these assets and hence covers a part where our model technically is not valid.
one market alone may be too small so that the economy remains constrained even when the central bank has bought all of them. As an example, consider Figure 8. Even once the central bank has absorbed the entire excess supply of foreign bonds from the home banks \( (e_0 A_{CB,0} = p_0 N X_0 \text{ and } e_0 A_{p,0} = 0) \), they are still credit constrained \( (\lambda > 0) \).

Regarding the foreign central bank, i.e., the central bank in the country facing an excess supply of its bonds, note that the set of unconventional monetary policy tools at its disposal to reverse the effects of financial frictions and portfolio flows is more restricted. In the case of frictions in the foreign investment market, which would trigger a depreciation and an outflow of funds, it can, like the home central bank, engage in credit easing, i.e., purchase (foreign) investment securities and issue (foreign) bonds in return. In the case of frictions in the international credit market and exogenous capital outflows, both triggering a foreign depreciation, the foreign central bank would need to purchase foreign bonds for home bonds to reverse the distorting effects. But this obviously requires that it is in the possession of such home bonds, i.e., has sufficient holdings of foreign exchange reserves. Furthermore, unlike in the home economy, intervening in one market does not automatically reduce the spreads in both the international credit and the foreign investment market. In contrast, a relaxation of financial conditions in the international credit market leads to a shift of funds towards the foreign country, increasing the spread in the foreign investment market if foreign banks are balance sheet constrained\(^{28}\). Hence, if there are frictions present in both markets, the foreign central bank should engage in both credit easing and foreign exchange interventions, as interventions in one market cannot substitute interventions in the other market.

6. Policy Response to Appreciation Pressure

Even though simple, our model aims to capture the fundamental structure of domestic and international bond markets. This allows

\(^{28}\)For illustration, see Figure D.3 in Appendix D. While the figure shows purchases of foreign bonds by the home central bank, the effects would be largely identical in the case of purchases of foreign bonds for home bonds by the foreign central bank.
us to draw a number of useful conclusions on the effectiveness of two important unconventional monetary policy tools, credit easing and FX interventions. In this last section, we shed light on three sources of external home appreciation pressure related to credit market frictions and discuss the policy options of the home central bank to reverse the effects on the exchange rate if judged to be unwanted.

Importantly, note that a home appreciation does not necessarily cause damage to the home country in our stylized setup. This is because the typical channels through which a home appreciation would lead to a depression of home output, employment, and income and downward pressure on home inflation are missing. However, we would like to emphasize that the main mechanism of how financial frictions affect the exchange rate in our model, namely the widening of the interest rate spread, would also be at work in a much richer setup and, hence, our conclusions regarding the effectiveness of unconventional monetary policy tools would remain valid.

In practice, a central bank may judge appreciation pressure to be unwanted for a number of reasons. Especially for economies with a high degree of openness, appreciation pressure can pose a major challenge, in particular during periods of economic slack and low inflationary pressure. A stronger currency not only reduces the relative competitiveness of domestic products in the international market, but also weighs on domestic consumer prices through imported

\footnote{For example, an increase in capital inflows can make consumption by home households increase in both periods (see Figure 6). This is because the higher holdings of foreign bonds and the higher excess return on these bonds boost the home banks’ profit and thereby the home households’ lifetime income.}

\footnote{The absence of systematically negative welfare effects is mainly because in our model, for simplicity, output is not produced but given exogenously in the first period and because there is only one traded good and no price or wage stickiness. Without these elements, there is, for example, no expenditure-switching effect after an exchange rate movement and no role for the international competitiveness.}

\footnote{Note that, despite the inefficiencies caused by exchange rate fluctuations, Cavallino (2019) finds that it is not optimal for the central bank to fully stabilize the exchange rate. The reason is that the higher excess return that banks earn on foreign assets after capital inflow shocks has positive wealth effects for the home country. By reducing the excess return, foreign exchange interventions reduce this wealth effect. From a welfare point of view, as there are very similar mechanisms at work in our model, in a New Keynesian version of it, a full stabilization of the exchange rate would probably not be optimal either.}
Figure 9. Overview of Possible Constellations of Parameters $\Delta$ and $\theta^*$ for a Given $\theta$

Note: The home banks’ capital constraint (CC) is binding in the shaded area only. International portfolio flows affect the general tightness of the CC (see left-side graph) and have an impact on excess returns whenever the CC is binding. Likewise, central bank interventions can be used to relax the CC (see right-side graph). Whenever the CC is binding, such interventions are effective in lowering excess returns. FX interventions (FXI) should be given priority over credit easing (CE) whenever $\Delta > \theta$.

The three cases of external appreciation pressure to the home country that we consider are financial frictions in the international credit market ($\Delta \uparrow$), portfolio inflows ($f^* \uparrow$), and financial frictions in the foreign investment market ($\theta^* \uparrow$). In Figure 9, we provide an overview of how these three factors and the two above-mentioned policies affect the capital constraint of the home banks, which lies at the core of our model, for different values of the friction parameters of the international credit and the foreign investment market. For a given level of $\theta$ (where by assumption $\theta > 0$), these figures show where in the $\theta^* \Delta$-space the home capital constraint is binding (shaded area), and how a change in the friction parameters $\theta^*$ and $\Delta$ or a change in portfolio inflows and central bank intermediation affect its general tightness. The line at the border of the shaded area corresponds to the critical values of $\Delta$ as a function of $\theta^*$ at which the capital constraint starts to bind and hence portfolio flows and central bank intervention become effective. For low levels of $\theta^*$, the capital constraint of the foreign banks is not binding and, hence,
the critical value of $\Delta$ does not depend on the level of $\theta^*$. Once foreign banks are constrained, home banks need to absorb a lower excess supply of foreign bonds, which makes them more risk resistant so that they can face higher levels of $\Delta$ before being restricted. In the area above this curve, the home capital constraint binds more tightly and this tightness increases further as the economy moves to the upper left.

The first source of external appreciation pressure that we focus on is financial frictions in the international credit market that can, for instance, arise when markets lose confidence in a country’s economic institutions or future economic development and as a result lose confidence in its currency. An example is the European sovereign debt crisis, which resulted in an appreciation of major currencies against the euro. Within the framework of our model, this scenario can be captured by a (further) increase in $\Delta$. As described in Sections 3.1 and 3.3, in this case, the home appreciation mainly results from an increase in the deviation from interest parity. A country facing appreciation pressure from this type of market imperfection has at least one policy tool it can rely on. As described in Section 5, FX interventions ($A_{CB,0}$) reduce the excess supply of these bonds that needs to be absorbed by private intermediaries. This relaxes the capital constraint of the home banks, which translates into an upward shift of the “critical-\(\Delta\)” curve in Figure 9 (right-side graph). This, in turn, results in a reduction of the deviation from interest parity and a home depreciation in the first period. Suppose now that, in addition to the international credit market, the home investment market exhibits limits to arbitrage as well. Such a situation may arise when financial frictions reach a global level. An example is the 2007–09 financial crisis, which raised fears of the potential collapse of large financial institutions in many countries. In this case, the home central bank has an additional policy tool at its disposal. As argued in Section 5, credit easing ($S_{CB,0}$) and FX interventions are close substitutes whenever financial frictions are present in both the home investment market and the international credit market. Both tools relax the capital constraint of banks, and the ensuing portfolio rebalancing of private intermediaries makes sure that the relative excess returns in the two markets remain constant. Hence, like FX interventions, credit easing leads to an upward shift of the “critical-\(\Delta\)” curve in Figure 9. In order to avoid an unnecessarily
large expansion of its balance sheet, the central bank should intervene in the market with higher frictions. Thus, should the limits to arbitrage in the home investment market be larger than in the international credit market, then credit easing is more desirable to reduce appreciation pressure than direct interventions in the international credit market. Note, however, that whichever type of intervention the central bank chooses, it will not be able to bring the economy back to its initial state, i.e., the state before the increase in frictions in the international credit market, unless this initial state corresponds to the frictionless state. The reason is that the increase in $\Delta$ has permanently altered the no-arbitrage relation (18) and hence the optimal allocation of funds, from the banks’ perspective, across investment market and international credit market.

A second external source of appreciation pressure is an increase in *portfolio inflows* $f^*$, as experienced by countries like Switzerland, Denmark, and Israel during the global financial crisis and the European sovereign debt crisis.\(^{32}\) As discussed in Section 4, this involves an increase in the excess supply of foreign bonds that the home banks need to absorb. This results in a larger deviation from interest parity and hence a home appreciation in period 0 if banks are constrained in the international credit market. In Figure 9 (left-side graph), portfolio inflows lead to a downward shift in the “critical-$\Delta$” curve, reflecting that banks will be subject to a higher general restrictiveness of the credit constraint for given levels of the parameters $\theta, \theta^*$ and $\Delta$. When facing this type of appreciation pressure, the policy options of the home central bank are the same as in the case of an increase in $\Delta$. Whenever capital flows have an impact on the exchange rate, FX interventions, which in the end are just another but special type of capital flow, will as well. By choosing $e_0A_{CB,0} = f^*$ (where $e_0$ is equal to the value before the increase in $f^*$), central bank purchases of foreign bonds can fully reverse the increase in the excess supply of foreign bonds and thus the appreciation. By absorbing all portfolio inflows, the central bank can shift the “critical-$\Delta$” curve in Figure 9 back to its original position. A prominent example of a central bank addressing capital inflows by

\(^{32}\)Obviously, a decrease in portfolio outflows $e_0f$ would have the same effects as an increase in portfolio inflows $f^*$, but would not be classified as an external source of appreciation pressure.
FX interventions is the Swiss National Bank. Starting in 2009, it purchased considerable amounts of foreign assets to counter the upward pressure on the Swiss franc and prevent a tightening of monetary conditions. Ten years later, the SNB’s foreign reserves amounted to 112 percent of GDP (average between 2017 and 2019) and thereby exceeded the size of Switzerland’s net foreign assets (94 percent).

Again, an interesting result is that credit easing can achieve exactly the same goal as FX interventions if banks are constrained in the home investment market as well ($\theta > 0$). The central bank’s acquisitions of home investment securities can likewise free up risk-bearing capacity of home banks, which these in turn can use to absorb the increased excess supply of foreign bonds, reducing the deviation from interest parity. As we can see from Equation (40), purchases of home investment securities to the amount of $p_0 S_{CB,0} = \frac{\Delta \Theta}{\theta} f^*$ (where $p_0$ is equal to the value before the increase in $f^*$) can even bring the economy back to the state prior to the increase in portfolio inflows. Because both policies act in the same manner, they are both able to shift the “critical-$\Delta$” curve in Figure 9 back to its original position and thereby offset the effect on interest rate spreads. Once again, it is more costly for the central bank to intervene in the market with lower limits to arbitrage, i.e., credit easing is the preferred policy whenever $\theta > \Delta$. However, there is a major limit to credit easing: It cannot exceed the level of home capital $p_0 K_1$—once all domestic capital is owned by the central bank, the policy is no longer feasible. As an example, consider again the case of Switzerland. In 2009, in addition to the interventions in the foreign exchange market, the Swiss National Bank also embarked on a bond purchasing program. The goal was to relax the conditions in capital markets and thereby improve the transmission of monetary policy. Compared with the programs of other countries, the amount of bonds purchased was rather small relative to GDP, though, and the program was stopped in 2010.

The third and last external source of appreciation pressure related to credit market frictions in our model is a financial crisis in the foreign country, taking the form of financial frictions in the foreign investment market and captured by an increase in $\theta^*$. During the 2007–09 global crisis, countries like Australia, Canada, and Norway did not experience financial crises themselves, but were negatively affected by the global consequences of the turmoil in U.S. and
European markets, i.e., by financial frictions abroad. As described in Sections 3.2 and 3.3, foreign financial frictions induce a reallocation of capital away from the foreign country towards the home country, leading to a relative increase in the home country’s lifetime resources and hence to an appreciation in both periods, i.e., a permanent home appreciation. If home banks are not constrained in the international credit market, there is no further effect on the exchange rate. If home banks are also constrained in the international credit market, an increase in $\theta^*$ and the resulting drop in net outflows may take the economy to a state where the home banks’ capital constraint is relaxed (see Figure 9, left-side graph) and therefore result in a decrease in excess returns in the international credit market.

Compared with the previous two cases, this type of appreciation pressure is solely driven by a change in economic fundamentals. In this respect, it is “justified,” as opposed to any (temporary) appreciation pressure resulting only from an increase in the deviation from interest parity, as is experienced when there are limits to arbitrage in the international credit market or an increase in capital inflows. Even if it wanted to, with the tools discussed here, the home central bank could not counter this last type of appreciation pressure. When home banks are not constrained in the international credit market ($\Delta = 0$ or $\Delta$ and $\theta$ small enough not to be binding), the central bank has no possibility to affect the exchange rate. When there are limits to arbitrage in the international credit market, it would have this option, but purchases of foreign bonds (or home investment securities, if $\theta > 0$) merely lead to a decrease in the safety premium on domestic bonds: they do reduce appreciation pressure in the first period, but cause additional upward pressure in the second period. Such purchases only address the capital misallocation and exchange rate distortion caused by the international credit market frictions, but not the appreciation pressure caused by the frictions in the foreign investment market.

While the focus of this section has been on appreciation pressure to the home country, our model also allows some interesting statements regarding the exchange rate and unconventional monetary policy tools from the point of view of the foreign country, i.e., the country facing an excess supply of its currency. As can be concluded from the previous sections, in the case of the foreign country, an appreciation is always the result of a relaxation of the (home or...
foreign) banks’ balance sheet constraints, either due to a decrease in the international credit market friction, a decrease in the foreign investment market friction, or exogenous capital inflows to the foreign country. Hence, for the foreign country, an appreciation always means shifting closer towards the frictionless state and the exchange rate explained by economic fundamentals. Obviously, if the appreciation is the result of a narrowing of the spread in the interest parity, the foreign central bank could always buy home bonds if it wanted to counter such an appreciation. However, the probably more relevant question to answer from the point of view of the foreign central bank is whether it can use interventions to reverse the depreciation pressure to the foreign country caused in the three cases discussed above. This issue was discussed in Section 5 where we argued that, as opposed to the home central bank, the foreign central bank’s set of unconventional monetary policy tools to respond to distortions caused by financial frictions is more restricted. Above all, it needs to have sufficient holdings of home bonds if it wishes to counter depreciation pressure resulting from frictions in the international credit market and exogenous capital outflows.

7. Conclusions

We provide a simple two-country framework with imperfect financial intermediation to analyze and compare the effectiveness of two unconventional monetary policy measures, foreign exchange interventions and credit easing. International portfolio flows and central bank interventions only have real effects when banks are financially constrained. Our focus is on cases where the financial frictions lead to appreciation pressure. Increased frictions in the international credit market and higher capital inflows both result in an increase in the safety premium on domestic bonds and hence a temporary home appreciation. In these two cases, foreign exchange interventions can reverse the appreciation. An increase in the limits to arbitrage in the foreign investment market also triggers an appreciation, but the appreciation is permanent in this case. It cannot be reversed by central bank purchases of foreign bonds.

Another interesting result concerns the relative effects of the two unconventional policy responses. If, in addition to frictions in the international credit market, there are also frictions in the home
investment market, credit easing is a substitute for foreign exchange interventions. However, the effectiveness of the two policies can differ. Interventions will come at a lower cost if they target the market that faces the highest excess returns.

Appendix A. Stylized Facts

Figure A.1 and Table A.1 suggest that for the United States and for Switzerland, changes in the nominal effective exchange rate (NEER) and changes in credit spreads are significantly correlated, at least in periods of high uncertainty (VIX > 20). The sample is monthly from 1994:1 to 2021:5 for the United States and 2008:2 to 2021:5 for Switzerland. Data on the NEER are from the Bank for International Settlements. Data on bond yields and the VIX are from Datastream. Credit spreads are defined as the yield spread between corporate bonds (AAA-rating, maturity of five to seven years) and government bonds (maturity of five to seven years).

Figure A.1. Changes in NEER versus Changes in Credit Spreads
Table A.1. Correlation between Changes in NEER and Changes in Credit Spreads

<table>
<thead>
<tr>
<th></th>
<th>United States</th>
<th>Switzerland</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>0.16***</td>
<td>0.21***</td>
</tr>
<tr>
<td>(N = 328)</td>
<td>(N = 155)</td>
<td></td>
</tr>
<tr>
<td>VIX &gt; 20</td>
<td>0.20**</td>
<td>0.29**</td>
</tr>
<tr>
<td>(N = 131)</td>
<td>(N = 55)</td>
<td></td>
</tr>
<tr>
<td>VIX ≤ 20</td>
<td>0.10</td>
<td>0.06</td>
</tr>
<tr>
<td>(N = 197)</td>
<td>(N = 100)</td>
<td></td>
</tr>
</tbody>
</table>

Note: ***, **, and * denote significance levels of 1, 5, and 10 percent, respectively, based on a t-test.

Appendix B. Baseline Model: Derivations, Equilibrium Equations, and Numerical Solution

B.1 Derivation of Equilibrium Equations

We can reduce the system of equations to one of 13 equations and 13 unknowns.

- **Euler conditions:**
  
  Since we set $Y_{NT,0} = Y_{NT,1} = \chi$, it follows from the combination of Equations (4), (6), (7), and the market clearing conditions for the non-traded goods that the domestic households’ Euler condition reduces to $R_1 = \frac{1}{\beta}$. Equivalently, we have $R_1^* = \frac{1}{\beta^*}$.

- **Home household’s intertemporal budget constraint:**
  
  The combination of Equations (2) and (3) yields the intertemporal budget constraint of the domestic household,

  $$C_{NT,1} + p_1 C_{T,1} = R_1(p_0 Y_{T,0} - p_0 N_0 + Y_{NT,0} - C_{NT,0} - p_0 C_{T,0}) + w_1 L + p_1 N_1 + Y_{NT,1}.$$ 

  Using the market clearing condition for non-traded goods, the market clearing condition for labor, and Equation (10), the intertemporal budget constraint simplifies to
\[
\frac{1}{p_1 C_{T,1}} = R_1 \left( p_0 Y_{T,0} - p_0 N_0 - \frac{1}{p_0 C_{T,0}} \right) \underbrace{B_0}_{w_1} + (1 - \alpha) \left( \frac{K_1}{L} \right)^\alpha p_1 L + p_1 N_1. \tag{B.1}
\]

Consumption expenditure in period 1 depends on the savings in domestic bonds in period 0, on the wage in period 1, and on the profit of the bank the household owns. Note that consumption expenditure on the traded good is constant and equal to 1.

- **Returns on investment securities:**
  
  Using Equation (9) and the market clearing condition for labor, we can rewrite the return on home securities as follows:

  \[
  R_{k,1} = \left( \alpha \left( \frac{L}{K_1} \right)^{1-\alpha} + (1 - \delta) \right) \frac{p_1}{p_0}. \tag{B.2}
  \]

  Equivalently, we can simplify the return on foreign securities:

  \[
  R_{k,1}^* = \left( \alpha \left( \frac{L^*}{K_1^*} \right)^{1-\alpha} + (1 - \delta) \right) \frac{p_1 e_0}{p_0 e_1}. \tag{B.3}
  \]

- **Value of the home bank’s equity capital:**
  
  The value of the home bank’s equity capital in period 1 is (from Equation (15))

  \[
p_1 N_1 = (R_{k,1} - R_1) p_0 S_{p,0} + \left( R_{L1}^* \frac{e_1}{e_0} - R_1 \right) e_0 A_{p,0} + R_1 p_0 N_0. \tag{B.4}
  \]

  Using the market clearing condition for domestic investment securities and Equation (29), the value of the home bank’s equity capital in period can be rewritten as
\[ p_1 N_1 = (R_{k,1} - R_1) p_0 K_1 \]
\[ + \left( \frac{R^*_1 e_1}{e_0} - R_1 \right) \frac{p_0 Y_{T,0} - p_0 K_1 - 1}{p_0 N X_0} + R_1 p_0 N_0. \] \hspace{1cm} (B.5)

- **Banks’ first-order conditions:**

  The first-order conditions of the domestic bank, Equations (16) and (17), can be slightly simplified to

  \[ \frac{1}{R_1} (R_{k,1} - R_1) = \frac{\lambda}{1 + \lambda \theta}, \] \hspace{1cm} (B.6)

  \[ \frac{1}{R_1} \left( \frac{R^*_1 e_1}{e_0} - R_1 \right) = \frac{\lambda}{1 + \lambda \Delta}, \] \hspace{1cm} (B.7)

  And the first-order condition of the foreign bank, Equation (23), is now

  \[ \frac{1}{R^*_1} (R^*_{k,1} - R^*_1) = \frac{\lambda^*}{1 + \lambda^* \theta^*}, \] \hspace{1cm} (B.8)

- **Home capital constraint:**

  Under the assumption that \( \theta > 0 \) and \( \Delta > 0 \), we can rewrite the domestic incentive constraint (13) as a capital constraint (CC) (use Equations (15)–(18)):

  \[ V_0 \geq \theta p_0 S_{p,0} + \Delta e_0 A_{p,0} \]

  \[ \iff \Lambda_{0,1} \left( (R_{k,1} - R_1) p_0 S_{p,0} + \left( \frac{R^*_1 e_1}{e_0} - R_1 \right) e_0 A_{p,0} + R_1 p_0 N_0 \right) \]

  \[ \geq \theta p_0 S_{p,0} + \Delta e_0 A_{p,0} \]

  \[ \iff \Lambda_{0,1} \left( \frac{R^*_1 e_1}{e_0} - R_1 \right) \left( \frac{\theta}{\Delta} p_0 S_{p,0} + e_0 A_{p,0} \right) \]

  \[ + \Lambda_{0,1} R_1 p_0 N_0 \geq \Delta \left( \frac{\theta}{\Delta} p_0 S_{p,0} + e_0 A_{p,0} \right) \]

  \[ \iff \Lambda_{0,1} R_1 p_0 N_0 \geq \left( \frac{\theta}{\Delta} p_0 S_{p,0} + e_0 A_{p,0} \right) \left( \Delta - \Lambda_{0,1} \left( \frac{R^*_1 e_1}{e_0} - R_1 \right) \right) \]
\begin{align*}
\L_0R_0N_0 \geq \frac{\theta p_0S_{p,0} + \e_0 A_{p,0}}{\D - \L_0 \left( \frac{R_{1}^{*} e_{1}}{e_{0}} - R_1 \right)} \\
\L_0R_0N_0 \geq \phi p_0N_0 \geq \theta p_0S_{p,0} + \D \e_0 A_{p,0}.
\end{align*}

(B.9)

where \( \phi = \frac{\D \L_0R_1}{\D - \L_0 \left( \frac{R_{1}^{*} e_{1}}{e_{0}} - R_1 \right)} = \frac{\theta \L_0R_1}{\D - \L_0 \left( R_{k,1} - R_1 \right)} \). If in one of the two markets the friction parameter is set to zero, then this inequality simplifies to

\begin{align*}
\frac{\L_0R_0}{\D - \L_0 \left( \frac{R_{1}^{*} e_{1}}{e_{0}} - R_1 \right)} p_0N_0 &\geq \e_0 A_{p,0} \quad \text{if } \theta = 0 \quad \text{(B.10)} \\
\frac{\L_0R_0}{\D - \L_0 \left( R_{k,1} - R_1 \right)} p_0N_0 &\geq p_0S_{p,0} \quad \text{if } \D = 0. \quad \text{(B.11)}
\end{align*}

Note that if \( \theta > 0 \) and/or \( \D > 0 \), this does not necessarily imply that the capital constraint is binding (\( \lambda > 0 \)), i.e., that there are limits to arbitrage in at least one market. The capital constraint is only binding if \( \theta > \bar{\theta} \) and/or \( \D > \bar{\D} \). We can summarize the capital constraint as follows:

\begin{align*}
\text{CC} = \begin{cases}
\frac{\D \L_0R_1}{\D - \L_0 \left( \frac{R_{1}^{*} e_{1}}{e_{0}} - R_1 \right)} p_0N_0 \geq \theta p_0S_{p,0} \\
+ \frac{\D \e_0 A_{p,0}}{\D - \L_0 \left( \frac{R_{1}^{*} e_{1}}{e_{0}} - R_1 \right)} p_0N_0 \geq \theta p_0S_{p,0} \\
\frac{\D \e_0 A_{p,0}}{\D - \L_0 \left( R_{k,1} - R_1 \right)} p_0N_0 \geq \theta p_0S_{p,0} \\
\text{no CC}
\end{cases}
\end{align*}

(B.12)

If \( \theta > 0 \) and \( \D > 0 \), it is irrelevant whether the first or the second Equation of (B.12) is considered.

Using the Euler condition, the market clearing condition for domestic investment securities, and Equation (29), we can simplify the capital constraint of the domestic bank as follows:
CC = \begin{cases} \Delta \left( \frac{\Delta}{\pi_1} \left( R^*_{k,1} - R^*_{1} \right) p_0 N_0 - \theta p_0 K_1 \right) + \Delta (p_0 Y_{T,0} - p_0 K_1 - 1) & \text{if } \theta \geq 0, \Delta > 0 \\ \frac{\theta}{\pi_1} \left( R_{k,1} - R_{1} \right) p_0 N_0 \geq \theta p_0 K_1 \\ \Delta (p_0 Y_{T,0} - p_0 K_1 - 1) & \text{if } \theta > 0, \Delta \geq 0 \\ \text{no CC} & \text{if } \theta = 0, \Delta = 0. \end{cases}

Keep in mind that \((p_0 Y_{T,0} - p_0 K_1 - 1) = p_0 N X_0\). For any parameter specification but \(\theta = \Delta = 0\), the Karush-Kuhn-Tucker (KKT) conditions need to hold. Define

\[ g = \begin{cases} \Delta \left( \frac{\Delta}{\pi_1} \left( R^*_{k,1} - R^*_{1} \right) p_0 N_0 - \theta p_0 K_1 \right) - \Delta (p_0 Y_{T,0} - p_0 K_1 - 1) & \text{if } \theta \geq 0, \Delta > 0 \\ \frac{\theta}{\pi_1} \left( R_{k,1} - R_{1} \right) p_0 N_0 - \theta p_0 K_1 \\ - \Delta (p_0 Y_{T,0} - p_0 K_1 - 1) & \text{if } \theta > 0, \Delta \geq 0, \end{cases} \tag{B.13} \]

where \(g\) is a function of domestic endogenous variables. Then, the KKT conditions for the inequality constraint of the home bank are

\[ g \geq 0 \tag{B.14} \]
\[ \lambda \geq 0 \tag{B.15} \]
\[ \lambda g = 0. \tag{B.16} \]

**Foreign capital constraint:**

Equivalently, for the foreign bank we have

\[ CC^* = \begin{cases} \frac{1}{\pi_1} \left( R^*_{k,1} - R^*_{1} \right) p_0 N^*_{0} \geq \frac{p_0}{e_0} K^*_{1} & \text{if } \theta^* > 0 \\ \text{no CC}^* & \text{if } \theta^* = 0, \end{cases} \tag{B.17} \]

where we have used the law of one price. For any parameter specification but \(\theta^* = 0\), the KKT conditions need to hold. Define

\[ g^* = \begin{cases} \frac{1}{\pi_1} \left( R^*_{k,1} - R^*_{1} \right) p_0 N^*_{0} - \frac{p_0}{e_0} K^*_{1} & \text{if } \theta^* > 0, \end{cases} \]
where $g^*$ is a function of foreign endogenous variables. Then, the KKT conditions for the inequality constraint of the foreign bank are

$$g^* \geq 0 \quad (B.18)$$
$$\lambda^* \geq 0 \quad (B.19)$$
$$\lambda^* g^* = 0. \quad (B.20)$$

- **Market clearing conditions for traded goods:**
  The market clearing condition for traded goods in period 0 simplifies to

$$Y_{T,0} + Y^*_{T,0} = \frac{1}{p_0} + \frac{e_0}{p_0} + K_1 + K^*_1. \quad (B.21)$$

The market clearing condition for traded goods in period 1 is

$$K_1^{\alpha} L^{1-\alpha} + K^*_1^{\alpha} L^{*1-\alpha} + (1 - \delta)K_1 + (1 - \delta)K^*_1 = \frac{1}{p_1} + \frac{e_1}{p_1}, \quad (B.22)$$

where we have used the production function of the domestic and foreign firm.

In sum, we reduce our system of equations to the following 13 equilibrium equations:

$$R_1 = \frac{1}{\beta} \quad (B.23)$$
$$R^*_1 = \frac{1}{\beta^*} \quad (B.24)$$

$$1 = R_1(p_0Y_{T,0} - p_0N_0 - 1) + (1 - \alpha) \left( \frac{K_1}{L} \right)^{\alpha} p_1L + p_1N_1 \quad (B.25)$$

$$R_{k,1} = \left( \alpha \left( \frac{L}{K_1} \right)^{1-\alpha} + (1 - \delta) \right) \frac{p_1}{p_0} \quad (B.26)$$

$$R^*_{k,1} = \left( \alpha \left( \frac{L^*}{K^*_1} \right)^{1-\alpha} + (1 - \delta) \right) \frac{p_1e_0}{p_0e_1} \quad (B.27)$$
\[ p_1 N_1 = (R_{k,1} - R_1) p_0 K_1 + \left( R_1^* \frac{e_1}{e_0} - R_1 \right) (p_0 Y_{T,0} - p_0 K_1 - 1) + R_1 p_0 N_0 \] (B.28)

\[ \frac{1}{R_1} (R_{k,1} - R_1) = \frac{\lambda}{1 + \lambda} \theta \] (B.29)

\[ \frac{1}{R_1^*} \left( R_1^* \frac{e_1}{e_0} - R_1 \right) = \frac{\lambda}{1 + \lambda} \Delta \] (B.30)

\[ \frac{1}{R_1^*} (R_1^* - R_1^*) = \frac{\lambda^*}{1 + \lambda^*} \theta^* \] (B.31)

\[
\begin{align*}
\text{CC} = & \begin{cases} 
\Delta - \frac{1}{R_1^* (R_1^* - R_0)} \ p_0 N_0 \geq \theta p_0 K_1 \\
+ \Delta (p_0 Y_{T,0} - p_0 K_1 - 1) & \text{if } \theta \geq 0, \Delta > 0 \\
+ \Delta (p_0 Y_{T,0} - p_0 K_1 - 1) & \text{if } \theta > 0, \Delta \geq 0 \\
\text{no CC, } \lambda = 0 & \text{if } \theta = 0, \Delta = 0
\end{cases}
\end{align*}
\] (B.32)

\[
\begin{align*}
\text{CC}^* = & \begin{cases} 
\frac{1}{\theta^* - \frac{1}{R_1^* (R_{k,1} - R_1^*)}} \ p_0 N_0^* \geq \frac{p_0}{e_0} K_1^* & \text{if } \theta^* > 0 \\
\text{no CC}^*, \lambda^* = 0 & \text{if } \theta^* = 0
\end{cases}
\end{align*}
\] (B.33)

\[
Y_{T,0} + Y_{T,0}^* = \frac{1}{p_0} + \frac{e_0}{p_0} + K_1 + K_1^* \] (B.34)

\[
K_1^* L^{1-\alpha} + K_1^{**} L^{*1-\alpha} + (1 - \delta) K_1 + (1 - \delta) K_1^* = \frac{1}{p_1} + \frac{e_1}{p_1} \] (B.35)

Furthermore, we have to take into account the remaining KKT conditions for the domestic country if the domestic friction parameters are non-zero (\( \theta \neq 0 \) and \( \Delta \neq 0 \)), Equations (B.15) and (B.16), and the remaining KKT conditions for the foreign country if the foreign friction parameter is non-zero (\( \theta^* \neq 0 \)), Equations (B.19) and (B.20). The 13 unknowns are \( e_0, e_1, p_0, p_1, K_1, K_1^*, R_1, R_1^*, R_{k,1}, R_{k,1}^*, N_1, \lambda, \lambda^* \).
B.2 Proof: Properties of the Model with \( \theta = \theta^* > 0 \) and \( \Delta = 0 \) (for ICs Binding)

First note that taking the ratio of the two expressions for the returns on the investment securities, \( R_{k,1} \) and \( R^*_{k,1} \) (see Equations (B.26) and (B.27)), yields the following (generally valid) relationship between \( K_1 \) and \( K^*_1 \) (for \( L = L^* \)):

\[
K_1 = \left( \frac{p_0 e_1}{p_1 e_0} R^*_{k,1} - (1 - \delta) \right)^{\frac{1}{1-\alpha}} K^*_1. \tag{B.36}
\]

For finding the relative level of capital for when \( \theta = \theta^* > 0 \) and \( \Delta = 0 \) conditional on the ICs being binding, combine each country’s (binding) capital constraints (see Equations (B.32) and (B.33)) with the respective expression for the return on the investment securities:

\[
\frac{1}{\theta - \frac{1}{R_1} \left( \left( \alpha \left( \frac{L}{K_1} \right)^{1-\alpha} + (1 - \delta) \right) \frac{p_1}{p_0} - R_1 \right)} N_0 = K_1 \tag{B.37}
\]

\[
\frac{1}{\theta^* - \frac{1}{R^*_1} \left( \left( \alpha \left( \frac{L^*}{K^*_1} \right)^{1-\alpha} + (1 - \delta) \right) \frac{p_1}{p_0} e_0 - R^*_1 \right)} N^*_0 = K^*_1. \tag{B.38}
\]

By assumption, \( L^* = L \) and \( N^*_0 = N_0 \). Furthermore, \( \Delta = 0 \) implies that \( R_1 = R^*_{k,1} \frac{e_1}{e_0} \), and we have \( \theta = \theta^* > 0 \). Thus, Equation (B.38) can be written as

\[
\frac{1}{\theta - \frac{1}{R^*_1} \left( \left( \alpha \left( \frac{L^*}{K^*_1} \right)^{1-\alpha} + (1 - \delta) \right) \frac{p_1}{p_0} e_0 - R^*_1 \right)} N_0 = K^*_1. \tag{B.39}
\]

Looking at Equations (B.37) and (B.39), it becomes clear that it must be the case that \( K_1 = K^*_1 \). From \( K_1 = K^*_1 \), in turn, it follows that \( R_{k,1} = R^*_{k,1} \frac{e_1}{e_0} \) (see Equation (B.36)), i.e., returns on the investment securities in terms of the home numéraire are equalized.

For evaluating the relative tightness of the two countries’ banks’ incentive constraints, note that the first-order conditions (16) and
(23) can be written as follows (remember that $\Lambda_{0,1} = \frac{1}{R_1}$ and $\Lambda_{0,1}^* = \frac{1}{R_1^*}$):

\[
\frac{R_{k,1}}{R_1} = 1 + \frac{\lambda}{1 + \lambda \theta} \quad \text{(B.40)}
\]

\[
\frac{R_{k,1}^*}{R_1^*} = 1 + \frac{\lambda^*}{1 + \lambda^* \theta^*} \quad \text{(B.41)}
\]

From $R_1 = R_{1,1}^* e_{1,0}$ and $R_{k,1} = R_{k,1,1}^* e_{1,0}$, it follows that $\frac{R_{k,1}}{R_1} = \frac{R_{k,1}^*}{R_1^*}$. Hence, and given that $\theta = \theta^*$, Equations (B.40) and (B.41) imply that $\lambda = \lambda^*$, i.e., the incentive constraints are equally binding in the two countries.

B.3 Proof: Properties of the Model with $\theta = \theta^* = \Delta > 0$
(for ICs Binding)

An interesting case to have a closer look at is the one where banks are equally constrained in all markets: $\theta = \theta^* = \Delta > 0$: From no-arbitrage relation (18), it follows that home banks will choose their portfolio such that excess returns in the domestic investment market and the international credit market are just equalized. Measured in terms of the home numéraire, we have $R_1 < R_{k,1} = R_{k,1,1}^* e_{1,0} < R_{k,1,1}^* e_{1,0}$, which implies that the level of investment is lower in the foreign country as compared with the home country. This results from the fact that home banks are constrained to hold less foreign bonds and hence intermediate fewer net exports relative to the frictionless case. Also note that the home banks’ incentive constraint is more binding than the one of the foreign banks ($\lambda > \lambda^*$). Intuitively, when banks are only constrained in the investment markets ($\theta = \theta^* > 0$ and $\Delta = 0$), the real interest rates in the two countries must be equalized and home and foreign banks hold the same amount in investment securities. Consequently, both incentive constraints are equally binding. Once $\Delta$ is larger than zero, the restriction of the home banks increases, while tension on the foreign banks is released as less funds flow into the country and demand for intermediation falls.
Formal proof: $\theta = \theta^* = \Delta > 0$ implies that $R_1 < R_{k,1} = R_{k,1}^{e_{1e_0}} < R_{k,1}^{e_{1e_0}}$ (conditional on the ICs being binding). From $R_{k,1} < R_{k,1}^{e_{1e_0}}$ and Equation (B.36), it follows that $K_1 > K_1^*$. For evaluating the relative tightness of the two countries’ banks’ incentive constraints, combine each country’s (binding) capital constraints (see Equations (B.32) and (B.33)) with first-order conditions (16) and (23) and the respective market clearing conditions:

$$\frac{1}{\theta - \frac{\lambda}{1+\theta}} N_0 = K_1 + N X_0 \tag{B.42}$$

$$\frac{1}{\theta^* - \frac{\lambda^*}{1+\lambda^*}\theta^*} N_0^* = K_1^*. \tag{B.43}$$

Again, by assumption, $N_0^* = N_0$. Furthermore, we have $\theta = \theta^*$. Thus, taking the ratio of Equations (B.42) and (B.43) yields

$$\frac{1 + \lambda}{1 + \lambda^*} = \frac{K_1 + N X_0}{K_1^*}. \tag{B.44}$$

We know that $K_1 > K_1^*$ and $N X_0 \geq 0$, from which it finally follows that $\lambda > \lambda^*$, i.e., the home banks’ incentive constraint binds tighter than the foreign banks’ incentive constraint.

B.4 Effect of Financial Frictions in the Home Investment Market

This section discusses the effects of financial frictions in the home investment market, which are captured by an increase in the home investment market friction parameter $\theta$. $\theta^*$ and $\Delta$ are set to zero. When the home investment market friction parameter is sufficiently large for the home incentive constraint and hence also the endogenous capital constraint to become binding ($\lambda > 0$), home banks are hindered to exploit all arbitrage opportunities and excess returns in the home investment market become positive: $R_{k,1} - R_1 > 0$ (see Equation (16)). Excess returns in the international credit market and the foreign investment market, however, remain zero (see Equations (17) and (23)). Combining the home banks’ capital constraint (19) and the investment market clearing condition (25) reveals that
Figure B.1. Financial Frictions in the Home Investment Market: $\theta > 0$

Note: The solid lines represent the frictionless equilibrium, the dashed lines the equilibrium with the fiction.

with the constraint starting to be binding, the level of capital in the home country will obviously be limited:

$$\frac{1}{\theta - \frac{1}{R_1} (R_{k,1} - R_1)} p_0 N_0 \geq p_0 K_1. \quad (B.45)$$

(As $\lambda > 0$, this equation will hold with equality.)

Graphically, financial frictions in the home investment market shift the home investment curve to the left (see Figure B.1). For a given real rate of return $\frac{p_0}{R_1} R_1$, investment in the home country decreases, as the home banks’ ability to intermediate funds in this market has decreased and they face limits to arbitrage. Costs of capital in the home market increase. In order to maintain the world equilibrium, the equilibrium real rate of return has to decrease. Due to the frictions in the home investment market, the home country will in equilibrium slightly decrease its savings, and invest a much larger part abroad: The credit constraint with respect to investments

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33 For a formal proof of how an increase in $\theta$ affects the two countries’ saving and investment schedules, see Appendix E.
in home capital makes the home banks reallocate their portfolio and invest a larger part in foreign bonds. Altogether, this causes an increase in the home country’s net exports. The foreign country, on the other hand, also decreases its savings, but at the same time can increase its investments as the foreign banks obtain a larger amount of funds, which leads to an increase in its net imports. Overall, there is a decrease in world savings and, consequently, world investments, implying a lower level of world output in the second period. Furthermore, the frictions in the home investment market also lead to a misallocation of capital: Now, a majority of capital is invested in the foreign country. This change in the allocation of capital implies that relative to the frictionless level, the home country’s output in the second period will decrease while the foreign country’s output in the second period will increase, implying that there is a change in the two countries’ fundamentals. The relative decrease in the home country’s lifetime resources induces a home depreciation in both periods.

Figure B.2 provides a numerical illustration of these results. Setting $\theta^*$ and $\Delta$ equal to zero and using the calibration of Table 1 for the remaining parameters, it shows how the model’s equilibrium evolves as the home investment market friction parameter $\theta$ increases. Note that $\theta = 1$ (i.e., banks can divert all home investment securities) does not imply that home banks do not hold domestic assets anymore: It just means that in case of misbehavior, the banks could divert and keep the proceeds of all these assets. If, however, the excess returns they can earn on the investment securities when not diverting them are large enough, they still have no incentive to misbehave and the financial markets will work even with $\theta = 1$.

### B.5 Numerical Illustration

As mentioned before, there exists no closed-form solution of the model; however, we can solve it numerically in MATLAB. The following figures provide numerical results—in particular, they show the evolution of the model’s equilibrium under different specifications of the friction parameters using the calibration of Table 1 for the remaining parameters.
Figure B.2. Effect of an Increase in $\theta$ ($\theta$ on x-axis)

Note: Evolution of the model’s equilibrium as the friction parameter in the home investment market increases, starting from a frictionless point. $\theta \geq 0$, $\theta^* = 0$, $\Delta = 0$. The remaining parameter values are summarized in Table 1. The red line represents the value in the frictionless equilibrium.
Figure B.3. Effect of an Increase in $\Delta$ ($\Delta$ on x-axis)

Note: Evolution of the model’s equilibrium as the friction parameter in the international credit market increases, starting from a point where there are limits to arbitrage in all financial markets. $\theta = 1/3$, $\theta^* = 1/3$, $\Delta \geq 1/3$. The remaining parameter values are summarized in Table 1. The red line represents the value in the frictionless equilibrium.
Figure B.4. Effect of an Increase in $\theta^*$ ($\theta^*$ on x-axis)

Note: Evolution of the model’s equilibrium as the friction parameter in the foreign investment market increases, starting from a point where there are limits to arbitrage in all financial markets. $\theta = 1/3$, $\theta^* \geq 1/3$, $\Delta = 1/3$. The remaining parameter values are summarized in Table 1. The red line represents the value in the frictionless equilibrium. Given that the setup of the banking sector is only valid when $e_0 A_{p,0} \geq 0$, the plots only cover a limited range of possible values for $\theta^*$. 
Figure B.5. Effect of an Increase in $\theta$ ($\theta$ on x-axis)

Note: Evolution of the model’s equilibrium as the friction parameter in the home investment market increases, starting from a point where there are limits to arbitrage in all financial markets. $\theta \geq 1/3$, $\theta^* = 1/3$, $\Delta = 1/3$. The remaining parameter values are summarized in Table 1. The red line represents the value in the frictionless equilibrium.
Appendix C. Analytical Solution of the Frictionless Model

In the frictionless model the parameters \( \theta, \Delta, \) and \( \theta^* \) are either equal to zero or sufficiently low for the banks’ incentive constraint not to be binding. In the system of 13 equations derived in Appendix B, Equation (B.28) reduces to \( p_1 N_1 = R_1 p_0 N_0 \) since all excess returns are zero and Equations (B.29)–(B.31) can be replaced by \( R_{k,1} = R_1, \ R_{k,1}^* = R_1 \) and \( R_{k,1}^* = R_1^* \), respectively. In a frictionless environment domestic and foreign banks face no capital constraint, therefore we can omit Equations (B.32) and (B.33), and the variables \( \lambda \) and \( \lambda^* \) and consider the resulting system of 11 equations and 11 unknowns. Under the assumption that \( \delta = 1 \), it is possible to find an analytical solution of the frictionless model. Given that \( Y_{T,0} = Y_{T,0}^* \) and \( L = L^* \), solving the system of equations yields

\[
\begin{align*}
e_0 &= \frac{1 + \beta}{1 + \beta^*}, e_1 = \frac{\beta^*}{\beta} e_0, K_1 = K_1^* = \gamma_1 Y_{T,0}, \\
p_0 &= \gamma_2 \frac{1}{Y_{T,0}}, p_1 = \gamma_3 \frac{1}{Y_{T,1}} = \gamma_3 \frac{1}{K_1^\alpha L^{1-\alpha}}, \tag{C.1}
\end{align*}
\]

where \( \gamma_1 \equiv \frac{\alpha \beta(1+\beta^*) + \alpha \beta^*(1+\beta)}{(1+\alpha\beta^*)(1+\beta) + (1+\alpha\beta)(1+\beta^*)} \), \( \gamma_2 \equiv \frac{(1+\alpha\beta^*)(1+\beta) + (1+\alpha\beta)(1+\beta^*)}{2(1+\beta^*)} \), and \( \gamma_3 \equiv \frac{\beta(1+\beta^*)^2 + \beta^*(1+\beta)}{2(1+\beta^*)} \). The exchange rate only depends on the discount factor of home and foreign agents. Investment, and hence production, is equally high in both countries and is increasing in endowment of traded goods in the first period \( Y_{T,0} \). The price of traded goods depends negatively on its supply. The remaining variables of the model can be derived from these six variables. Net exports, e.g., are

\[
e_0 A_{p,0} = p_0 N X_0 = \frac{\beta - \beta^*}{2(1+\beta^*)}. \tag{C.2}
\]

Appendix D. Equilibrium Equations under International Portfolio Flows and Central Bank Intermediation

Introducing international portfolio flows and central bank intermediation to the baseline model in Section 2 leads to the following changes in the system of 13 equations derived in Appendix B: Equation (B.25) is augmented by the returns on the portfolio flows and
\( \Pi_{CB,1} \), the profit of the central bank that is transferred to the domestic household:

\[
1 = R_1(p_0Y_{T,0} - p_0N_0 - 1) + (1 - \alpha)\left( \frac{K_1}{L} \right)^\alpha p_1L + p_1N_1 \\
+ \left( R_1^* \frac{e_1}{e_0} - R_1 \right) e_0f + \Pi_{CB,1} \tag{D.1}
\]

where

\[
\Pi_{CB,1} = (R_{k,1} - R_1)p_0S_{CB,0} + \left( R_1^* \frac{e_1}{e_0} - R_1 \right) e_0A_{CB,0}. \tag{D.2}
\]

The consolidation of the domestic household’s, bank’s and central bank’s budget constraint (Equation (37)) yields

\[
e_0A_{p,0} + e_0A_{CB,0} + e_0f - f^* = p_0NX_0. \tag{D.3}
\]

Equation (B.28) changes to

\[
p_1N_1 = (R_{k,1} - R_1)p_0(K_1 - S_{CB,0}) \\
+ \left( R_1^* \frac{e_1}{e_0} - R_1 \right) (p_0NX_0 - e_0f + f^* - e_0A_{CB,0}) + R_1p_0N_0 \tag{D.4}
\]

and Equation (B.32) now looks as follows:

\[
CC = \begin{cases} \\
\Delta - \frac{\Delta}{R_1^*} \left( R_1^* \frac{e_1}{e_0} - R_1 \right) p_0N_0 \geq \theta p_0(K_1 - S_{CB,0}) \\
+ \Delta(p_0NX_0 - e_0f + f^* - e_0A_{CB,0}) & \text{if } \theta \geq 0, \Delta > 0 \\
\frac{\theta}{R_1^*} (\theta - R_{k,1} - R_1) p_0N_0 \geq \theta p_0(K_1 - S_{CB,0}) \\
+ \Delta(p_0NX_0 - e_0f + f^* - e_0A_{CB,0}) & \text{if } \theta > 0, \Delta \geq 0 \\
\text{no CC, } \lambda = 0 & \text{if } \theta = 0, \Delta = 0,
\end{cases} \tag{D.5}
\]

where \( p_0NX_0 \) is substituted by \( (p_0Y_{T,0} - p_0K_1 - 1) \). The KKT conditions change accordingly. Along with these changes, we have to include one additional equation which is the profit of the central bank in period 1 (Equation (D.2)).
The following figures provide a numerical illustration of the model solution under capital inflows, credit easing, or foreign exchange interventions. They show the evolution of the model’s equilibrium under increasing values for one of these variables using different specifications of the friction parameters and the calibration of Table 1 for the remaining parameters.
Figure D.1. Effect of an Increase in $f^*$ ($f^*$ on x-axis)

Note: Evolution of the model’s equilibrium as capital inflows increase, starting from a frictionless point. $\theta = 0$, $\theta^* = 0$, $\Delta = 1/4$. The remaining parameter values are summarized in Table 1. Note that the home banks only get credit constrained once $f^*$ exceeds a certain value.
Figure D.2. Effect of an Increase in $S_{CB,0}$ ($p_0S_{CB,0}$ on x-axis)

Note: Evolution of the model’s equilibrium as central bank intermediation in the domestic investment market increases, starting from a point where there are limits to arbitrage in all financial markets. $\theta = 1/3$, $\theta^* = 1/3$, $\Delta = 1/3$. The remaining parameter values are summarized in Table 1. The red line represents the value in the frictionless equilibrium.
Note: Evolution of the model’s equilibrium as central bank intermediation in the international credit market increases, starting from a point where there are limits to arbitrage in all financial markets. $\theta = 1/3$, $\theta^* = 1/3$, $\Delta = 1/3$. The remaining parameter values are summarized in Table 1. The red line represents the value in the frictionless equilibrium. The dashed part of the lines captures the range of FX interventions where the latter would require the home banks to go short in foreign bonds in order to fulfill the central bank's demand for these assets and hence covers a part where our model technically is not valid.
Appendix E. Proof: Metzler Diagram

For convenience, Figures 4, 2, and B.1 depict the reaction of the economy for the case where the respective friction parameter passes from being non-binding ($\lambda = 0$) to being binding ($\lambda > 0$). Due to the banks’ positive equity capital, this always happens at some strictly positive value of $\theta$, $\theta^*$, or $\Delta$, respectively, denoted by $\overline{\theta}$, $\overline{\theta}^*$, or $\overline{\Delta}$, which represent the highest possible values where the friction parameters are still non-binding.\footnote{Intuitively, for very low values of $\theta$, $\theta^*$, or $\Delta$, respectively, the divertable part of a bank’s assets will inevitably be lower than the bank’s equity capital, which it would lose in case of misbehavior. Thus, the banks’ incentive constraint will not be binding.}

\textit{E.1 Investment Schedules}

The two countries’ investment schedules are given by market clearing in the investment markets ($S_{p,0} = K_1$ and $S_{p,0}^* = K_1^*$), by Equations (B.26) and (B.27), which relate the levels of capital and the (real) returns on the investment securities $\frac{p_0}{p_1} R_{k,1}$ and $\frac{p_0^*}{p_1^*} R_{K,1}^*$, and by Equations (5), (16), and (23), which define the relationships between the returns on the investment securities and the return on the bonds ($R_1$ and $R_1^*$, respectively) and result from the banks’ optimization problem. Thus, the investment schedules ($KK$) and ($KK^*$) are

\[ KK_1 = \left( \frac{1}{\alpha} \left( \frac{p_0}{p_1} R_{k,1} - (1 - \delta) \right) \right)^{\frac{1}{\alpha - 1}} L, \]

where $\frac{p_0}{p_1} R_{k,1} = \frac{p_0}{p_1} R_1 \left( 1 + \frac{\lambda}{1 + \lambda} \theta \right)$

\[ KK_1^* = \left( \frac{1}{\alpha} \left( \frac{p_0^*}{p_1^*} R_{K,1}^* - (1 - \delta) \right) \right)^{\frac{1}{\alpha - 1}} L^*, \]

where $\frac{p_0^*}{p_1^*} R_{K,1}^* = \frac{p_0^*}{p_1^*} R_1^* \left( 1 + \frac{\lambda^*}{1 + \lambda^*} \theta^* \right)$.\footnote{Intuitively, for very low values of $\theta$, $\theta^*$, or $\Delta$, respectively, the divertable part of a bank’s assets will inevitably be lower than the bank’s equity capital, which it would lose in case of misbehavior. Thus, the banks’ incentive constraint will not be binding.}
• **Effect of an increase in $\theta$:**

Using the concept of the total differential, one finds that for a given real interest rate $\frac{p_0}{p_1} R_1$, an increase in $\theta$ has the following effect on home capital:

$$
\left. \frac{dK_1}{d\theta} \right|_{\frac{p_0}{p_1} R_1 \text{ constant}} = \frac{1}{\alpha(\alpha - 1)} \left( \frac{1}{\alpha} \left( p_0 \left( \frac{R_{k,1}}{p_1} - (1 - \delta) \right) \right) \right)^{\frac{2-\alpha}{\alpha - 1}}
$$

$$
\left. \frac{d}{d\theta} \left( \frac{p_0}{p_1} R_{k,1} \right) \right|_{\frac{p_0}{p_1} R_1 \text{ constant}}
$$

$$
\left. \frac{d}{d\theta} \left( \frac{p_0}{p_1} R_{k,1} \right) \right|_{\frac{p_0}{p_1} R_1 \text{ constant}} = -\frac{1}{\alpha(1 - \alpha)} \left( \frac{L}{K_1} \right)^{\alpha-2} \frac{d}{d\theta} \left( \frac{p_0}{p_1} R_{k,1} \right) \left|_{\frac{p_0}{p_1} R_1 \text{ constant}} \right. 
$$

(E.1)

The term in front of the final derivative is negative as $0 < \alpha < 1$, while the derivative itself is equal to the following expression:

$$
\left. \frac{d}{d\theta} \left( \frac{p_0}{p_1} R_{k,1} \right) \right|_{\frac{p_0}{p_1} R_1 \text{ constant}}
$$

$$
\left. \frac{d}{d\theta} \left( \frac{p_0}{p_1} R_{k,1} \right) \right|_{\frac{p_0}{p_1} R_1 \text{ constant}} = \frac{d}{d\theta} \left( \frac{p_0}{p_1} R_1 \left( 1 + \frac{\lambda}{1 + \lambda} \theta \right) \right) \left|_{\frac{p_0}{p_1} R_1 \text{ constant}} \right.
$$

$$
\left. \frac{d}{d\theta} \left( \frac{p_0}{p_1} R_{k,1} \right) \right|_{\frac{p_0}{p_1} R_1 \text{ constant}} = \frac{p_0}{p_1} R_1 \left( \frac{1}{(1 + \lambda)^2} \theta \frac{d\lambda}{d\theta} \left|_{\frac{p_0}{p_1} R_1 \text{ constant}} \right. + \frac{\lambda}{1 + \lambda} \right)
$$

$$
\left. \frac{d}{d\theta} \left( \frac{p_0}{p_1} R_{k,1} \right) \right|_{\frac{p_0}{p_1} R_1 \text{ constant}} \geq 0,
$$

(E.2)

where the last steps follow from the fact that we look at the effect where the friction parameter passes from being non-binding to being binding ($\Rightarrow \lambda = 0$ and $\frac{d\lambda}{d\theta} \left|_{\frac{p_0}{p_1} R_1 \text{ constant}} \right. > 0$),
which happens at the strictly positive value $\theta = \bar{\theta}$. Altogether, this implies that for a given real interest rate $\frac{p_0}{p_1} R_1$, an increase in $\theta$ leads to a decrease in the level of home capital, which corresponds to a negative shift in the home investment curve. Likewise, one finds for foreign capital:

$$\frac{dK_1^*}{d\theta} \bigg|_{\frac{p_0^*}{p_1^*} R_1^* \text{ constant}} = -\frac{1}{\alpha(1 - \alpha)} \left( \frac{L^*}{K_1^*} \right)^{\alpha-2} L^* \frac{d}{d\theta} \left( \frac{p_0^*}{p_1^*} R_{K,1}^* \right) \bigg|_{\frac{p_0^*}{p_1^*} R_1^* \text{ constant}},$$

(E.3)

where

$$\frac{d}{d\theta} \left( \frac{p_0^*}{p_1^*} R_{K,1}^* \right) \bigg|_{\frac{p_0^*}{p_1^*} R_1^* \text{ constant}} = \frac{1}{\alpha(1 - \alpha)} \left( \frac{L^*}{K_1^*} \right)^{\alpha-2} L^* \frac{d}{d\theta} \left( \frac{p_0^*}{p_1^*} R_{K,1}^* \right) \bigg|_{\frac{p_0^*}{p_1^*} R_1^* \text{ constant}} = 0. \quad \text{(E.4)}$$

Hence, given a foreign real interest rate $\frac{p_0^*}{p_1^*} R_1^*$, an increase in $\theta$ has no effect on foreign investment (remember that $\theta^* = 0$). **Effect of an increase in $\theta^*$:**

By the same reasoning as above, one finds:

$$\frac{dK_1}{d\theta^*} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} = -\frac{1}{\alpha(1 - \alpha)} \left( \frac{L}{K_1} \right)^{\alpha-2} L \frac{d}{d\theta^*} \left( \frac{p_0}{p_1} R_{k,1} \right) \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}},$$

(E.5)
where

\[ \frac{d \left( \frac{p_0}{p_1} R_{k,1} \right)}{d \theta^*} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} = \frac{d}{d \theta^*} \left( \frac{p_0}{p_1} R_1 \left( 1 + \frac{\lambda}{1 + \lambda} \theta \right) \right) \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} = 0. \tag{E.6} \]

Hence, given a real interest rate \( \frac{p_0}{p_1} R_1 \), an increase in \( \theta^* \) has no effect on home investment (remember that \( \theta = 0 \)).

For the foreign investment curve, one finds:

\[ \frac{dK^*_1}{d \theta^*} \bigg|_{\frac{p_0^*}{p_1^*} R_1^* \text{ constant}} = -\frac{1}{\alpha(1 - \alpha)} \left( \frac{L^*}{K_1^*} \right)^{\alpha-2} L^* \frac{d \left( \frac{p_0^*}{p_1^*} R_{K,1}^* \right)}{d \theta^*} \bigg|_{\frac{p_0^*}{p_1^*} R_1^* \text{ constant}}, \tag{E.7} \]

where

\[ \frac{d \left( \frac{p_0^*}{p_1^*} R_{K,1}^* \right)}{d \theta^*} \bigg|_{\frac{p_0^*}{p_1^*} R_1^* \text{ constant}} = \frac{p_0^*}{p_1^*} R_1^* \frac{d \lambda^*}{d \theta^*} \bigg|_{\frac{p_0^*}{p_1^*} R_1^* \text{ constant}} > 0. \tag{E.8} \]

Hence, given a foreign real interest rate \( \frac{p_0^*}{p_1^*} R_1^* \), an increase in \( \theta^* \) leads to a decrease in the level of foreign capital, which corresponds to a negative shift in the foreign investment curve.
Effect of an increase in $\Delta$:

For the home investment curve, one finds:

$$\frac{dK_1}{d\Delta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = -\frac{1}{\alpha(1 - \alpha)} \left( \frac{L}{K_1} \right)^{\alpha - 2} L \frac{d\left(\frac{p_0}{p_1}R_{k,1}\right)}{d\Delta} \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}},$$

(E.9)

where (remember that $\theta = 0$):

$$d\left(\frac{p_0}{p_1}R_{k,1}\right) \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = \frac{d}{d\Delta} \left( \frac{p_0}{p_1}R_1 \left(1 + \frac{\lambda}{1 + \lambda \theta}\right) \right) \bigg|_{\frac{p_0}{p_1}R_1 \text{ constant}} = 0. \quad \text{(E.10)}$$

Likewise, one finds for foreign capital:

$$\frac{dK^*_1}{d\Delta} \bigg|_{\frac{p_0^*}{p_1^*}R^*_1 \text{ constant}} = -\frac{1}{\alpha(1 - \alpha)} \left( \frac{L^*}{K_1^*} \right)^{\alpha - 2} L^* \frac{d\left(\frac{p_0^*}{p_1^*}R^*_{K,1}\right)}{d\Delta} \bigg|_{\frac{p_0^*}{p_1^*}R^*_1 \text{ constant}},$$

(E.11)

where (remember that $\theta^* = 0$):

$$d\left(\frac{p_0^*}{p_1^*}R^*_{K,1}\right) \bigg|_{\frac{p_0^*}{p_1^*}R^*_1 \text{ constant}} = \frac{d}{d\Delta} \left( \frac{p_0^*}{p_1^*}R^*_1 \left(1 + \frac{\lambda^*}{1 + \lambda^* \theta^*}\right) \right) \bigg|_{\frac{p_0^*}{p_1^*}R^*_1 \text{ constant}} = 0. \quad \text{(E.12)}$$
Hence, given a real interest rate, an increase in \( \Delta \) does not have any effect on the level of investment of either country.

### E.2 Savings Schedules

The home country’s saving schedule is described by the Euler equation (expressed in terms of traded goods, see Equations (5) and (7)), where the households’ intertemporal budget constraint (given by \( C_{NT,1} + p_1 C_{T,1} = R_1(p_0 Y_{T,0} - p_0 N_0 + Y_{NT,0} - C_{NT,0} - p_0 C_{T,0}) + w_1 L + p_1 N_1 + Y_{NT,1} \)) is used to eliminate \( p_1 C_{T,1} \):

\[
\begin{align*}
p_0 C_{T,0} &= \frac{p_1 C_{T,1}}{\beta R_1} \\
&\Leftrightarrow p_0 C_{T,0} = \frac{1}{\beta R_1} (R_1(p_0 Y_{T,0} - p_0 N_0 + Y_{NT,0} - C_{NT,0} - p_0 C_{T,0}) \\
&+ w_1 L + p_1 N_1 + Y_{NT,1} - C_{NT,1}).
\end{align*}
\]

Market clearing in the non-traded goods’ sector implies that in equilibrium demand and endowment for non-traded goods always have to cancel each other out, and the non-financial firms’ technology and optimization behavior ensure that labor income is a constant share of (nominal) output \((w_1 L = p_1 (1 - \alpha) Y_{T,1})\). Finally, the value of the equity capital in period 1, \( p_1 N_1 \), is given by Equation (B.4), where by market clearing \( S_{p,0} = K_1 \) and \( e_0 K_{p,0} = p_0 N X_0 = p_0 (Y_{T,0} - K_1 - C_{T,0}) \). Thus, the home households’ savings schedule (SS) is defined by

\[
\begin{align*}
p_0 C_{T,0} &= \frac{1}{\beta R_1} \left( R_1(p_0 Y_{T,0} - p_0 C_{T,0}) + p_1 (1 - \alpha) Y_{T,1} \\
&+ (R_{k,1} - R_1) p_0 K_1 + \left( R_1^* \frac{e_1}{e_0} - R_1 \right) p_0 N X_0 \right) \\
&\Leftrightarrow C_{T,0} = \frac{1}{(1 + \beta)} \frac{p_0}{p_1} R_1 \left( \frac{p_0}{p_1} R_1 Y_{T,0} + (1 - \alpha) Y_{T,1} \\
&+ \frac{p_0}{p_1} (R_{k,1} - R_1) K_1 + \frac{p_0}{p_1} \left( R_1^* \frac{e_1}{e_0} - R_1 \right) N X_0 \right).
\end{align*}
\]
Likewise, the foreign country’s saving schedule ($SS^*$) is implicitly given by
\[
C_{T,0}^* = \frac{1}{(1 + \beta^*)} \left( \frac{p_0^*}{p_1^*} R_1^* Y_{T,0}^* + (1 - \alpha) Y_{T,1}^* \right) + \frac{p_0^*}{p_1^*} (R_{k,1}^* - R_1^*) K_1^* 
\]
(E.14)

- **Effect of an increase in $\theta$:**

The effect of an increase in $\theta$ on the home country’s saving curve can be found by differentiating Equation (E.13), holding $\frac{p_0}{p_1} R_1$ constant:
\[
\frac{dC_{T,0}}{d\theta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} = \frac{1}{(1 + \beta) \frac{p_0}{p_1} R_1} \left( (1 - \alpha) \frac{\partial Y_{T,1}}{\partial K_1} \frac{dK_1}{d\theta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} + \frac{d}{d\theta} \left( \frac{p_0}{p_1} R_1 e_{1,0} \right) \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} + \frac{d}{d\theta} \left( \frac{p_0}{p_1} R_1 e_{1,0} \right) \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} \right).
\]

As we consider the case where the countries are initially in the frictionless state, excess returns are zero and the two respective terms disappear. Furthermore, by using Equations (5) and (17), we find that
\[
\frac{d}{d\theta} \left( \frac{p_0}{p_1} R_1 \left( 1 + \frac{\lambda \Delta}{1 + \chi} \right) \right) \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} = 0 \quad \text{(remember that}
\]
\( \Delta = 0 \). The two terms that are then still left represent the effect of the decrease in the households’ second-period labor income (due to the lower level of capital) and the higher return on equity capital due to the increase in the excess return on home investment securities. Using Equation (E.1) to replace \( \frac{dK_1}{d\theta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} \) and substituting the marginal product to capital \( \left( \frac{\partial Y_{T,1}}{\partial K_1} = \alpha \left( \frac{L}{K_1} \right)^{1-\alpha} \right) \), one finds that they just cancel each other out:

\[
\frac{dC_{T,0}}{d\theta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} = 0.
\]
Hence, for a given real interest rate, an increase in $\theta$ has no effect on the level of consumption in the home country.

Likewise, by differentiating Equation (E.14), holding $\frac{p_0}{p_1}R_1^*$ constant, one finds the effect of an increase in $\theta$ on the foreign country’s saving curve:

$$\frac{dC^*_T,0}{d\theta} \bigg|_{\frac{p_0}{p_1}R_1^* \text{ constant}} = \frac{1}{(1 + \beta^*)} \frac{p_0}{p_1}R_1^* \left(1 - \alpha\right) \frac{\partial Y^*_{T,1}}{\partial K_1^*} \left| \frac{p_0}{p_1}R_1^* \text{ constant} \right| + \frac{1}{\frac{p_0}{p_1}R_1^* \text{ constant}} \frac{d}{d\theta} \left( \frac{p_0}{p_1}R_{k,1}^* \right) K_1^*$$

$$+ \frac{p_0}{p_1}R_1^* \left( R_{k,1}^* - R_1^* \right) \frac{dK_1^*}{d\theta} \left| \frac{p_0}{p_1}R_1^* \text{ constant} \right| \left( \frac{p_0}{p_1}R_1^* \text{ constant} \right)$$

$$= 0.$$

We know from the analysis of the foreign investment curves that the first two expressions in the big brackets are equal to zero (see Equations (E.3) and (E.4)), and given that the economy is initially in a frictionless state, excess returns are zero as well. Hence, for a given real interest rate, an increase in $\theta$ has no effect on the foreign country’s consumption.

- **Effect of an increase in $\theta^*$:**
  Following the same reasoning as in the case of an increase in $\theta$, one finds that

$$\frac{dC^*_{T,0}}{d\theta^*} \bigg|_{\frac{p_0}{p_1}R_1^* \text{ constant}} = 0.$$  

Thus, an increase in $\theta^*$ does not lead to a shift in the two countries’ saving schedules.

- **Effect of an increase in $\Delta$:**
  The effect of an increase in $\Delta$ on the home country’s consumption, holding $\frac{p_0}{p_1}R_1^*$ constant:
\[
\frac{dC_{T,0}}{d\Delta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} = \frac{1}{(1 + \beta) \frac{p_0}{p_1} R_1} \left( (1 - \alpha) \frac{\partial Y_{T,1}}{\partial K_1} \frac{dK_1}{d\Delta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} + \frac{d \left( \frac{p_0}{p_1} R_{k,1} \right)}{d\Delta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} \right) \\
K_1 + \frac{p_0}{p_1} (R_{k,1} - R_1) \frac{dK_1}{d\Delta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} + \frac{d \left( \frac{p_0}{p_1} R^*_1 e_1 \right)}{d\Delta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} \\
NX_0 + \frac{p_0}{p_1} \left( R^*_1 e_1 e_0 - R_1 \right) \frac{dNX_0}{d\Delta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} \bigg) .
\]

Again, excess returns are zero and the two respective terms disappear. From the analysis above, we know that \( \frac{dK_1}{d\Delta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} = 0 \) (see Equations (E.9) and (E.10)). Furthermore, by using Equations (5) and (16), we find that
\[
\frac{d \left( \frac{p_0}{p_1} R_{k,1} \right)}{d\Delta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} = \frac{d}{d\Delta} \left( \frac{p_0}{p_1} R_1 \left( 1 + \frac{1}{1+\lambda} \theta \right) \right) \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} = 0
\]
(remember that \( \theta = 0 \)). The one term that is still left represents the effect of the higher return on equity capital due to the increase in excess returns on foreign assets and equals (using Equation (17)):

\[
\frac{dC_{T,0}}{d\Delta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} = \frac{1}{(1 + \beta) \frac{p_0}{p_1} R_1} \left( \frac{d \left( \frac{p_0}{p_1} R^*_1 e_1 \right)}{d\Delta} \bigg|_{\frac{p_0}{p_1} R_1 \text{ constant}} \right) .
\]
\[ \begin{align*}
&= \frac{1}{1 + \beta} \frac{p_0}{p_1} R_1 \left( \frac{\partial}{\partial \Delta} \left( p_0 R_1 \left( 1 + \frac{\lambda}{1 + \Delta} \right) \right) \bigg|_{p_0 R_1 \text{ constant}} \right) NX_0 \\
&= \frac{1}{1 + \beta} \frac{p_0}{p_1} R_1 \left( \frac{p_0}{p_1} R_1 \Delta \frac{\partial \lambda}{\partial \Delta} \bigg|_{p_0 R_1 \text{ constant}} \right) NX_0 \\
&= \frac{1}{1 + \beta} \left( \Delta \frac{\partial \lambda}{\partial \Delta} \bigg|_{p_0 R_1 \text{ constant}} \right) NX_0 \\
&= \frac{1}{1 + \beta} \left( \Delta \frac{\partial \lambda}{\partial \Delta} \bigg|_{p_0 R_1 \text{ constant}} \right) (Y_{T,0} - K_1 - C_{T,0}) > 0.
\end{align*} \]

As we look at the effect where the friction parameter passes from being non-binding to being binding, we know that \( \frac{\partial \lambda}{\partial \Delta} \bigg|_{p_0 R_1 \text{ constant}} > 0 \). Hence, given a real interest rate, an increase in \( \Delta \) leads to an increase in consumption due to the higher return on home equity capital, which in turn corresponds to a negative shift in the home country’s saving curve.

On the other hand, as the foreign banks have no international portfolio and their equity capital is independent of the excess return on foreign international transactions, there is no shift in the foreign country’s saving curve:

\[ \begin{align*}
\frac{dC^*_{T,0}}{d\Delta} \bigg|_{p_0^* R_1^* \text{ constant}}
&= \frac{1}{1 + \beta^*} \frac{p_0^*}{p_1^*} R_1^* \left( 1 - \alpha \right) \frac{\partial Y^*_{T,1}}{\partial K^*_1} \left. \frac{\partial K^*_1}{\partial \Delta} \bigg|_{p_0^* R_1^* \text{ constant}} \right) \\
&+ \frac{d}{d\Delta} \left( \frac{p_0^*}{p_1^*} R_{k,1}^* \right) \bigg|_{p_0^* R_1^* \text{ constant}} \left( K^*_1 + \frac{p_0^*}{p_1^*} (R_{k,1}^* - R_1^*) \right) \\
\frac{dK^*_1}{d\Delta} \bigg|_{p_0^* R_1^* \text{ constant}} &= 0.
\end{align*} \]
Again, excess returns in the initial frictionless state are zero, and the first two terms drop out as well:

\[
\left. \frac{d}{d\Delta} \left( \frac{\hat{p}_0}{\hat{p}_1} R^*_1 \right) \right|_{\frac{\hat{p}_0}{\hat{p}_1} R^*_1 \text{ constant}} = \frac{d}{d\Delta} \left( \frac{\hat{p}_0}{\hat{p}_1} R^*_1 \left( 1 + \frac{\chi}{1 + \chi} \theta^* \right) \right) \bigg|_{\frac{\hat{p}_0}{\hat{p}_1} R^*_1 \text{ constant}} = 0
\]

(remember that \( \theta^* = 0 \)) and therefore \( \frac{dK^*_1}{d\Delta} \bigg|_{\frac{\hat{p}_0}{\hat{p}_1} R^*_1 \text{ constant}} \) (see Equation (E.11)).

### Appendix F. CPI-Based Real Exchange Rate

The exchange rate \( e_t \) is equal to the relative price of the non-traded goods, i.e., the numéraires, respectively, in our model. To find the CPI-based real exchange rate, we first need to derive the price indices. This is done by replacing \( C_{NT,t} \) and \( C_{T,t} \) in the consumption index by the demand functions resulting from the intratemporal optimization problem (see Equations (6) and (7)):

\[
C_t = \left( C_{NT,t}^\chi C_{T,t}^\chi \right)^{\frac{1}{1+\chi}}
\]

\[
= \left( \left( \frac{\chi}{1+\chi} \left( \frac{1}{P_t} \right)^{-1} C_t \right)^\chi \left( \frac{1}{1+\chi} \left( \frac{p_t}{P_t} \right)^{-1} C_t \right) \right)^{\frac{1}{1+\chi}}
\]

\[
= \frac{\chi^{\frac{1}{1+\chi}}}{1 + \chi} \left( \frac{1}{p_t} \right)^{\frac{1}{1+\chi}} P_tC_t.
\]

Hence,

\[
P_t = \frac{1 + \chi}{\chi^{\frac{1}{1+\chi}}} \frac{1}{p_t^{\frac{1}{1+\chi}}}. \tag{F.1}
\]

Similarly, the foreign price index is found to be

\[
P^*_t = \frac{1 + \chi}{\chi^{\frac{1}{1+\chi}}} \frac{1}{p^*_t^{\frac{1}{1+\chi}}}. \tag{F.2}
\]
It follows that the CPI-based real exchange rate, defined as the ratio of the price indices multiplied by the relative price of the two numéraires, is given by

$$\mathcal{E}_t \equiv \frac{P_t^*}{P_t} e_t = \left( \frac{p_t^*}{p_t} \right)^{\frac{1}{1+\chi}} e_t = \left( \frac{1}{e_t} \right)^{\frac{1}{1+\chi}} e_t$$

$$= e_t^{\frac{1}{1+\chi}}. \quad (F.3)$$

Thus, the exchange rate as we define it in our model is very closely related to the CPI-based real exchange rate. Whenever $e_t$ is larger (smaller) than 1, this also holds for $\mathcal{E}_t$.

References


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$^{35}$ $\mathcal{E}_t$ is defined to be the price of a foreign consumption bundle expressed in terms of home consumption bundles. Thus, if $\mathcal{E}_t < 1$, one consumption bundle in the home country gives more than one consumption bundle in the foreign country.


