Monetary Policy Implementation and Payment System Modernization*

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24/7 payment settlement may affect the demand for central bank reserves and thus could have an effect on monetary policy implementation. By modifying the standard workhorse model of monetary policy implementation (Poole 1968), we show that 24/7 payment settlement induces a precautionary demand for central bank reserves. Absent any changes or response by the central bank, this will put upward pressure on the overnight interest rate in frameworks with a low level of reserves.

JEL Codes: E, E4, E40, E42, E43.

1. Introduction

Payment, clearing, and settlement systems have undergone drastic changes since banks began accepting claims on each other (Norman, Shaw, and Speight 2011). Technological change and a regulatory interest in systemic risk oversight over the last decade or so has accelerated the pace of these changes. Now, several countries have retail payment systems that provide settlement in real time or near real time 24 hours a day, seven days a week (Tompkins and Olivares 2016). Other countries also plan on adopting such systems. Canada has planned for such a system, and the United States' FedNow system that will also offer 24/7 payment clearing services to retail customers is anticipated to be launched in 2023.

Payment systems are inextricably linked to the implementation of monetary policy—i.e., how the central bank sets overnight

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interbank rates. The demand for reserves in a standard model of monetary policy implementation (e.g., Poole 1968) is generated by how uncertainty over interbank payment flows affects the use of central bank borrowing and lending facilities. 24/7 payment settlement has the potential to change both the nature of payment uncertainty as well as the use of central bank facilities. For example, demand for reserves could be a function of whether the central bank provides access to its lending facilities (i.e., intraday credit) only during standard operational hours or always. This paper aims to understand how demand for reserves is a function of those hours. If the central bank does not provide access to an after-hours central bank lending facility, a bank needs to have positive reserve balances greater than the payment amount to process a given payment. While a bank could establish a credit line to borrow from another bank to meet the payment, that other bank would also be worried about an inability to process payments. In either case, an extra dollar of reserves in the after-hours market provides a benefit in that it helps banks avoid having insufficient funds to process payments in the after-hours market. Does this matter for overnight interbank interest rates? Under what conditions will this matter, and is the impact different for different implementation frameworks? To answer these questions, we adjust the standard model of monetary policy implementation to incorporate after-hours payment shocks.

Traditional models (e.g., Poole 1968 and Bech and Keister 2013) have a payment shock that occurs while banks have access to central bank facilities. Thus, in these models, the cost and probability of accessing these facilities influences the interbank rate. If a payment shock occurs when banks do not have access to this facility, then banks need to factor the cost of having insufficient funds in the after-hours period into their marginal benefit of an extra dollar of reserves. Once we start thinking about 24/7 settlement, it opens up several questions related to monetary policy implementation, which this paper attempts to answer.[1]

[1] For example, in the United States, Fedwire Funds Service operates until 6:30 pm and the National Settlement Service (NSS) operates until 5:30 pm. The Federal Reserve is considering expanding their hours to 24/7 to provide a liquidity management tool to support a 24/7 real-time gross settlement (RTGS) service.

Some models consider how differential access to central bank facilities and segmentation in the overnight market affect the interbank interest rate (Bech and Klee 2011; Martin et al. 2013; Armenter and Lester 2015; Williamson 2019). In these papers, access to central bank facilities is segmented by participant. In our model, all participants have the same access, but that access is segmented by time. Like these other models, segmentation affects interbank interest rates. We also extend the baseline model to two periods, an intraday trading period and an after-hours trading period. In different contexts, other models also extend the baseline model to multiple periods (e.g., by looking at reserve averaging over multiple trading periods as in Ennis and Keister 2008).

We provide the conditions under which after-hours payments can have an effect on interbank interest rates. When after-hours payment volatility is material relative to intraday payment volatility, banks will have an increased demand for reserves. This increased demand increases with the volatility of the after-hours payment shock and is precautionary in that banks want to hold extra reserves to avoid having insufficient funds in the after-hours session. When the expected penalty cost is sufficiently large, banks will want to borrow more than the minimum requirement from the central bank. How this all affects interbank rates depends on the monetary policy implementation framework. Interbank rates in monetary policy frameworks that naturally have large reserves will not be affected much by such a change. On the other hand, there will be upward pressure on interbank rates in frameworks that typically have zero or low levels of reserves.

While the central bank can intervene by providing more aggregate reserves to offset this upward pressure, this could be more challenging if the volatility of the after-hours payment shock fluctuates. This could happen, for instance, if the after-hours period is longer in certain periods such as weekends or holidays. We therefore investigate how a change in the volatility of the after-hours payment shock affects the volatility of the overnight rate. When reserves are sufficiently large, changes in after-hours payment volatility do not matter. When reserves are smaller, changes in after-hours payment volatility result in volatility in the overnight interbank rate, absent a central bank response.

Finally, we examine the impact on the interbank overnight rate of having payments spread across two payment systems—a traditional intraday system and a 24/7 system. Parallel payment systems exist in several jurisdictions. When there is not real-time settlement between the two systems, we show that the overnight rate could be higher or lower than the overnight rate in one single payment system. In the extreme, when settlement across the two systems is completely restricted, the central bank must supply the appropriate level of reserves in each of the two systems if it wants to implement its target rate in both systems.

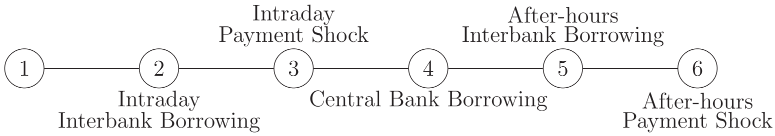
In practice, several central banks have already implemented payment systems with 24/7 retail payments, but overnight interbank rates still trade close to target in their jurisdictions. Our model would imply that either (i) uncertainty about retail payment flows in the after-hours session is small in these jurisdictions, or (ii) the level of reserves in these jurisdictions is large enough such that there is little chance that banks will have insufficient funds to process payment flows in the after-hours session. However, should more payment flows migrate to the 24/7 system, our model would suggest that this could lead to deviations from the target interest rate. Further, the implementation of a 24/7 payment system in countries that operate a system with low reserves or plan to return to such a framework could be very different than the experience thus far.

2. Model

2.1 Model Timing

Our model extends Bech and Keister (2013) and Boutros and Witmer (2019) and consists of six stages. We assume a continuum of perfectly competitive banks indexed by $i \in [0, 1]$. The first four stages presented in Figure 1 are standard in the literature. Banks borrow from (and lend to) each other during the day. After this borrowing and lending window is over, banks are subject to a payment shock. If they are short reserve balances after this payment shock, they must borrow from the central bank at rate r_X to make up the shortfall. If they have excess reserves, these get deposited with the central bank and earn interest r_R. We depart from the standard models by introducing after-hours trading in stage 5 and an after-hours

Figure 1. Model Timing



payment shock in stage 6. What distinguishes this after-hours payment shock from the intraday payment shock is that we assume that banks do not have recourse to the central bank borrowing facility in the after-hours market.

Banks begin the day in stage 1 with reserves, R^i, bond holdings, B^i, and deposit liabilities, D^i. Aggregate reserves are defined as $R = \int_i R^i \mathrm{d}i$. Bond holdings are exogenous and fixed throughout the day. We include bond holdings to be consistent with the literature (e.g., Bech and Keister 2013), but bonds are irrelevant to most of the analysis. Banks cannot choose the size of their deposits, the deposit rate (r_D) is fixed, and deposits only change when a bank experiences a payment shock.

In the standard model, a bank becomes a net lender ($\Delta^i_{intra} < 0$) or net borrower ($\Delta^i_{intra} > 0$) in stage 2 to position itself for the intraday payment shock it experiences in stage 3. In our model with after-hours payments shocks (i.e., shocks that happen after clearing and settlement of intraday balances), the banks' decisions are going to change. Specifically, a bank must also consider the effect of a penalty cost resulting from a large after-hours payment shock on its profitability. That is, it is not only minimizing the penal borrowing and lending rates associated with the central bank facilities, but is also minimizing penalty costs of having insufficient funds to process payments in the after-hours market.

In stage 3, after the trading session is closed, each bank experiences an intraday payment shock, $\epsilon^i_{intraday}$. This payment shock is independent and is identically and normally distributed with mean zero and standard deviation σ_G. We denote the cumulative distribution function of this shock $G(\epsilon^i_{intraday})$. This payment shock lowers the bank's reserves on the asset side of its balance sheet and correspondingly lowers its deposits on its liabilities side.

In stage 4, after the intraday payment shock, each bank borrows X_i from the central bank to meet or exceed its required level of

Table 1. Bank i's Stage 4 Balance Sheet

Assets	Liabilities
B^i Bonds $R^i + \Delta^i_{intra} - \varepsilon^i_{intraday} + X^i$ Reserves	$D^i - \varepsilon^i_{intraday}$ Deposits Δ^i_{intra} Interbank Borrowing X^i Central Bank Borrowing

reserves $K \geq 0$. We assume that each bank has the same required level of reserves, and that this required level of reserves is positive. The aggregate reserve requirement is defined as $K = \int_i K \mathrm{d}i$. We will focus our attention on frameworks with positive excess aggregate reserves (e.g., $R - K \geq 0$) that are commonly used in practice.

Assumption 1. *Aggregate reserves are greater than or equal to the aggregate reserve requirement.*

Banks that must borrow from the central bank do so at a rate of r_X. Each bank will borrow

$$X^i \geq \max\{0, K - (R^i + \Delta^i_{intra} - \epsilon^i_{intra})\}. \tag{1}$$

At a minimum, the bank must borrow at least enough to meet its minimum reserve requirement. We leave open the possibility that a bank may want to borrow more than this to help reduce the potential penalty cost of having insufficient funds in the after-hours session. At the end of stage 4, the bank earns r_K on its required reserves and $r_R < r_X$ on any reserves in excess of its required reserves. Table 1 illustrates bank i's balance sheet at the end of stage 4, before the after-hours payment shock.

In the stage 5 after-hours session, banks may be able to borrow from one another before the realization of the after-hours payment shock (similar to how they can borrow from each other during the day). We assume that banks can access interbank trading in the after-hours session with probability $1 - p$, $p \approx 0$ or $p = 1$. We make this assumption to see how the ability to trade in the after-hours market affects our model results. When $p = 1$, there is no after-hours trading session. When $p \approx 0$, the after-hours session resembles a session in which all banks can freely trade and lend from one another.

In this case, we assume $p \approx 0$ instead of $p = 0$ to generate a unique equilibrium when the penalty cost is high and banks may want to borrow more than the minimum from the central bank.

When they can access the after-hours market, a bank can be either a net lender ($\Delta^i_{after} < 0$) or a net borrower ($\Delta^i_{after} > 0$) in the after-hours market.

Assumption 2. *Each bank pays back both its intraday and after-hours loans at the beginning of the next day's session.*

In this sense, the intraday loan is an overnight loan, since it spans both before and after the central bank compensation of reserve balances in stage 4. The after-hours loan, in contrast, may be considered intraday because it is paid back before stage 4 the next day. The assumption about after-hours loans is for ease of exposition: when banks have access to an after-hours trading session ($p \approx 0$) we could instead assume banks make after-hours loans that are paid back after stage 4 the next day and obtain similar results.

In the final stage, each bank receives a payment shock in the after-hours market, ϵ^i_{after}. The after-hours payment shock is independent and is identically and normally distributed with mean zero and standard deviation σ_F. We denote the cumulative distribution function of this shock $F(\epsilon^i_F)$.

Assumption 3. *The volatility of the after-hours payment shock is smaller than the volatility of the intraday payment shock*: $\sigma_F \leq \sigma_G$.

This assumption reflects the fact that the after-hours time period is smaller than the intraday period. Also, most countries that offer 24/7 settlement do so for retail payment flows only, which would result in after-hours payment flows being smaller then intraday payment flows. Nonetheless, this assumption is inconsequential for most of the results in this paper, with the exception of Corollary 1.

The bank cannot meet its after-hours payment if

$$\epsilon^i_{after} \geq R^i + \Delta^i_{intra} + \Delta^i_{after} - \epsilon^i_{intra} + X^i. \tag{2}$$

If the bank cannot meet its payment, it suffers a penalty cost s on Z^i, each dollar of payment it is unable to make.

$$Z^i = \max\{0, \epsilon^i_{after} - (R^i + \Delta^i_{intra} + \Delta^i_{after} - \epsilon^i_{intra} + X^i)\} \tag{3}$$

ASSUMPTION 4. *Commercial banks can freely overdraft during the intraday session. During the after-hours session, in contrast, banks suffer a penalty cost, s, should they have insufficient funds to process a payment.*

This difference is a consequence of the central bank policy choice of not providing access to the central bank facilities in the after-hours period.[2] If access to central bank facilities and daylight credit was provided 24/7, our model would collapse to the standard model. We assume that the penalty cost of running out of funds during the day is zero. In reality there is a small cost to accessing intraday credit from the central bank (see Ennis and Weinberg 2007 for a model including intraday credit).[3] Our model reflects a situation where the after-hours penalty cost is materially larger than the small cost of intraday daylight credit.[4]

This penalty cost is modeled in reduced form as a negative profit if the bank is unable to make a payment. It could represent, for example, stigma associated with inability to make a payment, lost clients as a result of the inability to make a payment on their behalf, or late-payment charges embedded in contracts with clients.

2.2 Bank Behavior

Banks earn r_B on their bond holdings and pay r_D on their deposit holdings, both of which are exogenously determined. After the after-hours payment shock, bank i's realized profits are therefore

[2]The Federal Reserve is considering whether to provide intraday credit on a 24/7 basis with the implementation of its FedNow system. For more information, please refer to the following press release: https://www.federalreserve.gov/newsevents/pressreleases/files/other20190805a1.pdf.

[3]These costs include the opportunity cost of posting collateral with the central bank and any interest rates associated with daylight credit. Since 2011, the Federal Reserve assesses daylight overdraft charges on uncollateralized daylight overdrafts, which on average represent about 5 percent of daylight overdrafts. See https://www.federalreserve.gov/paymentsystems/psr_data.htm and https://www.federalreserve.gov/paymentsystems/files/psr_overview.pdf.

[4]We could adapt the model to include a non-zero intraday penalty cost if the bank's reserve position falls below zero in stage 4. This would not change the main insights from the model, so we assume that this penalty cost is zero during the day.

$$\pi^i = r_B B^i - r_D(D^i - \epsilon^i_{intra}) + r_K K - r_{intra}\Delta^i_{intra} - r_X X^i + r_R(R^i + X^i + \Delta^i_{intra} - \epsilon^i_{intra} - K) - r_{after}\Delta^i_{after} - s * Z^i. \quad (4)$$

We work backwards to find the bank's optimal behavior. In stage 5, when banks have access to the after-hours market they will choose their net interbank after-hours borrowing Δ^i_{after} to maximize their expected profits in the stage, $E_5[\pi^i_5]$ (i.e., expectations over the last two terms in the above equation). The numerical subscript on the expectations operator and profit variable indicates that these expectations are on bank profits as of stage 5. Banks will maximize:

$$\mathrm{E}_5[\pi^i_5] = -r_{after}\Delta^i_{after} - s\int_{\epsilon^i_Z}^{\infty}(\epsilon^i_{after} - \epsilon^i_Z)\mathrm{dF}(\epsilon^i_{after}). \quad (5)$$

In this equation, the threshold before the bank is expected to experience the penalty cost, $\epsilon^i_Z \equiv R^i + \Delta^i_{intra} + \Delta^i_{after} - \epsilon^i_{intra} + X_i$, is equal to the amount of reserves after after-hours trading is complete. This takes into account the amount of reserves at the beginning of the day, less the intraday payment shock, plus the amount of central bank borrowing after the intraday payment shock and the amount borrowed in the intraday and after-hours markets.

It will also be useful to define the same threshold in the absence of access to after-hours trading, $\epsilon^i_{Z,\Delta^0_{after}} \equiv R^i + \Delta^i_{intra} - \epsilon^i_{intra} + X_i$. This is the same as ϵ^i_Z, except that after-hours trading is set equal to zero. This is useful for examining the case where banks do not have the ability to trade in the after-hours session.

The value of Δ^i_{after} that maximizes the expected after-hours profits in Equation (5) is given by the following first-order condition:

$$r_{after} = s(1 - F(\epsilon^i_Z)). \quad (6)$$

Given that banks borrow and lend from each other at the same rate in the after-hours market (r_{after}), it follows from Equation (6) that banks will trade with each other such that they have the same $\epsilon^i_Z \equiv \epsilon_Z$. The bank's expected trading (in stage 4, before the after-hours payment shock) in the after-hours market can thus be written as

$$E_4[\Delta^i_{after}] = \epsilon_Z - (R^i + \Delta^i_{intra} - \epsilon^i_{intra} + X_i). \quad (7)$$

In stage 4, banks will take into account that they may be able to trade in the after-hours market, and will choose their central bank borrowing X_i to maximize the expected value of their profits in that stage, $E_4[\pi^i_4]$, subject to the constraint on central bank borrowing in Equation (1). They will maximize the Lagrangian:

$$\mathcal{L}^i = \mathrm{E}_4[\pi^i_4] + \lambda^i(X^i - \max\{0, K - (R^i + \Delta^i_{intra} - \epsilon^i_{intra})\}), \quad (8)$$

where

$$\begin{aligned}\mathrm{E}_4[\pi^i_4] = (r_R - r_X)X_i - (1-p)[r_{after}E_4[\Delta^i_{after}] \\ + s\int_{\epsilon_Z}^{\infty}(\epsilon^i_{after} - \epsilon_Z)\mathrm{dF}(\epsilon^i_{after})] \\ - ps\int_{\epsilon^i_{Z,\Delta^0_{after}}}^{\infty}(\epsilon^i_{after} - \epsilon^i_{Z,\Delta^0_{after}})\mathrm{dF}(\epsilon^i_{after}). \quad (9)\end{aligned}$$

This yields the following first-order condition and complementary slack condition with $\lambda_i \geq 0$:

$$r_X - r_R = (1-p)r_{after} + ps(1 - F(\epsilon^i_{Z,\Delta^0_{after}})) + \lambda_i \quad (10)$$

$$\lambda_i(X^i - \max\{0, K - (R^i + \Delta^i_{intra} - \epsilon^i_{intra})\}) = 0. \quad (11)$$

Because all banks face the same after-hours interbank rate, r_{after}, it follows from Equation (10) that if there are unconstrained banks (i.e., for whom $\lambda_i = 0$), they will borrow from the central bank until they have the same $\epsilon^i_{Z,\Delta^0_{after}} \equiv \epsilon^u_{Z,\Delta^0_{after}}$. We can define $\epsilon^u_{Z,\Delta^0_{after}}$ as the value of $\epsilon^i_{Z,\Delta^0_{after}}$ that solves Equation (10) with equality:

$$r_X - r_R = (1-p)r_{after} + ps(1 - F(\epsilon^u_{Z,\Delta^0_{after}})). \quad (12)$$

PROPOSITION 1. *No bank will borrow more (from the central bank) than the minimum required to meet its reserve requirement if*

$$r_X - r_R \geq (1-p)r_{after} + ps(1 - F(K)). \tag{13}$$

Proof. Because of the reserve requirement, all banks will be holding at least K in the after-hours session. Equations (10) and (13) imply that $\lambda_i > 0$ for all banks and hence all banks are constrained by the minimum reserve requirement. Intuitively, the marginal cost of borrowing from the central bank is greater than the marginal expected penalty cost in the after-hours session when all banks hold at least K, so no bank will want to borrow more than the minimum from the central bank. □

In this case, there will be no precautionary borrowing to alleviate the potential after-hours penalty cost. From Equations (12) and (13), it follows that all banks will be constrained if $K > \epsilon^u_{Z,\Delta^0_{after}}$.[5]

This means that banks will borrow from the central bank when they experience an intraday payment shock larger than the threshold ϵ^i_X:

$$\epsilon^i_{intra} \geq \epsilon^i_X \equiv R^i + \Delta^i_{intra} - max(\epsilon^u_{Z,\Delta^0_{after}}, K). \tag{14}$$

In stage 2, banks will take into account that they can borrow from the central bank and trade in the after-hours market. The expected amount of borrowing in the after-hours interbank market ($E_2[\Delta^i_{after}]$) is based on the expected borrowing from the central bank, which itself is a function of the bank's intraday trading (Δ^i_{intra}). However, borrowing funds in the intraday interbank market does not decrease after-hours borrowing one-for-one, since borrowing an extra dollar of reserves during the intraday session makes it less likely that the bank will need to borrow from the central bank at

[5]If $s < r_X - r_R$, there is no value of $\epsilon^u_{Z,\Delta^0_{after}}$ that will solve Equation (12), in which case we assume $K > \epsilon^u_{Z,\Delta^0_{after}}$. That is, no bank will want to borrow from the central bank and pay a higher cost, $r_x - r_R$, to avoid a probability of paying a lower cost s.

the end of the day. Taking expectations as of stage 2 of Equation (7) produces

$$E_2[\Delta^i_{after}] = \epsilon_Z - (R^i + \Delta^i_{intra} + \int_{\epsilon^i_X}^{\infty} (\epsilon^i_{intra} - \epsilon^i_X) \mathrm{dG}(\epsilon^i_{intra})). \tag{15}$$

By taking the derivative of Equation (15) with respect to Δ^i_{intra}, borrowing an additional dollar in the intraday market will reduce expected after-hours borrowing by $G(\epsilon^i_X)$.

Given this expected after-hours borrowing, banks will choose their net interbank intraday borrowing Δ^i_{intra} to maximize the expected value of their profits:

$$\begin{aligned}
\mathrm{E}_2[\pi^i] = {} & r_B B^i - r_D D^i + r_K K^i - r_{intra}\Delta^i_{intra} \\
& + r_R \epsilon^i_X + (r_R - r_X) \int_{\epsilon^i_X}^{\infty} (\epsilon^i_{intra} - \epsilon^i_X) \mathrm{dG}(\epsilon^i_{intra}) \\
& - (1-p)\Big[r_{after} E_2[\Delta^i_{after}] + s \int_{\epsilon_Z}^{\infty} (\epsilon^i_{after} - \epsilon_Z) \mathrm{dF}(\epsilon^i_{after}) \Big] \\
& - ps\Big[\int_{-\infty}^{\epsilon^i_X} \int_{\epsilon^i_{Z,\Delta^0_{after}}}^{\infty} (\epsilon^i_{after} - \epsilon^i_{Z,\Delta^0_{after}}) \mathrm{dF}(\epsilon^i_{after}) \mathrm{dG}(\epsilon^i_{intra}) \\
& + \int_{\epsilon^i_X}^{\infty} \int_{max(\epsilon^u_{Z,\Delta^0_{after}},K)}^{\infty} (\epsilon^i_{after} - max(\epsilon^u_{Z,\Delta^0_{after}}, K)) \\
& \quad \mathrm{dF}(\epsilon^i_{after}) \mathrm{dG}(\epsilon^i_{intra}) \Big].
\end{aligned} \tag{16}$$

Relative to a standard Poole (1968) model, the bank's expected profit in (16) includes two groups of extra terms. The first group (with terms multiplied by $1-p$) concerns the bank's expected profit when it has access to the after-hours trading session. It includes a term that accounts for the expected profit from its expected borrowing and lending in the after-hours market. It includes another term that accounts for the penalty cost, s, of falling short of funds in the after-hours session when the bank is able to trade with other banks in stage 5. This term has the same threshold for each bank since their after-hours trading will make their after-hours reserves position the same. As the integral suggests, the bank only pays this cost if the after-hours payment shock, ϵ^i_{after}, exceeds the threshold ϵ_Z. This term, unlike the first term in this group, is not affected by the bank's intraday interbank trading.

The second group of terms (with terms multiplied by p) represent the penalty cost of falling short of funds in the after-hours session when the bank is unable to trade with other banks in stage 5. The first term in this group represents the expected penalty cost if the intraday payment shock is small enough such that the bank does not borrow from the central bank. The second term in this group represents the expected penalty cost if the bank does borrow from the central bank. Banks borrowing from the central bank will borrow such that they all have the same threshold before the penalty cost affects their profitability.

Formally, banks will choose Δ^i_{intra} to maximize their expected profits in Equation (16), resulting in the following first-order condition:

$$\begin{aligned} r_{intra} &= r_R + (r_X - r_R)(1 - G(\epsilon^i_X)) + (1-p) r_{after} G(\epsilon^i_X) \\ &\quad + ps \int_{-\infty}^{\epsilon^i_X} [1 - F(\epsilon^i_{Z,\Delta^0_{after}})] \mathrm{dG}(\epsilon^i_{intra}). \end{aligned} \tag{17}$$

2.3 Equilibrium

2.3.1 Equilibrium Overnight Rate

DEFINITION. *An equilibrium consists of interest rates r_{intra} and r_{after} and individual bank net borrowing decisions (Δ^i_{intra}) and (Δ^i_{after}) such that*

(i) Banks choose Δ^i_{after} to maximize expected profits in the stage 5 after-hours session, as in (5).

(ii) Banks choose X^i to maximize expected profit when borrowing from the central bank in stage 4, as in (9).

(iii) Banks choose Δ^i_{intra} to maximize expected profit in stage 2, as in (16).

(iv) The interbank markets are closed systems that clear, that is, $\Delta_{intra} = \int_i \Delta^i_{intra}\mathrm{d}i = 0$ *and* $\Delta_{after} = \int_i \Delta^i_{after}\mathrm{d}i = 0$*. Moreover, for banks that do not have access to the after-hours trading session (*$i \in$ *no access),*

$$\int\limits_{i\ \in\ no\ \ access} \Delta^i_{intra}\mathrm{d}i = \int\limits_{i\ \in\ access} \Delta^i_{intra}\mathrm{d}i = \Delta_{intra} = 0.$$

By the first-order condition in (17), and by the fact that banks are all subject to the same reserve requirement K, they will have the same ϵ^i_X in equilibrium. That is, since r_{intra} is the same for all banks, there is only one value of ϵ^i_X that will solve (17).

By market clearing $\Delta_{intra} = \Delta_{after} = 0$, and it follows that

$$\epsilon_X = \int_i \epsilon^i_X \mathrm{d}i = R - max(\epsilon^u_{Z,\Delta^0_{after}}, K) \tag{18}$$

$$\epsilon_Z = \int_i \epsilon^i_Z \mathrm{d}i = R + \int_{\epsilon_X}^{\infty} (\epsilon^i_{intra} - \epsilon_X)\mathrm{dG}(\epsilon^i_{intra}). \tag{19}$$

Given that each bank trades to hold the same threshold amounts before the intraday payment shock, it follows from the equilibrium definition that $\epsilon_X = \epsilon^i_X$ and $\epsilon_Z = \epsilon^i_Z$.

There are two equilibrium possibilities, depending on the size of the penalty cost. In the first equilibrium, the penalty cost is small enough such that all banks borrow just the minimum from the central bank to meet their reserve requirement. This will be the case if $K \geq \epsilon^u_{Z,\Delta^0_{after}}$. In this case, the cost of borrowing from the central bank will be more than the expected penalty cost s in the after-hours session when the bank holds the required level of reserves (e.g., Equation (13)). In this low penalty cost equilibrium, the threshold amount in (18) is the same as in the standard Poole model (i.e., $\epsilon_X = R - K$).

In the second equilibrium, the penalty cost is sufficiently large such that some banks borrow more than the minimum from the central bank ($\epsilon^u_{Z,\Delta^0_{after}} > K$). In this high penalty cost equilibrium, the threshold for central bank borrowing (ϵ_X) in Equation (18) is lower than in the low penalty cost equilibrium, since banks will want to borrow more from the central bank to reduce the penalty cost in the after-hours session. And, since they borrow more from the central bank and thus will have more funds in the after-hours session, it follows that the threshold to experience the penalty cost ϵ_Z as in Equation (19) will be higher in the high penalty cost equilibrium.

The first-order condition in Equation (17) can now be written as a function of these aggregate threshold amounts, which themselves depend on the aggregate bank's balance sheet (i.e., as in Equations (18) and (19)).

$$\begin{aligned} r_{intra} &= (r_R + (1-p)r_{after})G(\epsilon_X) + r_X(1 - G(\epsilon_X)) \\ &\quad + ps\int_{-\infty}^{\epsilon_X} [1 - F(R - \epsilon^i_{intra})]\mathrm{dG}(\epsilon^i_{intra}) \end{aligned} \tag{20}$$

2.3.2 Comparison with Overnight Rate in the Absence of an After-Hours Payment Shock

Our second proposition compares the overnight rate in Equation (20) with the overnight rate in a standard model, $r_{Poole} \equiv r_R G(R-K) + r_X(1 - G(R-K))$.

Proposition 2. *The overnight rate in the presence of an after-hours payment shock will be weakly greater than the overnight rate in the absence of one:*

$$\begin{aligned} r_{intra} = r_{Poole} &+ (r_X - r_R)[G(R-K) - G(\epsilon_X)] \\ &+ (1-p)r_{after}G(\epsilon_X) \\ &+ ps\int_{-\infty}^{\epsilon_X} [1 - F(R - \epsilon^i_{intra})]\mathrm{dG}(\epsilon^i_{intra}). \end{aligned} \tag{21}$$

Equation (21), which substitutes the Poole rate into Equation (20), shows that the overnight rate with an after-hours payment shock is equal to the overnight rate in a standard model, r_{Poole}, plus three additional positive terms to account for the benefit of additional funds in the after-hours session in avoiding the expected penalty cost in the after-hours session. This holds in both the high penalty cost and low penalty cost equilibrium. The first additional term, $(r_X - r_R)[G(R-K) - G(\epsilon_X)]$, represents borrowing beyond the minimum reserve requirement and will put upward pressure on the overnight rate relative to the Poole model. In the low penalty cost equilibrium, this term is equal to zero. In the high penalty cost equilibrium, this term is positive and the overnight rate with a high penalty cost will be above the Poole rate. The second additional term accounts for the expected penalty cost when there is an after-hours trading session, while the third additional term accounts for the cost if there is no after-hours trading session.

Just like in the standard Poole model, the overnight rate in the intraday session will still be bounded by the central bank deposit and lending rates, r_R and r_X, in the presence of a penalty cost in the after-hours session.

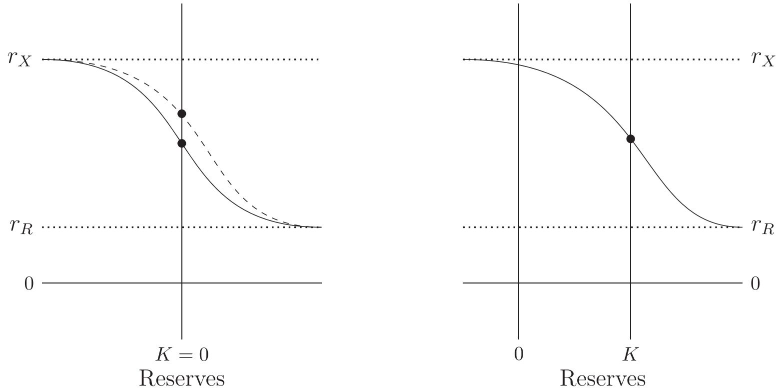
$$r_R \leq r_{intra} \leq r_X \tag{22}$$

It is easy to see that the overnight rate will be bounded from below by the central bank deposit rate. The standard Poole rate is bounded from below by the central bank deposit rate, and Proposition 2 shows that the overnight rate is higher than the Poole rate in the presence of an after-hours penalty cost. To see that it is bounded from above by the central bank lending rate, substitute Equation (10) into the first-order condition in Equation (20). This yields $r_{intra} = r_X - \int_{-\infty}^{\epsilon_X} \lambda^i \leq r_X$.

Figure 2 illustrates how the demand for reserves changes in the presence of an after-hours payment shock. In the graph on the left-hand side, where there are no required reserves, the demand for reserves in the presence of an after-hours payment shock (dashed line) is higher than the demand for reserves in the absence of this shock (solid line). This will, if anything, put upward pressure on the overnight interbank rate.

Interestingly, the demand for reserves is unaffected if aggregate reserves are very large or very small. When aggregate reserves are very large, there is almost zero probability that a bank would experience an after-hours payment shock that fully drains its reserves, so the expected penalty cost associated with having insufficient reserves is negligible. On the other hand, when aggregate reserves are very small (large, negative value), banks will almost surely borrow from the central bank at the end of the day. Therefore, trading away an additional dollar in the interbank market will have no effect on the probability of having insufficient funds in the after-hours period, since the bank would borrow an extra dollar from the central bank at

Figure 2. Demand for Overnight Reserves



Note: The left-side graph illustrates the case where the required level of reserves, K, is equal to zero. The solid line in this graph illustrates the demand for reserves when there is no after-hours payment shock (i.e., the traditional Poole model). The dashed line represents the demand for reserves when there is an after-hours payment shock. The dots represent the equilibrium allocation and rates when the level of reserves is also equal to zero, showing that the interbank rate could trade above the middle of the corridor when there is an after-hours payment shock. In the right-side graph, the required level of reserves, K, is a large positive number. In this graph, the demand for reserves is unaffected by the presence of an after-hours payment shock and the dashed line and the solid line coincide. The dot represents the equilibrium allocation and rate when the level of reserves is equal to the required level of reserves, showing that, with large required reserves, the interbank rate could still trade in the middle of the corridor when there is an after-hours payment shock.

the end of the day before the after-hours payment shock. As such, its interbank trading would have no effect on its reserve position before experiencing the after-hours payment shock.

2.3.3 Impact of After-Hours Payment Shock in Different Monetary Policy Implementation Frameworks

The following corollaries state the conditions under which the overnight rate will equal the Poole rate under different central bank operating frameworks. For these corollaries, we will focus on the equilibrium with a small penalty cost, since the high penalty cost equilibrium produces an overnight rate above the Poole rate. Specifically, we examine three different monetary policy implementation frameworks commonly used in practice, defined as follows:

(i) *A floor framework*: In this framework, reserves are sufficiently large such that the overnight rate in the standard Poole model (i.e., no after-hours payment shock) is equal to the deposit rate (i.e., $r_{Poole} = r_R$). For this to be the case, $G(R - K) \approx 1$.

(ii) *A zero-reserve corridor framework*: $R = K = 0$. The overnight rate in the standard Poole model will be equal to the midpoint of the central bank lending and deposit rates (i.e., $r_{Poole} = \frac{r_R + r_X}{2}$).

(iii) *A positive reserve corridor framework*: $R = K > 0$. The overnight rate in the standard Poole model will also be equal to the midpoint of the central bank lending and deposit rates (i.e., $r_{Poole} = \frac{r_R + r_X}{2}$).

COROLLARY 1. *In a floor framework with a low penalty cost ($K \geq \epsilon^u_{Z,\Delta^0_{after}}$), the overnight rate is equal to the Poole interest rate when*

- *participants can almost surely trade in the after-hours session ($p \approx 0$), or*
- *excess reserves are larger than a threshold value A: $R - K \geq A$, with $G(A) \approx 1$ and $F(R - A) \approx 1$.*

Proof. In the low penalty cost equilibrium, Equation (21) simplifies to

$$r_{intra} = r_{Poole} + (1-p) r_{after} G(\epsilon_X) + ps \int_{-\infty}^{\epsilon_X} [1 - F(R - \epsilon^i_{intra})] \mathrm{dG}(\epsilon^i_{intra}). \tag{23}$$

$F(\epsilon_Z) \geq F(R - K)$ and, by Assumption 3 and the definition of a floor, $F(R - K) \geq G(R - K) \approx 1$. Therefore, $F(\epsilon_Z) \approx 1$ and from Equation (6) $r_{after} = 0$, meaning the first additional term in the above equation is equal to zero. The second additional term in the above equation is equal to zero when either $p \approx 0$ by the first condition in this corrollary or $F(K) \approx 1$ by the second condition, leaving $r_{intra} = r_{Poole}$. □

The result is intuitive. In a floor framework, the expected value of the penalty cost is near zero since there is close to zero probability that the after-hours payment shock will reduce the bank's reserves below zero.

Visually, this can be illustrated with the central bank supplying a large quantity of reserves in Figure 2 (left-side graph), such that the supply of reserves is a vertical line that intersects with the demand for reserves at r_R. At this point, the demand for reserves is the same as it would be in the absence of an after-hours payment shock.

COROLLARY 2. *In a zero-reserve corridor framework ($R = K = 0$) with a low penalty cost, the overnight rate will equal the Poole rate only when*

- *the volatility of the overnight payment shock is relatively small (i.e., when F and G are normally distributed and $\Phi(\frac{\sigma G}{\sigma_F}\phi(0)) \approx 1$), and*
- *participants can almost surely trade in the after-hours session ($p \approx 0$).*

Proof. Substituting $R = K = 0$ and $p \approx 0$ into Equation (23) and writing in terms of standard normal distributions yields the result, $r_{intra} = r_{Poole} + \frac{s}{2}(1 - \Phi(\frac{\sigma G}{\sigma_F}\phi(0)))$. Since $\Phi(\frac{\sigma G}{\sigma_F}\phi(0)) \approx 1)$, $r_{intra} = r_{Poole}$. □

Intuitively, when the volatility of the intraday payment shock increases (holding σ_F constant), there is more borrowing from the central bank in stage 4. In particular, when G is a normal distribution, aggregate borrowing from the central bank increases linearly with the volatility of the intraday payment shock in a zero-reserve corridor. This aggregate borrowing increases the aggregate supply of reserves in the after-hours market. When this aggregate borrowing becomes sufficiently large and banks can trade in the after-hours market, the expected penalty cost of the after-hours payment shock for each bank is approximately 0. This means that $r_{after} = 0$ and thus $r_{intra} = r_{Poole}$.

Determining the optimal level of aggregate reserves to target the Poole rate in a zero-reserves corridor is more challenging in the presence of material after-hours payment shocks. In the absence of an after-hours payment shock, the central bank simply needs to target aggregate reserves equal to the aggregate reserve requirement, $R = K$. With a material after-hours payment shock, the central bank needs to understand the demand for reserves to determine the amount of aggregate reserves to supply to the market. This will depend on, among other things, the size of the penalty cost s and the magnitude of after-hours payment shocks, σ_F. This can be seen in Figure 2 (left-side graph). With $R = K = 0$, the equilibrium interest rate (the intersection of the supply and the dashed demand curve) will be higher than the equilibrium interest rate in the absence of an overnight payment shock (the intersection of the supply and the solid demand curve).

An alternative for the central bank could be to establish a higher required reserves amount. This leads to our next corollary.

COROLLARY 3. *In a positive-reserve requirement corridor system ($R = K > 0$), the overnight rate will equal the Poole rate when the aggregate reserve requirement is sufficiently large (e.g., $F(K) \approx 1$).*

Proof. Substituting $R = K$ and $F(K) \approx 1$ into Equation (6) shows that $r_{after} = 0$ and thus the first additional term in Equation (23) equals zero. The second additional term in Equation (23) is also equal to zero when $F(K) \approx 1$, which means $r_{intra} = r_{Poole}$. □

The intuition for this result is the same as the intuition for Corollary 1. When K is sufficiently large, all banks will hold enough reserves such that there is a near zero probability that the after-hours payment shock will bring the bank's level of reserves to zero, where it will begin to experience the penalty cost. The higher amount of required reserves shifts the demand curve for reserves to the right, as seen in Figure 2 (right-side graph). Thus, the demand for reserves is the same as in the standard case.

2.4 Effect of After-Hours Payment Shock Volatility on the Equilibrium

A good monetary policy implementation framework should be characterized by low volatility in the overnight rate (e.g., Bindseil 2016).

In our model, we can examine how the volatility of the overnight rate is affected by the volatility of σ_F, the volatility of the after-hours payment shock. After-hours payment volatility could fluctuate, for instance, if the length of the after-hours session fluctuates (i.e., over a weekend).

To see the impact on day-to-day volatility in the overnight rate, we now assume that the after-hours payment shock volatility is a random variable that is symmetrically distributed around its mean value. We label this mean value as $\overline{\sigma_F}$. Before stage 1 each day, all banks see the realization of the payment shock volatility random variable. Since the random variable is realized before trading begins, it will not change the model's results on a given day but will change results from one day to the next. The overnight rate is also now a random variable because it is a function of the after-hours payment volatility, which is itself a random variable.

The volatility of the overnight rate with respect to changes in the volatility of the after-hours payment shock can thus be expressed as

$$\sigma_{r_{intra}} = \sqrt{E[(r_{intra}(\sigma_F) - E[r_{intra}(\sigma_F)])^2]}. \tag{24}$$

Next, we can take a Taylor-series expansion of the overnight rate around the mean value of the after-hours payment shock volatility:

$$r_{intra}(\sigma_F) \approx r_{intra}(\overline{\sigma_F}) + \frac{\partial r_{intra}}{\partial \sigma_F}(\sigma_F - \overline{\sigma_F}). \tag{25}$$

This and the symmetric distribution of σ_F means we can approximate the expected overnight rate by $E[r_{intra}(\sigma_F)] \approx r_{intra}(\overline{\sigma_F})$. By substituting this and Equation (25) into Equation (24), the effect of volatility of the after-hours payment shock volatility on overnight rate volatility can be approximated as

$$\sigma_{r_{intra}} \approx \left|\frac{\partial r_{intra}}{\partial \sigma_F}\right| \sqrt{E[(\sigma_F - \overline{\sigma_F})^2]}. \tag{26}$$

This expression suggests that transmission of volatility of after-hours payment shock volatility ($\sigma_{\sigma_F} \equiv \sqrt{E[(\sigma_F - \overline{\sigma_F})^2]}$) to volatility of the overnight rate will depend on $\frac{\partial r_{intra}}{\partial \sigma_F}$. All else equal, an overnight rate that is more responsive to changes in after-hours payment shock volatility will be more volatile.

Some central banks could adjust aggregate reserves to limit the impact of changes in after-hours payment shock volatility on overnight rate volatility. In this case, aggregate reserves would also be a function of after-hours payment shock volatility (e.g., $R = R(\sigma_F)$). That is, the central bank may offset the effect of changes in after-hours payment shock volatility by changing aggregate reserves. Given this, we can take the derivative of the overnight rate with respect to after-hours payment volatility and substitute this derivative into Equation (26) to write overnight rate volatility as

$$\sigma_{r_{intra}} \approx \left| s\Phi\left(\frac{\epsilon_X}{\sigma_G}\right)\phi\left(\frac{\epsilon_Z}{\sigma_F}\right)\frac{\epsilon_Z}{\sigma_F^2} + \frac{\partial r_{intra}}{\partial R}\frac{\partial R}{\partial \sigma_F}\right| \sigma_{\sigma_F}. \tag{27}$$

In the absence of a central bank response ($\frac{\partial R}{\partial \sigma_F} = 0$), changes in after-hours payment volatility will weakly increase volatility of the overnight rate. Two sufficient circumstances where changes in after-hours payment volatility do not affect overnight rate volatility are as follows:

- A floor framework. In Corollary 1 we showed that $F(\epsilon_Z) \approx 1$ in a floor framework. When $F(\epsilon_Z) \approx 1$, $\phi(\frac{\epsilon_Z}{\sigma_F}) \approx 0$ and hence $\sigma_{r_{intra}} \approx 0$.
- A positive-reserve requirement corridor system, where the aggregate reserve requirement is sufficiently large (e.g., $F(K) \approx 1$). Similarly, when $F(K) \approx 1$, $\phi(\frac{\epsilon_Z}{\sigma_F}) \approx 0$ and hence $\sigma_{r_{intra}} \approx 0$.

In both of these cases, volatility of after-hours payment volatility has no effect on the overnight rate. In this case reserves are sufficiently large such that a change in after-hours payment volatility does not change the probability of running out of funds in the after-hours session.

In a zero-reserve requirement corridor system, on the other hand, volatility in after-hours payment volatility will increase the volatility of the overnight rate. While the central bank could offset this impact on overnight rate volatility by adjusting aggregate reserves in responses to changes in after-hours payment volatility, it may be difficult for the central bank to determine the necessary adjustment. The required adjustment requires knowledge of the demand for overnight funds (i.e., $\frac{\partial r_{intra}}{\partial R}$) as well as knowledge about the volatility of the after-hours payment shock and intraday payment shock.

3. Model Applications

3.1 Optimal Reserve Requirements in a Corridor System

The previous subsection suggests that the central bank can effectively maintain a corridor system if it chooses required reserves that are sufficiently high. With a high reserve requirement, the expected penalty cost of having insufficient funds in the after-hours market is negligible, and volatility of after-hours payment shock volatility does not affect the overnight rate.

However, a large positive-reserve requirement corridor system may not necessarily be optimal if the central bank has other concerns beyond the overnight rate setting. To illustrate this, we assume that a bank now faces a balance sheet cost that is an increasing function of balance sheet size, $c(D) \geq 0$, $c'(D) \geq 0$, and $c''(D) \geq 0$. We focus on the scenario where banks can trade in the after-hours session ($p \approx 0$) and banks are in the low penalty cost equilibrium. Bank bond holdings remain exogenous.[6] Given the balance sheet identity and the fact that $R = K$ in a corridor, aggregate deposits are equal to $B + K$. Then, suppose that the central bank chooses K to minimize costs to the banking system:

$$\text{Cost} = c(B+K) + s\int_{\epsilon_Z^i}^{\infty} (\epsilon_{after}^i - \epsilon_Z)\mathrm{dF}(\epsilon_{after}^i). \tag{28}$$

Then, it is simple to see that the central bank will choose a reserve requirement such that

$$c'(B+K) = s\left(1 - \Phi\left(\frac{K + \sigma_G\phi(0)}{\sigma_F}\right)\right). \tag{29}$$

This illustrates the trade-off where increasing reserve requirements reduces the expected after-hours penalty cost but increases

[6]This section is only meant to illustrate that there are other considerations beyond controlling the overnight rate. A general equilibrium analysis of social welfare where banks also choose their bond holdings is beyond the scope of the paper, but we point the interested reader to Martin et al. (2013), Canzonari, Cumby, and Diba (2017), and Williamson (2019).

the balance sheet cost of banks. Assuming it can't set negative reserve requirements, a central bank would prefer a zero-reserve requirement corridor to a positive-reserve requirement corridor if the marginal balance sheet cost is large relative to the penalty cost at $K = 0$:

$$c'(B) \geq s\left(1 - \Phi\left(\frac{\sigma_G \phi(0)}{\sigma_F}\right)\right). \tag{30}$$

3.1.1 Return on Required Reserves in a Corridor

If this social optimum requires a large reserve requirement, banks may see this large reserve requirement as a tax (e.g., see the discussion in Lipscomb, Martin, and Wiggins 2017). However, by adjusting the return on required reserves, we show how the central bank could make its choice of reserve requirement coincide with that which banks would choose themselves.[7]

We assume banks choose their required relative reserves before the start of intraday trading, under the assumption that the central bank will supply aggregate reserves equal to the aggregate reserve requirement ($R = K$). This is a slight departure from Baughman and Carapella (2018), given that in their model voluntary reserve targets adjust to central bank's supply of reserves rather than the other way around. In our model, the introduction of a balance sheet cost also allows us to avoid the problem of banks setting infinite targets. In the absence of a balance sheet cost, banks could make infinite profits if $r_D < r_K$ by increasing their reserve requirements and this would cause reserve targets to converge to $+\infty$.

The aggregate bank will choose K to maximize expected profits:

$$\begin{aligned} \mathrm{E}[\pi] = {} & r_B B - r_D(B + K) - c(B + K) + r_K K \\ & + (r_R - r_X) \int_0^{\infty} \epsilon^i_{intraday} \mathrm{dG}(\epsilon^i_{intraday}) \\ & - s \int_{\epsilon^i_Z}^{\infty} (\epsilon^i_{after} - \epsilon_Z) \mathrm{dF}(\epsilon^i_{after}). \end{aligned} \tag{31}$$

[7] Baughman and Carapella (2018) develop a model of voluntary reserve targets and show the potential advantages of such a model over other models.

The first-order condition is

$$r_D + c'(B + K) = r_K + s\left(1 - \Phi\left(\frac{K + \sigma_G \phi(0)}{\sigma_F}\right)\right). \qquad (32)$$

The left-hand side of the first-order condition represents the marginal cost of an additional dollar of required reserves. This is the bank's marginal funding cost, and it includes the deposit rate, as well as the cost associated with increasing the bank's balance sheet. The right-hand side of the equation is the marginal benefit of an additional dollar of required reserves. It consists of two components. The first is r_K, the rate at which the central bank compensates required reserves. The second is the reduction in the expected penalty costs associated with having insufficient funds to process after-hours transactions.

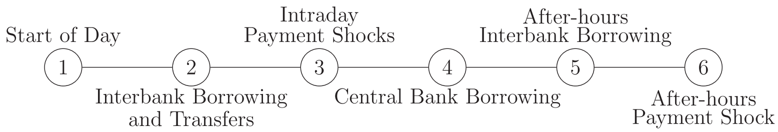
In this illustrative example, if the central bank could set the rate on required reserves equal to the deposit rate ($r_D = r_K$), the social optimum level of required reserves would coincide with that which the banks would choose themselves. In a more general equilibrium setup, r_D would not be fixed and it may be more challenging for the central bank to set $r_D = r_K$. Nonetheless, the main point of this subsection should still hold more generally: a central bank can adjust r_K so that its choice of reserve requirement coincides with what banks would choose themselves in the optimum.

3.2 Multiple Payment Systems

In this section, we analyze how the results are affected by the operation of two interlinked payment systems: one which operates only during the day (labeled a traditional system) and has access to the central bank borrowing and lending facilities during operating hours, and one which operates 24/7 and has access to the central bank deposit facility but not the lending facility (outside of operating hours).

Several jurisdictions currently have multiple payment systems, with one payment system providing 24/7 access. The 24/7 system is usually a system for retail payment flows, and there are only a few cases where 24/7 retail payment flows are accommodated within the large-value payment system or are settled in real time

Figure 3. Model Timing with Two Payment Systems



in the large-value payment system.[8] Tompkins and Olivares (2016) show that some jurisdictions, however, have infrequent settlement between the 24/7 retail payment system and the large-value payment system, with the ability to pre-fund the 24/7 system when the large-value payment system is closed. How does the presence of multiple payment systems affect monetary policy implementation? How will parameters such as the frequency of settlement between two interoperating systems affect the overnight rate?

To analyze this setup, we model infrequent settlement as a restriction on the ability to transfer between the two systems after trading is complete in stage 2. Figure 3 illustrates how the setup is modified. We focus on the low cost equilibrium and assume that all banks can trade in the after-hours session. For ease of notation, we assume that banks begin the day with zero reserves in the 24/7 payment system. This does not affect the results since, in stage 2, in addition to borrowing in the interbank market, we assume banks can also transfer funds between the two payment systems. We denote the net transfer from the traditional payment system to the 24/7 payment system $T^i \geq 0$. We interpret this transfer between systems as both a settlement between the systems as well as the ability to pre-fund the 24/7 system.

[8]See Tompkins and Olivares (2016), Figure 13 for a list of jurisdictions with 24/7 retail payment systems and non-24/7 wholesale payment systems, and the interoperability between these systems. Moreover, in 2016, the Committee on Payments and Market Infrastructures of the Bank for International Settlements listed 11 countries with retail payment systems that operated on a near-24/7 basis, plus an additional 8 countries that had plans to implement one (Committee on Payments and Market Infrastructures 2016). Canada, for instance, plans to introduce a 24/7 retail payments system that will operate alongside its large-value payment system, which operates with traditional hours (Payments Canada 2017).

In stage 3, after the trading session is closed, each bank experiences an intraday payment shock, $\epsilon^i_{intraday}$, in the traditional payment system. This is the same payment shock as in the baseline setup, with cumulative distribution function $G(\epsilon^i_{intraday})$. For simplicity and comparison with the earlier results, we assume there is no intraday payment shock in the 24/7 payment system.

After experiencing the intraday payment shock, as before, banks borrow from the central bank in stage 4 if they are in a negative excess reserve position in the traditional system.[9] Then, the banks earn r_R on positive balances they hold in either system. They also pay r_X on their borrowing from the central bank.

In stage 5, banks will still choose their net interbank after-hours borrowing Δ^i_{after} to maximize their expected profits in the after-hours market, given the payment shock it is exposed to in stage 6 in the 24/7 payment system:

$$\mathrm{E}_5[\pi^i_5] = -r_{after}\Delta^i_{after} - s\int_{\epsilon^i_{Z,T}}^{\infty}(\epsilon^i_{after} - \epsilon^i_{Z,T})\mathrm{dF}(\epsilon^i_{after}), \quad (33)$$

where the threshold, $\epsilon^i_{Z,T} \equiv T^i + \Delta^i_{after}$, is a little different because it accounts for the effect of transfers between the two systems. Since central bank borrowing only affects the reserve position in the traditional system, it does not affect the threshold for experiencing the penalty cost in the 24/7 system.

Like before, the first-order condition from Equation (33) provides the after-hours interbank rate that maximizes the expected after-hours profits:

$$r_{after} = s(1 - F(\epsilon^i_{Z,T})). \quad (34)$$

Equation (34) implies that banks will trade with each other until they have the same $\epsilon^i_{Z,T} \equiv \epsilon_{Z,T}$.

[9] If we assume that banks can transfer between systems in stage 4, that reserve requirements apply to balances in the 24/7 system, and that there is a reserve requirement of 0 in the traditional system at the end of the day, results will be identical to the baseline setup of a single system. Each bank will make intersystem transfers in stage 4 such that there will be a zero balance in the traditional payment system.

Each bank will choose their borrowing, lending, and transfer activity in stage 2 to maximize expected profits:

$$\begin{aligned}\mathrm{E}[\pi^i] = {} & r_B B^i - r_D D^i + r_K K^i - r_{intra}\Delta^i_{intra} \\ & + r_R(\epsilon^i_X - T^i) + (r_R - r_X) \\ & \quad \int_{\epsilon^i_X - T^i}^{\infty} (\epsilon^i_{intraday} - (\epsilon^i_X - T^i))\mathrm{dG}(\epsilon^i_{intraday}) \\ & - r_{after}(T^i + \Delta^i_{after}) - s\int_{\epsilon_{Z,T}}^{\infty} (\epsilon^i_{after} - \epsilon_{Z,T})\mathrm{dF}(\epsilon^i_{after}).\end{aligned} \tag{35}$$

Maximizing expected profits produces two first-order conditions. After combining these two conditions and aggregating across all banks, the optimality conditions can be written as

$$r_{intra} = r_R + (r_X - r_R)\left(1 - \Phi\left(\frac{R - K - T}{\sigma_G}\right)\right)$$

$$r_{intra} = r_R + s\left(1 - \Phi\left(\frac{T}{\sigma_F}\right)\right).$$

The first equation represents the marginal value of reserves in the traditional system, and the second equation represents the marginal value of reserves in the 24/7 system. Since both equations have the overnight interbank rate on the left-hand side, it suggests that banks will transfer funds between the two systems until the marginal value of reserves is equal across both systems.

The marginal value of reserves suggested by the first equation is a small departure from the standard model. Transfers to the 24/7 system will reduce reserves in the traditional system and increase the probability that the bank will have to borrow from the central bank. Hence, it will put upward pressure on the overnight rate relative to the Poole rate.

The marginal value of reserves in the 24/7 system is a function of two factors. First, banks will earn interest on reserves on funds held in the 24/7 system. Second, an extra dollar of reserves in the 24/7 system lowers the likelihood that the bank will be short of funds in the after-hours session, thus reducing the expected penalty cost of being short of funds.

Overall, the interbank rate in the two-system environment may be higher or lower than the interbank rate in a single system. On the one hand, banks will borrow more in the traditional system than they do in a single system, since transfers to the 24/7 system reduce reserves in the traditional system and hence increase the amount of interbank borrowing. Also, some banks will end up with positive balances at the end of the day in the traditional system that cannot be used to reduce the probability of experiencing the penalty cost in the 24/7 system (since post-intraday shock transfers are not allowed). This will put upward pressure on the interbank rate in a dual system, relative to a single system. On the other hand, in a single system, the marginal benefit of an additional dollar of reserves includes both the marginal benefit of reducing the expected cost of central bank borrowing, as well as the marginal benefit of reducing the expected penalty cost. Because it contains both of these marginal benefits, and the interbank rate reflects these marginal benefits, the interbank rate in a single system could be higher. Depending on which of these two competing effects dominates, the rate could be higher in a single system or in a dual system.

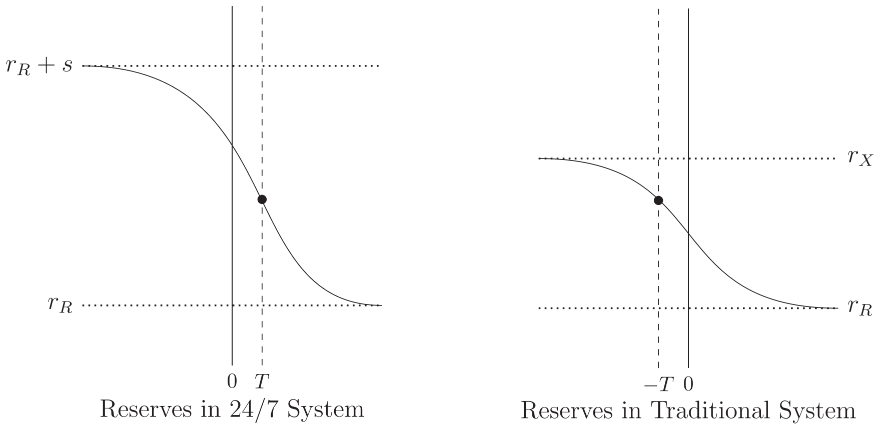
Figure 4 illustrates how these transfers affect the overnight interbank rate. When both the traditional system and the 24/7 system begin the day with zero reserves, the marginal value of funds in the 24/7 system (Figure 4, left-side graph) is greater than the marginal value of funds in the intraday system (Figure 4, right-side graph). Since the marginal value of funds is higher in the 24/7 system, participants will have incentives to move reserves into that system. They continue doing so until the marginal value of funds in the two systems is equal.

3.2.1 Tighter Restrictions on Transfers

We assumed that transfers can occur during the interbank trading period. An even more extreme assumption would be to completely restrict transfers across the two systems. This could represent, for example, those 24/7 retail payment systems that settle in the large-value payment system only once a day (Tompkins and Olivares 2016).

A complete restriction on transfers generates segmented markets across the two systems. Because transfers cannot take place in

Figure 4. Overnight Rate in Multiple Systems



Note: The graphs represent overnight trading in the two systems when the required level of reserves, K, and aggregate level of reserves, R, are both equal to zero. The left-side graph illustrates the equilibrium allocation and rate in the 24/7 system, while the right-side graph illustrates the equilibrium allocation and rates in the traditional system. Assuming that both systems begin the day with zero aggregate reserves, T represents the transfers between the two systems that occur in stage 2. Specifically, transfers occur until the overnight rates in the two different systems are equal.

stage 2 when trading occurs, the marginal value of funds in the two systems will (most likely) be different from each other. Specifically, there will be an overnight interbank rate for the traditional system ($r_{intra,traditional}$) and an overnight interbank rate for the 24/7 system ($r_{intra,24/7}$). Reserves in the two systems are now denoted $R_{traditional}$ and $R_{24/7}$, respectively. Thus, in stage 2, banks are either trading funds in the traditional system with each other, or trading funds in the 24/7 system with each other:

$$r_{intra,traditional} = r_R + (r_X - r_R)\left(1 - \Phi\left(\frac{R_{traditional} - K}{\sigma_G}\right)\right)$$

$$r_{intra,24/7} = r_R + s\left(1 - \Phi\left(\frac{R_{24/7}}{\sigma_F}\right)\right).$$

Restricting transfers completely insulates the traditional trading market from effects of the after-hours payment shock: the interbank rate in the traditional system is equal to the Poole rate. To implement the same rate in the 24/7 system, the central bank would need to determine the appropriate supply of reserves in the 24/7 system based on how these reserves affect $r_{intra,24/7}$ in the formula above.

4. Conclusion

Our paper shows how changes in the payment system could have implications for overnight interest rates. Specifically, monetary policy implementation frameworks that naturally have a large amount of reserves are less affected by a move to 24/7 payment settlement in our model. More broadly, while our model focuses on the effect of 24/7 payment settlement on interbank rates, it can also be applied to other factors that increase the benefits of reserves. For example, the penalty costs of having insufficient funds after hours could be interpreted as a cost of having insufficient reserves relative to a target level of reserves that could be driven by regulation or other factors.

Further, our model is derived in a centralized market, so it does not say anything about trading volumes or dispersion of traded rates. While we believe our model delivers the important implications of 24/7 settlement, a search model with a decentralized market (e.g., Afonso and Lagos 2015) could provide additional implications for trading activity.

Finally, we assume that the intraday shock process and other underlying features of the system, such as the number and characteristics of participants, are invariant to payment system modernization. In practice, these features may adjust to a lengthening of the payment period. We leave a more detailed exploration of these drivers to future work.

References

Afonso, G., and R. Lagos. 2015. "Trade Dynamics in the Market for Federal Funds." *Econometrica* 83 (1): 263–313.

Armenter, R., and B. Lester. 2015. "Excess Reserves and Monetary Policy Normalization." Working Paper No. 16-33, Federal Reserve Bank of Philadelphia.

Baughman, G., and F. Carapella. 2018. “Voluntary Reserve Targets.” Finance and Economics Discussion Series No. 2018-032, Board of Governors of the Federal Reserve System.

Bech, M., and T. Keister. 2013. “Liquidity Regulation and the Implementation of Monetary Policy.” Working Paper No. 432, Bank for International Settlements.

Bech, M., and E. Klee. 2011. “The Mechanics of a Graceful Exit: Interest on Reserves and Segmentation in the Federal Funds Market.” *Journal of Monetary Economics* 58 (5): 415–31.

Bindseil, U. 2016. “Evaluating Monetary Policy Operational Frameworks.” Remarks at the 2016 Economic Policy Symposium sponsored by the Federal Reserve Bank of Kansas City, held at Jackson Hole, Wyoming, August 25–27.

Boutros, M., and J. Witmer. 2019. “Monetary Policy Implementation in a Negative Rate Environment.” *Journal of Money, Credit and Banking* 52 (2–3): 441–70.

Canzonari, M., R. Cumby, and B. Diba. 2017. “Should the Federal Reserve Pay Competitive Interest on Reserves?” *Journal of Money, Credit and Banking* 49 (4): 663–93.

Committee on Payments and Market Infrastructures. 2016. “Fast Payments — Enhancing the Speed and Availability of Retail Payments.” Technical Report, Bank for International Settlements.

Ennis, H. M., and T. Keister. 2008. “Understanding Monetary Policy Implementation.” *Economic Quarterly* (Federal Reserve Bank of Richmond) 94 (3, Summer): 235–63.

Ennis, H. M., and J. A. Weinberg. 2007. “Interest on Reserves and Daylight Credit.” *Economic Quarterly* (Federal Reserve Bank of Richmond) 93 (2, Spring): 111–42.

Lipscomb, L., A. Martin, and H. Wiggins. 2017. “Why Pay Interest on Required Reserve Balances.” Blog Entry, Liberty Street Economics Blog, Federal Reserve Bank of New York, September 25.

Martin, A., J. McAndrews, A. Palida, and D. Skeie. 2013. “Federal Reserve Tools for Managing Rates and Reserves.” Staff Report No. 642, Federal Reserve Bank of New York.

Norman, B., R. Shaw, and G. Speight. 2011. “The History of Interbank Settlement Arrangements: Exploring Central Banks’ Role in the Payment System.” Working Paper No. 412, Bank of England.

Payments Canada. 2017. "Modernization Target State." Technical Report.

Poole, W. 1968. "Commercial Bank Reserve Management in a Stochastic Model: Implications for Monetary Policy." *Journal of Finance* 23 (5): 1177–95.

Tompkins, M., and A. Olivares. 2016. "Clearing and Settlement Systems from Around the World: A Qualitative Analysis." Staff Discussion Paper No. 2016-14, Bank of Canada.

Williamson, S. D. 2019. "Interest on Reserves, Interbank Lending, and Monetary Policy." *Journal of Monetary Economics* 101 (January): 14–30.