Online Appendix to "Monetary Policy and COVID-19"

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In this appendix we present a full list of equations making up the model. Variables with bars denote steady-state values.

A.1 Epidemic Block

Evolution of susceptible individuals:

$$S_{t+1} = (1 - \varpi_{I,t})S_t. \tag{A.1}$$

Evolution of symptomatic infected:

$$I_{t+1} = (1 - \varpi_R)I_t + \rho\varpi_{I,t}S_t. \tag{A.2}$$

Evolution of asymptomatic infected:

$$A_{t+1} = (1 - \varpi_R)A_t + (1 - \rho)\varpi_{I,t}S_t. \tag{A.3}$$

Evolution of formerly asymptomatic infected:

$$V_{t+1} = V_t + \varpi_R A_t. \tag{A.4}$$

Evolution of recovered individuals:

$$R_{t+1} = R_t + (\varpi_R - \varpi_{D,t})I_t. \tag{A.5}$$

Evolution of deceased individuals:

$$D_{t+1} = D_t + \varpi_{D,t}I_t. \tag{A.6}$$

Supposedly susceptible individuals:

$$\tilde{S}_t = S_t + A_t + V_t. \tag{A.7}$$

A.2 Infection and Death Probabilities

Probability of susceptible individuals becoming infected:

$$\varpi_{I,t} = \varpi_c \tilde{c}_t(\zeta I_t c_t^I + \kappa A_t \tilde{c}_t) + \varpi_n \tilde{n}_t \kappa A_t \tilde{n}_t + \varpi_t(I_t + \kappa A_t). \quad \text{(A.8)}$$

Probability of supposedly susceptible agent becoming symptomatic infected:

$$\tilde{\varpi}_{I,t} = \rho \varpi_{I,t} \frac{S_t}{\tilde{S}_t}. \quad \text{(A.9)}$$

Death probability:

$$\varpi_{D,t} = \min\left[\left(1 + \frac{I_t}{\nu_0}\right)\varpi_D, \nu_1 \varpi_D\right]. \quad \text{(A.10)}$$

A.3 Supposedly Susceptible Individuals

Utility:

$$\tilde{U}_t = \log \tilde{c}_t + \theta \log(1 - \tilde{n}_t) + \beta(1 - \tilde{\varpi}_{I,t})\tilde{U}_{t+1} + \beta \tilde{\varpi}_{I,t} U_{t+1}^I. \quad \text{(A.11)}$$

Budget constraint:

$$(1 + \tau_{c,t})\tilde{c}_t + \tilde{b}_{t+1} = (1 - \tau_{n,t}) w_t \tilde{n}_t + \frac{\mathcal{I}_{t-1}}{\pi_t}\tilde{b}_t(1 - \tilde{\varpi}_{I,t-1}) + \Gamma_t. \quad \text{(A.12)}$$

Optimality conditions:

$$\frac{1}{\tilde{c}_t} = \tilde{\lambda}_{S,t}(1 + \tau_{c,t}) - \tilde{\lambda}_{\varpi,t} \rho \frac{S_t}{\tilde{S}_t} \varpi_c(\zeta I_t c_t^I + \kappa A_t \tilde{c}_t) \quad \text{(A.13)}$$

$$\frac{\theta}{1 - \tilde{n}_t} = \tilde{\lambda}_{S,t}(1 - \tau_{n,t}) w_t + \tilde{\lambda}_{\varpi,t} \rho \frac{S_t}{\tilde{S}_t} \varpi_n \kappa A_t \tilde{n}_t \quad \text{(A.14)}$$

$$\tilde{\lambda}_{\varpi,t} = \beta[U_{I,t+1} - \tilde{U}_{t+1}] \quad \text{(A.15)}$$

$$\tilde{\lambda}_{S,t} = \beta[(1 - \tilde{\varpi}_{I,t})\tilde{\lambda}_{S,t+1} + \tilde{\varpi}_{I,t}\lambda_{I,t+1}]\frac{\mathcal{I}_t}{\pi_{t+1}}. \quad \text{(A.16)}$$

A.4 Symptomatic Infected Individuals

Utility:

$$U_t^I = \log c_t^I + \theta \log(1 - n_t^I) + \beta(1 - \varpi_R)U_{t+1}^I + \beta(\varpi_R - \varpi_{D,t})U_{t+1}^R + \beta\varpi_{D,t}U^D. \tag{A.17}$$

Budget constraint:

$$(1 + \tau_{c,t})c_t^I + b_{t+1}^I = w_t\xi n_t^I + \Gamma_t + \frac{\mathcal{I}_{t-1}}{\pi_t}\left[\frac{(1 - \varpi_R + \varpi_{D,t-1})I_{t-1}}{I_t}b_t^I + \frac{\rho\varpi_{I,t-1}S_{t-1}}{I_t}\tilde{b}_t\right]. \tag{A.18}$$

Optimality conditions:

$$\frac{1}{c_t^I} = \lambda_{I,t}(1 + \tau_{c,t}) \tag{A.19}$$

$$\frac{\theta}{1 - n_t^I} = \xi w_t \lambda_{I,t} \tag{A.20}$$

$$\lambda_{I,t} = \beta[(1 - \varpi_R)\lambda_{I,t+1} + (\varpi_R - \varpi_{D,t})\lambda_{R,t+1}]\frac{\mathcal{I}_t}{\pi_{t+1}(1 - \varpi_{D,t})}. \tag{A.21}$$

A.5 Symptomatic Recovered Individuals

Utility:

$$U_t^R = \log c_t^R + \theta \log(1 - n_t^R) + \beta U_{t+1}^R. \tag{A.22}$$

Budget constraint:

$$(1 + \tau_{c,t})c_t^R + b_{t+1}^R = (1 - \tau_{n,t})w_t n_t^R + \frac{\mathcal{I}_{t-1}}{\pi_t}\left[\frac{R_{t-1}}{R_t}b_t^R + (\varpi_R - \varpi_{D,t-1})\frac{I_{t-1}}{R_t}b_t^I\right] + \Gamma_t. \tag{A.23}$$

Optimality conditions:

$$\frac{1}{c_t^R} = \lambda_{R,t}(1 + \tau_{c,t}) \tag{A.24}$$

$$\frac{\theta}{1 - n_t^R} = (1 - \tau_{n,t}) w_t \lambda_{R,t} \tag{A.25}$$

$$\lambda_{R,t} = \beta \lambda_{R,t+1} \frac{\mathcal{I}_t}{\pi_{t+1}}. \tag{A.26}$$

A.6 Firms

Optimal price set by reoptimizing firms:

$$\tilde{p}_t = \frac{\Omega_t}{\Upsilon_t}. \tag{A.27}$$

Auxiliary functions for optimal price setting:

$$\Omega_t = (1 - \delta\beta) \frac{w_t}{Z} \frac{y_t}{\tilde{S}_t + I_t + R_t} \left(\frac{\tilde{S}_t}{\tilde{c}_t} + \frac{I_t}{c_t^I} + \frac{R_t}{c_t^R} \right) + \delta\beta\pi_{t+1}^{\varepsilon} \Omega_{t+1} \tag{A.28}$$

$$\Upsilon_t = (1 - \delta\beta) \frac{y_t}{\tilde{S}_t + I_t + R_t} \left(\frac{\tilde{S}_t}{\tilde{c}_t} + \frac{I_t}{c_t^I} + \frac{R_t}{c_t^R} \right) + \delta\beta\pi_{t+1}^{\varepsilon - 1} \Upsilon_{t+1}. \tag{A.29}$$

Price index:

$$1 = \delta \left(\pi_t\right)^{\varepsilon - 1} + (1 - \delta) \tilde{p}_t^{1-\varepsilon}. \tag{A.30}$$

Price dispersion:

$$\Delta_t = \delta \pi_t^{\varepsilon} \Delta_{t-1} + (1 - \delta) \, \tilde{p}_t^{-\varepsilon}. \tag{A.31}$$

A.7 Government and Central Bank

Containment policies:

$$\tau_{c,t} = \Phi_c I_t \tag{A.32}$$

$$\tau_{n,t} = \Phi_n I_t \tag{A.33}$$

$$\omega_t = \omega(1 - \tau_{c,t})^{\Phi_\omega}. \tag{A.34}$$

Monetary policy rule:

$$\frac{\mathcal{I}_t}{\bar{\mathcal{I}}} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\Phi_\pi} \left(\frac{y_t}{y_t^f}\right)^{\Phi_y} \exp(I_t)^{\Phi_I}. \tag{A.35}$$

A.8 Market Clearing

Final goods market:

$$\tilde{S}_t \tilde{c}_t + I_t c_t^I + R_t c_t^R = (\tilde{S}_t + I_t + R_t) y_t. \tag{A.36}$$

Labor market:

$$\tilde{S}_t \tilde{n}_t + I_t \xi n_t^I + R_t n_t^R = (\tilde{S}_t + I_t + R_t) n_t. \tag{A.37}$$

Aggregate output:

$$\Delta_t y_t = Z n_t. \tag{A.38}$$

Bond market:

$$\tilde{S}_t \tilde{b}_t + I_t b_t^I + R_t b_t^R = 0. \tag{A.39}$$

A.9 Flexible-Price Block

The flexible-price block consists of the same set of equations as above, except that $\delta = 0$. The equilibrium evolution of per capita output in this hypothetical economy, denoted by y_t^f, enters the monetary policy rule (A.35).