

# The Power of Forward Guidance and the Fiscal Theory of the Price Level\*

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Standard models predict implausible responses to forward-guidance announcements when interest rates are pegged (the “forward-guidance puzzle”). This paper develops conditions for regime-switching models to exhibit a forward-guidance puzzle, and shows when the fiscal theory of the price level does—and does not—resolve the forward-guidance puzzle. Forward guidance has reasonable effects when real fiscal revenues are very unresponsive to government debt, but in some empirically relevant cases, inflation *must* ensure debt stability and forward-guidance puzzles are still there. Cyclical variation in deficits also affects the power of forward guidance.

JEL Codes: E63, D84, E50, E52, E58, E60.

## 1. Introduction

In standard macroeconomic models with an interest rate peg, announcements about the path of future nominal interest rates can have unbounded effects on inflation today. Further, a promise to cut a future interest rate has *larger* effects on today’s inflation than the same cut in the current rate. This counterintuitive phenomenon, which Del Negro, Giannoni, and Patterson (2015) call the “forward-guidance puzzle,” implies that central bankers can strongly influence inflation simply by announcing future policy actions when interest rates are constrained by the zero lower bound (ZLB). Many papers

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offer solutions of the puzzle, i.e., models that only allow for bounded unexpected changes in inflation and therefore explain the limited effectiveness of real-life forward-guidance policies. Cochrane (2017) shows that the fiscal theory of the price level (FTPL) can rule out large unexpected jumps in inflation.<sup>1</sup> His logic suggests that debt-stabilizing or “Ricardian” fiscal policies provide the fiscal backing for forward guidance to have power, while non-Ricardian policies give inflation a debt-stabilizing role that may limit the magnitude of inflation responses to new information. However, Cochrane does not specify fiscal assumptions that lead to well-behaved forward-guidance announcements.

This paper shows when the FTPL does—and does not—resolve the forward-guidance puzzle in New Keynesian models. In the standard modeling environment we consider, fiscal policy is either Ricardian or non-Ricardian depending on how often and the extent to which real fiscal surpluses respond to debt in a “passive” (i.e., debt-stabilizing, à la Leeper 1991) manner versus an “active” manner. Sufficiently active policies, such as permanent active fiscal regimes, typically rule out unreasonable responses to forward-guidance announcements. Furthermore, *some* non-Ricardian fiscal policies involving recurring passive and active fiscal regimes are able to resolve forward-guidance puzzles (even when the forward-guidance announcement occurs during a passive fiscal regime). However, some non-Ricardian policies involving recurring fiscal regime changes do not resolve the puzzle. As examples, we show that some estimated non-Ricardian fiscal rules eliminate forward-guidance puzzles, while others do not.

We furthermore examine impulse responses to forward-guidance announcements in cases where the fiscal theory rules out forward-guidance puzzles. Forward-guidance announcements that lower expected nominal interest rates *eventually* lower inflation and

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<sup>1</sup>The FTPL literature highlights the role that *both* monetary and fiscal policy play in determining the price level (e.g., see Leeper and Leith 2016 or Cochrane 2020 for a thorough review). A major strand of the literature studies how the equilibrium properties of standard macro models depend on the conventional assumption that governments conduct a Ricardian fiscal policy, i.e., a policy that generates sufficient real fiscal revenues to ensure debt stability. Therefore, for our purposes, we use “FTPL” to refer to the analysis of the interaction of *non-Ricardian* fiscal policy and monetary policy.

output. This is true even when forward guidance is conducted during a transient passive fiscal regime. Whether inflation initially rises or falls in response to this kind of announcement depends on the maturity structure of debt, and also whether surpluses are procyclical or countercyclical. Countercyclical policy can mitigate the power of forward guidance, while forward guidance has ambiguous effects under a procyclical fiscal regime. For one special calibration of procyclical surpluses in a simple model, forward-guidance solutions diverge to negative or positive infinity, which highlights one caveat to our finding that permanent active regimes rule out unreasonable forward-guidance effects, and one potential peril of pairing monetary forward guidance with austerity policy.

This paper also presents a framework for studying the effects of anticipated structural changes. Specifically, we offer a solution algorithm for anticipated structural changes in a general class of Markov-switching dynamic stochastic general equilibrium (DSGE) models, and necessary and sufficient conditions for when a given model of this form exhibits a puzzle. This contribution directly extends that of Gibbs and McClung (2020), who provide the corresponding solution algorithm and set of conditions for ruling out forward-guidance puzzles in a general class of single-regime linearized DSGE models. The algorithm and conditions presented in this paper are applied to test the FTPL, but others could apply these techniques to study the effects of forward guidance, or other anticipated structural changes, in uncertain environments.

After a brief literature review, section 2 presents key analytical results in a simple model. Section 3 develops a general framework for studying forward guidance in regime-switching models. Section 4 studies the forward-guidance puzzle in New Keynesian models, including models with estimated fiscal rules, and models with rich maturity structure and cyclical policy. Section 5 concludes.

### *1.1 Literature Review*

We contribute to a literature examining monetary-fiscal interactions in zero lower bound environments. Cochrane (2017, 2018a, 2018b, 2020) extensively studies forward guidance and the fiscal theory, but his focus is not on the forward-guidance puzzle per se. Caramp and Silva (2018) argue that the direct response of fiscal transfers

to monetary policy generates wealth effects that explain the monetary forward-guidance puzzle, but their analysis abstracts from fiscal theory considerations, including issues of fiscal solvency and fiscal regimes.<sup>2</sup>

Chung, Davig, and Leeper (2007), Davig and Leeper (2011), Bianchi and Ilut (2017), Bianchi and Melosi (2017), Corhay, Kung, and Morales (2017), Cho and Moreno (2019), and Ascari, Florio, and Gobbi (2020) all examine the effects of recurring fiscal policy regimes on expectations. Additionally, Sims (2013), Leeper and Leith (2016), and Corhay, Kung, and Morales (2017) all examine the interaction of unconventional monetary policy, debt maturity structure, and non-Ricardian fiscal policy. Ascari, Florio, and Gobbi (2020) discuss forward guidance on future policy *regimes* and its implications for the efficacy of monetary and fiscal policy at the zero lower bound. Unlike the above papers, we identify the fiscal assumptions that preclude a forward-guidance puzzle.

This paper also adds to a large literature addressing the forward-guidance puzzle. Early motivation for the literature comes from papers such as Drumond, Martins, and Verona (2013), Carlstrom, Fuerst, and Paustian (2015), and Del Negro, Giannoni, and Patterson (2015), in which the authors observe large, even unbounded, responses of inflation and output to forward-guidance announcements in standard DSGE models. This theoretical prediction is not easily reconciled with the data:<sup>3</sup> the Federal Open Market Committee's forward-guidance announcements *modestly* affected long-term yields and interest rate expectations (Gürkaynak, Sack, and Swanson 2005, Moessner 2013, and Swanson and Williams 2014), inflation expectations (Del Negro, Giannoni, and Patterson 2015), or perhaps barely impacted yields and expectations at all (Kool and Thornton 2015). Announcements may also contain forecasts of many macrovariables, which lead to qualitatively different responses to forward guidance than Del Negro, Giannoni, and Patterson (2015)

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<sup>2</sup>We note that an exogenous fiscal transfers rule—a special case of a permanent active fiscal policy—cannot be endogenous to monetary policy. Consequently, a forward-guidance announcement in a model with exogenous surpluses does not trigger the movements in expected transfers that appear necessary to generate a forward-guidance puzzle in their framework.

<sup>3</sup>See Moessner, Jansen, and de Haan (2017) for a survey of the empirical literature on forward guidance.

predict (Campbell et al. 2012, Nakamura and Steinsson 2018). Some work even suggests that standard models make reasonable predictions: Bundick and Smith (2020) show that the predictions of Del Negro, Giannoni, and Patterson (2015) rely on unreasonably large forward-guidance shocks, and D'Amico and King (2015) show that empirical responses grow as the horizon of guidance is extended into the future.

Empirical findings notwithstanding, standard models predict counterintuitive effects of forward guidance. We seek a theoretical resolution to this phenomenon, and other resolutions of the puzzle include life-cycle considerations (Del Negro, Giannoni, and Patterson 2015), sticky information (Carlstrom, Fuerst, and Paustian 2015; Chung, Herbst, and Kiley 2015; Kiley 2016), borrowing constraints (McKay, Nakamura, and Steinsson 2016), bounded rationality (Cole 2020; Gabaix 2020), common knowledge considerations (Angeletos and Lian 2018), state of the economy (Keen, Richter, and Throckmorton 2017), credibility (Haberis, Harrison, and Waldron 2019), limited foresight (Garcia-Schmidt and Woodford 2019), level- $k$  thinking (Farhi and Werning 2018), and heterogeneous beliefs (Andrade et al. 2019).

## 2. Basic Results in a Simple Model

We only need an endowment economy with flexible prices to illustrate the basic results of the paper:

$$i_t = E_t \pi_{t+1} \tag{1}$$

$$b_t = \beta^{-1} (b_{t-1} - \pi_t) + i_t - \tau_t \tag{2}$$

$$i_t = \phi^{st} \pi_t + \epsilon_t^m \tag{3}$$

$$\tau_t = \gamma^{st} b_{t-1} + \epsilon_t^f, \tag{4}$$

where  $0 < \beta < 1$ ,  $i$  is the nominal interest rate,  $\pi$  is inflation,  $b$  is real government debt,  $\tau$  is real fiscal surpluses, and  $\epsilon^f$  and  $\epsilon^m$  are iid shocks. All variables are expressed in terms of percentage deviations from steady state.<sup>4</sup> (1) and (2) are the Fisher relation and

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<sup>4</sup>This model is a special case of the model presented in section 3 of Leeper and Leith (2016). Alternatively, we could linearize the fiscal variables,  $b$  and  $\tau$ , and

intertemporal government budget constraint, respectively, while (3) and (4) are the monetary and fiscal policy rules.

The variable  $s_t$  is an exogenous two-state Markov process that changes the monetary and fiscal “regime” over time, e.g., because the behavior of governments may vary over time. The current government’s response of surpluses to outstanding debt (i.e., the current fiscal “regime”) can be succinctly summarized in terms of  $\gamma$ : a one percentage deviation of debt from steady state triggers a  $\gamma$  percentage deviation of surpluses from steady state. This paper, and the FTPL at large, considers two types of fiscal regimes: (i) when  $s_t = M$ , fiscal policy is “passive” and  $|\beta^{-1} - \gamma^M| < 1$ ; and (ii) when  $s_t = F$ , fiscal policy is “active” and  $|\beta^{-1} - \gamma^F| > 1$ . For reasonable calibrations, this condition boils down to  $\gamma^F < \beta^{-1} - 1 < \gamma^M$ . The transition probabilities are  $Pr(s_t = M | s_{t-1} = M) = p_M$  and  $Pr(s_t = F | s_{t-1} = F) = p_F$ . Throughout this paper, agents have full-information rational expectations and observe all contemporaneous endogenous and exogenous variables, including  $s_t$ , such that  $E_t z_{t+1} = Pr(s_{t+1} = M | s_t) E(z_{t+1} | s_{t+1} = M, \mathcal{I}_t) + (1 - Pr(s_{t+1} = M | s_t)) E(z_{t+1} | s_{t+1} = F, \mathcal{I}_t)$ , where  $z$  is any model variable and  $\mathcal{I}_t$  is agents’ time- $t$  information set. Substituting (4) into (2) reveals the crucial difference between the two regimes:

$$b_t = (\beta^{-1} - \gamma^{s_t}) b_{t-1} + i_t - \beta^{-1} \pi_t - \epsilon_t^f. \quad (5)$$

If we set  $s_t = M$  and hold it fixed over time, then the autoregressive coefficient in (5),  $\beta^{-1} - \gamma^M$ , is less than one.<sup>5</sup> Thus, a permanent passive fiscal regime guarantees the dynamic stability of debt (around its steady state) over time given *any* sequence of  $\pi_t$ ,  $i_t$ , etc. In other words, a fixed (i.e., permanent) passive fiscal regime is a debt-stabilizing or “Ricardian” policy. Because inflation does not need to depend on debt or taxes when the government stabilizes debt using passive fiscal policy, a “Ricardian equilibrium” of the model exists in which  $\{\pi_t\}$  is entirely determined by (1) and (3). When monetary policy is active ( $\phi^M > 1$ ), the Ricardian equilibrium is the unique equilibrium. We can obtain this equilibrium solution

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log-linearize remaining variables in keeping with the approach of other papers in the literature, but this would not affect the qualitative results in this paper. Also, we assume that steady-state real debt is strictly positive throughout this paper.

<sup>5</sup>We tacitly assume that  $\gamma^M$  is small enough to satisfy  $|\beta^{-1} - \gamma^M| < 1$ .

by substituting (3) into (1) and solving the Fisher relation forward for  $\pi_t$ . Then  $b_t$  passively adjusts to satisfy the government budget constraint (2), taking the solution for  $\pi$  as given. The Ricardian equilibrium for  $\pi_t$  and  $b_t$  is then given by

$$\begin{aligned}\pi_t &= -(\phi^M)^{-1}\epsilon_t^m \\ b_t &= (\beta^{-1} - \gamma^M)b_{t-1} + (\beta^{-1}/\phi^M)\epsilon_t^m - \epsilon_t^f.\end{aligned}$$

The standard assumptions in macroeconomics, namely active monetary policy and passive fiscal policy, select the Ricardian equilibrium. The Ricardian equilibrium is therefore the “standard” equilibrium of this model. Its existence depends on the fiscal stance; if we instead choose to impose a fixed active fiscal regime, then the last equation becomes a dynamically *unstable* process for debt—active fiscal policy is non-Ricardian policy. Under non-Ricardian policy,  $\pi_t$  must bear the brunt of stabilizing debt around its steady state: high (low) debt calls for high (low) inflation. There is a good economic motivation for why debt-stabilizing inflation occurs under non-Ricardian policy: households view their bond holdings (government debt) as net wealth when Ricardian equivalence fails, and therefore anything that raises debt and the rate of return on debt can increase aggregate demand and inflation. This failure of Ricardian equivalence has implications for monetary policy as well: a policy that lowers interest rates for an extended period of time also lowers debt service cost (and hence debt) and interest receipts to bondholders, which reduces households’ net wealth and therefore reduces demand and inflation.

Since debt-stabilizing inflation is necessary in any stationary equilibrium with non-Ricardian fiscal policy, inflation, debt, interest rates and surpluses are jointly determined by the full system of equations (1)–(4). In other words, non-Ricardian fiscal policies *only* permit non-Ricardian (stationary) equilibria in which variation in the debt affects inflation, etc. When monetary policy is “passive” (i.e.,  $\phi^F < 1$ ),<sup>6</sup> a unique equilibrium exists under a fixed active fiscal regime. We can obtain this solution by substituting (4) into (2), solving (2) forward, taking expectations, and imposing the Fisher

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<sup>6</sup>We assume  $\phi^{st}$  is non-negative for all  $s_t$ .

**Table 1. Determinacy Conditions with Permanent Regimes**

	<b>AM: <math>\phi &gt; 1</math></b>	<b>PM: <math>\phi &lt; 1</math></b>
PF: $\gamma \in (\beta^{-1} - 1, \beta^{-1} + 1)$	Determinate	Indeterminate
AF: $\gamma \notin (\beta^{-1} - 1, \beta^{-1} + 1)$	No Stable Solution	Determinate

relation, the transversality condition, and interest rate rule which gives a solution for  $b_t$ .  $\pi_t$  passively adjusts to satisfy (2):<sup>7</sup>

$$\begin{aligned}
 b_t &= \phi^F b_{t-1} + \frac{\beta^{-1}}{\beta^{-1} - \gamma^F} \epsilon_t^m - \frac{\phi^F}{\beta^{-1} - \gamma^F} \epsilon_t^f \\
 \pi_t &= \frac{(\beta^{-1} - \gamma^F - \phi^F)}{(\beta^{-1} - \phi^F)} b_{t-1} - \frac{\gamma^F}{(\beta^{-1} - \gamma^F)(\beta^{-1} - \phi^F)} \epsilon_t^m \\
 &\quad - \frac{\beta^{-1} - \gamma^F - \phi^F}{(\beta^{-1} - \gamma^F)(\beta^{-1} - \phi^F)} \epsilon_t^f.
 \end{aligned}$$

Table 1 summarizes the determinacy properties of the simple model with fixed policy regimes. Consistent with the above discussion of Ricardian and non-Ricardian policy, we need to pair an active policymaker with a passive policymaker to ensure the existence of a unique equilibrium. Two passive regimes lead to indeterminacy—a familiar consequence of passive monetary policy, since standard macro analysis assumes passive fiscal policy.

Most papers assume a fixed fiscal regime, but this assumption ignores evidence of recurring fiscal regime change in the United States.<sup>8</sup> Recurring regime changes capture variation in the government's fiscal priorities over time, and they could reflect changes in the demand for spending (e.g., in war versus peacetime). Moreover, these recurring changes could affect households' expectations, and therefore the transmission of forward guidance.

Active and passive fiscal regimes recur provided that  $p_F < 1$ ,  $p_M < 1$ . As before, the question of identifying Ricardian policies

<sup>7</sup>Alternatively, we could solve (2) forward, take expectations, impose the transversality condition, etc., and obtain the same solution for  $\pi_t$ .

<sup>8</sup>See Davig and Leeper (2007a, 2011), Bianchi and Ilut (2017), Bianchi and Melosi (2017), and Chen, Leeper, and Leith (2018), among others.

boils down to identifying fiscal policies, indexed by  $(p_M, p_F, \gamma^M, \gamma^F)$ , that ensure the asymptotic stability of the debt process (5). We choose the mean square stability concept of Costa, Fragoso, and Marques (2005) to determine when debt is stable.<sup>9</sup> An application of techniques from Costa, Fragoso, and Marques (2005) gives the condition under which fiscal policy ensures mean square stability of the government debt.

**THEOREM 1.** *Let  $h_F = \beta^{-1} - \gamma^F$  and  $h_M = \beta^{-1} - \gamma^M$  and assume  $p_M + p_F \geq 1$ . A fiscal policy,  $(p_M, p_F, \gamma^M, \gamma^F)$ , is Ricardian if and only if*

$$(p_M + p_F - 1)h_F^2 h_M^2 < 1$$

$$p_M h_M^2 (1 - h_F^2) + p_F h_F^2 (1 - h_M^2) + h_M^2 h_F^2 < 1.$$

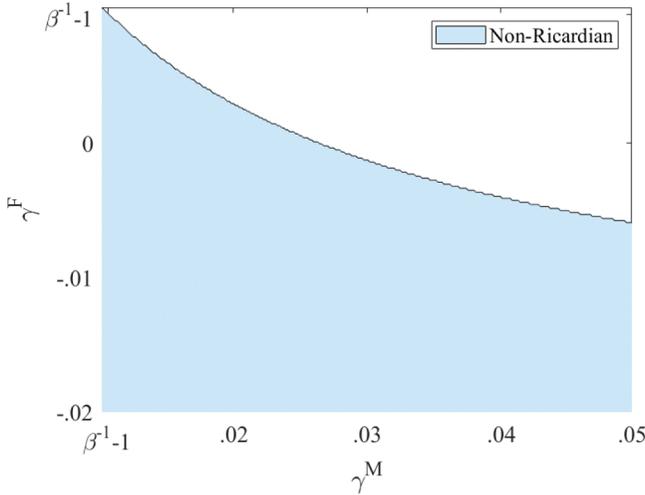
*Proof.* See appendix A.1. ■

Figure 1 displays combinations,  $(\gamma^M, \gamma^F)$ , that satisfy theorem 1 conditions for given  $p_M, p_F$ . Ascari, Florio, and Gobbi (2020) first used these conditions to select mean square stable solutions of a similar model of the FTPL. For the conditions in theorem 1 to be satisfied, active regimes must be sufficiently short-lived and/or timid (i.e.,  $p_F$  ( $\gamma^F$ ) must be relatively low (high)), and passive regimes must be sufficiently persistent and strong (i.e.,  $p_M$  and  $\gamma^M$  must be relatively high). If the conditions in theorem 1 are satisfied by fiscal policy, then fiscal policy is Ricardian and a mean square stable Ricardian equilibrium exists. If these conditions are violated, perhaps because passive fiscal regimes have short expected durations or because active regimes are highly persistent, then fiscal policy is non-Ricardian and *any* mean square stable equilibrium of the model must be a non-Ricardian equilibrium.

Theorem 1 implies that any fiscal policy is either Ricardian or non-Ricardian, even if said policy involves both passive and active regimes. The FTPL primarily focuses on the impact of non-Ricardian fiscal policy on equilibrium dynamics and, accordingly,

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<sup>9</sup>Farmer, Waggoner, and Zha (2009, 2011), Cho (2016), and Foerster et al. (2016), among others, have popularized the use of the mean square stability concept in Markov-switching DSGE environments. Interested readers are referred to those papers for discussions on the mean square stability concept.

**Figure 1. Ricardian vs. Non-Ricardian Policy**

**Note:** The white region is the set of Ricardian fiscal policies.

our focus is on the interaction of non-Ricardian policy and monetary forward guidance.

### 2.1 Forward Guidance

Forward guidance can come in the form of an announcement about future policy actions, including future interest rate policy. To see how policy *regimes* alter the effects of announcements in our model, suppose the public becomes aware of  $\epsilon_{t+j}^m \neq 0$  at time  $t$  (such that  $E_t \epsilon_{t+j}^m = \epsilon_{t+j}^m$ ). In the Ricardian solution the magnitude of the response of inflation,  $\partial \pi_t / \partial \epsilon_{t+j}^m = -(\phi^M)^{-j-1}$ , is strictly increasing in  $j$  if monetary policy is passive ( $\phi^M < 1$ ); passive monetary policy can induce responses to anticipated  $\epsilon_{t+j}^m$  that resemble qualities of the forward-guidance puzzle described in the introduction. As  $\phi^M \rightarrow 0$ , consistent with an interest rate peg ( $\phi^M = 0$ ), the response of inflation to any anticipated policy shock becomes unbounded. On the other hand,  $\partial \pi_t / \partial \epsilon_{t+j}^m = -\beta \gamma^F (\beta - \gamma^F)^{-j-1}$  in the non-Ricardian equilibrium (with  $\phi^F = 0$ ), which is bounded for all  $j$  and strictly decreasing (in magnitude) in  $j$  if fiscal policy

is permanently active.<sup>10</sup> This exercise suggests that (i) models with interest rate pegs, or passive monetary policy generally, are susceptible to a forward-guidance puzzle; (ii) non-Ricardian fiscal policy gives inflation a debt-stabilizing role that may preclude the puzzle.

Here we show that non-Ricardian fiscal policy does not always eliminate the forward-guidance puzzle. We follow Del Negro, Giannoni, and Patterson (2015) and much of the literature and model monetary forward guidance as credible news about the future path of nominal interest rates,  $\{i_t\}$ . For example, a forward-guidance announcement at  $t = 0$  could fix agents' expectations of the path of interest rates from  $t = T - j$  where  $T > t$  and  $1 \leq j \leq T$ . In this context, a forward-guidance puzzle emerges if the response of inflation explodes or otherwise fails to converge as  $T \rightarrow \infty$ . Given this definition, one can check if (1)–(4) is susceptible to a puzzle simply by examining what happens when the central bank announces a permanent peg:  $i_t = \bar{i}$  where, importantly,  $\bar{i}$  does not equal the steady-state interest rate ( $\bar{i} \neq 0$ , since  $i$  is log-linearized around steady state with zero inflation).<sup>11</sup> Some manipulations of (1)–(4) give us the system of equations for this exercise:

$$\bar{i} = E_t \pi_{t+1} \quad (6)$$

$$b_t = (\beta^{-1} - \gamma^{st}) b_{t-1} + \bar{i} - \beta^{-1} \pi_t. \quad (7)$$

The fiscal and monetary policy shocks are shut down in this exercise, because they do not matter for our assessment of the forward-guidance puzzle. Fiscal policy,  $(p_M, p_F, \gamma^M, \gamma^F)$ , is non-Ricardian, which means that any equilibrium of the model is non-Ricardian. As before, we attempt to find a non-Ricardian forward-guidance solution by solving (7) forward to obtain  $b_t$  (after taking expectations, imposing the Fisher relation, transversality condition) and

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<sup>10</sup>Derivatives in this paragraph come from the Ricardian solution,  $\pi_t = -\sum_{j \geq 0} (\phi^M)^{-j-1} E_t \epsilon_{t+j}^m$ , and the non-Ricardian solution,  $\pi_t = (1 - \beta \gamma^F) b_{t-1} - \sum_{j \geq 0} (\beta^{-1} - \gamma^F)^{-j-1} (\beta \gamma^F E_t \epsilon_{t+j}^m + (1 - \beta \gamma^F) E_t \epsilon_{t+j}^f)$ .

<sup>11</sup>More generally, we could assume that the central bank makes an announcement at  $t = 0$  about the future path of interest rates from  $t = 0$  to  $t = T \geq 0$ :  $\{i_t\}_{t \geq 0}^T$ . We assume  $i_t = \bar{i}$  and  $T \rightarrow \infty$  to simplify the exposition, but this simplifying assumption does not matter for the conditions governing the emergence of a forward-guidance puzzle.

forcing  $\pi_t$  to passively satisfy the government budget constraint at the end of each period. Begin by forwarding (7) one period and taking expectations

$$\begin{aligned} E_t b_{t+1} &= E_t \left( (\beta^{-1} - \gamma^{s_{t+1}}) b_t + \bar{i} - \beta^{-1} \pi_{t+1} \right) \\ &= \delta_t b_t + \bar{i} (1 - \beta^{-1}), \end{aligned}$$

where  $\delta_t = E_t (\beta^{-1} - \gamma^{s_{t+1}})$ . Now, expectations depend on  $s_t$ , and this fact allows us to use a regime-dependent representation of the above equation:<sup>12</sup>

$$\begin{aligned} b_t^M &= \delta_M^{-1} (p_M E_t b_{t+1}^M + (1 - p_M) E_t b_{t+1}^F + (\beta^{-1} - 1) \bar{i}) \\ b_t^F &= \delta_F^{-1} (p_F E_t b_{t+1}^F + (1 - p_F) E_t b_{t+1}^M + (\beta^{-1} - 1) \bar{i}), \end{aligned}$$

where  $b_t^M$  ( $b_t^F$ ) is  $b_t$  when  $s_t = M$  ( $s_t = F$ ) and  $\delta_M = p_M (\beta^{-1} - \gamma^M) + (1 - p_M) (\beta^{-1} - \gamma^F)$ ,  $\delta_F = p_F (\beta^{-1} - \gamma^F) + (1 - p_F) (\beta^{-1} - \gamma^M)$ . In matrix form,

$$\begin{aligned} \begin{pmatrix} b_t^M \\ b_t^F \end{pmatrix} &= \underbrace{\begin{pmatrix} p_M \delta_M^{-1} & (1 - p_M) \delta_M^{-1} \\ (1 - p_F) \delta_F^{-1} & p_F \delta_F^{-1} \end{pmatrix}}_A \begin{pmatrix} E_t b_{t+1}^M \\ E_t b_{t+1}^F \end{pmatrix} \\ &+ \begin{pmatrix} \delta_M^{-1} (\beta^{-1} - 1) \bar{i} \\ \delta_F^{-1} (\beta^{-1} - 1) \bar{i} \end{pmatrix}. \end{aligned} \quad (8)$$

We solve (8) forward to obtain the forward-guidance solution, which is only possible if the eigenvalues of  $A$  are inside the unit circle.<sup>13</sup> If this eigenvalue condition is satisfied, then the forward-guidance responses of  $b_t$  and therefore  $\pi_t$ , which passively adjusts to satisfy (7) given  $b_t$ , are bounded and convergent. Otherwise, a forward-guidance puzzle emerges: news about the distant future generates unbounded, or nonconvergent, responses in  $b_t$  and therefore  $\pi_t$ .

<sup>12</sup>Davig and Leeper (2007b) cast their model in a regime-dependent or “quasi-linear” form as well. Unlike Davig and Leeper (2007b), we do not cast the model in a quasi-linear form in order to study the uniqueness of equilibrium.

<sup>13</sup>Alternatively, we could obtain the forward-guidance solution by first getting an expression for  $b_t$  from (7), taking expectations, and solving  $b_t = E_t \{ (\beta - \gamma^{t+1})^{-1} (b_{t+1} - \bar{i} + \beta^{-1} \pi_{t+1}) \}$  forward. Appendix A.8 shows that this approach yields the same solution for  $b_t$  as solving (8) forward.

PROPOSITION 1. Consider (1)–(4) and suppose the fiscal policy is non-Ricardian. The model does not exhibit a forward-guidance puzzle if and only if

$$\begin{aligned} |(p_M + p_F - 1)/(\delta_F \delta_M)| &< 1 \\ |p_F/\delta_F + p_M/\delta_M| - (p_M + p_F - 1)/(\delta_F \delta_M) &< 1, \end{aligned}$$

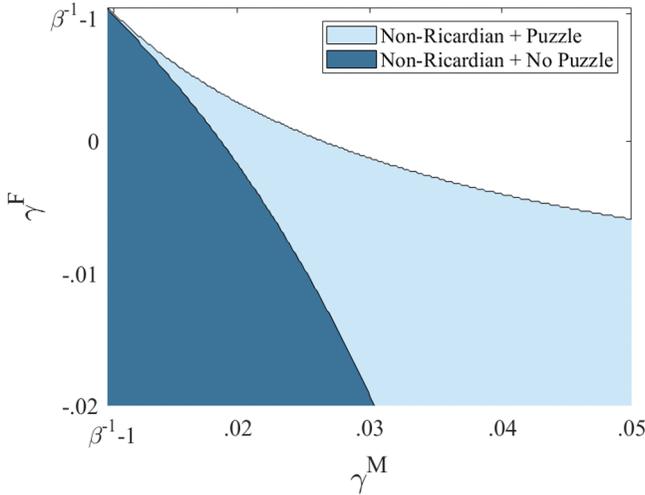
where  $\delta_M = p_M(\beta^{-1} - \gamma^M) + (1 - p_M)(\beta^{-1} - \gamma^F)$ ,  $\delta_F = p_F(\beta^{-1} - \gamma^F) + (1 - p_F)(\beta^{-1} - \gamma^M)$ .

*Proof.* See appendix A.2. ■

Proposition 1 gives us the condition for ruling out a forward-guidance puzzle in the simple model (1)–(4). The condition leads us to several important conclusions:

- Permanent active fiscal policy precludes forward-guidance puzzles. To see this, set  $p_F = 1$  and  $p_M = 0$ , so that the conditions in proposition 1 collapse down to  $|1/\delta_F| = |1/(\beta^{-1} - \gamma^F)| < 1$ .
- A permanent *absorbing* active fiscal regime is not sufficient for ruling out a forward-guidance puzzle in economies that allow for a transient passive fiscal spell. Again, set  $p_F = 1$  such that the first condition becomes  $|p_M/(\delta_M \delta_F)| < 1$ . Sufficiently high values of  $p_M$  and  $\gamma^M$  will violate this condition; forward-guidance puzzles can emerge in economies with strong and/or persistent passive regimes, even if agents anticipate an absorbing active fiscal regime.
- With recurring regimes ( $p_M < 1$  and  $p_F < 1$ ) the solution typically exists in both states, if it exists at all. This is a consequence of “expectations effects” of regime switching; even if a forward-guidance announcement occurs during an active fiscal regime, agents place probability on experiencing a passive government during the forward-guidance horizon. Thus forward guidance only has reasonable, bounded effects during an active fiscal regime if it has reasonable effects during a passive regime.

Figure 2 plots the eigenvalue condition in proposition 1 in  $(\gamma^M, \gamma^F)$ -space for  $p_F = p_M = .95$ . Values of  $(\gamma^M, \gamma^F)$  in the

**Figure 2. Forward-Guidance Puzzle in Benchmark Model**

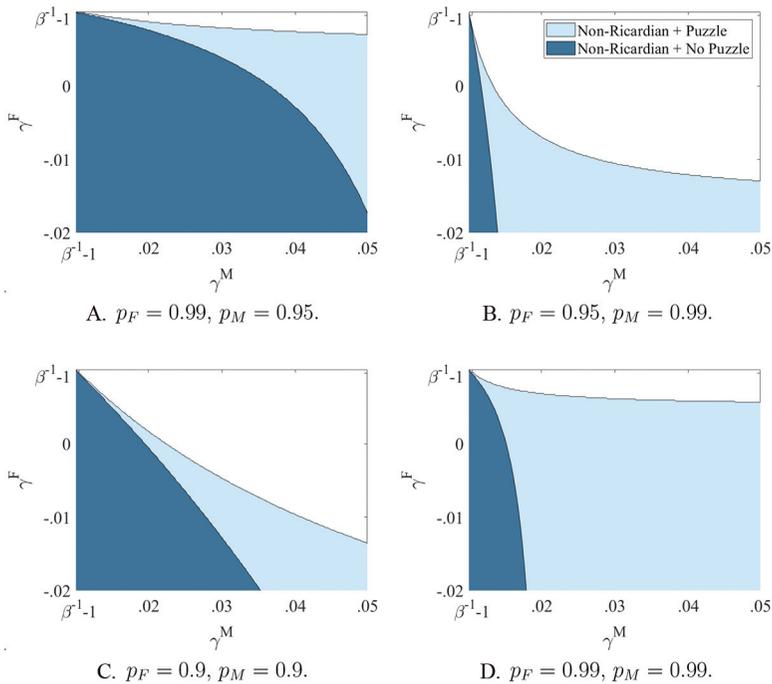
**Note:** The white region is the set of Ricardian fiscal policies.

northeast corner of figure 2 are the least active, most passive fiscal policies. Figure 2 shows that surpluses need to be sufficiently unresponsive to debt (i.e.,  $\gamma^F$  or  $\gamma^M$  must be sufficiently small or negative) to deliver well-behaved responses to a forward-guidance announcement. Importantly, figure 2 shows that some non-Ricardian policies are susceptible to forward-guidance puzzles.

Figure 3 plots the eigenvalue condition in proposition 1 for different values of  $p_M$ ,  $p_F$ . It is clear that the effects of forward guidance are sensitive to transition probabilities:

- More persistent active fiscal regimes (higher values of  $p_F$ ) shrink the region of  $(\gamma^M, \gamma^F)$ -space that leads to a forward-guidance puzzle (see figure 3, panel A). The opposite is true with more persistent passive regimes (see panel B).
- Increasing *both*  $p_F$  and  $p_M$  shrinks the Ricardian region of  $(\gamma^M, \gamma^F)$ -space, since persistent active fiscal regimes strain debt stability. At the same time, persistent passive fiscal regimes enlarge the non-Ricardian region of  $(\gamma^M, \gamma^F)$ -space that leads to a forward-guidance puzzle (see figure 3 panel D).

**Figure 3. Forward-Guidance Puzzle and Transition Probabilities**



**Note:** The white region is the set of Ricardian fiscal policies.

Decreasing the persistence of both regimes has the opposite effects (panel C).

We emphasize that it is generally possible to obtain a stable minimal state variable (MSV) equilibrium of (1)–(4) that assumes the form

$$\pi_t = \Omega_\pi(s_t)b_{t-1} + \Gamma_\pi(s_t)u_t \tag{9}$$

$$b_t = \Omega_b(s_t)b_{t-1} + \Gamma_b(s_t)u_t, \tag{10}$$

where  $u = (\epsilon^m, \epsilon^f)'$ , even in cases where (i) fiscal policy is non-Ricardian (i.e., theorem 1 conditions are not satisfied) and (ii) a forward-guidance puzzle emerges (i.e., proposition 1 conditions are not satisfied). As such, the forward-guidance puzzle can emerge even

when other stable equilibriums exist. Importantly, however, these MSV solutions are not forward-guidance solutions; (9)–(10) only relate equilibrium  $b$  and  $\pi$  to contemporaneous fundamental variables, and this may prevent agents who inhabit the MSV equilibrium from fully incorporating forward-guidance information in their decisionmaking. Because we seek to determine when forward-guidance puzzles emerge, section 2.1 and proposition 1 focus on rational expectations solutions in which agents fully utilize information about anticipated structural changes. Section 3 offers a general approach for obtaining these solutions in more sophisticated models.

### 3. A General Approach to Forward Guidance

Unlike the simple model of section 2, more advanced regime-switching models cannot be solved analytically. This section presents tractable numerical techniques for approaching forward guidance in a general class of regime-switching DSGE models.

Specifically, this section presents a solution technique for the responses of endogenous variables to a wide variety of policy announcements and anticipated structural changes, including monetary forward-guidance announcements. Our innovation generalizes techniques from Kulish and Pagan (2017) and Gibbs and McClung (2020) to Markov-switching DSGE models of the form

$$x_t = A^{(s_t)}E_t x_{t+1} + B^{(s_t)}x_{t-1} + C^{(s_t)}u_t + D^{(s_t)}, \quad (11)$$

where  $x_t$  is an  $n \times 1$  vector of endogenous variables,  $u_t$  is an  $m \times 1$  vector of iid mean-zero exogenous shocks, and  $s_t$  is an exogenous  $S$ -state Markov process with transition matrix,  $P$ , where  $p_{ij} = Pr(s_{t+1} = j | s_t = i)$  is the  $(i, j)$ -th element of  $P$ .<sup>14</sup> Notice that equations (1)–(4) jointly share the functional form (11). Moreover, (11) nests the class of linear DSGE models studied by Kulish and Pagan (2017) (i.e., when  $S = 1$ ).

We model any policy announcement or anticipated structural change as follows. Suppose at time 0 agents become aware of  $N$

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<sup>14</sup>We assume that shocks are iid for exposition's sake. We could straightforwardly generalize our results in this section to a model class with serially correlated shocks.

structural changes that occur at horizons  $0 \leq T_1 < T_2 < \dots < T_N$ . These anticipated structural changes could, for example, arise from monetary forward guidance as in Del Negro, Giannoni, and Patterson (2015), or from forward fiscal guidance as in Canzoneri et al. (2018).<sup>15</sup> The anticipated changes imply the following time-varying structural model:

$$x_t = A_i^{(s_t)} E_t x_{t+1} + B_i^{(s_t)} x_{t-1} + C_i^{(s_t)} u_t + D_i^{(s_t)} \quad \text{for } T_i \leq t < T_{i+1} \quad (12)$$

for  $i = 1, \dots, N$ , where  $T_0 = 0$  and  $T_{N+1} \rightarrow \infty$ . We assume that agents do not know the future path of  $s_t$  (i.e.,  $E_t x_{t+1} = E(x_{t+1} | x_t, u_t, s_t) = \sum_{j=1}^S p_{s_t, j} E(x_{t+1} | x_t, u_t, s_{t+1} = j)$ ). This assumption is not restrictive: if policymakers control  $s_t$ , they can simply announce a path for  $s_t$  in the form of  $N$  structural changes at horizons  $0 \leq T_1 < T_2 < \dots < T_N$ . Our use of the exogenous Markov process  $s_t$  therefore allows us to model any uncertainty about the economy's structure that remains after a sequence of future changes becomes anticipated. For example, section 2 studies the effects of an announcement about the path for future interest rates when agents are uncertain about the future fiscal regime.

Appendix A.3 presents a recursive solution technique that returns a solution of the system (12) (i.e., a forward-guidance solution). To use the solution method, follow these steps:

- (i) Select an MSV solution for  $t \geq T_N$ :<sup>16</sup>

$$x_t = \Omega^{(s_t)} x_{t-1} + \Gamma^{(s_t)} u_t + \xi^{(s_t)}. \quad (13)$$

The MSV solution (13) provides an “asymptotic” model of the economy after the anticipated changes end. Determinacy

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<sup>15</sup>Canzoneri et al. (2018) study announcements about changes in future government spending in a prototypical New Keynesian model with an interest rate peg. The anticipated structural changes they study assume the form of the anticipated changes we study here.

<sup>16</sup>See Farmer, Waggoner, and Zha (2009, 2011), Maih (2015), Cho (2016), Foerster et al. (2016), and Barthelemy and Marx (2017) for alternative solution techniques. Notice also that (12) assumes the form (11) for  $t \geq T_N$ . Finally, the models we study in section 4 satisfy  $D_N^{s_t} = \xi^{(s_t)} = 0^n$  for all  $s_t$ .

is *not* a requirement for our method to work (if multiple solutions (13) exist, choose one).

- (ii) Compute the matrix sequence,  $\{\Omega_t^{(s_t)}, \Gamma_t^{(s_t)}, \xi_t^{(s_t)}\}_{t=0}^{T_N-1}$ :

$$\Omega_t^{(s_t)} = \left( I - A_i^{(s_t)} E_t(\Omega_{t+1}^{(s_{t+1})}) \right)^{-1} B_i^{(s_t)} \tag{14}$$

$$\Gamma_t^{(s_t)} = \left( I - A_i^{(s_t)} E_t(\Omega_{t+1}^{(s_{t+1})}) \right)^{-1} C_i^{(s_t)} \tag{15}$$

$$\xi_t^{(s_t)} = \left( I - A_i^{(s_t)} E_t(\Omega_{t+1}^{(s_{t+1})}) \right)^{-1} \left( D_i^{(s_t)} + A_i^{(s_t)} E_t \left( \xi_{t+1}^{(s_{t+1})} \right) \right), \tag{16}$$

where  $(\Omega_{T_N}^{(s_t)}, \Gamma_{T_N}^{(s_t)}, \xi_{T_N}^{(s_t)}) = (\Omega^{(s_t)}, \Gamma^{(s_t)}, \xi^{(s_t)})$  is given by (13) in the first step and  $i \in \{0, 1, 2, \dots, N - 1\}$  depending on  $t$ . Note that the sequence is obtained by iterating on (14)–(16) backward in time from  $t = T_N - 1$  to  $t = 0$ . The derivation of this sequence (appendix A.3) is related to the method of undetermined coefficients.<sup>17</sup>

- (iii) Using  $\{\Omega_t^{(s_t)}, \Gamma_t^{(s_t)}, \xi_t^{(s_t)}\}_{t=0}^{T_N}$ , form the forward-guidance solution:

$$x_t = \Omega_t^{(s_t)} x_{t-1} + \Gamma_t^{(s_t)} u_t + \xi_t^{(s_t)}. \tag{17}$$

Our method recovers the solution for all conceivable realizations of the Markov chain,  $s_t$ , from time 0 to time  $T_N$ . Moreover, (17) is uniquely determined by (13).

**DEFINITION 1** (forward-guidance puzzle). *Suppose at time 0 agents become aware of structural changes that will occur at  $T_1, T_2, \dots, T_N$ . Then the model does not exhibit a forward-guidance puzzle if and only if  $\lim_{T_1 \rightarrow \infty} x_0$  exists for all  $s_0 \in \{1, \dots, S\}$ , for any given  $x_{-1} \in R^n$  and for any given  $u_0 \in R^m$ .*

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<sup>17</sup>Cho (2016) employs the same recursion to solve the model (11) forward to obtain solutions of the form (13). This paper contributes a backward application of that recursion to solve for rational expectations responses of  $x$  to forward-guidance announcements.

Intuitively,  $\lim_{T_1 \rightarrow \infty} x_0$  does not exist in two economically unreasonable cases. In the first case,  $\|\lim_{T_1 \rightarrow \infty} x_0\| \rightarrow \infty$ , which means that initial equilibrium responses are unboundedly responsive to policy changes that are scheduled to occur infinitely far in the future. The forward-guidance puzzle explored by Del Negro, Giannoni, and Patterson (2015) is a classic example of this case. In the second case,  $x_0$  oscillates as  $T_1$  increase, as in the models of Carlstrom, Fuerst, and Paustian (2015) that generate “reversals” in the sign of  $\pi_0$  as  $T_1$  increases. The two cases are not mutually exclusive, and we require the limit of  $x_0$  to exist for all  $x_{-1}$ ,  $u_0$ , and  $s_0$  to help us rule out solutions to the forward-guidance puzzle that could rely on restrictive assumptions such as  $x_{-1} = 0^n$  or  $u_0 = 0^m$ . We also note that if the  $\lim_{T_1 \rightarrow \infty} x_0$  does not exist for some  $s_0$ , then it will generally not exist for all  $s_0$ . Some intuition for this last claim follows from (14), which reveals that the coefficients in one regime depend on the equilibrium coefficients in all other regimes. See also section 2.1, where the solution in one policy regime depends on the other regime and vice versa.

PROPOSITION 2. *A model of the form (11) does not exhibit a forward-guidance puzzle if and only if*

- $\bar{\Omega}(s_0) = \lim_{T_1 \rightarrow \infty} \Omega_0(s_0)$  exists for all  $s_0$ .
- $r(\Psi_{\bar{F}}) < 1$ ,

where  $r(A)$  denotes the spectral radius of matrix  $A$  and

$$\Psi_{\bar{F}} = \left( \bigoplus_{s_0=1}^S \left( I_n - A_0^{(s_0)} E_0(\bar{\Omega}(s_1)) \right)^{-1} A_0^{(s_0)} \right) (P \otimes I_n).$$

*Proof.* See appendix A.4. ■

Intuitively, the matrix conditions introduced in proposition 2 tell us when the limit of (14)–(16) exists. If this limit exists, then  $x_0$  converges for any  $x_{-1}$ ,  $u_0$  as  $T_1 \rightarrow \infty$ . To check these conditions, we only need to iterate on (14) and compute  $r(\Psi_{\bar{F}})$  when (14) converges.<sup>18</sup> These matrix operations are easier than computing the full

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<sup>18</sup>If the model under study is a purely forward-looking model, then  $\bar{\Omega}(s_0) = 0_n$  for all  $s_0$ , and one only needs to compute  $r(\Psi_{\bar{F}})$  to determine when the model exhibits the forward-guidance puzzle.

forward-guidance solution and they obviate the need for extensive robustness testing involving many different forward-guidance experiments before one can claim that a model is not susceptible to a forward-guidance puzzle.<sup>19</sup>

### 3.1 Monetary Forward Guidance

Section 2.1 studies a special case of the section 3 method in which the central bank announces an interest rate peg at  $t = 0$  that is in effect from  $t = 0$  to  $T_1 \rightarrow \infty$ .<sup>20</sup> Many papers study the effects of a time- $t = 0$  forward-guidance announcement that pegs interest rates at the ZLB from  $t = 0$  to  $T_1 = T$ , after which a new monetary policy regime begins for  $t > T$ .<sup>21</sup> Other studies consider the effects of a time  $t = 0$  announcement that  $i$  will be pegged at steady state until  $T_1 = T$ , when interest rates are dropped to the ZLB for one period, after which a new monetary policy regime begins for  $t > T$ . Section 4 considers both experiments and applies proposition 2 to determine when a model subject to those experiments has a forward-guidance puzzle.

## 4. New Keynesian Model

Section 2 presents the paper's basic findings using a flexible-price model. While this model can be analytically solved, we turn to models with sticky prices to generate more interesting impulse responses to forward-guidance announcements, and to compare the effects of forward-guidance announcements in models with estimated fiscal rules. Moreover, we can compare the impact of other fiscal theory

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<sup>19</sup>Proposition 2 generalizes the main result of Gibbs and McClung (2020) to the class of regime-switching models (11). Gibbs and McClung (2020) show that the conditions stated in proposition 2 are a special case of the E-stability conditions that govern when adaptive learning agents can learn a solution of the form (13). Further, Gibbs and McClung (2020) apply this paper's section 3 methodology to a New Keynesian model with recurring active and passive fiscal regimes in order to better illustrate the idea that E-instability, and not indeterminacy, predicts whether a model and model solution is susceptible to a forward-guidance puzzle.

<sup>20</sup>Appendix A.8 shows how to recover proposition 1 using the general methodology of section 3 and proposition 2. For larger models that do not admit analytical solutions, we rely on the section 3 method for numerical solutions.

<sup>21</sup>In the models we study,  $i$  is defined in percent deviations from steady state. Therefore, throughout this paper, we peg  $i$  at the ZLB by setting  $i = -\bar{i}$  where  $\bar{i}$  is the steady-state nominal interest rate.

features, such as debt maturity structure, and procyclical and countercyclical fiscal policy, on the transmission of forward guidance in a sticky-price framework.<sup>22</sup>

The following comprises a simple New Keynesian model, augmented to include long-term debt and recurring policy regimes:

$$y_t = E_t y_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) \quad (18)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t \quad (19)$$

$$i_t = \phi_y^{s_t} y_t + \phi_\pi^{s_t} \pi_t \quad (20)$$

$$\tau_t = \gamma^{s_t} b_{t-1} + \gamma_y^{s_t} y_t \quad (21)$$

$$b_t = \beta^{-1} b_{t-1} - (1 - \rho) P_t^m - \tau_t - \beta^{-1} \pi_t \quad (22)$$

$$P_t^m = -i_t + \beta \rho E_t P_{t+1}^m, \quad (23)$$

where  $y$  is the output gap. (18) and (19) are the New Keynesian IS and Phillips equations, (20) and (21) are the policy rules, and (22) is the government's budget constraint.<sup>23</sup> If  $\kappa \rightarrow \infty$  (i.e., if prices become flexible) and if  $\rho = \phi_y^{s_t} = \gamma_y^{s_t} = 0$ , (18)–(23) collapses to a model of the form (1)–(4). We abstract from exogenous shocks in (18)–(23) because they do not matter for our results.<sup>24</sup>

Equation (23) gives the price of a “bond portfolio” that the government uses to finance deficits. The portfolio has a geometrically decaying maturity structure, as in Cochrane (2001) and Eusepi and Preston (2018), among others. In the true nonlinear model

<sup>22</sup>Sticky prices are not necessary to study how some of these features affect inflation responses to forward guidance, but we need sticky prices to study output responses.

<sup>23</sup>The fiscal variables in (21) and (22) are again log-linearized around their respective steady states, and we further let  $b$  equal real government debt. Alternatively, we could define  $b$  as debt to GDP, which would alter the derivation of (22), but it would not alter the main qualitative results in figures 4–5 and 7–9. Similarly as in section 2, we could linearize the fiscal variables,  $b$  and  $\tau$ , and log-linearize remaining variables in keeping with the approach of other papers in the literature, but this would not affect the qualitative results in this paper.

<sup>24</sup>To see this, consider section 3.1; we may model any forward-guidance announcement that changes  $\{E_0 i_t\}_{t=0}^T$  by selecting the right sequence of coefficients  $\{A_i^{(s_t)}, B_i^{(s_t)}, D_i^{(s_t)}\}_{i=0}^N$ , which agents are assumed to observe and incorporate in their expectations formation at time  $t = 0$ . If one wants to include demand, supply, policy shocks, etc., they may—and these shocks may matter for the empirical fit of the model—but these shocks do not determine whether a model exhibits a forward-guidance puzzle. As such, we abstract from them.

underlying (18)–(23), one unit of the portfolio purchased today pays one unit of nominal income tomorrow,  $\rho \in [0, 1]$  the period after,  $\rho^2$  the period after that, and so on. When  $\rho = 0$ , all debt is short term, and all debt is in consols when  $\rho = 1$ . The log-linearized conditions for the government budget constraint, and the bond price, which relates short-term interest rates to bond prices via a no-arbitrage condition, are, respectively, (22) and (23).

We include a richer maturity structure of debt because of its importance for monetary transmission in a non-Ricardian policy framework. In a model with all short-term debt ( $\rho = 0$ ), a policy that lowers the path of interest rates also lowers the debt level and interest receipts, which lowers demand since households view their bond holdings (i.e., government debt) as net wealth. When  $\rho > 0$ , however, the same policy also raises the bond price,  $P_t^m$ , which raises the market value of *outstanding* debt and induces a surprise increase in the ex post real rate of return on bonds and interest receipts at every level of inflation at the time of announcement.<sup>25</sup> This suggests that a forward-guidance announcement may either increase or decrease aggregate demand at the time of announcement, depending on the short or long maturity structure. Importantly, it is the *change* in bond prices that raises aggregate demand when the debt maturity is long; a policy that raises the bond price for extended periods eventually lowers interest receipts and aggregate demand, regardless of the maturity structure.

#### 4.1 Calibration and Determinacy

Table 1 describes the determinacy properties of (18)–(23) when regimes are permanent, with a slight modification: monetary policy is active if  $\phi_\pi > 1 - \frac{1-\beta}{\kappa}\phi_y$  and passive otherwise. To our knowledge, no general analytical determinacy conditions exist for

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<sup>25</sup>When  $\rho > 0$ , the ex post rate of return on debt,  $b_{t-1}$ , depends on both the policy rate,  $i_{t-1}$ , and on the price of bondholders' claims to the portfolio in time  $t$ ,  $P_t^m$ . One can show that the log-linearized expression for ex post real interest rate,  $r$ , is  $r_t = \beta\rho P_t^m - \pi_t - P_{t-1}^m$ . For given  $\pi_t$  and  $P_{t-1}^m$ , a rise in  $P_t^m$  increases  $r$ . However, a sustained rise in  $\{P_t^m\}$  (e.g., the rise implied by a forward-guidance policy that lowers  $i$  for extended periods of time) does not induce a sustained increase in  $r$ . To see this, suppose  $P_t^m = P_{t-1}^m = P^m$ , then  $r = (\beta\rho - 1)P^m - \pi_t$  which is decreasing in  $P^m$ .

**Table 2. Calibration**

	Benchmark		Davig and Leeper	Davig and Leeper
	1	2	(2007a)	(2011)
$p_M$	.95	.95	.9372	.94
$p_F$	.95	.95	.948	.95
$\gamma^M$	.0175	.0175	.0136	.071
$\gamma^F$	0	0	−.0094	−.025
$\gamma_y^M$	0	0	.4596	.498
$\gamma_y^M$	0	0	.2754	.324
$\rho$	.1	.9	0	0
$\phi_\pi^M$	1.1	1.1	1.1	1.1
$\phi_\pi^F$	.5	.5	.5	.5
$\phi_y^M$	.1	.1	.1	.1
$\phi_y^F$	.1	.1	.1	.1

**Notes:** We also assume  $\beta = .99$ ,  $\sigma = 1$ ,  $\kappa = .05$ , and  $\bar{b}/\bar{Y} = 1$  where  $\bar{b}(\bar{Y})$  is steady-state real debt (output). The Davig and Leeper (2007a) calibration is reported in tables 4.2 and 4.3 of Davig and Leeper (2007a), and the Davig and Leeper (2011) calibration is reported in table 2 of Davig and Leeper (2011).

(18)–(23) when  $p_M < 1$  and  $p_F < 1$ , but Cho and Moreno (2019), McClung (2019), and Ascari, Florio, and Gobbi (2020) study equilibrium multiplicity issues in models of the FTPL with recurring fiscal and monetary regimes.

Table 2 presents calibration details. We consider three different fiscal policy rule calibrations for the model (18)–(23). The “benchmark” calibration imposes  $\gamma^F = 0$ , such that fiscal surpluses evolve exogenously in the active regime, and a value of  $\gamma^M$  that is low enough to rule out a forward-guidance puzzle in the recurring-regimes model. To compare the effects of forward guidance across different debt maturity structures, we pair the benchmark rule with short maturity structure ( $\rho = .1$ ) and then separately with long maturity structure ( $\rho = .9$ ). We also consider calibrations of (21) that are motivated by the estimated fiscal rules in Davig and Leeper (2007a, 2011). Those papers estimate a two-state univariate model of fiscal surpluses that resembles (21) using U.S. data.<sup>26</sup> In table 2, we

<sup>26</sup>Davig and Leeper (2007a, 2011) specify fiscal surpluses rules of the form  $\tau = \gamma_0(s_t) + \gamma(s_t)b_{t-1} + \gamma_y(s_t)y_t + \gamma_g(s_t)g_t + \sigma(s_t)\epsilon_t^f$ , where  $g$  is government

choose monetary policy parameters that ensure determinacy after forward guidance, based on the determinacy criteria in Cho (2016, 2020).

## 4.2 The New Keynesian Forward-Guidance Puzzle

### 4.2.1 Benchmark Calibration

Here we apply proposition 2 to (18)–(23) to determine when fiscal policy selects puzzle-free equilibria in the model with sticky prices. Extensive numerical analysis confirms that the qualitative results in section 2 carry over to the model with sticky prices: some non-Ricardian fiscal policies rule out forward guidance, while others entail too much passive policy.<sup>27</sup> Also as before, a permanent active fiscal regime is sufficient to rule out a forward-guidance puzzle.

**PROPOSITION 3.** *Consider (18)–(23) and assume (i) a temporary interest rate peg (such that  $\phi_\pi = \phi_y = 0$  for  $0 \leq t < T_1$ ); (ii) long-run passive monetary policy ( $\phi_\pi^{st} = \phi_\pi$ ,  $\phi_y^{st} = \phi_y$ , for all  $t \geq T_N$ , where  $0 \leq \phi_\pi < 1 - \frac{1-\beta}{\kappa}\phi_y$ ); and (iii) permanent active fiscal policy ( $\gamma^{st} = \gamma$  for all  $t$  where  $\gamma \notin (\beta^{-1} - 1, \beta^{-1} + 1)$ ). Then the model does not exhibit a forward-guidance puzzle.*

*Proof.* See appendix A.5. ■

Proposition 3 shows that permanent active fiscal policy solves the forward-guidance puzzle in the sticky-price model. To our knowledge, no work has shown this formally. In some models with a permanent active fiscal regime, it is also possible to derive an analytical solution to a forward-guidance announcement (e.g., see appendix A.6). We note that the section 3 approach requires (13) to exist. Hence, proposition 2 (and proposition 3) only applies to calibrations that permit well-defined post-forward-guidance solutions.

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spending to output. We suppress  $\gamma_0(s_t)$ ,  $g_t$ , and  $\epsilon_t^f$  here, but extensive numerical analysis suggests that our results are unaffected by  $\gamma_0(s_t)$ ,  $g_t$ , and  $\epsilon_t^f$ .

<sup>27</sup>We find the sticky-price version of figure 2 closely resembles figure 2 for all values of  $\rho$  we consider. For example, see figure A.2 in the appendix.

### 4.2.2 *Estimated Models*

Figures 2–3 and propositions 1 and 3 inform us about the effects of forward guidance for many different calibrations of the fiscal policy rule. However, the data may only prefer some calibrations, and here we pay special attention to the effects of forward guidance in models with empirically relevant descriptions of the fiscal policy stance. First, we embed the estimated fiscal surplus rules from Davig and Leeper (2007a) and Davig and Leeper (2011), who estimate two univariate models of the form (21), into the model (18)–(23). The second exercise in this section applies the section 3 methodology to the full DSGE model of Bianchi and Ilut (2017) to determine whether they find evidence of sufficiently strong active fiscal policy to rule out a forward-guidance puzzle.<sup>28</sup>

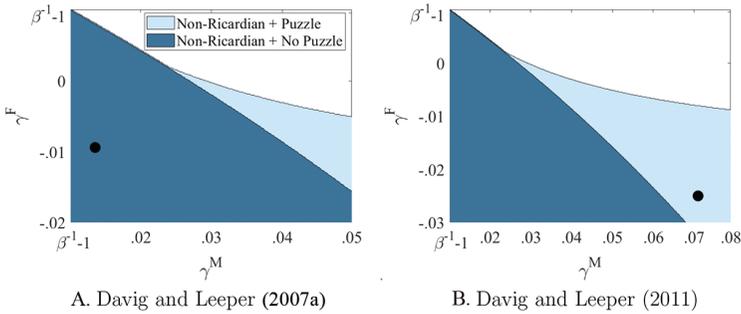
Figure 4 plots  $(\gamma^M, \gamma^F)$ -space for the Davig and Leeper (2007a) and Davig and Leeper (2011) calibrations. The contrast between the two rules is stark: the Davig and Leeper (2007a) passive regime is relatively weak (i.e.,  $\gamma^M$  is relatively low) and, consequently, the estimated fiscal rule from that paper does not induce a forward-guidance puzzle. On the other hand, the estimated rule in Davig and Leeper (2011) does provide the fiscal backing necessary for a forward-guidance puzzle to emerge. This is due to the fact that the estimated passive regime in Davig and Leeper (2011) is strongly passive (i.e.,  $\gamma^M = .072$ ). Both fiscal rules are non-Ricardian policies.

Figure 5 plots impulse responses of time  $t = 0$  inflation to a one-time anticipated monetary policy shock at different horizons,  $T$ . Loosely keeping with the notation of section 3, these experiments involve an announcement at  $t = 0$  that the nominal interest rate will fall to the ZLB between periods  $T - 1$  and  $T$ . The interest rate is fixed at steady state between  $t = 0$  and  $T - 1$ , and agents become

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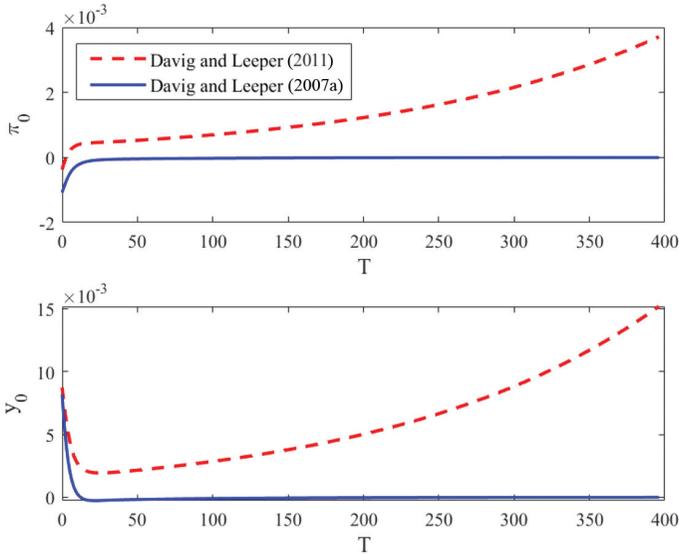
<sup>28</sup>See appendix A.7 for the system of equations comprising the Bianchi and Ilut (2017) model. We calibrate their model using the posterior mode estimates reported in table 1 of Bianchi and Ilut (2017). The model is a New Keynesian model with recurring active and passive fiscal and monetary regimes. We find that their estimated fiscal policy rule is non-Ricardian in the sense that it does not ensure mean square stable debt given any sequence of variables impacting the evolution of debt via the government’s budget constraint. See Bianchi and Melosi (2017) and Chen, Leeper, and Leith (2018) for other papers that find evidence of recurring regime change in the U.S. monetary-fiscal stance by estimating DSGE models.

**Figure 4. Forward-Guidance Puzzle and Estimated Fiscal Rules**



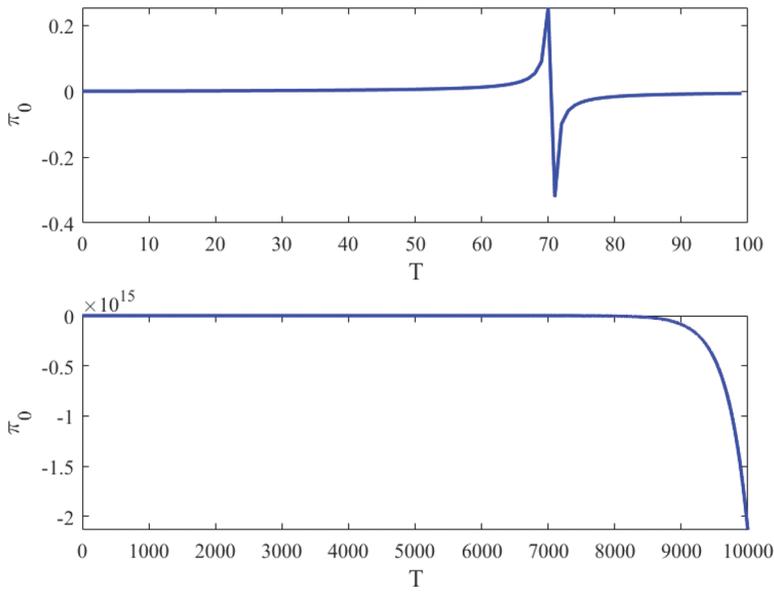
**Notes:** The black dot in panel A is the estimated value of  $(\gamma^M, \gamma^F)$  in Davig and Leeper (2007a). The black dot in panel B is the estimated value of  $(\gamma^M, \gamma^F)$  in Davig and Leeper (2011). The white region is the set of Ricardian fiscal policies.

**Figure 5. Davig and Leeper (2007a) vs. Davig and Leeper (2011)**



**Notes:** The dashed (solid) line shows the time- $t = 0$  responses of inflation and output to a time- $t = 0$  announcement of a one-time shock to  $i_T$  for different values of  $T$  in the model with the Davig and Leeper (2011) (Davig and Leeper 2007a) calibration. We assume  $i_t$  is in steady state (i.e.,  $i_t = 0$ ) for  $t = 0, \dots, T - 1$ . The vertical axes units are percent deviations from steady state (e.g., .01 is 1 percent).

**Figure 6. Forward Guidance in an Estimated DSGE Model: Bianchi and Ilut (2017)**



**Notes:** Each panel shows the time- $t = 0$  responses of inflation ( $\pi_0$ ) to a time- $t = 0$  announcement of a one-time shock to  $i_T$  in the model of Bianchi and Ilut (2017) (see appendix A.7). We assume  $i_t$  is in steady state (i.e.,  $i_t = 0$ ) for  $t = 0, \dots, T - 1$ . The vertical axes units are percent deviations from steady state (e.g., .01 is 1 percent). The top panel shows  $\pi_0$  for  $T = 0, \dots, 100$ ; the bottom panel shows  $\pi_0$  for  $T = 0, \dots, 10000$ .

fully aware of the time- $T$  shock at  $t = 0$ . The solid line shows the responses in the model with the Davig and Leeper (2007a) calibration; the dashed line shows the responses in the model with the Davig and Leeper (2011) calibration. The figure shows that the magnitude of  $\pi_0$  grows in  $T$  for the Davig and Leeper (2011) calibration, which is a sign of the forward-guidance puzzle in action. The opposite is true for the Davig and Leeper (2007a) calibration.

Figure 6 shows the time- $t = 0$  response of inflation to a one-time anticipated shock to nominal interest rates at horizons  $T$  in the model of Bianchi and Ilut (2017), calibrated at the posterior mode estimates reported in table 1 of their paper. As in the last exercise, the central bank announces a policy at  $t = 0$  that fixes

interest rates at steady state for  $t = 0, \dots, T - 1$  and sets  $i$  at the ZLB between  $T - 1$  and  $T$ , after which normal monetary policy is conducted according to the estimated policy rules.<sup>29</sup> Their estimates deliver a non-Ricardian fiscal policy, which suggests that there could be no forward-guidance puzzle in their model. However, the conditions in proposition 2 are not satisfied for the Bianchi and Ilut (2017) model, calibrated at the posterior mode estimates they report. As a result, a forward-guidance puzzle emerges in simulations: the top panel of figure 6 shows that  $\pi_0$  is subject to the above-mentioned “reversals” of Carlstrom, Fuerst, and Paustian (2015); the bottom panel shows that inflation is excessively responsive to news about far-away events (e.g., an announcement at  $t = 0$  about a one-time shock to  $i_{10000}$  causes  $\pi_0$  to drop to somewhere near  $-2 \times 10^{15} \times 100\%$ ). Though Bianchi and Ilut (2017) find evidence of fiscal theory features in the U.S. data, they do not find evidence that U.S. fiscal policy was sufficiently active to rule out forward-guidance puzzles. In particular, their estimate of the transition probability associated with their model’s passive fiscal regime is larger than 0.99—which implies that the average duration of passive fiscal regimes exceeds 100 quarters in their model—and this is one reason their model suggests a forward-guidance puzzle.

By comparing the estimated rules of Davig and Leeper (2007a, 2011), and the estimated model of Bianchi and Ilut (2017), we see that both puzzle-prone non-Ricardian fiscal policies and puzzle-free non-Ricardian policies are empirically plausible. One cannot simply disregard one non-Ricardian case in favor of the other on empirical grounds.

We emphasize one caveat to our treatment of Davig and Leeper (2007a, 2011) and Bianchi and Ilut (2017): these papers largely focus on pre-2008 U.S. data and, as such, their estimates of the persistence and strength of active and passive fiscal regimes depend on the behavior of governments during the latter half of the 20th century. It’s entirely possible that the policy response to the financial crisis (e.g., the American Recovery and Reinvestment Act (ARRA) and the Troubled Asset Relief Program (TARP)) “reset” expectations,

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<sup>29</sup>Additional details about the experiment are discussed in appendix A.7 as well.

thereby allowing for expectations of strongly active fiscal policy during the horizon over which the Federal Reserve conducted forward guidance. It's also possible that gridlock in U.S. Congress, debt ceiling debacles, and the "fiscal cliff" of 2013 encouraged expectations of strongly passive policy. Therefore, while these papers provide interesting evidence of fiscal theory effects in the U.S. data, future work could shed more light on fiscal expectations during the U.S. ZLB episode of 2008–15.

### 4.3 *Impulse Responses*

While we should expect models to generate reasonable responses to news about very distant structural changes, policymakers might also be interested in knowing the model-implied responses of inflation and output to some reasonable forward-guidance horizon, e.g., 12 quarters. This section displays impulse responses to forward-guidance announcements at  $t = 0$  that fix nominal interest rates at the ZLB from  $t = 0$  to  $t = T = 12$ .<sup>30</sup>

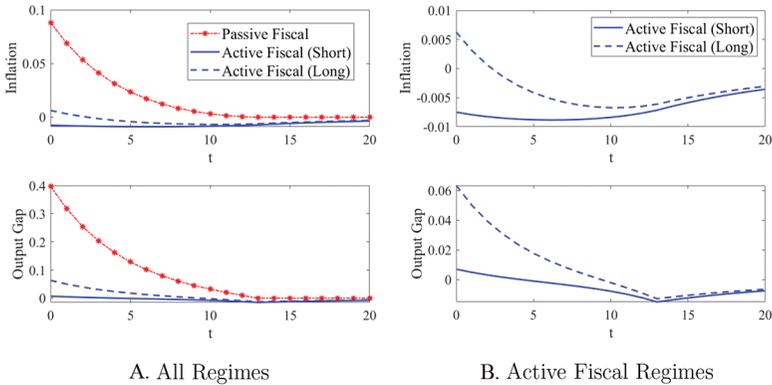
#### 4.3.1 *Permanent Regimes*

Figure 7 shows the responses to 12 quarters of forward guidance when fiscal regimes are assumed permanent.<sup>31</sup> From panel A of figure 7, a fixed passive regime paired with a rule that implements active monetary policy after forward guidance generates large inflation and output responses that reflect the intertemporal substitution effects of lower expected real interest rates. This is a prime example of the forward-guidance puzzle. In contrast, the fixed active fiscal regimes paired with passive monetary policy post-forward guidance generate much milder responses, in line with proposition 3. Moreover,

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<sup>30</sup>Throughout this section, we assume the economy is in steady state at  $t = -1$ .

<sup>31</sup>In the section 4.3 exercises, we calibrate the permanent regime models according to the benchmark parameter values in table 2 (e.g., for a permanent active fiscal regime model with short maturity, we calibrate the model according to the  $s_t = F$  parameter values and solve the resulting linearized DSGE model with fixed coefficients using standard techniques). We choose the calibration with short-term maturity for simulations with permanent passive fiscal regimes. Under a permanent passive fiscal regime, Ricardian equivalence holds and therefore debt maturity details are irrelevant.

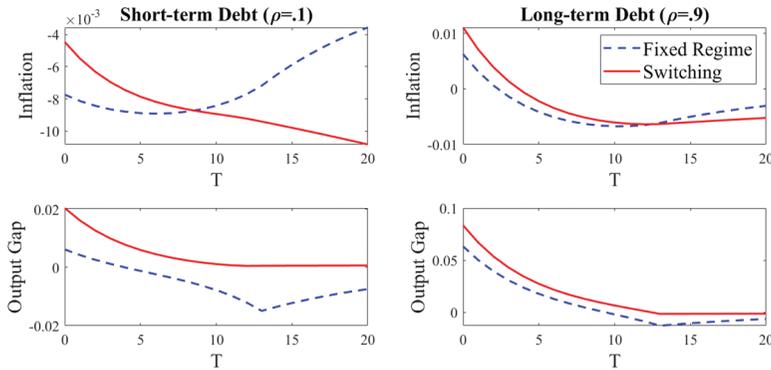
**Figure 7. Permanent Regimes**

**Note:** The vertical axes units are percent deviations from steady state (e.g., .01 is 1 percent).

the *eventual* deflation and negative output gap reflect the above-mentioned wealth effects of lower interest rates in economies with non-Ricardian fiscal policy; a prolonged period of low interest rates eventually lowers interest receipts and the level of bonds—which households view as net wealth when fiscal policy is non-Ricardian—and therefore demand and inflation fall over the forward-guidance horizon. These wealth effects do not arise in models with Ricardian policy, such as models with a permanent passive fiscal regime. However, whether inflation *initially* rises or falls depends on the debt maturity structure: when maturity is long, news about lower interest rates initially raises the market value of *outstanding* debt and the ex post real rate of return on bonds, which raises demand and inflation at the time of announcement. Panel B of figure 7 focuses on the two active fiscal regime impulse responses in panel A, and illustrates the last point.

#### 4.3.2 Recurring Regimes

When fiscal regime changes recur, and agents are consequently uncertain about the path of future fiscal regimes, the effects of forward guidance are similar to those displayed in figure 7—provided the fiscal policy at hand solves the forward-guidance

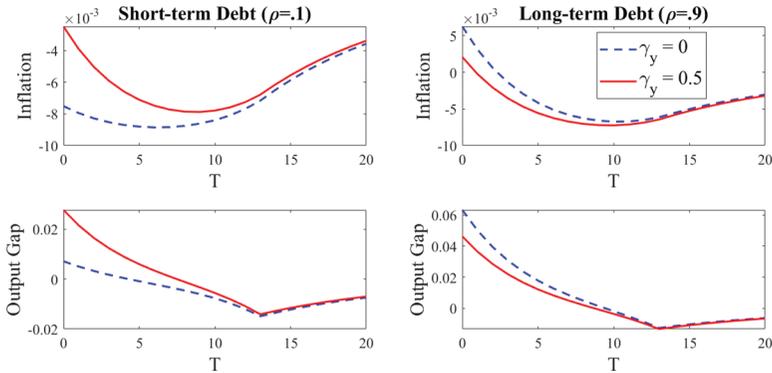
**Figure 8. Recurring Regimes**

**Note:** The vertical axes units are percent deviations from steady state (e.g., .01 is 1 percent).

puzzle according to the proposition 2 criteria. This is true even if the forward-guidance policy is carried out during a passive fiscal regime. Figure 8 displays the responses of forward guidance using the benchmark regime-switching parameterizations. As in the experiment of figure 7, we assume that the economy is in steady state when the central bank announces that it will drop  $i$  to the ZLB for the next 12 quarters. Unlike the experiments of figure 7, which assume permanent fiscal regimes, we must specify a path for the fiscal regime,  $s_t$ . We show impulse responses when  $s_t = M$  for  $t = 0, \dots, 20$  (i.e., we assume fiscal policy is passive during the forward-guidance horizon). Crucially, we also assume that agents do not know this path ex ante.<sup>32</sup>

Figure 8 also reproduces the impulse responses for the permanent active regime economies studied in figure 7. The impulse responses in the puzzle-free recurring-regimes models largely resemble the impulse responses in economies with permanent active fiscal

<sup>32</sup>That is to say, we assume that agents form rational expectations using the true transition probabilities (e.g., if  $s_t = M$  then  $E_t(s_{t+1} = M | s_t = M) = p_M$  where  $p_M$  is the probability of remaining in regime M). We choose this path for the fiscal regime because it attenuates the central fiscal theory mechanism in the model, which helps to prevent us from overstating the ability of the fiscal theory to rule out forward-guidance puzzles.

**Figure 9. Countercyclical Fiscal Policy**

**Note:** The vertical axes units are percent deviations from steady state (e.g., .01 is 1 percent).

policy, particularly in economies with longer maturity structures. For a model with shorter maturity, the initial responses across the two parameterizations are similar, but forward guidance has more persistent effects in the switching model due to a smaller initial fall in inflation (and hence, a larger fall in the real rate of return on bonds) in the economy with recurring regimes relative to the economy with permanent regimes (e.g., see upper left panel of figure 8). One can show that these persistent declines in inflation are quickly undone by a switch to active fiscal policy.

### 4.3.3 Cyclical Fiscal Surpluses

The Federal Reserve and other central banks practiced forward guidance in the aftermath of the Great Recession, at a time when some national governments had recently conducted countercyclical policies (e.g., ARRA in the United States), and other governments implemented austerity measures. In the simple model (18)–(23), we can study the interaction of monetary forward-guidance policy and cyclical fiscal policy by setting  $\gamma_y \neq 0$ . When  $\gamma_y > 0$ , surpluses are “countercyclical” in the sense that deficits rise as output falls. Otherwise, fiscal policy is procyclical. Automatic stabilizers are examples of countercyclical policy; austerity regimes are procyclical policies.

Figure 9 plots responses to the 12-quarter experiment described above when a permanent countercyclical active fiscal regime is in

place.<sup>33</sup> We set  $\gamma_y = 0.5$  such that a 1 percent increase in the output gap corresponds to a 0.5 percent increase in fiscal surpluses. This particular value of  $\gamma_y$  is similar to estimated values from Davig and Leeper (2007a, 2011) and Bianchi and Ilut (2017).

With short maturity structure, countercyclical policy raises the responses of inflation relative to the case with  $\gamma_y = 0$ . We interpret this finding as follows: forward guidance causes  $\pi_0 = \kappa \sum_{t \geq 0} \beta^t E_0 y_t$  to fall, and this triggers a countercyclical tax cut that counteracts the deflationary effects of forward guidance. Thus, for positive  $\gamma_y$  and shorter debt maturity, forward guidance has *less* deflationary effects than when  $\gamma_y = 0$ . With long maturity structure, the opposite result is true: forward guidance causes  $\pi_0 = \sum_{t \geq 0} E_0 \beta^t y_t$  to rise, and this triggers a countercyclical tax hike that counteracts the inflationary effects of forward guidance. Appendix A.9 contains a more detailed description of these results.

Procyclical policies have the opposite effects of countercyclical policies, with a caveat: if  $\gamma_y$  becomes too negative, then the sign of  $\pi_0$  “flips.” To be concrete, we derive an analytical forward-guidance solution of (18)–(23) (see appendix A.6) and show that the sign of  $\pi_0$  flips at some critical value,  $\bar{\gamma}_y < 0$ .<sup>34</sup> For the benchmark calibration,  $\bar{\gamma}_y = -.247$ , which is a policy that calls for a .247 percent reduction in deficits in response to a 1 percent fall in output. For  $\gamma_y$  near  $\bar{\gamma}_y$ , forward-guidance solutions diverge to negative or positive infinity.<sup>35</sup>

What should we value from this detour into the interaction of forward guidance and cyclical fiscal policy? For one, we learn that countercyclical policies can reduce the magnitude of inflation responses

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<sup>33</sup>As before, we use the benchmark calibration in table 2, except as otherwise noted in the main text.

<sup>34</sup>The derivation assumes  $\phi_\pi = \phi_y = \gamma = 0$  for all  $t$ , which simplifies the derivation without sacrificing basic fiscal theory mechanisms. The expression for the critical value is  $\bar{\gamma}_y = \beta^{-1} \kappa (\sigma \beta \kappa^{-1} (\lambda_1 - 1) - 1)$  and  $\gamma_0 = 1/\beta$ ,  $\gamma_1^* = \gamma_1 = (1 + \beta + \sigma^{-1} \kappa)/\beta$ ,  $\lambda_1 = .5(\gamma_1 - \sqrt{(\gamma_1^2 - 4\gamma_0)})$ , and where  $|\lambda_1| < 1$  and  $\gamma_0 = 1/\beta$ ,  $\gamma_1^* = \gamma_1 = (1 + \beta + \sigma^{-1} \kappa)/\beta$ ,  $\lambda_1 = .5(\gamma_1 - \sqrt{(\gamma_1^2 - 4\gamma_0)})$ , and where  $|\lambda_1| < 1$ .

<sup>35</sup>Importantly, a well-defined post-forward-guidance solution (13) does not exist at  $\gamma_y = \bar{\gamma}_y$ , and this prevents us from invoking proposition 3 findings when  $\gamma_y = \bar{\gamma}_y$ . However, by studying  $\gamma_y$  near  $\bar{\gamma}_y$  we see that procyclical regimes interact with forward guidance in volatile ways, even if there is no forward-guidance puzzle in the definition 1 sense.

to monetary forward guidance. Thus, countercyclical responses to the Great Recession may have mitigated the power of monetary forward guidance. The model also suggests that austerity regimes can interact with monetary policy in volatile ways, sometimes leading to explosive inflation or explosive deflation.

#### 4.4 *Discussion: Limitations*

Sections 2 and 4 show that the fiscal theory *can* resolve the forward-guidance puzzle, but some limitations of our approach are worth mentioning here. Our framework assumes perfect credibility and full-information rational expectations. These strong assumptions, which are standard in the fiscal theory literature and benchmark analyses of the puzzle, allow us to cleanly characterize the interaction of fiscal policy and forward guidance. However, many of the papers cited in the literature review show that deviations from the above assumptions resolve the forward-guidance puzzle, thus highlighting central roles for rationality and credibility in generating counterintuitive model predictions of the effects of forward guidance. Future work could examine interactions of deviations from rationality, non-Ricardian policy, and forward guidance.

Our calendar-based approach to modeling forward guidance, which is similar to the approach of Del Negro, Giannoni, and Patterson (2015), among others, also has limitations. This model strains credibility by assuming that central banks commit to pegging interest rates *regardless* of the magnitude of inflation and output over the forward-guidance horizon, and it fails to capture the many ways central banks give forward guidance in real life (e.g., see Moessner, Jansen, and de Haan 2017). Further, our model does not account for the fact that simultaneous announcements (e.g., quantitative easing, news about other macrovariables, etc.) could affect agents' information sets, thereby affecting the transmission of forward guidance. Reasonable alternatives to the standard model we consider include state-contingent or threshold-based promises, or communication of macroeconomic forecasts and the policy actions consistent with those forecasts ("Delphic" guidance). It is reasonable to conjecture that some of these alternatives, namely state-contingent or imperfectly credible announcements, are

less susceptible to the problems we consider in a fiscal theory framework.

Finally, with the exception of the richer model of Bianchi and Ilut (2017), we study simple New Keynesian models. Appendix A.10 shows that our results in the simple New Keynesian models are robust to distortionary labor taxes, but there is room for more analysis in larger models. In sum, this paper shows that the fiscal theory *can* resolve the forward-guidance puzzle, but our results do not imply that the fiscal theory is the most reasonable solution of the forward-guidance puzzle.

## 5. Conclusion

We develop conditions for a regime-switching model to exhibit a forward-guidance puzzle, and show when the fiscal theory of the price level does—and does not—resolve the forward-guidance puzzle. Importantly, we find that the fiscal theory rules out puzzles, provided fiscal policy is, or is expected to be, sufficiently active. Equally important, we illustrate some empirically relevant cases for which fiscal policy is non-Ricardian, the fiscal theory determines inflation, and forward-guidance puzzles are still there. Thus, one cannot simply ignore the forward-guidance puzzle when inflation serves a necessary debt-stabilizing purpose. We also simulate the effects of announcements that lower the path of interest rates in non-Ricardian economies. Inflation and output gaps *eventually* become negative over the forward-guidance horizon, regardless of the current fiscal regime, but the initial effects of the announcement depend on debt maturity structure and whether policy is countercyclical or procyclical. Countercyclical surpluses attenuate the effects of forward guidance across different maturity structures, whereas forward guidance has ambiguous (and potentially wildly volatile) effects when surpluses are procyclical.

We leave room for future work, particularly work that addresses the important limitations of our approach discussed in section 4.4. In addition, future work could address more complicated monetary-fiscal interactions, including the interaction of cyclical fiscal policy and forward guidance, in a larger estimated model.

## Appendix

### A.1 Proof of Theorem 1

Define  $h_F$  and  $h_M$  as in the main text. Using Costa, Fragoso, and Marques (2005), we can show that (5) is a mean square stable process for debt if

$$r(B) = r \begin{pmatrix} p_M h_M^2 & (1 - p_F) h_M^2 \\ (1 - p_M) h_F^2 & p_F h_F^2 \end{pmatrix} < 1,$$

where  $r(B)$  denotes the spectral radius of  $B$ . The eigenvalues of  $B$  solve the following characteristic polynomial:

$$\lambda^2 - \lambda(p_M h_M^2 + p_F h_F^2) + (p_M + p_F - 1) h_M^2 h_F^2 = 0.$$

Applying techniques from LaSalle (1986, p. 28), we conclude that both eigenvalues of  $B$  are inside the unit circle if and only if

$$|(p_M + p_F - 1) h_M^2 h_F^2| < 1 \tag{A.1}$$

$$|p_M h_M^2 + p_F h_F^2| < 1 + (p_M + p_F - 1) h_M^2 h_F^2. \tag{A.2}$$

If we further assume  $p_M + p_F \geq 1$ , then  $|(p_M + p_F - 1) h_M^2 h_F^2| = (p_M + p_F - 1) h_M^2 h_F^2$  and  $|p_M h_M^2 + p_F h_F^2| = p_M h_M^2 + p_F h_F^2$ , these two conditions can be rearranged into the conditions theorem 1 presents. ■

### A.2. Proof of Proposition 1

As argued in the main text, the forward-guidance puzzle does not emerge if the eigenvalues of the following matrix are inside in the unit circle:

$$A = \begin{pmatrix} p_M \delta_M^{-1} & (1 - p_M) \delta_M^{-1} \\ (1 - p_F) \delta_F^{-1} & p_F \delta_F^{-1} \end{pmatrix} < 1.$$

The eigenvalues of  $A$  solve the following characteristic polynomial:

$$\lambda^2 - \lambda(p_M \delta_M^{-1} + p_F \delta_F^{-1}) + (p_M + p_F - 1) \delta_M^{-1} \delta_F^{-1} = 0.$$

Applying techniques from LaSalle (1986, p. 28), we conclude that both eigenvalues of  $A$  are inside the unit circle if and only if

$$|(p_M + p_F - 1)\delta_M^{-1}\delta_F^{-1}| < 1 \tag{A.3}$$

$$|p_M\delta_M^{-1} + p_F\delta_F^{-1}| < 1 + (p_M + p_F - 1)\delta_M^{-1}\delta_F^{-1}, \tag{A.4}$$

which is the condition proposition 1 presents. ■

### A.3 Forward-Guidance Solution Technique and Proof

The solution technique applies to Markov-switching DSGE models that, in the absence of an announcement about future structural changes, assume the form

$$x_t = A^{(s_t)}E_t x_{t+1} + B^{(s_t)}x_{t-1} + C^{(s_t)}u_t + D^{(s_t)}, \tag{A.5}$$

where  $x_t$  is an  $n \times 1$  vector of endogenous variables,  $u_t$  is an  $m \times 1$  vector of iid mean-zero exogenous shocks, and  $s_t$  is an exogenous  $S$ -state Markov process with transition matrix,  $P$ , where  $p_{ij} = Pr(s_{t+1} = j | s_t = i)$  is the  $(i, j)$ -th element of  $P$ .<sup>36</sup>

Suppose at time 0 agents become aware of  $N$  structural changes that occur at horizons  $0 \leq T_1 < T_2 < \dots < T_N$ . The announcement implies the following model structure:

$$x_t = A_N^{(s_t)}E_t x_{t+1} + B_N^{(s_t)}x_{t-1} + C_N^{(s_t)}u_t + D_N^{(s_t)} \quad \text{for } t \geq T_N \tag{A.6}$$

$$\vdots \tag{A.7}$$

$$x_t = A_1^{(s_t)}E_t x_{t+1} + B_1^{(s_t)}x_{t-1} + C_1^{(s_t)}u_t + D_1^{(s_t)} \quad \text{for } T_1 \leq t < T_2 \tag{A.8}$$

$$x_t = A_0^{(s_t)}E_t x_{t+1} + B_0^{(s_t)}x_{t-1} + C_0^{(s_t)}u_t + D_0^{(s_t)} \quad \text{for } 0 \leq t < T_1. \tag{A.9}$$

We assume that agents do not know the future path of  $s_t$  (i.e.,  $E_t x_{t+1} = E(x_{t+1} | x_t, u_t, s_t) = \sum_{j=1}^S p_{s_t j} E(x_{t+1} | x_t, u_t, s_{t+1} = j)$ ).

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<sup>36</sup>We assume that shocks are iid for exposition's sake. We could straightforwardly generalize our results in this section to a model class with serially correlated shocks.

The following procedure gives solutions of (A.6)–(A.9), i.e., “forward guidance solutions”:

- (i) Select an MSV solution, which describes the equilibrium law of motion for  $t \geq T_N$ :<sup>37</sup>

$$x_t = \Omega^{(s_t)}x_{t-1} + \Gamma^{(s_t)}u_t + \xi^{(s_t)}. \tag{A.10}$$

The solution (A.10) provides an “asymptotic” model of the economy for  $t \geq T_N$ . Determinacy is *not* a requirement for our method to work (if multiple solutions (A.10) exist, choose one).

- (ii) Initiate the backward recursion by forming expectations of  $x_{T_N}$  at time  $T_N - 1$ . To simplify the notation, let  $T = T_N$ . Then,  $E_{T-1}x_T = E_{T-1}(\Omega^{(s_T)}x_{T-1} + \Gamma^{(s_T)}u_T + \xi^{(s_T)})$ , where  $E_{T-1}$  conditions on all time  $T - 1$  variables including  $x_{T-1}, u_{T-1}, s_{T-1}$ . Substitute  $E_{T-1}x_T$  into (A.5), where  $(A^{(s_t)}, B^{(s_t)}, C^{(s_t)}, D^{(s_T)}) = (A_{N-1}^{(s_t)}, B_{N-1}^{(s_t)}, C_{N-1}^{(s_t)}, D_{N-1}^{(s_T)})$ , and solve for  $x_{T-1}$  to recover the following rational expectations equilibrium (REE) matrices:

$$\begin{aligned} \Omega_{T-1}^{(s_{T-1})} &= \left( I - A_{N-1}^{(s_{T-1})} E_{T-1}(\Omega^{(s_T)}) \right)^{-1} B_{N-1}^{(s_{T-1})} \\ \Gamma_{T-1}^{(s_{T-1})} &= \left( I - A_{N-1}^{(s_{T-1})} E_{T-1}(\Omega^{(s_T)}) \right)^{-1} C_{N-1}^{(s_{T-1})} \\ \xi_{T-1}^{(s_{T-1})} &= \left( I - A_{N-1}^{(s_{T-1})} E_{T-1}(\Omega^{(s_T)}) \right)^{-1} \\ &\quad \left( D_{N-1}^{(s_{T-1})} + A_{N-1}^{(s_{T-1})} E_{T-1} \left( \xi^{(s_T)} \right) \right). \end{aligned}$$

Hence,  $x_{T-1} = \Omega_{T-1}^{(s_{T-1})}x_{T-2} + \Gamma_{T-1}^{(s_{T-1})}u_{T-1} + \xi_{T-1}^{(s_{T-1})}$ , where the REE matrices  $\Omega_{T-1}^{(s_{T-1})}$ ,  $\Gamma_{T-1}^{(s_{T-1})}$ , and  $\xi_{T-1}^{(s_{T-1})}$  are uniquely determined for all  $s_{T-1}$ .

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<sup>37</sup>See Farmer, Waggoner, and Zha (2009, 2011), Maih (2015), Cho (2016), Foerster et al. (2016), and Barthelémy and Marx (2017) for alternative solution techniques. The models we study satisfy  $D_N^{s_t} = \xi^{(s_t)} = 0^n$  for all  $s_t$ .

(iii) Repeat this undetermined coefficients logic to obtain the solution matrices  $\{\Omega_t^{(s_t)}, \Gamma_t^{(s_t)}, \xi_t^{(s_t)}\}_{t=0}^{T-1}$ :

$$\Omega_t^{(s_t)} = \left( I - A_i^{(s_t)} E_t(\Omega_{t+1}^{(s_{t+1})}) \right)^{-1} B_i^{(s_t)} \tag{A.11}$$

$$\Gamma_t^{(s_t)} = \left( I - A_i^{(s_t)} E_t(\Omega_{t+1}^{(s_{t+1})}) \right)^{-1} C_i^{(s_t)} \tag{A.12}$$

$$\xi_t^{(s_t)} = \left( I - A_i^{(s_t)} E_t(\Omega_{t+1}^{(s_{t+1})}) \right)^{-1} \left( D_i^{(s_t)} + A_i^{(s_t)} E_t \left( \xi_{t+1}^{(s_{t+1})} \right) \right) \tag{A.13}$$

given  $(\Omega_T^{(s_T)}, \Gamma_T^{(s_T)}, \xi_T^{(s_T)}) = (\Omega^{(s_t)}, \Gamma^{(s_t)}, \xi^{(s_t)})$ , where  $i \in \{0, 1, 2, \dots, N - 1\}$  depending on  $t$ . Note that the sequence of matrices,  $\{\Omega_t^{(s_t)}, \Gamma_t^{(s_t)}, \xi_t^{(s_t)}\}_{t=0}^T$ , is uniquely determined by  $(\Omega_T^{(s_T)}, \Gamma_T^{(s_T)}, \xi_T^{(s_T)}) = (\Omega^{(s_t)}, \Gamma^{(s_t)}, \xi^{(s_t)})$ .

(iv) The forward-guidance solution is

$$x_t = \Omega_t^{(s_t)} x_{t-1} + \Gamma_t^{(s_t)} u_t + \xi_t^{(s_t)}. \tag{A.14}$$

Given  $\{\Omega^{(s_t)}, \Gamma^{(s_t)}, \xi^{(s_t)}\}$ , (A.14) is unique. We did not impose any restrictions on the state history,  $\mathcal{S}^{T+1} = \{s_0, \dots, s_{T+1}\}$ , which means that our method provides the solution,  $x_t$ , for all conceivable realizations of the Markov chain,  $s_t$ , from time 0 to time  $T + 1$ .

#### A.4 Proof of Proposition 2

First, we show that  $\lim_{T_1 \rightarrow \infty} x_0$  exists for all  $s_0 \in \{1, \dots, S\}$ , given any  $x_{-1} \in \mathbb{R}^n$ ,  $u_0 \in \mathbb{R}^m$  if and only if  $\lim_{T_1 \rightarrow \infty} \xi_0^{(s_0)} = \bar{\xi}^{(s_0)}$ ,  $\lim_{T_1 \rightarrow \infty} \Gamma_0^{(s_0)} = \bar{\Gamma}^{(s_0)}$ , and  $\lim_{T_1 \rightarrow \infty} \Omega_0^{(s_0)} = \bar{\Omega}^{(s_0)}$  exist for all  $s_0$ .

$$\begin{aligned} \lim_{T_1 \rightarrow \infty} x_0 &= \lim_{T_1 \rightarrow \infty} \left( \xi_0^{(s_0)} + \Omega_0^{(s_0)} x_{-1} + \Gamma_0^{(s_0)} u_0 \right) \\ &= \left( \lim_{T_1 \rightarrow \infty} \xi_0^{(s_0)} \right) + \left( \lim_{T_1 \rightarrow \infty} \Omega_0^{(s_0)} \right) x_{-1} + \left( \lim_{T_1 \rightarrow \infty} \Gamma_0^{(s_0)} \right) u_0, \end{aligned}$$

which clearly exists for any given  $x_{-1}$ ,  $u_0$ ,  $s_0$  if and only if  $\lim_{T_1 \rightarrow \infty} \xi_0^{(s_0)} = \bar{\xi}^{(s_0)}$ ,  $\lim_{T_1 \rightarrow \infty} \Gamma_0^{(s_0)} = \bar{\Gamma}^{(s_0)}$ , and  $\lim_{T_1 \rightarrow \infty} \Omega_0^{(s_0)} =$

$\bar{\Omega}^{(s_0)}$  exist for all  $s_0$ . Second, we prove the following:  $\lim_{T_1 \rightarrow \infty} \xi_0^{(s_0)} = \bar{\xi}^{(s_0)}$ ,  $\lim_{T_1 \rightarrow \infty} \Gamma_0^{(s_0)} = \bar{\Gamma}^{(s_0)}$ , and  $\lim_{T_1 \rightarrow \infty} \Omega_0^{(s_0)} = \bar{\Omega}^{(s_0)}$  exist for all  $s_0$  if and only if  $\lim_{T_1 \rightarrow \infty} \Omega_0^{(s_0)} = \bar{\Omega}^{(s_0)}$  exists for all  $s_0$  and  $r(\Psi_{\bar{F}}) < 1$ , where

$$\Psi_{\bar{F}} = \left( \bigoplus_{s_0=1}^S \left( I_n - A_0^{(s_0)} E_0(\bar{\Omega}^{(s_1)}) \right)^{-1} A_0^{(s_0)} \right) (P \otimes I_n),$$

where  $\bigoplus_{s_0=1}^S \left( I_n - A_0^{(s_0)} E_0(\bar{\Omega}^{(s_1)}) \right)^{-1} A_0^{(s_0)} = \text{diag} \left( \left( I_n - A_0^{(1)} E_0(\bar{\Omega}^{(s_1)}) \right)^{-1} A_0^{(1)}, \dots, \dots, \left( I_n - A_0^{(S)} E_0(\bar{\Omega}^{(s_1)}) \right)^{-1} A_0^{(S)} \right)$ . Since (14) decouples from (15)–(16),  $\lim_{T_1 \rightarrow \infty} \Omega_0^{(s_0)} = \bar{\Omega}^{(s_0)}$  exists independently of (15)–(16), and (15) reveals that  $\bar{\Gamma}^{(s_0)}$  exists for all  $s_0$  if  $\bar{\Omega}^{(s_0)}$  exists for all  $s_0$ . Finally, define  $\Psi_{\bar{A}} = \left( \bigoplus_{s_0=1}^S \left( I_n - A_0^{(s_0)} E_0(\bar{\Omega}^{(s_1)}) \right)^{-1} \right)$ ,  $\xi_t = (\xi_t^{(1)'}, \dots, \xi_t^{(S)'})'$ , and  $D_0 = (D_0^{(1)'}, \dots, D_0^{(S)'})'$ . When  $\lim_{T_1 \rightarrow \infty} \Omega_0^{(s_0)} = \bar{\Omega}^{(s_0)}$  exists,  $\xi_t$  evolves backwards in time (i.e., as  $t \rightarrow -\infty$  and  $T_1 \rightarrow \infty$ ) according to<sup>38</sup>

$$\xi_t = \Psi_{\bar{F}} \xi_{t+1} + \Psi_{\bar{A}} D_0. \tag{A.15}$$

(A.15) is a system of linear difference equations that is globally stable around the unique steady state,  $\lim_{t \rightarrow -\infty} \xi_t = \lim_{T_1 \rightarrow \infty} (\xi_0^{(1)'}, \dots, \xi_0^{(S)'})' = (\bar{\xi}^{(1)'}, \dots, \bar{\xi}^{(S)'})'$ , if and only if  $r(\Psi_{\bar{F}}) < 1$ . This completes the second part of the proof. ■

### A.5. Proof of Proposition 3

Let  $\mathcal{U}$  denote the set of equilibriums of (14) when  $i = 0$ :  $\mathcal{U} = \{\Omega^* | \Omega^* = (I - A_0 \Omega^*)^{-1} B_0\}$ , where  $s_t$  is dropped to reflect the permanence of policy regimes. For any  $\Omega^* \in \mathcal{U}$  and corresponding  $F^* = (I - A_0 \Omega^*)^{-1} A_0$ ,  $\Omega^*$  is a locally stable equilibrium in  $\mathcal{U}$  if and only if  $r((\Omega^*)' \otimes F^*) = r(\Omega^*)r(F^*) < 1$ . Hence, if  $r(\Omega^*)r(F^*) > 1$ ,

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<sup>38</sup>Technically, our notation in the main text assumes  $t \geq 0$ , but we can redefine  $t$  to claim that (A.15) governs the evolution of  $\xi_0$  as  $T_1 \rightarrow \infty$ .

then  $\lim_{T_1 \rightarrow \infty} \Omega_0 \neq \Omega^*$ . Given that  $\gamma \notin (\beta^{-1} - 1, \beta^{-1} + 1)$  and an interest rate peg ( $\phi_\pi = \phi_y = 0$ ) is in place at  $t = 0$ , we know from the determinacy conditions reported in table 1 that there exists a unique equilibrium of (14),  $\bar{\Omega}$ , such that  $r(\bar{\Omega}) < 1$  (i.e.,  $r(\Omega^*) > 1$  for all  $\Omega^* \in \mathcal{U} \setminus \{\bar{\Omega}\}$ ). Furthermore, McCallum (2007) shows that  $r(\bar{F}) < 1$ , such that  $r(\bar{\Omega})r(\bar{F}) < 1$ , which implies that  $\bar{\Omega}$  is a locally stable equilibrium of (14). We show that it is the unique locally stable equilibrium using analysis from Cho (2020). Define  $D = \bar{\Omega} - \Omega^*$ . Since  $\Omega^*, \bar{\Omega} \in \mathcal{U}$  we can write the following:<sup>39</sup>

$$\begin{aligned} x_t &= \bar{\Omega}x_{t-1} \\ &= A_0\bar{\Omega}x_t + B_0x_{t-1} \\ &= A_0\Omega^*x_t + A_0Dx_t + B_0x_{t-1} \\ &= F^*Dx_t + \Omega^*x_{t-1} = F^*D\bar{\Omega}x_{t-1} + \Omega^*x_{t-1}. \end{aligned}$$

Therefore,  $D = F^*D\bar{\Omega}$ , which implies  $vec(D) = ((\bar{\Omega}') \otimes F^*) vec(D)$ , such that  $r(\bar{\Omega})r(F^*) \geq 1$ . It immediately follows that  $r(F^*) > 1$  and  $r(\Omega^*)r(F^*) > 1$  since  $r(\bar{\Omega}) < 1$  and  $r(\Omega^*) > 1$ . Hence,  $\lim_{T_1 \rightarrow \infty} \Omega_0$  can only be  $\bar{\Omega}$ , and  $r(\bar{F}) = r(\Psi_{\bar{F}}) < 1$ , which means that there is no forward-guidance puzzle. We also note that we rely on local stability conditions, but the assumption of passive monetary policy and active fiscal policy for  $t \geq T_N$  gives us small  $\phi_y, \phi_\pi$  that are local to  $\phi_y = \phi_\pi = 0$  (i.e.,  $\Omega$  is in a reasonably small neighborhood of  $\bar{\Omega}$ ). ■

### A.6 Analytical Solution

Suppose  $i_t = 0$  for  $t \geq T$  and let  $i_t$  be exogenous for  $0 \leq t \leq T - 1$ . Further suppose that at time  $t = 0$ , the central bank announces  $\{i_t\}_{t=0}^{T-1}$ . We solve for the response of inflation,  $\pi_0$ , to this forward-guidance announcement as follows.

First, using techniques demonstrated in Tan and Walker (2015), Leeper and Leith (2016), and Tan (2017), substitute (19) and the

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<sup>39</sup>For simplicity, we suppress exogenous shocks and intercept terms in the model so that we can write the model in the form  $x_t = A_0E_t x_{t+1} + B_0x_{t-1}$ . We can obtain the same results if we include shocks and intercept terms.

equations for  $i_t$  into (18) to obtain a second-order difference equations for expected inflation:

$$E_t\pi_{t+2} - \gamma_1 E_t\pi_{t+1} + \gamma_0\pi_t = -\kappa i_t/\sigma\beta = x_t, \quad (\text{A.16})$$

where  $\gamma_0 = 1/\beta$ ,  $\gamma_1 = (1 + \beta + \sigma^{-1}\kappa)/\beta$ , and  $x_t = -\kappa i_t/\sigma\beta$ . We factor (39):

$$(E_t\pi_{t+2} - \lambda_1 E_t\pi_{t+1}) - \lambda_2(E_t\pi_{t+1} - \lambda_1\pi_t) = x_t,$$

where  $\lambda_1 = .5(\gamma_1 - \sqrt{(\gamma_1^2 - 4\gamma_0)})$ , and  $\lambda_2 = .5(\gamma_1 + \sqrt{(\gamma_1^2 - 4\gamma_0)})$ . Some algebra shows that  $|\lambda_1| < 1$  and  $|\lambda_2| > 1$ . Following the literature, we solve the unstable root forward, which gives us

$$E_0\pi_{t+1} = \lambda_1^{t+1}\pi_0 - \sum_{k=0}^t \lambda_1^k \sum_{j=t-k}^{T-1} \lambda_2^{t-k-j-1} x_j \quad \text{for } 0 < t < T. \quad (\text{A.17})$$

Now, set  $\gamma^{st} = 0$  and  $\gamma_y^{st} = \gamma_y$ , substitute (21) into (22), solve (22) forward, and impose the transversality condition to obtain an expression for  $b_{-1}$ ,

$$b_{-1} = E_0 \sum_{t \geq 0} \beta^{t+1} (\gamma_y y_t + \beta^{-1}\pi_t + (1 - \rho)P_t^m). \quad (\text{A.18})$$

We have

$$\begin{aligned} E_0 \sum_{t \geq 0} \beta^{t+1} (1 - \rho)P_t^m &= -\beta\rho P_t^m - E_0 \sum_{t \geq 0} \beta^{t+1} i_t \\ E_0 \sum_{t \geq 0} \beta^{t+1} \gamma_y y_t &= \beta\gamma_y \pi_0 / \kappa \\ -\beta\rho P_t^m &= E_0 \sum_{t \geq 0} (\beta\rho)^{t+1} i_t. \end{aligned}$$

Substituting the above expressions into (A.18) gives

$$b_{-1} = (1 + \beta\gamma_y/\kappa)\pi_0 + E_0 \sum_{t \geq 0} (\beta^{t+1}(\pi_{t+1} - i_t) + (\beta\rho)^{t+1} i_t). \quad (\text{A.19})$$

We have  $E_0 \sum_{t \geq 0} \beta^{t+1} (\pi_{t+1} - i_t) = \sigma E_0 \sum_{t \geq 0} \beta^{t+1} (y_t - y_{t+1}) = \sigma \beta / \kappa (\pi_0 - E_0 \pi_1)$ , where the first equality holds by substituting (18) into the first sum, and the second equality follows from (19). We also have  $E_0 \sum_{t \geq 0} (\beta \rho)^{t+1} i_t = E_0 \sum_{t=0}^{T-1} (\beta \rho)^{t+1} i_t$ . Substituting these last expressions and  $E_0 \pi_1$  from (A.17) into (A.19) gives

$$\pi_0 = \Omega_\pi(T) b_{-1} + \xi(T) \tag{A.20}$$

$$\Omega_\pi(T) = (1 + \sigma \beta \kappa^{-1} (1 - \lambda_1) + \gamma_y \beta \kappa^{-1})^{-1} \tag{A.21}$$

$$\xi(T) = -\Omega_\pi(T) \left( \sum_{t=0}^{T-1} (\beta \rho)^{t+1} i_t + \beta \sigma \kappa^{-1} \sum_{j=0}^{T-1} \lambda_2^{t-j-1} x_j \right). \tag{A.22}$$

The solution for  $\pi_0$  exists for any  $T$  given that  $|\lambda_2| > 1$ ,  $|\lambda_1| < 1$ .

Finally, to solve for  $\bar{\gamma}_y$  from section 4, set  $\Omega_\pi(T)^{-1} = 0$  and solve for  $\gamma_y$ . Define the solution  $\bar{\gamma}_y$ . Intuitively,  $\bar{\gamma}_y$  is the value that causes  $\pi_0$  to change signs. Since  $\xi(T)/\Omega_\pi(T)$  does not depend on  $\gamma_y$ , the solution  $\pi_0$  can only change when  $\gamma_y$  changes via the effects of  $\gamma_y$  and  $\Omega_\pi(T)$ .

### A.7 Bianchi and Ilut (2017) Model

Bianchi and Ilut (2017) estimate the following log-linearized system of equations (reported in their online appendix; details about the posterior mode estimates, including estimated steady states, are reported in table 1 of Bianchi and Ilut 2017):

$$\begin{aligned} \hat{y}_t = & \tilde{g}_t - \frac{1}{1 + \Phi \gamma^{-1}} E_t(\tilde{g}_{t+1}) + \frac{\Phi \gamma^{-1}}{1 + \Phi \gamma^{-1}} (\hat{y}_{t-1} - \tilde{g}_{t-1} - a_t) \\ & - \frac{1 - \Phi \gamma^{-1}}{1 + \Phi \gamma^{-1}} \left[ \tilde{R}_t - E_t[\tilde{\pi}_{t+1}] - (1 - \rho_d) d_t \right] \\ & + \frac{1}{1 + \Phi \gamma^{-1}} [E_t[\hat{y}_{t+1}] + \rho_a a_t] \end{aligned} \tag{A.23}$$

$$\begin{aligned} \tilde{\pi}_t = & \frac{\kappa(1 - \Phi\gamma^{-1})^{-1}}{1 + \varsigma\beta} \left( \left[ 1 + \frac{\alpha}{1 - \alpha} (1 - \Phi\gamma^{-1}) \right] \hat{y}_t - \tilde{g}_t \right. \\ & \left. - \Phi\gamma^{-1}(\hat{y}_{t-1} - \tilde{g}_{t-1} - a_t) \right) \\ & + \frac{\varsigma}{1 + \varsigma\beta} \tilde{\pi}_{t-1} + \frac{\beta}{1 + \varsigma\beta} E_t[\tilde{\pi}_{t+1}] + \tilde{\mu}_t \end{aligned} \quad (\text{A.24})$$

$$\begin{aligned} \tilde{R}_t = & \rho_{R, \xi_t^{SP}} \tilde{R}_{t-1} + \left( 1 - \rho_{R, \xi_t^{SP}} \right) \left[ \gamma_{\pi, \xi_t^{SP}} \tilde{\pi}_t + \gamma_{y, \xi_t^{SP}} (\hat{y}_t - \hat{y}_t^*) \right] \\ & + \sigma_{R, \xi_t^{vo}} \epsilon_{R,t} \end{aligned} \quad (\text{A.25})$$

$$\tilde{\chi}_t = \rho_{\chi} \tilde{\chi}_{t-1} + (1 - \rho_{\chi}) l_y (\hat{y}_t - \hat{y}_t^*) + \sigma_{\chi, \xi_t^{vo}} \epsilon_{\chi,t} \quad (\text{A.26})$$

$$\begin{aligned} \tilde{\tau}_t = & \rho_{\tau, \xi_t^{SP}} \tilde{\tau}_{t-1} + (1 - \rho_{\tau, \xi_t^{SP}}) \left[ \delta_{b, \xi_t^{SP}} \tilde{b}_{t-1}^m + \delta_e (\tilde{c}_t^S + \tilde{c}_t^L) \right. \\ & \left. + \delta_y (\hat{y}_t - \hat{y}_t^*) \right] + \sigma_{\tau, \xi_t^{vo}} \epsilon_{\tau,t} \end{aligned} \quad (\text{A.27})$$

$$\begin{aligned} \tilde{b}_t^m = & \beta^{-1} \tilde{b}_{t-1}^m + b^m \beta^{-1} \left( \hat{R}_{t-1,t}^m - \hat{y}_t + \hat{y}_{t-1} - a_t - \tilde{\pi}_t \right) \\ & - \tilde{\tau}_t + \tilde{c}_t^S + \tilde{c}_t^L + \tilde{t}p_t \end{aligned} \quad (\text{A.28})$$

$$\hat{R}_{t,t+1}^m = R^{-1} \rho \hat{P}_{t+1}^m - \hat{P}_t^m \quad (\text{A.29})$$

$$\tilde{R}_t = E_t [R_{t,t+1}^m] \quad (\text{A.30})$$

$$\tilde{e}_t^S = \rho_{e^S} \tilde{e}_{t-1}^S + (1 - \rho_{e^S}) \phi_y (\hat{y}_t - \hat{y}_t^*) + \sigma_{e^S, \xi_t^{vo}} \epsilon_{e^S,t} \quad (\text{A.31})$$

$$\tilde{e}_t^L = \rho_{e^L} \tilde{e}_{t-1}^L + \sigma_{e^L, \xi_t^{vo}} \epsilon_{e^L,t} \quad (\text{A.32})$$

$$\tilde{t}p_t = \rho_{tp} \tilde{t}p_{t-1} + \sigma_{tp, \xi_t^{vo}} \epsilon_{tp,t} \quad (\text{A.33})$$

$$a_t = \rho_a a_{t-1} + \sigma_{a, \xi_t^{vo}} \epsilon_{a,t} \quad (\text{A.34})$$

$$d_t = \rho_d d_{t-1} + \sigma_{d, \xi_t^{vo}} \epsilon_{d,t} \quad (\text{A.35})$$

$$\begin{aligned} \hat{y}_t^* = & \left[ \frac{1}{1 - \Phi\gamma^{-1}} + \frac{\alpha}{1 - \alpha} \right]^{-1} \\ & \left( \frac{1}{1 - \Phi\gamma^{-1}} \tilde{g}_t + \frac{\Phi\gamma^{-1}}{1 - \Phi\gamma^{-1}} (\hat{y}_{t-1}^* - \tilde{g}_{t-1} - a_t) \right) \end{aligned} \quad (\text{A.36})$$

$$\tilde{\chi}_t = \frac{1}{g-1} \tilde{g}_t - e^{-1} \tilde{e}_t \quad (\text{A.37})$$

$$\tilde{\mu}_t = \rho_{\tilde{\mu}} \tilde{\mu}_{t-1} + \sigma_{\tilde{\mu}, \xi_t^{vo}} \epsilon_{\tilde{\mu},t}, \quad (\text{A.38})$$

where  $\kappa = (1 - v)/(v\gamma\Pi^2)$ ,  $g_t = 1/(1 - \zeta_t)$ ,  $\tilde{g}_t = \ln(g_t/g)$ ,  $\tilde{\mu}_t = \frac{\kappa}{1+\varsigma\beta} \log(\aleph_t/\aleph)$ , and  $\aleph_t = \frac{1/v_t}{1/v_t - 1}$ , where  $v_t$  is described in Bianchi and Ilut (2017). (A.23) describes the evolution of output,  $\hat{y}$ ; (A.24) is the Phillips curve describing inflation,  $\tilde{\pi}$ ; (A.25) is the monetary policy rule for nominal interest rate,  $\tilde{R}$ ; (A.26) gives the ratio of government purchases to total government expenditure,  $\tilde{\chi}$ ; (A.27) is the fiscal rule for real fiscal surpluses,  $\tilde{\tau}$ ; (A.28) is the government budget constraint for the debt portfolio,  $\tilde{b}^m$ , with geometrically decaying maturity structure; (A.29) gives the rate of return on long-term bonds,  $\tilde{R}_{t,t+1}^m$ ; (A.30) is a no-arbitrage condition linking the nominal short-term interest rate,  $\tilde{R}$ , to the return on long-term debt,  $\tilde{R}_{t,t+1}^m$ ; and (A.31)–(A.38) give us short-term federal expenditures,  $\tilde{e}^s$ , long-term federal expenditures,  $\tilde{e}^L$ , term premium shock,  $\tilde{t}p$ , technology shock,  $a$ , demand shock,  $d$ , potential output,  $\hat{y}^*$ , government purchases to expenditures ratio,  $\chi$ , and markup shock,  $\tilde{\mu}$ , respectively. The market clearing condition, relating output to consumption and government purchases, is substituted into the above equations. Finally,  $\xi^{SP}$  is a three-state exogenous Markov process describing the fiscal-monetary regime, and  $\xi^{vo}$  is a two-state exogenous Markov process that captures time variation in the volatility of exogenous shocks in the model. Interested readers are referred to Bianchi and Ilut (2017) for more detail.

To compute the impulse responses in figure 6, follow these steps:

- (i) Solve the model at the posterior mode reported in table 1 of Bianchi and Ilut (2017) using, e.g., techniques from Farmer, Waggoner, and Zha (2011) as in Bianchi and Ilut (2017).<sup>40</sup> The solution can be cast in the form (13), and it describes the evolution of the economy's variables for  $t > T$ , where  $T$  is the last period of the forward-guidance policy.
- (ii) Shut down exogenous processes in (A.32)–(A.35) and (A.38), since they do not determine whether the model is subject to a forward-guidance puzzle.

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<sup>40</sup>We set  $\alpha = .33$  and  $e = 1$ , since these parameters are not reported in table 1, but we find that our qualitative results are robust to changes in these parameter values.

- (iii) Apply techniques from section 3 to the system (A.23)–(A.38) to simulate the effects of a forward-guidance announcement that sets  $\tilde{R}_t = 0$  from the time of announcement  $t = 0$  until  $T - 1$ ; sets  $\tilde{R}_t = -\bar{i}$  at  $t = T$ , where  $\bar{i}$  is the steady-state *net* nominal interest rate; and sets  $\tilde{R}_t$  according to (A.25), where (A.25) is calibrated at the posterior mode for  $t > T$ .
- (iv) Vary  $T$  and compute the inflation response at the time of announcement,  $\pi_0$ , for each  $T$ .

We also confirm that the fiscal rule calibrated at the posterior mode estimate is non-Ricardian, which means that any stable solution of the model is non-Ricardian.

### A.8 Alternative Approaches to Proposition 1

This section details alternative ways of obtaining the forward-guidance solution in the simple model (1)–(4) of section 2. We explore two alternative approaches here, and show that both recover the conditions in proposition 1 that determine when the simple model is subject to a forward-guidance puzzle.

#### A.8.1 Alternative Approach 1: Section 3 Methodology

As in section 2, assume at  $t = 0$  the central bank announces that  $i_t = \bar{i} \neq 0$  for  $t = 0, \dots, T_1 - 1$ . For  $t \geq T_1$ :

$$i_t = E_t \pi_{t+1} \tag{A.39}$$

$$b_t = \beta^{-1} (b_{t-1} - \pi_t) + i_t - \tau_t \tag{A.40}$$

$$i_t = \phi^{st} \pi_t + \epsilon_t^m \tag{A.41}$$

$$\tau_t = \gamma^{st} b_{t-1} + \epsilon_t^f. \tag{A.42}$$

Let  $x_t = (\pi_t, b_t)'$  and  $u_t = (\epsilon_t^m, \epsilon_t^f)'$ . An MSV solution of (A.39)–(A.42) for  $t \geq T_1$  assumes the form

$$x_t = \Omega^{(st)} x_{t-1} + \Gamma^{(st)} u_t, \tag{A.43}$$

where

$$\Omega^{(s_t)} = \begin{pmatrix} 0 & \Omega_\pi(s_t) \\ 0 & \Omega_b(s_t) \end{pmatrix}.$$

Here we treat (A.43) as (13) from section 3 in this application. For  $t = 0, \dots, T_1 - 1$ :

$$x_t = A_0^{(s_t)} E_t x_{t+1} + B_0^{(s_t)} x_{t-1} + D_0^{(s_t)},$$

where

$$A_0^{(s_t)} = \begin{pmatrix} 0 & -\beta \delta_{s_t}^{-1} \\ 0 & \delta_{s_t}^{-1} \end{pmatrix} \quad B_0^{(s_t)} = \begin{pmatrix} 0 & 1 - \beta \gamma^{s_t} \\ 0 & 0 \end{pmatrix},$$

where  $\delta_{s_t} = E_t(\beta^{-1} - \gamma^{s_{t-1}})$  (as in the main text of section 2). We set  $u_t = 0$  for all  $t$  because these shocks are not anticipated at the time of announcement  $t = 0$ . Finally,  $D_0^{(s_t)}$  is an exogenous vector whose terms are functions of the exogenous path for  $i_t$ ,  $t = 0, \dots, T_1 - 1$ .

To determine whether the model described above has a forward-guidance puzzle, we first iterate on

$$\Omega_t^{(s_t)} = \left( I - A_0^{(s_t)} E_t(\Omega_{t+1}^{(s_{t+1})}) \right)^{-1} B_0^{(s_t)}. \tag{A.44}$$

If  $\lim_{T_1 \rightarrow \infty} \Omega_0^{(s_0)} = \lim_{t \rightarrow -\infty} \Omega_t^{(s_t)} = \bar{\Omega}^{(s_0)}$  exists for all  $s_0$ , then we need to check the condition

$$r(\Psi_{\bar{F}}) = r \left( \left( \bigoplus_{s_0=M}^F \left( I_n - A_0^{(s_0)} E_0(\bar{\Omega}^{(s_1)}) \right)^{-1} A_0^{(s_0)} \right) (P \otimes I_n) \right) < 1.$$

Initiate (A.44) by computing  $\Omega_{T-1}^{(s_{T-1})}$ :

$$\begin{aligned} \Omega_{T-1}^{(s_{T-1})} &= \left( I - A_0^{(s_{T-1})} E_{T-1}(\Omega_T^{(s_T)}) \right)^{-1} B_0^{(s_{T-1})} \\ &= \begin{pmatrix} 1 & -\frac{\beta \delta_{T-1}^{-1} E_{T-1} \Omega_b^{(s_T)}}{1 - \delta_{T-1}^{-1} E_{T-1} \Omega_b^{(s_T)}} \\ 0 & \frac{1}{1 - \delta_{T-1}^{-1} E_{T-1} \Omega_b^{(s_T)}} \end{pmatrix} \begin{pmatrix} 0 & 1 - \beta \gamma^{(s_{T-1})} \\ 0 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 1 - \beta \gamma^{(s_{T-1})} \\ 0 & 0 \end{pmatrix}. \end{aligned}$$

Iterating further gives us the following solution for  $t = 0, \dots, T - 2$ :

$$\Omega_t^{(s_t)} = \begin{pmatrix} 0 & 1 - \beta\gamma^{(s_t)} \\ 0 & 0 \end{pmatrix} = \bar{\Omega}^{(s_t)}.$$

Since  $\bar{\Omega}^{(s_t)}$  exists, we compute  $r(\Psi_{\bar{F}})$  to check whether a forward-guidance puzzle emerges. Since our previous work in this appendix implies  $A_0^{(s_0)} E_0(\bar{\Omega}^{(s_1)}) = 0_n$ ,  $(I_n - A_0^{(s_0)} E_0(\bar{\Omega}^{(s_1)}))^{-1} A_0^{(s_0)} = A_0^{(s_0)}$ . Therefore,

$$\begin{aligned} r(\Psi_{\bar{F}}) &= \begin{pmatrix} p_M A_0^M & (1 - p_M) A_0^M \\ (1 - p_F) A_0^F & p_F A_0^F \end{pmatrix} \\ &= \begin{pmatrix} 0 & -p_M \beta \delta_M^{-1} & 0 & -(1 - p_M) \beta \delta_M^{-1} \\ 0 & p_M \delta_M^{-1} & 0 & (1 - p_M) \delta_M^{-1} \\ 0 & -(1 - p_F) \beta \delta_F^{-1} & 0 & -p_F \beta \delta_F^{-1} \\ 0 & (1 - p_F) \delta_F^{-1} & 0 & p_F \delta_F^{-1} \end{pmatrix}. \end{aligned} \tag{A.45}$$

We have that the eigenvalues of (A.45),  $\lambda$ , solve

$$\lambda^2 (\lambda^2 - \lambda(p_M \delta_M^{-1} + p_F \delta_F^{-1}) + (p_M + p_F - 1) \delta_M^{-1} \delta_F^{-1}) = 0. \tag{A.46}$$

Since two of the four eigenvalues equal zero, we only need the roots of  $\lambda^2 - \lambda(p_M \delta_M^{-1} + p_F \delta_F^{-1}) + (p_M + p_F - 1) \delta_M^{-1} \delta_F^{-1} = 0$  to be in the unit circle in order for  $r(\Psi_{\bar{F}}) < 1$  to hold. The condition determining whether these roots are inside the unit circle is derived in appendix A.2. and presented in proposition 1. Therefore,  $r(\Psi_{\bar{F}}) < 1$  if and only if proposition 1 is satisfied.

### A.8.2 Alternative Approach 2

Section 2.1 obtains a forward-guidance solution of (1)–(4) by solving the following equation forward:

$$b_t = \delta_t^{-1} (E_t b_{t+1} - \bar{v} + \beta^{-1} E_t \pi_{t+1}), \tag{A.47}$$

where  $\delta_t = E_t(\beta^{-1} - \gamma^{s_{t+1}})$ . The last equation can be recast as (8). Alternatively, we could obtain a forward-guidance solution by solving the following equation forward:

$$b_t = E_t \{ \hat{\delta}_{t+1}^{-1} (b_{t+1} - \bar{v} + \beta^{-1} \pi_{t+1}) \}, \tag{A.48}$$

where  $\hat{\delta}_{t+1} = \beta^{-1} - \gamma^{s_{t+1}}$ . We recast (A.48) as

$$\begin{aligned} b_t^M &= p_M \hat{\delta}_M^{-1} (E_t b_{t+1}^M - \bar{v} + \beta^{-1} E_t (\pi_{t+1}^M | s_t = M)) + \dots \\ &\quad (1 - p_M) \hat{\delta}_F^{-1} (E_t b_{t+1}^F - \bar{v} + \beta^{-1} E_t (\pi_{t+1}^F | s_t = M)) \\ b_t^F &= (1 - p_F) \hat{\delta}_M^{-1} (E_t b_{t+1}^M - \bar{v} + \beta^{-1} E_t (\pi_{t+1}^M | s_t = F)) + \dots \\ &\quad p_F \hat{\delta}_F^{-1} (E_t b_{t+1}^F - \bar{v} + \beta^{-1} E_t (\pi_{t+1}^F | s_t = F)), \end{aligned}$$

where  $E_t(\pi_{t+1}^i | s_t = M) \neq E_t(\pi_{t+1}^i | s_t = F)$  for  $i = M, F$ , since  $\pi$  adjusts passively to satisfy the intertemporal government budget constraint (5) (however,  $E_t(b_{t+1}^i | s_t = M) = E_t(b_{t+1}^i | s_t = F)$  for  $i = M, F$  because  $b_t$  is solved forward and only depends on current and future variables). Now consider the following:

$$\begin{aligned} b_t^M &= p_M \hat{\delta}_M^{-1} (E_t b_{t+1}^M - \bar{v} + \beta^{-1} E_t (\pi_{t+1}^M | s_t = M)) + \dots \\ &\quad (1 - p_M) \hat{\delta}_F^{-1} (E_t b_{t+1}^F - \bar{v} + \beta^{-1} E_t (\pi_{t+1}^F | s_t = M)) \\ &= p_M \hat{\delta}_M^{-1} (E_t b_{t+1}^M - \bar{v} + \beta^{-1} E_t (\pi_{t+1}^M | s_t = M)) + (1 - p_M) b_t^M \\ b_t^M &= \hat{\delta}_M^{-1} (E_t b_{t+1}^M - \bar{v} + \beta^{-1} E_t (\pi_{t+1}^M | s_t = M)). \quad (\text{A.49}) \end{aligned}$$

Also, from the Fisher relation,

$$\begin{aligned} \bar{v} &= p_M E_t (\pi_{t+1}^M | s_t = M) + (1 - p_M) E_t (\pi_{t+1}^F | s_t = M) \\ &= p_M E_t (\pi_{t+1}^M | s_t = M) + (1 - p_M) \beta \left( -E_t b_{t+1}^F + \bar{v} + \hat{\delta}_F b_t^M \right) \\ \implies E_t (\pi_{t+1}^M | s_t = M) &= (p_M)^{-1} \left( \bar{v} - (1 - p_M) \beta \left( -E_t b_{t+1}^F + \bar{v} + \hat{\delta}_F b_t^M \right) \right). \quad (\text{A.50}) \end{aligned}$$

Substituting (A.50) into (A.49) gives

$$\begin{aligned} b_t^M &= \hat{\delta}_M^{-1} \left( E_t b_{t+1}^M - \bar{v} - \frac{1 - p_M}{p_M} \left( \hat{\delta}_F b_t^M - E_t b_{t+1}^F + \bar{v} \right) + (\beta p_M)^{-1} \bar{v} \right) \\ b_t^M &= p_M \delta_M^{-1} E_t b_{t+1}^M + (1 - p_M) \delta_M^{-1} E_t b_{t+1}^F + \delta_M^{-1} (\beta^{-1} - 1) \bar{v}, \quad (\text{A.51}) \end{aligned}$$

where the last equality follows from  $\delta_M = p_M \hat{\delta}_M + (1 - p_M) \hat{\delta}_F$  and  $\delta_F = p_F \hat{\delta}_F + (1 - p_F) \hat{\delta}_M$ . Notice that (A.51) is the first equation

of (8). Hence, the alternative solution approach presented here gives the same equation for  $b_t^M$  as (8). An analogous argument gives us  $b_t^F$ .

### A.9 Cyclical Fiscal Surpluses: Supplement

In section 4.3.3, we argue that countercyclical policy raises the responses of inflation relative to the case with  $\gamma_y = 0$  when the maturity structure is short. With longer maturity, the opposite effects obtain: countercyclical policy actually lowers inflation relative to the benchmark case. To make sense of these results, it's helpful to solve (22) forward (and impose the transversality condition, take expectations):

$$b_{-1} - \pi_0 + \beta \rho P_0^m = E_0 \sum_{t \geq 0} \beta^{t+1} (\tau_t - i_t + \pi_{t+1}). \quad (\text{A.52})$$

Equation (A.52) is an equilibrium condition that is satisfied at the time of announcement,  $t = 0$ .<sup>41</sup> The forward-guidance announcement we consider lowers time-0 expectations of interest rates, which raises the right-hand side of (A.52). Since  $b_{-1}$  is predetermined and  $P_0^m$  is partially determined by the forward-guidance announcement via (23),  $\pi_0$  adjusts to ensure (A.52) holds. In the benchmark active fiscal regime calibration,  $\gamma = 0$ , such that  $E_0 \sum_{t \geq 0} \beta^{t+1} \tau_t = \gamma_y E_0 \sum_{t \geq 0} \beta^{t+1} y_t = \beta \gamma_y \kappa^{-1} \pi_0$ , where the last equality is obtained by solving (19) forward and making suitable substitutions. Hence, when  $\gamma_y \neq 0$ , the right-hand side of (A.52) also depends directly on the initial jump in inflation.

To make things more precise, assume  $\rho = 0$ , such that the fall in expected nominal interest rates, which raises the right-hand side of (A.52), requires  $\pi_0$  to jump down in order to stabilize real debt around its steady-state value. If  $\gamma_y = 0$ , this fall may be relatively large. If  $\gamma_y > 0$ , however, a given fall in inflation *also* reduces the right-hand side of (A.52). In other words, the fall in inflation triggers a countercyclical tax cut, which helps reduce the amount of debt-stabilizing deflation needed in equilibrium. Thus, for positive  $\gamma_y$  and shorter debt maturity, forward guidance has less deflationary effects than when  $\gamma_y = 0$ .

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<sup>41</sup>Of course, (A.52) does not describe an equilibrium *solution*.

Now assume  $\rho > 0$  (e.g.,  $\rho = .9$  in the benchmark calibration). Again, the fall in expected inflation raises the right-hand side of (A.52), but it also raises  $\beta\rho P_t^m$  by an amount that is increasing in  $\rho$ . In the benchmark calibration with  $\gamma_y = 0$ , the left-hand side rises by more than the right-hand side, such that  $\pi_0$  has to rise to offset this increase in bond prices. When  $\gamma_y > 0$ , however, this rise in  $\pi_0$  also increases the right-hand side of (A.52), which reduces the magnitude of the jump in equilibrium inflation. In other words, the rise in inflation triggers a countercyclical tax hike, which helps reduce the amount of debt-stabilizing inflation needed in equilibrium.

Procyclical policies have the opposite effects, but only until a certain point: if  $\gamma_y$  becomes too negative, then the sign of  $\pi_0$  “flips.” To be specific, consider the analytical forward-guidance solution of (18)–(23) that appendix A.6 derives.<sup>42</sup> The closed-form solution shows that the sign of  $\pi_0$  flips when  $\gamma$  crosses  $\bar{\gamma}_y < 0$ , where

$$\bar{\gamma}_y = \beta^{-1}\kappa(\sigma\beta\kappa^{-1}(\lambda_1 - 1) - 1)$$

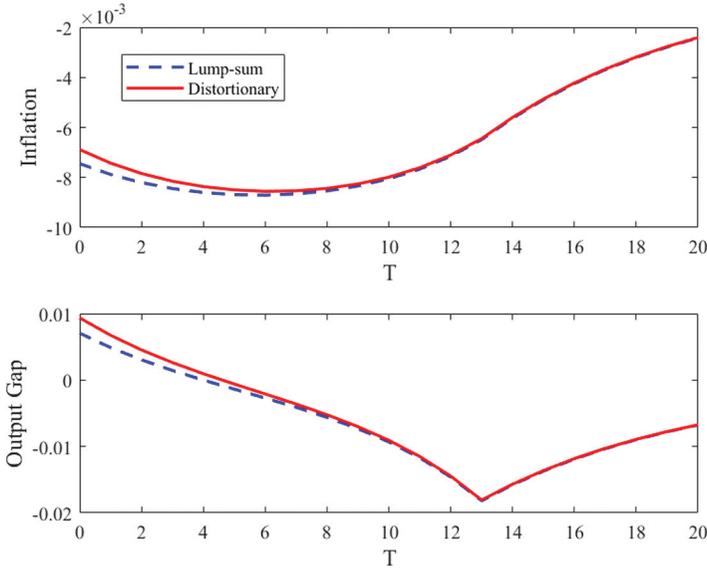
and  $\gamma_0 = 1/\beta$ ,  $\gamma_1^* = \gamma_1 = (1 + \beta + \sigma^{-1}\kappa)/\beta$ ,  $\lambda_1 = .5(\gamma_1 - \sqrt{(\gamma_1^2 - 4\gamma_0)})$ , and where  $|\lambda_1| < 1$ . For the benchmark calibration,  $\bar{\gamma}_y = -.247$ , which is a policy that calls for a .247 percent reduction in deficits in response to a 1 percent fall in output. For  $\gamma_y$  near  $\bar{\gamma}_y$ , economic responses to forward guidance diverge to positive or negative infinity. We have these extreme results near  $\bar{\gamma}_y$  because the solution has a discontinuity at  $\bar{\gamma}_y$ , and this discontinuity is a bifurcation. Importantly, a well-defined post-forward-guidance solution (13) does not exist at  $\gamma_y = \bar{\gamma}_y$ , and this prevents us from invoking proposition 3 findings when  $\gamma_y = \bar{\gamma}_y$ . However, by studying  $\gamma_y$  near  $\bar{\gamma}_y$  we see that procyclical regimes interact with forward guidance in volatile ways, even if there is no forward-guidance puzzle in the definition 1 sense.

### A.10 Distortionary Taxation

Figure A.1 illustrates the impact of the 12-quarter forward-guidance announcement studied throughout this section in a model with distortionary labor income taxes. As in the model with lump-sum taxes,

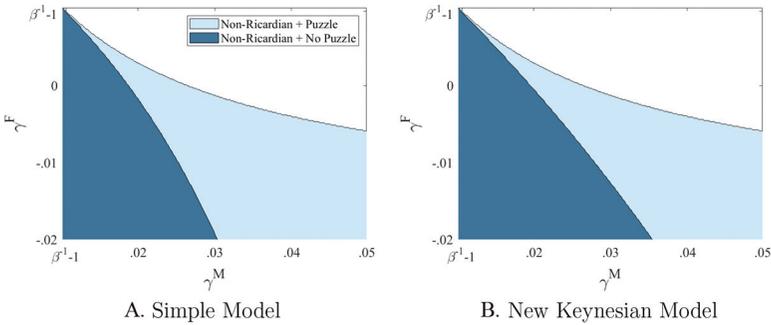
<sup>42</sup>The derivation assumes  $\phi_\pi = \phi_y = \gamma = 0$  for all  $t$ , which simplifies the derivation without sacrificing basic fiscal theory mechanisms.

**Figure A.1. Distortionary Tax**



**Note:** The vertical axes units are percent deviations from steady state (e.g., .01 is 1 percent).

**Figure A.2. Forward-Guidance Puzzle: Flexible vs. Sticky Prices**



**Notes:** The white region is the set of Ricardian fiscal policies.  $\beta = .99$ ,  $p_M = p_F = .95$  in panels A and B.

the system of equations for the log-linearized model includes (18), (20), and (23). However some modifications are made: first, (21) determines the marginal tax rate on labor income, and since the labor income tax affects marginal costs, (19) is modified to depend linearly on the marginal tax rate, (21). Finally, (22) is modified to reflect the dependence of current fiscal surpluses on both the tax rate and labor income.<sup>43</sup> Figure A.1 shows that the qualitative effects of forward guidance do not vary significantly between the model with lump-sum taxes only and the model with distortionary taxes.

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<sup>43</sup>The model we consider is a simplified version of Leith and Wren-Lewis (2013).

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