Optimal Monetary Policy in Small Open Economies: Producer-Currency Pricing*

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We establish the share of exports in production as a sufficient statistic for optimal noncooperative monetary policy. Under financial autarky, markups positively co-move with the export share. For complete markets, markups should be procyclical if the export share is procyclical. When central banks cooperate, markups are constant under complete markets, and countercyclical under financial autarky.

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1. Introduction

Although price stability is widely viewed as a benchmark monetary policy for central banks, various ingredients in the open economy drive optimal policy away from replicating the flexible-price allocation. Cooperation in the absence of commitment (Rogoff 1985), imperfect risk sharing (Corsetti, Dedola, and Leduc 2010), incomplete exchange rate pass-through (Devereux and Engel 2003), noncooperative policy (De Paoli 2009a, 2009b), and trade elasticities (Benigno and Benigno 2003) generate deviations from price stability. Because there are so many additional ingredients in the open economy relative to the closed economy, it is very difficult to suggest a one-size-fits-all optimal monetary policy like price stability.

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The precise optimal policy is sensitive to a variety of assumptions and specific parameter settings.

Our goal is to provide a coherent, tractable framework to examine optimal monetary policy in small open economies under producer-currency pricing. We establish a set of simple rules to guide central banks in four unique cases where monetary policy is either cooperative or noncooperative and markets are complete or cross-border trade in financial assets is prohibited. These four cases nest most of the key distortions in the open economy: nominal rigidities, terms-of-trade externalities, and incomplete cross-country risk sharing.

For optimal noncooperative policy, we find that central banks should generate markups that follow the share of exported goods in total production, unless these markup movements cause excess consumption volatility. For example, if the share of exported goods is procyclical, then monetary policy should generate procyclical markups. For optimal cooperative policy, markups should be constant when markets are complete, and countercyclical under financial autarky.

The paper makes several contributions to the literature. First, we provide a unified framework for studying cooperative and noncooperative optimal monetary policy under both complete markets and financial autarky. Second, our solution is analytical and covers the full range of parameter values instead of focusing on a particular calibration. Third, we are the first to study cooperative policy for small open economies under producer-currency pricing (PCP) and show how it differs from the noncooperative case. Fourth, in our study of the optimal policy, we do not restrict import and export trade elasticities to be equal to each other. Finally, we establish the export share of gross domestic product (GDP) as a sufficient statistic for noncooperative policy.

We consider a continuum of small open economies that are hit by asymmetric productivity shocks, following Galí and Monacelli (2005). We deviate from their paper in three ways. First, we do not restrict our analysis to the widely used Cole-Obstfeld (1991) specification where the coefficient of relative risk aversion and trade elasticities are set to unitary values but instead analyze the most general case analytically. Second, we extend their analysis to both cooperative and noncooperative policies under financial autarky. Finally, we utilize one-period-in-advance price rigidities used by Obstfeld and

Rogoff (2000, 2002), Corsetti and Pesenti (2001, 2005), Faia and Monacelli (2008), Dmitriev and Hoddenbagh (2019), and Egorov and Mukhin (2019) instead of a more traditional Calvo setup.

We frame monetary policy in terms of deviations from the flexible-price allocation using markups. For example, when the policymaker intends to decrease markups, she lowers the interest rate and depreciates the currency. Producer prices remain stable in the home currency and fall when expressed in foreign currencies. Thus, export volume increases. At the same time, import prices remain constant in foreign currencies and rise in the home currency. Therefore, the terms of trade depreciate. Also, locally produced goods become more competitive at home and crowd out imports in terms of volumes. Consumer prices, composed of higher import prices and stable local product prices, increase. Finally, output and employment, driven by higher demand for exports and domestic import substitution, tend to go up, raising wages and reducing markups. Thus, negative deviations of markups from the steady state are associated with expansionary monetary policy, which generates positive output gaps, currency depreciation, and price and wage inflation.

We begin our analysis by considering cooperative policy and complete markets. In this case, nominal rigidity is the only distortion present, and optimal monetary policy replicates the flexible-price allocation through a policy of price stability. While we are the first to consider cooperative policy under complete markets for small open economies, our contribution for this specification is mainly technical. For example, Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc (2010) have established that replicating the flexible-price allocation is optimal for cooperative policymakers under complete markets for two large open economies.

We next consider optimal cooperative policy under financial autarky. In this case, there are two distortions: nominal rigidities and incomplete risk sharing across countries. For empirically relevant parameter settings, we show that optimal cooperative monetary policy should generate countercyclical markups. These markup movements are designed to manipulate the terms of trade and redistribute resources from countries with positive supply shocks to countries hit by adverse shocks via terms-of-trade depreciation for the former and appreciation for the latter. As trade elasticities rise and monopoly power at the export level deteriorates, central banks lose their ability to influence the terms of trade, such that they focus more on replicating flexible prices and less on terms-of-trade adjustments.

The closest relevant study by Corsetti, Dedola, and Leduc (2010) is for large open economies. They set import and export elasticities to be equal. While their focus is when optimal policy replicates the flexible-price allocation or the first-best allocation, we study the full markup dynamics.

Next, we analyze optimal policy under financial autarky without cooperation. There are three distortions that drive the equilibrium away from the efficient allocation: nominal rigidities, terms-of-trade externalities, and market incompleteness. We give an explicit analytic expression for the optimal markup, and then establish the share of exports in production as a sufficient statistic for the optimal monetary policy. Optimal markups positively co-move with the export share. If the export share is constant, replication of the flexible-price allocation is optimal. Indeed, under noncooperative policy, risk sharing across countries becomes irrelevant for the policymaker. The policymaker desires to sell exports abroad with a positive markup and have no markup for products produced and consumed at home. Moreover, when prices are set one period in advance, policymakers cannot influence markups systematically. As a result, central banks tend to increase (decrease) markups when the export share goes up (down).

De Paoli (2009b) also analyzes noncooperative policy under financial autarky for the limiting case of two large open economies. We differ from her analysis in several ways. First, her study has a more quantitative focus so that she fixes most of the parameters to particular values and uses Calvo pricing. Instead, we consider markup movements for the broadest possible range of parameter values. Second, we provide a sufficient statistic for optimal monetary policy: the policymaker only needs the dynamics of the export share regardless of the underlying parameters. Third, we allow export and import elasticities to differ from each other.

Finally, we consider the case of complete markets and noncooperative policy. In this setting the optimal markup is procyclical whenever the export share is procyclical. A procyclical markup enables the policymaker to extract higher monopolistic rents from foreigners through terms-of-trade appreciation which stabilizes domestic consumption. On the other hand, when the export share is countercyclical, countercyclical markups generate stronger terms-of-trade externality rents and destabilize consumption. When the costs from destabilizing consumption exceed the benefits from the terms-of-trade externality, the optimal markups might be procyclical despite the countercyclical export share.

Noncooperative policy under complete markets and PCP has been studied by Faia and Monacelli (2008) and De Paoli (2009a, 2009b) for small open economies as the limiting case of two large economies. As mentioned before, we differ from these studies by differentiating between trade elasticities and by deriving analytical expressions for markups and other variables, instead of focusing on a particular calibration with Calvo pricing. We also consider a case where trade elasticities equal to each other.

Using the export share as a sufficient statistic helps to explain why under the Cole-Obstfeld (1991) specification, where trade elasticities and risk aversion are set to one, the flexible-price allocation is optimal under all four cases considered here. Under Cole-Obstfeld the export share is constant and terms-of-trade movements provide complete risk sharing. As a result, policymakers have no incentive to stabilize consumption or extract monopolistic rents from foreigners, as the export share is constant in all cases. Also, our principle explains why under noncooperative policy the flexible-price allocation is optimal for fully open economies as the export share remains equal to one over the cycle. The export share becomes less relevant for noncooperative policy only if the central bank loses its ability to influence the terms of trade. Then the optimal policy is to replicate the flexible-price allocation.

There are several reasons why we use producer-currency pricing, despite some recent empirical evidence supporting dominant-currency pricing, or DCP (Goldberg and Tille 2008; Gopinath, Itskhoki, and Rigobon 2010; Gopinath et al. 2016). First, both PCP and DCP imply equal sensitivity of import prices to the exchange rate. Second, the evidence by Amiti, Itskhoki, and Konings (2014) on export prices supports at least 50 percent exchange rate pass-through for large import-intensive exporters and full pass-through for smaller exporters, while DCP assumes no pass-through for export prices. Third, under PCP, high export elasticities allow terms of trade to be stable and independent from monetary policy, similar to

DCP. Finally, PCP, formally developed by Fleming (1962), Mundell (1963), Obstfeld and Rogoff (1995), and others, serves as the benchmark case for optimal monetary policy analysis in the open economy since other setups add extra distortions in addition to the ones present in PCP.

We differ from many papers in the field by having prices set one period in advance instead of Calvo pricing. This price setting allows us to arrive at the optimal conditions in a fully nonlinear manner. We linearize the equilibrium afterward, which makes our results robust to the Kim and Kim (2003) critique of linear-quadratic approximation.[1] Also, the Calvo approach to nominal rigidities introduces price dispersion in addition to standard output gap costs. Although price dispersion increases the complexity of the model, it is proportional to the output gap, which makes economic intuition for optimal policy similar between Calvo pricing and one-period-in-advance pricing. As our contribution is more analytical than quantitative, we use one-period-in-advance price setting for the sake of tractability.

We also differ from the optimal policy literature by studying a continuum of small open economies instead of considering a limit of two economies. The continuum allows us to differentiate import and export elasticities, which are equal in the case of two countries. In the absence of this differentiation, the analysis faces empirical and theoretical difficulties. For example, under inelastic export demand, an increase in the quantity of exports leads to lower export revenues, which makes positive supply shocks potentially welfare reversing. Second, under inelastic export demand with constant elasticity, an infinite export tax generates limitless export revenues and consumption.[2] To avoid these problems, we set the export elasticity to be greater than one, and allow the import elasticity to vary between zero and infinity. Forcing import and export elasticities to be equal allows us to nest the limiting case of two open economies. Also, a continuum of small open economies under cooperation nests two large open economies as a special case. For instance, if we divide the

[1]Note that Benigno and Woodford (2012) establish conditions under which linear-quadratic analysis is robust to the Kim and Kim (2003) critique.

[2]Infinite export tariffs are never optimal for two large open economies, where they generate a negative wealth effect through impoverishing the trading partner and provoking trade wars. However, potential welfare reversals under inelastic export demand are still present even for two large open economies.

countries in a continuum into two groups and allow them to have two realizations of the technology shocks, the framework is equivalent to two large open economies, as small open economies differ only by the realization of the technology shock. While we do not introduce symmetric shocks, the division of the asymmetric shocks into two groups of positive and negative values allows for a close approximation of two large open economies.[3]

Our research is also closely related to the old debate on the value of trade elasticities in the data. While the trade literature finds larger elasticities, with values ranging between four and five (Anderson and van Wincoop 2004; Simonovska and Waugh 2014a, 2014b; Imbs and Mejean 2015), the international macro and finance literature (a non-exhaustive list starting from Backus, Kehoe, and Kydland 1994; Stockman and Tesar 1995; and many others) often assumes these elasticities are smaller, with values ranging between 0.8 and 1.5. This debate has a strong effect on optimal monetary policy where trade elasticities play a central role. We show that this debate is not relevant for the policymaker since the effect of trade elasticities on monetary policy is expressed through the dynamics of the directly observed export share.

2. The Model

We consider a continuum of small open economies represented by the unit interval, as popularized in the literature by Galí and Monacelli (2005, 2008).[4] Each economy consists of a representative household and a representative firm. All countries are identical ex ante: they have the same preferences, technology, and price setting. Ex post, economies differ depending on the realization of their technology shock. Households are immobile across countries, while goods can move freely across borders. Each economy produces one final good, over which it exercises a degree of monopoly power. In particular, countries are able to manipulate their terms of trade even though

[3]Noncooperative policy for two large open economies differs from the noncooperative policy under a continuum due to the presence of strategic interactions.

[4]A similar version of this model appears in Dmitriev and Hoddenbagh (2019), where we employ wage rigidity instead of price rigidity and study the optimal design of a fiscal union within a currency union.

they are measure zero, similar to an individual producer in a model of monopolistic competition. However, because countries are small, they have no impact on world income or the world interest rate.

We use a one-period-in-advance price setting to introduce nominal rigidities. Monopolistic firms set the next period's nominal prices in terms of the domestic currency, before the next period's production and consumption decisions. These firms charge a constant markup in the flexible-price equilibrium, utilizing their monopoly power at the firm level. Given this preset price, firms supply as much output as demanded by households.

We lay out a general framework below and then focus on four particular cases: cooperative policy under complete markets and financial autarky, and noncooperative policy under complete markets and financial autarky.

Households. In each economy $i \in [0, 1]$, there is a representative household with lifetime expected utility

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{it+k}^{1-\sigma}}{1-\sigma} - \frac{N_{it+k}^{1+\varphi}}{1+\varphi} \right) \right\}, \tag{1}$$

where $\beta < 1$ is the household discount factor, C is the consumption basket, and N is household labor effort. Households face a general budget constraint that nests both complete markets and financial autarky; we will discuss the differences between the two in subsequent sections. For now, it is sufficient to simply write out the most general form of the budget constraint:

$$C_{it} = (1+\tau_i) \left(\frac{W_{it}}{P_{it}} \right) N_{it} + \frac{P_{F,it}}{P_{it}} \mathcal{D}_{it} + \Pi_{it} - \mathcal{T}_{it} + \frac{B_{it}}{P_{it}} - \frac{B_{it+1}}{R_{it} P_{it}}. \tag{2}$$

The distortionary subsidy on household labor income in country i is denoted by τ_i, while $\mathcal{T}_{it}$ is a lump-sum tax collected from the households to finance this subsidy. Overall, the government budget is balanced at every period. These subsidies and taxes are designed to enforce the efficient steady-state allocation. Π_{it} denotes profits from the monopolistic firms which are distributed lump sum to households. Without loss of generality, we assume that equities in the model are not traded. The consumer price index corresponds to P_{it}, while the nominal wage is W_{it}. The price index $P_{F,it}$ reflects the price of the basket composed from the imported goods expressed in units of currency i. $\mathcal{D}_{it}$ denotes net state-contingent portfolio payments expressed in the units of the imported goods, which are available to households under complete markets. This portfolio consists of state-contingent bonds, available for every state of the world. To simplify the exposition, we allow only domestic households to hold noncontingent bonds B_{it}. This is not a limitation, as foreigners have access to the portfolio of state-contingent securities. In equilibrium, noncontingent bonds are relevant only as a source of information to pin down interest rate dynamics. When international asset markets are complete, households perform all cross-border trades in contingent claims in period 0, insuring against all possible states in all future periods. Under financial autarky, households have access only to noncontingent bonds so that $\mathcal{D}_{it} = 0$.

Consumption and Price Indexes. The consumption basket for a representative small open economy i consists of home goods, $C_{H,it}$, and foreign goods, denoted by $C_{F,it}$. It is defined as follows:

$$C_{it} = \left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,it})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,it})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{3}$$

where the import basket $C_{F,it}$ is defined as

$$C_{F,it} = \left(\int_0^1 (C_{F,ijt})^{\frac{\gamma-1}{\gamma}} dj\right)^{\frac{\gamma}{\gamma-1}}. \tag{4}$$

The variable $C_{F,ijt}$ corresponds to consumption in country i of the variety produced in country j. The value of η reflects the elasticity of substitution between domestically produced goods and imported varieties in the aggregate consumption basket. The parameter γ corresponds to the elasticity of substitution between exported domestic and foreign exported varieties. The degree of consumption home bias is represented by the value $1-\alpha$. Therefore, in a fully closed economy $\alpha = 0$, and in the fully open economy $\alpha = 1$.

The consumer price index P_{it} is an aggregator of the domestic variety price $P_{H,it}$ and import price index $P_{F,it}$:

$$P_{it} = \left[(1-\alpha)P_{H,it}^{1-\eta} + \alpha P_{F,it}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{5}$$

Here the import price index $P_{F,it}$ is a constant elasticity of substitution aggregator that takes the following form:

$$P_{F,it} = \left(\int_0^1 (\mathcal{E}_{ijt} P_{H,jt})^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}, \tag{6}$$

where the variety produced in country j is sold in country i for a price $\mathcal{E}_{ijt} P_{H,jt}$. We define the nominal bilateral exchange rate $\mathcal{E}_{ijt}$ as units of currency i per one unit of currency j.

Household expenditure minimization yields the demand for the home variety $C_{H,it}$, the demand for imported varieties $C_{F,it}$, and the relative demand for the variety produced in country j and consumed in country i:

$$C_{H,it} = (1-\alpha) \left(\frac{P_{H,it}}{P_{it}} \right)^{-\eta} C_{it}, \tag{7}$$

$$C_{F,it} = \alpha \left(\frac{P_{F,it}}{P_{it}} \right)^{-\eta} C_{it}, \tag{8}$$

$$C_{F,ijt} = \left(\frac{P_{H,jt}}{P_{F,jt}} \right)^{-\gamma} C_{F,it}. \tag{9}$$

We assume that producer-currency pricing holds so that the law of one price applies. Put differently, the price of the same good is equal across countries when converted into a common currency. In this case, the good produced in country i has a price in country j equal to $\mathcal{E}_{jit} P_{H,it}$. Although the law of one price holds for individual varieties, purchasing power parity does not hold because of home bias in consumption. Nevertheless, import price indexes are identical across countries when converted to the same currency, such that $P_{F,it} = \mathcal{E}_{ijt} P_{F,jt}$. The terms of trade for country j, defined as the home-currency price of exports over the home-currency price of imports, is denoted $\tilde{P}_{H,jt} = \frac{P_{H,jt}}{P_{F,jt}}$. We define the aggregate consumer price index normalized by the import price index as $\tilde{P}_{it} = \frac{P_{it}}{P_{F,it}}$. In our model, log-deviations of $\tilde{P}_{it}$ from the steady state correspond to real exchange rate movements to a first-order approximation. Using the normalized price levels $\tilde{P}_{H,it}$ and $\tilde{P}_{it}$, we modify the demand expressions (10), (11), and (12) into

$$C_{H,it} = (1-\alpha)\tilde{P}_{H,it}^{-\eta}\tilde{P}_{it}^{\eta}C_{it}, \tag{10}$$

$$C_{F,it} = \alpha C_{it}\tilde{P}_{it}^{\eta}, \tag{11}$$

$$C_{F,ijt} = \tilde{P}_{H,it}^{-\gamma}C_{F,it}. \tag{12}$$

Goods market clearing requires that the supply of the domestic variety produced in country i equals demand from home consumers $C_{H,it}$ and foreign consumers $\int_0^1 C_{F,jit}dj$:

$$Y_{it} = C_{H,it} + \int_0^1 C_{F,jit}dj. \tag{13}$$

Substituting equations (10) and (12) into the goods market clearing condition (13) yields global demand for country i's unique variety:

$$Y_{it} = (1-\alpha)\left(\frac{P_{H,it}}{P_{it}}\right)^{-\eta} C_{it} + \left(\frac{P_{H,it}}{P_{F,it}}\right)^{-\gamma} \int_0^1 C_{F,jt}dj. \tag{14}$$

Given the symmetric structure of the model as well as the independence of idiosyncratic shocks across countries, the integral on the import basket in (14) is equivalent to the unconditional expectation, which corresponds to the ergodic mean of the import basket. More formally,

$$\int_0^1 C_{F,jt}dj = \mathbb{E}\{C_{F,it}\} = \alpha\mathbb{E}\left\{\left(\frac{P_{F,it}}{P_{it}}\right)^{-\eta} C_{it}\right\} = \alpha\mathbb{E}\{\tilde{P}_{it}^{\eta}C_{it}\}. \tag{15}$$

Noncooperative policymakers take global import $\int_0^1 C_{F,jt}dj$ as given. Substituting (15) into (14) yields the following goods market clearing condition:

$$Y_{it} = (1-\alpha)\tilde{P}_{H,it}^{-\eta}\tilde{P}_{it}^{\eta}C_{it} + \alpha\tilde{P}_{H,it}^{-\gamma}\mathbb{E}\{\tilde{P}_{it}^{\eta}C_{it}\}. \tag{16}$$

We define the share of the domestically produced variety in economy i that is exported goods as

$$E_{s,it} = \frac{\alpha\tilde{P}_{H,it}^{-\gamma}\mathbb{E}\{\tilde{P}_{it}^{\eta}C_{it}\}}{Y_{it}}. \tag{17}$$

Production. Each economy i consists of a group of intermediate goods producers, $h \in [0, 1]$, who exercise monopoly power over their unique variety, and a perfectly competitive final goods producer, who aggregates the intermediates in a constant elasticity of substitution fashion into a final good. For simplicity, we assume that intermediates are nontradable. Thus, each country bundles its intermediates into one final good, which is consumed both at home and abroad.[5]

Production of intermediates requires technology Z_{it}, which is common across firms within a country, and labor $N_{it}(h)$, which is unique to each firm. We assume that technology is independent across time and across countries (assumptions that can be easily relaxed) but is identical across firms within the same country. Given this, the production function of a representative intermediate goods firm h in country i is $y_{it}(h) = Z_{it}N_{it}(h)$, and aggregate output is described by

$$Y_{it} = Z_{it}N_{it}. \tag{18}$$

Because intermediate goods firms produce differentiated varieties, they exercise monopoly power and charge markups over their costs. A perfectly competitive final goods producer aggregates the intermediate input of each firm in the following way:

$$Y_{it} = \left[\int_0^1 Y_{it}(h)^{\frac{\varepsilon-1}{\varepsilon}} dh\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{19}$$

where ε is the elasticity of substitution between different intermediates. For country i, the price of the final good, $P_{H,it}$, is a

[5]We assume nontradable intermediates with a final tradable consumption good that aggregates those intermediates for simplicity. In Galí and Monacelli's (2005, 2008) setup, intermediate goods are tradable, such that every country's import consumption basket is made up of an infinite number of varieties imported from an infinite number of countries. This assumption requires integrating over two continuums. While it is straightforward for us to maintain their setup, we prefer the tractable alternative: a final goods producer bundles the domestically produced intermediates for export. In this way, each country produces only one unique variety, and we only need to integrate over one continuum. This assumption does not change the results in any way. In both cases the household consumption basket in each country is made up of imported goods from all i countries, which are themselves made up of intermediates produced domestically.

function of the nominal price for intermediate goods, $P_{H,it}(h)$: $P_{H,it} = \left[\int_0^1 P_{H,it}(h)^{1-\varepsilon}dh\right]^{\frac{1}{1-\varepsilon}}$. Cost minimization by the perfectly competitive final goods exporter leads to the following demand for intermediate variety h:

$$Y_{it}(h) = \left[\frac{P_{H,it}(h)}{P_{H,it}}\right]^{-\varepsilon} Y_{it}. \tag{20}$$

Intermediate goods firms choose the profit-maximizing price for their unique good one period in advance according to the following condition:

$$P_{H,it}(h) = \mu \frac{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma} Y_{it}(h) \frac{W_{it}}{Z_{it}P_{it}}\right\}}{\mathbb{E}_{t-1}\left\{\frac{C_{it}^{-\sigma} Y_{it}(h)}{P_{it}}\right\}}, \tag{21}$$

where $\mu = \frac{\varepsilon}{\varepsilon-1}$ is the markup, which defines the degree of monopoly power at the firm level.

Households maximize utility (1) subject to their budget constraint (2). The first-order condition with respect to labor gives the following household labor supply:

$$\frac{W_{it}}{P_{it}} = \frac{1}{1+\tau} N_{it}^{\varphi} C_{it}^{\sigma}. \tag{22}$$

Because firms are identical at the national level, in equilibrium $P_{H,it}(h) = P_{H,it}$ and $Y_{it}(h) = Z_{it}N_{it}(h) = Z_{it}N_{it} = Y_{it}$. To eliminate steady-state markups at the firm level and achieve the first-best steady state, we choose distortionary labor subsidies such that $\frac{\mu}{1+\tau} = 1$. Using the optimal pricing equation (21), the labor supply condition (22), and the fact that prices are preset at time $t-1$, the optimal pricing condition under PCP is

$$1 = \frac{\mathbb{E}_{t-1}\left\{N_{it}^{1+\varphi}\right\}}{\mathbb{E}_{t-1}\left\{C_{it}^{-\sigma} \frac{N_{it}Z_{it}\tilde{P}_{H,it}}{\tilde{P}_{it}}\right\}}. \tag{23}$$

We evaluate monetary policy in relation to markup fluctuations. In our setup, flexible prices are equivalent to constant markups. As marginal costs for one unit of the final good are equal to $\frac{W_{it}}{Z_{it}}$, we define the markup as

$$\mu_{it} = \frac{P_{H,it}}{\frac{W_{it}}{Z_{it}}}. \tag{24}$$

Substituting the labor supply condition (22), the terms of trade $\tilde{P}_{H,it} = \frac{P_{H,it}}{P_{F,it}}$ into (24), we obtain

$$\mu_{it} = \frac{Z_{it}\frac{\tilde{P}_{H,it}}{\tilde{P}_{it}}}{N_{it}^{\varphi} C_{it}^{\sigma}}. \tag{25}$$

Note, that under flexible prices we set $\mu_{it} = 1$, which fully corresponds to the optimal pricing condition (23) once we take out the expectations. The intuitive interpretation of (23) is that firms set up prices equal to expected marginal costs. Optimal pricing condition can also be formulated in terms of markups. Plugging (25) into (23) gives

$$1 = \frac{\mathbb{E}_{t-1}\{N_{it}^{1+\varphi}\}}{\mathbb{E}_{t-1}\{N_{it}^{1+\varphi}\mu_{it}\}}. \tag{26}$$

Intuitively, equation (26) says that markups on average are equal to one. In other words, the central bank chooses cyclical markup dynamics around one to maximize household welfare subject to market clearing constraints.

2.1 Complete Markets

In complete markets, agents in each economy have access to a full set of domestic and foreign state-contingent assets to insure against country-specific consumption risk. Households in all countries maximize their lifetime utility (1) choosing consumption, labor, and a complete set of nominal state-contingent portfolio payments, subject to the budget constraint (2). Since countries are symmetric ex ante, complete markets imply the following risk-sharing condition:

$$\frac{C_{it}^{-\sigma}}{\tilde{P}_{it}} = \frac{C_{jt}^{-\sigma}}{\tilde{P}_{jt}} \quad \forall i, j, \tag{27}$$

which states that the marginal utility from consumption of imported varieties $C_{F,it}$, which is equal to the ratio of marginal utility of consumption and normalized aggregate price index, must be equal across all countries.

When international asset markets are complete, households perform all cross-border trades in contingent claims in period 0 before the realization of any shocks, insuring against all possible states in all future periods. To ensure that there are no Ponzi schemes in issuing state-contingent securities, we impose the intertemporal asset constraint that all transactions in period 0 before the realization of shocks must be balanced. Payment for claims issued in subsequent periods must equal payment for claims received. In online appendix A (available at http://www.ijcb.org), we show that the intertemporal asset constraint for complete markets is

$$\mathbb{E}\left\{\sum_{t=1}^{\infty} \beta^t C_{it}^{-\sigma} \frac{\mathcal{D}_{it}}{\tilde{P}_{it}}\right\} = 0, \tag{28}$$

which corresponds to equation (A.6). Intuitively, the intertemporal asset constraint stipulates that the present discounted value of future earnings should be equal to the present discounted value of future consumption flows. Total state-contingent portfolio payments across countries in every period must sum to zero such that in the absence of worldwide uncertainty

$$\mathbb{E}\{\tilde{P}_{it} C_{it}\} = \mathbb{E}\{\tilde{P}_{H,it} Y_{it}\}. \tag{29}$$

In online appendix A we combine the risk-sharing condition (27) and balanced portfolio flows among all countries in the absence of symmetric shocks (29) to yield the following expression for consumption in country i:

$$\mathbb{E}\left\{\tilde{P}_{it}^{\frac{\sigma-1}{\sigma}}\right\} \tilde{P}_{it}^{\frac{1}{\sigma}} C_{it} = \mathbb{E}\left\{\tilde{P}_{H,it} Y_{it}\right\}, \tag{30}$$

which corresponds to (A.9).

Our treatment of complete markets appears somewhat different from the rest of the literature. For example, Galí and Monacelli (2005) do not use the intertemporal asset constraint (28). As a result, they assume that the value $\frac{C_{it}^{-\sigma}}{\tilde{P}_{it}}$ is constant and independent from monetary policy, which is not correct. While the expression $\frac{C_{it}^{-\sigma}}{\tilde{P}_{it}}$ is indeed independent of the realization of the shocks, our derivations show that it is a composite of ergodic means of the endogenous variables, which are not only affected by the monetary policy but can potentially have a first-order impact on the optimal policy rule itself.

2.2 Financial Autarky

In financial autarky there is no trade in state-contingent financial assets, such that $\mathcal{D}_{it} = 0 \ \ \forall i, t$. The aggregate resource constraint under financial autarky specifies that the nominal value of output in the home country must equal the nominal value of consumption in the home country:

$$P_{it}C_{it} = P_{H,it}Y_{it}. \tag{31}$$

Normalizing this expression by $P_{F,it}$ gives

$$\tilde{P}_{it}C_{it} = \tilde{P}_{H,it}Y_{it}. \tag{32}$$

2.3 Technology Shocks

We assume that technology shocks are independent and identical across time and countries, and have a log-normal distribution such that

$$\log Z_{it} \sim \mathcal{N}(0, \sigma_z^2). \tag{33}$$

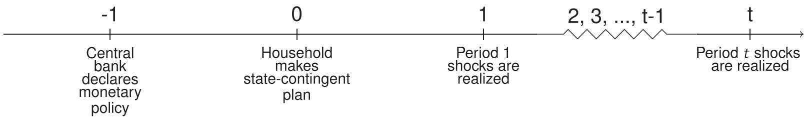
The model is formulated such that independence across time can be relaxed. It is also straightforward to relax the assumption of independence across countries by introducing a global aggregate component to technology. Our conclusions are robust to assumption of iid exogenous shock dynamics.

3. Optimal Monetary Policy

3.1 Setting Up the Optimization Problem

Without loss of generality, we assume a cashless limiting economy. Central banks use an interest rate rule to set monetary policy,

Figure 1. The Timeline of the Model



which affects the dynamics of the nominal exchange rate through uncovered interest rate parity. Under PCP, fluctuations in the nominal exchange rate pass through fully to import prices expressed in domestic currency $P_{F,it}$. As the price of the final domestic good is fixed one period in advance, nominal exchange rate fluctuations affect the terms of trade $\tilde{P}_{H,it}$. The terms of trade, in turn, affect the real exchange rate, consumption, and hours worked. Ultimately, a change in hours worked affects household disutility from labor, wages, and markups. We refer the reader to online appendix D, where the relationship between the interest rate and the terms of trade is formally established.

The timing of the model is described in figure 1. Before any shocks are realized, national central banks declare their policy for all states of the world. With this knowledge in hand, households lay out a state-contingent plan for consumption, labor hours, money, and asset holdings. After that, shocks hit the economy. Note that under financial autarky, no international asset trading occurs.

We summarize the optimization problem for the four cases we consider below. In each economy, the central bank maximizes the utility[6] of the representative household

$$\max_{\tilde{P}_{H,t},P_t,C_t,N_t} \mathbb{E}\left\{\sum_{t=1}^{\infty}\beta^t\left[\frac{C_t^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\varphi}}{1+\varphi}\right]\right\}, \quad (34)$$

subject to the optimal pricing condition (35), goods market clearing (36), aggregate consumer price index (37), and asset market clearing (38). The first-order conditions with respect to the terms of trade

[6] Pricing one period in advance allows us to derive fully nonlinear first-order conditions without relying on linear-quadratic approximations, similar to Egorov and Mukhin (2019).

$\tilde{P}_{H,t}$[7], the normalized price index $\tilde{P}_t$, consumption C_t, and labor N_t are

$$\mathbb{E}_{t-1}\left\{C_t^{-\sigma}\frac{N_t Z_t \tilde{P}_{H,t}}{\tilde{P}_t}\right\} = \mathbb{E}_{t-1}\left\{N_t^{1+\varphi}\right\}, \tag{35}$$

$$Z_t N_t = (1-\alpha)\tilde{P}_{H,t}^{-\eta} C_t P_t^{\eta} + \alpha \tilde{P}_{H,t}^{-\gamma}(\mathbb{1}_{CP}\mathbb{E}[C_t P_t^{\eta}] + (1-\mathbb{1}_{CP})C_F), \tag{36}$$

$$\tilde{P}_t^{1-\eta} = (1-\alpha)\tilde{P}_{H,t}^{1-\eta} + \alpha, \tag{37}$$

$$C_t = \mathbb{1}_{CM}\frac{\mathbb{E}\{Z_t N_t \tilde{P}_{H,t}\}}{\tilde{P}_t^{\frac{1}{\sigma}}\mathbb{E}[\tilde{P}_t^{\frac{\sigma-1}{\sigma}}]} + (1-\mathbb{1}_{CM})\frac{Z_t N_t \tilde{P}_{H,t}}{\tilde{P}_t}. \tag{38}$$

Without loss of generality subscript i is omitted in expressions (34)–(38), as the economies effectively differ only by the realization of the technology shock. The indicator function $\mathbb{1}_{CP}$ is equal to one when the policy is cooperative, and zero otherwise. The indicator function $\mathbb{1}_{CM}$ is equal to one when financial markets are complete, and zero under financial autarky. When the policy is cooperative, the domestic central bank takes into account the effect its policy on other countries through the average consumption of foreign goods $\int_0^1 C_{F,it}di = \mathbb{E}C_{F,it} = \mathbb{E}[C_t\tilde{P}_t]$. On the other hand, when the policy is noncooperative, central banks take aggregate world consumption of foreign goods as given so that $\int_0^1 C_{F,it} = C_F$.

Stochastic processes for the terms of trade $\tilde{P}_{H,t}$ and the technology shock Z_t fully define the equilibrium path for the endogenous variables $\tilde{P}_t$, C_t, N_t using (35)–(38). Under the optimal monetary policy, the terms of trade $\tilde{P}_{H,t}$ reacts endogenously to the technology shock Z_t so that all variables $\tilde{P}_{H,t}$, $\tilde{P}_t$, C_t, N_t can be expressed as

[7] As mentioned at the beginning of this section, the central bank affects the terms-of-trade path only indirectly through its choice of the interest rate path, which in turn affects the nominal exchange rate through uncovered interest rate parity. Under producer-currency pricing, the price of home exports is set one period in advance in domestic currency, while import prices move with the exchange rate, changing the terms of trade. The equivalence between the allocation resulting from direct central bank control of the terms of trade and the allocation resulting from the central bank choosing an interest rate path is established in online appendix D.

functions of the technology shock. Before we analyze optimal monetary policy, let us consider a few stochastic processes for the terms of trade.

3.2 Stochastic Processes for the Terms of Trade

Traditionally, monetary policy in closed economies is expressed using nominal interest rate rules such as the Taylor rule. In our model, the interest rate affects the rest of the economy through the terms of trade. For analytical convenience, we consider shocks to the terms of trade instead of shocks to the interest rate rule. In particular, we employ a class of interest rate paths that generate the following stochastic process for the terms of trade:

$$\tilde{P}_{H,t} = f(Z_t). \tag{39}$$

We focus on interest rate rules that are not history dependent, where the terms of trade react only to the technology shock in the current period. The log-linear equivalent of the process described by (39) is

$$\hat{\tilde{P}}_{H,t} = \alpha_T \hat{Z}_t. \tag{40}$$

After we combine (39) with the constraints (35)–(38), we can express the dynamics of the log-linearized endogenous variables $(\hat{Y}_t, \hat{C}_t, \hat{N}_t, \hat{\tilde{P}}_t,\ \hat{\mu}_t)$ in terms of the technology shock. The search for the optimal monetary policy rule is equivalent to searching for the coefficient α_T that gives the highest welfare. To compare different policy rules and values for α_T, we need to know how terms-of-trade fluctuations affect other endogenous variables for a given technology shock. Lemma 1 below summarizes the impact of the terms of terms of trade on other key endogenous variables for a given technology shock.

LEMMA 1. *For an equilibrium that is characterized by (35)–(39) and for a given realization of the technology shock, a monetary policy that generates lower (higher) terms of trade also leads to lower (higher) markups, and higher (lower) consumption, employment, and output. This result holds under financial autarky and complete markets, for cooperative and noncooperative policies.*

Proof. See online appendix C.1. ∎

Lemma 1 describes the effect of the terms-of-trade depreciation on the economy for a given technology shock. The transmission mechanism from expansionary monetary policy through terms-of-trade depreciation into markups, consumption, hours worked, and output is intuitive. Lower terms of trade lead to an increase in consumption, employment, and output through higher exports and import substitution, as currency depreciation makes domestic goods more competitive at home and abroad. Higher demand for labor drives up wages, and in the presence of price rigidities reduces markups.

It is natural to consider the stochastic process for the terms of trade that can replicate the flexible-price allocation.

LEMMA 2. *Under the flexible-price allocation, a positive technology shock increases consumption and output and leads to a decline in the terms of trade and a real exchange rate depreciation. This result holds under both financial autarky and complete markets.*

Proof. See online appendix C.2. ∎

Lemma 2 states that under flexible prices, a positive technology shock increases consumption and output, similar to a closed economy. An increase in productivity also leads to a decrease in the terms of trade as abundant home-produced goods become cheaper relative to foreign goods.

The optimal monetary policy under some circumstances differs from replication of the flexible-price allocation. We use markup dynamics to characterize monetary policy, while in the literature monetary policy is often characterized by the importance of flexible prices or exchange rate stabilization. In the corollary that follows, we show that the policymaker, who desires to increase markups relative to the flexible-price allocation, should appreciate the currency, increase terms of trade, and decrease the output gap.

LEMMA 3. *Under both financial autarky and complete markets, a monetary policy that generates procyclical (countercyclical) markups also gives rise to a countercyclical (procyclical) output gap and a more (less) stable real exchange rate.*

Proof. Lemma 3 establishes a monotonic relationship between the terms of trade, markups, and output. Since the output gap is zero when markups are equal to one, and for any given state of the world higher markups imply lower output, markups above one necessarily imply that output gaps are negative, and the terms of trade and real exchange rate are lower than they would be under flexible prices. ■

To summarize, lemmas 1, 2, and 3 state that the model we consider does not generate any nonstandard dynamics. The impact of monetary policy or productivity shocks is consistent with standard closed or open economy models. With these mechanisms established for simple monetary rules, we are now ready to consider optimal monetary policy.

3.3 Complete Markets and Cooperative Policy

In this section, we examine the optimal monetary policy for cooperative central banks under complete markets. In this setup, nominal rigidity is the only distortion present. Therefore, replicating flexible prices is an optimal policy that also implements the efficient allocation, which is stated more formally in the proposition below.

PROPOSITION 1. *In complete markets, cooperative central banks maximize (34) subject to (35), (36), (37), and (38), where* $\mathbb{1}_{CP} = 1$ *and* $\mathbb{1}_{CM} = 1$. *The indicators for cooperative policy* $\mathbb{1}_{CP}$ *and complete markets* $\mathbb{1}_{CM}$ *are set to one. The solution is*

$$\hat{\mu}_t = 0.$$

The resulting equilibrium allocation exactly coincides with the flexible-price allocation. Mimicking the flexible-price allocation is the optimal policy under cooperation, and corresponds to the social planner allocation.

Proof. See online appendix B.1. ■

In our setup, constant markups imply flexible prices. Moreover, since labor subsidies remove monopolistic distortions, and as complete markets provide full risk sharing, the resulting allocation is efficient. The finding that the flexible-price allocation is the optimal policy under cooperation, and corresponds to the social planner allocation, aligns with Benigno and Benigno (2006) and Corsetti, Dedola, and Leduc's (2010) results for two large economies. Our contribution here is mostly technical.

3.4 Financial Autarky and Cooperative Policy

In this section, we consider the optimal cooperative monetary policy under financial autarky. To our knowledge, we are the first to consider this case for small open economies. Corsetti, Dedola, and Leduc (2010) consider cooperative policy under financial autarky for two open economies. Before comparing our results, we formulate the optimal cooperative policy problem under financial autarky.

PROPOSITION 2. *In financial autarky, cooperative central banks maximize (34) subject to (35), (36), (37), and (38), where* $\mathbb{1}_{CP} = 1$ *and* $\mathbb{1}_{CM} = 0$*. The indicators for cooperative policy* $\mathbb{1}_{CP}$ *and complete markets* $\mathbb{1}_{CM}$ *are set to one and zero, respectively. The solution is given by*

$$\hat{\mu}_t = \frac{G_2}{F_2}\hat{Z}_t, \tag{41}$$

where

$$\begin{aligned} G_2 &= -\alpha(1+\varphi)((1-\alpha)(\eta\sigma-1)+\sigma(\gamma-1)), \\ F_2 &= \varphi+\sigma+\alpha\eta+\alpha\gamma-2\eta\sigma+\eta^2\sigma+\gamma^2\sigma+\alpha^2\eta^2\sigma-2\gamma\sigma-\alpha^2\eta \\ &\quad -2\alpha\eta^2\sigma-2\alpha\eta\gamma\sigma+2\eta\gamma\sigma+2\alpha\eta\sigma \\ &\quad -2\alpha\varphi-2\eta\varphi-2\gamma\varphi+\alpha^2\varphi+\eta^2\varphi+\gamma^2\varphi+\alpha^2\eta^2\varphi+4\alpha\eta\varphi \\ &\quad +2\alpha\gamma\varphi+2\eta\gamma\varphi-2\alpha\eta^2\varphi-2\alpha^2\eta\varphi-2\alpha\eta\gamma\varphi. \end{aligned}$$

The resulting equilibrium allocation differs from the flexible-price allocation.

Proof. See online appendix B.2. ∎

The resulting markup movement reported in equation (41) is a complicated function of openness, trade elasticities, labor disutility, and risk aversion. Although the monetary authority will deviate from replicating the flexible-price allocation in general, there are four specific calibrations where the flexible-price allocation will be optimal for the policymaker. Corollary 2.1 details each of these four cases.

COROLLARY 2.1. *Cooperative central banks under financial autarky implement the flexible-price allocation, whenever any of the four listed conditions below is satisfied:*

(i) under Cole-Obstfeld conditions, when $\sigma = \gamma = \eta = 1$*;*

(ii) under full home bias, $\alpha = 0$*;*

(iii) whenever $(\eta\sigma - 1)(1 - \alpha) + \sigma(\gamma - 1) = 0$*;*

(iv) for an economy with stable terms of trade, when $\gamma \rightarrow \infty$ *or* $\eta \rightarrow \infty$*.*

Proof. Under (i), (ii), and (iii) in equation (41) we have $G_2 = 0$, which implies constant markups and flexible prices. With respect to (iv), we know that as the export elasticity γ or the import elasticity η increase, the numerator F_2 grows faster than G_2. Or, more formally, the following relationships hold as export elasticity increases: $\lim_{\gamma \rightarrow \infty} \frac{G_2}{\gamma} \rightarrow -\alpha(1+\varphi)\sigma$, $\lim_{\gamma \rightarrow \infty} \frac{F_2}{\gamma^2} \rightarrow \sigma + \varphi$, thus $\lim_{\gamma \rightarrow \infty} \frac{G_2}{F_2} = \hat{\mu}_t \rightarrow 0$. Also, the following relationships apply as import elasticity grows: $\lim_{\eta \rightarrow \infty} \frac{G_2}{\eta} \rightarrow -\alpha(1-\alpha)(1+\varphi)\sigma$, $\lim_{\eta \rightarrow \infty} \frac{F_2}{\eta^2} \rightarrow (1-\alpha)^2(\sigma+\varphi)$, thus $\lim_{\eta \rightarrow \infty} \frac{G_2}{F_2} = \hat{\mu}_t \rightarrow 0$. ∎

Under cooperative policy and financial autarky policymakers face two distortions: nominal rigidities and incomplete cross-country risk sharing. Optimal monetary policy in this setting thus faces a trade-off between mitigating the distortionary impact of nominal rigidities through price stability as well as the mitigation of incomplete cross-country risk sharing via terms-of-trade adjustments. Under Cole-Obstfeld conditions, where $\sigma = \gamma = \eta = 1$, terms-of-trade movements provide full risk sharing, such that the optimal monetary policy is to mimic the flexible-price allocation and thereby eliminate distortions from nominal rigidities. In a closed economy

($\alpha = 0$), risk sharing cannot be improved by trade flows, and the central bank thus focuses on eliminating the internal distortion arising from nominal rigidities via replication of the flexible-price allocation. Finally, as the trade elasticities increase (η, γ), monopoly power at the national level is reduced, and policymakers are less able to influence the terms of trade through monetary policy. To summarize, policymakers focus on price stability when terms-of-trade movements provide full risk sharing under flexible prices, or monetary policy is powerless to reduce this risk sharing.

How do cooperative central banks improve risk sharing across countries? Under cooperative monetary policy, countries with positive productivity shocks reduce their markups and depreciate their currencies and terms of trade via lower interest rates in order to supply exports for the rest of the world at reduced prices. On the other hand, countries with negative productivity shocks increase their interest rate, markups, and terms of trade in order to sell exports at higher prices. This cooperative monetary policy response to asymmetric shocks stabilizes employment and consumption across countries.

Corollary 2.2 below summarizes formally the conditions required for countercyclical markups.

COROLLARY 2.2. *If* $\gamma > 1-\eta(1-\alpha)+\frac{1-\alpha}{\sigma}$, *then markup* $\hat{\mu}_t$ *negatively co-moves with output.*

Proof.

$$\hat{\mu}_t = -\alpha \frac{(1-\alpha)(\eta\sigma - 1) + \sigma(\gamma - 1)}{(1-\alpha)^2(\eta - 1)^2 + (\gamma - 1)^2 + 2\eta\gamma(1-\alpha) + 2\alpha\gamma - 1} \hat{Y}_t. \tag{42}$$

Since $\gamma \geq 1$, $\eta > 0$, and $\alpha > 0$, the denominator in equation (42) is positive. Indeed, the denominator monotonically increases with γ. One can also show that for $\gamma = 1$, the denominator is equal to $((1-\alpha)\eta - \alpha)^2 \geq 0$. Therefore, the denominator is positive for $\gamma > 1$. Under $\gamma > 1 - \eta(1-\alpha) + \frac{1-\alpha}{\sigma}$, the numerator in the fraction is positive as well. Therefore, overall coefficient on the right-hand side is negative. ∎

For example, if $\sigma \geq 2$, then $\gamma \geq 1.5$ guarantees that, regardless of other parameters, the policymaker reduces markups in response to higher output, further boosting production. Since in the data the trade elasticity η is positive and risk aversion σ is ≥ 2, markups will co-move negatively with output for almost all parameter values. As a result, cooperative monetary policy in financial autarky yields countercyclical markups and procyclical output gaps, and manipulates the terms of trade to increase cross-border risk sharing.

Corsetti, Dedola, and Leduc (2010) study cooperative policy under financial autarky for two open economies. We differ from their research in several dimensions. First, we use prices set one period in advance instead of Calvo pricing. Second, we allow trade elasticities to be different from each other. Third, they analyze first-order conditions without solving fully for each allocation. For example, they find out that the optimal policy under financial autarky faces a tradeoff between stabilizing output gaps, price dispersion, and global demand imbalances. However, their work does not provide an explicit roadmap for when the central bank should seek to depreciate or appreciate the exchange rate given certain parameter values.

Our analytical solution allows us to see the tradeoff between the output gap stabilization and imperfect risk sharing explicitly. In particular, manipulation of the terms of trade should deliver risk sharing, and the policymakers try to move international prices to deliver higher risk sharing relative to maximizing output gaps. We solve for the explicit policy rule under central bank cooperation: countries with a positive shock lower their interest rate and depreciate their currency and terms of trade relative to the flexible-price allocation in order to provide cheaper products to the rest of the world and stabilize consumption abroad.

Our analytical solution allows us to investigate the role of specific parameters. When trade elasticities are low (η and γ close to one), goods are less substitutable across countries, and central banks exert a stronger influence on the terms of trade because of monopoly power at the export level. However, under low trade elasticities the terms of trade also provide a high degree of risk sharing across countries, mitigating the need for central banks to deviate from the flexible-price allocation. On the other hand, as trade elasticities increase, the terms of trade provide less risk sharing across countries without policy intervention, and thus policymakers will manipulate the terms of trade to improve risk sharing and move the economy closer to the efficient allocation. The caveat is that as goods become more substitutable, policymakers' capacity to influence the terms of trade declines. As a result, under high trade elasticities, central banks will focus more on closing national output gaps and replicating the flexible-price allocation than improving risk sharing. Overall, optimal monetary policy will be closer to the flexible-price allocation for low and high trade elasticities, while deviations from flexible prices will be strongest for intermediate values of the trade elasticities.

3.5 Financial Autarky and Noncooperative Policy: Introduction of the Export Share

In this section, we study noncooperative monetary policy under financial autarky. There are three distortions that may drive the equilibrium away from the first-best allocation. In addition to nominal rigidities and incomplete risk sharing across countries, policymakers exploit terms-of-trade externalities to boost national welfare. Moreover, while incomplete cross-country risk sharing moves the equilibrium away from the first best, this distortion only has an indirect effect on the policymaker, who is disinterested in providing cross-country risk sharing. Nevertheless, market incompleteness affects the dynamics of the endogenous variables and has an effect on the tradeoff the policymaker faces between terms-of-trade externalities and nominal rigidities. In general, the policymaker chooses to deviate from the flexible-price equilibrium in this environment. Proposition 3 below formally establishes that optimal markups deviate from one, such that the flexible-price allocation is suboptimal.

Proposition 3. *In financial autarky, noncooperative central banks maximize (34) subject to (35), (36), (37), and (38), where the indicators for cooperative policy $\mathbb{1}_{CP}$ and complete markets $\mathbb{1}_{CM}$ are set to zero. The solution is*

$$\hat{\mu}_t = \frac{G_3}{F_3}\hat{Z}_t, \tag{43}$$

where

$$
\begin{aligned}
G_3 &= -\alpha(1-\alpha)(1+\varphi)(\eta-1)(\eta-1+\gamma),\\
F_3 &= 2\alpha\varphi - \sigma - \alpha\gamma - \varphi + \alpha\sigma + 3\eta\varphi + 3\gamma\varphi + 3\eta\sigma + 3\gamma\sigma + \alpha^2\eta\\
&\quad - \alpha^3\eta + \alpha\gamma^2 - \alpha^2\varphi - 3\eta^2\varphi + \eta^3\varphi - 3\gamma^2\varphi + \gamma^3\varphi\\
&\quad - 3\eta^2\sigma + \eta^3\sigma - 3\gamma^2\sigma + \gamma^3\sigma - \alpha^2\eta^2 + \alpha^3\eta^2 - 7\alpha^2\eta^2\varphi\\
&\quad + 3\alpha^2\eta^3\varphi + 2\alpha^3\eta^2\varphi - \alpha^3\eta^3\varphi - 5\alpha^2\eta^2\sigma + 3\alpha^2\eta^3\sigma + \alpha^3\eta^2\sigma\\
&\quad - \alpha^3\eta^3\sigma + \alpha\eta\gamma - 7\alpha\eta\varphi - 4\alpha\gamma\varphi - 5\alpha\eta\sigma - 2\alpha\gamma\sigma - 6\eta\gamma\varphi\\
&\quad - 6\eta\gamma\sigma - \alpha^2\eta\gamma + 8\alpha\eta^2\varphi + 5\alpha^2\eta\varphi - 3\alpha\eta^3\varphi - \alpha^3\eta\varphi\\
&\quad + 2\alpha\gamma^2\varphi + \alpha^2\gamma\varphi + 7\alpha\eta^2\sigma + 2\alpha^2\eta\sigma - 3\alpha\eta^3\sigma + \alpha\gamma^2\sigma + 3\eta\gamma^2\varphi\\
&\quad + 3\eta^2\gamma\varphi + 3\eta\gamma^2\sigma + 3\eta^2\gamma\sigma + 8\alpha\eta\gamma\sigma - 3\alpha\eta\gamma^2\varphi\\
&\quad - 6\alpha\eta^2\gamma\varphi - 4\alpha^2\eta\gamma\varphi - 3\alpha\eta\gamma^2\sigma - 6\alpha\eta^2\gamma\sigma - 2\alpha^2\eta\gamma\sigma\\
&\quad + 3\alpha^2\eta^2\gamma\varphi + 3\alpha^2\eta^2\gamma\sigma + 10\alpha\eta\gamma\varphi.
\end{aligned}
$$

Proof. See online appendix B.3. ∎

Equation (43) describes markup fluctuations as a function of technology shocks. While (43) is more complex than the relationship we can obtain by expressing markup dynamics in terms of the other endogenous variables, it clearly reveals under what conditions the flexible-price allocation is optimal. Corollary 3.1 lists the conditions when the flexible-price allocation is optimal.

COROLLARY 3.1. *Noncooperative central banks under financial autarky implement the flexible-price allocation whenever any of the four listed conditions below is satisfied:*

- *(i) under unitary import elasticity,* $\eta = 1$ *(including Cole-Obstfeld conditions* $\sigma = \gamma = \eta = 1$*);*
- *(ii) under full home bias,* $\alpha = 0$;
- *(iii) under no home bias,* $\alpha = 1$;
- *(iv) under stable terms of trade,* $\gamma \to \infty$ *or* $\eta \to \infty$*;*
- *(v) under extreme risk aversion,* $\sigma \to \infty$.

Proof. Under (i), (ii), and (iii) in equation (43) we have $G_2 = 0$, which implies constant markups and flexible prices. With respect to (iv), as the export elasticity γ increases, the numerator F_3 grows faster than G_3. More formally, $\lim_{\gamma\to\infty} \frac{G_3}{\gamma} \to -\alpha(1-\alpha)(1+\varphi)(\eta-1)$, $\lim_{\gamma\to\infty} \frac{F_3}{\gamma^3} \to \sigma+\varphi$. Thus, markups become more stable as the export elasticity increases and eventually become constant: $\lim_{\gamma\to\infty} \hat{\mu}_t = \lim_{\gamma\to\infty} \frac{G_3}{F_3} = 0$. We also know that as the import elasticity η increases, the numerator F_3 grows faster than G_3. More formally, $\lim_{\eta\to\infty} \frac{G_3}{\eta^2} \to -\alpha(1-\alpha)(1+\varphi)$, $\lim_{\eta\to\infty} \frac{F_3}{\eta^3} \to (1-\alpha)^3(\sigma+\varphi)$. Thus, markups become more stable as the import elasticity grows and eventually become constant: $\lim_{\eta\to\infty} \hat{\mu}_t = \lim_{\eta\to\infty} \frac{G_3}{F_3} = 0$. Finally, as risk aversion $\sigma \to \infty$, the numerator G_3 remains constant, while the denominator $F_3 \to \infty$. Thus, markups become more stable or $\lim_{\sigma\to\infty} \hat{\mu}_t = \lim_{\sigma\to\infty} \frac{G_3}{F_3} = 0$. ∎

In financial autarky, the intuition for case (ii) and (iv) in corollary 3.1 is similar under cooperative and noncooperative policy. In a closed economy (case (ii)), manipulation of the terms of trade does not generate any monopolistic rents, as there are no exports. Thus, optimal cooperative and noncooperative policy focuses on replicating the flexible-price allocation. Regarding case (iv), as goods become more substitutable, national monopoly power at the export level is reduced, and in the limit the central bank's ability to influence the terms of trade through monetary policy is eliminated. As a result, optimal monetary policy will focus on alleviating the distortion from nominal rigidities and the central bank will replicate the flexible-price allocation. However, this is where the similarities between optimal cooperative and noncooperative policies in financial autarky end.

Under extreme risk aversion in part (v), the flexible-price allocation stabilizes consumption. Under cooperative policy in financial autarky, joint manipulation of the terms of trade allows policymakers to overcome their inability to share risk through international asset markets via the adjustment of international prices to stabilize consumption across countries. In the absence of coordination, deviation from flexible prices causes consumption and labor to fluctuate, which is infinitely costly under extreme risk aversion. Thus, replication of the flexible-price allocation is optimal for noncooperative policymakers in financial autarky.

Also, the results on cooperative policies are no longer relevant for part (i) and (iii) of corollary 3.1. In a fully open economy and under unitary import elasticity, the flexible-price allocation is optimal. In both cases the share of goods exported is constant and independent from technology shocks. While in a fully open economy the export share equals one, under unitary import elasticities the export share is equal to the degree of openness α. We can formally show this by plugging (38) into (36), where we set $\eta = 1$, $\mathbb{1}_{CM} = 0$, and $\mathbb{1}_{CP} = 0$.

In corollary 3.1 we see that a constant export share implies constant markups, which implies a relationship between the dynamics of the export share and optimal markups. Corollary 3.2 below formally establishes this relationship.

COROLLARY 3.2. *Under the optimal noncooperative monetary policy in financial autarky, there is positive co-movement between the markup and the export share.*

Proof. We can express markup dynamics in terms of export share dynamics using the results from online appendix B.3, equation (B.77):

$$\hat{\mu}_t = \frac{\alpha(\eta + \gamma - 1)}{(\eta(1-\alpha) + \gamma - 1)^2 + \alpha(\eta(1-\alpha) + \gamma - 1)} \hat{E}_{s,t}. \tag{44}$$

For $\gamma \geq 1, \eta > 0, 0 < \alpha < 1$, we have $\frac{\alpha(\eta+\gamma-1)}{(\eta(1-\alpha)+\gamma-1)^2+\alpha(\eta(1-\alpha)+\gamma-1)} > 0$. ■

Corollary 3.2 holds that when the export share goes up, the markup should increase. Since markups negatively co-move with the output gap, the latter should also negatively correlate with the export share. In other words, monetary policy should be expansionary when the export share goes down, and contractionary when the share of goods exported goes up.

Why do optimal markups tend to move with the share of exports in production? In the steady state our model sets markups equal to one, which allows the implementation of the efficient cooperative steady state. However, under noncooperative policy, the goal of the policymaker is to charge only domestic households markups equal to one. Since the country has monopolistic power with an elasticity of foreign demand equal to γ, the policymakers try to maximize their monopolistic rents by selling goods abroad with markups $\frac{\gamma}{\gamma-1}$. But under the law of one price, the markup is the same regardless of whether producers sell goods abroad or at home. Thus, the optimal markup is an average between one and $\frac{\gamma}{\gamma-1}$. One can show that in the symmetric steady state the optimal noncooperative markup is equal to $1 + \frac{\alpha}{\gamma-1+(1-\alpha)\eta}$.

LEMMA 4. *In the steady state, noncooperative social planners maximize (34) subject to (36), (37), and (38), where the indicators for cooperative policy $\mathbb{1}_{CP}$ and complete markets $\mathbb{1}_{CM}$ are set to zero. The technology level Z is equal to one, as there are no technology shocks in steady state. The optimal markup for the social planner in the steady state is*

$$\mu = 1 + \frac{\alpha}{\gamma - 1 + (1-\alpha)\eta}.$$

Proof. See online appendix C.3. ∎

The optimal markup converges to one for the closed economy as α approaches zero, and converges to $\frac{\gamma}{\gamma-1}$ for the fully open economy as α approaches one. In addition, the optimal steady-state markup increases monotonically with α.

LEMMA 5. *The optimal markup for the noncooperative social planner in the steady state monotonically increases with openness:*

$$\frac{\partial \mu}{\partial \alpha} > 0.$$

Proof. $\frac{\partial \mu}{\partial \alpha} = \frac{\gamma+\eta-1}{(\gamma-1+(1-\alpha)\eta)^2} > 0$ for $0 < \alpha < 1$, $\gamma \geq 1$, and $\eta > 0$. ∎

Lemmas 4 and 5 illustrate that steady-state markups increase with openness α, which is equal to the export share in steady state. In our model, we set the steady-state markup to one. However, the optimal markup changes over the cycle since the export share changes. With a higher export share, the optimal monetary policy is to generate higher markups.

The closest relevant study was conducted by De Paoli (2009b), where she considered the small open economy as a limiting case of two large open economies. We differ from her paper in three dimensions. First, she uses Calvo pricing, while we utilize prices set one period in advance. Second, we differentiate between export and import elasticities. Finally, we analyze the solution for the global set of parameters, instead of focusing on a particular calibration. Overall, our results are consistent with De Paoli (2009b), and we confirm in the global parameter-space that the sign of the markup and the output gap depends on whether the import elasticity is greater or less than one. Our main contribution is the presentation of sufficient statistics for optimal policy, which holds whether we take trade elasticity estimates from the macro or trade literature.

3.6 Complete Markets and Noncooperative Policy

Optimal monetary policy for noncooperative central banks under complete markets is the most complex of the four cases we consider. Two distortions drive equilibrium away from the first-best allocation: nominal rigidities and terms-of-trade externalities. The central bank faces a tradeoff between replicating the flexible-price allocation or extracting terms-of-trade rents. The main complicating factor in complete markets is that consumption dynamics differ from output dynamics. The decoupling of consumption from output makes complete markets different from financial autarky or the steady state. As a result, optimal monetary policy is more complicated. Before moving to the intuition, we state the principles for the optimal policy formally.

PROPOSITION 4. *In complete markets, noncooperative central banks maximize (34) subject to (35), (36), (37), and (38), where the indicators for cooperative policy* $\mathbb{1}_{CP}$ *and complete markets* $\mathbb{1}_{CM}$ *are set to zero and one, correspondingly. The solution is given by*

$$\hat{\mu}_t = \frac{G_4}{F_4}\hat{Z}_t, \tag{45}$$

where

$$
\begin{aligned}
G_4 &= \sigma\alpha(1-\alpha)(1+\varphi)((1-2\eta)(\eta\sigma-1)\alpha+(\eta-1)^2 \\
&\quad + \eta\sigma(\gamma-1)+\eta^2(\sigma-1)), \\
F_4 &= 4\alpha\varphi - \sigma - \varphi + 3\alpha\sigma + \eta\varphi + \gamma\varphi + \eta\sigma + \gamma\sigma - 6\alpha^2\varphi + 4\alpha^3\varphi \\
&\quad - \alpha^4\varphi - 3\alpha^2\sigma + \alpha^3\sigma + 2\alpha\eta^2\sigma^2 + 3\alpha^2\eta\sigma^2 - \alpha^3\eta\sigma^2 + \alpha\gamma^2\sigma^2 \\
&\quad - 5\alpha\eta\varphi - 4\alpha\gamma\varphi - 5\alpha\eta\sigma - 2\alpha\gamma\sigma - 5\alpha^2\eta^2\sigma^2 + 3\alpha^3\eta^2\sigma^2 \\
&\quad + 10\alpha^2\eta\varphi - 10\alpha^3\eta\varphi + 5\alpha^4\eta\varphi - \alpha^5\eta\varphi + 6\alpha^2\gamma\varphi - 4\alpha^3\gamma\varphi \\
&\quad + \alpha^4\gamma\varphi - 2\alpha\eta\sigma^2 + 7\alpha^2\eta\sigma - 3\alpha^3\eta\sigma - \alpha\gamma\sigma^2 + \alpha^2\gamma\sigma - 2\alpha\eta\varphi\sigma \\
&\quad - 2\alpha\gamma\varphi\sigma - \alpha^2\eta^2\varphi\sigma^2 + \alpha^2\eta^3\varphi\sigma^2 + 2\alpha^3\eta^2\varphi\sigma^2 - 3\alpha^3\eta^3\varphi\sigma^2 \\
&\quad - \alpha^4\eta^2\varphi\sigma^2 + 3\alpha^4\eta^3\varphi\sigma^2 - \alpha^5\eta^3\varphi\sigma^2 - \alpha^2\gamma^2\varphi\sigma^2 + \alpha^2\gamma^3\varphi\sigma^2 \\
&\quad + 3\alpha\eta\gamma\sigma^2 + 2\alpha\eta^2\varphi\sigma + 6\alpha^2\eta\varphi\sigma - 6\alpha^3\eta\varphi\sigma + 2\alpha^4\eta\varphi\sigma \\
&\quad + 2\alpha\gamma^2\varphi\sigma + 4\alpha^2\gamma\varphi\sigma - 2\alpha^3\gamma\varphi\sigma - 3\alpha^2\eta\gamma\sigma^2 - 8\alpha^2\eta^2\varphi\sigma \\
&\quad + 12\alpha^3\eta^2\varphi\sigma - 8\alpha^4\eta^2\varphi\sigma + 2\alpha^5\eta^2\varphi\sigma - 4\alpha^2\gamma^2\varphi\sigma + 2\alpha^3\gamma^2\varphi\sigma \\
&\quad - 2\alpha^2\eta\gamma\varphi\sigma^2 + 2\alpha^3\eta\gamma\varphi\sigma^2 + 4\alpha\eta\gamma\varphi\sigma + 3\alpha^2\eta\gamma^2\varphi\sigma^2 \\
&\quad + 3\alpha^2\eta^2\gamma\varphi\sigma^2 - 3\alpha^3\eta\gamma^2\varphi\sigma^2 - 6\alpha^3\eta^2\gamma\varphi\sigma^2 + 3\alpha^4\eta^2\gamma\varphi\sigma^2 \\
&\quad - 12\alpha^2\eta\gamma\varphi\sigma + 12\alpha^3\eta\gamma\varphi\sigma - 4\alpha^4\eta\gamma\varphi\sigma.
\end{aligned}
$$

Proof. See online appendix B.4. ∎

Equation (45) describes markup dynamics as a function of technology shocks. While it is possible to express the markup in terms of output or other endogenous variables in a more transparent way, equation (45) shows the set of parameters under which the replication of flexible prices is optimal. Corollary 4.1 lists the conditions under which replication of flexible-price allocation is optimal.

COROLLARY 4.1. *Noncooperative central banks under complete markets implement the flexible-price allocation, whenever any of the four listed conditions below is satisfied:*

(i) under Cole-Obstfeld conditions ($\sigma = \gamma = \eta = 1$)*;*

(ii) under full home bias ($\alpha = 0$);

(iii) under no home bias ($\alpha = 1$);

(iv) for an economy with stable terms of trade or whenever $\gamma \to \infty$ or $\eta \to \infty$.

Proof. Under (i), (ii), and (iii) in equation (45) we have $G_4 = 0$, which implies constant markups and flexible prices. With respect to (iv), we know that as the export elasticity γ increases, the numerator F_4 grows faster than G_4. More formally, $\lim_{\gamma\to\infty} \frac{G_4}{\gamma} \to \alpha\sigma^2\eta(1-\alpha)(1+\varphi)$, $\lim_{\gamma\to\infty} \frac{F_4}{\gamma^3} \to \alpha^2\varphi\sigma^2$. Thus, markups become more stable as the export elasticity increases: $\lim_{\gamma\to\infty} \hat{\mu}_t = \lim_{\gamma\to\infty} \frac{G_4}{F_4} = 0$. As the import elasticity η increases, the denominator F_4 grows faster than the numerator G_4. More formally, $\lim_{\eta\to\infty} \frac{G_4}{\eta^2} \to -\sigma\alpha(1-\alpha)(1+\varphi)(1+\sigma-2\sigma\alpha)$, $\lim_{\eta\to\infty} \frac{F_4}{\eta^3} \to \alpha^2(1-\alpha)^3\sigma^2\varphi$. Thus, markups become more stable as the import elasticity increases and eventually become constant: $\lim_{\eta\to\infty} \hat{\mu}_t = \lim_{\eta\to\infty} \frac{G_4}{F_4} = 0$. ■

Similar to both cooperative and noncooperative policies under financial autarky, price stability is the optimal noncooperative policy in complete markets when the economy is fully closed (part (ii)) or when the terms of trade are always stable (part (iv)). In a closed economy, manipulation of the terms of trade does not generate any extra benefits because there are no exports, while for stable terms of trade the central bank cannot exploit the terms of trade externality to enhance domestic welfare.

Parts (i) and (iii) are similar to noncooperative policy under financial autarky. Under the Cole-Obstfeld calibration ($\sigma = \eta = \gamma = 1$) or in a fully open economy ($\alpha = 1$), mimicking the flexible-price allocation is optimal. In both cases the export share remains constant. In financial autarky there is a positive relationship between the markup and the export share, but unfortunately the relationship between the two is more complicated in complete markets. Using the solution for the markup from equation (45) and other endogenous variables from online appendix B.4, we obtain the following relationship between the optimal markup and export share:

$$\hat{\mu}_t = \frac{G_x}{F_x H_x} \hat{E}_{s,t}, \tag{46}$$

where

$$\begin{aligned}
G_x &= \alpha\sigma(\sigma(1-2\alpha)\eta^2 + \gamma\sigma\eta - (1-\alpha)\eta(\sigma+2) + 1 - \alpha), \\
F_x &= \alpha - 1 + \sigma(\gamma - \alpha\eta), \\
H_x &= (\eta(1-\alpha) + \gamma - 1)(1-\alpha)^2 \\
&\quad + \alpha\sigma[\eta(\eta(1-\alpha) + 2(2\gamma - 1))(1-\alpha) + \gamma(\gamma - 1)].
\end{aligned}$$

In (46) the relationship between the optimal markup and the export share is no longer monotonic. The key intuitive difference between financial autarky and complete markets is that the former behaves more like the steady state in comparative statics. The monetary policy tradeoff under complete markets is to adjust the markup to extract higher terms-of-trade rents without generating too much volatility in consumption and hours worked. If consumption and labor become too volatile when the central bank attempts to exploit terms-of-trade externalities, it may be optimal for the policymaker to move toward the flexible-price allocation and away from terms-of-trade manipulation. However, the more extracting higher terms-of-trade rents actually reduces the volatility of consumption and labor, the more optimal policy will deviate from replicating flexible prices.

As the parameter governing labor disutility is absent in (46), we focus now on the volatility of consumption. Consider a positive productivity shock. In general, under flexible prices the terms of trade should go down, consumption should go up, and the export share may increase or decrease. If the export share increases following a positive productivity shock, the terms-of-trade externality creates upward pressure on the markup, such that the policymaker will appreciate the terms of trade and the real exchange rate and decrease the level of consumption. Thus, a higher markup allows the policymaker to stabilize consumption and extract higher monopolistic rents, such that an increase in the markup is likely to be a dominant strategy when the export share is positively correlated with technology. On the other hand, if the export share declines after a positive productivity shock, the policymaker faces a different tradeoff. In this case, the terms-of-trade externality creates downward pressure on the markup. However, a lower markup will depreciate the terms of trade and the real exchange rate, and increase consumption. Since consumption increases on impact, a further increase is undesirable when households are risk averse due to higher consumption volatility. Corollary 4.2 below formally establishes the relationship we have described.

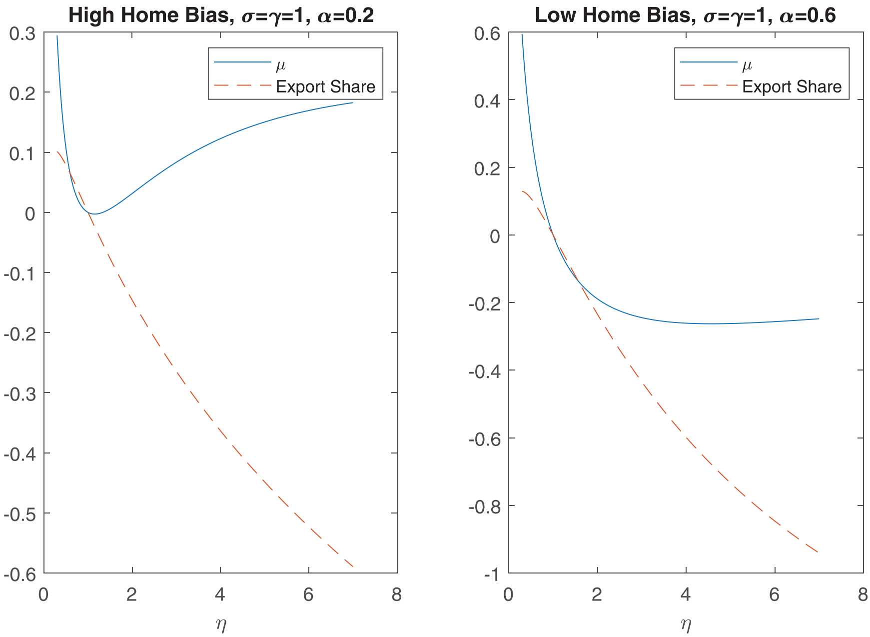
Corollary 4.2. *Under complete markets and noncooperative policy, markups are procyclical if the export share is procyclical and risk aversion (σ) is greater than one.*

Proof. See online appendix C.4. ∎

Under what conditions might the export share decline after a positive productivity shock? First, this might happen if domestic consumption goes up too strongly in response to a shock. For example, under complete markets, consumption is more sensitive to the real exchange rate for lower values of risk aversion, and a positive productivity shock typically leads to a depreciation in the terms of trade and the real exchange rate and a rise in consumption. The sensitivity of consumption to the real exchange is inversely proportional to the coefficient of risk aversion. However, while a lower degree of risk aversion leads to higher consumption volatility, it also decreases the welfare loss from increased volatility. The second factor is home bias. The real exchange rate becomes more sensitive to the terms of trade as home bias increases. Thus, for a given fall in the terms of trade, the real exchange rate will depreciate and consumption will rise more strongly as home bias increases. The third factor is the relative magnitude of the import and export elasticities. If the import elasticity is significantly larger than the export elasticity, a decrease in the terms of trade will generate a rise in demand for the home good from domestic consumers that outstrips the rise in demand from abroad, leading to a decline in the export share.

To generate a disconnect between the export share and the markup, we need $\eta > \gamma$ in combination with a low degree of consumption home bias. In figure 2 we deviate from the Cole-Obstfeld

Figure 2. Response in Percent of the Markup and Export Share to a 1 Percent Increase in Technology Level



calibration by increasing η and considering both a low and high degree of home bias.

Figure 2 plots the reaction of the markup and the export share for a particular combination of parameters following a 1 percent increase in technology. For example, the value of 0.2 on the vertical axis reflects that the markup or export share goes up by 0.2 percent in response to a 1 percent increase in the technology level. The subplot on the left corresponds to high home bias ($\alpha = 0.2$) while the subplot on the right corresponds to low home bias ($\alpha = 0.6$). Relative risk aversion and the elasticity of export substitution equal one in both subplots. When the import substitution elasticity η is equal to one, we return to the Cole-Obstfeld case, where the export share is constant for any realization of the technology shock and the flexible-price allocation is optimal. Thus, under the Cole-Obstfeld calibration, the markup and export share lines intersect at zero on the vertical axis and $\eta = 1$ on the horizontal axis as both the markup and the export share are constant in this case. Under high home bias and high elasticity of import substitution η, the markup responds positively to a technology shock, while the export share declines. On the other hand, for low home bias, the markup and export share decline in response to a positive shock when home and foreign goods are substitutes ($\eta > 1$), while they both increase after a positive technology shock when home and foreign goods are complements ($\eta < 1$).

The co-movement between the export share and markup is broken for high home bias and high η, but not for low home bias or low η. Why is this the case? With a positive technology shock, high home bias, and high η, the export share shrinks, and the terms of trade externality creates an incentive to reduce the markup. However, lowering the terms of trade in order to reduce markups has a strong effect on the real exchange rate and causes a substantial consumption increase when consumption is already high. This strong pressure from excess consumption volatility causes the policymaker to push markups in the opposite direction to stabilize consumption. Under low home bias, a reduction in the terms of trade has a small effect on the real exchange rate and causes only a small increase in consumption. In this case, markups can be countercyclical when the export share is countercyclical.

To summarize our findings on noncooperative policy in complete markets, under the most realistic calibrations the optimal markup and the export share are procyclical. Monetary policy will generate negative output gaps in response to positive technology shocks by appreciating the real exchange rate and the terms of trade. We prove that when the export share is procyclical, optimal monetary policy requires the central bank engineer procyclical markups and countercyclical output gaps.

The closest relevant study was conducted by De Paoli (2009a, 2009b), who considered the small open economy as a limiting case of two open economies. As in the previous section, we differ from her paper in three dimensions. First, she uses Calvo pricing, while we utilize prices set one period in advance. Second, we differentiate between export and import elasticities. Finally, we analyze the solution for a global set of parameters, instead of focusing on a particular calibration.

For noncooperative policy under complete markets, we find that differentiation between import and export elasticities plays a major role. Corollary 4.3 below shows that the markup and the export share are positively correlated.

COROLLARY 4.3. *If the import substitution elasticity* η *is equal to the export substitution elasticity* γ*, and* $\gamma > 1$*, then the markup and the export share co-move positively.*

Proof. See online appendix C.5. ■

Whenever relative risk aversion is greater than one, the co-movement between the markup and the export share disappears only if the import elasticity is greater than the export elasticity.

4. Conclusion

There is a long tradition in macroeconomics of explaining the data by introducing distortions into perfectly competitive and efficient markets. In the closed economy this strategy has been fruitful and brought some immediate results in the form of the divine coincidence: by stabilizing inflation the central bank eliminates distortions arising from nominal rigidities, closes the output gap, and removes price dispersion. Such a strategy has been less successful in the context of open economies, where imperfect cross-country risk sharing and terms-of-trade externalities in addition to nominal rigidities prevent the divine coincidence.

Our aim here is to establish some simple principles for optimal cooperative and noncooperative monetary policy in small open economies under complete markets and financial autarky. Relative to the literature, we do not consider the small open economy as the limiting case of two large economies, which allows us to differentiate between the export and import elasticity of substitution.

We are the first to consider cooperative optimal monetary policy for small open economies, which enables a clearer understanding of distortions in the absence of strategic interactions between countries. We find that mimicking the flexible-price allocation is the optimal cooperative monetary policy under complete markets, which aligns with studies focused on two large economies. We also establish that under most realistic calibrations, cooperative optimal monetary policy under financial autarky deviates from price stability in favor of countercyclical markups, procyclical output gaps, and volatile terms of trade.

We then examine noncooperative policy. Under financial autarky, markups set up by the monetary authority co-move positively with the share of the goods exported regardless of the degree of risk aversion, home bias, or product substitutability across countries. Under complete markets, optimal noncooperative monetary policy should generate procyclical markups and countercyclical output gaps whenever the export share is procyclical. This rule may be violated if consumption is strongly sensitive to monetary policy and is negatively correlated with the share of goods exported. In this case, central banks should restrain from lowering markups during a boom even if the export share falls, since responding to such movements with a lower markup might cause excess consumption volatility.

Across all four cases examined, the simple prescription for optimal monetary policy is to replicate flexible prices unless the export share is too volatile. If the export share is procyclical, then the optimal markup should also be procyclical. If the export share is countercyclical, then the optimal markup is countercyclical unless it leads to high consumption volatility.

To conclude, we find a simple monetary policy rule in the noncooperative case. Policymakers should set markups to react to the export share unless a deviation of the markup from zero causes excess consumption volatility.

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