Independent Central Banks: Low Inflation at No Cost? A Model with Fiscal Policy*

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In this article we extend the rational partisan model of Alesina and Gatti (1995) to include a second policy, fiscal policy, besides monetary policy. It is shown that the extent to which an independent central bank is successful in attaining price stability depends on the degree of conservativeness of the central bank in relation to the political parties and the private sector’s expectations on which party will win the elections. In addition, the inclusion of fiscal policy in Alesina and Gatti’s model implies that uncertainty about the course of policy is not a sufficient factor to ensure that, when supply shocks are not relevant, independent central banks bring about low inflation at no real cost.

JEL Codes: E58, E63.

1. Introduction

By appointing a conservative independent central bank to take control of monetary policy, Rogoff (1985) showed that average inflation would be reduced. Given the tradeoff between the objectives of output and inflation stabilization, a conservative central bank would prioritize fighting inflation with a theoretical cost of higher output variability. Alesina and Gatti (1995), based on the lack of empirical evidence of higher output variability shown by Alesina

*We thank three anonymous referees for their comments which helped improve the quality of this paper. We also acknowledge financial support from Ministerio de Economía y Competitividad (ECO2016-75410-P AEI/FEDER, UE), Universitat Rovira i Virgili, Generalitat de Catalunya (2019PFR-URV-B2-53 and 2017 SGR 770), and the Ministerio de Ciencia e Innovación (PID2019-105982GB-I00 AEI/FEDER, UE). Corresponding author (Manzano): carolina.manzano@urv.cat. Address: Departament d’Economia and CREIP, Av. Universitat 1, 43204-Reus (Spain).
and Summers (1993), developed a theoretical model to illustrate why a conservative independent central bank might not bring higher volatility of output.

The rational partisan model presented by Alesina and Gatti (1995) included two political parties running for office, with different views of the economy. Monetary policy, the only policy in their model, is either decided by the party that wins the elections or delegated to an independent central bank. The authors identify two sources of instability: the first source is due to the uncertainty about which party will be in office and the second one is due to exogenous shocks. Consequently, they decompose the variability of output into two components: the political volatility, introduced by the uncertainty about the future course of policy, and the economic volatility, induced by exogenous shocks. Alesina and Gatti (1995) show that by removing the conduct of monetary policy from the hands of the government, the first component of the variance of output is eliminated, allowing for the possibility that the overall volatility of output does not necessarily increase. In particular, these authors conclude that if the volatility of shocks is low enough, delegation of the conduct of monetary policy reduces the overall variance of output.

Monetary policy has been considered an ideal candidate for delegation (see, for instance, Drazen 2002 and Alesina and Tabellini 2007, 2008), due to its technical nature and the difficulty in judging the ability or talent of the person responsible for making the decisions. Fiscal policy, on the other hand, is not viewed as a clear candidate for delegation, mainly because of its redistributive impact. Moreover, as fiscal policy can secure a minimum number of voters, politicians will not willingly delegate such policy if they want to be re-elected. Therefore, fiscal and monetary policies are implemented in many countries by different authorities that are generally independent from each other. For this reason, an interesting extension of Alesina and Gatti (1995) would be the inclusion of fiscal policy in the model, in order to see whether an independent central bank responsible for monetary policy and presumably isolated from electoral cycles is still able to eliminate the politically induced volatility of output.

In this paper, we generalize Alesina and Gatti’s model by introducing a second policy, fiscal policy, that will be decided by the party in government. We initially consider a basic framework where
(i) the two parties running for office only differ in the relative weights assigned to output stabilization and (ii) the government and the central bank simultaneously choose their policy in case of delegation of monetary policy. In the next paragraphs we explain the main results obtained.

The benefits in terms of inflation (low and stable inflation) of the appointment of an independent central bank depend on the degree of conservativeness of the central bank in relation to the political parties and the private sector’s expectations on which party will win. An ultraconservative independent central bank (i.e., a central bank more conservative than the two parties) is always expected to achieve lower and more stable inflation. However, a moderately independent central bank (i.e., a central bank that has an inflation aversion intermediate between the two parties) is expected to attain lower and more stable inflation only if the probability of the less inflation-averse party winning the elections is high enough.

Further, the politically induced output variability is not removed by the introduction of the central bank—in fact, it may even increase. This last case occurs when the central bank is ultraconservative and political parties are not particularly concerned about achieving their public spending targets. In this case, when monetary policy is delegated to an independent ultraconservative central bank, the inflation rate is chosen almost regardless of output deviation. This will lead to a higher politically induced volatility of output compared with nondelegation, where outputs will be closer to the target and will differ less.

Similarly to Alesina and Gatti’s results, the appointment of an ultraconservative central bank unequivocally increases the economically induced variance of output. By contrast, when a moderately conservative independent central bank is responsible for monetary

\[\text{In Alesina and Gatti’s model, as there is no fiscal policy, the central bank chooses the same inflation under delegation, regardless of the party that is in office. This, in turn, leads to the same value of output and, therefore, one concludes that the institution of an independent central bank eliminates the politically induced output variability in their model. By contrast, when fiscal policy is introduced, the central bank will choose the inflation level depending on the party that is in office. This implies that the level of output obtained depends on the party that is in office and, therefore, the politically induced output variability is present in our model.}\]
policy, this component of the variance of output increases whenever the probability of the less inflation-averse party winning the elections is high enough.

In addition, the relationship between central bank independence and output variability depends on the degree of conservativeness of the central bank and on exogenous shocks. We will show the conditions that will lead to an increase or a decrease of the overall variance of output. In this way, our analysis suggests that if the volatility of shocks is low enough, delegation of the conduct of monetary policy may not reduce the overall variance of output and, hence, the above-mentioned conclusion reached by Alesina and Gatti is not robust.

The last result we would like to point out is that, in contrast to Alesina and Gatti (1995), the study of output stabilization in our model is not reduced to the study of the variance of output. Unlike their paper, we obtain that output is not necessarily more stable under delegation of monetary policy to an independent central bank when economic shocks are not relevant. For instance, when the parties’ preferences are not too different, output is less stable with an ultraconservative independent central bank.

To test the robustness of our results, we analyze two generalizations of the basic framework: (i) parties differ in their target for public spending, and (ii) the authorities choose their policies sequentially. In relation to the first extension of the model, recent empirical studies have provided evidence to support the popular view that left-wing parties are associated with higher public spending (see, for instance, Blomberg and Hess 2003, Pickering and Rockey 2011, 2013, among others). Consequently, we generalize the model, assuming that a left-wing party would prefer a higher target for government expenditure than a right-wing party would. In relation to the second generalization, one could argue that as the process of changing tax rates takes longer than the process to adjust monetary policy, a more appropriate description of fiscal–monetary interactions would involve a leader–follower game. In this second variation of the basic framework, the fiscal authority acts as the leader and the monetary authority acts as the follower. We show that the main results derived in the basic framework also hold in these new setups.

The different outcomes delivered by our model can account for the mixed results obtained by the empirical literature. The
initial evidence seemed to favor the existence of a negative relationship between central bank independence (CBI) and inflation in OECD countries (see, for instance, the surveys of Eijffinger and de Haan 1996 and Berger, de Haan, and Eijffinger 2001, or the meta-regression analysis of Klomp and de Haan 2010b), but for developing countries the situation is less clear. In fact, when large heterogeneous samples of countries are used, no general significant negative relation between CBI and inflation is found (Klomp and de Haan 2010a and Dincer and Eichengreen 2014).


One of the reasons suggested for the discordant empirical results is related to the variables used to measure central bank independence. The de facto independence of the central bank is not properly captured by the legal definition. In addition, the presence of a relationship between CBI and inflation or growth does not imply causality. Further, the distinction between independence and the degree of conservativeness or inflation aversion presented in theoretical models is not easily captured by the variables used empirically. It will be shown in this paper that the effects of CBI on inflation and output stabilization will be dependent on the degree of central bank conservativeness.

The present paper is related to the theoretical literature that focuses on the impact of delegating monetary policy to an independent central bank. Demertzis (2004) carries out numerical simulations of Alesina and Gatti’s (1995) model and shows that changing political uncertainty values could alter their results. Further, by introducing fiscal policy, this article can be associated to the

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2Dincer and Eichengreen (2014) obtain some—although statistically inconsistent—negative relationship.

3More on this can be found in, among others, Cukierman et al. (1993), Alpanda and Honig (2010), Klomp and de Haan (2010b), and Dincer and Eichengreen (2014).
literature that studies the interaction of monetary and fiscal policy (see, for instance, Alesina and Tabellini 1987, Debelle and Fisher 1994, Beetsma and Bovenberg 1997, Dixit and Lambertini 2003, among others). However, our model extends this literature by introducing electoral uncertainty. In a previous article, Ferré and Manzano (2014) have included electoral uncertainty in a model with two policymakers by extending Alesina’s (1987) rational partisan theory model. It is shown that the inclusion of a central bank can alter the predictions of the rational partisan theory, in the sense that the direct relationship predicted between inflation and output in Alesina (1987) does not hold.

The remainder of this paper is organized as follows. The next section will develop a rational partisan model where fiscal and monetary policies are initially under the control of the government and, then, monetary policy is delegated thereafter to an independent central bank. Additionally, the effects of the introduction of an independent central bank, responsible for monetary policy, on expected inflation and inflation stabilization are analyzed. Section 3 checks the robustness of the model by including two extensions of the basic setup. Finally, section 4 will present the conclusions.

2. The Benchmark Model

In this section, we will present a model that combines features from both Alesina and Tabellini (1987) and Alesina and Gatti (1995). We will assume that there are two parties competing for office, $L$ (a left-wing party) and $R$ (a right-wing party), and there is an exogenous probability $P$ that party $L$ wins the elections and takes office. Agents (wage setters) in this economy will not know what party will be in office when they form their inflation expectations, $\pi^e$. For this reason, their expectations embody electoral uncertainty: $\pi^e = PE(\pi_L) + (1 - P)E(\pi_R)$, where $E(\pi_j)$ represents expected inflation if party $j$ is in office ($j = L, R$). Once elections take place, the party in office will attempt to stabilize the economy after the shocks occur, and the optimal values of inflation and taxes will be revealed. This sequential structure of the game is static by nature, as the game ends once the policy instruments are chosen.
If party $j$ is in office, the output is given by

$$x_j = \pi_j - \pi^e - \tau_j - w^* + \varepsilon,$$

(1)

where $\pi_j$ is the actual inflation rate. Moreover, $\tau_j$ represents taxes levied on output, $w^*$ denotes the target real wage that workers seek to achieve, and $\varepsilon$ is a productivity shock such that $E(\varepsilon) = 0$ and $\text{var}(\varepsilon) = \sigma^2_{\varepsilon}$.

The budget constraint of government $j$ is

$$g_j = \tau_j + \pi_j,$$

(2)

where $g_j$ denotes the ratio of public expenditures over output when party $j$ is in office. Note that public spending will be financed by a distortionary tax (controlled by the fiscal authority) and/or by money creation (controlled by the authority responsible for monetary policy). Given the static nature of the model, debt is not included.

We assume that the loss function for party $j$ is given by

$$V_{Gj} = \frac{1}{2} \left( \pi_j^2 + \delta_j (x_j - x^*)^2 + \gamma (g_j - g^*)^2 \right),$$

(3)

where $\delta_j$ and $\gamma$ represent the relative weights assigned to output and public spending stabilization with respect to inflation, respectively, and $\delta_j, \gamma > 0$, while $x^*$ and $g^*$ denote the output and public spending targets, respectively. The government’s objective function is given by an augmented but otherwise conventional loss function (see, for instance, Alesina and Tabellini 1987, Debelle and Fisher 1994, Beetsma and Bovenberg 1997, Huang and Wei 2006, and Hefeker 2010, among others). This objective function reflects that the government aims to stabilize output and inflation simultaneously, as well as meet a spending target, which could reflect the

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4A detailed derivation of expressions (1) and (2) is given at the beginning of appendix A.

5By not including debt in the model we avoid the introduction of another interesting but separate issue, namely the manipulation of economic variables to influence the outcome of elections. The incumbent party could affect the result of the elections and/or the success of the mandate of the winning party by increasing spending and debt.
aim of being re-elected or other demands from interest groups that influence the government. Following the literature, we suppose that \( \delta_L > \delta_R \). In the benchmark case, we assume that parties have identical relative weights assigned to public spending stabilization and share the same goals. This framework will allow us to make a clear comparison between our results and the ones derived by Alesina and Gatti (1995)\(^6\).

In what follows we distinguish two frameworks: first, when monetary and fiscal policy are controlled by the government, and second, when monetary policy is delegated to an independent authority (central bank). The first framework will represent an economy with no (or very little) central bank independence, whereas the second one will refer to an economy that has granted independence to its central bank for the conduct of monetary policy. In both cases, the timing of events is as follows: expectations and, thus, wages are set first. Afterward, elections take place; party \( L \) wins with probability \( P \), and party \( R \) with probability \( 1 - P \). After the election, the shock \( \varepsilon \) occurs. In the first case, the government chooses both policies. In the second case, the government and the central bank will simultaneously choose their policy.

### 2.1 No Independent Monetary Policy

When monetary and fiscal policy are both under the control of the government, the party in government will attempt to minimize its loss function (3) by using two instruments, \( \pi \) and \( \tau \). The inflation rates chosen by the two parties if in office and the corresponding outputs are (where the superscript \( N \) indicates nondelegation of monetary policy)

\[
\pi^N_L = \frac{m_R + 2}{\Delta^N} A - \frac{\varepsilon}{m_L + 2},
\]

\[
\pi^N_R = \frac{m_L + 2}{\Delta^N} A - \frac{\varepsilon}{m_R + 2},
\]

\[
x^N_L = x^* - \frac{1}{2\delta_L} \pi^N_L,
\]

\( ^6 \)We would like to thank an anonymous referee for this suggestion.
\[ x^N_R = x^* - \frac{1}{2\delta_R} \pi^N_R, \]  

(7)

where \( m_L = \frac{1+\frac{1}{\delta_L}}{2} \), \( m_R = \frac{1+\frac{1}{\delta_R}}{2} \), \( \Delta^N = (m_L + 2)(m_R + 1) + P(m_L - m_R) \), and \( A = x^* + g^* + w^* \).

As it is indicated in Ferré and Manzano (2014), \( m_j \) represents a measure of party \( j \)'s inflation aversion. The assumption that \( \delta_L > \delta_R \), that is, party \( L \) gives more weight to output stabilization than party \( R \), implies that \( m_R > m_L \), i.e., the goal of stabilizing inflation is more important for party \( R \) than for party \( L \). Accordingly, taking expectations in expressions (4) and (5), it follows that \( E(\pi^N_L) > E(\pi^N_R) \) as \( m_R > m_L \), i.e., expected inflation will always be higher under an \( L \) administration.

The following lemma shows that in the benchmark model expected output will be also higher under an \( L \) administration.

**Lemma 1.** \( E(x^N_L) > E(x^N_R) \).

According to lemma 1, we expect a lower deviation of output when party \( L \) is in office. This is due to the fact that party \( L \) is more concerned about output stabilization than party \( R \).

2.2 *Introducing an Independent Monetary Authority*

We will now study the case where monetary policy is undertaken by an independent monetary authority. Independence refers to the extent to which the central bank determines monetary policy without political interference. Hence, when party \( j \) is in office, we will assume now that the central bank will have its own loss function to minimize given by

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7 A detailed derivation of the optimal policies under nondelegation and delegation of monetary policy to an independent central bank can be found in the appendix (see propositions A.1 and A.2, respectively).

8 Notice that 1 is the weight attributed to inflation in the parties’ loss functions. Thus, \( m_j \) is the arithmetic mean of the weight of inflation relative to output and public spending for party \( j \).

9 In models with only one policy (and, in particular, \( \gamma = 0 \)), it is assumed that \( \delta_L > \delta_R \)—see, for instance, Alesina (1987) and Alesina and Gatti (1995)—and, thus, in this case \( m_L = 1/\delta_L \) and \( m_R = 1/\delta_R \), which would also correspond to \( m_R > m_L \).
\[ V_{CB} = \frac{1}{2} \left( \pi_j^2 + \delta_{CB} (x_j - x^*)^2 \right), \] (8)

where \( \delta_{CB} > 0 \). In this case, the timing of events is the same, but after the shock \( \varepsilon \) occurs, the central bank will control inflation \( (\pi) \) to minimize its loss function \( (8) \), and the party in government will attempt to minimize its loss function \( (3) \) by using taxes \( (\tau) \). With this institutional specialization we obtain the following inflation rates and outputs (where superscript \( D \) indicates delegation of monetary policy):

\[ \pi^D_L = \frac{c_R m_R + 2}{\Delta^D} A - \frac{\varepsilon}{c_L m_L + 2}, \] (9)
\[ \pi^D_R = \frac{c_L m_L + 2}{\Delta^D} A - \frac{\varepsilon}{c_R m_R + 2}, \] (10)
\[ x^D_L = x^* - \frac{c_L}{2 \delta_L} \pi^D_L, \text{ and}\]
\[ x^D_R = x^* - \frac{c_R}{2 \delta_R} \pi^D_R, \] (12)

where \( \Delta^D = (c_L m_L + 2) (c_R m_R + 1) + P (c_L m_L - c_R m_R) \). In these expressions we have introduced two new variables, \( c_L \) and \( c_R \). The variable \( c_j, j = L, R, \) is a measure of the degree of the relative conservativeness of the central bank with respect to party \( j \):

\[ c_j = \frac{2 \delta_j}{\delta_{CB}}. \]

**Remark 1.** If \( c_L = 1 \) and \( c_R = 1 \), that is, the central bank is as conservative as both parties, then \( \pi^D_j = \pi^N_j \) and \( x^D_j = x^N_j \).

Using the expressions for \( m_L \), \( m_R \), \( c_L \), and \( c_R \), we have that \( c_L m_L > c_R m_R \). Hence, expressions (9) and (10) imply that \( E (\pi^D_R) > E (\pi^D_L) \). If party \( L \) is relatively more interested in stabilizing output than party \( R \), party \( L \) is expected to have more incentives to reduce taxes. This, in turn, has an effect on the behavior of the central bank: the decrease in taxes diminishes the incentives to inflate and, thus, \( E (\pi^D_R) > E (\pi^D_L) \).

\(^{10}\)The notion of conservativeness generally refers to the degree of the central bank’s inflation aversion. See Ferré and Manzano (2012) for a detailed explanation of the conservativeness measure \( c \).
If we compare expected outputs in the presence of an independent central bank, we obtain the following result:

**Lemma 2.** \( E(x^D_L) > E(x^D_R) \).

Rogoff (1985) showed that, in a model with only monetary policy, society’s welfare could be improved by appointing a more conservative central bank. We will follow the Rogoff tradition and assume that an agreement can be reached to appoint a central bank that is more conservative than both political parties: \( c_L > c_R \geq 1 \). We will refer to this central bank as being “ultraconservative.” Alesina and Gatti (1995) point out that if political parties are polarized, it might not be easy to reach an agreement to delegate the conduct of monetary policy to an independent institution. They argue, however, that such an agreement will be easier to reach when the independent institution has an inflation aversion that is intermediate. Following these authors, we will also analyze a central bank more conservative than the left-wing party and less conservative than the right-wing party. In our framework this assumption is represented by \( c_L > 1 > c_R \) and we label it “moderately conservative.”

According to Alesina and Gatti (1995), “the institution of an independent and inflation-averse central bank has two benefits: first, it reduces average inflation; second, it eliminates politically induced output variability.” In the following two subsections, we will analyze whether these results hold when the model is extended to consider two policies.

### 2.3 The Effects of an Independent Central Bank on Inflation

What is the effect of the introduction of an independent central bank, responsible for monetary policy, on expected inflation and inflation stabilization? The following proposition shows that it will depend on the degree of conservativeness of the central bank.

**Proposition 1.** (a) By appointing an ultraconservative \((c_L > c_R \geq 1)\) independent central bank responsible for monetary policy, the expected value of inflation is reduced and a higher degree of inflation stabilization is achieved. (b) By appointing a moderately conservative \((c_L > 1 > c_R)\) independent central bank, the expected value of inflation is reduced and a higher degree of inflation stabilization is achieved if and only if \( P \) is high enough.
In the presence of political uncertainty, inflation is generally lower (in expected terms) and more stable when monetary policy has been delegated to an independent and conservative central bank. In other words, this proposition indicates that when monetary policy is carried out by a conservative and independent central bank, agents expect inflation to be lower and more stable than if monetary policy was set by the parties. An exception arises in case (b), where the central bank is less conservative than party $R$. When the probability of party $R$ coming to power is high enough (that is, $P$ is low enough), we expect lower and more stable inflation without delegating monetary policy to an independent central bank, as this party is already very inflation averse.\footnote{Demertzis (2004) carries out numerical simulations on Alesina and Gatti’s model and also finds that, for an intermediate central bank, inflation might not always be lower.}

2.4 The Effects of an Independent Central Bank on Output

The theoretical research that followed Rogoff’s article (1985) suggested that central bank independence came at a cost of higher output variability. However, as empirical studies did not seem to find clear evidence of a higher variance of output, Alesina and Gatti (1995) developed a model where they decomposed the variance of output in two parts: the \textit{politically induced variance} ($\text{Var}_P$), which reflects the fluctuations in the variable induced by electoral uncertainty, and the \textit{economically induced variance} ($\text{Var}_E$), which is due to the exogenous shocks. In Alesina and Gatti (1995), removing the conduct of monetary policy from the government eliminates the politically induced variance of output, which explains why the variance of output might not necessarily increase with an independent conservative central bank. We will study how this result is altered with the introduction of fiscal policy in the analysis.

2.4.1 The Politically Induced Variance of Output

The politically induced variances of output when monetary policy is under the control of the government ($N$) and when it
is conducted by an independent central bank \((D)\) are given by

\[
Var_P(x^N) = P (1 - P) \left( E(x_L^N) - E(x_R^N) \right)^2 \quad \text{and} \quad (13)
\]

\[
Var_P(x^D) = P (1 - P) \left( E(x_L^D) - E(x_R^D) \right)^2 . \quad (14)
\]

The last expression implies that, in general, the politically induced variance of output does not vanish when monetary policy is delegated to an independent central bank. There will only be two scenarios in which this variance vanishes: when there is no political uncertainty \((P = 0, 1)\) and when \(E(x_L^D) = E(x_R^D)\), which occurs when \(\delta_L = \delta_R\) or \(\gamma = 0\). In these last cases both parties behave identically and, consequently, the political uncertainty introduced by elections does not play any role. These results are summarized in the following proposition:

**PROPOSITION 2.** The appointment of an independent central bank when there is more than one policy instrument does not eliminate the variance of output induced by political uncertainty, except when both parties behave identically \((i.e., \delta_L = \delta_R \text{ or } \gamma = 0)\).

Given that the politically induced variance of output is not automatically eliminated by introducing an independent central bank, in the next lines we will study whether this variance is at least reduced with delegation of monetary policy.

Notice that the comparison of \(Var_P(x^N)\) and \(Var_P(x^D)\) in (13) and (14) is equivalent to contrasting the distance between the expected values of output, \(\mid E(x^i_L) - E(x^i_R) \mid\), with \(i = N, D\). Consequently, the comparison of this type of variances is reduced to the study of expected outputs under both frameworks \(N\) and \(D\).

Further, nondelegation and delegation would coincide when \(c_L = c_R = 1\), i.e., when monetary policy is undertaken by a central bank that is as conservative as the two parties. We can then study the effect of moving toward a moderately conservative central bank \((c_L > 1 > c_R)\) by analyzing the impact of increasing the relative conservativeness of the central bank with respect to party \(L\) (an

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12Lemma A.1 in the appendix shows the derivation of these expressions.
increase in \( c_L \) and lowering the relative degree of conservativeness of the central bank with respect to party \( R \) (a decrease in \( c_R \)). Similarly, we can study the effect of introducing an ultraconservative central bank (\( c_L > c_R \geq 1 \)) by analyzing the consequences of increasing the relative conservativeness of the central bank with respect to both parties (an increase in both \( c_L \) and \( c_R \)). The following result will prove useful in explaining how expected outputs are affected in moving from \( N \) (no independent central bank) to \( D \) (independent central bank):

**Lemma 3.** Let \( i, j = L, R, i \neq j \). A change in the relative conservativeness of the central bank with respect to party \( j \) will have two effects on expected outputs: the direct effect \( \left( \frac{\partial}{\partial c_j} E(x^D_i) \right) \) and the indirect effect \( \left( \frac{\partial}{\partial c_j} E(x^D_j) \right) \). Moreover, it holds that \( \frac{\partial}{\partial c_L} E(x^D_L) < 0 \) and \( \frac{\partial}{\partial c_L} E(x^D_R) > 0 \).

The logic of lemma 3 is as follows. Without any loss of generality, let’s start from the initial nondelegation situation \( ((c_L, c_R) = (1,1)) \) and assume an increase in \( c_L \), keeping \( c_R \) constant \( (c_L > c_R = 1) \). This corresponds to a new situation identical to the initial one, except that now monetary policy is undertaken by a more conservative central bank if party \( L \) is in office. As lemma 3 points out, this change in \( c_L \) will have two effects on expected outputs: a direct effect \( \left( \frac{\partial}{\partial c_L} E(x^D_L) \right) \) and an indirect effect \( \left( \frac{\partial}{\partial c_L} E(x^D_R) \right) \).

Under the direct effect, as the authority in charge of monetary policy becomes more conservative, the difference between expected inflation and expected average inflation \( (E(\pi_L) - \pi^e) \) becomes smaller, and thus expected output under party \( L \)’s office will be lower. Hence, \( \frac{\partial}{\partial c_L} E(x^D_L) < 0 \). Under the indirect effect, the possibility that the authority in charge of monetary policy under the other party’s office (party \( L \)) is more conservative will bring expected average inflation \( (\pi^e) \) down. Thus, the difference between expected inflation and expected average inflation \( (E(\pi_R) - \pi^e) \) becomes larger and, hence, expected output under party \( R \)’s office will be higher. Therefore, \( \frac{\partial}{\partial c_L} E(x^D_R) > 0 \).\(^{13}\)

\(^{13}\)Similarly, a change in \( c_R \), keeping \( c_L \) constant, would imply a direct effect \( \left( \frac{\partial}{\partial c_R} E(x^D_R) < 0 \right) \) and an indirect effect \( \left( \frac{\partial}{\partial c_R} E(x^D_L) > 0 \right) \).
A Moderately Conservative Central Bank. Delegating monetary policy to a moderately conservative central bank implies that such policy will now be implemented by an authority that is more conservative than party $L$ and less conservative than party $R$. In other words, there will be an increase in $c_L$ and a decrease in $c_R$ with respect to the initial nondelegation situation $((c_L, c_R) = (1, 1))$. Notice that, in this case, the direct and indirect effects for expected output under each party work in the same direction. For party $L$, these effects bring a reduction in expected output and for party $R$ an increase in expected output. Consequently, $E(x^N_L) > E(x^D_L)$ and $E(x^N_R) < E(x^D_R)$. From lemmas 1 and 2, it follows that $E(x^N_L) > E(x^N_R)$ and $E(x^D_L) > E(x^D_R)$. Therefore, we obtain $E(x^N_L) < E(x^D_L) < E(x^D_R) < E(x^N_R)$, as shown in figure 1. In this case the politically induced variance of output is reduced by the presence of an independent and moderately conservative central bank, i.e., $\text{Var}_P(x^D) < \text{Var}_P(x^N)$.

An Ultraconservative Central Bank. Delegating monetary policy to an ultraconservative central bank implies that such policy will now be implemented by an authority that is more conservative than both parties. Formally, now $c_L > c_R \geq 1$, as there will have been an increase in both $c_L$ and $c_R$ with respect to the initial nondelegation situation $((c_L, c_R) = (1, 1))$. In this case, the direct and indirect effects on expected outputs work in opposite directions. Notice that the increase in $c_L$ is larger than the increase in $c_R$. For this reason, the direct effect always dominates for party $L$ and so expected output for this party falls $(E(x^N_L) > E(x^D_L))$. By contrast, for party $R$ the direct effect might not always dominate. For instance, the indirect effect will be more important for party $R$ when party $L$ is substantially less conservative than party $R$, or when the central bank is very similar in conservativeness to $R$. In this last case, i.e., when the indirect effect dominates for party $R$, expected
output for this party will increase \( (E(x^N_L) < E(x^D_R)) \). Now, as \( E(x^N_L) > E(x^D_L) \) and \( E(x^N_R) < E(x^D_R) \), the analysis related to the comparison of the politically induced variance of output would be identical to the moderately conservative central bank case.

When the direct effect dominates for party \( R \), we can have two possible cases, illustrated in the following two figures. Figure 2 illustrates the case in which the reduction in expected output for party \( L \) will be larger and, consequently, \( Var_P(x^N) > Var_P(x^D) \).

Figure 3 shows the case in which the reduction in expected output for party \( L \) will be smaller than for party \( R \). Therefore, \( Var_P(x^N) < Var_P(x^D) \).

The following proposition summarizes these results and identifies the parameter configurations in which the politically induced variance of output is reduced with delegation of monetary policy.

**Proposition 3.** (a) By appointing a moderately conservative independent central bank responsible for monetary policy, the politically induced variance of output is reduced. (b) By appointing an ultra-conservative independent central bank, the politically induced variance of output is reduced when both parties are concerned enough about public spending stabilization or when the central bank is not too ultra-conservative (i.e., when \( \gamma \) is high enough or when \( \delta_{CB} \) is high enough).
Proposition 3(b) shows that the politically induced variance of output is reduced with the introduction of an ultraconservative central bank whenever $\gamma$ is high enough. To understand this result, consider the limiting case when $\gamma$ converges to infinity. In this case the behavior of both parties would be identical since both would choose the tax rate such that $g_j = g^*$. Consequently, the central bank would select the same inflation rate and, hence, we would expect identical outputs under delegation, resulting in a null politically induced variance of output in this framework. By contrast, under nondelegation we expect different inflation rates due to different preferences between parties, which will generate a strictly positive $\text{Var}_P(x_N)$ even though we consider this limiting case ($\gamma \to \infty$).

Proposition 3(b) also indicates that when $\gamma$ is low enough (but not null) and $\delta_{CB}$ is also low, the opposite result is obtained, i.e., $\text{Var}_P(x_N) < \text{Var}_P(x_D)$\footnote{If $\gamma = 0$, then $\text{Var}_P(x_N) = \text{Var}_P(x_D) = 0$.} To understand the logic of this result, let us consider the limiting case in which $\delta_{CB} = 0$. Under delegation, the inflation rate would be chosen regardless of output deviations, which is not the case under nondelegation. In addition, for low values of $\gamma$, output stabilization is relatively important in the choice of $\pi$ under nondelegation. This causes outputs to be closer to the target and differ less under nondelegation when $\gamma$ is low enough and, hence, the politically induced variance of output is increased with delegation of monetary policy.

2.4.2 The Economically Induced Variance of Output

The economically induced variances of output are originated by exogenous shocks. In the model presented here with fiscal policy, these variances when monetary policy is controlled by the government and when it is delegated to an independent central bank are, respectively,

$$\text{Var}_E(x^N) = \left(P \left( \frac{1}{2\delta_L (m_L + 2)} \right)^2 \right.$$

$$+ (1 - P) \left( \frac{1}{2\delta_R (m_R + 2)} \right)^2 \left) \sigma^2_{\varepsilon} \right.$$ and
\[ \text{Var}_E(x^D) = \left( P \left( \frac{c_L}{2\delta L (c_L m_L + 2)} \right)^2 ight. \\
+ \left. (1 - P) \left( \frac{c_R}{2\delta R (c_R m_R + 2)} \right)^2 \right) \sigma^2_{\varepsilon}. \]

**Proposition 4.** The appointment of a moderately conservative independent central bank increases the economically induced variance of output whenever \( P \) is large enough. By contrast, the appointment of an ultraconservative central bank always increases the economically induced variance of output.

This result is in line with the previous literature: appointing an independent central bank more conservative than both parties increases the economically induced variance of output. However, if the central bank’s conservativeness is intermediate, then the economically induced variance of output is higher under delegation of monetary policy whenever \( P \) is large enough, that is, when the probability of party \( L \)—the less inflation-averse party—winning elections is high.

### 2.5 Output Stabilization

Alesina and Gatti (1995) find that if the volatility of shocks is low enough, then delegation of the conduct of monetary policy reduces the variance of output. This is so because, in this case, the relevant component of the volatility of output is the politically induced variance of output. However, the analysis developed in the previous section allows us to conclude that this result is not robust in our framework. Moreover, we would like to point out that there is another difference between the two models. In Alesina and Gatti (1995), the study of output stabilization coincides with the study of the variance of output. To see this, note that applying the standard statistics theory,

\[ E \left( (x^i - x^*)^2 \right) = (E(x^i - x^*))^2 + \text{var}(x^i), \quad i = N, D. \]

In Alesina and Gatti (1995), \( E(x^i - x^*) = 0 \), and hence, \( E \left( (x^i - x^*)^2 \right) = \text{var}(x^i) \), which indicates that in their model to
study the stabilization of output it suffices to analyze the variance of output. However, when \( E(x^i - x^*) \neq 0 \), as in our model, this will not be the case. We can rewrite the output stabilization term as follows:\(^{15}\)

\[
E \left( (x^i - x^*)^2 \right) = P(E(x^i_L - x^*)^2) + (1 - P)(E(x^i_R - x^*)^2)
+ \text{Var}_E \left( x^i \right), \ i = N, D.
\]

When \( \sigma^2_\varepsilon \) is large enough, the comparison of output stabilization is reduced to the comparison of the economically induced variance of output. By contrast, when \( \sigma^2_\varepsilon \) is low enough, the comparison of output stabilization under delegation and nondelegation involves the analysis of the sum of the first two terms in the previous expression.

We know from the analysis carried out previously that the introduction of a moderately conservative independent central bank will lower expected output under a left-wing party and increase it under a right-wing party (\( E(x^N_L) > E(x^D_L) \) and \( E(x^N_R) < E(x^D_R) \)).\(^{16}\) Moreover, given that all expected outputs are smaller than the output target, \( x^* \), then \( 0 < E(x^* - x^N_L) < E(x^* - x^D_L) \) and \( 0 < E(x^* - x^R_R) < E(x^* - x^N_R) \). Hence, \( (E(x^D_L - x^*))^2 > (E(x^N_L - x^*))^2 \) and \( (E(x^D_R - x^*))^2 < (E(x^N_R - x^*))^2 \). Therefore, we can conclude that when \( \sigma^2_\varepsilon \) is low enough and \( P \) is high enough, output is more stable under nondelegation, whereas the opposite result is obtained when \( P \) is low enough. In other words, whenever the supply shocks are not significant, output stabilization will be more effective without a moderately conservative independent central bank the more likely is the less inflation-averse party to win the elections.

When an ultraconservative central bank is appointed and the indirect effect dominates, we obtain the same result. If the direct effect dominates, then \( E(x^N_L) > E(x^D_L) \) and \( E(x^N_R) > E(x^D_R) \). Hence, \( 0 < E(x^* - x^N_L) < E(x^* - x^D_L) \) and \( 0 < E(x^* - x^N_R) < E(x^* - x^D_R) \). Accordingly, \( (E(x^D_L - x^*))^2 > (E(x^N_L - x^*))^2 \) and \( (E(x^D_R - x^*))^2 < (E(x^N_R - x^*))^2 \). Therefore, we can conclude that,

\(^{15}\)See lemma A.1 in the appendix.
\(^{16}\)See figure 1.
in this case, when $\sigma_\epsilon^2$ is low enough output is more stable under nondelegation of monetary policy.

3. **Robustness**

In this section, we test the robustness of the results derived in the benchmark model. We examine two possible variations of our initial model. In subsection 3.1, we extend the model by allowing parties to differ in their target for public spending. In subsection 3.2, we analyze the framework in which authorities choose their policies sequentially, where the fiscal authority acts as the leader and the central bank acts as the follower. As shown below, we conclude that the main results obtained in section 2 are maintained in these other frameworks.

3.1 **Different Targets for Public Spending**

Next, we will generalize the benchmark model by assuming that both parties differ in their target for public spending. It could be argued that left-wing parties would prefer a higher target for government expenditure than right-wing parties. For this reason, we will assume that party $L$ has a larger government expenditure target than party $R$: $g^*_L > g^*_R$. In this new setup, we obtain the following inflation rates and outputs under nondelegation:

$$
\pi^N_L = \frac{(P + m_R + 1) A_L + (1 - P) A_R}{\Delta^N} - \frac{\varepsilon}{m_L + 2},
$$

$$
\pi^N_R = \frac{(1 - P + m_L + 1) A_R + P A_L}{\Delta^N} - \frac{\varepsilon}{m_R + 2},
$$

$$
x^N_L = x^* - \frac{1}{2\delta_L^N} \pi^N_L, \quad \text{and} \quad x^N_R = x^* - \frac{1}{2\delta_R^N} \pi^N_R,
$$

where $A_j = g^*_j + w^* + x^*$, $j = L, R$, and under delegation:

$$
\pi^D_L = \frac{(c_Rm_R + 1 + P) A_L + (1 - P) A_R}{\Delta^D} - \frac{\varepsilon}{c_Lm_L + 2},
$$

$$
\pi^D_R = \frac{(c_Lm_L + 1 + 1 - P) A_R + P A_L}{\Delta^D} - \frac{\varepsilon}{c_Rm_R + 2},
$$

$$
x^D_L = x^* - \frac{c_L}{2\delta_L^D} \pi^D_L, \quad \text{and} \quad x^D_R = x^* - \frac{c_R}{2\delta_R^D} \pi^D_R.
The following proposition shows that the results obtained in the benchmark model in terms of expected inflation and inflation stabilization still hold in this new setup.

**Proposition 5.** (a) By appointing an ultraconservative independent central bank responsible for monetary policy, the expected value of inflation is reduced and a higher degree of inflation stabilization is achieved. (b) By appointing a moderately conservative independent central bank, the expected value of inflation is reduced and a higher degree of inflation stabilization is achieved provided that $P$ is high enough.

In what follows, we will show that, when parties differ in their views on the size of the government, the results obtained for output in the benchmark model are no longer robust. In particular, how much more concerned about output stabilization party $L$ is with respect to party $R$ will play a crucial role. The following lemma points out that in this general setup expected output might be higher or lower under an $L$ administration.

**Lemma 4.** (a) $E(x^N_L) > E(x^N_R)$ if and only if $\delta_L - \delta_R$ is large enough. (b) $E(x^D_L) > E(x^D_R)$ if and only if $\delta_L - \delta_R$ is large enough.

This lemma indicates that the comparison between expected outputs depends on the value of the difference $\delta_L - \delta_R$. Notice that when the difference $\delta_L - \delta_R$ is small, we could expect a lower output when party $L$ is in office. This occurs because, as party $L$ has a higher target for public spending ($g^*_L > g^*_R$), it will be more willing to increase taxes, which will lead to lower output.

Obviously, the discrepancy of results between the two models will also affect the results related to the politically induced variance of output, as shown in the following proposition.

**Proposition 6.** (a) By appointing a moderately conservative independent central bank responsible for monetary policy, the politically

---

\(^{17}\)The results derived in propositions 1 and 5 also hold in an extension of the benchmark model in which the assumption that all policymakers have identical output targets is relaxed by considering that the left-wing party has a higher target.
Figure 4. Expected Outputs When $g_L^* > g_R^*$, $\delta_L - \delta_R$ Is Low and the Central Bank Is Moderately Conservative

induced variance of output is reduced provided that the difference $\delta_L - \delta_R$ is large enough. (b) By appointing an ultraconservative independent central bank, the politically induced variance of output is reduced when $\delta_L - \delta_R$ is large enough for high values of $g_L^* - g_R^*$. For low values of $g_L^* - g_R^*$, it is required that $\delta_L - \delta_R$ is large enough and that (i) both parties are concerned enough about public spending stabilization or (ii) the central bank is not too ultraconservative.

Unlike the benchmark model, the introduction of a moderately conservative independent central bank increases the politically induced variance of output whenever $\delta_L - \delta_R$ is low enough. Remember that delegating monetary policy to a moderately conservative central bank decreases expected output for party $L$ and increases expected output for party $R$. As expected output for party $L$ will be smaller than party $R$’s under nondelegation, the overall effect is that $E(x_L^D) < E(x_L^N) < E(x_R^N) < E(x_R^D)$. Hence, the politically induced variance of output with a moderately conservative independent central bank is higher than with no independent monetary policy, i.e., $Var_P(x^N) < Var_P(x^D)$. We show this situation in figure 4.

Furthermore, when party $L$ places much more weight on output stabilization ($\delta_L - \delta_R$ is large enough), lemma 4 states that $E(x_L^N) > E(x_R^N)$ and $E(x_L^D) > E(x_R^D)$, as in the benchmark model, which results in the same comparison of the politically induced variance of output, i.e., $Var_P(x^D) < Var_P(x^N)$.

Proposition 6(b) shows that the introduction of an ultraconservative independent central bank will increase the politically induced variance of output whenever $\delta_L - \delta_R$ is low enough. In this case, expected output under party $L$ will always be lower than under party $R$ ($E(x_R^N) > E(x_L^N)$ and $E(x_R^D) > E(x_L^D)$). The fact
that party $L$ has a higher target for public spending will bring in a stronger response from the ultraconservative central bank to reduce inflation and, thus, it will result in a larger reduction in expected output for party $L$\textsuperscript{18} as shown in figure 5. Consequently, $\text{Var}_P(x^N) < \text{Var}_P(x^D)$.

By contrast, when $\delta_L - \delta_R$ is large enough, the comparison between the politically induced variances of output depends on the difference $g^*_L - g^*_R$. For low values of this difference, we obtain the same results as in the benchmark case, but for high values of $g^*_L - g^*_R$ we find that $E(x^N_L) - E(x^N_R) > E(x^D_L) - E(x^D_R)$. As both sides of the previous inequality are positive whenever $\delta_L - \delta_R$ is large enough, it holds that $\text{Var}_P(x^N) > \text{Var}_P(x^D)$.

In relation to the economically induced variance of output, notice that the terms that multiply the supply shock in the expressions for output under delegation and nondelegation derived in this extension and in the benchmark model coincide. Lemma A.1 implies that we obtain the same results in this setup as in the basic framework. Therefore, proposition 4 applies in this new setup.

Additionally, it can be shown that $E(x^N_L) > E(x^D_L)$ is always satisfied, while $E(x^N_R) < E(x^D_R)$ if the central bank is moderately conservative. As a result, the analysis included in subsection 2.5 also applies in this extension and, therefore, we can conclude that the results related to output derived in the benchmark model are robust.

\textsuperscript{18}If $\delta_L$ and $\delta_R$ are similar, the parties’ behavior basically differs because of the different target for public spending. As the central bank chooses the inflation rate without taking into account public spending, under nondelegation we expect a large change in output for the party that chooses inflation with the largest target for public spending (party $L$), i.e., $E(x^N_L) - E(x^D_L) > E(x^N_R) - E(x^D_R)$. 

Figure 5. Expected Outputs When $g^*_L > g^*_R$, $\delta_L - \delta_R$ Is Low and the Central Bank Is Ultraconservative

\[
\begin{array}{ccc}
E(x^N_L) & E(x^N_R) & E(x^D) \\
E(x^D_L) & E(x^D_R) & x^* \\
\end{array}
\]

\text{Expected output}
3.2 Fiscal Leadership

The assumption that the government and the central bank choose their policies simultaneously is rather unrealistic. Taxes tend to be changed when the yearly budget is approved or when there is a change in the government. Money supply or interest rates, on the other hand, can be changed throughout the year. Thus, changing taxes is in general a more time-consuming process than adjusting the stance of monetary policy. Therefore, a more appropriate description of fiscal–monetary interactions would involve a leader–follower game, in which the fiscal authority acts as the leader and the monetary authority acts as the follower. Therefore, in this case the fiscal authority would choose the tax rate anticipating the response of the central bank to its action.

The following lemma shows that expected output will also be higher under an $L$ administration when the government acts as a Stackelberg leader.

\[ \text{Lemma 5. } E\left(x_{D,L}^{S}\right) > E\left(x_{R}^{D,S}\right). \]

Lemma 5 also implies that the politically induced variance of output is not automatically eliminated by delegating the monetary policy to an independent central bank in this sequential game.

The next two propositions show that the effects on inflation and output of appointing an independent central bank are not altered when introducing sequentiality in policy actions.

**Proposition 7.** (a) By appointing an ultraconservative independent central bank responsible for monetary policy, the expected value of inflation is reduced and a higher degree of inflation stabilization is achieved. (b) By appointing a moderately conservative independent central bank, the expected value of inflation is reduced and a higher degree of inflation stabilization is achieved provided that $P$ is high enough.

\[ \text{19 The superscript } S \text{ indicates that we are considering a Stackelberg game. A detailed derivation of the optimal inflation rates and outputs under this new setup is given in proposition C.1.} \]
Proposition 8. (a) The appointment of a moderately conservative independent central bank reduces the politically induced variance of output. By contrast, the appointment of an ultraconservative central bank reduces the politically induced variance of output when both parties are concerned enough about public spending stabilization or when the central bank is not too ultraconservative (i.e., when \( \gamma \) is high enough or when \( \delta_{CB} \) is high enough). (b) The appointment of a moderately conservative independent central bank increases the economically induced variance of output whenever \( P \) is large enough. By contrast, the appointment of an ultraconservative central bank always increases the economically induced variance of output.

Finally, in this new setup it can also be shown that \( E(x^N_L) > E(x^{D,S}_L) \) is always satisfied, while \( E(x^N_R) < E(x^{D,S}_R) \) if the central bank is moderately conservative. Applying the same reasoning as in the previous extension of the benchmark case, we conclude that the results related to output stabilization derived in the benchmark model are robust.

4. Conclusions

The analysis presented in this article has shown that the extent to which an independent central bank is successful in attaining price and output stability depends on the degree of conservativeness of the central bank in relation to the political parties, the private sector’s expectations on which party will win, and the level of economic uncertainty.

Alesina and Stella (2010) point out that in an economic crisis the level of economic uncertainty is high. In such a case, according to the model presented here, we have shown that the appointment of an ultraconservative central bank always increases output instability. By contrast, the appointment of a moderately conservative independent central bank will increase output instability if the probability of the less inflation-averse party winning the elections is high enough.

Further, when there is little economic uncertainty, Alesina and Gatti’s (1995) result that delegation of the conduct of monetary policy increases output stabilization is not always valid. In particular, we have shown that a moderately conservative central bank
reduces output stabilization if the probability of the less inflation-averse party winning the elections is high enough. The same result applies for an ultraconservative central bank when the left-wing party is substantially less conservative than the right-wing party, or when the central bank is very similar in conservativeness to the right-wing party. In other cases, an ultraconservative central bank always reduces output stability.

Focusing on the European case, the Maastricht Treaty determined that the European Central Bank should primarily be concerned with price stability in the euro area. Our analysis suggests that the creation of such an ultraconservative central bank has resulted in a lower and more stable inflation, but likely at a cost of more output instability, even in the case when there is little economic uncertainty. In particular, Martínez-Martín, Saiz, and Stoekyll (2018) show how growth volatility across the euro-area countries was substantially higher than for the G-7 countries in the period after the global financial crisis (2009 to 2012).

We have checked the robustness of our results considering some variations of our initial model. There are some other possible interesting avenues of research. For instance, a natural extension would be to endogenize the probability of a party being elected. In this potentially dynamic setting, public debt could also be introduced in order to consider how the incumbent party could affect the probability of being elected.

Finally, we would like to point out that our key results depend on the manner in which taxes are introduced in Alesina and Gatti’s model. The model presented in this article follows the related literature where taxes lower aggregate supply. However, taxes could alternatively be incorporated through aggregate demand. In this case, the inflation rate and the level of output chosen by a central bank would not depend on the party in office, which would be aligned with Alesina and Gatti’s (1995) result.

Further, the authors mention that Ireland was excluded “to avoid distortions in the analysis caused by the high volatility of Irish GDP.”

We would like to thank an anonymous referee for pointing out this issue.
Appendix A

We initially derive the expressions for output and public spending when government $j$ is in office. To ease the notation, we omit the subscript $j$ in the following two proofs.

Derivation of Expression (1)

Output of a representative firm is given by $X = L^\lambda \exp(\varepsilon/2)$, where $X$ denotes the real output, $L$ represents labor, $\lambda$ indicates the output elasticity, and $\varepsilon$ represents a supply shock. We assume that $\varepsilon$ has zero mean and variance $\sigma_\varepsilon^2$. Distortionary taxes are levied on production. The firm maximizes profit, given by $(1 - \tau)PL^\lambda \exp(\varepsilon/2) - WL$, where $\tau$ denotes the tax rate on total revenue of firms, $P$ represents the price level, and $W$ is the nominal wage. Solving for the firm’s labor demand, assuming it can hire the labor it demands at the given nominal wage, taking logs, we have $x = a(p - \tau - w) + b + \frac{\varepsilon}{2(1 - \lambda)}$, where lowercase letters denote logs of nominal variables, $a = \lambda/(1 - \lambda)$, $b = \lambda \ln \lambda/(1 - \lambda)$, and $\ln (1 - \tau) \approx -\tau$. Following Debelle and Fischer (1994), for simplicity, we set $\lambda = 0.5$, so that $a = 1$ and we approximate $\ln \lambda$ to 0. Hence, $x = p - \tau - w + \varepsilon$.

In addition, following Alesina and Tabellini (1987), Debelle and Fischer (1994), and Beetsma and Bovenberg (2001), among others, we assume that workers are represented by a centralized trade union which seeks to minimize deviations of the real wage rate from a particular target $w^*$, hence it sets the nominal wage (in logs) to achieve the target $w^*$. The trade union chooses the nominal wage in advance of the actions of the two policymakers, but knowing their objective functions, i.e., it minimizes the objective function: $E \left( (w - p - w^*)^2 \right) / 2$. The first-order condition of this optimization problem immediately yields $w = p^e + w^*$. Therefore, $x = p - p^e - \tau - w^* + \varepsilon$. Finally, approximating $p - p^e$ by $\pi - \pi^e$, where $\pi$ represents inflation rate and $\pi^e$ expected inflation rate, we get the aggregate supply equation of the model, i.e., $x = \pi - \pi^e - \tau - w^* + \varepsilon$. ■
Derivation of Expression (2)

The government’s budget constraint in nominal terms at \( t \) is given by

\[ \Pi_t G_t = \tau \Pi_t X_t + M_t - M_{t-1}, \]

where \( G_t \) denotes the public spending and \( M_t \) the nominal money supply at \( t \). Following Canzoneri (1985), money demand depends only on an output level that is independent of fiscal policy (taxes), \( M_t = \Pi_t \bar{X} \). Dividing the government’s budget constraint by nominal income, \( \Pi_t \bar{X} \), yields

\[ g_t = \frac{\tau X_t}{\bar{X}} + \frac{M_t - M_{t-1}}{\Pi_t \bar{X}}, \]

where \( g_t \) is public spending as a share of the (nondistortionary) output \( \bar{X} \). Taking into account the money demand function, whenever \( X_t \) is close to \( \bar{X} \) and approximating \( \frac{\Pi_t - \Pi_{t-1}}{\Pi_t} \) to \( \pi_t \) (as in Alesina and Tabellini 1987 and Dimakou 2013), we get the government budget constraint at \( t \), i.e., \( g_t = \tau + \pi_t \).

In the following two propositions, we derive the policies chosen by the two parties, if in office, under nondelegation and under delegation in the benchmark model, where parties differ only in the relative weight assigned to output stabilization.

**Proposition A.1.** The policies chosen by the two parties, if in office, under nondelegation are given by

\[
\begin{align*}
\pi_l^N &= \frac{m_R + 2}{\Delta^N} A - \frac{\varepsilon}{m_L + 2}, \\
\pi_r^N &= \frac{m_L + 2}{\Delta^N} A - \frac{\varepsilon}{m_R + 2}, \\
\tau_l^N &= g^* - \left(1 + \frac{1}{2\gamma}\right) \pi_l^N, \text{ and } \tau_r^N = g^* - \left(1 + \frac{1}{2\gamma}\right) \pi_r^N,
\end{align*}
\]

where \( \Delta^N = P(m_R + 2)(m_L + 1) + (1 - P)(m_R + 1)(m_L + 2) \) and \( A = g^* + w^* + x^* \).

**Proof of Proposition A.1.** Under nondelegation the party in office, denoted by \( j \), chooses \( \pi_j \) and \( \tau_j \) in order to solve the following problem:

\[
\min_{\pi_j, \tau_j} V_{G_j} = \frac{1}{2} \left( \pi_j^2 + \delta_j (x_j - x^*)^2 + \gamma (g_j - g^*)^2 \right).
\]

\(^{22}\)To ease the notation, we drop the superscript \( N \) and \( D \) in the proofs of propositions A.1 and A.2, respectively.
The first-order conditions (FOC) of this problem are given by
\begin{align*}
\frac{\partial}{\partial \pi_j} V_{Gj} &= \pi_j + \delta_j (x_j - x^*) + \gamma (g_j - g^*) = 0 \\
\frac{\partial}{\partial \tau_j} V_{Gj} &= -\delta_j (x_j - x^*) + \gamma (g_j - g^*) = 0.
\end{align*}

Using expressions (1) and (2) in the previous two equalities, we get
\begin{align*}
\pi_j &= \frac{\pi^e + A - \varepsilon}{m_j + 2} \quad \text{and} \\
\tau_j &= g^* - \frac{\delta_j (2\gamma + 1)}{\gamma + \delta_j + 4\gamma\delta_j} (\pi^e + A - \varepsilon),
\end{align*}
where \( m_j = \frac{1}{\delta_j + \gamma} \) and \( A = g^* + w^* + x^* \). Rewriting (A.1) for the two parties, we have \( \pi_L = \frac{\pi^e + A - \varepsilon}{m_L + 2} \) and \( \pi_R = \frac{\pi^e + A - \varepsilon}{m_R + 2} \). Moreover, recall that \( \pi^e = PE(\pi_L) + (1 - P)E(\pi_R) \). Taking expectations in the previous expressions and solving for \( \pi^e \), we get
\begin{equation}
\pi^e = \frac{\frac{P}{m_L + 2} + \frac{1-P}{m_R + 2}}{1 - \left( \frac{P}{m_L + 2} + \frac{1-P}{m_R + 2} \right)} A. \tag{A.3}
\end{equation}

Substituting this expression into (A.1) and (A.2) for \( j = L, R \), and after some algebra, we obtain the expressions for \( \pi_L, \pi_R, \tau_L, \) and \( \tau_R \).

\textbf{PROPOSITION A.2.} Under delegation, the policies chosen by the central bank and the party, if in office, are given by
\begin{align*}
\pi^D_L &= \frac{c_R m_R + 2}{\Delta^D} A - \frac{\varepsilon}{c_L m_L + 2}, \quad \pi^D_R = \frac{c_L m_L + 2}{\Delta^D} A - \frac{\varepsilon}{c_R m_R + 2}, \\
\tau^D_L &= g^* - \left( 1 + \frac{c_L}{2\gamma} \right) \pi^D_L, \quad \text{and} \quad \tau^D_R = g^* - \left( 1 + \frac{c_R}{2\gamma} \right) \pi^D_R,
\end{align*}

Direct computations yield that the objective function is strictly convex. Therefore, the first-order conditions are necessary and sufficient to obtain a minimum. The same comment applies to the remaining optimization problems.
where
\[ \Delta^D = P(c_{LmL} + 1)(c_{RmR} + 2) + (1 - P)(c_{RmR} + 1)(c_{LmL} + 2). \]

Proof of Proposition A.2. Suppose that party \( j \) is in office. Under delegation, the central bank chooses \( \pi_j \) in order to solve the following problem:

\[
\min_{\pi_j} V_{CB} = \frac{1}{2} \left( \pi_j^2 + \delta_{CB} (x_j - x^*)^2 \right).
\]

The FOC of this problem is given by \( \frac{\partial}{\partial \pi_j} V_{CB} = \pi_j + \delta_{CB} (x_j - x^*) = 0 \).

In this setup the fiscal authority chooses \( \tau_j \) in order to solve the following problem:

\[
\min_{\tau_j} V_{Gj} = \frac{1}{2} \left( \pi_j^2 + \delta_j (x_j - x^*)^2 + \gamma (g_j - g^*)^2 \right).
\]

The FOC of this problem is given by \( \frac{\partial}{\partial \tau_j} V_{Gj} = -\delta_j (x_j - x^*) + \gamma (g_j - g^*) = 0 \). Using expressions (1) and (2) in the FOC of the authorities' problems, it follows that

\[
\pi_j = \frac{\delta_{CB}\gamma}{\gamma + \delta_j + 2\gamma\delta_{CB}} \left( \pi^e + A - \varepsilon \right) \quad \text{and} \quad (A.4)
\]

\[
\tau_j = g^* - \frac{\delta_j + \gamma\delta_{CB}}{\gamma + \delta_j + 2\gamma\delta_{CB}} \left( \pi^e + A - \varepsilon \right). \quad (A.5)
\]

Using them in the expression for \( \pi^e \) and solving for \( \pi^e \), we get

\[
\pi^e = \frac{\delta_{CB}\gamma \left( \frac{P}{\gamma + \delta_L + 2\gamma\delta_{CB}} + \frac{1-P}{\gamma + \delta_R + 2\gamma\delta_{CB}} \right) A}{1 - \delta_{CB}\gamma \left( \frac{P}{\gamma + \delta_L + 2\gamma\delta_{CB}} + \frac{1-P}{\gamma + \delta_R + 2\gamma\delta_{CB}} \right)}.
\]

Substituting this expression into (A.4) and (A.5) for \( j = L, R \), and after some algebra, we obtain the desired expressions.

Proof of Lemma 1

Combining the FOC of the optimization problem given in the proof of proposition A.1, (6) and (7) are derived. Using the expressions of \( \pi^N_L \) and \( \pi^N_R \) given in the statement of proposition A.1, we get
\[ x^N_L = x^* - \frac{1}{2\delta_L} \left( \frac{m_R + 2}{\Delta^N} A - \frac{\varepsilon}{m_L + 2} \right) \] and \( (A.6) \)

\[ x^N_R = x^* - \frac{1}{2\delta_R} \left( \frac{m_L + 2}{\Delta^N} A - \frac{\varepsilon}{m_R + 2} \right). \] \( (A.7) \)

Taking expectations, we have \( E(x^N_L) = x^* - \frac{m_R + 2}{2\delta_L \Delta^N} A \) and \( E(x^N_R) = x^* - \frac{m_L + 2}{2\delta_R \Delta^N} A \). Using the expressions of \( m_L \) and \( m_R \), we have \( E(x^N_L) - E(x^N_R) = \frac{(4\gamma + 1)(\delta_L - \delta_R)}{4\gamma \delta_L \delta_R \Delta^N} A \). As \( \delta_L > \delta_R \), we conclude that \( E(x^N_L) > E(x^N_R) \).

**Proof of Lemma 2**

From the FOC of the optimization problem of the central bank given in the proof of proposition A.2, we have (11) and (12). Using the expressions of \( \pi^D_L \) and \( \pi^D_R \) given in the statement of proposition A.2, it follows that

\[ x^D_L = x^* - \frac{c_L}{2\delta_L} \left( \frac{c_R m_R + 2}{\Delta^D} A - \frac{\varepsilon}{c_L m_L + 2} \right) \] and \( (A.8) \)

\[ x^D_R = x^* - \frac{c_R}{2\delta_R} \left( \frac{c_L m_L + 2}{\Delta^D} A - \frac{\varepsilon}{c_R m_R + 2} \right). \] \( (A.9) \)

Taking expectations, we have

\[ E(x^D_L) = x^* - \frac{c_L (c_R m_R + 2)}{2\delta_L \Delta^D} A \] and \( (A.10) \)

\[ E(x^D_R) = x^* - \frac{c_R (c_L m_L + 2)}{2\delta_R \Delta^D} A. \] \( (A.11) \)

From the expressions of \( m_L, m_R, c_L, \) and \( c_R, E(x^D_L) - E(x^D_R) = \frac{\delta_L - \delta_R}{\delta_{CEB} \gamma \Delta^D} A \), which implies \( E(x^D_L) > E(x^D_R) \) since \( \delta_L > \delta_R \). \( \blacksquare \)

Next, we derive a lemma which will be useful to prove some of the results that follow.

**Lemma A.1.** Consider a random variable \( z \) that, conditional on the realization of the shock, takes two possible values given by \( z_L = E(z_L) + F_L \varepsilon \) and \( z_R = E(z_R) + F_R \varepsilon \). Then, the politically induced
The variance of \( z \) is given by \( \text{Var}_P(z) = P(1-P)(E(z_L) - E(z_R))^2 \), and the economically induced variance of \( z \) by \( \text{Var}_E(z) = (P(F_L)^2 + (1-P)(F_R)^2)\sigma^2_\varepsilon \). Moreover,

\[
E(z^2) = P(E(z_L))^2 + (1-P)(E(z_R))^2 \\
+ \left( P(F_L)^2 + (1-P)(F_R)^2 \right) \sigma^2_\varepsilon.
\] (A.12)

**Proof of Lemma A.1.** Notice that \( \text{Var}(z) = PE((z_L - E(z))^2) + (1-P)E((z_R - E(z))^2) \). As \( E((z_L - E(z))^2) = (1-P)^2(E(z_L) - E(z_R))^2 + (F_L)^2\sigma^2_\varepsilon \) and \( E((z_R - E(z))^2) = P^2(E(z_L) - E(z_R))^2 + (F_R)^2\sigma^2_\varepsilon \), we have \( \text{Var}(z) = P(1-P)(E(z_L) - E(z_R))^2 + (P(F_L)^2 + (1-P)(F_R)^2)\sigma^2_\varepsilon \). The first term corresponds to the politically induced variance of \( z \), whereas the second term corresponds to the economically induced variance of \( z \). Finally, expression (A.12) follows from the fact that \( E(z^2) = (E(z))^2 + \text{Var}(z) \).

**Proof of Proposition 1**

Direct computations yield

\[
E(\pi^N) = \frac{P(m_R + 2) + (1-P)(m_L + 2)}{\Delta^N} A \text{ and} \\
E(\pi^D) = \frac{P(c_R m_R + 2) + (1-P)(c_L m_L + 2)}{\Delta^D} A.
\]

In addition, applying lemma A.1 for \( z = \pi^N \) and \( z = \pi^D \), it follows that

\[
E((\pi^N)^2) = \frac{P(m_R + 2)^2 + (1-P)(m_L + 2)^2}{(\Delta^N)^2} A^2 \\
+ \left( P \left( \frac{1}{m_L + 2} \right)^2 + (1-P) \left( \frac{1}{m_R + 2} \right)^2 \right) \sigma^2_\varepsilon \text{ and} \\
E((\pi^D)^2) = \frac{P(c_R m_R + 2)^2 + (1-P)(c_L m_L + 2)^2}{(\Delta^D)^2} A^2 \\
+ \left( P \left( \frac{1}{c_L m_L + 2} \right)^2 + (1-P) \left( \frac{1}{c_R m_R + 2} \right)^2 \right) \sigma^2_\varepsilon.
\]
(a) Let \( f(c_L, c_R) = E(\pi^D) \) and \( g(c_L, c_R) = E((\pi^D)^2) \). Notice that \( f(c_L, c_R) \) and \( g(c_L, c_R) \) are decreasing functions in \( c_L \) and \( c_R \). Moreover, \( f(1, 1) = E(\pi^N) \) and \( g(1, 1) = E((\pi^N)^2) \). The combination of these results allows us to conclude that \( E(\pi^N) > E(\pi^D) \) and \( E((\pi^N)^2) > E((\pi^D)^2) \) whenever \( c_L > c_R \geq 1 \).

(b) Suppose now that \( c_L > 1 > c_R \). First, we focus on the comparison of the expected inflation. Let \( h(P) = E(\pi^N) - E(\pi^D) \). Differentiating,

\[
\frac{\partial}{\partial P} h(P) = \frac{(m_L + 2)(m_R + 2)(m_R - m_L)}{(\Delta^N)^2} A - \frac{(c_L m_L + 2)(c_R m_R + 2)(c_R m_R - c_L m_L)}{(\Delta^D)^2} A.
\]

Direct computations yield \( \frac{\partial}{\partial c_L} \left( \frac{\partial}{\partial P} h(P) \right) > 0 \) and \( \frac{\partial}{\partial c_R} \left( \frac{\partial}{\partial P} h(P) \right) < 0 \). Hence, \( \frac{\partial}{\partial P} h(P) > \frac{\partial}{\partial P} h(P) \big|_{c_L=1} = 0 \) since \( c_L > 1 > c_R \). Therefore, \( h(P) \) is an increasing function in \( P \). Moreover, \( h(1) > 0 \) and \( h(0) < 0 \) whenever \( c_L > 1 > c_R \). This implies that there exists a unique value \( \overline{P} \) such that \( h(P) > 0 \) (or equivalently, \( E(\pi^N) > E(\pi^D) \)) if and only if \( P > \overline{P} \).

In relation to the comparison of the term related to inflation stabilization, notice that as \( c_L > 1 > c_R \), \( E((\pi^N)^2)|_{P=1} > E((\pi^D)^2)|_{P=1} \) and \( E((\pi^N)^2)|_{P=0} < E((\pi^D)^2)|_{P=0} \). Moreover, direct computations yield: (1) \( E((\pi^N)^2) \) increases in \( P \) since \( m_R > m_L \), and (2) \( E((\pi^D)^2) \) decreases in \( P \) since \( c_R m_R < c_L m_L \). Therefore, we can conclude that there exists a value \( \overline{P} \) such that \( E((\pi^D)^2) < E((\pi^N)^2) \) if and only if \( P > \overline{P} \).

\[\square\]

**Proof of Lemma 3**

Differentiating (A.10) and (A.11), we have the results stated in the statement of this lemma.
Proof of Proposition 3

Direct computations yield

\[
E(x_N^L) - E(x_N^R) = \frac{(4\gamma + 1)(\delta_L - \delta_R)\gamma}{\left( P (\gamma + \delta_L + 2\gamma\delta_L) (\gamma + \delta_R + 4\gamma\delta_R) + (1 - P) (\gamma + \delta_R + 2\gamma\delta_R) (\gamma + \delta_L + 4\gamma\delta_L) \right)} \tag{A}
\]

and

\[
E(x_D^L) - E(x_D^R) = \frac{(\delta_L - \delta_R)\gamma}{\left( P (\gamma + \delta_L + \gamma\delta_{CB}) (\gamma + \delta_R + 2\gamma\delta_{CB}) + (1 - P) (\gamma + \delta_R + \gamma\delta_{CB}) (\gamma + \delta_L + 2\gamma\delta_{CB}) \right)} \tag{A}
\]

Using lemma 1 and lemma 2, we have that \( \text{Var}_P(x_N^L) > \text{Var}_P(x_D^L) \) if and only if \( E(x_N^L) - E(x_N^R) > E(x_D^L) - E(x_D^R) \), which is equivalent to \( g(\delta_{CB}) > 0 \), with

\[
g(\delta_{CB}) = 2\gamma^2\delta_{CB}^2 + \delta_{CB} (\gamma + \delta_L) + 2\gamma (\gamma + \delta_R) + P\gamma (\delta_L - \delta_R) + 2\gamma\left(\frac{2\gamma^2 + (\delta_R + P\delta_L - P\delta_R - 4\delta_L\delta_R)\gamma - \delta_L\delta_R}{4\gamma + 1}\right).
\]

Note that \( g \) is increasing in \( \delta_{CB} \). Next, we distinguish two cases:

Case 1: The central bank is moderately conservative \((2\delta_R < \delta_{CB} < 2\delta_L)\). Combining the monotonicity property of \( g(\delta_{CB}) \) and the fact that \( g(2\delta_R) > 0 \), we have that \( g(\delta_{CB}) > 0 \) whenever \( 2\delta_R < \delta_{CB} < 2\delta_L \) and, hence, \( \text{Var}_P(x_N^L) > \text{Var}_P(x_D^L) \).

Case 2: The central bank is ultraconservative \((\delta_{CB} \leq 2\delta_R)\). Then, we consider two subcases:

- Subcase 2.1: \( g(0) \geq 0 \). In this case \( 2\gamma^2 + \gamma(\delta_R + P\delta_L - P\delta_R - 4\delta_L\delta_R) - \delta_L\delta_R \geq 0 \). Descartes’ rule tells us that there exists a unique value of \( \gamma \), denoted by \( \gamma_1 \), such that the previous inequality is satisfied whenever \( \gamma \geq \gamma_1 \). Combining the monotonicity property of \( g(\delta_{CB}) \) and the fact that \( g(0) \geq 0 \), we have that \( g(\delta_{CB}) > 0 \) whenever \( \delta_{CB} > 0 \) and,
hence, \( \text{Var}_P(x^N) > \text{Var}_P(x^D) \). Therefore, we conclude that if \( \gamma \geq \overline{\gamma} \), then \( \text{Var}_P(x^N) > \text{Var}_P(x^D) \).

- **Subcase 2.2:** \( g(0) < 0 \) (or, equivalently, \( \gamma < \overline{\gamma} \)). In this case, the monotonicity property of \( g(\delta_{CB}) \) implies that there exists a unique value of \( \delta_{CB} \), denoted by \( \delta_{CB}^* \), such that \( g(\delta_{CB}^*) > 0 \) whenever \( \delta_{CB} > \delta_{CB}^* \). Consequently, \( \text{Var}_P(x^N) > \text{Var}_P(x^D) \) if and only if \( \delta_{CB} > \delta_{CB}^* \).

**Proof of Proposition 4**

Combining (A.6), (A.7), and lemma A.1,

\[
\text{Var}_E(x^N) = \left( P \left( \frac{1}{2\delta_L (m_L + 2)} \right)^2 + (1 - P) \left( \frac{1}{2\delta_R (m_R + 2)} \right)^2 \right) \sigma_\varepsilon^2.
\]

Analogously, combining (A.8), (A.9), and lemma A.1,

\[
\text{Var}_E(x^D) = \left( P \left( \frac{c_L}{2\delta_L (c_Lm_L + 2)} \right)^2 + (1 - P) \left( \frac{c_R}{2\delta_R (c_Rm_R + 2)} \right)^2 \right) \sigma_\varepsilon^2.
\]

Hence, \( \text{Var}_E(x^N) - \text{Var}_E(x^D) \) is a linear function in \( P \). Next we distinguish two cases:

**Case 1:** \( c_R < 1 < c_L \). It is easy to see that \( \text{Var}_E(x^N)|_{P=1} - \text{Var}_E(x^D)|_{P=1} < 0 \) and that \( \text{Var}_E(x^N)|_{P=0} - \text{Var}_E(x^D)|_{P=0} > 0 \) whenever \( c_R < 1 < c_L \). Hence, we can conclude that there exists a value of \( P \), denoted by \( \overline{P} \), such that \( \text{Var}_E(x^D) > \text{Var}_E(x^N) \) if and only if \( P > \overline{P} \).

**Case 2:** \( c_L > c_R \geq 1 \). Direct computations yield that \( \text{Var}_E(x^D) > \text{Var}_E(x^N) \) whenever \( c_L > c_R \geq 1 \).

**Appendix B**

In the following two propositions, we derive the policies chosen by the two parties, if in office, under nondelegation and under delegation in the generalized model, where parties differ both in the
relative weight assigned to output stabilization and in their target for public spending.

**Proposition B.1.** The policies chosen by the two parties, if in office, under nondelegation are given by

\[
\begin{align*}
\pi_N^L &= \frac{(P + m_R + 1) A_L + (1 - P) A_R}{\Delta^N} - \frac{\epsilon}{m_L + 2}, \\
\pi_N^R &= \frac{(1 - P + m_L + 1) A_R + P A_L}{\Delta^N} - \frac{\epsilon}{m_R + 2}, \\
\tau_N^L &= g^*_L - \left(1 + \frac{1}{2\gamma}\right) \pi_N^L, \quad \text{and} \quad \tau_N^R = g^*_R - \left(1 + \frac{1}{2\gamma}\right) \pi_N^R,
\end{align*}
\]

where \(\Delta^N = P(m_R + 2)(m_L + 1) + (1 - P)(m_R + 1)(m_L + 2)\) and \(A_j = g_j^* + w^* + x^*, \ j = L, R.\)

**Proof of Proposition B.1.** Under nondelegation, the party in office, denoted by \(j\), chooses \(\pi_j\) and \(\tau_j\) in order to solve the following problem:

\[
\min_{\pi_j, \tau_j} V_{Gj} = \frac{1}{2}\left(\pi_j^2 + \delta_j(x_j - x^*)^2 + \gamma(g_j - g_j^*)^2\right).
\]

The FOCs of this problem are given by

\[
\begin{align*}
\frac{\partial}{\partial \pi_j} V_{Gj} &= \pi_j + \delta_j(x_j - x^*) + \gamma(g_j - g_j^*) = 0 \quad \text{and} \\
\frac{\partial}{\partial \tau_j} V_{Gj} &= -\delta_j(x_j - x^*) + \gamma(g_j - g_j^*) = 0.
\end{align*}
\]

Using expressions (1) and (2) in the previous two equalities, we get

\[
\begin{align*}
\pi_j &= \frac{1}{m_j + 2} (\pi^e + A_j - \varepsilon) \quad \text{and} \quad (B.1) \\
\tau_j &= g_j^* - \frac{\delta_j(2\gamma + 1)}{\gamma + \delta_j + 4\gamma\delta_j} (\pi^e + A_j - \varepsilon), \quad (B.2)
\end{align*}
\]

where \(A_j = g_j^* + w^* + x^*\). Rewriting (B.1) for the two parties, we have \(\pi_L = \frac{\pi^e + A_L - \varepsilon}{m_L + 2}\) and \(\pi_R = \frac{\pi^e + A_R - \varepsilon}{m_R + 2}\). Moreover, recall that
\[ \pi^e = PE(\pi_L) + (1 - P)E(\pi_R). \]

Taking expectations in the previous expressions and solving for \( \pi^e \), we get
\[
\pi^e = \frac{P}{m_L+2}A_L + \frac{1-P}{m_R+2}A_R \quad \frac{1}{1 - \left( \frac{P}{m_L+2} + (1 - P)\frac{1}{m_R+2} \right)}.
\]

Substituting this expression into (B.1) and (B.2) for \( j = L, R \), and after some algebra, we obtain the expressions for \( \pi_L, \pi_R, \tau_L, \) and \( \tau_R \).

**Proposition B.2.** Under delegation, the policies chosen by the central bank and the party, if in office, are given by

\[
\begin{align*}
\pi^D_L &= \left( \frac{c_R m_R + 1 + P}{\Delta^D} \right) A_L + \left( \frac{1 - P}{\Delta^D} \right) A_R - \frac{\varepsilon}{c_L m_L + 2}, \\
\pi^D_R &= \left( \frac{c_L m_L + 1 + 1 - P}{\Delta^D} \right) A_R + \left( \frac{1 - P}{\Delta^D} \right) A_L - \frac{\varepsilon}{c_R m_R + 2}, \\
\tau^D_L &= g^*_L - \left( 1 + \frac{c_L}{2\gamma} \right) \pi^D_L, \quad \text{and} \quad \tau^D_R = g^*_R - \left( 1 + \frac{c_R}{2\gamma} \right) \pi^D_R,
\end{align*}
\]

where \( \Delta^D = P(c_L m_L + 1)(c_R m_R + 2) + (1 - P)(c_R m_R + 1)(c_L m_L + 2) \).

**Proof of Proposition B.2.** Suppose that party \( j \) is in office. Under delegation, the central bank chooses \( \pi_j \) in order to solve the following problem:

\[
\min_{\pi_j} V_{CB} = \frac{1}{2} \left( \pi_j^2 + \delta_{CB} (x_j - x^*)^2 \right).
\]

The FOC of this problem is given by \( \frac{\partial}{\partial \pi_j} V_{CB} = \pi_j + \delta_{CB}(x_j - x^*) = 0 \).

In this setup the fiscal authority chooses \( \tau_j \) in order to solve the following problem:

\[
\min_{\tau_j} V_{G_j} = \frac{1}{2} \left( \tau_j^2 + \delta_j (x_j - x^*)^2 + \gamma (g_j - g^*_j)^2 \right).
\]

The FOC of this problem is given by \( \frac{\partial}{\partial \tau_j} V_{G_j} = -\delta_j (x_j - x^*) + \gamma (g_j - g^*_j) = 0 \).

Using expressions (1) and (2) in the FOC of the authorities’ problems, it follows that
\[
\pi_j = \frac{\gamma \delta_{CB}}{\gamma + \delta_j + 2\gamma \delta_{CB}} (\pi^e + A_j - \varepsilon) \quad \text{and} \quad (B.3)
\]
\[
\tau_j = g_j^* - \frac{\delta_j + \gamma \delta_{CB}}{\gamma + \delta_j + 2\gamma \delta_{CB}} (\pi^e + A_j - \varepsilon). \tag{B.4}
\]

Using them in the expression for \(\pi^e\) and solving for \(\pi^e\), we get

\[
\pi^e = \frac{\delta_{CB} \gamma \left( \frac{P}{\gamma + \delta_L + 2\gamma \delta_{CB}} A_L + \frac{1-P}{\gamma + \delta_R + 2\gamma \delta_{CB}} A_R \right)}{1 - \delta_{CB} \gamma \left( \frac{P}{\gamma + \delta_L + 2\gamma \delta_{CB}} + \frac{1-P}{\gamma + \delta_R + 2\gamma \delta_{CB}} \right)}.
\]

Substituting this expression into (B.3) and (B.4) for \(j = L, R\), and after some algebra, we obtain the desired expressions.

**Proof of Proposition 5**

Direct computations yield

\[
E(\pi^N) = \frac{P(m_R + 2)L + (m_L + 2)(1-P)L}{\Delta^N}
\]

and

\[
E(\pi^D) = \frac{P(c_rm_R + 2)L + (c_lm_L + 2)(1-P)L}{\Delta^D}.
\]

In addition, applying lemma A.1 for \(z = \pi^N\) and \(z = \pi^D\), it follows that

\[
E((\pi^N)^2) = \frac{P ((P + m_R + 1)L + (1 - P)R)^2 + (1 - P)(((1 - P + m_L + 1)L + P)R)^2}{(\Delta^N)^2}
\]

and

\[
E((\pi^D)^2) = \frac{P ((c_rm_R + 1 + P)L + (1 - P)R)^2 + (1 - P)(((c_lm_L + 2 - P)L + P)R)^2}{(\Delta^D)^2}
\]

(a) Let \(f(c_L, c_R) = E(\pi^D)\) and \(g(c_L, c_R) = E((\pi^D)^2)\). Notice that \(f(c_L, c_R)\) and \(g(c_L, c_R)\) are decreasing functions in \(c_L\) and \(c_R\). Moreover, \(f(1, 1) = E(\pi^N)\) and \(g(1, 1) = E((\pi^N)^2)\). The combination of these results allows us to conclude that \(E(\pi^N) > E(\pi^D)\) and \(E((\pi^N)^2) > E((\pi^D)^2)\) whenever \(c_L > c_R \geq 1\).
(b) Suppose now that $c_L > 1 > c_R$. First, we focus on the comparison of the expected inflation. Let $h(P) = E(\pi^N) - E(\pi^D) = \frac{P(m_R + 2)A_L + (m_L + 2)(1-P)A_R}{\Delta^N} - \frac{P(c_R m_R + 2)A_L + (c_L m_L + 2)(1-P)A_R}{\Delta^D}$. Differentiating,

$$\frac{\partial}{\partial P} h(P) = \frac{(m_L + 2) (m_R + 2) (A_L (m_R + 1) - A_R (m_L + 1))}{(\Delta^N)^2} - \frac{\left((c_L m_L + 2) (c_R m_R + 2) \times (A_L (c_R m_R + 1) - A_R (c_L m_L + 1))\right)}{(\Delta^D)^2}.$$ 

Now, we distinguish two cases:

**Case 1:** $A_L (c_R m_R + 1) \leq A_R (c_L m_L + 1)$. In this case $\frac{\partial}{\partial P} h(P) > 0$. Therefore, $h(P)$ is an increasing function in $P$. Moreover, $h(1) > 0$ and $h(0) < 0$ whenever $c_L > 1 > c_R$. This implies that there exists a unique value $\bar{P}$ such that $h(P) > 0$ (or equivalently, $E(\pi^N) > E(\pi^D)$) if and only if $P > \bar{P}$.

**Case 2:** $A_L (c_R m_R + 1) > A_R (c_L m_L + 1)$. In this case $\frac{\partial^2}{\partial P^2} h(P) > 0$. Therefore, $h(P)$ is a convex function in $P$. Moreover, $h(1) > 0$ and $h(0) < 0$ whenever $c_L > 1 > c_R$. This implies that there exists a unique value $\bar{P}$ such that $h(P) > 0$ (or equivalently, $E(\pi^N) > E(\pi^D)$) if and only if $P > \bar{P}$.

In relation to the comparison of the term related to inflation stabilization, notice that as $c_L > 1 > c_R$, $E((\pi^N)^2) |_{P=1} > E((\pi^D)^2) |_{P=1}$ and $E((\pi^N)^2) |_{P=0} < E((\pi^D)^2) |_{P=0}$. Hence, we can conclude that when $P$ is large enough, then $E((\pi^D)^2) < E((\pi^N)^2)$.

**Proof of Lemma 4**

(a) Substituting the expressions of $\pi^N_L$ and $\pi^N_R$ into the expressions of $x^N_L$ and $x^N_R$, and taking expectations, we have $E(x^N_L) = x^* - \frac{(P + m_R + 1)A_L + (1-P)A_R}{2\delta_L \Delta^N}$ and $E(x^N_R) = x^* - \frac{(1-P + m_L + 1)A_R + PA_L}{2\delta_R \Delta^N}$. Using the expressions of $m_L$ and $m_R$,
Proof of Proposition 6

(a) Recall that $\text{Var}_P(x^N) > \text{Var}_P(x^D)$ is equivalent to $|E(x^N_L) - E(x^N_R)| > |E(x^D_L) - E(x^D_R)|$. Moreover, when monetary policy is delegated to a moderately conservative central bank, we have

$$\begin{align*}
E(x^N_L) &> E(x^D_L) \quad \text{(B.5)} \\
E(x^N_R) &< E(x^D_R). \quad \text{(B.6)}
\end{align*}$$

Now we distinguish three cases: (1) $\delta_L \leq \delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_L - A_R)}{(4\gamma + 1)A_R + 2P\gamma(A_L - A_R)}$, (2) $\delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_L - A_R)}{(4\gamma + 1)A_R + 2P\gamma(A_L - A_R)} < \delta_L \leq \delta_R + \frac{(\gamma + \delta_R + \gamma \delta_{CB})(A_L - A_R)}{A_R}$, and (3) $\delta_L > \delta_R + \frac{(\gamma + \delta_R + \gamma \delta_{CB})(A_L - A_R)}{A_R}$.

**Case 1:** $\delta_L \leq \delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_L - A_R)}{(4\gamma + 1)A_R + 2P\gamma(A_L - A_R)}$. Using the proof of lemma 4, we have $E(x^N_L) \leq E(x^N_R)$. Combining this inequality, (B.5) and (B.6), it follows that $E(x^D_L) < E(x^D_R)$, $E(x^N_L) < E(x^D_L)$, and $E(x^N_R) > E(x^D_R)$. Hence, $E(x^D_R) - E(x^D_L) > E(x^N_R) - E(x^N_L) \geq 0$, which implies that $\text{Var}_P(x^D) > \text{Var}_P(x^N)$.

**Case 2:** $\delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_L - A_R)}{(4\gamma + 1)A_R + 2P\gamma(A_L - A_R)} < \delta_L \leq \delta_R + \frac{(\gamma + \delta_R + \gamma \delta_{CB})(A_L - A_R)}{A_R}$. Using the proof of lemma 4, we know
that $E(x^N_L) > E(x^N_R)$ and $E(x^D_L) \leq E(x^D_R)$. Therefore, to show $\text{Var}_P(x^N) < \text{Var}_P(x^D)$, it suffices to prove

$$E(x^N_L) - E(x^N_R) < E(x^D_L) - E(x^D_R). \quad (B.7)$$

Substituting the expressions of $E(x^N_L)$, $E(x^N_R)$, $E(x^D_L)$, and $E(x^D_R)$ in (B.7) and doing some algebra, we have that (B.7) is equivalent to $f(\delta_L) < g(\delta_L)$, where

$$f(\delta_L) = \frac{\gamma (\gamma (A_R + 4\gamma A_R + 2\gamma P(A_L - A_R)) (\delta_L - \delta_R))}{P (\gamma + \delta_R + 4\gamma \delta_R) (\gamma + \delta_L + 2\gamma \delta_L)}$$

and

$$g(\delta_L) = \frac{\gamma (\gamma (A_R (\delta_L - \delta_R) - (A_L - A_R) (\gamma + \delta_R + \gamma \delta_{CB}))}{P (\gamma + \delta_R + 2\gamma \delta_{CB}) (\gamma + \delta_L + \gamma \delta_{CB}) + (1 - P) (\gamma + \delta_R + \gamma \delta_{CB}) (\gamma + \delta_L + 2\gamma \delta_{CB})}.$$  

Direct computations yield that $f(\delta_L)$ is an increasing function in $\delta_L$, while $g(\delta_L)$ is a decreasing function in $\delta_L$. Furthermore,

$$f \left( \delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_R - A_L)}{(4\gamma + 1)A_R + 2P\gamma(A_L - A_R)} \right) < g \left( \delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_R - A_L)}{(4\gamma + 1)A_R + 2P\gamma(A_L - A_R)} \right)$$

and

$$f \left( \delta_R + \frac{(\gamma + \delta_R + \gamma \delta_{CB})(A_R - A_L)}{A_R} \right) > g \left( \delta_R + \frac{(\gamma + \delta_R + \gamma \delta_{CB})(A_R - A_L)}{A_R} \right).$$

Hence, we conclude that there exists a unique value $\bar{\delta}_L$ belonging to the interval
such that \( f(\delta_L) < g(\delta_L) \) if and only if \( \delta_L \in \left( \delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_L - A_R)}{(4\gamma + 1) A_R + 2P\gamma(A_L - A_R)}, \delta_L \right) \). Therefore, \( \text{Var}_P(x^N) < \text{Var}_P(x^D) \) if and only if \( \delta_L \in \left( \delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_L - A_R)}{(4\gamma + 1) A_R + 2P\gamma(A_L - A_R)}, \delta_L \right) \).

**Case 3:** \( \delta_L > \delta_R + \frac{(\gamma + \delta_R + \gamma \delta_{CB})(A_L - A_R)}{A_R} \). In this case we know that \( \text{E}(x^*_L) > \text{E}(x^*_R) \) and \( \text{E}(x^*_D) > \text{E}(x^*_R) \). Therefore, in order to show \( \text{Var}_P(x^N) > \text{Var}_P(x^D) \), it suffices to prove \( \text{E}(x^*_L) - \text{E}(x^*_R) > \text{E}(x^*_L) - \text{E}(x^*_R) \) or, equivalently, \( \text{E}(x^*_N) - \text{E}(x^*_D) > \text{E}(x^*_R) - \text{E}(x^*_R) \). Using (B.5) and (B.6), we know that the left-hand side of the previous inequality is positive and the right-hand side is negative. Hence, \( \text{Var}_P(x^N) > \text{Var}_P(x^D) \).

(b) Consider now an ultraconservative central bank. Again, \( \text{Var}_P(x^N) > \text{Var}_P(x^D) \) if and only if \( |\text{E}(x^*_N) - \text{E}(x^*_R)| > |\text{E}(x^*_L) - \text{E}(x^*_R)| \). We distinguish three cases: (1) \( \delta_L \leq \delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_L - A_R)}{(4\gamma + 1) A_R + 2P\gamma(A_L - A_R)} \), (2) \( \delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_L - A_R)}{(4\gamma + 1) A_R + 2P\gamma(A_L - A_R)} < \delta_L \leq \delta_R + \frac{(\gamma + \delta_R + \gamma \delta_{CB})(A_L - A_R)}{A_R} \), and (3) \( \delta_L > \delta_R + \frac{(\gamma + \delta_R + \gamma \delta_{CB})(A_L - A_R)}{A_R} \).

**Case 1:** \( \delta_L \leq \delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_L - A_R)}{(4\gamma + 1) A_R + 2P\gamma(A_L - A_R)} \). In this case we know that \( \text{E}(x^*_L) \leq \text{E}(x^*_R) \) and \( \text{E}(x^*_D) < \text{E}(x^*_R) \). Therefore, in order to show \( \text{Var}_P(x^N) < \text{Var}_P(x^D) \) it suffices to prove \( \text{E}(x^*_R) - \text{E}(x^*_L) < \text{E}(x^*_R) - \text{E}(x^*_R) \) or, equivalently, \( \text{E}(x^*_N) - \text{E}(x^*_R) < \text{E}(x^*_L) - \text{E}(x^*_R) \). (B.8)

Notice that the right-hand side of the previous inequality is positive since it always holds \( \text{E}(x^*_R) > \text{E}(x^*_L) \). Next, we distinguish two subcases: (1.1) \( \text{E}(x^*_N) \leq \text{E}(x^*_R) \), and (1.2) \( \text{E}(x^*_N) > \text{E}(x^*_R) \).
• **Subcase 1.1:** \( (E(x^N_R) \leq E(x^D_R)) \). In this case (B.8) holds since the right-hand side of (B.8) is positive, whereas the left-hand side of (B.8) is negative. Consequently, \( \text{Var}_P(x^N) < \text{Var}_P(x^D) \).

• **Subcase 1.2:** \( (E(x^N_R) > E(x^D_R)) \). In this case, substituting the expressions of \( E(x^N_L), E(x^N_R), E(x^D_R) \), and \( E(x^D_L) \) and after some algebra, we have that (B.8) is equivalent to

\[
\frac{\delta_L}{\delta_R} < \frac{(P + m_R + 1)A_L + (1 - P)A_R}{(1 - P + m_L + 1)A_R + PA_L} + \frac{\Delta^N F(c_L, c_R)}{\left( (c_R \Delta^N \left((1 - P + c_L m_L + 1)A_R + PA_L\right) - \Delta^D \left((1 - P + m_L + 1)A_R + PA_L\right) \right) \times \left((1 - P + m_L + 1)A_R + PA_L\right)},
\]

where

\[ F(c_L, c_R) = c_R(c_R - 1) \times (P(A_L - A_R)(A_L m_R - A_R m_L) + A_R(2A_L m_R - A_L m_L - A_R m_L)) + (c_L - c_R) \times \left( m_L(c_R - 1)(A_L - A_R)(A_R + P(A_L - A_R)) \right) + (2A_R + P(A_L - A_R)) \times \left( A_L + A_R + P(A_L - A_R) + A_L m_L \right) + A L c_R(m_R - m_L)(2A_R + P(A_L - A_R)) \times \left( \right). \]

In this case the second term of the right-hand side of the previous inequality is positive. Then, we can conclude that this inequality is satisfied whenever \( \frac{\delta_L}{\delta_R} \leq \frac{(P + m_R + 1)A_L + (1 - P)A_R}{(1 - P + m_L + 1)A_R + PA_L} \). Using the expressions of \( m_L \) and \( m_R \), the previous inequality is equivalent to

\[
\frac{\delta_L}{\delta_R} \leq \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_L - A_R)}{A_R(4\gamma + 1) + 2P\gamma(A_L - A_R)}. \]

Consequently, we have that in this case \( \text{Var}_P(x^N) < \text{Var}_P(x^D) \).

**Case 2:** \( \delta_R + \frac{(\gamma + \delta_R + 2\gamma \delta_R)(A_L - A_R)}{A_R(4\gamma + 1) + 2P\gamma(A_L - A_R)} < \delta_L \leq \delta_R + \frac{(\gamma + \delta_R + \gamma \delta_C)(A_L - A_R)}{A_R} \). This proof is omitted since it is identical to the proof of case 2 in part (a).
Case 3: $\delta_L > \delta_R + \frac{(\gamma+\delta_R+\gamma\delta_{CB})(A_L-A_R)}{A_R}$. In this case we know that $E(x^N_L) > E(x^N_R)$ and $E(x^D_L) > E(x^D_R)$. Therefore, to show $Var_P(x^N) > Var_P(x^D)$, it suffices to prove $E(x^N_L) - E(x^N_R) > E(x^D_L) - E(x^D_R)$ or, equivalently, $E(x^N_L) - E(x^D_L) > E(x^N_R) - E(x^D_R)$. Substituting the expressions of $E(x^N_L)$, $E(x^N_R)$, $E(x^D_R)$, and $E(x^D_L)$ and after some algebra, we have that

$$E(x^N_L) - E(x^D_L) - (E(x^N_R) - E(x^D_R)) = \frac{p(\delta_{CB})}{4\gamma^2\delta_L\delta_R\delta_{CB}^2 \Delta^N \Delta^D},$$

with

$$p(\delta_{CB}) = 2A_R(\delta_L - \delta_R)(P\gamma(\delta_L - \delta_R)$$

$$+ (2\gamma^2 + \gamma\delta_R - \delta_L\delta_R(4\gamma + 1)))$$

$$+ 2\delta_L(A_L - A_R)(\gamma + \delta_R)(2\gamma + 2\delta_R(2\gamma + 1)$$

$$+ P(\delta_L - \delta_R))$$

$$+ \delta_{CB} \left( \frac{\delta_L^2}{A_R} (4\gamma + 1) + 2P\gamma (A_L - A_R) (P + 1) \right)$$

$$+ \frac{\delta_L}{A_R} (4\gamma + 1) (3\gamma + \delta_R - 2P\delta_R) +$$

$$\times (-4\gamma\delta_R P^2 + (4\gamma^2 - \gamma - \delta_R) P$$

$$+ 4\gamma (\gamma + \delta_R + 2\gamma\delta_R))$$

$$+ \delta_R A_R (4\gamma + 1) (P\delta_R - 2\delta_R - 3\gamma)$$

$$+ (\gamma + \delta_R + 2\gamma\delta_R + 2P\gamma\delta_R)$$

$$\times (2\gamma + 2\delta_R - P\delta_R) (A_R - A_L)$$

$$+ \delta_{CB} \left( 2 \left( \delta_L - \left( \delta_R + \frac{(\gamma+\delta_R)(A_L-A_R)}{A_R} \right) \right) \right.$$}

$$\times (2\gamma P (A_L - A_R) + 4\gamma A_R + A_R) \gamma$$

$$\left. + \delta_R P (A_L - A_R) \right) (A_L - A_R)^2$$

$$+ 4 \left( \frac{(\gamma P (A_L - A_R) + 2\gamma A_R + A_R\delta_R}{A_R}$$

$$+ \delta_R P (A_L - A_R)) (A_L - A_R)^2 \right) \gamma^2.\)
Taking into account that $\delta_L > \delta_R + \frac{(\gamma + \delta_R)(A_L - A_R)}{A_R}$ and $A_L > A_R$, computations yield that the coefficients of $\delta_{CB}^2$ and $\delta_{CB}$ are positive. In addition if $A_L - A_R$ is very close to 0, then we obtain the same results as in the benchmark case. Otherwise, i.e., if $A_L - A_R$ is high enough, then we obtain that the independent coefficient is positive. Thus, in this case we have that $p(\delta_{CB}) > 0$ and, hence, $\text{Var}_P(x^N) > \text{Var}_P(x^D)$. □

Appendix C

PROPOSITION C.1. Under delegation in the sequential game, in equilibrium the inflation rates and outputs are given by

\[
\begin{align*}
\pi_{D,S}^L &= \frac{\gamma (2\delta_{CB} + 1) \delta_{CB} (\gamma + \delta_R + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2)}{\Delta_{D,S} A} \\
&\quad - \frac{\delta_{CB} \gamma (2\delta_{CB} + 1)}{\gamma + \delta_L + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2} \varepsilon, \\
\pi_{D,S}^R &= \frac{\gamma (2\delta_{CB} + 1) \delta_{CB} (\gamma + \delta_R + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2)}{\Delta_{D,S} A} \\
&\quad - \frac{\delta_{CB} \gamma (2\delta_{CB} + 1)}{\gamma + \delta_R + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2} \varepsilon, \\
x_{D,S}^L &= x^* - \frac{1}{\delta_{CB}} \pi_{D,S}^L \quad \text{and} \quad x_{D,S}^R = x^* - \frac{1}{\delta_{CB}} \pi_{D,S}^R,
\end{align*}
\]

where

\[
\Delta_{D,S} = P \left( \gamma + \delta_R + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2 \right) \\
\times \left( \gamma + \delta_L + 3\gamma \delta_{CB} + 2\gamma \delta_{CB}^2 + \delta_{CB}^2 \right) \\
+ (1 - P) \left( \gamma + \delta_R + 3\gamma \delta_{CB} + 2\gamma \delta_{CB}^2 + \delta_{CB}^2 \right) \\
\times \left( \gamma + \delta_L + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2 \right).
\]

Proof of Proposition C.1. We solve by backward induction. In the second step, the central bank chooses the inflation rate after observing the tax rate chosen by the government in office. Suppose that
party $j$ is in office. Under delegation, the central bank chooses $\pi_j$ in order to solve the following problem after observing $\tau_j$:

$$
\min_{\pi_j} V_{CB} = \frac{1}{2} \left( \pi_j^2 + \delta_{CB} (x_j - x^*)^2 \right).
$$

As in the simultaneous setup, the FOC is given by \( \frac{\partial}{\partial \pi_j} V_{CB} = \pi_j + \delta_{CB} (x_j - x^*) = 0 \), which implies

$$
\pi_j = \frac{\delta_{CB}}{\delta_{CB} + 1} (w^* + x^* + \pi^e + \tau_j - \varepsilon). \quad (C.1)
$$

In the first step, the fiscal authority chooses $\tau_j$ taking into account (C.1). Thus,

$$
\min_{\tau_j} V_{G_j} = \frac{1}{2} \left( \pi_j^2 + \delta_j (x_j - x^*)^2 + \gamma (g_j - g^*)^2 \right)
$$

s.t. $\pi_j = \frac{\delta_{CB}}{\delta_{CB} + 1} (w^* + x^* + \pi^e + \tau_j - \varepsilon)$.

The FOC of this problem is given by

$$
\frac{\partial}{\partial \tau_j} V_{G_j} = \pi_j \frac{\delta_{CB}}{\delta_{CB} + 1} + \delta_j (x_j - x^*) \left( \frac{\delta_{CB}}{\delta_{CB} + 1} - 1 \right)
$$

$$
+ \gamma (g_j - g^*) \left( 1 + \frac{\delta_{CB}}{\delta_{CB} + 1} \right) = 0.
$$

Using expressions (C.1), (1), and (2), it follows that

$$
\tau_j = g^* - \frac{\delta_j + \gamma \delta_{CB} + 2 \gamma \delta_{CB}^2 + \delta_{CB}^2}{\gamma + \delta_j + 4 \gamma \delta_{CB} + 4 \gamma \delta_{CB}^2 + \delta_{CB}^2} (A + \pi^e - \varepsilon).
$$

Substituting the previous expression in (C.1),

$$
\pi_j = \frac{\delta_{CB} \gamma (2 \delta_{CB} + 1)}{\gamma + \delta_j + 4 \gamma \delta_{CB} + 4 \gamma \delta_{CB}^2 + \delta_{CB}^2} (A + \pi^e - \varepsilon). \quad (C.2)
$$
Using the previous formula in the expression for $\pi^e$ and solving for $\pi^e$, we get

$$\pi^e = \left( \frac{\gamma (2\delta_{CB} + 1) \delta_{CB} \left( \frac{P}{\gamma + \delta_{L} + 4\gamma \delta_{CB} + 4\gamma^2 \delta_{CB}^2 + \delta_{CB}^2} \right) 1 - P}{1 - \gamma (2\delta_{CB} + 1) \delta_{CB} \left( \frac{P}{\gamma + \delta_{L} + 4\gamma \delta_{CB} + 4\gamma^2 \delta_{CB}^2 + \delta_{CB}^2} \right)} \right) A. \quad \text{(C.3)}$$

Substituting this expression into (C.2), for $j = L, R$, and after some algebra we obtain the desired expressions of the inflation rates. Finally, the expressions of outputs follow from the FOC of the optimization problem of the monetary authority.

Proof of Lemma 5

Using the expressions of $x_{D,S}^L$ and $x_{D,S}^R$ given in proposition C.1 and taking expectations, it follows that $E \left( x_{D,S}^L \right) - E \left( x_{D,S}^R \right) = \frac{\gamma(\delta_L - \delta_R)(2\delta_{CB} + 1)}{\Delta_{D,S}} A$. Hence, we can conclude that $E \left( x_{D,S}^L \right) > E \left( x_{D,S}^R \right)$ since $\delta_L > \delta_R$.

Proof of Proposition 7

Let $h^S(P) = E \left( \pi^N \right) - E \left( \pi^{D,S} \right)$ and $f^S(P) = E \left( (\pi^N)^2 \right) - E \left( (\pi^{D,S})^2 \right)$. Using (A.3) and (C.3), it follows that $\frac{\partial}{\partial P} \left( E \left( \pi^N \right) \right) > 0$ and $\frac{\partial}{\partial P} \left( E \left( \pi^{D,S} \right) \right) < 0$. Therefore, $h^S(P)$ is an increasing function in $P$. Applying lemma A.1, we have

$$E \left( (\pi^N)^2 \right) = \left( \frac{P}{\frac{1}{s_L^2} + \frac{1}{\gamma} + 2} + \frac{1 - P}{\frac{1}{s_R^2} + \frac{1}{\gamma} + 2} \right)^2 A$$

$$+ \left( P \left( \frac{1}{\frac{1}{s_L^2} + \frac{1}{\gamma} + 2} \right)^2 + (1 - P) \left( \frac{1}{\frac{1}{s_R^2} + \frac{1}{\gamma} + 2} \right)^2 \right) \sigma_\varepsilon^2$$
\[
E \left( (\pi^{D,S})^2 \right) = \\
\left( \frac{\gamma (2\delta_{CB} + 1) \delta_{CB} \left( \frac{P}{\gamma + \delta_L + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2} \right)^{1-P}}{\gamma + \delta_R + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2} \right) \left( 1 - \gamma (2\delta_{CB} + 1) \delta_{CB} \left( \frac{P}{\gamma + \delta_L + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2} \right)^{1-P} \right) A \\
+ \left( P \left( \frac{\delta_{CB} \gamma (2\delta_{CB} + 1)}{\gamma + \delta_L + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2} \right)^2 \right) \sigma^2_\varepsilon.
\]

Direct computations yield that \( \frac{\partial}{\partial P} \left( E \left( (\pi^N)^2 \right) \right) > 0 \) and \( \frac{\partial}{\partial P} \left( E \left( (\pi^{D,S})^2 \right) \right) < 0 \). Therefore, \( f^S(P) \) is an increasing function in \( P \). Next, we distinguish two cases:

(a) The central bank is ultraconservative (\( \delta_{CB} \leq 2\delta_R \)). Combining the monotonicity property of \( h^S(P) \) and \( f^S(P) \) and the fact that \( h^S(0) \geq 0 \) and \( f^S(0) \geq 0 \) when \( \delta_{CB} \leq 2\delta_R \), we have that \( h^S(P) > 0 \) and \( f^S(P) > 0 \) or, equivalently, \( E \left( \pi^N \right) > E \left( \pi^{D,S} \right) \) and \( E \left( (\pi^N)^2 \right) > E \left( (\pi^{D,S})^2 \right) \), whenever \( P > 0 \).

(b) The central bank is moderately conservative (\( 2\delta_R < \delta_{CB} < 2\delta_L \)). Combining the monotonicity property of \( h^S(P) \) and the fact that \( h^S(0) < 0 \), \( f^S(0) < 0 \), \( h^S(1) > 0 \), and \( f^S(1) > 0 \) whenever \( 2\delta_R < \delta_{CB} < 2\delta_L \), it follows that there exists a unique value \( \overline{P}^S \) such that \( h^S(P) > 0 \) and \( f^S(P) > 0 \) or, equivalently, \( E \left( \pi^N \right) > E \left( \pi^{D,S} \right) \) and \( E \left( (\pi^N)^2 \right) > E \left( (\pi^{D,S})^2 \right) \) whenever \( P > \overline{P}^S \).
Proof of Proposition 8

(a) Recall that $\text{Var}_P(x^N) > \text{Var}_P(x^{D,S})$ if and only if $E(x_L^N) - E(x_R^N) > E(x_L^{D,S}) - E(x_R^{D,S})$. Let $g^S(\delta_{CB}) = E(x_L^N) - E(x_R^N) - \left( E(x_L^{D,S}) - E(x_R^{D,S}) \right)$. Direct computations yield that $g^S$ is increasing in $\delta_{CB}$. Next, we distinguish two cases:

Case 1: The central bank is moderately conservative ($2\delta_R < \delta_{CB} < 2\delta_L$). Combining the monotonicity property of $g^S(\delta_{CB})$ and the fact that $g^S(2\delta_R) > 0$, we have that $g^S(\delta_{CB}) > 0$ whenever $2\delta_R < \delta_{CB} < 2\delta_L$ and, hence, $\text{Var}_P(x^N) > \text{Var}_P(x^{D,S})$.

Case 2: The central bank is ultraconservative ($\delta_{CB} \leq 2\delta_R$). Then, we consider two subcases:

- Subcase 2.1: $g^S(0) \geq 0$. In this case $2\gamma^2 + \gamma(\delta_R + P\delta_L - P\delta_R - 4\delta_L\delta_R) - \delta_L\delta_R \geq 0$. Descartes’ rule tells us that there exists a unique value of $\gamma$, denoted by $\gamma^S$, such that the previous inequality is satisfied whenever $\gamma \geq \gamma^S$. Combining the monotonicity property of $g^S(\delta_{CB})$ and the fact that $g^S(0) \geq 0$, we have that $g^S(\delta_{CB}) > 0$ whenever $\delta_{CB} > 0$ and, hence, $\text{Var}_P(x^N) > \text{Var}_P(x^{D,S})$. Therefore, we conclude that if $\gamma \geq \gamma^S$, then $\text{Var}_P(x^N) > \text{Var}_P(x^{D,S})$.

- Subcase 2.2: $g^S(0) < 0$ (or, equivalently, $\gamma < \gamma^S$). In this case, the monotonicity property of $g^S(\delta_{CB})$ implies that there exists a unique value of $\delta_{CB}$, denoted by $\delta_{CB}^S$, such that $g^S(\delta_{CB}) > 0$ whenever $\delta_{CB} > \delta_{CB}^S$. Consequently, $\text{Var}_P(x^N) > \text{Var}_P(x^{D,S})$ if and only if $\delta_{CB} > \delta_{CB}^S$.

(b) From the proof of proposition C.1, it follows that

$$x_{L}^{D,S} = x^* - \frac{1}{\delta_{CB}} \left( \frac{\gamma(2\delta_{CB} + 1)\delta_{CB}}{\Delta^{D,S}} \times (\gamma + \delta_R + 4\gamma\delta_{CB} + 4\gamma\delta_{CB}^2 + \delta_{CB}^2) \right) A$$

Note that $\gamma^S$ coincides with $\gamma$ given in the proof of proposition 3.
\[ x_{R}^{D,S} = x^* - \frac{1}{\delta_{CB}} \left( \frac{\gamma (2\delta_{CB} + 1)}{\gamma + \delta + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2} \right) A \]

\[ + \frac{\gamma (2\delta_{CB} + 1)}{\gamma + \delta + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2} \epsilon. \]

Applying lemma A.1, we have that

\[ \text{Var}_E (x^{D,S}) \]

\[ = \left( P \left( \frac{\gamma (2\delta_{CB} + 1)}{\gamma + \delta + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2} \right)^2 \right) \]

\[ + (1 - P) \left( \frac{\gamma (2\delta_{CB} + 1)}{\gamma + \delta + 4\gamma \delta_{CB} + 4\gamma \delta_{CB}^2 + \delta_{CB}^2} \right)^2 \sigma^2 \epsilon. \]

In addition, in the proof of proposition 4 we have obtained

\[ \text{Var}_E (x^N) \]

\[ = \left( P \left( \frac{1}{2\delta_{L} (m_{L} + 2)} \right)^2 + (1 - P) \left( \frac{1}{2\delta_{R} (m_{R} + 2)} \right)^2 \right) \sigma^2 \epsilon \]

or, using the expressions of \( m_{L} \) and \( m_{R} \),

\[ \text{Var}_E (x^N) \]

\[ = \left( P \left( \frac{\gamma}{\gamma + \delta + 4\gamma \delta_{L}} \right)^2 + (1 - P) \left( \frac{\gamma}{\gamma + \delta + 4\gamma \delta_{R}} \right)^2 \right) \sigma^2 \epsilon. \]

Hence, \( \text{Var}_E (x^N) - \text{Var}_E (x^D) \) is a linear function in \( P \). Next we distinguish two cases:

**Case 1:** \( c_{R} < 1 < c_{L} \) (i.e., \( 2\delta_{R} < \delta_{CB} < 2\delta_{L} \)). Direct computations yield that \( \text{Var}_E (x^N) |_{P=1} - \text{Var}_E (x^D) |_{P=1} < 0 \) and that \( \text{Var}_E (x^N) |_{P=0} - \text{Var}_E (x^D) |_{P=0} > 0 \) whenever \( 2\delta_{R} < \delta_{CB} < 2\delta_{L} \). Hence, we can conclude that there exists
a unique value of \( P \), denoted by \( \overline{PS} \), such that \( \text{Var}_E(x^D) > \text{Var}_E(x^N) \) if and only if \( P > \overline{PS} \).

**Case 2:** \( c_L > c_R \geq 1 \) (i.e., \( \delta_{CB} \leq 2\delta_R < 2\delta_L \)). Note that \( \text{Var}_E(x^N)|_{P=1} - \text{Var}_E(x^D)|_{P=1} < 0 \) and that \( \text{Var}_E(x^N)|_{P=0} - \text{Var}_E(x^D)|_{P=0} \leq 0 \) whenever \( \delta_{CB} \leq 2\delta_R < 2\delta_L \). Therefore, for all \( 0 < P < 1 \), it holds that \( \text{Var}_E(x^D) > \text{Var}_E(x^N) \) whenever \( \delta_{CB} \leq 2\delta_R < 2\delta_L \). 

References


