Finite Horizons and the Monetary/Fiscal Policy Mix

Kostas Mavromatis
De Nederlandsche Bank and University of Amsterdam

Fiscal policy in the United States has been documented to have been the leading authority in the '70s (active fiscal policy), while having switched to fiscal discipline following Volcker’s appointment (passive fiscal policy) onward. Most papers in the literature focus on taxes as the main instrument to stabilize debt when fiscal policy is passive. I extend the existing analysis by also allowing federal spending to adjust. I construct a New Keynesian Markov-switching DSGE model with a Blanchard-Yaari structure and estimate it for the United States. I find that the U.S. economy has switched regimes over time. In line with the existing literature, I show that the economy spent the '70s in a regime where fiscal policy was active while monetary policy was passive. Interestingly, I show that the economy spent a short period in the early '80s where both policies were passive, before switching to a persistent regime where monetary policy has been fighting inflation aggressively and fiscal policy ensured that both tax revenues and federal spending adjusted to stabilize debt.

JEL Codes: E31, E58, E62.

1. Introduction

Regime switches in the monetary/fiscal policy mix in dynamic stochastic general equilibrium (DSGE) models have attracted much

*The views expressed do not represent the position of De Nederlandsche Bank or the Eurosystem. I am very grateful for the comments and suggestions provided by two anonymous referees and Boragan Aruoba. I am also grateful to Christophe Kamps, Roel Beetsma, Jacopo Cimadomo, Massimo Giuliodori, Damjan Phajfar, and Harald Uhlig. I am also grateful to seminar participants at the European Central Bank, the University of Amsterdam, and Tilburg University as well as participants at several conferences. Author e-mail: k.mavromatis@dnb.nl.
attention over the last years. The interactions between monetary and fiscal policy are crucial for the determination of inflation and output dynamics as well as debt and, more importantly, of expectations. Whether or not monetary and fiscal policy coordinate or whether they switch in a joint manner is a question that many papers in the literature have addressed and are still trying to address. However, what is also important is not only whether the policy mix is such that it allows for higher inflation and lower debt, or the other way around, but also whether the fiscal authority changes the composition of its instruments as a means to consolidate debt regardless of the monetary policy stance. In this paper, I address those issues. I estimate a DSGE model for the U.S. economy allowing for joint switching in monetary and fiscal policy. In deviation from existing papers, I allow fiscal policy to switch not only between a passive and an active regime but also between using tax revenues or spending as a means to stabilize debt.

The current literature has restricted its attention to two regimes, namely an active monetary/passive fiscal (AM/PF) regime and a passive monetary/active fiscal (PM/AF) regime.\footnote{Bianchi and Iliut (2017) estimate a Markov-switching DSGE for the U.S. economy and find that apart from the AM/PF and PM/AF regimes, the U.S. economy went through a short period (in the early years of Ronald Reagan’s presidency) during which both monetary and fiscal policy were active. Following Leeper (1991), the model has no solution under this policy mix. For this reason, in this paper I restrict my attention to policy mixes that yield a unique bounded solution.} Figure 1 (panel A) plots the debt-to-GDP ratio along with inflation and the real interest rate. Clearly, the debt-to-GDP ratio was lower during the ’70s. There is mixed evidence in the literature regarding that period. According to the findings of Bianchi (2012), Bianchi and Iliut (2017), Davig and Leeper (2011), and Traum and Yang (2011), this period corresponds to the passive monetary (PM) and an active fiscal (AF) policy regime and is associated with persistently high inflation and a low real interest rate.\footnote{The definition of monetary and fiscal regimes is borrowed from Leeper (1991).} As shown in Leeper (1991), lack of fiscal discipline in a rational expectations model implies dynamics that depend on the joint behavior of monetary and fiscal authorities. More importantly, the effects of policy interventions differ substantially from those when fiscal discipline is in place. On the contrary,
Bhattarai, Lee, and Park (2012a) find that both policies have been passive during that period, while Traum and Yang (2011) find that monetary policy has been active and fiscal policy has been passive during the ‘70s in the United States. The literature seems to converge only on the events following the appointment of Volcker onward. That is, the policy mix has since the early ’80s been characterized by an active monetary and a passive fiscal policy. When it comes to the ’70s, the literature does not seem to be converging to a specific policy mix.

A natural measure of fiscal stance is the primary surplus as a fraction of outstanding debt. The reluctance of the fiscal authorities, from the mid-’70s until the late ’70s, to make the necessary

fiscal adjustments in order to stabilize debt can be seen in panel C of figure 1. In particular, the first period of sustained primary deficits begins in the mid-'70s (Ford tax cut and tax rebate). As panel C shows, the high deficit persisted for a couple of years. On the other hand, from the early '80s onward (Volcker’s appointment), inflation started declining steadily while the real interest rate started to rise substantially. At the same time, the debt-to-GDP increased gradually until the early '90s, which can be partly attributed to the rising interest payments and the subsequent recession. In fact, as panel B shows and as also argued by Sims (2011), the interest expense was a small fraction of the budget until the late '70s but started to shoot up from the early '80s onward and stayed there for several years. The steady decline in deficits in the middle of the Reagan Administration was not enough to offset the sharp rise in interest expenses. According to the existing literature, the period from the '80s onward corresponds to an AM/PF regime.

Historical evidence regarding U.S. fiscal policy shows that it has fluctuated not only between a passive and an active regime but also between alternative instruments (tax revenues or federal expenditure) in its effort to stabilize debt. During the Reagan presidency, the Tax Equity and Fiscal Responsibility Act of 1982 was approved amid concerns regarding the growing primary deficits. The ongoing recession of the early '80s caused a short-term fall in tax revenues. The Act was approved with the ultimate purpose of closing the growing budget gaps. At the same time, the cuts in spending concerned cuts in projected spending. On the other hand, during the Clinton Administration sustained cuts in federal expenditure were implemented in an effort to bring the budget deficit into a surplus and to reduce federal debt. Panel C depicts the sustained and growing primary surpluses over that period, while panel A depicts the substantial decline in debt-to-GDP ratio. During those years, federal spending fell to almost 18.2 percent of GDP from 22.1 percent in 1992 (Balanced Budget Act of 1997). On the other hand, taxes were raised in 1993 but were cut in 1997 (Taxpayer Relief Act of 1997). The current literature on Markov-switching DSGE

\[^3\] According to the Balanced Budget Act of 1997, the targeted cut in spending would amount to $160 billion between 1998 and 2002.
(MS-DSGE) models remains relatively silent on the effects of switches between alternative fiscal instruments.\(^4\)

I address the events described above by developing an MS-DSGE model where monetary policy switches between an active and a passive regime. Not only does fiscal policy switch between being active and passive, but also, while passive, it is allowed to change the composition of its fiscal instruments. That is, the model allows the reactions of tax revenues and federal spending to debt fluctuations to vary so long as fiscal policy stays passive. With this specification (i.e., allowing for possible switching between fiscal instruments) I deviate from the current literature.\(^5\) I deviate from the current literature on estimated MS-DSGE models for the United States further by introducing a Blanchard-Yaari structure similar to Richter (2015).\(^6\) Agents are no longer infinitely lived. With this structure fiscal shocks have real effects in the economy, implying inflationary pressures following tax cuts or increases in federal expenditure regardless of the current regime and agents’ beliefs.

A number of authors attribute the lower inflation and output volatility in the post-Volcker era to lower shock volatility instead of better policy (see Cogley and Sargent 2005; Primiceri 2005; Sims and Zha 2006; Stock and Watson 2002). For this reason I also allow the variances of shocks to vary over time according to a Markov-switching process and independently of switches in monetary and fiscal policy.

I perform a Bayesian estimation of the model using quarterly data. In line with estimated closed- and open-economy MS-DSGE

---

\(^4\) A number of papers employ Bayesian estimation to estimate fiscal policy rules and to explore the economic effects of fiscal policy (Forni, Monteforte, and Sessa 2009; Kamps 2007; López-Salido and Rabanal 2007; Marco, Werner, and Jan 2006; Straub and Coenen 2005). However, they do not account for regime switches of the nature considered in this paper.

\(^5\) The importance of the fiscal instrument on the dynamics of economic variables following fiscal shocks has been studied in simple real business cycle models in the literature. Leeper, Plante, and Traum (2010) and Leeper and Yang (2008) show that the dynamics and the impulse responses of key variables following both fiscal policy and nonpolicy shocks depend on what fiscal instrument finances debt. However, those papers do not include the interactions between monetary and fiscal policies. Moreover, they do not analyze the effects of switches across different instruments on expectations.

\(^6\) Richter (2015) does not focus on switching between different fiscal instruments during an AM/PF regime.
models, I find that the monetary policy of the Federal Reserve has indeed varied over time and is characterized by a weaker response of the federal funds rate to inflation fluctuations during the ’70s mainly (passive monetary policy), and was associated with an active fiscal policy in the spirit of Leeper (1991) and in line with Bianchi (2012, 2013) and Bianchi and Ilut (2017). I find that the probability of a switch to a regime where the reaction of monetary policy to inflation is strong and where fiscal policy is concerned with debt stabilization (passive fiscal policy) increases substantially from the mid-’80s onward and stays persistently high until recently. Interestingly, I find that the economy spent a short period in a regime where both monetary and fiscal policy were passive before switching to the persistent regime which characterized the evolution of the economy from the mid-’80s till today. Importantly, I find that during this short-lived regime, the fiscal authority used federal spending heavily as a means to stabilize debt. As regards the policy mix from the mid-’80s onward, I find that, while the Federal Reserve has been committed to fighting inflation, the fiscal authority has been using both tax revenues and federal spending in order to stabilize debt. This finding contributes further to the existing literature on estimated MS-DSGE models which restrict federal spending not to react to debt fluctuations.

The paper is organized as follows. Section 2 presents some of the existing literature on the monetary fiscal policy mix as well as on existing models using a perpetual youth structure. In section 3, a closed-economy model with a Blanchard-Yaari structure is developed. Section 4 includes the solution algorithm, the estimation technique, and the data set. Section 5 summarizes the estimation results, while section 6 concludes.

2. Literature Review

The monetary/fiscal policy mix has been analyzed in the literature extensively. On the theoretical side, the landmark paper of Leeper (1991) analyzed the stability properties in an economy featuring monetary and fiscal policy. Canzoneri, Cumby, and Diba (2010) provide a game-theoretic approach to the interactions between the monetary and fiscal authorities in an attempt to analyze positive and normative issues in those. Since the seminal paper of Leeper, a
number of papers have been written in an attempt to estimate the policy mix in the United States. Most of these papers have made use of a closed-economy DSGE model. Some of those studies estimate fixed-coefficients models, where monetary and fiscal policy are not allowed to change behavior over time, whereas others consider changes in the behavior of the two authorities over time. Bhattarai, Lee, and Park (2012a) estimate a closed-economy DSGE model for the United States using postwar data and find evidence in favor of changes in the monetary/fiscal policy mix over time. Their model has fixed coefficients and they estimate it in different subsamples. They find that pre-Volcker, a passive monetary and fiscal policy regime prevailed, while post-Volcker, an active monetary and passive fiscal policy regime was dominating. Hence, according to the terminology by Leeper (1991), since both monetary and fiscal policies were passive pre-Volcker, there was equilibrium indeterminacy. Along the same lines, Bhattarai, Lee, and Park (2012b) using a similar model also find evidence in favor of passive monetary and fiscal policy in the pre-Volcker era and an active monetary and passive fiscal policy in the post-Volcker era. Traum and Yang (2011) also estimate a closed-economy DSGE model for the United States for different subsamples and find that the data in the pre-Volcker period strongly prefer an AM/PF regime, even with a prior centered in the PM/AF region. Contrary to Bhattarai, Lee, and Park (2012a, 2012b), Traum and Yang (2011) allow both government purchases and tax revenues to react to government debt. In this paper, I deviate from Traum and Yang (2011) by introducing an overlapping-generations structure into the model as well as allowing for time variation in the monetary and fiscal feedback rules. This allows me to estimate the model for the whole sample period and detect whether or not the economy has indeed been switching across regimes over time, instead of estimating a time-invariant model for different subsamples as Traum and Yang (2011) do.

---

7 A number of papers employ Bayesian estimation to estimate fiscal policy rules and to explore the economic effects of fiscal policy (Forni, Monteforte, and Sessa 2009; Kamps 2007; López-Salido and Rabanal 2007; Marco, Werner, and Jan 2006; Straub and Coenen 2005). However, they do not account for regime switches of the nature considered in this paper.
At the moment there is also large evidence in favor of joint regime switching in monetary and fiscal policy. Specifically, there are a number of papers which estimate time-varying models allowing for switches in the policy mix, instead of estimating a fixed-coefficients model for different subsamples. Bianchi (2012) and Bianchi and Ilut (2017) estimate an MS-DSGE for the United States where monetary and fiscal policy switch jointly regimes. Both papers find evidence in favor of a passive monetary/active fiscal policy mix in the ’70s and an active monetary/passive fiscal mix from the late ’80s onward. Importantly, both papers find that fiscal policy did not switch to become passive once monetary policy started committing to a better anchoring of inflation expectations. As a result, both papers find that the U.S. economy spent most of the early ’80s in a regime where both policies were active. Besides models allowing for joint monetary and fiscal policy interactions, Favero and Monaco (2005) estimate fiscal policy feedback rules in the United States for the period 1960–2002, and find that fiscal policy in the United States has been active between 1960 and 1980, while from 1980 onward it switched to become passive. Apart from estimated DSGE models or single-equation estimates, there are a number of papers calibrating regime-switching models and looking at the interactions between monetary and fiscal policy. Davig and Leeper (2011) and Davig, Leeper, and Chung (2007) construct an MS-DSGE model, look at how the interactions between monetary and fiscal policy affect agents’ decisions following monetary and fiscal shocks, and also analyze the stability properties of the model. However, little evidence exists in the MS-DSGE literature as regards switches of the fiscal authorities between using tax revenues and spending as a means to stabilize debt. As discussed in the introduction, there have been periods where the U.S. government focused more on spending cuts than on tax hikes in order to control debt. In this paper, I account for this possibility by estimating a three-regime model for the United States where the fiscal authority is allowed to also vary the composition of its fiscal policy while being passive.

The interaction between monetary and fiscal policy has also been analyzed through the lens of the perpetual youth model due to Blanchard (1985) and Yaari (1965). Leith and Wren-Lewis (2000) use a version of the perpetual youth model to analyze the implications of the fiscal stability pact in the European Economic and
Monetary Union for monetary policy. Importantly, Leith and Wren-Lewis (2000) consider two scenarios of fiscal discipline—one in which the fiscal authorities use taxes to control debt and another in which they use government spending—and they analyze the stability properties of their model. Annicchiarico, Giammarioli, and Piergallini (2012) construct a New Keynesian model with capital accumulation and finite horizons to analyze the macroeconomic implications of fiscal policy. They find that fiscal expansions generate a tradeoff in output dynamics between short-term gains and medium-term losses. They also show that the effects of fiscal shocks crucially depend upon the conduct of monetary policy. Along similar lines, Annicchiarico, Marini, and Piergallini (2008) also build a New Keynesian model where agents have finite horizons to analyze the performance of monetary policy under Ricardian and non-Ricardian fiscal regimes. Interestingly, they show that, within the class of Ricardian fiscal rules, active monetary policies are not necessary for equilibrium determinacy. Richter (2015) constructs a DSGE model with a fiscal limit, also making use of the perpetual youth setup in order to analyze how intergenerational redistributions, debt maturity, and entitlement reforms affect the consequences of explosive federal transfers. He finds that the finite horizon structure is crucial in generating more severe and persistent stagflation than representative-agent models. Moreover, the interactions between debt maturity and finite horizons may dampen the short-run effects of explosive transfers.

Analyzing fiscal consolidations, Bi, Leeper, and Leith (2013) show that expenditure-based consolidations are more successful in stabilizing debt. Although they focus on the uncertainty about the timing and the nature of fiscal consolidations, they conclude that tax-based fiscal consolidations are not as successful as expenditure-based ones. Moreover, they compute the effects of fiscal consolidations in the periods following each specific consolidation to show that tax-based consolidations are expansionary. In this paper, however, I do not consider cases of short-horizon fiscal consolidations.

The current paper contributes to the existing literature in the following respects. First, it introduces the perpetual youth model and combines it with Markov switching in monetary and fiscal policy and then estimates the resulting MS-DSGE model for the United States. Second, it assumes three regimes, namely two regimes where monetary policy is active and fiscal policy is passive and one in which
monetary policy is passive and fiscal policy is active. Specifically, while being passive, fiscal policy can vary the extent to which tax revenues and federal spending react to fluctuations of the debt ratio.

3. The Model

I adopt the specification of the Blanchard (1985) and Yaari (1965) model of perpetual youth in discrete time similar to Devereux (2011), Richter (2015), and Smets and Trabandt (2012). Households die with probability $1 - \delta$ each period, and every period a newborn generation $i$ represents a fraction $1 - \delta$ of total population, where $0 \leq \delta \leq 1$. In other words, $\delta$ captures the probability of survival from one period to the next. Therefore, $\sum_{i=0}^{\infty} \delta^t = \frac{1}{1 - \delta}$ represents the average household lifetime. As pointed out by Smets and Trabandt (2012), an alternative and empirically more plausible interpretation of $1/(1 - \delta)$ is that it reflects the effective planning horizon of households. In this paper, I adopt the planning horizon interpretation. Households have no bequest motive and the usual Ricardian equivalence breaks down.

Households derive utility from the consumption of goods and supply labor to firms. They are assumed to have external habits in consumption. Each household is the owner of a firm producing a differentiated good. Households receive a wage from labor and profits from firm ownership. Firms operate in a monopolistically competitive market with price stickiness as in Calvo (1983). Monetary policy is described by an interest rate rule whose coefficients on inflation and output gap vary over time according to a Markov-switching process. In particular, there are periods where monetary policy satisfies the Taylor principle—active monetary policy—and periods where it does not—passive monetary policy. Fiscal policy is conducted by the government and fluctuates between three regimes. The first and the third regime are regimes in which the fiscal authority commits to fiscal discipline through targeting tax revenues and/or spending—passive fiscal policy. Crucially, the extent to which tax revenues and spending are adjusted to stabilize debt can vary between those two regimes. The second regime is one in which it does not make the necessary fiscal adjustments in order to stabilize debt—active fiscal policy.
3.1 Households

In every period a new generation/cohort is born. The size of generation $i$ at time $t$ is $(1 - \delta) \delta^{t-i}$ and total population is of measure 1. The representative household $l$ in each generation $i$ maximizes the expected lifetime utility function:

$$U_t = E_t \sum_{s=t}^{\infty} (\beta \delta)^{s-t} e^{ds} \left[ \frac{(C_i^s - \kappa C_{s-1})^{1-\sigma}}{1 - \sigma} - \frac{(H_i^s)^{1+\gamma}}{1 + \gamma} \right],$$

where $\beta \in (0, 1)$ is the subjective discount factor; $C_i^i$, $H_i^i$, and $B_i^i$ are consumption, labor supply, and government bond holdings of households of generation $i$; $C_t$ represents the average level of consumption in the economy. The parameter $\kappa$ captures the degree of external habit. The preference shock, $d_t$, is assumed to have a mean zero and follows a stationary $AR(1)$ process:

$$d_t = \rho d_{t-1} + \sigma d_{t-1} \epsilon_{d,t} \epsilon_{d,t} \sim N(0, 1).$$

The composite consumption good $C_i^i$ is differentiated across a continuum of individual goods, such that

$$C_i^i = \left[ \int_0^1 c_i^i(j)^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}},$$

where $\theta$ denotes the intratemporal elasticity of substitution. The household in each generation $i$ chooses $c_i^i(j)$ to minimize its total expenditure, which implies a demand function for each good $j$ described by

$$c_i^i(j) = \left( \frac{p_t(j)}{P_t} \right)^{-\theta} C_i^i,$$

where $P_t$ is the aggregate price index defined as

$$P_t = \left[ \int_0^1 p_t(j)^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}}.$$

The flow budget constraint for members of generation $i$ is summarized as

$$P_tC_i^i + \frac{B_i^i}{R_t} = \frac{1}{\delta} B_{i-1}^i + W_t H_i^i + A_i^i - T_t + TR_t,$$
where $B_i^t$ denotes one-period riskless government bonds paying one unit of numeraire in period $t$. $R_t$ is the gross nominal interest rate on bonds purchased in period $t$. $W_t$ is the nominal wage, $\Lambda_t^i$ are nominal profits that generation $i$ receives lump sum, $T_t^i$ are lump-sum taxes imposed by the government to generation $i$, while $TR_t^i$ are nominal lump-sum transfers. As in Blanchard (1985), I assume a full annuities market. This implies that rates of return are grossed up to cover the probability of death. The representative household in generation $i$ chooses at time $t$ the set \(\{C_i^t, H_i^t, B_i^t\}\) and the sequences of contingency plans \(\{C_i^s, H_i^s, B_i^s\}_{s=0}^{\infty}\) in order to maximize (1) subject to (5). The first-order conditions at an interior solution are written as

\[
1 = \beta e^{d_{t+1}} E_t \left[ \frac{R_t P_t}{P_{t+1}} \left( \frac{\lambda_{t+1}^i}{\lambda_t^i} \right) \right] (H_s^i)^{\gamma} = w_t \lambda_t^i, \tag{6}
\]

where $\lambda_t^i = (C_t^i - \kappa C_{t-1})^{-\sigma}$ and $w_t$ is the real wage. The budget constraint holds with equality at the optimum and the transversality condition needs to be satisfied:

\[
\lim_{T \to \infty} E_t \{ \delta^{T-t} Q_{t,T}^i B_t^i \} = 0, \tag{7}
\]

where $Q_{t,T}^i(l)$ is the stochastic discount factor of the representative household $l$ of generation $i$ defined as

\[
Q_{t,t+s}^i = \beta^s e^{d_{t+s}} E_t \left[ \frac{C_t^i - \kappa C_{t-1}}{C_{t+s}^i - \kappa C_{t+s-1}} \right]^{\sigma}. \tag{8}
\]

Following Castelnuovo and Nisticò (2010), Del Negro, Giannoni, and Patterson (2012), Nisticò (2012), and Piergallini (2006), one can show that the consumption policy function is described by

\[
C_t^i = (1 - \beta \delta) \left( \frac{1}{\delta} \frac{B_t^i}{\pi_t^i} + D_t^i \right), \tag{9}
\]

where

\[
D_t^i = E_t \left[ \sum_{s=0}^{\infty} \delta^s Q_{t,t+s}^i \left( \frac{W_{t+s} H_{t+s} + \Lambda_{t+s}^i - T_{t+s} + TR_{t+s}}{P_{t+s}} \right) \right]. \tag{10}
\]
describes the present discounted value of future disposable income adjusted for past consumption.

3.2 Aggregation

As in Annicchiarico, Giammarioli, and Piergallini (2012), given the overlapping-generations structure of the model, the aggregate value $z_t$ of a generic economic $z_t^i(l)$ is obtained as a sum across generations:

$$z_t = \sum_{i=-\infty}^{t} \left( \int_0^{(1-\delta)\delta^{-i}} z_t^i(l) dl \right). \quad (12)$$

Define $Q_{t,t+s}$ as the population weighted average of the generation-specific stochastic discount factors:

$$Q_{t,t+s} = \sum_{i=-\infty}^{t} (1-\delta) \delta^{-i} Q_{t,t+s}^i. \quad (13)$$

Since (10) is linear in generation-specific variables, I can express it as follows:

$$C_t = (1 - \beta \delta) \left( \frac{1}{\delta} \frac{B_{t-1}}{\pi_t} + D_t \right), \quad (14)$$

where

$$D_t = E_t \left[ \sum_{s=0}^{\infty} \delta^s Q_{t,t+s} \left( \frac{W_{t+s} H_{t+s} + \Lambda_{t+s} - T_{t+s} + T R_{t+s}}{P_{t+s}} \right) \right]. \quad (15)$$

In appendix section A.3.1, I show that aggregation across all generations at time $t$ yields the following expression for the aggregate budget constraint:

$$P_tC_t + \frac{B_t}{R_t} = B_{t-1} + W_t H_t + \Lambda_t - T_t + T R_t. \quad (16)$$
Following the same procedure as in Smets and Trabandt (2012), in appendix section A.3.2 I derive the aggregate representation of the Euler which receives the following form:

\[
\beta \frac{e_{t+1}}{e_t} \frac{R_t}{\Pi_{t+1}} \frac{1}{\lambda_t} = \frac{1 - \delta}{\delta \mu_{t+1}} \frac{1}{\lambda_{t+1}^\sigma} B_t + \left( \frac{\lambda_t}{\lambda_{t+1}} \right)^{\frac{\sigma - 1}{\sigma}} \frac{1}{\lambda_{t+1}}. \tag{17}
\]

Note that for \( \delta < 1 \), government debt affects consumption spending. This leads to a breakdown of Ricardian equivalence even though taxes are lump sum. Moreover, under log-preferences, equation (17) receives a more intuitive form:

\[
\beta \frac{e_{t+1}}{e_t} \frac{R_t}{\Pi_{t+1}} \frac{1}{\lambda_t} = \frac{1 - \delta}{\delta \mu_{t+1}} B_t + \frac{1}{\lambda_{t+1}}. \tag{18}
\]

### 3.3 Firms

Final goods are produced by monopolistically competitive firms which employ only labor and use a linear production technology,

\[
Y_t(j) = A_t H_t(j), \tag{19}
\]

where \( A_t \) is an aggregate productivity shock at date \( t \) which is assumed to follow a log-stationary AR(1) process: \( a_t = \rho_a a_{t-1} + \sigma_{a,\varepsilon} \varepsilon_{a,t} \sim N(0,1) \). Firm profits are distributed to households at the end of each period. Each firm is the only producer of its good and sets its price in a staggered way as in Calvo (1983). In each period, a firm faces a constant probability of being able to reoptimize its nominal price, \( 1 - \omega \), regardless of the time elapsed since it last adjusted its price. Following Christiano, Eichenbaum, and Evans (2005), I assume that firms that do not reoptimize their price set their price according to the price that has been previously set, also accounting for past inflation (partial indexation). Specifically, if a firm \( j \) does not reoptimize, it sets the price of its good according to the following rule:

\[
P_t(j) = \pi_{t-1} P_{t-1}(j). \tag{20}
\]

This structure adds lagged inflation in the Phillips curve. Let \( \hat{p}_t(j) \) denote the price that is set by a firm that can reoptimize at date \( t \).
Given the Calvo price-setting mechanism, the price level can be summarized as

\[ P_t = \left[ \omega (\pi_{t-1} P_{t-1})^{1-\theta} + (1 - \omega) \tilde{p}_t(j)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \]  

(21)

If firm \( j \) reoptimizes, it chooses the price that maximizes the expected discounted sum of its profits. Profit maximization thus solves

\[
\max_{p_t^*(j)} E_{t} \sum_{s=0}^{\infty} (\delta \beta \omega)^s Q_{t+s} \{ \tilde{p}_t(j) X_{ts} - P_{t+s} m c_{t+s} \} y_{t+s}(j)
\]  

(22)

subject to

\[ y_{t+s}(j) = \left( \frac{p_t^*(j)}{P_t} \right)^{-\theta} Y_{t+s} \]

and

\[
X_{ts} = \begin{cases} 
\pi_t \times \pi_{t+1} \times \cdots \times \pi_{t+s-1} & \text{for } s \geq 1 \\
1 & \text{for } s = 0,
\end{cases}
\]  

(23)

where \( mc \) is the real marginal cost specified as

\[ mc_t = \frac{W_t}{A_t P_t}. \]  

(24)

The first-order condition associated with the firm’s choice of \( \tilde{p}_t \) is

\[
E_{t} \sum_{s=0}^{\infty} (\delta \beta \omega)^s Q_{t+s} \left\{ \tilde{p}_t(j) X_{ts} - \frac{\theta}{\theta - 1} P_{t+s} m c_{t+s} \right\} y_{t+s}(j).
\]  

(25)

Obviously, for \( \omega = 0 \), the firm sets its price equal to a markup over the current real marginal cost, \( mc \). By definition, for \( s = 0 \), the term \( X_{ts} \) is equal to 1.

### 3.4 Fiscal Authority

The flow budget constraint of the federal government is given by

\[ B_t = B_{t-1} (R_{t-1}) - T_t + S_t + TP_t, \]
where \( B_t \) is government debt, \( T_t \) is lump-sum taxes, and \( S_t \) is federal expenditures given by the sum of government purchases and transfers, \( S_t = P_t G_t + TR_t \). As in Bianchi and Ilut (2017) \( TP_t \) is a shock that captures a series of features that are not explicitly modeled here, such as changes in the term premium. As Bianchi and Ilut point out, this shock is necessary to avoid stochastic singularity when estimating the model given that I treat debt, taxes, and expenditures as observables. Expressing the variables as a fraction of output, the flow budget constraint receives the following form:

\[
b_t = b_{t-1} R_{t-1}/(\Pi_t Y_t/Y_{t-1}) - \tau_t + s_t + tp_t,
\]

where \( b_t = B_t/P_t Y_t \), \( \tau_t = T_t/P_t Y_t \), and \( s_t = S_t/P_t Y_t \), while \( \Pi_t \) is CPI inflation. I define variable \( \chi_t = g_t/s_t \) as the fraction of federal expenditure devoted to government purchases, where both variables are expressed as a share of GDP (\( g_t = G_t/Y_t \) and \( s_t = S_t/P_t Y_t \)). As in Bianchi and Ilut (2017), I assume that variable \( \chi_t \) has the following law of motion:

\[
\tilde{\chi}_t = \rho \tilde{\chi}_{t-1} + (1 - \rho) i_y \hat{Y}_t + \sigma_{\chi, s} \varepsilon_{\chi, t}, \varepsilon_{\chi, t} \sim N(0, 1),
\]

where \( Y^*_t \) is the flexible-price equilibrium output.

### 3.5 Market Clearing

Market clearing in the goods market requires

\[
Y_t(j) = C_t(j) + G(j)
\]

for all \( j \in [0, 1] \) and all \( t \). Defining aggregate output as \( Y_t = \left( \int_0^1 Y_t(j) \frac{\theta - 1}{\sigma} dj \right)^{\frac{\theta}{\sigma - 1}} \) and accounting for the fact that I have

---

\(^8\)In Bianchi and Ilut (2017) the government issues long-term debt only. Therefore, the term \( TP_t \) can also capture changes in the maturity structure of federal debt or changes in the term premium.

\(^9\)In what follows, for all the variables normalized with respect to GDP (debt, government purchases, federal expenditure, tax revenues) \( \tilde{x}_t \) denotes a linear deviation (\( \tilde{x}_t = X_t - \bar{X} \)) from its steady state. Instead, for all other variables \( \hat{x}_t \) denotes a percentage deviation (\( \hat{x}_t = \log(X_t/\bar{X}) \)) from its steady state. This distinction avoids having the percentage change of a percentage.
expressed fiscal variables as fractions of nominal GDP, the aggregate resource constraint is as follows:

\[ Y_t = C_t + g_t Y_t. \]

4. Markov Switching

4.1 Solution and Estimation Technique

In this section, I describe how Markov switching is introduced into the model and how the resulting model is estimated. I follow the approach of Chen (2017), Chen, Kirsanova, and Leith (2017), and Liu and Mumtaz (2011). First, I log-linearize the model and then I introduce Markov switching. In appendix section A.1, I present the full model log-linearized. The algorithm that is used to solve the model and obtain a time-varying minimum state variable solution is that of Farmer, Waggoner, and Zha (2011). I assume that both monetary and fiscal policy are subject to regime shifts. Specifically, as regards monetary policy, I assume that the parameters in the Taylor rule of the Federal Reserve are subject to regime shifts. As regards fiscal policy, I assume that the fiscal authority uses tax revenues and federal spending whenever it seeks to stabilize debt and that the parameters on each of the two fiscal feedback rules are subject to regime shifts. Monetary and fiscal policy switch regimes simultaneously. Moreover, I allow the variances of all the shocks in the model to be subject to regime shifts as well. I allow for independent regime switching in the volatility of the structural shocks that the model features. To specify the MS-DSGE model, I partition the parameter vector \( \Phi \) into three blocks,

\[ \Phi = \{ \Phi^Z; \Sigma^\zeta; \bar{\Phi} \}, \]

where \( \Phi^Z \) is the set of parameters subject to regime shifts, \( \Sigma^\zeta \) is the variance of the regime-switching volatilities, and \( \bar{\Phi} \) denotes the remaining time-invariant parameters. The Markov-switching interest rate and fiscal feedback rules are specified as

\[
\begin{align*}
\hat{R}_t &= \rho_{R,Z_t} \hat{R}_{t-1} + (1 - \rho_{Z_t}) \left( \phi_{\pi,Z_t} \pi_t + \phi_{y,Z_t} \hat{Y}_t \right) \\
&\quad + \sigma_{R,\zeta_t} \varepsilon_{R,t} \varepsilon_{R,t} \sim N (0, 1), \quad (26)
\end{align*}
\]
\[
\tilde{\tau}_t = \rho_{\tau,Z} \tilde{\tau}_{t-1} + (1 - \rho_{\tau,Z}) \left( \gamma_{b,Z} \tilde{b}_{t-1} + \gamma_y \tilde{Y}_t \right) \\
+ \sigma_{\tau,\xi} \varepsilon_{\tau,t} \varepsilon_{\tau,t} \sim N(0,1), \tag{27}\]
and
\[
\tilde{s}_t = \rho_{s,Z} \tilde{s}_{t-1} + (1 - \rho_{s,Z}) \left( -\delta_{b,Z} \tilde{b}_{t-1} - \delta_y \tilde{Y}_t \right) \\
+ \sigma_{s,\xi} \varepsilon_{s,t} \varepsilon_{s,t} \sim N(0,1), \tag{28}\]

where \( \bar{\pi} \) is steady-state inflation. All shocks considered in this paper are independent of one another. I allow all shock volatilities \( \sigma_{d,\xi}, \sigma_{a,\xi}, \sigma_{c,\xi}, \sigma_{R,\xi}, \sigma_{\tau,\xi}, \sigma_{s,\xi}, \) and \( \sigma_{tp,\xi} \) to be time varying. The superscript \( Z \) denotes the unobserved regime associated with the monetary and fiscal policy parameters taking on values 1, 2, or 3. Monetary and fiscal policy regime follows a Markov process with transition probabilities \( p_{ij} = P[Z_t = i | Z_{t-1} = j] \), where \( i, j = 1, 2, 3 \). Hence, I assume three regimes for the monetary/fiscal policy mix. For convenience, from now on I will refer to those regimes which concern switches in policy parameters only as Regime-Pol.

In the MS-DSGE literature, it has been documented that U.S. monetary policy has been passive (i.e., weak reaction to inflation fluctuations) during the '70s (see Bianchi 2012, 2013; Bianchi and Ilut 2017) and fiscal policy has been active (i.e., no reaction of taxes or expenditure to stabilize debt). Absent regime switches, Leeper (1991) distinguishes four regions of the parameter space according to existence and uniqueness of a solution. In the linearized version of the model, the monetary and fiscal policy rule in practice determine the existence and the uniqueness of an equilibrium. Hence, there are two policy mixes that yield a determinate equilibrium. The first is an active monetary/passive fiscal (AM/PF) mix where monetary policy satisfies the Taylor principle and fiscal policy passively accommodates monetary policy by guaranteeing debt stability. Hence, the inflation coefficient in the interest rate rule satisfies \( \phi_{\tau,Z} > 1 \) and the coefficient on debt in the tax rule satisfies \( \gamma_{b,Z} > (\bar{R} - 1) / (1 - \rho_{r,Z}) \) while that on the spending rule satisfies \( \delta_{b,Z} > (\bar{R} - 1) / (1 - \rho_{s,Z}) \), where \( \bar{R} \) is the steady-state interest rate, derived in appendix section A.2, which is a function of the subjective discount factor, \( \beta \),
and the survival probability, \( \delta \). The second is a passive monetary/active fiscal (PM/AF) mix, where monetary policy does not satisfy the Taylor principle and fiscal authority is not committed to stabilizing the process for debt. Formally, the policy coefficients in this regime satisfy \( \phi_{\pi,Z_t} < 1, \gamma_{b,Z_t} < (\tilde{R} - 1) / (1 - \rho_{\tau,Z_t}), \) and \( \delta_{b,Z_t} < (\tilde{R} - 1) / (1 - \rho_{\tau,Z_t}) \), respectively. Finally, no stationary equilibrium exists when both authorities are active (AM/AF), whereas when both of them are passive (PM/PF) the economy is subject to multiple equilibriums.

I restrict the policy parameters such that one regime (Regime-Pol 2), \( j = 2 \), is characterized by an active fiscal policy and by a weaker reaction of monetary policy to inflation fluctuations compared with Regime-Pols 1 and 3. Specifically, as I describe in the prior specification below, I center the mean of the prior for the inflation coefficient in the Taylor rule in Regime-Pol 2 at \( \phi_{\pi,2} = 1 \) and also impose the restriction \( \phi_{\pi,2} < \phi_{\pi,1} \) and \( \phi_{\pi,2} < \phi_{\pi,3} \). As regards fiscal policy, I set the reaction to debt in the tax and the federal spending rule equal to zero, \( \gamma_{b,2} = 0 \) and \( \delta_{b,2} = 0 \), similar to Bianchi and Ilut (2017). In Regime-Pols 1 and 3, I place no restrictions on the coefficients on debt in the two fiscal feedback rules. In other words, I allow those two to be freely estimated. The reason behind this approach is to allow the data to identify whether, while monetary policy has been active (i.e. \( \phi_{\pi,1} > 1 \) and \( \phi_{\pi,3} > 1 \)), the fiscal authority has been through periods during which tax revenues were the main instrument to stabilize debt and through other periods where federal spending was the main instrument. Obviously, I interpret a higher coefficient on debt in the spending rule, \( \delta_{b,Z_t} \), compared with its counterpart in the tax rule, \( \gamma_{b,Z_t} \), as an indication that spending is the main instrument to stabilize debt, and vice versa. The superscript \( \zeta = 1, 2 \) in variances denotes the unobserved regime associated with the volatilities and which evolves independently of \( S \). For convenience, from now on I denote the volatility regime which concerns switches in volatilities only as Regime-Vol. The two state variables \( Z \) and \( \zeta \) follow a first-order Markov chain with the following transition probability matrices, respectively:

\[ \text{Bianchi and Ilut (2017) assume that the reaction of tax revenues to the debt ratio is zero whenever fiscal policy is active.} \]
\[
P = \begin{bmatrix}
P_{11} & 1 - P_{11} & 0 \\
1 - P_{21} & P_{22} & 1 - P_{21} - P_{22} \\
1 - P_{33} & 0 & P_{33}
\end{bmatrix},
\]

\[
Q = \begin{bmatrix}
Q_{11} & 1 - Q_{11} \\
1 - Q_{22} & Q_{22}
\end{bmatrix},
\]

where \( P_{ji} = p[Z_t = i|Z_{t-1} = j] \), where \( i, j = 1, 2, 3 \), and \( Q_{mn} = p[\zeta_t = n|\zeta_{t-1} = m] \), where \( m, n = 1, 2 \). The specification of the transition probability matrix \( P \) is similar to that in Bianchi and Ilut (2017), the only difference being that, in Regime-Pol 3, the economy is restricted to switch to Regime-Pol 1 instead of Regime-Pol 2. The model can be written in a matrix form as

\[
A(Z_t)X_{t+1} = B(Z_t)X_t + \Psi(Z_t)\epsilon_t + \Pi(Z_t)\eta_t, \quad \text{where} \quad \epsilon_t \sim N(0, Q_\zeta),
\]

where \( X_{t+1} = [\hat{C}_{t+1}, \pi_{t+1}, \hat{C}_t, \pi_t, \hat{Y}_t, \hat{R}_t, \tilde{r}_t, \tilde{s}_t, \tilde{g}_t, \tilde{x}_t, \alpha_t, u_t, d_t, \tilde{f}_t] \) \( \epsilon_t \) is a 8×1 vector of structural shocks of mean zero and whose variance is allowed to vary over time as specified above. \( \eta_t \) is a 2×1 vector of endogenous random variables.

I solve the model using the approach of Farmer, Waggoner, and Zha (2011). Farmer, Waggoner, and Zha (2011) show that if a unique solution exists, then this can be cast as a Markov-switching VAR of the following form:

\[
X_t = g_1, Z_t, X_{t-1} + g_2, Z_t, \epsilon_t. \tag{30}
\]

11I have also estimated the model with \( P_{13} > 0 \) and the results are robust. I have set it to zero in order to reduce the parameter space given the complexities of the model. As regards \( P_{32} \), I have set it to zero in order to account for switches in the composition of fiscal tools only while fiscal policy stays passive.

12As in Justiniano and Preston (2010), when log-linearizing the model I add an import cost-push shock, denoted by \( \epsilon_{cp,t} \), to the Phillips curve. The reason is mainly to have a number of structural shocks equal to the number of observables in the state-space representation of the model.

13I log-linearize the model around the symmetric zero-inflation steady state. The log-linearized model is summarized in appendix section A.1.
Equation (30) above can be combined with an observation equation giving the following state-space model with Markov switching:

\[ X_t = g_{1,Z_t}X_{t-1} + g_{2,Z_t}\varepsilon_t, \quad \text{where} \quad \varepsilon_t \sim N(0, Q_\zeta) \]

\[ D_t = HX_t. \]  

Equation (31)

I assume no measurement errors. Specifically, the data for the variables and the log-linearized variables are linked by the following equations.

\[
\begin{bmatrix}
dlGDP_t \\
Inflation_t \\
FedFunds_t \\
d(Debt/GDP)_t \\
d(G/GDP)_t \\
d(TaxRev/GDP)_t \\
d(S/GDP)_t
\end{bmatrix}
= 
\begin{bmatrix}
\hat{Y}_t - \hat{Y}_{t-1} \\
\pi_t \\
\hat{R}_t \\
\hat{b}_t - \hat{b}_{t-1} \\
\hat{g}_t - \hat{g}_{t-1} \\
\hat{\tau}_t - \hat{\tau}_{t-1} \\
\hat{s}_t - \hat{s}_{t-1}
\end{bmatrix}
\]  

(32)

As described above, the Markov states \( Z \) and \( \zeta \) evolve independently with transition probability matrices \( P \) and \( Q \), respectively. \( D_t \) represents the observed data and matrix \( H \) is the loading matrix. As Liu and Mumtaz (2011) point out, the presence of the unobserved DSGE states \( X_t \) and the unobserved Markov states makes the standard Kalman filter not operational in order to provide inference on \( X_t \) and to calculate the value of the likelihood. Inference now has to be conditioned on both current and past values of \( Z \) and \( \zeta \). Following their approach, I define a new state variable \( Z^* \) indexing both \( Z_t \) and \( \zeta_t \) and which has a four-state transition matrix, \( P^* = PQ \).

As in Davig and Doh (2014) and Kim and Nelson (1999), I track \( Z^*_t \), \( Z^*_{t-1} \), and \( Z^*_{t-2} \), which means that I account for \( 6^3 = 216 \) possible paths for the state variables at each point in time.  

As mentioned above, I follow a Bayesian approach to estimate the model, where I combine the approximate likelihood function with the assumed prior distributions and use a random-walk Metropolis-Hastings algorithm with 200,000 replications in order to approximate the posterior.

---

4.2 Priors

A summary of the prior distributions and the relevant bounds for the model parameters is provided in table 1. Parameters with a dash are fixed according to the specification and thus are not estimated. I specify the prior distributions following the literature on either closed- or open-economy models (see, e.g., Bianchi 2013; Bianchi and Ilut 2017; Lubik and Schorfheide 2007; Smets and Wouters 2003, 2007). As regards the parameters that are not estimated, I calibrate the steady-state government purchases to GDP, $g_Y$, and federal spending to GDP, $s_Y$, to 0.21 and 0.25, respectively, while I set the steady-state debt-to-GDP ratio, $b$, equal to 1 as in Bianchi (2012). I set the intratemporal elasticity of substitution across varieties, $\rho$, equal to 8. Finally, I set parameter $\iota_y$ in the law of motion for $\tilde{\chi}_t$ equal to 0.1.

In the literature, there are not many models that estimate the survival probability, $\delta$. I assume a beta prior with mean 0.95 and standard deviation 0.1. In the interest rate rule, I assume that the smoothing parameter $\rho_R$ follows a beta distribution with a mean of 0.5 and standard deviation equal to 0.2 in all regimes, and the inflation coefficient $\phi_\pi$ follows a gamma distribution with a mean of 1.5 and a standard deviation of 0.1 in Regime-Pols 1 and 3 and a gamma distribution with mean 1 and standard deviation 0.1 in Regime-Pol 2. The coefficient on output follows also a gamma distribution with a mean of 0.4 and a standard deviation of 0.2 in Regime-Pols 1 and 3 and a gamma distribution with mean 0.15 and standard deviation 0.1 in Regime-Pol 2. As regards the fiscal feedback rules, I assume that the reaction of tax revenues to lagged debt-to-GDP ratio, $\gamma_{b,1}$, and the reaction of federal spending to lagged debt-to-GDP, $\delta_{b,3}$, both follow a gamma distribution with mean 0.07 and standard deviation 0.025. Finally, regarding the transition probabilities, I follow Sims and Zha (2006) in assuming a Dirichlet prior, with the scale matrix chosen to reflect the belief that regimes are persistent. The relevant parameters for the Dirichlet distribution imposed are $\alpha_1 = 18$ and $\alpha_2 = 1$.

4.3 Data

The sample consists of quarterly data spanning from 1969:Q1 to 2012:Q4. The real per capita GDP was constructed using data on
### Table 1. Prior Distributions and Posterior Estimates

| Model: No Switching in Spending | Model: Infinite | Benchmark Model | Model: Infinite
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Log Marginal</td>
<td>Log Marginal</td>
<td>Log Marginal</td>
</tr>
<tr>
<td></td>
<td>Likelihood: –1,283</td>
<td>Likelihood: –1,374</td>
<td>Likelihood: –1,311</td>
</tr>
<tr>
<td><strong>Prior Specification</strong></td>
<td><strong>Mean</strong></td>
<td><strong>Median</strong></td>
<td><strong>95%</strong></td>
</tr>
<tr>
<td><strong>Parameters</strong></td>
<td><strong>Distrib.</strong></td>
<td><strong>5%</strong></td>
<td><strong>95%</strong></td>
</tr>
<tr>
<td>δ</td>
<td>0.9750</td>
<td>0.9760</td>
<td>0.9950</td>
</tr>
<tr>
<td></td>
<td>0.95</td>
<td>0.95</td>
<td>—</td>
</tr>
<tr>
<td>φ_π,1</td>
<td>0.9059</td>
<td>0.9490</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.9719</td>
<td>0.9719</td>
<td>—</td>
</tr>
<tr>
<td>φ_π,2</td>
<td>0.9559</td>
<td>0.9490</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.9719</td>
<td>0.9719</td>
<td>—</td>
</tr>
<tr>
<td>φ_π,3</td>
<td>0.9559</td>
<td>0.9490</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.9719</td>
<td>0.9719</td>
<td>—</td>
</tr>
<tr>
<td>φ_y,1</td>
<td>0.4749</td>
<td>0.3262</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.4292</td>
<td>0.4292</td>
<td>—</td>
</tr>
<tr>
<td>φ_y,2</td>
<td>0.4749</td>
<td>0.3262</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.4292</td>
<td>0.4292</td>
<td>—</td>
</tr>
<tr>
<td>φ_y,3</td>
<td>0.4749</td>
<td>0.3262</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.4292</td>
<td>0.4292</td>
<td>—</td>
</tr>
<tr>
<td>φ_ρ,1</td>
<td>0.5959</td>
<td>0.6780</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.6780</td>
<td>0.6780</td>
<td>—</td>
</tr>
<tr>
<td>φ_ρ,2</td>
<td>0.5959</td>
<td>0.6780</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.6780</td>
<td>0.6780</td>
<td>—</td>
</tr>
<tr>
<td>φ_ρ,3</td>
<td>0.5959</td>
<td>0.6780</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.6780</td>
<td>0.6780</td>
<td>—</td>
</tr>
<tr>
<td>φ_δ,1</td>
<td>0.0415</td>
<td>0.0415</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.0415</td>
<td>0.0415</td>
<td>—</td>
</tr>
<tr>
<td>φ_δ,2</td>
<td>0.0415</td>
<td>0.0415</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.0415</td>
<td>0.0415</td>
<td>—</td>
</tr>
<tr>
<td>φ_δ,3</td>
<td>0.0415</td>
<td>0.0415</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.0415</td>
<td>0.0415</td>
<td>—</td>
</tr>
<tr>
<td>γ_b,1</td>
<td>0.0794</td>
<td>0.0794</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.0794</td>
<td>0.0794</td>
<td>—</td>
</tr>
<tr>
<td>γ_b,2</td>
<td>0.0794</td>
<td>0.0794</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.0794</td>
<td>0.0794</td>
<td>—</td>
</tr>
<tr>
<td>γ_b,3</td>
<td>0.0794</td>
<td>0.0794</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.0794</td>
<td>0.0794</td>
<td>—</td>
</tr>
<tr>
<td>ρ_τ,1</td>
<td>0.5338</td>
<td>0.5338</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.5338</td>
<td>0.5338</td>
<td>—</td>
</tr>
<tr>
<td>ρ_τ,2</td>
<td>0.5338</td>
<td>0.5338</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.5338</td>
<td>0.5338</td>
<td>—</td>
</tr>
<tr>
<td>ρ_τ,3</td>
<td>0.5338</td>
<td>0.5338</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.5338</td>
<td>0.5338</td>
<td>—</td>
</tr>
<tr>
<td>ρ_β,1</td>
<td>0.4956</td>
<td>0.4956</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.4956</td>
<td>0.4956</td>
<td>—</td>
</tr>
<tr>
<td>ρ_β,2</td>
<td>0.4956</td>
<td>0.4956</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.4956</td>
<td>0.4956</td>
<td>—</td>
</tr>
<tr>
<td>ρ_β,3</td>
<td>0.4956</td>
<td>0.4956</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.4956</td>
<td>0.4956</td>
<td>—</td>
</tr>
<tr>
<td>P_12</td>
<td>0.4573</td>
<td>0.4573</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.4573</td>
<td>0.4573</td>
<td>—</td>
</tr>
<tr>
<td>P_21</td>
<td>0.0356</td>
<td>0.0356</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.0356</td>
<td>0.0356</td>
<td>—</td>
</tr>
<tr>
<td>P_23</td>
<td>0.0232</td>
<td>0.0232</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.0232</td>
<td>0.0232</td>
<td>—</td>
</tr>
<tr>
<td>P_31</td>
<td>0.0725</td>
<td>0.0725</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>0.0725</td>
<td>0.0725</td>
<td>—</td>
</tr>
</tbody>
</table>

(continued)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Q11</td>
<td>0.6678</td>
<td>0.9859</td>
<td>0.8184</td>
<td>Dirichlet</td>
</tr>
<tr>
<td>Q22</td>
<td>0.5277</td>
<td>0.5106</td>
<td>0.7597</td>
<td>Dirichlet</td>
</tr>
<tr>
<td>σ</td>
<td>2.5979</td>
<td>1.8849</td>
<td>1.6563</td>
<td>Normal</td>
</tr>
<tr>
<td>ω</td>
<td>0.7523</td>
<td>0.9103</td>
<td>0.7478</td>
<td>Beta</td>
</tr>
<tr>
<td>γ</td>
<td>1.3927</td>
<td>2.3131</td>
<td>1.2814</td>
<td>Normal</td>
</tr>
<tr>
<td>κ</td>
<td>0.4380</td>
<td>0.7934</td>
<td>0.6801</td>
<td>Beta</td>
</tr>
<tr>
<td>γy</td>
<td>0.0530</td>
<td>0.0977</td>
<td>0.1219</td>
<td>Normal</td>
</tr>
<tr>
<td>δy</td>
<td>0.0069</td>
<td>0.0214</td>
<td>0.1426</td>
<td>Normal</td>
</tr>
<tr>
<td>ρα</td>
<td>0.7105</td>
<td>0.7851</td>
<td>0.7174</td>
<td>Beta</td>
</tr>
<tr>
<td>ρu</td>
<td>0.2835</td>
<td>0.1831</td>
<td>0.4243</td>
<td>Beta</td>
</tr>
<tr>
<td>ρd</td>
<td>0.4774</td>
<td>0.8085</td>
<td>0.5238</td>
<td>Beta</td>
</tr>
<tr>
<td>ρχ</td>
<td>0.9688</td>
<td>0.6767</td>
<td>0.9343</td>
<td>Beta</td>
</tr>
<tr>
<td>ρtp</td>
<td>0.2992</td>
<td>0.5131</td>
<td>0.9663</td>
<td>Beta</td>
</tr>
<tr>
<td>100σ₁</td>
<td>3.8327</td>
<td>3.7632</td>
<td>0.8673</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₂</td>
<td>1.3610</td>
<td>9.6442</td>
<td>2.6450</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₃</td>
<td>1.1682</td>
<td>0.6367</td>
<td>1.4882</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₄</td>
<td>2.7940</td>
<td>5.9296</td>
<td>8.7578</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₅</td>
<td>3.0398</td>
<td>0.5540</td>
<td>6.0144</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₆</td>
<td>7.8448</td>
<td>5.6665</td>
<td>7.2595</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₇</td>
<td>0.0215</td>
<td>0.0190</td>
<td>4.8039</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₈</td>
<td>2.5164</td>
<td>7.4701</td>
<td>7.9932</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₉</td>
<td>2.4507</td>
<td>0.0412</td>
<td>2.3780</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₁₀</td>
<td>4.8961</td>
<td>0.7009</td>
<td>4.9122</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₁₁</td>
<td>0.1856</td>
<td>0.0136</td>
<td>0.1680</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₁₂</td>
<td>9.4111</td>
<td>0.1212</td>
<td>3.4925</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₁₃</td>
<td>2.3717</td>
<td>2.4630</td>
<td>4.1466</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₁₄</td>
<td>2.1127</td>
<td>2.4038</td>
<td>5.4969</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₁₅</td>
<td>2.2114</td>
<td>1.3362</td>
<td>0.7017</td>
<td>Inv. Gamma</td>
</tr>
<tr>
<td>100σ₁₆</td>
<td>4.7957</td>
<td>2.4849</td>
<td>4.7187</td>
<td>Inv. Gamma</td>
</tr>
</tbody>
</table>
nominal GDP (NIPA table 1.1.5, line 1) over the GDP deflator (NIPA table 1.1.4, line 1) and the civilian non-institutional population over 16 (given by LNU00000000Q, at the Bureau of Labor Statistics). The table and line numbers refer to the NIPA (national income and product accounts) tables on the Bureau of Economic Analysis website. The data for consumption, government purchases, federal spending, tax revenues, and debt were linearly detrended to get stationary series. Specifically, private consumption consists of the sum of personal consumption expenditures on nondurable goods (table 1.1.5, line 5) and services (table 1.1.5, line 6) divided by the GDP deflator and by population. I construct inflation as the first difference of the GDP deflator. GDP deflator data are obtained as described above. I use monthly federal funds rate series from the Federal Reserve Bank of St. Louis’s FRED (Federal Reserve Economic Data) website, averaged to receive quarterly series of the federal funds rate. Government purchases consist of government consumption expenditures and gross investment (table 1.1.5, line 22) divided by the GDP deflator and by population. Federal expenditure is the sum of government purchases plus net purchases of nonproduced assets (table 3.2, line 43), minus consumption of fixed capital (table 3.2, line 44), minus wage accruals less disbursements (table 3.2, line 33) plus net current transfer payments (table 3.2, lines 22–16), subsidies (table 3.2, line 32), and net capital transfers (table 3.2, lines 42–38) divided by the GDP deflator and by population. Tax revenues are given by the difference between current receipts (table 3.2, line 37) and current transfer receipts (table 3.2, line 16) divided by the GDP deflator and by population. Interest payments are measured using total interest payments to persons and businesses and the rest of the world (table 3.2, line 29). Finally, I compute government debt by taking the market value of privately held gross federal debt from the Federal Reserve Bank of Dallas website. The quarterly series is constructed by summing up the monthly series.

5. Parameter Estimates

Table 1 reports the posterior parameter estimates. Since the focus of the paper is on the finite horizon of agents and on the interactions between monetary and fiscal policy across regimes, I will
focus on the parameters describing those eventualities. The posterior mean of the survival probability parameter is equal to 0.9760, implying a planning horizon of agents equal to 41 quarters (or 10 years), approximately. Recall that in this paper I interpret \( \delta \) as a parameter determining the effective planning horizon of households, as mentioned in the description of household decisions in section 2.

The estimation of the monetary/fiscal policy parameters reveals the following facts. The monetary/fiscal policy mix has switched three regimes over time. Specifically, the estimation reveals that the U.S. economy has been through two determinate regimes and one indeterminate regime. In line with the existing literature, a passive monetary/active fiscal policy mix (Regime-Pol 2) is the most recurrent during the ’70s until the early ’80s (see figure 2). From the mid-’80s onward the economy switched to a regime (Regime-Pol 3) where monetary policy was active and fiscal policy was passive, adjusting both taxes and spending in reaction to debt fluctuations. I display the estimated posterior distributions of the reaction coefficients in the monetary policy rule in figure 3. As regards the fiscal feedback rules, the reaction coefficients on lagged debt-to-GDP ratios are close to each other (\( \gamma_{b,3} = 0.0794 \) and \( \delta_{b,3} = 0.0677 \)). However, the economy spent a very short period during the early ’80s where both monetary and fiscal policy were passive, with the latter adjusting spending heavily in order to stabilize debt (Regime-Pol 1). In fact, the coefficient on debt in the spending rule is \( \delta_{b,1} = 0.1117 \), while that in the tax rule is \( \gamma_{b,1} = 0.0415 \). During that period the median of the posterior of the coefficient on inflation in the Taylor rule is estimated at \( \phi_{\pi,1} = 0.9059 \). This result is in sharp contrast with existing evidence on estimated MS-DSGE models. In particular, Bianchi and Ilut (2017) find that in the early ’80s the U.S. economy spent a short period where both policies were active until it later switched to an AM/PF policy mix. However, Bianchi and Ilut do not account for the possibility of federal spending being used as an additional or alternative instrument to stabilize debt. Moreover, I find that over that period monetary policy was passive, although the posterior distribution of \( \phi_{\pi,1} \) lies to the right of \( \phi_{\pi,2} \) (see figure 3). This could

\[15\] In fact, the mode of the posterior distribution of the inflation coefficient is above 1 in Regime-Pol 3, while in Regime-Pol 1 and 2 it is below 1 and much lower.
indicate that monetary policy in the United States started to adjust gradually, in the face of the persistent inflationary pressures of the previous decade, until it switched to become active from the mid-'80s onward. The model estimation clearly identifies that the reaction of the Federal Reserve has changed over time with the mass of the estimated Regime-Pol 2 distribution of $\phi_{\pi,2}$ lying to the left of the distribution of Regime-Pols 1 and 3 coefficients, $\phi_{\pi,1}$ and $\phi_{\pi,3}$.

The estimation of the model also shows that there have been changes in the volatilities of the shocks. Specifically, most the shock volatilities are lower in Regime-Vol 1 than in Regime-Vol 2, with the productivity and the government purchases share, $\tilde{\chi}_t$, shocks to be the only exceptions. For those two shocks, their volatilities are
Figure 3. Estimated Posterior Distribution of Inflation Reaction Coefficient in Both Regimes

Notes: The distribution in Regime-Pol 1 is displayed by the dashed line, the distribution in Regime-Pol 2 by the solid line, and the distribution in Regime-Pol 3 by the dashed-dotted line.

estimated to be higher in Regime-Vol 1. As shown in the bottom panel in figure 2, the probability of Regime-Vol 1 increases in the early to mid-’80s and stays high until the wake of the recent financial crisis.\footnote{Bianchi and Ilut (2017) estimate an MS-DSGE model for the United States and also find evidence for a high-volatility regime in the early ’80s along with an increase in the probability of this regime almost during the same period in the early 2000s. Bianchi (2013) finds that the high-volatility regime is the dominant regime during the ’70s until the mid-’80s, with a small break in the late ’70s.}

5.1 Impulse Responses

In this section, I look at the impulse responses in each regime individually. The model features eight shocks, namely a preference
shock, technology shock, cost-push shock, tax shock, federal spending shock, government purchases shock, monetary policy shock, and term-premium shock. In order to save space, I look at the responses to a monetary policy, a tax shock, a federal spending shock, and a preference shock. Specifically, I look at a contractionary monetary policy shock, a tax hike shock, a positive federal spending shock, and a positive preference shock. The responses of inflation, output, the federal funds rate, and the debt-to-GDP are displayed in figure 4. In each case, I compute the median impulse response function by solving the model using the median posterior estimate of each parameter. Everything thus is evaluated at the posterior median.

Let me compare the responses in the two determinate regimes, namely Regime-Pols 2 and 3. Inflation and output revert back to the steady state faster in Regime-Pol 3 than in Regime-Pol 2 in all cases, the only exception being after a preference shock. As regards output, the responses in Regime-Pols 2 and 3 following a preference shock look very similar. Notice the change in sign in the responses of inflation in these two regimes following a monetary policy shock. Inflation increases in Regime-Pol 2, while it falls in Regime-Pol 3. In this case, higher inflation and a weak response of monetary policy to inflation keep the real rate and hence the debt ratio lower in Regime-Pol 2 than in Regime-Pol 3. In general, the weak response of monetary policy to inflation in Regime-Pol 2 leads to higher inflation volatility regardless of the shock hitting the economy. As a result, when shocks are inflationary, the real interest rate stays persistently lower compared with Regime-Pol 3, which implies lower debt service costs. This allows the debt ratio to fluctuate at lower levels in Regime-Pol 2 than in Regime-Pol 3.

Given the regime-switching environment, inflation expectations stay anchored in Regime-Pol 2 even though monetary policy responds weakly to inflation fluctuations. Specifically, the specification of the transition matrix implies that while the economy lies in Regime-Pol 2, agents are aware of the fact that the economy may switch to Regime-Pol 3 with probability $P_{23} = 1 - P_{21} - P_{22}$. Therefore, this possibility allows for some anchoring of inflation expectations. However, the anchoring of inflation expectations is weaker than in the case where agents are infinitely lived, $\delta = 1$. This is because agents discount the future more under a Blanchard-Yaari
Figure 4. Impulse Response Functions

Notes: This figure displays the responses of inflation, output, real interest rate, and debt-to-GDP to a monetary, tax, federal expenditure, and preference shock. The solid lines display the median of the impulse responses in Regime-Pol 1, where both monetary and fiscal policy is passive; the dashed-dotted lines display the median in Regime-Pol 2, where monetary policy is passive and fiscal policy is active; and the dashed lines display the median in Regime-Pol 3, where monetary policy is active and fiscal policy is passive.

structure, which implies that they place less weight on a future strong monetary response to inflation fluctuations when forming their expectations.\(^{17}\) Additionally, the anchoring of inflation expectations is further weakened, compared with the \(\delta = 1\) case, by the

\(^{17}\)In fact, Del Negro, Giannoni, and Patterson (2012) embed a Blanchard-Yaari structure in the medium-scale model of Smets and Wouters (2007) and show that announcements of policy changes in the future generate smaller effects on current aggregate variables compared with a model with infinitely lived agents.
possibility of a switch to the indeterminate Regime-Pol 1. This would not happen if Regime-Pol 2 stayed forever. Let me now look at the responses following fiscal shocks. Clearly, after a tax increase or a federal spending increase, inflation is better controlled in Regime-Pol 3. The strong reaction of the central bank to inflation fluctuations in this regime keeps inflation expectations anchored. The finite lifetimes of households allow the effect of the monetary policy stance on inflation expectations to dominate the negative effect stemming from the probability of a switch to Regime-Pol 1 where monetary policy is passive. The same argument holds in Regime-Pol 2. The weak reaction of monetary policy to inflation dominates the effect of a possible switch to Regime-Pol 3 in the future. In Regime-Pol 1 the responses of inflation and output also seem to be mainly affected by the current monetary policy and fiscal policy stance rather than by the probability of a future switch to Regime-Pol 2. In fact, even though passive, the monetary reaction to inflation is stronger in Regime-Pol 1 than Regime-Pol 2. All in all, it seems that the finite-lifetime structure of the model makes the current monetary/fiscal policy stance more important in households’ decisions than the probability of a future switch to another regime. These effects become clearer in the next section, where I estimate the same model but with infinite lifetimes, $\delta = 1$.

5.2 Alternative Specifications

5.2.1 The Model with Infinite Lifetimes: $\delta = 1$

In this section I reestimate the model under the assumption that agents are infinitely lived. Specifically, I fix the survival probability

\[ \delta = 1 \]

\[ \text{Bianchi and Ilut (2017) perform beliefs counterfactuals to show the anchoring of inflation expectations when agents anticipate a switch to an active monetary/passive fiscal policy mix in the future. In their approach, though, agents do not face a probability of death, as such an active monetary/passive fiscal policy mix can have zero effects on inflation because agents anticipate a switch to a Ricardian regime in the future, as long as the probability of the latter is high enough. In the framework of the current paper, though, this can never happen. This is because I have assumed that agents face a probability of death which leads to a debt-to-GDP ratio having real effects on output even though taxes are lump sum.}\]
The estimation results reveal some crucial differences between the benchmark model with finite lifetimes and the one with infinite lifetimes. First, the estimation of the model favors two regimes. Specifically, as shown in figure 5, the economy switches between two regimes, namely a regime where monetary policy is passive and fiscal policy is active (Regime-Pol 2) during the ’70s and a regime where monetary policy is active and fiscal is passive (Regime-Pol 3) from the ’80s onward. Interestingly, the economy switches
earlier to Regime-Pol 3 when lifetimes are infinite compared with the benchmark case (see figure 2). In the benchmark model with finite lifetimes, the economy switched to Regime-Pol 3 around 1984 after having spent a short time in Regime-Pol 1. The parameters of Regime-Pol 1 show both policies to be passive in that regime. However, the estimated smoothed state probabilities indicate that the economy has spent nearly no time in this regime, as opposed to the benchmark model.

The second important difference between the two models is that in the model with infinite lifetimes tax revenues are substantially more sensitive to debt fluctuations ($\gamma_{b,3} = 0.1347$) than federal expenditure ($\delta_{b,3} = 0.013$) in Regime-Pol 3. In the benchmark model, instead the sensitivity of the two fiscal tools in that regime was similar ($\gamma_{b,3} = 0.079$ and $\delta_{b,3} = 0.067$). As far as monetary policy is concerned, the coefficients on inflation across regimes do not seem to differ much in the two models.

Third, some key parameters are also different in the two models. The Phillips curve is flatter in the model with infinite lifetimes, with a posterior mode of the Calvo parameter equal to $\omega = 0.9102$, contrary to $\omega = 0.6565$ in the benchmark model. Additionally, output is more persistent than in the benchmark case, with a posterior median of the degree of habits $\kappa = 0.7934$, compared with $\kappa = 0.4380$. Finally, households appear to be less risk averse when lifetimes are infinite, with a posterior median for the degree of relative risk aversion of $\sigma = 1.8548$ relative to $\sigma = 2.5979$ in the benchmark case.

Fourth, the responses of the variables differ substantially in some cases. However, those differences should be treated with caution, as not only the lifetimes differ but also the probabilities of the regimes, the policy parameters, and the rest of the deep parameters. In figure 6, I compare the responses of the variables from the two models following a monetary policy shock (in both cases a one-standard-deviation shock for valid comparisons). The responses of inflation and output when lifetimes are infinite are either dampened

\[ \text{The model estimation reveals also two volatility regimes, namely a high-volatility regime and a low-volatility regime. The former dominates from the early '80s onward, while the latter dominates in the '70s. This is in line with the benchmark model. I do not display in figure 5 the estimated filtered probabilities of the volatility regimes in order to save space. The results are available though upon request.} \]
or revert back to the steady state faster than in the benchmark model, in all three regimes. This is mainly because inflation expectations are better anchored when agents have an infinite lifetime. In this case, agents discount the future less and are aware that the economy may switch to the most recurrent Regime-Pol 3 where monetary policy reacts aggressively to inflation fluctuations. As such, they expect lower inflation on average. Instead, when agents have finite lifetimes, they put more weight on the current contractionary monetary policy and less weight on the higher likelihood to stay in a regime where the central bank commits to keep inflation low (i.e., Regime-Pol 3). The fact that inflation expectations are better anchored when lifetimes are infinite can be observed more clearly...
when looking at the responses in Regime-Pol 2. The response of inflation in the benchmark model is substantially more amplified than in the estimated model with $\delta = 1$. In the benchmark model, inflation stays persistently higher because agents discount the future more than when $\delta = 1$. As such, they put less weight on the possibility of a future switch to the most recurrent Regime-Pol 3 where monetary policy reacts aggressively to inflation. Agents with infinite horizons instead place a higher weight on that event to happen in the future, which keeps their expectations well anchored.

5.2.2 No Switching in Federal Expenditure

In this section, I estimate another version of the model with the following features. I allow households to face a probability of death in their maximization problem, as in the benchmark model, but now I assume that the fiscal authority uses only tax revenues to stabilize debt. In the process for federal spending, I set the reaction to the debt ratio, $\delta_b$, equal to zero in all regimes. However, I allow the autoregressive coefficient, $\rho_{s,Z_t}$, to vary across regimes. The new process is now specified as follows:

$$\tilde{s}_t = \rho_{s,Z_t} \tilde{s}_{t-1} - (1 - \rho_{s,t}) \delta_y \hat{Y}_t + \sigma_{s,\zeta_t} \varepsilon_{s,t} \varepsilon_{s,t} \sim N(0,1). \quad (33)$$

The model estimation reveals that the U.S. economy has spent the '70s in Regime-Pol 2 where monetary policy was passive and fiscal policy was active, in line with the literature and the previous estimates. The economy now seems to switch to Regime-Pol 1 from the mid-to-late '70s until the mid-'80s. The probability of that regime increases above that of Regime-Pol 3 during that period. This could well mean that the persistently high inflation of the late '70s till the very first quarters of 1980 could still be attributed to a passive monetary policy, although fiscal policy had already started to stabilize debt. In fact the latter started to slightly increase in the mid-'70s until it started to increase abruptly in the early to mid-'80s owing to higher real rates. Contrary to the benchmark model, the economy is likely to have shifted to Regime-Pol 1 after Regime-Pol 2 earlier and is likely to have stayed there longer than what the benchmark model suggests. However, from the mid-'80s onward the probability of Regime-Pol 3 increases continuously above that of Regime-Pol 1. This indicates that the likelihood that the economy
As far as the rest of the parameters are concerned, the posterior median of the survival probability $\delta = 0.9717$ is close to the benchmark estimation. Monetary policy is reacting to inflation in a similar way as in the benchmark model in Regime-Pol 2 with a posterior median of $\phi_{pi,2} = 0.7754$. In Regime-Pol 3, although active, the coefficient on inflation is lower than in the benchmark case, with a posterior median of $\phi_{pi,2} = 1.2132$. As regards Regime-Pol 1, monetary policy is more passive in the model where federal spending does not react to debt than it is in the other two models.
5.2.3 Comparisons across Models and Key Findings

The estimation of the three models leads to three important observations. First, the data seem to support the shorter-horizon approach. Both the benchmark model and the model without switching in spending have a higher likelihood than the model with infinite planning horizons. Both models point toward similar values of the survival probability $\delta$ (9 to 10 years). As mentioned in section 3, I adopt the planning horizon interpretation of $1/(1-\delta)$, which means that in both models households have a planning horizon of 9 to 10 years. This can be considered as a plausible number of years to plan ahead.

Second, of the two models with finite lifetimes, the one with switching in federal spending to debt fluctuations fits the data better. Hence, it seems that accounting for regime switches in spending engineered by a varying response to debt improves the model fit. Third, the two models with finite lifetimes find that the economy has also spent a period in an indeterminate regime where both monetary and fiscal policy have been passive. When federal spending also reacts to debt in a time-varying fashion (benchmark model), the economy seems to have spent a very short period in the early ’80s, when both policies are passive. When federal spending does not react to debt, the model also indicates a switch to a regime where both policies have been passive in the early ’80s. However, in that very model the economy seems to have stayed longer in that regime before switching to a determinate regime where monetary policy was active and fiscal policy was passive.

Third, all models agree that the economy spent most of the ’70s in a regime where monetary policy was passive and fiscal policy was active. All models also find that the economy switched to a regime where monetary policy has been active while fiscal policy has been passive at least from the early ’80s onward.

6. Concluding Remarks

Monetary policy and fiscal policy in the United States have been widely documented to have switched over time. There have been periods during which fiscal policy has been the leading authority, while monetary policy has been accommodating (pre-Volcker era). On the other hand, there have been periods over which monetary
policy is leading (active) and fiscal policy commits to fiscal discipline (passive). Most studies have focused on tax revenues being the only or the main fiscal instrument. However, evidence shows that, while passive, fiscal policy has at times switched toward using federal expenditure instead of taxes in order to stabilize debt. One example is the continuous spending cuts during the Clinton Administration.

I have addressed the above facts using a Markov-switching DSGE model with a Blanchard-Yaari structure. The model features three regimes. I estimate the model for the United States and find that the U.S. economy experienced regime shifts in the monetary/fiscal policy mix. In particular, in line with the existing literature, I have found that monetary policy was passive during the '70s, while fiscal policy was active. The estimation of the model showed that this regime prevailed until the early '80s where the economy switched to an indeterminate regime where both policies were passive. From the mid-'80s onward, I found that the economy switched to a regime associated with an active monetary and a passive fiscal policy.

I have contributed to the current literature by allowing federal spending to also react to debt fluctuations. I provided empirical evidence motivating this approach. The estimation revealed that the U.S. government has used both tax revenues and spending almost equally, from the mid-'80s onward, in order to stabilize debt. Interestingly, though, I found that the U.S. economy spent a very short period during the early '80s in an indeterminate regime, where the U.S. government used spending heavily as a means to consolidate. Finally, by estimating alternative specifications I have shown, first, that the model with finite lifetimes fits the data better than a model with infinite lifetimes and, second, that allowing for federal spending to also react to debt fluctuations improves the performance of the model.

Appendix

A.1 The Linearized Model

The model is linearized with respect to taxes, government expenditure, and debt, whereas it is log-linearized with respect to all the other variables. I obtain a system of equations:
• Aggregate Euler equation:

\[
\hat{C}_t = \varpi_1 \hat{C}_{t-1} + \varpi_2 E_t \hat{C}_{t+1} - \varpi_3 (\hat{R}_t - E_t \pi_{t+1} + (\rho_d - 1) d_t) \\
+ \varpi_4 \left( \hat{b}_t + \hat{\mu}_{t+1} + \hat{b} \bar{Y}_t \right),
\]

(A.1)

where

\[
\varpi_1 = \kappa \left( 1 + \frac{1}{\varsigma} \cdot \frac{\sigma - 1}{\sigma} \left( (1 - \kappa) \bar{C} \right)^{1 - \sigma} \right)
\]

(A.2)

\[
\varpi_2 = \sigma - 1 \sigma + \frac{1}{\nu} (1 - \kappa)^2 \bar{C}
\]

(A.3)

\[
\varpi_3 = \frac{1 - \kappa}{\varsigma} \cdot \frac{1}{\sigma}
\]

(A.4)

\[
\varpi_4 = \frac{(1 - \kappa)(1 - \delta)}{\varsigma \nu \delta \mu} \cdot \bar{Y} \sigma
\]

(A.5)

with

\[
\varsigma = 1 + \frac{1}{\nu} \cdot \frac{\sigma - 1}{\sigma} \left( (1 - \kappa) \bar{C} \right)^{1 - \sigma} + \frac{\kappa}{\nu} (1 - \kappa)^2 \bar{C} + \frac{\kappa}{\sigma} \left( 1 - \kappa \bar{C} \right)^{1 - \sigma}
\]

(A.6)

\[
\nu = \frac{(1 - \delta)}{\delta \mu} \bar{b} \bar{Y} + (1 - \kappa) \bar{C}.
\]

(A.7)

The process for \( \hat{\mu}_t \) is specified as follows:

\[
\hat{\mu}_t = \frac{\bar{\mu} - 1}{\bar{\mu}} \times \left[ \hat{\mu}_{t+1} + \frac{\sigma - 1}{1 - \kappa} \left( (1 + \kappa) \hat{C}_t - \hat{C}_{t+1} - \kappa \hat{C}_{t-1} \right) + \Delta \hat{a}_{t+1} \right],
\]

(A.8)

where \( \bar{\mu} \) is equal to

\[
\bar{\mu} = \frac{1}{1 - \delta \beta}.
\]

(A.9)
• Phillips curve:

\[
\pi_t = \frac{1}{1 + \beta \delta} E_t \pi_{t-1} + \frac{\beta \delta}{1 + \beta \delta} E_t \pi_{t+1} + \frac{(1 - \omega)(1 - \omega \beta \delta)}{\omega (1 + \beta \delta)} \times \left( \gamma \hat{Y}_t + \sigma \hat{C}_t - \sigma \kappa \hat{C}_{t-1} - (\gamma + \sigma) \alpha_t \right) + u_t. \tag{A.10}
\]

• Government budget constraint:

\[
\tilde{b}_t = \tilde{R} \tilde{b}_{t-1} + \tilde{b} \tilde{R} (\tilde{R}_{t-1} - \pi_t - \hat{Y}_t + \hat{Y}_{t-1} - a_t) - \tilde{\tau}_t + \tilde{s}_t + \tilde{t}_p_t. \tag{A.11}
\]

• Market clearing:

\[
\hat{Y}_t = \hat{C}_t + \frac{1}{1 - g_Y} \hat{g}_t. \tag{A.12}
\]

• Monetary policy rule:

\[
\hat{R}_t = \rho_{R,Z_t} \hat{R}_{t-1} + (1 - \rho_{R,Z_t}) \left( \phi_{y,Z_t} \hat{y}_t + \phi_{\pi,Z_t} \pi_t \right) + \sigma_{R,\zeta_t} \varepsilon_{R,t}. \tag{A.13}
\]

• Ratio between government purchases and federal expenditure:

\[
\tilde{\chi}_t = \rho_{\chi} \tilde{\chi}_{t-1} + (1 - \rho_{\chi}) \tilde{y}_t + \sigma_{\chi,\zeta_t} \varepsilon_{\chi,t}. \tag{A.14}
\]

• Tax rule:

\[
\tilde{\tau}_t = \rho_{\tau,Z_t} \tilde{\tau}_{t-1} + (1 - \rho_{\tau,Z_t}) \left( \gamma_{b,Z_t} \tilde{b}_{t-1} - \gamma_{y} \hat{Y}_t \right) + \sigma_{\tau,\zeta_t} \varepsilon_{\tau,t}. \tag{A.15}
\]

• Federal spending rule:

\[
\tilde{s}_t = \rho_{s,Z_t} \tilde{s}_{t-1} + (1 - \rho_{s,Z_t}) \left( -\delta_{b,Z_t} \tilde{b}_{t-1} - \delta_{y} \hat{Y}_t \right) + \sigma_{s,\zeta_t} \varepsilon_{s,t}. \tag{A.16}
\]

• Technology:

\[
\alpha_t = \rho_\alpha \alpha_{t-1} + \sigma_{\alpha,\zeta_t} \varepsilon_{\alpha,t}. \tag{A.17}
\]
• Cost-push shock:
\[ u_t = \rho_u u_{t-1} + \sigma_{u,\zeta} \varepsilon_{u,t}. \]  
(A.18)

• Preference shock:
\[ d_t = \rho_d d_{t-1} + \sigma_{d,\zeta} \varepsilon_{d,t}. \]  
(A.19)

• Risk premium shock:
\[ \tilde{t}_p_t = \rho_{d,\tilde{t}_p} t_{p-1} + \sigma_{tp,\zeta} \varepsilon_{tp,t}. \]  
(A.20)

• Definition of \( \chi_t \):
\[ \tilde{\chi}_t = \frac{1}{g_Y - 1} \tilde{g}_t - \frac{1}{s} \tilde{s}_t. \]  
(A.21)

A.2 Steady State

The zero-inflation steady state of the model is summarized as follows. From the representative household’s labor supply decision, I have for each country that
\[ \bar{H} = \bar{w} \bar{\lambda}, \]  
(A.22)

while from the firms’ production function in each country, I have that
\[ \bar{Y} = \bar{A} \bar{H}. \]  
(A.23)

Using the demand for each good and the market clearing condition, the steady-state level of aggregate output is specified as
\[ \bar{Y} = \frac{1}{1 - g_Y} \bar{C}. \]  
(A.24)

Using the aggregate Euler equation, the steady-state gross real interest rate is equal to
\[ \bar{R} = \frac{1}{\beta} + \frac{(1 - \delta)}{\delta \beta \mu} \bar{b} \bar{Y} \bar{\lambda}^{1/\sigma}. \]  
(A.25)

Finally, from the government budget constraint, the steady-state level of debt-to-GDP ratio is specified as follows:
\[ \bar{b} = \frac{\bar{s} - \bar{\tau}}{1 - \bar{R}}. \]  
(A.26)
A.3 Deriving Aggregate Budget Constraint and Euler Equation

A.3.1 Deriving Aggregate Budget Constraint

The budget constraints of the different generations living in the economy at a given time \( t \) are given as follows:

\[
(1 - \delta) \left( P_t C_t^i + \frac{B_t^i}{R_t} \right) = (1 - \delta) \left( W_t H_t^i + \Lambda_t^i - T_t + TR_t \right)
\]

\[
(1 - \delta) \frac{\delta}{\delta} \left( P_t C_t^i + \frac{B_t^i}{R_t} \right) = (1 - \delta) \frac{\delta}{\delta} \left( W_t H_t^i + \Lambda_t^i - T_t + TR_t \right)
\]

\[
+ (1 - \delta) \frac{\delta}{\delta} B_{t-1}^i
\]

\[
(1 - \delta) \frac{\delta^2}{\delta^2} \left( P_t C_t^i + \frac{B_t^i}{R_t} \right) = (1 - \delta) \frac{\delta^2}{\delta^2} \left( W_t H_t^i + \Lambda_t^i - T_t + TR_t \right)
\]

\[
+ (1 - \delta) \frac{\delta^2}{\delta^2} B_{t-1}^i
\]

\[
(1 - \delta) \frac{\delta^3}{\delta^3} \left( P_t C_t^i + \frac{B_t^i}{R_t} \right) = (1 - \delta) \frac{\delta^3}{\delta^3} \left( W_t H_t^i + \Lambda_t^i - T_t + TR_t \right)
\]

\[
+ (1 - \delta) \frac{\delta^3}{\delta^3} B_{t-1}^i
\]

\[
\ldots
\]

Let me denote the relation between generation-specific variable \( x_t^i \) and aggregate variable \( x_t \) as follows:

\[
x_t = \sum_{i=-\infty}^{t} (1 - \delta) \delta^{t-i} x_t^i
\]

\[
x_{t-1} = \sum_{i=-\infty}^{t} (1 - \delta) \delta^{t-1-i} x_{t-1}^i,
\]

so that the aggregate budget constraint reads as follows:

\[
P_t C_t + \frac{B_t}{R_t} = B_{t-1} + W_t H_t + \Lambda_t - T_t + TR_t. \quad (A.27)
\]
A.3.2 Deriving Aggregate Euler Equation

The flow budget constraint for members of generation \(i\) is summarized as

\[
P_t C_i^t + \frac{B_i^t}{R_t} = \frac{1}{\delta} B_{i-1}^t + W_t H_t^i + \Lambda^i_t - T_t + TR_t.
\]

Define variable \(\Delta^i_t\) as

\[
\Delta^i_t = W_t H_t^i + \Lambda^i_t - T_t + TR_t. \quad (A.28)
\]

Writing the budget constraint above for periods \(t + 1, t + 2, \ldots\), I receive

\[
\begin{align*}
&\left[ P_{t+1} C_{t+1}^i + \frac{B_{t+1}^i}{R_{t+1}} - \Delta_{t+1}^i \right] \delta = B_i^t \\
&\left[ P_{t+2} C_{t+2}^i + \frac{B_{t+2}^i}{R_{t+2}} - \Delta_{t+2}^i \right] \delta = B_{t+1}^i.
\end{align*}
\]

Substituting for \(B_i^t\) in the generation \(i\)’s budget constraint in period \(t\), I receive

\[
P_t C_i^t + \frac{\delta}{R_t} P_{t+1} C_{t+1}^i + \frac{\delta}{R_t R_{t+1}} B_i^t - \frac{\delta}{R_t R_{t+1}} \Delta_{t+1}^i - \Delta_i^t = \frac{1}{\delta} B_{i-1}^t.
\]

Substituting for \(B_{i+1}^t\) in the equation above, I receive

\[
P_t C_i^t + \frac{\delta}{R_t} P_{t+1} C_{t+1}^i + \frac{\delta}{R_t R_{t+1}} \left[ P_{t+2} C_{t+2}^i + \frac{B_{t+2}^i}{R_{t+2}} - \Delta_{t+2}^i \right] \\
\times \delta - \frac{\delta}{R_t R_{t+1}} \Delta_{t+1}^i - \Delta_i^t = \frac{1}{\delta} B_{i-1}^t
\]

or

\[
P_t C_i^t + \frac{\delta^2}{R_t} P_{t+1} C_{t+1}^i \frac{\delta^2}{R_t R_{t+1}} P_{t+2} C_{t+2}^i + \frac{\delta^2}{R_t R_{t+1} R_{t+2}} B_{t+2}^i \\
- \frac{\delta^2}{R_t R_{t+1}} \Delta_{t+2}^i - \frac{\delta}{R_t R_{t+1}} \Delta_{t+1}^i - \Delta_i^t = \frac{1}{\delta} B_{i-1}^t.
\]
By iterating forward, I receive
\[
\sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} P_{t+s} C_{t+s}^i \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} - \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} \Delta^i_{t+s} + \lim_{s \to \infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} \frac{B^i_{t+s}}{R_{t+s}} = \frac{1}{\delta} B^i_{t-1}.
\]

By imposing the transversality condition,
\[
\lim_{s \to \infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} \frac{B^i_{t+s}}{R_{t+s}} = 0,
\]
I receive
\[
\sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} P_{t+s} C_{t+s}^i - \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} \Delta^i_{t+s} = \frac{1}{\delta} B^i_{t-1}. \tag{A.29}
\]

Now I need to work with the first term on the left-hand side of the equation above. Using the first-order condition (6) and iterating forward, I receive the following expression:
\[
\frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} \frac{P_{t+s}}{\lambda^i_{t+s}} = \frac{P_t}{\lambda^i_t} \left( \frac{1}{(\delta \beta)^s} \frac{e^{d_t}}{e^{d_t+s}} \right)^{-1}. \tag{A.30}
\]

Using the definition \(\lambda^i_t = (C^i_t - \kappa C_{t-1})^{-\sigma}\), equation (A.29) can be rewritten as follows:
\[
\sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} P_{t+s} C_{t+s-1} \frac{\lambda^i_{t+s}}{\lambda^i_{t+s}}^{\sigma-1} + \kappa \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} P_{t+s} C_{t+s-1} - \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} \Delta^i_{t+s} = \frac{1}{\delta} B^i_{t-1}. \tag{A.31}
\]

Substituting (A.30) into (A.31), I receive the following:
\[
\frac{P_t}{\lambda^i_t} \sum_{s=0}^{\infty} (\delta \beta)^s \left( \frac{e^{d_{t+s}}}{e^{d_t}} \right) \left( \frac{\lambda^i_{t+s}}{\lambda^i_{t+s}} \right)^{\sigma-1} + \kappa \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} P_{t+s} C_{t+s-1} - \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} \Delta^i_{t+s} = \frac{1}{\delta} B^i_{t-1}. \tag{A.32}
\]
By rearranging and aggregating, I receive

\[
\frac{P_t}{\lambda_t} = e^{d_t} \frac{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1}} \\
\left( B_{t-1} - \kappa \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} P_{t+s} C_{t+s-1} + \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s}} \Delta_{t+s} \right). \tag{A.33}
\]

Adding and subtracting \(\frac{\delta}{R_t} e^{d_t} \sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1} B_t\), I receive

\[
\frac{P_t}{\lambda_t} = e^{d_t} \frac{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1}} (B_{t-1} - \kappa P_t C_{t-1} + \Delta_t) \\
- \frac{\delta}{R_t} e^{d_t} \frac{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1}} B_t + \frac{\delta}{R_t} e^{d_t} \frac{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1}} \left( B_t - \kappa \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s+1}} P_{t+s} C_{t+s} \right. \\
\left. + \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s+1}} \Delta_{t+s+1} \right). \tag{A.34}
\]

Note that

\[
\frac{P_{t+1}}{\lambda_{t+1}} = e^{d_{t+1}} \frac{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s+1}} \lambda_{t+s+1}^{\sigma-1}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s+1}} \lambda_{t+s+1}^{\sigma-1}} \\
\times \left( B_t - \kappa \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s+1}} P_{t+s} C_{t+s} \right. \\
\left. + \sum_{s=0}^{\infty} \frac{\delta^s}{\prod_{s=0}^{s-1} R_{t+s+1}} \Delta_{t+s+1} \right). \tag{A.35}
\]
Substituting (A.35) in (A.36), I receive
\[
P_t \frac{\lambda_t}{\lambda_t} = \frac{e^{d_t}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1}}
\times \left[ B_{t-1} - \kappa P_t C_{t-1} + \Delta_t - \frac{\delta}{R_t} B_t 
+ \frac{\delta}{R_t} \sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s+1}} \lambda_{t+s+1}^{\sigma-1} \frac{P_{t+1}}{\lambda_{t+1}} \right]. \tag{A.36}
\]

Using the aggregate budget constraint (A.27) to substitute out for \( B_{t-1} \), I get
\[
P_t \frac{\lambda_t}{\lambda_t} = \frac{e^{d_t}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1}}
\times \left[ P_t C_t - \kappa P_t C_{t-1} + \frac{1 - \delta}{R_t} B_t 
+ \frac{\delta}{R_t} \sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s+1}} \lambda_{t+s+1}^{\sigma-1} \frac{P_{t+1}}{\lambda_{t+1}} \right]. \tag{A.37}
\]

Note that \( C_t - \kappa C_{t-1} = \lambda_t^{-\frac{1}{\sigma}} \). Substituting this expression into (A.38), I receive
\[
P_t \frac{\lambda_t}{\lambda_t} = \frac{e^{d_t}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \lambda_{t+s}^{\sigma-1}}
\times \left[ P_t \lambda_t^{-\frac{1}{\sigma}} + \frac{1 - \delta}{R_t} B_t 
+ \frac{\delta}{R_t} \sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s+1}} \lambda_{t+s+1}^{\sigma-1} \frac{P_{t+1}}{\lambda_{t+1}} \right]. \tag{A.38}
\]
Multiplying and dividing the first term of the right-hand side above with \( \lambda_{\cdot t}^{\sigma-1} \), I receive

\[
\frac{P_t}{\lambda_t} = \frac{e^{dt}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{dt+s} \lambda_{t+s}^{\sigma-1}}
\times \left[ \frac{P_t \lambda_t^{\sigma-1}}{\lambda_t} + \frac{1 - \delta}{\frac{R_t}{\delta}} B_t + \frac{\delta}{\frac{R_t}{\delta}} \sum_{s=0}^{\infty} (\delta \beta)^s e^{dt+s+1} \lambda_{t+s+1}^{\sigma-1} \frac{P_{t+1}}{\lambda_{t+1}} \right].
\]

(A.39)

Rearranging the equation above, I get

\[
\left[ 1 - \frac{e^{dt} \lambda_t^{\sigma-1}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{dt+s} \lambda_{t+s}^{\sigma-1}} \right] \frac{P_t}{\lambda_t} = \frac{e^{dt} \lambda_t^{\sigma-1}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{dt+s} \lambda_{t+s}^{\sigma-1}}
\times \left[ \frac{1 - \delta}{\frac{R_t}{\delta}} B_t + \frac{\delta}{\frac{R_t}{\delta}} \sum_{s=0}^{\infty} (\delta \beta)^s e^{dt+s+1} \lambda_{t+s+1}^{\sigma-1} \frac{P_{t+1}}{\lambda_{t+1}} \right].
\]

(A.40)

Multiplying and dividing the right-hand side of (A.40) by \( \lambda_{\cdot t}^{\sigma-1} \), I receive

\[
\left[ 1 - \frac{e^{dt} \lambda_t^{\sigma-1}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{dt+s} \lambda_{t+s}^{\sigma-1}} \right] \frac{P_t}{\lambda_t} = \frac{e^{dt} \lambda_t^{\sigma-1}}{\sum_{s=0}^{\infty} (\delta \beta)^s e^{dt+s} \lambda_{t+s}^{\sigma-1}}
\times \left[ \frac{1 - \delta}{\frac{R_t}{\delta}} B_t + \frac{\delta}{\frac{R_t}{\delta}} \sum_{s=0}^{\infty} (\delta \beta)^s e^{dt+s+1} \lambda_{t+s+1}^{\sigma-1} \frac{P_{t+1}}{\lambda_{t+1}} \right].
\]

(A.41)

Multiplying and dividing the last term of the right-hand side in (A.41) by \( \lambda_{t+1}^{\sigma-1} \), I receive

\[
\frac{(\mu_t - 1)}{\delta} \frac{P_t}{\lambda_t} R_t = \frac{1 - \delta}{\frac{\sigma-1}{\delta}} B_t + \frac{P_{t+1}}{\lambda_{t+1}} \frac{\mu_{t+1}}{\lambda_{t+1}},
\]

(A.42)

where

\[
\mu_t = \sum_{s=0}^{\infty} (\delta \beta)^s e^{dt+s} \lambda_{t+s}^{\sigma-1} \lambda_{t+s}^{\sigma-1}
\]
\[
\mu_t = 1 + \sum_{s=1}^{\infty} (\delta \beta)^s e^{d_{t+s}} \frac{\sigma^{-1}}{\lambda_{t+s}^{\sigma}} \\
\mu_t = 1 + e^{d_{t+1}} \delta \beta \sum_{s=0}^{\infty} (\delta \beta)^s e^{d_{t+s}} \frac{\sigma^{-1}}{\lambda_{t+s}^{\sigma}} \\
\mu_t = 1 + \frac{e^{d_{t+1}} \lambda_{t+1}^{\sigma-1}}{e^{d_t} \lambda_t^{\sigma}} \delta \beta \mu_{t+1}.
\]

(A.43)

Finally, multiplying the left-hand side and the right-hand side of (A.44) by \( \lambda_t^{\sigma-1} \) and using (A.43) to substitute out for \( \mu_t \), I receive expression (17):

\[
\beta \frac{e^{d_{t+1}}}{e^{d_t}} \frac{R_t}{\Pi_{t+1}} \frac{1}{\lambda_t} = 1 - \delta \frac{1}{\delta \mu_{t+1}} B_t + \left( \frac{\lambda_t}{\lambda_{t+1}} \right)^{\sigma-1} \frac{1}{\lambda_{t+1}}.
\]

(A.44)

References


