Narrow Banking with Modern Depository Institutions: Is There a Reason to Panic?

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What would be the effect of imposing a 100 percent reserve requirement to depository institutions? This paper contends that reserves do not compete with loans on the asset side of banks’ balance sheets. Thus, they only affect liquidity provision by banks indirectly through their impact on the cost of loan and deposit creation. This cost could be driven to zero if, as the Eurosystem does, central banks remunerated required reserves at the same rate of their refinancing operations. The paper argues that the crucial constraint imposed by a fully backed banking system is collateral availability by depository institutions.

JEL Codes: E4, E5, G21.

1. Introduction

This paper evaluates the benefits and costs of narrow banking—in particular, the suggestion that depository institutions should maintain a reserve requirement of 100 percent of their deposits. This proposition can be found in broader proposals of financial reform.

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such as the Chicago plan (see Phillips 1995 or Benes and Kumhof 2012 for excellent surveys) or the limited-purpose banking of Chamley, Kotlikoff, and Polemarchakis (2012). It is also reminiscent of the “run-free” financial system proposed by Cochrane (2014). Nowadays narrow banking has even entered into the legislative process. On June 10, 2018, Switzerland carried out a referendum to vote on this issue, and the government of Iceland is seriously considering reforming its monetary system in this direction.\footnote{In Switzerland, the narrow banking initiative was rejected by 75.7 percent of the votes (see \url{https://www.vollgeld-initiative.ch/english/}). On the Icelandic proposal see The Telegraph (2015).}

As pointed out by its advocates, the main advantage of having fully reserve-backed depository institutions would be the complete elimination of bank runs, which should significantly contribute to financial stability.\footnote{Throughout the paper, the terms “depository institutions,” “monetary financial institutions,” “commercial banks,” and, more generally, “banks” are treated indistinctly to mean any institution engaged in the business of receiving deposits which are insured as defined, for instance, by section 3 of the Federal Deposit Insurance Act of the United States.} On the other hand, critics of full reserve banking warn of the unbearable inefficiencies such system would bring to our economies. For example, Diamond and Dybvig (1986) state that

“100% reserve banking is a dangerous proposal that would do substantial damage to the economy by reducing the overall amount of liquidity. Furthermore, the proposal is likely to be ineffective in increasing stability since it will be impossible to control the institutions that will enter in the vacuum left when banks can no longer create liquidity. Fortunately, the political realities make it unlikely that this radical and imprudent proposal will be adopted.”

Similar criticisms can be found in Wallace (1996), Goodhart (1987) or Kashyap, Rajan, and Stein (2002).

The arguments put forward by both proponents and critics of narrow banking can be understood with a simplified version of the model by Diamond and Dybvig (1983).\footnote{See Freixas and Rochet (2008, chapter 7).} In that model, banks act as intermediaries between savers and investment opportunities (or borrowers). In so doing, banks obtain funds from savers in exchange...
for deposits. In principle, this deposit contract allows deposits to be fully convertible into the assets initially provided. Then, banks have the choice between a short-term, low-return, liquid investment and a long-term, high-return, illiquid investment. Clearly, as long as the bank invests some of the deposits in the long-term technology, it will not be able to serve deposit convertibility should a sufficiently large number of depositors decide to exercise that right before the long-term investment matures. The prospects that there may not be enough funds available support an inefficient equilibrium in which it is in the interest of any individual depositor to withdraw his funds given that everyone else is also running to withdraw their funds. This possibility of a bank run, and the inefficiencies associated with this form of bank instability, summarizes the main argument of supporters of 100 percent reserve systems. If all assets were invested in the liquid technology, nobody would have incentives to coordinate into a bank run.

Following the previous argument, supporters of a fractional reserve system emphasize that narrow banking will automatically separate financial institutions into two types: one offering money-like deposits but investing all their funds in low-return, short-term assets, and another one dedicated to high-return, long-term investments but unable to be used for payment purposes. For these authors, the costs associated with permanently maintaining a sizable fraction of the economy’s assets idle in an inferior investment, and/or without taking advantage of the synergies produced when the same institution provides loans and deposits simultaneously, will be enormous. These costs could be avoided with a system that lets banks produce money-like liabilities and decide on their optimal asset portfolio, but that tackles the perils of bank runs with cheaper (so they claim) alternatives such as deposit insurance or suspension of convertibility.

Despite their initial role as bank run backstops, deposit insurance and suspension of convertibility have also been criticized in the literature. Kareken and Wallace (1978) and Freeman (1988) show how deposit insurance creates a moral hazard problem for banks in that they end up taking too much risk. This prediction has been corroborated empirically by Demirguc-Kunt and Detragiache (2002) and Anginer, Demirguc-Kunt, and Zhu (2014). Furthermore, Wallace (1988) illustrates how deposit insurance is preferred to suspension of convertibility if the fraction of agents with a liquidity shock is
random. This is because deposit insurance allows for contingent allocations. Engineer (1989) demonstrates that suspension of convertibility does not prevent bank runs in longer-horizon models. Samartin (2002), however, indicates that the choice between deposit insurance and suspension of convertibility depends on the level of risk aversion, the intertemporal discount factor, or the existence of moral hazard. Finally, Bruche and Suarez (2010) point out that the presence of deposit insurance can contribute to a money market freeze in the event of increased counterparty risk.

 Needless to say, the seminal paper by Diamond and Dybvig (1983) and all the subsequent work based on this type of analysis have fundamentally improved our understanding of the role financial intermediaries play in our economies. However, this traditional view of modeling commercial banks as middlemen, obtaining assets from savers that are then channeled to borrowers, misses several important features of depository institutions we find in actual financial systems. In particular, the model developed in this paper includes the following elements that characterize a modern monetary system. First, banks create deposits when providing loans to customers. In fact, the bulk of deposits in developed economies is created this way. Second, reserves do not compete with loans in the asset side of banks’ balance sheets. Reserve demand and loan provision, although endogenously linked through the optimization problem of banks, are two magnitudes that, in principle, could be set independent of each other.

 The contribution of this paper is to show that incorporating this realistic description of the functioning of a modern monetary system produces a set of results that are at odds with those of traditional models, at least regarding the role required reserves play and their impact on economic activity and welfare. Specifically, I show that, unlike traditional models of bank intermediation, (i) reserve requirements do not directly affect liquidity provision by depository institutions to the nonfinancial sector, (ii) reserve requirements only have an indirect effect on loan and deposit creation through its impact on the borrowing–deposit rate spread, and (iii) this indirect effect can be driven to zero if required reserves are remunerated at the refinancing rate set by the central bank. According to the model, an important consequence of these results is that 100 percent reserve requirements does not automatically imply the separation of
financial institutions mentioned above. Commercial banks could go on with their long-term lending in a fully reserve-backed system as they do now with fractional reserves and take advantage of any synergies arising from combining deposit and loan production.

Although reserve requirements may not necessarily influence liquidity provision by banks, I also show that collateral regulations by central banks in their refinancing operations could effectively constrain the implementation of a fully backed deposit system. Furthermore, provided the collateral constraint is not binding, one could ask what role deposit insurance would have as the 100 percent reserve system is implemented. Below I provide some insights on how to go around this collateral constraint as well as the implications regarding the role of deposit insurance.

To elaborate on the argument, section 2 describes the main features of modern depository institutions incorporated in the model below. Section 3 presents the model. The main point of the paper can be made with a partial equilibrium model for the liquidity management problem of a bank. However, I would also like to measure the effects of imposing 100 percent reserve requirements when the conditions that isolate liquidity provision by banks from these requirements are not met. For that, the second part of section 3 closes the model, incorporating a solvency problem for banks. Section 4 characterizes the equilibrium and tests the robustness of the results of the model by discussing several extensions, including the existence of collateral regulations by central banks. Section 5 shows numerical simulations to quantitatively measure the effects of different reserve requirement schemes. Finally, section 6 concludes and discusses policy implications, implementation, and the role of deposit insurance.

2. Describing a Modern Monetary System

This section reviews some of the features of modern depository institutions not included in traditional models of banking that I believe are crucial to understand their contribution in our economies. To help fix the main idea, I start with a simple example in which I isolate a string of financial operations and follow their accounting.

Imagine I am about to make you a payment of $A$ dollars in exchange for the provision of a service or for buying a particular
good or asset. Furthermore, imagine I lack these funds so, to obtain them, I have to ask for a loan from my bank. In real life, and unlike how it is represented in traditional models of banking, if the loan is approved, my bank does not search for existing deposits to channel them to me. Instead, what my bank does is just create these deposits on the spot, out of the thin air, at the stroke of a computer key. This deposit-creation power is the distinguishing characteristic of depository institutions.\textsuperscript{4} Thus, at the time the loan is provided, my bank will have the same amount $A$ on its asset side as a loan as it has on its liability side as a deposit. These two entries will both be under my name. Now, by the time I want to dispose of my deposits to make you the promised payment, two things may happen. If you have an account in the same bank, our bank just renames my deposits under your name and the payment is made. However, if you have an account in a different bank, funds need to be transferred to that depository institution. These funds are usually reserves, that is, current accounts of depository institutions at the central bank. In general, these reserves are obtained through a loan from the central bank. Thus, imagine my bank asks for a loan equal to $A$, the amount to be transferred. Then, before the payment is made, my bank will have $A$ in reserves plus $A$ in loans on the assets and $A$ in deposits plus $A$ in borrowing from the central bank on its liability side. Next, as the payment is made, an amount $A$ of deposits and reserves is transferred from my bank to your bank. At this point, my bank will have my loan $A$ in its assets and the borrowing $A$ from the central bank in its liabilities. Your bank will have $A$ in reserves in its assets and your deposit of $A$ in its liabilities. After that, you could use your deposits to make further payments to other agents, and these deposits together with the reserves associated with them will be circulating in the economy. This way, the value in deposits each loan creates can potentially move around from agent to agent, as long as the original loan does not mature.\textsuperscript{5}

\textsuperscript{4}The accounting conventions that allow banks to create money out of nothing are described in Werner (2014a, 2014b).

\textsuperscript{5}Notice this circulation of deposits should also include deposit competition by which one bank tries to attract other banks’ deposits in order to “fund” their loans.
This example highlights a set of features of modern monetary and payment systems that are at odds with the representation provided in traditional models of banking. First, as a matter of practice, commercial banks create money, in the form of bank deposits, when making new loans. This is how the bulk of deposits we use to make payments is originated. If you could trace back the life of a deposit someone has recently transferred to you, invariably it was born with a loan to someone somewhere in the past. This view in which money is created through credit is shared by “orthodox” academics (see Goodhart 1987) as well as central bankers (see, among others, McLeay, Radia, and Thomas 2014 from the Bank of England, Holmes 1972 from the Federal Reserve Bank of New York, or Constancio 2011 from the European Central Bank), regulators (see Turner 2016), and market practitioners (see Sheard 2013). Importantly, this observation calls for the abandonment of the traditional view of banking, as it is loans, and the incentives to create credit, that originates deposits and not the other way around, as this traditional view contends.

The second challenge of the traditional view of banking is the role reserves play in the process of money creation. In the traditional view, reserves are just deposited assets that are left idle or invested in an inferior technology that allows full recovery at any time. In reality, reserves, in the form of current accounts at the central bank, are a completely different object than customers’ deposits at commercial banks or the loans these banks provide to their borrowers. Reserves are produced by the central bank, while loans and deposits are produced by commercial banks. Banks maintain reserves for two reasons. As seen above, the first reason is to satisfy depositors’ payments demand. Whenever a client wishes to make a payment to be done at another bank, this payment is usually done with reserves. The second reason is to satisfy reserve requirements wherever these requirements are in place. Thus, reserve demand is driven both by regulation and by the netting of payments derived from the loan and deposit creation to finance economic activity. Reserve supply, on the other hand, is characterized by the monetary policy stance of the central bank. This monetary policy stance is typically defined

\footnote{See McLeay, Radia, and Thomas (2014).}
as a target on very short rates (i.e., overnight) in money markets. In implementing its monetary policy, monetary authorities are usually ready to supply, at the target rate, as much reserves as depository institutions demand. When the monetary policy stance changes, the central bank modifies its interest rate target but still “reads” the amount of reserves needed to support that new target from demand by commercial banks.

An important conclusion can be drawn from the description in the previous paragraphs. In modern monetary systems there is no sense in which holding central bank reserves represents maintaining idle financing capacity or investing society’s assets in an inferior technology. Reserves are not competing with other financial investments on the asset side of banks’ balance sheets. As seen above, in normal times reserves are, in fact, a consequence of banks looking for productive loan opportunities and the deposit creation this provision of loans generates. Even without reserve requirements, banks need to hold them for payment settlement purposes. In the example above, the problem of maintaining a 100 percent reserve ratio (so that the bank kept an equal amount of reserves and deposits) was not that it made it impossible for the bank to provide loans. The problem appeared because these reserves had to be borrowed from the central bank at a cost. Thus, reserve requirements impose a tax in the banking business of loan and deposit provision. This tax should appear as a wedge in the lending–deposit rate spread and affect the efficiency of bank intermediation between borrowers and depositors. However, these costs could be driven to zero if, as the Eurosystem does, required reserves are remunerated at the same rate banks have to pay to obtain them in the first place. In such a case, reserve requirements could be set to any level, including 100 percent of deposits, without this cost channel affecting liquidity creation or how much the nonfinancial sector is financed through banks.

Another way to state the previous conclusion is that neither deposits nor reserves holdings should constrain loan production by commercial banks. Deposits are automatically produced when creating new loans. On the other hand, as explained above, reserve

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7See Keister (2016).
8Since 2008 the Federal Reserve System also remunerates required reserves to eliminate this implicit tax.
demand is determined by reserve requirements and the netting of payments, both of which depend on loan and deposit creation. Thus, it is not clear whether forcing banks to hold more reserves will directly dry out the overall amount of liquidity the nonfinancial sector uses to finance real economic activity. Reserves and deposits are two different layers of liquidity, overlapping each other, and are used by different economic agents (banks and the nonfinancial sector, respectively). To see how imposing restrictions on one layer affects the other calls for including in our models the relation between these two concepts of liquidity we find in modern financial systems.

At this point, we could now ask: if official reserves and existing deposits do not constrain bank lending and money creation, what does? Banks face several constraints, some exogenous to the banking system and some endogenous. The most obvious exogenous constraint is capital requirements. Banks have to maintain a fraction of their risky assets in the form of capital. As capital is expensive to collect, the potential to create new loans (and the money associated with them) is impaired by the amount of capital banks hold. Another limit is loan demand. To create new money, someone has to agree to take a loan. Furthermore, banks themselves constrain their lending behavior as they look for profitable opportunities, at a reasonable level of risk, to place their loans. Finally, all these constraints are also affected by monetary policy, as it influences the opportunity cost of money together with the amount of liquidity with which to fund these loans and how the interbank market distributes it among depository institutions. All these dimensions in which endogenous money is created will be present in the model below.

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9 An important element to the description in the main text is the possible connections between the amount of reserves and the amount of deposits. These connections are exemplified by the Treasury accounts at the central bank and cash holdings by the nonfinancial sector. For example, as we withdraw cash from automated teller machines, banks use their reserves to get the bank notes needed to replenish their cash machines. In normal times, however, these movements are not significant and the central bank usually accommodates them to restore the levels of reserves held previously by banks. In any case, to analyze the implications of a hypothetical situation in which the nonfinancial sector massively transforms deposits into cash, one needs a model that distinguishes between deposits and reserves as different objects, as it is done here.
Regarding monetary policy, and as pointed out above, one important element in the whole money-creation process, and in the relevance on reserve requirements in particular, is whether reserve requirements are remunerated or not, as well as the level of their remuneration. Goodfriend (2002), Keister, Martin, and McAndrews (2008), and the literature on corridor systems (see, for instance, Pérez Quirós and Rodriguez Mendizábal 2012), among others, have analyzed the effect of paying interest on reserves and how this monetary policy instrument can be used for the simultaneous control of prices and quantities in the market for daily funds. In the model presented here, the remuneration of required reserves (in fact, the difference between the rate of the central bank’s open market operations and the interest paid back to banks on required reserves) affects reserve demand in a trivial way and cannot be used to separate the control of interest rates from that of quantities. The model shows that it is the remuneration of excess reserves that matters for this separation. On the other hand, the remuneration of required reserves eliminates the distortionary taxation on liquidity creation by depository institutions, as Friedman (1959) advocated, which represents a factor altering the efficient supply of liquidity to the nonfinancial sector.

Another important item regarding how monetary policy affects money creation by banks is the collateral regulations of central banks. Typically, when banks borrow reserves from the central bank, they have to pledge enough assets as guarantees for that loan. Given existing regulations in developed countries, the overall amount of assets that are eligible to serve as collateral in the refinancing operations of central banks is, to a large extent, exogenous to the banking system. Thus, existing eligible assets could effectively constrain the amount of reserves to be supplied to the banking system and, therefore, the amount of deposits that these reserves could back up.

The endogenous view of the process of money creation developed in this paper is slowly entering mainstream economics. For

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10Nowadays, this statement could be refined. One of the consequences of the 2007–08 financial crisis has been the general expansion of the lists of eligible collateral for most central banks which include debt instruments issued by credit institutions, asset-backed securities, or nonsecurities such as loans. See Bank for International Settlements (2013) or European Central Bank (2013) for a description of these modifications observed across different central banks.
example, Disyatat (2011) uses this approach when revisiting the bank lending channel of monetary policy. His model, however, is partial equilibrium. Goodfriend and McCallum (2007) also include some form of endogenous deposit creation by banks. Their model, however, assumes binding reserve requirements as a constraint on the deposit production by banks. This is to say, the reserves the bank possesses at the time of intermediating limits the amount of loans and deposits the bank can create. As explained above, under the institutional arrangements we observe in reality, this is not the case and reserves do not constrain loan and deposit production in this way. Furthermore, the models in Jakab and Kumhof (2015) and Bianchi and Bigio (2014) are closer to the research in this paper, but there are several differences. Developing fully dynamic models, both these papers cannot explicitly find solutions and ought to rely on simulations instead. Here, I take advantage of the model being static and am able to present explicit solutions. Jakab and Kumhof (2015) compare the response to various shocks under the traditional view and the endogenous money-creation view (what they call “financing through money creation”) and conclude that the way we model bank intermediation may have significant quantitative effects. These authors argue that reserves do not constrain loan behavior by commercial banks and so they abstract from reserve demand and interbank markets. As in this paper, Bianchi and Bigio (2014) also include deposit creation through loan production together with an interbank market and monetary policy instruments such as open market operations, reserve requirements, and standing facilities. In their model, however, banks produce deposits and loans in real terms. Finally, Chari and Phelan (2014) and Benes and Kumhof (2012) use this endogenous view to explicitly evaluate the virtues of fractional reserve banking and/or fully reserve-backed deposits. Unlike my model, Chari and Phelan (2014), as Goodfriend and McCallum (2007), assume binding reserve requirements as a constraint on the deposit production by banks. On the other hand, Benes and Kumhof (2015), as Jakab and Kumhof (2015), do not solve the model explicitly and also abstract from reserve demand and interbank markets.

Compared with this recent literature, my model includes a seemingly realistic representation of payment systems which explicitly encompasses monetary policy instruments (as summarized by open market operations, standing facilities, reserve requirements, and
reserve remuneration), bidding behavior by banks at the open market operations, and net positions in the interbank market, and it links these items to the funding of decisions on real economic activity. This connection is done by solving the solvency and liquidity problems of banks in their financing of the nonfinancial sector.

3. A Model of Modern Depository Institutions

This static model is characterized by an economy consisting of a continuum of identical islands with measure 1. Each island is represented by a circle with circumference length equal to 1. On each point of the circle there is a location which includes a measure 1 of risk-averse agents. Thus, the total population of the economy is equal to 1. Each island includes a market in which agents need to make payments to buy some good or service. For the time being, I assume that agents find whatever they demand in the island they live in. Thus, no payment between agents leaves each island. I make this assumption and the ones below to simplify the exposition of the model but, as shown in section 4.2, most of them could be relaxed without affecting the main results of the paper.

The problem these agents face is that they lack the funds to make these payments. The role of banks in this model is to create deposits that serve as a medium of exchange. The following assumptions introduce some structure in the banking sector.

Assumption 1. Each island is equally divided into \( \beta \geq 2 \) regions of contiguous locations. Each region is served by a different bank.

Assumption 1 means that each bank serves a measure \( \delta = 1/\beta \leq 1/2 \) of contiguous locations. The way in which a particular bank serves the locations in its region is as follows. Whenever an agent wants to make a payment, he will ask for a loan to his local bank. Then, after the loan is made, the buyer will order the bank to transfer these funds to the account of the seller. Notice that assumption 1 introduces market segmentation in banking services. Agents can only ask for a loan to and hold deposits from the single bank servicing their location.

Next, I include two assumptions to incorporate liquidity and solvency risks in the allocation problems of banks.
Assumption 2. The fraction of loans repaid to a bank is a random variable $x \in [0, 1]$ with probability distribution $\Xi(x)$ and density $\xi(x)$. This random variable is identical and independently distributed across banks in the economy. The realization of $x$ is private information of each bank.

This assumption implies that banks face solvency risks. If the realization of $x$ is low enough, the bank becomes insolvent in that the value of its assets will be below that of its liabilities. Because this fraction of solvent loans provided by the bank is private information, there will be no insurance market among banks to insure this risk away. The way the economy deals with the solvency risk faced by banks is twofold. On the one hand, banks accumulate capital provided by investors. As owners of the bank, investors will have access to the information on the nonperforming loan rate (the realization of $1 - x$) of the bank. The existence of capital will decrease but not completely eliminate the possibility of bank failure so that depositors (sellers) still risk losing their deposits. The prospects of a bank being insolvent could trigger a bank run if deposits are transferable into cash. Notice that this bank run is not necessarily connected to illiquidity of the bank (that would depend on the liquidity provisions of the central bank, to be explained below) but on the possibility that the bank becomes insolvent. Thus, the second way to deal with banks’ solvency risk, the one faced by depositors, is for the economy to create a deposit insurance scheme.

The way the deposit insurance scheme works is as follows. The bank now has two types of liabilities. On the one hand, there is capital in the hands of investors with information about the performance of the loans made by the bank. On the other hand, there are deposits owned by agents without that information. In this regard, I assume next the existence of a costly monitoring technology to audit banks.

Assumption 3. Each individual depositor could learn the realization of $x$ for a particular bank by spending an amount of deposits equal to a fraction $\phi$ of the total value of loans of the bank. This cost is paid for with cash withdrawn from deposits and is used to pay monitors.

This setup resembles the costly state verification framework of Townsend (1979), Gale and Hellwig (1985), and Williamson (1987).
The information asymmetry between banks and depositors creates a moral hazard problem, as banks have incentives to misreport the fraction of loans defaulting and to claim not to have enough resources to pay back depositors. The optimal financial contract should be structured so as to induce the bankers to truthfully report the realization of $x$. These authors show that in this setup the optimal contract has the form of risky debt. This contract is characterized by a threshold level for $x$, call it $x_0$, that separates its support $[0, 1]$ in two sections. In section $[x, 1]$, there is no monitoring and banks make a constant repayment to depositors, independent of the realization of $x$. In section $[0, x_0)$, there is monitoring, the realization of $x$ is learned by depositors, and these depositors appropriate all assets of the bank net of monitoring costs. Clearly, having all depositors sign for this contract duplicates monitoring costs unnecessarily. Thus, a deposit insurance (DI) scheme is introduced in the economy that centralizes all bank-monitoring activities. This DI insures the face value of deposits (not the interest) and is the one negotiating the terms of the deposit contracts with the banks. The details of the relation between the DI and the depositors will be spelled out below. Notice that the optimal contract is risky debt in the relation between banks and the DI but has the form of insured deposits from the point of view of depositors.

Finally, I make the last assumption:

Assumption 4. Apart from the transfer of deposits from buyers to sellers, banks face a random net outflow of funds (deposits and reserves), equal to a random fraction $\varepsilon \in (-\infty, \infty)$ of deposits. This shock is distributed according to probability distribution $\Psi(\varepsilon)$ and corresponding density $\psi(\varepsilon)$. This random variable is identical and independently distributed across banks in the economy with mean 0 and standard distribution $\sigma_\varepsilon$.

Assumption 4 incorporates liquidity risks to the problem of banks and provides an active role for the interbank market. Banks are transferring deposits among themselves as buyers are paying sellers for their purchases. Banks settle those transfers by paying among themselves the net of the deposit flows. At this point notice that the realization of $x$ still is private information of banks. This means loans to buyers are not a valid asset to be used as general means
of payment among banks. In settling their accounts, a bank can always transfer bad loans and claim the receiver of the payment to have bad luck. Furthermore, I assume the initial assets provided by investors to be illiquid too (they could be invested in physical capital or some intangible). Thus, banks need a homogeneous, generally acceptable means of payments to be used among themselves. This is the role reserves play in the model. Banks obtain these reserves through a loan from the central bank and settle with them their payment accounts. The liquidity shock of assumption 4 causes the final reserve position of banks to be random and opens up the possibility of further trades in reserves through the interbank market. This liquidity shock could be related to deposit competition, float, or any other random event affecting the flow of funds between banks.

Along all this process, banks also have to satisfy reserve and capital requirements imposed by the central bank. These requirements will be specified below.

3.1 Timing and Balance Sheets

To see the whole flow of funds, take a general bank with size $\delta$, meaning serving a measure $\delta$ of contiguous locations. At the very beginning of period 0 this bank has no resources. Then, first, investors provide outside illiquid nominal assets in the form of equity. Let $Q$ be the amount of equity per location this bank receives from investors\textsuperscript{11} Thus, the initial balance sheet of this bank will read as shown in table 1.

Once this investment is made, the bank provides loans, $L$, to buyers on each of the locations it serves. When providing these loans, the

\textsuperscript{11}As will be clear below, all locations are identical at the time agents, banks, and investors make their decisions. For some variables, it is more convenient to work with the corresponding variable per location.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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<tbody>
<tr>
<td>Assets</td>
<td>$\delta Q$</td>
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</table>

Table 1. Balance Sheet of Bank When Portfolio Decisions Are Made by Investors at Period 0
bank makes a double entry in its books. On the asset side, the bank annotates the right associated with the loan taken by the buyer. On the other hand, deposits (i.e., means of payments) are created, and the liability side reflects the right of buyers (obligation for the bank) to dispose of those deposits to make payments. Thus, the balance sheet of the bank at the time loans are granted on period 0 is as shown in table 2.

These loans are used by buyers to pay sellers for their purchases. This means buyers order the transfer of the property of these deposits to sellers. To face the associated liquidity needs, the bank seeks liquidity at an open market operation (OMO) at the central bank. Let $M$ be the allotment of such an operation. At this point, and contrary to what happens in reality, I assume this operation is done without collateral. Below I will discuss collateral issues, as they represent the main drawback regarding implementation of 100 percent reserve requirements. Thus, the balance sheet will be as shown in table 3.

After the OMO, only the net flow of funds between banks, $FF(\varepsilon)$, exchanges hands in the form of changes in deposits and reserves. This net flow of funds will depend on the realization of the liquidity shock $\varepsilon$. The balance sheet now reads as shown in table 4.

### Table 2. Balance Sheet of Bank When Loans Are Made at Period 0

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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<tbody>
<tr>
<td>Assets</td>
<td>$\delta Q$</td>
</tr>
<tr>
<td>Loans (Buyers)</td>
<td>$\delta L$</td>
</tr>
<tr>
<td>Deposits (Buyers)</td>
<td>$\delta L$</td>
</tr>
<tr>
<td>Equity</td>
<td>$\delta Q$</td>
</tr>
</tbody>
</table>

### Table 3. Balance Sheet of Bank When the Open Market Operation Takes Place at Period 0

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves</td>
<td>$M$</td>
</tr>
<tr>
<td>Assets</td>
<td>$\delta Q$</td>
</tr>
<tr>
<td>Loans (Buyers)</td>
<td>$\delta L$</td>
</tr>
<tr>
<td>Loan from OMO</td>
<td>$M$</td>
</tr>
<tr>
<td>Deposits (Buyers)</td>
<td>$\delta L$</td>
</tr>
<tr>
<td>Equity</td>
<td>$\delta Q$</td>
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</tbody>
</table>

### Table 4.
Table 4. Balance Sheet of Bank When Transfers Are Made at Period 0

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves</td>
<td>$M + FF(\varepsilon)$</td>
</tr>
<tr>
<td>Assets (Buyers)</td>
<td>$\delta Q$</td>
</tr>
<tr>
<td>Loans (Buyers)</td>
<td>$\delta L$</td>
</tr>
<tr>
<td>Loan from OMO</td>
<td>$\delta L + FF(\varepsilon)$</td>
</tr>
<tr>
<td>Deposits (Sellers)</td>
<td>$\delta Q$</td>
</tr>
<tr>
<td>Equity</td>
<td>$\delta Q$</td>
</tr>
</tbody>
</table>

Table 5. Balance Sheet of Bank When Interbank Market Opens at Period 0

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves</td>
<td>$M - I(\varepsilon) + FF(\varepsilon)$</td>
</tr>
<tr>
<td>Assets (Interb.)</td>
<td>$\delta Q$</td>
</tr>
<tr>
<td>Loans (Interb.)</td>
<td>$I(\varepsilon)$</td>
</tr>
<tr>
<td>Loans (Buyers)</td>
<td>$\delta L$</td>
</tr>
<tr>
<td>Loan from OMO</td>
<td>$\delta L + FF(\varepsilon)$</td>
</tr>
<tr>
<td>Deposits (Sellers)</td>
<td>$\delta Q$</td>
</tr>
<tr>
<td>Equity</td>
<td>$\delta Q$</td>
</tr>
</tbody>
</table>

Once liquidity uncertainty is resolved, banks know whether they have an excess or deficit of reserves and access the interbank market to compensate it. Let $I(\varepsilon)$ be the interbank lending of the bank (borrowing if negative), which will depend upon the realization of the liquidity shock, $\varepsilon$. Thus, the balance sheet of the bank right before the solvency shock $x$ hits is as shown in table 5.

It is at this time that the solvency shock is resolved, which causes a random fraction $1 - x$ of loans in each location to return nothing. Depending on the locations each bank serves, these defaults may make some banks go bankrupt. Auditors audit bankrupt banks while solvent banks pay returns on deposits and equity. Because there is no aggregate uncertainty, the fraction of insolvent banks at the end is known at all times. Thus, since the beginning of this timing, agents know the amount of deposits to be used to pay for auditors.

Notice that $\delta L$ appears on both the asset and the liability sides of the balance sheet not because the bank is intermediating between depositors (sellers) and borrowers (buyers). In fact, when the loans were created, buyers appeared on both sides of the bank’s balance. It is the loan creation to make payments that causes different agents to be on both sides of the bank’s balance sheet. It is in this sense...
that loans create deposits and not the other way around. Also notice that interbank loans and loans to buyers are very different objects. Interbank loans are done with reserves, which are supplied by the central bank. Thus, an interbank loan does not change the size of the balance sheet of the bank. It just increases an asset category (loans in the interbank market) at the expense of the reduction in another category (reserves). In contrast, loans to agents are produced by creating deposits and, therefore, change the size of the balance sheet by increasing both an asset category (loans to buyers) and a liability category (deposits). Finally, notice that reserve holdings are used to face liquidity risks from payments while equity is used to deal with solvency risks. These two risks are related through the total amount of loans, $\delta L$, which is a decision of banks. In solving these problems, the bank will first decide on the amount of loans, $L$, considering these risks, and then will work out its implications for reserve demand. Because all banks are ex ante identical, they will all demand the same amount of reserves. After the liquidity shock realizes, some banks will be net recipients of funds and will make them available in the interbank market, while other banks will face net outflows of funds and will demand them in the interbank market.

3.2 The Liquidity Problem of Banks

3.2.1 Net Worth

As seen above, there are a number $\beta$ of ex ante identical banks evenly distributed around each island. By the time one of these banks starts functioning, it has collected a total of $\delta Q$ of assets from investors in the form of equity. Banks take these funding choices as given. At this point, the bank makes a total of $\delta L$ loans to the buyers on the locations that bank serves. As seen above, a random fraction $x$ of these loans will be paid back to the bank. Absent any other uncertainty, conditional on the realization of $x$, the ex post net worth of a solvent bank would be

$$x \left(1 + i^l\right) \delta L + \delta Q - \left(1 + i^d\right) \delta L \geq 0,$$ (1)

where $i^l$ is the interest rate on loans and $i^d$ is the interest rate on deposits.
Providing loans and deposits on the amount $\delta L$ has implications for liquidity, though, which will also affect ex post net worth. These loans $\delta L$ are initially held by buyers as deposits to be transferred to the sellers they buy things from. If all sellers lived in the very same section of the circle served by the bank providing loans to the buyers, the transfer of deposits between buyers and sellers would just be a renaming of the bank’s liabilities. However, with an island-wide market, buyers can buy goods or services from any part of the circle. Because there is a continuum of locations in each island, under the law of large numbers, on average, a fraction $\delta$ of those purchases will live in the section of the circle served by the bank and the other fraction $(1 - \delta)$ will live in outside locations served by the other banks. This means, when transferring deposits to pay for expenditures, funds equal to $(1 - \delta)\delta L$ will leave the bank. This is a fraction $(1 - \delta)$ of the total payments, $\delta L$. On the other hand, buyers in the locations not served by the bank will probably buy goods and services from sellers whose accounts are in that bank. Buyers in those outside locations will go to the other banks and ask for a loan of an average size $\hat{L}$ to pay for their purchases (so their total bill is $(1 - \delta)\hat{L}$). Then, a fraction $\delta$ of those transfers will end up in our segment of the circle. This means an inflow of funds to the bank equal to $\delta(1 - \delta)\hat{L}$.\footnote{Notice that in equilibrium $L$ and $\hat{L}$ will be equal, as all banks are identical. However, our individual bank takes $\hat{L}$ as given when making decisions.}

Thus, when considering its liquidity needs, the bank foresees it will have a total outflow of funds equal to $(1 - \delta)\delta L$ and a total inflow of funds equal to $\delta(1 - \delta)\hat{L}$. The problem is that, apart from these deterministic payments, banks face uncertain fund flows, as described in assumption 4. I take this uncertainty to be proportional to total loan creation, with this proportion governed by the random variable $\varepsilon$. With this in mind, the average change in liquidity for the bank would be

$$FF(\varepsilon) = \delta(1 - \delta) \left(\hat{L} - L\right) - \varepsilon\delta L.$$  

This change in liquidity is assumed to be permanent from the bank’s point of view. As mentioned above, banks deal with these changes in reserves by obtaining $M$ units of reserves at the central bank’s
OMO at the interest rate $i^o$. Thus, the ex post net worth of the bank (once both $x$ and $\varepsilon$ are realized) taking into account liquidity risks and reserve demand from the OMO would be

$$x (1 + i^d) \varepsilon + \delta Q - i^o M + FF(\varepsilon) - (1 + i^d) \left[ \delta L + FF(\varepsilon) \right] \geq 0.$$  

(3)

Furthermore, assume that banks have to satisfy a reserve requirement of $\rho \delta L$, where $0 \leq \rho \leq 1$. Then, the realized reserve position of the bank would be

$$M - \rho \delta L + \delta (1 - \delta) \left( \hat{L} - L \right) - \varepsilon \delta L.$$  

(4)

It is this position that the central bank checks to see whether the bank has fulfilled its reserve obligation. By inspecting (4) one can see that, given the OMO allotment, $M$, the reserve requirement, $\rho \delta L$, and the planned loan and deposit productions, $L$ and $\hat{L}$, the larger the realization of $\varepsilon$, the more likely it is for the average reserve position to be negative. In particular, for

$$-\infty \leq \varepsilon \leq \varepsilon \equiv \frac{M + \delta (1 - \delta) \left( \hat{L} - L \right)}{\delta L} - \rho$$  

(5)

the average reserve position will be positive, while it will be negative for

$$\varepsilon < \varepsilon \leq \infty.$$  

(6)

Assume the central bank offers two permanent standing facilities. First, there is a marginal lending facility in which commercial banks can obtain as much liquidity as needed at a penalty rate $i^{LF} > i^o$. Second, there is a deposit facility in which commercial banks can place any excess liquidity remunerated at the

\[\text{Notice that reserve requirements are defined over deposits created, } \delta L, \text{ but then its fulfillment includes net flows later in the period as expressed in (4). This could be interpreted as the lagged requirements typically defined by central bank regulations.}\]

\[\text{Actually, when in place, commercial banks can borrow from marginal lending facilities against the presentation of eligible collateral, as they do with the OMO.}\]
rate $i^{DF} < i^o$.\footnote{Nowadays many central banks offer explicitly such facilities with the aim of setting a corridor for overnight rates. Such corridors also implicitly exist whenever these facilities are not provided. In such cases, the rate of the discount window (plus a valuation of any nonpecuniary costs associated with its use) can provide an upper bound (corresponding to the marginal lending rate), while zero provides the lower bound (corresponding to the deposit rate).} Now assume that an economy-wide interbank market, connecting all islands, opens right after the uncertainty about $\varepsilon$ is resolved but before the realization on $x$ is known. In this market, banks with excess reserves can provide liquidity to banks with liquidity deficits. Let $I(\varepsilon)$ be the lending of the bank in the interbank market (borrowing if negative) for a particular realization of $\varepsilon$. Clearly, under no uncertainty about the liquidity position at the end of period 0, which is the case here once the value of $\varepsilon$ is known, if the return on lending in the interbank market is larger than the rate of the deposit facility, $i^{DF}$, and the cost of borrowing in the interbank market is smaller than the rate of the marginal lending facility, $i^{LF}$, it must be the case that the supply of funds in the interbank market will equal (4), that is,

$$I(\varepsilon) = M - \rho \delta L + \delta(1 - \delta) \left( \hat{L} - L \right) - \varepsilon \delta L. \quad (7)$$

Notice that a particular bank in a particular island will be a lender or a borrower in the interbank market, depending on its particular realization of $\varepsilon$ as compared with its threshold (5). With the continuum of banks producing different realizations for $\varepsilon$, there will be a continuum of liquidity positions within the interbank market. This is the purpose of modeling the physical setup of the economy with a continuum of islands.

Because there are solvency risks in this economy and because loans in the interbank market are unsecured, a bank demanding reserves will have to pay a spread or risk premium—call it $s > 0$—to lenders. With this premium, lenders are sure to obtain on average a rate $i$ on that interbank loan. This premium will be computed below once the solvency risks are determined. Thus, the condition for trade in the interbank market to happen is $i^{DF} < i < i + s < i^{LF}$.

The implications for net worth of accessing the interbank market will depend on the reserve position of the bank, which itself will
depend upon the realization of $\varepsilon$. For $\varepsilon \in (-\infty, \underline{\varepsilon}]$ the bank will have excess reserves that can be loaned out in the interbank market, obtaining a rate $i$, while for $\varepsilon \in (\varepsilon, \infty)$ the bank will have a reserve deficit that will be borrowed from the interbank market at the rate $i + s$. Then ex post net worth for a solvent bank will equal

$$x \left(1 + i^l\right) \delta L + \delta Q + FF(\varepsilon) - \left(1 + i^d\right) \left[\delta L + FF(\varepsilon)\right] - i^o M + i^\rho \rho \delta L + i I(\varepsilon; \varepsilon \leq \underline{\varepsilon}) + (i + s) I(\varepsilon; \varepsilon > \underline{\varepsilon}) \geq 0,$$

where $I(\varepsilon)$ is displayed in (7). Notice that the expression includes the possibility that the central bank remunerates required reserves at the rate $i^\rho$.

For a given realization of the fraction of entrepreneurs paying back the loan, $x$, the expected net worth with respect to the liquidity shock, $\varepsilon$, would be

$$x \left(1 + i^l\right) \delta L + \delta Q - \left(1 + i^d\right) \delta L + \Pi(L),$$

where $\Pi(L)$ represents the optimized expected profits from liquidity management which only depend on the amount of loans produced by the bank, $L$, and equals

$$\Pi(L) = \max_M - i^d \delta (1 - \delta) \left(\hat{L} - L\right) - i^o M + i^\rho \rho \delta L + i \int_{-\infty}^{\underline{\varepsilon}} I(\varepsilon) d\Psi(\varepsilon) + (i + s) \int_{\underline{\varepsilon}}^{\infty} I(\varepsilon) d\Psi(\varepsilon),$$

where $I(\varepsilon)$ is described in (7).

This setup is convenient, as it separates the liquidity problem from the solvency problem in the following way. Given a choice of loan provision by the bank, $L$, problem (8) determines the reserve demand at the central bank’s OMO together with the supply of funds in the interbank market. Once this problem is solved, $\Pi(L)$ summarizes the liquidity implications on the net worth of the bank streaming from its loan provision. The bank will then choose its loan supply in order to maximize expected net worth. These two steps are followed in the next two sections.
3.2.2 Solving the Liquidity Problem

Taking first-order conditions with respect to \( M \) in (8) implies the condition

\[
i^o = i + s \left[ 1 - \Psi(\varepsilon) \right].
\]

The intuition behind this expression is simple. The bank will demand reserves at the OMO up to the point in which the marginal cost of borrowing from the central bank \( (i^o) \) equals the expected marginal value from using these reserves later on in the period. This expected value is the interbank lending rate, \( i \), if the bank has excess reserves, which happens with probability \( \Psi(\varepsilon) \), and is the interbank borrowing rate, \( i + s \), if the bank has a reserve deficit, which happens with probability \( 1 - \Psi(\varepsilon) \). Then, using (5) produces the following demand for reserves at the OMO:

\[
M = \rho \delta L - \delta(1 - \delta) \left( \hat{L} - L \right) + \Psi^{-1} \left( \frac{i + s - i^o}{s} \right) \delta L,
\]

where \( \Psi^{-1}(.) \) represents the inverse of the distribution function of the liquidity shock. Notice that this expression determines reserve demand to be decreasing in the OMO rate, \( i^o \), and increasing in the interbank rate, \( i \), and the risk premium, \( s \). Substituting this demand in (7) yields the supply of funds in the interbank market

\[
I(\varepsilon) = (\varepsilon - \varepsilon) \delta L.
\]

Expressions (9) and (10) make evident the role of reserve requirements in the liquidity management problem of the bank. These requirements are just incorporated deterministically in the reserve demand at the OMO and do not directly affect the supply of funds in the interbank market. Substituting these expressions in the objective function (8) yields the expected profits from liquidity management, which now is only a function of the amount of loans the bank has decided to extend:

\[
\Pi(L) = (i^o - i^d)\delta(1 - \delta) \left( \hat{L} - L \right) - (i^o - i^o)\rho \delta L - s \delta L \int_\varepsilon^\infty \varepsilon d\Psi(\varepsilon).
\]
Notice that these liquidity-management profits are a linear function of loan production, \( L \), which means marginal profits per additional loan produced,

\[
\frac{\partial \Pi(L)}{\partial L} = -(i^o - i^d)\delta(1 - \delta) - (i^o - i^\rho)\rho\delta - s \int_\varepsilon^\infty \varepsilon d\Psi(\varepsilon),
\]

are a constant. Also notice the role reserve requirements, \( \rho \), play in the determination of these profits. In particular, assuming central banks do not remunerate required reserves at a rate larger than their refinancing rate, \( i^\rho \leq i^o \), required reserves have a negative impact on marginal liquidity profits. There are two ways to drive this effect to zero, though—by eliminating required reserves, \( \rho = 0 \), or by equating the remuneration of required reserves to the refinancing rate of the central bank, \( i^\rho = i^o \).

3.3 Closing the Model

3.3.1 Solving the Solvency Problem

The previous section makes it clear that if required reserves are remunerated at the same rate as the refinancing operations of the central bank, so that \( i^\rho = i^o \), reserve requirements disappear from the problem and, therefore, cannot affect loan and deposit provision by commercial banks. Therefore, unless there is another channel in which reserve requirements affect allocations, one can argue, without the need of modeling the rest of the economy, that the level of reserve requirements is completely innocuous. With this result, the main point of the paper is made. However, it may be the case that a central bank is not able or is unwilling to remunerate required reserves at that level. For that reason, this section solves the model for the general case and later presents a calibration in which the effects of changing reserve requirements are computed.

In this economy, the fraction of loans that are paid back to the bank, \( x \in [0, 1] \), is a random variable with marginal distribution denoted by \( \Xi(x) \), corresponding density \( \xi(x) \), and mean \( \mu \). Since all buyers served by this very bank are ex ante identical, they will all ask for the same loan amount and the nonperforming loan (NPL) ratio of that bank will be \( 1 - x \), which, of course, is also
random. When providing loans, banks need to satisfy the capital requirement

\[ \delta Q \geq \kappa \delta L, \]

where \( 0 < \kappa < 1 \) is a parameter determined by regulation.

As seen above, conditional on the realization of \( x \), the expected net worth of a solvent bank from the point of view of deciding the amount of loans to be produced, \( L \), would be

\[ x (1 + i^l) \delta L + \delta Q + \Pi(L) - (1 + i^d) \delta L \geq 0, \]

where \( i^l \) is the interest rate on loans, \( i^d \) is the interest rate on deposits, and \( \Pi(L) \) is the expected profits from liquidity management determined by (11). Thus, the bank will be solvent as long as the loan recovery rate satisfies

\[ x \geq \frac{(1 + i^d) \delta L - \delta Q - \Pi(L)}{(1 + i^l) \delta L} \equiv \bar{x}. \]  

Expression (12) defines the minimum loan recovery rate, \( \bar{x} \), to ensure solvency of the bank. As all banks start identical and face the same prices, the fraction of insolvent banks will be \( \Xi(\bar{x}) \).

Then, the expected net worth, per location, of a solvent bank will be

\[ \int_{\bar{x}}^{1} [x (1 + i^l) \delta L + \delta Q + \Pi(L) - (1 + i^d) \delta L] d\Xi(x). \]

Using the definition of \( \bar{x} \) in (12), the expected net worth becomes

\[ (1 + i^l) \delta L \left\{ \int_{\bar{x}}^{1} xd\Xi(x) - [1 - \Xi(\bar{x})] \bar{x} \right\} \equiv (1 + i^l) \delta L [\mu - F(\bar{x})], \]

where \( \mu \) is the mean of the distribution \( \Xi(x) \),

\[ F(\bar{x}) = \bar{x} [1 - \Xi(\bar{x})] + G(\bar{x}), \]

and

\[ G(\bar{x}) = \int_{0}^{\bar{x}} xd\Xi(x). \]
As in Bernanke, Gertler, and Gilchrist (1999), $0 \leq F(x) \leq 1$. Furthermore,

\[ F'(x) = 1 - \Xi(x) > 0; \quad F''(x) = -\xi(x) < 0 \]

\[
\lim_{x \to 0} F(x) = 0; \quad \lim_{x \to 1} F(x) = \mu \\
\lim_{x \to 0} G(x) = 0; \quad \lim_{x \to 1} G(x) = \mu.
\]

Depositors will get back their entire deposits only if the bank serving their location is solvent. Thus, they face default risk. As described before, because depositors are risk averse, a deposit insurance scheme is introduced in the economy. This DI insures the face value of deposits (not the interest) and is the one negotiating the terms of the deposit contracts with the banks. The details of the relation between the DI and the depositors will be spelled out below. When bargaining the terms of the deposit contract, I have assumed that the realization of $x$ is private information of investors (as equity holders) and banks and is not known either by the DI or the depositors (sellers). The DI can learn the true value of $x$ by hiring auditors. The nominal cost of the audit is borne by the depositors and is equal to a proportion $0 < \phi < 1$ of the amount to be audited, that is, $\phi x (1 + \delta L)$. This cost is paid for with deposits. As in Carlstrom and Fuerst (1997) or Bernanke, Gertler, and Gilchrist (1999), this information asymmetry creates a moral hazard problem, as banks have incentives to misreport the fraction of loans defaulting. The optimal financial contract should be structured so as to induce the bankers to truthfully report the realization of $x$. Carlstrom and Fuerst (1997) and Bernanke, Gertler, and Gilchrist (1999) use the findings of Townsend (1979), Gale and Hellwig (1985), and Williamson (1987) to show that in this setup the optimal contract has the form of risky debt. Then, the expected revenues for the DI, per location, will be

\[
\int_0^x \left[ (1 - \phi)x(1 + \delta L) + \delta Q + \Pi(L) \right] d\Xi(x) + \left[ 1 - \Xi(x) \right](1 + \delta L).
\]

Again, using the definition of $x$, these revenues become

\[
\delta Q + \Pi(L) + (1 + \delta L) [F(x) - \phi G(x)].
\]
In this case, notice that
\[
\lim_{x \to 0} [F(x) - \phi G(x)] = 0; \quad \lim_{x \to 1} [F(x) - \phi G(x)] = \mu (1 - \phi)
\]
and
\[
F'(x) - \phi G'(x) = 1 - \Xi(x) - \phi x \xi(x) = [1 - \Xi(x)] [1 - \phi h(x)],
\]
with \(h(x)\) defined as
\[
h(x) = \frac{x \xi(x)}{1 - \Xi(x)}.
\]
Thus, \(\mu - F(x)\) represents the fraction of expected bank revenues going to equity holders, \(F(x) - \phi G(x)\) represents the fraction of expected bank revenues going to depositors, and \(\phi G(x)\) represents the fraction of expected bank revenues remunerating auditors. Notice that
\[
[\mu - F(x)] + [F(x) - \phi G(x)] + \phi G(x) = \mu,
\]
i.e., the expected fraction of loans being repaid.

As locations are ex ante identical, we can define the optimization problem of each bank for a representative location among the ones it serves. Banks take \(Q, i^o, i^p, i^l,\) and \(i^d\) as given and choose loan supply, \(L,\) and the threshold value, \(x,\) to maximize the return for shareholders (investors) over their opportunity cost
\[
\frac{(1 + i^l) L [\mu - F(x)]}{(1 + i^f) Q},
\]
subject to the participation constraint for depositors
\[
\delta Q + \Pi(L) + (1 + i^l) \delta L [F(x) - \phi G(x)] \geq (1 + i^f) \delta L \quad (13)
\]
and the capital requirement
\[
Q \geq \kappa L. \quad (14)
\]
Here \(i^f\) is the nominal risk-free rate, which I take as exogenous and given from the point of view of both banks and the DI. Notice that
I am assuming that rate to be the opportunity cost of both equity and deposits. Because investors are diversifying their equity holdings across all banks in the economy, they will not be facing any risk from this investment.\footnote{16}

When solving this maximization problem, I will not include the constraint (14). After solving the problem, I will check whether this constraint binds or not. Let \( \lambda \) be the leverage of the bank defined as

\[
\lambda = \frac{L}{Q}.
\]

Recalling the expression for profits from liquidity management (11), these profits are

\[
\Pi(L) = (i^o - i^d)\delta(1 - \delta)\left(\hat{L} - L\right) - (i^o - i^p)\rho\delta L - s\delta L \int_\varepsilon^\infty \varepsilon d\Psi(\varepsilon).
\]

With this in mind, the problem of the bank can be rewritten as choosing leverage \( \lambda \) and the threshold \( x \) to maximize the equity premium

\[
\left(\frac{1 + i^l}{1 + i^f}\right) [\mu - F(x)] \lambda,
\]

subject to the participation constraint for the DI

\[
(1 + i^f)\lambda - (1 + i^l) (F(x) - \phi G(x)) \lambda = 1 + \pi(\lambda),
\]

where

\[
\pi(\lambda) = (i^o - i^d)(1 - \delta)\left(\frac{\hat{L}}{Q} - \lambda\right) - (i^o - i^p)\rho\lambda - s\lambda \int_{-\infty}^\varepsilon \varepsilon d\Psi(\varepsilon)
\]

are liquidity management profits per unit of bank equity. Notice that these liquidity management profits are a linear function of leverage, \( \lambda \), which means marginal profits per additional loan produced equal

\[
\frac{d\pi(\lambda)}{d\lambda} = -(i^o - i^d)(1 - \delta) - (i^o - i^p)\rho - s \int_{-\infty}^\varepsilon \varepsilon d\Psi(\varepsilon).
\]

\footnote{16}{Although in the model the risk-free rate \( i^f \) and the official rate \( i^o \) are independent, they could be made to depend on each other.}
Let $\eta$ be the Lagrange multiplier associated with (16). The first-order conditions determining bank leverage, $\lambda$, the bank solvency threshold level, $\bar{x}$, and the Lagrange multiplier, $\eta$, are, respectively,

\[
\left(\frac{1 + i_l}{1 + i_f}\right) [\mu - F(\bar{x})] = \eta \left[ 1 + i_f - \frac{d\pi(\lambda)}{d\lambda} - (1 + i_l) (F(\bar{x}) - \phi G(\bar{x})) \right],
\]

(19)

\[
F'(\bar{x}) = \eta(1 + i_f) [F'(\bar{x}) - \phi G'(\bar{x})],
\]

(20)

and

\[
(1 + i_f) \lambda = \frac{1 + \pi(\lambda)}{1 - \left(\frac{1 + i_l}{1 + i_f}\right) [F(\bar{x}) - \phi G(\bar{x})]}.
\]

(21)

To fully characterize this system, I need the following assumption, also included in Bernanke, Gertler, and Gilchrist (1999):

**Assumption 5. The hazard rate**

\[
h(x) = \frac{x\xi(x)}{1 - \Xi(x)}
\]

is a strictly increasing function of $x$.

This is a relatively weak condition satisfied by many of the distributions used in economics including the log-normal. As shown in the appendix, with this assumption, the system (19)–(21) defines a positive relationship between the cutoff value, $\bar{x}$, bank leverage, $\lambda$, the Lagrange multiplier, $\eta$, and the loan rate, $i_l$, for a given level of the official rates, $i_o$ and $i^o$. Furthermore, this relationship determines, for a given level of initial equity, $Q$, a positively sloped loan supply function

\[
L^s = Q \times \lambda.
\]
Larger loan rates \( i^l \) induce banks to increase leverage through providing more loans to buyers. Furthermore, it can be shown (see the appendix) that the deposit rate is

\[
i^d = i^f + (1 + i^l) \left[ x - F(x) + \phi G(x) \right].
\]

Also, the capital constraint (14) imposes an upper bound on leverage

\[
\lambda \leq \frac{1}{\kappa},
\]

and, therefore, for a given level of equity \( Q \), there is a limit on the amount of loans banks can provide, \( L \). Notice that this leverage bound is common to all banks. Because \( \lambda \) is a choice of each bank, and because of the agency problem between banks and the DI, limiting bank leverage, the constraint (14), or its equivalent (22), may not be binding.

Finally, given that the default probability of a bank is \( \Xi(x) \), the spread paid by these banks if they borrow in the interbank market should satisfy

\[
1 + i = [1 - \Xi(x)] (1 + i + s)
\]

or

\[
s = \frac{\Xi(x)}{1 - \Xi(x)} (1 + i).
\]

### 3.3.2 Investors

As mentioned above, in the economy there is a measure 1 of identical, risk-neutral investors. These investors are not assigned to any island in particular. They start period 0 with nominal assets \( A \), which are distributed between bank equity, \( Q \), earning a gross nominal return \( 1 + \iota^g \), or the risk-free investment, \( B \), paying interest \( i^f \), i.e.,

\[
Q + B \leq A.
\]

In general, total equity is distributed equally across all banks in the economy. Being risk neutral, investors make these portfolio decisions to maximize their net worth at period 1, \( A' \), that is,

\[
A' = (1 + \iota^g)Q + (1 + i^f)B.
\]
Because investors diversify across all banks and islands, the total return on bank equity is deterministic even though some of the banks are insolvent along the way.

3.3.3 The Deposit Insurance Scheme

The DI stands between banks and households, negotiating the terms of the deposit contract with banks and providing insurance to depositors. The DI scheme is as follows. The DI insures the face value of deposits. It collects all deposit revenues from all banks and returns $L$ to each depositor. Any excess of the deposit revenues from banks with respect to the insurance claims from depositors, after monitoring costs are paid, is transferred to all agents in the economy, independently of whether they are buyers or sellers, in a lump-sum fashion. Let this nominal lump-sum transfer be $T$.

On the one hand, notice that the DI, as well as all agents, knows how much the auditors will cost in terms of deposits. As mentioned above, the total bill to pay auditors is

$$\phi(1 + i^l)LG(x).$$

On the other hand, all agents in any location receive from the DI an aggregate revenue of

$$Q + \frac{\Pi(L)}{\delta} + (1 + i^l)L[F(x) - \phi G(x)],$$

of which $L$ is the deposit insurance, to be paid to sellers, and

$$T = Q(\delta) + \frac{\Pi(L)}{\delta} + (1 + i^l)L[F(x) - \phi G(x)] - L$$

is the lump-sum interest transfer to be paid to all agents, sellers, and buyers per location.

3.3.4 The Central Bank

The central bank is in charge of setting the policy rates, $i^o$, $i^{DF}$, $i^{LF}$, and $i^\rho$. It also determines regulation such as the capital to loan ratio, $\kappa$, and reserve requirement, $\rho$. I take these rates as parameters and look at how the equilibrium changes when the rate of the OMO,
\(i^o\), and the remuneration of required reserves, \(i^\rho\), move together with reserve requirements, \(\rho\). As mentioned above, the rates of the deposit and lending facilities satisfy

\[i^{DF} \leq i < i + s \leq i^{LF},\]

where \(i\) is the rate in the interbank market and \(s\) is the borrower risk premium. Thus, I assume commercial banks do not use the standing facilities. I also assume the central bank is ready to supply all reserves that banks demand at the official rates. Seigniorage by the central bank will then equal (per bank)

\[S(i^o, i^\rho) = i^o M - i^\rho \rho \delta L.\]

4. Equilibrium

4.1 Characterization

As mentioned above, on each island there are \(\beta \geq 2\) identical banks in the economy with sizes \(0 < \delta < 1/2\). To characterize the equilibrium of this economy, start with investors. These agents split their nominal assets, \(A\), between equity in all banks in the economy. As there is a measure 1 of locations served by these banks,

\[Q = A.\]

For all banks to be financed through equity, and therefore function, it must be the case that investors should receive the same expected return on all types of equity. Furthermore, competition in the market for bank equity will drive down the return on equity to its opportunity cost, \(i^f\). Then,

\[
\left(\frac{1 + i^f}{1 + i^f}\right) \left[\mu - F(x)\right] \lambda = 1. \tag{26}
\]

This condition, together with the first-order conditions of the banks’ solvency problem (19) through (21), provides the equilibrium loan
recovery threshold $x$, leverage $\lambda$, loan rate $i^l$, and Lagrange multiplier $\eta$ for each bank

$$
\left(\frac{1 + i^l}{1 + i^f}\right) \left[\mu - F(x)\right]
= \eta \left[1 + i^f - \frac{d\pi(\lambda)}{d\lambda} - (1 + i^l) (F(x) - \phi G(x))\right],
$$

(27)

$$
F'(x) = \eta (1 + i^f) [F'(x) - \phi G'(x)],
$$

(28)

and

$$
(1 + i^f) \lambda = \frac{1 + \pi(\lambda)}{1 - \left(\frac{1 + i^l}{1 + i^f}\right) [F(x) - \phi G(x)]}.
$$

(29)

Leverage $\lambda$, together with bank equity $Q$, generates the supply of loans to be produced in each of the locations served by each bank

$$
L^s = \lambda \times Q.
$$

Additionally, the deposit rate $i^d$ is computed from

$$
i^d = i^f + (1 + i^l) [x - F(x) + \phi G(x)].
$$

Because there is a continuum of islands, and because the liquidity shock $\varepsilon$ is idiosyncratic to each bank and island, equilibrium in the interbank market implies

$$
\int_{-\infty}^{\infty} I(\varepsilon) d\Psi(\varepsilon) = 0,
$$

where, from (10),

$$
I(\varepsilon) = (\varepsilon - \varepsilon) \delta L.
$$

Because the liquidity shock $\varepsilon$ has zero mean, market clearing then implies

$$
\varepsilon = 0,
$$
which determines the equilibrium interbank rate
\[ i = i^o - s\Psi(0) = i^o - \frac{s}{2}. \]  
(30)

Using (23), the risk spread equals
\[ s = \frac{2\Xi(x)}{2 - \Xi(x)}(1 + i^o). \]  
(31)

Substituting back this rate in (10) produces the net positions in the interbank market:
\[ I(\varepsilon) = -\varepsilon\delta L. \]  
(32)

Notice that although the interbank rate is below the official rate, \( i < i^o \) (see (30)), the rate paid by borrowers, which includes the risk premium, exceeds the official rate, i.e.,
\[ i + s = i^o + s[1 - \Psi(0)] > i^o. \]

Furthermore, substituting the equilibrium rates in (9) and the fact that all banks are identical (\( \hat{L} = L \)) produces the equilibrium bids at the OMO by banks:
\[ M = \rho\delta L, \]  
(33)

that is, the reserve requirement. At equilibrium, average nominal profits from liquidity management become (see (11) evaluated at \( \hat{L} = L \))
\[ \Pi(L) = -(i^o - i^l)\rho\delta L - s\delta L \int_0^\infty \varepsilon d\Psi(\varepsilon). \]

The first term corresponds to seigniorage by the central bank and the second to the losses associated with borrowing banks being bankrupt. Thus, of all nominal revenue generated in this economy, \( (1 + i^l)[\mu - F(x)]L = (1 + i^l)Q \) remunerates investors, \( Q + \Pi(L) + (1 + i^l)L[F(x) - \phi G(x)] = (1 + i^l)L \) is obtained by risk-averse agents, \( (1 + i^l)L\phi G(x) \) is earned by auditors, \( -(i^o - i^l)\rho L \) is collected by the central bank as seigniorage, and
\[ (1 - \mu)(1 + i^l)L + \frac{\delta(1 - \delta)}{4} \left( \frac{\Xi(x)}{2 - \Xi(x)} \right)(1 + i^o)L \]
is lost from either the loan or interbank market due to failures by entrepreneurs.

Equilibrium allocations and prices depend on the givens of the problem, namely, initial assets from investors, $A$; the monetary policy parameters, $i^o$, $i^ρ$, and $ρ$; and the risk-free rate, $i^f$. Notice that changes in investors’ initial assets, $A$, only translate into the amount of liquidity provision $L$ without affecting loan rates or any other variable in the model. On the contrary, in general, monetary policy parameters will affect general monetary conditions such as rates as they influence the solvency and/or the liquidity problem of banks.

4.2 Discussion of Model and Extensions

The characterization of the equilibrium shows how reserve requirements affect the provision of liquidity at the two layers described in section 2, namely, reserves, used by monetary financial institutions, and deposits, used by the nonfinancial sector. Regarding reserves, reserve requirements are translated directly into the bid at the OMO by all banks, $M$, and do not have a direct impact on the interbank positions of banks, $I(ε)$. These interbank positions are only affected indirectly by reserve requirements to the extent that these requirements modify deposit and loan creation, $L$, which by themselves influence reserve demand. Clearly, if excess reserves were to be remunerated instead, the central bank could change the liquidity management of the bank and will be able to move the overall demand for reserves. It is in this sense that excess reserve remuneration helps with the separation of prices and quantities in monetary policy, but only remunerating required reserves does not do the job.

Regarding the second liquidity layer, deposit and loan creation, $L$, reserve requirements only affect it through its impact on profits from liquidity management, $π(λ)$. These profits feed into the first-order conditions of the solvency problem faced by banks (expressions (27) through (29) above) and affect leverage, $λ$, the solvency ratio of banks, $Ξ(ξ)$, and the lending rate, $ι^l$. This effect is canceled, though, either if there are no reserve requirements, $ρ = 0$, or if required reserves are remunerated at the rate of the OMO, $ι^ρ = ι^o$. These two situations are different in terms of equilibrium allocations though. Not imposing reserve requirements, $ρ = 0$, supports a bank run equilibrium in which, fearing bank insolvency, agents rush to the bank to
withdraw their deposits. This situation could be resolved by imposing a deposit insurance scheme, as was done in the model. Notice that this DI scheme could be financed by taxes on deposit rates. On the contrary, if required reserves are remunerated at the OMO rate, \( i^p = i^o \), the bank could maintain a 100 percent reserve ratio and there will never be a run on a solvent bank. In the last section I discuss the role the DI plays with fully backed deposits.

As an aside, the model links these two layers of liquidity. In particular, we see that in the model reserve requirements, if imposed, are binding, as seen in expression (33). That is, reserve demand by banks, \( M \), represents a fraction of deposits created, \( \delta L \), equal to the reserve requirement, \( \rho \). However, unlike Goodfriend and McCallum (2007) or Chari and Phelan (2014), this fact does not arise from a situation in which a fixed amount of existing reserves limits the size of deposit creation by banks through a binding reserve requirement. Instead, both deposit creation and reserve demand are jointly and endogenously determined thanks, among other features, to an elastic supply of reserves at the official rate \( i^o \). In this sense, a binding reserve requirement does not necessarily impose, per se, a limit to the size of banks’ balance sheets.

Do these conclusions survive extensions of the model? First, it is easy to see that the initial assumption of separating the islands in terms of the payment streams is completely innocuous. Even if agents are making payments across islands, banks will always demand at the OMO the deterministic part of these flows, which includes reserve requirements. Thus, if these requirements are remunerated at the cost of the OMO, the result will still go through. This will also be true if we assumed banks to be risk averse instead of risk neutral. With risk aversion, banks will demand excess reserves for precautionary reasons associated with payment uncertainty but, again, still will demand the deterministic reserve requirements at the OMO. We could also dispose of the assumed market segmentation in banking payment services by allowing agents to deal with several banks as long as individual banks still faced solvency and liquidity risks associated with their loan and deposit provision.

Two possible caveats to the previous argument are related to the central bank. First, one problem of remunerating required reserves could be the elimination of one source of revenues for central banks. However, the fact that many central banks function nowadays
without reserve requirements and, therefore, without income of this sort suggests that this does not seem to be a major concern for their operations. In general, that income could be brought about through demand for excess reserves or bank notes which have nothing to do with reserve requirements.

As a second qualification, although by remunerating required reserves central banks could reduce to zero the implicit tax associated with reserve requirements, there is a channel by which these requirements can affect loan provision by depository institutions. As a whole, the banking system needs collateral to borrow reserves from the central bank. Imposing a 100 percent reserve requirement would call for the banking system to maintain eligible assets in an amount at least equal to their loan and deposit production. Because the overall supply of assets accepted as collateral is exogenous to the banking system, current collateral regulations could effectively constrain the implementation of narrow banking. The obvious way out of this situation would be to reform these collateral frameworks, expanding the set of eligible collateral. One possibility is to securitize bank loans after checking on compliance with some minimum standards and to use these asset-backed securities as collateral for the central bank’s refinancing operations.

Chailloux, Gray, and McCaughrin (2008), Bindseil (2016), and Nyborg (2017), among others, discuss the arguments on which to base the definition of the collateral framework of central banks. Relevant issues evolve around counterparty access, adverse selection, market distortions, and the like. In fact, as the recent financial crisis has shown, the collateral framework could be significantly broadened to facilitate counterparty access to refinancing operations without much concern about the financial risks central banks were assuming. Together with this list of items to consider when defining eligible collateral, this paper adds another one, namely, the possibility of backing up the deposit base of commercial banks to avoid runs.

The model is intentionally fuzzy about what the payment streams among agents in each island were, so it seems hard to grasp the consequences different reserve requirements could have on real outcomes and welfare. We could think of these payments as wage bills to be paid in advance of workers providing labor services, or payments for consumption durables or for schooling. At the end of the day, if there is a connection between the banking and the real
sectors, this has to go through the amount, \( L \), and/or the price, \( i^l \) and \( i^d \), of the financial intermediation. To the extent that an institution such as reserve requirements does not change the terms of this financing, it will be very hard to argue that it will have an impact on real outcomes even without specifying what these banks finance.

Finally, the model is static. Arguably, this may not seem an adequate assumption, as dynamics plague the financial problem of banks. On the one hand, reserve requirements are usually lagged in time. This should not be a problem, since banks will anticipate the costs from future reserve requirements associated with current loan and deposit provision. Because these requirements are weekly or monthly at most, these future requirements will barely be discounted and expressions in the current static model will be very close to those of the dynamic model. Another issue has to do with the life of a loan and the deposits it creates. As banks are making and receiving payments, the deposits they will hold and the reserves they will have to demand to meet reserve requirements will be random during the whole maturity of the loan that created the original deposits. However, with 100 percent reserve requirements, deposits will always travel across banks with their own reserves so that these requirements will be always fulfilled at zero cost. The only significant cost is the one borne by the bank originating the initial loan and deposits. As discussed, this cost could be driven to zero if \( i^p = i^o \).

5. Simulation

5.1 Calibration

The previous sections have discussed the extent to which reserve requirements affect bank liquidity provision to the nonfinancial sector. In particular, it has been shown that reserve requirements should not have any effect on real outcomes if they are remunerated at the refinancing rate of the central bank. However, with the exception of the Eurosystem, the Federal Reserve, and a few other cases, the majority of central banks do not remunerate required reserves (see Gray 2011). In this section I calibrate the model to assess quantitatively the real effects of imposing nonremunerated required reserves.

The model includes two parameters: the size of the average bank, \( \delta \), and the cost of bank monitoring, \( \phi \). Furthermore, there are two
probability distributions, namely, the distribution of the performing loan ratio for the average bank, $\Xi(x)$, and the distribution of liquidity shocks, $\Psi(\varepsilon)$. Finally, there are three policy rates (the refinancing rate of the central bank, $i^o$, the remuneration of required reserves, $i^p$, and the risk-free rate, $i^f$) together with two regulation parameters (the reserve ratio, $\rho$, and the capital requirement, $\kappa$).

To calibrate the size of the average bank and the distribution of the performing loan ratio, I use the Statistics on Depository Institutions database provided by the Federal Deposit Insurance Corporation (FDIC) and available at http://www.fdic.gov. The data are obtained from the Federal Financial Institution Examination Council (FFIEC) Consolidated Report of Condition and Income (also known as Call Reports) and the Office of Thrift Supervision (OTS) Thrift Financial Reports submitted by all FDIC-insured depository institutions. The data set spans from the last quarter of 1992 until the second quarter of 2016, which represents 95 periods of data. This data set includes information on the amount of loans and leases produced by each depository institution in the United States together with their noncurrent loans and leases, defined as the fraction of loans and leases 90 days or more past due plus loans in nonaccrual status. The size of the average bank, $\delta$, is calibrated by the time average of the share of loans and leases by each bank. The resulting value is $\delta = 0.0001$, or 0.01 percent. To set the value of the cost of bank monitoring, $\phi$, I look at the value of losses from bank failures between 1992 and 2016 as a fraction of deposits. The data are provided by the FDIC. For failed banks in this period, the losses represented approximately 22 percent of deposits. Thus, I take $\phi = 0.22$.

On the other hand, data on noncurrent loans and leases are pooled for all banks and all periods to produce the empirical counterpart for the distribution of $x$, $\Xi(x)$. This distribution is then approximated by a log-normal truncated at $x = 1$. The approximation finds the mean and standard deviation of the log-normal to match the mean and standard deviation of nonperforming loan ratios in the sample. The resulting mean and standard deviation are $\mu_x = 5.860$ and $\sigma_x = 0.36$. To check that this theoretical distribution fits the

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17 The appendix provides information about the data used in the calibration.
empirical one, I run a Kolmogorof-Smirnof test. The test cannot reject the null hypothesis of equality of distributions at the 10 percent significant level.

The capital requirement is set to $\kappa = 0.08$, as was required by the Basel I Accord, which was in place for most of the periods in the sample. In any case, in the computations done below, this requirement is never binding, as banks end up having larger capital ratios. The refinancing rate is approximated as the time average of the federal funds rate during the sample covered by the FDIC Call Reports. This average equals $i^o = 0.0278$. The risk-free rate is computed equivalently by using the three-month Treasury-bill (T-bill) rate, which produces a value of $i^f = 0.0257$. Because the exercise is to compute the effects of nonremunerated required reserves, this rate is set to $i^o = 0$ and the reserve requirement $\rho$ is set to vary between 0 and 100 percent.

Finally, the liquidity shock, $\varepsilon$, is assumed to follow a normal distribution, $\Psi(\varepsilon)$, with mean $\mu_\varepsilon = 0$. Its standard deviation is calibrated to $\sigma_\varepsilon = 0.22$ to match the average of federal funds trade as a fraction of total loans by banks. The FDIC Call Reports also include information about the net federal funds market position for each bank. I then compute the average of that position as a fraction of loan provision both for banks selling as well as for banks borrowing funds in the interbank market. The average is between 8.13 percent (for banks selling funds) and 8.64 percent (for banks borrowing funds).

5.2 Results

Figures 1 and 2 present information about leverage and interest rates produced by the model for different values for the reserve requirement ranging from $\rho = 0$ to $\rho = 1$. Figure 1 includes leverage. Moving from no reserve requirement (which computationally is equivalent to remunerating required reserves at the refinancing rate of the central bank) to 100 percent reserve requirements reduces leverage by roughly 2 percent (from 8.70 to 8.54). Interestingly, these numbers are in line with data for the United States where average leverage for the period considered equals 7.84 with a standard deviation of 0.54. As figure 2 shows, this reduction in leverage is associated with a significant increase of lending rates, from 5.67 percent with
Figure 1. Leverage ($\lambda$) as a Function of Reserve Requirement ($\rho$)

Figure 2. Lending Rate ($i^l$, solid line) and Deposit Rate ($i^d$, dashed line) as a Function of Reserve Requirement ($\rho$)
\( \rho = 0 \) to 8.54 percent with \( \rho = 1 \). Because deposit rates are basically constant at around 2.59 percent, raising reserve requirements also means increasing the spread between the two rates.

6. Concluding Comments

This paper develops a new model of banking and payment systems to evaluate the benefits and costs of imposing a 100 percent reserve requirement to depository institutions. When developing the model, care has been taken in reproducing actual institutions present in our monetary systems. In particular, broad money (deposits) is created by commercial banks and used by the nonfinancial sector of the economy to finance real activity. At the same time, narrow money (reserves) is created by the central bank, used by commercial banks to face the net payments derived from the creation of broad money, and exchanged between these banks through the interbank market. The connection between these two layers of liquidity (narrow and broad) is provided by the endogenous loan and deposit creation of commercial banks.

With this model in hand, the paper shows that, contrary to traditional banking models existing in the literature, a fully reserve-backed monetary system does not necessarily have to reduce the amount of liquidity produced by depository institutions. In particular, required reserves do not force banks to maintain deposited assets idle in an inferior investment. Required reserves only affect loan and deposit creation indirectly through it effect on the costs of liquidity provision. These costs could be driven to zero, and, therefore, isolate liquidity provision from the level of reserve requirements if required reserves were to be remunerated at the rate the central banks supply liquidity at its OMOs.

The model has been calibrated to assess the quantitative impact of changing reserve requirements whenever central banks do not remunerate required reserves. Moving from no reserve requirements to a fully backed depository system has significant effects on lending rates and, to some extent, on bank leverage.

The model presented here casts doubts on the traditional approach to narrow banking, based on the seminal paper by Diamond and Dybvig (1983), by which an increase in reserves automatically means a reduction in the lending activity of banks. It
also questions a more general argument by which narrow banking would suppress banks’ money-creation power, making credit to the private sector scarcer and more costly. In such a scenario, nonfinancial agents would need to fund their activities directly through more expensive nondemandable debt or nondebt instruments. Since, by regulation, these instruments cannot be used as money, narrow banking will also reduce liquidity in the system. As has been discussed here, both of these arguments seem to be misplaced. This is because narrow banking does not necessarily imply a reduction in the amount of intermediation processed by banks, so that neither liquidity nor credit needs to be reduced when deposits are fully backed by reserves.

Of course, this result is derived from the idea that banks create their own liabilities in the process of loan production, a feature of the banking system repeatedly reported by a part of the literature. If, instead, we stick to the traditional view, by which the asset side of banks’ balance sheets are bounded by the amount of deposits previously accumulated, this result will not go through, as in that scenario reserves will compete with loans.

The model has immediate implications regarding banking regulations and the role of the central bank as a lender of last resort. The first important implication has to do with the role of collateral. Because banks have to pledge collateral when demanding reserves at the central bank open market operation, for the narrow banking proposal to be implementable, depository institutions should maintain a stock of eligible assets at least equal to their deposit liabilities. Although this point was obviated in the model above by assuming refinancing operations of the central bank to be unsecured, clearly, this could prove to be a binding constraint in deposit and liquidity production. This is because the stock of available collateral is beyond the control of the banking system and, therefore, could fall short of the amount needed to fully back deposits with reserves. For some critics of narrow banking, this argument is described as there are not enough safe assets available to support the deposit base of the banking system as a whole. But the real constraint is not the supply of safe assets but the supply of the assets included in the collateral framework of central banks. As witnessed in the recent financial crisis, the set of eligible assets may not be limited to riskless securities if central banks decide to incorporate assets with some degree of risk.
Thus, one possible solution to this scarcity problem could be to expand the list of eligible collateral. This, however, would mean making central banks take on more risk, which they may not be willing to do even after applying the appropriate haircuts. Another possibility is to use some government-sponsored enterprise to securitize bank loans after checking on compliance with some minimum standards and use these asset-backed securities as collateral for the open market operations.

If the collateral constraints described in the previous paragraph can be overcome in the aggregate and make possible the implementation of a fully backed deposit system, a second implication of the model is that, as long as the central bank provides liquidity to needed banks, depositors do not have any incentive to coordinate in a bank run on solvent banks. This is because deposits dominate cash as a medium of exchange since it pays interest and has smaller holding costs. But liquidity provision by the central bank does not solve insolvency problems as long as the monetary authority is reluctant to take on losses of the assisted commercial banks. This means that whenever depositors cannot distinguish between solvent and insolvent banks, there exists the possibility of a bank run. If the bank turns out to be solvent, depositors will leave with their deposits converted into cash and the balance sheet of the bank will be left with their clients’ loans in the asset side and the borrowing from the central bank in the liability side. Since the bank is solvent, as long as the central bank rolls over its lending until the assets of the bank mature, the run should not imply the need for the bank to be liquidated. This is the role of the central bank as a lender of last resort. However, if the bank is insolvent, the run will leave the bank with insufficient funds to pay back the central bank reserve loan, even taking into account possible haircuts (remember, the bank is insolvent and, therefore, cannot meet all its obligations). In such a case, the deposit insurance is needed to avoid the run and guarantee that the central bank, which is senior to other obligations of depository institutions, recovers its investment.

As a third policy implication of the model, it is not clear that the existence of a deposit insurance scheme introduces a moral hazard problem with respect to the situation without it, as Kareken and Wallace (1978) and Freeman (1988) have pointed out. The reason is that the DI acts as an intermediary between banks and depositors.
However, from the point of view of the bank, its deposit liabilities have always the form of unsecured short-term debt. The DI does not change that. It only changes the agent holding that debt. The issue here is whether the existence of this type of liability introduces a moral hazard problem as compared with other types of liabilities such as equity.

Appendix

A.1 The Data Used in the Calibration

I have used several databases for the calibration and simulation exercises. First, I use the Statistics on Depository Institutions database provided by the FDIC and available at http://www.fdic.gov. The data are obtained from the FFIEC Consolidated Report of Condition and Income (also known as Call Reports) and the OTS Thrift Financial Reports submitted by all FDIC-insured depository institutions. The data set spans from the last quarter of 1992 until the second quarter of 2016, which represents 95 periods of data. From this data set I have collected the following variables (acronyms in parentheses):

- Total Assets (asset): The sum of all assets owned by the institution, including cash, loans, securities, bank premises, and other assets. This total does not include off-balance-sheet accounts.
- Net Loans and Leases (lnlsnet): Total loans and lease financing receivables minus unearned income and loan loss allowances.
- Noncurrent Loans and Leases (nclnls): Assets past due 90 days or more, plus assets placed in nonaccrual status.
- Federal Funds Sold and Reverse Repurchase (frepo): Total federal funds sold and securities purchased under agreements to resell in domestic offices. Includes only federal funds sold for TRF Reporters before March 1998.
- Federal Funds Purchased and Repurchase Agreements (frepp): Total federal funds purchased and securities sold under agreements to repurchase in domestic offices. Thrift Financial Reports include only federal funds purchased.
From the FDIC I also used bank failure data produced by the corresponding report on their webpage.

From the Federal Reserve Economic Data (FRED) database of the Federal Reserve Bank of St. Louis, I used the effective federal funds rate (FEDFUNDS) to approximate \(i^o\) and the three-month T-bill rate (TB3MS) to compute \(i^f\).

**A.2 The Loan Supply Schedule**

Here I show how to derive the properties of the loan supply schedule from expressions (19)–(21) in the text. For convenience, these expressions are reproduced here:

\[
\left(\frac{1 + i^l}{1 + i^f}\right) \left[\mu - F(x)\right] = \eta \left[1 + i^f - \frac{d\pi(\lambda)}{d\lambda} - (1 + i^l) (F(x) - \phi G(x))\right], \tag{A.1}
\]

\[
F'(x) = \eta (1 + i^f) [F'(x) - \phi G'(x)], \tag{A.2}
\]

and

\[
(1 + i^f) \lambda = \frac{1 + \pi(\lambda)}{1 - \left(\frac{1 + i^l}{1 + i^f}\right) [F(x) - \phi G(x)]}. \tag{A.3}
\]

The argument closely follows the derivations in Bernanke, Gertler, and Gilchrist (1999). First, notice that, for given rates \(i^o\), \(i^f\), and \(i^l\),

\[
F'(x) - \phi G'(x) = 1 - \Xi(x) - \phi x \xi(x) = [1 - \Xi(x)] [1 - \phi h(x)]
\]

with

\[
h(x) = \frac{x \xi(x)}{1 - \Xi(x)}
\]

being the hazard rate. By assumption 5, \(h(x)\) is a strictly increasing function of \(x\). This means there must be a level of \(x\), (call it \(x^*\))
such that $F(x) - \phi G(x)$ is maximum, which implies that leverage, $\lambda$, defined by (21) is also maximum. Thus, the relevant range to choose $x$ from should be $x \in [0, x^*]$. No bank will choose a higher cutoff level $x$ if, at the same leverage, there is a lower level for $x$ such as depositors are indifferent between the two. This is because the lower level for $x$ saves on monitoring costs.

Use (20) to define the Lagrange multiplier $\eta$ as a function of the cutoff level $x$:

$$\eta(x) = \frac{F'(x)}{(1 + i') [F'(x) - \phi G'(x)]}.$$  

Taking derivatives, we have

$$\frac{d\eta(x)}{dx} = \frac{\phi [F'(x)G''(x) - F''(x)G'(x)]}{(1 + i') [F'(x) - \phi G'(x)]} > 0$$

for all $x \in [0, x^*]$. Furthermore, notice

$$\lim_{x \to 0} \eta(x) = \frac{1}{1 + i'}; \lim_{x \to x^*} \eta(x) = \infty.$$  

Thus, expression (20) defines a one-to-one increasing mapping between the cutoff level $x$ and the Lagrange multiplier $\eta$.

Next, from (19) define the function

$$1 + i^l = R^l(x) \equiv \frac{\eta(x) \left(1 + i^f - \frac{d\pi(\lambda)}{d\lambda}\right)}{\frac{\mu - F(x)}{1 + i^f} + \eta(x) [F(x) - \phi G(x)]}.$$  

Again, it can be shown that

$$\frac{dR^l(x)}{dx} = \left[\frac{R^l(x)\eta'(x)}{(1 + i^o)\eta(x)}\right] \frac{\mu - F(x)}{\frac{\mu - F(x)}{1 + i^f} + \eta(x) [F(x) - \phi G(x)]} > 0$$

for all $x \in [0, x^*]$. Also, notice

$$\lim_{x \to 0} R^l(x) \frac{1 + i^f}{\mu}; \lim_{x \to x^*} R^l(x) = \frac{1 + i^f - \frac{d\pi(\lambda)}{d\lambda}}{\mu}.$$
Thus, expression (19), together with (20), defines a one-to-one increasing mapping between the cutoff level \( x \) and the loan rate \( i^l \). Notice that the previous expression bounds the loan rate from below and above.

Furthermore, use (21) to define the function

\[
\Lambda(x) \equiv \frac{\lambda(x)}{1 + \pi [\lambda(x)]}
\]

\[
= \frac{1}{1 + i^f} \left[ 1 + \eta(x) \left( 1 + i^f - \frac{d\pi(\lambda)}{d\lambda} \right) \frac{(F(x) - \phi G(x))}{\mu - F(x) + \eta(x) \frac{d\pi(\lambda)}{d\lambda} (F(x) - \phi G(x))} \right]
\]

\[
= \frac{\left( 1 + i^f - \frac{d\pi(\lambda)}{d\lambda} \right)}{(1 + i^f) \frac{d\pi(\lambda)}{d\lambda}} \times \left[ 1 - \frac{\mu - F(x)}{\mu - F(x) + \eta(x) \frac{d\pi(\lambda)}{d\lambda} (F(x) - \phi G(x))} \right] + \frac{1}{1 + i^f}.
\]

Notice that the left-hand side is an increasing function of \( \lambda \).

Taking derivatives, one can be shown that

\[
\Lambda'(x) = \frac{\left( 1 + i^f - \frac{d\pi(\lambda)}{d\lambda} \right)}{(1 + i^f) \frac{d\pi(\lambda)}{d\lambda}} \left[ \mu - F(x) + \eta(x) \frac{d\pi(\lambda)}{d\lambda} (F(x) - \phi G(x)) \right]^{2}
\]

\[
\times \left[ F'(x) \eta(x) \frac{d\pi(\lambda)}{d\lambda} (F(x) - \phi G(x)) \right] + (\mu - F(x)) \eta'(x) \frac{d\pi(\lambda)}{d\lambda} (F(x) - \phi G(x))
\]

\[
+ (\mu - F(x)) \eta(x) \frac{d\pi(\lambda)}{d\lambda} (F'(x) - \phi G'(x))
\]

\[
> 0
\]
for all $\bar{x} \in [0, \bar{x}^*]$ with

$$\lim_{\bar{x} \to 0} \Lambda(\bar{x}) = \frac{1}{1 + i^f}; \quad \lim_{\bar{x} \to \bar{x}^*} \Lambda(\bar{x}) = \infty.$$  

Thus, expression (21), together with (19) and (20), defines a one-to-one increasing mapping between the cutoff level $\bar{x}$ and the leverage $\lambda$.

Also, using (A.1) and (A.3), the excess return of bank equity over its opportunity cost can be written as

$$\left(\frac{1 + i^f}{1 + i^f}\right) [\mu - F(\bar{x}; \delta)]$$

$$\times \lambda = \eta \left(1 - \frac{d\pi(\lambda)}{d\lambda} \left(1 + i^f - (1 + i^f)(F(\bar{x}) - \phi G(\bar{x}))\right) \right) \left[1 + \pi(\lambda)\right].$$

Finally, using (12),

$$\bar{x} = \frac{(1 + i^d) \delta L - \delta Q - \Pi(L)}{(1 + i^d) \delta L},$$

$$1 + i^d = \bar{x} (1 + i^l) + \frac{Q}{L} + \frac{\Pi(L)}{\delta L}$$

$$= 1 + i^f + (1 + i^l) \left[\bar{x} - F(\bar{x}) + \phi G(\bar{x})\right]$$

so that the deposit rate is

$$i^d = i^f + (1 + i^l) \left[\bar{x} - F(\bar{x}) + \phi G(\bar{x})\right].$$

References


