Real Term Structure and New Keynesian Models*

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Recently some authors have argued that a New Keynesian model with simple modifications can match the nominal term structure of interest rates. In this paper, I investigate how well these models do in matching the term structure of real rates using TIPS data. I find that a standard New Keynesian model that is successful in matching nominal term structure properties cannot match real yield curve features. Then I investigate the model's relative success in fitting the nominal term structure and show that the model generates implausibly volatile inflation expectations and an implausibly high inflation risk premium to fit the nominal yield curve to compensate for the lack of fit to real yields. I study various potential extensions of the benchmark model and find that incorporating labor market frictions, long-run productivity risks, and preference shocks is not helpful in matching real term structure features.

JEL Codes: E4, E43, E44.

1. Introduction

Dynamic New Keynesian models are widely used by academics and policymakers. There are two main reasons for their popularity: They tell an internally consistent story about the dynamics of the macroeconomy and they can match the moments of the aggregate variables fairly well (e.g., Smets and Wouters 2007). These two points are used as a first-pass test for the success or failure of the models with

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the New Keynesian core, which are examined in great detail in the literature.

One of the main problems with this test is that many models fit to a small number of macroeconomic stylized facts well (Taylor and Wieland 2012, Gürkaynak and Wright 2013). Hence, these models are put to a different test: Can they match asset prices—in particular, the nominal yield curve properties? Specifically, can these models generate a time-varying and positive term premium? This is an important question because if the models are indeed successful in matching the key moments of the yield curve, then both policymakers and researchers would have an internally consistent model for the connection between the macroeconomic fundamentals and yields. Furthermore, it would be a leap forward in the macro-finance literature because, as Duffee (2013) suggests, asset pricing models are incapable of explaining this relationship. On the other hand, if these models fail to explain the connection between asset prices and the economic fundamentals, then this shows that the models are flawed in some dimension and need refining.

In recent contributions De Graeve, Emiris, and Wouters (2009), Rudebusch and Swanson (2012), and Van Binsbergen et al. (2012) show that New Keynesian models with standard extensions have the potential to match the properties of the nominal term structure. Even though these studies use different approaches, they attribute the positive nominal term premium to the combination of negative technology shocks and long-run inflation risk. The intuition behind their result is that if shocks to inflation are bad news for future consumption growth, the negative long-run covariance must be compensated by higher returns. This result is related to the empirical evidence in Piazzesi and Schneider (2007), who show that bad times are associated with low consumption growth and high inflation. They argue that consumers are worried about stagflation and they want to be compensated for holding long-term bonds. Even though Duffee (2013) and Albuquerque et al. (2016) show that the long-run consumption–inflation correlation is poorly identified, the models that try to match the nominal yield curve stick to explanations in this vein and, by doing so, match the average shape of the nominal yield curve.

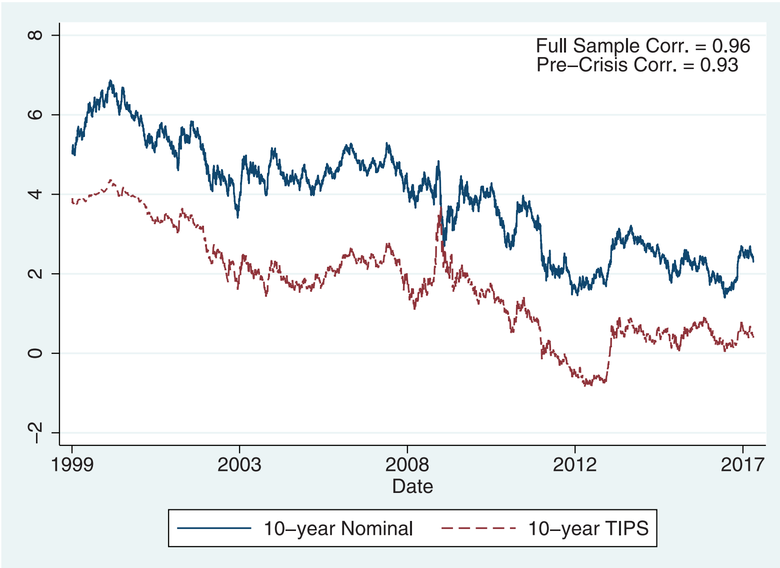
In this paper I ask whether a canonical New Keynesian model is consistent with real term structure properties or not. Specifically, I ask how well these models can match the mean and volatility of the slope, the excess returns, and the real term premiums of the TIPS (Treasury Inflation-Protected Securities) term structure. A yield curve is characterized not only by its shape but also by the properties of term premium and excess returns. Differently from the literature (e.g., Piazzesi and Scheneider 2007, Swanson 2016) the focus of this paper is not only the average shape of the real yield curve or the volatility of the real yields, but on top of these moments, I focus on the size and volatility of real term premium and the excess returns as well.

First, I show that a standard New Keynesian model with Epstein-Zin preferences and long-run inflation risk (following Rudebusch and Swanson 2012) under standard calibration has difficulty in jointly matching macroeconomic, nominal, and real term structure properties for the United States. Then I estimate the benchmark model and show that the model requires a high coefficient of relative risk aversion (CRRA) and a low intertemporal elasticity of substitution (IES) to match nominal term structure moments and to get the model-implied real term structure moments closer to data. Even then the model is unable to match the real term structure features.

Armed with this result, I investigate why New Keynesian models are relatively more successful in matching the nominal term structure. I show that the fit to nominal term structure moments comes at the cost of unreasonably volatile long-term inflation expectations and high inflation risk premiums. Since the model is incapable of generating enough real risk, inflation risk has to be high to make sure that nominal bonds are risky. Finally, I search for extensions that have the potential to increase quantity of real risk. Specifically, I incorporate labor market frictions, long-run real risk, and preference shocks in turn and show that these extensions are not helpful in matching real term structure features.

This failure of the canonical macro-finance model in explaining ex ante real rates is worrisome, especially given that the correlation between 10-year nominal and real zero-coupon bond yields in the United States is more than 90 percent (as shown in figure 1). Moreover, there is high-frequency evidence that monetary policy and macroeconomic announcements generate movement in the real term structure that accounts for most of the variation in nominal yield curve in the United States (Beechey and Wright 2009, Hanson and

Figure 1. Ten-Year TIPS and Nominal Yield Correlation



Stein 2015). At lower frequencies, there is similar evidence that the variation in nominal bond yields can be partly explained by the variation in the real rates (Pflueger and Viceira 2011; Haubrich, Penacchi, and Ritchken 2012; Duffee 2018). For these empirical results to be reconciled, either expected real rates or the real risk premiums (or both) needs to be volatile. Because having volatile long-term real rate expectations is not plausible (the Blue Chip survey implies that the long-term real rate expectations are very close to 2 percent at all times), almost all variation in real yields should be a result of the volatility of the real risk premiums. Hence, we would like our macro models to capture the dynamics of real risk premiums, but this paper shows that they do not.

This question also has policy implications because monetary policy works through real interest rates. Analyzing this relationship in a structural model will enable researchers to have a better understanding of monetary policy transmission mechanism through its impact on the yield curve, particularly the real yield curve. Given the high-frequency evidence (Hanson and Stein 2015, for instance), a successful model should explain why monetary policy has effects on the real term premium on top of its effects on average expected real short rates.

The main goal of this paper is to show that even though the literature made some headway in making models compatible with nominal yield curve moments, it seems that implausible expected inflation and inflation risk premium dynamics are the crucial components of these results. The usual mechanism that would help the model to match the upward-sloping nominal yield curve would imply a downward-sloping real yield curve. In an economy with well-anchored long-run inflation expectations, economic fundamentals will affect real short- and long-term yields and, in turn, long-term nominal bond yields. If these models cannot match the real bond yields, then we should not be comfortable with their success in matching the nominal yield curve, as a major component must be misspecified. In this sense, this paper is a call for action for further study of the macroeconomic drivers of yield curve dynamics.

This paper is organized as follows. Section 2 presents the benchmark model; section 3 calculates the U.S. TIPS yield curve moments that the model is required to match. In section 4 I take the model to the data and show that this model is not capable of reproducing real yield curve properties. In section 5 I explore the reasons for the model's inability to fit the real term structure and ask why the model is more successful in matching the nominal term structure. Section 6 considers different extensions of the benchmark model and examines their implication on the real term structure. In this section I show that these extensions are not helpful either. Section 7 concludes.

2. The Benchmark Model

The benchmark model closely follows Rudebusch and Swanson (2012), which is the generalization of the dynamic New Keynesian model of Woodford (2003) with Epstein and Zin (1989) preferences and a time-varying inflation target. Rudebusch and Swanson (2012) show that a model with such characteristics can match nominal term structure properties. An important reason why I chose a small-scale New Keynesian model is to be able to clearly examine the role of nonlinearities and the contribution of various mechanisms to fit term structure properties.[1] Without properly understanding why the models are unsuccessful in matching the real term structure properties, it will not be possible to build better models.

2.1 Households

The representative agent maximizes the discounted lifetime expected utility by choosing consumption, c_t, and labor, l_t. The maximand of the lifetime expected utility is denoted by V_t and given by

$$V_t \equiv \; u(c_t, l_t) + \beta E_t V_{t+1}, \tag{1}$$

where $u(c_t, l_t)$ is the instantaneous utility at time t and β is the discount factor.

An important ingredient of this model is Epstein-Zin, or recursive, preferences. This specification allows for the separation between intertemporal elasticity of substitution and the coefficient of relative risk aversion. The former parameter governs smoothing over time and the latter parameter governs smoothing over states. There is no reason for these two parameters to be tightly linked to each other like expected utility suggests. Standard New Keynesian models that use expected utility specification need risk aversion to be very high to match risk premiums (see Rudebusch and Swanson 2008), implying very low intertemporal elasticity of substitution. If the IES is very low, this implies that consumption is too smooth and short-term real interest rates are too volatile. By generalizing the preferences, I can change the coefficient of relative risk aversion to match the real term structure moments and keep the intertemporal elasticity

[1] There are medium-scale models in the literature that try to reconcile macroeconomic dynamics and the properties of nominal/real yield curves. For instance, Hsu, Li, and Palomino (2016) aim to match the average excess returns and real/nominal, and first and second moments of macroeconomic variables by approximating the model to a second order. Dew-Becker (2014) has a medium-scale dynamic stochastic general equilibrium (DSGE) model (similar to Smets and Wouters 2007) with time-varying relative risk aversion that tries to match the nominal yield curve moments. In his model, risk aversion and labor-neutral technology shocks are crucial for the nominal term structure. The latter is important essentially because it generates inflation risk. In such a model, the real term structure would be downward sloping. Therefore, it is not clear whether a medium-scale DSGE model would help with matching the term structure of real rates given that stagflation risk is the main driver in these models.

of substitution parameter the same, which is crucial for matching macro moments.

Following Rudebusch and Swanson (2012), the recursive utility specification is given by

$$V_t \equiv u(c_t, l_t) + \beta(E_t V_{t+1}^{1-\alpha})^{1/1-\alpha}, \tag{2}$$

where the period utility function is given by

$$u(c_t, l_t) \equiv \frac{c_t^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-l_t)^{1-\chi}}{1-\chi}, \tag{3}$$

where φ, χ, and χ_0 are positive.[2] In this specification, intertemporal elasticity of substitution is given by $1/\varphi$ and Frisch labor supply elasticity is given by $(1-l)/\chi l$, where l is the steady-state labor supply. When α is 0, recursive utility specification reduces to expected utility specification.

Households maximize the Epstein-Zin functional with respect to a flow budget constraint, given by

$$p_t a_t + P_t c_t = w_t l_t + d_t + p_t a_{t-1}, \tag{4}$$

where p_t is the price of the real bond at time t, a_t is the amount of the asset that the household chooses to hold, w_t is the wage rate, d_t is the lump-sum transfer from the firms owned by the households, and P_t is the aggregate price index at time t.[3]

The optimality conditions are given by

$$-\frac{\chi_0(1-l_t)^{-\chi}}{c_t^{-\varphi}} = \frac{w_t}{P_t} \tag{5}$$

$$c_t{}^{-\varphi} = \beta E_t \left\{ (E_t V_{t+1}^{1-\alpha})^{\alpha/1-\alpha} V_{t+1}^{-\alpha} c_{t+1}^{-\varphi} \frac{p_{t+1}}{p_t} \frac{P_t}{P_{t+1}} \right\}. \tag{6}$$

[2]Rudebusch and Swanson (2012) add a stochastic trend for productivity to make the preferences consistent with the balanced growth path. Moreover, they use the stochastic trend in their model of long-run real risk. Having this feature of the model is not crucial for the results that will be presented below.

[3]All the variables in this model are state contingent. For notational simplicity the states are dropped from the equations.

Further manipulation of the Euler equation gives the nominal stochastic discount factor from period t to $t+1$:

$$m^{nom}_{t,t+1} \equiv \left(\frac{V_{t+1}}{(E_t V^{1-\alpha}_{t+1})^{1/1-\alpha}}\right)^{-\alpha} \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\varphi} \frac{P_t}{P_{t+1}}. \tag{7}$$

Since the nominal stochastic discount factor is a combination of the real stochastic discount factor and one-period-ahead inflation, we can rewrite it as

$$m^{nom}_{t,t+1} = \frac{m^{real}_{t,t+1}}{\pi_{t+1}}, \tag{8}$$

where $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the one-period-ahead inflation and

$$m^{real}_{t,t+1} \equiv \left(\frac{V_{t+1}}{(E_t V^{1-\alpha}_{t+1})^{1/1-\alpha}}\right)^{-\alpha} \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\varphi}. \tag{9}$$

The first term in the stochastic discount factor implies that news at $t+1$ about consumption in $c_{t+2}, c_{t+3}, \ldots$ affects marginal utility of c_{t+1} relative to marginal utility of c_t, where all of the marginal utilities are with respect to agent's time-t lifetime utility function V_t.[4]

The key parameter that I am interested in is the coefficient of relative risk aversion, given its implications for asset prices. Following Rudebusch and Swanson (2012) and Swanson (2012), I define the "effective" coefficient of relative risk aversion, which includes endogenous labor choice, as

$$CRRA = \frac{\varphi}{1 + \frac{\varphi}{\chi}\frac{1-l}{l}} + \alpha \frac{1-\varphi}{1 + \frac{1-\varphi}{1-\chi}\frac{1-l}{l}}. \tag{10}$$

Note that the coefficient of relative risk aversion is no longer the inverse of intertemporal elasticity of substitution due to the

[4]The net effect of good news on the discount factor is ambiguous: Due to intertemporal consumption smoothing, this positive shock raises the marginal utility of c_{t+1} and this effect increases with φ. On the other hand, because of intratemporal risk aversion, a high certainty equivalent term lowers marginal utility of c_{t+1}. The net effect of good news on the discount factor depends on the parameter values of φ and α. If $|\alpha| > |\varphi|$, then positive news about future consumption growth is a negative shock to the real stochastic discount factor. If the inequality is reversed, the relationship between consumption growth and the real discount factor is reversed as well.

recursive utility specification. This "effective" coefficient of relative risk aversion is inversely related with labor supply since the household can endogenously change its labor supply to hedge against unexpected changes in its income in a production economy. On the other hand, in an endowment economy, households have fixed labor supply and their consumption is equal to their endowment. As Swanson (2012) shows, in a production economy the endogenous choice of labor supply changes how one defines coefficient of relative risk aversion. Furthermore, this new definition of the risk-aversion parameter helps to match the equity premium. Thus the "effective" coefficient of relative risk aversion will be our focus below.

2.2 Firms

Following Woodford (2003), I assume that the economy contains a continuum of monopolistically competitive intermediate-goods firms indexed by f that set prices according to Calvo contracts. Firms hire labor from households, and capital is firm specific. Woodford (2003, chapter 5, subsection 3) shows that a model with firm-specific fixed capital and a model with endogenous capital stock and investment adjustment costs have very similar business cycle dynamics.[5] I follow this specification because adding endogenous capital with investment adjustment costs would make estimation computationally inefficient (Rudebusch and Swanson 2012). Therefore, I assume that $k_t(f) = \bar{k}$.

All the firms in the economy have identical Cobb-Douglas production functions. The production function of a given firm f is given by

$$y_t(f) = A_t k_t(f)^{1-\eta}(l_t(f))^{\eta}, \tag{11}$$

where A_t is a stationary aggregate productivity shock that affects all the firms and follows an AR(1) process:

$$\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon_t^A, \tag{12}$$

where $|\rho_A| < 1$ and $\varepsilon_A \sim N(0, \sigma_A^2)$.

[5] Altig et al. (2011) show that firm-specific capital stock does help to generate empirically relevant inflation persistence, which is important in matching the standard deviation of inflation.

The prices are set by Calvo contracts, and the duration of these contracts are determined by ξ. The firm chooses a new price $p_t(f)$ when it is its turn to update the prices. These prices are set such that they would maximize the expected discounted profits of the monopolistic firm. The objective function of this optimization problem is given by

$$E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j}^{nom} [p_t(f) y_{t+j}(f) - w_{t+j} l_{t+j}(f)], \tag{13}$$

where $m_{t,t+j}^{nom}$ is the nominal discount factor from period t to $t+j$.

The firm's optimality condition is given by

$$p_t(f) = \frac{(1+\theta) E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j}^{nom} mc_{t+j}(f) y_{t+j}(f)}{E_t \sum_{j=0}^{\infty} \xi^j m_{t,t+j}^{nom} y_{t+j}(f)}. \tag{14}$$

The marginal cost of the firm f is given by

$$mc_t(f) = \frac{w_t l_t(f)}{\eta y_t(f)}. \tag{15}$$

2.3 Nominal and Real Bonds

The price of a default-free n-period nominal zero-coupon bond that pays one dollar at maturity satisfies

$$p_{nom,t}^{(n)} = E_t [m_{t,t+1}^{nom} p_{nom,t+1}^{(n-1)}]. \tag{16}$$

In a model with a nominal stochastic discount factor, one can price any asset, including real bonds. Using the real stochastic discount factor and the no-arbitrage condition, we can price a real bond that pays one unit of consumption good:

$$p_{real,t}^{(n)} = E_t [m_{t,t+1}^{real} p_{real,t+1}^{(n-1)}]. \tag{17}$$

Then the continuously compounded nominal and real yield to maturity on an n-period nominal and real zero-coupon bond are given as[6]

$$y_t^{(n)} = -\frac{1}{n}\log p_{nom,t}^{(n)} \tag{18}$$

$$r_t^{(n)} = -\frac{1}{n}\log p_{real,t}^{(n)}, \tag{19}$$

where $y_t^{(n)}$ is the n-period nominal zero-coupon bond and $r_t^{(n)}$ is the n-period real zero-coupon bond.

I calculate the n-period term premium as the difference between the n-period yield and average expected short rates:

$$\psi_{nom,t}^{(n)} = y_t^{(n)} - \frac{1}{n}E_t\left(\sum_{j=0}^{n-1} i_{t+j}\right) \tag{20}$$

$$\psi_{real,t}^{(n)} = r_t^{(n)} - \frac{1}{n}E_t\left(\sum_{j=0}^{n-1} r_{t+j}^{(1)}\right), \tag{21}$$

where $\psi_{nom,t}^{(n)}$ is the nominal term premium and $\psi_{real,t}^{(n)}$ is the real term premium.[7] Differently from the affine term structure literature, this definition of real and nominal term premium includes Jensen's inequality term as a part of the real term premium. Empirically, the inequality term is too small to make a difference in the model's predictions below (see D'Amico, Kim, and Wei 2018).

2.4 Aggregation and Resource Constraints

Aggregate output and price index is given by

$$Y_t = \left[\int_0^1 y_t(f)^{1/1+\theta} df\right]^{1+\theta} \tag{22}$$

[6]This definition of the term premium is consistent with Rudebusch and Swanson (2012) and Swanson (2016), where in those studies term premium is calculated as the difference between the risk-neutral yield and yield to maturity.

[7]This definition of nominal and real term premium is consistent with Rudebusch and Swanson (2012).

$$P_t = \left[\int_0^1 p_t(f)^{-1/\theta} df\right]^{-\theta}. \tag{23}$$

Next I define cross-sectional price dispersion in this economy. This is useful since one of the channels through which inflation affects real economy is by creating a dispersion of output. Introducing cross-sectional dispersion into the model helps the model to match the business cycle moments better. Following Rudebusch and Swanson (2012), I define cross-sectional dispersion as

$$\Delta_t^{1/\eta} = (1-\xi)\sum_{j=0}^{\infty} \xi^j p_{t-j}(f)^{-(1+\theta)/\theta\eta}. \tag{24}$$

Aggregate labor in the economy is

$$L_t = \int_0^1 l_t(f) df. \tag{25}$$

Then the aggregate production function is

$$Y_t = \Delta_t^{-1} A_t K_t^{1-\eta}(Z_t L_t)^{\eta}, \tag{26}$$

where $K_t = \bar{k}$ is the aggregate capital stock.

I assume a fiscal authority that levies lump-sum taxes G_t on households and destroys the resource it collects. Government consumption follows a stationary AR(1) process:

$$\log(G_t/\bar{G}) = \rho_G \log(G_{t-1}/\bar{G}) + \varepsilon_t^G, \tag{27}$$

where $\varepsilon_G \sim N(0, \sigma_G^2)$.

Then the aggregate resource constraint becomes

$$Y_t = C_t + I_t + G_t, \tag{28}$$

where $I_t = \bar{k}(1-\delta)$, where δ is depreciation.

To close the model, I have to specify how the nominal interest rates are determined in the economy. There is a monetary authority that sets the short-term nominal interest rates following a Taylor rule:

$$\begin{aligned} i_t = \rho_i i_{t-1} + (1-\rho_i)[r^* + \log\overline{\pi}_t + g_y((Y_t - Y^*/Y^*)) \\ + g_\pi(\log\overline{\pi}_t - \log\pi_t^*)] + \varepsilon_t^i, \end{aligned} \tag{29}$$

where r^* denotes the steady-state real interest rate, Y^* is the steady-state level of output, π_t^* is the long-run inflation, and ε_t^i is an iid monetary policy shock with variance σ_i^2. The variable $\overline{\pi}_t$ denotes a geometric moving average given by

$$\log \overline{\pi}_t = \theta_\pi \log \overline{\pi}_{t-1} + (1 - \theta_\pi) \log \pi_t, \tag{30}$$

where inflation is $\pi_t = P_t/P_{t-1}$ and $\theta_\pi = 0.7$ so the geometric average has a duration of four quarters. I assume that the central bank has a time-varying inflation target, which has the following specification (Rudebusch and Swanson 2012):

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \nu_{\pi^*} (\bar{\pi}_t - \pi_t^*) + \varepsilon_t^*, \tag{31}$$

where $\varepsilon_{\pi^*} \sim N(0, \sigma_{\pi^*}^2)$. Gürkaynak, Sack, and Swanson (2005) show that this specification is crucial in matching the high-frequency responses of nominal bond yields to macroeconomic news and monetary policy announcements.

Finally, the ex ante short-term real interest rate is given by

$$E_t(r_{t+1}) = i_t - E_t(\pi_{t+1}). \tag{32}$$

3. Macroeconomic, Nominal, and Real Yield Curve Moments

A natural question is whether the benchmark model presented in the previous section can jointly match the properties of macroeconomic variables and the term structure of nominal and real interest rates. To answer this question, I calibrate and estimate the model to generate unconditional moments from the benchmark model and compare them with their empirical counterparts.

The macroeconomic moments that I am going to match are the second moments of consumption, hours worked, real wage, inflation, and nominal and real short-term interest rates.[8] I use the Edge, Gürkaynak, and Kısacıkoğlu (2013) data set for macroeconomic moments.[9] Standard deviations for consumption, hours worked, and

[8]In this model, consumption and output have similar dynamics.

[9]The details of the macroeconomic data used in this paper can be found in the data appendix of Edge, Gürkaynak, and Kısacıkoğlu (2013).

real wage were computed for the (quarterly) deviations from the Hodrick-Prescott trend. For inflation and nominal and real short-term rates, the (annualized) standard deviations are calculated from the level of these variables.[10] Moments for macro variables are calculated using quarterly data from 1985 to 2007.

I calculate the mean and the standard deviation of the nominal term premium using the end-of-quarter values of nominal term premium estimates provided by Adrian, Crump, and Moench (2013). The mean and the standard deviation of the 10-year nominal bond, three-month nominal excess returns, and the slope of the nominal yield curve is calculated using the Gürkaynak, Sack, and Wright (2007) data set. I use end-of-quarter values of nominal yields to calculate these moments. The sample for nominal term structure moments is 1985–2007.

The real yield curve moments that I am interested in are the first and second moments of slope of the TIPS term structure, one-period real excess returns, and the estimates of the real term premium, which is estimated using a vector autoregression in real yield curve factors (more on this below). TIPS moments are calculated using end-of-quarter TIPS yields from 1999 to 2007.[11]

In order to be able calculate the real term premium, the slope of the real yield curve, and real excess returns, I have to measure short-term ex ante real rates (i.e., three-month TIPS yields), which do not exist.[12] Hence I measure the short-term real rate as the short-term nominal bond yield minus the one-quarter-ahead headline consumer price index (CPI) forecast from the Survey of Professional Forecasters.[13]

[10]Here the short-term nominal rate is the three-month nominal Treasury-bill yield. The federal funds rate and three-month nominal Treasury-bill yields have a correlation around 99 percent. From the model's perspective, these are indistinguishable. I follow Rudebusch and Swanson (2012) and use Treasury-bill yields.

[11]All the results presented below are similar if I used a 1999–2007 sample for macroeconomic and nominal yield curve moments.

[12]The reason why Treasury never issues real bills is because the indexation lag would be overwhelmingly important.

[13]Unfortunately, core CPI forecasts in the Survey of Professional Forecasters do not go back to 1999. So I used headline CPI forecasts instead. However, using headline CPI forecasts makes the implied short rates consistent with the TIPS since they are indexed by the headline CPI.

Using one-period ex ante real rates and one-period nominal interest rates, I can compute nominal and real excess returns:

$$x_{nom,t}^{(n)} = ln\left(\frac{p_{nom,t}^{(n-1)}}{p_{nom,t-1}^{(n)}}\right) - y_{t-1}^{(1)} \tag{33}$$

$$x_{real,t}^{(n)} = ln\left(\frac{p_{real,t}^{(n-1)}}{p_{real,t-1}^{(n)}}\right) - r_{t-1}^{(1)}, \tag{34}$$

where $y_{t-1}^{(1)} = i_{t-1}$.

One reasonable concern about the TIPS yield curve moments is the liquidity premium associated with the TIPS. Gürkaynak, Sack, and Wright (2010), Pflueger and Viceira (2011), and D'Amico, Kim, and Wei (2018) show that when TIPS were first introduced, the yields had a substantial liquidity premium, but it decreased as the market for TIPS grew. Moreover, they show that the liquidity premium increased again during the Great Recession. Pflueger and Viceira (2011) attribute this increase to high uncertainty in the economy and the disruption of the financial system. Since I am not explicitly modeling the liquidity premium, I have to adjust the TIPS yields accordingly. I follow Gürkaynak, Sack, and Wright (2010) to estimate the liquidity premium and use the 10-year TIPS liquidity-adjusted measure.[14]

3.1 Estimating the Real Term Premium

As equation (21) shows, there are two ingredients that are needed to calculate the real term premiums: 10-year TIPS yields and the average expected future short-term real interest rates. I follow the methodology of Joslin, Le, and Singleton (2013) to construct average expected real short rates.[15] Subtracting average expected real short-term rates from the liquidity-adjusted 10-year TIPS gives the real term premium, which has an average of 1.3 percent. This estimate of the real term premium is in line with the literature. Kim and Wright (2005) find the real term premium to be 1.4 percent

[14] The details can be found in the online appendix; see http://www.ijcb.org.
[15] The details can be found in the online appendix.

on average, whereas Haubrich, Penacchi, and Ritchken (2012) and D'Amico, Kim, and Wei (2018) find it to be 1.25 percent and 2 percent on average, respectively.[16]

Although this paper focuses only on the U.S. real term structure, the United States is not the only country that issues index-linked bonds. A prominent example is the United Kingdom, a country that has been issuing index-linked bonds since 1983, a longer time span than other countries (and the United States) with similar markets. The slope of the U.K. index gilt yield curve in different time periods has been subject to debate (Anderson and Sleath 2001), where differences in sample periods may play a crucial role in the differences among yield curve slope estimates. Evans (1998) estimates the zero curve to be downward sloping for 1983–95, whereas Bank of England data shows that for the 1985–95 sample, the curve is, on average, upward sloping (average slope is around 50 basis points).[17] However, after 1997 the U.K. real yield curve is, on average, downward sloping (consistent with the evidence of Piazzesi and Schneider 2007), with a slope of –30 basis points.

Some studies in the literature attribute the reversal in the shape of the U.K. real yield curve to the pension fund reforms undertaken by the United Kingdom, which increased the demand for real bonds by the pension funds and decreased real yields through preferred-habitat effects. This point has been brought up by the Bank of England May 1999 Inflation Report, McGrath and Windle (2006), Shen (2006), Campbell, Shiller, and Viciera (2009), Joyce, Lildtholt, and Sorensen (2010), Vayanos and Vila (2010), and Andreasen (2012),

[16]For comparison, a model-free estimate of the 10-year real term premiums can be calculated using survey forecasts. One can subtract 5-by-5-year real interest rate forecasts from the 5-by-5-year real forward rates to gauge the magnitude of the real term premiums. To do so, I used the median Blue Chip long-term (6 to 10 years) nominal short rate and inflation forecasts to calculate long-term real rate forecasts, which I use as a proxy for average expected real rates in 5 to 10 years. After correcting for the liquidity premium, which is found to be around 40 basis points on average in the literature (D'Amico, Kim, and Wei 2018 and Gürkaynak, Sack, and Wright 2010), the model-free risk premium is around 50 basis points for the United States. This naive estimate shows that a positive real term premium survives in the data.

[17]Swanson (2016) shows that the U.K. real yield curve is slightly upward sloping (34 basis points) for the 1985–2017 sample and almost flat (slope with 1 basis point) for the 1990–2007 sample.

among others.[18] However, the points brought up by the literature are about the average slope of the U.K. index gilt yield curve, implying that the market could be sufficiently segmented in the past 20 years to generate a downward-sloping real yield curve. Moreover, the literature finds a sizable and volatile real term premium in the United Kingdom (Joyce, Lildtholt, and Sorensen 2010, Andreasen 2012, and Abrahams et al. 2016). The uncertainty about the estimates of the real yield curve slope justifies a focus on other important features of the real term structure such as excess returns and the real term premium. Even if one were to take the average slope to be negative and find that the baseline model matches this, as will be evident below for the United States, the model grossly misses the other main moments (such as size and the volatility of the real term premium and excess returns[19]) that should be matched to understand the indexed yield

[18] Pension fund reform in the United Kingdom proceeded in two steps. The first step was in 1995 (effective April 1997) and the second one was in 2004. Vayanos and Vila (2010) point out that the funding requirements instated with the Pension Act of 1995 coincided with accounting reforms, implying higher demand from pension funds for long-term U.K. bonds, including inflation-indexed bonds. The Bank of England's February 1999 Inflation Report mentions that "institutional factors, such as the minimum funding requirement for pension funds, may also have put upward pressure on the price of UK index-linked bonds by raising demand relative to supply" (p. 9). Joyce, Lildtholt and Sorensen (2010) mention the act in 1995 as a major contributor to the fall in the longer-term real yields in the United Kingdom due to an increase in the pension fund demand after the minimum funding requirements. Campbell, Shiller, and Viceira (2009) indicate that pension reforms in the United Kingdom had effects on the real term structure.

For the Pension Act of 2004, Vayanos and Vila (2010) do an event-study analysis where they show that after the reform, cumulative net purchases of long-term bonds by the pension funds went up substantially compared with equities and short-term assets. Higher demand from the pension funds made the 10-year/3-year spread negative and the real yield curve downward sloping. Similarly, Joyce, Lildtholt, and Sorensen (2010) stress that there are reasons to believe that the index-linked gilt market in the United Kingdom might have become more segmented between 2005 and 2007 due to the Pension Act of 2004, which replaced the minimum funding requirement implemented in the Pension Act of 1995. They show that this segmentation might result with lower long-term yields than short-term yields. Similar ideas are explored in McGrath and Windle (2006), where they show that both the supply and the demand to index-linked gilts increased; however, pension funds rebalanced their portfolios towards index-linked gilts, which created a demand much higher than the supply.

[19] Pflueger and Viceira (2016) show that real excess returns for the United Kingdom are as volatile as real excess returns for the United States.

curve.[20] A successful model is expected to generate a sizable real term premium (and real excess returns) with enough volatility in these features of the real term structure. Therefore, the discussions below will mostly focus on the properties of the real term premium.

There are recent studies for two of the other index-linked bond issuers, Australia and France, which show that in these countries the indexed-linked bond yield curve is upward sloping, on average, with a positive real risk premium. Hambur and Finlay (2018) estimate the zero-coupon real term structure for Australia and show that the average yield curve is slightly upward sloping, with a slope (10-year minus 2-year) of 23 basis points. They show that the 10-year real term premium was mostly positive in Australia between 1997 and 2011, then became negative after 2011. For France, Hördahl and Tristani (2014) show that the average real term premium is around 150 basis points for the sample 1999–2006. Though not conclusive, there is some evidence that (at least for these countries) the real term premium is positive on average, implying an upward-sloping real yield curve on average.[21]

4. Taking the Model to Data

The baseline calibration is reported in table 1 and is very standard in the literature. I set the discount factor β to 0.99 (consistent with 4 percent annualized real interest rates), the depreciation rate δ to 0.02, the share of government spending in output at the steady state ($\frac{\bar{G}}{\bar{Y}}$) to 17 percent, and the capital-output ratio at the steady state ($\frac{\bar{K}}{4\bar{Y}}$) to 2.5 (Rabanal and Rubio-Ramirez 2005, Smets and Wouters 2007, Rudebusch and Swanson 2012, and Swanson 2016).

Following Rudebusch and Swanson (2012) and Del Negro, Giannoni, and Schorfheide (2015), χ_0 is chosen such that at the steady

[20] To make this point more clear, I calculated the mean (2.66 percent) and the standard deviation (12.86 percent) of the U.K. excess returns using zero-coupon index-linked gilt yields and asked the model to match it. The model cannot match the size and the volatility of the real excess returns. Therefore, the results below are robust to the United Kingdom.

[21] Obviously there are many other countries that issue index-linked bonds. Unfortunately these data are neither public (or do not have studies analyzing the properties) nor have a long enough time series to be able to make meaningful analysis.

Table 1. Baseline Calibration

β	0.99	χ_0	1.224	η	2/3	ρ_i	0.73	σ_a	0.005
δ	0.02	χ	3	θ_π	0.2	g_π	0.53	σ_g	0.004
$\frac{\bar{G}}{\bar{Y}}$	0.17	ϕ	0.5	ξ	0.75	g_y	0.93	σ_i	0.003
$\frac{\bar{K}}{4\bar{Y}}$	2.5	γ	75	π^*	0	$\rho_{\pi*}$	0.99	$\nu_{\pi*}$	0.01
$\sigma_{\pi*}$	0.0005								

state, hours worked is equal to one-third of the time endowment. I choose χ to have Frisch elasticity of 2/3, consistent with microeconomic estimates (see Chetty et al. 2011 for a survey). The intertemporal elasticity of substitution, ϕ, is calibrated to 0.5, which is consistent with the micro-evidence (see Havránek 2015 for meta-analysis). Following Rudebusch and Swanson (2012), the coefficient of relative risk aversion, γ, is calibrated to 75.

Firms' elasticity with respect to labor η is set to 2/3 and firms' steady-state markup θ_π is set to 0.2, which are standard in the literature. Calvo contract duration ξ is set to 0.75, implying a duration of four quarters, similar to the estimates of Rabanal and Rubio-Ramirez (2005), Altig et al. (2011), Del Negro, Giannoni, and Schorfheide (2015), and Swanson (2016).

The monetary policy rule coefficients ρ_i, g_π, and g_y are taken from Rudebusch (2002). I normalize the steady-state inflation rate π^* to 0. Both technology shock persistence ρ_a and government spending shock persistence ρ_g are set to 0.95, whereas shock standard deviations σ_a, σ_g, and σ_i are set to 0.005, 0.004, and 0.003, respectively, consistent with the estimates in Rudebusch (2002) and Smets and Wouters (2007). For the long-term inflation, following Rudebusch and Swanson (2012), I set $\nu_{\pi^*} = 0.01$, $\rho_{\pi^*} = 0.99$, and $\sigma_{\pi^*} = 5$ basis points.

Since the model is highly nonlinear, I use the Perturbation AIM (Swanson, Anderson, and Levin 2005) algorithm to solve and estimate the benchmark model.[22] I compute the model-implied moments for the nominal and the real term premium by approximating the model up to the third order around the nonstochastic

[22] The Perturbation AIM package can be found at Eric Swanson's website: http://www.socsci.uci.edu/~swanson2/perturbation.html.

steady state. In the first- and second-order approximations of the model, the nominal and real term premium is either zero or constant, respectively. Using a third-order approximation gives a time-varying and sizable term premium because third-order approximation incorporates endogenous conditional hetereoskedasticity to the nominal and the real stochastic discount factor. In other words, this approximation generates time-varying quantity of risk.[23]

Results of the calibration exercise are given in the second column of table 2. The model can match macroeconomics moments well, which is not surprising. This is an important reason why New Keynesian models are widely used. Under calibrated values, the benchmark model can generate a sizable and somewhat volatile positive nominal term premium and an upward-sloping average nominal yield curve. Model-implied mean nominal excess returns are close to their empirical counterpart, whereas the standard deviation is high but still away from the data. The positive nominal term premium in the model is generated by negative technology shocks, and with Epstein-Zin preferences and a time-varying long-run inflation target, the effects of these shocks are magnified. A negative technology shock increases inflation and marginal utility of consumption, which in turn decreases long-term nominal bond prices. Since low nominal bond prices coincide with high marginal utility, nominal bonds command a positive risk premium. For the nominal term premium to be large, the covariance needs to be sufficiently positive over the life of the bond. This long-run positive covariance is generated by the time-varying inflation target. For a positive ν_{π^*}, a negative technology shock raises inflation and, in turn, long-term inflation. This mechanism generates positive covariance between marginal utility and inflation in the long run, increasing riskiness of long-term bonds, implying higher risk compensation. Epstein-Zin preferences play a crucial role in this model by making high marginal utility in the future being priced in the nominal bond yields today. Therefore, the model can generate a high nominal risk premium, and

[23]There is a large body of evidence that the expectations hypothesis fails for nominal bonds (see Campbell and Shiller 1991 and Cochrane and Piazzesi 2005, among many others), implying a time-varying and sizable nominal term premium. Recently, similar evidence emerged in the literature for real bonds (Pflueger and Viceira 2011).

Table 2. Macroeconomic, Nominal, and Real Term Structure Moments Implied by the Estimated Benchmark New Keynesian Model

	Data	Baseline	Best Fit
sd(C)	0.79	1.63	0.81
sd(L)	1.29	1.79	2.29
sd(w^r)	1.02	1.44	1.34
sd(π)	0.93	2.81	3.59
sd(i)	1.97	2.68	4.07
sd($r^{(3)}$)	1.51	1.17	2.08
sd($y^{(40)}$)	1.73	1.88	2.78
sd($r^{(40)}$)	0.68	0.25	0.90
mean($\psi_{nom}^{(40)}$)	1.93	0.56	1.89
sd($\psi_{nom}^{(40)}$)	1.01	0.32	0.72
mean($y^{(40)} - i$)	1.77	0.50	1.68
sd($y^{(40)} - i$)	1.28	1.01	1.63
mean($x_{nom}^{(40)}$)	1.67	0.99	3.27
sd($x_{nom}^{(40)}$)	26.77	15.14	29.21
mean($\psi_{real}^{(40)}$)	1.32	−0.02	0.33
sd($\psi_{real}^{(40)}$)	0.42	0.02	0.17
mean($r^{(40)} - r^{(3)}$)	1.32	−0.10	0.10
sd($r^{(40)} - r^{(3)}$)	1.13	1.15	1.93
mean($x_{real}^{(40)}$)	2.90	0.06	0.87
sd($x_{real}^{(40)}$)	14.20	0.97	6.52
IES		0.5	0.07
CRRA		75	125
Frisch		0.66	0.23
ξ		0.75	0.71
ρ_A		0.95	0.95
σ_A		0.005	0.005
$\nu_{\pi*}$		0.01	0.014
$\rho_{\pi*}$		0.99	0.96
$\sigma_{\pi*}$		0.0005	0.0008

nominal excess returns and a positive slope for the nominal term structure.

However, the benchmark model is not as successful in matching the real term structure. The model-implied real term premium is negative, with a standard deviation of nearly 0. The model also does badly in matching other real term structure moments. The model-implied average real yield curve is slightly downward sloping. Real excess returns are far too small and way too stable compared with their empirical counterparts. From the model's perspective, these results are intuitive. Bad news about current consumption is also bad news for future consumption. This bad news about future consumption increases the real stochastic discount factor over the course of the bond's maturity. Since real rates are low in a bad state, the price of the real bond will be high. So the bond pays well when consumption growth is low, or when the real stochastic discount factor is high due to high future marginal utility. Thus the long-term investors see these bonds as a hedge and are willing to pay more to have these bonds.

Next I ask if there is a different parameter combination that would match the nominal and real term structure moments without sacrificing the fit of the macroeconomic moments. To answer this question, I find the values that minimize the following generalized method of moments objective function:

$$\widehat{\Phi} = \underset{\Phi}{\operatorname{argmin}}[\Psi(\Phi) - \Psi_D]'\Sigma_D^{-1}[\Psi(\Phi) - \Psi_D], \tag{35}$$

where $\Psi(\Phi)$ is the vector of unconditional first and second moments computed from the model, Ψ_D is the vector of unconditional first and second moments of the data, and Σ_D^{-1} is the weighting matrix.[24] I search the parameter values for intertemporal elasticity of substitution (IES), risk aversion, Frisch labor supply elasticity, Calvo contract duration, technology shock persistence, and variance (I vary the monetary policy rule parameters in subsection 6.1).[25] Varying

[24]I follow Rudebusch and Swanson (2012) and divide the weight of nominal and real excess returns by 10. I use equal weights for all other moments.

[25]Treating these parameters as calibrated and varying them makes a difference in the fit, but does not improve over the estimated parameters.

other parameters—in particular, the steady-state markup, standard deviations to shocks to government spending, persistence in government spending, and the geometric average parameter given in equation (27)—do not make a consequential change for the fit to the real term structure.[26] The third column of table 2 gives the results for the model-implied moments and the parameter values that are needed to match the data.

Estimation results show that macroeconomic and nominal term structure moments are matched fairly well. The benchmark model performs better in matching the real term structure moments than the baseline calibration. But the fit is still poor; the model matches the sign of the averages but not the magnitudes. The model-implied real term premium is low and stable compared with the data. The slope of the real term structure is positive, but the real yield curve is too flat compared with the data. Real expected excess returns are too low and too stable compared with their empirical counterparts. To jointly match macroeconomic, nominal, and real yield curve moments, the model requires a high coefficient of relative risk aversion (125) and low intertemporal elasticity of substitution (0.07) relative to their calibrated values.

The finding that a high coefficient of relative risk aversion is needed to match asset prices in a production economy is an established result in macro-finance. In matching the risk premium, Epstein-Zin preferences improve standard models' fit, but these models still need a high coefficient of relative risk aversion to match the properties of asset prices (Tallarini 2000, Weitzman 2007, and Barillas, Hansen, and Sargent 2009). The reason is that standard macroeconomic models cannot generate enough quantity of risk. Malloy, Maskowitz, and Vissing-Jorgensen (2009) and Campanale et al. (2010) show that high consumption volatility (or high quantity of risk) is needed to fit the risk premium on assets. If the model

[26] I experimented with varying parameters individually and in groups in calibration and estimation, but did not improve over the original fit to the real term structure except for policy parameters, which were discussed above. To be sure that my calibration does not drive the results in a way that hurts the model's fit, I estimated the entire model without calibrating any parameters. The estimated parameters were close to the values for calibration I had picked, hence the model fit was not affected due to estimation either.

cannot generate the required quantity of risk, to match the risk premium, the price of risk (risk aversion) needs to be very high. The results show that this conclusion extends to bond pricing.[27]

In power utility models, a high relative risk aversion implies that the households are extremely unwilling to substitute current consumption with future consumption. However, the results show that the estimated intertemporal elasticity of substitution is low even with Epstein-Zin preferences. An important reason for this result is that real risk-free rates are volatile with a low covariance with the expected aggregate consumption growth (Duffee 2013). To match this empirical result, the model estimate of intertemporal elasticity of substitution is substantially below one, implying very smooth consumption and volatile short-term real risk-free rates. Even though the estimated value is in contrast with the typical value for the IES used in macroeconomic models (which is between 0.5 and 1), the estimated value of the IES may not be inconsistent with the literature given the uncertainty around its estimates. For instance, Hall (1988) and Yogo (2004) find it to be very small (close to 0), Vissing-Jorgensen (2002) finds it to be between 0.8 and 1 for bond holders, whereas Hansen and Singleton (1982) and Attanasio and Weber (1995) find it to be well above 1.5. Braun and Nakajima (2012) show ways of making a macroeconomic model consistent with estimates between 0.35 and 0.5. Havránek (2015) performs a meta-analysis of the literature and finds that the mean of the estimates is around 0.6 (for the United States). Therefore, the estimated intertemporal elasticity of substitution lies in the battery of estimates found in the literature.

[27] For robustness, I considered using 5-to-10-year and 3-to-10-year nominal and real forward premiums as targeted moments instead of the real term premium calculated using the ex ante real rate. The 5-to-10 year average real forward premium is 1.85 percent with a standard deviation of 0.34 percent, whereas the average of the 5-to-10-year nominal forward rate is 2.31 percent with a standard deviation of 2.44 percent. I used the benchmark model both under calibration and with the best-fit estimates. The benchmark model generates 5-to-10-year nominal and real forward premiums around 28 basis points. Then I estimated the model by removing yield term premiums from the target moments and plugging in 5-to-10-year forward nominal and real term premiums and their standard deviations. The results show that the model can match nominal forward rates but has trouble matching real forward rates. Results for 3-to-10-year forward premiums are similar.

In summary, to jointly match macroeconomic and term structure moments, the model requires a high coefficient of relative risk aversion and low intertemporal elasticity of substitution. The coefficient of relative risk aversion is relevant for the nominal and real term structure moments, especially for real and nominal term premiums, whereas intertemporal elasticity of substitution is crucial for matching the macroeconomic moments through its effects on consumption. Even though this parameter combination is helpful in matching macroeconomic and nominal term structure moments, it is not as helpful in matching real term structure moments. In section 5 I analyze this issue in more detail.

5. Decomposing the Model Fit

This section explores the benchmark model's inability to fit the real term structure properties and the reasons behind its relative success in matching the nominal term structure features. Nominal bond yields have three components: real yields (measured using TIPS term structure), expected inflation, and the nominal term premium. If the model can match the nominal term structure moments without matching the real term structure features, this has to be because the model misses other components of nominal yields. I first explain why the model is not capable of matching the real term structure properties, then taking the results as given, I ask what helps the model to match the nominal term structure.

5.1 Why Does the Model Fail to Match the Real Term Structure?

The results in the previous section showed that even with a high coefficient of relative risk aversion, the model-implied real term premium and real excess returns are low and stable. Due to a low model-implied real term premium, the real yield curve is flatter than the data suggest. An important reason for this result is that conditional heteroskedasticity in the real stochastic discount factor is low, implying a low and very stable real term premium.

This model implication can be understood in the context of Hansen-Jagannathan bounds. Using the no-arbitrage condition for real bond prices, the Hansen-Jagannathan bound for the real stochastic discount factor can be defined as follows (Backus, Boyarchenko, and Chernov 2018):

$$\frac{E_t(x^{(n)}_{real,t+1})}{\sigma_t(x^{(n)}_{real,t+1})} \leq \frac{\sigma_t(m^{real}_{t,t+1})}{E_t(m^{real}_{t,t+1})}, \tag{36}$$

where $E_t(x^{(n)}_{real,t+1})$ is the conditional expectations of one-period real excess returns, $\sigma_t(x^{(n)}_{real,t+1})$ is the conditional standard deviation of one-period real excess returns, $E_t(m^{real}_{t,t+1})$ is the conditional expectation of the one-period real stochastic discount factor, and $\sigma_t(m^{real}_{t,t+1})$ is the conditional standard deviation of the one-period real stochastic discount factor. Note that the right-hand side of this inequality is mainly determined by $\sigma_t(m^{real}_{t,t+1})$, since $1/E_t(m^{real}_{t,t+1})$ is equal to the return of the one-period real bond. Hence to match the observed risk premium (or the Sharpe ratio), the conditional standard deviation of the real stochastic discount factor needs to be high enough. The magnitude and volatility of $\sigma_t(m^{real}_{t,t+1})$ depend on the coefficient of relative risk aversion, intertemporal elasticity of substitution, and volatility of macroeconomic variables, which in turn determine the size and the volatility of the real term premium. In essence, the real risk premium is a function of the price of risk (coefficient of relative risk aversion) and the quantity of real risk determined by the volatility of macroeconomic variables and intertemporal elasticity of substitution (affecting the real stochastic discount factor through consumption smoothing). The model cannot generate enough quantity of real risk and therefore requires a high coefficient of relative risk aversion to increase volatility in the real stochastic discount factor to get a better, but still poor, fit for the real term premium and, in turn, real term structure features.

5.2 What Helps the Model Fit the Nominal Term Structure?

If the model is not capable of matching the real term structure properties, then it should be compensating this poor performance through the other components of nominal bond yields such as long-term inflation expectations and the inflation risk premium. The no-arbitrage condition implies that inflation expectations matter for bond pricing through the nominal stochastic discount factor, whereas the inflation risk premium matters for the nominal term premium. To derive model implications, I treat the moments of these components as untargeted moments in the model, i.e., the model is not asked to make an effort to match them. Therefore, the properties needed by the model can be directly compared with their data counterparts.

In the benchmark model, π_t^* proxies for model-implied long-term inflation expectations. In a standard New Keynesian model, if the inflation target is known and credible, long-term inflation expectations will be formed around the long-term inflation target of the central bank. An implication of this result is that time variation in the target captures the time variation in long-term inflation expectations.[28] For the same reason, in macro-finance, a time-varying inflation target is used as a proxy for time-varying long-term inflation expectations. In particular, Kozicki and Tinsley (2001) show that time-varying "endpoints" (or limiting conditional forecasts) of inflation, which are proxied by long-term inflation expectations, are a crucial mechanism to reconcile long-term short rate forecasts generated from the reduced-form models with the long-term short rate expectations implied by the bond yields. Following this result, the time-varying endpoint for inflation is modeled as the time-varying inflation target to better fit short rate expectations, which captures the dynamics of long-term inflation expectations.[29] An alternative way of modeling long-term inflation expectations is to link observed long-term inflation expectations to the average of one-period forecasts over the forecast horizon as in Aruoba, Cuba-Borda, and Schorfheide (2018). I leave this interesting and viable alternative for future work to keep the model tightly linked to the macro-finance

[28]See Smets and Wouters (2007), Cogley and Sbordone (2008), Del Negro and Schorfheide (2013), Campbell et al. (2017), and Carvalho et al. (2017). There is game-theoretic justification for this mechanism as well. Demertzis and Viegi (2008) describe monetary policy as an information game and show that explicit quantitative objectives provide better anchors for coordinating agents' expectations.

[29]See Hördahl, Tristani, and Vestin (2006), Rudebusch and Wu (2007, 2008), and Rudebusch and Swanson (2012). Del Negro and Schorfheide (2013) show that having time-varying inflation target improves long-term interest rate forecasts of standard DSGE models, consistent with shifting endpoints interpretation.

literature by keeping the standard mechanisms common to the macro-finance models.

As discussed in the previous subsection, the benchmark model cannot generate enough quantity of real risk, implying a low real term premium. However, with the estimated coefficient of relative risk aversion, the model can match the nominal term premium and other nominal term structure features. This implies that the model should generate a high quantity of nominal risk through high conditional heteroskedasticity of the nominal stochastic discount factor. To achieve high conditional heteroskedasticity, the model requires counterfactually high volatility in inflation expectations. Table 3 shows this result, where model-implied moments are derived from the estimated model (referred to as "Best Fit" in table 2). Table 3, panel A shows the standard deviation of model-implied long-term inflation expectations and its empirical counterpart. The model-implied standard deviation of long-term inflation expectations is 236 basis points, which is around 80 basis points between 1983 and 2007 in the Blue Chip (around 8 basis points between 1999 and 2007 in the Blue Chip surveys and 11 basis points based on TIPS (Aruoba 2018) in the data). Generating long-run inflation risks one way or another is the standard way in the literature to make the model-based nominal term structure slope upward. Since the model-implied real risk is very small, inflation risk has to be unreasonably high to make nominal bonds risky assets.

If the inflation risk should be very high to make nominal bonds risky, agents holding long-term nominal bonds should require high compensation to bear that high inflation risk. The compensation for inflation risk is measured by the inflation risk premium, which is defined as the difference between the nominal and the real term premium.[30] Table 3, panel B shows the model-implied average risk premium (i.e., implied by the estimated model referred to as "Best Fit" in table 2) and estimates of the average 10-year inflation risk

[30] In a consumption-based model, the inflation risk premium is determined by the covariance between the real stochastic discount factor and inflation times the risk premium. For a positive inflation risk premium, the covariance needs to be negative. As Andreasen (2012) shows, the nominal term premium is equal to the real term premium plus the inflation risk premium.

Table 3. Model-Implied Unconditional Standard Deviation of Inflation Expectations and Mean of Inflation Risk Premium

	Sample	**Estimate**
A. Std. (10-Year Inflation Expectations)		
Blue Chip Survey	1983–2007	80 bps
	1999–2007	8 bps
Aruoba (2018)	1999–2007	11 bps
Benchmark Model	1985–2007	236 bps
B. Mean (10-Year Inflation Risk Premium)		
Buraschi and Jiltsov (2005)	1960–2000	70 bps
Chernov and Mueller (2012)	1971–2008	67 bps
Haubrich, Penacchi, and Ritchken (2012)	1982–2010	45 bps
Aruoba (2018)	1999–2007	30 bps
D'Amico, Kim, and Wei (2018)	1990–2013	29 bps
Ajello, Benzoni, and Chyruk (2016)	1985–2011	24 bps
Abrahams et al. (2016)	1999–2013	10 bps
Fleckenstein, Longstaff, and Lustig (2017)	2004–2014	2.8 bps
Grischenko and Huang (2013)	2000–2008	–12 bps
Benchmark Model	1985–2007	155 bps

Notes: The estimates are in basis points. Panel A shows the model-implied unconditional standard deviation of 10-year inflation expectations, standard deviation of 5-to-10-year inflation expectations calculated from the Blue Chip survey, and the standard deviation of 10-year inflation expectations from the data set of Aruoba (2018). Panel B shows the model-implied unconditional mean of inflation risk premium and various estimates of inflation risk premium that are present in the literature (the listed papers except for Aruoba 2018 are from table 8 of D'Amico, Kim, and Wei 2018) for the United States. Here "Benchmark Model" refers to the estimated version of the benchmark model (referred to as "Best Fit" in table 2).

premium in the literature.[31] The model-implied inflation risk premium is around 155 basis points, which is high compared with the

[31]The moments that are shown in this table (except for Aruoba 2018) are given in table 8 of D'Amico, Kim, and Wei (2018). Moments for Aruoba (2018) are calculated using the data available on the Federal Reserve Bank of Philadelphia's website. The sample for the benchmark model is given as 1985–2007, which is the sample that covers all the targeted moments used in estimation. However, the results are very similar if the model is estimated for the 1999–2007 sample.

estimates of the premium based on TIPS breakeven inflation, inflation expectations from surveys, and inflation swaps. As the table shows, these estimates are in the range of –12 to 70 basis points for different sample periods.[32] Studies that include the Great Recession in the sample tend to estimate a lower inflation risk premium (due to high deflation probability) whereas studies with the 1970s and early 1980s in the sample tend to estimate a high inflation risk premium due to hyperinflation risks (D'Amico, Kim, and Wei 2018). Considering that the sample period used in this paper coincides with the Great Moderation, one would expect inflation risk premium estimates to fall between these various estimates.[33] However, the model-implied inflation risk premium is very high—twice as large the highest estimate.

Intuitively, long-run inflation risk increases the riskiness of long-term nominal bonds through higher volatility in the nominal stochastic discount factor. However, due to the stability of the real stochastic discount factor, the model requires very volatile long-run inflation, which in turn implies counterfactually high average inflation risk premium. The reason is, the higher the uncertainty about future inflation, the riskier the nominal bonds are going to be and the more compensation households will ask for inflation risk. The model makes up for the missing real risk through counterfactually volatile inflation expectations and a high inflation risk premium to make the nominal term premium sizable.

6. Robustness and Extensions of the Model

The previous section demonstrated that the key issue with the benchmark model is that the real stochastic discount factor is too smooth, implying a low quantity of real risk. Therefore to match the nominal term structure, the model requires very high inflation risk through counterfactually volatile inflation expectations, which in turn implies a high inflation risk premium required by investors.

[32]The studies shown in the table (except for Aruoba 2018) are from D'Amico, Kim, and Wei (2018).

[33]Recently, Gomez-Cram and Yaron (2018) show that in the past two decades inflation-related risk factors did not play a dominant role in determining the nominal term premium and nominal excess returns.

In this section, I search for extensions that might increase the quantity of real risk, therefore decreasing the inflation risk needed to match nominal term structure properties.[34] I first analyze the effects of monetary policy on real rates. Then I analyze the effects of labor market frictions, long-run real risks, and preference shocks on the model's fit for the real term structure. The bottom line of the results presented in this section is that none of these extensions are helpful and they imply very volatile long-term inflation expectations along with a high inflation risk premium. To make that point, I estimate the extended models in turn but only report the implications for the real term structure moments, standard deviation of inflation expectations, and average inflation risk premium with the estimates for CRRA and IES. The full set of results (along with results under calibration) can be found in the online appendix.

6.1 Monetary Policy and the Real Term Structure

Monetary policy affects the macroeconomy through real interest rates. Therefore it is worth exploring the effects of monetary policy on the real term structure. If the central bank responds more aggressively to inflation and less so to the output gap relative to the benchmark calibration, then a negative technology shock would increase real interest rates and decrease real bond prices. Hence real bonds pay less when the marginal utility is high, implying a positive real term premium. Moreover, more aggressive monetary policy toward inflation implies higher volatility in real rates.

To approach the problem, I do a comparative statics exercise where I change the monetary policy rule parameters. I simultaneously change the response of monetary policy to inflation and output gap (keeping other parameters at their calibrated values) to investigate the effects on the real term structure moments. Second, to isolate the effects, I change the response to inflation and output gap individually. I compare the effects under high and low degrees of smoothing as well.

[34]Another missing feature can be uncertainty. Barillas, Hansen, and Sargent (2009) show that a model with moderate uncertainty and low risk aversion and a model with high risk aversion are isomorphic. I do not take that route and leave it to future research.

Table 4. Effects of a Change in Policy Parameters on Real Term Structure

	Data	Baseline	$g_\pi = 2$, $g_y = 0.45$	$g_\pi = 2$, $g_y = 0.93$	$g_\pi = 0.53$, $g_y = 0.45$
A. $\rho_i = 0.73$					
mean($\psi^{(40)}_{real}$)	1.32	−0.02	0.13	0.10	0.02
sd($\psi^{(40)}_{real}$)	0.42	0.02	0.01	0.006	0.005
mean($r^{(40)} - r^{(3)}$)	1.32	−0.10	0.10	0.06	−0.03
sd($r^{(40)} - r^{(3)}$)	1.13	1.15	0.53	0.57	0.79
mean($x^{(40)}_{real}$)	2.90	0.06	0.24	0.21	0.12
sd($x^{(40)}_{real}$)	14.20	0.97	2.44	2.15	1.47
B. $g_\pi = 2, g_y = 0.45$					
	$\rho_i = 0.4$	$\rho_i = 0.95$			
mean($\psi^{(40)}_{real}$)	0.15	0.05			
sd($\psi^{(40)}_{real}$)	0.02	0.004			
mean($r^{(40)} - r$)	0.13	0.007			
sd($r^{(40)} - r$)	0.45	0.99			
mean($x^{(40)}_{real}$)	0.27	0.15			
sd($x^{(40)}_{real}$)	2.60	2.91			

Notes: Panel A shows the model-implied unconditional moments for the real term structure with different output gap and inflation responses. Panel B shows the effects of higher and lower interest rate smoothing conditional on high response to inflation and low response to the output gap. "Baseline" refers to the moments implied by the calibrated version of the benchmark model.

Table 4, panel A shows the results of this calibration exercise in terms of monetary policy's implication on the real term structure, where "Baseline" refers to the calibrated version of the benchmark model. In this exercise I choose extreme values for the responses to make the point clear. In particular, I double the total long-run response to inflation from 1.53 (i.e., $g_\pi = 0.53$) to 3 (i.e., $g_\pi = 2$) and simultaneously halve the response to output gap from 0.93 to 0.45. The second column of the table shows the empirical moments, and the third column shows the model-implied moments under benchmark calibration. The fourth column shows that a high response to inflation and a low response to the output gap help the model to generate a positive real term premium and an upward-sloping real term structure. However, same improvement does not materialize with other model-implied moments. The model-implied real term premium and slope are too stable. Similarly, model-implied excess returns are too small and too stable. Columns 5 and 6 show that a positive real term premium and real yield curve slope is implied by higher response to inflation, and lowering the response to the output gap has minimal effects on the real term structure. An implication of this exercise is that the monetary policy response to inflation is a crucial determinant of the sign of the real yield curve slope. The stronger the response to inflation, the more positive the real yield curve slope is.

Does varying interest rate smoothing improve the model's fit to the real term structure? To explore this possibility I reduce the interest rate smoothing to 0.4 and increase to 0.95 in turn, and keep the high inflation and low output gap response (i.e., $g_\pi = 2, g_y = 0.45$). The results in the second column of table 4, panel B show that lower smoothing combined with a high response to inflation and a low response to the output gap generates slightly larger and slightly more volatile real term premium and real excess returns. However, the gains are modest. On the contrary, higher interest rate smoothing, as shown in the third column of table 4, panel B worsens the model's performance. For this model to generate a plausible real term premium and yield curve slope, monetary policy response to inflation has to be too high and interest rate smoothing has to be too low. Even then, it is doubtful that the model can generate enough volatility in the real term premium and excess returns.[35]

6.2 Labor Market Frictions

One of the main differences between endowment and production economies is that in a production-based economy, the households can freely choose their labor–consumption tradeoff. In a production economy, the agents have the ability to insure themselves against adverse shocks by endogenously varying their labor supply. As a result of this feature of the model, the benchmark model may not

[35]Estimation of the parameters does not alter the conclusion. The results can be found in the online appendix.

generate enough risk. To overcome this problem, I consider labor market frictions: quadratic labor adjustment costs (following Uhlig 2007 and Rudebusch and Swanson 2008) and real wage rigidities (following Blanchard and Galí 2007). The results in the following two subsections are consistent with Rudebusch and Swanson (2008) that labor market frictions are not helpful in matching risk premiums.

6.2.1 Quadratic Labor Adjustment Costs

For this extension, I assume that the households have to pay a quadratic cost to change their labor supply, $\kappa(\log(l_t/l_{t-1}))^2$, which is parameterized by κ. In the benchmark calibration, I assume that $\kappa = 50\overline{Y}$, where $\overline{Y}$ is the steady state of aggregate output. This calibration implies that a 1 percent change in labor supply from the previous quarter costs households 0.5 percent of quarterly steady-state output.

The third column of table 5 shows the model-implied moments for the real term structure.[36] The results are consistent with Rudebusch and Swanson (2008). Even though the model can generate a slightly higher quantity of real risk by making fluctuations less desirable, the result found in the previous section cannot be overturned. That is, the volatility of the real stochastic discount factor is still very low to match excess returns and the real risk premium.

6.2.2 Real Wage Rigidities

Another way of introducing labor market frictions to the model is to consider real wage rigidities following Blanchard and Galí (2007). In particular, I assume that the real wage is no longer equal to the marginal rate of substitution between consumption and leisure, but rather it is a function of the lagged real wage, the marginal rate of substitution (which is the frictionless real wage), and a steady-state wedge between households' marginal rate of substitution and the real wages. That is, the real wage is given by

$$w_t = (w_{t-1})^{\mu}(mrs_t)^{(1-\mu)}\exp(\omega), \tag{37}$$

[36] See the online appendix for the full set of estimates and the results under calibration.

Table 5. Real Term Structure Moments Implied by the Extensions of the Benchmark Model

	Data	Adjustment Costs	Real Wage Rigidity	Long-Run Risks	Preference Shocks
mean($\psi^{(40)}_{real}$)	1.32	0.51	0.33	0.13	0.59
sd($\psi^{(40)}_{real}$)	0.42	0.55	0.28	0.07	0.68
mean($r^{(40)} - r^{(3)}$)	1.32	0.10	0.12	0.06	0.22
sd($r^{(40)} - r^{(3)}$)	1.13	1.32	1.67	1.05	2.20
mean($x^{(40)}_{real}$)	2.90	1.48	0.99	0.61	1.77
sd($x^{(40)}_{real}$)	14.20	9.09	6.20	3.62	9.77
mean(10-Year Inflation Risk Premium)	–12–70 bps	295	200	123	350
sd(10-Year Inflation Expectations)	8–80 bps	618	378	479	769
CRRA		115	130	140	125
IES		0.07	0.05	0.07	0.04

Notes: Model-implied average 10-year inflation risk premium and standard deviation of 10-year inflation expectations are given in basis points. Empirical moments for average 10-year inflation risk premium and standard deviation of 10-year inflation expectations represent the range of estimates given in table 3.

where w_t is the real wage, mrs_t is the marginal rate of substitution between leisure and consumption derived from the household's optimization problem, and ω is the steady-state wedge between w_t and mrs_t. In this specification, μ captures the sluggishness of the real wage: the closer μ is to one, the higher the wage stickiness. Blanchard and Galí (2007) use this specification as a simple way of capturing the properties of wage bargaining.

The results are given in the fourth column of table 5 for the estimated model. However, real wage rigidities do not change the conclusions about the model's fit for the real term structure moments. Even this high coefficient of relative risk aversion is not enough to have a good fit to the real term structure.

6.3 Long-Run Productivity Risks

A promising extension is to introduce long-run productivity risks in the spirit of Bansal and Yaron (2005). Epstein-Zin preferences play a crucial role in long-run productivity risk models because under recursive preferences future uncertainty about the expected consumption growth is priced, which in turn may increase the conditional heteroskedasticity of the real stochastic discount factor.[37]

Consistent with Bansal and Yaron (2005) and others, I assume that productivity shocks have two components: a persistent and predictable component and a transitory component.[38] To this end, I replace technology process with the following:

$$\log(A_t/A) = \rho^{A^*} \log(A^*_{t-1}/A) + \sigma_{A^*}\varepsilon_t^{A^*} + \sigma_A \varepsilon_t^A. \tag{38}$$

The fifth column of table 5 shows the implications of the model for the real term structure. As consistent with other extensions, adding long-run real risk to the model is not helpful in matching the real term structure moments. An important reason is that negative technology shocks increase marginal utility but decrease real interest rates. The positive covariance of marginal utility and real bond prices makes real bonds a hedge.

6.4 Preference Shocks

The previous subsection showed that long-run productivity risks are not helpful in jointly matching the nominal and real yield curve properties. Recently, Albuquerque et al. (2016) showed that preference shocks help a model with an endowment economy match real term structure properties. I incorporate their specification to the production economy and ask the model to jointly match nominal and real term structure features along with macroeconomic moments.[39]

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[37]Ulrich (2011) explores the implications of long-run real risk and ambiguity in an endowment economy for the real term structure. He shows that the interaction between ambiguity and long-run risks is the crucial mechanism to generate an upward-sloping real term structure. I leave this extension for future work.

[38]The long-run real risk specification in this paper and the papers cited above is different. In this paper I assume that the technology follows a stationary process that is highly persistent, whereas in the long-run productivity risk literature, the productivity is assumed to follow a random walk. In other words, the long-run risk is incorporated in the models through shocks to the long-run growth trend of the real variables. Since I do not have long-run trends in the model, I choose a stationary productivity specification. This specification follows the earlier working paper version of Rudebusch and Swanson (2012).

[39]See Albuquerque et al. (2016) for the details of the preference shocks considered in this subsection. See the online appendix for the details of how the preference shocks are incorporated into the baseline model.

With preference shocks, we can write the real stochastic discount factor as

$$m_{t,t+1}^{real} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t} (E_t V_{t+1}^{1-\alpha})^{\alpha/1-\alpha} V_{t+1}^{-\alpha} \left(\frac{c_{t+1}}{c_t} \right)^{-\varphi}. \tag{39}$$

In this specification, a preference shock increases real rates and the marginal utility of consumption, generating a negative covariance between the real stochastic discount factor and real bond yields. If the preference shocks are very volatile, they can increase the conditional heteroskedasticity of the real stochastic discount factor. The last column of table 5 shows the implications of the extended model for the real term structure properties. The model-implied moments are slightly closer to their empirical counterparts than are other extensions, but the gains are modest. Therefore, like other extensions, this extension is not helpful in a production economy.

7. Conclusion

In this paper I have shown that a standard New Keynesian model with Epstein-Zin preferences has trouble matching macroeconomic, nominal, and real term structure properties simultaneously. The dynamic New Keynesian model can match macroeconomic moments and the nominal term structure, but it is unable to match the real term structure. The success of matching the nominal term structure comes at the cost of counterfactually volatile inflation expectations and a very high inflation risk premium. It further shows that the mechanisms that are proposed in the asset pricing literature, which are argued to be helpful in matching asset pricing facts, are not useful in matching the real term structure properties. An important reason is that these extensions cannot generate enough real risk to match the real term structure features. Either New Keynesian models and their natural extensions are misspecified or there is a substantial mispricing of index-linked bonds.

What other mechanisms could have helped increase the real risk in the model? One approach could be rare disasters. Recently Kozlowski, Veldkamp, and Venkateswaran (2019) use a similar idea to explain why riskless rates are depressed after the recession in developed countries. Contrary to Gabaix (2012) (who matches nominal term structure through inflation risk), in Kozlowski, Veldkamp, and Venkateswaran (2019) tail risk is incorporated through capital quality shocks in a real business cycle model. The rise in tail risk makes agents invest and produce less, leading to lower output and capital because investing today is riskier. This negatively affects riskless rates due to precautionary savings (future consumption is riskier) and through higher demand for more liquid assets. Other approaches could be to analyze the effects of market segmentation (Fisher 2015; Andreasen, Fernandez-Villaverde, and Rubio-Ramirez 2018) or examine the implications of heterogeneous agents for the real term structure in the spirit of Constantinides and Duffie (1996). The macro-finance implications of these extensions are left for future research.

This paper has shown that there should indeed be further research, as the existing canonical macro-finance models do not satisfactorily fit the key moments of the data.

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