What Are the Effects of Changes in Taxation and New Types of Mortgages on the Real Economy? The Case of Denmark during the 2000s*

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What are the effects of introducing interest-only, flexible-rate mortgage contracts and a tax freeze on housing wealth for the real and financial economy? I study this within a DSGE model with housing, banking, and financial frictions, and the coexistence of flexible-rate, fixed-rate, and interest-only mortgage contracts. I find that the introduction and the adoption of flexible-rate and interest-only mortgage contracts together with a freeze of taxation of the housing wealth can explain 8 percent of the increase in the real house price in Denmark during the period 2004–06 and 0.75 percentage point of the output gap. The household debt-to-GDP ratio would have been almost constant instead of increasing by 20 percentage points. The analysis points to a more volatile Danish economy after the implementation of the structural changes to the economy.

JEL Codes: E17, E32, E62, E65, F41.

1. Introduction

Danish house prices were booming in the 2000s, rising by more than 20 percent in 2006. They subsequently showed comparable

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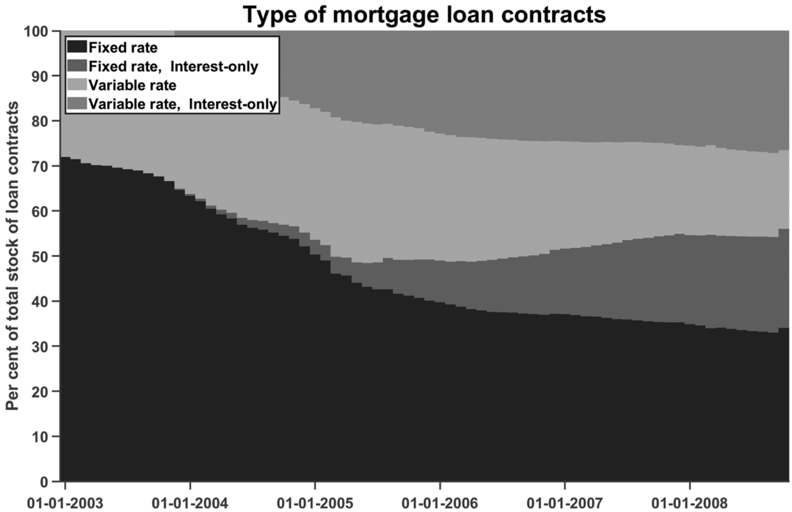
Figure 1. Changes in the Outstanding Stock of Mortgage Loan Contracts



Source: Danmarks Nationalbank.
Note: The figure shows the development in the stock of outstanding mortgage contracts.

negative growth rates after the onset of the financial crisis in 2008. This coincided with a boom-bust cycle in the Danish economy with large increases in residential investments, private consumption, and GDP. A similar development can be found in other developed economies.

In what follows, I study the role played by two structural changes for these developments: the introduction and adoption of new mortgage contracts and a freeze of the effective tax on housing wealth. These two changes are shown in figures 1 and 2. Figure 1 shows the outstanding stock of mortgage debt contracts distributed by type of contract: flexible-rate and fixed-rate contracts with and without principal payments. From this figure it can be seen that pre-2002, almost all mortgage contracts were fixed rate with amortization and thirty-year maturity. In 2007, this was only the case for around 40 percent of the outstanding contracts. The remainder constituted interest-rate-only contracts and/or flexible-rate mortgage contracts.

Figure 2. Changes in the Real House Price and the Effective Tax Rate on Housing Wealth

Source: Danmarks Nationalbank.
Note: The figure shows the effective tax rate on housing together with the real house price.

The introduction and adoption of new mortgage contracts coincided with a change in the taxation of housing wealth shown in figure 2. Starting from January 1, 2002, the nominal amount Danish households have to pay in taxes on the value of the property has been frozen at its 2002 level. The effect of this way of taxing housing wealth is that when house prices rose during the mid-2000s, the effective tax rate on housing fell. The opposite was true during the bust.

What are the effects of these structural changes on real house prices, household debt, and real GDP? To answer this question, I use a medium-size small open-economy DSGE model with banks, financial frictions, and housing for Denmark building on the work in Pedersen (2016). One contribution of this paper is the extensions made to this model. Specifically, I expand this model with long-term debt contracts with and without amortization and the coexistence of flexible-rate and fixed-rate contracts. I do this through a reinterpretation of the Calvo setup, usually used to model sticky prices and/or wages. I reinterpret the probability of being able to reset interest rates as instead determining how often mortgage lending rates are set on the borrowers' side, thus effectively determining the average duration of the mortgage portfolio in the economy. This avoids a large number of state variables, while still being able to model the coexistence of fixed-rate and flexible-rate mortgage contracts of varying maturity. Further, through the explicit modeling of the amortization schedule of these contracts, I allow for interest-only mortgage contracts (IO loans). I use this framework to analyze the permanent and transitory effects, and the effects on second-order moments from these changes.

The results point to a combined effect from changes in the taxation of housing wealth and the introduction and adoption of new types of mortgage contracts as follows. Without these changes, the real house price would have increased by around 8 percentage points less during the period 2002–06, and the construction-to-GDP ratio would have been around 1 percentage point lower. The output gap in 2005 would have been around 0.75 percentage point lower. This leaves a large part unexplained. This is not too surprising. Here I only consider the two mentioned structural changes to the economy, while in reality also other shocks hit the Danish economy during this period.[1]

On the financial side, the debt-to-GDP ratio for households would have been almost constant without these two changes, while it in fact increased by 20 percentage points. Further, these changes have had implications for the business cycle—equivalent shocks propagate stronger after the changes to the housing market. This concerns both the real and the financial side of the economy. Specifically, using the coefficient of variation of the changes in the variables in the model as a measure of the business cycle, the real house price and construction can be expected to be, respectively, 25 and 15 percent more volatile, while debt to households can be expected to be

[1]In Pedersen (2016), the real house price is decomposed into structural shocks. There I find that foreign shocks, such as demand for Danish exports and foreign monetary policy shocks, as well as domestic productivity shocks, etc. contributed to the house price gap during the period under study. Further, housing preference shocks played a large part. These shocks can be interpreted as the consumers getting more utility of a given level of housing or that, given fundamentals, the household demanded more housing. This is thus a sign of a house price bubble.

50 percent more volatile. Private consumption can in the future be expected to move by 10 percent more.

While I find that the main drivers for first-order moments of the main variables in the model have been taxation of housing wealth, the adoption of new types of mortgage contracts has had permanent effects on especially debt. That is, if the composition of the stock of outstanding mortgage contracts in 2008 had been constant and taxation kept frozen in all future, then in the new regime household debt would have been around 20 percent higher than in the old regime. Likewise, the real house price would have been 3 percent higher and residential investments around 10 percent higher. Also, while I find that the interest-only mortgage contracts are not likely to have had a large effect on either, this is not the case for these permanent effects. I find that they contribute to around half of the buildup of private debt in the new regime relative to the situation in 2002.[2]

In the case of Denmark, Dam et al. (2011) study similar questions. They find that, without interest-only mortgage contracts, the real house price would have been around 15 percent lower in 2006–07. Likewise, without flexible-rate loans, the house price would have been around 15 percent lower in 2006–07. Without changes to the taxation, the real house price would have been around 7.5 percent lower in 2006–07. In total, Dam et al. (2011) find that the three changes to the housing market can explain the 40 percentage point increase in the real house price in Denmark during the period from the fourth quarter of 1999 to the first quarter of 2007.

In Dam et al. (2011), a traditional macroeconometric model is used. While it is a standard well-tried tool with data-consistent framework, the model lacks forward-looking economic agents, and simulating the model and calculating higher-order moments is not straightforward.

[2]These findings are not a case against financial liberalization. In the current study, I do not consider the effects on welfare. To this end, I need both a heterogeneous agent model and endogenous defaults. As an example, differences in risk aversion could make it welfare improving to finance housing using flexible-rate contracts, while interest-only loans provide an insurance against, as an example, uninsurable unemployment risk. This is not the focus of this paper.

I introduce long-term debt with and without amortization, building upon the work of Gelain, Lansing, and Natvik (2015) and Kydland, Rupert, and Sustek (2012). I reinterpret the financial market structure introduced by Gerali et al. (2010). Specifically, the markup which arises due to monopolistic competition in the banking sector is, in this model, interpreted as risk premiums inherent and observed in data for fixed-rate, long-maturity mortgage contracts. The stickiness of the interest rates is in turn interpreted as a measure of the average duration of the mortgage contract and hence for how long these interest rates are fixed in the contracts.

Other papers have considered long-term debt contracts with fixed rate and flexible rate, but not simultaneously. Calza, Monacelli, and Stracca (2013) consider the effects of monetary policy under fixed- or flexible-rate mortgage contracts. However, in their model, the two types of mortgages do not coexist. Rubio (2011) considers a model in which fixed-rate and flexible-rate mortgage contracts coexist. Her model captures that, all else being equal, shocks which transmit to the economy through interest rates are mitigated under fixed-rate mortgages. But her model does not include amortization and long-term debt contracts. Brzoza-Brzezina, Gelain, and Kolasa (2014) expand on the framework in Calza, Monacelli, and Stracca (2013) to long-term debt contracts and consider both fixed-rate and flexible-rate mortgage contracts. Also, flexible-rate contracts are not one-period contracts, and they do not consider the coexistence of all types of contracts. In a parallel study, Alpanda and Zubairy (2017) have a slightly different focus—namely, how to address household indebtedness. They too have a model with long-term debt contracts with amortization and fixed interest rates. Though the models share some key characteristics, the focus in this paper is on the effects on the economy of financial liberalization, while their focus is on policies which can reduce household indebtedness.

This paper proceeds as follows. Section 2 provides an overview of the Danish mortgage system. In section 3, the model is presented; the scenarios are shown in section 4; and the results are shown in section 5. I analyze welfare in section 5.4. I compare the results found in this paper with findings from a similar study using a macroeconometric model in section 6. The results are discussed in section 7, and section 8 concludes.

2. Main Characteristics of the Danish Mortgage Market

To begin with, it is useful to provide an overview of the main characteristics of the Danish market for mortgages.[3] Danish households, and to some extent, also Danish firms, finance purchases of housing and buildings mainly through mortgage banks. In terms of volume, the market is large. The outstanding stock of mortgage debt to households amounts to more than 100 percent of GDP in 2017 and 150 percent if mortgages to firms are included. This is equivalent to approximately 70 percent of total credit to non-financial firms and households provided by banks and mortgage credit institutions.

In the Danish system, mortgages are provided through specialized mortgage institutions. These institutions assume only minor risks other than credit risk. For the question addressed here, the following points are worth mentioning. The mortgage banks are specialized institutions. Their main activity is to provide loans collateralized against real property and financed by issuing bonds. Like banks, mortgage banks must meet, e.g., capital requirements.

Five factors in particular ensure that mortgage bond investment is associated with very low credit risk. As an example, during the financial crisis, impairments reached a level of only 0.2 percent in 2009, from which they declined. First, the balance principle and the close link between loans and bonds mean that mortgage banks do not assume significant market risks. The statutory balance principle sets the limits for the financial risks that mortgage banks can assume, including interest rate, option, liquidity, and exchange rate risk, and it imposes strict limits on the relationship between payment flows for loans and their funding. This means that the most significant risk for the mortgage banks is credit risk. Second, the credit risk that the mortgage banks can assume is limited by fixed loan-to-value ratios and rules on valuation of the collateral. Third, mortgage banks can strengthen their capital base on an ongoing basis by generally increasing their administration margins. Fourth, due to a strong legal framework, they have reliable access to fast realization of the collateral relating to a non-performing loan. If a borrower defaults on a loan, the mortgage bank can call the loan and sell the mortgaged property, and if the mortgage bank's claim is not met,

[3] A useful overview can be found in Gundersen, Hesselberg, and Hove (2011).

it retains a claim on the borrower. This gives the borrowers strong incentives to service their loans. Fifth, mortgage bond investors may file a claim against the mortgage bank and, in the event of compulsory liquidation, they have priority over other investors in relation to the underlying collateral.

Low liquidity and funding risk in the Danish mortgage-credit system imply that the lack of these elements in the model is not of major concern with regards to the results in this paper. Likewise, the low default risk implies no need to model defaults in the banking sector. Also, as loans can only be granted against collateral in the form of real property, and the requirements for the collateral underlying the loans are laid down by statute, the existence of debt-constrained households who need to post collateral is thus a reasonable assumption. Lastly, the Danish mortgage system is centralized, with five main credit institutions. Hence, an assumption of monopolistic competition on the lending market applies well to the Danish financial system.

3. The Model

The questions I seek to answer in this paper will be analyzed using the model in Pedersen (2016) with some extensions explained below. The model documented in Pedersen (2016) is a small open-economy model with a fixed exchange rate regime, housing, construction, a financial sector, and financial frictions. The fixed exchange rate regime implies that the policy rate is exogenous; the monetary policy rate reacts neither to changes in the output gap nor to inflation. Instead, the Danish CPI price level is anchored through a purchasing power parity condition, and the monetary policy rate is determined by developments in the euro area. The model is estimated on quarterly Danish data running from 1995 to 2016. The data set includes information on house prices, loans, and loan rates. I will refer to Pedersen (2016) for detail in what follows. Here, I only analyze and present the changes made to that model.

The extensions made to the model in Pedersen (2016) are long-term debt with varying amortization, and flexible-rate and fixed-rate mortgages. On the household side, the idea is to set up a representative mortgage contract with an interest rate, an amortization rate, and duration which reflects the average mortgage contract in

Denmark.[4] On the banking side, the idea is to use the standard Calvo setup for interest rates normally applied to model sticky prices; see Galí (2008). The advantage of this approach is that it is memoryless and hence there is no need to follow all cohorts of fixed-rate and flexible-rate mortgage contracts.

3.1 Model Extension: Long-Term Debt Contracts and Amortization

In the model in Pedersen (2016) all debt contracts are one period. This is standard in the literature, but the questions addressed in this paper cannot be answered within such a framework. There can be no meaningful distinction between interest-only loans and debt contracts with amortizations if all debt is rolled over each period. Similarly, if the contract only runs for one period, then the distinction between fixed-rate and flexible-rate mortgages becomes less interesting. Therefore, I introduce long-term debt contracts into the model in Pedersen (2016) with a process for the amortization rate.

I will follow Gelain, Lansing, and Natvik (2015) for the introduction of long-term debt contracts. I will follow Kydland, Rupert, and Sustek (2012) for the specification of the amortization of the long-term debt. Let B_t^I be the stock of real debt outstanding at the beginning of the period, let $B_t^{I,New}$ be new borrowing incurred in period t, and let $\delta_t^{\mathcal{A}}$ be the amortization rate. The law of motion for debt is given by

$$B_t^I = \left(1 - \delta_{t-1}^{\mathcal{A}}\right) \frac{B_{t-1}^I}{\pi_t^{DK}} + B_t^{I,New}. \tag{1}$$

π_t^{DK} is the change in the Danish PPI deflator. If $\delta_t^{\mathcal{A}} = 0\ \forall t$, then no debt is amortized and all debt contracts are perpetuities. If $\delta_t^{\mathcal{A}} = 1 \forall t$, the model collapses into a model with one-period debt contracts only.

I follow Kydland, Rupert, and Sustek (2012) in the specification of $\delta_t^{\mathcal{A}}$. One approach is to simply let the process be a constant, $\delta^{\mathcal{A}}$.

[4] A setup with, as an example, four representative (impatient) households, which each finance their housing purchases using fixed-rate or flexible-rate mortgages with or without amortization, would a priori yield similar conclusions but with the cost of having a much larger model.

However, that would not take into account the actual amortization schedule for Danish mortgage contracts, which to a large degree are annuities. Kydland, Rupert, and Sustek (2012) propose a process for $\delta_t^{\mathcal{A}}$ as follows:

$$\delta_t^{\mathcal{A}} = \left(1 - \frac{B_t^{I,New}}{B_t^I}\right) \left(\delta_{t-1}^{\mathcal{A}}\right)^{\alpha^\delta} + \frac{B_t^{I,New}}{B_t^I} \left(1 - \alpha^\delta\right)^{\kappa^\delta}, \tag{2}$$

where $\alpha^\delta \in [0,1)$ and $\kappa^\delta > 0$ are constants. By setting $\alpha^\delta = 0$, amortization becomes equal to one, $\delta_t^{\mathcal{A}} = 1$, and the model collapses into a model in which all debt is rolled over each period. As shown in Kydland, Rupert, and Sustek (2012), the parameters $(\alpha^\delta, \kappa^\delta)$ can be calibrated to match approximately the amortization of a thirty-year mortgage contract, which is standard in Denmark.[5]

Equations (1) and (2) can be combined as follows:

$$\delta_t^{\mathcal{A}} = \left(1 - \alpha^\delta\right)^{\kappa^\delta} + \frac{B_{t-1}^I}{\pi_t^{DK} B_t^I} \left(1 - \delta_{t-1}^{\mathcal{A}}\right) \left(\left(\delta_{t-1}^{\mathcal{A}}\right)^{\alpha^\delta} - \left(1 - \alpha^\delta\right)^{\kappa^\delta}\right), \tag{3}$$

which only depends on the stock of debt.[6] I treat the amortization process as being taken as given for the households. Consequently, they do not take into account that the level of debt affects the amortization rate when determining their borrowing in equilibrium. The budget constraint for the impatient households is given by[7]

$$\begin{aligned} &\left(1 + \tau_t^{VAT}\right) \frac{P_t^C}{P_t} C_t^I + Q_t^H \left(H_t^I - \left(1 - \delta^H\right) H_{t-1}^I\right) \\ &\quad + \frac{R_{t-1}^{L,I}}{\pi_t^{DK}} B_{t-1}^I + (1 - \omega) T_t \\ &= B_t^I + \left(1 - \tau_t^N\right) W_t^I N_t^I - \tau_t^H Q_t^H H_t^I. \end{aligned} \tag{4}$$

[5] I refer to Kydland, Rupert, and Sustek (2012) for the details on how this can be done.

[6] The amortization process depends on the change in the stock of debt, as, by the nature of an annuity, principal payments are lower and interest payments higher at the initiation of the debt contract.

[7] Compared with the model in Pedersen (2016), I disregard tax deduction on interest rate payments.

Here P_t is the overall price level (producer price), P_t^C is the consumer price, Q_t^H denotes the real house price, and $\delta^H > 0$ is the depreciation rate of the housing stock. The parameter ω denotes the share of impatient households in the economy. W_t^I and N_t^I are, respectively, employment and the real wage rate. I let τ_t^{VAT}, τ_t^B, τ_t^N, and τ_t^H be the tax rates on consumption (i.e., a value-added tax, VAT), interest income, labor, and housing property, respectively. T_t denotes real lump-sum taxes. C_t^I denotes consumption. Using (1), the budget constraint can, with long-term debt, be written as

$$\begin{aligned}&\left(1+\tau_t^{VAT}\right)\frac{P_t^C}{P_t}C_t^I + Q_t^H\left(H_t^I - \left(1-\delta^H\right)H_{t-1}^I\right) - B_t^{I,New}\\&\quad + \frac{\left(R_{t-1}^{L,I} - 1 + \delta_{t-1}^{\mathcal{A}}\right)}{\pi_t^{DK}}B_{t-1}^I\\&\quad = -\left(1-\omega\right)T_t + \left(1-\tau_t^N\right)W_t^I N_t^I - \tau_t^H Q_t^H H_t^I. \qquad (5)\end{aligned}$$

The terms in parentheses in front of the stock of loans in the budget constraint, $(R_{t-1}^{L,I} - 1 + \delta_{t-1}^{\mathcal{A}})$, are net interest payments and principal on the stock of loans.

The introduction of long-term debt contracts does not change the nature of the budget constraint per se. And long-term debt contracts only matter for the equilibrium dynamics if financial frictions are introduced into the model. Following Iacoviello (2005) and the setup in Pedersen (2016), I assume the existence of a collateral constraint. That is, the outstanding debt of the household cannot exceed some fraction, Θ_t^I, of the value of the housing stock, $H_t^I Q_t^H$,

$$B_t^{I,New} = \Theta_t^I\left(\frac{E_t\left[Q_{t+1}^H \pi_{t+1}^{DK}\right]H_t^I}{R_t^{L,I}} - B_t^I\right). \qquad (6)$$

Hence, the collateral constraint is imposed on new debt; the household can only take on new debt if the collateral value, the first term in (6), is higher than the existing stock of debt, B_t^I.[8]

[8] In what follows, I assume that the collateral constraint is always and everywhere binding. In the case of one-period debt, this question has been studied in depth; see, e.g., Iacoviello (2005).

Assuming that the collateral constraint binds, I can rewrite equation (6) in terms of the stock of debt only using equation (1):

$$B_t^I = \frac{\Theta_t^I}{1+\Theta_t^I} E_t \left[\pi_{t+1}^{DK} Q_{t+1}^H\right] H_t^I \frac{1}{R_t^{L,I}} + \frac{1-\delta_{t-1}^{\mathcal{A}}}{1+\Theta_t^I} \frac{B_{t-1}^I}{\pi_t^{DK}}. \quad (7)$$

I notice that the parameter Θ_t^I is the loan-to-value, LTV, ratio on net worth, and the steady-state LTV ratio is equal to $\frac{\Theta^I}{\Theta^I+\delta^{\mathcal{A}}} = \frac{Q^H H^I \pi^{DK}}{R^{L,I}}$. In the calibration of the model, Θ_t^I is set such that the steady-state loan-to-value ratio equals 0.8. This implies that the household can borrow up to four times its net worth in steady state. I have left an analysis of how the changes to the model change propagation in the model to appendix 1.

3.2 Model Extension: Flexible-Rate and Fixed-Rate Mortgage Contracts

The next step is to introduce flexible-rate and fixed-rate mortgage contracts. Clearly, these contracts need to coexist. I apply the representative mortgage contract framework in Pedersen (2016). I reinterpret the markup in the competitive monopolistic framework for the banking sector as risk premiums reflecting duration risk from financing housing through fixed-rate/long-maturity contracts. This is a shortcut in modeling risk and risk premiums, but it is a tractable way forward that introduces a way to implement the change from fixed-rate contracts to flexible-rate contracts, which is the aim of this analysis. I next reinterpret sticky lending rates to households as reflecting the duration of the mortgage rate contract instead: The longer the duration of the contract, the less often the bank can change the interest rate on the outstanding loan portfolio, and this translates into different stickiness of lending rates. The Calvo parameter is thus interpreted as a measure of the average duration of the mortgage contract, and hence for how long interest rates are fixed.[9]

[9] A drawback of this approach is that, strictly speaking, the individual household does not know the duration of its mortgage contract. In the aggregate, this is, however, not important and does not change the results in this paper.

In detail, in the banking sector I assume that the banks set a markup over the lending rate, as in the original model in Pedersen (2016). In the steady state the markup is such that the average interest rate on the average contract is 5 percent per year. The markup reflects both monopolistic competition in the banking sector, as in Pedersen (2016), and, generally, the existence of an interest rate spread between interest rates on mortgage loans and the policy rate. It is hence a shortcut to model term premiums. It is also, as explained next, a shortcut to model the empirical spread between fixed-rate and flexible-rate mortgages and hence introduces a way forward to implement scenarios in which the outstanding stock of loans are changed from fixed- to flexible-rate contracts.

Hence, I will in this paper assume that the banks cannot change the interest rate set on its portfolio of mortgage contracts every period. Specifically, a Calvo-style setup is applied to the lending rates from the banks to the households. Therefore, the banks cannot change the interest rate on the mortgage contracts every period. In the steady state, the optimal lending rate, $\widetilde{R^{L,I}}$, is equal to

$$\widetilde{R^{L,I}} = \frac{\epsilon_R}{\epsilon_R - 1} R_t^L, \tag{8}$$

in which the first term, $\frac{\epsilon_R}{\epsilon_R - 1}$, is the markup, or spread, between the lending rate from the wholesale bank to the branch, R_t^L. The actual representative or average mortgage rate evolves as

$$R_t^{L,I} = \left(\theta_R \, {R_{t-1}^{L,I}}^{1-\epsilon_R} + (1-\theta_R) \; {\widetilde{R^{L,I}}_t}^{1-\epsilon_R} \right)^{\frac{1}{1-\epsilon_R}} . \tag{9}$$

One big computational advantage of this setup is that it is memoryless. This implies a reduction in the number of state variables, as there is no need to follow all the mortgage contracts through time. The Calvo parameter for the mortgage contract is denoted by θ_R, and is calibrated such that it reflects the average length of the representative mortgage contract; the expected duration of the contract in this framework is $\frac{1}{1-\theta_R}$.

The interpretation of θ_R is therefore slightly different from the sticky-price setup; see Galí (2008). In that framework, the Calvo parameter denotes the probability of being able to change prices for the firms. This is also so in this framework mathematically, but here the Calvo parameter is interpreted as determining the duration of the average mortgage contract. The implications are the same: The bank is not able to change the interest rate in the contract on existing loans, but must wait until the contract is refinanced and/or new loans come in. If $\theta_R = 0$, the model collapses into a model in which all loans are flexible-rate loans. When $\theta_R \rightarrow 1$, once the interest rate is set it cannot be changed, reflecting a fixed-rate mortgage contract.

4. Scenarios

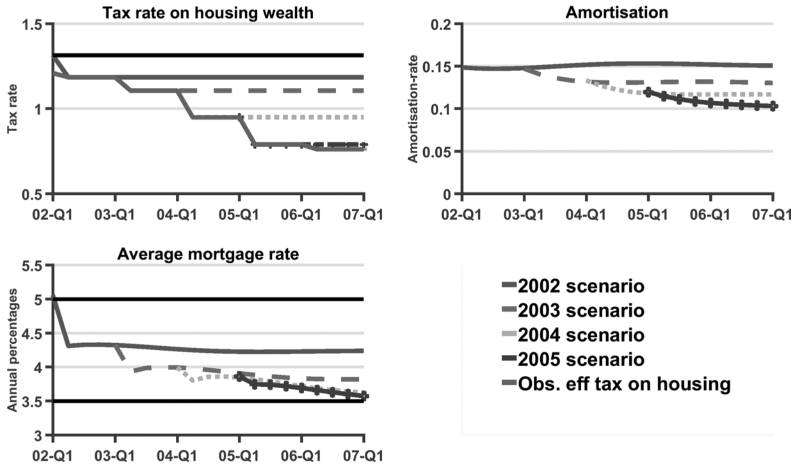
Next, I explain how I include in the model the changes to the Danish housing market experienced from approximately 2002 until the outbreak of the financial crisis shown in figures 1 and 2. The scenarios are shown in figure 3. Specifically, figure 3 shows the structural changes implemented in the model for each year. The 2002 scenario is thus the change in the outstanding mortgage stock in that year together with the change in taxation. The total scenario for the period 2002–06 is the lower envelope of the lines in the figure.

I analyze the transition path to the new steady state from these changes, focusing on the initial part from 2002 to 2007.[10] The effect from, say, taxation is the combined effect over the years, from the 2002 scenario to the 2005 scenario. I denote the effects from this analysis the *temporary effects*. I next look at the effects from these changes on the business cycle. That is, I analyze how these changes to the housing market have affected the transmission of shocks in the economy when the new steady state has been reached. Lastly, I will analyze the long-run or permanent effects from the changes to taxation and the introduction of new mortgage contracts. Here, I compare the level in the old steady state with the level of the variables in the new steady state.

Changes to the amortization rate are implemented through the process for $\delta_t^{\mathcal{A}}$. As an example, starting from 2004, the amortization rate is changed permanently such that the principal payments on the mortgage contract in the model reflect the average contract: In

[10] I thus simulate the model under certainty equivalence. Hence, households and firms know and can calculate their optimal response to the changes once these are revealed. I consider a sequence of simulations, each starting from the transition path from the previous change.

Figure 3. Changing Taxation on Housing Wealth and New Loan Types



Note: This figure shows the scenarios presented in section 4.

2004, the amortization rate in the scenario falls to approximately 15 percent, as in the data shown in figure 1. When the scenarios end in 2006, the rate has fallen by around 35 percent. Starting from 2002, almost all mortgage contracts were amortized over a thirty-year period. Therefore, the process for amortization, equation (3), is set such that it resembles a thirty-year mortgage annuity before 2002.

When I calculate second-order moments and impulse response functions (IRFs), I assume that the tax rate, τ_t^H, is constant but that the base is fixed to the steady-state value of the housing stock and the real house price. This implies that tax payments do not change with changes to the value of the housing stock, and hence the effective tax rate falls. I make permanent shocks to the tax rate each year with the size of the shock set such that the shock resembles as closely as possible the effective tax rate on housing in figure 1. In this way, the agents in the economy face a declining tax rate.[11]

[11]Naturally, this is in fact not how the tax rule works; it is only a permanent lower tax payment if the real house price increases above its level in 2002. In a

The change from fixed-rate mortgages to flexible-rate mortgages is implemented through changes in the markup, $\mathcal{M}_R$, on the lending rate for households, $R_t^{I,L}$, over the lending rate in the bank, $R_t^{I,L}$: $R_t^{I,L} = \mathcal{M}_R R_t^L$. In the steady state, the (annualized) lending rate to households is 5 percent, and this is taken as the starting point in 2002, the average interest rate paid on fixed-rate mortgages. In 2003, as an example, around 30 percent of the outstanding mortgage debt was already flexible-rate contracts, with the remaining being fixed rate; see figure 2. Empirically, the spread between fixed-rate and flexible-rate mortgages has been around $2\frac{1}{2}$ percent. Hence, in the scenario for the year 2003, I consequently lower the markup such that the lending rate to households is $4\frac{1}{4}$ percent.[12] I further change the parameter, θ_R, determining the average duration of the loan portfolio for the banks such that it reflects how often the interest rate can be changed.

To isolate effects from the sequence of changes made, I present four scenarios:

1. The tax rule for housing is changed from being paid on the actual value of the housing stock to the value on the steady-state value.

2. The change from fixed-rate to flexible-rate mortgage is introduced.

3. The change from amortization to interest-only loans is introduced.

4. All changes are introduced (scenario 1 to scenario 3).

The effect from these four scenarios is thus the sum of the changes to the housing market implemented each year.[13]

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model like the one used here with forward-looking agents, the permanent change in the tax rate will be capitalized into the real house price today, and hence, the effects shown later can be thought of as an upper bound given model uncertainty and its specification.

[12] $(70\% * 5\% + 30\% * (5\% - 2\frac{1}{2}\%)$ equals $4\frac{1}{4}\%)$.

[13] Changes to fiscal policy within a DSGE model also necessitate an instrument to stabilize public debt. I assume throughout that public debt is stabilized using

5. Results

5.1 Temporary Effects—The Period 2002–07

The immediate effects, defined as the effect during the period starting in the first quarter of 2002 and ending in the fourth quarter of 2006, are shown in figure 4 for the real house price, debt to households, and construction-to-GDP ratio. Here, I have also included the individual effects from the four scenarios to be able to understand the main drivers of the results. I compare the results with actual data.

Overall, according to the model, the changes in taxation of housing and the adoption of new mortgage contracts played an economically significant role in the development of both the real economy and financial markets. Take the real house price first. From figure 4 it can be seen that the model in fact explains too much of the developments in the real house price through the period 2001–04. From around 2004:Q2, the real house price rose too much to be explained by the change in taxation and the adoption of new mortgage contracts.

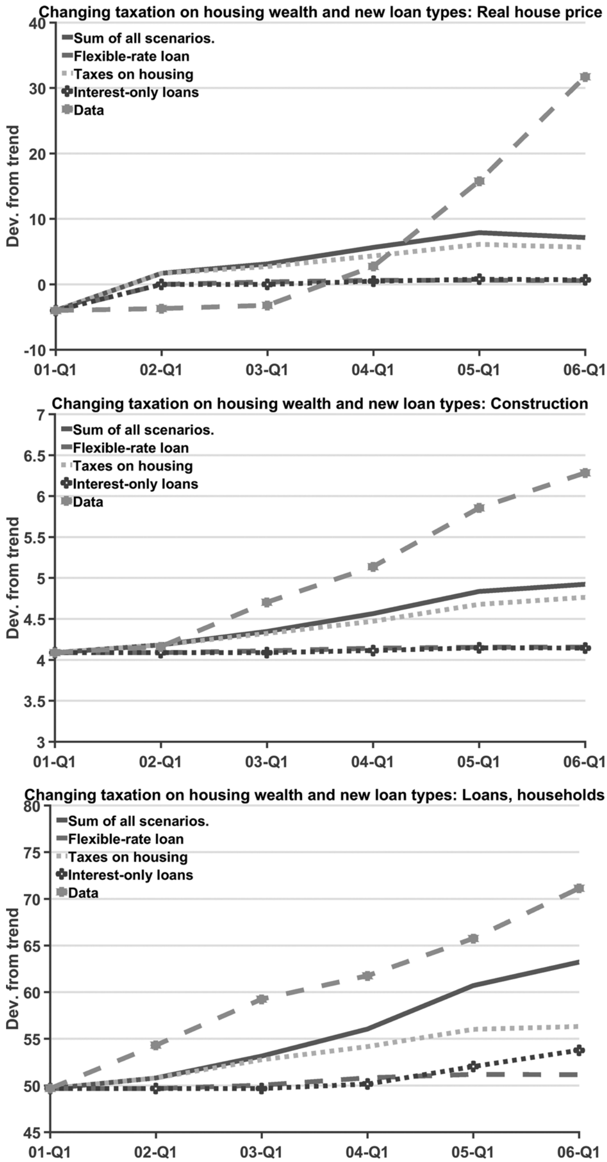
On the financial side, the debt-to-GDP ratio for households would have been almost constant. The main economic mechanisms were explained in section 3: Interest-only loans and flexible-rate loans make borrowing cheaper and relax the collateral constraint, fueling demand for both housing and consumption for impatient households. Taxation of housing alters relative after-tax prices of housing versus consumption goods.

In figure 5, the effects are shown as deviations from the initial steady state. I interpret these deviations as gaps. For the house price gap, the difference between the real house price and its steady state is 8 percentage points. Depending on the method to detrend the house price, the observed real house price gap was between 3 and 5 percentage points in 2004 and around 15 percentage points in 2005.[14] Hence, the model can explain around half of the real house price gap during this period. In 2006, the observed gap widens to close to

lump-sum taxation paid by patient and impatient households according to their relative size in the economy.

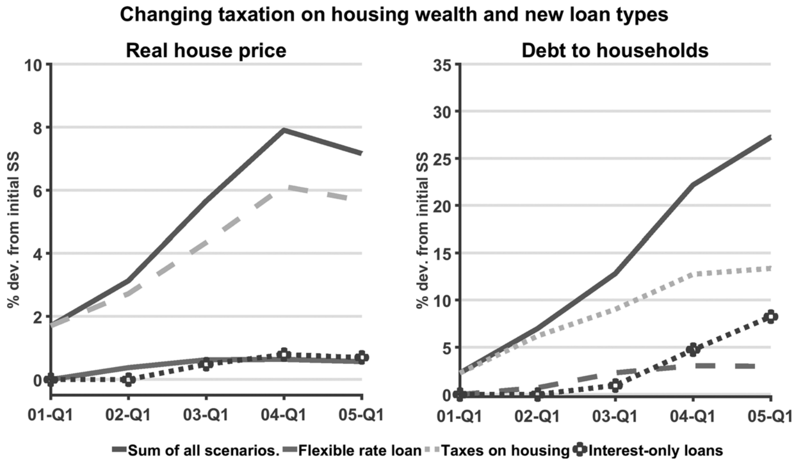
[14] In figure 4 is shown the detrended real house price using a linear trend and normalizing the series to 1 in 2001.

Figure 4. The Effect on the Real House Price, Construction, and Household Debt



Notes: The figure shows the effect from the scenarios presented in section 4 on the real house price, construction, and household debt. Only the temporary effects are shown corresponding to the period under study, which is 2002–06. In the figures are also shown data. To be able to compare data with the model simulations, the following calculations are done. For the real house price is shown detrended data normalized to 1 in 2000. The construction-to-GDP ratio is normalized in the model to 4 percent as it was in data, while the household debt-to-GDP ratio likewise is normalized to 50 percent.

Figure 5. The Effect on the Real House Price and Household Debt



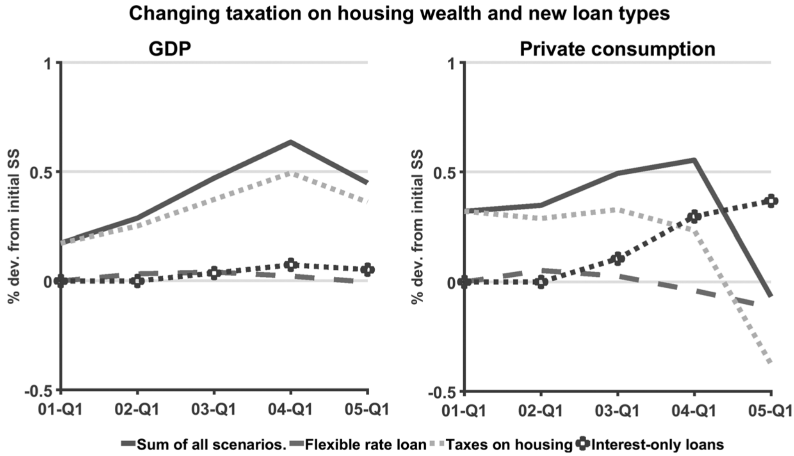
Notes: The figure shows the effect from the scenarios presented in section 4 on the real house price and household debt. Only the temporary effects are shown corresponding to the period under study, which is 2002–06. In the figures are also shown data. To be able to compare data with the model simulations, the following calculations are done. For the real house price is shown detrended data normalized to 1 in 2000. The construction-to-GDP ratio is normalized in the model to 4 percent as it was in data, while the household debt-to-GDP ratio likewise is normalized to 50 percent.

30 percentage points; the model thus cannot explain the subsequent movements in the real house price.[15]

As expected, the increase in the real house price stimulates construction, pushing the construction-to-GDP ratio up by 1 percentage point. The observed ratio was just above 6 percent, increasing from

[15]Naturally, this leaves a large part of the gaps unexplained. To explain this, I stress that I look at the impact of the two structural changes to the Danish housing market and nothing more; a lot of shocks hit the Danish economy during the period under study. In Pedersen (2016), the real house price is decomposed into structural shocks. There I find that foreign shocks, such as demand for Danish exports and foreign monetary policy shocks, as well as domestic productivity shocks, etc., contributed to the house price gap during the period under study. Further, housing preference shocks played a large part. These shocks can be interpreted as the consumers getting more utility of a given level of housing or that, given fundamentals, the households demanded more housing.

Figure 6. The Effect on the GDP and Total Private Consumption



Notes: The figure shows the effect from the scenarios presented in section 4 on GDP and total private consumption (sum of the consumption of patient and impatient households and the entrepreneur). Only the temporary effects are shown corresponding to the period under study, which is 2002–06. In the figures are also shown data. To be able to compare data with the model simulations, the following calculations are done. For the real house price is shown detrended data normalized to 1 in 2000. The construction-to-GDP ratio is normalized in the model to 4 percent as it was in data, while the household debt-to-GDP ratio likewise is normalized to 50 percent.

around 4 percent. Hence, the scenarios can explain almost half of the increase in the construction-to-GDP ratio. GDP is also stimulated, pushing the output gap in the model up by around 0.6 percentage point in 2004; see figure 6. The actual output gap was, as an example, $\frac{1}{4}$ percentage point in 2004 and $4\frac{1}{2}$ percentage points in 2006; see, as an example, Andersen and Rasmussen (2011). Hence, the changes in the scenarios can explain around one-third of the actual output gap. Total private consumption can explain part of this increase in the output gap: Total private consumption is $\frac{1}{2}$ percent above its steady state in 2004 in response to the changes considered in this study. The aggregate response of consumption covers the fact that consumption of patient households falls, while the consumption of impatient households increases due to the relaxation of the borrowing constraint.

Looking at the individual scenarios, the main driver has, according to the model, been the changes to taxation. This is so for the financial variables, the real house price and debt, as well as for the real variables, GDP, construction, and consumption. The reason why taxation plays such a large role is that it affects the user cost of housing for *both* types of households in combination with the collateral constraint. Lower taxes on housing wealth cannot be smoothed for the impatient households. These would, if they could, try to smooth the extra lump-sum tax payments needed to stabilize public debt.

The introduction of interest-only loans contributed significantly to the buildup of debt and consumption in the late part of the period under study. The reason why interest-only loans affect debt relatively strongly is due to their direct effect on the user cost of borrowing, which falls, as discussed in section 3.1.

5.2 Effects on the Business Cycle

In this section, I analyze the effect of the policy changes on the business cycle defined as the volatility of the endogenous variables in the model. I use the coefficient of variation as a measure of volatility or "uncertainty." To this end, I calculate the coefficient of variation, the standard deviation divided by the mean, of a subset of the macroeconomic variables in the model before any changes were made to the housing market in Denmark in 2002. I compare these estimates with the coefficient of variation calculated at the new steady state reached after implementation of the scenarios above. I simulate the model using the calibrated parameters set to the values estimated in Pedersen (2016) and the estimated standard deviations of the innovations to the structural shocks. I note that I keep these innovations fixed in all the scenarios such that it is not the size of the shocks that changes but the transmission mechanism in the model. In the case of taxation, I simply change the tax rule such that the tax rate and the base are kept constant at their steady-state value in 2002.

I will focus on the case in which all changes are made to the housing market. For comparison and for analyzing the effects from the different changes, I will also calculate the standard deviations in the other three scenarios presented in section 4. The results are shown in table 1.

Table 1. Theoretical Moments Before and After Changes to Market Structure for Housing

Variable	$\boldsymbol{Co\text{-}Var(j)^{OLD}}$	$\boldsymbol{Co\text{-}Var(j)^{NEW}}$	**Change (%)**	$\frac{Co\text{-}Var(j)^{OLD}}{Co\text{-}Var(Y_t)}$	$\frac{Co\text{-}Var(j)^{NEW}}{Co\text{-}Var(Y_t)}$	**ΔSS%**
		Scenario 1: Taxation				
GDP (Y_t)	1.04	1.04	−0.34	1.00	1.00	0.15
Real House Price (Q_t^H)	1.15	1.29	12.35	1.10	1.24	3.00
Residential Inv. (X_t)	3.01	3.28	8.98	0.13	0.14	9.25
Household Debt (B_t^I)	5.76	6.62	14.80	4.17	4.80	10.50
Total Private Cons. (C_t^{tot})	1.37	1.41	3.37	0.65	0.67	−0.50
Consumption, Patient (C_t^P)	0.81	0.82	1.65	0.29	0.30	−0.75
Consumption, Impatient (C_t^I)	4.80	5.29	10.26	0.39	0.43	0.25
Investment (I_t^Y)	2.64	2.62	−0.89	0.35	0.35	0.10
		Scenario 2: Fixed to Floating				
GDP (Y_t)	1.04	1.07	2.34	1.00	1.00	0.00
Real House Price (Q_t^H)	1.15	1.27	10.73	1.10	1.19	0.30
Residential Inv. (X_t)	3.01	3.42	13.46	0.13	0.14	1.00
Household Debt (B_t^I)	5.76	6.54	18.61	4.17	4.73	3.00
Total Private Cons. (C_t^{tot})	1.37	1.38	1.30	0.65	0.64	−0.15
Consumption, Patient (C_t^P)	0.81	0.88	7.78	0.29	0.31	−0.50
Consumption, Impatient (C_t^I)	4.80	5.79	22.97	0.39	0.46	1.30
Investment (I_t^Y)	2.64	2.77	5.02	0.35	0.36	−0.35

(*continued*)

Table 1. (Continued)

Variable	$\boldsymbol{Co\text{-}Var(j)^{OLD}}$	$\boldsymbol{Co\text{-}Var(j)^{NEW}}$	**Change (%)**	$\boldsymbol{\frac{Co\text{-}Var(j)^{OLD}}{Co\text{-}Var(Y_t)}}$	$\boldsymbol{\frac{Co\text{-}Var(j)^{NEW}}{Co\text{-}Var(Y_t)}}$	**ΔSS%**
Scenario 3: Interest-Only Loans						
GDP (Y_t)	1.04	1.04	−0.41	1.00	1.00	0.00
Real House Price (Q_t^H)	1.15	1.13	−1.56	1.10	1.09	−0.10
Residential Inv. (X_t)	3.01	3.03	0.59	0.13	0.13	−0.25
Household Debt (B_t^I)	5.76	5.45	−6.32	4.17	3.94	7.25
Total Private Cons. (C_t^{tot})	1.37	1.36	−0.03	0.65	0.65	0.00
Consumption, Patient (C_t^P)	0.81	0.81	−0.44	0.29	0.29	0.10
Consumption, Impatient (C_t^I)	4.80	4.86	2.71	0.39	0.40	−0.40
Investment (I_t^Y)	2.64	2.63	−0.21	0.35	0.35	−0.10
Scenario 4: All Changes						
GDP (Y_t)	1.04	1.06	1.36	1.00	1.00	0.05
Real House Price (Q_t^H)	1.15	1.37	19.70	1.10	1.30	3.00
Residential Inv. (X_t)	3.01	3.69	22.59	0.13	0.16	9.75
Household Debt (B_t^I)	5.76	6.92	23.55	4.17	5.01	21.50
Total Private Cons. (C_t^{tot})	1.37	1.43	4.53	0.65	0.67	−0.50
Consumption, Patient (C_t^P)	0.81	0.90	10.14	0.29	0.32	−1.25
Consumption, Impatient (C_t^I)	4.80	6.26	34.75	0.39	0.51	1.50
Investment (I_t^Y)	2.64	2.75	4.07	0.35	0.36	0.00

Notes: This table reports theoretical moments calculated in the model. "Co-Var" refers to the coefficient of variation (the ratio of the standard deviation to the mean). "Old" refers to the conditions for the housing market before the changes to taxation of housing and the introduction of new mortgage loans. Public debt is stabilized using lump-sum taxation. In the calculation of the moments, the estimated standard deviations of the shocks are taken from Pedersen (2016). ΔSS% refers to the difference between the steady state before the implementation of the scenario and after the new steady state has been reached. In the text, this is referred to as the permanent effects of the policy changes. "New" refers to the model in which the changes in the scenarios are made as described in the text.

Compared with the case before 2002, the real house price can be expected to vary by close to 20 percent more at the new steady state. This change in the volatility can be attributed mostly to the change in taxation and the introduction of flexible-rate mortgage contracts. The explanation behind the fact that house prices vary by more under a tax freeze is that tax payments do not change with the real house price, and hence, an automatic stabilizer is taken away. In the next section, I will explain in greater detail why flexible-rate mortgage contracts affect the volatility of house prices.

Naturally, the greater volatility of house prices also affects residential investments, which also have become around 20 percent more volatile. This also affects the volatility of debt to households, which has become 25 percent more volatile. Private consumption can, in the future, be expected to move by 5 percent more, which is mainly driven by the consumption of the impatient households; they are exposed to a collateral constraint, which has now become more volatile.[16]

I also calculate the relative standard deviation with respect to GDP. This is another measure of volatility in the economy. The real house price, construction, and especially household debt vary more with GDP after the changes to the housing market than before.

5.2.1 Effects on the Business Cycle—The Role Played by Flexible-Rate Contracts

While the change from fixed- to flexible-rate mortgage contracts has not had large immediate effects on the economy, it does contribute significantly to the change in the second-order moments. The explanation is that with a larger proportion of flexible-rate mortgage

[16]It is perhaps a bit surprising that the volatility of GDP does not change by more. Public consumption and exports are 26 and 38 percent of steady-state output, respectively, but the volatility of exports naturally does not change much in response to the changes in the economy, while public consumption is exogenous. Further, investments and consumption for the entrepreneur are small parts of output in steady state and their volatility does not change much; the changes on the housing market do not spill over to that part of the economy. Also, both construction and private consumption have a non-significant import content. Hence, although these variables become relatively more volatile, output is affected relatively little.

contracts in the economy, shocks which propagate through interest rates propagate stronger.

This result can perhaps on the outset seem counterintuitive. The shocks and parameters are estimated on Danish data for a period in which the Danish business cycle has followed closely the business cycle in the euro area, and therefore Denmark has on average imported a monetary policy stance which has been close to what the Danish economy would call for in terms of stabilization of inflation and output. Based on this insight, a higher degree of interest rate sensitivity should, all else being equal, be stabilizing and not destabilizing for the Danish economy as shown here. However, this insight only concerns the imported changes in interest rates and not domestically generated shocks. Even though the monetary policy rate is imported from the euro area, the interest rate faced by borrowers on their mortgage contracts does not vary one-to-one with the imported monetary policy rate, and it is not orthogonal to domestic shocks.

In detail, the lending rate faced by the households and firms is determined by the banks in the economy, and through the frictions in the model, specifically the capital ratio, all shocks which affect the borrowing decision for banks will affect lending rates. The wholesale branch of the bank chooses the overall amounts of lending and deposits of the bank so as to maximize profits. In addition, the wholesale branch takes into account a capital requirement imposed on banks: In any given period, it is costly for a bank to deviate from a target value $\kappa^B > 0$ for the bank's capital-to-assets ratio, $K_t^B\left(j^b\right)/B_t\left(j^b\right)$. The first-order condition yields

$$R_t^L = R_t^D - \Phi^B\left(\frac{K_t^B}{B_t} - \kappa^B\right)\left(\frac{K_t^B}{B_t}\right)^2. \tag{10}$$

The parameter $\Phi^B > 0$ is the cost of deviating from the capital ratio, κ^B; R_t^L denotes the gross interest rate charged by the wholesale branch on the loans it makes to the loan branch; and R_t^D is the gross interest rate paid by the wholesale branch on the funds it receives from the deposit branch.[17] In equilibrium, that interest

[17] Here is considered a symmetric equilibrium, such that I can continue with a representative bank in what follows.

rate is equal to the European Central Bank (ECB) monetary policy rate.

Condition (10) shows that if the capital-to-asset ratio (or inverse leverage ratio) of the bank falls short of its target ratio, the wholesale branch will charge a lending rate that exceeds the deposit rate, R_t^D, at which it remunerates its deposit branch so as to increase its capital ratio. The further away the actual capital ratio is from the target ratio and the more costly it is to do so, determined by the parameter Φ^B, the higher the spread. Equation (10) can therefore be interpreted as a loan supply schedule: When loans, B_t, increase, the capital-to-asset ratio falls below target, κ^B, and the bank is induced to raise the lending rate to balance the extra income from giving the loan with the cost of deviating from the optimal capital ratio.

The monetary policy rate imported from the euro zone affects the first term, R_t^D, one-for-one.[18] But all shocks which affect either desired borrowing or bank capital will affect the second term, $\Phi^B \left(\frac{K_t^B}{B_t} - \kappa^B\right)\left(\frac{K_t^B}{B_t}\right)^2$, lending rates to bank lending branches or mortgage institutions, and thus affect the interest rate faced by borrowers in the economy. Clearly, the more these lending rates respond to shocks, the greater are the effects on borrowing costs.

5.3 Permanent Effects

Lastly, I look at the permanent effects. I define permanent effects as the difference between the value of a variable in the model in the old steady state with respect to its value in the new steady state when reached. These values can be found in table 1.

I point to the following observations. The real house price is around 3 percent higher in the new steady state and construction has increased by almost 10 percent. Further, debt to households has increased by 20 percent. Firstly, this is due to the increasing demand for housing due to the relaxation of the collateral constraint, which has made the impatient household able to buy more housing financed by borrowing—the financial accelerator on the household side of the model—and, secondly, it is due to taxation, which has made the

[18]For simplicity, I disregard a small risk premium on the Danish net foreign asset position needed to ensure stationarity of that variable.

patient household increase its purchases of housing. Equilibrium has been restored through an increase in the house price, which stimulates construction. In the end, consumption for impatient households is almost constant in the new steady state: They pay fewer taxes on their housing stock and more interest and amortization on a now higher debt level. Consumption for patient households falls due to crowding out: GDP increases by a little, while construction increases by a lot, taking resources away from non-durable consumption and directing them to consumption of housing, and patient households are the providers of funds to the banking sector.

Investments do not change in the new steady state. Even though the entrepreneurs face slightly higher lending rates, which push borrowing and investments down, they can also post more collateral due to higher land rents. But these two effects effectively cancel out.[19]

The main driver for first-order moments has been taxation of housing wealth, and this is also so for the permanent effects except for interest-only loans on especially debt. The adoption of interest-only contracts has pushed borrowing up by 7 percent; in the model, the impatient household used the lower borrowing costs to take on more debt and thus come close to satisfying its desire to consume more now relative to the future.

5.4 Welfare Analysis

The welfare analysis in this paper is based on the standard approach used in the DSGE literature.[20] The aim is to evaluate the impact on welfare from changing the structure on the housing market in Denmark based on second-order approximations to the solution of the model, not to maximize welfare. The welfare function for each household is given by the expectation of lifetime utility. I augment

[19]Notice that if I had assumed that patient households were the landowners, then investment would fall in the economy and there would have been a traditional crowding out of real investments from higher residential investments. See Pedersen (2016) for a full presentation of the model used here.

[20]See, e.g., Lambertini, Mendicino, and Punzi (2013), Rubio (2011), and Schmitt-Grohé and Uribe (2006, 2007). I compute the welfare conditional on the initial states being the deterministic steady state, and I calculate the theoretical moments.

Table 2. The Impact on Welfare from the Changes to the Housing Market

Relative Welfare	V_t^P	V_t^I	W_t
Change to Taxation	1.32	1.18	1.16
Flexible-Rate Mortgage	1.81	1.72	1.70
Interest-Only Mortgage	1.08	0.97	0.95
After 2008	2.15	1.79	1.74

Notes: This table reports relative welfare with respect to the 2002 scenario. That is the case before the taxation of housing wealth was changed and before the introduction of new mortgage contracts. Numbers above 1 imply that welfare is lower than in the 2002 scenario.

the equilibrium conditions of the model with two equations and two variables denoting the welfare of each agent,

$$V_t^j = U\left(C_t^j, H_t^j, N_t^j\right) + \beta^j E_t\left[V_{t+1}^j\right], j = P, I.$$

Due to the heterogeneity of the households, I explore the implications for the welfare for each household. I also aggregate individual welfare in a social welfare function,

$$W_t \equiv \omega_W^P V_t^P + \omega_W^I V_t^I.$$

Here ω_W^P ω_W^I are weights. I follow Rubio (2011) and assume that they are given such that, given a constant consumption stream, the patient and impatient households achieve the same level of utility. To achieve this objective, the weights are set as $\omega_W^P = 1 - \beta^P$ and $\omega_W^I = 1 - \beta^I$. I stress that this is only one criterion out of many to define a social welfare function. The results are shown in table 2.

The results point to a welfare loss for both households in response to changes in taxation of housing wealth, and when flexible-rate mortgage contracts are introduced in the economy. The reason for this was discussed in section 5.2 and section 5.2.1 above. Interestingly, the introduction of interest-only loans is welfare improving for impatient households, which reflects that these households come closer to satisfying their desire for consumption today. The combined effect from changes in taxation and new mortgage contracts points to a welfare loss for both agents.

6. Results from a DSGE Model Compared with Results from a Traditional Macroeconometric Model

The introduction of new mortgage contracts has, according to this analysis, not had large effects. I am not the first to analyze the questions addressed here. Dam et al. (2011) use Danmarks Nationalbank's traditional backward-looking macroeconometric model, MONA.[21] Dam et al. (2011) find that the real house price without the introduction of interest-only mortgage contracts would have been around 15 percent lower in 2006–07, and 15 percent lower if the flexible-rate contracts had not been introduced. Without changes to the taxation, the real house price would have been around 7.5 percent lower in 2006–07. In total, the analysis in Dam et al. (2011) finds the combined effect to be around 28.5 percentage points. The current study finds similar effects with regards to the effect of taxation, while the introduction of new types of mortgage contracts is markedly lower, around 1 percent.

What can explain these large differences in these findings? First, it is important to point out that the model used in Dam et al. (2011) leaves 22.8 percentage points of the increase in the real house price from 1999:Q4 to 2007:Q1 unexplained. That is, the total increase in the real house price in Denmark during that period is 71 percent. The model in Dam et al. (2011) can explain in total 48.2 percentage points, leaving a residual of 22.8 percentage points unexplained. When the authors divide the total real house price increase into the three components addressed in this paper, they divide this residual of 22.8 equally among the different components, and this way they find that the three components addressed in this paper can explain a 40 percentage point increase in the real house price in the period from 1999:Q4 to 2007:Q1.[22]

[21] See Nationalbanken (2003) for a documentation of the model.

[22] The implicit assumption made in Dam et al. (2011) is that without the changes to the housing market addressed here, taxation and new types of mortgage contracts, the unexplained part of the real house price growth, the residual of 22.8 percentage points, would have been zero.

The actual contribution from the three components is thus lower, i.e., if the residual of 22.8 is not divided equally among the three components. Nevertheless, it still leaves a large difference in the results.

Which model—the structural DSGE model or a traditional macroeconometric model, MONA—is the most appropriate model for studying the questions put forth in this paper? It is well known that each type of model has its strengths and weaknesses. One big advantage of using traditional models is that they are flexible. It it thus relatively easy to, e.g., include and test whether interest-only loans affect the real house price using econometric techniques. However, this can also be a weakness, as there is no need to put a structure on why a variable has been included. Do the households react to the introduction of interest-only loans because they are short-sighted, because they are irrational, or because they are credit constrained? This can be important for policy analysis. In contrast, with microfounded models, rational expectations, and general equilibrium it is relatively more challenging to model why a new type of mortgage should affect the house price. But once the modeler has thought about why and how, say, interest-only mortgage contracts should and could affect the real house price, it is possible to test and discuss the exact assumptions in the model, in this case frictions on the financial markets.

While it is fairly easy to compute moments in a DSGE model, it is difficult in a traditional model to consider the impact from changes on second-order moments. That is, it is not straightforward to analyze the effect on the business cycle of changes to the economy. Also, typically, the traditional models lack forward-looking economic agents, which is also the case in MONA. This can be a serious shortcoming during the study of the real house price, which is a forward-looking variable with asset pricing behavior. Further, rational expectations, forward-looking behavior, and general equilibrium make it relatively more challenging to model large effects compared with the traditional macroeconometric models; as an example, lower taxation of housing wealth needs to be funded in the model used in this study. Lastly, the main reason why the DSGE models were developed is the Lucas critique, which is likely to be binding in the case of the questions addressed in this paper, as these questions concern policy changes.

6.1 What Can Explain the Differences between the Findings in a Traditional Model and the Results in the DSGE Model? Theoretical Insight

The most important financial friction in the model used in this paper is the presence of a borrowing constraint. As explained above, without that constraint, the presence of long-term debt contracts and amortization would have an effect on neither the real house price nor the real economy. As shown in Iacoviello (2005), this friction has the ability to match data for the wealth effect from housing on consumption for the United States. The approach used in the model is standard in the DSGE literature (see, e.g., Iacoviello and Neri 2010), but it has some inconveniences (see Barsky, House, and Kimball 2007 and Sterk 2010).

The house price is effectively determined by patient households via their Euler equation for housing,

$$\varsigma^{HP}\mathcal{H}_t\frac{1}{H_t^P}+\beta^P\left(1-\delta^H\right)E_t\left(\lambda_{t+1}^P Q_{t+1}^H\right)=\left(1+\tau_t^H\right)\lambda_t^P Q_t^H. \quad (11)$$

λ_t^P denotes the Lagrange multiplier associated with the budget constraint. $0<\beta^P<1$ is the discount factor for the patient household. $\mathcal{H}_t$ is a housing demand (or "taste") shock. The parameter $\varsigma^{HP}>0$ is used to calibrate the steady-state level of housing, $\delta^H>0$ is the depreciation rate of the housing stock, and τ_t^H is the tax rate housing property.

Let $U_t^H\equiv\varsigma^{HP}\mathcal{H}_t\frac{1}{H_t^P}$ denote the marginal utility of housing for the patient household. Equation (11) can be rewritten as

$$\lambda_t^P Q_t^H=\sum_{j=0}^{\infty}\left(\beta^P\left(1-\delta^H\right)\right)^j E_t\left[\frac{U_{t+j}^H}{\prod_{i=0}^{j}\left(1+\tau_{t+i}^H\right)}\right]. \quad (12)$$

Equation (12) shows that the real house price is effectively determined by patient households via their Euler equation for housing. Specifically, the real house price is tied to the marginal utility of consumption for patient households, λ_t^P, as the the right-hand side of equation (12) is almost constant.[23] The reason for this is that the

[23]This was first pointed out in Barsky, House, and Kimball (2007).

marginal utility of housing depends on the *stock* of housing. The stock of housing is not much affected by variations in the flow of durables, as housing is a very durable good: δ^H is small, or the flow of housing is small compared with the stock of housing ($\frac{I_t}{K_t}$ is small, where I_t and K_t are investments and the capital stock, respectively). Also, the shadow value of housing, the right-hand side of equation (12), depends for a great part on the marginal utility of housing in the distant future.

This implies that the marginal utility of housing does not move much in response to shocks, and this means that these agents' housing-demand schedule is almost flat: Since they display a very high intertemporal elasticity of substitution—the willingness to substitute housing through time—they are willing to sell part of their stock of housing to the impatient households, and therefore the housing stock shifts between the two types of households. This implies that, while the real house *price* is relatively insensitive to shocks including temporary shocks to the interest rate, the *quantity* of housing held by the patient and impatient households responds strongly.

The relation also implicitly shows why taxation can have a relatively large impact on the real house price. Changes in taxation, changes to τ_t^H, affect the user cost of housing both today and in all future periods, and this holds for *both* types of households. As they are rational and forward looking, the full change in taxation is taken into account today.[24] On the contrary, changes in financial conditions only affect impatient households, who wish to add to their housing stock, if they can, to relax the borrowing constraint. For the real house price to move, patient households need to increase consumption of consumption goods. The extent to which they do so depends on the stimulative effects from the actions of the impatient households through their higher demand for housing and consumption. The analysis below will compare how large this effect is compared with the traditional model.

[24]Notice that this points towards a strength of using forward-looking structural models and against the use of a traditional backward-looking model, as the latter cannot take into account the full capitalized effects of changes to taxation of housing wealth. It also points, using similar arguments as for the change in taxation, towards a large role for the housing demand shock, $\mathcal{H}_t$.

6.2 What Can Explain the Differences between the Findings in a Traditional Model and the Results in the DSGE Model? Looking at Correlations

Next I look into further explanations for the differences between the findings in this paper and the ones in Dam et al. (2011), keeping in mind the caveat put forth above—that the residuals in the regression in Dam et al. (2011) are included in their results. Is the difference between the reduced-form models and the DSGE that the real house price is much less responsive to various variables in the DSGE model? Or is the issue that the explanatory variables move less in the DSGE model than in the real world? I will focus on the interest rate and the effect of amortization on the real house price, as the differences between the findings with regards to the effect of changes to taxation are relatively small across the two studies.

To this end, I take the mean of 1,000 simulated data series of length 400 quarters from the DSGE model using the estimated structural shocks from Pedersen (2016).[25] I estimate the house price relation from MONA on this simulated data set.[26] To analyze how the coefficients vary, I change the degree of flexible-rate loan contracts and interest-only contracts, and I introduce the tax freeze in the model. I then compare the coefficients in MONA's house price relation with coefficients estimated using simulated data from the DSGE model.

The results are shown in table 3. The coefficients vary through the different scenarios in an intuitively compelling way. As an example, the interest rate pass-through is higher in scenarios 2, 4, and 6, where the interest rate on the mortgage contracts are flexible rate. And this is more so when debt is one-period debt only, as is the case in scenarios 4 and 6.[27]

Comparing the coefficients with the estimated coefficients in MONA, there are differences and similarities.

Firstly, the size of the income effect on housing, coefficient β_7, is roughly three times as large in the DSGE model as the coefficient

[25] This is standard in the DSGE literature; see, e.g., Iacoviello and Neri (2010).

[26] In appendix 2, details of the house price relation in MONA are presented.

[27] It also shows one advantage of using structural models, as discussed at the beginning of this section, namely the ease with which structural changes to the economy can be made and analyzed in a transparent and straightforward way.

Table 3. Estimating House Price Relation

ΔQ_t^H	α	$\beta_1 \times \Delta Q_{t-1}^H$	$\beta_2 \Delta((1-\kappa_t^{RI}) \times R_t^{L,I,long} + \tau_t^H)$	$\beta_3 Q_{t-1}^H$	$\beta_4((1-\kappa_{t-1}^{RI}) \times R_{t-1}^{L,I} + \tau_{t-1}^H - \pi_{t-1}^e)$	$\beta_5((1-\kappa_{t-1}^{RI}) \times R_{t-1}^{L,Imin} + \tau_{t-1}^H + \delta_{t-1}^A)$	$\beta_6 \Delta Q_{t-1}^{H,e}$	$\beta_7 Y_{t-1}^{income}$
MONA	0.09253	0.37519	3.82471	0.03960	0.18606	−0.35146	0.09830	0.07653
	(1.95393)	(6.20071)	(10.7836)	(4.18644)	(2.14953)	(3.22649)	(3.48939)	(1.63624)
DSGE (1)	1.2501	0.48496	1.6757	−0.0012225	−18.7092	−1.1752	0.1695	0.29958
	(4.7157)	(6.9153)	(3.7833)	(−0.69154)	(−3.6962)	(−4.5117)	(0.73261)	(6.7258)
DSGE (2)	0.7705	0.31341	−0.085165	−0.0013482	−5.8955	−2.1944	0.56882	0.29377
	(5.3281)	(3.9243)	(−0.30147)	(−0.83805)	(−1.7721)	(−5.4666)	(2.8135)	(7.7327)
DSGE (3)	−37.1727	0.19262	1.4482	−0.0019643	−4.4928	—	1.0611	0.3503
	(−6.2316)	(2.2872)	(4.6849)	(−1.3265)	(−1.2649)	—	(5.1141)	(9.0026)
DSGE (4)	−20.1149	0.29797	0.032954	−0.0011698	−5.3293	—	1.0087	0.29936
	(−4.4774)	(3.4879)	(0.13088)	(−0.78738)	(−1.8993)	—	(4.9207)	(8.1742)
DSGE (5)	1.8306	0.50322	1.8351	−0.00089977	−29.301	−1.2315	−0.16661	0.3821
	(5.9029)	(7.6997)	(3.5221)	(−0.52354)	(−5.1148)	(−5.1021)	(−0.73825)	(7.2644)
DSGE (6)	−24.4727	0.25718	0.16394	−0.00052715	−6.3403	—	1.0088	0.36017
	(−5.0094)	(3.0591)	(0.60877)	(−0.36765)	(−2.1185)	—	(5.0358)	(8.8082)

Notes: The table shows the estimates from the house price regression in MONA and the coefficients estimated in a corresponding regression using simulated data from the DSGE model. Scenario (1): Thirty-year fixed-rate mortgages and no tax freeze. Scenario (2): Thirty-year flexible-rate mortgages and no tax freeze. Scenario (3): One-quarter fixed-rate mortgages and no tax freeze. Scenario (4): One-quarter flexible-rate mortgages and no tax freeze. Scenario (5): Thirty-year fixed-rate mortgages and tax freeze. Scenario (6): One-quarter flexible-rate mortgages and tax freeze.

in MONA's house price regression. The source of the larger income effect in the DSGE model is collateral-constrained households; if the share of the credit-constrained households decreases from 0.4 to 0.1, the coefficient in scenario 1 falls to roughly 0.09 with a t-statistic of 3.8.[28]

Secondly, the DSGE model finds larger coefficients on the future expected house price, β_6, than in MONA. When using the DSGE model's expected house price, Q^H_{t+1}, in scenario 1, and not the adaptive structure used in MONA, the coefficient changes to almost 2.[29] At the same time, lagged house price changes become negative. This can reflect the importance of forward-looking households in the housing market.

Thirdly, the level of user cost in the DSGE model plays a relatively large role for the determination of the real house price. This can be seen from coefficients β_4 and β_5, which are more negative than in MONA.[30]

Fourthly, while the change in the (after-tax) interest rate in MONA, coefficient β_2, is statistically significant and equal to around 3.8 in MONA, this coefficient varies a lot more over the various scenarios. And overall, the coefficient is around half the size as in MONA, reflecting lower interest rate sensitivity. I conclude and confirm the theory setup above that the main difference between the coefficients estimated using actual data and simulated data is the interest rate sensitivity.

6.3 Comparing the Simulated House Price Series with Data

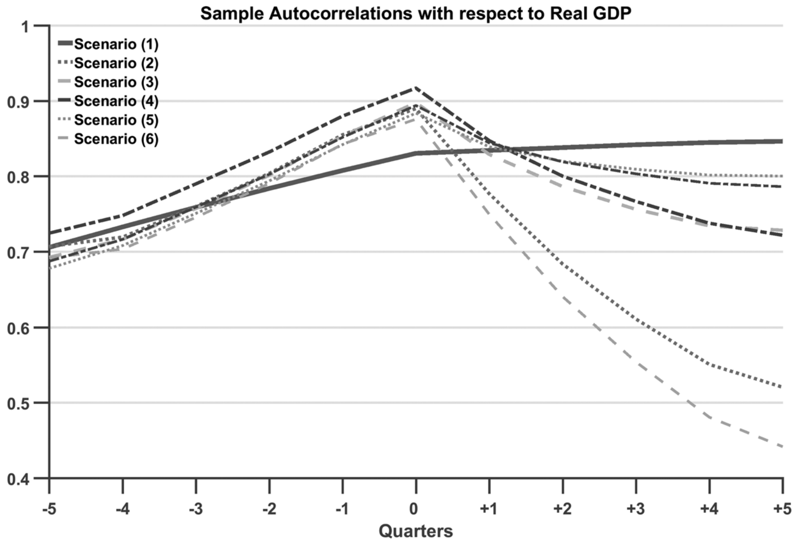
The previous section looked at the size of the correlations between explanatory variables and the real house price. Next I look at the consequences for the real house price from lower interest rate sensitivity. Firstly, comparing moments of the simulated house price series with the actual house price series, the simulated series for the real house price show less persistence and correlate less with

[28]This scenario is not presented in table 3 for brevity.

[29]This scenario is not presented in table 3 for brevity.

[30]I have not included variable 5 in the regressions in scenarios 3, 4, and 6. This is to avoid multicollinearity. In the DSGE model there is no distinction between short and long interest rates. Thus, when the amortization rate is constant, variables 4 and 5 become highly correlated.

Figure 7. Cross-Autocorrelations between Real House Price and GDP



Notes: The figure shows the cross-autocorrelations between GDP and the real house price for various leads and lags. In the figure is shown the empirical correlations together with the correlations calculated in the six scenarios presented in table 3.

the business cycle compared with data; see figure 7.[31] This points toward a lack of persistence of key macroeconomic variables in the DSGE model.

Next I simulate the house price relation from MONA using actual data, changing the coefficient on the interest changes, β_2, to the estimated value in scenario 1; see table 3. The result is shown in figure 8. MONA had trouble fitting the large house price increases during the 2000s in Denmark.[32] Use of the coefficients from scenario 1 worsens the fit. Specifically, the residual changes from around 10 percentage points at the peak of the bubble to around 25 percentage points. This is an indication that the data need a higher interest rate sensitivity than provided by the DSGE model.

[31]That the DSGE model has problems fitting the persistence of the house price is not specific for the model in use; see, e.g., WGEM Team on Real and Financial Cycles (2018).

[32]See the in-depth discussion of this in Dam et al. (2011).

Figure 8. Actual and Simulated Real House Prices

Notes: The figure shows actual real house price and the fitted house price using the relation for the real house price in MONA. The last house price series is constructed using MONA relation, but changing the coefficient for the change in the tax-adjusted interest rate, β_2, in relation (16), with the corresponding estimated value in scenario 1 shown in table 3.

I conclude that using a DSGE model to study the implications of policy changes to the housing market has clear merits. Firstly, the model, being structural, should be robust to the Lucas critique. And forward-looking agents can be important when looking at house prices, which is clearly a forward-looking variable. Further, the DSGE model can provide an answer to the effects of the changes studied to the Danish economy on the business cycle in a clear and straightforward manner. However, I have also shown that the house price in the model used in the current study is relatively less sensitive to changes in interest rates and shows less persistence than a traditional macroeconometric model.[33] As shown in section 6.1, the problem in the DSGE model is the straitjacket imposed on the real house price from the first-order condition for the patient

[33] This is not specific to the current model, as it is built upon a widely used setup with regards to the housing sector.

households—shocks need to be very persistent to generate movements in the real house price. This feature of the model helps to explain the outcome of the regressions conducted in section 6.2. Here I found smaller coefficients on the change in the (after-tax) interest rate on the change in the real house price in the traditional macroeconometric model used at Danmarks Nationalbank compared with the DSGE model. This lower interest rate sensitivity means that, compared with the findings using a traditional macroeconometric model, the current study finds relatively small effects of the introduction of new mortgage contracts, while the effect of changes to the taxation of housing wealth is within the same range. This points towards further research into how to model interplays between the housing market and the real economy.

7. Reflections Outside the Model and Policy

As emphasized previously, according to the model, GDP is not affected greatly by the changes to the market structure for housing. However, this is likely to be a lower bound for the estimate of these effects. As shown in this study, both the volatility in debt and the real house price can be expected to be higher in the future after the introduction of new mortgage contracts and the tax freeze. And large swings in asset prices a priori lead to increases in default rates as house prices and economic activity fall after negative shocks to the economy. In a model with household default, bank default, and/or default in the production sector, this can be expected to lead to larger effects on output.

Should the introduction of new loan types have been postponed to after a bust or indefinitely? In practice, though costly, it was possible to get interest-only loans before 2002 in Denmark. This could be done by refinancing the mortgage contracts whenever the value of the debt was below the LTV ratio. As such, the introduction of interest-only loans can be regarded as "legalizing" an already widely used practice with the added value for the households that it has become cheaper to get interest payments only. What the model does not capture is that the amount not used to pay principal on an interest-only loan can be used to pay down other types of debt, which often are more expensive, as they cannot be used as collateral, such as credit card debt, student loans, or car loans. And interest-only loans can further play the role of a buffer to unexpected temporary shocks to income, such as spells of unemployment. Neither of these effects are modeled here.

While the focus in this paper is narrow, considering only Denmark, the results can be of interest for countries considering liberalizing financial markets and for countries which conduct macroprudential policies to limit growth in house prices. As an example, the macroprudential regulator can use stricter requirements for amortization and/or requirements for the type of loans that borrowers can get in terms of the length of the interest rate fixing to limit speculative behavior on the housing markets.

Should Danish policymakers have responded to the buildup of debt and rising house prices? The question about pricking asset bubbles through leaning against the wind, or cleaning up after the bubble has burst, remains unanswered. In a model which shares some characteristics of the model in the current study, Gelain, Lansing, and Natvik (2015) study the question about leaning against the credit cycle. They show that a policy in which the monetary policy rate responds to household debt induces equilibrium indeterminancy. I leave an analysis of macroprudential tools within the current setup to future research.

8. Conclusion

In this paper I have asked: What are the effects from the introduction and adoption of new types of mortgage contracts and a freeze on the nominal tax paid on housing wealth on the real house price, household debt, and real GDP during the period 2002–08? I found that the two changes to the structure of the housing market in Denmark significantly affected both the financial side and the real side of the Danish economy during the period of study. Specifically, the combined effect of the changes can explain around an 8 percentage point increase in the real house price and 0.75 percent of the output gap. I further showed that these changes have had implications for the business cycle—equivalent shocks propagate stronger after the changes to the housing market. Using the coefficient of variation of the changes in the variables as a measure of the business cycle, the real house price and construction have become 25 percent and 15 percent more volatile, respectively, while debt to households has become 50 percent more volatile.

I compared the results from this analysis with results from a similar study using a traditional macroeconometric model instead of a DSGE model. I have shown that the DSGE model provides a relatively low sensitivity of the real house price to changes in interest rates. I point to a need for further research. Firstly, the term premium is not explicitly modeled; it is simply assumed to be the markup over the wholesale lending rate. It therefore reflects neither credit nor duration or liquidity risk. This leads to the second point: There are no defaults in the model. Hence, the model cannot be used to study the welfare effects of financial stability, e.g., high levels of debt. Within the model used in this paper, I could only address the volatility of high leverage, but not, e.g., risk of default and its dependence on the debt level. Thirdly, future research could analyze microfoundations for the choice of mortgage contracts and include it within a DSGE model.

Appendix 1. How Do Amortization and Flexible-Rate/Fixed-Rate Contracts Affect the Transmission of Shocks in the Economy?

The aim of this appendix is to show how temporary shocks to the amortization rate and the risk premiums affect the economy. This is done both through the analysis of the respective impulse response functions and through the analytical expressions in the model. Lastly, I show how the change to flexible-rate interest-only contracts from a situation with fixed-rate contracts with amortization affects the transmission of shocks in the economy.

Firstly, I look at the effect in the model of a shock to the amortization rate, which is seen as the effect of increasing the part of the stock of outstanding mortgage contracts which are interest-only contracts. Secondly, I will look at the effect in the model of a shock to the risk premium reflecting a change in the stock of outstanding mortgage contracts which are flexible-rate contracts.

I will calibrate the parameters to the values estimated in Pedersen (2016) and keep them fixed throughout the scenarios. I will then vary new parameters introduced in the model extensions. The idea is to calibrate the model under the structure of the housing market before 2002, in which all mortgage contracts were fixed rate with amortization and thirty-year maturity, and the situation around

2008, in which a part of the debt contracts was flexible rate without amortization. In the 2002 calibration, I calibrate the parameters in the amortization process, κ^{δ} and α^{δ}, such that the amortization rate reflects a mortgage contract with a maturity of thirty years. For the 2008 calibration, these parameters are set such that the average loan contract is of one-period maturity, basically collapsing the model into a standard DSGE model with one-period debt contracts. That was not the case in 2008, but this is done to highlight the mechanism played by long-term debt.[34] I change the degree of how often the banks can change the interest rate on their loans, θ_R, or, equivalently, the degree of flexible-rate contracts in the portfolio from approximately 0.95 to $\frac{2}{3}$. Everything else is kept constant, including the taxation of housing wealth.

The Effect of a Shock to the Amortzation Rate

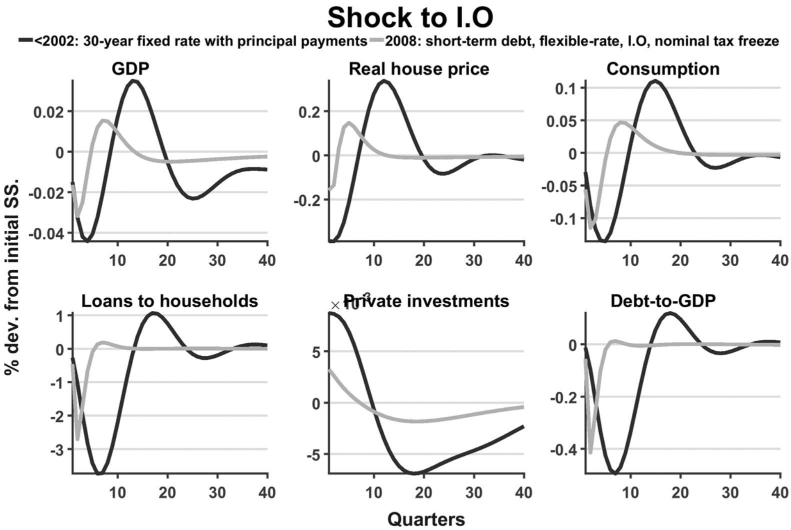
Figure 9 shows a temporary shock to the amortization rate keeping the maturity of the contracts fixed. Here is considered a higher amortization rate; i.e., $\delta_t^{\mathcal{A}}$ increases. Recalling that higher amortization implies that a larger part of the outstanding debt is paid back each period, this, all else being equal, implies that the household is forced to pay more during the period of the shock before the household can borrow more. The increase in principal payments drags down borrowing, and the borrowing constraint tightens. One unit of housing is less valuable in the collateral constraint, and the house price falls. As can be seen in the figure, this pushes consumption and output down.

These effects can be explained through the collateral constraint, equation (7), and the first-order equation for the impatient households with respect to borrowing,

$$\lambda_t^I - \mu_t^I = \beta^I E_t \left[\lambda_{t+1}^I \frac{1}{\pi_{t+1}^{DK}} \right] R_t^{L,I} - \beta^I E_t \left[\frac{\mu_{t+1}^I}{\pi_{t+1}^{DK} \left(1 + \Theta_{t+1}^I\right)} \right] \left(1 - \delta_t^{\mathcal{A}}\right). \tag{13}$$

[34]To see how the model collapses into a model with one-period debt contracts, see section 3.1.

Figure 9. Impulse Response Functions



Notes: The figures show impulse response functions for a subset of the macroeconomic variables with respect to shock to amortization rate. The size of the shock is held constant. The parameters are calibrated to the estimated and calibrated parameters in Pedersen (2016) except for a subset of parameters to be able to define a 2002 calibration and a 2008 calibration. These two calibrations reflect the structure for the housing market in 2002, respectively, with a thirty-year fixed-rate mortgage contract with principal payments, and in 2008 with a flexible-rate, short-term debt contract and a nominal tax freeze.

Introducing the collateral constraint in equation (13), the first-order condition with respect to borrowing for the impatient households can be written as

$$\lambda_t^I - \mu_t^I = \beta^I E_t \left[\lambda_{t+1}^I R_t^{L,I} \frac{1}{\pi_{t+1}^{DK}} \right] - \beta^I E_t \left[\frac{\mu_{t+1}^I}{\pi_{t+1}^{DK} \left(1 + \Theta_{t+1}^I\right)} \right] \left(1 - \delta_t^{\mathcal{A}}\right) \Leftrightarrow$$

$$\frac{\mu_t^I}{\lambda_t^I} \equiv \psi_t = 1 - E_t \left[\frac{\lambda_{t+1}^I}{\lambda_t^I} \beta^I \frac{1}{\pi_{t+1}^{DK}} R_t^{L,I} \right] + E_t \left[\beta^I \frac{\mu_{t+1}^I \lambda_{t+1}^I}{\lambda_t^I \lambda_{t+1}^I \pi_{t+1}^{DK} \left(1 + \Theta_{t+1}^I\right)} \right] \left(1 - \delta_t^{\mathcal{A}}\right) \Leftrightarrow$$

$$\psi_t = 1 - E_t\left[M_{t+1}R_t^{L,I}\right] + E_t\left[M_{t+1}\psi_{t+1}^I \frac{\left(1-\delta_t^{\mathcal{A}}\right)}{\left(1+\Theta_{t+1}^I\right)}\right]. \quad (14)$$

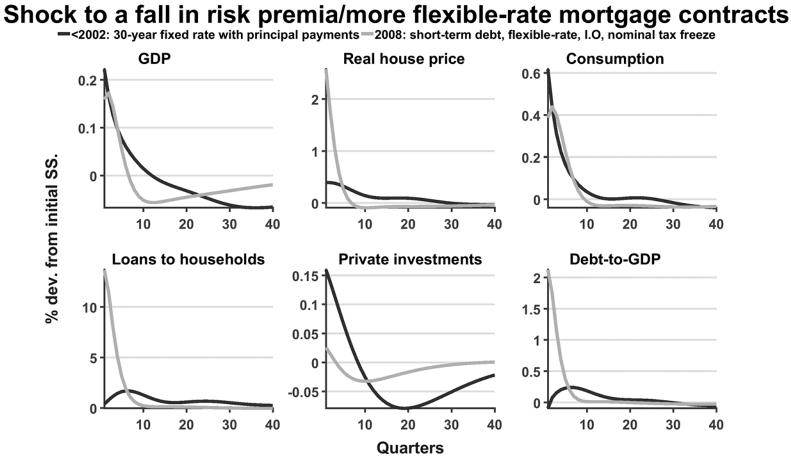
Here I have introduced the stochastic discount factor, $E_t\left[M_{t+1}\right] \equiv E_t\left[\beta^I \frac{\lambda_{t+1}^I}{\lambda_t^I}\frac{1}{\pi_{t+1}^{DK}}\right]$. It can be seen from this expression that the marginal benefit in terms of utility of acquiring an extra unit of debt, ψ_t, depends on the utility gain from consuming the proceeds minus the cost of doing so, $1 - E_t\left[M_{t+1}R_t^{L,I}\right]$, plus the value of the stock of debt not amortized discounted to the present, the last term, $E_t\left[M_{t+1}\psi_{t+1}^I \frac{\left(1-\delta_t^{\mathcal{A}}\right)}{\left(1+\Theta_{t+1}^I\right)}\right]$. This term is decreasing in the amortization rate and in the loan-to-value rate.

Long-term debt and the collateral constraint introduce two new channels relative to the model with one-period debt contracts only: Firstly, when choosing how much to borrow, the household takes into account that new borrowing today is long term and is hence committed to pay interest rate and principal on the stock of debt not only today but also in the future. This is reflected in the last term in equation (14). And secondly, the stock of debt in the future, and not only in the present, determines the amount of new debt the household can take on; the impatient households smooth their borrowing due to this forward-looking behavior.

This also implies that a tighter collateral constraint, a strictly positive value of the multiplier, $\mu_{t+j}^I, j > 1$, is not reflected one-to-one in new borrowing; the problem for the households of borrowing has become more dynamic in the sense that when households optimize their borrowing, they need to take into account the state of the economy today and in the future. This makes shocks which affect debt more persistent. It can also be seen that increases in the amortization rate, $\delta_t^{\mathcal{A}}$, translate directly into a tightening of the collateral constraint, which will allow the household to borrow less. This will in turn move house prices, and a financial accelerator mechanism kicks in from house prices to borrowing. Naturally, the quantitative effects depend in general equilibrium on the behavior of savers and banks.

From equation (13) it can be seen that if the consumer does not have a borrowing limit, i.e., when the collateral constraint does

Figure 10. Impulse Response Functions



Notes: The figures show impulse response functions for a subset of the macroeconomic variables with respect to shock to the mortgage rate. The size of the shock is held constant. The parameters are calibrated to the estimated and calibrated parameters in Pedersen (2016) except for a subset of parameters to be able to define a 2002 calibration and a 2008 calibration. These two calibrations reflect the structure for the housing market in 2002, respectively, with a thirty-year fixed-rate mortgage contract with principal payments, and in 2008 with a flexible-rate, short-term debt contract and a nominal tax freeze.

not bind and the multiplier always is zero, $\mu_t^I = 0 \forall t$, the first-order condition for borrowing collapses into the standard Euler equation. Hence, it can explicitly be seen, as suggested above, that it is the introduction of financial frictions which gives the potential to alter the equilibrium dynamics under long-term debt contracts. If all debt is one-period debt, $\forall t \ \delta_t^{\mathcal{A}} = 0$, then the model collapses into the standard collateral constraint as in Iacoviello (2005) or Pedersen (2016).

The Effect of a Shock to Flexible-Rate/Fixed-Rate Mortgage Contracts

Figure 10 shows the effect of a shock to the risk premium between fixed- and flexible-rate mortgage contracts in the two calibrations, or states, of the economy. The shock is a temporary shock. Here is considered a lower risk premium; $\frac{\epsilon_R}{\epsilon_R - 1}$ decreases. This shock can be interpreted as a higher degree of flexible-rate mortgage contracts in the economy.

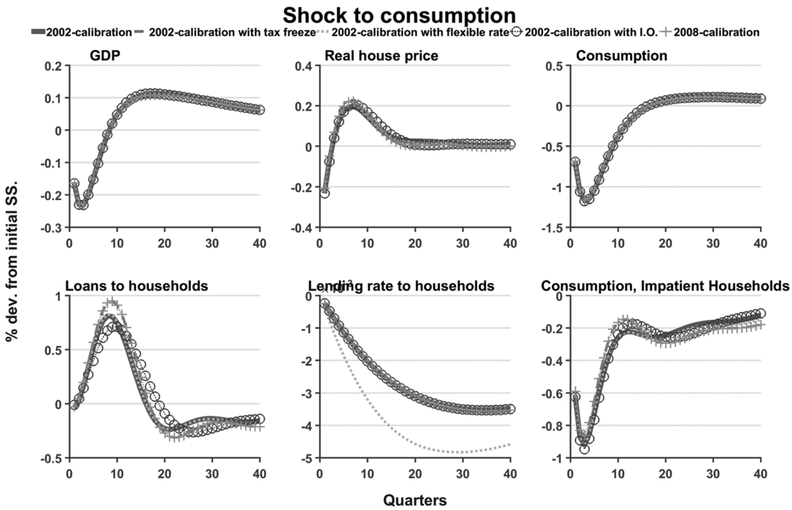
A decrease in the markup, here interpreted as borrowers facing a lower mortgage rate, increases borrowing for both firms and households, as borrowing has become cheaper. This naturally fuels debt: The impatient household can get closer to satisfying its desire for consumption now by buying more housing, taking on more debt, and consuming the proceeds. Consumption increases in both calibrations. This is due to the financial accelerator effect on the household side: The real house price increases, which relaxes the collateral constraint further, leading to more borrowing whose proceeds can be consumed.[35]

Borrowing increases by more in the 2008 calibration. This is due to debt being short term and especially because the shock to the lending rate is transmitted much faster due to flexible rates. The extra housing is met partly by extra residential investments and partly by patient households, who sell part of their housing stock. The largest effect on the real house price is in the 2008 calibration, which can be explained through the same mechanism for the response of consumption: Loans increase by more in the 2008 calibration due to a relatively lower duration of the outstanding mortgage contracts. Hence, borrowing can respond faster to changes in mortgage rates and consequently the real house price can increase by more.

The major differences between the two calibrations are exactly the response of debt. Specifically in the 2008 calibration, debt responds much faster than in the 2002 calibration. And interestingly, the debt-to-GDP ratio actually falls in the 2002 calibration due to a denominator effect, while this ratio increases in the 2008 calibration. The response to leveraging from changes in interest rates has been debated recently in the literature. The motivation has been the question about what monetary policy should do in response to increases in debt and asset prices; see Svensson (2013).

[35]Here, the assumption that the collateral constraint always binds becomes crucial. This issue is discussed in depth in, e.g., Iacoviello (2005).

Figure 11. Impulse Response Functions



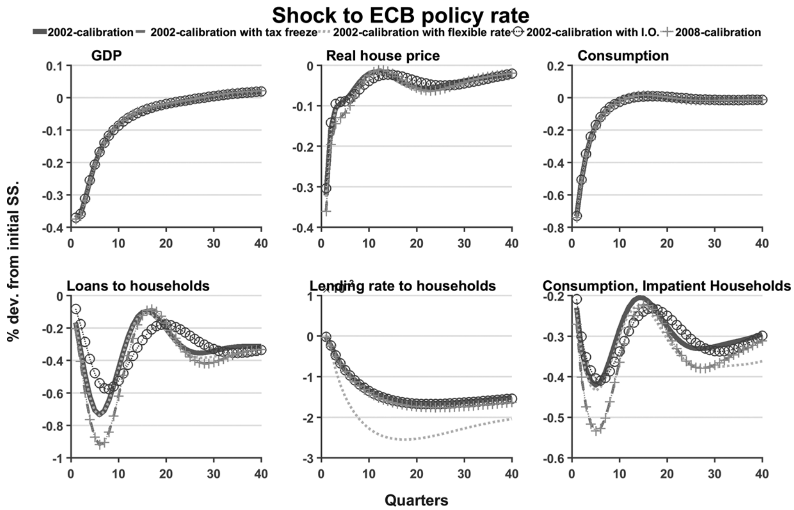
Notes: The figures show impulse response functions for a subset of the macroeconomic variables with respect to an intertemporal preference shock. I consider the following different calibrations of the model: (i) 2002 calibration: long-term fixed-rate debt and no tax freeze; (ii) 2002 calibration with tax freeze; (ii) 2002 calibration with flexible-rate loans; (iv) 2002 calibration with interest-only loans; (v) 2008 calibration: short-term debt with flexible rate, tax freeze.

How Do Long-Term Debt Contracts and Fixed-Rate/Flexible-Rate Mortgages Change the Transmission of Shocks?

How do long-term debt and fixed-rate/flexible-rate mortgages change the transmission of shocks in the economy? That is, how does, say, a shock to the preferences for housing propagate through the economy under flexible-rate, fixed-rate, long-term, and short-term mortgage debt contracts?

To answer this, I look at a set of IRFs for a set of variables and for a set of shocks. Figures 11–14 show the effect of a shock to four representative shocks—a risk shock, $\epsilon^{\mathcal{R}}$, a temporary productivity shock, $\epsilon^{A_{Y,T}}$, a housing preference shock, ϵ^{H}, and an intertemporal consumption shock, $\epsilon^{\mathcal{C}}$—on GDP, the real house price, consumption, loans to households, debt-to-GDP ratio, and inflation.

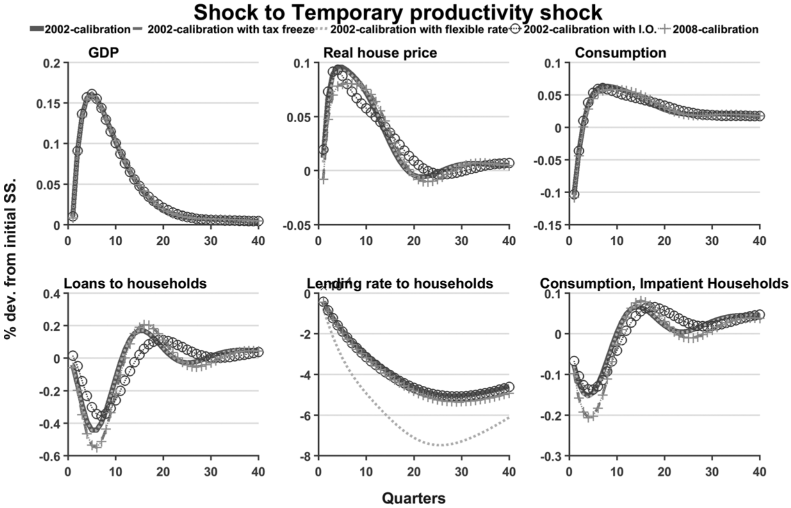
Figure 12. Impulse Response Functions



Notes: The figures show impulse response functions for a subset of the macroeconomic variables with respect to a shock to the ECB monetary policy rate. I consider the following different calibrations of the model: (i) 2002 calibration: long-term fixed-rate debt and no tax freeze; (ii) 2002 calibration with tax freeze; (ii) 2002 calibration with flexible-rate loans; (iv) 2002 calibration with interest-only loans; (v) 2008 calibration: short-term debt with flexible rate, tax freeze.

Pedersen (2016) provides a detailed discussion presentation of how the shocks considered here propagate through the model. I will consequently only be brief. The risk shock, $\epsilon^{\mathcal{R}}$, can be interpreted as a funding shock for banks, increasing the marginal costs of providing loans. A positive risk shock will thus increase the interest rate on all newly issued loans, or refinancing of existing loans, and thus make credit more expensive for the borrower. A temporary productivity shock, $\epsilon^{A_{Y,T}}$, works as a standard temporary productivity shock, while a housing preference shock, ϵ^{H}, makes households demand more housing, both for given expectations of the future and for given user costs. An intertemporal consumption shock, $\epsilon^{\mathcal{C}}$, makes households more impatient, lifting consumption from the future to the present.

Figure 13. Impulse Response Functions



Notes: The figures show impulse response functions for a subset of the macroeconomic variables with respect to a temporary productivity shock. I consider the following different calibrations of the model: (i) 2002 calibration: long-term fixed-rate debt and no tax freeze; (ii) 2002 calibration with tax freeze; (ii) 2002 calibration with flexible-rate loans; (iv) 2002 calibration with interest-only loans; (v) 2008 calibration: short-term debt with flexible rate, tax freeze.

To isolate the effects of the various new channels in the model, I consider the following different calibrations of the model:

(i) 2002 calibration: long-term fixed-rate debt and no tax freeze;

(ii) 2002 calibration with tax freeze;

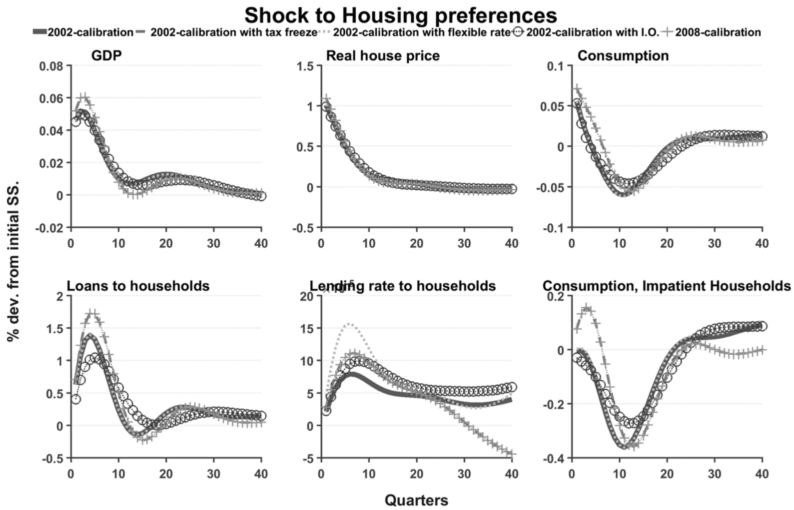
(iii) 2002 calibration with flexible-rate loans;

(iv) 2002 calibration with interest-only loans;

(v) 2008 calibration: short-term debt with flexible rate and tax freeze.

Overall shocks are transmitted in fairly similar ways across the models with regard to GDP. Long-term debt, as expected, introduces undershooting and overshooting in the responses. The

Figure 14. Impulse Response Functions



Notes: The figures show impulse response functions for a subset of the macroeconomic variables with respect to a housing preference shock. I consider the following different calibrations of the model: (i) 2002 calibration: long-term fixed-rate debt and no tax freeze; (ii) 2002 calibration with tax freeze; (ii) 2002 calibration with flexible-rate loans; (iv) 2002 calibration with interest-only loans; (v) 2008 calibration: short-term debt with flexible rate, tax freeze.

economy reacts quicker and by more in response to the shocks if debt is short-term debt. Clearly, flexible-rate mortgage contracts make the reaction even faster. On top of this, the tax freeze makes the transmission of shocks greater. That is the case both in the 2002 calibration with long-term, fixed-rate debt contracts and in the case of short-term debt with flexible rates.

The response of debt differs across the four models. I will use the housing preference shock to explain why household debt reacts differently across the five calibrations. With regard to the housing preference shock, the effects on GDP are largest in model (iv). Short-term debt implies that the stock of household debt increases quickly; this was explained in the previous section. If debt is one-period debt, when the impatient households determine their borrowing, they need not take into account that debt in the next period can be posted as collateral; the impatient household can reoptimize all periods without having to take into account the stock of outstanding debt. This also implies that debt returns much quicker to the steady state in comparison with a model with long-term debt contract.

A higher stock of debt implies more collateral, which the impatient household can borrow against and consume the proceeds. Consumption is consequently higher in models (iii) and (iv). On top of this, under a tax freeze, the consumer need not take into account extra tax payments when purchasing more housing stock, and consequently, the response of borrowing and consumption is largest in model (iv).

With long-term debt contracts, the response of debt is firstly muted and secondly more persistent. Again, this is because the households smoothen their borrowing over time and rationally see that the boom does not last forever, and that they need not to carry over to the following period the stock of debt; the consumer cannot reoptimize on the existing stock of debt. This naturally also has consequences for consumption, which inherits the cyclical response of debt.

Appendix 2. The Regression for the Real House Price in the Macroeconometric Model MONA

The regression for the real house price in the macroeconometric model MONA can be written in the following manner using the same notation as in the DSGE model:

$$\begin{aligned}\Delta Q_t^H = {} & 0.37519 \times \Delta Q_{t-1}^H - 3.82471 \\ & \times \Delta \left(\left(1 - \kappa_t^{RI}\right) R_t^{L,I,long} + \tau_t^H \right) \\ & - 0.03960 \times Q_{t-1}^H - 0.18606 \\ & \times \left(\left(1 - \kappa_{t-1}^{RI}\right) R_{t-1}^{L,I} + \tau_{t-1}^H - \pi_{t-1}^e \right) \\ & - 0.35146 \times \left(\left(1 - \kappa_{t-1}^{RI}\right) R_{t-1}^{L,Imin} + \tau_{t-1}^H + \delta_{t-1}^{\mathcal{A}} \right) \\ & + 0.09830 \times \Delta Q_{t-1}^{H,e} + 0.07653 \times \frac{Y_{t-1}^{income}}{H_{t-1}^{ToT}}. \qquad (15)\end{aligned}$$

The long-run relationship can be written as

$$0.03960 \times Q^{H}_{t-1} = usc_t + 0.07653 \times \frac{Y^{income}_{t-1}}{H^{ToT}_{t-1}} \Leftrightarrow$$

$$Q^{H}_{t-1} = \frac{1}{0.03960} \times usc_t + \frac{0.07653}{0.03960} \times \frac{Y^{income}_{t-1}}{H^{ToT}_{t-1}} \Leftrightarrow$$

$$Q^{H}_{t-1} = \underbrace{\frac{1}{0.03960}}_{\equiv\beta} \times usc_t + \underbrace{\frac{0.07653}{0.03960}}_{\equiv\alpha} \times \frac{Y^{income}_{t-1}}{H^{ToT}_{t-1}}.$$

Here I have defined the user cost as $usc_t \equiv -0.18606 \times \left(\left(1-\kappa^{RI}_{t-1}\right) R^{L,I}_{t-1} + \tau^{H}_{t-1} - \pi^{e}_{t-1}\right) - 0.35146 \times \left(\left(1-\kappa^{RI}_{t-1}\right) R^{L,Imin}_{t-1} + \tau^{H}_{t-1} + \delta^{\mathcal{A}}_{t-1}\right)$.

Here it is implicitly assumed that logs have been taken of the house price. $\pi^{e}_{t-1}, \Delta Q^{H,e}_{t-1}$ denotes the backward-looking inflation expectations and house price inflation, respectively; $R^{L,I,long}_{t}$ denotes the interest rate on mortgage on long-duration debt; and $R^{L,I,min}_{t-1}$ denotes the lowest interest rate available. The numbers in front of the variables denote estimates of the coefficient using Danish data. Finally, Y^{income}_{t} denotes real income for the two households defined in the models as $Y^{income,P}_{t} \equiv (1-\tau_t)\left(W^{P}_{t} N^{P}_{t}\right) - \omega T_t - \tau^{B}_{t-1} D^{P}_{t-1} \frac{\left(R^{D}_{t-1}-1\right)}{\Pi^{DK}_{t}}$ and $Y^{income,I}_{t} \equiv (1-\tau_t)\left(W^{I}_{t} N^{I}_{t}\right) - (1-\omega) T_t - \kappa^{RI}_{t-1} B^{I}_{t-1} \frac{\left(R^{L,I}_{t-1}-1\right)}{\Pi^{DK}_{t}}$.

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