Online Appendix to Macroeconomic Effects of Banking-Sector Losses across Structural Models

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List of Tables

1. Matteo Iacoviello: An Estimated Model of Banks with Financing Frictions

This appendix contains the complete set of equations for the model described in section 3 of the paper "Macroeconomic Effects of Banking-Sector Losses across Structural Models." The material borrows heavily on the technical appendix of the paper "Financial Business Cycles," described in Iacoviello (2015).

1.1 The Dynamic Model

1.1.1 Household Savers

Savers (denoted with subscript $_H$) choose consumption C, housing H, and hours N to solve

$$\max \sum_{t=0}^{\infty} \beta_H^t \left(A_{p,t} \left(1-\eta\right) \log \left(C_{H,t} - \eta C_{H,t-1}\right) \right.$$
$$\left. + j A_{j,t} A_{p,t} \log H_{H,t} + \tau \log \left(1 - N_{H,t}\right)\right)$$

subject to

$$C_{H,t} + \frac{K_{H,t}}{A_{K,t}} + D_t + q_t \left(H_{H,t} - H_{H,t-1}\right) + ac_{KH,t} + ac_{DH,t}$$
$$= \left(R_{M,t} z_{KH,t} + \frac{1-\delta_{KH,t}}{A_{K,t}}\right) K_{H,t-1} + R_{H,t-1} D_{t-1} + W_{H,t} N_{H,t}, \tag{1.1}$$

where the adjustment costs take the following form,

$$ac_{KH,t} = \frac{\phi_{KH}}{2} \frac{\left(K_{H,t} - K_{H,t-1}\right)^2}{K_H}$$
$$ac_{DH,t} = \frac{\phi_{DH}}{2} \frac{\left(D_t - D_{t-1}\right)^2}{D},$$

and the depreciation function is

$$\delta_{KH,t} = \delta_{KH} + b_{KH} \left(0.5 \zeta_H' z_{KH,t}^2 + \left(1 - \zeta_H'\right) z_{KH,t} + \left(0.5 \zeta_H' - 1\right)\right),$$

where $\zeta'_H = \frac{\zeta_H}{1-\zeta_H}$ is a parameter measuring the curvature of the utilization rate function. $\zeta_H = 0$ implies $\zeta'_H = 0$; ζ_H approaching 1 implies ζ'_H approaches infinity and $\delta_{KH,t}$ stays constant. $b_{KH} = \frac{1}{\beta_H} + 1 - \delta_{KH}$ and implies a unitary steady-state utilization rate. ac measures a quadratic adjustment cost for changing the quantity i between time t and time $t+1$. The adjustment cost is external. Habits are external too.

The household problem yields, denoting with $u_{CH,t} = \frac{A_{p,t}}{C_{H,t}-\eta C_{H,t-1}}$ and $u_{HH,t} = \frac{jA_{j,t}A_{p,t}}{H_{H,t}}$ the marginal utilities of consumption and housing,

$$u_{CH,t}\left(1+\frac{\partial ac_{DH,t}}{\partial D_t}\right) = \beta_H R_{H,t} u_{CH,t+1} \tag{1.2}$$

$$W_{H,t}u_{CH,t} = \frac{\tau_H}{1-N_{H,t}} \tag{1.3}$$

$$\begin{aligned}&\frac{1}{A_{K,t}}u_{CH,t}\left(1+\frac{\partial ac_{KH,t}}{\partial K_{H,t}}\right)\\ &\quad = \beta_H\left(R_{M,t+1}z_{KH,t+1}+\frac{1-\delta_{KH,t+1}}{A_{K,t+1}}\right)u_{CH,t+1}\end{aligned} \tag{1.4}$$

$$q_t u_{CH,t} = u_{HH,t} + \beta_H q_{t+1} u_{CH,t+1} \tag{1.5}$$

$$R_{M,t} = \delta'\left(z_{KH,t}\right), \tag{1.6}$$

where $A_{K,t}$ is an investment shock, $A_{p,t}$ is a consumption preference shock, and $A_{j,t}$ is a housing demand shock.

1.1.2 Household Borrowers

They solve

$$\begin{aligned}\max \sum_{t=0}^{\infty}\beta_S^t\,&\left(A_{p,t}\left(1-\eta\right)\log\left(C_{S,t}-\eta C_{S,t-1}\right)\right.\\ &\left.+\,jA_{j,t}A_{p,t}\log H_{S,t}+\tau\log\left(1-N_{S,t}\right)\right)\end{aligned}$$

subject to

$$\begin{aligned}&C_{S,t}+q_t\left(H_{S,t}-H_{S,t-1}\right)+R_{S,t-1}L_{S,t-1}-\varepsilon_{H,t}+ac_{SS,t}\\ &\quad = L_{S,t}+W_{S,t}N_{S,t}\end{aligned} \tag{1.7}$$

and to

$$L_{S,t} \leq \rho_S L_{S,t-1} + (1 - \rho_S)\, m_S A_{MH,t} \frac{q_{t+1}}{R_{S,t}} H_{S,t} - \varepsilon_{H,t}, \tag{1.8}$$

where $\varepsilon_{H,t}$ is the borrower repayment shock; $A_{M,t}$ is a loan-to-value ratio shock. The adjustment cost is

$$ac_{SS,t} = \frac{\phi_{SS}}{2} \frac{(L_{S,t} - L_{S,t-1})^2}{L_S}.$$

The first-order conditions are, denoting with $u_{CS,t} = \frac{A_{p,t}}{C_{S,t}}$ and $u_{HS,t} = \frac{jA_{j,t}A_{p,t}}{H_{S,t}}$ the marginal utilities of consumption and housing, and with $\lambda_{S,t} u_{CS,t}$ the (normalized) multiplier on the borrowing constraint,

$$\left(1 - \frac{\partial ac_{SS,t}}{\partial L_{S,t}} - \lambda_{S,t}\right) u_{CS,t} = \beta_S (R_{S,t} - \rho_S \lambda_{S,t+1})\, u_{CS,t+1} \tag{1.9}$$

$$W_{S,t} u_{CS,t} = \frac{\tau_S}{1 - N_{S,t}} \tag{1.10}$$

$$\left(q_t - \lambda_{S,t} (1 - \rho_S)\, m_S A_{MH,t} \frac{q_{t+1}}{R_{S,t}}\right) u_{CS,t} = u_{HS,t} + \beta_S q_{t+1} u_{CS,t+1}. \tag{1.11}$$

1.1.3 Bankers

Bankers solve

$$\max \sum_{t=0}^{\infty} \beta_B^t \log (C_{B,t} - \eta C_{B,t-1})$$

subject to

$$\begin{aligned} &C_{B,t} + R_{H,t-1} D_{t-1} + L_{E,t} + L_{S,t} + ac_{DB,t} + ac_{EB,t} + ac_{SB,t} \\ &\quad = D_t + R_{E,t} L_{E,t-1} + R_{S,t} L_{S,t-1} - \varepsilon_{E,t} - \varepsilon_{S,t}, \end{aligned} \tag{1.12}$$

where $\varepsilon_{E,t}$ is the entrepreneur repayment shock. The adjustment costs are

$$ac_{DB,t} = \frac{\phi_{DB}}{2}\frac{(D_t - D_{t-1})^2}{D}$$
$$ac_{EB,t} = \frac{\phi_{EB}}{2}\frac{(L_{E,t} - L_{E,t-1})^2}{L_E}$$
$$ac_{SB,t} = \frac{\phi_{SB}}{2}\frac{(L_{S,t} - L_{S,t-1})^2}{L_S}.$$

Denote $\varepsilon_t = \varepsilon_{E,t}+\varepsilon_{S,t}$. Let $L_t = L_{E,t}+L_{S,t}$. The banker's constraint is a capital adequacy constraint of the form

$$\underbrace{(L_t - D_t - \varepsilon_t)}_{\text{bank equity}} \geq \rho_D (L_{t-1} - D_{t-1} - \varepsilon_{t-1}) + (1-\gamma)(1-\rho_D)\underbrace{(L_t - \varepsilon_t)}_{\text{bank assets}}$$

stating that bank equity (after losses) must exceed a fraction of bank assets, allowing for a partial adjustment in bank capital given by ρ_D. Such constraint can be rewritten as a leverage constraint of the form

$$D_t \leq \rho_D (D_{t-1} - (L_{E,t-1} + L_{S,t-1} - (\varepsilon_{E,t-1} + \varepsilon_{S,t-1}))) + (1-(1-\gamma)(1-\rho_D))(L_{Et} + L_{S,t} - (\varepsilon_{E,t} + \varepsilon_{S,t})). \quad (1.13)$$

The first-order conditions to the banker's problem imply, choosing D, L_E, L_S and letting $\lambda_{B,t}u_{CB,t}$ be the normalized multiplier on the borrowing constraint,

$$\left(1 - \lambda_{B,t} - \frac{\partial ac_{DB,t}}{\partial D_t}\right) u_{CB,t} = \beta_B (R_{H,t} - \rho_D \lambda_{B,t+1}) u_{CB,t+1} \quad (1.14)$$

$$\left(1 - (\gamma_E(1-\rho_D) + \rho_D)\lambda_{B,t} + \frac{\partial ac_{EB,t}}{\partial L_{E,t}}\right) u_{CB,t} = \beta_B (R_{E,t+1} - \rho_D \lambda_{B,t+1}) u_{CB,t+1} \quad (1.15)$$

$$\left(1 - (\gamma_S(1-\rho_D) + \rho_D)\lambda_{B,t} + \frac{\partial ac_{SB,t}}{\partial L_{S,t}}\right) u_{CB,t} = \beta_B (R_{S,t} - \rho_D \lambda_{B,t+1}) u_{CB,t+1}. \quad (1.16)$$

1.1.4 Entrepreneurs

Entrepreneurs obtain loans and produce goods (including capital). Entrepreneurs hire workers and demand capital supplied by the household sector,

$$\max \sum_{t=0}^{\infty} \beta_E^t \log\left(C_{E,t} - \eta C_{E,t-1}\right),$$

subject to

$$\begin{aligned} C_{E,t} &+ \frac{K_{E,t}}{A_{K,t}} + q_t H_{E,t} + R_{E,t} L_{E,t-1} + W_{H,t} N_{H,t} + W_{S,t} N_{S,t} \\ &+ R_{M,t} z_{KH,t} K_{H,t-1} \\ &= Y_t + \frac{1-\delta_{KE,t}}{A_{K,t}} K_{E,t-1} + q_t H_{E,t-1} + L_{E,t} + \varepsilon_{E,t} \\ &+ ac_{KE,t} + ac_{EE,t} \end{aligned} \tag{1.17}$$

and to

$$\begin{aligned} Y_t = A_{Z,t} &\left(z_{KH,t} K_{H,t-1}\right)^{\alpha\mu} \left(z_{KE,t} K_{E,t-1}\right)^{\alpha(1-\mu)} \\ &\times H_{E,t-1}^{\nu} N_{H,t}^{(1-\alpha-\nu)(1-\sigma)} N_{S,t}^{(1-\alpha-\nu)\sigma}, \end{aligned} \tag{1.18}$$

where $A_{Z,t}$ is a shock to total factor productivity. The adjustment costs are

$$\begin{aligned} ac_{KE,t} &= \frac{\phi_{KE}}{2} \frac{\left(K_{E,t} - K_{E,t-1}\right)^2}{K_E} \\ ac_{EE,t} &= \frac{\phi_{EE}}{2} \frac{\left(L_{E,t} - L_{E,t-1}\right)^2}{L_E}. \end{aligned}$$

Note that symmetrically to the household problem entrepreneurs are subject to an investment shock, can adjust the capital utilization rate, and pay a quadratic capital adjustment cost. The depreciation rate is governed by

$$\delta_{KE,t} = \delta_{KE} + b_{KE}\left(0.5\zeta_E' z_{KE,t}^2 + \left(1-\zeta_E'\right) z_{KE,t} + \left(0.5\zeta_E' - 1\right)\right),$$

where setting $b_{KE} = \frac{1}{\beta_E} + 1 - \delta_{KE}$ implies a unitary steady-state utilization rate.

Entrepreneurs are subject to a borrowing/pay-in-advance constraint that acts as a wedge on the capital and labor demand. The constraint is

$$\begin{aligned} L_{E,t} &= \rho_E L_{E,t-1} + (1-\rho_E) A_{ME,t} \\ &\quad \times \left(m_H \frac{q_{t+1}}{R_{E,t+1}} H_{E,t} + m_K K_{E,t} - m_N \left(W_{H,t} N_{H,t} + W_{S,t} N_{S,t} \right) \right). \end{aligned} \tag{1.19}$$

Letting $u_{CE,t}$ be the marginal utility of consumption and $\lambda_{E,t} u_{CE,t}$ the normalized borrowing constraint, the first-order conditions for L_E, K_E, and H_E are

$$\left(1 - \lambda_{E,t} + \frac{\partial ac_{LE,t}}{\partial L_{E,t}} \right) u_{CE,t} = \beta_E \left(R_{E,t+1} - \rho_E \lambda_{E,t+1} \right) u_{CE,t+1} \tag{1.20}$$

$$\begin{aligned} &\left(1 + \frac{\partial ac_{KE,t}}{\partial K_{E,t}} - \lambda_{E,t} (1-\rho_E) m_K A_{ME,t} \right) u_{CE,t} \\ &\quad = \beta_E \left(1 - \delta_{KE,t+1} + R_{K,t+1} z_{KE,t+1} \right) u_{CE,t+1} \end{aligned} \tag{1.21}$$

$$\begin{aligned} &\left(q_t - \lambda_{E,t} (1-\rho_E) m_H A_{ME,t} \frac{q_{t+1}}{R_{E,t+1}} \right) u_{CE,t} \\ &\quad = \beta_E q_{t+1} \left(1 + R_{V,t+1} \right) u_{CE,t+1}. \end{aligned} \tag{1.22}$$

Additionally, these conditions can be combined with those of the “production arm” of the firm, giving

$$\alpha \mu Y_t = R_{K,t} z_{KE,t} K_{E,t-1} \tag{1.23}$$

$$\alpha (1-\mu) Y_t = R_{M,t} z_{KH,t} K_{H,t-1} \tag{1.24}$$

$$\nu Y_t = R_{V,t} q_t H_{E,t-1} \tag{1.25}$$

$$(1-\alpha-\nu)(1-\sigma) Y_t = W_{H,t} N_{H,t} \left(1 + m_N A_{ME,t} \lambda_{E,t} \right) \tag{1.26}$$

$$(1-\alpha-\nu) \sigma Y_t = W_{S,t} N_{S,t} \left(1 + m_N A_{ME,t} \lambda_{E,t} \right) \tag{1.27}$$

$$R_{K,t} = \delta' \left(z_{KE,t} \right). \tag{1.28}$$

1.1.5 Equilibrium

Market clearing is implied by Walras's law by aggregating all the budget constraints. For housing, we have the following market clearing condition:

$$H_{H,t} + H_{S,t} + H_{E,t} = 1. \tag{1.29}$$

The model dynamics (except for the stochastic properties of the exogenous shocks, described separately below) are fully described by equations (1.1) to (1.29). These equations—together with the definition of the depreciation rate functions and the adjustment cost functions given above—represent a dynamic system in the following twenty-nine endogenous variables:

- Fourteen quantities: Y, H_E, H_H, H_S, K_E, K_H, N_H, N_S, C_B, C_E, C_H, C_S, z_{KH}, z_{KE}.
- Three loans and deposits: L_E, L_S, D.
- Three prices: q, W_H, W_S.
- Six interest rates: R_K, R_M, R_V, R_E, R_S, R_H.
- Three Lagrange multipliers: λ_E, λ_S, λ_B.

1.1.6 Shocks

The shocks obey the following stochastic processes:

$$\varepsilon_{E,t} = \rho_{be}\varepsilon_{E,t-1} + u_{E,t}, \quad u_E \sim N(0, \sigma_{be})$$
$$\varepsilon_{H,t} = \rho_{bh}\varepsilon_{H,t-1} + \upsilon_{H,t}, \quad u_H \sim N(0, \sigma_{bh})$$
$$\log A_{j,t} = \rho_j \log A_{j,t-1} + \upsilon_{j,t}, \quad u_j \sim N(0, \sigma_j)$$
$$\log A_{K,t} = \rho_K \log A_{K,t-1} + \upsilon_{K,t}, \quad u_K \sim N(0, \sigma_k)$$
$$\log A_{ME,t} = \rho_{me} \log A_{ME,t-1} + \upsilon_{ME,t}, \quad u_{ME} \sim N(0, \sigma_{me})$$
$$\log A_{MH,t} = \rho_{mh} \log A_{MH,t-1} + \upsilon_{MH,t}, \quad u_{MH} \sim N(0, \sigma_{mh})$$
$$\log A_{p,t} = \rho_p \log A_{p,t-1} + \upsilon_{p,t}, \quad u_p \sim N(0, \sigma_p)$$
$$\log A_{Z,t} = \rho_z \log A_{Z,t-1} + \upsilon_{z,t}, \quad u_z \sim N(0, \sigma_z).$$

1.2 Calibration

Table 1.1 Calibrated Parameters for the Extended Model

Calibrated Parameter		**Value**
Household-Saver (HS) Discount Factor	β_H	0.9925
Household-Borrower (HB) Discount Factor	β_S	0.94
Banker Discount Factor	β_B	0.945
Entrepreneur (E) Discount Factor	β_E	0.94
Total Capital Share in Production	α	0.35
Loan-to-Value Ratio on Housing, HB	m_S	0.9
Loan-to-Value Ratio on Housing, E	m_H	0.9
Loan-to-Value Ratio on Capital, E	m_K	0.9
Wage Bill Paid in Advance	m_N	1
Liabilities-to-Assets Ratio for Banker	γ_E, γ_S	0.9
Housing Preference Share	j	0.075
Capital Depreciation Rates	δ_{KE}, δ_{KH}	0.035
Labor Supply Parameter	τ	2

Table 1.2 Estimated Structural Parameters and Shock Processes

Estimated Parameter		**Value**
A. Estimated Structural Parameters		
Habit in Consumption	η	0.46
D Adj. Cost, Banks	ϕ_{DB}	0.14
D Adj. Cost, Household Saver (HS)	ϕ_{DH}	0.10
K Adj. Cost, Entrepreneurs (E)	ϕ_{KE}	0.59
K Adj. Cost, Household Saver (HS)	ϕ_{KH}	1.73
Loan to E Adj. Cost, Banks	ϕ_{EB}	0.07
Loan to E Adj. Cost, E	ϕ_{EE}	0.06
Loan to HB Adj. Cost, Banks	ϕ_{SB}	0.47
Loan to HB Adj. Cost, HH Borrower HB	ϕ_{SS}	0.37
Capital Share of E	μ	0.46
Housing Share of E	ν	0.04
Inertia in Capital Adequacy Constraint	ρ_D	0.24
Inertia in E Borrowing Constraint	ρ_E	0.65
Inertia in HB Borrowing Constraint	ρ_S	0.70
Wage Share HB	σ	0.33
Curvature for Utilization Function E	ζ_E	0.42
Curvature for Utilization Function HS	ζ_H	0.38
B. Estimated Shock Processes		
Autocorrelation E Default Shock	ρ_{be}	0.932
Autocorrelation HB Default Shock	ρ_{bh}	0.969
Autocorrelation Housing Demand Shcok	ρ_j	0.992
Autocorrelation Investment Shock	ρ_k	0.916
Autocorrelation LTV Shock, E	ρ_{me}	0.839
Autocorrelation LTV Shock, HB	ρ_{mh}	0.873
Autocorrelation Preference Shock	ρ_p	0.994
Autocorrelation Technology Shock	ρ_z	0.988
St. Dev., Default Shock, E	σ_{be}	0.0011
St. Dev., Default Shock, HB	σ_{bh}	0.0013
St. Dev., Housing Demand Shock	σ_j	0.0346
St. Dev., Investment Shock	σ_k	0.0081
St. Dev., LTV Shock, E	σ_{me}	0.0204
St. Dev., LTV Shock, HB	σ_{mh}	0.0115
St. Dev., Preference Shock	σ_p	0.0205
St. Dev., Technology Shock	σ_z	0.0070

2. Francisco Covas and John Driscoll: A Non-linear Model of Borrowing Constraints

2.1 Introduction

In this appendix, we describe the setup of the model by Covas and Driscoll included in "Macroeconomic Effects of Banking-Sector Losses across Structural Models." We construct a general equilibrium model augmenting that of Aiyagari (1994) by having three types of agents that face uninsurable risks: workers, entrepreneurs, and bankers. Workers supply labor to entrepreneurs and face labor productivity shocks which dictate their earning potential. Entrepreneurs can invest in their own technology and face investment risk shocks which determine their potential profitability. Bankers play the role of financial intermediaries in this economy by accepting deposits from workers and making loans to entrepreneurs. In addition, bankers can also invest in riskless securities. Bankers are subject to revenue shocks that determine their potential profitability. An important feature of the banker's problem is the presence of occasionally binding capital and liquidity constraints.

2.2 The Model

The model includes three groups of agents: workers, entrepreneurs, and bankers. We describe the economic problems faced by each group of agents below.

2.2.1 Workers

As in Aiyagari (1994) workers are heterogeneous with respect to wealth holdings and earnings ability. Since there are idiosyncratic shocks, the variables of the model will differ across workers. To simplify notation, we do not index the variables to indicate this cross-sectional variation. Let c_t^w denote the worker's consumption in period t, d_t^w denote the deposit holdings, and a_t^w denote the worker's asset holdings in the same period, and ϵ_t is a labor-efficiency process which follows a first-order Markov process. Workers choose consumption to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta_w^t u(c_t^w, d_{t+1}^w),$$

subject to the following budget constraint:

$$c_t^w + d_{t+1}^w + a_{t+1}^w = w\, \epsilon_t + R^D d_t^w + R\, a_t^w,$$

where $0 < \beta_w < 1$ is the worker's discount factor, w is the worker's wage rate, R^D is the gross rate on deposits, and R is gross return on capital. We assume workers are subject to an ad hoc borrowing constraint; that is, $a_{t+1}^w \geq \underline{a}$, where $\underline{a} \leq 0$. The wage rate and the return on capital are determined in general equilibrium such that labor and corporate capital markets clear in the steady state. Note that we have introduced a demand for deposits by assuming that their holdings bring utility to the worker. However, the deposit rate is assumed to be exogenous since, as described later, bankers take as given the stock of deposits supplied by the workers.

Let $v^w(\epsilon, x_w)$ be the optimal value function for a worker with earnings ability ϵ and cash on hand x_w.[1] The worker's optimization problem can be specified in terms of the following dynamic programming problem:

$$\begin{aligned} v^w(\epsilon, x_w) = \max_{c_w, d'_w, a'_w} \; & u(c_w, d'_w) + \beta_w E[v(\epsilon', x'_w)|\epsilon], \qquad (2.1)\\ \text{s.t.} \quad & c_w + d'_w + a'_w = x_w, \\ & x'_w = w\, \epsilon' + R^D d'_w + R\, a'_w, \\ & a'_w \geq \underline{a}. \end{aligned}$$

The full list of parameters of the worker's problem is shown at the top of table 2.1.

2.2.2 Entrepreneurs

Entrepreneurs are also heterogeneous with respect to wealth holdings and productivity of the individual-specific technology that they

[1] Because the worker's problem is recursive, the subscript t is omitted in the current period, and a prime denotes the value of the variables one period ahead.

Table 2.1 Parameter Values under Baseline Calibration

Parameter	Description	Value
Workers' Parameters		
β_w	Discount Factor	0.96
γ_w	Coefficient of Relative Risk Aversion	2.0
ω	Weight on Consumption	0.97
ρ_ϵ	Persistence of Earnings Risk	0.80
σ_ϵ	Unconditional s.d. of Earnings Risk	0.16
$\underline{a}$	Borrowing Constraint	0.0
η_w	Mass of Workers	0.666
Entrepreneurs' Parameters		
β_e	Discount Factor	0.95
γ_e	Coefficient of Relative Risk Aversion	2.0
ρ_z	Persistence of Productivity Risk	0.70
σ_z	Unconditional s.d. of Productivity Risk	0.22
κ	Borrowing Constraint	0.50
α	Capital Share	0.45
ν	Labor Share	0.35
δ	Depreciation Rate	0.08
η_e	Mass of Entrepreneurs	0.333
Bankers' Parameters		
β_b	Discount Factor	0.95
γ_b	Coefficient of Relative Risk Aversion	1.0
$\underline{\chi}$	Capital Requirements	0.06
$\bar{\delta}$	Loan Maturity	0.24
α_b	Curvature of Loan Revenues	0.75
ρ_θ	Persistence of Shock to Loan Revenues	0.70
σ_θ	Unconditional s.d. of Shock to Loan Revenues	0.09
ρ_d	Persistence of Shock to Deposits	0.80
σ_d	Unconditional s.d. of Shock to Deposits	0.15
ϕ_b	Intermediation Cost	0.15
ν^-	Adjustment Cost for Decreasing Loans	0.04
ν^+	Adjustment Cost for Increasing Loans	0.02
Corporate Sector's Parameters		
α_c	Capital Share	0.36
δ_c	Depreciation Rate	0.08

operate. Entrepreneurs choose consumption to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta_e^t u(c_t^e),$$

where $0 < \beta_e < 1$ is the entrepreneur's discount factor. Each period, the entrepreneur can invest in an individual-specific technology (risky investment) or invest its savings in securities. The risky technology available to the entrepreneur is represented by

$$y_t = z_t f(k_t, l_t),$$

where z_t denotes productivity, k_t is the capital stock in the risky investment, and l_t is labor. This investment is risky because the stock of capital is chosen before productivity is observed. The labor input is chosen after observing productivity. The idiosyncratic productivity process follows a first-order Markov process. As is standard, capital depreciates at a fixed rate δ.

In addition, the entrepreneur is allowed to borrow to finance consumption and the risky investment. Let b_{t+1}^e denote the amount borrowed by the entrepreneur and R^L denote the gross rate on bank loans. The loan rate is determined in general equilibrium. Borrowing is constrained, for reasons of moral hazard and adverse selection that are not explicitly modeled, to be no more than a fraction of entrepreneurial capital:

$$b_{t+1}^e \geq -\kappa\, k_{t+1},$$

where κ represents the fraction of capital that can be pledged at the bank as collateral. Entrepreneurs that are not borrowing to finance investment can save through a riskless security, denoted by s^e with a gross return R^S which will also be determined in general equilibrium.

Under this set of assumptions, the entrepreneur's budget constraint is as follows:

$$\begin{aligned} c_t^e + k_{t+1} + b_{t+1}^e + s_{t+1}^e &= x_t^e, \\ x_{t+1}^e &= z_{t+1} f(k_{t+1}, l_{t+1}) + (1 - l_{t+1}) w \\ &\quad + (1-\delta) k_{t+1} + R^L b_{t+1}^e + R^S\, s_{t+1}^e, \end{aligned}$$

where x_t^e denotes the entrepreneur's period-t wealth. It should be noted that the entrepreneur can also supply labor to the corporate sector or other entrepreneurial businesses.

Let $v^e(z, x_e)$ be the optimal value function for an entrepreneur with productivity z and wealth x_e.[2] The entrepreneur's optimization problem can be specified in terms of the following dynamic programming problem:

$$\begin{aligned} v^e(z, x_e) = \max_{c_e, k', b_e', s_e'} & \; u(c_e) + \beta_e E[v(z', x_e')|z], \qquad (2.2)\\ \text{s.t.} \quad & c_e + k' + s_e' + b_e' = x_e,\\ x_e' = & \; \pi(z', k'; w) + (1-\delta)k' + R^L b_e' + R^S s_e',\\ & 0 \geq b_e' \geq -\kappa\, k',\\ & s_e' \geq 0,\\ & k' \geq 0, \end{aligned}$$

where $\pi(z', k'; w)$ represents the operating profits of the entrepreneur's and incorporates the static optimization labor choice. From the properties of the utility and production functions of the entrepreneur, the optimal levels of consumption and the risky investment are always strictly positive. The constraints that may be binding are the choices of bank loans, b_e', and security holdings, s_e'. The full list of parameters of the entrepreneur's problem is shown in the middle panel of table 2.1.

2.2.3 Bankers

Bankers are heterogeneous with respect to wealth holdings, loan balances, deposit balances, and productivity. Bankers choose consumption to maximize expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta_b^t u(c_t^b),$$

where $0 < \beta_b < 1$ is the banker's discount factor.

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[2]Because the entrepreneur's problem is recursive, the subscript t is omitted in the current period, and we let the prime denote the value of the variables one period ahead.

Bankers hold two types of assets—risky loans (b) and riskless securities (s)—and fund those assets with deposits (d) and equity (e). Loans can also be funded by short-selling securities—implying s can be negative.

Each period, the banker chooses the amount of loans it makes to the entrepreneurs, denoted by b_{t+1}. Loans, which are assumed to mature at a rate $\bar{\delta}$, yield both interest and non-interest income (the latter arises, for example, from fees, which in practice are a substantial part of bank income). Banks may differ in their ability to extract net revenue from loans due to (unmodeled) differences in their ability to screen applicants or monitor borrowers, or in market power. For analytical convenience, we represent net revenue in period t from the existing stock of loans b_t as

$$y_t^b = (R^L - \phi_b)b_t + \theta_t g(b_t),$$

where θ_t denotes the idiosyncratic productivity of the bank, the function $g(b_t)$ exhibits decreasing returns to scale, and ϕ_b is the cost of operating the loan technology.

The banks also face adjustment costs in changing the quantity of loans, which allows us to capture the relative illiquidity of such assets. The adjustment costs are parametrized by

$$\Psi(b_{t+1}, \bar{\delta}b_t) \equiv \frac{\nu_t}{2}\left(\frac{b_{t+1} - \bar{\delta}b_t}{b_t}\right)^2 b_t,$$

where

$$\nu_t \equiv \nu^+ 1_{\{b_{t+1} \geq \bar{\delta}b_t\}} + \nu^- 1_{\{b_{t+1} < \bar{\delta}b_t\}}.$$

In our calibration, we will assume that the cost of adjusting the stock of loans downwards is much greater than the cost of adjusting it upwards—reflecting the idea that calling in or selling loans is more costly than originating loans.

Gross returns from the bank's securities holdings is given by

$$y_t^s = R^S s_t,$$

which may be negative if the bank is short-selling securities. The banker's budget constraint is written as follows:

$$c_t^b + b_{t+1} + s_{t+1} + d_{t+1} = x_t^b - \Psi(b_{t+1}, \bar{\delta}b_t),$$

$$x_{t+1}^b = (R^L - \phi_b)b_{t+1} + \theta_{t+1} g(b_{t+1}) + R^S s_{t+1} + R^D d_{t+1},$$

where x_t^b denotes the banker's period-t wealth and d_{t+1} the stock of deposits. The bank borrows through deposits that it receives from the workers, but it can also borrow by selling securities to other bankers or entrepreneurs. For simplicity, we assume the share of deposits received by each bank is exogenous and follows a four-state first-order Markov chain (see section 2.4.3 of this appendix for further details). However, borrowing from entrepreneurs and other bankers is endogenous and is constrained by capital requirements. Letting e_{t+1} denote banks' equity, the capital requirement may be written as

$$e_{t+1} \geq \chi b_{t+1},$$

which is equivalent to a risk-based capital requirement, giving a zero risk weight to securities. The capital requirement may in turn be rewritten in terms of securities holdings as follows (since $e_{t+1} = x_t^b - \Psi(b_{t+1}, \bar{\delta} b_t) - c_t^b$):

$$s_{t+1} \geq (\chi - 1) b_{t+1} - d_{t+1}.$$

We also impose a liquidity requirement, in which we assume that cash on hand—which consists of the return on existing securities holdings, $R^S s_{t+1}$m, and the net revenue from paydowns on existing loans, $\bar{\delta} b_{t+1}$—must be sufficient to satisfy demand for deposit withdrawals under a liquidity stress scenario and interest payments on deposits. This can be represented as

$$R^S s_{t+1} + \bar{\delta} b_{t+1} \geq (d_{\{s-1,1\}^+} - R^D d_{t+1}), \tag{2.3}$$

where $d_{\{s-1,1\}^+}$ represents a decline in the stock of deposits (note that $d < 0$). Since d_t follows a Markov chain, if in period t the bank is in state s, then deposit withdrawals correspond to state $\{s-1,1\}^+$. The stringency of the liquidity requirement is given by the assumption about the relative size of the bad deposits realization.[3] It will be calibrated through an assumption of how quickly deposits would run off in a crisis situation.

[3]When not in a crisis, the deposits runoff will be smaller, and the constraint will not bind.

Let $v^b(\theta, x^b, b, d')$ be the optimal value function for a banker with wealth x^b, loans b, deposits d', and productivity θ. The banker's optimization problem can be specified in terms of the following dynamic programming problem:

$$v^b(\theta, x_b, b, d') = \max_{c_b, b', s'} u(c^b) + \beta_b E[v^b(x'_b, b', d'', \theta')|\theta, d'], \qquad (2.4)$$

$$\text{s.t.} \quad c_b + b' + s' + d' = x_b - \Psi(b', \bar{\delta}b),$$

$$x'_b = (R^L - \phi_b)b' + \theta' g(b') + R^S s' + R^D d',$$

$$e' \geq \chi b',$$

$$R^S s' + \bar{\delta} b' \geq (d_{\{s-1,1\}^+} - R^D d').$$

Banker's Capital Constraint. The balance sheet constraint of the banker is given by

$$b' + s' = x_b - c_b - \Phi(b', \bar{\delta}b) - d',$$

where the left-hand side of this expression is the banker's assets, $b' + s'$, and the right-hand side is the banker's equity, $e_b \equiv x_b - c_b - \Phi(b', \bar{\delta}b)$, and debt, $-d'$. The capital constraint can be written as

$$e_b \geq \chi b'$$

$$b' + s' + d' \geq \chi b'$$

$$d' \geq (\chi - 1)b' - s'.$$

Banker's First-Order Conditions. The first-order conditions for b' and s' are as follows:

$$\left[1 + \frac{\partial \Phi(b', b)}{\partial b'}\right] u_c(c) = \beta_b E\left[\frac{\partial v_b}{\partial x_b}\frac{\partial x_b}{\partial b'} + \frac{\partial v_b}{\partial b'}\middle|\theta, d'\right] + (1-\chi)\lambda + \bar{\delta}\mu$$

$$u_c(c) = \beta_b E\left[\frac{\partial v_b}{\partial x_b}\frac{\partial x_b}{\partial s'}\middle|\theta, d'\right] + \lambda + \mu R_S,$$

where λ is the Lagrange multiplier associated with the capital constraint and μ is the Lagrange multiplier associated with the liquidity constraint. Note that the envelope conditions are

$$\frac{\partial v_b}{\partial x_b} = u_c(c)$$
$$\frac{\partial v_b}{\partial b} = -u_c(c)\frac{\partial \Phi}{\partial b}.$$

Using the envelope condition on the set of first-order conditions, one obtains

$$\begin{aligned}
&\left[1 + \frac{\partial \Phi(b', b)}{\partial b'}\right] u_c(c) \\
&\quad = \beta_b E\left[\left(\theta' g_b(b') + R_L - \phi_b - \frac{\partial \Phi(b'', b')}{\partial b'}\right) u_c(c') \middle| \theta, d'\right] \\
&\qquad + (1-\chi)\lambda + \bar{\delta}\mu \\
&\qquad\quad u_c(c) = \beta_b E\left[R_S u_c(c') \middle| \theta, d'\right] + \lambda + \mu R_S.
\end{aligned}$$

2.2.4 Corporate Sector

In this economy there is also a corporate sector that uses a constant-returns-to-scale Cobb-Douglas production function, which uses the capital and labor or workers and entrepreneurs as inputs. The aggregate technology is represented by

$$Y_t = F(K_t, L_t),$$

and aggregate capital, K_t, is assumed to depreciate at rate δ.

2.2.5 Equilibrium

Definition 1 summarizes the steady-state equilibrium in this economy.

DEFINITION 1. *The steady-state equilibrium in this economy is a value function for the worker,* $v^w(\epsilon, x^w)$, *for the entrepreneur* $v^e(z, x^e)$, *and for the banker,* $v^b(\theta, x_b, b, d')$; *the worker's policy functions* $\{c^w(\epsilon, x^w), d^w(\epsilon, x^w), a^w(\epsilon, x^w)\}$; *the entrepreneur's policy functions* $\{c^e(z, x_e), k(z, x_e), l(z, x_e), b^e(z, x_e), a^e(z, x_e)\}$; *the banker's policy functions* $\{c^b(x_b, b, \theta, d')$, $b^b(x_b, b, \theta, d')$, $s(x_b, b, \theta, d')$, $d(x_b, b, \theta, d')\}$; *a constant cross-sectional distribution of worker's* *characteristics,* $\Gamma_w(\epsilon, x^w)$ *with mass* η_w*; a constant cross-sectional distribution of entrepreneur's characteristics,* $\Gamma_e(z, x^e)$ *with mass* η_e*; a constant cross-sectional distribution of banker's characteristics,* $\Gamma_b(x_b, b, \theta, d')$*, with mass* $(1 - \eta_w - \eta_e)$*; and prices* (R^D, R^L, R^S, R, w)*, such that*

(i) Given R^D*,* R*, and* w*, the worker's policy functions solve the worker's decision problem (2.1).*

(ii) Given R*,* R^L*, and* w*, the entrepreneur's policy functions solve the entrepreneur's decision problem (2.2).*

(iii) Given R^D*,* R^L*, and* R^S*, the banker's policy functions solve the banker's decision problem (2.4).*

(iv) The loan, securities, and deposit markets clear:

$$\eta_e \int b^e \, d\Gamma_e + (1 - \eta_w - \eta_e) \int b^b \, d\Gamma_b = 0, \qquad \textit{(Loan market)}$$

$$\bar{S} = \eta_e \int s^e \, d\Gamma_e + (1 - \eta_w - \eta_e) \int s^b \, d\Gamma_b, \qquad \textit{(Securities market)}$$

$$\eta_w \int d^w \, d\Gamma_w + (1 - \eta_w - \eta_e) \int d^b \, d\Gamma_b = 0. \qquad \textit{(Deposit market)}$$

(v) Corporate-sector capital and labor are given by

$$K = \eta_w \int a^w \, d\Gamma_w$$

$$L = (\eta_w + \eta_e) - \eta_e \int l \, d\Gamma_e.$$

(vi) Given K *and* L*, the factor prices are equal to factor marginal productivities:*

$$R = 1 + F_K(K, L) - \delta,$$

$$w = F_L(K, L).$$

(vii) *Given the policy functions of workers, entrepreneurs, and bankers, the probability measures of workers, Γ_w, entrepreneurs, Γ_e, and bankers, Γ_b, are invariant.*

2.3 Calibration

The properties of the model can be evaluated only numerically. We assign functional forms and parameters values to obtain the solution of the model and conduct comparative statics exercises. We choose one period in the model to represent one year.

2.3.1 Workers' and Entrepreneurs' Problems

The parameters of the workers' and entrepreneurs' problems are fairly standard, with the exception of the discount factor of entrepreneurs, which is chosen to match the loan rate. The period utility of the workers is assumed to have the following form:

$$u(c_e, d'_w) = \omega\left(\frac{c_w^{1-\gamma_w}}{1-\gamma_w}\right) + (1-\omega)\ln(d'_w),$$

where ω is the relative weight on the marginal utility of consumption and deposits and γ_w is the risk-aversion parameter. We set γ_w to 2, a number often used in representative-agent macroeconomic models. We set ω equal to 0.97 to match the ratio of banking assets relative to output, since this parameter controls the stock of deposits in our economy. The discount factor of workers is set at 0.96, which is standard.

We adopt a constant relative risk-aversion (CRRA) specification for the utility function of entrepreneurs:

$$u(c_e) = \frac{c_e^{1-\gamma_e}}{1-\gamma_e}.$$

We set γ_e to 2, close to that of Quadrini (2000). The idiosyncratic earnings process of workers is first-order Markov with the serial correlation parameter, ρ_ϵ, set to 0.80 and the unconditional standard deviation, σ_ϵ, set to 0.16. Although we lack direct information to calibrate the stochastic process for entrepreneurs, we make the reasonable assumption that the process should be persistent and consistent with the evidence provided by Hamilton (2000) and Moskowitz and Vissing-Jørgensen (2002) that the idiosyncratic risk facing entrepreneurs is larger than the idiosyncratic risk facing workers. Hence, we set the serial correlation of entrepreneurs to 0.70 and the unconditional standard deviation to 0.22.

As is standard in the business cycle literature, we choose a depreciation rate δ of 8 percent for the entrepreneurial as well as the corporate sector. The degree of decreasing returns to scale for entrepreneurs is equal to 0.80—slightly less than Cagetti and De Nardi (2006)—with capital and labor shares of 0.45 and 0.35, respectively. As in Aiyagari (1994), we assume workers are not allowed to have negative assets, and let the maximum leverage ratio of entrepreneurs be at about 50 percent, which corresponds to κ set to 0.5.[4]

The discount factor of entrepreneurs is chosen to match the average loan rate between 1997 and 2012. Based on bank holding company and Call Report data, the weighted average real interest rate charged on loans of all types was 4.6 percent. By setting β_e to 0.95, we obtain approximately this calibration.

2.3.2 Bankers' Problem

We divide the set of parameters of the bankers' problem into two parts: (i) parameters set externally, and (ii) parameters set internally. The parameters set externally are taken directly from outside sources. These include the loan maturity, $\bar{\delta}$, and the capital constraint parameter, χ. In addition, we assume the banker has log utility to minimize the amount of precautionary savings induced by the occasionally binding capital constraint. The remaining nine parameters of the banker's problem are determined so that a set of nine moments in the model are close to a set of nine moments available in the bank holding company and commercial bank Call Reports. The lower panel in table 2.1 reports the parameter values assumed in the parametrization of the banker's problem.

We now describe the parameters set externally. For the capital constraint we assume that the minimum capital requirement in the

[4]Leverage is defined as debt to assets, that is, $-b/k$. At the constraint $b = -\kappa k$, the maximum leverage in the model is equal to $\kappa = 0.50$.

model is equal to 6 percent, which corresponds to the minimum tier 1 ratio a bank must maintain to be considered well capitalized. Thus, χ equals 0.06. The loan maturity parameter, $\bar{\delta}$, is set to 0.24 so that the average maturity of loans is 4.2 years based on the maturity buckets available on banks' Call Reports.

The parameters set internally—namely the banker's discount factor, the intermediation cost, the parameters of the banker's loan technology, the persistence and standard deviation of the shock to deposits, and the adjustment cost parameters—are chosen to match a set of nine moments calculated from regulatory reports. The moments selected are (i) tier 1 capital ratio, (ii) the fraction of capital-constrained banks, (iii) leverage ratio, (iv) adjusted return on assets, (v) the cross-sectional volatility of adjusted return on assets, (vi) the share of assets with a zero or 20 percent Basel I risk weight, (vii) the share of interest income relative to total revenues, (viii) the share of non-interest expenses, and (ix) the return on securities. The upper panel of table 2.2 presents a comparison between the data and the model for this selected set of moments. Given the relatively large number of parameters and that we are solving the model using non-linear methods, it is difficult to match closely the moments of the model with those in the data.

As discussed above, the supplies of certain types of safe assets such as U.S. Treasury securities, Agency debt, and municipal bonds are not directly modeled in our framework. We capture the supply of these assets using the parameter $\overline{S}$. We calibrate this parameter using the estimates of the share of safe assets provided by Gorton, Lewellen, and Metrick (2012). Specifically, that paper estimates that during the post-war period the safe-asset share has fluctuated between 30 and 35 percent. In the model we define the safe-asset share as follows. The numerator includes bank deposits, the exogenous amount of safe assets, $\overline{S}$, and the amount of borrowing by banks in the securities market. The denominator includes all assets in the economy for each of the three types of agents: workers' deposits and corporate-sector assets; entrepreneurs' capital and securities; and bankers' loans and securities. By setting $\overline{S}$ to 9, we obtain a safe-asset share of 33 percent in our calibrated model. The solution of the model is obtained via computational methods and additional details are provided in section 2.4 below.

Table 2.2 Selected Moments

Moment	Data	Model
Tier 1 Capital Ratio	10.0	9.7
Share of Constrained Banks	0.1	0.3
Leverage Ratio	7.0	6.3
Adjusted Return on Assets	2.9	3.4
Cross-Sectional Volatility of Adjusted Return on Assets	1.3	1.4
% Safe Assets Held by Banks	33.1	34.4
Share of Interest Income in Revenues	1.3	0.3
Share of Non-interest Expenses	3.0	8.5
Return on Securities	0.5	0.5
Loan Rate	4.0	4.1
Consumption to Output	0.7	0.7
Banking Assets to Output	0.9	1.2
Safe-to-Total Assets	0.3	0.3
Memo: Deposit Rate	0.1	0.1
% Labor in Entrepreneurial Sector	—	37.6
% Labor in Corporate Sector	—	62.4
% Output of Entrepreneurial Sector	—	48.6
% Output of Corporate Sector	—	44.0
% Output of Banking Sector	—	7.5

Notes: Moments are based on sample averages using quarterly observations between 1997:Q1 and 2012:Q3, with the exception of the percentage of safe assets held by banks, which is only available starting in 2001:Q1, and averages for share of interest income in revenues and banking assets to output are calculated only for the period after the fourth quarter of 2008 when investment banks became bank holding companies. The adjusted return on assets is defined as net income excluding income taxes and salaries and employee benefits. The percentage of safe assets held by banks includes all assets with a zero and with a 20 percent risk weight. The deposit rate is a parameter. The sample includes all banking holding companies and commercial banks that are not part of a BHC, or that are part of a BHC which does not file the Y-9C report. The share of constrained banks is based on banks' responses in the Senior Loan Officer Opinion Survey. The safe-asset share is obtained from Gorton, Lewellen, and Metrick (2012).

2.4 Solution Techniques

2.4.1 Numerical Solution

The numerical algorithm solves the banker's problem by solving for a fixed point in the consumption function by time iteration as in

Coleman (1990). The policy function $c_b(\theta, x_b, b, d')$ is approximated using piecewise bilinear interpolation of the state variables x_b and b. The variables x_b and b are discretized in a non-uniformly spaced grid points with 100 nodes each. More grid points are allocated to lower levels of each state variable. The two stochastic processes, θ and d', are discretized into five and four states, respectively, using the method proposed by Tauchen (1986). The policy functions of consumption for workers and entrepreneurs are also solved by time iteration. Because the state space is smaller, the variables x_w and x_e are discretized in a non-uniformly spaced grid with 900 nodes. The invariant distributions of bankers, workers, and entrepreneurs are derived by computing the inverse decision rules on a finer grid than the one used to compute the optimal decision rules. Finally, the equilibrium prices are determined using a standard quasi-newton method.

2.4.2 Transitional Dynamics

The transition to the new stationary equilibrium is calculated assuming the new steady state is reached after sixty periods ($T = 60$). We take as inputs the steady-state distribution of agents in period $t = 1$ (prior to the change in policy); guesses for the path of R^L, R^S, and K/L between $t = 1$ and $t = T$; and the optimal decision functions at the new steady state. Using those guesses we solve the problem of each agent backwards in time, for $t = T - 1, \ldots, 1$. With the time-series sequence of decision rules for each agent, we simulate the dynamics of the distribution for workers, entrepreneurs, and bankers and check if the loan market, the deposit market, and goods market clear. If these markets are not in equilibrium, we update the path of R^L, R^S, and K/L using a simple linear updating rule. Finally, after convergence of the algorithm, we compare the simulated distribution at $T = 60$ with the steady-state distribution of each agent type obtained after the change in the policy parameters.

2.4.3 Markov Chains

Both the revenue and deposit shocks of the banker follow a first-order Markov process with five and four states, respectively. The Markov-chain process for the revenue process is as follows:

$$\bar{\theta} = [0.69; 0.83; 1.0; 1.21; 1.46]$$

$$\Pi(\theta', \theta) = \begin{bmatrix} 0.42 & 0.55 & 0.03 & 0.00 & 0.00 \\ 0.05 & 0.62 & 0.33 & 0.00 & 0.00 \\ 0.00 & 0.15 & 0.70 & 0.15 & 0.00 \\ 0.00 & 0.00 & 0.33 & 0.62 & 0.05 \\ 0.00 & 0.00 & 0.03 & 0.55 & 0.42 \\ \end{bmatrix}.$$

As for the deposit shock process, we assume

$$\bar{d} = [0.47; 0.78; 1.28; 2.12]$$

$$\Pi(d'|d) = \begin{bmatrix} 0.75 & 0.25 & 0.00 & 0.00 \\ 0.02 & 0.89 & 0.09 & 0.00 \\ 0.00 & 0.09 & 0.89 & 0.02 \\ 0.00 & 0.00 & 0.25 & 0.75 \\ \end{bmatrix}.$$

3. Michael Kiley and Jae Sim: Intermediary Leverage, Macroeconomic Dynamics, and Macroprudential Policy

This appendix provides the description of the structure of the model and the estimation/calibration strategy used in Kiley and Sim (2015). Since the focus of the analysis is on the financial intermediary, the description is more in detail for the sector. However, the description of the other sectors will be very brief.

3.1 Model without Pigovian Tax

The model economy consists of (i) a representative household, (ii) a representative firm producing intermediate goods, (iii) a continuum of monopolistically competitive retailers, (iv) a representative firm producing investment goods, and (v) a continuum of financial intermediaries.

3.1.1 The Financial Intermediary Sector

Financial intermediaries fund investment projects by issuing debt and equity securities. Debt is tax advantaged and is subject to default, while equity issuance is associated with a sizable issuance cost. We adopt the following timing convention: a time period is split into two subperiods where lending and borrowing (e.g., asset and liability) decisions have to be made in the first half of period t; idiosyncratic shocks to the returns of the projects are realized in the second half of period t, at which point lending and borrowing decisions cannot be reversed (until period $t+1$).

3.1.2 Debt Contract

We denote the return on intermediary project by $1 + r^F_{t+1} = \epsilon_{t+1}(1 + r^A_{t+1})$, where r^A_{t+1} is the aggregate component and ϵ_{t+1} is the idiosyncratic component. The latter follows a time-varying log-normal distribution: $\log \epsilon_t \sim N(-0.5\sigma_t^2, \sigma_t^2)$. The time-varying volatility follows:

$$\log \sigma_t = (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log \sigma_{t-1} + \sigma_\sigma v_{\sigma t}, \quad v_{\sigma t} \sim N(0, 1). \tag{3.1}$$

We let $F_t(\cdot) = F(\cdot|\sigma_t)$ denote the cumulative distribution function of ϵ given the realization of σ_t. We also denote the fraction of balance sheet asset funded through equity by m_t. $1 - m_t$ then represents the fraction funded by debts. For each unit of debt financing, the financial intermediary owes $1 + (1 - \tau_c)r^B_{t+1}$, where r^B_{t+1} is the borrowing rate and τ_c is a flat-rate corporate income tax rate. The intermediary is insolvent when the realized return is below its debt obligation:

$$\epsilon_{t+1}(1 + r^A_{t+1}) \leq [1 + (1 - \tau_c)r^B_{t+1}](1 - m_t).$$

We define the default threshold shock as

$$\epsilon^D_{t+1} \equiv \frac{1 + (1 - \tau_c)r^B_{t+1}}{1 + r^A_{t+1}}(1 - m_t). \tag{3.2}$$

Using the default threshold, investors' participation constraint can be expressed as

$$\begin{aligned} 1 - m_t \leq \mathbb{E}_t \Bigg\{ M_{t,t+1} \Bigg[(1 - \eta) \int_0^{\epsilon^D_{t+1}} \epsilon(1 + r^A_{t+1}) dF_{t+1}(\epsilon) \\ + \int_{\epsilon^D_{t+1}}^{\infty} (1 - m_t)(1 + r^B_{t+1}) dF_{t+1} \Bigg] \Bigg\}, \end{aligned} \tag{3.3}$$

where the default recovery is discounted by a factor $1 - \eta$ owing to bankruptcy costs and $M_{t,t+1}$ is the stochastic discount factor of the representative household.

3.1.3 Intermediary Equity Finance

We denote the dividend payouts of the intermediary by D_t. When D_t is negative, it should be interpreted as equity issuance. We express equity-related cash flow $\bar{\varphi}(D_t)$ as

$$\bar{\varphi}(D_t) = \begin{cases} D_t & \text{if } D_t \geq 0 \\ -(1-\varphi)D_t & \text{if } D_t < 0. \end{cases} \tag{3.4}$$

Note that $-(1-\varphi_t)D_t < -D_t$ when D_t is negative. This implies that the actual cash flow from the equity issuance of $-D_t$ is strictly less than $-D_t$ owing to equity dilution cost $\varphi \in (0,1)$. The dilution cost is a transfer from old shareholders to new shareholders. In general equilibrium, both are an identical entity. As a result, investors, as a whole, do not gain from this dilution cost. In the extreme of $\varphi = 1$, this would be equivalent to the assumption that the intermediary cannot issue equities. We denote the number of claims that the intermediary purchases by S_t and its unit price by Q_t. The flow of funds constraint for the intermediary is

$$\begin{aligned} Q_t S_t = {} & \max\{0, \epsilon_t(1 + r_t^A) - [1 + (1-\tau_c)r_t^B](1 - m_{t-1})\} Q_{t-1} S_{t-1} \\ & + (1 - m_t) Q_t S_t - \bar{\varphi}(D_t). \end{aligned} \tag{3.5}$$

We define an equity-financing trigger ϵ_t^E as the level of idiosyncratic shock below which financial intermediary must raise external funds. The shock threshold can be found by setting $\bar{\varphi}(D_t) = 0$ and solves (3.5) for ϵ_t, guessing that at this level of shock, the intermediary does not default, i.e., $\epsilon_t^E > \epsilon_t^D$:

$$\begin{aligned} \epsilon_t^E &= (1 - m_{t-1}) \frac{1 + (1-\tau_c) r_t^B}{1 + r_t^A} + \frac{m_t Q_t}{(1 + r_t^A) Q_{t-1} S_{t-1}} \\ &= \epsilon_t^D + \frac{m_t Q_t}{(1 + r_t^A) Q_{t-1} S_{t-1}}. \end{aligned} \tag{3.6}$$

(3.6) shows that $\epsilon_t^E > \epsilon_t^D$ indeed.

3.1.4 Value Maximization

The intermediary problem is presented in two stages. We denote the ex ante value of the intermediary by J_t prior to the realization of the idiosyncratic shock. We denote the ex post value $V_t(N_t)$ after the realization. Before the realization of the idiosyncratic shock, the intermediary solves

$$J_t = \max_{Q_t S_t, m_t, \epsilon^B_{t+1}} \left\{ \mathbb{E}^{\epsilon}_t [D_t] + \mathbb{E}_t \left[M_{t,t+1} \mathbb{E}^{\epsilon}_{t+1} [V_{t+1}(N_{t+1})] \right] \right\}$$
$$\text{s.t. (3.3) and (3.5),} \tag{3.7}$$

where the expectation operator $\mathbb{E}^{\epsilon}_t[\cdot]$ is defined with respect to ϵ. After the realization of the idiosyncratic shock, the intermediary solves

$$V_t(N_t) = \max_{D_t} \left\{ D_t + \mathbb{E}_t \left[M_{t,t+1} J_{t+1} \right] \right\} \quad \text{s.t. (3.5).} \tag{3.8}$$

We denote the shadow value of the flow of funds constraint (3.5) by λ_t. The first-order condition for (3.8) is

$$\lambda_t = \begin{cases} 1 & \text{if } D_t \geq 0 \\ 1/(1-\varphi) & \text{if } D_t < 0. \end{cases} \tag{3.9}$$

What matters for the investment problem is not λ_t, but its expected value $\mathbb{E}^{\epsilon}_t[\lambda_t]$. Using (3.6) and (3.9), one can evaluate the expected value as

$$\mathbb{E}^{\epsilon}_t[\lambda_t] = 1 - F_t(\epsilon^E_t) + \frac{F_t(\epsilon^E_t)}{1-\varphi} = 1 + \mu F_t(\epsilon^E_t) > 1, \quad \mu \equiv \frac{\varphi}{1-\varphi}. \tag{3.10}$$

We define standardized default and equity issuance thresholds as $s^D_{t+1} \equiv \sigma^{-1}_{t+1}(\log \epsilon^D_{t+1} + 0.5\sigma^2_{t+1})$ and $s^E_{t+1} \equiv \sigma^{-1}_{t+1}(\log \epsilon^E_{t+1} + 0.5\sigma^2_{t+1})$, respectively. The appendix of Kiley and Sim (2015) derives the first-order conditions of problem (3.7) as

$$Q_t S_t : 1 = \mathbb{E}_t \left\{ M^B_{t,t+1} \frac{1}{m_t} \left[1 + \tilde{r}^A_{t+1} - (1-m_t)[1 + (1-\tau_c) r^B_{t+1}] \right] \right\}, \tag{3.11}$$

$$m_t : \mathbb{E}_t^{\epsilon}[\lambda_t] = \theta_t \Bigg\{ 1 - \mathbb{E}_t \Bigg[M_{t,t+1} \Bigg((1-\eta) r_t^m \Phi(s_{t+1}^D) - \frac{\tau_c - r_t^m}{1-\tau_c} [1 - \Phi(s_{t+1}^D)] \Bigg) \Bigg] \Bigg\}, \tag{3.12}$$

$$\begin{aligned} \epsilon_{t+1}^D : 0 = {} & \mathbb{E}_t \Bigg[M_{t,t+1} \Bigg(\frac{\Phi(s_{t+1}^D)}{1-\varphi_{t+1}} - [1 + \mu_{t+1}\Phi(s_{t+1}^E)] \Bigg) (1 + r_{t+1}^A) \Bigg] \\ & + \theta_t \mathbb{E}_t \Bigg\{ M_{t,t+1} \Bigg[(1-\eta) \frac{\phi(s_{t+1}^D - \sigma_{t+1})}{\sigma_{t+1}\epsilon_{t+1}^D} \\ & + \frac{1}{1-\tau_c} \Bigg(1 - \Phi(s_{t+1}^D) - \frac{\phi(s_{t+1}^D)}{\sigma_{t+1}} \Bigg) \Bigg] (1 + r_{t+1}^A) \Bigg\} \\ & + \theta\,(1 - m_t)\, \mathbb{E}_t \Bigg[M_{t,t+1} \frac{\phi(s_{t+1}^D)}{\sigma_{t+1}\epsilon_{t+1}^D} \frac{\tau_c}{1-\tau_c} \Bigg], \end{aligned} \tag{3.13}$$

where θ_t is the shadow value of the constraint (3.3), the intermediary asset pricing kernel is given by

$$M_{t,t+1}^B \equiv M_{t,t+1} \frac{\mathbb{E}_{t+1}^{\epsilon}[\lambda_{t+1}]}{\mathbb{E}_t^{\epsilon}[\lambda_t]} = M_{t,t+1} \frac{1 + \mu\Phi(s_{t+1}^E)}{1 + \mu\Phi(s_t^E)}, \tag{3.14}$$

and the modified asset return $1 + \tilde{r}_{t+1}^A$ is defined as

$$\begin{aligned} 1 + \tilde{r}_{t+1}^A \equiv {} & \Bigg[\frac{1 + \mu_{t+1}\Phi(s_{t+1}^E - \sigma_{t+1})}{1 + \mu\Phi(s_{t+1}^E)} \\ & + \frac{\epsilon_{t+1}^D \Phi(s_{t+1}^D) - \Phi(s_{t+1}^D - \sigma_t)}{(1-\varphi)[1 + \mu_{t+1}\Phi(s_{t+1}^E)]} \Bigg] (1 + r_{t+1}^A). \end{aligned} \tag{3.15}$$

The appendix of Kiley and Sim (2015) further shows that the analytical solution for (3.15) is given by

$$\begin{aligned} 1 + \tilde{r}_{t+1}^A \equiv {} & \Bigg[\frac{1 + \mu_{t+1}\Phi(s_{t+1}^E - \sigma_{t+1})}{1 + \mu\Phi(s_{t+1}^E)} \\ & + \frac{\epsilon_{t+1}^D \Phi(s_{t+1}^D) - \Phi(s_{t+1}^D - \sigma_t)}{(1-\varphi)[1 + \mu_{t+1}\Phi(s_{t+1}^E)]} \Bigg] (1 + r_{t+1}^A). \end{aligned} \tag{3.16}$$

3.1.5 Production and Investment

There is a competitive industry that produces intermediate goods using a constant-returns-to-scale technology; without loss of generality, we assume the existence of a representative firm. The firm combines capital (K) and labor (H) to produce the intermediate goods using a Cobb-Douglas production function,

$$Y_t^M = a_t H_t^{\alpha} K_t^{1-\alpha}, \tag{3.17}$$

where the technology shock follows a Markov process,

$$\log a_t = \rho_a \log a_{t-1} + \sigma_a v_{at}, \quad v_{at} \sim N(0,1). \tag{3.18}$$

The intermediate goods producer issues state-contingent claims S_t to a financial intermediary and uses the proceeds to finance capital purchases, $Q_t K_{t+1}$. No arbitrage implies that the price of the state-contingent claim must be equal to Q_t such that $Q_t S_t = Q_t K_{t+1}$. The firm's static profit per unit of capital is determined by the capital share of revenue, i.e., $r_t^K = (1-\alpha) P_t^M Y_t / K_t$, where P_t^M is the relative price of the intermediate goods. The aggregate return on asset is given by

$$1 + r_t^A = \frac{(1-\tau_c)(1-\alpha) P_t^M Y_t / K_t + [1-(1-\tau_c)\delta] Q_t}{Q_{t-1}}. \tag{3.19}$$

To endogenize the price of capital, we introduce a competitive investment-goods industry, which produces investment goods by combining and consumption goods and undepreciated capital using a quadratic adjustment cost of investment, $\chi_t/2(I_t/I_{t-1} - 1)^2 I_{t-1}$, where χ_t follows a Markov process,

$$\log \chi_t = (1-\rho_\chi) \log \bar{\chi} + \rho_\chi \log \chi_{t-1} + \sigma_\chi v_{\chi t}, \quad v_{\chi t} \sim N(0,1). \tag{3.20}$$

The optimization condition of the investment-goods firm leads to a well-known investment Euler equation.

3.1.6 Households

The preferences of the representative household is specified as

$$\sum_{s=0}^{\infty} \beta^s \left[\frac{1}{1-\gamma} [(C_{t+s} - hC_{t+s-1})^{1-\gamma} - 1] - \frac{1}{1+\nu} H_{t+s}^{1+\nu} \right], \tag{3.21}$$

where C_t is consumption, H_t is hours worked, β is the time discount factor, γ governs the curvature in the utility function, h is the external habit, and ν is the inverse of the Frisch elasticity of labor supply. The household problem is to optimize over the choices of intermediary bond holdings, intermediary equity holdings, risk-free nominal bond holdings, and labor hours. Of these we skip the static optimizing condition for hours.

The household invests in a perfectly diversified portfolio of intermediary debts, $B_t = \int [1 - m_{t-1}(i)] Q_{t-1} S_{t-1} di$. The optimization condition for bond investment leads to the participation constraint (3.3).

The appendix of Kiley and Sim (2015) shows that the optimization condition of equity investment in intermediary shares satisfies

$$1 = \mathbb{E}_t \left[M_{t,t+1} \frac{\mathbb{E}_{t+1}^{\epsilon} [\max\{D_{t+1}, 0\} + (1 - \varphi_{t+1}) \min\{D_{t+1}, 0\}] + P_{t+1}^S}{P_t^S} \right], \tag{3.22}$$

where P_t^S is the ex-dividend price of an intermediary share. In our symmetric equilibrium, $P_t^S(i) = P_t^S$ for all $i \in [0, 1]$ because $\mathbb{E}_t[M_{t,t+1} \cdot J_{t+1}]$ does not depend on intermediary-specific variables.[5]

Finally, the household's optimizing condition for risk-free bond holding leads to the well-known consumption Euler equation:

$$1 = \mathbb{E}_t \left[M_{t,t+1} R_t \Xi_t \right]. \tag{3.23}$$

We assume that the "risk premium" follows a Markov process,

$$\log \Xi_t = \rho_{\Xi} \log \Xi_{t-1} + \sigma_{\Xi} \upsilon_{\Xi t}, \quad \upsilon_{\Xi t} \sim N(0, 1). \tag{3.24}$$

[5]In general equilibrium, the existing shareholders and the investors in the new shares are the same entity, the representative household. Hence, costly equity financing does not create a wealth effect for the household, but affects the aggregate allocation through the marginal efficiency conditions of the intermediaries.

3.1.7 Nominal Rigidity and Monetary Policy

To generate nominal rigidity, we assume that the retailers face a quadratic cost in adjusting their prices $P_t(i)$ given by $\chi^p/2\left(P_t(i)/P_{t-1}(i) - (\bar{\Pi}^{1-\kappa}\Pi_{t-1}^{\kappa})\right)^2 Y_t$, where Y_t is the CES aggregate of the differentiated products with an elasticity of substitution ε_t, which follows a Markov process,

$$\log \varepsilon_t = (1-\rho_\varepsilon)\log\bar{\varepsilon} + \rho_\varepsilon \log\varepsilon_{t-1} + \sigma_\varepsilon v_{\varepsilon t}, \quad v_{\varepsilon t} \sim N(0,1). \tag{3.25}$$

κ is the inflation indexation parameter. The optimal pricing decision leads to a well-known Phillips curve, which is both backward and forward looking.

Monetary policy is specified by the following Taylor rule:

$$R_t = R_{t-1}^{\rho_R}\left[\frac{\bar{\Pi}}{\beta}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{r_\Pi}\left(\frac{Y_t - Y^*}{Y^*}\right)^{r_{y^*}}\left(\frac{Y_t}{Y_{t-1}}\right)^{r_{\Delta y}}\right]^{1-\rho_R}\exp(e_t^R), \tag{3.26}$$

where e_t^R is iid monetary policy shock.

3.1.8 Fiscal Policy

The fiscal policy is simply dictated by the period-by-period balanced budget constraint. The revenues for government come from two sources: corporate income tax of the financial intermediaries and lump-sum tax on households. The proceeds from the corporate income tax are assumed to be transferred back to the financial intermediaries in a lump-sum fashion. We also assume that the distortionary subsidies on product prices and wages are funded by the lump-sum tax on the households. In addition, fluctuations in government purchases are a source of autonomous demand shocks, as in Smets and Wouters (2007).

3.2 Pigovian Tax

When the Pigovian tax is introduced, the flow of funds constraint facing the intermediaries becomes

$$
\begin{aligned}
Q_tS_t = {} & \max\{0, \epsilon_t(1+r_t^A) - [1+(1-\tau_c)r_t^B](1-m_{t-1})\}Q_{t-1}S_{t-1} \\
& + (1-\tau_t^m)(1-m_t)Q_tS_t - \bar{\varphi}(D_t),
\end{aligned} \tag{3.27}
$$

where T_t is the lump-sum transfer of the proceeds from the leverage taxation. In equilibrium $\tau_t^m(1-m_t)Q_tS_t = T_t$, though T_t is taken as given by the intermediaries. The default threshold is now given by

$$
\epsilon_{t+1} \leq \epsilon_{t+1}^D \equiv (1-m_t)\left[\frac{1+(1-\tau_c)r_{t+1}^B}{1+r_{t+1}^A}\right]. \tag{3.28}
$$

Following the same steps, one can derive the following efficiency conditions:

$$
\begin{aligned}
Q_tS_t : 1 = {} & \mathbb{E}_t\left[M_{t,t+1}^B\frac{1}{m_t+\tau_t^m(1-m_t)}\right. \\
& \left.\times\left[1+\tilde{r}_{t+1}^A - (1-m_t)[1+(1-\tau_c)r_{t+1}^B]\right]\right]
\end{aligned} \tag{3.29}
$$

$$
m_t : 1 - \tau_t^m)\mathbb{E}_t^{\epsilon}[\lambda_t] = \theta_t\left\{1+\frac{\tau_c}{1-\tau_c}\mathbb{E}_t\left[M_{t,t+1}[1-\Phi(s_{t+1}^D)]\right]\right\} \tag{3.30}
$$

$$
\begin{aligned}
\epsilon_{t+1}^D : 0 = {} & \mathbb{E}_t\left\{M_{t,t+1}\left[\frac{\Phi(s_{t+1}^D)}{1-\varphi_{t+1}} - [1+\mu_{t+1}\Phi(s_{t+1}^E)]\right](1+r_{t+1}^A)\right\} \\
& + \theta_t\mathbb{E}_t\left\{M_{t,t+1}\left[(1-\eta)\frac{\phi(s_{t+1}^D-\sigma_{t+1})}{\sigma_{t+1}\epsilon_{t+1}^D}\right.\right. \\
& \left.\left. + \frac{1}{1-\tau_c}\left(1-\Phi(s_{t+1}^D) - \frac{\phi(s_{t+1}^D)}{\sigma_{t+1}}\right)\right](1+r_{t+1}^A)\right\} \\
& + \theta_t(1-m_t)\mathbb{E}_t\left\{M_{t,t+1}\frac{\tau_c}{1-\tau_c}\frac{\phi(s_{t+1}^D)}{\sigma_{t+1}\epsilon_{t+1}^D}\right\}.
\end{aligned} \tag{3.31}
$$

3.3 Calibration/Estimation of Key Parameters

Our approach involves calibration of certain parameters and estimation of others—we assign parameters to each category based on the degree to which observed fluctuations in the data are likely to be informative about parameter values. Our estimation is informed by eight macroeconomic time series. The first six are among those in Smets and Wouters (2007), given below.

$$\text{Change in output per capita} = \hat{y}_t - \hat{y}_{t-1}$$
$$\text{Change in consumption per capita} = \hat{c}_t - \hat{c}_{t-1}$$
$$\text{Change in investment per capita} = \hat{\imath}_t - \hat{\imath}_{t-1}$$
$$\text{Change in nominal wage per capita} = \hat{w}_t - \hat{w}_{t-1}$$
$$\text{Change in hours worked per capita} = \hat{l}_t - \hat{l}_{t-1}$$
$$\text{GDP price inflation} = \hat{\pi}_t$$
$$\text{Nominal federal funds rate} = \hat{r}_t$$

In each case, lowercase letters refer to the natural logarithm of a variable, and we remove the mean from the series prior to estimation.

The last two time series used in estimation are data on long-run expected inflation from the Survey of Professional Forecasters and the excess bond premium from Gilchrist and Zakrajsek (2012), which we link to the model by

$$\text{Expected inflation} = \frac{1}{40}\sum_{j=1}^{40} E_t[\hat{\pi}_{t+j}]$$
$$\text{Excess bond premium} = \frac{1}{20}\sum_{j=1}^{40} E_t[\hat{R}^L_{t+j} - \hat{R}_{t+j}].$$

Table 3.1 summarizes the calibrated parameters. Tables 3.2 and 3.3 report the key estimated parameters and the variance decomposition implied by the estimation results. Our estimation sample spans the periods from 1965 to 2008.

Table 3.1 Baseline Calibration

Description	**Calibration**
Preferences and Production	
Time Discounting Factor	$\beta = 0.985$
Value-Added Share of Labor	$\alpha = 0.600$
Depreciation Rate	$\delta = 0.025$
Financial Frictions	
Liquidation Cost	$\eta = 0.050$
Corporate Income Tax	$\tau_c = 0.200$
Long-Run Level of Uncertainty	$\bar{\sigma} = 0.030$

Table 3.2 Posterior Moments of Key Parameters

Parameter	**Mean**	**[0.05, 0.95]**
Preferences		
γ	1.57	[1.41, 1.72]
h	0.37	[0.30, 0.44]
ν	0.95	[0.63, 1.27]
Financial Friction		
$\bar{\varphi}$	0.24	[0.20, 0.28]
$\bar{\chi}$	4.44	[3.76, 5.13]
Nominal Rigidities		
$\bar{\varepsilon}$	51.69	[41.14, 59.06]
κ	0.07	[0.01, 0.12]
Monetary Policy		
ρ_R	0.72	[0.68, 0.75]
r_{y*}	0.02	[–0.01, 0.06]
$r_{\Delta y}$	0.53	[0.41, 0.64]
r_{Π}	0.72	[0.59, 0.84]

Table 3.3 Variance Decomposition for Observable Variables

	Fin. Vol.	**Nat. Rate of Int.**	**Tobin's Q**	**Technology**	**Markup**	**Nominal Int. Rate**	**Infl. Target**	**Auto. Dem.**
Δy	2.2	15.5	17.2	3.3	27.0	7.5	0.4	27.0
Δc	10.6	38.0	5.3	5.8	13.3	18.3	0.6	8.0
Δi	11.5	4.0	50.6	1.6	29.1	1.9	1.3	0.1
Δh	1.2	11.8	13.3	27.7	20.2	5.7	0.2	20.0
EBP	41.0	51.7	0.0	0.1	2.2	4.3	0.6	0.0
R	0.1	0.1	0.1	0.0	0.0	0.0	99.7	0.0
Π	0.0	0.0	0.0	0.0	0.1	0.0	99.9	0.0
$E\Pi^{40}$	0.0	0.0	0.0	0.0	0.0	0.0	100.0	0.0

4. Albert Queralto: Banks and Outside Equity

This appendix provides details on the model by Gertler, Kiyotaki, and Queralto (2012) included in "Macroeconomic Effects of Banking-Sector Losses across Structural Models." Section 4.1 describes the agents' optimization problems. Section 4.2 contains the model's full set of equilibrium conditions. Section 4.3 describes the calibration of the model parameters.

4.1 Model Setup

4.1.1 Households

The household chooses consumption, labor supply, riskless debt, and outside equity $(C_t, L_t, D_{h,t}, \overline{e}_t)$ to maximize

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\gamma} \left(C_\tau - hC_{\tau-1} - \frac{\chi}{1+\varphi} L_\tau^{1+\varphi} \right)^{1-\gamma} \tag{4.1}$$

subject to

$$\begin{aligned} C_t + D_{h,t} + q_t \overline{e}_t = W_t L_t + \Pi_t - T_t + R_t D_{h,t-1} \\ + \left[Z_t + (1-\delta) q_t \right] \psi_t \overline{e}_{t-1}. \end{aligned} \tag{4.2}$$

Here q_t is the price of a unit of outside equity, normalized so that each equity is a claim to the future returns of one unit of the asset that the bank holds. Z_t is the flow returns generated by one unit of the bank's asset, δ is the depreciation rate of capital, and ψ_t is the capital quality shock. Thus, the total payoff at t for a share of outside equity acquired at $t-1$ is $[Z_t + (1-\delta)q_t]\psi_t$.

W_t is the wage rate, T_t is lump-sum taxes, and Π_t is net profit from both banks and non-financial firms.

4.1.2 Non-financial Firms

There are two types of non-financial firms: goods producers and capital producers.

4.1.3 Goods Producers

Competitive goods producers use capital K_t and labor L_t as inputs to produce final goods. They operate a production function given by

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}. \tag{4.3}$$

Good producers purchase capital one period in advance. To finance their capital purchases, they issue state-contingent securities to banks, at price Q_t (the price of a unit of physical capital). Then, given capital, in period t firms choose labor to satisfy

$$W_t = (1-\alpha)\frac{Y_t}{L_t}. \tag{4.4}$$

Gross profits per unit of capital, Z_t, are then

$$Z_t \equiv \frac{Y_t - W_t L_t}{K_t} = \alpha \left(\frac{L_t}{K_t}\right)^{1-\alpha}. \tag{4.5}$$

Since there are no financial frictions between firms and banks, through perfect competition the (gross) return on goods firms' securities is $\psi_t \left[Z_t + (1-\delta)Q_t\right]$, and these firms earn zero residual profits state by state.

4.1.4 Capital Producers

Capital producers make new capital goods using final output as input, and are subject to adjustment costs given by $f(I_t/I_{t-1})I_t$, with $f(1) = f'(1) = 0$ and $f''(I_t/I_{t-1}) > 0$. A capital producer chooses I_t to solve

$$\max \quad \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_\tau I_\tau - \left[1 + f\left(\frac{I_\tau}{I_{\tau-1}}\right)\right] I_\tau \right\}. \tag{4.6}$$

Above, $\Lambda_{t,\tau}$ is the household's discount factor between periods t and τ.

4.1.5 Banks

Each bank raises funds by issuing deposits d_t and outside equity to purchase producers' equity, s_t:

$$Q_t s_t = n_t + q_t e_t + d_t. \tag{4.7}$$

The evolution of a bank's net worth (or inside equity), n_t, is

$$n_t = [Z_t + (1-\delta)Q_t]\,\psi_t s_{t-1} - [Z_t + (1-\delta)q_t]\psi_t e_{t-1} - R_t d_{t-1} - \epsilon_t n_{t-1}. \tag{4.8}$$

Above, $\epsilon_t n_{t-1}$ is a capital transfer which subtracts from the bank's resources at the beginning of the period. We assume that the transfer is equal to fraction ϵ_t of previous-period inside equity n_{t-1}, where ϵ_t is an exogenous stochastic process.

The value of the bank at the end of period t is

$$V_t = V(s_t, x_t, n_t) = \mathbb{E}_t \sum_{\tau=t+1}^{\infty} (1-\sigma)\sigma^{\tau-t}\Lambda_{t,\tau} n_\tau, \tag{4.9}$$

where $x_t \equiv \frac{q_t e_t}{Q_t s_t}$, and σ is the banker's survival probability. After obtaining funds, the banker may default on its debt and divert a fraction $\Theta(x_t)$ of assets. The incentive constraint for the bank not to steal is

$$V(s_t, x_t, n_t) \geq \Theta(x_t) Q_t s_t. \tag{4.10}$$

The divertable fraction is

$$\Theta(x_t) = \theta\left(1 + \epsilon x_t + \frac{\kappa}{2} x_t^2\right). \tag{4.11}$$

The bank's problem is to choose assets and outside equity, (s_t, x_t), to maximize (4.9) subject to (4.7), (4.8), and (4.10). To solve the problem, we first conjecture that the bank's value function takes the following form:

$$V_t(s_t, x_t, n_t) = (\mu_{s,t} + x_t \mu_{e,t}) Q_t s_t + \nu_t n_t, \tag{4.12}$$

where $\mu_{s,t}$, $\mu_{e,t}$ and ν_t are coefficients to be determined, which do not depend on the bank's individual state. The Lagrangian for the bank's problem, $\mathcal{L}_t$, is then

$$\mathcal{L}_t = [(\mu_{s,t} + x_t \mu_{e,t})\, Q_t s_t + \nu_t n_t]\,(1+\lambda_t) - \lambda_t \theta \left(1 + \epsilon x_t + \frac{\kappa}{2} x_t^2\right), \tag{4.13}$$

where λ_t is the multiplier on (4.10).

As shown in the working paper version of Gertler, Kiyotaki, and Queralto (2012), the bank's optimality conditions are as follows:

$$Q_t s_t = \phi_t n_t \tag{4.14}$$

$$\phi_t = \frac{\nu_t}{\Theta(x_t) - (\mu_{s,t} + x_t \mu_{e,t})} \tag{4.15}$$

$$\begin{aligned} x_t &= -\frac{\mu_{s,t}}{\mu_{e,t}} + \left[\left(\frac{\mu_{s,t}}{\mu_{e,t}}\right)^2 + \frac{2}{\kappa}\left(1 - \epsilon\frac{\mu_{s,t}}{\mu_{e,t}}\right)\right]^{1/2} \\ &\equiv x\left(\frac{\mu_{s,t}}{\mu_{e,t}}\right), \quad \text{where } x' > 0 \text{ given } \kappa > \frac{1}{2}\epsilon^2 \end{aligned} \tag{4.16}$$

with

$$\nu_t = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{t+1} - \epsilon_{t+1}\right)\right] \tag{4.17}$$

$$\mu_{s,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{k,t+1} - R_{t+1}\right)\right] \tag{4.18}$$

$$\mu_{e,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{t+1} - R_{e,t}\right)\right] \tag{4.19}$$

$$\Omega_{t+1} = 1 - \sigma + \sigma\left[\nu_{t+1} + \phi_{t+1}\left(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1}\right)\right]. \tag{4.20}$$

Note that the marginal value of inside equity, ν_t, includes the term $-\epsilon_{t+1}$, capturing the inside equity transfer in period $t+1$. Above, we have defined the rates of return to non-financial firms' securities and to banks' outside equity, $R_{k,t}$ and $R_{e,t}$, respectively, as

$$R_{k,t} \equiv \psi_t \frac{Z_t + (1-\delta)Q_t}{Q_{t-1}} \tag{4.21}$$

$$R_{e,t} \equiv \psi_t \frac{Z_t + (1-\delta)q_t}{q_{t-1}}. \tag{4.22}$$

4.2 Equilibrium Conditions

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{4.23}$$

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - \mathbb{E}_t\left[\Lambda_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right)\right] \tag{4.24}$$

$$Q_t S_t = \phi_t N_t \tag{4.25}$$

$$S_t = I_t + (1-\delta)K_t \tag{4.26}$$

$$K_{t+1} = \psi_{t+1} S_t \tag{4.27}$$

$$N_t = \sigma\left\{\left[R_{k,t} - R_t - (R_{e,t} - R_t)x_{t-1}\right] Q_{t-1} S_{t-1} + R_t N_{t-1}\right\} + (1-\sigma)\xi Q_{t-1} S_{t-1} - \epsilon_t N_{t-1} \tag{4.28}$$

$$1 = \mathbb{E}_t\left(\Lambda_{t+1} R_{t+1}\right) \tag{4.29}$$

$$0 = \mathbb{E}_t\left[\Lambda_{t+1}\left(R_{e,t+1} - R_{t+1}\right)\right] \tag{4.30}$$

$$\Lambda_t = \beta \frac{u_{c,t}}{u_{c,t-1}} \tag{4.31}$$

$$\phi_t = \frac{\nu_t}{\theta\left(1 + \epsilon x_t + \frac{\kappa}{2} x_t^2\right) - \left(\mu_{s,t} + x_t \mu_{e,t}\right)} \tag{4.32}$$

$$\nu_t = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{t+1} - \epsilon_{t+1}\right)\right] \tag{4.33}$$

$$\mu_{s,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{k,t+1} - R_{t+1}\right)\right] \tag{4.34}$$

$$\mu_{e,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{t+1} - R_{e,t}\right)\right] \tag{4.35}$$

$$x_t = -\frac{\mu_{s,t}}{\mu_{e,t}} + \left[\left(\frac{\mu_{s,t}}{\mu_{e,t}}\right)^2 + \frac{2}{\kappa}\left(1 - \epsilon\frac{\mu_{s,t}}{\mu_{e,t}}\right)\right]^{1/2} \tag{4.36}$$

$$\Omega_{t+1} = 1 - \sigma + \sigma\left[\nu_{t+1} + \phi_{t+1}\left(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1}\right)\right] \tag{4.37}$$

$$R_{k,t} = \psi_t \frac{\alpha\left(\frac{L_t}{K_t}\right)^{1-\alpha} + (1-\delta)Q_t}{Q_{t-1}} \tag{4.38}$$

$$R_{e,t} = \psi_t \frac{\alpha\left(\frac{L_t}{K_t}\right)^{1-\alpha} + (1-\delta)q_t}{q_{t-1}} \tag{4.39}$$

$$(1-\alpha)\frac{Y_t}{L_t} u_{C,t} = \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi} L_t^{1+\varphi}\right)^{-\gamma} \chi L_t^{\varphi} \tag{4.40}$$

$$u_{C,t} = \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi}\right)^{-\gamma} - \beta h \mathbb{E}_t \left(C_{t+1} - hC_t - \frac{\chi}{1+\varphi}L_{t+1}^{1+\varphi}\right)^{-\gamma} \quad (4.41)$$

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \quad (4.42)$$

The twenty equilibrium conditions (4.23)–(4.42) determine the twenty endogenous variables Y_t, C_t, I_t, Q_t, q_t, ϕ_t, N_t, S_t, K_{t+1}, $R_{k,t}$, $R_{e,t}$, R_{t+1}, x_t, Λ_t, $u_{C,t}$, ν_t, $\mu_{s,t}$, $\mu_{e,t}$, Ω_t, L_t. The exogenous variables are the capital quality shock, ψ_t, and the bank capital transfer, ϵ_t.

4.3 Calibration

Table 4.1 contains the values assigned to the model's parameters. We choose conventional values for the standard preference and technology parameters: $\gamma, \beta, \alpha, \delta, \chi, \varphi, h$, and the elasticity of investment to Q.

There are five parameters specific to our model: σ, ξ, θ, ϵ, and κ. We set the survival rate of bankers, σ, to 0.9685, implying that bankers survive for eight years on average. We set the remaining four parameters to hit four targets. The first three targets involve characteristics of the low-risk economy, which is meant to capture the "Great Moderation" period. In particular, we target an aggregate leverage ratio (assets to the sum of inside and outside equity) of 4, an average credit spread of 100 basis points annually, and a ratio of outside to inside equity of two-thirds. The final target is having the aggregate leverage ratio fall by a third as the economy moves from low to high risk. The choice of an aggregate leverage of 4 represents a first-pass attempt to average across sectors with vastly different financial structures, from housing finance (featuring very large leverage ratios) to other sectors of the economy where leverage is clearly lower. The target for the spread is based on a rough average of the following spreads over the Great Moderation period: mortgage rates relative to government bonds rates, BAA corporate rates versus government bond rates, and commercial paper rates versus T-bill rates. The target of outside to inside equity approximates the ratio of common equity to the sum of preferred equity and subordinate debt in the banking sector prior to the crisis. Finally, the

Table 4.1 Calibration

γ	2	Risk Aversion
β	0.99	Discount Factor
α	0.33	Capital Share
δ	0.025	Depreciation Rate
χ	0.25	Utility Weight of Labor
φ	1/3	Inverse Frisch Elasticity of Labor Supply
If''/f	1	Inverse Elasticity of Investment to the Price of Capital
h	0.75	Habit Parameter
σ	0.9685	Survival Rate of Bankers
ξ	0.0289	Transfer to Entering Bankers
θ	0.264	Parameter in Asset Diversion Function (1)
ϵ	−1.21	Parameter in Asset Diversion Function (2)
κ	13.41	Parameter in Asset Diversion Function (3)

drop in the aggregate leverage ratio of a third as the economy moves from low to high risk is a rough estimate of what would occur if the financial system undid the buildup of leverage over the last decade.

5. Luca Guerrieri and Mohammad Jahan-Parvar: Capital Shortfalls in a Two-Sector Production Economy

In this appendix we describe the setup of the model by Guerrieri and Jahan-Parvar included in "Macroeconomic Effects of Banking-Sector Losses across Structural Models."

We build the model in layers. We start with a frictionless real business cycle (RBC) model, decentralized in a way that firms operate for only two periods. In the first period they plan and raise equity from households to buy capital and produce the following period. The next layer puts financial intermediaries between households and firms introducing the same principal-agent problem considered by Gertler and Karadi (2011). Building up, we show how to introduce a transfer shock from banks to households. Expanding the one-sector model, we consider an environment in which a fraction of firms can access equity markets directly, without having to reach them through banks. Finally, we layer on nominal rigidities and monetary policy.

5.1 Asset Pricing in a Basic RBC Model

5.1.1 Production

The production technology of the representative firm is

$$Y_t = A_t {K_t}^{\alpha} L_t^{1-\alpha}. \tag{5.1}$$

Firms operate for only one period, but some of the planning for production is done one period in advance. To operate capital in period $t+1$, a firm must purchase it in period t. To do so, the firm issues shares in period t. There are as many shares S_t as units of capital purchased. By arbitrage, the current value of capital equals the value of shares. Thus,

$$Q_t K_{t+1} = Q_t S_t. \tag{5.2}$$

Let π_{t+1} denote the revenue of firms in period $t+1$ net of expenses. Revenues include proceeds from the sale of output as well as from the sale of the undepreciated fraction of capital. Expenses include obligations connected with the servicing of shares and with the compensation for labor services. Thus,

$$\pi_{t+1} = Y_{t+1} + Q_{t+1}(1-\delta)K_{t+1} - W_{t+1}L_{t+1} - (1+R^s_{t+1})Q_t S_{t.} \tag{5.3}$$

At time t the problem of firms is to choose S_t and K_{t+1} to maximize the expected profits in period $t+1$, knowing that the firms will be able to choose the optimal quantity of labor in that period. The firm takes Q_t, Q_{t+1}, R^s_{t+1}, and W_{t+1} as given. This maximization problem can be expressed as

$$\max_{S_t, K_{t+1}} E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \max_{L_{t+1}} \pi_{t+1} \tag{5.4}$$

subject to constraints of the production technology $Y_t = A_t {K_t}^{\alpha} L_t^{1-\alpha}$ and financing $Q_t K_{t+1} = Q_t S_t$. The solution of $\max_{L_{t+1}} \pi_{t+1}$ implies that

$$W_{t+1} = (1-\alpha)\,\frac{Y_{t+1}}{L_{t+1}} \tag{5.5}$$

$$L_{t+1} = (1-\alpha)\,\frac{Y_{t+1}}{W_{t+1}} \tag{5.6}$$

under all states of nature. Accordingly, $\max_{S_t,K_{t+1}} E_t \max_{L_{t+1}} \pi_{t+1}$ collapses to

$$\max_{S_t K_{t+1},L_{t+1}} E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}[A_{t+1}{K_{t+1}}^{\alpha}L_{t+1}^{1-\alpha}$$
$$+ Q_{t+1}(1-\delta)K_{t+1} - W_{t+1}L_{t+1} - (1+R^s_{t+1})Q_tS_{t.} + \tag{5.7}$$
$$\lambda_{lt+1t}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\,\frac{A_{t+1}{K_{t+1}}^{\alpha}L_{t+1}^{1-\alpha}}{W_{t+1}} - L_{t+1}\right]$$
$$+ \lambda_{st}\left(Q_tS_{t.} - Q_tK_{t+1}\right). \tag{5.8}$$

Notice that there is no expectation operator on the Lagrangian multipliers because those constraints hold under every state of nature.

The problem implies the following conditions:

$$\frac{\partial}{S_t} = -E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^s_{t+1})Q_t + \lambda_{st}Q_t = 0 \tag{5.9}$$

$$\frac{\partial}{\partial K_{t+1}} = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)\right]$$
$$+ \lambda_{lt+1}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\,\frac{\alpha\frac{Y_{t+1}}{K_{t+1}}}{W_{t+1}}\right] - \lambda_{st}Q_t \tag{5.10}$$

$$\frac{\partial}{\partial L_{t+1}} = \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\,\frac{Y_{t+1}}{L_{t+1}} - W_{t+1}\right] +$$
$$\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\lambda_{lt+1}\left[(1-\alpha)^2\,\frac{Y_{t+1}}{L_{t+1}W_{t+1}} - 1\right]. \tag{5.11}$$

Working on $\frac{\partial}{S_t}$,

$$\lambda_{st} = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^s_{t+1}). \tag{5.12}$$

From $\frac{\partial}{\partial K_{t+1}}$,

$$\lambda_{st}Q_t = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)\right] + \lambda_{lt+1}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\frac{\alpha\frac{Y_{t+1}}{K_{t+1}}}{W_{t+1}}\right] \quad (5.13)$$

$$E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^s_{t+1})Q_t = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\alpha\frac{Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)\right] + \lambda_{lt+1}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\frac{\alpha\frac{Y_{t+1}}{K_{t+1}}}{W_{t+1}}\right] \quad (5.14)$$

$$E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^s_{t+1}) = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right] + E_t\lambda_{lt+1}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\frac{\alpha\frac{Y_{t+1}}{K_{t+1}}}{W_{t+1}}\right]. \quad (5.15)$$

Next work on $\frac{\partial}{\partial L_{t+1}}$. Again, since $(1-\alpha)\frac{Y_{t+1}}{L_{t+1}} = W_{t+1}$,

$$\frac{\partial}{\partial L_{t+1}} = \lambda_{lt+1}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)^2\frac{Y_{t+1}}{L_{t+1}W_{t+1}} - 1\right] = 0. \quad (5.16)$$

Substituting $(1-\alpha)\frac{Y_{t+1}}{L_{t+1}} = W_{t+1}$ again in the equation above, one can see that

$$\frac{\partial}{\partial L_{t+1}} = \lambda_{lt+1}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha) - 1\right] = 0 \quad (5.17)$$

$$\lambda_{lt+1} = 0. \quad (5.18)$$

Then, combining the implications of $\frac{\partial}{\partial L_{t+1}} = 0$ and $\frac{\partial}{\partial K_{t+1}} = 0$ yields

$$E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^s_{t+1}) = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha\frac{Y_{t+1}}{K_{t+1}} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right]. \quad (5.19)$$

We can also think of this equation as determining the demand for capital K_{t+1} (or loans S_t). Remembering that K_{t+1} is in the information set at time t, and rearranging,

$$\begin{aligned} K_{t+1}E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^s_{t+1}) \\ = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha Y_{t+1}+(1-\delta)\frac{Q_{t+1}}{Q_t}K_{t+1}\right] \end{aligned} \tag{5.20}$$

$$K_{t+1}E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1+R^s_{t+1})-(1-\delta)\frac{Q_{t+1}}{Q_t}\right]=E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha Y_{t+1}\right] \tag{5.21}$$

$$K_{t+1}=\frac{E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha Y_{t+1}\right]}{E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1+R^s_{t+1})-(1-\delta)\frac{Q_{t+1}}{Q_t}\right]}. \tag{5.22}$$

Notice that firms will make zero profits under all states of nature (and that's why we can drop the expectation operator). Thus,

$$0=Y_{t+1}+Q_{t+1}(1-\delta)K_{t+1}-W_{t+1}L_{t+1}-(1+R^s_{t+1})Q_tS_t \tag{5.23}$$

$$(1+R^s_{t+1})Q_tS_{t.}=Y_{t+1}+Q_{t+1}(1-\delta)K_{t+1}-W_{t+1}L_{t+1} \tag{5.24}$$

$$(1+R^s_{t+1})=\frac{Y_{t+1}+Q_{t+1}(1-\delta)K_{t+1}-W_{t+1}L_{t+1}}{Q_tS_t} \tag{5.25}$$

$$(1+R^s_{t+1})=\frac{Y_{t+1}+Q_{t+1}(1-\delta)K_{t+1}-W_{t+1}L_{t+1}}{Q_tK_{t+1}} \tag{5.26}$$

$$(1+R^s_{t+1})=\frac{Y_{t+1}+Q_{t+1}(1-\delta)K_{t+1}-W_{t+1}\left(1-\alpha\right)\frac{Y_{t+1}}{W_{t+1}}}{Q_tK_{t+1}} \tag{5.27}$$

$$(1+R^s_{t+1})=\frac{1}{Q_t}\frac{\alpha Y_{t+1}}{K_{t+1}}+\frac{(1-\delta)}{Q_t}Q_{t+1}. \tag{5.28}$$

This condition will also imply $E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^s_{t+1}) = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha\frac{Y_{t+1}}{K_{t+1}}+(1-\delta)\frac{Q_{t+1}}{Q_t}\right]$ derived above (if profits are always zero, it does not matter how you discount them). To interpret the zero-profit condition, notice that if Q_t is the price of capital normalized by the price of consumption, then $\frac{1}{Q_t}$ must be the capital obtained by giving up one unit of consumption. That quantity of capital $\frac{1}{Q_t}$ obtains a rental rate $\frac{\alpha Y_{t+1}}{K_{t+1}}$. After production takes place, the underpreciated portion can be resold at price Q_{t+1}, so the same quantity of capital $\frac{1}{Q_t}$ obtains additionally capital gains equal to $(1-\delta)Q_{t+1}$. Also note that because the condition above holds under every state of nature, it can be written as

$$(1+R_t^s) = \frac{1}{Q_{t-1}}\frac{\alpha Y_t}{K_t} + \frac{(1-\delta)}{Q_{t-1}}Q_t. \tag{5.29}$$

Firms sell their output to households, to the government, and to investment-goods producers. Consequently, the resource constraint can be expressed as

$$Y_t = C_t + I_t^g + G_t. \tag{5.30}$$

5.1.2 Households

A representative household maximizes utility given by

$$\max_{C_{t+i},L_{t+i},S_t,B_t} E_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon} L_{t+i}^{1+\varepsilon}\right]. \tag{5.31}$$

In the absence of financial frictions, households buy shares of firms directly. Then, the budget constraint of households takes the following form:

$$C_t = W_t L_t - T_t - Q_t S_t + (1+R_t^s)Q_{t-1}S_{t-1} - B_t + (1+R_{t-1})B_{t-1}. \tag{5.32}$$

There is a riskless government bond B_t. In period t households purchase B_t of the riskless bond and earn $(1+R_{t-1})B_{t-1}$ from previous purchases. Households take R_t^s, R_t, W_t, and T_t as given.

5.1.3 Capital-Producing Firms

The evolution of capital takes the form

$$K_{t+1} = I_t^n + (1-\delta)\, K_t. \tag{5.33}$$

Net investment is simply given by

$$I_t^n = K_{t+1} - (1-\delta) K_t. \tag{5.34}$$

The production technology for investment involves a quadratic adjustment for current production relative to past production, thus the supply of investment goods is given by

$$I_t^n = \left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_t^g. \tag{5.35}$$

Capital-producing firms solve the problem

$$\max_{I_{t+i}^g} E_t \sum_{i=0}^{\infty} \psi_{t,t+i} \left[Q_{t+i}\left[1 - \frac{\phi}{2}\left(\frac{I_{t+i}^g}{I_{t+i-1}^g} - 1\right)^2\right] I_{t+i}^g - I_{t+i.}^g\right]. \tag{5.36}$$

In the maximization, Q_t is taken as given and $\psi_{t,t+i}$ is the stochastic discount factor of households who own the capital-producing firms (defined below).

5.1.4 Necessary Conditions for an Equilibrium

From the side of firms,

$$K_{t+1} = S_{t.} \tag{5.37}$$

$$Y_t = A_t {K_t}^{\alpha} L_t^{1-\alpha}. \tag{5.38}$$

From the solution of $\max_{L_{t+1}} \pi_{t+1}$,

$$L_t = (1-\alpha)\frac{Y_t}{W_t}. \tag{5.39}$$

From the zero-profit condition for firms,

$$(1 + R_t^s) = \frac{1}{Q_{t-1}}\frac{\alpha Y_t}{K_t} + \frac{(1-\delta)}{Q_{t-1}} Q_t. \tag{5.40}$$

From the problem for households,

$$\max_{C_{t+i},L_{t+i},S_{t+i},B_{t+i}} U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon} L_{t+i}^{1+\varepsilon} \right]$$
$$+ \beta^i \lambda_{ct+i} \left(-C_{t+i} + W_{t+i} L_{t+i} - T_{t+i} \right.$$
$$- Q_{t+i} S_{t+i} + (1 + R_{t+i}^s) Q_{t-1+i} S_{t-1+i} - B_{t+i}$$
$$\left. + (1 + R_{t-1+i}) B_{t-1+i} \right) \tag{5.41}$$

$$\frac{\partial U_t}{\partial C_t} = \frac{1}{C_t - \gamma C_{t-1}} - \lambda_{ct} - E_t \beta \frac{\gamma}{C_{t+1} - \gamma C_t} = 0 \tag{5.42}$$

$$\frac{\partial U_t}{\partial L_t} = -\chi L_t^{\varepsilon} + \lambda_{ct} W_t = 0 \tag{5.43}$$

$$\frac{\partial U_t}{\partial S_t} = -\lambda_{ct} Q_t + E_t \beta \lambda_{ct+1} Q_t (1 + R_{t+1}^s) = 0 \tag{5.44}$$

$$\lambda_{ct} = E_t \beta \lambda_{ct+1} (1 + R_{t+1}^s) \tag{5.45}$$

$$\frac{\partial U_t}{\partial B_t} = -\lambda_{ct} + E_t \beta \lambda_{ct+1} (1 + R_t) = 0 \tag{5.46}$$

$$\lambda_{ct} = E_t \beta \lambda_{ct+1} (1 + R_t) \tag{5.47}$$

$$E_t \frac{\lambda_{ct+1}}{\lambda_{ct}} = \frac{1}{\beta(1 + R_t)}. \tag{5.48}$$

Define the stochastic discount factor $\psi_{t,t+i}$ as $E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} = \frac{1}{1+R_t}$.

The evolution of capital takes the form

$$K_{t+1} = I_t^n + (1 - \delta) K_t. \tag{5.49}$$

From the maximization problem for capital-producing firms,

$$\max_{I_{t+i}^g} E_t \sum_{i=0}^{\infty} \psi_{t,t+i} \left[Q_{t+i} \left[1 - \frac{\phi}{2} \left(\frac{I_{t+i}^g}{I_{t+i-1}^g} - 1 \right)^2 \right] I_{t+i}^g - I_{t+i.}^g \right] \tag{5.50}$$

$$\frac{\partial}{\partial I_t^g} = Q_t\left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] - Q_t\phi\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)\frac{I_t^g}{I_{t-1}^g} - 1$$
$$+ \psi_{t,t+1}Q_{t+1}\phi\left(\frac{I_{t+1}^g}{I_t^g} - 1\right)\frac{I_{t+1}^g}{I_t^{g2}}I_{t+1}^g. \tag{5.51}$$

And from the resource constraint,

$$Y_t = C_t + I_t^g + G_t. \tag{5.52}$$

Finally, G_t is set as a fixed share of Y_t and the government's budget is balanced every period.

5.2 Introducing Financial Constraints Following Gertler and Karadi (2011)

The problem of the firms is unchanged, but they are prevented from issuing shares to households directly. Instead, they need to use financial intermediaries, which are dubbed "banks" and are described below.

5.2.1 Households

The representative household has a continuum of members. A fraction $1 - f$ of members in this continuum supplies labor to firms and returns the wage earned to the household. A fraction f of members in the continuum works as bankers. The consumption of workers and bankers within the household is equalized. As before, the utility function is

$$U_t = E_t\sum_{i=0}^{\infty}\beta^i\left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon}L_{t+i}^{1+\varepsilon}\right]. \tag{5.53}$$

However, in this case, the budget constraint takes the form

$$C_t = W_tL_t + \Pi_t - T_t - D_t + (1 + R_{t-1})D_{t-1}. \tag{5.54}$$

The term D_t represents the amount of deposits with banks (not owned by the household).

Because banks may be financially constrained, they have an incentive to retain earnings. To avoid making the financial constraint irrelevant with iid probability $1-\theta$, a banker exits next period. Upon exiting, bankers transfer retained earnings back to the households and become workers. Each period $(1-\theta)\,f$ workers are selected to become bankers. These new bankers receive a startup transfer from the family. By construction, the fraction of household members in each group is constant over time. Π_t is net funds transferred to the household from its banker members; that is, funds transferred from existing bankers minus the funds transferred to new bankers.

5.2.2 Banks

Banks lend funds obtained from households to non-financial firms. Let $N_t(j)$ be the amount of wealth—or net worth—that a banker j has at the end of period t.

$$Q_t S_t(j) = N_t(j) + D_t(j) \tag{5.55}$$

As noted earlier, deposits $D_t(j)$ pay the non-state-contingent return $(1+R_t)$ at time $t+1$. Thus $D_t(j)$ may be thought of as the debt of bank j, and $N_t(j)$ as its capital. As seen above, the shares $S_t(j)$ earn the stochastic return $(1+R^s_{t+1})$ at time $t+1$.

Over time, the banker's equity capital evolves as the difference between earnings on assets and interest payments on liabilities:

$$N_{t+1}(j) = (1+R^s_{t+1})Q_t S_t(j) - (1+R_t)D_t(j) \tag{5.56}$$

$$D_t(j) = Q_t S_t(j) - N_t(j) \tag{5.57}$$

$$N_{t+1}(j) = (1+R^s_{t+1})Q_t S_t(j) - (1+R_t)\left(Q_t S_t(j) - N_t(j)\right) \tag{5.58}$$

$$N_{t+1}(j) = \left[(1+R^s_{t+1}) - (1+R_t)\right] Q_t S_t(j) + (1+R_t)N_t(j) \tag{5.59}$$

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right) Q_t S_t(j) + (1+R_t)N_t(j). \tag{5.60}$$

Let $\psi_{t,t+j} = \beta^j \frac{\lambda_{ct+j}}{\lambda_{ct}}$ be the stochastic discount factor between periods t and $t+i$. The banker's objective is to maximize expected terminal wealth, given by

$$\max_{s_{t+i}(j)} V_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i} \left[\left(R^s_{t+1+i} - R_{t+i} \right) Q_{t+i} S_{t+i}(j) \right.$$
$$\left. + (1 + R_{t+i}) N_{t+i}(j) \right]. \tag{5.61}$$

Notice that there is an asymmetry between period 0 and all subsequent periods. If a bank has to quit in period 0, it does not conduct any operations and revenues are 0. Since the banker will not fund assets with a discounted return less than the discounted cost of borrowing, for the bank to operate in period $t+i$, it must be that $E_t \psi_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i} \right) \geq 0$, i.e., there are expected positive excess returns from holding stocks even after discounting and adjusting for risk through $\psi_{t,t+1+i}$. In the absence of financial frictions, when $E_t \psi_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i} \right)$ is positive, the bank will want to expand its balance sheet by attracting additional deposits from households.

To limit the ability of banks to attract deposits indefinitely, consider the following agency problem. At the beginning of each period, a banker can choose to transfer a fraction λ of assets (in period t those assets equal $Q_t S_t(j)$) back to his household. If the banker makes the transfer, depositors will force the bank into bankruptcy and recover the remaining fraction $1 - \lambda$ of assets. Thus, households are willing to make deposits only if the incentive-compatibility constraint is satisfied:

$$V_t(j) \geq \lambda Q_t S_t(j). \tag{5.62}$$

This constraint says that the expected terminal wealth for period t needs to be at least as large as the fraction of assets that can be diverted in that period. The left-hand side is the cost of diverting assets; the right-hand side is the benefit. When the constraint binds, it affects the ability to raise deposits and will imply expected positive excess returns in equilibrium. Next we show that the ability of the banks to attract deposits is related to their net worth. For this purpose, it is useful to separate the recursive form of net worth into a component that depends on total assets $v_t(j)$ and a component that depends on net worth $\eta_t(j)$. The form we are after is the following:

$$V_t(j) = v_t Q_t S_t(j) + \eta_t N_t(j) \tag{5.63}$$

$$v_t(j) = E_t \left(1-\theta\right) \psi_{t,t+1} \left(R^s_{t+1} - R_t\right) + \psi_{t,t+1}\theta \frac{Q_{t+i}S_{t+i}(j)}{Q_t S_t(j)} v_{t+1}(j) \tag{5.64}$$

$$\eta_t(j) = E_t \left(1-\theta\right) + \psi_{t,t+1}\theta \frac{N_{t+1}(j)}{N_t(j)} \eta_{t+1}(j). \tag{5.65}$$

Notice that

$$\begin{aligned} V_t(j) &= E_t \sum_{i=0}^{\infty} \left(1-\theta\right) \theta^i \psi_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i}\right) Q_{t+i} S_{t+i}(j) \\ &\quad + E_t \sum_{i=0}^{\infty} \left(1-\theta\right) \theta^i \psi_{t,t+1+i} (1+R_{t+i}) N_{t+i}(j). \end{aligned} \tag{5.66}$$

Define

$$v_t(j) = E_t \sum_{i=0}^{\infty} \left(1-\theta\right) \theta^i \psi_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i}\right) \frac{Q_{t+i}S_{t+i}(j)}{Q_t S_t(j)} \tag{5.67}$$

$$\eta_t(j) = E_t \sum_{i=0}^{\infty} \left(1-\theta\right) \theta^i \psi_{t,t+1+i} (1+R_{t+i}) \frac{N_{t+i}(j)}{N_t(j)}. \tag{5.68}$$

Then

$$V_t(j) = v_t(j) Q_t S_t(j) + \eta_t(j) N_t(j). \tag{5.69}$$

Next write $v_t(j)$ and $\eta_t(j)$ recursively. Start by pulling out the first term in each summation,

$$\begin{aligned} v_t(j) &= E_t \left(1-\theta\right) \psi_{t,t+1} \left(R^s_{t+1} - R_t\right) \frac{Q_t S_t(j)}{Q_t S_t(j)} \\ &\quad + \sum_{i=1}^{\infty} \left(1-\theta\right) \theta^i \psi_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i}\right) \frac{Q_{t+i}S_{t+i}(j)}{Q_t S_t(j)} \end{aligned} \tag{5.70}$$

$$\eta_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t)\frac{N_t(j)}{N_t(j)} + \sum_{i=1}^{\infty}(1-\theta)\,\theta^i\psi_{t,t+1+i}(1+R_{t+i})\frac{N_{t+i}(j)}{N_t(j)}. \tag{5.71}$$

Now transform the summations so that they start from 0:

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) + \theta\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t,t+2+i}\left(R^s_{t+2+i}-R_{t+1+i}\right)\frac{Q_{t+1+i}S_{t+1+i}(j)}{Q_tS_t(j)} \tag{5.72}$$

$$\eta_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) + \theta\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t,t+2+i}(1+R_{t+1+i})\frac{N_{t+1+i}(j)}{N_t(j)}. \tag{5.73}$$

Express $\psi_{t,t+2+i}$ as a function of $\psi_{t+1,t+2+i}$. Remember that $\psi_{t,t+j} = \beta^j\frac{\lambda_{ct+j}}{\lambda_{ct}}$. Thus, $\psi_{t+1,t+2+i} = \beta^{1+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct+1}}$ and $\psi_{t,t+2+i} = \beta^{2+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct}}$. Notice that

$$\psi_{t,t+2+i} = \beta\beta^{1+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct}}\frac{\lambda_{ct+1}}{\lambda_{ct+1}} \tag{5.74}$$

$$= \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\beta^{1+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct+1}} \tag{5.75}$$

$$= \psi_{t,t+1}\psi_{t+1,t+2+i}. \tag{5.76}$$

Substituting $\psi_{t,t+2+i} = \psi_{t,t+1}\psi_{t+1,t+2+i}$ into the last equations for $v_t(j)$ and for $\eta_t(j)$, one can see that

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) + \theta\psi_{t,t+1}\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t+1,t+2+i}\left(R^s_{t+2+i}-R_{t+1+i}\right) \times\frac{Q_{t+1+i}S_{t+1+i}(j)}{Q_tS_t(j)} \tag{5.77}$$

$$\eta_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) + \theta\psi_{t,t+1}\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t+1,t+2+i}(1+R_{t+1+i})\frac{N_{t+1+i}(j)}{N_t(j)}. \tag{5.78}$$

But the above equations can also be written as

$$\begin{aligned} v_t(j) &= E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) \\ &\quad + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t+1,t+2+i} \\ &\quad \times\left(R^s_{t+2+i}-R_{t+1+i}\right)\frac{Q_{t+1+i}S_{t+1+i}(j)}{Q_{t+1}S_{t+1}(j)} \\ \eta_t(j) &= E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) \\ &\quad + \theta\psi_{t,t+1}\frac{N_{t+1}(j)}{N_t(j)}\sum_{i=0}^{\infty}(1-\theta)\,\theta^i\psi_{t+1,t+2+i}(1+R_{t+1+i}) \\ &\quad \times\frac{N_{t+1+i}(j)}{N_{t+1}(j)}, \end{aligned}$$

which yields

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}v_{t+1}(j) \tag{5.79}$$

$$\eta_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) + \theta\psi_{t,t+1}\frac{N_{t+1}(j)}{N_t(j)}\eta_{t+1}(j), \tag{5.80}$$

but remember that from the households' problem $E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}} = \frac{1}{(1+R_t)}$

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}v_{t+1}(j) \tag{5.81}$$

$$\eta_t(j) = E_t\,(1-\theta) + \theta\psi_{t,t+1}\frac{N_{t+1}(j)}{N_t(j)}\eta_{t+1}(j). \tag{5.82}$$

Q.E.D.

Since all banks have access to the same investment opportunities, $\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}$ will be equalized across all j and similarly for $\frac{N_{t+1+i}(j)}{N_{t+1}(j)}$. Consequently, we can drop the dependence on j and simply carry around v_t and η_t. Notice that v_t and η_t have an interesting interpretation: v_t is the expected discounted marginal gain of expanding assets Q_tS_t by one unit holding net worth constant; η_t is the expected discounted value of having another unit of net worth $N_t(j)$ holding Q_tS_t constant. Notice that v_t is zero in a frictionless market without the agency problem.

Substituting

$$V_t(j) = v_tQ_tS_t(j) + \eta_tN_t(j) \tag{5.83}$$

into the incentive-compatibility constraint

$$V_t(j) \geq \lambda Q_tS_t(j), \tag{5.84}$$

one obtains that

$$v_tQ_tS_t(j) + \eta_tN_t(j) \geq \lambda Q_tS_t(j). \tag{5.85}$$

When this constraint binds,

$$Q_tS_t(j) = \frac{\eta_t}{(\lambda - v_t)}N_t(j). \tag{5.86}$$

Therefore, $\frac{\eta_t}{(\lambda-v_t)}$ is the ratio of assets to equity. This constraint limits the leverage ratio of the intermediary to the point where the banker's incentive to cheat is exactly balanced by the costs. Holding $N_t(j)$ constant, expanding $S_t(j)$ raises the banker's incentive to divert funds. To prove this, I need to show that $\frac{\partial V_t(j)}{\partial S_t(j)} < \frac{\partial \lambda Q_tS_t(j)}{\partial S_t(j)} = \lambda Q_t$. From

$$v_tQ_tS_t(j) + \eta_tN_t(j) \geq \lambda Q_tS_t(j), \tag{5.87}$$

given that $\eta_tN_t(j) > 0$, it must be that the constraint binds if $v_t < \lambda$. Additionally, we know that if the constraint binds, $v_t > 0$. Hence, for the constraint to bind, it must be that $\lambda > 0$.

Using $Q_tS_t(j) = \frac{\eta_t}{(\lambda-v_t)}N_t(j)$ and the evolution of net worth derived above,

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right)Q_tS_t(j) + (1 + R_t)N_t(j) \tag{5.88}$$

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - \upsilon_t)} N_t(j) + (1 + R_t) N_t(j)$$
$$= \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - \upsilon_t)} + (1 + R_t)\right] N_t(j). \quad (5.89)$$

It also follows that $\frac{N_{t+1}(j)}{N_t(j)}$, conditional on surviving, as used above, is given by

$$\frac{N_{t+1}(j)}{N_t(j)} = \left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - \upsilon_t)} + (1 + R_t). \quad (5.90)$$

In turn, $\frac{Q_{t+1} S_{t+1}(j)}{Q_t S_t(j)}$ is given by

$$\frac{Q_{t+1} S_{t+1}(j)}{Q_t S_t(j)} = \frac{\frac{\eta_{t+1}}{(\lambda - \upsilon_{t+1})}}{\frac{\eta_t}{(\lambda - \upsilon_t)}} \frac{N_{t+1}(j)}{N_t(j)}$$
$$= \frac{\frac{\eta_{t+1}}{(\lambda - \upsilon_{t+1})}}{\frac{\eta_t}{(\lambda - \upsilon_t)}} \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - \upsilon_t)} + (1 + R_t)\right]. \quad (5.91)$$

Consequently, υ_t and η_t are equalized across all j and evolve according to

$$\upsilon_t = E_t \left(1 - \theta\right) \psi_{t,t+1} \left(R^s_{t+1} - R_t\right)$$
$$+ \theta \psi_{t,t+1} \frac{\frac{\eta_{t+1}}{(\lambda - \upsilon_{t+1})}}{\frac{\eta_t}{(\lambda - \upsilon_t)}} \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - \upsilon_t)} + (1 + R_t)\right] \upsilon_{t+1}(j) \quad (5.92)$$

$$\eta_t = E_t \left(1 - \theta\right) + \theta \psi_{t,t+1} \left[\left(R^s_{t+1} - R_t\right) \frac{\eta_t}{(\lambda - \upsilon_t)} + (1 + R_t)\right] \eta_{t+1}(j). \quad (5.93)$$

Since $\frac{\eta_{t+1}}{(\lambda - \upsilon_{t+1})}$ is independent of j, one can aggregate across banks to obtain

$$\int_j Q_t S_t(j) dj = \int_j \frac{\eta_t}{(\lambda - \upsilon_t)} N_t(j) dj \quad (5.94)$$

$$Q_t S_t = \frac{\eta_t}{(\lambda - \upsilon_t)} N_t. \quad (5.95)$$

Finally, recognize that there is a distinction between the net worth of continuing and new bankers. Aggregate net worth is the sum the two types: Bankers that survive from period $t-1$ to period t will have aggregate net worth equal to

$$\theta\left[\left(R_t^s - R_{t-1}\right)\frac{\eta_{t-1}}{\left(\lambda - \upsilon_{t-1}\right)} + \left(1 + R_{t-1}\right)\right] N_{t-1}. \tag{5.96}$$

Assume that new bankers receive as endowment a fixed fraction of the current value of the assets intermediated by exiting bankers in the previous period, amounting to $(1-\ \theta)\, Q_t S_{t-1}$. Assume that the household transfers the fraction $\frac{\omega}{(1-\ \theta)}$ of that amount to new bankers. Thus, in the aggregate,

$$N_t^n = \frac{\omega}{(1-\ \theta)}\,(1-\ \theta)\, Q_t S_{t-1} = \omega Q_t S_{t-1}. \tag{5.97}$$

Then, current net worth is the sum of net worth carried from the previous period by surviving firms $\theta\big[\,(R_t^s - R_{t-1})\,\frac{\eta_{t-1}}{(\lambda-\upsilon_{t-1})} + (1 + R_{t-1})\big] N_{t-1}$, plus the net worth of new entrants, $\omega Q_t S_{t-1}$, i.e.,

$$N_t = \theta\left[\left(R_t^s - R_{t-1}\right)\frac{\eta_{t-1}}{\left(\lambda - \upsilon_{t-1}\right)} + \left(1 + R_{t-1}\right)\right] N_{t-1} + \omega Q_t S_{t-1}. \tag{5.98}$$

5.3 Introducing Transfer Shocks between Banks and Households

Change the problem of the households to be

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon} L_{t+i}^{1+\varepsilon}\right]. \tag{5.99}$$

However, in this case, the budget constraint takes the form

$$C_t = W_t L_t + \Pi_t - T_t + \tau_t N_t - D_t + (1 + R_{t-1}) D_{t-1}. \tag{5.100}$$

Notice that BT_t is a transfer shock from banks back to households in a lump-sum fashion.

5.3.1 Banks

Banks lend funds obtained from households to non-financial firms. Let $N_t(j)$ be the amount of wealth—or net worth—that a banker j has at the end of period t.

$$Q_t S_t(j) = N_t(j)\,(1-\tau_t) + D_t(j) \tag{5.101}$$

As noted earlier, deposits $D_t(j)$ pay the non-state-contingent return $(1 + R_t)$ at time $t + 1$. Thus $D_t(j)$ may be thought of as the debt of bank j, and $N_t(j)$ as its capital. As seen above, the shares $S_t(j)$ earn the stochastic return $(1 + R^s_{t+1})$ at time $t + 1$.

Over time, the banker's equity capital evolves as the difference between earnings on assets and interest payments on liabilities:

$$N_{t+1}(j) = (1 + R^s_{t+1})Q_t S_t(j) - (1 + R_t)D_t(j) \tag{5.102}$$

$$D_t(j) = Q_t S_t(j) - N_t(j)\,(1-\tau_t) \tag{5.103}$$

$$N_{t+1}(j) = (1 + R^s_{t+1})Q_t S_t(j) - (1 + R_t)\,(Q_t S_t(j) - N_t(j)\,(1-\tau_t)) \tag{5.104}$$

$$N_{t+1}(j) = \left[(1 + R^s_{t+1}) - (1 + R_t)\right] Q_t S_t(j) + (1 + R_t)N_t(j)\,(1-\tau_t) \tag{5.105}$$

$$N_{t+1}(j) = \left(R^s_{t+1} - R_t\right) Q_t S_t(j) + (1 + R_t)N_t(j)\,(1-\tau_t)\,. \tag{5.106}$$

Let $\psi_{t,t+j} = \beta^j \frac{\lambda_{ct+j}}{\lambda_{ct}}$ be the stochastic discount factor between periods t and $t + i$. The banker's objective is to maximize expected terminal wealth, given by

$$\begin{aligned}\max_{s_{t+i}(j)} V_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i}[\left(R^s_{t+1+i} - R_{t+i}\right) Q_{t+i} S_{t+i}(j) \\ + (1 + R_{t+i})N_{t+i}(j)\,(1-\tau_{t+i})].\end{aligned} \tag{5.107}$$

Since the banker will not fund assets with a discounted return less than the discounted cost of borrowing, for the bank to operate in period $t + i$, it must be that $E_t \psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right) \geq 0$, i.e., there are expected positive excess returns from holding stocks even after discounting and adjusting for risk through $\psi_{t,t+1+i}$. In the absence of financial frictions, when $E_t \psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right)$ is positive, the bank will want to expand its balance sheet by attracting additional deposits from households.

To limit the ability of banks to attract deposits indefinitely, consider the following agency problem. At the beginning of each period, a banker can choose to transfer a fraction λ of assets (in period t those assets equal $Q_t S_t(j)$) back to his household. If the banker makes the transfer, depositors will force the bank into bankruptcy and recover the remaining fraction $1 - \lambda$ of assets. Thus, households are willing to make deposits only if the incentive-compatibility constraint is satisfied:

$$V_t(j) \geq \lambda Q_t S_t(j). \tag{5.108}$$

This constraint says that the expected terminal wealth for period t needs to be at least as large as the fraction of assets that can be diverted in that period. The left-hand side is the cost of diverting assets; the right-hand side is the benefit. When the constraint binds, it affects the ability to raise deposits and will imply expected positive excess returns in equilibrium. Next we show that the ability of the banks to attract deposits is related to their net worth. For this purpose, it is useful to separate the recursive form of net worth into a component that depends on total assets $v_t(j)$ and a component that depends on net worth $\eta_t(j)$.

Notice that

$$\begin{aligned} V_t(j) = {} & E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i}\right) Q_{t+i} S_{t+i}(j) \\ & + E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i} (1 + R_{t+i}) N_{t+i}(j) \left(1 - \tau_{t+i}\right). \end{aligned} \tag{5.109}$$

Define

$$v_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i} \left(R^s_{t+1+i} - R_{t+i}\right) \frac{Q_{t+i} S_{t+i}(j)}{Q_t S_t(j)} \tag{5.110}$$

$$\eta_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i}(1+R_{t+i}) \frac{N_{t+i}(j)\,(1-\tau_{t+i})}{N_t(j)\,(1-\tau_t)}. \tag{5.111}$$

Then

$$V_t(j) = v_t(j) Q_t S_t(j) + \eta_t(j) N_t(j)\,(1-\tau_t)\,. \tag{5.112}$$

Next write $v_t(j)$ and $\eta_t(j)$ recursively. Start by pulling out the first term in each summation,

$$\begin{aligned} v_t(j) = {} & E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) \frac{Q_t S_t(j)}{Q_t S_t(j)} \\ & + \sum_{i=1}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right) \frac{Q_{t+i} S_{t+i}(j)}{Q_t S_t(j)} \end{aligned} \tag{5.113}$$

$$\begin{aligned} \eta_t(j) = {} & E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) \frac{N_t(j)\,(1-\tau_t)}{N_t(j)\,(1-\tau_t)} \\ & + \sum_{i=1}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i}(1+R_{t+i}) \frac{N_{t+i}(j)\,(1-\tau_{t+i})}{N_t(j)\,(1-\tau_t)}. \end{aligned} \tag{5.114}$$

Now transform the summations so that they start from 0:

$$\begin{aligned} v_t(j) = {} & E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) \\ & + \theta \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+2+i}\left(R^s_{t+2+i} - R_{t+1+i}\right) \frac{Q_{t+1+i} S_{t+1+i}(j)}{Q_t S_t(j)} \end{aligned} \tag{5.115}$$

$$\begin{aligned} \eta_t(j) = {} & E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) \\ & + \theta \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+2+i}(1+R_{t+1+i}) \frac{N_{t+1+i}(j)\,(1-\tau_{t+i})}{N_t(j)\,(1-\tau_t)}. \end{aligned} \tag{5.116}$$

Express $\psi_{t,t+2+i}$ as a function of $\psi_{t+1,t+2+i}$. Remember that $\psi_{t,t+j} = \beta^j \frac{\lambda_{ct+j}}{\lambda_{ct}}$. Thus, $\psi_{t+1,t+2+i} = \beta^{1+i} \frac{\lambda_{ct+2+i}}{\lambda_{ct+1}}$ and $\psi_{t,t+2+i} = \beta^{2+i} \frac{\lambda_{ct+2+i}}{\lambda_{ct}}$. Notice that

$$\begin{aligned}\psi_{t,t+2+i} &= \beta\beta^{1+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct}}\frac{\lambda_{ct+1}}{\lambda_{ct+1}} \\ &= \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\beta^{1+i}\frac{\lambda_{ct+2+i}}{\lambda_{ct+1}} \\ &= \psi_{t,t+1}\psi_{t+1,t+2+i}. \end{aligned} \tag{5.117}$$

Substituting $\psi_{t,t+2+i} = \psi_{t,t+1}\psi_{t+1,t+2+i}$ into the last equations for $v_t(j)$ and for $\eta_t(j)$, one can see that

$$\begin{aligned} v_t(j) = {} & E_t\left(1-\theta\right)\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) \\ & + \theta\psi_{t,t+1}\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^i\psi_{t+1,t+2+i}\left(R^s_{t+2+i}-R_{t+1+i}\right) \\ & \times \frac{Q_{t+1+i}S_{t+1+i}(j)}{Q_tS_t(j)} \end{aligned} \tag{5.118}$$

$$\begin{aligned} \eta_t(j) = {} & E_t\left(1-\theta\right)\psi_{t,t+1}(1+R_t) \\ & + \theta\psi_{t,t+1}\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^i\psi_{t+1,t+2+i}(1+R_{t+1+i})\frac{N_{t+1+i}(j)}{N_t(j)} \\ & \times \frac{(1-\tau_{t+i})}{(1-\tau_t)}. \end{aligned} \tag{5.119}$$

But the above equations can also be written as

$$\begin{aligned} v_t(j) = {} & E_t\left(1-\theta\right)\psi_{t,t+1}\left(R^s_{t+1}-R_t\right) \\ & + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^i\psi_{t+1,t+2+i} \\ & \times \left(R^s_{t+2+i}-R_{t+1+i}\right)\frac{Q_{t+1+i}S_{t+1+i}(j)}{Q_{t+1}S_{t+1}(j)} \\ \eta_t(j) = {} & E_t\left(1-\theta\right)\psi_{t,t+1}(1+R_t) \\ & + \theta\psi_{t,t+1}\frac{N_{t+1}(j)}{N_t(j)}\frac{(1-\tau_{t+1})}{(1-\tau_t)}\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^i\psi_{t+1,t+2+i} \\ & \times (1+R_{t+1+i})\frac{N_{t+1+i}(j)}{N_{t+1}(j)}\frac{(1-\tau_{t+1+i})}{(1-\tau_{t+1})}, \end{aligned}$$

which yields

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}v_{t+1}(j) \tag{5.120}$$

$$\eta_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}(1+R_t) + \theta\psi_{t,t+1} \times \frac{N_{t+1}(j)}{N_t(j)}\frac{(1-\tau_{t+1})}{(1-\tau_t)}\eta_{t+1}(j), \tag{5.121}$$

but remember that from the households' problem $E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}} = \frac{1}{(1+R_t)}$,

$$v_t(j) = E_t\,(1-\theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) + \theta\psi_{t,t+1}\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}v_{t+1}(j) \tag{5.122}$$

$$\eta_t(j) = E_t\,(1-\theta) + \theta\psi_{t,t+1}\frac{N_{t+1}(j)}{N_t(j)}\frac{(1-\tau_{t+1})}{(1-\tau_t)}\eta_{t+1}(j). \tag{5.123}$$

Since all banks have access to the same investment opportunities, $\frac{Q_{t+1}S_{t+1}(j)}{Q_tS_t(j)}$ will be equalized across all j and similarly for $\frac{N_{t+1+i}(j)}{N_{t+1}(j)}$. Consequently, we can drop the dependence on j and simply carry around v_t and η_t.

Substituting

$$V_t(j) = v_tQ_tS_t(j) + \eta_tN_t(j)\,(1-\tau_t) \tag{5.124}$$

into the incentive-compatibility constraint

$$V_t(j) \geq \lambda Q_tS_t(j), \tag{5.125}$$

one obtains that

$$v_tQ_tS_t(j) + \eta_tN_t(j)\,(1-\tau_t) \geq \lambda Q_tS_t(j). \tag{5.126}$$

When this constraint binds,

$$Q_tS_t(j) = \frac{\eta_t}{(\lambda - v_t)}N_t(j)\,(1-\tau_t)\,. \tag{5.127}$$

Therefore, $\frac{\eta_t}{(\lambda - v_t)}$ is the ratio of assets to equity. This constraint limits the leverage ratio of the intermediary to the point where the banker's incentive to cheat is exactly balanced by the costs. Next derive $\frac{N_{t+1}(j)}{N_t(j)}$ and $\frac{Q_{t+1}S_{t+1}(j)}{Q_t S_t(j)}$.

$$N_{t+1}(j) = \left[(R^s_{t+1} - R_t)\frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] N_t(j)(1 - \tau_t)$$

$$\frac{N_{t+1}(j)}{N_t(j)} = \left[(R^s_{t+1} - R_t)\frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right](1 - \tau_t)$$

Taking the lead of $Q_t S_t(j) = \frac{\eta_t}{(\lambda - v_t)} N_t(j)\,(1 - \tau_t)$ and dividing it by $Q_t S_t(j)$, one can see that

$$\begin{aligned}\frac{Q_{t+1}S_{t+1}(j)}{Q_t S_t(j)} &= \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})} N_{t+1}(j)\,(1 - \tau_{t+1})}{\frac{\eta_t}{(\lambda - v_t)} N_t(j)\,(1 - \tau_t)} \\ &= \frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}}\left[(R^s_{t+1} - R_t)\frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] \\ &\quad \times (1 - \tau_{t+1})\,.\end{aligned}$$

Accordingly,

$$\begin{aligned} v_t(j) = E_t\,&(1 - \theta)\,\psi_{t,t+1}\left(R^s_{t+1} - R_t\right) \\ &+ \theta\psi_{t,t+1}\frac{\frac{\eta_{t+1}}{(\lambda - v_{t+1})}}{\frac{\eta_t}{(\lambda - v_t)}}\left[(R^s_{t+1} - R_t)\frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] \\ &\times (1 - \tau_{t+1})\,v_{t+1}(j) \end{aligned} \tag{5.128}$$

$$\begin{aligned}\eta_t(j) = E_t\,&(1 - \theta) + \theta\psi_{t,t+1}\left[(R^s_{t+1} - R_t)\frac{\eta_t}{(\lambda - v_t)} + (1 + R_t)\right] \\ &\times (1 - \tau_{t+1})\,\eta_{t+1}(j).\end{aligned} \tag{5.129}$$

Since $\frac{\eta_{t+1}}{(\lambda - v_{t+1})}$ is independent of j, one can aggregate across banks to obtain

$$\int_j Q_t S_t(j)dj = \int_j \frac{\eta_t}{(\lambda - v_t)} N_t(j)(1 - \tau_t)dj \tag{5.130}$$

$$Q_t S_t = \frac{\eta_t}{(\lambda - v_t)} N_t(1 - \tau_t). \tag{5.131}$$

Finally, recognize that there is a distinction between the net worth of continuing and new bankers. Aggregate net worth is the sum the two types: Bankers that survive from period $t-1$ to period t will have aggregate net worth equal to

$$\theta\left[\left(R_t^s - R_{t-1}\right)\frac{\eta_{t-1}}{\left(\lambda - v_{t-1}\right)} + \left(1 + R_{t-1}\right)\right] N_{t-1}\left(1 - \tau_{t-1}\right). \quad (5.132)$$

Assume that new bankers receive as endowment a fixed fraction of the current value of the assets intermediated by exiting bankers in the previous period, amounting to $(1-\ \theta)\,Q_t S_{t-1}$. Assume that the household transfers the fraction $\frac{\omega}{(1-\ \theta)}$ of that amount to new bankers. Thus, in the aggregate,

$$N_t^n = \frac{\omega}{(1-\ \theta)}\,(1-\ \theta)\,Q_t S_{t-1} = \omega Q_t S_{t-1}. \quad (5.133)$$

Then, current net worth is the sum of net worth carried from the previous period by surviving firms $\theta\big[\,(R_t^s - R_{t-1})\,\frac{\eta_{t-1}}{(\lambda - v_{t-1})} + (1 + R_{t-1})\big]$ $N_{t-1}\,(1-\tau_t)$, plus the net worth of new entrants, $\omega Q_t S_{t-1}\,(1-\tau_t)$, i.e.,

$$\begin{aligned} N_t = \theta&\left[\left(R_t^s - R_{t-1}\right)\frac{\eta_{t-1}}{\left(\lambda - v_{t-1}\right)} + \left(1 + R_{t-1}\right)\right] \\ &\times N_{t-1}\left(1 - \tau_{t-1}\right) + \omega Q_t S_{t-1}. \end{aligned} \quad (5.134)$$

5.4 Introducing Heterogenous Firms

Now suppose that a fraction of firms can access equity markets directly, without having to reach them through banks. Call the type of such firms u. The other firms have to rely on banks to fund their capital purchases. Call the type of such firms b. The cost structure of the two types of firms will be different, and their products will have different prices in equilibrium. Both types of firms will coexist in equilibrium because the final consumption and investment goods are assumed to be a composite of both types of intermediate goods (possibly in different proportions).

5.4.1 Households

As before, the representative household has a continuum of members. A fraction $1-f$ of members in this continuum supplies labor to firms and returns the wage earned to the household. A fraction f of members in the continuum works as bankers. The consumption of workers and bankers within the household is equalized. As before, the utility function is

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon} L_{t+i}^{1+\varepsilon} \right]. \qquad (5.135)$$

However, in this case, the budget constraint takes the form

$$\begin{aligned} C_t = {} & W_t L_t + \Pi_t - T_t + Q_t S_t^u - (1 + R_t^{su}) Q_{t-1} S_{t-1}^u + D_t \\ & - (1 + R_{t-1}) D_{t-1}. \end{aligned} \qquad (5.136)$$

The term D_t represents the amount of deposits with banks (not owned by the household). R_{t-1} is non-state contingent. When the price of consumption is chosen to be the numeraire, the interest rate on deposits is "risk free" (under other normalization of prices deposits would not insure against the risk of changes in the price of consumption). The term S_t^u represents the shares issued by final product firms that have direct access to equity markets. Shares acquired the previous period pay the risky rate R_t^{su}.

The division between bankers and workers within the representative family remains unchanged relative to the setup considered before.

Households allocate consumption between two goods produced by firms of type u and by firms of type b. The production of final goods takes place through perfectly competitive firms. Their production technology is

$$Y_t = (Y_t^u)^{\alpha} \left(Y_t^b\right)^{1-\alpha}. \qquad (5.137)$$

Each period they minimize the cost of production subject to meeting demand:

$$\min_{Y_t^u, Y_t^b, P_t^F} P_t^u Y_t^u + P_t^b Y_t^b + P_t^F \left(Y_t - (Y_t^u)^{\alpha^F} \left(Y_t^b\right)^{1-\alpha^F} \right). \qquad (5.138)$$

We are using the prices of final goods to be the numeraire units, hence the Lagrange multiplier on the technology of production P_t^F is set to 1.

First-order conditions:

$$P_t^u + P_t^F \left(-\alpha^F \left(Y_t^u\right)^{\alpha^F - 1} \left(Y_t^b\right)^{1-\alpha^F} \right) = 0$$

$$P_t^u = P_t^F \alpha^F \frac{Y_t}{Y_t^u}$$

$$Y_t^u = \alpha^F Y_t \frac{P_t^F}{P_t^u}.$$

But $P_t^F = 1$:

$$Y_t^u = \alpha^F Y_t \frac{1}{P_t^u}.$$

5.4.2 Output-Producing Firms

There are two kinds of firms: firms that have direct access to equity markets and firms that have to use banks for their financing requirements. Both have production technologies

$$Y_t^j = A_t K_t^{j\,\alpha} L_t^{j 1-\alpha}, \tag{5.139}$$

where j is either u for the firms that have access to equity markets or b for the firms that have to use banks. Firms operate for only one period, but some of the planning for production is done one period in advance. To operate capital in period $t+1$, a firm must purchase it in period t. To do so, the firm issues shares in period t. There are as many shares S_t^j as units of capital purchased. By arbitrage, the current value of capital equals the value of shares. Thus,

$$Q_t K_{t+1}^j = Q_t S_{t.}^j. \tag{5.140}$$

Let π_{t+1} denote the revenue of firms in period $t+1$ net of expenses. Revenues include proceeds from the sale of output as well as from the sale of the undepreciated fraction of capital. Expenses include obligations connected with the servicing of shares and with the compensation for labor services. Thus,

$$\pi_{t+1}^j = P_{t+1}^j Y_{t+1}^j + Q_{t+1}(1-\delta) K_{t+1}^j - W_{t+1} L_{t+1}^j - (1 + R_{t+1}^{js}) Q_t S_{t.}^j \tag{5.141}$$

At time t the problem of firms is to choose S_t^j and K_{t+1}^j to maximize the expected profits in period $t+1$, knowing that the firms will be able to choose the optimal quantity of labor in that period. The firm takes Q_t, Q_{t+1}, R_{t+1}^{js}, and W_{t+1} as given. This maximization problem can be expressed as

$$\max_{S_t, K_{t+1}^j} E_t \max_{L_{t+1}^j} \pi_{t+1}^j. \tag{5.142}$$

Notice that the equalization of Q_t and W_{t+1} across types of firms arises because of the absence of sector-specific frictions in physical markets for labor and capital.

At time t the problem of firms is to choose S_t and K_{t+1} to maximize the expected profits in period $t+1$, knowing that the firms will be able to choose the optimal quantity of labor in that period. The firm takes Q_t, Q_{t+1}, R_{t+1}^s, and W_{t+1} as given. This maximization problem can be expressed as

$$\max_{S_t^j, K_{t+1}^j} E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \max_{L_{t+1}^j} \pi_{t+1}^j \tag{5.143}$$

subject to constraints of the production technology $Y_t^j = A_t K_t^{j\,\alpha} L_t^{j\,1-\alpha}$ and financing $Q_t K_{t+1}^j = Q_t S_t^j$. The solution of $\max_{L_{t+1}^j} \pi_{t+1}^j$ implies that

$$W_{t+1} = (1-\alpha)\frac{P_{t+1}^j Y_{t+1}^j}{L_{t+1}^j} \tag{5.144}$$

$$L_{t+1}^j = (1-\alpha)\frac{P_{t+1}^j Y_{t+1}^j}{W_{t+1}} \tag{5.145}$$

under all states of nature. Accordingly, $\max_{S_t, K_{t+1}} E_t \max_{L_{t+1}} \pi_{t+1}$ collapses to

$$\begin{aligned}\max_{S_t K_{t+1}, L_{t+1}} E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} \Big[& P_{t+1}^j A_{t+1} K_{t+1}{}^{\alpha} L_{t+1}^{1-\alpha} + Q_{t+1}(1-\delta) K_{t+1}^j \\ & - W_{t+1} L_{t+1}^j - (1 + R_{t+1}^{sj}) Q_t S_{t.} \Big]\end{aligned}$$

$$
\begin{aligned}
&+ \lambda^j_{lt+1t}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\,\frac{P^j_{t+1}A_{t+1}{K^j_{t+1}}^{\alpha}{L^{j}_{t+1}}^{1-\alpha}}{W_{t+1}} - L_{t+1}\right]\\
&+ \lambda^j_{st}\left(Q_t S_{t.} - Q_t K^j_{t+1}\right). \qquad (5.146)
\end{aligned}
$$

Notice that there is no expectation operator on the Lagrangian multipliers because those constraints hold under every state of nature.

The problem implies the following conditions:

$$
\frac{\partial}{S_t} = -E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^{sj}_{t+1})Q_t + \lambda_{st}Q_t = 0 \qquad (5.147)
$$

$$
\begin{aligned}
\frac{\partial}{\partial K_{t+1}} &= E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\alpha\frac{P^j_{t+1}Y_{t+1}}{K_{t+1}} + Q_{t+1}(1-\delta)\right]\\
&\quad + \lambda^j_{lt+1}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\,\frac{\alpha P^j_{t+1}\frac{Y_{t+1}}{K_{t+1}}}{W_{t+1}}\right] - \lambda^j_{st}Q_t \qquad (5.148)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial L_{t+1}} &= \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\,\frac{P^j_{t+1}Y_{t+1}}{L_{t+1}} - W_{t+1}\right]\\
&\quad + \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\lambda_{lt+1}\left[(1-\alpha)^2\,\frac{P^j_{t+1}Y_{t+1}}{L_{t+1}W_{t+1}} - 1\right]. \qquad (5.149)
\end{aligned}
$$

Working on $\frac{\partial}{S_t}$,

$$
\lambda^j_{st} = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^{js}_{t+1}). \qquad (5.150)
$$

From $\frac{\partial}{\partial K_{t+1}}$,

$$
\begin{aligned}
E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^{js}_{t+1}) &= E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha\frac{P^j_{t+1}Y^j_{t+1}}{K^j_{t+1}} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right]\\
&\quad + E_t\lambda_{lt+1}\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[(1-\alpha)\,\frac{\alpha\frac{P^j_{t+1}Y^j_{t+1}}{K^j_{t+1}}}{W_{t+1}}\right].
\end{aligned}
$$

Next work on $\frac{\partial}{\partial L_{t+1}}$. Again, since $(1-\alpha)\frac{P^j_{t+1}Y^j_{t+1}}{L^j_{t+1}} = W_{t+1}$,

$$\frac{\partial}{\partial L_{t+1}} = \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}[0] + \beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\lambda_{lt+1}[(1-\alpha)-1] = 0 \tag{5.151}$$

$$\lambda_{lt+1} = 0. \tag{5.152}$$

Then, combining the implications of $\frac{\partial}{\partial L_{t+1}} = 0$ and $\frac{\partial}{\partial K_{t+1}} = 0$ yields

$$E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^{js}_{t+1}) = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha\frac{P^j_{t+1}Y^j_{t+1}}{K^j_{t+1}} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right]. \tag{5.153}$$

Notice that firms will make zero profits under all states of nature (and that's why we can drop the expectation operator). Thus,

$$0 = P^j_{t+1}Y^j_{t+1} + Q_{t+1}(1-\delta)K^j_{t+1} - W_{t+1}L^j_{t+1} - (1+R^{sj}_{t+1})Q_tS^j_{t.} \tag{5.154}$$

$$(1+R^{sj}_{t+1}) = \frac{1}{Q_t}\frac{\alpha P^j_{t+1}Y^j_{t+1}}{K^j_{t+1}} + \frac{(1-\delta)}{Q_t}Q_{t+1}. \tag{5.155}$$

This condition will also imply $E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}(1+R^{sj}_{t+1}) = E_t\beta\frac{\lambda_{ct+1}}{\lambda_{ct}}\left[\frac{1}{Q_t}\alpha\frac{P^j_{t+1}Y^j_{t+1}}{K^j_{t+1}} + (1-\delta)\frac{Q_{t+1}}{Q_t}\right]$ derived above (if profits are always zero, it does not matter how you discount them).

5.4.3 Capital-Producing Firms

The evolution of capital takes the form

$$K_{t+1} = I^n_t + (1-\delta)K_t. \tag{5.156}$$

Net investment is simply given by

$$I^n_t = K_{t+1} - (1-\delta)K_t. \tag{5.157}$$

The production technology for investment involves a quadratic adjustment for current production relative to past production, thus the supply of investment goods is given by

$$I_t^n = \left[1 - \frac{\phi}{2}\left(\frac{I_t^g}{I_{t-1}^g} - 1\right)^2\right] I_{t.}^g \tag{5.158}$$

Capital-producing firms solve the problem

$$\max_{I_{t+i}^g} E_t \sum_{i=0}^{\infty} \psi_{t,t+i} \left[Q_{t+i}\left[1 - \frac{\phi}{2}\left(\frac{I_{t+i}^g}{I_{t+i-1}^g} - 1\right)^2\right] I_{t+i}^g - P_t^i I_{t+i.}^g\right]. \tag{5.159}$$

In the maximization, Q_t is taken as given and $\psi_{t,t+i}$ is the stochastic discount factor of households who own the capital-producing firms (defined below).

5.4.4 Banks

Banks lend funds obtained from households to non-financial firms. Let $N_t(j)$ be the amount of wealth—or net worth—that a banker j has at the end of period t.

$$Q_t S_t^b(j) = N_t(j) + D_t(j) \tag{5.160}$$

As noted earlier, deposits $D_t(j)$ pay the non-state-contingent return $(1 + R_t)$ at time $t + 1$. Thus $D_t(j)$ may be thought of as the debt of bank j, and $N_t(j)$ as its capital. As seen above, the shares $S_t^b(j)$ earn the stochastic return $(1 + R_{t+1}^{bs})$ at time $t + 1$.

Over time, the banker's equity capital evolves as the difference between earnings on assets and interest payments on liabilities:

$$N_{t+1}(j) = (1 + R_{t+1}^{bs})Q_t S_t(j) - (1 + R_t)D_t(j) \tag{5.161}$$

$$D_t(j) = Q_t S_t(j) - N_t(j) \tag{5.162}$$

$$N_{t+1}(j) = (1 + R_{t+1}^{bs})Q_t S_t(j) - (1 + R_t)\left(Q_t S_t(j) - N_t(j)\right) \tag{5.163}$$

$$N_{t+1}(j) = \left[(1 + R_{t+1}^{bs}) - (1 + R_t)\right] Q_t S_t^b(j) + (1 + R_t)N_t(j) \tag{5.164}$$

$$N_{t+1}(j) = \left(R_{t+1}^{bs} - R_t\right) Q_t S_t^b(j) + (1 + R_t)N_t(j). \tag{5.165}$$

Let $\psi_{t,t+i} = \beta^i \frac{\lambda_{ct+i}}{\lambda_{ct}}$ be the stochastic discount factor between periods t and $t + i$. The banker's objective is to maximize expected terminal wealth, given by

$$\max_{S^b_{t+i}(j)} V_t(j) = E_t \sum_{i=0}^{\infty} (1-\theta)\,\theta^i \psi_{t,t+1+i}[\left(R^{bs}_{t+1+i} - R_{t+i}\right) Q_{t+i} S^b_{t+i}(j) + (1+R_{t+i}) N_{t+i}(j)]. \tag{5.166}$$

Since the banker will not fund assets with a discounted return less than the discounted cost of borrowing, for the bank to operate in period $t+i$, it must be that $E_t \psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right) \geq 0$, i.e., there are expected positive excess returns from holding stocks even after discounting and adjusting for risk through $\psi_{t,t+1+i}$. In the absence of financial frictions, when $E_t \psi_{t,t+1+i}\left(R^s_{t+1+i} - R_{t+i}\right)$ is positive, the bank will want to expand its balance sheet by attracting additional deposits from households.

To limit the ability of banks to attract deposits indefinitely, now impose the external requirement λ_t:

$$N_t(j) \geq \lambda_t Q_t S^b_t(j). \tag{5.167}$$

Log-linearizing,

$$\lambda_t = \frac{N_t}{Q_t S^b_t}. \tag{5.168}$$

As before, then

$$N_t = \theta \left[\left(R^{bs}_t - R_{t-1}\right) \frac{1}{\lambda_{t-1}} + (1+R_{t-1})\right] N_{t-1} + \omega Q_t S^b_{t-1}. \tag{5.169}$$

5.5 Introducing Nominal Rigidities

Modify the problem of households to be

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left[\log(C_{t+i} - \gamma C_{t+i-1}) - \frac{\chi}{1+\varepsilon} L^{1+\varepsilon}_{t+i}\right]. \tag{5.170}$$

However, in this case, the budget constraint takes the form

$$P_t C_t = P_t W_t L_t + P_t \Pi_t - P_t T_t + P_t Q_t S^u_t - (1+R^{su}_t) P_t Q_{t-1} S^u_{t-1} + P_t D_t - (1+R_{t-1}) P_t D_{t-1}. \tag{5.171}$$

Note that, despite writing the budget constraint in nominal terms, we are guaranteeing a real return R_t. In this respect, deposits are akin to indexed bonds.

Consider the first order-condition with respect to deposit holdings:

$$\lambda_{ct}^N P_t - E_t \beta \lambda_{ct+1}^N (1 + R_{t+1}) P_{t+1} = 0$$
$$\lambda_{ct}^N P_t = E_t \beta \lambda_{ct+1}^N (1 + R_{t+1}) P_{t+1}$$
$$E_t \beta \frac{\lambda_{ct+1}^N}{\lambda_{ct}^N} \frac{P_{t+1}}{P_t} (1 + R_{t+1}) = 1$$
$$E_t \beta \frac{\lambda_{ct+1}}{\lambda_{ct}} (1 + R_{t+1}) = 1.$$

Proceed as before, but modify the problem of producing firms.

The production technology of the representative firm is $Y_t = (Y_t^u)^\alpha \left(Y_t^b\right)^{1-\alpha}$. Firms operate for only one period, but some of the planning for production is done one period in advance. To operate capital in period $t+1$, a firm must purchase it in period t. To do so, the firm issues shares in period t. There are as many shares S_t as units of capital purchased. By arbitrage, the current value of capital equals the value of shares. Thus,

$$P_t Q_t K_{t+1} = P_t Q_t S_{t.} \tag{5.172}$$

Let π_{t+1} denote the revenue of firms in period $t+1$ net of expenses. Revenues include proceeds from the sale of output as well as from the sale of the undepreciated fraction of capital. Expenses include obligations connected with the servicing of shares and with the compensation for labor services. Thus,

$$\pi_{t+1} = \sigma_{t+1} Y_{t+1} + P_{t+1} Q_{t+1} (1 - \delta) K_{t+1} - P_{t+1} W_{t+1} L_{t+1} \\ - (1 + r_{t+1}^s) P_t Q_t S_{t.} \tag{5.173}$$

At time t the problem of firms is to choose S_t and K_{t+1} to maximize the expected profits in period $t+1$, knowing that the firms will be able to choose the optimal quantity of labor in that period. The firm takes Q_t, Q_{t+1}, R_{t+1}^s, and W_{t+1} as given. This maximization problem can be expressed as

$$\max_{S_t, K_{t+1}} E_t m_{t+1/t} \max_{L_{t+1}} \pi_{t+1} \tag{5.174}$$

subject to constraints of the production technology $Y_t = A_t {K_t}^{\alpha} L_t^{1-\alpha}$ and financing $Q_t P_t K_{t+1} = Q_t P_t S_t$. The solution of $\max_{L_{t+1}} \pi_{t+1}$ implies that

$$P_{t+1} W_{t+1} = (1-\alpha) \frac{\sigma_{t+1} Y_{t+1}}{L_{t+1}} \tag{5.175}$$

$$L_{t+1} = (1-\alpha) \frac{Y_{t+1}}{W_{t+1}} \frac{\sigma_{t+1}}{P_{t+1}} \tag{5.176}$$

under all states of nature. Accordingly, $\max_{S_t, K_{t+1}} E_t \max_{L_{t+1}} \pi_{t+1}$ collapses to

$$\begin{aligned} \max_{S_t K_{t+1}, L_{t+1}} & E_t m_{t+1/t} \left[\sigma_{t+1} A_{t+1} {K_{t+1}}^{\alpha} L_{t+1}^{1-\alpha} + P_{t+1} Q_{t+1} (1-\delta) K_{t+1} \right. \\ & \left. - P_{t+1} W_{t+1} L_{t+1} - (1 + r_{t+1}^s) P_t Q_t S_{t.} \right] \\ & + \lambda_{lt+1t} m_{t+1/t} \left[(1-\alpha) \frac{A_{t+1} {K_{t+1}}^{\alpha} L_{t+1}^{1-\alpha}}{W_{t+1}} \frac{\sigma_{t+1}}{P_{t+1}} - L_{t+1} \right] \\ & + \lambda_{st} \left(Q_t S_{t.} - Q_t K_{t+1} \right). \end{aligned} \tag{5.177}$$

Subject to the modifications above, the derivations follow closely what we had in the absence of nominal rigidities. The conditions for an equilibrium from the side of producing firms are as follows.

From the zero-profit condition,

$$\begin{aligned} 0 = {} & \sigma_{t+1} Y_{t+1} + P_{t+1} Q_{t+1} (1-\delta) K_{t+1} - P_{t+1} W_{t+1} L_{t+1} \\ & - (1 + r_{t+1}^s) P_t Q_t S_{t.} \end{aligned} \tag{5.178}$$

$$\begin{aligned} (1 + r_{t+1}^s) P_t Q_t S_{t.} = {} & \sigma_{t+1} Y_{t+1} + Q_{t+1} P_{t+1} (1-\delta) K_{t+1} \\ & - P_{t+1} W_{t+1} L_{t+1} \end{aligned} \tag{5.179}$$

$$(1 + r_{t+1}^s) = \frac{\sigma_{t+1} Y_{t+1} + Q_{t+1} P_{t+1} (1-\delta) K_{t+1} - P_{t+1} W_{t+1} L_{t+1}}{P_t Q_t S_{t.}} \tag{5.180}$$

$$(1 + r_{t+1}^s) = \frac{\sigma_{t+1} Y_{t+1} + Q_{t+1} P_{t+1} (1-\delta) K_{t+1} - P_{t+1} W_{t+1} L_{t+1}}{P_t Q_t K_{t+1.}} \tag{5.181}$$

$$(1+r^s_{t+1}) = \frac{\begin{array}{c}\sigma_{t+1}Y_{t+1} + Q_{t+1}P_{t+1}(1-\delta)K_{t+1} \\ -P_{t+1}W_{t+1}\left(1-\alpha\right)\frac{\sigma_{t+1}Y_{t+1}}{P_{t+1}W_{t+1}}\end{array}}{P_tQ_tK_{t+1.}} \tag{5.182}$$

$$(1+r^s_{t+1}) = \frac{1}{Q_t}\frac{\alpha\sigma_{t+1}Y_{t+1}}{P_tK_{t+1}} + \frac{(1-\delta)}{P_tQ_t}P_{t+1}Q_{t+1} \tag{5.183}$$

$$(1+r^s_{t+1}) = \frac{1}{Q_t}\frac{\alpha\sigma_{t+1}Y_{t+1}}{P_{t+1}K_{t+1}}\frac{P_{t+1}}{P_t} + \frac{(1-\delta)}{Q_t}\frac{P_{t+1}}{P_t}Q_{t+1} \tag{5.184}$$

$$(1+r^s_{t+1}) = \frac{1}{Q_t}\frac{\alpha\sigma_{t+1}Y_{t+1}}{P_{t+1}K_{t+1}}\frac{P_{t+1}}{P_t} + \frac{(1-\delta)}{Q_t}\frac{P_{t+1}}{P_t}Q_{t+1} \tag{5.185}$$

$$\frac{(1+r^s_{t+1})}{\frac{P_{t+1}}{P_t}} = \frac{1}{Q_t}\frac{\alpha\sigma_{t+1}Y_{t+1}}{P_{t+1}K_{t+1}} + \frac{(1-\delta)}{Q_t}Q_{t+1}. \tag{5.186}$$

Define

$$(1+R^s_t) = \frac{(1+r^s_t)}{\frac{P_t}{P_{t-1}}}.$$

Accordingly,

$$(1+R^s_{t+1}) = \frac{1}{Q_t}\frac{\alpha\sigma_{t+1}Y_{t+1}}{P_{t+1}K_{t+1}} + \frac{(1-\delta)}{Q_t}Q_{t+1}, \tag{5.187}$$

and from above,

$$L_{t+1} = \left(1-\alpha\right)\frac{Y_{t+1}}{W_{t+1}}\frac{\sigma_{t+1}}{P_{t+1}}. \tag{5.188}$$

The problem of the final firms is

$$\max_{P_{t+i(}(f)} E_t\sum_{i=0}^{\infty}\psi_{t,t+i}\left\{\left(1+\tau_p\right)P_{t+i}\left(f\right) - \sigma_{t+i}\right\}\left(1-\phi_{P,t+i}\left(f\right)\right)Y_{t+i} \times\left(\frac{P_{t+i}\left(f\right)}{P_{t+i}}\right)^{-\frac{1+\theta_p}{\theta_p}},$$

where

$$\phi_{P,t} = \frac{\phi_p}{2}\left(\frac{P_t\left(f\right)}{\pi P_{t-1}\left(f\right)} - 1\right)^2.$$

The first-order conditions are

$$E_t \left[\begin{array}{c} (1+\tau_p)(1-\phi_{P,t}(f))\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \\ -\frac{1+\theta_p}{\theta_p}\{(1+\tau_p) P_t(f) - \sigma_t\}(1-\phi_{P,t}(f))\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}-1} \frac{1}{P_t} \\ -\{(1+\tau_p) P_t(f) - \sigma_t\}\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \frac{\partial \phi_{P,t}(f)}{\partial P_t(f)} \\ -\psi_{t,t+1}\{(1+\tau_p) P_{t+1}(f) - \sigma_{t+1}\}\, Y_{t+1} \left(\frac{P_{t+1}(f)}{P_{t+1}}\right)^{-\frac{1+\theta_p}{\theta_p}} \frac{\partial \phi_{P,t+1}(f)}{\partial P_t(f)} \end{array} \right] = 0$$

$$E_t \left[\begin{array}{c} (1+\tau_p)(1-\phi_{P,t}(f))\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \\ -\frac{1+\theta_p}{\theta_p}\left\{(1+\tau_p) - \frac{\sigma_t}{P_t(f)}\right\}(1-\phi_{P,t}(f))\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}-1} \frac{P_t(f)}{P_t} \\ -\{(1+\tau_p) P_t(f) - \sigma_t\}\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \frac{\partial \phi_{P,t}(f)}{\partial P_t(f)} \\ -\psi_{t,t+1}\{(1+\tau_p) P_{t+1}(f) - \sigma_{t+1}\}\, Y_{t+1} \left(\frac{P_{t+1}(f)}{P_{t+1}}\right)^{-\frac{1+\theta_p}{\theta_p}} \frac{\partial \phi_{P,t+1}(f)}{\partial P_t(f)} \end{array} \right] = 0$$

$$E_t \left[\begin{array}{c} (1+\tau_p)(1-\phi_{P,t}(f))\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \\ +\left\{-\frac{1+\theta_p}{\theta_p}(1+\tau_p) + \frac{1+\theta_p}{\theta_p}\frac{\sigma_t}{P_t(f)}\right\}(1-\phi_{P,t}(f))\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \\ -\{(1+\tau_p) P_t(f) - \sigma_t\}\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \frac{\partial \phi_{P,t}(f)}{\partial P_t(f)} \\ -\psi_{t,t+1}\{(1+\tau_p) P_{t+1}(f) - \sigma_{t+1}\}\, Y_{t+1} \left(\frac{P_{t+1}(f)}{P_{t+1}}\right)^{-\frac{1+\theta_p}{\theta_p}} \frac{\partial \phi_{P,t+1}(f)}{\partial P_t(f)} \end{array} \right] = 0$$

$$E_t \left[\begin{array}{c} +\left\{\left(1-\frac{1+\theta_p}{\theta_p}\right)(1+\tau_p) + \frac{1+\theta_p}{\theta_p}\frac{\sigma_t}{P_t(f)}\right\}(1-\phi_{P,t}(f)) \\ Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} - \{(1+\tau_p) P_t(f) - \sigma_t\}\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \frac{\partial \phi_{P,t}(f)}{\partial P_t(f)} \\ -\psi_{t,t+1}\{(1+\tau_p) P_{t+1}(f) - \sigma_{t+1}\}\, Y_{t+1} \left(\frac{P_{t+1}(f)}{P_{t+1}}\right)^{-\frac{1+\theta_p}{\theta_p}} \frac{\partial \phi_{P,t+1}(f)}{\partial P_t(f)} \end{array} \right] = 0$$

$$E_t \left[\begin{array}{c} \left[-\frac{1}{\theta_p}(1+\tau_p) + \frac{1+\theta_p}{\theta_p}\frac{\sigma_t}{P_t(f)}\right](1-\phi_{P,t}(f))\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \\ -\{(1+\tau_p) P_t(f) - \sigma_t\}\, Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\frac{1+\theta_p}{\theta_p}} \frac{\partial \phi_{P,t}(f)}{\partial P_t(f)} \\ -\psi_{t,t+1}\{(1+\tau_p) P_{t+1}(f) - \sigma_{t+1}\}\, Y_{t+1} \left(\frac{P_{t+1}(f)}{P_{t+1}}\right)^{-\frac{1+\theta_p}{\theta_p}} \frac{\partial \phi_{P,t+1}(f)}{\partial P_t(f)} \end{array} \right] = 0.$$

Due to symmetry,

$$E_t \left[\begin{array}{c} \left[-\frac{1}{\theta_p}\left(1+\tau_p\right)+\frac{1+\theta_p}{\theta_p}\frac{\sigma_t}{P_t}\right]\left(1-\phi_{P,t}\right)Y_t \\ -\left\{\left(1+\tau_p\right)-\frac{\sigma_t}{P_t}\right\}Y_t P_t \frac{\partial \phi_{P,t}}{\partial P_t} \\ -\psi_{t,t+1}\left\{\left(1+\tau_p\right)-\frac{\sigma_{t+1}}{P_{t+1}}\right\}Y_{t+1}P_{t+1}\frac{\partial \phi_{P,t+1}(f)}{\partial P_t(f)} \end{array} \right] = 0$$

with the adjustment costs

$$\phi_{P,t} = \frac{\phi_p}{2}\left(\frac{P_t\left(f\right)}{\pi P_{t-1}\left(f\right)}-1\right)^2$$

$$\frac{\partial \phi_{P,t}}{\partial P_t} = \phi_p\left(\frac{P_t\left(f\right)}{\pi P_{t-1}\left(f\right)}-1\right)\frac{1}{\pi P_{t-1}\left(f\right)}$$

$$\frac{\partial \phi_{P,t}}{\partial P_{t-1}\left(f\right)} = -\phi_p\left(\frac{P_t\left(f\right)}{\pi P_{t-1}\left(f\right)}-1\right)\frac{P_t\left(f\right)}{\pi P_{t-1}\left(f\right)}\frac{1}{P_{t-1}\left(f\right)}$$

or

$$\phi_{P,t} = \frac{\phi_p}{2}\left(\frac{\pi_t}{\pi}-1\right)^2$$

$$\frac{\partial \phi_{P,t}}{\partial P_t}P_t = \phi_p\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}$$

$$\frac{\partial \phi_{P,t}}{\partial P_{t-1}}P_t = -\phi_p\left(\frac{\pi_t}{\pi}-1\right)\frac{\pi_t}{\pi}\pi_t.$$

As a small detour, let's map the parameter ϕ_p into the parameterization of sticky price contracts following the Calvo scheme.

Let $\hat{\pi}_t = \pi_t - \pi$. Let $\hat{\sigma}_t = \frac{\frac{\sigma_t}{P_t}-\frac{\sigma}{P}}{\frac{\sigma}{P}}$. But notice that with $P = 1$, in our model $\sigma = 1$ (since we impose $\tau_p = \theta_p$), so $\hat{\sigma}_t = \frac{\frac{\sigma_t}{P_t}-\frac{\sigma}{P}}{\frac{\sigma}{P}} = \frac{\sigma_t}{P_t}-\frac{\sigma}{P} = \frac{\sigma_t}{P_t}-1$. Standard results are that, under Calvo contracts, the first-order approximation of the firms' pricing equation yields

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \kappa_p\hat{\sigma}_t,$$

where $\kappa_p = \frac{(1-\beta\xi)(1-\xi)}{\xi}$, and where $1-\xi$ is the probability that a firm will be allowed to reoptimize its price. Now, consider the pricing condition for Rotemberg contracts:

$$E_t \begin{bmatrix} \left[-\frac{1}{\theta_p}(1+\tau_p) + \frac{1+\theta_p}{\theta_p}\frac{\sigma_t}{P_t}\right](1-\phi_{P,t})Y_t \\ -\left\{(1+\tau_p) - \frac{\sigma_t}{P_t}\right\} Y_t \phi_p \left(\frac{\pi_t}{\pi} - 1\right)\frac{\pi_t}{\pi} \\ \psi_{t,t+1}\left\{(1+\tau_p) - \frac{\sigma_{t+1}}{P_{t+1}}\right\} Y_{t+1}\phi_p\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{\pi_{t+1}}{\pi}\pi_{t+1} \end{bmatrix} = 0,$$

with $P = 1$ in steady state. Using the first-order Taylor series expansion around the steady-states π and σ, we find

$$0 = \frac{1+\theta_p}{\theta_p} Y\hat{\sigma}_t - [(1+\tau_p) - \sigma] Y\phi_p\hat{\pi}_t + \beta[(1+\tau_p) - \sigma] Y\phi_p\hat{\pi}_{t+1}.$$

But remembering $\tau_p = \theta_p$,

$$0 = \frac{1+\theta_p}{\theta_p} Y\hat{\sigma}_t - [(1+\theta_p) - \sigma] Y\phi_p\hat{\pi}_t + \beta[(1+\theta_p) - \sigma] Y\phi_p\hat{\pi}_{t+1}.$$

Remembering that $\sigma = 1$ in steady state,

$$\theta_p Y\phi_p\hat{\pi}_t = \beta\theta_p Y\phi_p\hat{\pi}_{t+1} + \frac{1+\theta_p}{\theta_p} Y\hat{\sigma}_t$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{1+\theta_p}{\theta_p^2\phi_p}\hat{\sigma}_t.$$

$$\hat{\pi}_t = \beta\hat{\pi}_{t+1} + \frac{(\varepsilon-1)\varepsilon}{\phi_p}\hat{\sigma}_t.$$

Matching the coefficients on marginal costs from Calvo and Rotemberg contracts, we obtain $\frac{1+\theta_p}{\theta_p^2\phi_p} = \kappa_p$ or

$$\phi_p = \frac{1+\theta_p}{\theta_p^2\kappa_p}.$$

Finally, monetary policy is set according to a interest rate reaction function of the following form:

$$R_t = \phi_R(R_{t-1} - \bar{R}) + (1-\phi_R)(\pi_t - \bar{pi}).$$

Table 5.1 Calibration

Parameter	Description	Sector
$\alpha = 0.33$	Share of Capital in Production	Production
$\rho = 0.95$	Autoregressive Coefficient of the Productivity Growth Process	
$\delta = 0.025$	Capital Depreciation Rate	Capital-Producing Firms
$\phi = 1.5$	Investment Adjustment Coefficient	Capital-Producing Firms
$\beta = 0.99$	Household Subjective Discount Factor	Households
$\gamma = 0.82$	Habit Persistence Parameter	
$\epsilon = 1.00$	Inverse Frisch Elasticity of Labor Supply	
$\theta = 0.97$	Expected Number of Periods as Banker = 30	Banks
$\alpha_{fp} = 0.60$	Share of Bank-Financed Firms	
$\xi_p = 0.88$	Coefficient of Average Contract Duration	
$\theta_p = 0.1$	Steady-State Markup	Nominal Rigidities
$\xi_p = 0.88$	Calvo Probability of Not-Adjusting Price	
$\phi_R = 0.7$	Interest Rate Smoothing	Monetary Policy Rule
$\phi_\pi = 3$	Weight on Inflation	

5.6 Calibration

The share of output devoted to government spending is 20 percent. The fraction of time spent working is 0.5 in steady state. Following Gertler and Karadi (2011), the parameter θ is set to deliver an expected duration of a banker's assignment of thirty-five quarters. The steady-state loan-to-equity ratio is set to 4 and the steady-state spread is 0.5 percent, or 2 percent when annualized. These latter two steady-state choices are achieved by setting λ to 0.60 and ω to 0.0011. The persistence of the transfer shock to households is 0.9. All the other calibrated parameters are shown in table 5.1.

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