Online Appendixes to External Shocks, Banks, and Optimal Monetary Policy: A Recipe for Emerging Market Central Banks*

Yasin Mimir[a] and Enes Sunel[b]
[a]Norges Bank
[b]Sunel & Sunel Law Firm

Appendix A. Model Derivations

A.1 Households

There is a large number of infinitely lived identical households, which derive utility from consumption c_t, leisure $(1 - h_t)$, and real money balances $\frac{M_t}{P_t}$. The consumption good is a constant-elasticity-of-substitution (CES) aggregate of domestically produced and imported tradable goods as in Galí and Monacelli (2005) and Gertler, Gilchrist, and Natalucci (2007),

$$c_t = \left[\omega^{\frac{1}{\gamma}}(c_t^H)^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}}(c_t^F)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \tag{A1}$$

*We would like to thank Leonardo Gambacorta, Nobuhiro Kiyotaki, Giovanni Lombardo, Hyun Song Shin, Philip Turner, the seminar participants at the Bank for International Settlements, Norges Bank, the Georgetown Center for Economic Research Biennial Conference 2015, the CBRT-BOE Joint Workshop 2015 on "The International Monetary and Financial System — Long-term Challenges, Short-term Solutions," the 21st International Conference on Computing in Economics and Finance, the 11th World Congress of the Econometric Society, and the 47th Money, Macro and Finance Research Group Annual conference for their helpful comments and suggestions. This paper circulated earlier under the title "External Shocks, Banks and Optimal Monetary Policy in an Open Economy." Yasin Mimir completed this project while visiting the Bank for International Settlements under the Central Bank Research Fellowship program. The views expressed in this paper are those of the authors only and do not necessarily reflect the views of Norges Bank or of the Bank for International Settlements (BIS). The usual disclaimer applies. Corresponding author: Yasin Mimir, Norges Bank, Monetary Policy Department, Bankplassen 2, 0151 Oslo, Norway; Tel.: +4722316811; E-mail: yasin.mimir@norges-bank.no; Personal homepage: http://www.yasinmimir.com.

where $\gamma > 0$ is the elasticity of substitution between home and foreign goods, and $0 < \omega < 1$ is the relative weight of home goods in the consumption basket, capturing the degree of home bias in household preferences. Let P_t^H and P_t^F represent domestic-currency-denominated prices of home and foreign goods, which are aggregates of a continuum of differentiated home and foreign good varieties, respectively.

The expenditure minimization problem of households

$$\min_{c_t^H, c_t^F} P_t c_t - P_t^H c_t^H - P_t^F c_t^F$$

subject to (A1) yields the demand curves $c_t^H = \omega \left(\frac{P_t^H}{P_t}\right)^{-\gamma} c_t$ and $c_t^F = (1-\omega)\left(\frac{P_t^F}{P_t}\right)^{-\gamma} c_t$ for home and foreign goods, respectively.

The final demand for home consumption good c_t^H is an aggregate of a continuum of varieties of intermediate home goods along the [0,1] interval. That is, $c_t^H = \left[\int_0^1 (c_{it}^H)^{1-\frac{1}{\epsilon}} di\right]^{\frac{1}{1-\frac{1}{\epsilon}}}$, where each variety is indexed by i, and ϵ is the elasticity of substitution between these varieties. For any given level of demand for the composite home good c_t^H, the demand for each variety i solves the problem of minimizing total home goods expenditures, $\int_0^1 P_{it}^H c_{it}^H di$, subject to the aggregation constraint, where P_{it}^H is the nominal price of variety i. The solution to this problem yields the optimal demand for c_{it}^H, which satisfies

$$c_{it}^H = \left(\frac{P_{it}^H}{P_t^H}\right)^{-\epsilon} c_t^H,$$

with the aggregate home good price index P_t^H being

$$P_t^H = \left[\int_0^1 (P_{it}^H)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}.$$

Therefore, the expenditure minimization problem of households subject to the consumption aggregator (A1) produces the domestic consumer price index (CPI),

$$P_t = \left[\omega (P_t^H)^{1-\gamma} + (1-\omega)(P_t^F)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \tag{A2}$$

and the condition that determines the optimal demand frontier for home and foreign goods,

$$\frac{c_t^H}{c_t^F} = \frac{\omega}{1-\omega}\left(\frac{P_t^H}{P_t^F}\right)^{-\gamma}. \tag{A3}$$

We assume that each household is composed of a worker and a banker who perfectly insure each other. Workers consume the consumption bundle and supply labor h_t. They also save in local currency assets which are *deposited* within financial intermediaries owned by the banker members of *other* households.[1] The balance of these deposits is denoted by B_{t+1}, which promises to pay a net nominal risk-free rate r_{nt} in the next period. There are no inter-bank frictions, hence r_{nt} coincides with the policy rate of the central bank. Furthermore, the borrowing contract is *real* in the sense that the risk-free rate is determined based on the expected inflation. By assumption, households cannot directly save in productive capital, and only banker members of households are able to borrow in foreign currency.

Preferences of households over consumption, leisure, and real balances are represented by the lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, h_t, \frac{M_t}{P_t}\right), \tag{A4}$$

where U is a CRRA-type period utility function given by

$$U\left(c_t, h_t, \frac{M_t}{P_t}\right) = \left[\frac{\left(c_t - h_c c_{t-1}\right)^{1-\sigma} - 1}{1-\sigma} - \frac{\chi}{1+\xi} h_t^{1+\xi} + \upsilon \, \log\left(\frac{M_t}{P_t}\right)\right]. \tag{A5}$$

E_t is the mathematical expectation operator conditional on the information set available at t, $\beta \in (0,1)$ is the subjective discount rate, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $h_c \in [0,1)$ governs the degree of habit formation, χ is the

[1]This assumption is useful in making the agency problem that we introduce in section 3.2 more realistic.

utility weight of labor, and $\xi > 0$ determines the Frisch elasticity of labor supply. We also assume that the natural logarithm of real money balances provides utility in an additively separable fashion with the utility weight υ.[2]

Households face the flow budget constraint,

$$c_t + \frac{B_{t+1}}{P_t} + \frac{M_t}{P_t} = \frac{W_t}{P_t}h_t + \frac{(1+r_{nt-1})B_t}{P_t} + \frac{M_{t-1}}{P_t} + \Pi_t - \frac{T_t}{P_t}. \tag{A6}$$

On the right-hand side are the real wage income $\frac{W_t}{P_t}h_t$, real balances of the domestic currency interest-bearing assets at the beginning of period t $\frac{B_t}{P_t}$, and real money balances at the beginning of period t $\frac{M_{t-1}}{P_t}$. Π_t denotes real profits remitted from firms owned by the households (banks, intermediate home goods producers, and capital goods producers). T_t represents nominal lump-sum taxes collected by the government. On the left-hand side are the outlays for consumption expenditures and asset demands.

Households choose c_t, h_t, B_{t+1}, and M_t to maximize preferences in (A5) subject to (A6) and standard transversality conditions imposed on asset demands B_{t+1} and M_t. The first-order conditions of the utility maximization problem of households are given by

$$\varphi_t = (c_t - h_c c_{t-1})^{-\sigma} - \beta h_c E_t (c_{t+1} - h_c c_t)^{-\sigma}, \tag{A7}$$

$$\frac{W_t}{P_t} = \frac{\chi h_t^{\xi}}{\varphi_t}, \tag{A8}$$

$$\varphi_t = \beta E_t \left[\varphi_{t+1}(1+r_{nt})\frac{P_t}{P_{t+1}}\right], \tag{A9}$$

$$\frac{\upsilon}{M_t/P_t} = \beta E_t \left[\varphi_{t+1} r_{nt}\frac{P_t}{P_{t+1}}\right]. \tag{A10}$$

Equation (A7) defines the Lagrange multiplier φ_t as the marginal utility of consuming an additional unit of income. Equation (A8)

[2]The logarithmic utility used for real money balances does not matter for real allocations, as it enters into the utility function in an additively separable fashion and money does not appear in any optimality condition except the consumption–money optimality condition.

equates marginal disutility of labor to the shadow value of real wages. Finally, equations (A9) and (A10) represent the Euler equations for bonds, the consumption–savings margin, and money demand, respectively.

First-order conditions (A7) and (A9) that come out of the utility maximization problem can be combined to obtain the consumption–savings optimality condition,

$$\begin{aligned}
&(c_t - h_c c_{t-1})^{-\sigma} - \beta h_c E_t\,(c_{t+1} - h_c c_t)^{-\sigma} \\
&= \beta E_t \left[\left\{ (c_{t+1} - h_c c_t)^{-\sigma} - \beta h_c\,(c_{t+2} - h_c c_{t+1})^{-\sigma} \right\} \frac{(1 + r_{nt+1}) P_t}{P_{t+1}} \right].
\end{aligned}$$

The consumption–money optimality condition,

$$\frac{\upsilon / m_t}{\varphi_t} = \frac{r_{nt}}{1 + r_{nt}},$$

on the other hand, might be derived by combining first-order conditions (A9) and (A10) with m_t denoting real balances held by consumers.

A.2 Banks' Net Worth Maximization

Bankers solve the following value maximization problem:

$$\begin{aligned}
V_{jt} &= \max_{l_{jt+i}, b^*_{jt+1+i}} E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \Lambda_{t,t+1+i}\; n_{jt+1+i} \\
&= \max_{l_{jt+i}, b^*_{jt+1+i}} E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \Lambda_{t,t+1+i} \Big(\left[R_{kt+1+i} - \hat{R}_{t+1+i} \right] \\
&\quad \times q_{t+i} l_{jt+i} + \left[R_{t+1+i} - R^*_{t+1+i} \right] b^*_{jt+1+i} + \hat{R}_{t+1+i} n_{jt+i} \Big),
\end{aligned}$$

subject to the constraint (7). Since

$$V_{jt} = \max_{l_{jt+i}, b^*_{jt+1+i}} E_t \sum_{i=0}^{\infty} (1-\theta)\theta^i \Lambda_{t,t+1+i}\; n_{jt+1+i}$$

$$= \max_{l_{jt+i}, b^*_{jt+1+i}} E_t \Bigg[(1-\theta)\Lambda_{t,t+1} n_{jt+1} + \sum_{i=1}^{\infty} (1-\theta)\theta^i \Lambda_{t,t+1+i}\ n_{jt+1+i} \Bigg],$$

we have

$$V_{jt} = \max_{l_{jt}, b^*_{jt+1}} E_t \Big\{ \Lambda_{t,t+1} [(1-\theta) n_{jt+1} + \theta V_{jt+1}] \Big\}.$$

The Lagrangian which solves the bankers' profit maximization problem reads

$$\max_{l_{jt}, b^*_{jt+1}} L = \nu^l_t q_t l_{jt} + \nu^*_t b^*_{jt+1} + \nu_t n_{jt} + \mu_t \left[\nu^l_t q_t l_{jt} + \nu^*_t b^*_{jt+1} + \nu_t n_{jt} - \lambda \left(q_t l_{jt} - \omega_l \left[\frac{q_t l_{jt} - n_{jt}}{1 - rr_t} - b^*_{jt+1} \right] \right) \right], \tag{A11}$$

where the term in square brackets represents the incentive compatibility constraint (7) combined with the balance sheet (2), to eliminate b_{jt+1}. The first-order conditions for l_{jt}, b^*_{jt+1}, and the Lagrange multiplier μ_t are

$$\nu^l_t (1 + \mu_t) = \lambda \mu_t \left(1 - \frac{\omega_l}{1 - rr_t} \right), \tag{A12}$$

$$\nu^*_t (1 + \mu_t) = \lambda \mu_t \omega_l, \tag{A13}$$

and

$$\nu^l_t q_t l_{jt} + \nu^*_t \left[\frac{q_t l_{jt} - n_{jt}}{1 - rr_t} - b_{jt+1} \right] + \nu_t n_{jt} - \lambda (q_t l_{jt} - \omega_l b_{jt+1}) \geq 0, \tag{A14}$$

respectively. We are interested in cases in which the incentive constraint of banks is always binding, which implies that $\mu_t > 0$ and (A14) holds with equality.

An upper bound for ω_l is determined by the necessary condition for a positive value of making loans $\nu_t^l > 0$, implying $\omega_l < 1 - rr_t$. Therefore, the fraction of non-diverted domestic deposits has to be smaller than one minus the reserve requirement ratio, as implied by (A12).

Combining (A12) and (A13) yields

$$\frac{\frac{\nu_t^*}{1-rr_t}}{\nu_t^l + \frac{\nu_t^*}{1-rr_t}} = \frac{\omega_l}{1-rr_t}.$$

Rearranging the binding version of (A14) leads to equation (9).

We replace V_{jt+1} in equation (6) by imposing our linear conjecture in equation (8) and the borrowing constraint (9) to obtain

$$\tilde{V}_{jt} = E_t\Big\{\Xi_{t,t+1} n_{jt+1}\Big\}, \tag{A15}$$

where $\tilde{V}_{jt}$ stands for the optimized value.

Replacing the left-hand side to verify our linear conjecture on bankers' value (8) and using equation (5), we obtain the definition of the augmented stochastic discount factor $\Xi_{t,t+1} = \Lambda_{t,t+1}\Big[1 - \theta + \theta\Big(\zeta_{t+1}\kappa_{t+1} + \nu_{t+1} - \frac{\nu_{t+1}^*}{1-rr_{t+1}}\Big)\Big]$ and find that ν_t^l, ν_t, and ν_t^* should consecutively satisfy equations (10), (11), and (12) in the main text.

Surviving bankers' net worth n_{et+1} is derived as described in the main text and is equal to

$$\begin{aligned} n_{et+1} = \theta\Bigg(& \left[R_{kt+1} - \hat{R}_{t+1} + \frac{R_{t+1} - R_{t+1}^*}{1-rr_t}\right]\kappa_t \\ & - \left[\frac{R_{t+1} - R_{t+1}^*}{1-rr_t}\right] + \hat{R}_{t+1}\Bigg) n_t \\ & + \Bigg(\left[R_{kt+1} - \hat{R}_{t+1} + \frac{R_{t+1} - R_{t+1}^*}{1-rr_t}\right]\omega_l \\ & - \left[\frac{R_{t+1} - R_{t+1}^*}{1-rr_t}\right]\Bigg) b_{t+1}. \end{aligned}$$

A.3 Capital Producers

Capital producers play a profound role in the model, since variations in the price of capital drives the financial accelerator. We assume that capital producers operate in a perfectly competitive market, purchase investment goods, and transform those goods into new capital. They also repair the depreciated capital that they buy from the intermediate-goods-producing firms. At the end of period t, they sell both newly produced and repaired capital to the intermediate goods firms at the unit prices of q_t and p_t^I, respectively. Intermediate goods firms use this new capital for production at time $t+1$. Capital producers are owned by households and return any earned profits to their owners. We also assume that they incur investment adjustment costs while producing new capital, given by the following quadratic function of the investment growth:

$$\Phi\left(\frac{i_t}{i_{t-1}}\right) = \frac{\Psi}{2}\left[\frac{i_t}{i_{t-1}} - 1\right]^2.$$

Capital producers use an investment good that is composed of home and foreign final goods in order to repair the depreciated capital and to produce new capital goods

$$i_t = \left[\omega_i^{\frac{1}{\gamma_i}}\left(i_t^H\right)^{\frac{\gamma_i-1}{\gamma_i}} + (1-\omega_i)^{\frac{1}{\gamma_i}}\left(i_t^F\right)^{\frac{\gamma_i-1}{\gamma_i}}\right]^{\frac{\gamma_i}{\gamma_i-1}},$$

where ω_i governs the relative weight of home input in the investment composite good and γ_i measures the elasticity of substitution between home and foreign inputs. Capital producers choose the optimal mix of home and foreign inputs according to the intratemporal first-order condition

$$\frac{i_t^H}{i_t^F} = \frac{\omega_i}{1-\omega_i}\left(\frac{P_t^H}{P_t^F}\right)^{-\gamma_i}.$$

The resulting aggregate investment price index P_t^I is given by

$$P_t^I = \left[\omega_i(P_t^H)^{1-\gamma_i} + (1-\omega_i)(P_t^F)^{1-\gamma_i}\right]^{\frac{1}{1-\gamma_i}}.$$

Capital producers require i_t units of investment good at a unit price of $\frac{P_t^I}{P_t}$ and incur investment adjustment costs $\Phi\left(\frac{i_t}{i_{t-1}}\right)$ per unit of investment to produce new capital goods i_t and repair the depreciated capital, which will be sold at the price q_t. Therefore, a capital producer makes an investment decision to maximize its discounted profits represented by

$$\max_{i_{t+i}} \sum_{i=0}^{\infty} E_0 \left[\Lambda_{t,t+1+i} \left(q_{t+i} i_{t+i} - \Phi\left(\frac{i_{t+i}}{i_{t+i-1}}\right) q_{t+i} i_{t+i} - \frac{P^I_{t+i}}{P_{t+i}} i_{t+i} \right) \right]. \tag{A16}$$

The optimality condition with respect to i_t produces the following Q-investment relation for capital goods:

$$\frac{P^I_t}{P_t} = q_t \left[1 - \Phi\left(\frac{i_t}{i_{t-1}}\right) - \Phi'\left(\frac{i_t}{i_{t-1}}\right) \frac{i_t}{i_{t-1}} \right] + E_t \left[\Lambda_{t,t+1} q_{t+1} \Phi'\left(\frac{i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2 \right].$$

Finally, the aggregate physical capital stock of the economy evolves according to

$$k_{t+1} = (1-\delta_t) k_t + \left[1 - \Phi\left(\frac{i_t}{i_{t-1}}\right) \right] i_t, \tag{A17}$$

with δ_t being the endogenous depreciation rate of capital determined by the utilization choice of intermediate goods producers.

A.4 Final Goods Producers

Finished goods producers combine different varieties $y^H_t(i)$, which sell at the monopolistically determined price $P^H_t(i)$, into a final good that sells at the competitive price P^H_t, according to the constant-returns-to-scale technology,

$$y^H_t = \left[\int_0^1 y^H_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{1}{1-\frac{1}{\epsilon}}}.$$

The profit maximization problem of final goods producers is represented by

$$\max_{y^H_t(i)} \; P^H_t \left[\int_0^1 y^H_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{1}{1-\frac{1}{\epsilon}}} - \left[\int_0^1 P^H_t(i) y^H_t(i) di \right]. \tag{A18}$$

The profit maximization problem, combined with the zero profit condition, implies that the optimal variety demand is

$$y_t^H(i) = \left(\frac{P_t^H(i)}{P_t^H}\right)^{-\epsilon} y_t^H,$$

with $P_t^H(i)$ and P_t^H satisfying

$$P_t^H = \left[\int_0^1 P_t^H(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}.$$

We assume that imported intermediate good varieties are repackaged via a similar technology with the same elasticity of substitution between varieties as in domestic final good production. Therefore, $y_t^F(i) = \left(\frac{P_t^F(i)}{P_t^F}\right)^{-\epsilon} y_t^F$ and $P_t^F = \left[\int_0^1 P_t^F(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$ hold for imported intermediate goods.

A.5 Intermediate Goods Producers

There is a large number of intermediate goods producers, indexed by i, who produce variety $y_t^H(i)$ using the constant-returns-to-scale production technology,

$$y_t^H(i) = A_t \Big(u_t(i) k_t(i)\Big)^{\alpha} h_t(i)^{1-\alpha}.$$

As shown in the production function, firms choose the level of capital and labor used in production, as well as the utilization rate of the capital stock. A_t represents the aggregate productivity level and follows an autoregressive process given by

$$\ln(A_{t+1}) = \rho^A \ln(A_t) + \epsilon_{t+1}^A,$$

with zero mean and constant variance innovations ϵ_{t+1}^A.

Part of $y_t(i)$ is sold in the domestic market as $y_t^H(i)$, in which the producer i operates as a monopolistically competitor. Accordingly, the nominal sales price $P_t^H(i)$ is chosen by the firm to meet the aggregate domestic demand for its variety,

$$y_t^H(i) = \left(\frac{P_t^H(i)}{P_t^H}\right)^{-\epsilon} y_t^H,$$

which depends on the aggregate home output y_t^H. Apart from incurring nominal marginal costs of production MC_t, these firms additionally face Rotemberg (1982)-type quadratic menu costs of price adjustment in the form of

$$P_t \frac{\varphi^H}{2}\left[\frac{P_t^H(i)}{P_{t-1}^H(i)} - 1\right]^2.$$

These costs are denoted in nominal terms, with φ^H capturing the intensity of the price rigidity.

Domestic intermediate goods producers choose their nominal price level to maximize the present discounted real profits. We confine our interest to symmetric equilibrium, in which all intermediate producers choose the same price level, that is, $P_t^H(i) = P_t^H \ \forall i$.

Domestic intermediate goods producers' profit maximization problem can be represented as follows:

$$\max_{P_t^H(i)} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j}\left[\frac{D_{t+j}^H(i)}{P_{t+j}}\right] \tag{A19}$$

subject to the nominal profit function

$$\begin{aligned} D_{t+j}^H(i) &= P_{t+j}^H(i) y_{t+j}^H(i) + S_{t+j} P_{t+j}^{H*} c_{t+j}^{H*}(i) - MC_{t+j} y_{t+j}^H(i) \\ &\quad - P_{t+j}\frac{\varphi^H}{2}\left[\frac{P_{t+j}^H(i)}{P_{t+j-1}^H(i)} - 1\right]^2 \end{aligned} \tag{A20}$$

and the demand function $y_t^H(i) = \left(\frac{P_t^H(i)}{P_t^H}\right)^{-\epsilon} y_t^H$. Since households own these firms, any profits are remitted to consumers and future streams of real profits are discounted by the stochastic discount factor of consumers, accordingly. Notice that the sequences of the nominal exchange rate and export prices in foreign currency $\{S_{t+j}, P_{t+j}^{H*}\}_{j=0}^{\infty}$ are taken exogenously by the firm, since it acts as a price taker in the export market. The first-order condition to this problem becomes

$$(\epsilon-1)\left(\frac{P_t^H(i)}{P_t^H}\right)^{-\epsilon}\frac{y_t^H}{P_t}=\epsilon\left(\frac{P_t^H(i)}{P_t^H}\right)^{-\epsilon-1}MC_t\frac{y_t^H}{P_tP_t^H}$$
$$-\varphi^H\left[\frac{P_t^H(i)}{P_{t-1}^H(i)}-1\right]\frac{1}{P_{t-1}^H(i)}$$
$$+\varphi^H E_t\left\{\Lambda_{t,t+1}\left[\frac{P_{t+1}^H(i)}{P_t^H(i)}-1\right]\frac{P_{t+1}^H(i)}{P_t^H(i)^2}\right\}. \tag{A21}$$

Imposing the symmetric equilibrium condition to the first-order condition of the profit maximization problem and using the definitions $rmc_t=\frac{MC_t}{P_t}$, $\pi_t^H=\frac{P_t^H}{P_{t-1}^H}$, and $p_t^H=\frac{P_t^H}{P_t}$ yields

$$p_t^H=\frac{\epsilon}{\epsilon-1}rmc_t-\frac{\varphi^H}{\epsilon-1}\frac{\pi_t^H(\pi_t^H-1)}{y_t^H}$$
$$+\frac{\varphi^H}{\epsilon-1}E_t\left\{\Lambda_{t,t+1}\frac{\pi_{t+1}^H(\pi_{t+1}^H-1)}{y_t^H}\right\}. \tag{A22}$$

Notice that even if prices are flexible—that is, $\varphi^H=0$—the monopolistic nature of the intermediate goods market implies that the optimal sales price reflects a markup over the marginal cost, that is, $P_t^H=\frac{\epsilon}{\epsilon-1}MC_t$.

The remaining part of the intermediate goods is exported as $c_t^{H*}(i)$ in the foreign market, where the producer is a price taker. To capture the foreign demand, we follow Gertler, Gilchrist, and Natalucci (2007) and impose an autoregressive exogenous export demand function in the form of

$$c_t^{H*}=\left[\left(\frac{P_t^{H*}}{P_t^*}\right)^{-\Gamma}y_t^*\right]^{\nu^H}(c_{t-1}^{H*})^{1-\nu^H},$$

which positively depends on foreign output that follows an autoregressive exogenous process,

$$\ln(y_{t+1}^*)=\rho^{y^*}\ln(y_t^*)+\epsilon_{t+1}^{y^*},$$

with zero mean and constant variance innovations. The innovations to the foreign output process are perceived as export demand shocks by the domestic economy. For tractability, we further assume that the small open economy takes $P_t^{H*} = P_t^* = 1$ as given. We think that this model feature is realistic for Turkey and other emerging market economies in general considering the observed export structure of these economies.

Imported intermediate goods are purchased by a continuum of producers that are analogous to the domestic producers except that these firms face exogenous import prices as their marginal cost. In other words, the law of one price holds for the import prices, so that $MC_t^F = S_t P_t^{F*}$. Since these firms also face quadratic price adjustment costs, the domestic price of imported intermediate goods is determined as

$$p_t^F = \frac{\epsilon}{\epsilon - 1} s_t - \frac{\varphi^F}{\epsilon - 1} \frac{\pi_t^F(\pi_t^F - 1)}{y_t^F} + \frac{\varphi^F}{\epsilon - 1} E_t \left\{ \Lambda_{t,t+1} \frac{\pi_{t+1}^F(\pi_{t+1}^F - 1)}{y_t^F} \right\}, \tag{A23}$$

with $p_t^F = \frac{P_t^F}{P_t}$, $s_t = \frac{S_t P_t^{F*}}{P_t}$, and $P_t^{F*} = 1$ $\forall t$ is taken exogenously by the small open economy.

For a given sales price, optimal factor demands and utilization of capital are determined by the solution to a symmetric cost minimization problem, where the cost function shall reflect the capital gains from market valuation of firm capital and resources that are devoted to the repair of the worn-out part of it. Consequently, firms minimize

$$\min_{u_t, k_t, h_t} q_{t-1} r_{kt} k_t - (q_t - q_{t-1}) k_t + p_t^I \delta(u_t) k_t + w_t h_t + rmc_t \left[y_t^H - A_t \left(u_t k_t \right)^\alpha h_t^{1-\alpha} \right] \tag{A24}$$

subject to the endogenous depreciation rate function,

$$\delta(u_t) = \delta + \frac{d}{1+\varrho} u_t^{1+\varrho}, \tag{A25}$$

with $\delta, d, \varrho > 0$. The first-order conditions to this problem govern factor demands and the optimal utilization choice as

$$p_t^I \delta'(u_t) k_t = \alpha \left(\frac{y_t^H}{u_t} \right) rmc_t, \tag{A26}$$

$$R_{kt} = \frac{\alpha\left(\frac{y_t^H}{k_t}\right) rmc_t - p_t^I \delta(u_t) + q_t}{q_{t-1}}, \tag{A27}$$

and

$$w_t = (1-\alpha)\left(\frac{y_t^H}{h_t}\right) rmc_t. \tag{A28}$$

A.6 Resource Constraints

The resource constraint for home goods equates domestic production to the sum of domestic and external demand for home goods and the real domestic price adjustment costs, so that

$$y_t^H = c_t^H + c_t^{H*} + i_t^H + g_t^H y_t^H + \left(p_t^H\right)^{-\gamma} \frac{\varphi^H}{2}\left(\pi_t \frac{p_t^H}{p_{t-1}^H} - 1\right)^2. \tag{A29}$$

A similar market clearing condition holds for the domestic consumption of the imported goods, that is,

$$y_t^F = c_t^F + i_t^F + \left(p_t^F\right)^{-\gamma} \frac{\varphi^F}{2}\left(\pi_t \frac{p_t^F}{p_{t-1}^F} - 1\right)^2. \tag{A30}$$

Finally, the balance of payments vis-à-vis the rest of the world defines the trade balance as a function of net foreign assets,

$$R_t^* b_t^* - b_{t+1}^* = c_t^{H*} - y_t^F. \tag{A31}$$

A.7 Definition of Competitive Equilibrium

A competitive equilibrium is defined by sequences of prices $\left\{p_t^H, p_t^F, p_t^I, \pi_t, w_t, q_t, s_t, R_{kt+1}, R_{t+1}, R_{t+1}^*\right\}_{t=0}^{\infty}$; government policies $\{r_{nt}, rr_t, M_{0t}, T_t\}_{t=0}^{\infty}$; allocations $\Big\{c_t^H, c_t^F, c_t, h_t, m_t, b_{t+1}, b_{t+1}^*, \varphi_t, l_t,$ $n_t, \kappa_t, \nu_t^l, \nu_t^*, \nu_t, i_t, i_t^H, i_t^F, k_{t+1}, y_t^H, y_t^F, u_t, rmc_t, c_t^{H*}, D_t^H, \Pi_t, \delta_t\Big\}_{t=0}^{\infty}$; initial conditions $b_0, b_0^*, k_0, m_-, n_0$; and exogenous processes $\left\{A_t, g_t^H, \psi_t, r_{nt}^*, y_t^*\right\}_{t=0}^{\infty}$ such that

(i) Given exogenous processes, initial conditions, government policy, and prices, the allocations solve the utility maximization problem of households (A5)–(A6); the net worth maximization problem of bankers (6)–(7); and the profit maximization problems of capital producers (A16), final goods producers (A18), and intermediate goods producers (A19)–(A20) and (A24)–(A25).

(ii) Home and foreign goods, physical capital, investment, security claims, domestic deposits, money, and labor markets clear. The balance of payments identity (A31) holds.

Appendix B. Impulse Responses under Domestic and Other External Shocks

Figures B1–B4 in this section present the impulse responses of real, financial, and external variables under the productivity, government spending, U.S. policy rate, and export demand shocks. For brevity, we do not explain the dynamics of model variables under each shock in detail. We note that most of the endogenous variables and the policy instruments respond to each shock in a fairly standard way, which was already extensively studied in the previous literature. In this section, we also report one-quarter-ahead and one-year-ahead variance decomposition results (see table B1 below), which were not reported in the main text.

Table B1. Variance Decomposition in the Decentralized Economy (%)

One Quarter Ahead	TFP	Gov't. Spending	Country Risk Premium	U.S. Interest Rate	Export Demand
Output	16.41	67.89	13.05	2.38	0.27
Consumption	17.69	0.12	69.81	12.36	0.03
Investment	5.11	0.00	80.86	13.98	0.05
Credit	96.45	2.54	0.77	0.12	0.11
Liability Composition (Foreign)	1.64	0.01	83.97	14.32	0.05
Loan–Domestic Deposit Spread	0.04	0.00	84.50	15.36	0.10
Loan–Foreign Deposit Spread	0.75	0.00	84.70	14.49	0.05
Real Exchange Rate	1.12	0.01	86.41	12.45	0.01
CA Balance to GDP	0.04	0.00	79.93	13.72	6.31
Trade Balance to GDP	0.18	0.25	79.87	13.73	5.98
Inflation Rate	36.25	0.21	56.00	7.55	0.00
Policy Rate	36.25	0.21	56.00	7.55	0.00
One Year Ahead					
Output	37.33	15.59	39.67	7.14	0.27
Consumption	15.98	0.10	71.23	12.66	0.03
Investment	6.17	0.00	79.89	13.90	0.05
Credit	70.40	1.40	23.71	4.19	0.31
Liability Composition (Foreign)	9.34	0.08	77.57	12.98	0.04
Loan–Domestic Deposit Spread	0.91	0.00	84.48	14.58	0.03
Loan–Foreign Deposit Spread	0.28	0.02	85.06	14.61	0.02
Real Exchange Rate	1.90	0.02	85.50	12.56	0.02
CA Balance to GDP	1.30	0.02	84.33	14.17	0.18
Trade Balance to GDP	0.94	0.02	83.12	14.46	1.45
Inflation Rate	27.13	0.14	64.14	8.60	0.00
Policy Rate	23.32	0.09	67.55	9.05	0.00

Figure B1. Impulse Response Functions Driven by a One-Standard-Deviation Increase in Productivity

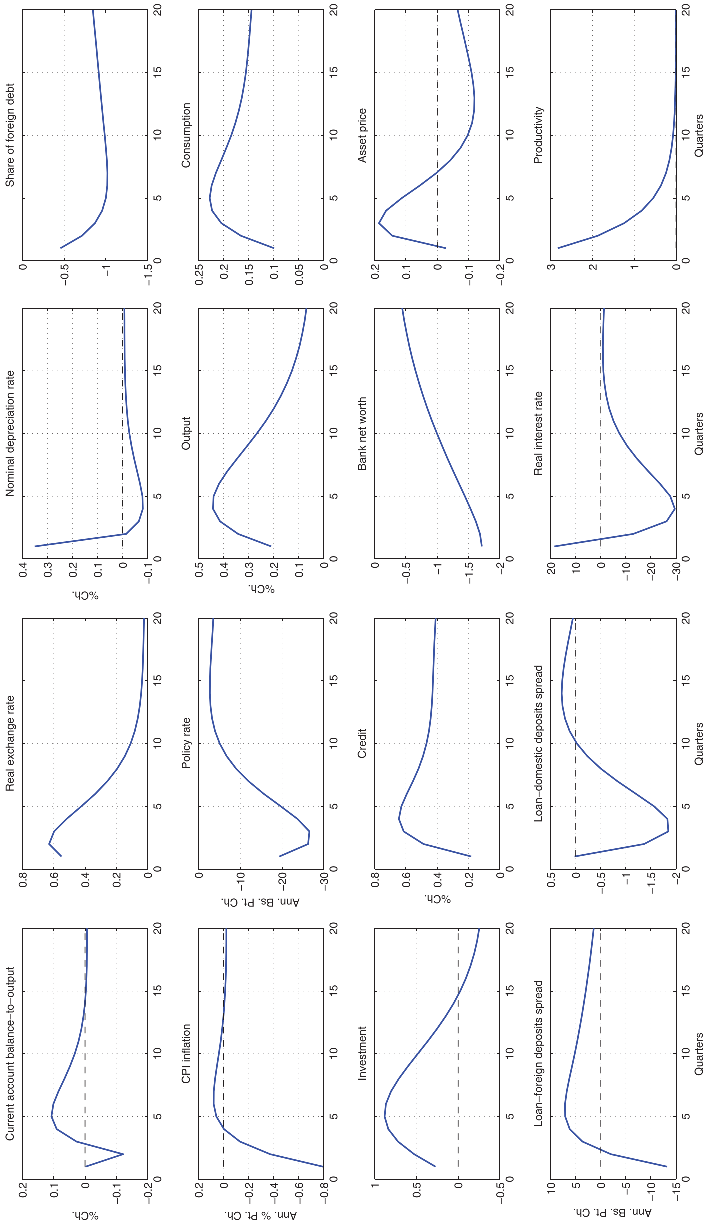


Figure B2. Impulse Response Functions Driven by a One-Standard-Deviation Increase in Government Spending

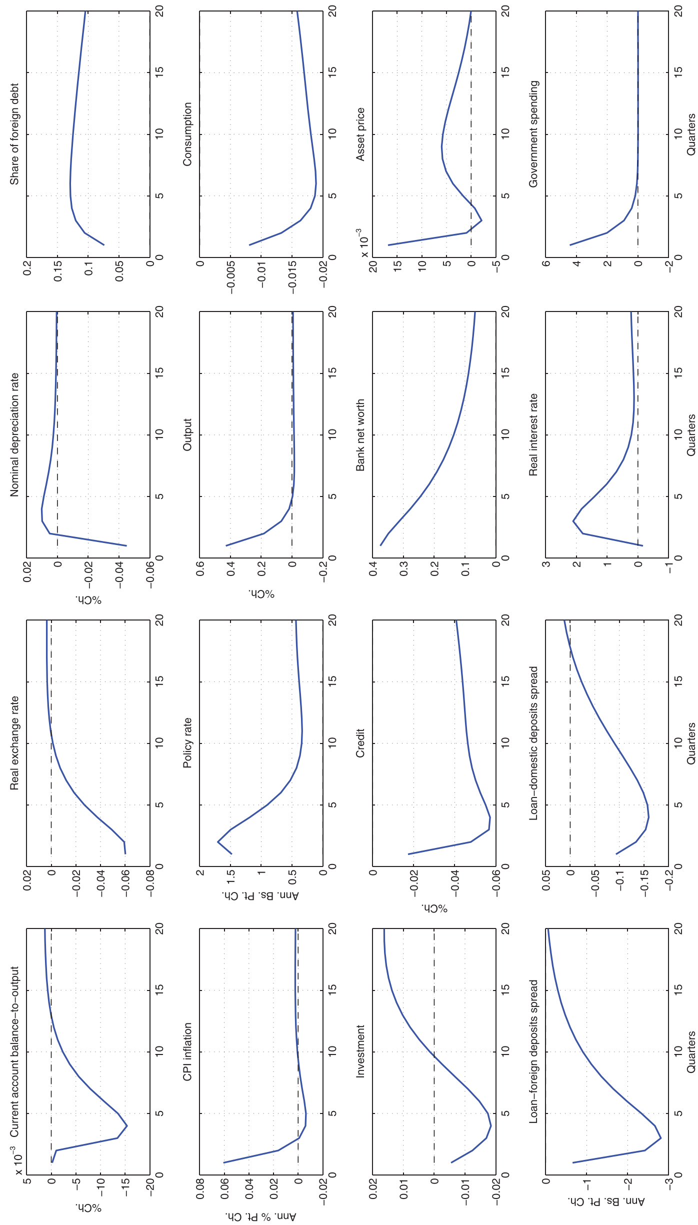


Figure B3. Impulse Response Functions Driven by a 40 Annualized Basis Point Increase in the U.S. Policy Rate

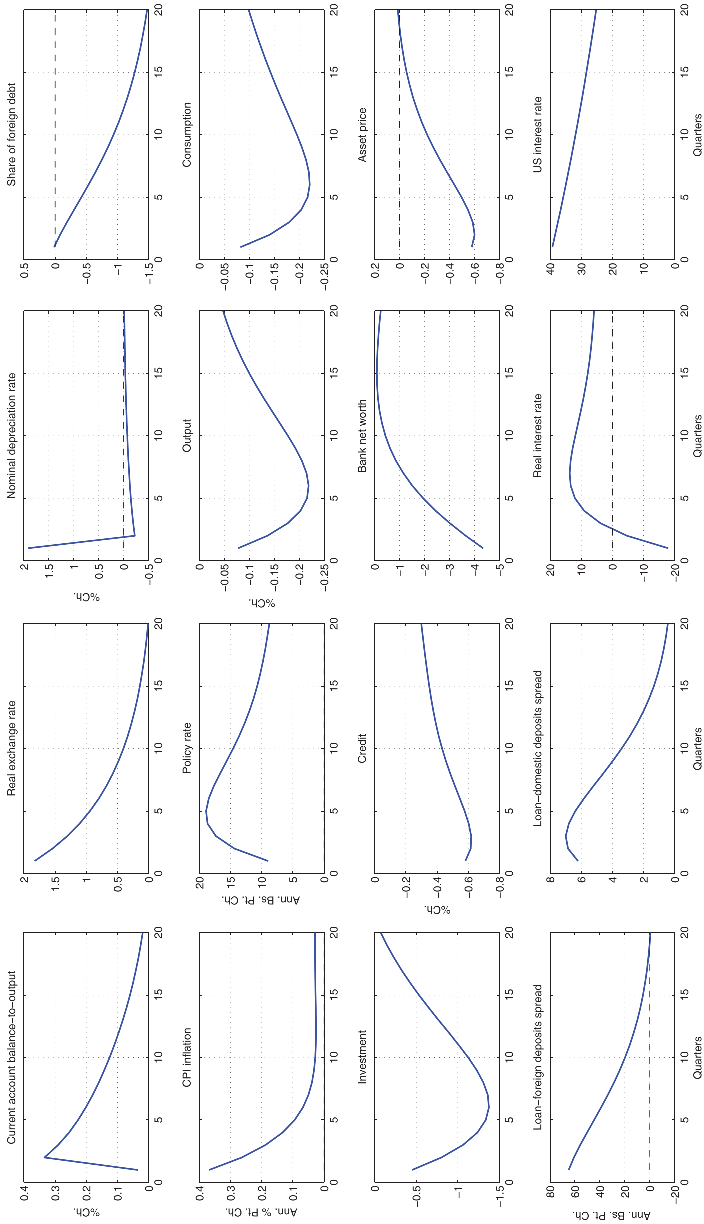
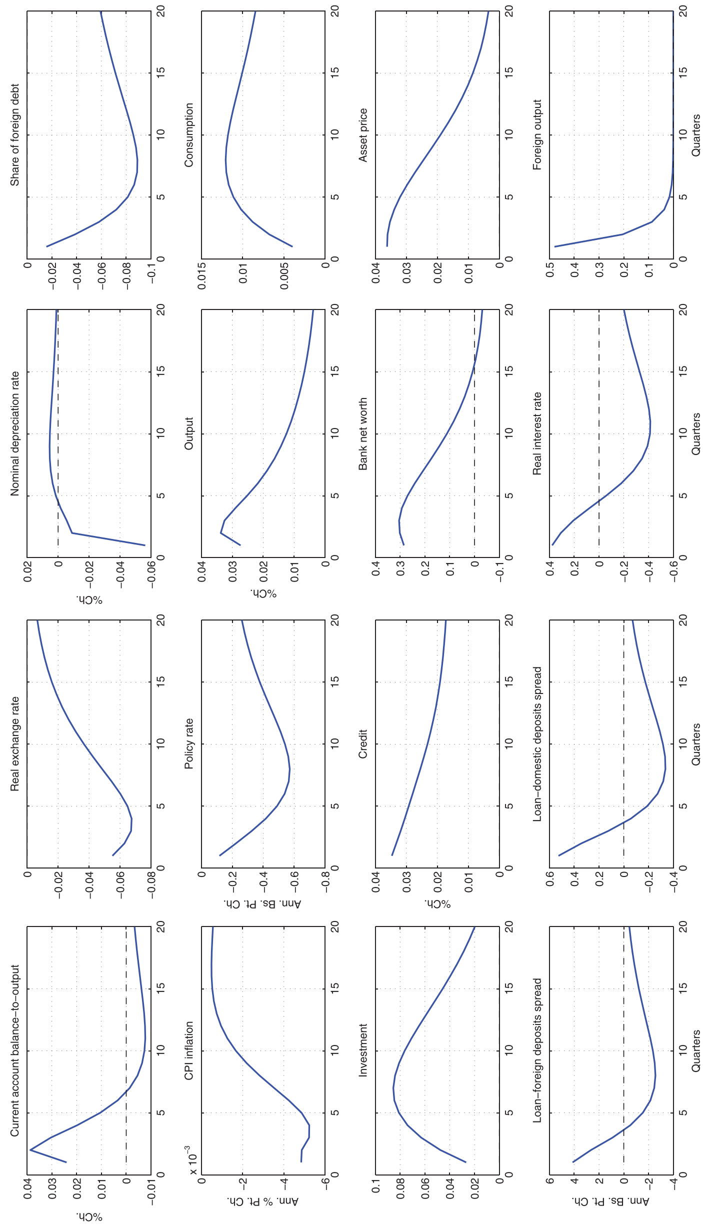


Figure B4. Impulse Response Functions Driven by a One-Standard-Deviation Increase in Foreign Output



Appendix C. Optimal Simple Rules under Domestic Shocks

Table C1 reports the response coefficients of optimized conventional and augmented Taylor rules, the absolute volatilities of policy rate, inflation, lending spread over foreign debt and the real exchange rate (RER) under these rules, and the corresponding consumption-equivalent welfare costs relative to the Ramsey-optimal policy under productivity and government spending shocks.

The optimized Taylor rules with and without smoothing feature no response to output variations under productivity shocks, which is consistent with the results of Schmitt-Grohé and Uribe (2007) in a canonical closed-economy New Keynesian model without credit frictions. The optimal simple rule with smoothing displays a large degree of inertia and a limited response to the CPI inflation. The latter result is in line with Schmitt-Grohé and Uribe (2007) in the sense that the level of the response coefficient of inflation plays a limited role for welfare and it matters to the extent that it affects the determinacy. It is also in line with Monacelli (2005, 2013) and Faia and Monacelli (2008), since open-economy features such as home bias and incomplete exchange rate pass-through may cause the policymaker to deviate from strict domestic markup stabilization and resort to some degree of exchange rate stabilization. Higher volatilities of inflation and credit spreads together with a lower volatility of asset prices compared with the Ramsey policy indicate that these two optimized Taylor rules can only partially stabilize the intratemporal and intertemporal wedges, explaining the welfare losses associated with these rules.

Under productivity shocks, optimized augmented Taylor rules suggest that a negative response to credit spreads together with a moderate response to inflation and a strong response to output deviations achieve the highest welfare possible. In response to a 100 basis points increase in credit spreads, the policy should be reduced by 150 basis points, all else equal. This policy substantially reduces the volatility of spreads in comparison with that in the Ramsey policy. Moreover, the optimized augmented Taylor rules that respond to credit, asset prices, or the real exchange rate also achieve a level of welfare very close to that implied by the spread-augmented Taylor rule. Both rules feature a lower degree of volatility of the real

Table C1. Optimal Simple Policy Rules under Domestic Shocks

TFP	ρ_{r_n}	φ_π	φ_y	φ_f	ρ_{rr}	φ_{rr}	σ_{r_n}	σ_π	σ_{spread}	σ_{RER}	σ_q	CEV(%)[a]
Optimized Taylor Rules (TR)												
Standard (without smoothing)	–	1.5721	0	–	–	–	0.2701	0.3071	0.0830	0.4714	0.4155	2.2690
Standard (with smoothing)	0.9950	1.0010	0	–	–	–	0.0015	0.2559	0.1183	0.9274	0.6568	1.3583
Optimized Augmented TR												
Credit	0.7818	1.2866	2.7857	−1.7143	–	–	0.4441	0.4697	0.1008	1.1231	1.0792	0.0010
Asset Price	0.4264	3.0000	2.7857	−3.0000	–	–	2.9289	0.7777	0.4339	4.5204	3.5392	0.0009
Credit Spread	0.9239	1.2866	1.2857	−1.5000	–	–	0.2268	0.3903	0.0234	1.3792	0.4942	0.0001
Real Exchange Rate	0.7107	1.0010	1.9286	2.7857	–	–	0.3718	0.4431	0.8391	1.1026	5.2674	0.0014
Optimized TR and RRR	0.1421	2.8572	1.0714	–	0.7107	−2.7857	0.4204	0.5282	14.5650	12.8479	4.3218	0.0008
Ramsey Policy	–	–	–	–	–	–	0.2248	0.1703	0.0623	1.5168	1.1261	0
Gov't. Spending	ρ_{r_n}	φ_π	φ_y	φ_f	ρ_{rr}	φ_{rr}	σ_{r_n}	σ_π	σ_{spread}	σ_{RER}	σ_q	CEV(%)[a]
Optimized Taylor Rules (TR)												
Standard (without smoothing)	–	1.0010	0.2143	–	–	–	0.0725	0.0427	0.0290	0.1186	0.1641	2.0672
Standard (with smoothing)	0.9950	1.0010	0.4286	–	–	–	0.0009	0.0145	0.0064	0.0949	0.0616	2.0106
Optimized Augmented TR												
Credit	0.9950	1.1438	1.5000	−0.8571	–	–	0.0089	0.2327	0.2649	0.4117	1.7141	0.0332
Asset Price	0.9950	2.4289	1.2857	−2.3571	–	–	0.0278	0.1991	0.2551	0.4078	1.6741	0.0010
Credit Spread	0.3554	1.2866	0.2143	−2.5714	–	–	0.3014	0.4829	0.0007	0.6864	0.2662	0.0147
Real Exchange Rate	0.1421	1.2866	2.3571	−2.7857	–	–	0.3445	0.3008	0.4325	1.2370	3.3088	0.0005
Optimized TR and RRR	0.9950	1.1438	0.8571	–	0.2843	−3.0000	0.0040	0.1182	0.0521	0.2786	0.8837	0.0001
Ramsey Policy	–	–	–	–	–	–	0.0144	0.0146	0.0060	0.0961	0.0508	0

[a]The reported welfare figures include both long-run and dynamic costs.

exchange rate in comparison with that in the Ramsey policy. We also observe that it is optimal to negatively respond to bank credit under productivity shocks. In addition, comparing volatilities of key variables under these augmented Taylor rules displays the nature of the policy tradeoffs that the central bank faces. For instance, although the spread-augmented rule features a lower volatility of the credit spreads relative to the Ramsey policy as can be expected, it displays a much higher volatility in the CPI inflation rate. In addition, the optimized RER-augmented rule features a lower volatility in the real exchange rate as expected, but it displays much larger variations in the inflation rate and the credit spread against the Ramsey-optimal policy.

Optimized augmented Taylor rules under the government spending shock suggest similar results in general when compared to those under the TFP shock, except the following. In response to this domestic demand shock which pushes inflation and output in the same direction, the optimized standard Taylor rules with and without smoothing display a positive response to output. Moreover, the best policy is to respond to the RER instead of credit spreads in the case of the TFP shock.

References

Faia, E., and T. Monacelli. 2008. "Optimal Monetary Policy in a Small Open Economy with Home Bias." *Journal of Money, Credit and Banking* 40 (4): 721–50.

Galí, J., and T. Monacelli. 2005. "Monetary Policy and Exchange Rate Volatility in a Small Open Economy." *Review of Economic Studies* 72 (3): 707–34.

Gertler, M., S. Gilchrist, and F. Natalucci. 2007. "External Constraints on Monetary Policy and the Financial Accelerator." *Journal of Money, Credit and Banking* 39 (2/3): 295–330.

Monacelli, T. 2005. "Monetary Policy in a Low Pass-Through Environment." *Journal of Money, Credit and Banking* 37 (6): 1047–66.

———. 2013. "Is Monetary Policy in an Open Economy Fundamentally Different?" *IMF Economic Review* 61 (1): 6–21.

Rotemberg, J. J. 1982. "Sticky Prices in the United States." *Journal of Political Economy* 90 (6): 1187–1211.

Schmitt-Grohé, S., and M. Uribe. 2007. "Optimal Simple and Implementable Monetary and Fiscal Rules." *Journal of Monetary Economics* 54 (6): 1702–25.