

# A Shadow Policy Rate to Calibrate U.S. Monetary Policy at the Zero Lower Bound\*

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The recent global financial crisis, the Great Recession, and the subsequent implementation of a variety of unconventional policy measures have raised the issue of how to correctly measure monetary policy when short-term nominal interest rates reach the zero lower bound (ZLB). In this paper, we propose a new “shadow policy rate” for the U.S. economy, using a large set of data representing the various facets of the U.S. Federal Reserve’s policy actions. We document that our shadow rate tracks the effective federal funds rate very closely before the crisis. More importantly, it provides a reasonable gauge of monetary policy when the ZLB becomes binding. This facilitates the assessment of U.S. monetary policy stance against familiar Taylor-rule benchmarks. Finally, we show that in structural vector autoregressive (VAR) models, the shadow policy rate helps identify monetary policy shocks that better reflect the Federal Reserve’s unconventional policy measures.

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## 1. Introduction

Following the recent global financial crisis and the onset of the Great Recession, central banks of major advanced economies quickly reduced policy rates close to zero and have since implemented a growing variety of unconventional policy measures, referred to by some as “quantitative easing” (QE), with the objective of further easing monetary conditions and restoring credit flows.<sup>1</sup> A main staple of such unorthodox measures has been the large-scale asset purchases, as well as a significant lengthening of the maturity of central bank asset holdings. In the current low-inflation environment and with policy rates practically stuck at the zero lower bound (ZLB) in most major advanced economies, it has become very difficult for central banks and market participants to assess accurately the monetary policy stance. Given the nature and diversity of recent central bank balance sheet policies, no single indicator is seen as a consistent representation of monetary policy stance in both the pre- and post-crisis periods.

In the pre-recession period, there was a growing consensus in both academia and the central banking community that a short-term interest rate such as the federal funds rate would provide a good measure of U.S. monetary policy, as well as the most relevant policy instrument.<sup>2</sup> Bernanke and Blinder (1992) conclude that the federal funds rate is “extremely informative about future movements of real macroeconomic variables” and is a “good indicator of monetary policy actions” that is “mostly driven by policy decisions.” Therefore, it became commonplace to use overnight rates to proxy monetary policy in macroeconomic models, as well as to use shocks to them to study the transmission and the ultimate effects of monetary policy.<sup>3</sup> But this approach would obviously produce misleading results once the ZLB becomes binding, when such overnight rates lose their

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<sup>1</sup>For ease of exposition, we use the terminologies “quantitative easing,” “central bank balance sheet policy,” “unconventional monetary policy,” and “asset purchase programs” interchangeably wherever the circumstances are clear.

<sup>2</sup>Earlier discussions on the most appropriate measure of monetary policy focused on monetary aggregates; see, e.g., Havrilesky (1967) and Froyen (1974). However, monetary aggregates are likely to be affected by other endogenous factors.

<sup>3</sup>See, for example, Bernanke and Blinder (1992), Christiano, Eichenbaum, and Evans (1996), or Kim (1999).

information content and non-standard measures are implemented to provide additional stimulus.

Still, a precise and consistent measure of U.S. monetary policy is crucial for analyzing the effectiveness of QE measures, i.e., their impact on economic activity, and for calibrating further policy measures. As Romer and Romer (2004) suggested, “the accuracy of estimates of the effects of monetary policy depends crucially on the validity of the measure of monetary policy that is used.” This is borne out in recent policy and academic debates: as the transmission channels of unconventional policies are not yet well understood, the lack of one single and consistent policy indicator constitutes a major hurdle.<sup>4</sup> In addition, in the absence of proper quantification of the size of stimulus provided by today’s unconventional policies, it would be hard to answer the question of whether the current policy stance is appropriate, too tight, or too loose.

Lacking such a measure, most of the recent attempts to gauge unconventional policies resorted to the impact of announcements and asset purchases on financial market prices, with a special focus on the term structure of interest rates. Among others, Meaning and Zhu (2011, 2012) used changes in the size and maturity of the Federal Reserve asset holdings to estimate the impact of central bank asset purchases on the yield curve. Along similar lines, Chadha, Turner, and Zampolli (2013) examined the effects of government debt maturity on long-term interest rates. Such measures may indeed be useful for gauging monetary policy and assessing its impact during times of unconventional policies, but they typically changed little before the ZLB on nominal interest rates became binding. Without an indicator that is consistent over a long period of time, it becomes difficult to quantify the effects of unconventional policies against historical benchmarks. Consequently, much of the recent empirical work has taken the event-study approach by measuring financial market responses to QE announcements.<sup>5</sup>

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<sup>4</sup>Woodford (2012) discusses various unconventional policies, including forward guidance and asset purchases. Cúrdia and Woodford (2011) provide a model-based assessment of the balance sheet of the central bank as an instrument of monetary policy.

<sup>5</sup>See, for example, Meaning and Zhu (2011) and references therein. Notable exceptions are Peersman (2011), Chen et al. (2012, 2016), and Gambacorta, Hofmann, and Peersman (2014), who attempt to pin down unconventional monetary policy shocks in a structural or global VAR framework.

Still, having a single measure of monetary policy that could successfully capture non-standard policy actions at the ZLB and remain consistent when the ZLB is no longer binding would enable the use of all existing modeling devices introduced in the pre-ZLB era, as well as a comparative assessment of the effectiveness of conventional versus unconventional measures.

A popular approach to the construction of such a measure is that of shadow rates. The concept of shadow rate was first introduced in a seminal paper by Black (1995), and corresponds to the (unobservable) nominal interest rate that would have prevailed had the ZLB not been binding. In a nutshell, shadow rates à la Black (1995) are based on a certain model for the term structure of interest rates, whereby the long end of the yield curve determines the behavior of shorter-term rates that hit the ZLB.

Shadow rates à la Black (1995) were already employed in the 2000s to track monetary policy in Japan, where the uncollateralized overnight call rate faced the ZLB. Gorovoi and Linetsky (2004) model the shadow rate as a Vasicek process and apply it to the Japanese government bonds (JGB) data, reporting a good fit of the Japanese term structure. Ueno, Baba, and Sakurai (2006) examine the Bank of Japan (BoJ) zero interest rate policy during the quantitative easing between 2001 and 2006, reporting a negative estimate of the policy rate throughout the period.<sup>6</sup>

In the wake of the Great Recession, as the ZLB became an issue for several central banks other than the BoJ, shadow rates have seen their popularity grow, and a number of different implementations of the original Black (1995) approach have appeared. Krippner (2012, 2013a) adds an explicit function of maturity to the shadow rate forward curve; this leads to more tractable models with closed-form solutions. Bauer and Rudebusch (2016) propose an implementation of the pricing kernel based on simulations, and suggest that shadow rate models could be more informative on monetary policy expectations than standard dynamic term structure models that ignore the ZLB. Wu and Xia (2016) construct an analytical approximation

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<sup>6</sup>More recently, Kim and Singleton (2012) find that their two-factor affine, quadratic-Gaussian, and shadow rate models capture some key features of JGB data and that the shadow rate models outperform other models in terms of the fit to realized excess returns.

for the forward rates to make the model tractable, and they show that the effects of their shadow rate on macroeconomic variables are similar to those of the federal funds rate.

While all these approaches build on the same theoretical framework—a certain breed of term structure model—they often provide very different results in practice.<sup>7</sup> The reason is that implementation choices that may appear rather innocuous, like the number of factors to be used in modeling the term structure, or the type of approximations chosen, play a big role. Christensen and Rudebusch (2015), for example, show that estimates of the shadow rate are very sensitive to model specifications.<sup>8</sup> Krippner (2015) also investigates the robustness of shadow rates with respect to the number of factors, showing that a two-factor structure should be preferred over the three-factor one employed by Wu and Xia (2016). A similar point is made in Krippner (2013b), which provides an alternative indicator of monetary conditions and shows that it is more robust to model specification compared with other shadow rates à la Black (1995).<sup>9</sup>

On top of this sensitivity, an embedded and somewhat hidden assumption of shadow rates à la Black (1995) is that every unconventional monetary policy action only matters to the extent that it affects the term structure of government bond yields, especially its long end. This may have undesired effects: the early easing programs deployed by the Federal Reserve (e.g., the purchases of mortgage-backed securities) did actually boost the size of its balance sheet and arguably provided sizable monetary stimulus through portfolio rebalancing and providing relief to banks overburdened with toxic assets on their balance sheets. Yet they had a rather limited impact on the longer-term government bond yields compared with asset purchases explicitly targeting U.S. Treasury securities.<sup>10</sup> This

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<sup>7</sup>See section 3.2 for a comparison.

<sup>8</sup>For instance, the shadow market rates à la Black (1995) estimated by Krippner (2012, 2013a) and Wu and Xia (2016) are significantly different for the period when the ZLB on nominal interest rates is binding.

<sup>9</sup>Krippner (2013b) constructs his “effective monetary stimulus” by aggregating the current and estimated expected path of interest rates, and shows that it is consistent and comparable across conventional and unconventional monetary policy frameworks.

<sup>10</sup>The Wu-Xia (2016) estimates indeed point to a shadow rate estimate above zero until early 2009 (see figure 6).

should not necessarily imply that purchases of assets other than U.S. Treasury securities would provide less monetary accommodation.

We aim at constructing a monetary policy indicator, in the form of a “shadow policy rate,” but our approach takes a very different, essentially statistical perspective. Instead of tying our indicator to a specific term structure model, we rely on a comprehensive data set with a much wider range of variables that could potentially reflect most, if not all, monetary policy actions, and we let the data speak while selecting the best econometric specification.<sup>11</sup> More precisely, after pooling together a data set of variables that are closely associated with monetary policy measures, both conventional and unconventional, we summarize their information content using a dynamic factor model, where the estimated factors represent different aspects of monetary policy. Finally, we use these factors to reconstruct a shadow policy rate by treating the federal funds rate as unobserved after it hit the ZLB. The resulting shadow policy rate can be consistently applied in the pre- and post-ZLB periods.

We illustrate this using two standard monetary VAR models, and show that it is possible to measure monetary policy shocks with a shadow policy rate and to study the impact and transmission of QE measures. More importantly, the shadow rate allows us to examine to what extent various unconventional monetary policy measures have managed to fill the “policy gap” that opened between the federal funds rate when it reached the ZLB and the levels suggested by the rules of Taylor (1993, 1999), Ball (1999), and Yellen (2012). We find that policymakers have been reasonably successful in trying to achieve the prescribed Taylor rates with QE measures. We also use this approach to evaluate Bullard’s (2012, 2013) assessment of U.S. monetary policy stance.

Our estimates lie within the range of shadow interest rates provided by the term structure models à la Black (1995) described above. But given that our shadow policy rate is strongly and directly influenced by the changes in the Federal Reserve’s balance sheet, it better reflects the various aspects of U.S. quantitative easing, including the Federal Reserve’s large-scale purchases of asset-backed

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<sup>11</sup> Johannsen and Mertens (2016) employ a different econometric methodology to estimate a shadow rate. Babecká Kucharčuková, Claeys, and Vašíček (2016) build on our work and apply it to the euro area.

securities. Our approach precludes an explicit theoretical structure and is purely data driven. In our view, allowing for a more direct and explicit role for the size and the composition of the central bank balance sheet is a step in the right direction to better understand the economics of the different balance sheet policies that have been deployed by the Fed.

The rest of the paper is structured as follows. In the next section, we give a detailed account of our empirical strategy, as well as the set of monetary variables we employ. In section 3, we present our shadow policy rate and investigate its properties through several robustness checks. We also examine the evolution of the estimated factors and relate these to different aspects of U.S. monetary policy. In section 4, we evaluate the overall monetary policy stance by comparing the shadow policy rate with the levels of federal funds rate prescribed by alternative Taylor rules, so as to assess the extent to which the policy gap opened between the actual federal funds rate and benchmark Taylor-rule levels since the crisis has been filled by unconventional measures. We then use two standard monetary VAR models to study the properties of monetary policy shocks based on the estimated shadow policy rate in the post-crisis period. Section 5 concludes.

## 2. Measuring Monetary Policy at the Zero Lower Bound

### 2.1 A Factor-Based Shadow Interest Rate

Measuring monetary policy and estimating its effects on economic activity is one of the most active research topics in macroeconomics. An accurate indicator of monetary policy that can be easily computed in real time is essential for gauging and calibrating current policy stance: Is it appropriate or neutral? Too tight or too loose? Is policy adjustment needed and, if so, how much? A good indicator also allows central banks to better measure policy effects on financial markets and real activity, and to better design and implement monetary policy.

To construct a reliable indicator of the overall stance of monetary policy that ideally would work in both conventional and unconventional monetary policy environments, we interpret monetary policy as an unobserved variable, which one can estimate using a variety

of methods. This idea is not new and dates back to Avery (1979). He interprets monetary policy as a “single dimensioned unobserved variable” and estimates an “index of monetary policy” by extracting a common factor out of real and monetary variables. Since then, other synthetic indicators of monetary policy have been proposed. The Divisia index of money proposed by Barnett (1980), for example, weighs different assets by the value of the monetary services they provide. Barnett, Fisher, and Serletis (1992) show that the Divisia index predicts GNP at least as well as basic monetary aggregates such as M1 and M2. The Federal Reserve Bank of St. Louis (see, e.g., Thornton and Yue 1992) has developed a monetary services index (MSI) based on such monetary aggregation theory. Rotemberg, Driscoll, and Poterba (1995) find that a utility-based currency-equivalent aggregate proposed by Rotemberg (1991) remains valid even as asset characteristics change and predicts output movements better than conventional monetary aggregates.

Our approach also aims at providing a synthetic indicator of monetary policy. Rather than producing a complicated index hard to understand and interpret, we map the monetary data reflecting all different facets of policy onto the simple metric of a policy interest rate, using estimated dynamic factors. The resulting index can therefore be directly compared to the federal funds rate—the Federal Reserve’s operational target and instrument of choice. It is in this sense that we name our monetary policy indicator a shadow rate. But unlike the shadow rates derived from Black (1995), our indicator takes an entirely different, essentially statistical approach. We let the data speak for itself, so our shadow policy rate is model free in the sense that it does not depend on any specific term structure model, or indeed any formal economic model. Furthermore, our approach uses as much information as possible on monetary operations. The resulting shadow rate estimate thus reflects information contained in interest rates, monetary aggregates, reserves, and the Federal Reserve’s asset holdings, and not only the yield curve.

To construct our shadow rate, we first pool together a data set comprising variables that are closely associated with different types of monetary policy operations. Then, we estimate a dynamic factor model based on the data set up until the time when the effective federal funds rate hits the zero lower bound, and select an optimal

model specification (based on statistical criteria) for the number of factors and the number of lags.

Next, we treat the federal funds rate as missing, and rerun the dynamic factor model to obtain the shadow rate series when the ZLB becomes binding. The “missing” elements are replaced by their best estimates given the evolution of the observed series. In fact, our shadow federal funds rate is a “weighted average” of all monetary information contained in the original data set, with weights determined on the basis of the historical correlations of the federal funds rate with the other variables. In other words, we map changes in all other monetary policy variables onto a single shadow federal funds rate, based on the estimated historical relationships. As such, the shadow rate indicates how the funds rate would have behaved had the policymakers been able to drive it negative, providing a federal funds rate equivalent for the unconventional measures implemented so far.<sup>12</sup>

## 2.2 A Dynamic Factor Model with Missing Observations

Dynamic factor models are useful in the analysis of very large data sets: they reduce the data dimension by extracting a small number of common components out of a large amount of available information. The common components, or factors, are chosen in such a way as to maximize the proportion of total variability of the data set they can explain.

Let  $X_{1:T}$  be an  $N$ -dimensional multiple time series with  $T$  observations, some of which are missing. We write its factor representation as

$$X_t = \Lambda F_t + e_t, \quad e_t \sim N(0, R), \quad (1)$$

where  $F_t$  is an  $r \times 1$  vector of factors,  $\Lambda$  is the  $N \times r$  matrix which contains the factor loadings, and the errors  $e_t$  are idiosyncratic

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<sup>12</sup>The approach is similar in spirit to Bernanke and Mihov (1998). They construct a VAR model in which they include the federal funds rate and borrowed and non-borrowed reserves to measure monetary policy. Their indicator of the overall policy stance is a linear combination of these variables, with weights being based on the VAR parameter estimates. However, Bernanke and Mihov’s approach cannot be directly employed to account for unconventional measures: the relevant variables would be far too many to be included in a single monetary VAR.

components orthogonal to the factors  $F_t$ ; their covariance matrix is assumed to be diagonal.<sup>13</sup> The factors  $F_t$  are unobserved and must be estimated. We assume that the common factors follow a VAR process of order  $p$ :

$$F_t = \sum_{i=1}^p A_i F_{t-i} + u_t, \quad u_t \sim N(0, Q) \quad (2)$$

so that the resulting dynamic factor model can be written in the state-space form. Let  $\Theta = (\Lambda, A, R, Q)$  be a vector of the unknown parameters, where  $A$  is a vector stacking all  $A_i$ 's,  $i = 1, \dots, p$ ; then the log-likelihood function takes the form

$$\begin{aligned} \ell(X_{1:T}, F_{1:T}, \Theta) &= k - \frac{T}{2} \log |Q| - \frac{1}{2} \sum_{t=1}^T \left( F_t - \sum_{i=1}^p A_i F_{t-i} \right)' \\ &\quad \times Q^{-1} \left( F_t - \sum_{i=1}^p A_i F_{t-i} \right) \\ &\quad - \frac{T}{2} \log |R| - \frac{1}{2} \sum_{t=1}^T (X_t - \Lambda F_t)' R^{-1} (X_t - \Lambda F_t). \end{aligned} \quad (3)$$

The log-likelihood function (3) can in normal circumstances be evaluated using the Kalman filter, and maximized to obtain estimates of the unknown parameters (see Engle and Watson 1981; Doz, Giannone, and Reichlin 2011).

Yet evaluating the likelihood function (3) is not possible when the data matrix  $X_{1:T}$  in (1) has missing entries. To overcome this problem, Baíbura and Modugno (2014) propose the use of the generalized expectation maximization (EM) algorithm of Dempster, Laird, and Rubin (1977).<sup>14</sup>

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<sup>13</sup>Note that this assumption corresponds to an exact factor model. While in practice the error components may not be orthogonal to each other and can contain residual correlations that are not explained by the factors, Doz, Giannone, and Reichlin (2012) show that in the presence of weak cross-correlations the estimation of factors is still consistent. Baíbura and Modugno (2014) also provide algorithms to deal with serially correlated error terms.

<sup>14</sup>Watson and Engle (1983) first suggested the use of the EM algorithm for the estimation of unobserved-components models without missing entries.

The EM algorithm proceeds as follows. First, one substitutes the missing entries in  $X_{1:T}$  with arbitrary initial values  $z^{(0)}$  and constructs the matrix  $\tilde{X}_{1:T}^{(0)}$ , which is subject to the standard treatment of unobserved-components models. It is therefore possible to apply the Kalman filter, based on an arbitrary initial parameter vector  $\Theta^{(0)}$ , on  $\tilde{X}_{1:T}^{(0)}$  to filter out the unobservable factors. More precisely, the Kalman filter provides the expected value of the latent factors, conditional on the available observations and  $z^{(0)}$ :

$$\hat{F}_t^{(0)} = E_{\hat{\Theta}^{(0)}} \left[ F_t | \tilde{X}_{1:T}, z^{(0)} \right].$$

This allows evaluating and maximizing of the likelihood function—which also turns out to be conditional on the arbitrary starting values  $z^{(0)}$ —to produce a first estimate of the parameter vector  $\hat{\Theta}^{(1)}$ . This is sometimes referred to as the initialization step.

One can then replace the initial guess for the missing observations  $z^{(0)}$  with their expected values, which are obtained by evaluating (1) at the parameter estimates  $\hat{\Theta}^{(1)}$ ; this is known as the expectations step. Equivalently, this amounts to computing the expected value of the likelihood, conditional on the available data  $\tilde{X}_{1:T}$ . This can be written as

$$\ell \left( \Theta, \hat{\Theta}^{(1)} \right) = E_{\hat{\Theta}^{(1)}} \left[ \ell \left( X_{1:T}, F_{1:T}, \Theta \right) | \tilde{X}_{1:T} \right].$$

The expectations step produces a new guess for the missing observations  $z^{(1)}$ , which enables the construction of a new full data matrix  $X_{1:T}^{(1)}$ . We apply the Kalman filter again and maximize the likelihood function to obtain  $\hat{\Theta}^{(2)}$  in the maximization step:

$$\hat{\Theta}^{(2)} = \arg \max_{\Theta} \ell \left( \Theta, \hat{\Theta}^{(1)} \right).$$

The process is iterated until convergence at the  $j$ -th iteration (i.e., until the distance between  $\hat{\Theta}^{(j)}$  and  $\hat{\Theta}^{(j-1)}$ , becomes negligible), yielding a vector of parameter estimates  $\hat{\Theta}^*$ . Conditional on  $\hat{\Theta}^*$ , one can run once again the Kalman filter and obtain the moments of the latent factors, notably their expected value:

$$\hat{F}_t^* = E_{\hat{\Theta}^*} \left[ F_t | \tilde{X}_{1:T} \right].$$

One can then obtain the matrix of smoothed observations by applying the measurement equation (1):

$$\hat{X}_t = E_{\hat{\Theta}^*} \left[ X_t | \tilde{X}_{1:T} \right] = \hat{\Lambda}^* \hat{F}_t^*.$$

The entries of this matrix that were originally missing from the observed data set are therefore replaced by their expected values, conditional on the estimated parameters and latent factors. Hence this approach provides also estimates of the missing observations, which we denote as  $\hat{Z}^*$ .

### *2.3 Data and Variables*

We first pin down an appropriate set of variables that should provide useful information on U.S. monetary policy and contribute to the estimation of our factor-based shadow federal funds rate. The variables are selected based on their close association with the Federal Reserve's policy actions. For instance, the implementation of monetary policy via changes in the federal funds rate or large-scale asset purchase programs would both lead to changes in the size and composition of the Federal Reserve's asset holdings. As in Woodford (2012), we start with the basic blocks of interest rates and monetary aggregates, and then include other variables that could reflect a wide range of unconventional monetary policy measures.

We construct the monetary policy data set based on the following four major building blocks.

#### **Block 1. Interest Rates:**

- Effective federal funds rate (FFR)
- Rates of U.S. Treasury bills with maturities of one, three, and six months
- Yields of U.S. Treasury bonds with maturities of one, two, five, ten, and twenty years
- Overnight indexed swap (OIS)—three-month LIBOR spread

The different interest rates we include in block 1 reflect the Federal Reserve's policy actions, which affect the future interest rate path and the entire yield curve. The OIS spread provides information

on the market expectations of the federal funds rate. The interest rate block is therefore also likely to contain useful information on the Federal Reserve's forward guidance, broadly defined as the central bank's communications about its future policy intentions.

### **Block 2. Monetary Aggregates:**

- Monetary base or M0
- M1, M2, and Money Zero Maturity (MZM) money stock of the Federal Reserve Bank of St. Louis

Besides the federal funds rate, the various monetary aggregates included in block 2 are traditional monetary policy indicators. Monetary analysis based on monetary aggregates remains a cornerstone of policymaking in a number of central banks.

### **Block 3. Federal Reserve Balance Sheet (Assets):**

- Total assets
- Total Federal Reserve securities held outright
- Average maturity of Federal Reserve securities held outright
- Percentage of long-term U.S. Treasury securities (<5 years, <10 years, >10 years)

### **Block 4. Federal Reserve Balance Sheet (Liabilities):**

- Reserves: Total, excess and required reserve balances

Blocks 3 and 4 focus, respectively, on the asset and liability sides of the Federal Reserve's balance sheet, which provide important information on a wide range of the central bank's unconventional measures, especially the large-scale asset purchases and maturity extension programs. While the policy rate can be considered as the price of the reserves commercial banks hold with a central bank, it does not incorporate the information of special lending programs or the size changes and maturity transformation resulting from the central bank's large-scale asset purchases (and sales), especially at the ZLB. In fact, a central bank's balance sheet contains useful quantity information on almost all its monetary operations.

Our sample of monetary data for the United States ranges from January 1970 to June 2016. We use monthly data for our analysis, as we think they better reflect major monetary policy changes. The Federal Reserve holds eight Federal Open Market Committee (FOMC) meetings per calendar year, and some significant changes may also be adopted between these meetings, particularly in a crisis period. A monetary policy indicator constructed at quarterly frequency seems to be inadequate and untimely for our purpose of reporting the evolution of monetary policy. On the other hand, historically, we do not observe frequent major policy changes occurring at higher-than-monthly frequencies, so we do not consider weekly or biweekly estimates even in cases where data are available.<sup>15</sup>

## 2.4 *Estimation Results*

Based on the Baíbura-Modugno (2014) algorithm described in section 2.2, we estimate a dynamic factor model using the data set described in the previous subsection. Since the federal funds rate and other short-term interest rates have become practically constrained by the ZLB since late 2008, they have largely lost their information content, especially on further monetary policy actions.<sup>16</sup> Other variables, especially those directly related to the implementation of unconventional measures, might have taken over this role. Reflecting this, we treat the short-term interest rates as missing series once they drop to the proximity of the ZLB.<sup>17</sup> We treat the federal funds

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<sup>15</sup>Decisions on the federal funds rate are usually made by the FOMC in the eight regularly scheduled meetings each year, and policy changes tend to be less frequent in normal times. In the most recent three U.S. recession episodes, the federal funds rate was lowered fourteen times from October 29, 1990 to December 20, 1991; eleven times from January 3 to December 11, 2001; and ten times from September 18, 2007 to December 16, 2008. More frequent changes in the federal funds rate can be found in earlier decades. For example, the federal funds rate was raised seven times between August 16 and September 29, 1978, but this was rare.

<sup>16</sup>The federal funds rate target was reduced to 0–0.25 percent on December 16, 2008, when the effective funds rate dropped to 0.16 percent.

<sup>17</sup>In practice, the observed market rates usually stay above the ZLB, so one has to make a decision on the threshold below which rates are taken as constrained by the ZLB. We take a descriptive approach and look for substantial dips near ZLB followed by leveling-off of the rates. We also experiment with an alternative approach, taken by Wu and Xia (2016), to set an arbitrary threshold of, say, 25 basis points; the results are virtually unchanged.

rate and the three-month and six-month Treasury-bill rates as missing since December 2008, when they hit lows of 0.16 percent, 0.03 percent, and 0.26 percent, respectively. The one-year and two-year Treasury yields are assumed to be missing since November 2009, when they reached lows of 0.31 percent and 0.80 percent, respectively. The yields of Treasury securities of one-year and two-year maturities were low and could be considered to be close to zero for policy purposes, taking into account the term and risk premiums.

As the input series need to be stationary, we use the year-on-year rates of change for the quantity variables in blocks 2, 3, and 4. Admittedly, this apparently innocuous modeling choice may suggest that our shadow rate reflects more the rates of change rather than absolute sizes of the balance sheet items. In other words, a growing balance sheet will translate into a reduction in the shadow rate, whereas a large but static balance sheet will not.<sup>18</sup>

One critical issue in the estimation of the dynamic factor model is how to correctly select the “optimal” number of factors to adequately represent the underlying data set. The use of the Hallin and Liska (2007) criterion suggests that eight factors would be appropriate in our empirical analysis.<sup>19</sup> In fact, these factors can explain 90.5 percent of the total variance of the variables included in our data set, just above the commonly used 90 percent rule of thumb for lag selection. Another choice we need to make is the optimal number of lags  $p$  in the estimated dynamic factor model, and applying the Schwarz information criterion (SIC) yields two lags. Both the number of factors and the number of lags are selected based on the pre-crisis data sample, so as to ensure that the selection of the model structure is based on a sample in which all variables and, consequently, their joint dynamics are fully observed.

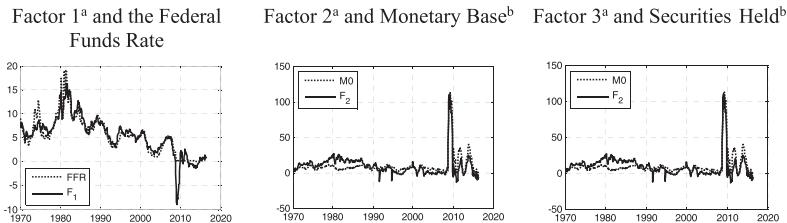
We report the parameter estimates of equations (1) and (2) in tables 1 and 2, respectively, in the appendix. In terms of the factor dynamics, the diagonal coefficients on the  $A$  matrices in (2) reveal

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<sup>18</sup>One alternative approach is to use differences of the interest rates and then back out the levels. By doing so, our focus moves towards the size of the balance sheet items rather than their changes (e.g., new purchases). Our choice reflects the fact that large balance sheets will stay for an extended period of time, likely well after the liftoff of policy rates.

<sup>19</sup>We analyze and discuss the robustness of our results to alternative selection criteria for the optimal number of factors as well as the lag order in section 3.1.

### Figure 1. Factors and Observed Variables



<sup>a</sup>The factors are rescaled (and in the case of factors 1 and 3, multiplied by -1) to match observed series.

<sup>b</sup>Year-on-year rate of change.

<sup>c</sup>Year-on-year rate of change of the outstanding amount of Treasury securities held by the Federal Reserve.

a strong degree of persistence, apparently near unit roots, especially for the first factor. Examining the factor loadings matrix  $\Lambda$  reveals the observed variables that most likely contribute to the factor dynamics: unsurprisingly, the first factor is mainly driven by the series contained in the interest rates block. The strong persistence of the first factor is hence a reflection of the inherent persistence of the interest rate series. The second factor seems more strongly associated with the monetary base, and the third factor with the size of the balance sheet. Factor 4 appears to be strongly linked with M1 and M2, while factor 5 seems to capture the dynamics of the maturity structure of the Federal Reserve's balance sheet.

An easier way to see this is to plot the dynamic factors against a number of key variables which best reflect the Federal Reserve's policy actions. In figure 1, we plot the three largest factors, which taken together account for almost 70 percent of the total variance of the data set. The first factor, which alone explains about 38 percent of the total variance, is strongly associated with the federal funds rate up to the moment when the federal funds rate effectively hit the ZLB (figure 1, left-hand panel). The second factor, which explains around 20 percent of the total variance, appears instead to be mainly driven by the monetary base (figure 1, center panel). The third factor, which accounts for around 11 percent of total variance, is more correlated with the growth rate of the Federal Reserve's outright Treasury securities holdings (figure 1, right-hand panel). This factor adds additional information on unconventional measures associated

with changes in the size of the central bank's securities holdings, which is especially useful during the period when the ZLB is binding.

### 3. Filling the Gap: Did Unconventional Measures Help?

The dynamic factor model introduced above has the advantage of providing estimates of the missing values for key variables based on the expectation maximization algorithm. These EM-based estimates are driven by the evolution of the fully observed series and historical patterns of their correlations with the series with missing observations. This is essential to our approach to constructing a monetary policy indicator in the form of a shadow federal funds rate, which captures changes in the monetary policy framework and the implementation of alternative policy measures once the ZLB becomes binding. In other words, we retrieve a shadow federal funds rate series that maps the changes in other indicators of monetary policy onto it. As such, it can be interpreted as an estimate of how the federal funds rate would have behaved had the ZLB not been binding, based on the evolution of a variety of observable indicators of unconventional policy actions.

One significant advantage of using the estimated shadow federal funds rate to measure monetary policy is that it preserves continuity and consistency. As noted earlier, the bulk of the existing literature has focused on the federal funds rate as the main indicator of monetary policy. Having a shadow policy rate that behaves in an almost identical manner as the federal funds rate in normal times, and yet continues to work in the ZLB environment, is useful in that respect. Such a shadow policy rate can then be simply included in any existing quantitative model for monetary policy analysis. We formally examine the consistency issue and elaborate on the use of the estimated shadow policy rate in monetary models in section 4.3.

One important caveat and limitation of our approach is that the shadow rate is estimated based on the correlation structure prevailing during the sample period when the effective federal funds rate was by no means constrained by the ZLB.<sup>20</sup> A possible concern is that the shadow rate, at a time when the ZLB became binding, could be driven by a different pattern of correlations, as some variables in

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<sup>20</sup>This caveat of course also applies to alternative shadow market rates based on term structure models.

**Figure 2. Shadow<sup>a</sup> and Actual Federal Funds Rate<sup>b</sup>**



<sup>a</sup>Solid black line; the dotted lines represent the 95 percent confidence interval.

<sup>b</sup>Grey line.

our data set grew in relevance as monetary policy instruments at the expense of the more conventional tools.

Nevertheless, the shadow rate estimate's performance in the sample period mitigates this concern. Indeed the shadow rate tracked very well the actual effective federal funds rate before the ZLB became binding (figure 2). This suggests that in the pre-crisis period without a binding ZLB, the shadow policy rate is as good a monetary policy indicator as the federal funds rate. In a few cases where we do observe discernible deviations of the shadow rate from the effective federal funds rate—i.e., in 1974–75 and 1982—the federal funds rate happened to be much higher than its historical average. The episodes were preceded by recessions, and monetary policy appeared to have been looser than what the actual rate would suggest. This indicates that the federal funds rate might not accurately reflect the full extent of policy actions at times of high inflation and monetary policy uncertainty.

Looking at the period of binding ZLB after the actual federal funds rate declined to close to zero towards the end of 2008, the shadow policy rate turned negative, reflecting additional monetary stimulus provided by unconventional measures. In particular, the shadow rate picked up the two major rounds of balance sheet policy measures: the first phase of the large-scale asset purchase program (LSAP1) announced in November 2008, which focused on mortgage-backed securities and was reinforced with purchases of

longer-term Treasury securities in March 2009; and LSAP2 put in place in November 2010, followed by the Maturity Extension Program (MEP) announced in September 2011.<sup>21</sup>

Specifically, the estimated shadow federal funds rate suggests that U.S. monetary policy became less accommodative following the completion of LSAP1 in March 2010. The shadow rate gradually edged back to around zero before a second dip associated with LSAP2 in November 2010. Once LSAP2 was terminated in mid-2011, the rise in the shadow rate arising from the halt in the Federal Reserve's outright asset purchases was not sufficiently addressed by the maturity extension of the Federal Reserve's asset holding through the subsequent MEP, also known as Operation Twist.<sup>22</sup> But the Federal Reserve's September 2012 decision to add purchases of agency mortgage-backed securities at a pace of \$40 billion per month via its LSAP3, and the December decision to continue to purchase longer-term Treasury bonds at a rate of \$45 billion per month upon the completion of the MEP, helped drive the shadow rate lower. LSAP3 provided additional stimulus throughout the first half of 2013, but it provided a much smaller stimulus than LSAP2.

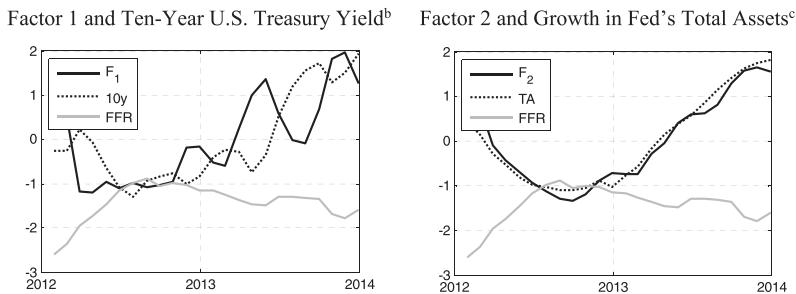
To a large extent, the smaller LSAP3 impact as reflected in the shadow rate can be explained by the timing, size, and pace of the Federal Reserve's large-scale asset purchases. This becomes clear when we examine the first two estimated dynamic factors, which account for about 60 percent of the total variance (figure 3). First, we notice that factor 1 declined in the first half of 2012, in line with a decline in the ten-year Treasury-bond yield, which might be attributed to the MEP implementation (figure 3, left-hand panel). Yet such a decline did not drive down the shadow federal funds rate, as it was more than compensated by a continued deceleration and eventual decline in the Federal Reserve's asset purchases (figure 3, right-hand panel). As the outright asset purchases were halted, the shadow rate drifted towards zero. The rise in the shadow rate was only reversed following the implementation of LSAP3 announced in

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<sup>21</sup>For further details on the Federal Reserve's asset purchase programs, see Meaning and Zhu (2011, 2012).

<sup>22</sup>Indeed, Chen et al. (2016) find that the impact on U.S. and global asset prices of MEP announcements differed markedly from that of LSAP announcements, and the MEP impact was quite similar to the tightening effects of the Federal Reserve's tapering announcements in 2013.

**Figure 3. Shadow FFR and the First Two Dynamic Factors since 2012<sup>a</sup>**



<sup>a</sup>The vertical dotted line corresponds to January 2013.

<sup>b</sup>The solid black line refers to the first dynamic factor (an increase corresponds to a policy tightening); the dotted line is the yield on ten-year Treasury bills and the grey line is the estimated shadow rate.

<sup>c</sup>The solid black line refers to the second dynamic factor (an increase corresponds to a policy tightening); the dotted line is the year-on-year growth of the Fed's total assets and the grey line is the estimated shadow rate.

September 2012, which exerted strong downward pressures on the shadow rate.

As U.S. long-term yields started to rise after Chairman Ben Bernanke's tapering suggestions in the June 19, 2013 press conference, the pace of the decline in the shadow rate moderated.<sup>23</sup> Following the announcement of the tapering of asset purchases in January 2014, the shadow rate started drifting towards zero. Shortly after the halt of purchases in October 2014, the shadow rate reached zero and started hovering around the target federal funds rate of 25 basis points. Eventually our shadow rate rose in tandem with the effective federal funds rate after the long-expected rate liftoff in December 2015.

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<sup>23</sup>On May 22, 2013, Chairman Ben Bernanke testified to the U.S. Congress that the Federal Reserve would likely start to taper the pace of its bond purchases later in 2013, subject to economic conditions. On June 19, he suggested that “the Committee currently anticipates that it would be appropriate to moderate the monthly pace of purchases later this year.” Subsequently, U.S. long-term bond yields and the U.S. dollar rose significantly, and several episodes of market turmoils ensued, which became known as the “taper tantrum.”

### 3.1 Robustness Analysis

In this subsection, we conduct a number of robustness checks to examine whether the estimated shadow federal funds rate remains a good indicator if the model specification changes. Indeed, in the context of their work on a shadow market rate à la Black (1995), Christensen and Rudebusch (2015) find that the sensitivity to model specifications is of particular concern. We examine the robustness of our approach in three dimensions: the number of lags in the dynamic factor model; the number of factors; and the inclusion or exclusion of certain variables from the underlying data set.<sup>24</sup>

First, correctly choosing the lag order is essential to the estimation of any time-series model. In our baseline dynamic factor model, the “optimal” lag order is selected based on the well-known Schwarz information criterion (SIC), which suggests a two-lag structure.<sup>25</sup> Indeed, the use of an arbitrary lag order affects the results only marginally: the shadow federal funds rates estimated with one or twelve lags (instead of two) both almost always fall within the 95 percent confidence band of the baseline two-lag specification (figure 4, left-hand panel). The more parsimonious AR(1) model is nearly identical to the baseline, while the AR(6) model estimate shows very similar dynamics despite some differences in terms of the magnitude.

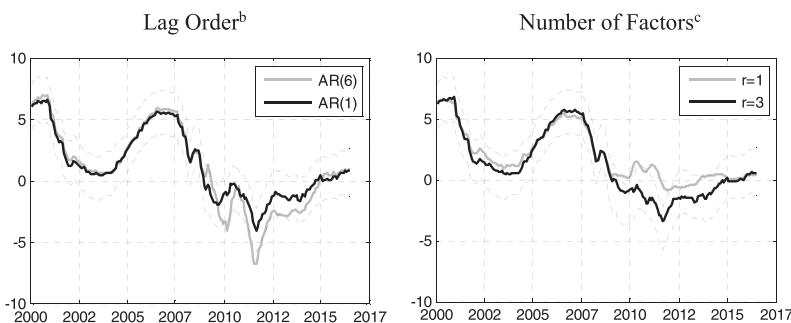
Second, choosing the right number of factors is a crucial step in the estimation of any dynamic factor model. The Hallin and Liska (2007) criterion, on which we rely, suggests eight factors as “optimal.” This result is clearly in line with the usual rule of thumb of retaining as many factors as possible to explain at least 90 percent of the total variance of the underlying data set. The alternative Bai and Ng (2007) criterion suggests three factors, which would account for around 70 percent of the total variance. Figure 4 (right-hand panel) reports two shadow federal funds rate series—one based on three factors, and the other on just one factor. Apparently, the shadow rate based on three factors somewhat underestimates the significant stimulus from LSAP1 as revealed by our benchmark estimate, though it stays within the confidence bands and manages to account

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<sup>24</sup>Another possible relevant dimension is that of the sample size used for the estimation. We tried to exclude the seventies and start the estimation in 1985, and results, not reported here for the sake of space, were virtually unchanged.

<sup>25</sup>The Akaike information criterion points to the same number of lags.

**Figure 4. Robustness of the Shadow FFR to Model Specification<sup>a</sup>**



<sup>a</sup>The dotted lines represent the 95 percent confidence interval for the baseline estimate (specification with two lags and eight factors).

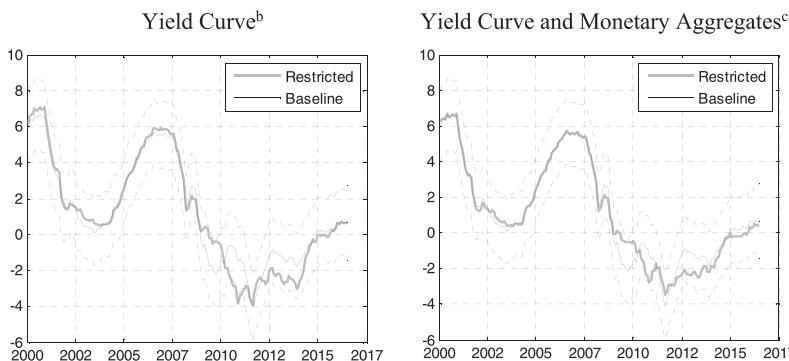
<sup>b</sup>The solid black line refers to the AR(1) estimate, the solid grey line to the AR(6) estimate.

<sup>c</sup>The solid line refers to the three-factor estimate, based on Bai-Ng (2007) criterion, and the solid grey line to the one-factor estimate.

for the stimulus provided by LSAP2 in 2011. By contrast, the use of just one factor is clearly inadequate to properly characterize U.S. monetary policy dynamics in the ZLB period: the estimate does not display enough variation, and fails to capture the extent of most easing programs. This is not surprising, given that the first factor—which we have shown to be closely associated with the near end of the yield curve—explains less than 40 percent of the total variance.

Last but not least, a key element for the success of any monetary policy indicator is its information content, and in our approach, the proper selection of variables to be included in the data set is essential. Essentially, the information content of the shadow rate is limited by the data on which the dynamic factor model is estimated. Whether the shadow policy rate reflects the full range of the Federal Reserve monetary policy operations—and nothing more than that—depends on the quality of the underlying monetary data set we put together in the first place. As we argue in section 2.2, our choice of variables is based on sound economic reasoning: we try to be comprehensive and make sure that our data represent all different aspects of monetary policy measures over the entire sample period,

**Figure 5. Robustness of the Shadow FFR to Data Selection<sup>a</sup>**



<sup>a</sup>The solid black lines refer to the baseline specification on the full data set, together with 95 percent confidence bands.

<sup>b</sup>The solid grey line refers to a restricted data set which only includes the interest rates.

<sup>c</sup>The solid grey line refers to a restricted data set which includes the interest rates and monetary aggregates, but not the Federal Reserve balance sheet items.

which include post-crisis unconventional policies. Yet what would happen had we left out some important elements?

To answer this question, we reestimate the dynamic factor model using two “amputated” data sets.<sup>26</sup> In the first exercise, we only include the elements of the yield curve (i.e., block 1 in section 2.3). This is of particular interest, as it makes our shadow rate directly comparable with those based on models of the term structure. In a sense, the dynamic factor model underlying our shadow rate would be an unrestricted version of the term structure model underlying the shadow rates à la Black (1995). Interestingly, the statistical criteria we adopt for “optimal” model specification seem to be able to capture this, since they suggest the use of three factors—a most common choice for existing term structure models.

The results are reported in the left-hand panel of figure 5. The magnitude and the dynamics of the alternative, restricted-information shadow rate is somewhat similar to the baseline case and lies mostly within the confidence bands. A notable exception is the

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<sup>26</sup>In both cases, the number of factors and the lag order are chosen according to the same statistical criteria as in the baseline case.

rate's behavior following the Federal Reserve's decision to implement LSAP1, which apparently fails to capture the effects of the large-scale asset purchases. This confirms the intuition that a monetary policy indicator that only relies on Treasury yields may miss the big picture and underestimate the extent of monetary accommodation coming from programs targeting different types of assets. It is also interesting to note that the dynamics of this restricted-information shadow rate is much more similar to the estimates based on term structure models, and lies somehow in between the Krippner (2013a) and Wu and Xia (2016) rates (see next section).

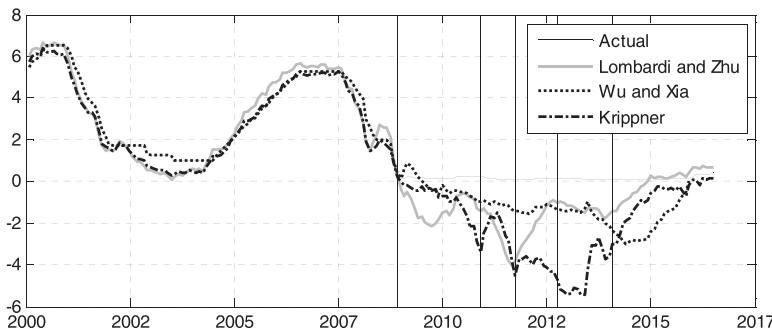
In the second exercise (figure 5, right-hand panel), we estimate a shadow rate based on a restricted data set including only the interest rates and monetary aggregates (i.e., blocks 1 and 2 in section 2.3). The dynamics of this restricted-information shadow rate estimate is more similar to the baseline case, especially after 2011. Nevertheless, excluding information on the Federal Reserve's balance sheet, unsurprisingly, downplays the extent of stimulus provided by the Federal Reserve's large-scale asset purchases. Despite the fact that it remains inside the confidence band for our baseline estimate, the restricted-information estimate can only provide information on non-standard measures to the extent that U.S. interest rates and monetary aggregates are affected by these.

### 3.2 Alternative Shadow Rates

We now investigate how our monetary policy indicator compares with other existing shadow rate estimates in the literature. We focus on what we consider to be the more widely rates used so far, namely the Krippner (2013a) and Wu and Xia (2016) shadow rates. We plot both rates against our shadow rate, as well as the effective federal funds rate, in figure 6.

Both alternative estimates suggest sizable monetary stimulus in the period when the ZLB is binding; this is in line with the behavior of our shadow federal funds rate, considering significant uncertainties surrounding the estimates in this period. But the dynamics is substantially different. First, our rate indicates a large stimulus provided by the first round of asset purchases (LSAP1) carried out by the Federal Reserve since late 2008, but both alternative shadow rates seem to miss this: the Wu and Xia (2016) shadow rate actually points to

**Figure 6. Alternative Estimates of the Shadow Federal Funds Rate<sup>a</sup>**



<sup>a</sup>The vertical lines correspond to the dates of introduction of the major asset purchase programs implemented by the Federal Reserve: LSAP1 (November 2008), LSAP2 (November 2010), MEP (September 2011), LSAP3 (September 2012), and the tapering of asset purchases (January 2014).

a monetary tightening, and the Krippner (2013a) rate shows a slow and moderate decline. This should not come as a surprise: both these alternative shadow rates are based on a term structure model using only the U.S. yield curve as input.<sup>27</sup> The early asset purchases targeted toxic assets held by banks, so they arguably had a more limited impact on U.S. Treasury yields. As the LSAP started targeting government bonds, the shadow rate estimates began to move in the same direction, following similar dynamics.

Second, while our shadow rate indicates a continued sharp expansion of monetary stimulus following the implementation of LSAP1 starting in November 2010, the Krippner (2013a) rate points to a strong tightening of monetary policy, while the Wu and Xia (2016) rate shows a rather slow and moderate decline.

Third, our shadow rate reveals that U.S. monetary policy tightened following the start of MEP, or Operation Twist, in September 2011 and the tapering of asset purchases in January 2014, as the expansion of the Federal Reserve asset holdings slowed or stopped

<sup>27</sup>Results indeed were similar to those provided by our “restricted” estimate in figure 5.

altogether.<sup>28</sup> While the Krippner (2013a) rate shows some similar dynamics, the Wu and Xia (2016) rate only reacts to these apparent policy changes with a significant time lag. Indeed, following the Federal Reserve's tapering of asset purchases in January 2014, both our rate and the Krippner (2013a) rate suggest a tightening of monetary policy, while the Wu and Xia (2016) rate continued to fall, reaching its lowest point of -2.9 percent in August 2014, well into the tapering process. At the point when asset purchases were halted altogether by end-October 2014, our estimate already reached 0 percent, while the Wu and Xia (2016) stayed at -2.8 percent, indicating a much easier monetary policy, even in comparison with the previous LSAP rounds.

#### 4. Effectiveness of Unconventional Policies

In this section, we provide some substantive analysis to demonstrate how one can employ our shadow federal funds rate in standard monetary policy analyses. The purpose of our exercise here is purely illustrative: assessing the stance and effectiveness of monetary policy is a daunting task beyond the scope of this paper, and countless possible approaches have been proposed for this purpose, each one with its pros and cons. Here we explicitly focus on some simple and well-known approaches. These simple approaches allow one to more easily appreciate how the use of our shadow rate could affect the analysis. However, this does not imply that the empirical evidence we obtain should be taken as conclusive.

First, we demonstrate that our shadow policy rate is a stable measure of monetary policy. Next, we check our shadow policy rate against the policy benchmarks prescribed by standard Taylor rules.<sup>29</sup> Finally, we examine, in canonical monetary VAR models, whether our shadow policy rate provides a better description of the underlying shocks to monetary policy in the post-ZLB period.

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<sup>28</sup>As illustrated in figure 3, our shadow rate rose due to the fact that under the MEP program put in place at that time, the reduction in long-term yields was insufficient to offset the effect of the slowdown in purchases.

<sup>29</sup>We also compared our shadow federal funds rates with the updated series of the Laubach-Williams (2003) equilibrium real interest rate, which is a useful benchmark for policy.

Such exercises are intended to validate our dynamic factor model-based shadow policy rate procedure and to gauge the overall monetary stance, especially the effectiveness of unconventional monetary policies in the aftermath of the global financial crisis.

#### *4.1 The Shadow FFR as a Stable Measure of Monetary Policy*

As anticipated in the introduction, one of the main advantages of shadow rates as gauges for monetary policy is that, unlike alternative approaches that aim at measuring explicitly unconventional policies, they provide a single policy gauge that can be used before and after the ZLB becomes binding. To do so, however, one needs to show that our shadow policy rate is a stable measure of monetary policy—i.e., that at the ZLB its dynamic properties in relation to key macroeconomic series are the same as those of the federal funds rate in pre-ZLB times.

We test that proposition in the context of a very simple VAR model that jointly models real GDP, inflation, and the federal funds rate—either the observed or the shadow one. Admittedly, this is a very simplified model, but we remark that it is the building block of the Bernanke and Blinder (1992) monetary VAR, which we will also use as a benchmark later, in section 4.3. Moreover, it implies a standard policy rule according to which the central bank sets its policy rate as a function of inflation and output, and also features interest rate persistence; we believe this is a fair, although simplified, characterization of monetary policymaking in the pre-ZLB times.

The stability test is akin to that employed by Wu and Xia (2016): the null hypothesis is that the coefficients related to the federal funds rate do not change in the post-ZLB period. More formally, we estimate the VAR:

$$\begin{bmatrix} y_t \\ p_t \end{bmatrix} = \mu + \sum_{i=1}^4 \beta_i \begin{bmatrix} y_{t-i} \\ p_{t-i} \end{bmatrix} + (1 - \mathbf{I}_Z) \sum_{i=1}^4 \rho_i^N r_{t-i} + \mathbf{I}_Z \sum_{i=1}^4 \rho_i^Z r_{t-i} + \varepsilon_t,$$

where  $y_t$  is (log) real GDP,  $p_t$  is the (log) price deflator,  $r_t$  is the federal funds rate, and  $\mathbf{I}_Z$  is an indicator function taking a value of one during the ZLB time; and we test the null hypothesis  $H_0: \rho^N = \rho^Z$  by means of standard likelihood-ratio test:

$$LR = (T - k)[\log |\Sigma_0 \Sigma'_0| - \log |\Sigma_1 \Sigma'_1|],$$

where  $T$  is the sample size;  $k$  is the number of parameters;  $\Sigma_0$  and  $\Sigma_1$  are the (estimated) covariance matrices of the error term  $\varepsilon$  under, respectively, the (constrained) null and (unconstrained) alternative hypotheses; and the test has a  $\chi^2$  distribution with  $q$  degrees of freedom, where  $q$  is the number of constraints.

We conduct the stability test described above for both the actual federal funds rate (FFR) and its shadow counterpart. Using the observed FFR leads to the rejection of the null hypothesis ( $LR = 15.77$ ,  $p$ -value 0.00): this signals that the dynamic relationship between the federal funds and the macro variables was changed by the ZLB, which is not surprising, as the FFR no longer reflected monetary policy actions. By contrast, by using our shadow rate one cannot reject the null ( $LR = 1.94$ ,  $p$ -value 0.38): this signals that, once one accounts for unconventional policy actions by means of our shadow FFR, the historical dynamic relationship with the key macro variables appears unaltered.

#### 4.2 *The Shadow FFR and Taylor Rules*

One critical issue in the design and deployment of the vast arsenal of unconventional monetary policy measures is to what extent such measures are effective and have eventually eliminated, or at least reduced, the policy gap that emerged since the ZLB on short interest rates set in. Due to the sharp economic downturn during the Great Recession, Taylor rules would suggest a very loose policy stance with negative nominal federal funds rates. To check whether this has been the case, one needs to summarize the non-standard measures in terms of the federal funds rate. In other words, if, for instance, the Federal Reserve's purchases of \$600 billion of longer-term Treasury bonds during LSAP2 were equivalent to a reduction of the federal funds rate by 50 basis points, would these purchases have been sufficient for the "implicit" shadow federal funds rate to become negative enough to attain the levels suggested by standard Taylor rules?

We answer this question by means of a simple exercise.<sup>30</sup> We first compute the levels of federal funds rates recommended by

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<sup>30</sup>Note that we do not address here the issue of whether central banks target both macroeconomic and financial stability, nor the possible implications of an extended period of low interest rates for financial stability.

simple Taylor rules, and then compare the estimated shadow funds rate with the Taylor benchmark rates. The Taylor rates are computed based on the latest U.S. data as follows:

$$i = \pi + 0.5y + 0.5(\pi - 2) + 2,$$

where  $i$  is the federal funds rate,  $\pi$  is the current rate of inflation, and  $y$  is a measure of output gap. The inflation target or equilibrium rate of inflation is set to be 2, and so is the equilibrium real interest rate. The parameterization of this rule follows Taylor (1993), who shows that the rule closely tracked the effective federal funds rate movements in his original sample period from 1987 to 1990. A second rule, analyzed by Taylor (1999) and recommended by Ball (1999) and by Yellen (2012), who renamed it the “balanced-approach rule,” takes a slightly different parameterization:

$$i = \pi + y + 0.5(\pi - 2) + 2.$$

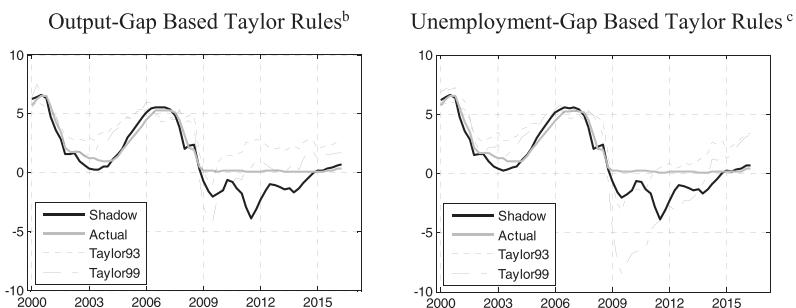
In this case the central bank responds more aggressively to movements in the output gap. The Taylor rules are simple and straightforward and are believed to provide a good description of the Federal Reserve’s behavior in much of the post-war era, including the period when the Federal Reserve targeted monetary aggregates.

We compute the Taylor (1993, 1999) benchmark rates using inflation rates based on CPI, PCE, and the GDP deflator, and measures of economic slack based on the output gap and unemployment gap, based on the U.S. Congressional Budget Office’s (CBO’s) estimates of potential output and long-term NAIRU.<sup>31</sup> The shadow federal funds rate is averaged to the quarterly frequency, and the rates derived from Taylor rules (1993, 1999) are based on the latest data.<sup>32</sup>

<sup>31</sup>We also experiment with the Hodrick-Prescott filter, as well as the output gap and potential growth series based on Laubach and Williams (2003). Results are broadly similar, but we do not report them for the sake of space.

<sup>32</sup>The estimated Taylor-rule (1993, 1999) rates based on the latest data can be different from previous estimates due to significant data revisions in the past. This can lead to different implications for monetary policy. Orphanides (2003) and Kahn (2012) compared Taylor rules estimated using different vintages of data and uncovered substantial differences among these, even within the original sample period of Taylor (1993).

**Figure 7. Shadow FFR and Taylor-Rule Prescriptions<sup>a</sup>**



<sup>a</sup>The black lines refer to the estimated shadow federal funds rate and the grey line refers to the actual effective federal funds rate. Numbers on the horizontal scale are placed to indicate the start of the year.

<sup>b</sup>The comparison is made against Taylor (1993) and Taylor (1999) rules based on CBO estimates of potential output.

<sup>c</sup>The comparison is made against Taylor (1993) and Taylor (1999) rules based on CBO estimates of long-term NAIRU.

Results are reported in figure 7 and suggest that monetary policy stance, as indicated by both the shadow and actual federal funds rates, was too loose or expansionary between 2001 and 2006.<sup>33</sup> Once the federal funds rate was rapidly cut to near zero, unconventional measures provided additional monetary stimulus and the shadow rate turned negative in early 2009. The amount of stimulus provided was broadly in line with the prescriptions of the Taylor (1993), but not sufficient according to Taylor (1999), especially when the CBO unemployment gap is used as a measure of slack.<sup>34</sup>

However, the shadow federal funds rate rebounded quickly to above zero in May 2010 as LSAP1 was wrapped up. While this was in line with the levels suggested by the Taylor (1999) rule based on the CBO output gap, it would have left a large policy gap if one were

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<sup>33</sup>Taylor (2007) suggests that U.S. monetary policy could have been too loose from 2000 to 2006, when the actual federal funds rate is compared with a counterfactual scenario based on a Taylor rule.

<sup>34</sup>Notably, policy rates suggested by Taylor (1993) rules appear to be much higher than those implied by Taylor (1999) rules from late 2008 on, as one would expect.

to look at the unemployment gap. On this measure, the monetary policy stance became less accommodative through much of 2010, until it was loosened sharply with the implementation of LSAP2, which was announced in November 2010: this brought the shadow rate in line with the prescriptions of the Taylor (1999) rule based on the unemployment gap. This last finding raises some questions on Bullard's (2012, 2013) claim, based on Krippner's (2012) shadow rate, that the Federal Reserve's policy stance became excessively loose in 2012.

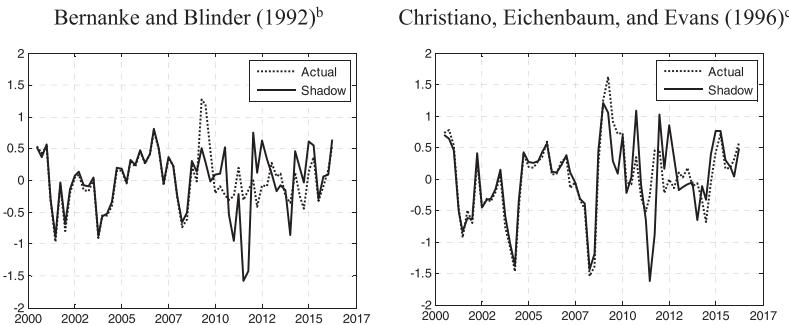
#### *4.3 Shadow FFR, Policy Shocks, and Monetary VARs*

In this section, we discuss the use of the shadow federal funds rate in standard VAR models for monetary policy analysis, focusing on the use of interest rate shocks to measure monetary policy.<sup>35</sup> Applying these models mechanically to samples that include periods when the ZLB is binding and policy rates are stuck at close to zero exposes researchers to the risk of grossly underestimating the true extent of policy stimulus provided by a central bank engaged in non-standard measures. We make the point by employing our shadow federal funds rate in two canonical monetary VAR models, and we show that the shadow rate provides a more appropriate measure of the post-ZLB unconventional stimulus.

We focus on two standard model specifications, the VAR model of Bernanke and Blinder (1992) and a more complex monetary VAR by Christiano, Eichenbaum, and Evans (1996). The model by Bernanke and Blinder (1992) consists of three key macro variables: the log of real GDP, the log of the real GDP deflator, and the federal funds rate. The identification of the structural shocks is based on a recursive Cholesky identification scheme: real GDP is postulated to react only with a lag to inflation and monetary policy, and inflation reacts only with a lag to monetary policy shocks. We estimate both VAR

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<sup>35</sup>Unlike monetary analysis based on policy rules, i.e., the anticipated or endogenous part of policy, Bernanke and Mihov (1998) point out the importance of studying innovations to the federal funds rate as it enables one to assess monetary policy effects with “minimal identifying assumptions.”

**Figure 8. Monetary Policy Shocks<sup>a</sup>**

<sup>a</sup>Structural shocks are extracted using recursive Cholesky schemes; estimation is based on data from January 1970 until March 2016. The dashed line corresponds to a model estimated with actual FFR, while the solid line refers to the shadow FFR series.

<sup>b</sup>The model features (in order) the log of real GDP, the log of the GDP deflator, and the FFR.

<sup>c</sup>The model features (in order) the log of real GDP, the log of the GDP deflator, the log of commodity prices, the log of non-borrowed reserves, the FFR, and the log of total reserves.

systems using quarterly data ranging from January 1970 to March 2016, using four lags in this exercise, as is common practice.<sup>36</sup>

Based on the estimation results, we then extract, in two separate exercises, two monetary policy shocks—one based on the actual federal funds rate, another based on our estimated shadow federal funds rate. The results suggest that the use of the actual federal funds rate would lead to the (wrong) conclusion that too little monetary stimulus was provided since late 2008, as shocks to the actual funds rate were rather small and only mildly negative (figure 8, left panel). In contrast, monetary shocks estimated using our shadow

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<sup>36</sup>This exercise is of course subject to a number of caveats, in particular that the macroeconomy and monetary policy may both have been subject to a structural break in the aftermath of the Great Recession, or even at the beginning of the Great Moderation period. Furthermore, small-scale VARs often give rise to price puzzles (Sims 1992), an occurrence which is at times accommodated by including commodity price indexes. We stress, however, that our objective here is not to pin down the most appropriate VAR model for post-crisis monetary policy. Rather, we aim at illustrating how, given a certain model specification, the identified monetary policy shocks differ based on the FFR series one employs.

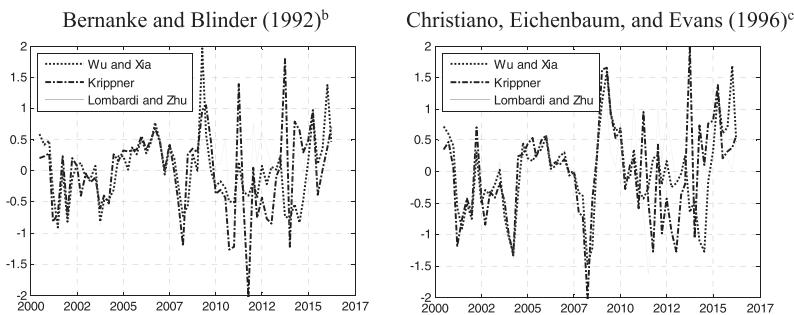
funds rate series clearly indicate sizable easing, its timing corresponding to the implementation of LSAP2 and LSAP3. The shocks suggest that LSAP2 provided unprecedented easing to the economy. It is also remarkable that, following the tapering of asset purchases in 2014, the shadow rate yields a positive shock—i.e., a tightening of the policy stance—which the observed federal funds rate is unable to capture.

The Christiano, Eichenbaum, and Evans (1996) VAR model is more elaborate in that it contemplates a broader range of measures of money policy. More specifically, the model features the log of real GDP, the log of the GDP deflator, the log of a commodity price index, the log of non-borrowed reserves, the federal funds rate, and the log of total reserves. As in Bernanke and Blinder (1992), the identification of structural shocks is based on a recursive Cholesky ordering.

The estimated monetary policy shocks based on both the shadow and actual federal funds rates are presented in the right-hand panel of figure 8. Given that this model includes, in addition to the federal funds rate, two monetary aggregates that are potentially more revealing of unconventional measures (i.e., total and non-borrowed reserves), it is not surprising that the difference between monetary policy shocks based on the actual and the shadow federal funds rates becomes less pronounced over much of the sample period. Nevertheless, the benefits from a policy indicator based on more comprehensive monetary information become clear in the post-crisis period, when unconventional policies are the norm. In fact, shocks to our shadow funds rate again reveal sizable monetary stimulus with the implementation of LSAP2 and LSAP3, while the stimulus appears to be minor if we assess monetary policy based on shocks to the actual federal funds rate constrained by the ZLB. Monetary policy shocks estimated in the standard way would therefore severely underestimate the true extent of monetary expansion afforded by non-standard policy measures implemented after the breakout of the financial crisis.

As a parallel to section 3.2, we also compared the monetary shocks identified with our shadow rate with those one would have obtained using either Wu and Xia (2016) or Krippner (2013a); this is visualized in figure 9. Unsurprisingly, the three alternative shadow rates yield similar monetary policy shocks during the pre-ZLB

**Figure 9. Monetary Policy Shocks from Alternative Shadow Rates<sup>a</sup>**



<sup>a</sup>Structural shocks are extracted using recursive Cholesky schemes; estimation is based on data from January 1970 until March 2016. The dotted line corresponds to a model estimated with the Wu and Xia (2016) shadow rate, the dashed line to a model estimated with the Krippner (2013a) rate, while the thin grey line refers to our shadow FFR.

<sup>b</sup>The model features (in order) the log of real GDP, the log of the GDP deflator, and the FFR.

<sup>c</sup>The model features (in order) the log of real GDP, the log of the GDP deflator, the log of commodity prices, the log of non-borrowed reserves, the FFR, and the log of total reserves.

period. As the ZLB binds, instead, the results differ markedly. First of all, it emerges that the Krippner (2013a) shadow rate produces more volatile shocks than both ours and Wu and Xia (2016), with a notable tightening spike at the end of 2013. According to the Wu and Xia (2016) shadow rate, instead, monetary policy has been much more stable and neutral, and only provided a large stimulus in mid-2014; as already remarked in section 3.2, this is rather surprising, as by then the tapering of asset purchases had already been announced and was about to be implemented.

Another important feature that speaks in favor of our purely econometric approach is that both term-structure-based shadow rates point to significant tightening of monetary policy in 2009. As we noted already in section 3.2, this follows from the fact that the first wave of Fed asset purchases did not target government bonds, and therefore may have had a much smaller impact on the yield curve.

## 5. Concluding Remarks

This paper introduces an intuitive and, we believe, easy-to-use measure of the U.S. monetary policy stance that incorporates the effects of changes in the Federal Reserve's balance sheet.

Our measure is based on a comprehensive set of information on monetary policy operations, and comes in the form of a shadow federal funds rate that has the advantage of being unaffected by the zero lower bound on nominal interest rates. Unlike shadow rates estimated à la Black (1995), our indicator directly measures U.S. monetary policy, in that it gives an explicit and direct role to the variables closely linked to the Federal Reserve's monetary operations.

We showed that our shadow policy rate is robust to different specifications and works well both before the crisis and after the zero lower bound became binding. This single policy indicator gives a good representation of unconventional policies. In fact, our measure can be used across different monetary policy regimes, as it includes information on monetary aggregates, interest rates, and, crucially, the Federal Reserve's balance sheet.

Furthermore, our analysis suggests that the unconventional policies were able to fill in a substantial portion of the policy gap between the ZLB and Taylor-rule rates. Monetary policy provided the greatest stimulus over 2011, with the shadow policy rate dropping to below -5 percent in August, but it became less accommodative since then, before loosening again starting in October 2012. We also applied the shadow funds rate in standard VAR models, and we found that monetary policy shocks estimated this way provided a far more realistic picture of U.S. monetary policy in the post-crisis period than those based on the actual federal funds rate.

Table 1. Factor Representation: Equation (1)

$\Lambda$	$F_1$	$F_2$	$F_3$	$F_4$	$F_5$	$F_6$	$F_7$	$F_8$	$R$ (Diagonal)
FFR	-0.40	0.13	0.18	-0.08	-0.04	0.04	-0.08	-0.26	0.03
TB3m	-0.40	0.14	0.14	-0.09	-0.03	0.01	-0.05	-0.22	0.01
TB6m	-0.45	0.14	0.16	-0.08	-0.01	0.06	-0.06	-0.03	0.01
TB1y	-0.39	0.15	0.10	-0.07	-0.01	0.03	-0.04	-0.12	0.00
TB2y	-0.37	0.15	0.05	-0.05	-0.01	0.03	0.01	-0.05	0.00
TB5y	-0.36	0.15	0.01	-0.02	0.00	0.06	0.08	0.13	0.00
TB10y	-0.36	0.16	-0.02	0.00	-0.01	0.08	0.13	0.17	0.00
TB20y	-0.41	0.17	-0.16	0.01	-0.02	0.18	-0.52	0.73	0.00
OIS3m	-0.14	-0.06	0.10	-0.07	-0.09	0.02	-2.16	-0.42	0.00
M1	-0.04	0.23	-0.24	0.45	-0.07	0.27	0.44	-0.13	0.01
M2	-0.08	0.16	0.08	0.47	0.48	-0.47	0.08	0.08	0.00
MZM	0.15	0.41	-0.15	-0.17	-0.02	-0.05	-0.02	0.01	0.00
M0	0.15	0.41	-0.16	-0.18	-0.01	-0.06	0.06	0.03	0.00
Total Reserves	0.17	0.41	0.04	-0.16	0.02	0.03	-0.04	0.08	0.10
Excess Res.	-0.09	-0.17	-0.13	0.06	-0.07	-0.03	0.03	-0.09	0.83
Req. Res.	0.05	0.24	-0.21	0.36	-0.10	0.24	-0.11	-0.21	0.11
Securities Held	-0.03	-0.05	-0.59	-0.03	0.00	-0.27	0.13	-0.22	0.02
Total Assets	0.17	0.40	-0.02	-0.14	0.02	0.02	-0.01	0.07	0.11
% of Treas. <5y	0.00	-0.05	-0.13	-0.27	0.76	0.47	-0.04	-0.19	0.03
% of Treas. <10y	0.12	0.12	0.10	-0.03	-0.17	-0.07	0.00	0.04	0.81
% of Treas. >10y	0.06	0.10	0.33	0.27	0.20	0.00	-0.09	0.07	0.54

**Table 2. Factor Dynamics: Equation (2)**

<b>A<sub>1</sub></b>	<b>F<sub>1</sub></b>	<b>F<sub>2</sub></b>	<b>F<sub>3</sub></b>	<b>F<sub>4</sub></b>	<b>F<sub>5</sub></b>	<b>F<sub>6</sub></b>	<b>F<sub>7</sub></b>	<b>F<sub>8</sub></b>
<i>F<sub>1</sub></i>	1.44	0.01	0.05	-0.18	-0.02	0.09	-0.16	0.03
<i>F<sub>2</sub></i>	0.11	1.43	-0.18	-0.04	0.08	0.12	0.11	0.04
<i>F<sub>3</sub></i>	0.05	0.03	1.56	-0.06	0.29	0.53	0.09	-0.03
<i>F<sub>4</sub></i>	0.19	-0.09	0.02	1.21	0.28	0.12	-0.41	-0.08
<i>F<sub>5</sub></i>	-0.01	0.12	0.27	0.22	0.82	-0.71	0.53	0.49
<i>F<sub>6</sub></i>	0.00	0.08	0.20	0.14	-0.45	1.33	0.18	0.29
<i>F<sub>7</sub></i>	0.05	0.00	-0.05	0.04	0.00	0.05	2.00	0.11
<i>F<sub>8</sub></i>	0.07	-0.06	0.05	-0.12	0.14	0.09	-0.36	1.35
<b>A<sub>2</sub></b>	<b>F<sub>1</sub></b>	<b>F<sub>2</sub></b>	<b>F<sub>3</sub></b>	<b>F<sub>4</sub></b>	<b>F<sub>5</sub></b>	<b>F<sub>6</sub></b>	<b>F<sub>7</sub></b>	<b>F<sub>8</sub></b>
<i>F<sub>1</sub></i>	-0.45	-0.04	-0.05	0.18	0.05	-0.08	0.14	-0.02
<i>F<sub>2</sub></i>	-0.12	-0.49	0.20	0.05	0.02	-0.04	-0.11	-0.06
<i>F<sub>3</sub></i>	-0.07	-0.05	-0.63	0.02	-0.08	-0.43	-0.12	-0.05
<i>F<sub>4</sub></i>	-0.19	0.07	-0.06	-0.27	-0.10	-0.06	0.40	0.03
<i>F<sub>5</sub></i>	0.03	-0.09	-0.20	-0.14	-0.35	0.43	-0.48	-0.34
<i>F<sub>6</sub></i>	0.01	-0.06	-0.17	-0.10	0.18	-0.50	-0.13	-0.21
<i>F<sub>7</sub></i>	-0.05	0.00	0.05	-0.04	0.00	-0.05	-1.02	-0.11
<i>F<sub>8</sub></i>	-0.07	0.06	-0.06	0.10	-0.05	-0.05	0.35	-0.40
<b>Q</b>	<b>F<sub>1</sub></b>	<b>F<sub>2</sub></b>	<b>F<sub>3</sub></b>	<b>F<sub>4</sub></b>	<b>F<sub>5</sub></b>	<b>F<sub>6</sub></b>	<b>F<sub>7</sub></b>	<b>F<sub>8</sub></b>
<i>F<sub>1</sub></i>	0.09	0.05	0.00	0.01	-0.02	-0.02	0.00	0.01
<i>F<sub>2</sub></i>	0.05	0.18	0.03	0.03	-0.10	-0.06	0.00	0.00
<i>F<sub>3</sub></i>	0.00	0.03	0.14	0.08	-0.26	-0.14	0.01	0.04
<i>F<sub>4</sub></i>	0.01	0.03	0.08	0.12	-0.19	-0.10	0.00	0.03
<i>F<sub>5</sub></i>	-0.02	-0.10	-0.26	-0.19	0.61	0.33	-0.01	-0.11
<i>F<sub>6</sub></i>	-0.02	-0.06	-0.14	-0.10	0.33	0.19	-0.01	-0.06
<i>F<sub>7</sub></i>	0.00	0.00	0.01	0.00	-0.01	-0.01	0.00	0.00
<i>F<sub>8</sub></i>	0.01	0.00	0.04	0.03	-0.11	-0.06	0.00	0.04

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