Online Appendix to The Effects of Government Bond Purchases on Leverage Constraints of Banks and Non-financial Firms*

Michael Kühl
Deutsche Bundesbank

A. Derivation of the Model

A.1 Households

The household consists of three different groups: bankers, entrepreneurs, and the remaining household members. Similar to Gertler and Kiyotaki (2010) or Gertler and Karadi (2011), the share of household members in the banking sector in each period is $s_B$. In order to keep the shares constant over time, I assume that the number of workers who become bankers is exactly the same as bankers who return to the goods-producing sector. The probability of staying a banker $p_B$ is exogenously fixed and does not change over time. The profits each bank manager potentially earns are not transferred to the household before the bank manager leaves the bank, which happens with a probability of $(1−p_B)$. In addition, a specific share $s_e$ of households becomes entrepreneurs. Like bank managers, entrepreneurs survive with a probability of $p^{E,c}$. During the time they are

---

*I am grateful to John C. Williams (the editor) and two anonymous referees for their valuable comments and suggestions. I would also like to thank Andrea Gerali, Ulrich Grosch, Sandra Gomes, Josef Hollmayr, Peter Karadi, Stéphane Moyen, Dominic Quint, and Ansgar Rannenberg as well as seminar participants at the Deutsche Bundesbank, the winter 2015 WGEM meeting, and the 12th Dynare Conference. The paper represents the author’s personal opinions and does not necessarily reflect the views of the Deutsche Bundesbank. Author contact: Wilhelm-Epstein-Str. 14, 60431 Frankfurt. E-mail: michael.kuehl@bundesbank.de.

1The reason why bankers exit lending banks is to guarantee that the lending banks do not accumulate equity indefinitely (see Gertler and Kiyotaki 2010 or Gertler and Karadi 2011).
entrepreneurs, household members accumulate net wealth, which is transferred back to the household when they leave the entrepreneurial sector.

A.1.1 Utility Maximization

The economy is populated by a continuum of households which are indexed by $h$ with $h \in (0, 1)$. Each $h$-th household decides on the supply of labor, how much to consume and to save, and on the allocation of its wealth. Households’ utility function is given in equation (35):

$$U_0 = E_0^t \sum_{j=0}^{\infty} \beta^j \left[ \ln \left( C_{h,t+j} - h^C C_{h,t-1+j} \right) - \kappa \nu_{t+j} N_{h,t+j}^{1+\phi} \right]$$

with discount factor $\beta$. The term $h^C$ reflects the internal habits in consumption with $h^C \in (0, 1)$. The budget constraint in real terms becomes

$$(1 + i_{t-1}) B_{h,t-1}^{IA} P_t + \left( 1 + r_t^{B-gov} \right) \frac{Q_{t-1}^{B-gov} B_{h,t-1}^{n,gov,H}}{P_t}$$

$$+ \left( 1 + r_t^{D} \right) \frac{D_{h,t-1}^{n}}{P_t} + (1 - \tau^w) \frac{W_{h,t}}{P_t} N_{h,t} + \frac{D_i h_{h,t} + \Xi_{h,t}}{P_t}$$

$$\geq (1 + \tau^C) C_{h,t} + T_t + \frac{D_{h,t}^{n}}{P_{t+j}} + \frac{B_{h,t}^{IA}}{P_t} + \frac{Q_{t}^{B-gov} B_{h,t}^{n,gov,H}}{P_t} + \Theta_{t}^{gov,H},$$

where the superscript $n$ denotes nominal terms. Households pay taxes on their labor income and on their consumption expenditures, $\tau^w$ and $\tau^C$, respectively.

Resulting from utility maximization, I obtain the marginal utility of consumption

$$\frac{\partial U_0}{\partial C_{h,t}} : (1 + \tau^C) \lambda_{h,t} = \left( C_{h,t} - h^C C_{h,t-1} \right)^{-1}$$

$$- \beta h^C \left( C_{h,t+1} - h^C C_{h,t} \right)^{-1},$$

the Euler equation for short-term bonds of the intervention authority

$$\frac{\partial U_0}{\partial B_{h,t}^{IA}} : \frac{1}{\lambda_{h,t}} = E_t \beta \frac{\lambda_{h,t+1}}{\lambda_{h,t}} (1 + i_t) \frac{1}{\pi_{t+1}},$$

(36)
the Euler equation for deposits
\[
\frac{\partial U_0}{\partial D_{h,t}} : 1 = E_t \beta \frac{\lambda_{h,t+1}}{\lambda_{h,t}} \frac{(1 + r^D_t)}{\pi_{t+1}}, \tag{38}
\]
and the Euler equation for long-term government bonds
\[
\frac{\partial U_0}{\partial B_{gov,H}^{h,t}} : 1 + \upsilon B_{gov,H}^{h,t} \left( B_{gov,H}^{h,s} - B_{gov,H}^{h,s} \right) + \tau_{B,gov}^{h,t+1} = E_t \beta \frac{\lambda_{h,t+1}}{\lambda_{h,t}} \frac{(1 + r^{B,gov}_{t+1})}{\pi_{t+1}}. \tag{39}
\]

A.1.2 Wage Setting

The households supply differentiated labor services \((N_{h,t})\) to the intermediate goods sector. Because of a monopolistically competitive labor market in which labor services are imperfect substitutes, each household has market power to set its nominal wage \((W_t)\). Following Erceg, Henderson, and Levin (2000), I assume, by analogy with Calvo pricing, that the household is not able to renegotiate its nominal wage in each period. Instead, it can only reoptimize with a specific probability \((1 - \gamma_w)\). In periods in which the household cannot renegotiate, it follows an indexation rule \(\bar{W}_t = \bar{\pi}_{w,t} W_{t-1}\), with
\[
\bar{\pi}_{w,t} = (\pi_{t-1})^{\xi_w} \left( \pi_t \right)^{1-\xi_w} \left( z_t \right)^{\xi_z} \left( z_s \right)^{1-\xi_z},
\]
where \(\xi_w\) is the weighting parameter for the past rate of inflation and \(\xi_z\) the weighting parameter for the shock to the growth rate of technology \(z_t\). Related to this, \(z_s\) is the steady-state growth rate of a non-stationary productivity shock. Furthermore, the (stationary) real wage is defined as
\[
\tilde{w}_t \equiv \frac{W_t}{Z_t P_t}
\]
and the growth rate of nominal wages by taking technology growth into account becomes
\[
\pi_{w,t+1} \equiv \pi_{t+1} \tilde{z}_{t+1} \frac{\tilde{w}_{t+1}}{\tilde{w}_t}.
\]
A labor agency is introduced that buys differentiated labor from households and pays the individual wage in order to produce a representative labor aggregate as output

\[ N_t = \left[ \int_0^1 N_{h,t} \frac{1}{\lambda w} dh \right]^{\lambda w}, \]  

(40)

where \( \lambda w \) is the degree of substitution and represents the markup of the wage over the household’s marginal rate of substitution. By minimizing the costs of producing this aggregator, the labor agency takes the wage rates of each differentiated labor input as given. From this optimization problem there follows the demand for labor of household \( h \) for use in goods production

\[ N_{h,t} = N_t \left( \frac{W_{h,t}}{W_t} \right)^{\lambda w}. \]  

(41)

By combining equations (40) and (41), one obtains the aggregate wage index

\[ W_t = \left[ \int_0^1 W_{h,t} \frac{1}{\lambda w} dh \right]^{1-\lambda w}. \]  

(42)

With the knowledge of demand for its labor, the household can proceed with determining the optimal wage rate \( (W^*_{h,t}) \) and the optimal labor supply \( (N^*_{h,t}) \). Thus, it maximizes

\[ \max_{\{W_{h,t}\}} E_t \sum_{s=0}^{\infty} (\beta \gamma w)^s \left[ -\kappa \nu_t N^*_{h,t+s} \left( N^*_{h,t+s} \right)^{1+\varphi} \right. \]

\[ + \varphi \]  

\( \Psi_{t+s}^w \left( 1-\tau w \right) P_{t+s} \]  

\[ \left. \left( 1+\varphi \right) \right] \left( \frac{1-\tau w}{P_{t+s}} \right) N^*_{h,t+s} \]  

(43)

by making use of equation (41). The term \( \varphi \) reflects the inverse Frisch elasticity. The term \( \Psi_{t+s}^w \) in equation (43) corrects the nominal wage for inflation and technology growth, i.e., \( \Psi_{t+s}^w = (\prod_{s=0}^{\infty} \pi_{t-1+s}^s)^{\xi w} \) \( (\pi^s)^{(1-\xi w)} \) \( (\prod_{s=0}^{\infty} \frac{Z_{t-1+s}}{Z_{t-2+s}})^{\zeta w} \) \( (\prod_{s=0}^{\infty} \tilde{\pi}_{w,t+s}^s) \). Before utility maximization is carried out, the optimal nominal wage emerges from a sub-problem in which the household minimizes its disutility of labor by choosing its nominal wage given the labor
demand of firms. With definitions \( \Psi^{w}_{t+s} \equiv \frac{\Psi^{w}_{t+s}}{P_{t+s} Z_{t+s}} \) and \( \lambda_{h,t} \equiv \lambda_{h,t} Z_{t} \), one obtains for the optimal (stationary) real wage \( (\tilde{w}_{h,t}^{*}) \)

\[
\frac{\partial}{\partial W} (\tilde{w}_{h,t}^{*}) = \frac{E_{t} \sum_{s=0}^{\infty} (\beta \gamma)^{s} \left[ \lambda_{h,t+s} Z_{t+s} \left( 1 - \tau^{N} \right) \tilde{w}_{t+s} \right]}{E_{t} \sum_{s=0}^{\infty} (\beta \gamma)^{s} \left[ \lambda_{h,t+s} (1 - \tau^{N}) \tilde{w}_{t+s} \right] 1 - \lambda_{w} (1 + \varphi)} N_{t+s} \left( \frac{1}{\lambda_{w}} \right),
\]

where the term \( \lambda_{w} \) is the wage markup and \( \varphi \) reflects the inverse Frisch elasticity. The optimal real wage can be expressed as

\[
\tilde{w}_{t}^{*} = \left( \lambda_{w} \kappa NW_{t,t} \right) \frac{1 - \lambda_{w}}{1 - \lambda_{w}^{1 + \varphi}},
\]

(44)

with

\[
NW_{t} = \nu_{t+s}^{N} N_{t}^{1 + \varphi} + (\beta \gamma) \left( \frac{\pi_{w,t+1}}{\pi_{w,t+1}} \right) 1 - \lambda_{w} \left( \frac{1}{1 - \lambda_{w}} \right) NW_{t+1}
\]

and

\[
DW_{t} = \tilde{\lambda}_{t} (1 - \tau^{N}) N_{t} + \gamma_{w} / \beta \left( \frac{\tilde{w}_{t+1}}{\tilde{w}_{t}} \right) 1 + \lambda_{w} \left( \frac{1}{1 - \lambda_{w}} \right) DW_{t+1}.
\]

The law of motion for the real wage is

\[
\frac{1}{w_{t}^{1 - \lambda_{w}}} = (1 - \gamma_{w}) w^{*}_{t} \frac{1}{1 - \lambda_{w}} + \gamma_{w} \left( \frac{\tilde{w}_{t}}{\tilde{w}_{t}} \right) \frac{1}{1 - \lambda_{w}}.
\]

A.2 Intermediate Goods Firms

Before intermediate goods firms maximize profits, they solve the sub-problem and minimize costs

\[
\min_{\{K^{A}_{i,t}, K^{B}_{i,t}, \tilde{N}_{i,t}\}} \left( r^{k,A}_{t} K^{A}_{i,t} + r^{k,B}_{t} K^{B}_{i,t} + w_{t} \tilde{N}_{i,t} \right)
\]

subject to the production function

\[
Y_{i,t} = A_{t} \left( \tilde{K}_{i,t} \right)^{\alpha} \left( Z_{t} \tilde{N}_{i,t} \right)^{1 - \alpha} - Z_{t} \Omega_{i},
\]

(45)
and
\[
\tilde{K}_{i,t} = \left( \left( \zeta^K \right)^{1/K} \left( \tilde{K}_{i,t}^A \right)^{\gamma_{K-1}/\gamma_K} + (1 - \zeta^K) \left( \tilde{K}_{i,t}^B \right)^{\gamma_{K-1}/\gamma_K} \right)^{\gamma_K/\gamma_{K-1}}.
\]

The term represents a stationary technology shock, i.e., a shock to total factor productivity which is an AR(1) process
\[
\log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t}, \quad \epsilon_{A,t} \sim N \left( 0, \sigma_A^2 \right)
\]
while \( Z_t \) is a labor-augmenting technology process with a stationary growth rate \( z_t \equiv Z_t - Z_{t-1} \):
\[
\log \left( z_t \right) = \left( 1 - \rho_z \right) \log z_s + \rho_z \log \left( z_{t-1} \right) + \epsilon_{z,t}, \quad \epsilon_{z,t} \sim N \left( 0, \sigma_z^2 \right).
\]
The first-order conditions are
\[
\frac{\partial}{\partial \tilde{K}^A_{i,t}} : r^A_{t,A} - \varrho_{i,t} A_t \alpha \left( \left( \zeta^K \right)^{1/K} \left( \tilde{K}_{i,t}^A \right)^{\gamma_{K-1}/\gamma_K} \right) + (1 - \zeta^K) \left( \tilde{K}_{i,t}^B \right)^{\gamma_{K-1}/\gamma_K} \right)^{\gamma_K/\gamma_{K-1}} = 0;
\]
\[
\frac{\partial}{\partial \tilde{K}^B_{i,t}} : r^B_{t,B} - \varrho_{i,t} A_t \alpha \left( \left( \zeta^K \right)^{1/K} \left( \tilde{K}_{i,t}^B \right)^{\gamma_{K-1}/\gamma_K} \right) + (1 - \zeta^K) \left( \tilde{K}_{i,t}^A \right)^{\gamma_{K-1}/\gamma_K} \right)^{\gamma_K/\gamma_{K-1}} = 0;
\]
\[
\frac{\partial}{\partial N_{i,t}} : w_t - \varrho_{i,t} A_t \alpha \left( \left( \zeta^K \right)^{1/K} \left( \tilde{K}_{i,t}^A \right)^{\gamma_{K-1}/\gamma_K} \right) + (1 - \zeta^K) \left( \tilde{K}_{i,t}^B \right)^{\gamma_{K-1}/\gamma_K} \right)^{\gamma_K/\gamma_{K-1}} = 0,
\]
\[
\frac{\partial}{\partial \varrho_{i,t}} = A_t \left[ \left( \zeta^K \right)^{\frac{1}{\gamma_K}} \left( \tilde{K}_{i,t}^A \right)^{\frac{K-1}{\gamma_K}} + \left( 1 - \zeta^K \right)^{\frac{1}{\gamma_K}} \left( \tilde{K}_{i,t}^B \right)^{\frac{K-1}{\gamma_K}} \right]^{\frac{\alpha K}{\gamma_K - 1}} \\
\times (Z_t N_{i,t})^{1-\alpha} - Z_t \Omega - Y_{i,t} = 0.
\] (52)

In equations (46), (48), (50), and (52), \( \varrho_{i,t} \) is the Lagrangian multiplier related to the production function. By combining the derived conditions, I obtain

\[
\frac{\tilde{K}_{i,t}^A}{N_{i,t}} = \frac{\alpha}{(1-\alpha)} \left( \zeta^K \right)^{\frac{1}{\gamma_K}} + \left( 1 - \zeta^K \right)^{\frac{1}{\gamma_K}} \left( \frac{1 - \zeta^K}{\zeta^K} \right)^{\frac{K-1}{\gamma_K}} \\
\times \left( \frac{r_t^{k,A}}{r_t^{k,B}} \right)^{\gamma_K - 1} \left( \zeta^K \right)^{\frac{1}{\gamma_K}} \left( \frac{w_t}{r_t^{k,A}} \right). 
\] (53)

and

\[
\frac{\tilde{K}_{i,t}^B}{N_{i,t}} = \frac{\alpha}{(1-\alpha)} \left( \zeta^K \right)^{\frac{1}{\gamma_K}} \left( \frac{\zeta^K}{1 - \zeta^K} \right) \left( \frac{r_t^{k,B}}{r_t^{k,A}} \right)^{\gamma_K} \\
\times \left( \zeta^K \right)^{\frac{1}{\gamma_K}} \left( 1 - \zeta^K \right)^{\frac{1}{\gamma_K}} \left( \frac{1}{r_t^{k,B}} \right) \left( \frac{w_t}{r_t^{k,A}} \right). 
\] (54)

The capital-to-capital ratio is

\[
\frac{\tilde{K}_{i,t}^B}{\tilde{K}_{i,t}^A} = \left( \frac{1 - \zeta^K}{\zeta^K} \right) \left( \frac{r_t^{k,A}}{r_t^{k,B}} \right)^{\gamma_K}. 
\] (55)

By integration over all \( i \) individuals, it is easy to see that all indexes can be dropped. After expressing equations (53) and (54) in terms of the other respective variables (\( K_{i,t}^A, K_{i,t}^B, \) and \( N_t \)), including the resulting expressions in the production function, and solving for the other variables which are included in the cost function, an expression for the marginal costs can be derived:
\[
\begin{align*}
\left( \frac{r_{k,A}}{t} \right)^{\alpha} \left( \frac{\alpha}{1-\alpha} \right) \left( \zeta^k \right)^{\gamma_K} + (1 - \zeta^k)^{\frac{1}{\gamma_K}} \\
\times \left( \frac{1-\zeta^k}{\zeta^k} \right)^{\gamma_K-1} \left( \frac{r_{k,A}}{t - \alpha} \right) \left( \frac{r_{k,B}}{t - \alpha} \right) \left( \zeta^k \right)^{\gamma_K-1} \left( \frac{1}{\zeta^k} \right)^{\frac{1}{\gamma_K}} \\
+ \left( \frac{r_{k,B}}{t} \right)^{\alpha} \left( \frac{\alpha}{1-\alpha} \right) \left( \zeta^k \right)^{\gamma_K} \left( \frac{(r_{k,B} - r_{k,A})}{1-\zeta^k} \right) \left( \frac{1}{\zeta^k} \right)^{\gamma_K},
\end{align*}
\]

(56)

\[
mc_t = \left( \zeta^k \right)^{\gamma_K} \left( \left( \zeta^k \right)^{\gamma_K} + (1 - \zeta^k)^{\frac{1}{\gamma_K}} \left( \frac{1-\zeta^k}{\zeta^k} \right)^{\gamma_K-1} \left( \frac{1}{\zeta^k} \right)^{\frac{1}{\gamma_K}} \right)^{-\frac{\alpha}{\gamma^k}} \left( \frac{1}{\gamma^k - 1} \right) \left( \frac{1}{\gamma^k - 1} \right)^{\frac{1}{\gamma^k}} \left( \frac{1}{\gamma^k - 1} \right)^{\frac{1}{\gamma^k}} \left( \frac{1}{\gamma^k - 1} \right)^{\frac{1}{\gamma^k}}.
\]

(57)

Intermediate goods firms \( i \) maximize their profits

\[
\max_{\{P_{i,t}^*\}} E_t \sum_{j=0}^{\infty} \beta^j \gamma^j \left[ Y_{i,t} \left( P_{i,t}^* - mc_{i,t+j} P_{t+j} \right) \right]
\]

(58)

subject to

\[
Y_{i,t} = Y_t \left( \frac{P_{i,t}}{P_t} \right)^{\frac{\lambda_p}{1-\alpha}}.
\]

From profit maximization one obtains

\[
\partial / \partial P_{i,t}^* : P_{i,t}^* = \lambda_{p,t} \frac{E_t \sum_{k=0}^{\infty} \beta^{k+1} P_{i,t+k} Y_{i,t+k} m c_{i,t+k}}{E_t \sum_{k=0}^{\infty} \beta^{k+1} Y_{i,t+k}};
\]

(59)

where \( \psi_{i,t+k} \) captures the price indexation, i.e., \((\pi_t)_{1-\xi} (\pi_t)^{1-\xi}\) with indexation parameter \( \xi \), and \( mc_t \) the marginal costs. Equation (59)
can be rewritten with $\pi_{i,t}^* \equiv \frac{P_{i,t}^*}{P_t}$

$$\pi_{i,t}^* = \lambda_{p,t} \frac{NP_{i,t}}{DP_{i,t}}, \quad (60)$$

whereas

$$NP_{i,t} = mc_{i,t} \lambda_t Y_t + \beta \gamma E_t \left( \frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^\lambda_{p,t+1} NP_{i,t+1}, \quad (61)$$

$$DP_{i,t} = \lambda_t Y_t + \beta \gamma E_t \left( \frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{1-\lambda_{p,t+1}} DP_{i,t+1}, \quad (62)$$

$$\tilde{\pi}_t = \pi_{t-1}^{\xi} \pi^{1-\xi}_s, \quad (63)$$

and

$$1 = \gamma \left( \frac{\tilde{\pi}_t}{\pi_t} \right)^{1-\lambda_{p,t}} + (1 - \gamma) \pi_t^{\lambda_{p,t}} \pi_{t-1}^{\frac{1}{1-\lambda_{p,t}}}, \quad (64)$$

whereas $P_{i,t}^* = P_t^*$. Since all firms that can adjust the price optimally have the same optimum, I can drop the indexes.

### A.3 Final Goods Firms

Final goods producers maximize profits

$$\max_{\{Y_{i,t}\}} P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} \, di \quad (65)$$

with

$$Y_t = \left[ \int_0^1 Y_{i,t}^{\lambda_{p,t}} \, di \right]^{\frac{1}{\lambda_{p,t}}} \quad (66)$$

and the demand for good $i$ results as

$$Y_{i,t} = Y_t \left( \frac{P_{i,t}}{P_t} \right)^{\lambda_{p,t}} \quad (67)$$

with which help the price aggregator can be derived:

$$P_t = \left[ \int_0^1 P_{i,t}^{\frac{1}{1-\lambda_{p,t}}} \, di \right]^{1-\lambda_{p,t}} \quad (68)$$
A.4 Capital Goods Producers

Capital producers maximize their profits

$$\max_{\{I^e_t, I^\mu_t\}} E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} D i v_{t+j}^I$$

with

$$\Psi \left( \frac{I^e_t}{I^e_{t-1}} \right) = \frac{1}{2} \left[ \exp \left[ \sqrt{\Psi''} \left( \frac{I^e_t}{I^e_{t-1}} - 1 \right) \right] + \exp \left[ -\sqrt{\Psi''} \left( \frac{I^e_t}{I^e_{t-1}} - 1 \right) - 2 \right] \right]. \quad (69)$$

whereas \( e \in [A, B] \), \( \Psi(1) = \Psi'(1) = 0 \), and \( \Psi'' > 0 \) are satisfied. This results in

$$Q^K_{t,e} = 1 - E_t \left( \frac{b_{t+1} \lambda_{t+1} \beta Q^e_{t+1}}{b_{t} \lambda_{t}} \right) \nu \left( \frac{E_t(I^e_{t+1})}{I^e_t} \right)^2 \left( \frac{E_t(I^e_{t+1})}{I^e_t} - 1 \right)$$

$$\left( 1 - \frac{\nu}{2} \left( \frac{I^e_t}{I^e_{t-1}} - 1 \right)^2 - \frac{\nu I^e_t}{I^e_{t-1}} \right). \quad (70)$$

The law of motion for capital is

$$K^e_t = K^e_{t-1} (1 - \delta^e) + I^e_t \left[ 1 - \Psi \left( \frac{I^e_t}{I^e_{t-1}} \right) \right] \mu_{I,t}, \quad (71)$$

where \( \mu_{I,t} \) is a shock to the marginal efficiency of investment and follows an AR(1) process

$$\log \mu_{I,t} = \rho_I \log \mu_{I,t-1} + \epsilon_{I,t} \epsilon_{I,t} \sim N \left( 0, \sigma_I^2 \right).$$

The shock hits both sectors at the same time.

A.5 Entrepreneurs

Entrepreneurs borrow from financial intermediaries and combine the external funds with internal funds (net worth). They conduct capital processing, which means that they buy capital and transform it with their own individual skill into new units of capital. The skills
are randomly and independently distributed over time and across entrepreneurs. As a consequence, the shock also affects the realized return on capital, as denoted by $R_{k,e}^{m,t}$ and given in

$$1 + R_{k,e}^{m,t} = \pi_t \frac{(1 - \tau^K) \left( r_{k,e}^{m,t} u_t^e - \Gamma(u_{m,t}^e) \right) + Q_t^e (1 - \delta^e) + \tau^K \delta^e Q_t^e}{Q_{t-1}^e} \omega_{m,t}^e$$

$$= (1 + R_{k,e}^{m,t}) \omega_{m,t}^e. \tag{72}$$

Since the shock controls the repayment capacity of debt, there is a value for $\omega_{m,t}^e$ (which shall be denoted with $\omega_{m,t}^e + 1$) below which defaults occur. With the contractual rate $Z_t^e$, this threshold becomes

$$\omega_{m,t}^e = \frac{Z_{t-1}^e I_{m,t-1}^e}{(1 + R_{k,e}^{m,t}) K_{m,t-1}^e}. \tag{73}$$

The expected earnings of the $m$-th entrepreneur ($E_{m,t}^e$) can be calculated based upon the expected capital return and the ex ante productivity threshold as

$$E_{m,t}^e = E_t \left\{ \left( \int_{\omega_{m,t+1}^e}^\infty \omega^e dF(\omega^e) \right) \left( 1 - \frac{1}{[1 - F(\omega_{m,t+1}^e)] \omega_{m,t+1}^e} \right) \right\} \left( 1 + R_{k,e}^{m,t+1} \right) K_{m,t+1}^e. \tag{74}$$

The first term on the right-hand side characterizes the expected earnings from the project by taking all realizations for $\omega_{m,t+1}^e \geq \omega_{m,t+1}^e$ into account, and the second term on the right-hand side reflects the payments to satisfy the debt contract. For $\omega_{m,t+1}^e < \omega_{m,t+1}^e$, the entrepreneur would be left with no earnings. The function $F(\omega_{m,t+1}^e)$ in equation (74) is the cumulative density function for realization of $\omega_{m,t}^e$, which means that its value for $\omega_{m,t+1}^e$ is the related ex ante default probability.

$$E_t \left( \frac{\omega_{m,t+1}^e}{(1 + \omega_{m,t+1}^e)} \right) \left( 1 + E_t \left( R_{k,e}^{m,t+1} \right) \right) Q_t^e K_{m,t}^e = Z_t^e L_{m,t}^e \tag{74}$$

Given the expected gross return of the project and its value as well as the borrowed amount, this threshold is linked to the default-free risky bond rate with $Z_t^e$ as the gross contract rate. The term
\( \omega_{m,t} \) denotes the realized threshold value, while \( E_t(\omega_{m,t+1}) \) is its expected value.\(^2\)

One can make use of the following definitions. The expected profits of the financial intermediary related to the realization of the productivity shock can be expressed as

\[
\Theta(\omega) = \omega \int_{\omega}^{\infty} f(\omega) d\omega + \int_{0}^{\omega} \omega f(\omega) d\omega.
\]

The first term on the right-hand side is the return stemming from all non-default cases, from which contractual payments result, while the expected payoff in the case of defaults is captured by

\[
G(\omega) = \int_{0}^{\omega} \omega f(\omega) d\omega,
\]

which is the last term on the right-hand side. Consequently, expected monitoring costs of the financial intermediary are

\[
\mu G(\omega) = \mu \int_{0}^{\omega} \omega f(\omega) d\omega.
\]

The expected net profits of financial intermediaries after paying for monitoring become \( \Theta(\omega) - \mu G(\omega) \). The expected profits of the entrepreneurs from all no-default cases are \( 1 - \Theta(\omega) \). Furthermore, I have the definitions

\[
\Theta_{\omega}(\omega) = 1 - F(\omega) = G_{\omega}(\omega) = \omega f(\omega) = f(\omega) = F_{\omega}(\omega),
\]

whereas \( X_Y \) is the first derivative of \( X \) with respect to \( Y \).

The debt contract for the lenders reveals that the lenders want to earn as much as they receive by investing in a risk-free asset. That is the reason why lenders’ opportunity costs must be equal to

\(^2\)An important difference between the original Bernanke, Gertler, and Gilchrist (1999) (BGG) setting and my setting is that the intermediaries expect zero profits in the model, while in the BGG model zero profits hold every period (see, for instance, Benes and Kumhof 2015).
the risk-free rate. Where the idiosyncratic shock exceeds the cutoff value, the lender receives the contractual interest payments. The converse probability of the default probability at the cutoff value of the shocks yields the probability of contractual payments. For the range of realizations of the shocks that are below the ex ante cutoff value $\omega_{m,t}$, the assets of the borrower are expected to be liquidated in order to partly redeem the debt contract. Before collecting the remaining assets, the lender has to observe the state of the borrower. Information is asymmetrically distributed, however. While the entrepreneur can always assess its situation, the financial intermediary cannot observe the state of the entrepreneur at no charge. As a consequence, the creditor has to pay transaction costs, which lower its repayments in the case of a default. It is assumed that the transaction costs are proportional to the realizable assets. These considerations are summarized in equation (75)

$$
[1 - F \left( E_t (\omega_{m,t+1}) \right)] \ Z_{m,t}^e L_{m,t}^e \\
+ (1 - \mu^e) \int_0^{E_t(\omega_{m,t+1})} \omega^e \left( 1 + E_t \left( R_{t+1}^{k,e} \right) \right) Q_t^e K_{m,t}^e dF (\omega^e) \\
= (1 + r_t^e) L_{m,t}^e 
$$

or

$$
[1 - F \left( E_t (\omega_{m,t+1}) \right)] \ E_t (\omega_{m,t+1}) \left( 1 + E_t \left( R_{t+1}^{k,e} \right) \right) Q_t^e K_{m,t}^e \\
+ (1 - \mu^e) \int_0^{E_t(\omega_{m,t+1})} \omega^e \left( 1 + E_t \left( R_{t+1}^{k,e} \right) \right) Q_t^e K_{m,t}^e dF (\omega^e) \\
= (1 + r_t^e) L_{m,t}^e
$$

with $r_t^A = E_t \left( r_{t+1}^{B,corp} \right)$ and $r_t^B = r_t^L$.

Entrepreneurs have a long-run perspective and maximize the expected utility of continuing their business by choosing the quantity of capital $K_{m,t}^e$ and selecting the expected productivity threshold below which they default $E_t (\omega_{m,t+1})$.

$$V_{m,t}^{E,e} = \max_{\left\{ K_{m,t}^e, \omega_{m,t+1}^e \right\}} \ E_t \left[ \sum_{i=1}^{\infty} \Lambda_{t,t+i} \left( 1 - p_t^{E,e} \right) \left( p_t^{E,e} \right)^{i-1} \Pi_{m,t+i}^{E,e} \right],$$
whereas $p_{t}^{E,e}$ is the probability that an entrepreneur stays in business, and $\Pi_{t+i}^{E,e}$ are the terminal funds available for exiting entrepreneurs at $t+i$ which is simply their net worth at that period in time, i.e., $\Pi_{m,t}^{E,e} = NW_{m,t}^{E,e}$. The terminal funds are redistributed to the households. The variable $\Lambda_{t,t+j}$ represents the discount factor which is households’ pricing kernel $\beta^{\frac{\lambda_{t+j}}{M}}$. The (expected) profits of entrepreneur (in nominal terms) are

$$
(1 - \Theta \left(E_{t} (\omega_{m,t+1}^{e})\right)) E_{t} \left(\bar{R}_{t+1}^{k,e}\right) Q_{m,t}^{e} K_{m,t}^{e},
$$

while the (expected) profits for intermediaries (in nominal terms) are

$$
(\Theta \left(E_{t} (\omega_{m,t+1}^{e})\right) - \mu^{e} G \left(E_{t} (\omega_{m,t+1}^{e})\right)) E_{t} \left(\bar{R}_{t+1}^{k,e}\right) Q_{m,t}^{e} K_{m,t}^{e},
$$

where I used the gross return on capital $E_{t} \bar{R}_{t+1}^{k,e} = E_{t} \left(1 + R_{t+1}^{k,e}\right)$. The participation constraint for intermediaries in real terms becomes

$$
(\Theta \left(E_{t} (\omega_{m,t+1}^{e})\right)) - \mu^{e} G \left(E_{t} (\omega_{m,t+1}^{e})\right)) \frac{\bar{R}_{t+1}^{k,e}}{\pi_{t+1}} Q_{m,t}^{e} K_{m,t}^{e} \geq \frac{R_{t+1}^{e}}{\pi_{t+1}} L_{m,t}^{e}.
$$

The value function in recursive form becomes

$$
V_{m,t}^{E,e} = \left(1 - p_{t}^{E,e}\right) NW_{m,t}^{E,e} + E_{t} \Lambda_{t,t+1} p_{t}^{E,e} V_{m,t+1}^{E,e}.
$$

(77)

Next, I solve the optimization problem and follow Carlstrom, Fuerst, and Paustian (2016). First, I guess that the individual value function is a linear combination of an aggregate value function multiplied by individual net worth

$$
V_{m,t}^{E,e} = V_{t}^{E,e} NW_{m,t}^{E,e}.
$$

With its help, equation (77) can be rewritten to obtain

$$
V_{t}^{E,e} NW_{m,t}^{E,e} = \left(1 - p_{t}^{E,e}\right) NW_{m,t}^{E,e} + \Lambda_{t,t+1} p_{t}^{E,e} E_{t} V_{t+1}^{E,e} NW_{m,t+1}^{E,e}.
$$
The maximization problem becomes

\[ \max_{\{\kappa_{m,t}, \bar{\omega}_{m,t+1}\}} V_t^{E,e} \]

\[ = E_t \left[ (1 - p_t^{E,e}) + \Lambda_{t,t+1} p_t^{E,e} \left(1 - \Gamma (\bar{\omega}_{m,t+1})\right) \frac{\tilde{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t} V_{t+1}^{E,e} \right] \]

s.t. \( E_t \left[ (\Theta(\bar{\omega}_{m,t+1}) - \mu^e G(\bar{\omega}_{m,t+1})) \frac{\tilde{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t} N W_{m,t}^{E,e} \right] \]

\[ \geq \frac{R_t^e}{\pi_{t+1}} (\kappa_{m,t} - 1) N W_{m,t}^{E,e} \]

where \( \kappa_{m,t} = \frac{Q_t^e \kappa_{m,t}^{n,e}}{N W_{m,t}^{E,e}} \) is the leverage ratio.

The first-order conditions with \( \phi_{m,t} \) as a Lagrangian multiplier are

\[ \frac{\partial}{\partial \omega_{m,t}^e} : -E_t \left[ \Lambda_{t,t+1} p_t^{E,e} \Theta_\omega (\bar{\omega}_{m,t+1}) \frac{\tilde{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t} V_{t+1}^{E,e} \right] \]

\[ + \phi_{m,t+1}^e \left( \Theta_\omega (\bar{\omega}_{m,t+1}) - \mu^e G_\omega (\bar{\omega}_{m,t+1}) \frac{\tilde{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t} \right) = 0 \] (78)

\[ \frac{\partial}{\partial \kappa_{m,t}^e} : E_t \left[ \Lambda_{t,t+1} p_t^{E,e} \left(1 - \Theta (\bar{\omega}_{m,t+1})\right) \frac{\tilde{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t} V_{t+1}^{E,e} \right] \]

\[ + \phi_{m,t}^e \left( \Theta(\bar{\omega}_{m,t+1}) - \mu^e G(\bar{\omega}_{m,t+1}) \frac{\tilde{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t} \right) \Lambda_{t,t+1} = 0 \] (79)

\[ \frac{\partial}{\partial \phi_{m,t}^e} : E_t \left[ (\Theta(\bar{\omega}_{m,t+1}) - \mu^e G(\bar{\omega}_{m,t+1})) \frac{\tilde{R}_{t+1}^{k,e}}{\pi_{t+1}} \kappa_{m,t}^{n,e} \right] \]

\[ - \frac{R_t^e}{\pi_{t+1}} (\kappa_{m,t}^e - 1) = 0. \] (80)

The equations to include in the model are

\[ E_t \left[ (\Theta(\bar{\omega}_{m,t+1}) - \mu^e G(\bar{\omega}_{m,t+1})) \frac{\tilde{R}_{t+1}^{k,e}}{\pi_{t+1}} \right] = \frac{\kappa_t^e - 1}{\kappa_t^e} \] (81)
\[
0 = E_t \left[ \Lambda_{t,t+1} p_{t}^{E,e} (1 - \Theta (\bar{\omega}^{e}_{m,t+1})) \frac{\bar{R}_{t+1}^{k,e}}{R_{t+1}^{e}} V_{t+1}^{E,e} 
\right. \\
\left. + \Lambda_{t,t+1} p_{t}^{E,e} \Theta \omega (\bar{\omega}^{e}_{m,t+1}) V_{t+1}^{E,e} 
\right. \\
\left. \times \left( \Theta (\bar{\omega}^{e}_{m,t+1}) - \mu^{e} G_{\omega} (\bar{\omega}^{e}_{m,t+1}) \right) \frac{\bar{R}_{t+1}^{k,e}}{R_{t+1}^{e}} - 1 \right] \tag{82}
\]

\[
NW_{t}^{E,e} = (1 - \Theta (\bar{\omega}^{e}_{m,t})) \frac{\bar{R}_{t}^{k,e}}{\pi_{t}} Q_{t-1}^{e} K_{t-1}^{e} \\
= (1 - \Theta (\bar{\omega}^{e}_{m,t})) \frac{\bar{R}_{t}^{k,e}}{\pi_{t}} \kappa_{t-1}^{e} NW_{t-1}^{E,e} \tag{83}
\]

\[
V_{t}^{E,e} = \left(1 - p_{t}^{E,e}\right) + E_t \Lambda_{t,t+1} p_{t}^{E,e} (1 - \Theta (\bar{\omega}^{e}_{m,t+1})) \frac{\bar{R}_{t+1}^{k,e}}{\pi_{t+1}} V_{t+1}^{E,e} \kappa_{t}^{e}. \tag{84}
\]

The external finance premium (or credit spread) can be defined as

\[
FP_{t}^{e} = \frac{E_{t} \left( \bar{R}_{t+1}^{k,e} \right)}{E_{t} \left( R_{t+1}^{e} \right)},
\]

with \(FP_{t}^{A} = \frac{1+E_{t} \left( R_{t+1}^{k,e} \right)}{1+E_{t} \left( r_{t+1}^{e} \right)}\) for type A and \(FP_{t}^{B} = \frac{1+E_{t} \left( R_{t+1}^{k,e} \right)}{1+r_{t}^{e}}\) for type B entrepreneurs. Ex post losses can occur if the realization of the shock leaves the realized capital return below its expected value, so that the risky contract rate is not sufficient to compensate the intermediaries for all defaults.

\[
\Upsilon_{t}^{B,e} \approx \left(1 - \mu^{f,e}\right) \left( K_{t-1}^{e} Q_{t-1}^{e} G_{\omega} (\bar{\omega}_{t}^{e}) \left(1 + E_{t-1} (R_{t}^{k,e})\right) \right) \\
- K_{t-1}^{e} Q_{t-1}^{e} \left(1 + R_{t}^{k,e}\right) G_{\omega} (\bar{\omega}_{t}^{e}) \tag{85}
\]

The losses can be split into two parts: the additional losses, because the realized default rate \(F(\bar{\omega}_{t}^{e})\) is above its ex ante value \(F (E_{t-1} (\bar{\omega}_{t}^{e}))\) (upper line on the right-hand side), and the reduced
amount of realizable assets through defaults (lower line on the right-hand side). To complete the description of the financial contracting between the entrepreneurs and the financial intermediaries, I introduce a time-varying standard deviation $\sigma^e_t$ regarding the distribution of the productivity parameter in each sector by assuming an AR(1) process for the standard deviation in every sector:

$$\log \sigma^e_t = (1 - \rho^{\sigma,e}) \log \sigma^e_{t-1} + \epsilon^{e}_{\sigma,t} e^{e}_{\sigma,t} \sim N \left(0, \sigma^2_{\sigma,e}\right), e \in (A, B).$$

Thus, I have $F(\bar{\omega}^e; \sigma^e_{t-1})$, $\Theta(\bar{\omega}^e_{m,t}; \sigma^e_{t-1})$, and $G(\bar{\omega}^e_{m,t}; \sigma^e_{t-1})$, which is also applicable to the first derivatives.

After choosing the amount of capital and the (expected) cutoff point given the corresponding intermediaries’ expected zero-profit conditions, entrepreneurs decide on the capital utilization. Based on the assembled capital, they supply to intermediate goods producers and they decide on the utilization of capital at the second state

$$\tilde{K}^{e}_{m,t+1} = u^e_{m,t} \tilde{K}^{e}_{m,t+1}. \quad (86)$$

They maximize the profits

$$\max_{\{u^e_{m,t}\}} \left[ r^{k,e}_{t} u^e_{m,t} - \Gamma(u^e_{m,t}) \right] K^{e}_{m,t+1+j}$$

given the costs

$$\Gamma(u^e_{m,t}) = \frac{r^{k,e}}{\psi^k} \left( \exp \left[ \psi^k \left( u^e_{m,t} - 1 \right) \right] - 1 \right). \quad (87)$$

In the optimum, the real cost of capital ($r^{k,e}_{t}$) is related to the adjustment costs on capital utilization ($u^e_{t}$)

$$\partial / \partial r^{k,e}_{t} : r^{k,e}_{t} = r^{k,e} \exp \left[ \psi^k \left( u^e_{t} - 1 \right) \right] \quad (88)$$

and is free of individual characteristics, i.e., every entrepreneur chooses the same utilization rate.

### A.6 Mutual Funds

In a decentralized market structure, different banks could earn different ex post returns (see equation (74)) because some banks hold
bonds issued by entrepreneurs that do not default, while some have purchased bonds from entrepreneurs that do. By introducing mutual funds, I easily circumvent this problem because I assume that mutual funds manage the market portfolio. Losses are redistributed to the bondholders via a reduced payoff.\footnote{This setting is discussed in greater depth in Kühl (2014).}

\section*{A.7 Banks}

The balance sheet constraint of the bank is

\begin{equation}
A^B_{n,t} = L_{n,t} + Q^B_{t}B^\text{corp}_{n,t} + Q^B_{t}B^\text{gov,}B_{n,t} = E^I_{n,t} + D_{n,t}
\end{equation}

and the law of motion for bank equity is

\begin{equation}
E^I_{n,t} = \left(1 + r^L_{t-1}\right)L_{n,t-1}\frac{1}{\pi_t} + \left(1 + r^B_{t,\text{corp}}\right)Q^B_{t-1}B^\text{corp}_{n,t-1}\frac{1}{\pi_t} \\
+ \left(1 + r^B_{t,\text{gov}}\right)Q^B_{t-1}B^\text{gov,}B_{n,t-1}\frac{1}{\pi_t} \\
- \left(1 + r^D_{t-1}\right)\frac{1}{\pi_t}D_{n,t-1}\frac{1}{\pi_t} - \gamma^L_{n,t} + \mu_{EI,t},
\end{equation}

which can be rewritten as

\begin{equation}
E^I_{n,t} = R^A_{n,t-1}A^B_{n,t-1}\frac{1}{\pi_t} - R^D_{t-1}D_{n,t-1}\frac{1}{\pi_t} - \gamma^L_{n,t} + \mu_{EI,t}
\end{equation}

by using with $R^A_t$ and $R^D_t$ the gross interest rates and defining $R^A_t = 1 + r^A_t$ as the average (gross) return on assets held by banks

\begin{equation}
1 + r^A_t = \left(1 + r^L_{t-1}\right)\varsigma^L_{n,t} + \left(1 + r^B_{t,\text{corp}}\right)\varsigma^B_{n,t} + \left(1 + r^B_{t,\text{gov}}\right)\varsigma^B_{n,t},
\end{equation}

with $\varsigma^L_{n,t} = \frac{L_{n,t-1}}{A^B_{n,t-1}}$, $\varsigma^B_{n,t} = \frac{Q^B_{t-1}B^\text{corp}_{n,t-1}}{A^B_{n,t-1}}$, $\varsigma^B_{n,t} = \frac{Q^B_{t-1}B^\text{gov,}B_{n,t-1}}{A^B_{n,t-1}}$, and $\varsigma^B_{n,t} = 1 - \varsigma^L_{n,t} - \varsigma^B_{n,t}$. The term $\mu_{EI,t}$ in equation (91) is a shock to bank equity and is assumed to follow the exogenous process

$$
\mu_{EI,t} = \epsilon_{EI,t}, \epsilon_{EI,t} \sim N\left(0, \sigma^2_{EI}\right).
$$
Bank managers maximize the franchise value of the bank, which is equivalent to their terminal available funds $\Pi_{n,t}^B = E_{n,t}^I$, by choosing the portfolio composition of their assets and the external funds

$$V_{n,t}^B = \max_{\{L_{n,t}, B_{n,t}^{corp}, B_{n,t}^{gov}, D_{n,t}\}} \sum_{i=1}^\infty \Lambda_{t,t+i} (1 - p^B) (p^B)^{i-1} E_{n,t+i}^I. \tag{93}$$

Bank managers like to divert a specific fraction of assets, from which the incentive constraint can be derived:

$$V_{n,t}^B \geq \theta^{IC} \left( L_{n,t} + \Delta B^{corp} Q_{t}^{B^{corp} B_{n,t}^{corp}} + \Delta B^{gov} Q_{t}^{B^{gov} B_{n,t}^{gov,B}} \right). \tag{94}$$

Similar to Gertler and Karadi (2013), the assets can be diverted to different degrees; $\Delta B^{corp}$ and $\Delta B^{gov}$ denote the specific relative shares which can be diverted related to corporate and government bonds, respectively. The franchise value of the banks can be expressed in a linear form, which is

$$V_{n,t}^B = v_t^L L_{n,t} + v_t^{B^{corp}} Q_{t}^{B^{corp} B_{n,t}^{corp}} + v_t^{B^{gov}} Q_{t}^{B^{gov} B_{n,t}^{gov,B}}$$

$$- \nu_t^D D_{n,t} - \eta_{n,t}^A E_{n,t}^I \tag{95}$$

or

$$V_{n,t}^B = v_t^L L_{n,t} + v_t^{B^{corp}} Q_{t}^{B^{corp} B_{n,t}^{corp}} + v_t^{B^{gov}} Q_{t}^{B^{gov} B_{n,t}^{gov,B}} + \eta_t E_{n,t}^I,$$

where $\eta_{n,t}^A$ catches the losses from the loan portfolio and is expressed in terms of banks’ equity. Thus, bank managers’ optimization problem is

$$\max_{\{L_{n,t}, B_{n,t}^{corp}, B_{n,t}^{gov}\}} v_t^L L_{n,t} + v_t^{B^{corp}} Q_{t}^{B^{corp} B_{n,t}^{corp}} + v_t^{B^{gov}} Q_{t}^{B^{gov} B_{n,t}^{gov,B}} + \eta_t E_{n,t}^I$$

$$\text{s.t. } V_{n,t}^B \geq \theta^{IC} \left( L_{n,t} + \Delta B^{corp} Q_{t}^{B^{corp} B_{n,t}^{corp}} + \Delta B^{gov} Q_{t}^{B^{gov} B_{n,t}^{gov,B}} \right) \Lambda_{n,t}^B.$$
The first-order conditions become

\[
\frac{\partial}{\partial L_{n,t}} : v_t^L = \frac{\lambda_{n,t}^{IC}}{(1 + \lambda_{n,t}^{IC})} \theta_L,
\]

\[
\frac{\partial}{\partial B_{n,t}^{corp}} : v_t^{B,corp} = \frac{\lambda_{n,t}^{IC}}{(1 + \lambda_{n,t}^{IC})} \theta^{IC} \Delta B_{n,t}^{corp},
\]

\[
\frac{\partial}{\partial B_{n,t}^{gov}} : v_t^{B,gov} = \frac{\lambda_{n,t}^{IC}}{(1 + \lambda_{n,t}^{IC})} \theta^{IC} \Delta B_{n,t}^{gov},
\]

and

\[
\frac{\partial}{\partial \lambda_{n,t}^{IC}} : \frac{A_{n,t}^{B}}{E_{n,t}^{L}} = \frac{\eta_t}{\theta^{IC} \left( \frac{L_{n,t} + \Delta B_{n,t}^{corp}}{E_{n,t}^{L}} + \Delta B_{n,t}^{gov} \frac{B_{n,t}^{gov}}{E_{n,t}^{L}} \right)} - \left( v_t^L \gamma_{n,t}^L + v_t^{B,corp} \gamma_{n,t}^{B,corp} + v_t^{B,gov} \gamma_{n,t}^{B,gov} \right),
\]

With the help of the method of undetermined coefficients, I get

\[
v_t^L = E_t \Omega_{t,t+1} A_{t,t+1} \left( R_t^L - R_t^D \right) \frac{1}{\pi_{t+1}},
\]

\[
v_t^{B,corp} = E_t \Omega_{t,t+1} A_{t,t+1} \left( R_{t+1}^{B,corp} - R_t^D \right) \frac{1}{\pi_{t+1}},
\]

\[
v_t^{B,gov} = E_t \Omega_{t,t+1} A_{t,t+1} \left( R_{t+1}^{B,gov} - R_t^D \right) \frac{1}{\pi_{t+1}},
\]

\[
\eta_t = E_t \Omega_{t,t+1} A_{t,t+1} \left( R_t^D - \frac{Y_{n,t}^L}{E_{n,t}^{L}} \right) \frac{1}{\pi_{t+1}},
\]

\[
v_t^L = E_t \Omega_{t,t+1} A_{t,t+1} R_t^L \frac{1}{\pi_{t+1}},
\]

\[
v_t^{B,corp} = E_t \Omega_{t,t+1} A_{t,t+1} R_{t+1}^{B,corp} \frac{1}{\pi_{t+1}},
\]

\[
v_t^{B,gov} = E_t \Omega_{t,t+1} A_{t,t+1} R_{t+1}^{B,gov} \frac{1}{\pi_{t+1}},
\]
\[ \nu_t^D = E_t \Omega_{t,t+1} \Lambda_{t,t+1} \left( R_t^D \right) \frac{1}{\pi_{t+1}}, \]

\[ \eta^A_{n,t} = E_t \Omega_{t,t+1} \Lambda_{t,t+1} \left( \frac{\Upsilon_{n,t}^L}{E_{n,t}} \right) \frac{1}{\pi_{t+1}}, \]

and

\[ \Omega_{t,t+1} = (1 - p^B) + p^B \left[ (v_{t+1}^L \Upsilon_{n,t+1}^L + v_{t+1}^{B,corp} \Upsilon_{n,t+1}^{B,corp} + v_{t+1}^{B,gov} \Upsilon_{n,t+1}^{B,gov}) \phi_{t+1}^B + \eta_{t+1} \right]. \]

A.8 Aggregation

By combining all first-order conditions of the banking sector, it can be shown that they are free from individual characteristics as long as \( \frac{\Upsilon_{n,t}^L}{E_{n,t}} \) is identical to all lending banks. One necessary condition for this to hold is that the Lagrangian multiplier for the enforcement constraint is identical across all individuals. It is easy to show by forward iteration and the validity of the transversality condition that the term is (nearly) identical to all individuals in the neighborhood of the steady state, i.e., as long as the sum of assets does not vary too much. Thus, I can drop all indexes.

Knowing that the leverage ratio \( \frac{A_t^B}{E_t^I} \) is identical to all lending banks, I see from the first-order conditions resulting from portfolio managers’ maximization problem that the portfolio shares related to the asset classes depend solely on the respective spreads between the (expected) lending rate and the expected return on government bonds, which is consequently identical to all individuals. Hence, the portfolio composition is the same across all lending banks. Thus, aggregation of quantities across the individuals can simply be conducted by integration.

For the sum of assets I get \( A_t^B = \int_0^1 A_{n,t}^B dn = \phi_t^I E_t^I \int_0^1 E_{n,t}^I dn = \phi_t^I E_t^I \) and for each asset class I get \( L_t = \int_0^1 L_{n,t} dn = \zeta_t^L \phi_t^I E_t^I, \) \( Q_t^{B,corp} B_t^{corp} = \int_0^1 Q_{n,t}^{B,corp} B_{n,t}^{corp} dn = \zeta_t^{B,corp} \phi_t^I E_t^I, \) and

---

This is a conventional assumption, particularly in models that work with collateral constraints; see Iacoviello (2005), for instance.
\[
Q_t^{B,\text{gov}} B_t^{\text{gov}} = \int_0^T Q_t^{B,\text{gov}} B_{n,t}^{\text{gov},B} \, dn = \zeta_t^{B,\text{gov}} \phi_t^I E_t^I, \quad \text{respectively. The aggregation of liabilities works similarly and the aggregate amount of external finance evolves as } D_t = (\phi_t^I - 1) \int_0^T E_{n,t}^I \, dn.
\]

**A.9 Exogenous Shocks**

- **Price markup shock:** It follows an exogenous stochastic process
  \[
  \log \lambda_{p,t} = (1 - \rho_{\lambda_p}) \log \lambda_p + \rho_{\lambda_p} \log \lambda_{p,t-1} + \epsilon_{\lambda_p},
  \]
  with \(\lambda_p\) as the steady-state value and \(\epsilon_{\lambda_p} \sim i.i.d. N(0, \sigma_{\lambda_p}^2)\).

- **Bank equity shock:**
  \[
  \mu_{EI,t} = \epsilon_{EI,t}.
  \]

- **Risk shock for type A entrepreneurs with innovations** \(\epsilon_{\sigma^A,t}\), where the variance of \(\log \omega\) is \(\sigma_t^2\):
  \[
  \log (\sigma_t^A) = (1 - \rho_{\sigma^A}) \log (\sigma^A) + \rho_{\sigma^A} \log (\sigma_{t-1}^A) + \epsilon_{\sigma^A,t}.
  \]

- **Risk shock for type B entrepreneurs with innovations** \(\epsilon_{\sigma^B,t}\), where the variance of \(\log \omega\) is \(\sigma_t^2\):
  \[
  \log (\sigma_t^B) = (1 - \rho_{\sigma^B}) \log (\sigma^B) + \rho_{\sigma^B} \log (\sigma_{t-1}^B) + \epsilon_{\sigma^B,t}.
  \]

- **Net worth shock for type A entrepreneurs with innovations** \(\epsilon_{p^E,A}\):
  \[
  \log \left( p_t^{E,A} \right) = \log \left( p_{s,t}^{E,A} \right) + \epsilon_{p^E,A}.
  \]

- **Net worth shock for type B entrepreneurs with innovations** \(\epsilon_{p^E,B}\):
  \[
  \log \left( p_t^{E,B} \right) = \log \left( p_{s,t}^{E,B} \right) + \epsilon_{p^E,B}.
  \]

- **Government expenditures:**
  \[
  \log G_t = (1 - \rho_G) \log G_{ss} + \rho_G \log G_{t-1} + \epsilon_{G,t}.
  \]

- **Investment-specific technology shock:**
  \[
  \log \mu_{I,t} = \rho_I \log \mu_{I,t-1} + \epsilon_{I,t}.
  \]
• Monetary policy shock: $\epsilon_{M,t}$.
• Stationary shock to total factor productivity:
  \[ \log A_t = \rho_A \log A_{t-1} + \epsilon_{A,t} \]
• Non-stationary shock to technology $Z_t$ with stationary growth rate:
  \[ \log (z_t) \equiv \log (Z_t/Z_{t-1}) = (1 - \rho_z) \log (z_s) + \rho_z \log (Z_{t-1}/Z_{t-2}) + \epsilon_{z,t}, \]
• Shock to labor supply:
  \[ \log \nu^N_t = \rho_N \log \nu^N_{t-1} + \epsilon_{N,t}. \]
• Shock to lump-sum taxes:
  \[ \mu_{T,t} = \rho_T \mu_{T,t-1} + \epsilon_{T,t}. \]
• Measurement errors on growth of entrepreneurs’ franchise value, $\epsilon_{V_{E,A}^{measurement}}$, the yield on government bonds $\epsilon_{B_{go,1}^{measurement}}$, and the volume of government bonds held by banks $\epsilon_{r_{B,go}^{measurement}}$.

B. Data

B.1 Observables

• GDP growth:
  \[ dGDP_{obs} = \log (I_t + C_t + G_t) - \log (I_{t-1} + C_{t-1} + G_{t-1}) + \log z_t - \log z_s \]
• Investment growth:
  \[ dI_{obs} = \log I_t - \log I_{t-1} + \log z_t - \log z_s \]
• Consumption growth:
  \[ dC_{obs} = \log C_t - \log C_{t-1} + \log z_t - \log z_s \]
• Rate of inflation:

\[ \pi_{\text{obs}} = (\pi_t - \pi_s) \times 400 \]

• Corporate bond rate:

\[ Z^A_{\text{obs}} = (Z^A_t - Z^A_s) \times 400 \]

• Loans rate:

\[ Z^B_{\text{obs}} = (Z^B_t - Z^B_s) \times 400 \]

• Policy rate:

\[ i_{\text{obs}} = (i_t - i_s) \times 400 \]

• Growth in real wages:

\[ dw_{\text{obs}} = \log w_t - \log w_{t-1} + \log z_t - \log z_s \]

• Growth in bank loans:

\[ dL_{\text{obs}} = \log L_t - \log L_{t-1} + \log z_t - \log z_s \]

• Total employment:

\[ E_{\text{obs}} = \log E_t - \log E_s \]

• Growth in franchise value of entrepreneurs in sector A:

\[ dV^{E,A}_{\text{obs}} = \log V^{E,A}_t - \log V^{E,A}_t + \log z_t - \log z_s + \epsilon_{V^{E,A}_t}^{\text{measurement}} \]

• Growth in franchise value of entrepreneurs:

\[ dV^E_{\text{obs}} = \log V^E_t - \log V^E_t + \log z_t - \log z_s + \epsilon_{V^E_t}^{\text{measurement}}, \]

with \( V^E_t = V^{E,A}_t + V^{E,B}_t \).

• Growth in bank holdings of corporate bonds:

\[ d \left( Q^{B,\text{corp}} B^{\text{corp}} \right)_{\text{obs}} = \log Q^{B,\text{corp}} B^{\text{corp}}_t - \log Q^{B,\text{corp}} B^{\text{corp}}_{t-1} + \log z_t - \log z_s \]
- Growth in bank holdings of government bonds:
  \[
  d \left( Q^{B,\text{gov}} B^{gov,B} \right)_{\text{obs}} = \log Q^{B,\text{gov}} B^{gov,B}_t - \log Q^{B,\text{gov}} B^{gov,B}_{t-1} + \log z_t - \log z_s + \epsilon^{\text{measurement}}_{B^{gov,B}_t}
  \]
- Growth in stock of government bonds outstanding:
  \[
  d \left( Q^{B,\text{gov}} B^{gov} \right)_{\text{obs}} = \log Q^{B,\text{gov}} B^{gov}_t - \log Q^{B,\text{gov}} B^{gov}_{t-1} + \log z_t - \log z_s
  \]
- Yield on government bonds:
  \[
  r^{B,\text{gov}}_{\text{obs}} = \left( r^{B,\text{gov}}_{\text{ytm},t} - r^{B,\text{gov}}_{\text{ytm},s} \right) \times 400 + \epsilon^{\text{measurement}}_{r^{B,\text{gov}}_t},
  \]
  with \( r^{B,\text{gov}}_{\text{ytm},t} = \left(1 + r^{B,\text{gov}}_t\right) \frac{Q^{B,\text{gov}}_{t-1}}{Q^{B,\text{gov}}_t} - 1. \)
- Growth in bank equity:
  \[
  dE^{I}_{\text{obs}} = \log E^{I}_t - \log E^{I}_{t-1} + \log z_t - \log z_s
  \]

B.2 Data Sources

- **Consumption (real):** Individual consumption expenditure — Euro area 18 (fixed composition) — World (all entities, including reference area, including IO), Households and non-profit institutions serving households (NPISH), Euro, Chain linked volume (rebased), Non-transformed data, Calendar and seasonally adjusted data. Source: European Central Bank, MNA.Q.Y.17.W0.S1M.S1.D.P31.Z.Z.T.EUR.LR.N.
- **Investment (real):** Gross fixed capital formation — Euro area 18 (fixed composition) — World (all entities, including reference area, including IO), Total economy, Fixed assets by type of asset (gross), Euro, Chain linked volume (rebased), Non-transformed data, Calendar and seasonally adjusted data. Source: European Central Bank, MNA.Q.Y.17.W0.S1.S1.D.P51G.N11G.T.Z.EUR.LR.N.
- **Gross domestic product:** Gross domestic product at market prices — Euro area 18 (fixed composition) — Domestic (home or reference area), Total economy, Euro, Chain linked
volume (rebased), Non-transformed data, Calendar and seasonally adjusted data. Source: European Central Bank,

- **GDP deflator**: Gross domestic product at market prices — Euro area 18 (fixed composition) — Domestic (home or reference area), Total economy, Index, Deflator (index), Non-transformed data, Calendar and seasonally adjusted data. Source: European Central Bank,

- **Bank equity**: Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector — Capital and reserves, All currencies combined — World not allocated (geographically) counterpart, Unspecified counterpart sector, denominated in euro, data neither seasonally nor working day adjusted. Source: European Central Bank,
BSI.M.U2.N.A.L60.X.1.Z5.0000.Z01.E.

- **Loans**: Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector — Loans, Total maturity, All currencies combined — Euro area (changing composition) counterpart, Non-financial corporations (S.11) sector, denominated in euro, data neither seasonally nor working day adjusted. Source: European Central Bank,

- **Corporate bond holdings in banking sector**: Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector — Debt securities held, Total maturity, All currencies combined — Euro area (changing composition) counterpart, Non-MFIs excluding general government sector, denominated in euro, data neither seasonally nor working day adjusted. Source: European Central Bank,

- **Government bond holdings in banking sector**: Euro area (changing composition), Outstanding amounts at the end of the period (stocks), MFIs excluding ESCB reporting sector — Debt securities held, Total maturity, All currencies combined — Euro area (changing composition) counterpart,
General government sector, denominated in euro, data neither seasonally nor working day adjusted. Source: European Central Bank,


- **Outstanding amount of government bonds:** Outstanding amounts of euro-denominated long-term debt securities issued by general government in euro area (changing composition), Outstanding amounts at the end of the period (stocks), Long-term securities other than shares, Nominal value, General government issuing sector, Euro, denominated in euro, Euro area (changing composition) Source: European Central Bank,

SEC.M.U2.1300.F33200.N.1.EUR.E.Z.

- **Yields on government bonds:** BOFA ML GERMAN FED GVT ALL MATS(E) — RED. YIELD. Source: Datastream, MLBDAME(RY).

- **Yields on non-financial corporate bonds:** BOFA ML EUR NON-FIN. BBB (E) — RED. YIELD. Source: Datastream, MLNF3BE(RY).

- **Loan rates:** Euro area (changing composition), Annualized agreed rate (AAR)/Narrowly defined effective rate (NDER), Credit and other institutions (MFI except MMFs and central banks) reporting sector — Loans, Total original maturity, Outstanding amount business coverage, Non-financial corporations (S.11) sector, denominated in euro, MIR.M.U2.B. A20.A.R.A.2240.EUR.O.

  Monetary Union (MU), Credit institutions and other MFIs reporting sector — Interest rate (unspecified rate type) on loans, Over one-year maturity, New business coverage, denominated in all currencies combined — Non-financial corporations (S.11) counterpart sector, RIR.M.U2.A.A20.K.2240.Z01. N.Z.R.

  Monetary Union (MU), Credit institutions and other MFIs reporting sector — Interest rate (unspecified rate type) on loans, Up to one-year maturity, Unspecified business coverage, denominated in all currencies combined — Non-financial corporations (S.11) counterpart sector. Source: European Central Bank, RIR.M.U2.A.A20.F.2240.Z01.Z.Z.R.
• **Entrepreneurial net worth (aggregate):** Equity/index; Dow Jones Euro Stoxx Price Index, Average of observations through period (A). Source: European Central Bank, FM.M.U2.EUR.DS.EI.DJEURST.HSTA.

• **Entrepreneurial net worth (sector A):** Equity/index; Dow Jones Euro Stoxx 50 Price Index, Average of observations through period (A). Source: European Central Bank, FM.M.U2.EUR.DS.EI.DJES50I.HSTA.

C. Calibration and Prior Selection of Standard Parameters

This section reports the calibration of the standard parameters. Hours worked in the steady state, $N_s$, are normalized to unity. Following results from Smets and Wouters (2003), I set the inverse Frisch elasticity, $\varphi$, to 2.5. The depreciation rates in both sectors, $\delta^e$, at 0.025, take the same value that is usually applied in the literature. Following Christiano, Motto, and Rostagno (2010), I choose 0.999 as a value for the time-preference rate, $\beta$. By analogy with the same reference, I calibrate the tax rates on capital, $\tau^K$, on consumption, $\tau^C$, and on labor, $\tau^N$, to 0.28, 0.2, and 0.45, respectively. I fix the steady-state rate of inflation, $\pi_s$, at 1.8 percent annualized and the steady technology growth, $z_s$, at 1.5 percent annualized, which corresponds roughly to historical averages.

Regarding the monetary policy rule, I assign a value of 0.8 to the mean and 0.15 to the standard deviation of the beta distribution for the interest rate smoothing parameter $\rho_\pi$. The weight on inflation, $\phi_\pi$, is given a mean of 1.7 and a standard deviation of 0.1, while the prior mean for the weight on output growth $\phi_y$ is set to 0.1 with a standard deviation of 0.05. These values are largely in line with Smets and Wouters (2003). The means for price and wage stickiness—$\gamma$ and $\gamma_w$, respectively—with values of 0.7 are a bit smaller than values from the literature. Since the sample starts at the end of the 1990s, this choice reflects the fact that price and wage stickiness might have changed over the past decades. The standard deviation under the beta distribution is 0.05 in each case. In addition to the stickiness parameters, I also estimate the price and wage markup, $\lambda_p$ and $\lambda_w$, respectively. In this respect, I start from values for the mean which are close to calibrations in Smets and Wouters.
and Christiano, Motto, and Rostagno (2010). I choose values of 1.2 for the means of a beta distribution, which is bounded to be within the range of 1 and 2 with a standard deviation of 0.1. To the weights on past inflation in price and wage indexation—$\xi$ and $\xi_w$, respectively—I assign means of 0.15 with a standard deviation of 0.15 under the beta distribution, which is in line with Christiano, Motto, and Rostagno (2010). The same is true of the weight on technology growth $\xi_z$ in the indexation rule for wages. The prior mean for habit persistence in consumption $h^C$ is set to be 0.7 with a standard deviation of 0.15, which is close to the literature. Analogously, the means for investment adjustment costs $\Psi''$ and costs related to varying the capital utilization are set. The former takes the value of 4 with a standard deviation of 1.5, while the latter is 5 with a standard deviation of 2. I also estimate the power on capital in the production function $\alpha$ and choose the conventional value of 0.3 as a mean with a related standard deviation of 0.15. For the Calvo employment parameter $\gamma_E$ to match total employment with hours worked, I follow Smets and Wouters (2003) and work with a mean of 0.5 and a standard deviation of 0.15.

For the autoregressive parameters of the shock processes, I opt for conventional priors of 0.75 as means and 0.15 as standard deviations. For the standard deviations of the shocks, I make use of the inverse-gamma distribution with means of 0.005 with the exceptions of the monetary policy shock, the price markup shock, and the consumer preference shock, with 0.002 for each, as well as for the labor supply shock with a mean of 0.01.

### D. Additional Results

#### D.1 Portfolio Rebalancing Effects from Government Bond Purchases

In figure 14 I show how the purchases of government bonds by the intervention authority affect the portfolio holdings of government bonds in the household sector and the banking sector and how they influence the supply of government bonds. The purchases of government bonds by the intervention authority lead to a redistribution of government bond holdings from the private sector to the public sector. The lower returns of government bonds as a consequence of
the purchases causes households and banks to reduce their holdings, whereas the majority of the intervention authority’s purchases stem from the banking sector. At the same time, the government bond supply is lower for two reasons. On the one hand, the fiscal agent collects more taxes as consumption, capital, and labor increase. On the other hand, the fiscal authority also benefits from lower borrowing costs.

### D.2 Limits to Arbitrage in the Banking and the Household Sector

In general, a necessary condition for public asset purchases to be effective is that the Wallace irrelevance proposition of full arbitrage does not hold, i.e., limits to arbitrage exist (Eggertsson and Woodford 2003; Chen, Cúrdia, and Ferrero 2012; Christiano and Ikeda 2013). Hence, the main channel through which asset purchases work

---

It should be noted that there is a large degree of uncertainty around the median responses of the household sector, reflected by the highest posterior density intervals covering positive and negative values.
is by influencing the relative price of assets which are imperfect substitutes (Andrés, Lopez-Salido, and Nelson 2004). As a reflection of the bond purchases, the quantity available to investors changes and the returns of the target asset diminish with the consequence that other assets seem to be preferable in terms of returns. Since limits to arbitrage render the assets imperfect substitutes, the related adjustment processes also reduce the returns of the other assets under consideration. This argument can also be applied to term premiums as long as market fragmentation across maturities causes deviations from the expectation hypothesis (Vayanos and Vila 2009). In the present model, portfolio costs in the household sector and financial frictions in the banking sector are the source of market segmentation.

Regarding government bonds, limits to arbitrage arise from two distinct domains. As discussed in section 3.1, households are faced with portfolio costs and have to bear costs if their portfolio holdings of government bonds deviate from the desired level (see equation (7)). Both frictions make government bonds imperfect substitutes for short-term assets and prevent full arbitrage, i.e., cause spreads between the yield on government bonds and interest rates of short-term assets.

\[
E_t \beta_{t+1}^\lambda b_{t+1}^\lambda b_t \left( \frac{r_{t+1}^{B, gov} - r_t^D}{\pi_t+1} \right) = \nu^{B, gov} \left( B_t^{gov,H} - B_s^{gov,H} \right) + \tau^{B, gov} \]

(96)

In addition to the leverage constraint, different diversion shares related to the assets held by banks also prevent full arbitrage between the (expected) returns on corporate bonds, government bonds and loans, and the interest rate on short-term assets.\(^6\)

\[
\theta^{IC} \left( L_t + \Delta^{B, corp} Q_t B_t^{corp} + \Delta^{B, gov} Q_t B_t^{gov} \right) \\
= E_t \Lambda_{t+1} \Omega_{t+1} \left( \frac{1}{\pi_t+1} \right) \left( R_t^L L_t + R_t^{B, corp} Q_t B_t^{corp} + R_t^{B, gov} Q_t B_t^{gov} \right) \\
+ R_t^{B, gov} Q_t B_t^{gov} - (R_t^D E_t^l - \gamma_t^L) \right) \]

(97)

\(^6\)This is the source of the effects from portfolio rebalancing.
Different diversion shares, however, also drive wedges between the returns on assets held by the banks. By combining the first-order conditions from the optimization problem in the banking sector, it turns out that the spreads between (expected) returns on corporate and government bonds and between the loan rate and the (expected) return on government bonds differ by a ratio comprising the diversion shares directed to each asset.

\[
E_t \Lambda_{t+1} \Omega_{t+1} \left( \frac{r_{t+1}^{B, corp} - r_{t+1}^{B, gov}}{\pi_{t+1}} \right)
\]
\[
= \frac{(\Delta B, corp - \Delta B, gov)}{(1 - \Delta B, gov)} E_t \Lambda_{t+1} \Omega_{t+1} \left( \frac{r_{t+1}^{L} - r_{t+1}^{B, gov}}{\pi_{t+1}} \right)
\] (98)

As can be seen in equations (96), (97), and (98), the frictions in both sectors drive wedges between the returns on short-term assets and on government bonds and between the returns on loans, corporate bonds, and government bonds. In order to assess the relative strength of the portfolio rebalancing channel and the balance sheet channel, it is important to have knowledge about the parameters which drive these channels.

In figure 15 I depict the present-value gains over one year (left-hand side) and ten years (right-hand side) in output by varying the diversion share in the banking sector related to government bonds on the y-axis and the market segmentation in the household sector on the x-axis. Consistent with earlier findings, the present-value gains rise with higher frictions in both sectors. For the shorter horizon, the present-value gains in output grow quickly for large frictions in both sectors. With a low level of frictions in one sector, the present-value gains are nearly independent of the level of frictions in the other sector. For a longer horizon, the present-value gain starts to rise very quickly with larger frictions in the banking sector. Except for the case of nearly no frictions in the household sector (small values for \(v^{B, gov}\)), frictions in the banking sector dominate the long-run

---

7I do not report results for the case where the frictions in one sector are deactivated since government bond purchases are ineffective in this case. I refer for this case to Christiano and Ikeda (2013) or Gertler and Karadi (2013).
Figure 15. Impact of Limits to Arbitrage from Frictions in the Banking Sector and the Household Sector on the Present-Value Gain in Output over One Year (Left-Hand Side) and Ten Years (Right-Hand Side) Resulting from Government Bond Purchases

Notes: The figure shows the present-value gains in output following government bond purchases, which are conducted entirely in one period, by varying the diversion share related to government bonds in the banking sector (y-axis) and portfolio costs in the household sector (x-axis). The gains in output are expressed as percentage deviations from steady state and are weighted with the time-preference rate. The present value is defined as \( \text{gain} = \sum_{k=1}^{K} \beta^k (X_{t+k-1} - X_s) / \sum_{k=1}^{K} \beta^k (Z_{t+k-1}) \cdot 100 \), with \( X \) as output and \( Z \) the stock of government bonds held by the central bank.

effects of government bond purchases. This means that it is predominantly the banking sector which affects the pricing of government bonds and therefore controls the medium-run effects on output.

D.3 The Stock and Flow Effects in Announced and Anticipated Programs

Next, I compare the cases of distributing the purchases across a specific period in time. In figure 16 I treat four cases, with two of them stemming from figure 3. The black solid lines and the blue dashed lines refer to the cases in which purchases are announced in advance to occur in four and eight quarters, respectively. After the purchases
Figure 16. Comparison of Responses to a One-Period Government Bond Purchase Program (Black Solid Lines) and Previously Announced Programs Distributed over One Year (Blue Dashed Lines) and Three Years (Red Dashed Lines with Dots)

Notes: The figure presents the effects of government bond purchases which are induced as “purchase shocks” as presented in equation (33). The purchases are scaled to achieve a maximal stock of 2.5 percent of GDP in every case. The responses are median responses from the estimated model.

have stopped, the stock dissipates over time. In addition, I present results for cases treated before, i.e., cases in which the purchases are distributed across four quarters (red dashed lines with dots) and eight quarters (magenta dotted lines), respectively. The maximum stock of government bonds held by the intervention authority is in all cases the same, but the time profile of holdings is different. In the two new cases the purchases are not distributed across periods, which means that the purchases in every period are obviously larger than for the two other programs. By investigating an announced one-off, I am basically able to treat an anticipation effect related to the expected stock of government bonds held by the authority. With
the two other programs there is also an anticipation effect, but it is additionally related to the expected stock and to the expected path of purchases. By comparing the two different programs for each of the two cases, I am able to identify flow effects of purchases. As can be seen in figure 16, the qualitative responses of the real economy do not change. Nevertheless, the maximum effects on output and investment are slightly stronger, which can be related solely to the anticipation effects. Thus, the stronger boost in investment results mainly from the expectational effect of agents. Since agents anticipate the drop in borrowing conditions, the real sector starts to produce more capital. Hence, output is driven by more investment and the wealth effect stimulates consumption. However, there are notable changes in the financial sector. A stock effect again pushes bank profits upwards until the period in which the purchases end. Obviously, flow effects through the purchases contract bank equity more quickly.

For the announcement cases including the purchases’ distribution over time, a small amount of purchases is already being conducted in the first period; however, the lion’s share of the volume is reached in later periods. With a longer duration of purchases, the anticipation effect more than offsets the portfolio rebalancing effect again, which depresses banks’ profit margins on impact and weakens bank net worth. Only once the entire volume is realized does the portfolio rebalancing channel start to dominate, weakening the balance sheet of the banks. It seems that distributing the purchases over time while reaching the same maximum stock produces larger stabilization gains in the real economy for a longer duration of the program. However, the evaluation of the quantitative effects on output in figure 16 can be misleading because the time profile of the intervention authority’s balance sheet is different. For taking a size effect into account, I refer again to present-value gains which are shown table 6 (in the main paper) and also draw on the present-value gains given in table 7 for the other cases. In the case where purchases are distributed across the periods, the long-run gain in output is not stronger compared to the announced purchases in one period. An output gain of 1.479 percent in terms of a cumulated balance sheet can be realized in ten years for the distribution of purchases over eight quarters, while an output gain of 1.483 percent is achieved for purchases conducted in eight periods. For the two
Table 7. Present-Value Gains in Output Following from Announced One-Off Programs

<table>
<thead>
<tr>
<th>In %</th>
<th>One Year</th>
<th>Ten Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchases in Four Quarters, Announced</td>
<td>2.38</td>
<td>1.479</td>
</tr>
<tr>
<td>Purchases in Eight Quarters, Announced</td>
<td>1.483</td>
<td>1.483</td>
</tr>
</tbody>
</table>

Notes: The table shows the present-value gains in output over a specified period for government bond purchases. The gains in output are expressed as percentage deviations from steady state and are weighted with the time-preference rate. The present-value gain is defined as:

\[ \text{gain} = \frac{\sum_{k=1}^{K \delta^k (X_{t+k-1} - X_s)} / \sum_{k=1}^{K \delta^k (Z_{t+k-1})} \cdot 100,} \]

with \( X \) as output and \( Z \) the stock of government bonds held by the central bank. Since the balance sheet is zero for announced purchases taking place in the eighth quarter, the present-value gains for the first year are not computed.

announced ad hoc programs, the long-run effects are quantitatively very similar. It turns out that the present-value gain in output is larger for the announced one-off programs compared to the distribution of the purchases over time. This indicates that a flow effect is responsible for the differences among the cases which materializes in a quicker decline in bank equity in the case of distributing the purchases over time.

D.4 Government Bond Purchases and a Lower Bound on the Interest Rate

The results in the main text are derived by letting the policy rate adjust endogenously according to the Taylor rule. Since government bond purchases eventually raise the rate of inflation, the policy rate rises as a consequence. However, government bond purchases have been introduced by central banks in an environment where the policy rate reached its effective lower bound. In order to investigate the effects of government bond purchases for a lower-bound scenario, I approximate this environment by keeping the policy rate constant for a specific time period. In figure 17 I compare the benchmark case from the main text (surprise program) with the generic effective

---

8Technically, this is imposed by anticipated shocks to the policy rate, as done by Laseen and Svensson (2011). Agents consequently know the exact time profile for the policy rate for building expectations accordingly.
Notes: The figure presents the effects of government bond purchases which are induced as “purchase shocks” as presented in equation (33) by keeping the policy rate (blue dashed lines) for two years constant. These cases are contrasted with the unconstrained benchmark case (black solid lines). The purchases are scaled to achieve a maximal stock of 2.5 percent of GDP in every case. The responses are based on the simulation of the model at its mode.

lower-bound case (denoted by ZLB). The results from the main text do not change qualitatively by introducing a period of a constant policy rate. As a result of preventing the policy rate from rising, consumption improves on impact in the lower-bound case, which boosts output by more than in the benchmark case, supported by a stronger rise in investment. This is also the reason why inflation rises by more. Investment behaves in a qualitatively similar manner in both cases. Lending to the entrepreneurs is first on a lower trajectory in the lower-bound case because entrepreneurs’ leverage ratio falls by more, showing that investment is financed to a greater extent through internal funds, i.e., net worth. The buildup of net worth is
stronger because the rise in the policy rate in the benchmark case slightly offsets the improvement in borrowing conditions.

E. Robustness of Results

E.1 Nominal and Real Frictions

Section 5.2 has shown how important it is to have a parameterization for the central financial frictions that is backed by an estimation of the model for correctly gauging the quantitative and qualitative effects of government bond purchases. It turned out that the intensity of financial frictions in both the banking and the non-financial sectors are important for exerting a sizable impact on the real economy. The propagation through the banking sector heavily depends on the importance of the relevant transmission channel. Without an important role for the balance sheet channel, the financial soundness of the banking sector deteriorates in the medium run following government bond purchases. However, it is vital to evaluate what role nominal and real frictions play in the transmission process.

Below I will present five cases accompanying the benchmark model in which I vary central nominal and real frictions. For each case the responses following government bond purchases of selected variables are shown in figure 18. Regarding nominal frictions, I remove price stickiness, i.e., $\gamma = 0$, in one case (blue dashed lines) and wage stickiness, $\gamma_w = 0$, in another case (red dashed lines with dots). Furthermore, I reduce the parameter $\psi^{k,e}$ to a very low value in each sector to give virtually no role to capital utilization costs (turquoise dotted lines). Habit formation in consumption is switched off in the fourth case, i.e., $h$ is set to zero (fewer purple dotted lines). As a last case, I start from the benchmark model but remove all nominal and real frictions, except investment adjustment costs, to obtain a purely standard real business cycle (RBC) model on the real side of the economy. Furthermore, I remove corporate bonds from the model so that only loans remain. Banks are the sole holders of government bonds, which means that households hold only short-term assets. In addition, tax income is heavily simplified by just allowing for lump-sum taxes which replace taxes on consumption, labor, and capital. This model has the minimal features to investigate government bond purchases but lacks the richness to capture business cycle
Figure 18. Robustness of Results: Varying Nominal and Real Frictions

Notes: The figure presents the effects of government bond purchases which are induced as “purchase shocks” as presented in equation (33). The purchases are scaled to achieve a maximal stock of 2.5 percent of GDP in each case. The responses are median responses from the estimated model. For the “no price stickiness” case, the Calvo parameter for prices ($\gamma$) is set to zero, while this is done for the Calvo parameter for wages ($\gamma_w$) in the “no wage stickiness” case. For the case with “no capital utilization costs,” the corresponding parameters ($\psi_{k,e}$) are set to a very small value of $1 \times 10^{-7}$. For “no habit formation in consumption,” $h$ is set to zero. The case with “simple RBC models” reflects an RBC model without real frictions except investment adjustment costs (whereas the related parameter is set to the estimated value from the New Keynesian model), which only features the loan sector. Furthermore, there are only lump-sum taxes in this model and households do not hold long-term government bonds.

It turns out that nominal and real frictions in the benchmark model mainly affect the propagation of shocks to the real sector, whereas the financial variables’ responses are mainly unaffected across the cases. The main conclusion regarding the transmission of government bond purchases against the backdrop of a dominance
of non-market-based debt in banks’ balance sheets also holds when using a model which is simpler than my benchmark model. The simple RBC model yields the same qualitative effects as the medium-size benchmark model. Bank net worth drops in the medium run, which contributes to the following increase in bank leverage. Non-financial firms’ leverage also increases, which lowers entrepreneurial leverage. The difference from the New Keynesian model is that the financial sector becomes more volatile following the purchases. Thus, conclusions drawn from the benchmark model regarding the effects of government bond purchases on leverage in the financial and non-financial sector still hold.

E.2 Role of the Intervention Authority

In the model it is assumed that an intervention authority which belongs to the public sector conducts the government bond purchases. Since there is a positive term premium resulting from portfolio costs and financial frictions, the intervention authority makes profits by buying government bonds as long as it does not push the long-term interest rate below the short rate. As described in the main text, the role of the intervention authority is to allow for a reallocation of profits as discussed by Christiano and Ikeda (2013). In reality, government bond purchases are conducted by the central bank, which generates profits mainly through seigniorage. Profits (or losses) from a portfolio of government bonds held for monetary policy purposes are consequently charged against seigniorage. The profits of the central bank are then distributed to the fiscal authority. Since there is no explicit role for money in the model, and therefore for seigniorage, the introduction of the intervention authority provides clarity in this regard. Without a loss of generality, it is a synonym for the central bank.

The role for distributing the profits from the intervention authority to the fiscal authority is presented in figure 19. This figure gives in panel A the pure responses of selected variables following purchases of government bonds (one-off program) for the case of profits’ redistribution and the case without the redistribution of profits (blue dashed lines). Regarding the redistribution of profits, I consider two cases: in the first case, the benchmark case, the profits are used to reduce the debt burden (black solid lines). In the second case, profits
Figure 19. Role of Redistributing the Net Profits from the Intervention Authority to the Fiscal Authority

Notes: The figure presents in panel A the responses following unanticipated government bond purchases for two cases: the benchmark case from the main text (black solid line), a case in which profits are used for government expenditures (blue dashed lines), and a case in which the net profits from the intervention authority are not redistributed to the fiscal authority (red dotted lines). Panel B shows the differences between responses of these two cases relative the no-redistribution case.

are used one-to-one for government expenditures (blue dashed lines). In panel B the differences between the two redistribution cases relative to the no-redistribution cases are given so that the net effect of the redistribution can be visualized. As can be seen by inspecting the responses in panel A, there is no qualitative and virtually no quantitative relevant difference between the no-distribution case and the benchmark case. This automatically means that the conclusions from the main text are not strongly influenced by the assumption about the redistribution of profits. Nevertheless, inspection of panel B yields an additional positive impact on real economic
activity through the redistribution of profits. The reason for this is that the intervention authority has full credibility and is able to finance the purchases at the short rate, which creates the profits. The redistribution of the profits allows the fiscal authority to lower the debt burden, which makes it possible to reduce taxes according to equations (29) and (30). This produces the additional expansionary (marginal) effect on output. In the case in which the profits are used for government expenditures, this expansionary effect is larger because of the multiplier effects stemming from boosting aggregate demand. Nevertheless, the qualitative responses of financial variables are alike. Quantitatively, the use profits for government expenditures marginally improves both the financial soundness of the banking and the non-financial sectors relative to the no-distribution case.

F. Discussion of Model Performance

The aim of this section is to connect the responses of bank leverage and bank equity to the discussion in the literature. For the United States it is well known that bank leverage is procyclical (Adrian and Shin 2010; Adrian, Colla, and Shin 2013). Nuño and Thomas (2017) show that the procyclicality of financial leverage is a robust finding for the United States, as it holds for different financial sectors and regardless of the filter which is used to remove a trend from the leverage series. Several models are available in the literature which are able to produce the procyclicality of bank leverage (see Chen 2001, Meh and Moran 2010, Rannenberg 2016, and Nuño and Thomas 2017, for example). Among these papers, Rannenberg’s (2016) model is the only one which is able to match the empirical properties of the external finance premium (a credit spread which captures the costs for expected defaults) and bank leverage. In his model, bank leverage is procyclical, while bank equity is countercyclical. My model is closely related to his model and shares relevant features like the costly enforcement problem between banks and depositors and the costly state verification problem between entrepreneurs and banks. Nevertheless, my model produces, as a result of the estimation based on euro-area data, a countercyclicality of bank leverage to quarterly GDP growth. I will give more insight into how this should be interpreted.
Table 8. Cyclical Properties of Bank Leverage in the Euro Area

<table>
<thead>
<tr>
<th></th>
<th>GDP$<em>t$/GDP$</em>{t-1}$</th>
<th>HP-Filtered GDP</th>
<th>BK-Filtered GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>0.222</td>
<td>0.016</td>
<td>0.069</td>
</tr>
<tr>
<td>Linearly Detrended</td>
<td>-0.156</td>
<td>0.100</td>
<td>0.145</td>
</tr>
<tr>
<td>Baxter-King Filter</td>
<td>-0.380</td>
<td>0.020</td>
<td>0.028</td>
</tr>
<tr>
<td>Hodrick-Prescott Filter</td>
<td>-0.371</td>
<td>0.073</td>
<td>0.046</td>
</tr>
</tbody>
</table>

Firstly, I will discuss the cyclical properties of bank leverage in the euro area before I turn to the performance of my model in this regard. For bank equity, I draw again on the series also used for the estimation of the model (ECB balance sheet items: BSI.M.U2.N.A.L60.X.1.Z5.0000.Z01.E). A series for total assets is taken from the same source (BSI.M.U2.N.A.T00.A.1.Z5.0000.Z01.E). Table 8 presents the correlation coefficients between bank leverage (total assets over bank equity) and real GDP (per capita). Before I discuss the results, several comments are necessary regarding the transformation of bank leverage data. Since bank leverage has a downward trend in my sample, bank leverage is linearly detrended as a starting point. Furthermore, I apply a band-pass filter (Baxter-King) and the Hodrick-Prescott filter similar to Nuño and Thomas (2017). The last two filtering techniques are also applied to real GDP complementing quarterly GDP growth as it is used in the estimation of the model. As it turns out, bank leverage is only slightly procyclical in the euro area for the period 1997:Q4 to 2013:Q3 when looking at GDP data where a trend is removed from levels. Except for bank leverage in levels, its detrended series are countercyclical relative to quarterly GDP growth. The correlation of linearly detrended bank leverage ratio to quarterly GDP growth comes very close to the model-implied correlation of −0.1482.

In order to show that the model-implied cyclical properties mainly depend on the estimated parameters, i.e., that the model is also able to reproduce stylized facts collected in the United States, I compare in figure 20 the responses of output, bank leverage, and bank equity for the benchmark model with those of a model calibration which is closer to Rannenberg (2016). Among others, one
important distinction between Rannenberg’s (2016) calibration for the United States and my estimation together with the calibration for the euro area is the intensity of frictions in the banking sector controlled by the diversion share \( \theta^{IC} \). In Rannenberg (2016) this share becomes 0.2351, while in my model the parameterization provides an overall value of 0.0938. I draw on the shocks discussed in Rannenberg (2016) and present responses following (i) a monetary policy shock, (ii) a shock to total factor productivity, and (iii) a shock to entrepreneurial net worth in my sector B. I provide two cases: in the first case, the simulation is based upon the modes resulting from my estimation (black solid lines in figure 20) and for the second case I set the diversion share to a value of 0.24 to all assets in my model (blue dashed lines in figure 20). For the latter case, the responses are very close to those reported by Rannenberg (2016) in his figures 1, 2, and 5. Bank leverage becomes more procyclical and bank equity more countercyclical. Hence, my model is able to generate the procyclicality of bank leverage, although it is weak for euro-area data in my sample.

G. The Optimality of the Debt Contract. . .

G.1 . . . between Entrepreneurs and Banks

For the contract described in Bernanke, Gertler, and Gilchrist (1999), it is assumed that financial intermediaries are risk neutral and earn zero profits. The participation constraint of the financial intermediaries states in this case that they need to earn as much as they have to pay for their debt. Given the participation constraint and the verification requirement for defaults statements which are related to monitoring costs, entrepreneurs have no incentive to deviate from the conditions of the contract. From this point of view, the contract is globally optimal. Although banks are risk neutral in my case, they do not earn zero profits as a result of financial frictions between the banking sector and households. I argue that the contract is locally optimal from the banks’ point of view. Regarding the loan and the corporate bond portfolio, bank managers can earn a rate which is ex ante free from default risk as a result of perfect portfolio diversification. As a consequence of the bank-specific financial frictions related to different shares of diversion, the default-free loan
Figure 20. The Benchmark Model and the Procyclicality of Bank Leverage

Notes: The figure presents the responses to (i) a monetary policy shock, (ii) a shock to total factor productivity, and (iii) an entrepreneurial net worth shock (in sector B).

rates differ, whereas the relationship is governed by the first-order conditions. In order to take the portfolio diversification issues and the constraints regarding their diverting behavior into account, bank managers demand different loan rates to participate in the contract with the entrepreneurs. For this reason, bank managers try to maximize the payoff resulting from each individual contract. The value of each loan contract between the \( n \)-th bank and the \( m \)-th entrepreneur becomes

\[
V^B,L_{n,m,t} = E_t \sum_{j=1}^{\infty} \Lambda_{t,t+1} \left( \tilde{R}^L_{m,t-1} - R^L_{t-1} \right) L_{n,m,t-1}
\]
and for purchases of corporate bonds

\[ V_{n,m,t}^{B,B_{corp}} = E_t \sum_{j=1}^{\infty} \Lambda_{t,t+1} \left( \tilde{R}_{m,t}^{B,corp} - R_t^{B,corp} \right) Q_{t+1}^{B,corp} B_{n,m,t-1} \]

The gross returns with tildes are the rates the bank managers require to participate. Maximization yields

\[ \frac{\partial V_{n,m,t}^{B,L}}{\partial L_{n,m,t}} = E_t \Lambda_{t,t+1} \left( \tilde{R}_{m,t}^{L} - R_t^{L} \right) = 0 \]

and

\[ \frac{\partial V_{n,m,t}^{B,L}}{\partial B_{corp}^{n,m,t}} = E_t \Lambda_{t,t+1} \left( \tilde{R}_{m,t}^{B,corp} - R_t^{B,corp} \right) Q_t^{B,corp} = 0. \]

Up to first order, this means that \( \tilde{R}_{m,t}^{L} = R_t^{L} \) and \( E_t \left( \tilde{R}_{m,t+1}^{B,corp} \right) = E_t \left( R_t^{B,corp} \right) \). Hence, bank managers need to participate in the contract to earn the group-specific default-free rate, which means that they participate in the contract only if \( R_t^{L} \) or \( E_t \left( \tilde{R}_{t+1}^{B,corp} \right) \), depending on the group, can be earned as a minimum.

G.2 . . . between Banks and Households

I formulate a two-sided agency problem between non-financial firms and banks and between banks and households. The way in which I solve the contracting problem between banks and households, and the agency problem between the non-financial sector and banks, does not seem to be a major factor. However, the two problems could be interrelated because the outcome of the contract in the non-financial sector might affect the bankers’ decision to run. If the incentive constraint holds, bankers will not run. In this section, I show that the combination of the two agency problems does not alter the general conclusion.

The value of the bank under the no-default case is \( V_t^{B,\text{no}} \), which is equivalent to the value that appears for continuing business. The value under running is
\[ V_t^{B,\text{run}} = E_t^I - (1 - \theta^{IC}) \left( L_{n,t} + \Delta_{B,\text{corp}} Q_{t}^{B,\text{corp}} B_{t}^{\text{corp}} + \Delta_{B,\text{gov}} Q_{t}^{B,\text{gov}} B_{t}^{\text{gov}} \right) + D_t, \]

where \( \Delta_{B,\text{corp}} \) and \( \Delta_{B,\text{gov}} \) denote asset-specific diversion shares. With the help of banks' balance sheet I can write

\[ V_t^{B,\text{run}} = \theta^{IC} \left( L_t + \Delta_{B,\text{corp}} Q_t^{B,\text{corp}} B_{t}^{\text{corp}} + \Delta_{B,\text{gov}} Q_t^{B,\text{gov}} B_{t}^{\text{gov}} \right). \]

Thus, there is no incentive for the bankers to run if

\[ V_t^{B,\text{no-run}} \geq \theta^{IC} \left( L_t + \Delta_{B,\text{corp}} Q_t^{B,\text{corp}} B_{t}^{\text{corp}} + \Delta_{B,\text{gov}} Q_t^{B,\text{gov}} B_{t}^{\text{gov}} \right) \]

holds.

As described in the previous section, entrepreneurs from the non-financial sector sign a contract without the incentive to deviate from its conditions. For this reason, potential misbehavior by entrepreneurs can be ruled out and does not affect the decision of bankers to run. However, ex post losses from the loan portfolio or from the bond holdings can have an impact on bankers' decision to run, as they reduce the realized value of the bank. Following the arguments of Iacoviello (2005) raised for collateral constraints, the incentive constraint is binding in the neighborhood of the steady state, i.e., as long as the shocks are not too large. This is true in my case because unexpected losses play a minor role compared to expected losses, which are completely captured through the external finance premium.

H. Relationship between the Yield to Maturity and the Period Return

In this section, I derive the relationship between the yield to maturity and the period return given long-term bonds with a coupon of \( \rho^B \) which is exponentially decaying over time following Woodford (2001). For the sake of simplicity, the assets are denominated in real terms below. I start from households' budget constraint by summarizing all expenditures in \( X_t \) and all revenues in \( Y_t \) except those
related to government bonds. Households invest in long-term bonds
with the amount of $Q_t^B B_t$ and receive $\left(1 + \rho^B Q_t^B \right) B_{t-1}$
\begin{equation}
X_t + \left(1 + \rho^B Q_t^B \right) B_{t-1} \leq Y_t + Q_t^B B_t, \tag{99}
\end{equation}
where $Q_t^B$ is the price of the bond and $B_t$ the quantity held. The second term on the left-hand side can be rewritten as
\((\rho^B + \frac{1}{Q_t^B}) Q_t^B B_{t-1}\), where the term in brackets reflects the yield to maturity as Chen, Cúrdia, and Ferrero (2012), for example, show. By using the yield to maturity, equation (99) becomes
\begin{equation}
X_t + R_{ytm,t}^B Q_t^B B_{t-1} \leq Y_t + Q_t^B B_t, \tag{100}
\end{equation}
where $R_{ytm,t}^B$ is the gross yield to maturity.

Maximizing utility by choosing $B_t$ given equation (99) yields
\[
\frac{\partial}{\partial B_t} : \lambda_t Q_t^B - E_t \lambda_{t+1} \beta \left(1 + \rho^B Q_t^B \right) = 0,
\]
with $\lambda_t$ as marginal utility. After rewriting this first-order condition, the well-known form of the Euler equation appears:
\[
1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1 + \rho^B Q_t^{B_{t+1}}}{Q_t^B}.
\]

The right-hand side can now be interpreted as the (gross) period return, which is defined as $1 + r_{t+1}^B = \frac{(1 + \rho^B Q_t^{B_{t+1}})}{Q_t^B}$. Now, I start from the budget constraint in which I include the period return on holdings of government bonds
\begin{equation}
X_t + \left(1 + r_t^B \right) Q_{t-1}^B B_{t-1} \leq Y_t + Q_t^B B_t. \tag{101}
\end{equation}
Utility maximization now yields
\[
\frac{\partial}{\partial B_t} : -\lambda_t Q_t^B + E_t \lambda_{t+1} \beta \left(1 + r_t^B \right) Q_t^B = 0,
\]
which can be rewritten to obtain
\[
1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{1 + \rho^B Q_t^{B_{t+1}}}{Q_t^B}.
\]
Based upon these considerations, the relationship between the yield to maturity and the period return arises as

\[ r^B_t = R^B_{ytm,t} \frac{Q^B_t}{Q^B_{t-1}} - 1. \]

I. Additional Figure

Figure 21. Overview of Central Relationships in the Model

References


———. 2013. “QE 1 vs. 2 vs. 3...: A Framework for Analyzing Large-Scale Asset Purchases as a Monetary Policy Tool.” International Journal of Central Banking 9 (S1): 5–53.


