Welfare Effects of Tax Policy in Open Economies: Stabilization and Cooperation*  

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This paper studies welfare implications of a simple operational tax policy (under which tax rates respond to changes in productivity) by employing an open-economy dynamic stochastic general equilibrium model with incomplete asset markets. We investigate the possibility of welfare-improving tax policies on factor incomes and consumption. Simulation results show that, in the closed economy, optimal tax policies are countercyclical since such policies would stabilize the economy by increasing the tax rates in a boom. However, in the open economy, optimal tax policies become less countercyclical and under certain cases can even become procyclical—in particular, for capital income tax. A two-country exercise suggests that tax policy cooperation on capital and labor income would yield only small welfare gains, while consumption tax policy cooperation would produce sizable welfare gains.

JEL Codes: F4, E6.

1. Introduction

Many economists have emphasized the role of fiscal policy as an effective tool to enhance the stability of the economy under certain

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circumstances. For example, in a monetary union such as the European Union, monetary policy cannot be used for stabilization purpose against regional or national shocks. Another case when monetary policy becomes less effective is when nominal interest rates are close to zero, such as Japan in the late 1990s and many other developed economies during the Great Recession. In order to properly use active fiscal policy under such circumstances, it is important to obtain accurate welfare implications of business-cycle-stabilizing fiscal policies.

This paper studies welfare implications of a simple operational tax policy rule using a dynamic stochastic general equilibrium (DSGE) model. In our model, a government levies taxes on capital and labor income as well as on consumption to meet exogenous government purchase requirements. The government uses state-contingent tax policy by setting tax rates as a linear function of productivity level. We then numerically derive the optimal feedback coefficients of tax rates to productivity and calculate welfare gains from such optimal contingent tax policy against fixed (exogenous) tax policy. Since we focus on simple linear tax rules and calibrate the steady-state levels of tax rates from the data instead of calculating from the fully fledged Ramsey problem, our tax policy would not necessarily be optimal in the class of all tax rules but could arguably provide realistic policy implications.

1See, for example, Debrun and Kapoor (2010), Fatas and Mihov (2012), and McKay and Reis (2016).
2See, for example, Galí and Perotti (2003) and Galí (2005).
3See Feldstein (2002) for the discussion on the positive role of discretionary fiscal policy in this case.
4Some considered active tax policy unrealistic because it would take too much time to change statutory tax rates in response to stochastic shocks. However, in this paper, we rely on the fact that active tax policy can be rather easily implemented through changes in effective tax rates by using tax credits, deductions, and exemptions—without necessarily changing statutory tax rates.
5Our search for “optimal” tax policy assumes a certain parametric family of tax policy rules and optimizes over the parameters of the rule. Such an optimizing procedure—common in monetary policy analysis as in Bergin, Shin, and Tchakarov (2007)—is different from the Ramsey approach, which defines optimal tax policy as the best possible tax rate responses to disturbances and all the state variables, as in Chari, Christiano, and Kehoe (1994) and Galí and Monacelli (2008).
We study both closed- and open-economy models to examine how optimal tax policies would behave without and with open capital markets. Our open-economy models feature incomplete financial markets with international bonds; two versions we analyze are a small open-economy model with an exogenously given interest rate and a two-country model where the interest rate is endogenously determined. Using the two-country model, we also examine welfare effects of domestic tax policies on both domestic and foreign countries and derive the non-cooperative Nash equilibrium and cooperative equilibrium for optimal tax policies. If non-cooperative and cooperative equilibriums are significantly different, then there is room for welfare improvement via tax policy cooperation. These results can provide plausible implications on potential welfare gains of international tax policy cooperation.

This paper contributes to the literature in the following ways. First, we adopt an open-economy framework. The literature on welfare analysis of tax policy has mostly focused on the closed economy. However, these results can significantly change in an open economy because tax policies can have effects on other countries through various channels such as the world interest rate and capital flows. Second, we analyze tax policies in a stochastic setup, which has been used extensively for the analysis of monetary policy (e.g., Obstfeld and Rogoff 2002 and Canzoneri, Cumby, and Diba 2005). Most existing papers have analyzed tax policies in a deterministic setup and focused on the effects of permanent changes in tax policies or tax policy reform. However, certain economic phenomena should be analyzed under the stochastic framework. For example, recent

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6Papers with the closed-economy setup include Greenwood and Huffman (1991), Chari, Christiano, and Kehoe (1994), McGrattan (1994), and Kletzer (2006). In many cases, tax policies aiming for the stabilization of the economy produce allocation distortions that outweigh the stabilization gains and therefore reduce welfare. Tax policies can be welfare improving if the economy is already subject to other distortions such as imperfect competition or externalities, e.g., Easley, Kiefer, and Possen (1993) and Hairault, Langot, and Portier (2001).

7For example, Baxter (1997) and Kollmann (1998) examined the effects of taxes as well as government spending to explain the twin deficits and the U.S. trade balance, respectively.

discussion in the European Union about the role of fiscal policies as absorbers of asymmetric shocks must deal with the stochastic nature of such shocks. Finally, in order to capture the non-linear dynamics of the model which matters for welfare analysis, we solve the model using a second-order accurate solution method based on Kim et al. (2008).

Our main findings are as follows. In the closed economy, optimal tax policy is countercyclical for all three types of taxes. In this paper, countercyclical tax policy is defined as an increase in tax rate when productivity rises, so that output can be stabilized. This definition is different from another conventional definition that refers to the adjective “countercyclical” to indicate a negative correlation between tax rates and output levels. Under our definition, countercyclical tax policy produces stabilization gains by reducing volatility of the economy, which would improve welfare.

In an open economy, optimal tax policies in general become less countercyclical than in the closed-economy case; current account plays a stabilization role, which reduces the role of countercyclical tax policies in stabilizing the economy. More importantly, optimal capital income tax policy becomes procyclical in an open economy under some parameter values, in the sense that increasing the capital income tax rate when facing negative productivity shocks increases welfare.

Two-country analysis shows that both optimal capital and labor income tax policies generate negative spillovers to foreign countries. Under the non-cooperative Nash equilibrium, both countries become worse off by adopting active tax policies due to negative spillovers. Even under the cooperative equilibrium when both countries maximize world welfare, active income tax policies generate negligible welfare gains. On the other hand, optimal consumption tax policy generates positive spillovers to foreign countries and both countries gain under the Nash equilibrium; moreover, cooperative equilibrium produces large welfare gains over the Nash equilibrium.

The remainder of this paper proceeds as follows. Section 2 describes a DSGE model with linear tax policies. Section 3 reports simulation results for welfare implications of optimal tax policy in both closed and open economies. In order to help interpret the welfare results, we examine impulse responses to a positive productivity shock with countercyclical and procyclical tax policies. Section 3 also
provides the results of tax policy transmission and cooperation; by comparing the non-cooperative Nash equilibrium and the cooperative equilibrium, we calculate potential welfare gains from tax policy cooperation. Finally, section 4 concludes.

2. The Model

This section explains the two-country open-economy model. Two countries are symmetric with identical preference and production technology. There is a single non-durable tradable good serving as the numeraire. Each country consists of a representative household, a representative firm, and a government. Households decide the level of consumption, leisure, investment, and bond holdings subject to budget constraints. Bond holdings and investment are subject to adjustment costs. We assume that the international financial market is incomplete in the sense that agents can trade only non-state-contingent bonds.

The government is described as a sequence of government spending and tax rates on consumption, capital income, and labor income. The entire amount of tax revenue, net of fixed government spending, is distributed to households as lump-sum transfers in each period. The transfers can be negative, and in this case they operate as lump-sum taxes. The use of lump-sum transfers allows us to avoid potential additional distortions from adjusting other tax rates to balance the budget. The only source of disturbances in the economy is productivity shocks which can be correlated across countries. Foreign variables are denoted by asterisks, and their behavior is symmetric to the home country when not specified.

2.1 Households and Firms

Households in each country maximize the expected lifetime utility given by

\[ E_0 \sum_{t=0}^{\infty} \beta^t U_t, \text{ where } U_t = \frac{C_t^\theta (1 - L_t)^{1-\theta}}{1 - \sigma}, \]  

(1)

and \( C_t \) and \( (1 - L_t) \) denote the level of consumption and the amount of leisure, respectively. Households in both countries have the same discount factor \( \beta \).
The budget constraint of households is given by
\[
(1 + \tau_{ct})C_t + I_t + B_t + \frac{\zeta}{2} (B_t)^2
= (1 - \tau_{tt})w_t L_t + [(1 - \tau_{kt})r_t + \tau_{kt}\delta] K_t + R_{t-1} B_{t-1} + T_t, \tag{2}
\]
where $B_t$ denotes the quantity of international bonds purchased in period $t$ maturing in $t + 1$, $R_t$ is the gross interest rate on bonds, $r_t$ is the rental rate, $w_t$ is the wage rate, and $\tau$ represents tax rates ($\tau_c =$ consumption tax rate, $\tau_k =$ capital income tax rate, and $\tau_l =$ labor income tax rate). Note that there is a depreciation allowance, $\tau_{kt}\delta K_t$, and bond holdings are subject to quadratic holding costs, $\frac{\zeta}{2} (B_t)^2$. $T_t$ is the lump-sum transfer (tax) to the household which amounts to the budget surplus (deficit).

As in Kim (2003), households accumulate capital according to the following equation:
\[
K_{t+1} = \left[\delta (I_t / \delta)^{1-\phi} + (1 - \delta) K_t^{1-\phi} \right]^{1/1-\phi}. \tag{3}
\]
A zero $\phi$ implies no adjustment costs. A positive $\phi$ implies the presence of adjustment costs, and $\phi = 1$ corresponds to a log-linear capital accumulation equation.

For firms, the production function follows a Cobb-Douglas form with labor and capital,
\[
Y_t = A_t L_t^\alpha K_t^{1-\alpha}. \tag{4}
\]
While labor cannot move across countries, investment in the domestic country can be financed by foreign capital.

Productivity variables $A_t$ and $A^*_t$, representing stochastic components of the production functions of the two countries, follow a symmetric vector Markov process:
\[
\begin{bmatrix}
\log(A_t) \\
\log(A^*_t)
\end{bmatrix} = \begin{bmatrix}
\rho & \nu \\
\nu & \rho
\end{bmatrix}
\begin{bmatrix}
\log(A_{t-1}) \\
\log(A^*_{t-1})
\end{bmatrix} + \begin{bmatrix}
\varepsilon_t \\
\varepsilon^*_t
\end{bmatrix}, \tag{5}
\]
where $E(\varepsilon_t) = E(\varepsilon^*_t) = 0$, $E(\varepsilon_t^2) = \sigma^2_\varepsilon$, $E((\varepsilon^*_t)^2) = \sigma^2_{\varepsilon^*}$, and $\rho(\varepsilon_t, \varepsilon^*_t) = \psi$ for all $t$. $\rho$ is the persistence of productivity shocks.

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9Using bond holding adjustment costs allows us to avoid the non-stationarity problem in the small open-economy model with incomplete markets. See Kim and Kose (2003) for a detailed discussion on this issue.
and $\nu$ represents the spillover effects. A non-zero $\psi$ means that the innovations are contemporaneously correlated across countries.

### 2.2 Government

Government income includes tax revenues as well as bond holding adjustment costs, and government spending $G_t$ is assumed to be fixed and unproductive. The government does not issue any debt and balances its budget in each period by rebating all the tax revenue. That is, the level of the government transfer satisfies

$$\tau_{ct} C_t + \tau_{lt} w_t L_t + \tau_{kt}(r_t - \delta)K_t + \zeta \left( B_t \right)^2 = \bar{G} + T_t. \quad (6)$$

In the benchmark case of exogenous tax policy, the tax rates are fixed at the steady-state level (denoted with $\bar{\tau}$). Note that we do not solve the Ramsey problem in this paper, as the steady-state tax rates are taken from the data rather than from an optimization problem. Active and fully committed tax policy means that governments change tax rates contingent on the current-period productivity. That is, tax policies are represented by the parameter $\eta$ in

$$\tau_t = \bar{\tau} + \eta \left( \log (A_t) - \log (\bar{A}) \right), \quad (7)$$

where $\bar{A}$ is the steady-state value of productivity which is fixed at 1, and the sign of $\eta$ indicates whether the tax policies are countercyclical (if positive) or procyclical (if negative). The absolute value of $\eta$ represents the sensitivity of tax policy (i.e., how much the tax rate should be changed to a unit change in productivity).

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10. We assume that bond holding adjustment costs work as domestic taxes on international borrowing and lending. Alternatively, one can assume that bond holding costs are collected by an international authority and disappear from the national income accounting. Effects of bond holding costs on welfare results are negligible because we set the bond holding costs quite low.

11. An alternative form of tax policy rule is to set the tax rate in response to the changes in directly observable variables such as the level of output—as is conventional in the literature on monetary policy rules. However, since output is an endogenous variable, optimal tax policy may become a non-convex problem under certain cases, and it would be hard to interpret the simulation results in an intuitive way. Some papers with nominal rigidities have set tax policy as a function of the ratio between government bond and output (Schmitt-Grohé and Uribe 2007, and Kollmann 2008). In this paper, we do not consider this form of tax policy given that we have lump-sum transfers and international bonds.
The country’s resource constraint is

\[ Y_t + R_{t-1} B_{t-1} = C_t + I_t + G + B_t. \]  

(8)

For the world equilibrium, the model requires the bond market clearing condition that bonds should be in zero net supply:

\[ B_t + B^*_t = 0. \]  

(9)

The equations describing the equilibrium are listed in the appendix.

We measure welfare gains by calculating the change in welfare when the government implements active tax policies relative to the benchmark economy where both countries face stochastic productivity shocks but tax rates are fixed at the steady-state level (\( \eta = 0 \) for all three taxes). Welfare is measured in terms of consumption units, a common measure in business cycle literature, as in Lucas (1987). The certainty-equivalence consumption is based on the conditional expectation of lifetime utility.\footnote{It is important to use the conditional mean, instead of the unconditional mean, in order to correctly capture the dynamic transition effects of policy changes. See Kim et al. (2008) and Kim and Kim (2018) for more on this.} In order to correctly calculate the level of welfare, we use a second-order accurate solution method following Kim et al. (2008).

2.3 Calibration

As for calibration, we use the conventional parameter values for annual data. We use the annual data because tax rates do not vary much on a quarterly basis. The capital depreciation rate, \( \delta \), is 0.1 per year. Labor share, \( \alpha \), is 0.6 and the consumption share parameter, \( \theta \), is set to match the steady-state share of time devoted to market activities, 0.4. The representative agent’s discount factor, \( \beta \), is 0.95 so that the steady-state annual real interest rate is 5 percent. We set the utility curvature parameter, \( \sigma \), which determines the household’s coefficient of relative risk aversion, at 2. The elasticity of bond holding costs, \( \zeta \), is set at \( 10^{-3} \) to allow only minimal effects from holding costs. Finally, we need to decide the parameter value for \( \phi \) in capital adjustment costs. We set it at 0.2 to match the volatility of
investment in the data. Most previous studies reported that productivity measures are highly persistent. For volatility of productivity shocks, we follow Backus, Kehoe, and Kydland (1992) and Baxter and Crucini (1995) and assume that $\sigma_\varepsilon = 0.852\%$. We experiment with different values for other productivity parameters ($\rho, \nu, \psi$) in simulations.

Measuring aggregate tax rates is a complex and difficult task, and there is little consensus on measures of effective tax rates. In this paper, we use the aggregate effective tax rates calculated by Mendoza, Razin, and Tesar (1994). They calculate effective tax rates for the G-7 countries by dividing actual tax payments by corresponding national accounts. For example, the consumption tax rate is measured by dividing the actual consumption tax payment (adding general taxes on goods and services, excise taxes, and import duties) by the pre-tax value of consumption. For labor and capital income tax rates, they first measure the general income tax rate of the household, assuming that all sources of the household income are taxed at the same rate. Then, labor income tax rate is calculated by dividing total labor income tax payment (income tax rate multiplied by wage and salaries plus all social security contributions) by the tax base (wage and salaries, employer’s social security contributions, and payroll taxes). The capital income tax payment is the sum of all corporate tax payments (including taxes on immovable properties as well as financial and capital transactions). The tax base for the capital income tax is the operating surplus of all corporations. These effective tax rates reflect government policies on tax credits, deductions, and exemptions as well as information on statutory tax rates. The estimated tax rates of the G-7 countries are on average 12 percent, 36 percent, and 31 percent for consumption, capital income tax, and labor income tax, respectively. We calibrate our model’s steady-state tax rates at these rates. Accordingly, government spending is fixed at the level that allows a balanced budget under these steady-state tax rates.

\[13\] Their method follows Lucas (1990) and Razin and Sadka (1994). A number of papers have used this method to construct data on tax rates. See, for example, Mendoza and Tesar (1998), Carey and Tchilinguirian (2000), and Carey and Rabesona (2004). Another widely used alternative for tax rate data is aggregate marginal tax rates. See Mendoza, Razin, and Tesar (1994) for a detailed explanation and comparison of different computation methods.
3. Welfare Implications of Tax Policy

This section analyzes welfare implications of active (i.e., contingent on the state of the economy) tax policy under both closed and open economies. In particular, we derive the optimal response of tax rates to productivity shocks and measure maximum welfare gains relative to the fixed tax rates. We use two types of open-economy models. One model is a small open economy where the world interest rate is exogenously given, and we also analyze the two-country setup where the interest rate is endogenously determined by clearing the bond market between the two countries. We then use the two-country model to analyze the international spillover of tax policy effects and the international cooperation between tax authorities.

3.1 A Closed Economy

In the closed economy, active tax policy can be welfare improving because governments should finance fiscal spending (which is positive and exogenously given) by collecting distortionary taxes. Since the steady-state tax rates are positive, taxes would introduce distortions in the static and intertemporal optimality conditions. Therefore, contingent tax policies can improve welfare by redistributing such distortions in a better way. We first calculate the level of welfare when tax rates are fixed at the steady-state level and then measure potential welfare gains when governments adopt active tax policy relative to the benchmark fixed-tax case.

Table 1 reports optimal \( \eta \) for each tax with different values of \( \rho \) (persistence of productivity shock).\(^{14}\) First, optimal tax policy is countercyclical for all three taxes: consumption tax (2.5 ∼ 2.7), capital income tax (0.8 ∼ 1.6), and labor income tax (0.04 ∼ 0.15). As mentioned in the Introduction, we define a tax policy as countercyclical when governments lower tax rates when the economy is hit by a negative productivity shock.

Welfare gains from active consumption tax policy is the largest of the three, while labor income tax policy brings almost negligible gains. When the productivity shock is very persistent (\( \rho = 0.95 \)),

\(^{14}\)Other parameters than \( \rho \) also affect optimal \( \eta \), but the effects are not significant in most cases.
Table 1. Optimal Tax Policies in Closed and Open Economies

<table>
<thead>
<tr>
<th></th>
<th>$\rho = 0.85$</th>
<th>$\rho = 0.9$</th>
<th>$\rho = 0.95$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Consumption Tax</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autarky</td>
<td>Optimal $\eta$</td>
<td>2.5</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>Welfare Gains</td>
<td>0.008</td>
<td>0.01</td>
</tr>
<tr>
<td>Small Open</td>
<td>Optimal $\eta$</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>Welfare Gains</td>
<td>0.0002</td>
<td>0.001</td>
</tr>
<tr>
<td>Two Country</td>
<td>Optimal $\eta$</td>
<td>0.1</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>Welfare Gains</td>
<td>0.00003</td>
<td>0.0005</td>
</tr>
<tr>
<td><strong>Capital Income Tax</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autarky</td>
<td>Optimal $\eta$</td>
<td>1.6</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Welfare Gains</td>
<td>0.015</td>
<td>0.003</td>
</tr>
<tr>
<td>Small Open</td>
<td>Optimal $\eta$</td>
<td>−1.6</td>
<td>−0.5</td>
</tr>
<tr>
<td></td>
<td>Welfare Gains</td>
<td>0.006</td>
<td>0.001</td>
</tr>
<tr>
<td>Two Country</td>
<td>Optimal $\eta$</td>
<td>−1.2</td>
<td>−0.3</td>
</tr>
<tr>
<td></td>
<td>Welfare Gains</td>
<td>0.002</td>
<td>0.0004</td>
</tr>
<tr>
<td><strong>Labor Income Tax</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Autarky</td>
<td>Optimal $\eta$</td>
<td>0.04</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>Welfare Gains</td>
<td>0.00004</td>
<td>0.0002</td>
</tr>
<tr>
<td>Small Open</td>
<td>Optimal $\eta$</td>
<td>0</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>Welfare Gains</td>
<td>0</td>
<td>0.0001</td>
</tr>
<tr>
<td>Two Country</td>
<td>Optimal $\eta$</td>
<td>0.19</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>Welfare Gains</td>
<td>0.001</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Notes: Small open: Small open-economy model with fixed world interest rate. Two country: Two-country model with endogenously determined world interest rate. Italic numbers in this table are optimal $\eta$’s. Welfare gains are measured as percentage changes in certainty-equivalent consumption over the benchmark case with fixed tax policy, while the certainty-equivalent consumption is calculated based on conditional welfare changes with labor fixed at the steady state.

maximum welfare gains from active tax policy are 0.03 percent, 0.005 percent, and 0.001 percent (in terms of permanent consumption) for consumption tax, capital income tax, and labor income tax, respectively. These gains decrease as shocks become less persistent. Even though the absolute magnitude of these welfare gains seems to be
small, the size of the welfare gains is comparable to the maximum possible welfare gains from removing business cycles in the economy, which is around $0.01 \sim 0.05$ percent of permanent consumption (Lucas 1987).

### 3.2 A Small Open Economy

The second rows in each panel of table 1 report the results of a small open-economy model with an exogenously fixed interest rate. First, optimal $\eta_c$ for consumption tax becomes less countercyclical, decreasing to $0.3 \sim 1.4$ (relative to $2.5 \sim 2.7$ in the closed economy), and welfare gains dramatically decrease compared to the closed-economy model. Optimal tax response $\eta$ for capital income tax becomes procyclical when shocks are not very persistent. In particular, optimal $\eta_k$ decreases to $-1.6$ when $\rho = 0.85$, and to $-0.5$ when $\rho = 0.9$. Welfare gains from optimal capital income tax policy are around $0.001\% \sim 0.006\%$, similar to the closed-economy case. Optimal $\eta_l$ for labor income tax and the amount of welfare gains are similar in both closed- and open-economy cases. This similarity is due to the fact that there is no labor mobility across countries, while consumption and capital goods are traded across countries.

In an open economy, the current account works as a buffer against productivity shocks and plays a role in consumption smoothing (other than the investment channel, which exists in the closed economy as well). The level of consumption smoothing achieved in the open economy is larger than that in the closed economy, and therefore the role of business-cycle-stabilizing tax policies is reduced. In the case of consumption tax where the optimal tax policy is countercyclical in the closed economy, governments—when facing positive shocks—do not have to increase tax rates as much as in the closed-economy case to stabilize business cycles. With positive shocks, agents can smooth consumption by accumulating international bonds (i.e., lending to other countries). Therefore, optimal consumption tax policy becomes less countercyclical and the amount of welfare gains significantly decrease in the open economy because of a decrease in stabilization gains.

Another channel of welfare gains is through improving efficiency. This channel becomes most evident in the case of capital income tax policy. The results in table 1 show that optimal tax policy for capital
income tax becomes procyclical in the open economy when shocks are not very persistent. Lowering tax rates with positive productivity shocks generates efficiency gains by stimulating agents to produce more in a more productive country and lend additional output to foreign countries. This channel is not available in the closed-economy model where extra output should be consumed domestically. In the closed-economy model, efficiency gains from procyclical policy are always outweighed by stabilization loss, resulting in welfare loss.

3.2.1 Sensitivity Analysis

We perform several sensitivity exercises in this section. First, instead of assuming that distortions are generated by all three types of taxes, we now assume that only one tax is used to finance government spending in order to analyze each tax policy individually. Figure 1 plots how the optimal tax policy $\eta$ changes with the amount of distortions, in both closed and small open economies. Government spending (as a ratio of output) and the corresponding steady-state tax rates (that satisfy a balanced budget at the steady state) are on the x-axis, while the y-axis represents optimal $\eta$. The figure shows that the results in table 1 hold in most cases. For all three taxes, optimal tax policy is countercyclical in the closed economy (i.e., positive $\eta$) and the absolute value of $\eta$ increases with the amount of distortions (as represented by the steady state $\bar{\tau}$). Optimal tax policy in the open economy becomes less countercyclical than that in the closed economy in all cases except for consumption tax when distortions are low ($G/Y$ is less than 15 percent). For capital income tax and labor income tax with low distortions ($G/Y$ is less than 15 percent), optimal policy is procyclical in an open economy.

Second, in order to understand the mechanism behind welfare gains, we compare welfare gains from procyclical and countercyclical tax policies when there are significant distortions ($G/Y = 20\%$), as reported in table 2. For each tax, we set $\eta$ at 0.4 (countercyclical) and $-0.4$ (procyclical) and calculate welfare gains, which are decomposed into the mean effect (generated by changes in the conditional mean of the variables) and the variance effect (generated by changes in the conditional variance of the variables). We further decompose the mean effects into consumption mean effect and labor mean effect. The results show that countercyclical tax policy generates positive
Figure 1. Optimal Tax Rate Response: Sensitivity Analysis

**Consumption Tax**

- **Closed**
- **Small open**

**Capital Income Tax**

- **Closed**
- **Small open**

**Labor Income Tax**

- **Closed**
- **Small open**

**Notes:** Government spending is financed by only one tax in each panel. Numbers in parentheses are the steady-state tax rate that balances the budget.

variance effects and negative mean effects in all cases, while procyclical policy generates opposite results (negative variance effects and positive mean effects) in all cases. Kollmann (2002) and Bergin, Shin, and Tchakarov (2007) used the terms “efficiency gains” for
Table 2. Decomposition of Welfare Gains ($G/Y = 20\%$)

<table>
<thead>
<tr>
<th></th>
<th>$\eta$</th>
<th>Welfare Gains</th>
<th>Mean Effect (Cons, Labor)</th>
<th>Variance Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Closed Economy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C-tax</td>
<td>−0.4</td>
<td>−0.002</td>
<td>0.009 (0.009, −0.0004)</td>
<td>−0.011</td>
</tr>
<tr>
<td>$\bar{\tau} = 37%$</td>
<td>0.4</td>
<td>0.001</td>
<td>−0.008 (−0.006, −0.002)</td>
<td>0.009</td>
</tr>
<tr>
<td>K-tax</td>
<td>−0.4</td>
<td>−0.011</td>
<td>0 (0.002, −0.002)</td>
<td>−0.011</td>
</tr>
<tr>
<td>$\bar{\tau} = 74%$</td>
<td>0.4</td>
<td>0.006</td>
<td>−0.003 (−0.004, 0.001)</td>
<td>0.009</td>
</tr>
<tr>
<td>L-tax</td>
<td>−0.4</td>
<td>−0.008</td>
<td>0.018 (0.019, −0.001)</td>
<td>−0.026</td>
</tr>
<tr>
<td>$\bar{\tau} = 33.5%$</td>
<td>0.4</td>
<td>−0.003</td>
<td>−0.022 (−0.026, 0.004)</td>
<td>0.019</td>
</tr>
</tbody>
</table>

| **Small Open Economy** |        |               |                           |                 |
| C-tax          | −0.4   | −0.002        | 0.007 (0.010, −0.002)     | −0.009          |
| $\bar{\tau} = 37\%$ | 0.4    | −0.001        | −0.007 (−0.007, 0.0001)   | 0.008           |
| K-tax          | −0.4   | −0.001        | 0.014 (0.015, −0.001)     | −0.015          |
| $\bar{\tau} = 74\%$ | 0.4    | −0.011        | −0.024 (−0.026, 0.002)    | 0.012           |
| L-tax          | −0.4   | −0.006        | 0.017 (0.011, 0.006)      | −0.023          |
| $\bar{\tau} = 33.5\%$ | 0.4    | −0.005        | −0.022 (−0.020, −0.002)   | 0.016           |

**Notes:** This table corresponds to figure 1, where government spending is financed by only one tax at a time. Mean effect is defined as welfare changes due to changes in the mean (first-order terms) of utility, while variance effect is welfare changes in the variance (second-order terms) of utility. Mean effect is decomposed into the mean effect due to changes in the conditional mean of consumption and labor. Since utility is a negative function of labor, positive mean effect from labor implies that the conditional mean of labor (leisure) decreases (increases).

Mean effects and “stabilization gains” for variance effects in analyzing welfare gains of monetary policy.\textsuperscript{15} Countercyclical tax policy reduces volatility of the variables and stabilizes the economy. These stabilization gains exceed the size of negative mean effects.

To better understand the mean and variance effects, we draw impulse responses.\textsuperscript{16} Figures 2–4 present impulse responses to a positive productivity shock of the economy with procyclical ($\eta = −0.4$)

\textsuperscript{15}Our decomposition follows their convention of defining the gains in term of the original variables rather than a transformation such as a logarithmic one.

\textsuperscript{16}These impulse responses are based on the “pruned” solution of the second-order perturbation method, as suggested in Kim et al. (2008).
and countercyclical ($\eta = 0.4$) tax policy in both closed and small open economies. All countercyclical tax policies lower the magnitude of responses of consumption and labor to the shock, which would lower volatility of consumption and labor. This generates a positive variance effect. On the other hand, procyclical tax policy generates more volatility of consumption and labor, resulting in negative variance effects. In particular, figure 3 demonstrates how procyclical capital income tax policy can improve welfare. In the open economy with a positive productivity shock, procyclical capital income tax policy increases investment by almost 50 percent more than the case with fixed-tax policy. Consumption also rises more than in the fixed-tax policy case. With procyclical tax policy, agents can
take advantage of positive productivity in a more aggressive manner without sacrificing consumption because of the possibility of international borrowing and lending. These efficiency gains exceed stabilization losses from procyclical tax policy under certain parameter values. On the other hand, in the closed economy, procyclical capital income tax policy increases investment by only 20 percent relative to the fixed-tax case. Increases in investment are constrained by domestic resource constraints and should be financed by sacrificing consumption. The amount of efficiency gains of procyclical capital income tax policy is less than the amount of stabilization losses.
As our third sensitivity analysis, we generalize the linear tax policy rule by including the previous period’s tax rate on the right-hand side of the equation (7) as follows:

$$\tau_t - \bar{\tau} = \omega(\tau_{t-1} - \bar{\tau}) + \eta \left( \log (A_t) - \log (\bar{A}) \right).$$

(10)

As the lagged tax rate enters the tax policy equation with more weight (higher $\omega$), the optimal tax response $\eta$ moves closer to zero. That is, the optimal tax rate does not need to respond to changes in total factor productivity (TFP) as much as under the case of less persistence.\(^{17}\)

\(^{17}\)The numerical results are available from the authors upon request.
Finally, in order to analyze the case when an economy starts rather away from the model’s steady state, we run simulations when initial bond-output ratios are not zero. We experimented when initial bond-output ratio is 5 percent, 1 percent, –1 percent, and –5 percent. In all cases, the optimal tax response $\eta$ stays the same. In this experiment, changes in initial asset holding positions (or other variables away from their steady state) affect the steady-state values of the model but not model dynamics expressed in terms of percentage changes from the steady state.

3.3 A Two-Country Model

In the two-country world, the interest rate is endogenously determined by the bond market clearing condition. It is well known that interest rate is a negative function of current world output. When world output increases temporarily, the interest rate decreases as illustrated in Kim, Kim, and Levin (2003). With positive shocks, agents would accumulate bonds to smooth consumption. However, increasing demand for bonds pushes up bond price (lowers interest rate), which lowers the amount of bond traded. Under the benchmark parameter values, endogenous interest rate (in the two-country model) reduces the amount of bond trading to about one-third of the level achieved in the case of fixed interest rate (as in the small open-economy model).

The last rows in each panel of table 1 show optimal tax policies derived in the two-country model. For all three types of taxes, optimal $\eta$’s are similar to those in the small open-economy case. Welfare gains significantly decrease in the case of consumption and capital income tax. Table 3 shows how optimal $\eta$’s and maximum welfare gains change when parameter values for capital mobility and shock correlation change. The following parameter values are used for the benchmark two-country model: $\rho$ (shock persistence) = 0.9, $\zeta$ (bond holding adjustment cost parameter) = 0.001, $\nu$ (shock spillover) = 0, and $\psi$ (contemporaneous cross-country correlation of shocks) = 0. We first examine the case when the bond holding adjustment cost parameter increases to ($\zeta = 0.1$). With higher adjustment costs, agents do not trade bonds as much as in the benchmark case and the behavior of the economy approaches that of the closed economy. Therefore, optimal $\eta$ increases (becomes more countercyclical or less
Table 3. Sensitivity Analysis in a Two-Country Case

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Optimal $\eta_c$</th>
<th>Optimal $\eta_k$</th>
<th>Optimal $\eta_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-Country (Benchmark)</td>
<td>0.4 (0.0005)</td>
<td>-0.3 (0.0004)</td>
<td>0.2 (0.002)</td>
</tr>
<tr>
<td>Low Capital Mobility</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low Capital Mobility $\zeta = 0.1$</td>
<td>2.3 (0.01)</td>
<td>0.8 (0.002)</td>
<td>0.1 (0.0003)</td>
</tr>
<tr>
<td>Positive Spillovers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Positive Spillovers $\nu = 0.08$</td>
<td>1.3 (0.01)</td>
<td>0.4 (0.003)</td>
<td>0.2 (0.005)</td>
</tr>
<tr>
<td>Correlated Shocks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correlated Shocks $\psi = 0.5$</td>
<td>1.0 (0.003)</td>
<td>0.2 (0.0001)</td>
<td>0.2 (0.001)</td>
</tr>
</tbody>
</table>

Notes: Benchmark economy is the two-country model with $\rho = 0.9$, taken from table 1. Numbers in parentheses are welfare gains.

procyclical) towards the value of the closed-economy model. Next, we experiment by increasing spillover of productivity shocks across countries (positive $\nu$) and contemporaneous correlation of shocks ($\psi = 0.5$). Both changes imply that home and foreign countries now face similar productivity shocks as before. Therefore, less amount of bond trading is required for consumption smoothing compared to the benchmark case when shocks are neither correlated nor transferred. As a result, optimal tax policies move closer to the closed-economy case (more countercyclical), while optimal labor income tax does not change by much.

3.3.1 Non-cooperative and Cooperative Equilibriums

In the simulation results reported above, tax rates in the foreign country are assumed to be fixed when the domestic country implements an active tax policy. In this section, we relax this assumption and analyze optimal tax policy of the domestic country when the foreign country also adopts an active tax policy. Two types of exercises are implemented. We first vary the reaction of the foreign country’s tax policy and find the non-cooperative Nash equilibrium using the best response curves of the two countries. Next, we calculate the cooperative equilibrium and analyze welfare gains from tax policy cooperation. We set the shock persistence parameter $\rho$ at 0.9 in this section. Note that we assume that the governments credibly commit
Figure 5. Welfare Effects of Tax Policy: Two-Country Case

Figure 5 shows the welfare gains (of home and foreign countries) of active tax policy when the foreign tax rate is fixed ($\eta^* = 0$). In the top panel with C-tax policy, domestic welfare is maximized to the tax rate rule (perfect commitment). In the case of imperfect commitment, the results can be somewhat different, as discussed in Nason (this issue).

Figure 5 shows the welfare gains (of home and foreign countries) of active tax policy when the foreign tax rate is fixed ($\eta^* = 0$). In the top panel with C-tax policy, domestic welfare is maximized.
Table 4. Welfare Effects of Tax Policy Cooperation

<table>
<thead>
<tr>
<th></th>
<th>Optimal ($\eta, \eta^*$)</th>
<th>Country</th>
<th>Welfare Gains</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>(Mean Effect, Variance Effect)</td>
</tr>
<tr>
<td>C-tax</td>
<td>(0.4, 0)$^a$</td>
<td>Home</td>
<td>0.0005 (–0.0121, 0.0126)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Foreign</td>
<td>0.0023 (0.003, –0.0007)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>World</td>
<td>0.0014 (–0.0045, 0.0059)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H,F,W</td>
<td>0.003 (–0.009, 0.012)</td>
</tr>
<tr>
<td></td>
<td>(0.4, 0.4)$^b$</td>
<td></td>
<td>0.006 (–0.025, 0.031)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H,F,W</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.5, 1.5)$^c$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>K-tax</td>
<td>(–0.3, 0)$^a$</td>
<td>Home</td>
<td>0.004 (0.0027, –0.0023)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Foreign</td>
<td>–0.0009 (–0.0011, 0.0002)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>World</td>
<td>–0.0002 (0.0008, –0.0011)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H,F,W</td>
<td>–0.0005 (0.0016, –0.0021)</td>
</tr>
<tr>
<td></td>
<td>(–0.3, –0.3)$^b$</td>
<td></td>
<td>0.00003 (–0.00065, 0.00068)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H,F,W</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1, 0.1)$^c$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L-tax</td>
<td>(0.2, 0)$^a$</td>
<td>Home</td>
<td>0.0016 (–0.0086, 0.0103)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Foreign</td>
<td>–0.0027 (–0.0035, 0.0008)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>World</td>
<td>–0.0005 (–0.0061, 0.0056)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H,F,W</td>
<td>–0.001 (–0.012, 0.011)</td>
</tr>
<tr>
<td></td>
<td>(0.2, 0.2)$^b$</td>
<td></td>
<td>0 (0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H,F,W</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0)$^c$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$Domestic tax policy only.
$^b$Non-cooperative Nash equilibrium.
$^c$Cooperative equilibrium

**Note:** For $^b$ and $^c$, home, foreign, and world welfare gains are identical due to the symmetry of countries.

when $\eta_c = 0.4$, an increase in consumption tax rate by 0.4 percent in response to a 1 percent increase in productivity. The maximum welfare gains are quite small at 0.0005 percent of permanent consumption, as shown in table 4. Countercyclical consumption tax policy generates positive spillovers to the foreign country as its welfare increases by 0.002 percent. Positive welfare gains are due to positive mean effects that exceed negative variance effects. We can derive the non-cooperative Nash equilibrium by drawing best-response curves of the two countries. For all three types of taxes, the best-response curves turn out to be vertical or horizontal, which implies that optimal $\eta$ does not depend on foreign tax policy. Therefore, the Nash equilibrium is achieved when $\eta_c = \eta_c^* = 0.4$ and the welfare gains are 0.003 percent, which is higher than the domestic welfare gains.
when the foreign country does not implement any tax policy. This is due to positive spillover effects.

In most cases, however, this non-cooperative Nash equilibrium does not maximize the world welfare. We define the cooperative equilibrium as the outcome when both countries use their tax policy to maximize the sum of domestic and foreign welfare. For consumption tax, the cooperative equilibrium is achieved when $\eta_c = \eta^*_c = 1.5$, suggesting that the consumption tax policy should be more countercyclical than the Nash equilibrium for the maximization of world welfare. The welfare gains at the cooperative equilibrium are 0.006 percent. We measure the welfare gains from cooperation by taking the difference of welfare level between the Nash solution and the cooperative solution. In the case of consumption tax, the gains from cooperation are 0.003 percent of permanent consumption.

The middle panel in figure 5 plots the welfare gains of the two countries when the domestic government changes $\eta_k$ holding $\eta^*_k$ constant at zero. The maximum welfare gains are quite small at 0.0004 percent of permanent consumption, and they are achieved when $\eta_k = -0.3$, interpreted as a decrease in capital income tax rate by 0.3 percent with a 1 percent positive productivity shock. In this case, the procyclical capital income tax policy (negative $\eta_k$) decreases the level of foreign welfare, mostly due to negative mean effects. The Nash equilibrium is achieved when $\eta_k = \eta^*_k = -0.3$. Because of the large size of negative spillovers, welfare of each country actually decreases at the Nash equilibrium. The cooperative equilibrium is achieved when the two countries implement slightly countercyclical tax policy at $\eta_k = \eta^*_k = 0.1$, but the size of welfare gain is negligible. The bottom panel in figure 5 shows the welfare gains of labor income tax policy. With no foreign tax policy ($\eta^*_l = 0$), optimal $\eta_l$ is at 0.2 with welfare gains of 0.0016 percent. The Nash equilibrium is at $\eta_l = \eta^*_l = 0.2$ with welfare loss of 0.001 percent due to negative spillovers. There is no welfare gain under the cooperative equilibrium in the case of labor income tax.

Summarizing, when foreign countries also implement an active tax policy, optimal tax policies on capital and labor income lower welfare of both countries at the non-cooperative Nash equilibrium. Tax policy cooperation produces a higher level of welfare compared to the Nash equilibrium, but the actual welfare gains are minimal relative to the fixed-tax policy case. In the case of consumption
tax, active consumption tax policy generates positive spillovers and, therefore, both countries gain at the Nash equilibrium. Furthermore, the cooperative equilibrium produces quite large welfare gains compared to the Nash equilibrium. However, as evidenced by recent experiences in European Union countries, cooperation among different fiscal authorities is hard to achieve. It has been shown in the game-theory literature (for example, Fudenberg and Tirole 1991) that reputation and repeated games as mechanisms—when combined with appropriate penalty structure—could yield cooperative outcomes, but such outcomes have not yet been achieved in the real world.

3.4 Two Possible Extensions

One of the ongoing policy debates in the United States is whether tax reform would boost productivity growth and the growth rate of potential output. In this paper, this possibility is ruled out because TFP is modeled as an exogenous AR(1) process. One potential extension is to let the tax policy have a potential to affect TFP—perhaps by affecting the adoption rate of new technologies or via firms’ entry and exit. For example, lowering capital income tax or increasing investment tax credit would encourage companies to increase investment spending on technological improvement that can positively affect the level of TFP. In such a case, the degree of cyclical for optimal tax policy would change, but this type of analysis is beyond the scope of this paper.

It would also be interesting to investigate our results on tax policy in the context of the optimal monetary policy literature. A number of studies have shown that optimal monetary policy is procyclical with supply shocks (productivity shocks), while the optimal policy is countercyclical with demand shocks. Procyclical interest rate policy improves welfare by reducing distortions from rigidities in the economy, when hit by supply shocks. In this paper, the sources of distortions are different since our model has no nominal rigidities and the only distortions are from distortionary taxes. Even with different sources of distortions, this model produces the same implication

\[18\] See, for example, Ireland (1996) and Obstfeld and Rogoff (2002).
as the monetary policy literature that optimal capital income tax policy is procyclical with supply shocks.

If the model has nominal rigidities in addition to distortionary taxes, optimal tax responses would change. Current literature using the model with nominal rigidities (and with monetary and fiscal policies) focuses on either (i) Ramsey-optimal fiscal policy (Schmitt-Grohé and Uribe 2004, and Siu 2004), (ii) government spending as a fiscal policy tool—combined with lump-sum taxation—as in Beetsma and Jensen (2005) and Galí and Monacelli (2008), or (iii) tax policy instruments responding to fiscal liabilities (Schmitt-Grohé and Uribe 2007, and Kollmann 2008). Among these three related areas, our paper is mostly related to the papers in the third area. However, it would be hard to directly infer how our results would change under nominal rigidities because tax rules and model features are specified in a different way.\footnote{We conjecture that, in the presence of nominal rigidities, additional distortions coming from nominal rigidities would affect the degree of countercyclicality of optimal tax policy; however, we also think that the main conclusion of the paper that optimal tax policy becomes less countercyclical in open economies is likely to stay.}

\par

4. Conclusion

A conventional wisdom is that optimal tax policy is probably countercyclical rather than procyclical. We have shown that this proposition—though true in the closed economy—may not necessarily hold in an open economy where countries can trade international bonds for the consumption-smoothing purpose. Optimal tax polices in the open economy become less countercyclical compared to the closed economy due to the consumption-smoothing role of the current account. More importantly, in the case of capital income tax, optimal tax policy can even be procyclical. Procyclical tax policy stimulates agents to produce more in a more productive state, and agents can take advantage of this extra output through international lending and borrowing. For capital income tax, the efficiency gains

\footnote{For example, the papers in the third area typically use a model with domestic bonds for risk sharing, and tax rates playing a passive role to match fiscal balance.}
from procyclical tax policy outweigh stabilization losses, improving overall welfare. We also show that positive welfare gains of active tax policy may disappear when foreign countries use active policy, in particular for the capital and labor income taxes. International tax policy cooperation does not generate significant welfare gains, except for the consumption tax.

In general, welfare gains from active tax policies are quite small compared to welfare gains of tax policy reform that changes tax rates permanently, as considered in Mendoza and Tesar (1998, 2005). This is because the tax policies considered in this paper are designed to be fine-tuning in the sense that tax rates can only respond to business cycles (fluctuations in productivity) in an economy. However, it is less difficult to implement such policies compared to the permanent changes in tax rates. Moreover, active tax policies can play an important role in stabilizing an economy where monetary policy cannot be used for the stabilization purpose in many instances, such as the case of zero lower bound for nominal interest rate and the lack of different monetary policy responses among the member countries of the European Union.

Appendix

The First-Order Conditions

The domestic economy is described by the following twelve equations together with equations for productivity shocks and tax processes:

\[ 0 = (1 - \sigma)U_t - \left[ C_t^\theta (1 - L_t)^{1-\theta} \right]^{1-\sigma}, \]

\[ 0 = Y_t - A_t L_t^\alpha K_t^{1-\alpha}, \]

\[ 0 = \lambda_t C_t (1 + \tau_{ct}) - \theta (1 - \sigma) U_t, \]

\[ 0 = (1 - \tau_{lt}) \lambda_t w_t (1 - L_t) - (1 - \theta)(1 - \sigma) U_t, \]

\[ 0 = K_{t+1} - \left[ \delta (I_t/\delta)^{1-\phi} + (1 - \delta) K_t^{1-\phi} \right]^{1-\sigma}, \]

\[ 0 = \beta R_t E_t (\lambda_{t+1}) - \lambda_t (1 + \zeta B_t), \]

\[ 0 = \tilde{G} + T_t - \tau_{ct} C_t - \tau_{lt} w_t L_t - \tau_{kt} (r_t - \delta) K_t - \frac{\zeta}{2} (B_t)^2, \]
\[ 0 = Y_t + R_{t-1}B_{t-1} - C_t - I_t - G_t - B_t, \]
\[ 0 = w_tL_t - \alpha Y_t, \]
\[ 0 = r_tK_t - (1 - \alpha)Y_t, \]
\[ 0 = \lambda_t - \mu_t \left[ \delta (I_t/\delta)^{1-\phi} + (1 - \delta)K_t^{1-\phi} \right]^{1/\phi} \left( \frac{I_t}{\delta} \right)^{-\phi}, \]
\[ 0 = \mu_t - \beta E_t \left[ (1 - \delta)\lambda_{t+1}(I_{t+1}/\delta)^{\phi}(K_{t+1})^{-\phi} + \lambda_{t+1}(1 - \tau_{k,t+1} + \delta \tau_{k,t+1}) \right], \]

where \( \lambda_t \) and \( \mu_t \) are Lagrangian multipliers for the budget constraint and capital accumulation equation, respectively. There are foreign country analogues to the above equations. The world equilibrium is achieved by imposing the world resource constraint.

References


