# Sovereign Default and Monetary Policy Tradeoffs*

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The paper is organized around the following question: when the economy moves from a debt-GDP level where the probability of default is nil to a higher level—the "fiscal limit"—where the default probability is non-negligible, how do the effects of routine monetary operations designed to achieve macroeconomic stabilization change? We find that the specification of the monetary policy rule plays a critical role. Consider a central bank that targets the risky rate. When the economy is near its fiscal limit, a transitory monetary policy contraction leads to a sustained rise in inflation, even though monetary policy actively targets inflation and fiscal policy passively adjusts taxes to stabilize debt. If the central bank targets the risk-free rate, on the other hand, the same transitory monetary contraction keeps inflation under control but leads output to contract for a prolonged period of time. The comparison shows that sovereign default risk puts into sharp relief the tradeoff between inflation and output stabilization.

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## 1. Introduction

Economies may behave significantly differently in times of crisis. In this paper we are interested in assessing, when the economy moves from a debt-GDP level where the probability of default is nil to a higher level where that default probability is non-negligible, how the effects of routine monetary operations change.

We find that the specification of monetary policy rule—whether a central bank targets an interest rate rule defined in terms of the risky or the risk-free nominal interest rate—plays a critical role. If the central bank targets the risky rate and when the economy is near its fiscal limit, a transitory monetary policy contraction leads to a sustained rise in inflation, even though monetary policy actively targets inflation and fiscal policy passively adjusts taxes to stabilize debt. If the central bank targets the risk-free rate, on the other hand, the same monetary policy contraction doesn't raise inflation but leads to a sustained lower path for output for a prolonged period of time. The intuition is following. If the central bank targets the risky rate, to satisfy a Taylor rule is equivalent to postulating a risk-free rule that includes the usual positive response to inflation and a negative response to the expected default rate. Higher expected default rates are accommodated with lower risk-free rates and, therefore, higher inflation. These effects arise even though fiscal policy passively adjusts taxes to stabilize debt.

The literature on the interactions between sovereign default and monetary policy rule typically defines that rule in terms of the yield on risky government debt. In this paper, we show that such a specification implicitly assumes that the monetary authority accommodates the rise in sovereign default risk by lifting the inflation target. There are strong theoretical and empirical reasons to expect causal links between sovereign default and the fragility of the banking system. For instance, Sosa-Padilla (2015) argues that the main costs of sovereign default arise from the impact on the balance sheets of banks holding that debt, as they sharply contract corporate lending, while Brunnermeier et al. (2016) suggest that there may be a "diabolic loop" linking soveriegn and bank credit risk. Such links can motivate why the central bank chooses to accommodate sovereign risk. We contrast such a description of monetary policy with an alternative where the central bank defines its policy rule in terms of the risk-free rate and is not seeking to offset default risk, ceteris paribus. In a related paper, Reis (2017) shows that during a fiscal crisis, quantitative easing policies can have a powerful impact on macroeconomic outcomes through changing both the composition of the privately held public debt and the costs of government default in banking sector.

To explore this issue, we develop a model of the "fiscal limit" in the context of a conventional New Keynesian model, with monetary and fiscal policy interactions. Monetary policy has real effects in our sticky-price economy, which implies that, in addition to the usual impacts on intertemporal consumption decisions, it also influences the size of the tax base and real debt service costs. On top of this rich mix of monetary and fiscal policy interactions we introduce a fiscal limit as in Bi (2012), whereby there is a partial default on outstanding government debt when the economy breaches a certain debt-output threshold. This threshold depends upon the underlying fundamental shocks to the economy, as well as stochastic fluctuations in political risk, so that bondholders demand significant risk premiums on government debt prior to hitting the fiscal limit.

The plan of the paper is as follows. The next section outlines our general model. In section 3, we use a simple endowment economy to explore how the possibility of debt default can cause the monetary authorities to lose control of inflation when they target the risky rate, even though monetary policy remains active and fiscal policy passive in the sense of Leeper (1991). The quantitative impact of this channel is then analyzed in section 4, where we first describe the setup of fiscal limits and then discuss the dynamic impacts of an exogenous monetary contraction with different specifications of monetary policy rules. Given that the loss of inflation control would have real effects in our sticky-price economy, we also explore the local determinacy properties of our model in appendix 1 and find that a high rate of default could matter for equilibrium determinacy, but it isn't a concern for our benchmark calibration.

## 2. A General Model

As our aim is to explore how the possibility of sovereign debt default interacts with monetary and fiscal policies, we consciously use a conventional New Keynesian model of the kind typically used to explore monetary and fiscal policy interactions (see, for example, Benigno and Woodford 2004), modified only by allowing government debt to be risky. Specifically, households in our economy supply labor to imperfectly competitive intermediate-goods-producing firms. Facing costly Rotemberg (1982)-style price adjustment, these firms do not completely adjust prices in the face of shocks. Moreover, rather than rendering fiscal policy redundant by balancing the budget through lump-sum taxes, we assume that households' labor and profit income is taxed. The distortionary taxes influence households' labor supply decisions, which in turn affects firms' marginal costs and pricing decisions. Taken together, the model allows a relatively rich set of monetary and fiscal policy interactions. Monetary policy has real effects due to the assumption of price stickiness, which in turn affects both the size of the tax base and real debt service costs. While fiscal policy has the obvious fiscal consequences, it also influences inflation either through the labor supply response to distortionary taxation or the aggregate demand effect of changes in government spending.[1]

We then further extend this model to allow for the possibility of government default: a haircut is applied whenever the economy hits its "fiscal limit." This fiscal limit is defined as the present value of maximum future primary surpluses, which depends upon both the state of the economy and the political constraints on taxing at the peak of Laffer curve. Given exogenous fluctuations in productivity, government spending, and political risk, the fiscal limit is stochastic and investors may demand risk premiums on government debt before reaching the fiscal limit. The resulting debt dynamics may imply quite different monetary and fiscal impacts, in comparison to an economy operating well away from its fiscal limit.

## *2.1 Households*

Our cashless economy is populated by a large number of identical households of size 1, who have preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, n_t\right),$$

[1] Reis (forthcoming) looks at additional linkages between monetary and fiscal policy.

where $\beta \in (0,1)$ is the households' subjective discount factor, $c_t$ is consumption, and $n_t$ is the households' labor supply. The household receives nominal wages $W_t$ and monopoly profits $\Upsilon_t$ from the firm, both of which are taxed at the rate $\tau_t$, and lump-sum transfers $z_t$ from the government. The household maximizes utility subject to the budget constraint,

$$P_t c_t + \frac{B_t}{R_t} = (1-\delta_t)B_{t-1} + (1-\tau_t)\left(W_t n_t + P_t \Upsilon_t\right) + P_t z_t. \qquad (1)$$

At each period, if the existing debt level is above a fiscal limit, the government partially defaults at rate $\delta_t = \delta$; otherwise, it repays in full amount with $\delta_t = 0$. Section 4 provides further discussion on the fiscal limit. This probability of default is endogenous to the model but is taken as given by households. Bonds, therefore, pay a risky yield of $R_t$. First-order conditions for this optimization problem are

$$\frac{1}{R_t} = \beta E_t \frac{u_c(t+1)}{u_c(t)} \frac{1-\delta_{t+1}}{\pi_{t+1}} \qquad (2)$$

$$-\frac{u_n(t)}{u_c(t)} = w_t \left(1-\tau_t\right), \qquad (3)$$

where $w_t \equiv W_t/P_t$ is the real wage. The first condition describes the household's optimal allocation of consumption over time, and the second, their optimal labor supply decision. Notice in the case of the latter, labor income is taxed so that changes in the tax rate will influence households' desire to work.

Households can also trade a risk-free bond, which is, however, in zero net supply, so that where Euler equation (2) determines the risky nominal interest rate,

$$\frac{1}{R_t^f} = \beta E_t \frac{u_c(t+1)}{u_c(t)} \frac{1}{\pi_{t+1}} \qquad (4)$$

defines the risk-free nominal interest rate.

## *2.2 Final Goods Production*

Final goods production is for the purposes of private and public consumption, and competitive final goods firms buy the differentiated products produced by intermediate goods producers in order to construct consumption aggregates, which have the usual constant elasticity of substitution (CES) form,

$$Y_t = \left( \int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \tag{5}$$

where $Y_t$ is aggregate output, $y_t(i)$ is the output of intermediate good firm $i$, and $\theta > 1$ is the elasticity of demand for each firm's product. Cost minimization on the part of final goods producers results in the following demand curve for intermediate good $i$:

$$y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t \tag{6}$$

and an associated price index for final goods,

$$P_t = \left( \int_0^1 p_t(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}. \tag{7}$$

### *2.3 Intermediate Goods Production*

The imperfectly competitive intermediate goods firms enjoy some monopoly power in producing a differentiated product. They face a downward-sloping demand curve, (6), but are also subject to Rotemberg (1982) quadratic adjustment costs in changing prices, $\frac{\phi}{2}\left(\frac{p_t(i)}{p_{t-1}(i)}\frac{1}{\pi^*} - 1\right)^2 P_t Y_t$. Therefore, large price changes in excess of steady-state inflation rates are particularly costly. The quadratic price adjustment costs render the firm's problem dynamic,

$$\max \quad \sum_{t=0}^{\infty} R_{0,t} \left( p_t(i) y_t(i) - mc_t P_t y_t(i) - \frac{\phi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} \frac{1}{\pi^*} - 1 \right)^2 P_t Y_t \right) \tag{8}$$

$$s.t. \quad y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} Y_t, \tag{9}$$

where $mc_t = w_t/A_t$ is the real marginal cost implied by a linear production function, $y_t(i) = A_t n_t(i)$. Productivity, $A_t$, is common to all firms and follows an $AR(1)$ process:

$$\log \frac{A_t}{A^*} = \rho_A \frac{A_{t-1}}{A^*} + \varepsilon_t^A \qquad \varepsilon_t^A \sim iid\, \mathcal{N}(0, \sigma_A^2).$$

The first-order condition, after imposing symmetry across firms, is

$$(1-\theta) + \theta mc_t - \phi \left( \frac{\pi_t}{\pi^*} - 1 \right) \frac{\pi_t}{\pi^*} + \beta \phi E_t \frac{u_c(t+1)}{u_c(t)} \left( \frac{\pi_{t+1}}{\pi^*} - 1 \right) \frac{\pi_{t+1}}{\pi^*} \frac{Y_{t+1}}{Y_t} = 0,$$

which represents the non-linear New Keynesian Phillips curve (NKPC) under Rotemberg pricing. Upon linearization, it would correspond to the standard NKPC under Calvo (1983) pricing.

The associated monopoly profit, which is taxed by the government when received by households, is

$$\Upsilon_t = Y_t - mc_t Y_t - \frac{\phi}{2} \left( \frac{\pi_t}{\pi^*} - 1 \right)^2 Y_t. \tag{10}$$

### 2.4 Government

The government collects distortionary taxes and issues bonds to finance its expenditures. It faces the following budget constraint:

$$\frac{B_t}{R_t} + \tau_t \left( W_t n_t + \Upsilon_t \right) = (1 - \delta_t) B_{t-1} + P_t g_t + P_t z_t. \tag{11}$$

In our model, default is costless in the sense that the defaulting government is neither forced to reform its policies by dramatically reducing deficits nor locked out of credit markets for some period. While fiscal policy changes in the form of tax, transfers, and government spending will obviously affect debt dynamics, monetary policy and risk premiums will also have a role to play, especially when debt stocks are large and the economy approaches its fiscal limit. The government's budget constraint can be rewritten in terms of real variables:

$$b_{t-1} \frac{1-\delta_t}{\pi_t} = \frac{b_t}{R_t} + T_t - g_t - z_t,$$

where $b_t = B_t/P_t$ is real government debt. $z_t$ is assumed to be fixed at its steady-state level, and $g_t$ follows an $AR(1)$ process:

$$\log \frac{g_t}{g^*} = \rho_g \frac{g_{t-1}}{g^*} + \varepsilon_t^g \qquad \varepsilon_t^g \sim iid\, \mathcal{N}(0, \sigma_g^2). \tag{12}$$

We assume that fiscal and monetary policy follow simple rules of the form

$$\tau_t - \tau^* = \gamma_\tau (b_t^d - b^*) \tag{13}$$

$$R_t^i - R^* = \alpha(\pi_t - \pi^*) + \varepsilon_t^R \qquad \varepsilon_t^R \sim iid\, \mathcal{N}(0, \sigma_R^2), \tag{14}$$

where $b_t^d = (1 - \delta_t) b_{t-1}$. The interest rate rule is defined in terms of $R_t^i$, which represents either the interest-rate-containing default risk premiums, $R_t$, or the risk-free rate of interest, $R_t^f$. Uribe (2006), Schabert (2010), and Matveev (2012) all consider a monetary policy rule defined in terms of the interest rate on risky government bonds, which we shall show in the next section amounts to accommodating default risk by effectively raising the inflation target, ceteris paribus.

### *2.5 Aggregate Resource*

The aggregate resource constraint is

$$c_t + g_t = A_t n_t \left(1 - \frac{\phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2\right). \tag{15}$$

In the aggregate, deviations of inflation from target generate real resource costs.

## 3. Simple Analytics: Default and Inflation

This section uses a simple analytical model to describe the link between default and inflation. Consistent with the economy described in section 2, we assume that monetary policy is active and fiscal policy is passive. This policy combination makes the linkages between inflation and default quite different from those described in Uribe (2006), who considers a similar economy but assumes that fiscal policy does not seek to stabilize government debt. In our setup, depending on the specification of the monetary rule, default may make it difficult for the monetary authority to hit its inflation target, even if monetary policy actively targets inflation and fiscal policy passively adjusts surpluses to stabilize debt.

Consider a constant endowment, cashless economy in which the equilibrium real interest rate, $1/\beta$, is also constant. Government default is the sole source of uncertainty and for current purposes, the decision to default by the fraction $\delta_t \in [0,1]$ on outstanding debt carried into period $t$ is exogenous and follows a known stochastic process. Let $R_t$ be the gross risky rate of return on nominal government debt and $\pi_t = P_t/P_{t-1}$ be the inflation rate. Household optimization yields the Fisher relation,

$$\frac{1}{R_t} = \beta E_t \left[\frac{1-\delta_{t+1}}{\pi_{t+1}}\right]. \tag{16}$$

Trading in risk-free bonds (assumed to be in zero net supply) gives an analogous relation for the risk-free interest rate, $R_t^f$,

$$\frac{1}{R_t^f} = \beta E_t \left[\frac{1}{\pi_{t+1}}\right]. \tag{17}$$

The government's budget constraint is

$$\frac{B_t}{P_t} + s_t = \frac{(1-\delta_t)}{\pi_t} R_{t-1} \frac{B_{t-1}}{P_{t-1}}, \tag{18}$$

where $s_t$ is the primary surplus. Write this constraint at $t+1$, take expectations conditional on information at $t$, impose the Euler equation $\beta^{-1} = E_t(1-\delta_{t+1})R_t/\pi_{t+1}$, and solve for $B_t/P_t$ to yield

$$\frac{B_t}{P_t} = \beta E_t \frac{B_{t+1}}{P_{t+1}} + \beta E_t s_{t+1}. \tag{19}$$

When the real interest rate is fixed, both the nominal rate and the inflation rate reflect default, so that the expected default rate drops out once expectations are taken. This implies that only surprises in default directly affect the evolution of real government debt in this flexible-price endowment economy. In light of this, we obtain, by iterating on (19) and imposing the household's transversality condition,

$$\frac{B_t}{P_t} = \sum_{j=1}^{\infty} \beta^j E_t s_{t+j}. \tag{20}$$

Expression (20) is the usual intertemporal equilibrium condition that equates the value of government debt to the expected present value of "cash flows," which are primary surpluses.

Fiscal policy sets the surplus in order to stabilize the post-default value of government debt,

$$s_t - s^* = \gamma \left[ (1 - \delta_t) \frac{B_{t-1}}{P_{t-1}} - b^* \right], \tag{21}$$

where $s^*$ and $b^*$ are target and steady-state values for the surplus and real debt. Substituting (21) into (18) and taking expectations at time $t$ yields the evolution of expected debt,

$$E_t b_{t+1} + (s^* - \gamma b^*) = [\beta^{-1} - \gamma(1 - E_t \delta_{t+1})] b_t. \tag{22}$$

One result that emerges immediately from (22) is that stability of the debt process in the face of debt default requires that

$$\gamma > \frac{\beta^{-1} - 1}{1 - E_t \delta_{t+1}}. \tag{23}$$

This condition is potentially far more demanding than the usual one that $\gamma > \beta^{-1} - 1$, particularly when substantial default rates are possible. Provided this condition is fulfilled, however, fiscal policy remains passive and capable of stabilizing the real value of government debt.

When specifying monetary policy behavior, we must choose which interest rate to adopt in defining the policy rule. If the central bank targets the risk-free rate, then the monetary policy rule is[2]

$$\frac{1}{R_t^f} = \frac{1}{R^*} + \alpha \left( \frac{1}{\pi_t} - \frac{1}{\pi^*} \right). \tag{24}$$

[2]The monetary policy rule is written in terms of $\frac{1}{R_t}$ instead of $R_t$ in order to obtain a closed-form solution without linearizing the model. Upon linearization this would be isomorphic with more standard representations of the Taylor rule.

Combining the policy rule with the Fisher relation, (17), yields the dynamic equation for inflation,

$$\frac{1}{\pi_t} - \frac{1}{\pi^*} = \frac{\beta}{\alpha} E_t \left( \frac{1}{\pi_{t+1}} - \frac{1}{\pi^*} \right). \tag{25}$$

Monetary policy can hit its target inflation rate, provided the policy behavior is sufficiently active, $\beta/\alpha < 1$.[3] Taken together, fiscal policy is passive if it satisfies (23), even though default can weaken its passivity; active monetary policy can successfully target inflation when the central bank's instrument is the risk-free nominal rate.

If the central bank targets the risky interest rate, on the other hand, the monetary policy rule becomes

$$\frac{1}{R_t} = \frac{1}{R^*} + \alpha \left( \frac{1}{\pi_t} - \frac{1}{\pi^*} \right). \tag{26}$$

This is our benchmark case, following Uribe (2006). Monetary policy aims to stabilize inflation by setting $\alpha/\beta > 1$. In defining this rule we are not saying that the policymaker has direct control over this rate as a policy instrument, but that through open market operations it adjusts the policy rate to achieve this relationship between the rate of return on government bonds and inflation. In the transmission from the very short-term rates targeted through open market operations to the wider economy, the central bank would expect to see a significant degree of pass-through to the contractual interest rates employed throughout the economy.[4]

In this case, default influences the ability of the monetary authority to target inflation, even if fiscal policy remains passive and monetary policy is active. To see this, combine the monetary policy rule in (26) with the Fisher relation to yield the dynamic equation for inflation,

$$\frac{1}{\pi_t} - \frac{1}{\pi^*} = \frac{\beta}{\alpha} E_t \left( \frac{1-\delta_{t+1}}{\pi_{t+1}} - \frac{1}{\pi^*} \right), \tag{27}$$

[3] In this paper, we restrict attention to locally bounded solutions, recognizing the argument in Cochrane (2011) that there is a continuum of explosive solutions to expressions like (25).

[4] Empirical evidence suggests that the rate at which policy interest rates pass through to bank interest rates is quite high—about 90 percent within a quarter (Gambacorta 2008). We are implicitly assuming similarly high rates of pass-through to government bond yields.

which now depends on the expected default rate. Active monetary policy implies that the unique locally bounded solution for inflation is

$$\frac{1}{\pi_t} = \frac{1}{\pi^*}\left(1 - \frac{\beta}{\alpha}\right)\left\{1 + E_t \sum_{i=1}^{\infty} \left(\frac{\beta}{\alpha}\right)^i \prod_{j=1}^{i} (1 - \delta_{t+j})\right\}. \tag{28}$$

In the absence of default, $\delta_t \equiv 0$, monetary policy achieves its inflation target exactly, $\pi_t = \pi^*$. Higher expected default rates in the future raise current inflation. The farther into the future default is expected, the more it is discounted by $\beta/\alpha < 1$, and the smaller is its impact on inflation at time $t$.

Consider a stylized experiment. At time $t$ news arrives that raises the expected default rate at $t + 1$, $E_t\delta_{t+1} > 0$, but all subsequent expected default rates are zero, $E_t\delta_{t+j} = 0$ for $j > 1$. Then (28) reduces to

$$\pi_t = \pi^* \left[\frac{1}{1 - \frac{\beta}{\alpha} E_t(\delta_{t+1})}\right] > \pi^*. \tag{29}$$

We see that higher expected default raises inflation, but the extent to which it does so is mitigated by a more aggressive monetary response to inflation in the form of a higher $\alpha$.

If the default rate is constant, $\delta_t \equiv \delta \in [0, 1]$, then more-aggressive monetary policy enhances the central bank's control of inflation. A constant default rate yields the solution for inflation,

$$\pi_t = \pi^* \left[\frac{1 - (1 - \delta)\frac{\beta}{\alpha}}{1 - \frac{\beta}{\alpha}}\right], \tag{30}$$

so that $\pi_t \to \pi^*$ as $\alpha \to \infty$. Again, a more-aggressive monetary policy response to inflation reduces the inflationary consequences of default. This simple example with a constant default rate is also useful in illustrating why a monetary rule with a risk-free rate, coupled with a passive fiscal policy, can successfully target inflation, while a rule with a risky rate cannot. Rewrite (26) in terms of the risk-free rate as

$$\frac{1}{R_t^f} = \frac{1}{R^*} + \frac{\alpha}{1 - \delta}\left[\frac{1}{\pi_t} - \left(\frac{1}{\pi^*} - \frac{\delta}{\alpha R^*}\right)\right]. \tag{31}$$

The monetary policy rules defined in terms of the risky rate of interest can be transformed into a rule of the same form as that defined in terms of the risk-free rate, but with two important differences. First, default does not make monetary policy less active; in fact, it raises the coefficient on excess inflation, $\frac{\alpha}{1-\delta} > \alpha$. Second, default raises the effective inflation target from $\pi^*$ to $\frac{\pi^*}{1-\delta\beta/\alpha}$.

Intuitively, a higher rate of default creates partial monetary policy accommodation: in the presence of default, the monetary authority must allow the risky rate of interest to rise to induce bondholders to continue holding the stock of government bonds. Given the monetary policy rule, the monetary authority will not raise interest rates without a rise in inflation. Bondholders attempt to sell bonds and increase their consumption paths, which pushes up the price level until bondholders are being compensated for their default risk and inflation and interest rates are consistent with the monetary rule. Stronger responsiveness of policy to inflation, higher $\alpha$, reduces the effective rise in the inflation target needed to achieve the rise in interest rates desired by bondholders. Another perspective on default is that higher expected default reduces the desirability of holding bonds, increasing aggregate demand. Higher demand drives up inflation. The central bank reacts to inflation by raising the risky nominal interest rate on government bonds.

As a general proposition, the possibility of default can undermine the central bank's control of inflation: there is a tight connection between expected default rates and inflation in our model, as in Uribe (2006), but the mechanism differs from Uribe's. Uribe obtains his result through a standard fiscal theory of the price-level mechanism by coupling an active monetary policy rule like (26) with an active fiscal rule akin to setting $\gamma = 0$ in (21), just as in Loyo (1999) and, more recently, Sims (2011). Those analyses highlight that a tight monetary policy can ultimately generate a worsening inflation situation if fiscal policy does not adjust to stabilize government debt. In contrast, our results stem from the monetary policy response to default, even though the monetary policy rule remains active and fiscal policy passive. Putting it differently, although we also find a positive link between default and inflation, that link differs in crucial aspects. For example, in Uribe (2006) delaying default supports unstable inflation dynamics for longer, making it more difficult for the monetary authorities to hit their inflation target. In our active monetary/passive fiscal regime, though, the impact of future default on prices is discounted so that delaying default reduces the immediate inflationary consequences of default. Furthermore, in Uribe (2006) raising $\alpha$ and making monetary policy more active further destabilizes inflation dynamics and moves the economy farther from its inflation target. More-active monetary policy in our environment reduces deviations from the inflation target due to default.

## 4. Quantitative Analysis

The simple model in section 3 illustrates that the specification of the monetary policy rule—risky versus risk-free nominal rate—matters for the inflation consequences of sovereign default risk and the ability of active monetary policy to target inflation in the face of such risk. In the rest of this paper, we explore the quantitative impact of this channel. To start, we first explain how to model sovereign default via fiscal limits.

### *4.1 Fiscal Limits*

Fiscal limits are defined, following Bi (2012), as the present value of maximum future primary surpluses. Laffer curves provide a natural starting point for quantifying the fiscal limit from the tax revenue side of the government's budget constraint. At the peak of the Laffer curve, tax revenues reach their maximum and, given some minimum level of total government expenditures, the expected present value of primary surpluses and, therefore, the value of government debt, are maximized. Revenues, expenditures, and discount rates, of course, vary with the shocks hitting the economy, generating a distribution for the maximum debt-GDP level that can be supported.

#### *4.1.1 Laffer Curve*

Assume the utility function is $u(c_t, n_t) = \log c_t + \chi_N \log(1 - n_t)$. Labor supply can be solved analytically as a function of $(\tau_t, \pi_t, A_t, g_t)$ using the first-order conditions. Work effort is given by

$$n_t = \frac{w_t X_{1,t} + \chi_n g_t}{w_t X_{1,t} + \chi_n X_{2,t}} \tag{32}$$

$$\text{with} \quad X_{1,t} = 1 - \tau_t \tag{33}$$

$$X_{2,t} = A_t \left(1 - \frac{\phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2\right). \tag{34}$$

Total tax revenue is

$$\begin{aligned} T_t &= (w_t n_t + \Upsilon_t)\,\tau_t \\ &= A_t n_t \tau_t \left(1 - \frac{\phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2\right). \end{aligned} \tag{35}$$

When the monetary authority keeps the inflation rate at its target ($\pi_t = \pi^*$) and transfers are at their steady-state level ($z_t = z^*$), the peak of the Laffer curve is a function only of the exogenous state of the economy $(A_t, g_t)$.

$$\tau_t^{\max} = \tau^{\max}(A_t, g_t) \tag{36}$$

$$T_t^{\max} = \mathcal{T}^{\max}(A_t, g_t) \tag{37}$$

Evidently, the stochastic processes governing the exogenous states induce stochastic processes for both the tax rate that maximizes revenues and the level of revenues.

### *4.1.2 Computing Fiscal Limit*

Importantly, the notion of a fiscal limit that we develop is *the private sector's perception* of the limit.

$$\mathcal{B}^* \sim \sum_{t=0}^{\infty} \beta^t \underbrace{\beta_t^p}_{\text{political factor}} \frac{u_c^{\max}(A_t, g_t)}{u_c^{\max}(A_0, g_0)} \left(\mathcal{T}^{\max}(A_t, g_t) - g_t - z\right) \tag{38}$$

Calculation of the fiscal limit uses the stochastic discount factor that obtains when tax rates are at the peak of the Laffer curve, $\beta^t u_c^{\max}(A_t, g_t)/u_c^{\max}(A_0, g_0)$, but modified to allow for a regime-switching political risk parameter $\beta_t^p \in \{\beta_L^p, \beta_H^p\}$ with transition matrix of $\begin{bmatrix} p_{LL} & 1 - p_{LL} \\ 1 - p_{HH} & p_{HH} \end{bmatrix}$. Higher political risk—lower $\beta_t^p$—lends itself to multiple interpretations that reflect the private sector's beliefs about policy. Most straightforward is the idea that policymakers are believed to have effectively shorter planning horizons than the private sector (see, for example, Acemoglu, Golosov, and Tsyvinski 2008). To see this, rewrite the discount factor in (38) as $(\beta^p\beta)^t/(\beta^p)^{t-1}$, so that a lower value of $\beta^p$ reduces the present value of maximum surpluses. An alternative interpretation is that a lower $\beta^p$ implies that private agents place probability mass on both the maximum surpluses, $s^{\max}$ reflected in (38), and on surpluses being zero. Rewrite the surpluses as $\beta^p s^{\max} + (1-\beta^p)\cdot 0$ for this interpretation.

Nothing we do hinges on the precise interpretation attached to $\beta^p$. As a practical matter, setting $\beta^p < 1$ serves to shift down the distribution of the fiscal limit, which generates risk premiums at lower levels of debt as observed in data. Uncertainty about $\beta^p$, generated by the Markov process, increases the dispersion of the fiscal limit, which is also important in generating plausible movements in risk premiums. Since there exists a unique mapping between the exogenous state space, $(A_t, g_t)$, to $\tau_t^{\max}$ and $T_t^{\max}$, the unconditional distribution of the fiscal limit, $f(\mathcal{B}^*)$, can be derived from a Markov chain Monte Carlo simulation following the steps that appendix 3 describes.

The government adopts a fixed-rate default rule. At each date, an effective fiscal limit, $\mathcal{B}_t$, is drawn from the normal density depicted in figure 1, given by $\mathcal{N}(\bar{\mathcal{B}}^*, \sigma^2_{\mathcal{B}})$. Government defaults on a fraction, $\delta_t$, of outstanding debt according to the rule (39), below. If the real value of debt at the beginning of period $t$, $b_{t-1}$, exceeds the effective fiscal limit, then the government partially defaults and debt outstanding at the beginning of period $t$ becomes $b_t^d = (1-\delta)b_{t-1}$.

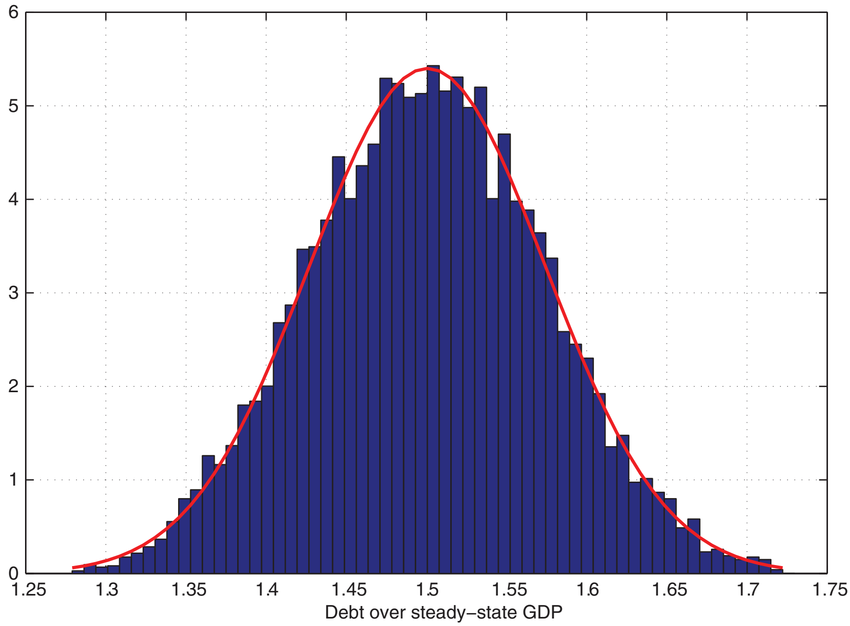
$$\delta_t = \begin{cases} \delta & \text{if } b_{t-1} > \mathcal{B}_t \text{ (Above Effective Fiscal Limit)} \\ 0 & \text{if } b_{t-1} \leq \mathcal{B}_t \text{ (Below Effective Fiscal Limit)} \end{cases} \tag{39}$$

The choice of $\mathcal{B}_t$, which we treat as random, is determined by political considerations that are driven by the policymakers' assessments of the costs associated with fully meeting the country's debt obligations.

#### *4.1.3 Interpretation*

The fiscal limit describes the stochastic upper bound on how much government debt the economy can support given the economic and political constraints. Policy rules that make fiscal instruments react

**Figure 1. Distribution of the Fiscal Limit as Computed from the Expression for $\mathcal{B}^*$ in (38)**




strongly enough to the state of government indebtedness serve to anchor fiscal expectations. Rather than making the default decision a strategic choice of an optimizing government, we opt to treat the intrinsically political decision as a random draw from the distribution of fiscal limits.

Although the theoretical analysis of fiscal policy in developed countries has largely abstracted from sovereign default risk, there are some exceptions. Uribe (2006) analyzes a flexible-price model in which sovereign default is inevitable, as the central bank targets the price level and the fiscal authority maintains a constant tax rate. By setting an ad hoc and fixed default threshold, he shows how the default scheme affects equilibrium dynamics. Similarly, Daniel and Shiamptanis (2012) assume government debt is constrained by an ad hoc fiscal limit to study a small open economy in a monetary union under alternative fiscal policy responses to a fiscal crisis. Instead of modeling sovereign default explicitly, Corsetti et al. (2013) assume that the risk premium on government debt depends on the expected level of government debt, as in Garcia-Cicco, Pancrazi, and Uribe (2010). Corsetti et al. (2013) show how the timing of fiscal retrenchment and the size of the risk premium affect economic outcomes at the zero lower bound for nominal interest rates.

### *4.2 Calibration*

The model is calibrated at a quarterly frequency. The household discount rate is 0.99. The utility function is assumed to be $u(c, L) = \log c + \chi \log(1 - n)$. The leisure preference parameter, $\chi$, is calibrated in such a way that the household spends 25 percent of its time working and the Frisch elasticity of labor supply is 3. Time endowment and the productivity level at the steady state are normalized to 1.

Parameterizations of the shock processes for $A_t$ and $g_t$ follow the literature. For instance, Schmitt-Grohé and Uribe (2007) assume $\rho_A$ to be 0.8556, $\sigma_A$ to be 0.0064, $\rho_g$ to be 0.87, and $\sigma_g$ to be 0.016. The price elasticity of demand, $\theta$, is assumed to be 11 and the Rotemberg adjustment parameter, $\phi$, is 100, which implies that 26.7 percent of the firms reoptimize each quarter (see Keen and Wang 2007). The gross inflation rate is calibrated to 1.03 at annual rate and the Taylor-rule parameter is assumed to be 2.5. As explained in section 3, a higher Taylor-rule parameter is needed when the haircut $\delta$ is sizable.[5]

The fiscal parameters are roughly calibrated to match Greek data from 1971 to 2007. In steady state, government purchases are 16.7 percent of GDP, lump-sum transfers are 13.34 percent of GDP, and the tax rate is 0.315.[6] The tax adjustment parameter, $\gamma$, is calibrated to 0.75 at annual rate. The default rate, $\delta$, is assumed to be 0.05, implying 20 percent annual default rate. We use a relatively small default rate to underscore that even small rates can generate quantitatively important effects.[7]

[5] Appendix 1 provides more details on determinacy conditions.

[6] The government debt is 35.26 percent of GDP at steady state, which is slightly lower than 40 percent, the average share of government debt over the GDP in Greece.

[7] Significantly higher values for $\delta$ tend to cause stability problems in the model. Risk premiums depend on current and expected default rates. Substantially higher default rates would drive risk premiums and inflation much higher. With Rotemberg (1982) costs to price adjustment, spikes in inflation carry real resource costs that, if too large, can actually make cost-adjusted output negative.

The International Country Risk Guide's (ICRG) index of political risk offers one way to calibrate the political factor, $\beta^p$ (see Arteta and Galina 2008). The ICRG index of political risk for Greece appears to follow a regime-switching process. It stayed low and stable during the period between 1984 and 1993, then rose to a higher level between 1994 and 1996, and stayed at the high level until the financial crisis erupted in 2008. In this model, we calibrate $\beta_t^p$ to be a two-state symmetric Markov regime-switching process. The low state, $\beta_L^p$, is calibrated to 0.4 and the high state, $\beta_H^p$, is 0.6. We assume that the probability of switching between the two states is 0.1. Under the calibration in table 1, the distribution of the fiscal limit has $\bar{\mathcal{B}}^* = 1.5$ (150 percent of GDP annually) and $\sigma_b = 0.0739$, shown in figure 1.

We solve the full non-linear model laid out in section 2, coupled with the fiscal limit described in section 4.1, using the monotone map method, which discretizes the state space and finds fixed points in the space of decision rules. Details appear in appendix 4.

## *4.3 Policy Effects*

We examine how the impact of monetary policy shocks changes when the economy moves from being far from its fiscal limit to within striking distance of the limit. Several European countries operated at or near their fiscal limits during the European debt crisis. The set of experiments in this section addresses precisely this situation by tracing out the impacts of an exogenous monetary contraction. We explore two types of monetary policy rules that are defined in terms of either risky or risk-free nominal rates.

### *4.3.1 Taylor Rule with Risky Rate*

Monetary and fiscal policy always must interact in specific ways to ensure that a unique equilibrium exists. When monetary policy targets inflation, even when there is no possibility of default, a monetary contraction engineered through an open market sale of bonds increases the public's bond holdings, raises nominal and real interest rates, and requires expected surpluses to rise. Higher outstanding debt and higher interest rates raise debt service. To prevent debt from exploding, fiscal policy must raise future taxes or

**Table 1. Model Calibration**

| | Parameter | Calibration |
|---|---|---|
| Discount Factor | $\beta$ | 0.99 |
| Elasticity of Substitution | $\theta$ | 11 |
| Rotemberg Adjustment Parameter | $\phi$ | 100 |
| Inflation Rate | $\pi^*$ | 1.03 (Annual) |
| Labor Supply | $n^*$ | 0.25 |
| Government Spending-GDP | $g^*/y^*$ | 0.167 |
| Government Transfer-GDP | $z^*/y^*$ | 0.134 |
| Tax Rate | $\tau^*$ | 0.315 |
| Fiscal Rule Parameter | $\gamma_\tau$ | 0.5/4 |
| Default Rate | $\delta$ | 0.05 (20% Annually) |
| Taylor-Rule Parameter | $\alpha$ | 2.5 |
| Technology | $A^*$ | 1 |
| Technology Shock Persistence | $\rho_A$ | 0.85 |
| Technology Shock Variance | $\sigma_A^2$ | $0.01^2$ |
| Monetary Shock Variance | $\sigma_R^2$ | $0.005^2$ |
| Fiscal Shock Persistence | $\rho_g$ | 0.85 |
| Fiscal Shock Variance | $\sigma_g^2$ | $0.015^2$ |
| Political Factor (Low) | $\beta_L^p$ | 0.4 |
| Political Factor (High) | $\beta_H^p$ | 0.6 |
| Political Factor Transition Matrix | $p_{LL}$ | 0.9 |
| | $p_{HH}$ | 0.9 |
| Fiscal Limit | $\bar{\mathcal{B}}^*$ | 1.5 |
| Fiscal Limit Variance | $\sigma_b^2$ | $0.0739^2$ |

reduce future non-interest spending. These interactions permit the monetary contraction to reduce inflation and economic activity.

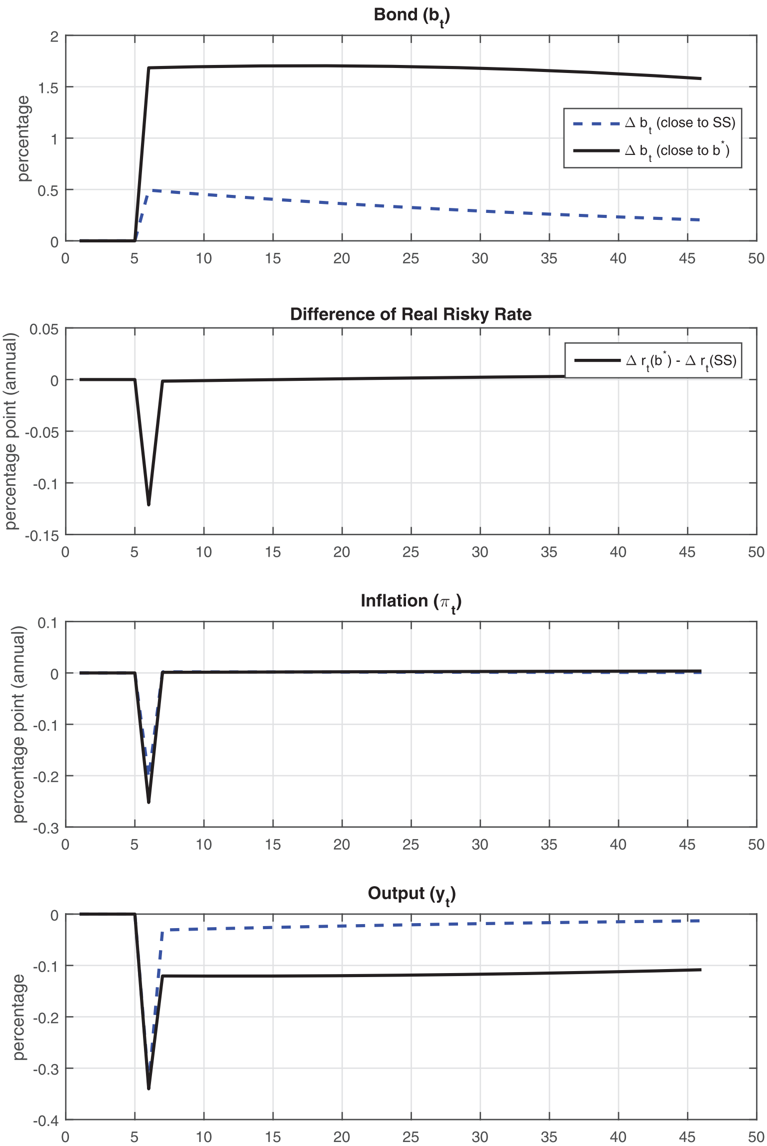
The possibility of debt default adds further complications to the interactions—complications that can dramatically alter the effects of monetary policy. A monetary policy contraction triggers three distinct sources of dynamics: those produced by the initial increase in the policy interest rate itself; the intrinsic dynamics that arise when debt is well above its steady-state level, as even in the absence of default the dynamics following a monetary contraction may be different at high levels of debt than at low levels; and the additional dynamics that stem from changes in the probability of default and the risk premium on sovereign debt.

This subsection assumes the central bank's policy rule is defined in terms of the risky nominal interest rate. Consider an iid contractionary monetary policy disturbance that raises the nominal interest rate by 2 percentage points at an annual rate. When the economy is at its steady state and the probability of default is nil, the contraction has the usual effects: nominal and real interest rates rise only in the period of the shock; output, inflation, and wages fall initially.

The contraction in the tax base this implies raises debt, bringing forth higher tax rates, which keep output persistently below steady state while debt is stabilized. These latter effects will be exacerbated when debt levels are high, even in the absence of default. Additionally, the higher tax rates employed to stabilize debt serve to raise marginal costs and fuel inflation, resulting in a monetary policy response. We can see the impact of this additional channel of monetary and fiscal policy interaction by contrasting the response to the monetary policy shock at high and low debt levels. Figure 2 shows that the intrinsic dynamics associated with debt being far from steady state show up primarily through chronically higher debt and tax rates, which allow a monetary contraction to have persistent negative effects on output.

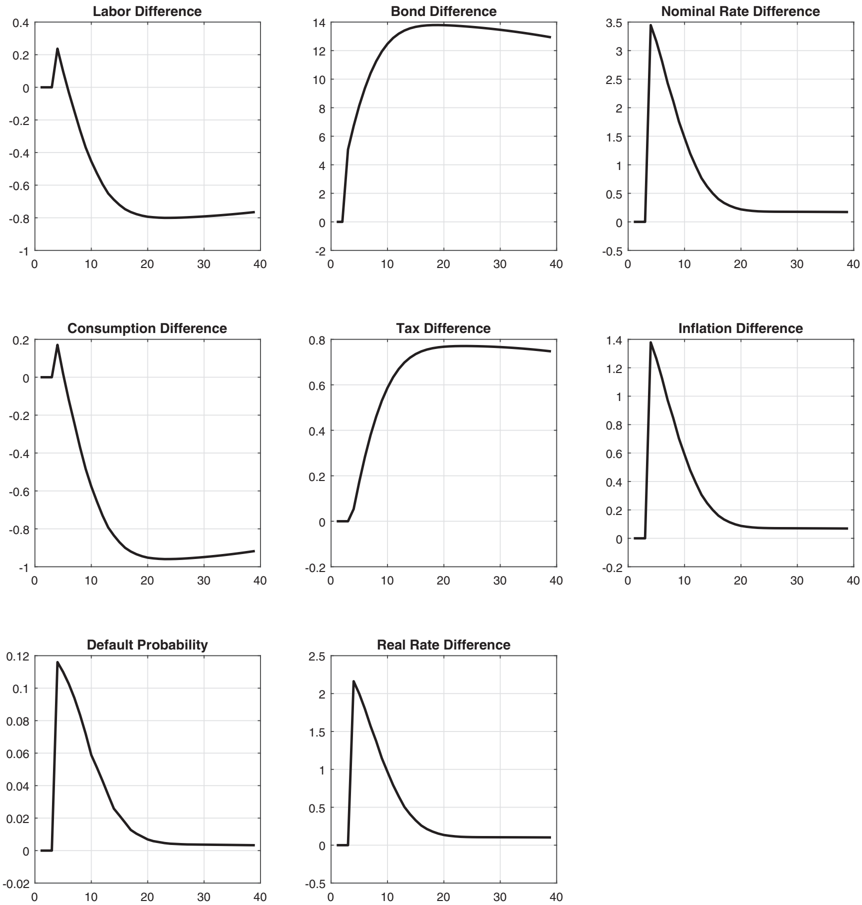
The third layer of dynamics is triggered by the possibility of debt default. To isolate the effects of default, figure 3 reports the difference that allowing for default makes to the time paths of variables when debt is near the fiscal limit and there is no monetary policy shock. This marginal effect of default is computed by solving the model first with the default rule in expression (39), then with $\delta_t \equiv 0$, and calculating the difference in the time paths from these two solutions, conditional on debt being near the fiscal limit. Government debt near the fiscal limit creates a probability of default, which produces a risk premium in real bond yields. Higher real rates raise debt service, which further increases debt and actual tax rates, as dictated by the tax rule in (13). Higher realized tax rates reduce hours worked and consumption. Inflation and, through the active monetary policy rule, nominal interest rates rise. This is the same phenomenon highlighted in the analytics of section 3.

**Figure 2. Impulse Responses under an iid Contractionary Monetary Policy Shock in Model that Rules Out Default: Initial Government Debt at Steady State (dashed lines) vs. Close to Fiscal Limit (solid lines)**




**Note:** Time periods are quarters.

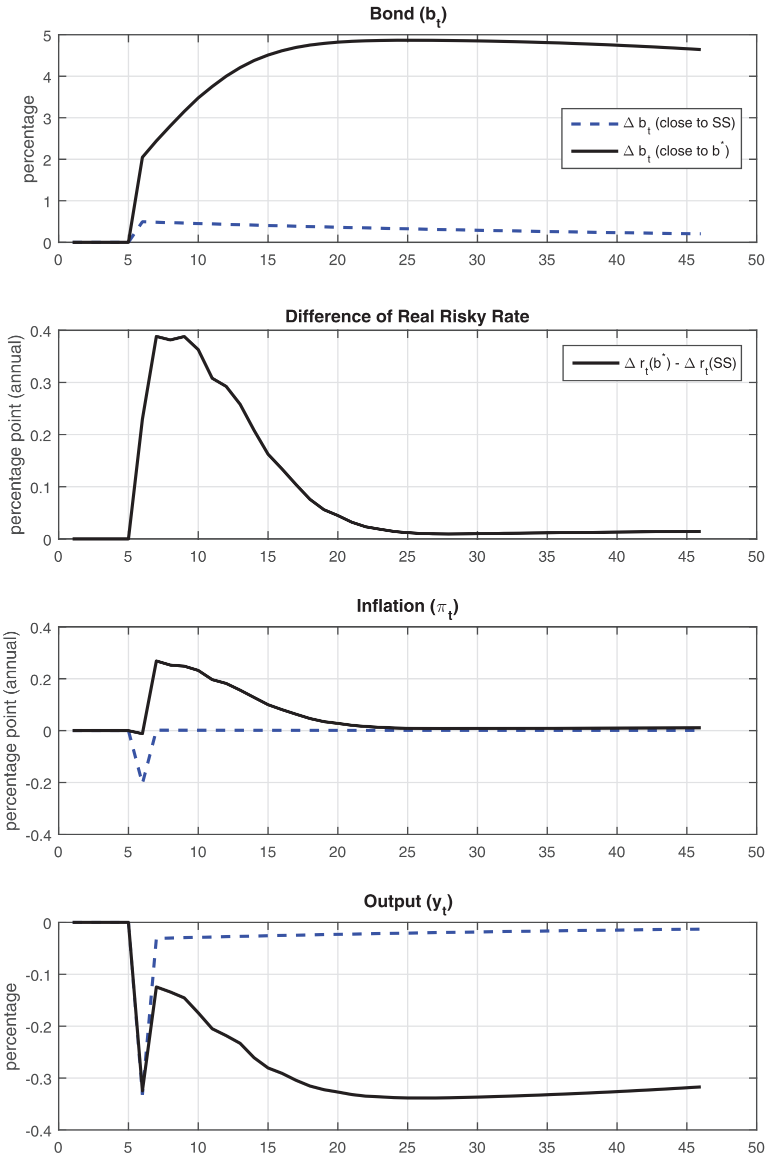
**Figure 3. Marginal Effect of Possibility of Default**




**Notes:** Difference in time paths from solving with the default rule in (39) and with $\delta_t \equiv 0$, conditional on debt being near the fiscal limit. All variables show deviations in percentage except that inflation, nominal, and real rates are in percentage points.

Pulling all these dynamics together, we obtain the overall effect of a monetary contraction when debt default is permitted. Figure 4 reports time paths following a serially uncorrelated monetary contraction, contrasting those when the economy is far from the fiscal limit—dashed lines—to those when the economy is staring at the

**Figure 4. Impulse Responses under an iid Contractionary Monetary Policy Shock in Model where Default Is Possible: Initial Government Debt at Steady State (dashed lines) vs. Close to Fiscal Limit (solid lines)**




**Note:** Time periods are quarters.

limit—solid lines. Away from the limit, tighter monetary policy has the usual effects because the probability of debt default is essentially zero. As the debt level is close to fiscal limits, monetary contraction barely lowers inflation even in the very short run, and soon the impact of expected default dominates and inflation rises dramatically and persistently. The effects of higher debt service manifest in sharply higher debt, which brings with it higher tax rates and persistently lower output.

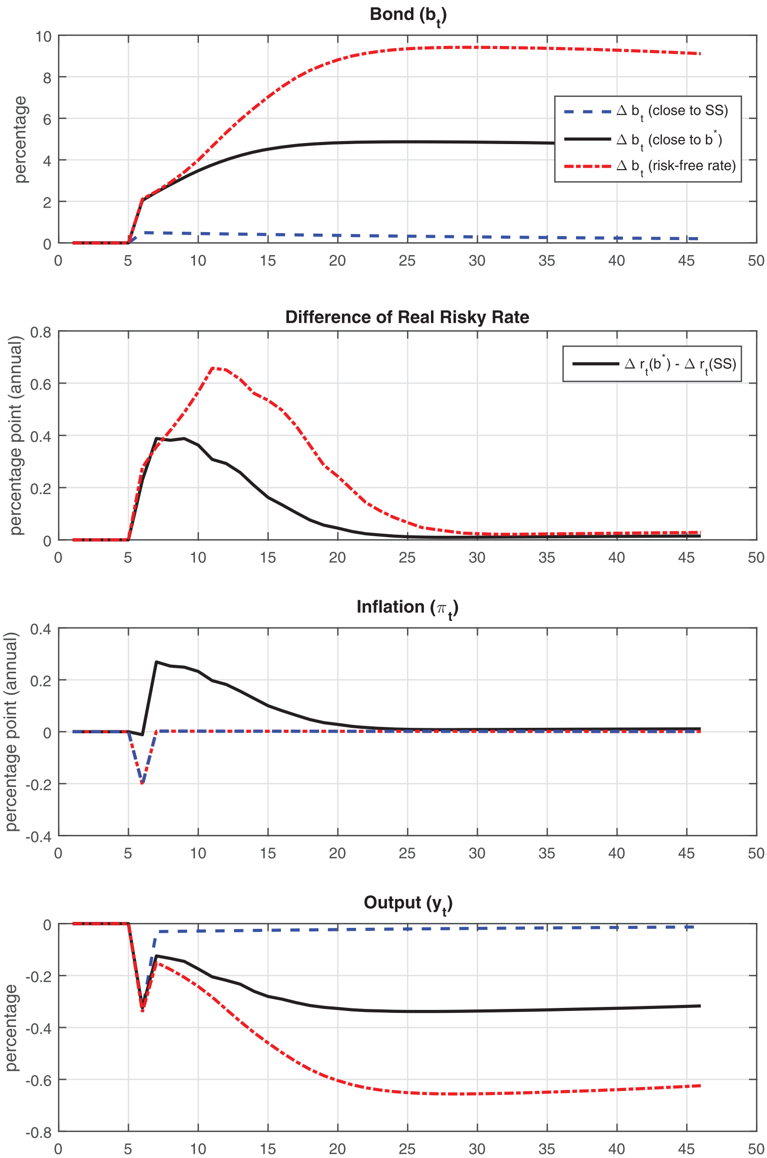
#### *4.3.2 Taylor Rule with Risk-Free Rate*

Analytics in section 3 made clear the importance of the instrument the central bank is assumed to target. Here we repeat the monetary contraction experiment of section 4.3.1 but now assume that the central bank sets the risk-free interest rate, $R_t^f$, according to a Taylor rule.

Analytical results suggest that the increase in inflation observed in figure 4 when monetary policy targets the risky interest rate may no longer occur once it is the risk-free interest rate the monetary authority focuses on. Figure 5 confirms that the analytical result holds in the New Keynesian model—the specification of monetary policy rule is a critical step in determining the inflationary consequences of sovereign debt risk. The figure overlays results when the risk-free interest rate enters the Taylor rule with those from figure 4 for the risky interest rate.

When government debt is far from the fiscal limit, the probability of default is close to zero, and there is no distinction between the risk-free and the risky rates. Therefore, the impacts of a monetary contraction are identical under both cases. Near the fiscal limit, however, when monetary policy adjusts the risk-free rate, it combats inflation without accommodating the increases in default probabilities. As a result, there is no tendency for inflation to rise, but policy ends up raising the real interest rate a lot more (dotted-dashed lines). Output contracts for a prolonged period of time following persistently higher debt and taxes. The striking comparison between the two cases with different interest rate instruments highlights that sovereign default risk puts into sharp relief the tradeoff between inflation and output stabilization.

**Figure 5. Monetary Policy Sets the Risk-Free Interest Rate (dotted-dashed lines) Overlaid with Figure 4**




**Notes:** The figure shows impulse responses under an iid contractionary monetary policy shock in a model where default is possible: initial government debt at steady state (dashed lines) vs. close to fiscal limit under risky rate (solid lines) vs. close to fiscal limit under risk-free rate (dotted-dashed lines). Time periods are quarters.

## 5. Concluding Remarks

While central banks may stress that they are solely concerned with the medium-term stabilization of inflation, there are reasons to expect that they have incentives to, at least partially, accommodate any rise in sovereign default risk. We contrast such a description of monetary policy with an alternative where the central bank defines its policy rule in terms of the risk-free rate.

At low levels of debt, sovereign default is a distant possibility, and the response to a monetary contraction under these two descriptions of policy is very similar and entirely conventional—a fall in inflation and output for as long as the monetary contraction persists. At higher debt levels, however, the policy response to a monetary contraction is quite different. The tightening of policy worsens debt dynamics, prompting a sustained rise in distortionary taxation in an attempt to stabilize that debt. These dynamics extend the impact of the monetary contraction well beyond the date of the initial shock. When the policy rule is defined in terms of the risk-free rate, the central bank tightens policy to offset the rise in inflation, resulting in output bearing the brunt of the macroeconomic cost of the ongoing fiscal adjustment. In contrast, if the central bank targets the risky rate on government debt, the worsening debt dynamics raise default risk; by accommodating that risk, the central bank implicitly amounts to a relaxation in the inflation target. Inflation rises significantly even though the initial shock driving this process was a monetary contraction. This analysis suggests that the monetary policy transmission mechanism is likely to be quite different in low- and high-debt environments, crucially depending on the central bank's attitude towards containing any emerging sovereign default risk premiums. While the possibility that banks are too big to fail may give rise to expectations of bailouts at low debt levels, at higher debt levels economic agents may anticipate that the sovereign debt-banking crisis nexus is too important for the central bank not to accommodate.

## Appendix 1. Simple Analytics: Default and Determinacy

This section examines the implications of default for the determinacy of equilibrium in a simplified version of the baseline model in section 2. To derive analytical results, we assume that government spending and transfers are zero, and taxes are lump sum. We also assume there is an exogenous and constant rate of default every period, $\delta_t \equiv \delta \in [0, 1]$. Varying this fixed default rate only affects the dynamics of our system if it affects the steady state of the system, and it only does that through the monetary policy rule. As noted above, in the presence of default, it is as if the inflation target has risen from

$$\pi^* \quad \text{to} \quad \bar{\pi}^* \equiv \frac{\pi^*}{1 - \delta\beta/\alpha}. \tag{40}$$

This implies that as we approach the fiscal limit, default risk premiums emerge and, given the specification of the interest rate rule with risky rate, this leads to monetary accommodation of default risk that is effectively the same as raising the inflation target. We now explore the implications for determinacy of increasing the fixed default rate, which raises risk premiums.

Appendix 2 details the log-linearization of our dynamic system. The appendix shows that the dynamic system can be written as

$$\begin{bmatrix} E_t\widehat{\pi}_{t+1} \\ E_t\widehat{y}_{t+1} \\ \widehat{b}_t \end{bmatrix} = \begin{bmatrix} \frac{\phi_1}{\beta\phi_2} & -\frac{\phi_3}{\beta\phi_2} & 0 \\ \gamma_2 - \gamma_1\frac{\phi_1}{\beta\phi_2} & 1 + \frac{\gamma_1\phi_3}{\beta\phi_2} & 0 \\ \beta^{-1}(\alpha\beta - 1) & 0 & \beta^{-1}(1 - \gamma\bar{\pi}^*) \end{bmatrix} \times \begin{bmatrix} \widehat{\pi}_t \\ \widehat{y}_t \\ \widehat{b}_{t-1} \end{bmatrix},$$

where a hat denotes the percentage deviation of that variable from its steady-state value; $\phi_i > 0$ for $i = 1, 2, 3$ are bundles of parameters contained in the New Keynesian Phillips curve defined in the appendix. The other parameter bundles are found in the consumption-Euler equation and are defined as

$$\gamma_1 = 1 - \phi(\bar{\pi}^* - 1)\frac{y^*}{c^*}$$
$$\gamma_2 = \alpha - \phi(\bar{\pi}^* - 1)\frac{y^*}{c^*},$$

where $\phi \geq 0$ determines the costs of adjustment in Rotemberg (1982) pricing. Parameters $\gamma_1$ and $\gamma_2$ are positive unless default raises inflation to such an extent that the second term dominates the first in either definition.

Since taxes are lump sum, the debt dynamics are decoupled from the dynamics of the rest of the system. Debt will be dynamically stable if

$$\beta^{-1}(1 - \gamma\bar{\pi}^*) < 1,$$

which requires

$$\gamma > \frac{(1-\beta)}{\bar{\pi}^*}.$$

Fiscal policy must passively raise lump-sum taxes by more than debt service costs in order to stabilize the debt.

We can now set aside debt dynamics to analyze the determinacy of the rest of the dynamic system. This part of the system can be represented as

$$\begin{bmatrix} E_t\widehat{\pi}_{t+1} \\ E_t\widehat{y}_{t+1} \end{bmatrix} = \begin{bmatrix} \widetilde{\beta}^{-1} & -\widetilde{\beta}^{-1}\kappa \\ \gamma_1(\widetilde{\gamma}_2 - \widetilde{\beta}^{-1}) & 1 + \gamma_1\widetilde{\beta}^{-1}\kappa \end{bmatrix} \begin{bmatrix} \widehat{\pi}_t \\ \widehat{y}_t \end{bmatrix},$$

where

$$\widetilde{\beta}^{-1} = \frac{\phi_1}{\beta\phi_2},\ \kappa = \frac{\phi_3}{\phi_1},\ \widetilde{\gamma}_2 = \frac{\gamma_2}{\gamma_1},$$

which is in exactly the same form as the model considered in Woodford (2003).

Consider the trace of the transition matrix

$$trA = 1 + \widetilde{\beta}^{-1}(1 + \kappa\gamma_1)$$

and determinant

$$\det A = \widetilde{\beta}^{-1}(1 + \gamma_2\kappa),$$

which imply

$$\begin{aligned} \det A - trA &= \widetilde{\beta}^{-1}(1 + \gamma_2\kappa) - 1 - \widetilde{\beta}^{-1}(1 + \kappa\gamma_1) \\ &= \widetilde{\beta}^{-1}\kappa(\alpha - 1) - 1 \end{aligned}$$

and

$$\det A + trA = \widetilde{\beta}^{-1}(1 + \gamma_2\kappa) + 1 + \widetilde{\beta}^{-1}(1 + \kappa\gamma_1)$$
$$= \widetilde{\beta}^{-1}(2 + (\gamma_1 + \gamma_2)\kappa) + 1.$$

The following set of determinacy conditions in Woodford (2003) are relevant for our model:

$$\det A > 1$$
$$\det A - trA > -1$$
$$\det A + trA > -1.$$

Provided all parameter bundles are positive, it is only the second condition that bites, requiring

$$\widetilde{\gamma}_2 = \frac{\alpha - \phi(\bar{\pi}^* - 1)\frac{y^*}{c^*}}{1 - \phi(\bar{\pi}^* - 1)\frac{y^*}{c^*}} > 1.$$

This reduces to the usual Taylor principle: $\alpha > 1$. Therefore, provided default rates are not too high, the usual combination of active monetary policy and passive fiscal policy will ensure determinacy of our sticky-price economy.

As steady-state inflation rises—as a result of steady-state increases in the default rate, $\delta$—the parameter combinations $\gamma_1$ and $\gamma_2$ can turn negative and may overturn the necessary conditions for stability and imply indeterminacy. Notice that $\det A - trA > -1$ irrespective of the steady-state rates of default and inflation, so the conditions outlined above remain the relevant case for determinacy. The other conditions for determinacy may be breached when

$$\gamma_2 = \alpha - \frac{\phi(\bar{\pi}^* - 1)}{1 - \frac{\phi}{2}(\bar{\pi}^* - 1)^2} < \frac{\widetilde{\beta} - 1}{\kappa}$$

or when

$$\gamma_1 + \gamma_2 = 1 + \alpha - \frac{2\phi(\bar{\pi}^* - 1)}{1 - \frac{\phi}{2}(\bar{\pi}^* - 1)^2} < -\frac{2}{\kappa},$$

either of which may occur for an active interest rate rule and a high enough default rate. In other words, when we move to our sticky-price economy, at high default rates the accommodation of risk premiums through rising inflation can result in the backward bending of the Phillips curve detailed in Ascari and Ropele (2005), which can render a standard active/passive policy mix indeterminate.

Finally, we note that this indeterminacy arising from default is different from that reported in Schabert (2010). By assuming that the government exogenously imposes a default rate (which applies at every period in this simplified model, but applies only upon hitting the fiscal limit in the baseline model in section 2) we circumvent the indeterminacy that arises when the rate of default, $\delta_t$, is endogenously determined by the need to satisfy the government's intertemporal budget constraint. Uribe (2006) similarly avoids indeterminacy by imposing a default rate which ensures the inflation target holds. Instead, the potential indeterminacy we have identified comes from the resource costs of rising inflation in a sticky-price economy, where default results in a monetary accommodation that raises inflation.

This section has shown that a monetary policy rule with risky rate, which implicitly raises the inflation target in the presence of default risk, can induce indeterminacy if the default rate is sufficiently high. However, for the rates of default considered upon hitting the fiscal limit in the main body of the paper, the model remains locally determinate for the policy rule parameters adopted.

## Appendix 2. Log-Linearized System

The log-linearized system includes

$$\hat{c}_t = \frac{c^* + g^*}{c^*}\hat{y}_t - \phi(\pi^* - 1)\frac{y^*}{c^*}\hat{\pi}_t \tag{41}$$

$$\hat{w}_t = \frac{n^*}{1-n^*}\left(\hat{y}_t - \hat{A}_t\right) + \frac{c^* + g^*}{c^*}\hat{y}_t - \phi(\pi^* - 1)\frac{y^*}{c^*}\hat{\pi}_t + \frac{\tau^*}{1-\tau^*}\widehat{\tau}_t \tag{42}$$

$$\hat{R}_t = E_t\left(\hat{c}_{t+1} + \hat{\pi}_{t+1}\right) - \hat{c}_t + E_t\frac{\delta}{1-\delta}\hat{\delta}_{t+1} \tag{43}$$

$$-\frac{\delta}{1-\delta}\hat{\delta}_t + \widehat{b}_{t-1} - \hat{\pi}_t = \beta\widehat{b}_t + (1-\beta)\widehat{T}_t - \beta\widehat{R}_t \tag{44}$$

$$\begin{aligned}\theta\frac{w^*}{A^*}\left(\hat{w}_t - \hat{A}_t\right) &- \phi(\pi^* - 1)\hat{\pi}_t - \phi\pi^*\hat{\pi}_t \\ &+ \beta\phi(\pi^* - 1)\pi E_t\left(\hat{c}_t - \hat{c}_{t+1} + \hat{y}_{t+1} - \hat{y}_t\right) \\ &+ \beta\phi(2\pi^* - 1)E_t\hat{\pi}_{t+1} = 0.\end{aligned} \tag{45}$$

If taxes are distortionary and applied to labor income and monopoly profits, then

$$T_t = \tau_t y_t (1 - \frac{\phi}{2}(\pi_t - 1)^2),$$

which log-linearizes as

$$\widehat{T}_t = \widehat{\tau}_t + \widehat{y}_t - \phi(\pi^* - 1)\frac{y\tau}{T}\hat{\pi}_t.$$

In this case our log-linearized fiscal rule is given by

$$\widehat{\tau}_t = \gamma \widehat{b}_{t-1},$$

while if taxes are lump sum, $\widehat{\tau}_t = 0$, and the fiscal rule is

$$T_t - T^* = \gamma((1 - \delta_t)b_{t-1} - b^*)$$

and total tax revenues log-linearize as

$$\widehat{T}_t = \frac{\gamma\pi^*}{(1 - \beta)}\widehat{b}_{t-1}.$$

In conjunction with the log-linearized monetary policy rule,

$$\widehat{R}_t = \alpha\widehat{\pi}_t,$$

these conditions can be combined to yield the dynamic systems described in the text.

## Appendix 3. Simulating the Fiscal Limit

The fiscal limit $\mathcal{B}^*$ can be obtained using Markov chain Monte Carlo simulation:

- First, for each simulation, we randomly draw the shocks of political factor, productivity, and government purchases for 1,000 periods. Assuming that the tax rate is always at the peak of the dynamic Laffer curves, we compute the paths of all other variables using the household first-order conditions and the budget constraints. According to equation (38), we compute the discounted sum of maximum fiscal surplus by discarding the first 200 draws as a burn-in period.

- Second, we repeat the simulation for 100,000 times and obtain the distribution of the fiscal limit, which is then approximated to a normal distribution $\mathcal{N}(b^*, \sigma_b^2)$.
- At each period of time, the effective fiscal limit $b_t^*$ is a random draw from the distribution.

## Appendix 4. Solving the Non-linear Model

Following the household first-order conditions, the labor supply and consumption can be solved in terms of $(w_t, \tau_t, \pi_t, A_t, g_t)$:

$$n_t = \frac{w_t X_{1,t} + \chi_n g_t}{w_t X_{1,t} + \chi_n X_{2,t}} \tag{46}$$

$$c_t = X_{2,t} n_t - g_t \tag{47}$$

$$\text{with} \quad X_{1,t} = 1 - \tau_t \tag{48}$$

$$X_{2,t} = A_t \left(1 - \frac{\phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2\right). \tag{49}$$

The complete model also consists of the following non-linear equations:

$$\begin{aligned}(1-\theta) + \theta\frac{w_t}{A_t} = \phi\left(\frac{\pi_t}{\pi^*} - 1\right)\frac{\pi_t}{\pi^*} \\ - \beta\phi E_t \frac{u_c(t+1)}{u_c(t)}\left(\frac{\pi_{t+1}}{\pi^*} - 1\right)\frac{\pi_{t+1}}{\pi^*}\frac{Y_{t+1}}{Y_t}\end{aligned} \tag{50}$$

$$Y_t = A_t n_t \tag{51}$$

$$\Upsilon_t = Y_t - \frac{w_t}{A_t} Y_t - \frac{\phi}{2}\left(\frac{\pi_t}{\pi^*} - 1\right)^2 Y_t \tag{52}$$

$$b_{t-1}\frac{1-\delta_t}{\pi_t} = \frac{b_t}{R_t} + \tau_t(w_t n_t + \Upsilon_t) - g_t - z^* \tag{53}$$

$$\tau_t - \tau^* = \gamma_\tau(b_t^d - b^*) \tag{54}$$

$$R_t^i - R^* = \alpha(\pi_t - \pi^*) + \varepsilon_t^R \tag{55}$$

$$\log\frac{g_t}{g^*} = \rho_g \frac{g_{t-1}}{g^*} + \varepsilon_t^g \tag{56}$$

$$\log \frac{A_t}{A^*} = \rho_A \frac{A_{t-1}}{A^*} + \varepsilon_t^A. \tag{57}$$

The solution method, based on Coleman (1991) and Davig (2004), conjectures candidate decision rules that reduce the system to a set of expectation first-order difference equations. In this model, the decision rule maps the state at period $t$ into the stock of government debt, the real wage, and the inflation rate in the same period. The state is denoted as $\psi_t = \left\{b_t^d, A_t, g_t\right\}$, while the mapping is denoted as $b_t = f^b(\psi_t), w_t = f^w(\psi_t), \pi_t = f^\pi(\psi_t)$. The state variable of the post-default government liability ($b_t^d$) incorporates the information of the effective fiscal limit at time $t$ ($b_t^*$) and the pre-default government liability ($b_{t-1}$).

The conjectured rules can be substituted into the non-linear system, in which the expectation terms are evaluated using a numerical quadrature. The model is solved for each set of state variables defined over a discrete partition of the state space. The decision rules are updated at every node of the state space. The procedure is repeated until the iterations update the current decision rules by less than some $\epsilon > 0$ (set to $1e - 6$).

After finding the decision rules, we can solve the pricing rule ($q_t = f^q(\psi_t)$) using the government budget constraint. The interest rate on government bonds can also be solved using $R_t = 1/q_t$, denoted as $f^R(\psi_t)$.

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