Appendix A. The Model

The model is based on Benigno and Benigno (2003) but with incomplete financial markets as in Benigno (2009). The relatively detailed modeling of the fiscal side follows Woodford (2001) and Leeper and Leith (2016), allowing for variable maturity of government debt.

Specifically, the world economy is populated by a continuum of agents on the interval of [0; 1]. The population on the segment [0; n) belongs to country H (home), while the rest of the population on [n; 1] belongs to country F (foreign). Each economy is populated by households and firms. Households’ preferences reflect home bias in consumption. Firms are monopolistically competitive and only use labor to produce differentiated tradable goods. The law of one price holds. Each country has an independent fiscal authority, which finances spending by bonds and distortionary taxes. The government debt is tradable and has geometric maturity structure. We look at implications of complete and incomplete financial markets: in the latter case the portfolio allocation is determined by transaction

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1Baele et al. (2004) argues that in the beginning of the 2000s the public debt market was fairly integrated. However, since the Greece debt restructuring there is a perceived non-zero probability of a sovereign debt default of an individual country.
costs. All profits received by home-country firms and financial intermediaries are rebated to home households. Countries are subject to technology and cost-push shocks.

A.1 Preferences

We assume home bias in consumption. Home households’ consumption, $C_t$, is an aggregate of the continuum of goods $i \in [0, 1]$ produced in the home country and abroad,

$$C_t = \left( (1 - \gamma) \frac{1}{\eta} C_{Ht}^{n-1} + \gamma \frac{1}{\eta} C_{Ft}^{n-1} \right)^{\frac{\eta}{n-1}},$$

where $\eta > 0$ is the intratemporal elasticity of substitution between home and foreign consumption goods, and $\gamma = (1-n)\alpha$ is the import share. The import share depends on $(1-n)$, which is the relative size of foreign economy, and on $\alpha$, which is the degree of trade openness. Similarly, for the foreign country $C^*_t$ is an aggregate of the continuum of goods $i \in [0, 1]$ produced in the foreign country and abroad and consumed by foreign households:

$$C^*_t = \left( (1 - \gamma^*) \frac{1}{\eta} C_{Ft}^{n-1} + \gamma^* \frac{1}{\eta} C_{Ht}^{n-1} \right)^{\frac{\eta}{n-1}},$$

where $\gamma^* = n\alpha$.

Each consumption bundle $C_{Ht}$ and $C_{Ft}$ is composed of imperfectly substitutable varieties with elasticity of substitution $\epsilon > 1$: The optimal allocation within each variety of goods yields household-level relationships

$$c_{Ht}(z) = \frac{1}{n} \left( \frac{p_{Ht}(z)}{P_{Ht}} \right)^{-\epsilon} C_{Ht} \quad \text{and} \quad c_{Ft}(z) = \frac{1}{1-n} \left( \frac{p_{Ft}(z)}{P_{Ft}} \right)^{-\epsilon} C_{Ft},$$

where $z$ denotes the good’s type or variety and $p_{Ht}(z), p_{Ft}(z)$ are prices of individual goods. Throughout the paper we use lowercase letters to denote individual (household) labor, consumption, output, and government spending, while uppercase letters are used to denote country aggregates.
The aggregation at the country level yields

$$C_{Ht} = \left( \frac{1}{n} \int_0^n c_{Ht}(z)^{\frac{\epsilon-1}{\epsilon}} \, dz \right)^{1-\frac{1}{\epsilon}}$$

and

$$C_{Ft} = \left( \frac{1}{1-n} \int_n^1 c_{Ft}(z)^{\frac{\epsilon-1}{\epsilon}} \, dz \right)^{1-\frac{1}{\epsilon}}.$$

Optimal allocation of expenditures between domestic and foreign bundles yields

$$C_{Ht} = (1 - \gamma) \left( \frac{P_{Ht}}{P_t} \right)^{-\eta} C_t \quad \text{and} \quad C_{Ft} = \gamma \left( \frac{P_{Ft}}{P_t} \right)^{-\eta} C_t.$$

$P_{Ht}, P_{Ft}$ are domestic and foreign goods price indexes and

$$P_t = \left( (1 - \gamma) P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta} \right)^{\frac{1}{1-\eta}},$$

where price indexes are computed as

$$P_{Ht} = \left( \frac{1}{n} \int_0^n p_{Ht}(z)^{1-\epsilon} \, dz \right)^{\frac{1}{1-\epsilon}}$$

and

$$P_{Ft} = \left( \frac{1}{1-n} \int_n^1 p_{Ft}(z)^{1-\epsilon} \, dz \right)^{\frac{1}{1-\epsilon}}.$$

Similarly, for the foreign country

$$C_{Ht}^* = \frac{1}{n} \left( \frac{P_{Ht}^*}{P_{Ht}^*} \right)^{-\epsilon} C_{Ht}^* \quad \text{and} \quad C_{Ft}^* = \frac{1}{1-n} \left( \frac{P_{Ft}^*}{P_{Ft}^*} \right)^{-\epsilon} C_{Ft}^*,$$

$$C_{Ft}^* = (1 - \gamma^*) \left( \frac{P_{Ft}^*}{P_t^*} \right)^{-\eta} C_t^* \quad \text{and} \quad C_{Ht}^* = \gamma^* \left( \frac{P_{Ht}^*}{P_t^*} \right)^{-\eta} C_t^*,$$

$$P_{Ht}^* = \left( \frac{1}{n} \int_0^n p_{Ht}^*(z)^{1-\epsilon} \, dz \right)^{\frac{1}{1-\epsilon}}$$

and

$$P_{Ft}^* = \left( \frac{1}{1-n} \int_n^1 p_{Ft}^*(z)^{1-\epsilon} \, dz \right)^{\frac{1}{1-\epsilon}},$$

$$P_t^* = \left( (1 - \gamma^*) P_{Ft}^{1-\eta} + \gamma^* P_{Ht}^{1-\eta} \right)^{\frac{1}{1-\eta}}.$$
A.2 Law of One Price, the Terms of Trade, and Relative Prices

We assume that the law of one price holds, implying \( p_{Ft}(z) = E_t p_{Ft}^*(z) \), \( p_{Ht}(z) = E_t p_{Ht}^*(z) \) for all \( z \in [0, 1] \) where \( E_t = [H]/[F] \) is the nominal exchange rate—that is, the price of foreign currency in terms of home currency—and \( p_{Ft}^*(z) \) is the price of foreign good \( z \) denominated in foreign currency. We define the terms of trade as the relative price of imported goods:

\[
S_t = \frac{P_{Ft}}{P_{Ht}}.
\]

From here it follows that

\[
\frac{P_t}{P_{Ht}} = \left( (1 - \gamma) + \gamma S_t^{1-\eta} \right)^{\frac{1}{1-\eta}} \quad \text{and} \quad \frac{P_t^*}{P_{Ht}^*} = \left( (1 - \gamma^*) S_t^{1-\eta} + \gamma^* \right)^{\frac{1}{1-\eta}}.
\]

A.3 Households

A.3.1 Home Households

The economy is populated by a continuum of homogeneous households maximizing the expected lifetime utility \( w_H^t = E_0 \sum_{t=0}^{\infty} \beta^t U (c_t, g_t, n_t) \), with period household utility

\[
U (c_t, g_t, n_t) \equiv U_t = \frac{c_t^{1-\sigma}}{1-\sigma} \xi_t + \varpi g_t^{1-\sigma} \xi_t - d n_t^{1+\varsigma} \delta_t,
\]

where \( \varsigma \geq 0 \) measures the labor supply elasticity, \( \sigma \geq 0 \) measures the elasticity of consumption, \( d \) is a preference parameter, \( c_t \) is consumption, \( g_t \) is household consumption of public goods, \( n_t \) is labor, \( \beta \) is the discount factor, and \( \xi_t \) is a taste shock. Households hold non-contingent bonds issued by governments in home and foreign countries. Each household contributes to the production of all home goods \( z \). Each household is also a firm producing home goods.

We assume that all households in the same country have the same level of initial wealth. As they face the same labor demand and own equal share of all firms, they face identical budget constraints. They all will have identical consumption paths, so we do not use an individual index within each country.
Each household invests in home-currency-denominated home government short- and long-term nominal bonds $B_{Ht}^S$ and $B_{Ht}^M$ and also in foreign-currency-denominated foreign government long-term nominal bonds $B_{Ft}^M$. The nominal individual intertemporal budget constraint at time $t$ for country $H$ is given by

$$(1 - \tau_w^t) W_t n_t + \frac{B_{Ht}^{S-1}}{n} + (1 + \rho H P_{Ht}^M) \frac{B_{Ht}^{M-1}}{n}$$

$$+ (1 + \rho F P_{Ft}^M) \frac{B_{Ft}^{M-1}}{n} E_t + P_{Ht} T_t$$

$$\geq P_{Ht}^S \frac{B_{Ht}^S}{n} + P_{Ht}^M \frac{B_{Ht}^M}{n} + \frac{P_{Ft}^M B_{Ft}^M E_t}{n \phi\left(\frac{P_{Ft}^M B_{Ft}^M E_t}{P_t c_t}\right)} + P_t c_t,$$

where $W_t$ is the nominal wage, $T_t$ are country-level lump-sum taxes/transfers, and $\tau_w^t$ is a country-specific tax on nominal wage income. Here $c_t = \frac{1}{n} C_t$ and $n_t = \frac{1}{n} N_t = \frac{1}{n} \int_0^n n_t (z) dz$, where $n_t (z)$ is individual labor supplied to each individual firm-household.

In the main case we assume that financial markets are incomplete, as in Benigno (2009). Households of country H can trade in four nominal bonds. In each country the government issues two forms of bonds. The first is the familiar one-period debt, $B_{Ht}^S$, which has a price equal to the inverse of the gross nominal interest rate, $P_{Ht}^S = R_t^{-1} = \frac{1}{1+i_t}$. The second type of bond is actually a portfolio of many bonds which, following Cochrane (2000), pay a declining premium of $\rho^j$, $j$ periods after being issued, where $0 < \rho < \beta^{-1}$. The duration of the bond is $\frac{1}{1-\beta \rho}$, which means that $\rho$ can be varied to capture changes in the maturity structure of debt. We only need to price a single bond, since any existing bond issued $j$ periods ago is worth $\rho^j$ of new bonds.

Bonds $B_{Ht}^S$ and $B_{Ht}^M$ are issued by the home government and are denominated in home currency; bonds $B_{Ft}^S$ and $B_{Ft}^M$ are issued by the foreign government and are denominated in foreign currency.

We assume that only long-term bonds are tradable. Households belonging to country H have to pay an intermediation cost if they want to trade in the foreign bond. These costs are determined by the function $\phi(\cdot)$. Function $\phi(\cdot)$ depends on the real holdings of the foreign assets in the entire economy, and therefore is taken as given by the domestic households. If a home household is in a “borrowing
position,” it will be charged with a premium on the foreign interest rate, and if the household is in a “lending position,” it receives a rate of return lower than the foreign interest rate. We impose the following restrictions on \( \phi(\cdot): \phi(x) = 1 \) only if \( x \) is in steady state. Furthermore, \( \phi(\cdot) \) has to be a differentiable, decreasing function in the neighborhood of steady state, and we denote \( \phi'(x) = -\chi \) in steady-state \( x \).

The Lagrangian can be written as

\[
\Gamma_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t, g_t, n_t) + \frac{\Lambda_t}{n} \left( (1 + \rho_H P^M_{Ht}) B^M_{Ht-1} + (1 + \rho_F P^M_{Ft}) B^M_{Ft-1} E_t + n (1 - \tau^w_t) W_t n_t + B^S_{Ht-1} + P_{Ht} T_t \right) \right. \\
- P^S_{Ht} B^S_{Ht} - P^M_{Ht} B^M_{Ht} - \left. \frac{P^M_{Ft} B^M_{Ft} E_t}{\phi(P^M_{Ft} B^M_{Ft} E_t)} - nP_t c_t \right\}.
\]

And the first-order conditions in the text are taken with respect to \( c_t, n_t, B^S_{Ht}, B^M_{Ht}, B^M_{Ft}, \) and \( \Lambda_t \).

We assume that the net supply of short-term bonds is zero, aggregate all first-order conditions, and arrive at the following system:

\[
\frac{W_t}{P_t} = -\frac{U_{n,t}}{(1 - \tau^w_t) U_{c,t}}, \quad (1) \\
\frac{1}{(1 + i_t)} = \beta \mathbb{E}_t \frac{P_t U_{c,t+1}}{P_{t+1} U_{c,t}}, \quad (2) \\
P^M_{Ht} = \beta \mathbb{E}_t \frac{U_{c,t+1} P_t}{U_{c,t} P_{t+1}} \left( 1 + \rho_H P^M_{Ht+1} \right), \quad (3) \\
P^M_{Ft} = \beta \mathbb{E}_t \frac{U_{c,t+1} P_t E_{t+1}}{U_{c,t} P_{t+1} E_t} \phi \left( \frac{P^M_{Ft} B^M_{Ft} E_t}{P_t} \right) \left( 1 + \rho_F P^M_{Ft+1} \right), \quad (4) \\
P^M_{Ht} B^M_{Ht} + \frac{P^M_{Ft} B^M_{Ft} E_t}{\phi \left( \frac{P^M_{Ft} B^M_{Ft} E_t}{P_t} \right)} = (1 + \rho_H P^M_{Ht}) B^M_{Ht-1} + (1 + \rho_F P^M_{Ft}) B^M_{Ft-1} E_t + (1 - \tau^w_t) W_t N_t + P_{Ht} T_t - P_t C_t, \quad (5)
\]

where the budget constraint is written in an aggregated form.
### A.3.2 Foreign Households

Similarly, the foreign households maximize the intertemporal utility $w_t^F = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^*, g_t^*, n_t^*)$, where

$$U(c_t^*, g_t^*, n_t^*) \equiv U^* = \frac{c_t^{*\frac{1-\frac{1}{\sigma}}{\frac{1-\frac{1}{\sigma}}{1}} - \frac{1-\frac{1}{\sigma}}{1-1}} - \frac{1-\frac{1}{\sigma}}{1-1}} - \frac{1-\frac{1}{\sigma}}{1-1} \xi_t^* - d_t^{*\frac{1+\frac{1}{1+\frac{1}{\delta}}}{1+\frac{1}{\delta}}}$$

and $\psi \geq 0$ measures the labor supply elasticity, $\sigma \geq 0$ measures the elasticity of consumption, $d^*$ is a preference parameter, $c^*_t$ is consumption, $n^*_t$ is labor, and $\beta$ is the discount factor in country F. $\xi_t^*$ is a taste shock. The individual household budget constraint in foreign currency is given by

$$(1 - \tau_t^w) W_t^* n_t^* + \left( \frac{1 + \rho_H P_{Ht}^M}{E_t} \right) \frac{B_{Ht-1}^M}{1 - n} + \frac{B_{Fl-1}^S}{1 - n}$$

$$+ \left( 1 + \rho_F P_{Fl}^M \right) \frac{B_{Fl-1}^M}{1 - n} + P_{Fl}^T \frac{T_t^*}{1 - n} \geq \frac{P_{Ht}^M B_{Ht}^M}{(1 - n) E_t \phi^* \left( \frac{P_{Ht}^M B_{Ht}^M}{E_t P_t} \right)} + P_{Fl}^S \frac{B_{Fl}^S}{1 - n} + P_{Fl}^M \frac{B_{Fl}^M}{1 - n} + P_{Fl}^* c_t^*,$$
Similarly, assuming zero net supply of short-term bonds and aggregating across all households, we obtain the following system of first-order conditions:

\[
\frac{W_t^*}{P_t^*} = -\frac{U_{n,t}^*}{U_{c,t}^* (1 - \tau_t^* w^*)}, \quad (6)
\]

\[
\frac{1}{(1 + i_t^*)} = \beta \mathbb{E}_t \frac{U_{c,t+1}^* P_t^*}{U_{c,t}^* P_{t+1}^*}, \quad (7)
\]

\[
P_{Ft}^M = \beta \mathbb{E}_t \frac{U_{c,t+1}^* P_t^*}{U_{c,t}^* P_{t+1}^*} (1 + \rho_F P_{Ft+1}^M), \quad (8)
\]

\[
P_{Ht}^M = \beta \mathbb{E}_t \frac{U_{c,t+1}^* P_t^*}{U_{c,t}^* P_{t+1}^*} (1 + \rho_H P_{Ht+1}^M) \phi^* \left( \frac{P_{Ht}^M B_{Ht}^{M*}}{E_t P_t^*} \right), \quad (9)
\]

\[
P_{Ft}^M B_{Ft}^{M*} = (1 + \rho_F P_{Ft}^M) B_{Ft-1}^{M*} + \frac{(1 + \rho_H P_{Ht}^M) B_{Ht-1}^{M*}}{E_t} + (1 - \tau_t^* w^*) W_t^* N_t^* + P_t^* T_t^* - P_t^* C_t^* - \frac{P_{Ht}^M B_{Ht}^{M*}}{E_t \phi^* \left( \frac{P_{Ht}^M B_{Ht}^{M*}}{E_t P_t^*} \right)}. \quad (10)
\]

A.4 Firms

A.4.1 Home Firms

The production side in this model is entirely conventional. An intermediate-good-producing firm in each sector only uses labor as a factor of production. Labor is mobile between firms so wages are equalized across all firms. The firm’s optimization problem is standard: a firm \( i \) chooses employment and prices to maximize profit

\[
\max \left\{ N_t(i), p_{Hs}(i) \right\} \prod_{s=t}^{\infty} \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s}(y_s(i) p_{Hs}(i) - W_s N_s(i)),
\]

where \( Q_{t,s} \) is the household’s stochastic discount factor subject to the production constraint

\[
y_t(i) = AZ_t N_t(i),
\]
demand constraint
\[ y_t(i) = Y_t \left( \frac{p_{Ht}(i)}{P_{Ht}} \right)^{-\varepsilon}, \]
and price rigidity
\[ p_{Ht}(i) = p_{Ht}^x(i), \]
\[ p_{Ht+1}(i) = \begin{cases} p_{Ht}^x(i), \text{ with prob } 1 - \theta \\ p_{Ht+1}(i), \text{ with prob } \theta. \end{cases} \]

The profit-maximization problem can be split into two separate problems: choose labor to minimize cost intratemporally and choose prices to maximize future profits. We deal with each of these problems separately.

**Aggregate Employment.** Cost minimization of firm \( i \) is
\[ \min_{N_t(i)} (W_t N_t(i)) \]
subject to the production constraint
\[ y_t(i) = AZ_t N_t(i). \]
Write down the Lagrangian:
\[ L = W_t N_t(i) - P_{Ht} \Xi_t (AZ_t N_t(i) - y_t(i)) \]
and obtain the expression for the marginal cost
\[ \Xi_t = \frac{W_t}{P_{Ht}AZ_t} = mc_t. \]

Aggregation of labor yields (we denote price dispersion
\[ \Delta_t \langle n, \varepsilon \rangle = \frac{1}{n} \int_0^n \left( \frac{p_{Ht}(i)}{P_{Ht}} \right)^{-\varepsilon} di \]
\[ N_t = \int_0^n N_t(i) di = \int_0^n \frac{y_t(i)}{AZ_t} di = \frac{1}{n} \int_0^n Y_t AZ_t \left( \frac{p_{Ht}(i)}{P_{Ht}} \right)^{-\varepsilon} di \]
\[ = \frac{Y_t}{AZ_t} \Delta_t \langle n, \varepsilon \rangle, \]
so the “aggregated” production function can be written as

\[ Y_t = N_t \frac{A Z_t}{\Delta_t}. \]

**Price Setting.** The following is price setting:

\[
\max_{\{ p_{Hs}^x(i) \}_{s=t}} \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \left( y_s (i) p_{Hs} (i) - W_s N_s (i) \right)
\]

\[
= \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \left( y_s (i) p_{Hs} (i) - W_s \frac{y_s (i)}{AZ_s} \right)
\]

\[
= \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s} \left( y_s (i) p_{Hs} (i) - y_s (i) MC_s \right),
\]

where

\[ MC_s = P_{Ht}mc_t. \]

The constraints are

\[ y_t (i) = Y_t \left( \frac{p_{Ht} (i)}{P_{Ht}} \right)^{-\varepsilon_t} \]

\[ p_{Ht} (i) = p_{Ht}^x (i) \]

\[ p_{Ht+1} (i) = \begin{cases} 
 p_{Ht+1}^x (i), \text{ with prob } 1 - \theta \\
 p_{Ht+1} (i), \text{ with prob } \theta.
\end{cases} \]

The problem for the optimal price setting at time \( t \) can, equivalently, be written as

\[
\max_{\{ p_{Ht}^x (i) \}_{s=t}} \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} \left( y_s (i) p_{Ht}^x (i) - y_s (i) MC_s \right),
\]

so we only consider profits of those firms that fix prices at time \( t \). Substitute demand

\[
\max_{\{ p_t^x (i) \}_{s=t}} \mathbb{E}_t \sum_{s=t}^{\infty} \theta^{s-t} Q_{t,s} P_{Hs} Y_s \left( \left( \frac{p_{Ht}^x (i)}{P_{Hs}} \right)^{1-\varepsilon} - \left( \frac{p_{Ht}^x (i)}{P_{Hs}} \right)^{-\varepsilon} \frac{MC_s}{P_{Hs}} \right).
\]
First-order conditions are

\[ 0 = E_t \sum_{s=t}^{\infty} \theta^s Q_{t,s} P_{Hs} y_s(i) \left( \frac{p_{Ht}^\times(i)}{P_{Hs}} - \mu mc_s \right), \]

where \( \mu = -\frac{\varepsilon}{1-\varepsilon} = \frac{\varepsilon}{\varepsilon-1} \).

We can rewrite these conditions in the form of the three equations:

\[
\begin{align*}
    p_{Ht}^\times(i) &= \frac{H_t}{F_t} \\
    F_t &= E_t \sum_{s=t}^{\infty} (\theta \beta)^{s-t} f_s \left( \frac{P_{Hs}}{P_{Ht}} \right)^{\varepsilon-1} \\
    H_t &= E_t \sum_{s=t}^{\infty} (\theta \beta)^{s-t} h_s \left( \frac{P_{Hs}}{P_{Ht}} \right)^{\varepsilon}
\end{align*}
\]

with

\[
\begin{align*}
    f_t &= U_C(C_t, n_t) \frac{P_{Ht}}{P_t} Y_t = C_t^{\frac{1}{\sigma}} \xi_t^{\frac{1}{\sigma}} P_{Ht} P_t Y_t \\
    h_t &= \mu U_C(C_t, n_t) \frac{P_{Ht}}{P_t} Y_t m c_t \\
    &= \mu C_t^{\frac{1}{\sigma}} \xi_t^{\frac{1}{\sigma}} P_{Ht} P_t W_t P_{Ht} AZ_t.
\end{align*}
\]

We can aggregate over \( i \) and remove index \( i \):

\[
\frac{p_{Ht}^\times}{P_{Ht}} = \frac{H_t}{F_t}.
\]

Price in the sector is determined as

\[
P_{Ht} = \left[ (1-\theta) \left( p_{Ht}^\times \right)^{1-\varepsilon} + \theta P_{Ht-1}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}},
\]

from where

\[
\Pi_{Ht}^{1-\varepsilon} = \left( \frac{P_{Ht}}{P_{Ht-1}} \right)^{1-\varepsilon} = (1-\theta) \left( \frac{p_{Ht}^\times}{P_{Ht-1}} \frac{P_{Ht}}{P_{Ht}} \right)^{1-\varepsilon} + \theta
\]
and so

\[
\left( \frac{p_{Ht}}{Ht} \right)^{1-\varepsilon} = \frac{\Pi_{Ht}^{1-\varepsilon} - \theta}{(1 - \theta) \Pi_{Ht}^{1-\varepsilon}} = \frac{1 - \theta \Pi_{Ht}^{\varepsilon-1}}{(1 - \theta)} = \left( \frac{Ht}{Ft} \right)^{1-\varepsilon}.
\]

We obtain the following first-order conditions:

\[Ht = \mu \nu t U_{C,t} P_{Ht} Yt \frac{Wt}{P_{Ht} AZt} + \theta \beta E_t \Pi_{Ht+1}^{\varepsilon} Ht+1, \tag{11}\]

\[Ft = U_{C,t} P_{Ht} Yt + \theta \beta E_t \Pi_{Ht+1}^{\varepsilon-1} Ft+1, \tag{12}\]

\[\left( \frac{Ht}{Ft} \right)^{1-\varepsilon} = \frac{(1 - \theta \Pi_{Ht}^{\varepsilon-1})}{(1 - \theta)} \tag{13},\]

\[Yt = Nt AZt \Delta_t, \tag{14}\]

where equations (11)–(13) define optimal price setting and \(\nu t\) represents a shock to the desired flexible-price markup \(\mu = \frac{\varepsilon}{\varepsilon-1}\). Equation (14) defines aggregate labor demand.

We derive the evolution of the price dispersion equation. Each period, a measure of \((1 - \theta)n\) of firms resets prices and sets \(p_{Ht} (i) = p_{Ht}^*\), which is the same for all such firms. A measure of \(\theta\) does not change prices, and for them \(p_{Ht} (i) = p_{Ht-1} (i)\). Therefore,

\[\Delta_t = \frac{1}{n} \int_{0}^{(1-\theta)n} \left[ \frac{p_{Ht} (i)}{P_{Ht}} \right]^{-\varepsilon} di + \frac{1}{n} \int_{(1-\theta)n}^{n} \left[ \frac{p_{Ht} (i)}{P_{Ht}} \right]^{-\varepsilon} di
\]

\[= \frac{1}{n} \int_{0}^{(1-\theta)n} \left[ \frac{p_{Ht}^*}{P_{Ht}} \right]^{-\varepsilon} di + \frac{1}{n} \int_{(1-\theta)n}^{n} \left[ \frac{p_{Ht-1} (i)}{P_{Ht}} \right]^{-\varepsilon} di.\]

\(^2\)We could have assumed a variable elasticity \(\varepsilon_t\). This would result in the identical linearized system but would not allow us to write down non-linear first-order conditions in the compact form (11)–(13). The derivation of linearized first-order conditions would be identical to the one in Leith and Wren-Lewis (2013).
We compute
\[
\int_0^{(1-\theta)n} \left[ \frac{p^*_H}{P_H} \right]^{-\xi} \, di = \left[ \frac{p^*_H}{P_H} \right]^{-\xi} \int_0^{(1-\theta)n} \, di = (1-\theta)n \left[ \frac{p^*_H}{P_H} \right]^{-\xi}
\]
and
\[
\int_{(1-\theta)n}^n \left[ \frac{p_{Ht}(i)}{P_{Ht}} \right]^{-\xi} \, di = \int_{(1-\theta)n}^n \left[ \frac{p_{Ht-1}(i)}{P_{Ht}} \right]^{-\xi} \, di = \int_{(1-\theta)n}^n \left[ \frac{p_{Ht-1}(i) P_{Ht-1}}{P_H P_{Ht-1}} \right]^{-\xi} \, di = \theta n \left( \frac{P_{Ht-1}}{P_H} \right)^{-\xi} 1_{\theta n} \int_{(1-\theta)n}^n \left[ \frac{p_{Ht-1}(i)}{P_{Ht-1}} \right]^{-\xi} \, di = \theta n \left( \frac{P_{Ht-1}}{P_H} \right)^{-\xi} \Delta_{t-1}.
\]

Collect two terms together to obtain
\[
\Delta_t = (1-\theta) \left[ \frac{p^*_H}{P_H} \right]^{-\xi} + \theta \Delta_{t-1} \Pi_{Ht}^{\xi}
\]
and with \(\xi = \epsilon\) we obtain
\[
\Delta_t = (1-\theta) \left( \frac{1 - \theta \Pi_{Ht}^{\epsilon-1}}{1 - \theta} \right)^{\frac{\epsilon}{\epsilon-1}} + \theta \Pi_{Ht}^{\epsilon} \Delta_{t-1}. \quad (15)
\]
Aggregate intraperiod nominal profit of monopolistically competitive firms is
\[
P_{Ht} \Phi_t = P_{Ht} Y_t - W_t N_t.
\]
We assume that the profit is 100 percent taxed by the government and redistributed to households:
\[
P_{Ht} \Phi_t = P_{Ht} \bar{T}_t. \quad (16)
\]
A.4.2 Flexible Price

\[
\max E_t \sum_{s=t}^{\infty} Q_{t,s} \left( y_s (i) \frac{p_{Hs} (i)}{P_{Hs}} - W_s N_s (i) \right)
\]

\[
= E_t \sum_{s=t}^{\infty} Q_{t,s} \left( P_{Hs} Y_s \left( \frac{p_{Hs} (i)}{P_{Hs}} \right)^{1-\varepsilon} - \frac{W_s}{AZ_s} Y_s \left( \frac{p_{Hs} (i)}{P_{Hs}} \right)^{-\varepsilon} \right)
\]

The constraints are

\[
y_t (i) = Y_t \left( \frac{p_{Ht} (i)}{P_{Ht}} \right)^{-\varepsilon_t}.
\]

First-order conditions are

\[
0 = (1 - \varepsilon) Y_s \left( \frac{p_{Hs} (i)}{P_{Hs}} \right)^{-\varepsilon} + \varepsilon \frac{W_s}{P_{Hs} AZ_s} Y_s \left( \frac{p_{Hs} (i)}{P_{Hs}} \right)^{-\varepsilon - 1},
\]

where \( \mu = -\frac{\varepsilon}{1-\varepsilon} = \varepsilon^{-1} \).

\[
0 = \frac{(1 - \varepsilon)}{\varepsilon} \left( \frac{p_{Hs} (i)}{P_{Hs}} \right) + \frac{W_s}{P_{Hs} AZ_s}
\]

\[
\frac{(\varepsilon - 1)}{\varepsilon} = \frac{W_s}{P_{Hs} AZ_s}
\]

\[
W_t \frac{1}{P_t} = -\frac{U_{n,t}}{(1 - \tau_t^w) U_{c,t}},
\]

\[
P_t \frac{1}{P_{Ht}} = \left( (1 - \gamma) + \gamma S_t^{1-\eta} \right)^{\frac{1}{1-\eta}} = \Upsilon_t \text{ and }
\]

\[
P_t^* \frac{1}{P_{Ht}^*} = \left( (1 - \gamma^*) S_t^{1-\eta} + \gamma^* \right)^{\frac{1}{1-\eta}} = \Gamma_t.
\]

\[
\frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon} = -\frac{1}{AZ_t (1 - \tau_t^w) U_{c,t}} \Upsilon_t
\]

\[
= \frac{1}{AZ_t} \frac{dn_t \delta_t^{-\frac{1}{\sigma}}}{(1 - \tau_t^w) c_t^{\frac{1}{\sigma}} \xi_t^{\frac{1}{\sigma}}} \Upsilon_t = \frac{\Upsilon_t}{AZ_t} \frac{dn_t \left( \frac{1}{\sigma} + \varsigma_t \right) N_t^{\frac{1}{\sigma}} \delta_t^{-\frac{1}{\sigma}}}{(1 - \tau_t^w) C_t^{\frac{1}{\sigma}} \xi_t^{\frac{1}{\sigma}}}
\]
A.4.3 Foreign Firms

Similarly, foreign firm $i$ chooses employment and prices to maximize profit:

$$\max_{\{N_t^*(i), p_{Ft}^*(i)\}} \mathbb{E}_t \sum_{s=t}^{\infty} Q_{t,s}^* (y_s^* (i) p_{Fs}^* (i) - W_s^* N_s^* (i)),$$

subject to constraints

$$y_t^* (i) = A^* Z_t^* N_t^* (i),$$
$$y_t^* (i) = Y_t^* \left( \frac{p_{Ft}^* (i)}{P_t^*} \right)^{-\varepsilon},$$
$$p_{Ft}^* (i) = p_{Ft}^* (i),$$
$$p_{Ft+1}^* (i) = \begin{cases} p_{Ft+1}^* (i), \text{ with prob } 1 - \theta \\ p_{Ft+1}^* (i), \text{ with prob } \theta. \end{cases}$$

The first-order conditions are

$$H_t^* = \mu u_t^* U_{C,t}^* \frac{P_{Ft}^* Y_t^*}{P_t^*} \frac{W_t^*}{P_{Ft}^* A^* Z_t^*} + \theta \beta \mathbb{E}_t \Pi_{Ft+1}^* H_{t+1}^*, \quad (17)$$
$$F_t^* = U_{C,t}^* \frac{P_{Ft}^* Y_t^*}{P_t^*} + \theta \beta \mathbb{E}_t \Pi_{Ft+1}^* F_{t+1}^*, \quad (18)$$
$$\left( \frac{H_t^*}{F_t^*} \right)^{1-\varepsilon} = \frac{(1 - \theta) \Pi_{Ft}^{\varepsilon-1}}{(1 - \theta)}, \quad (19)$$
$$Y_t^* = N_t^* \frac{A^* Z_t^*}{\Delta_t^*}, \quad (20)$$
$$\Delta_t^* = (1 - \theta) \left( \frac{1 - \theta \Pi_{Ft}^{\varepsilon-1}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \theta \Pi_{Ft}^\varepsilon \Delta_{t-1}^*. \quad (21)$$

The profit is

$$\Phi_t^* = Y_t^* - \frac{W_t^*}{P_{Ft}^*} N_t^*. \quad (22)$$
A.4.4 Flexible Price

\[
\max \mathbb{E}_t \sum_{s=t}^\infty Q_{t,s} (y^*_s(i) p_{F,s}^* (i) - W_s^* N_s^* (i)) = \mathbb{E}_t \sum_{s=t}^\infty Q_{t,s} \left( \frac{p_{F,t}^* (i)}{P_t^*} \right)^{\frac{1}{\varepsilon}} - \frac{W_s^*}{AZ_s^*} \left( \frac{p_{F,t}^* (i)}{P_t^*} \right)^{\frac{1}{\varepsilon}}
\]

The constraints are

\[
y^*_t(i) = Y_t^* \left( \frac{p_{F,t}^* (i)}{P_t^*} \right)^{\frac{1}{\varepsilon}}.
\]

First-order conditions are

\[
0 = (1 - \varepsilon) Y_t^* \left( \frac{p_{F,t}^* (i)}{P_t^*} \right)^{\frac{1}{\varepsilon}} + \varepsilon \frac{W_s^*}{AZ_s^*} \left( \frac{p_{F,t}^* (i)}{P_t^*} \right)^{\frac{1}{\varepsilon} - 1},
\]

where \( \mu = -\frac{\varepsilon}{1 - \varepsilon} = \frac{\varepsilon}{\varepsilon - 1} \).

\[
0 = \frac{(1 - \varepsilon)}{\varepsilon} \left( \frac{p_{F,t}^* (i)}{P_t^*} \right) + \frac{W_s^*}{AZ_s^* P_t^*}.
\]

\[
\left( \frac{\varepsilon - 1}{\varepsilon} \right) = \frac{W_s^*}{AZ_s^* P_t^*}
\]

\[
W_t^* = \frac{U_{n,t}^*}{U_{c,t}^* (1 - \tau^*_w)},
\]

\[
\frac{P_t}{P_{H,t}} = \left( (1 - \gamma) + \gamma S_t^{1-\eta} \right)^{\frac{1}{1-\eta}} = \Upsilon_t \quad \text{and}
\]

\[
\frac{P_t^*}{P_{H,t}^*} = \left( (1 - \gamma^*) S_t^{1-\eta} + \gamma^* \right)^{\frac{1}{1-\eta}} = \Gamma_t.
\]

\[
\frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon} = -\frac{U_{n,t}^*}{AZ_t^* U_{n,t}^* (1 - \tau^*_w)} \frac{\Gamma_t}{S_t} = \frac{1}{AZ_t^* \Gamma_t} \frac{dn_t^* \delta_t^{\frac{-1}{\sigma}}}{\theta_t} \frac{\delta_t^{\frac{-1}{\sigma}}}{\xi_t^{\frac{-1}{\sigma}}} S_t
\]

\[
= \frac{1}{AZ_t^* S_t} \frac{\Gamma_t}{(1 - \tau^*_w)} C_t^{\frac{-1}{\sigma}} \frac{\xi_t^{\frac{-1}{\sigma}}}{\xi_t^{\frac{-1}{\sigma}}}
\]
A.5 Financial Intermediaries

Financial intermediaries in each country help to sell foreign bonds to residents of their own country. The profit of intermediaries is

\[ P_{Ht} \Psi_t = \frac{P_{Mt} B_{Mt}^E_t}{\phi \left( \frac{P_{Mt}^E B_{Mt}^E_t}{P_t} \right)} - P_{Mt} B_{Mt}^E_t, \]

\[ P_{*Mt} \Psi_{*t} = \frac{P_{Ht}^* B_{Ht}^*}{E_t \phi^* \left( \frac{P_{Ht}^* B_{Ht}^*}{E_t F_{*t}} \right)} - \frac{P_{Ht}^* B_{Ht}^*}{E_t}. \]

A.6 Governments

The home-country government buys home goods \( G_{Ht} \), taxes labor income \( \tau_{lt} \), raises lump-sum taxes \( \tilde{T}_t \), pays an employment subsidy \( \tau_{st} \), expropriates monopoly profit \( \Phi_t \) and profit of financial intermediaries \( \Psi_t \), pays transfers to households

\[ T_t = \tilde{T}_t - \tilde{T}_t, \]

and issues nominal debt \( B_{Mt} \). The value \( B_{Mt} \) of end-of-period public debt then evolves according to the following law of motion:

\[ P_{Ht}^M \left( B_{Ht}^M + B_{Ht}^{*M} \right) = (1 + \rho_H P_{Ht}^M) \left( B_{Ht-1}^M + B_{Ht-1}^{*M} \right) + P_{Ht} G_t + P_{Ht} T_t \]

\[ - \tau_{wt} W_t n_t - P_{Ht} \Phi_t - P_{Ht} \Psi_t, \]

(23)

where we assumed that the aggregate stock of one-period bonds is in zero net supply, \( B_{t}^S = 0 \).

Lump-sum transfers \( \tilde{T}_t \) rebate monopolistic profit, \( \tilde{T}_t = \Phi_t + \Psi_t \), while lump-sum taxes \( \tilde{T}_t \) are used to pay the steady-state employment subsidy \( \tau^s \), the size of which will be determined later:

\[ P_{Ht} \tilde{T}_t = \tau^s W_t N_t, \]

so we define \( \tau_{lt} \) as \( \tau_{wt}^l = \tau_{lt} - \tau^s \).

For analytical convenience we introduce the home-country real debt \( B_{Ht} = B_{Ht}^M / P_{Ht} \), so that (23) becomes
\[ P_{Ht}^M (B_{Ht} + B_{Ht}^*) = (1 + \rho_HP_{Ht}^M) \frac{(B_{Ht-1} + B_{Ht-1}^*)}{1 + \pi_H} + G_t - \tau_t \frac{W_t}{P_{Ht}} N_t. \]  

(24)

Similarly, the foreign-country budget constraint becomes (using \( B_{Ft} = B_{Ft}^M/P_{Ft}^* \))

\[ P_{Ft}^M (B_{Ft} + B_{Ft}^*) = (1 + \rho_F P_{Ft}^M) \frac{(B_{Ft-1} + B_{Ft-1}^*)}{1 + \pi_F^*} + G_t^* - \tau_{t*} \frac{W_{t*}}{P_{Ft}^*} N_{t*} \]

(25)

and

\[ T_t^* = \bar{T}_t^* - \tilde{T}_t^*, \quad \bar{T}_t^* = \Phi_t^* + \Psi_t^*, \quad P_{Ft}^* \tilde{T}_t^* = \tau_{s*} W_{t*} N_{t*}. \]

A.7 Market Clearing and Evolution of the Economy

The two market clearing conditions for countries H and F are

\[ Y_t P_{Ht} = C_{Ht} P_{Ht} + C_{Ht}^* P_{Ht}^* E_t + G_t P_{Ht} \]  

(26)

\[ Y_t^* P_{Ft}^* = C_{Ft}^* P_{Ft}^* + C_{Ft} \frac{P_{Ft}}{E_t} + G_t^* P_{Ft}^*. \]  

(27)

Aggregating budget constraints, we get two financial constraints, and using market clearing conditions, we obtain the net foreign assets (NFA) equation:

\[ \gamma^* \left( (1 - \gamma^*) S_{t}^{1-\eta} + \gamma^* \right) \frac{\eta}{1-\eta} C_t^* - \gamma S_{t}^{1-\eta} \left( (1 - \gamma) + \gamma S_{t}^{1-\eta} \right) \frac{\eta}{1-\eta} C_t \]

\[ = \left( P_{Ft}^M B_{Ft} - \left( 1 + \rho_F P_{Ft}^M \right) \frac{B_{Ft-1}}{1 + \pi_F^*} \right) S_t \]

\[ - \left( P_{Ht}^M B_{Ht}^* - \left( 1 + \rho_H P_{Ht}^M \right) \frac{B_{Ht-1}^*}{1 + \pi_{Ht}} \right). \]  

(28)

Eight households’ first-order conditions (1)–(4) and (6)–(9), ten firms’ first-order conditions (11)–(15) and (17)–(21), two market clearing conditions (26)–(27), two government budget constraints,

\[ P_{Ht}^M (B_{Ht} + B_{Ht}^*) = (1 + \rho_H P_{Ht}^M) \frac{(B_{Ht-1} + B_{Ht-1}^*)}{1 + \pi_{Ht}} + G_t - \tau_t \frac{W_t}{P_{Ht}} N_t, \]
\[ P_{Ft}^M (B_{Ft} + B_{Ft}^*) = (1 + \rho_F P_{Ft}^M) \frac{(B_{Ft-1} + B_{Ft-1}^*)}{1 + \pi_{Ft}^*} \]
\[ + \ G_t^* - \tau_t^* \frac{W_t^*}{P_{Ft}^*} N_t^* , \]

one net foreign assets equation (28), and the definition of the nominal exchange rate

\[ S_t = \frac{E_t \ P_{Ft}^*}{P_{Ht}} \]

describe the evolution of the economy.

A.8 Private-Sector Equilibrium

To summarize the system as a whole,

\[ \frac{W_t}{P_t} = - \frac{U_n(c_t, n_t)}{U_c(c_t, n_t) (1 - \tau^w)} \]

\[ \frac{1}{1 + i_t} = \beta \mathbb{E}_t U_c(C_{t+1}, n_{t+1}) \left(\frac{1 - \gamma + \gamma S_{t+1}^{1-\eta}}{1 - \gamma + \gamma S_{t+1}^{1-\eta}}\right)^{\frac{1}{1-\eta}} \frac{1}{1 + \pi_{Ht+1}} \]

\[ P_{Ht}^M = \beta \mathbb{E}_t U_c(C_{t+1}, n_{t+1}) \left(\frac{1 - \gamma + \gamma S_{t+1}^{1-\eta}}{1 - \gamma + \gamma S_{t+1}^{1-\eta}}\right)^{\frac{1}{1-\eta}} \frac{1}{1 + \pi_{Ht+1}} \]

\[ \times (1 + \rho_H P_{Ht+1}^M) \]

\[ P_{Ft}^M = \beta \mathbb{E}_t U_c(C_{t+1}, n_{t+1}) \left(\frac{1 - \gamma + \gamma S_{t+1}^{1-\eta}}{1 - \gamma + \gamma S_{t+1}^{1-\eta}}\right)^{\frac{1}{1-\eta}} \frac{S_{t+1}}{S_t} \frac{1}{1 + \pi_{Ft+1}^*} \]

\[ \times (1 + \rho_F P_{Ft+1}^M) \phi \left( P_{Ft}^M B_{Ft} S_t \left( (1 - \gamma) + \gamma S_t^{1-\eta} \right)^{-\frac{1}{1-\eta}} \right) \]

\[ Y_t = (1 - \gamma) \left( (1 - \gamma) + \gamma S_t^{1-\eta} \right)^{\frac{1}{1-\eta}} C_t \]
\[ + \ \gamma^* \left( (1 - \gamma^*) S_t^{1-\eta} + \gamma^* \right)^{\frac{1}{1-\eta}} C_t^* + G_t \]
\[
\frac{1 - \theta \Pi_H^{e-1}}{1 - \theta} = \left( \frac{H_t}{F_t} \right)^{1-\varepsilon} \tag{33}
\]

\[
H_t = \mu v_t U_{C,t} \frac{P_{Ht}}{P_t} Y_t \frac{W_t}{P_{Ht} AZ_t} + \theta \beta \mathbb{E}_t \Pi_H^{e-1} H_{t+1} \tag{34}
\]

\[
F_t = U_{C,t} \frac{P_{Ht}}{P_t} Y_t + \theta \beta \mathbb{E}_t \Pi_H^{e-1} F_{t+1} \tag{35}
\]

\[
\Delta_t = (1 - \theta) \left( \frac{1 - \theta \Pi_H^{e-1}}{1 - \theta} \right)^{\frac{\varepsilon - 1}{\varepsilon}} + \theta \Pi_H \Delta_{t-1} \tag{36}
\]

\[
Y_t = N_t \frac{AZ_t}{\Delta_t} \tag{37}
\]

\[
P^M_{Ht} (B_{Ht} + B^*_{Ht}) = (1 + \rho_H P^M_{Ht}) \frac{\left( B_{Ht-1} + B^*_{Ht-1} \right)}{1 + \pi_{Ht-1}}
\]

\[+ G_t - \tau_t W_t \frac{N_t}{P_{Ht}} \tag{38}\]

\[
\frac{W^*_t P^*_{Ht} P^*_t}{P^*_t P^*_t P^*_t} = \frac{U_n (c^*_t, n^*_t)}{U(c^*_t, n^*_t)} (1 - \tau^*_w) \tag{39}\]

\[
\frac{1}{(1 + i^*_t)} = \beta \mathbb{E}_t \frac{U_C (c^*_{t+1}, n^*_{t+1})}{U_C (c^*_t, n^*_t)} \left( \frac{(1 - \gamma^*) S^1_{t-\eta} + \gamma^*}{(1 - \gamma^*) S^1_{t+1-\eta} + \gamma^*} \right)^{\frac{1}{1-\eta}}
\times \frac{S_{t+1}}{S_t} \frac{1}{1 + \pi^*_{Ft+1}} \tag{40}\]

\[
P^M_{Ft} = \beta \mathbb{E}_t \frac{U_C (c^*_{t+1}, n^*_{t+1})}{U_C (c^*_t, n^*_t)} \left( \frac{(1 - \gamma^*) S^1_{t-\eta} + \gamma^*}{(1 - \gamma^*) S^1_{t+1-\eta} + \gamma^*} \right)^{\frac{1}{1-\eta}}
\times \frac{S_{t+1}}{S_t} \frac{1}{1 + \pi^*_{Ft+1}} (1 + \rho_F P^M_{Ft+1}) \tag{41}\]

\[
P^M_{Ht} = \beta \mathbb{E}_t \frac{U_C (c^*_{t+1}, n^*_{t+1})}{U_C (c^*_t, n^*_t)} \left( \frac{(1 - \gamma^*) S^1_{t-\eta} + \gamma^*}{(1 - \gamma^*) S^1_{t+1-\eta} + \gamma^*} \right)^{\frac{1}{1-\eta}} \frac{1}{1 + \pi_{Ht+1}}
\times (1 + \rho_H P^M_{Ht+1}) \phi^* \left( P^M_{Ht} B^*_H \left( (1 - \gamma^*) S^1_{t-\eta} + \gamma^* \right)^{-\frac{1}{1-\eta}} \right) \tag{42}\]
\[ Y_t^* = (1 - \gamma^*) S_t^{-\eta} \left( (1 - \gamma^*) S_t^{1-\eta} + \gamma^* \right)^{\frac{\eta}{1-\eta}} C_t^* \]
\[ + \gamma S_t^{-\eta} \left( 1 - \gamma + \gamma S_t^{1-\eta} \right)^{\frac{\eta}{1-\eta}} C_t + G_t^* \] (43)

\[ \frac{1 - \theta \Pi_{Ft}^{\varepsilon-1}}{1 - \theta} = \left( \frac{H_t^*}{F_t^*} \right)^{1-\varepsilon} \] (44)

\[ H_t^* = \mu v_t^* U_{C,t}^* \frac{P_{Ft}^*}{P_t^*} Y_t^* \frac{W_t^*}{P_{Ft}^* A_t^* Z_t^*} + \theta \beta E_t \Pi_{Ft+1}^* H_{t+1}^* \] (45)

\[ F_t^* = U_{C,t}^* \frac{P_{Ft}^*}{P_t^*} Y_t^* + \theta \beta E_t \Pi_{Ft+1}^{\varepsilon-1} F_{t+1}^* \] (46)

\[ \Delta_t^* = (1 - \theta) \left( \frac{1 - \theta \Pi_{Ft}^{\varepsilon-1}}{1 - \theta} \right)^{\frac{\varepsilon}{1-\varepsilon}} + \theta \Pi_{Ft}^{\varepsilon} \Delta_{t-1}^* \] (47)

\[ Y_t^* = N_t^* \frac{A_t^* Z_t^*}{\Delta_t^*} \] (48)

\[ P_{Ft}^M (B_{Ft} + B_{Ft}^*) = \left( 1 + \rho_F P_{Ft}^M \right) \frac{B_{Ft-1} + B_{Ft-1}^*}{1 + \pi_{Ft}^*} \]
\[ + G_t^* - \tau_{t}^* \frac{W_t^*}{P_{Ft}^*} N_t^* \] (49)

\[ 0 = \gamma S_t^{1-\eta} \left( 1 - \gamma \right) + \gamma S_t^{1-\eta} \left( (1 - \gamma^*) S_t^{1-\eta} + \gamma^* \right)^{\frac{\eta}{1-\eta}} C_t^* \]
\[ + \left( P_{Ft}^M B_{Ft} - \left( 1 + \rho_F P_{Ft}^M \right) \frac{B_{Ft-1}}{1 + \pi_{Ft}^*} \right) S_t \]
\[ - \left( P_{Ht}^M B_{Ht}^* - \left( 1 + \rho_H P_{Ht}^M \right) \frac{B_{Ht-1}}{1 + \pi_{Ht}^*} \right) \] (50)

\[ S_t = \frac{E_t P_{Ft}^*}{P_{Ht}} \] (51)

\[ Q_t = \frac{E_t P_{Ht}^*}{P_t} = \frac{\left( (1 - \gamma^*) S_t^{1-\eta} + \gamma^* \right)^{\frac{1}{1-\eta}}}{\left( 1 - \gamma + \gamma S_t^{1-\eta} \right)^{\frac{1}{1-\eta}}} \] (52)
\[ D_t = \frac{P_t}{E_t P^*_t} \frac{c_t^{\frac{\sigma_0}{\sigma}} \xi_t^{\frac{1}{\sigma_0}}}{c_t^{\frac{1}{\sigma}} \xi_t^{\frac{1}{\sigma}}} = \frac{1}{Q_t} \frac{c_t^{\frac{\sigma_0}{\sigma}} \xi_t^{\frac{1}{\sigma}}}{c_t^{\frac{1}{\sigma}} \xi_t^{\frac{1}{\sigma}}} \times, \quad (53) \]

where we assumed steady-state lump-sum employment subsidy financed by labor taxes

\[
\tau_t^w = \tau_t^l - \tau_t^s \\
\tau_t^{*w} = \tau_t^{*l} - \tau_t^{*s}.
\]

The system of twenty-six equations determines twenty-six variables: \( C_t, N_t, W_t, Y_t, H_t, F_t, \Delta_t, \pi_H t, P_H^M, B_{Ht}, B_{Ft}, G^*_t, N^*_t, W^*_t, Y^*_t, H^*_t, F^*_t, \Delta^*_t, \pi^*_F, P_F^M, B^*_{Ht}, B^*_{Ft}, E_t, S_t, Q_t, \) and \( D_t \).

Policy instruments are \( i_t, i_t^*, G_t, G_t^* \), and it remains to describe policy.

A.9 Parameterization

Individual utility and its derivatives are given by

\[ U(c_t, g_t, n_t) = \frac{c_t^{1-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \varpi \frac{g_t^{1-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - d \frac{n_t^{1+\varsigma}}{1+\varsigma} \delta_t^{-\frac{1}{\sigma}} \\
U_c(c_t, g_t, n_t) = c_t^{-\frac{1}{\sigma}} \xi_t^{-\frac{1}{\sigma}} \\
U_n(c_t, g_t, n_t) = -dn_t^\varsigma \delta_t^{-\frac{1}{\sigma}} \\
U(c^*_t, g^*_t, n^*_t) = \frac{c_t^{*1-\frac{1}{\sigma}} \xi_t^{*\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \varpi \frac{g_t^{*1-\frac{1}{\sigma}} \xi_t^{*\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - d \frac{n_t^{*1+\varsigma}}{1+\varsigma} \delta_t^{-\frac{1}{\sigma}} \\
U_c(c^*_t, g^*_t, n^*_t) = c_t^{*\frac{1}{\sigma}} \xi_t^{*\frac{1}{\sigma}} \\
U_n(c^*_t, g^*_t, n^*_t) = -dn_t^{*\varsigma} \delta_t^{*\frac{1}{\sigma}}. \]

A.10 Steady State and Calibration

The fiscal side of the model is characterized by government-spending-to-output ratios \( \Theta_H = G_H/Y \) and \( \Theta_F = G_F/Y \), labor income taxes \( \tau^l \) and \( \tau^{*l} \), maturity of debt \( m_H \) and \( m_F \), annualized steady-state debt-to-output ratios \( \delta_H = \beta m_H (B_{Ht} + B_{*Ht}) \) and
\(\delta_F = \beta m_F \frac{(B_F + B^*_F)}{4 Y^*},\) the share of home-issued debt held by non-residents \(\varrho = \frac{B^*_H}{B_H + B^*_H},\) and the share of home-held foreign debt to foreign-held home debt \(\omega = \frac{m_H B_F S}{m_F B^*_H}\). Here \(B_H\) are home-issued bonds held by home residents, \(B^*_H\) are home-issued bonds held by foreign residents, \(B^*_F\) are foreign-issued bonds held by foreign residents, and \(B_F\) are foreign-issued bonds held by home residents.

The share of government spending to GDP, \(\Theta_H\) and \(\Theta_F\), is set to 0.20 for each country, in line with the EMU data. In our benchmark calibration we assume that the small home country has a total-debt-to-annual-output ratio of \(\delta_H = 1.10\), which is consistent with the employment-weighted average debt level in Portugal, Greece, Ireland, Italy, and Spain. The large foreign country has \(\delta_H = 0.6,\) consistent with the debt level in the rest of the EMU. The currently observed domestic debt levels are treated as steady-state values rather than initial condition, partly because the current projections (see International Monetary Fund Fiscal Monitor data) suggest that this level of government debt is expected to persist for at least a decade, thus making these values an (implicit) target of policy authorities, as it is expected that all variables are to return to these (steady-state) values in the long run.

As a significant proportion of the government debt is held by non-residents (see tables C2 and C3 in appendix C), we set the benchmark value of the small home-country government debt held by non-residents \(\varrho = 0.5.\) The IMF survey data suggest that the imbalances in long-term debt holdings imply \(\omega = 0.5,\) so that the small home country is a net debtor. Finally, we calibrate the adjustment cost parameter \(\chi = \chi^* = 0.01\) following Benigno (2009).

Calibration of parameters \(\delta_H, \delta_F, m_H, m_F, \Theta_H, \Theta_F, \varrho,\) and \(\omega\) yields the steady-state tax level needed to service debt

\[
\frac{\tau^l}{\mu} = \Theta_H + 4 \left(1 - \beta \right) \delta_H, \quad \frac{\tau^{*l}}{\mu} = \Theta_F + 4 \left(1 - \beta \right) \delta_F,
\]

and steady-state values of all debt components

\[
\frac{B_H}{Y} = 4 \left(1 - \varrho \right) \frac{\delta_H}{\beta m_H}, \quad \frac{B^*_H}{Y} = 4 \varrho \frac{\delta_H}{\beta m_H},
\]

\[
\frac{B_F}{Y^*} = 4 \omega \varrho \frac{Y}{Y^*} \frac{\delta_F}{\beta m_F}, \quad \frac{B^*_F}{Y^*} = 4 \left(1 - \omega \varrho \frac{Y}{Y^*} \right) \frac{\delta_F}{\beta m_F}.
\]
The other variables can be determined from

\[
1 = (1 - \gamma) \frac{\Upsilon}{y} y + \gamma \Gamma \frac{c^* y^*}{y^* y} + \frac{g}{y} y
\]

\[
1 = (1 - \gamma^*) S^{-\eta} \Gamma \frac{c^*}{y^*} y + \gamma^* S^{-\eta} \Upsilon \frac{c}{y^* y} + \frac{g^*}{y^*}
\]

\[
c^{-\frac{1}{\sigma}} \Upsilon = dy^c \left( \frac{\delta}{\xi} \right)^{-\frac{1}{\sigma}}
\]

\[
c^{*^{-\frac{1}{\sigma}}} = \frac{\Gamma}{S} dy^c \left( \frac{\delta^*}{\xi^*} \right)^{-\frac{1}{\sigma}}
\]

\[
0 = \gamma S^{1-\eta} \Upsilon \frac{c}{y} - \gamma \Gamma \frac{c^*}{y} + \frac{4 \varrho (1 - \beta)}{\beta} \left[ \delta_H - \omega S \delta_F \right],
\]

where

\[
\Upsilon = \Upsilon(S) = \left( (1 - \gamma) + \gamma S^{1-\eta} \right)^{\frac{1}{1-\eta}} = \frac{P}{P_H}
\]

\[
\Gamma = \Gamma(S) = \left( (1 - \gamma^*) S^{1-\eta} + \gamma^* \right)^{\frac{1}{1-\eta}} = \frac{P^*}{P_H^*}
\]

\[
\Xi = \frac{\Gamma}{S}.
\]

Following Benigno (2009), we assume steady-state employment subsidy such that

\[
\frac{(1 - \tau^w)}{\mu} = \frac{(1 - \tau^* w)}{\mu} = 1
\]

and require the steady-state ratio of taste levels to satisfy

\[
\frac{\Gamma}{\Upsilon} \frac{c^{-\frac{1}{\sigma}}}{c^{*^{-\frac{1}{\sigma}}}} = \left( \frac{\xi^*}{\xi} \right)^{-\frac{1}{\sigma}}.
\]

The steady state becomes

\[
\frac{c}{y} = \frac{\frac{S^{-\eta} (1 - \gamma^*) (1 - \Theta_H) - \frac{y^*}{y^*} \gamma (1 - \Theta_F)}{((1 - \gamma^*) (1 - \gamma) - \gamma \gamma^*) S^{-\eta} \Upsilon \eta}}
\]
The following are several important cases:

(i) Non-zero debt-to-output ratio \((B_H + B_H^*) \neq 0, (B_F + B_F^*) \neq 0\). Financial autarky in the steady state: \(B_F = B_H^* = 0\):

\[\varrho = 0, \omega < 1.\]

Here NFA in the steady state is zero.

(ii) Non-zero debt-to-output ratio \((B_H + B_H^*) \neq 0, (B_F + B_F^*) \neq 0\). Zero NFA in the steady state but no financial autarky, \(B_F \neq 0, B_H^* \neq 0\):

\[\omega = 1, \varrho > 0.\]

(iii) Zero debt-to-output ratio in country H: \(0 = \left(\frac{\sigma}{\mu} - \Theta_H\right) = \left(\frac{B_H + B_H^*}{Y_H}, \frac{B_F + B_F^*}{Y_F}\right) \neq 0\):

\[\omega \varrho = 0.\]

It implies financial autarky in the steady state: \(B_F = B_H^* = 0\).
(iv) Zero debt-to-output ratio in each country: $0 = \left( \frac{\tau^l}{\mu} - \Theta_H \right) = \left( \frac{\tau^*}{\mu} - \Theta_F \right) = \frac{(B_H + B_H^*)}{Y} = \frac{(B_F + B_F^*)}{Y^*}$: $\omega$ and $\varrho$ are irrelevant for the calibration of the model. These parameters do not affect equations. This case implies financial autarky with $\text{NFA} = 0$ in the steady state.

**A.11 Linearized Model**

In order to tractably solve the model in the presence of potential strategic interactions between the monetary and fiscal policymakers, we recast the policy problem in a linear-quadratic (LQ) form. In doing so we employ the device of a steady-state employment subsidy which ensures that the deterministic steady-state is efficient. We design this subsidy by contrasting the social planner’s allocation with what would be achieved in our decentralized economy in the absence of nominal inertia and with perfect risk sharing, and then choosing a steady-state employment subsidy which equates the two. This ensures that the steady state of our economy is equivalent to the steady state of the flexible-price economy with risk sharing which, in turn, is efficient. This allows us to generate a valid LQ approximation to the underlying policy problem across all the types of policy we consider.\(^3\)

Log-linearization of the system about the deterministic steady state, assuming the fixed exchange rate regime, yields

\[
\begin{align*}
\hat{C}_t &= E_t \hat{C}_{t+1} + \sigma \gamma (\pi^*_{Ft+1} - \pi_{Ht+1}) - \sigma \hat{r}_t \\
\hat{C}^*_t &= E_t \hat{C}^*_{t+1} - \sigma \gamma^* (\pi^*_{Ft+1} - \pi_{Ht+1}) - \sigma \hat{r}^*_t \\
\hat{r}_t &= E_t \hat{R}_{t+1} \\
\hat{r}^*_t &= E_t \hat{R}^*_{t+1}
\end{align*}
\]

\(^3\)If we did not adopt an efficient steady state, then the second-order approximation to social welfare would include linear terms which would prevent us calculating a second-order approximation to welfare using a linearized model and would also introduce an inflationary bias to our policy problem.
\[ \pi_{Ht} = \lambda \left( \hat{\varphi}_t + \gamma \hat{S}_t + \varsigma \left( \hat{Y}_t - \hat{Z}_t \right) + \frac{1}{\sigma} \hat{C}_t + \frac{\tau^w}{\mu} \hat{\tau}_t^w - \hat{Z}_t \right) + \beta \mathbb{E}_t \pi_{Ht+1} \]  

(58)

\[ \pi^*_{Ft} = \lambda \left( \hat{\varphi}_t^* - \gamma^* \hat{S}_t + \varsigma \left( \hat{Y}_t^* - \hat{Z}_t^* \right) + \frac{1}{\sigma} \hat{C}_t^* + \frac{\tau^w}{\mu} \hat{\tau}_t^w - \hat{Z}_t^* \right) \]

+ \beta \mathbb{E}_t \pi^*_{Ft+1} \]  

(59)

\[ \hat{Y}_t = \eta \left( \frac{C}{Y} (1 - \gamma) \gamma + \frac{C^*}{Y} \gamma^* (1 - \gamma^*) \right) \hat{S}_t + (1 - \gamma) \frac{C}{Y} \hat{C}_t \]

+ \gamma^* \frac{C^*}{Y} \hat{C}_t + G^* \hat{G}_t \]  

(60)

\[ \hat{Y}^*_t = - \frac{Y}{Y^*} \eta \left( \frac{C}{Y} (1 - \gamma) \gamma + \frac{C^*}{Y} \gamma^* (1 - \gamma^*) \right) \hat{S}_t + (1 - \gamma^*) \frac{C^*}{Y^*} \hat{C}_t^* \]

+ \gamma C^* \hat{C}_t + G^* \hat{G}_t^* \]  

(61)

\[ P^M_H \frac{B_H}{Y} d_{Ht} + \frac{Y^*}{Y} P^M_H \frac{B_H^*}{Y^*} d_{Ht} = \frac{1}{\beta} \hat{R}_t \frac{P^M_H B_H + P^M_H B_H^*}{Y} \]

+ \frac{1}{\beta} P^M_H \frac{B_H}{Y} d_{Ht-1} + G^* \hat{G}_t + \frac{1}{\beta} P^M_H \frac{B_H^*}{Y} d_{Ht-1} \]

\[ - \frac{N}{Y^*} \omega^l \left( \hat{\tau}_t^l + \gamma \hat{S}_t + (\varsigma + 1) \left( \hat{Y}_t - \hat{Z}_t \right) + \frac{1}{\sigma} \hat{C}_t + \frac{\tau^w}{\mu} \hat{\tau}_t^w \right) \]  

(62)

\[ P^M_F \frac{B_F}{Y^*} d_{Ft} + P^M_F \frac{B_F^*}{Y^*} d_{Ft} = \frac{1}{\beta} \hat{R}_t \frac{P^M_F B_F + P^M_F B_F^*}{Y^*} \]

+ \frac{1}{\beta} \frac{Y}{Y^*} P^M_F \frac{B_F}{Y} d_{Ft-1} + G^* \hat{G}_t + \frac{1}{\beta} P^M_F \frac{B_F^*}{Y^*} d_{Ft-1} - \frac{N^*}{Y^*} \omega^l \tau_t^* \]  

(63)

\[ \chi^* P^M_H B_H^* \left( d^*_{Ht} - (1 - \gamma^*) \hat{S}_t \right) = - \chi P^M_F B_F \left( d_{Ft} + (1 - \gamma) \hat{S}_t \right) \]

(64)

\[ \hat{r}_t - \hat{r}_t^* = \mathbb{E}_t \left( \pi^*_{Ft+1} - \pi_{Ht+1} \right) - \chi^* (1 - \gamma^*) \chi^* \hat{S}_t + \chi^* P^M_H B_H^* d_{Ht} \]

(65)
\[ 0 = \left( \gamma C \left( (1 - \eta)(1 - \gamma) + \gamma S^{1-\eta} \right) - C^* \eta \gamma^* \left( 1 - \gamma^* \right) \right) \hat{S}_t \]
\[ + \gamma C \hat{C}_t - \gamma^* C^* \hat{C}^*_t + P^M_B \left( d_{Ft} + \hat{S}_t - \frac{1}{\beta} \left( \hat{R}^*_t + d_{Ft-1} + \hat{S}_t \right) \right) \]
\[- P^M_H B^*_H \left( d^*_{Ht} - \frac{1}{\beta} \left( \hat{R}_t + d^*_{Ht-1} \right) \right) \]
\[ (66) \]

\[ \hat{i}_t = \hat{r}_t + \mathbb{E}_t \pi_{Ht+1} \]
\[ \hat{i}^*_t = \hat{r}^*_t + \mathbb{E}_t \pi^*_{Ft+1} \]
\[ \hat{S}_t - \hat{S}_{t-1} + \pi_{Ht} - \pi^*_{Ft} = 0. \]
\[ (69) \]

Here and everywhere below, unless stated differently, for each variable \( X_t \) with non-zero steady-state value \( X \), we use the notation \( \hat{X}_t = \ln \left( \frac{X_t}{X} \right) \). Here \( \hat{B}_t = \frac{B}{\hat{B}} \hat{B}_t \), \( \hat{\pi}_t = \ln \frac{1+i_t}{1+i_{t-1}} \), and \( \pi_t = \Pi_t - 1 \). Parameter \( \lambda = \frac{(1-\theta)(1-\theta^\beta)}{\theta} \) is the slope of the Phillips curve. Finally, \( \hat{P}^M_H, \hat{P}^M_F, \hat{B}^*_H, \hat{B}^*_F, \hat{P}^M_F, \hat{B}^*_F \) can be obtained from
\[ \hat{R}_{t+1} = \beta \rho_H \hat{P}^M_{Ht+1} - \hat{P}^M_{Ht} - \pi_{Ht+1} \]
\[ \hat{R}^*_{t+1} = \beta \rho_F \hat{P}^M_{Ft+1} - \hat{P}^M_{Ft} - \pi^*_{Ft+1} \]
\[ d_{Ht} = \hat{B}_{Ht} + \hat{P}^M_{Ht} \]
\[ d^*_{Ht} = \hat{B}^*_H + \hat{P}^M_{Ht} \]
\[ d_{Ft} = \hat{P}^M_{Ft} + \hat{B}_{Ft} \]
\[ d^*_{Ft} = \hat{B}^*_F + \hat{P}^M_{Ft}. \]

Without loss of generality, we can assume that the central bank controls \( \hat{i}_t \), and then the portfolio adjustment cost implies that households face different interest rates in two economies with incomplete financial markets. There are also two independent fiscal authorities in countries H and F. Each of them controls government spending (\( \hat{G}_t \) and \( \hat{G}^*_t \)) and the income tax rate (\( \hat{\tau}_t \) and \( \hat{\tau}^*_{t} \)).

**Appendix B. Quadratic Representation of Social Welfare**

To derive the welfare objective for policy analysis, we complete the following steps. First, we consider the social planner’s problem.
We then determine the level of the steady-state subsidy required to ensure that the model’s initial steady state is socially optimal. Finally, we construct a quadratic approximation to utility in the economy with sticky prices and distortionary taxes. The representation of utility assesses the extent to which the economic variables deviate from the efficient equilibrium levels due to price stickiness and tax distortions present in our model.

B.1 Welfare Components

Worldwide social welfare is written as

\[
W = \sum_{t=0}^{\infty} \beta^t \left( n \left( \frac{\xi_{t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \frac{\varpi}{1-\frac{1}{\sigma}} g_{t}^{1-\frac{1}{\sigma}} \xi_{t}^{1-\frac{1}{\sigma}} - d n_{i}^{1+\varsigma} \delta_{t}^{1-\frac{1}{\sigma}} \right) \right) + (1-n) \left( \frac{\xi_{t}^{*1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \xi_{t}^{*1-\frac{1}{\sigma}} + \varpi \frac{\xi_{t}^{*1-\sigma}}{1-\sigma} \xi_{t}^{*1-\sigma} - d n_{i}^{*1+\varsigma} \delta_{t}^{*1-\frac{1}{\sigma}} \right) \right).
\]

Use aggregate variables

\[
W = \sum_{t=0}^{\infty} \beta^t \left( n \left( \frac{\xi_{t}}{1-\frac{1}{\sigma}} \xi_{t}^{\frac{1}{\sigma}} + \frac{\varpi}{1-\frac{1}{\sigma}} g_{t}^{1-\frac{1}{\sigma}} \xi_{t}^{1-\frac{1}{\sigma}} - d n^{1+\varsigma} \delta_{t}^{1-\frac{1}{\sigma}} \right) \right) + (1-n) \left( \frac{\xi_{t}^{*}}{1-\frac{1}{\sigma}} \xi_{t}^{*\frac{1}{\sigma}} + \varpi \frac{\xi_{t}^{*1-\sigma}}{1-\sigma} \xi_{t}^{*1-\sigma} - d n^{*1+\varsigma} \delta_{t}^{*1-\frac{1}{\sigma}} \right) \right)
\]

and substitute out

\[
N_{t} = \frac{Y_{t} \Delta_{t}}{AZ_{t}}
\]

\[
N_{t}^{*} = \frac{Y_{t}^{*} \Delta_{t}^{*}}{A^{*} Z_{t}^{*}}.
\]

Linearization yields

\[
\left( \frac{Y_{t} \Delta_{t}}{AZ_{t}} \right)^{1+\varsigma} \delta_{t}^{1-\frac{1}{\sigma}} = \frac{1}{A^{1+\varsigma}} Y_{t}^{1+\varsigma} Z_{t}^{-(1+\varsigma)} \Delta_{t}^{1+\varsigma}
\]

\[
= \left( \frac{Y}{A} \right)^{1+\varsigma} \delta^{1-\frac{1}{\sigma}} \left( 1 + (1+\varsigma) \hat{\Delta}_{t} + (1+\varsigma) \hat{Y}_{t} - (1+\varsigma)^{2} \hat{Y}_{t} \hat{Z}_{t} + \frac{1}{2} (1+\varsigma)^{2} \hat{Y}_{t}^{2} - \frac{1}{\sigma} (1+\varsigma) \hat{Y}_{t} \hat{\delta}_{t} \right) + \text{tip}
\]
so that
\[
W = \sum_{t=0}^{\infty} \beta^t \left( \frac{n_{\frac{1}{\sigma}} G^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \left( 1 + \left( 1 - \frac{1}{\sigma} \right) \hat{C}_t - \left( 1 - \frac{1}{\sigma} \right) \frac{1}{\sigma} \hat{C}_t \hat{\xi}_t \right) + \frac{1}{2} \left( 1 - \frac{1}{\sigma} \right)^2 \hat{C}_t^2 \right) \\
+ \omega n_{\frac{1}{\sigma}} G^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left( 1 + \left( 1 - \frac{1}{\sigma} \right) \hat{G}_t - \frac{1}{\sigma} \left( 1 - \frac{1}{\sigma} \right) \hat{G}_t \hat{\xi}_t \right) + \frac{1}{2} \left( 1 - \frac{1}{\sigma} \right)^2 \hat{G}_t^2 \right) \\
- \frac{dn_{-\frac{1}{\sigma}}}{T_{\frac{1}{\sigma}}} \left( \frac{Y}{A^*} \right)^{\frac{1}{\sigma}} \delta^{-\frac{1}{\sigma}} \left( 1 + (1 + \varsigma) \hat{\Delta}_t + (1 + \varsigma) \hat{Y}_t \right) - \frac{1}{\sigma} (1 + \varsigma) \hat{Y}_t \hat{\delta}_t \\
+ \frac{(1-n)^{\frac{1}{\sigma}} C^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \left( 1 + \left( 1 - \frac{1}{\sigma} \right) \hat{C}_t^* - \frac{1}{\sigma} \left( 1 - \frac{1}{\sigma} \right)^2 \hat{C}_t^* \right) \\
- \frac{1}{\sigma} (1 + \varsigma) \hat{C}_t^* \hat{\xi}_t \\
+ \omega (1-n)^{\frac{1}{\sigma}} G^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left( 1 + \left( 1 - \frac{1}{\sigma} \right) \hat{G}_t^* - \frac{1}{\sigma} \left( 1 - \frac{1}{\sigma} \right)^2 \hat{G}_t^* \right) \\
- \frac{1}{\sigma} (1 + \varsigma) \hat{G}_t^* \hat{\xi}_t \\
- \frac{d(1-n)^{-\frac{1}{\sigma}}}{T_{\frac{1}{\sigma}}} \left( \frac{Y^*}{A^*} \right)^{\frac{1}{\sigma}} \delta^{-\frac{1}{\sigma}} \left( 1 + (1 + \varsigma) \hat{\Delta}_t^* + (1 + \varsigma) \hat{Y}_t^* \right) \\
- \frac{1}{\sigma} (1 + \varsigma) \hat{Y}_t^* \hat{\delta}_t \\
\right) \\
= \sum_{t=0}^{\infty} \beta^t \left( WQ_t + WL_t + WP_t \right),
\]

where

\[
WL_t = n_{\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left( C^{1-\frac{1}{\sigma}} \hat{C}_t + \omega G^{1-\frac{1}{\sigma}} \hat{G}_t \right) - dn_{-\frac{1}{\sigma}} \left( \frac{Y}{A} \right)^{\frac{1}{\sigma}} \delta^{-\frac{1}{\sigma}} \hat{Y}_t \\
+ \frac{1}{\sigma} \left( 1 - n \right)^{\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left( C^{1-\frac{1}{\sigma}} \hat{C}_t^* + \omega G^{1-\frac{1}{\sigma}} \hat{G}_t^* \right) \\
- \frac{d(1-n)^{-\frac{1}{\sigma}}}{T_{\frac{1}{\sigma}}} \left( \frac{Y^*}{A^*} \right)^{\frac{1}{\sigma}} \delta^{-\frac{1}{\sigma}} \hat{Y}_t^* 
\]

\[
WQ_t = n_{\frac{1}{\sigma}} C^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left( \frac{1}{\sigma} \left( 1 - \frac{1}{\sigma} \right) \hat{C}_t^2 - \frac{1}{\sigma} \hat{C}_t \hat{\xi}_t \right) \\
+ \omega n_{\frac{1}{\sigma}} G^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left( \frac{1}{\sigma} \left( 1 - \frac{1}{\sigma} \right) \hat{G}_t^2 - \frac{1}{\sigma} \hat{G}_t \hat{\xi}_t \right) \\
- \frac{dn_{-\frac{1}{\sigma}}}{T_{\frac{1}{\sigma}}} \left( \frac{Y}{A^*} \right)^{\frac{1}{\sigma}} \delta^{-\frac{1}{\sigma}} \left( - (1 + \varsigma) \hat{Y}_t \hat{\delta}_t + \frac{1}{\sigma} (1 + \varsigma) \hat{Y}_t^2 - \frac{1}{\sigma} \hat{Y}_t \hat{\delta}_t \right) \\
+ \frac{1}{\sigma} \left( 1 - n \right)^{\frac{1}{\sigma}} C^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left( \frac{1}{\sigma} \left( 1 - \frac{1}{\sigma} \right) \hat{C}_t^2 - \frac{1}{\sigma} \hat{C}_t \hat{\xi}_t \right) \\
+ \omega \left( 1 - n \right)^{\frac{1}{\sigma}} G^{1-\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left( \frac{1}{\sigma} \left( 1 - \frac{1}{\sigma} \right) \hat{G}_t^2 - \frac{1}{\sigma} \hat{G}_t \hat{\xi}_t \right) \\
- \frac{d(1-n)^{-\frac{1}{\sigma}}}{T_{\frac{1}{\sigma}}} \left( \frac{Y^*}{A^*} \right)^{\frac{1}{\sigma}} \delta^{-\frac{1}{\sigma}} \left( - (1 + \varsigma) \hat{Y}_t \hat{\delta}_t \right. \\
+ \frac{1}{\sigma} (1 + \varsigma) \hat{Y}_t^2 - \frac{1}{\sigma} \hat{Y}_t \hat{\delta}_t \right) 
\]
\[ WP_t = -dn^{-\xi} \left( \frac{Y}{A} \right)^{1+\xi} \xi^{-\frac{1}{2}} \hat{\Delta}_t - d(1-n)^{-\xi} \left( \frac{Y^*}{A^*} \right)^{1+\xi} \xi^{\frac{1}{2}} \hat{\Delta}_t^* \]

are linear and quadratic components of welfare. We separate price-dispersion components.

Now, the aim is to transform \( WL_t \) using second-order approximation of the system of constraints.

We also use second-order approximation to the two market clearing conditions,

\[
Y_t = (1 - \gamma) \left( (1 - \gamma) + \gamma S_t^{1-\eta} \right)^{\frac{\eta}{1-\eta}} C_t
\]
\[
+ \gamma^* \left( (1 - \gamma^*) S_t^{1-\eta} + \gamma^* \right)^{\frac{\eta}{1-\eta}} C_t^* + G_t
\]
\[
Y_t^* = (1 - \gamma^*) S_t^\eta \left( (1 - \gamma^*) S_t^{1-\eta} + \gamma^* \right)^{\frac{\eta}{1-\eta}} C_t^*
\]
\[
+ \gamma S_t^{-\eta} \left( (1 - \gamma) + \gamma S_t^{1-\eta} \right)^{\frac{\eta}{1-\eta}} C_t + G_t^*,
\]

which read

\[
\hat{Y}_t = HL_t + HQ_t
\]
\[
\hat{Y}_t^* = FL_t + FQ_t,
\]

where

\[
HL_t = (1 - \gamma) \frac{C}{Y} Y^\eta \left( \hat{C}_t + \eta \gamma \left( \frac{S}{Y} \right)^{1-\eta} \hat{S}_t \right)
\]
\[
+ \gamma^* S^\eta \frac{C^* Y^*}{Y} \left( \hat{C}_t^* + \eta \frac{(1 - \gamma^*)}{\Xi^{1-\eta}} \hat{S}_t \right) + G \frac{C}{Y} \hat{G}_t
\]
\[
HQ_t = (1 - \gamma) \frac{C}{Y} Y^\eta \left( \frac{1}{2} \hat{C}_t^2 + \eta \gamma \left( \frac{S}{Y} \right)^{1-\eta} \hat{S}_t \hat{C}_t \right)
\]
\[
+ \frac{\eta \gamma}{2} \left( \frac{(1 - \eta)(1 - \gamma)}{Y^{1-\eta}} + \eta \gamma \left( \frac{S}{Y} \right)^{1-\eta} \left( \frac{S^*}{Y^*} \right)^{1-\eta} \hat{S}_t^2 \hat{C}_t^* \right)
\]
\[
+ \gamma^* S^\eta \frac{C^* Y^*}{Y} \left( \frac{1}{2} \hat{C}_t^{*2} + \eta \frac{(1 - \gamma^*)}{\Xi^{1-\eta}} \hat{S}_t \hat{C}_t^* \right)
\]
\[
\begin{align*}
&\quad + \frac{\eta (1 - \gamma^*)}{2} \left( \frac{(1 - \eta) \gamma^*}{\Xi^{1-\eta}} + \frac{\eta (1 - \gamma^*)}{\Xi^{1-\eta}} \right) \hat{S}_t^2 \\
&\quad + \frac{1}{2} \frac{G}{\bar{Y}} \hat{G}_t^2 - \frac{1}{2} \hat{Y}_t^2 \\
&FL_t = (1 - \gamma^*) \Xi \eta \frac{C^*}{Y^*} \left( \hat{C}_t^* - \eta \frac{\gamma^*}{\Xi^{1-\eta}} \hat{S}_t \right) \\
&\quad + \gamma S^{-\eta} Y \eta \frac{C}{\bar{Y} Y^*} \left( \hat{C}_t - \eta \gamma \frac{1 - \gamma}{\bar{Y}^{1-\eta}} \hat{S}_t \right) + \frac{G^*}{\bar{Y}^*} \hat{G}_t^* \\
&FQ_t = (1 - \gamma^*) \Xi \eta \frac{C^*}{Y^*} \left( \frac{1}{2} \hat{C}_t^2 - \eta \frac{\gamma^*}{\Xi^{1-\eta}} \hat{S}_t \hat{C}_t^* \\
&\quad + \frac{\eta \gamma^*}{2} \left( \frac{(1 - \eta) (1 - \gamma^*)}{\Xi^{1-\eta}} + \frac{\eta \gamma^*}{\Xi^{1-\eta}} \right) \frac{1}{\Xi^{1-\eta}} \hat{S}_t^2 \right) \\
&\quad + \gamma S^{-\eta} Y \eta \frac{C}{\bar{Y} Y^*} \left( \frac{1}{2} \hat{C}_t^2 - \eta \gamma \frac{1 - \gamma}{\bar{Y}^{1-\eta}} \hat{S}_t \hat{C}_t \\
&\quad + \frac{\eta (1 - \gamma)}{2 \bar{Y}^{1-\eta}} \left( 1 - \eta \gamma \left( \frac{S}{\bar{Y}} \right)^{1-\eta} + \eta (1 - \gamma) \right) \hat{S}_t^2 \right) \\
&\quad + \frac{G^*}{\bar{Y}^*} \frac{1}{2} \hat{G}_t^2 - \frac{1}{2} \hat{Y}_t^2.
\end{align*}
\]

It follows that

\[
WL_t = n \frac{1}{\pi} \xi^{-\frac{1}{\pi}} \left( C^{1-\frac{1}{\pi}} \hat{C}_t + \omega G^{1-\frac{1}{\pi}} \hat{G}_t \right) \\
- d\delta^{-\frac{1}{\pi}} n^{-\varsigma} \left( \frac{Y}{A} \right)^{1+\varsigma} HL_t \\
+ (1 - n)^{\frac{1}{\pi}} \xi^{*-\frac{1}{\pi}} \left( C^{*-\frac{1}{\pi}} \hat{C}_t + \omega^* G^{*-\frac{1}{\pi}} \hat{G}_t \right) \\
- d\delta^{*-\frac{1}{\pi}} (1 - n)^{-\varsigma} \left( \frac{Y^*}{A^*} \right)^{1+\varsigma} FL_t \\
- dn^{-\varsigma} \delta^{-\frac{1}{\pi}} \left( \frac{Y}{A} \right)^{1+\varsigma} HQ_t - d (1 - n)^{-\varsigma} \delta^{*-\frac{1}{\pi}} \left( \frac{Y^*}{A^*} \right)^{1+\varsigma} FQ_t
\]
$$= A_{H,t} + A_{F,t} - dn^{-\varsigma} \delta^{-\frac{1}{\sigma}} \left( \frac{Y}{A} \right)^{1+\varsigma} HQ_t$$

$$- d (1 - n)^{-\varsigma} \delta^{*^{-\frac{1}{\sigma}}} \left( \frac{Y^*}{A^*} \right)^{1+\varsigma} FQ_t,$$

where linear terms

$$A_{H,t} = n^{\frac{1}{\sigma}} \xi^{-\frac{1}{\sigma}} \left( C^{1-\frac{1}{\sigma}} \hat{C}_t + \varpi G^{1-\frac{1}{\sigma}} \hat{G}_t \right)$$

$$- d \delta^{-\frac{1}{\sigma}} n^{-\varsigma} \left( \frac{Y}{A} \right)^{1+\varsigma} \left( 1 - \gamma \right) \frac{C}{Y} \Upsilon^\eta \left( \hat{C}_t + \eta \gamma \left( S^1 - \eta \hat{S}_t \right) \right)$$

$$\gamma^* S^\eta \Xi^\eta Y^* C Y \left( \hat{C}_t + \eta \gamma \left( \hat{S}_t \right) \right)$$

$$A_{F,t} = (1 - n)^{\frac{1}{\sigma}} \xi^{\star^{-\frac{1}{\sigma}}} \left( C^{1-\frac{1}{\sigma}} \hat{C}_t + \varpi G^{1-\sigma} \hat{G}_t \right)$$

$$- d \delta^{\star^{-\frac{1}{\sigma}}} (1 - n)^{-\varsigma} \left( \frac{Y^*}{A^*} \right)^{1+\varsigma}$$

$$\times \left( 1 - \gamma^* \right) \Xi^\eta \Upsilon^* C^* Y^* \left( \hat{C}_t - \eta \gamma^* \left( \hat{S}_t \right) \right)$$

$$\gamma^* S^\eta \Upsilon^\eta C Y \left( \hat{C}_t - \eta \gamma^* \left( \hat{S}_t \right) \right)$$

$$+ \gamma S^\eta \Upsilon^\eta C Y \left( \hat{C}_t - \eta \gamma^* \left( \hat{S}_t \right) \right)$$

and their sum

$$A_{H,t} + A_{F,t} = C \left( \xi^{-\frac{1}{\sigma}} c^{-\frac{1}{\sigma}} - \xi^{-\frac{1}{\sigma}} c^{-\frac{1}{\sigma}} \left( 1 - \tau^w \right) \mu \right) \left( 1 - \gamma \right) \Upsilon^{\eta-1}$$

$$- \xi^* \left( \frac{c^*}{\mu} \left( 1 - \gamma^* \right) \Upsilon^\eta \right) \hat{C}_t$$

$$+ C^* \left( \xi^* \left( \frac{c^*}{\mu} \left( 1 - \gamma^* \right) \Upsilon^\eta \Xi \right) \hat{C}_t^*$$

$$- \xi^* \left( \frac{c^*}{\Xi \mu} \left( 1 - \gamma^* \right) \Xi^\eta \right) \hat{C}_t^*$$
+ G\xi^{-\frac{1}{\sigma}} \left( \varpi g^{-\frac{1}{\sigma}} - \frac{c^{-\frac{1}{\sigma}}}{\Upsilon \mu} (1 - \tau^w) \right) \hat{G}_t \\
+ \left( \xi^* - \frac{1}{\sigma} c^* \frac{1}{\mu} \frac{c - \frac{1}{\sigma} (1 - \gamma^*) \Xi \eta^{1-\eta} \gamma^* C^*}{\Upsilon \mu} \right) + \xi^* - \frac{1}{\sigma} c^* \frac{1}{\mu} \frac{(1 - \gamma^*) \Xi S \eta^{1-\eta} \gamma (\frac{\xi}{\Xi})^1 - \eta C}{\Upsilon \mu} \\
+ \left( \xi^* - \frac{1}{\sigma} c^* \frac{1}{\mu} \frac{(1 - \gamma^*) \Xi S \eta^{1-\eta} \gamma (\frac{\xi}{\Xi})^1 - \eta C^*}{\Upsilon \mu} \right) + \left( \xi^* - \frac{1}{\sigma} c^* \frac{1}{\mu} \frac{(1 - \tau^w) \Xi S \eta^{1-\eta} \gamma (\frac{\xi}{\Xi})^1 - \eta C^*}{\Upsilon \mu} \right) \hat{G}_t^* \\

If \\
\frac{S \Xi}{\Upsilon} \xi^{-\frac{1}{\sigma}} c^{-\frac{1}{\sigma}} = \xi^* - \frac{1}{\sigma} c^* - \frac{1}{\sigma} \\
\frac{(1 - \tau^w)}{\mu} = \frac{(1 - \tau^w)}{\mu} = 1 \\
\varpi g^{-\frac{1}{\sigma}} = \frac{c^{-\frac{1}{\sigma}}}{\Upsilon} \\
\varpi^* g^{*-\sigma} = \frac{c^* - \frac{1}{\sigma}}{\Xi},

then \\
A_{H,t} + A_{F,t} = 0.

This determines the subsidy.

**B.2 Quadratic Representation**

\[ W = \sum_{t=0}^{\infty} \beta^t (WQ_t + WL_t + WP_t) \]

Note that 
\[ \sum_{t=0}^{\infty} \beta^t WP_t = -d \sum_{t=0}^{\infty} \beta^t \left( n^{-\varsigma} \left( \frac{Y}{A} \right)^{1+\varsigma} \xi^{-\frac{1}{\sigma}} \hat{\Delta}_t \right) + (1 - n)^{-\varsigma} \left( \frac{Y^*}{A^*} \right)^{1+\varsigma} \xi^{* - \frac{1}{\sigma}} \hat{\Delta}_t^* \]
\[
\begin{aligned}
&= -\frac{1}{2} \left(\frac{\theta \epsilon}{(1 - \theta) (1 - \beta \theta)}\right) d \sum_{t=0}^{\infty} \beta^t \left( n \left( \frac{y}{A} \right)^{1+\varsigma} \xi^{-\frac{1}{\sigma}} \right) \\
&\quad + (1 - \epsilon) \left( \frac{(y^*)^{1+\varsigma}}{(A^*)^{1+\varsigma}} \xi^{*^{-\frac{1}{\sigma}}} \right).
\end{aligned}
\]

Therefore, quadratic approximation to the household’s utility can be written as

\[
W = \sum_{t=0}^{\infty} \beta^t \left( n \xi_t^{-\frac{1}{\sigma}} \left( \frac{c_t^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \frac{\varpi}{1-\frac{1}{\sigma}} g_t^{1-\frac{1}{\sigma}} - d_{1+\varsigma} \right) \\
+ (1 - \epsilon) \xi_t^{*^{-\frac{1}{\sigma}}} \left( \frac{c_t^{*1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} + \varpi g_t^{*1-\sigma} - d_{1+\varsigma} \right) \right) \\
= -\frac{1}{2} \xi^{-\frac{1}{\sigma}} \left( \frac{y}{A} \right)^{\varsigma} d \frac{Y \epsilon}{A \lambda} \sum_{t=0}^{\infty} \beta^t V^H_t \\
- \xi^{*^{-\frac{1}{\sigma}}} \frac{1}{2} \left( \frac{y^*}{A^*} \right)^{\varsigma} d \left( \frac{SY^*}{A^*} \right) \frac{\epsilon}{\lambda} \sum_{t=0}^{\infty} \beta^t V^F_t + \text{tip} + O(2),
\]

where where \(\text{tip}\) denotes “terms independent of policy” and \(O(2)\) captures terms of order higher than two in the approximation of social welfare, and

\[
V^H_t = \hat{\Pi}^2_{Ht} + \frac{\lambda C}{\epsilon Y} \left( \frac{1}{\sigma} - 1 \right) \left( \hat{C}_t + \frac{1}{(1 - \sigma)} \hat{\xi}_t \right)^2 + \frac{\lambda G}{\epsilon \sigma Y} \left( \hat{G}_t + \hat{\xi}_t \right)^2 \\
+ \frac{\varsigma \lambda}{\epsilon} \left( \hat{Y}_t - \left( \frac{1}{\varsigma} \hat{Z}_t + \frac{1}{1-\frac{1}{\sigma}} \hat{\xi}_t \right) \right)^2 \\
+ \frac{\lambda}{\epsilon} (1 - \gamma) \frac{C}{Y} \left( \hat{C}_t + \gamma \hat{S}_t \right)^2 + \frac{\lambda}{\epsilon} \frac{C}{Y} \left( \hat{C}^*_t + \eta (1 - \gamma^*) \hat{S}_t \right)^2 \\
+ \frac{\lambda C}{\epsilon Y} \eta (1 - \eta) \gamma \left( (1 - \gamma)^2 + (1 - \gamma^*) \gamma^* \right) \hat{S}_t^2 \\
V^F_t = \hat{\Pi}^2_{Ft} + \frac{\lambda (1 - \vartheta)}{\epsilon} \left( \frac{1}{\sigma} - 1 \right) \left( \hat{C}^*_t + \frac{1}{(1 - \sigma)} \hat{\xi}^*_t \right)^2 \\
+ \frac{\lambda \vartheta}{\epsilon \sigma} \left( \hat{G}^*_t + \hat{\xi}^*_t \right)^2 + \frac{\varsigma \lambda}{\epsilon} \left( \hat{Y}^*_t - \left( \frac{1}{\varsigma} \hat{Z}^*_t + \frac{1}{1-\frac{1}{\sigma}} \hat{\xi}^*_t \right) \right)^2 \\
+ \frac{\lambda}{\epsilon} (1 - \gamma^*) (1 - \vartheta) \left( \hat{C}^*_t - \eta \gamma^* \hat{S}_t \right)^2
\]
\[ \begin{align*} &+ \frac{\lambda}{\epsilon} \gamma^* (1 - \vartheta) \left( \hat{C}_t - \eta (1 - \gamma) \hat{S}_t \right)^2 \\ &+ \frac{\lambda}{\epsilon} (1 - \vartheta) \eta (1 - \eta) \gamma^* (1 - \gamma^*)^2 + \gamma (1 - \gamma) \hat{S}_t^2, \end{align*} \]

where

\[
\lambda = \frac{(1 - \theta) (1 - \beta \theta)}{\theta}.
\]

Parameters are chosen to guarantee \( S = 1 \) in the steady state. In the main text all taste shocks are assumed to be zero.
Table C1. Selected Statistics for EMU Member Countries

<table>
<thead>
<tr>
<th></th>
<th>Employment¹ (% of Total)</th>
<th>Government Expenditures² (% of GDP)</th>
<th>Import of Goods and Services³ (% of GDP)</th>
<th>Net Government Debt⁴ (% of GDP)</th>
<th>Average Term to Maturity⁴ (Years)</th>
<th>Government Debt Held by Non-residents (% of Debt)</th>
<th>GDP Growth Rate¹ (%)</th>
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Sources:
¹OECD, data series ETONC.
²OECD, current government expenditures as share of GDP, data series GP132R and B1_GA.
³World Bank; see https://data.worldbank.org/indicator/NE.IMP.GNFS.ZS.
⁴IMF Fiscal Monitor October 2013, table 12b.
Table C2. Portfolio Investment Assets by Economy of Non-resident Issuer (top row),
Long-Term Debt Securities, 2011, Mln. U.S. Dollars

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Source: IMF Coordinated Portfolio Investment Survey and authors’ calculations.
Table C3. Portfolio Investment Assets by Group of Non-resident Issuer (top row), Long-Term Debt Securities, 2011, Mln. U.S. Dollars

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Source: IMF Coordinated Portfolio Investment Survey and authors’ calculations.

Appendix D. Discretion with Leadership

To illustrate the type of problems we are facing, in this appendix we consider non-cooperative discretionary interactions of three policymakers, labeled A, B, and C, with intraperiod leadership of one of them, but where the other two act simultaneously. Specifically, we assume that policymaker A moves first, then policymakers B and C make decisions simultaneously, and then the private sector reacts. The general principles illustrated here can be used to solve any leadership problem, provided equilibrium exists. The section is based on Oudiz and Sachs (1985), Backus and Driffill (1986), and Currie and Levine (1993), and their approach is used in, e.g., Le Roux and Kirsanova (2013) and here it is a natural extension to a three-agents setting.

We solve the system in a state-space form:

\[
\begin{bmatrix}
Y_{t+1} \\
H^E_t X_{t+1}
\end{bmatrix} = A \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + B \begin{bmatrix} U^B_t \\ U^C_t \end{bmatrix} + DU^A_t + C\varepsilon_{t+1},
\]

(70)

where \(Y_t\) is an \(n_1\)-vector of predetermined state variables, \(Y_0\) is given, and \(X_t\) are the effective instruments of the private sector, an \(n_2\)-vector of non-predetermined or jump variables \((n = n_1 + n_2)\). The policy instruments are represented by \(U^A_t\), \(U^B_t\), and \(U^C_t\). \(\varepsilon_{t+1}\) is an \(n_\varepsilon\)-vector of exogenous zero-mean iid shocks. Matrices A, B, C, and D are partitioned conformably with \(Y_t\) and \(X_t\) as

\[
A \equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix};
B \equiv \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix};
D \equiv \begin{bmatrix} D_1 \\ D_2 \end{bmatrix};
C \equiv \begin{bmatrix} C_1 \\ \textbf{O} \end{bmatrix}.
\]
This system describes the evolution of the economy as observed by policymakers.

D.1 Private Sector’s Reaction Function

The forward-looking variables must be linear functions of the state variables,

\[ X_t = -N_t Y_t. \]

Take this one step forward, substitute into the upper block of (70), and get

\[ \mathbb{E}_t X_{t+1} = -N_{t+1} \left[ A_{11} Y_t + A_{12} X_t + B_{11} U_t^B + B_{12} U_t^C + D_1 U_t^A \right] \]

while the lower block of (70) is

\[ H \mathbb{E}_t X_{t+1} = A_{21} Y_t + A_{22} X_t + B_{21} U_t^B + B_{22} U_t^C + D_2 U_t^A. \]

Multiplying the former equation by \( H \), setting the result equal to the latter equation, and solving for \( X_t \), we obtain

\[
X_t = -\left( A_{22} + H N_{t+1} A_{12} \right)^{-1} (A_{21} + H N_{t+1} A_{11}) Y_t \\
- \left( A_{22} + H N_{t+1} A_{12} \right)^{-1} (B_{21} + H N_{t+1} B_{11}) U_t^B \\
- \left( A_{22} + H N_{t+1} A_{12} \right)^{-1} (B_{22} + H N_{t+1} B_{12}) U_t^C \\
- \left( A_{22} + H N_{t+1} A_{12} \right)^{-1} (D_2 + H N_{t+1} D_1) U_t^A \\
= -J_t Y_t - K_t^A U_t^A - K_t^B U_t^B - K_t^C U_t^C, \tag{71}
\]

where \( J_t \) is \( n_2 \times n_1 \), \( K_t^A \) is \( n_2 \times k_A \), \( K_t^B \) is \( n_2 \times k_B \), and \( K_t^C \) is \( n_2 \times k_C \) \((k_A, k_B, and k_C \) stand, respectively, for the number of fiscal policy instruments of policymakers A, B, and C, respectively.\(^4\)

\(^4\)It is assumed that \( A_{22} + H N_{t+1} A_{12} \) is invertible.
We use (71) in the first \( n_1 \) equations in the system (70) to get the reduced-form evolution of the predetermined variables

\[
Y_{t+1} = \left[ A_{11} - A_{12} J_t \right] Y_t + \left[ B_{11} - A_{12} K_t^B \right] U_t^B
\]

\[
+ \left[ B_{12} - A_{12} K_t^C \right] U_t^C + \left[ D_1 - B_{12} L_t^A \right] U_t^A + C_1 \varepsilon_{t+1}
\]

\[
= O_{Y_t} Y_t + O_{A_t} U_t^A + O_{B_t} U_t^B + O_{C_t} U_t^C + C_1 \varepsilon_{t+1}.
\] (72)

Being a follower, authority B observes authority A’s actions and reacts to them. In a linear-quadratic setup, the optimal solution belongs to the class of linear feedback rules of the form

\[
U_t^B = -F_t^B Y_t - L_t^B U_t^A,
\] (73)

where \( F_t^B \) denotes feedback coefficients on the predetermined state variables and \( L_t^B \) is the leadership parameter. The other fiscal authority solves a similar problem and gets

\[
U_t^C = -F_t^C Y_t - L_t^C U_t^A.
\] (74)

Moving simultaneously, authorities B and C do not respond to each other’s actions.

The monetary leadership authority takes into account these fiscal policy reaction functions as well as the private sector’s optimal conditions when solving its optimization problem. Thus, the leader can manipulate the follower by changing its policy instrument. The monetary leadership reaction function takes the form of

\[
U_t^A = -F_t^A Y_t.
\] (75)

**D.2 The Followers’ Optimization Problem**

In the discretionary case, the three policymakers reoptimize every period by taking the process by which private agents form their expectations as given—and where the expectations are consistent with actual policies. Policymakers B and C minimize their loss functions treating policy instrument A parametrically but incorporating
the reaction functions of the private sectors. Policymaker B has the following objective function:

$$
\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left( G_t^B Q^A G_t^B \right) = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left( Z_t^B Q^B Z_t + 2Z_t^B P^B U_t + U_t^B R^B U_t \right), \quad (76)
$$

where $G_t^B$ is the target variables for authority B while $Q^B$ is the corresponding matrix of weights. The target variables can be rewritten in terms of the predetermined and non-predetermined state variables collected on vector $Z_t$, in terms of the policy instruments collected in $U_t = [U_t^A, U_t^B, U_t^C]'$ and their cross-terms.

Authority B optimizes every period, taking into account that she will be able to reoptimize next period. The model is linear quadratic, thus the solution in $t+1$ gives a period return which is quadratic in the state variables, $W_{t+1}^B \equiv Y_{t+1}' S_{t+1}^B Y_{t+1} + w_{t+1}^B$, where $S_{t+1}^B$ is a positive semidefinite matrix and $w_{t+1}^B$ is a scalar independent of $Y_{t+1}$.

Hence, the value function of authority B in $t$ will then satisfy the Bellman equation:

$$
W_t^B = \min_{U_t^B} \frac{1}{2} \left( (Z_t^B Q^B Z_t + Z_t^B P^B U_t + U_t^B R^B U_t) \right) + \beta E_t \left( W_{t+1}^B \right) \quad (77)
$$

s.t. $E_t X_{t+1} = -N_{t+1} E_t Y_{t+1}$,

$$
W_{t+1}^B \equiv Y_{t+1}' S_{t+1}^B Y_{t+1} + w_{t+1}^B,
$$

and $Y_t$ given.

Therefore we can substitute equations (71) and (72) into (77) to obtain an equivalent minimization problem$^5$:

$$
2\tilde{W}_t^B \equiv \min_{U_t^B} \left\{ Y_t' \left[ Q^B + \beta O_{t+1}^B S_{t+1}^B O_{t+1}^B \right] Y_t + U_t^B \left[ U_t^B \left[ S_{t+1}^B + \beta O_{t+1}^B S_{t+1}^B O_{t+1}^B \right] + U_t^C \left[ S_{t+1}^C + \beta O_{t+1}^C S_{t+1}^C O_{t+1}^C \right] \right] \right\} Y_t
$$

$^5$We have made use of the fact that $w_{t+1}^B$ is independent of $Y_{t+1}$ and $E_t \varepsilon_{t+1} = 0$. 
\begin{align}
+ Y_t \left[ U_{C}^{S,B} + \beta O_{Y} S_{B}^{t+1} O_{Y_t} \right] U_{C}^{A} + U_{A}^{B} \left[ U_{A}^{S,B'} + \beta O_{A}^{t} S_{B}^{t+1} O_{Y_t} \right] Y_t \\
+ Y_t \left[ U_{A}^{S,B} + \beta O_{Y} S_{B}^{t+1} O_{A_t} \right] U_{A}^{B} + U_{B}^{A} \left[ R_{B}^{S,B} + \beta O_{B}^{t} S_{B}^{t+1} O_{B_t} \right] U_{B}^{B} \\
+ U_{C}^{A} \left[ R_{B}^{S,B} + \beta O_{C} S_{B}^{t+1} O_{C_t} \right] U_{C}^{C} + U_{B}^{A} \left[ R_{A}^{S,B} + \beta O_{A}^{t} S_{B}^{t+1} O_{A_t} \right] U_{B}^{A} \\
+ U_{C}^{B} \left[ P_{C A}^{S,B} + \beta O_{C} S_{B}^{t+1} O_{C_t} \right] U_{C}^{A} + U_{A}^{B} \left[ P_{C A}^{S,B'} + \beta O_{A}^{t} S_{B}^{t+1} O_{C_t} \right] U_{C}^{C} \right),
\end{align}

where

\[ Q_{B}^{S} = Q_{11}^{B} - J_{t}^{'} Q_{21}^{B} - Q_{12}^{B} J_{t} + J_{t}^{'} Q_{22}^{B} J_{t}, \]

\[ U_{B}^{S,B} = J_{t}^{'} Q_{22}^{B} K_{t}^{B} - Q_{12}^{B} K_{t}^{B} + \beta O_{B}^{t} P_{22}^{B}, \]

\[ U_{F}^{S,B} = J_{t}^{'} Q_{22}^{B} K_{t}^{A} - Q_{12}^{B} K_{t}^{A} + \beta O_{F}^{t} P_{23}^{B}, \]

\[ U_{A}^{S,B} = J_{t}^{'} Q_{22}^{B} K_{t}^{A} - Q_{12}^{B} K_{t}^{A} + \beta O_{A}^{t} P_{21}^{B}, \]

\[ R_{B}^{S,B} = K_{t}^{B} Q_{22}^{B} K_{t}^{B} - K_{t}^{B} P_{22}^{B} - P_{22}^{B} K_{t}^{B} + \beta O_{B}^{t} P_{22}^{B}, \]

\[ R_{C}^{S,B} = K_{t}^{C} Q_{22}^{B} K_{t}^{C} - K_{t}^{C} P_{23}^{B} - P_{23}^{B} K_{t}^{C} + \beta O_{C}^{t} P_{23}^{B}, \]

\[ R_{A}^{S,B} = K_{t}^{A} Q_{22}^{B} K_{t}^{A} - K_{t}^{A} P_{21}^{B} - P_{21}^{B} K_{t}^{A} + \beta O_{A}^{t} P_{21}^{B}, \]

\[ P_{B C}^{S,B} = K_{t}^{B} Q_{22}^{B} K_{t}^{C} - K_{t}^{B} P_{23}^{B} - P_{23}^{B} K_{t}^{C} + \beta O_{B}^{t} P_{23}^{B}, \]

\[ P_{B A}^{S,B} = K_{t}^{B} Q_{22}^{B} K_{t}^{A} - K_{t}^{B} P_{21}^{B} - P_{21}^{B} K_{t}^{A} + \beta O_{B}^{t} P_{21}^{B}, \]

\[ P_{C A}^{S,B} = K_{t}^{C} Q_{22}^{B} K_{t}^{A} - K_{t}^{C} P_{21}^{B} - P_{21}^{B} K_{t}^{A} + \beta O_{C}^{t} P_{21}^{B}. \]

Hence, the problem faced by authority B has been transformed to a standard linear-quadratic regulator problem without forward-looking variables but with time-varying parameters. The first-order condition is

\[ 0 = \left[ U_{B}^{S,B'} + \beta O_{B}^{t} S_{B}^{t+1} O_{Y_t} \right] Y_t + \left[ R_{B}^{S,B} + \beta O_{B}^{t} S_{B}^{t+1} O_{B_t} \right] U_{B}^{B} \]

\[ + \left[ P_{B C}^{S,B} + \beta O_{B}^{t} S_{B}^{t+1} O_{C_t} \right] U_{C}^{C} + \left[ P_{B A}^{S,B} + \beta O_{B}^{t} S_{B}^{t+1} O_{A_t} \right] U_{B}^{A}. \]

Since \( U_{B}^{B} = -F_{t}^{B} Y_t - L_{t}^{B} U_{t}^{A} \) and \( U_{B}^{C} = -F_{t}^{C} Y_t - L_{t}^{C} U_{t}^{A} \), the first-order condition can be solved for the feedback coefficients of the reaction function of authority B:
where the system under control evolves as described by Oudiz and Sachs (1985) and Backus and Driffill (1986) This part of the problem is the standard optimization problem

\[ D.3 \text{ The Leader’s Optimization Problem} \]

This part of the problem is the standard optimization problem described by Oudiz and Sachs (1985) and Backus and Driffill (1986) where the system under control evolves as

\[
F_t^B \equiv \left[ R_{Bt}^{S,B} + \beta O_{Bt}^C S_{Bt}^{t+1} O_{Bt} \right]^{-1} \left\{ \begin{array}{l}
[ U_{Bt}^{S,B'} + \beta O_{Bt}^C S_{Bt}^{t+1} O_{Yt} ] \\
- \left[ P_{BC}^{S,B} + \beta O_{Bt}^C S_{Bt}^{t+1} O_{Ct} \right] F_t^C 
\end{array} \right.
\]

\[
L_t^B \equiv \left[ R_{Bt}^{S,B} + \beta O_{Bt}^C S_{Bt}^{t+1} O_{Bt} \right]^{-1} \left\{ \begin{array}{l}
[ P_{BM}^{S,B} + \beta O_{Bt}^C S_{Bt}^{t+1} O_{Mt} ] \\
- \left[ P_{BC}^{S,B} + \beta O_{Bt}^C S_{Bt}^{t+1} O_{Ct} \right] L_t^C. 
\end{array} \right.
\]

Substituting the decision rules (73), (74), and (75) into (78), we obtain the recursive equations for

\[
S_t^B \equiv T_{0,t}^B + \beta T_{t}^B S_{Bt}^{t+1} T_{t}^B
\]

\[
T_{0,t}^B = Q_{Bt}^S - U_{Bt}^{S,B} (F_t^B - L_t^B F_t^A) - (F_t^B - L_t^B F_t^A)' U_{Bt}^{S,B'}
- U_{Ct}^{S,B} (F_t^C - L_t^C F_t^A) - (F_t^C - L_t^C F_t^A)' U_{Ct}^{S,B'} - U_{A,B}^S F_t^A
- F_t^C U_{A'^B}^S (F_t^B - L_t^B F_t^A) + (F_t^C - L_t^C F_t^A)' R_{Ct}^{S,B} (F_t^B - L_t^B F_t^A)
+ (F_t^C - L_t^C F_t^A)' R_{Bt}^{S,B} (F_t^B - L_t^B F_t^A)
+ (F_t^C - L_t^C F_t^A)' P_{BC}^{S,B} (F_t^B - L_t^B F_t^A)
+ (F_t^B - L_t^B F_t^A)' P_{BC}^{S,B'} (F_t^B - L_t^B F_t^A)
\]

and

\[
T_{t}^B = O_{Yt} - O_{Bt} (F_t^B - L_t^B F_t^A) - O_{Ct} (F_t^C - L_t^C F_t^A) - O_{A,t} F_t^A.
\]

Similar formulas can be derived for authority C.
\[
\begin{bmatrix}
Y_{t+1} \\
H\mathbb{E}_tX_{t+1}
\end{bmatrix} = \begin{bmatrix}
A_{11} - B_{11}F^B_t - B_{12}F^C_t & A_{12} \\
A_{21} - B_{21}F^B_t - B_{22}F^C_t & A_{22}
\end{bmatrix} \begin{bmatrix}
Y_t \\
X_t
\end{bmatrix} \\
+ \begin{bmatrix}
D_{11} - B_{11}L^B_t - B_{12}L^C_t \\
D_{21} - B_{21}L^A_t - B_{22}L^B_t
\end{bmatrix} U^A_t + C\varepsilon_{t+1}. \quad (82)
\]

Authority A’s loss function is

\[
\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(G^A_t'Q^A_tG^A_t\right).
\]

But, since the leadership integrates the followers’ reaction functions—\(U^B_t = -F^B_tY_t - L^B_tU_t^A\) and \(U^F_t = -F^F_tY_t - L^F_tU_t^A\)—into its optimization problem, the leadership’s loss function has to be rewritten in terms of the relevant variables for the leadership authority. Since

\[
\begin{bmatrix}
Y_t \\
X_t \\
U_t^A \\
U_t^B \\
U_t^C
\end{bmatrix} = \begin{bmatrix}
I & 0 & 0 \\
0 & I & 0 \\
0 & 0 & I \\
-F^B_t & 0 & -L^B_t \\
-F^C_t & 0 & -L^C_t
\end{bmatrix} \begin{bmatrix}
Y_t \\
X_t \\
U_t^A
\end{bmatrix},
\]

we can set \(G^A_t'Q^A_tG^A_t = [Y'_t \hspace{1cm} X'_t \hspace{1cm} U'_t^A] \tilde{\mathcal{K}}^A [Y_t \hspace{1cm} X_t \hspace{1cm} U_t^A]\), where \(\tilde{\mathcal{K}}^A = C^A' C^A' C^A\) and \(\tilde{\mathcal{K}}^A\) have to be partitioned conformably with \((Y'_t \hspace{1cm} X'_t \hspace{1cm} U'_t^A)'\).

**D.4 The Iterative Procedure**

We start with initial approximation for policy rules, with \(F^B_{(0)}, F^C_{(0)}, F^A_{(0)}, L^B_{(0)},\) and \(L^C_{(0)}\), and solve the follower’s problem, using equations (79)–(81) for authority B and equivalent equations for authority C. We will improve \(F^B_{(1)}\) and \(L^B_{(1)}\), as well as \(F^C_{(1)}\) and \(L^C_{(1)}\), but not \(F^A_{(0)}\). We then take into account the policy reaction functions of B and C and compute new matrices in equation (82), update
the target variable, and solve the problem for A. This will give us
new optimal reaction $F^A_{(1)}$. Then we again solve the problem for the
authorities B and C to update $F^B_{(2)}, L^B_{(2)}, F^B_{(2)},$ and $L^B_{(2)}$ and so on.

Appendix E. Financial Integration

The results on policy coordination under international risk sharing
are given in tables E1 and E2. They are different from those for
incomplete financial markets on two accounts.

First, under technology shocks the losses to all policymakers
are smaller under international risk sharing than under the incom-
plete financial markets, while the opposite happens under the cost-
push shocks. This is consistent with Auray and Eyquem (2014),
who showed that incomplete markets (autarky) may produce lower
welfare costs than complete markets, primarily because of inflation
volatility costs.

To understand this result, consider an asymmetric cost-push
shock, negative in country H and positive in country F, which is
plotted in figure 1 in the main text. Complete financial markets
allow for international borrowing and risk sharing among households
of the two countries. Real marginal utilities from consumption are
equated across countries, periods, and states. Thus, since the real
exchange rate increases due to the presence of home bias in con-
sumption, relative consumption also increases, but by a relatively
small amount.

Incomplete financial markets do not allow that H households lend
(F households borrow) abroad as much as they would under com-
plete financial markets, and thus relative consumption increases by
much more. This helps to stabilize producer inflation rates and the
terms of trade. In effect, larger consumption at country H reduces
the marginal utility of consumption and lowers labor supply, exert-
ing an upward pressure on wages and mitigating deflation at country
H. Hence, compared to complete markets, producer inflation rates
and terms of trade are less volatile, while consumptions are more
volatile and relative consumption is inefficiently high.

Therefore, under international risk sharing and relative to the
incomplete financial markets, tax rates and government spending
need to move by more to stabilize more volatile producer inflation
Table E1. Policy Equilibriums under International Risk Sharing in a Symmetric Monetary Union

\[ \begin{array}{|c|c|c|}
\hline
\text{A. Equilibrium under Technology Shock} & \text{Foreign Country} & \text{Home Country} \\
\hline
\text{Foreign Country} & L_N & F_N \\
\hline
L_N & (0.522,0.522) & (0.521,0.544) \\
F_N & (0.544,0.521) & (0.522,0.522) \\
\hline
\end{array} \]

\[ \begin{array}{|c|c|c|}
\hline
\text{B. Equilibrium under Cost-Push Shock} & \text{Foreign Country} & \text{Home Country} \\
\hline
\text{Foreign Country} & L_N & F_N \\
\hline
L_N & (1.064,0.997) & (0.997,1.064) \\
F_N & (1.070,1.070) & (1.070,1.070) \\
\hline
\end{array} \]

\[ \begin{array}{|c|c|c|c|}
\hline
\text{C. Equilibrium under Technology Shock} & \text{Foreign Country} & \text{Home Country} \\
\hline
\text{Foreign Country} & L_N & L_U & F_N & F_U \\
\hline
L_N & (0.522,0.522) & (0.489,0.544) & (0.521,0.544) & (0.484,0.577) \\
L_U & (0.544,0.489) & (0.510,0.510) & (0.534,0.498) & (0.510,0.510) \\
F_N & (0.544,0.521) & (0.498,0.534) & (0.522,0.522) & (0.516,0.544) \\
F_U & (0.577,0.484) & (0.510,0.510) & (0.544,0.516) & (0.510,0.510) \\
\hline
\end{array} \]

\[ \begin{array}{|c|c|c|c|}
\hline
\text{D. Equilibrium under Cost-Push Shock} & \text{Foreign Country} & \text{Home Country} \\
\hline
\text{Foreign Country} & L_N & L_U & F_N & F_U \\
\hline
L_N & (1.070,1.070) & (1.073,1.052) & (0.997,1.064) & (0.978,1.054) \\
L_U & (1.052,1.073) & (1.029,1.029) & (0.990,1.070) & (1.029,1.029) \\
F_N & (1.064,0.997) & (1.070,0.990) & (1.070,1.070) & (1.073,1.052) \\
F_U & (1.054,0.978) & (1.029,1.029) & (1.052,1.073) & (1.029,1.029) \\
\hline
\end{array} \]

\text{Note:} Losses to (home, foreign); losses in Nash equilibriums are shown in bold fonts.

rates and terms of trade. The higher volatility of inflation, terms of trade, and government spending outweighs the smaller volatility of consumption and output under cost-push shocks.

Second, under technology shocks there are no qualitative differences with the case of incomplete financial markets, but there are differences under cost-push shocks.
Table E2. Policy Equilibriums under International Risk Sharing in an Asymmetric Monetary Union

<table>
<thead>
<tr>
<th>A. Equilibrium under Technology Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Large Foreign Country</strong></td>
</tr>
<tr>
<td>(L_N)</td>
</tr>
<tr>
<td>(0.701, 0.413)</td>
</tr>
<tr>
<td>(0.721, 0.413)</td>
</tr>
<tr>
<td>(F_N)</td>
</tr>
<tr>
<td>(0.701, 0.416)</td>
</tr>
<tr>
<td>(0.701, 0.413)</td>
</tr>
<tr>
<td><strong>Small Home Country</strong></td>
</tr>
<tr>
<td>(L_N)</td>
</tr>
<tr>
<td>(0.701, 0.416)</td>
</tr>
<tr>
<td>(F_N)</td>
</tr>
<tr>
<td>(0.701, 0.413)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Equilibrium under Cost-Push Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Foreign Country</strong></td>
</tr>
<tr>
<td>(L_N)</td>
</tr>
<tr>
<td>(1.297, 0.944)</td>
</tr>
<tr>
<td>(1.286, 0.888)</td>
</tr>
<tr>
<td>(F_N)</td>
</tr>
<tr>
<td>(1.267, 0.944)</td>
</tr>
<tr>
<td>(1.297, 0.944)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Equilibrium under Technology Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Large Foreign Country</strong></td>
</tr>
<tr>
<td>(L_N)</td>
</tr>
<tr>
<td>(0.701, 0.413)</td>
</tr>
<tr>
<td>(L_U)</td>
</tr>
<tr>
<td>(0.710, 0.407)</td>
</tr>
<tr>
<td>(U_N)</td>
</tr>
<tr>
<td>(0.635, 0.429)</td>
</tr>
<tr>
<td>(n_N)</td>
</tr>
<tr>
<td>(0.701, 0.417)</td>
</tr>
<tr>
<td>(F_N)</td>
</tr>
<tr>
<td>(0.705, 0.409)</td>
</tr>
<tr>
<td>(F_U)</td>
</tr>
<tr>
<td>(0.649, 0.422)</td>
</tr>
<tr>
<td>(0.649, 0.426)</td>
</tr>
<tr>
<td>(0.701, 0.413)</td>
</tr>
<tr>
<td>(0.710, 0.407)</td>
</tr>
<tr>
<td>(0.649, 0.422)</td>
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<tr>
<td>(0.649, 0.422)</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>D. Equilibrium under Cost-Push Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Large Foreign Country</strong></td>
</tr>
<tr>
<td>(L_N)</td>
</tr>
<tr>
<td>(1.297, 0.944)</td>
</tr>
<tr>
<td>(L_U)</td>
</tr>
<tr>
<td>(1.295, 0.933)</td>
</tr>
<tr>
<td>(F_N)</td>
</tr>
<tr>
<td>(1.267, 0.944)</td>
</tr>
<tr>
<td>(F_U)</td>
</tr>
<tr>
<td>(1.213, 0.934)</td>
</tr>
<tr>
<td>(1.297, 0.919)</td>
</tr>
<tr>
<td>(1.281, 0.956)</td>
</tr>
<tr>
<td>(1.297, 0.944)</td>
</tr>
<tr>
<td>(1.297, 0.919)</td>
</tr>
</tbody>
</table>

Under cost-push shocks, equilibrium \((F_N, L_N)\) is the unique Nash equilibrium under complete financial markets in a country-size asymmetric monetary union, while it is only one of the three Nash equilibriums if markets are incomplete. Moreover, as we discussed above, it
only exists under incomplete financial markets if there are external imbalances and the large foreign country is a net foreign creditor, creating bigger “asymmetry” and the desire for the smaller country to choose national objectives.

More generally, incentives to adopt union-wide objectives are smaller the larger the asymmetries (different country size and/or different steady-state external imbalances) within the monetary union. Since the risk sharing leads to larger demand shifts across countries, it further amplifies existing asymmetries under incomplete financial markets. Country-size asymmetry reveals to be high enough to rule out any equilibriums where at least one national authority adopts the union-wide objectives. Thus, \((F_N, L_N)\) is the only surviving equilibrium in the fiscal sequential leadership where the large country leads.

Appendix F. Heterogeneous Monetary Union

F.1 Country-Size Effects

F.1.1 Technology Shocks

In the case of technology shocks, the country-size effects on the equilibrium outcome in table 2 in the main text are limited. The outcome is determined by the individual ranking of outcomes in non-cooperative regimes [HF]M, HFM, and FHM. Regardless of size, the leader gains and the follower loses in the sequential regimes HFM and FHM relative to their payoffs in [HF]M.

The large country, however, gains more than under cooperation. This result is the consequence of the size asymmetry, as in the case of symmetric countries the cooperative outcome was the best for each fiscal authority. The larger country has a stronger effect on monetary policy and is able to reduce the volatility of its fiscal instruments, in particular of costly spending; see panel A in figure F1.

Both countries gain because of less negative terms of trade gap, and lose due to higher inflation volatility. Since the interest rate needs to react by more than cooperation, output and consumption are better stabilized for the large country, while the reverse occurs for the small country. As a consequence, the total loss for the small home
Figure F1. Country-Size Effects, Impulse Responses to Asymmetric Shocks, Monetary Union with $n = 0.3$, $\varrho = 0$, $\omega = 1$, $\delta_H = \delta_F = 0.6$
country is higher than under cooperation, while [HF]M is preferred to cooperation for the large foreign country.\footnote{Here and below in all simulations we also assume that there are no international holdings of debt in the steady state, \( \varrho = 0 \). We relax this assumption later in the text.}

**F.1.2 Cost-Push Shocks**

If there are no international holdings of debt in the steady state, \( \varrho = 0 \), then under cost-push shocks the country-size effects result in only two Nash equilibriums in the extended 4x4 game, \( (L_U, L_U) \) and \( (L_N, F_U) \).

Neither \( (F_N, L_N) \) nor \( (F_U, L_N) \) is a Nash equilibrium. In contrast to the country-size symmetric union discussed above, if the intraperiod follower country is small, then it does not want to adopt the union-wide objective unilaterally. On the one hand, when the small H country adopts union-wide objectives, it now attaches a large relative weight to the objectives of the large F country. On the other hand, since cross-border effects from H are too small, F does not change its policy reaction meaningfully. As country H fiscal policy produces a small negative cross-border effect and F country has a large weight in the union, H will react by much less than if it cares only for its own (national) objectives. Since H is too small to get enough stabilization feedback from monetary policy, its weaker fiscal policy reaction produces larger welfare stabilization costs than otherwise. Panel B of figure F1 shows that, except for government spending, welfare-relevant variables at H display higher volatility under \( (F_U, L_N) \) than under \( (F_N, L_N) \). As a result, \( (F_U, L_N) \) is not a Nash equilibrium. Furthermore, while the small country H, being a follower, prefers keeping national objectives, the large country F, being a leader, prefers to adopt the union-wide welfare, aligning with the monetary policymaker, and so \( (F_N, L_N) \) is also not a Nash equilibrium.

**F.2 International Exposure, Imbalances, and Debt Levels**

The previous analysis assumed steady-state financial autarky, with zero external debt \( \varrho = 0 \). Greater international exposure of both countries with positive steady-state external debt \( \varrho > 0 \) and equal
steady-state external debts ($\omega = 1$) can produce stabilization welfare gains under both technology and cost-push shocks.

With a positive steady-state external debt, price levels and the terms of trade have first-order effects on real external debt and, thus, on intermediation costs. Under either cost-push or technology shocks, consumption becomes more volatile while inflation and terms of trade become more stabilized than in the case of steady-state financial autarky. In the face of an asymmetric technology shock (positive in H and negative in F), H becomes the net lender and real external debt is now worse stabilized through the price-level effect. Consequently, interest rate at H decreases by more than under steady-state financial autarky (see panel A in figure F2). In turn, in the face of an asymmetric cost-push shock (negative in H and positive in F), H becomes the net borrower and real external debt is now better stabilized through the price-level effect. Consequently, the interest rate at H increases by less than under steady-state financial autarky (see panel B in figure F2). In both cases, consumption increases by more at H, depleting stabilization under cost-push shocks (larger deviation from the efficient level of consumption) while improving stabilization under technology shocks. However, overall welfare improves under both shocks, since higher external debt exposures promotes a better stabilization of inflation.

Suppose that, in addition to $\varrho > 0$, we reduce $\omega < 1$ and make the home country the net external debtor. In our example of symmetric monetary union with $n = 0.5$, but with $\varrho = 0.5$ and $\omega = 0.5$, the steady-state consumption and spending at home are lower than they are in the foreign country, because the net external debtor country has to generate current account surplus to pay for debt service. The union is not symmetric any more, with an effectively “smaller” home country, and an asymmetric shock now produces aggregate effects dominated by the effects of the shocks at the “larger” F country. In fact, country H bears larger stabilization costs as monetary policy reacts to averages so that it helps to stabilize F inflation while destabilizing H inflation. Terms of trade further increase, reducing the negative terms of trade gap under technology shocks, while increasing positive terms of trade gap under cost-push shocks; see figure F3. This results in better stabilization under technology shocks but worse stabilization under cost-push shocks for the net lender F country.
Figure F2. Effect of Greater International Exposure in Regime [HF]M, Monetary Union with $n = 0.3$ and $\omega = 1$, $\delta_H = \delta_F = 0.6$

A. Technology Shock

B. Cost-Push Shock
Figure F3. Effect of Greater Imbalances in Regime [HF]M, Monetary Union with $n = 1/2$ and $\varrho = 0.5$, $\delta_{H} = \delta_{F} = 0.6$

A. Technology Shock

B. Cost-Push Shock
Table F1. Emergence of Equilibrium \((F_N, L_N)\) with Positive Net External Debt of the Home Country

<table>
<thead>
<tr>
<th>A. Equilibriums under Cost-Push Shock, (q = 0.5; \omega = 1; \delta_H = \delta_F = 0.6)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Large Foreign Country</strong></td>
</tr>
<tr>
<td>(L_N)</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Small Home Country (L_N)</td>
</tr>
<tr>
<td>(L_U)</td>
</tr>
<tr>
<td>(F_N)</td>
</tr>
<tr>
<td>(F_U)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Equilibriums under Cost-Push Shock, (q = 0.5; \omega = 0.5; \delta_H = \delta_F = 0.6)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Large Foreign Country</strong></td>
</tr>
<tr>
<td>(L_N)</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Small Home Country (L_N)</td>
</tr>
<tr>
<td>(L_U)</td>
</tr>
<tr>
<td>(F_N)</td>
</tr>
<tr>
<td>(F_U)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Equilibriums under Cost-Push Shock, (q = 0.5; \omega = 0.5; \delta_H = 1.2; \delta_F = 0.6)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Large Foreign Country</strong></td>
</tr>
<tr>
<td>(L_N)</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Small Home Country (L_N)</td>
</tr>
<tr>
<td>(L_U)</td>
</tr>
<tr>
<td>(F_N)</td>
</tr>
<tr>
<td>(F_U)</td>
</tr>
</tbody>
</table>

Under cost-push shocks, this additional asymmetry results in the third Nash equilibrium \((F_N, L_N)\), which existed in the 2x2 game but disappeared in the 4x4 game in the country-size asymmetric monetary union, reappearing; see panels A and B in table F1. As a net debtor, the “smaller” H country has now fewer incentives to adopt the union-wide objectives; see the relative ranking of outcomes in the first column in panel B, table F1. As discussed above, when
a small H country is a follower, it achieves a worse stabilization outcome under union-wide objectives than under national objectives. Indeed, as cross-border effects of its policy are small, and the H country attaches a large relative weight to the objectives of the large F country under union-wide metrics, H reacts much less than it does under its national objectives, and optimal actions of the larger country have destabilizing effects on H. These asymmetries become sufficiently large when the small-size country is also a net debtor country, H country optimally chooses the national objective, and \((F_N, L_N)\) emerges as a Nash equilibrium.

Finally, a unilateral increase of steady-state debt-to-output ratio in small country H yields it higher stabilization losses for both types of shocks, while large country F gains under technology shocks and loses under cost-push shocks; see panel C in table F1. However, the relative change is numerically small, without implications for the ranking of regimes and for stability of policy equilibriums.

**Appendix G. Coordinating Role of a Central Bank**

Countries’ budget constraints can be written as

\[
P_{Ht}^M (B_{Ht} + B_{Ht}^*) = \left(1 + \rho_H P_{Ht}^M\right) \frac{B_{Ht-1} + B_{Ht-1}^*}{1 + \pi_{Ht}} + G_t - \tau_t^l \frac{W_t}{P_{Ht}} N_t + [T_{t}^{CB} - I_{t}^{CB}]
\]

\[
P_{Ft}^M (B_{Ft} + B_{Ft}^*) = \left(1 + \rho_F P_{Ft}^M\right) \frac{B_{Ft-1} + B_{Ft-1}^*}{1 + \pi_{Ft}^*} + G_t^* - \tau_t^l \frac{W_t^*}{P_{Ft}^*} N_t^* + [T_{t}^{CB} - I_{t}^{CB}].
\]

Here,

- \(B_{Ht}\) are home-issued bonds held by home residents,
- \(B_{Ht}^*\) are home-issued bonds held by foreign residents,
- \(B_{Ft}\) are foreign-issued bonds held by foreign residents, and
- \(B_{Ft}^*\) are foreign-issued bonds held by home residents.
Two terms in square brackets are zero in the benchmark model. Now $T_{t}^{CB}$ and $T_{t}^{CB*}$ are extra taxes (transfers) to the fund. Variables $I_{t}^{CB}$ and $I_{t}^{CB*}$ are the net borrowing from the fund by H and F governments. However, they are not independent, as

$$I_{t}^{CB} + I_{t}^{CB*} = T_{t}^{CB} + T_{t}^{CB*}. \tag{83}$$

Equation (83) assumes that all funds are loaned to/borrowed from countries H and F.

Let

$$L_{t} = T_{t}^{CB} - I_{t}^{CB}.$$ 

We can substitute out $I_{t}^{CB*}$ to get governments’ budget constraints:

$$P_{Ht}^{M} (B_{Ht} + B_{Ht}^{*}) = (1 + \rho_{H} P_{Ht}^{M}) \frac{(B_{Ht-1} + B_{Ht-1}^{*})}{1 + \pi_{Ht}}$$

$$+ G_{t} - \tau_{t}^{l} W_{t}^{l} N_{t} + L_{t}$$

$$P_{Ft}^{M} (B_{Ft} + B_{Ft}^{*}) = (1 + \rho_{F} P_{Ft}^{M}) \frac{(B_{Ft-1} + B_{Ft-1}^{*})}{1 + \pi _{Ft}^{*}}$$

$$+ G_{t}^{*} - \tau_{t}^{l*} W_{t}^{l*} N_{t}^{*} - L_{t}. $$

In other words, the net lending of one country is net borrowing of another.

Table G1 reports results if a small penalty (0.01) on volatility of $L_{t}$ is imposed for cost-push shocks only.

The net lending $L_{t}$ can be an additional instrument of the central bank. There can be constraints on its use, like the intertemporal budget constraint. There can be access criteria for fiscal authorities, based on fiscal conditions. This will affect the price of the central bank bonds. The detailed and more realistic modeling of such policy instrument is left for future research.
Table G1. Choice of Leadership, Central Bank Has Two Instruments

<table>
<thead>
<tr>
<th>A. Choice of Leadership</th>
<th>A1. Technology Shocks, %C×10²</th>
<th>A2. Cost-Push Shocks, %C×10³</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Foreign Country</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$L_N$</td>
<td>$F_N$</td>
</tr>
<tr>
<td>Home Country</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_N$</td>
<td>(0.547,0.547)</td>
<td>(0.544,0.568)</td>
</tr>
<tr>
<td>$F_N$</td>
<td>(0.568,0.544)</td>
<td>(0.547,0.547)</td>
</tr>
<tr>
<td>Foreign Country</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_N$</td>
<td>(0.11662,0.11662)</td>
<td>(0.11660,0.11370)</td>
</tr>
<tr>
<td>$F_N$</td>
<td>(0.11662,0.11662)</td>
<td>(0.11662,0.11662)</td>
</tr>
</tbody>
</table>

| B. Choice of Leadership and the Type of Objective           |                                |                             |
|                                                            | B1. Technology Shocks, %C×10² |                             |
|                                                            | Foreign Country               |                             |
|                                                            | $L_N$                         | $L_U$                       |
| Home Country                                               |                               |                             |
| $L_N$                                                      | (0.547,0.547)                 | (0.516,0.570)               |
| $L_U$                                                      | (0.570,0.516)                 | (0.538,0.538)               |
| $F_N$                                                      | (0.568,0.544)                 | (0.525,0.559)               |
| $F_U$                                                      | (0.602,0.510)                 | (0.538,0.538)               |
|                                                            |                               |                             |
|                                                            | B2. Cost-Push Shocks, %C×10³  |                             |
|                                                            | Foreign Country               |                             |
|                                                            | $L_N$                         | $L_U$                       |
| Home Country                                               |                               |                             |
| $L_N$                                                      | (0.11662,0.11662)             | (0.11638,0.11621)           |
| $L_U$                                                      | (0.11621,0.11638)             | (0.11497,0.11497)           |
| $F_N$                                                      | (0.11660,0.11370)             | (0.11650,0.11416)           |
| $F_U$                                                      | (0.11622,0.11247)             | (0.11497,0.11497)           |
References


