Online Appendix to Fiscal Policy Stabilization: Purchases or Transfers?

Neil R. Mehrotra
Federal Reserve Bank of Minneapolis

Extensions

In this section, I briefly consider fiscal policy in a model where a subset of the population operates as rule-of-thumb agents who simply consume current income each period and a model where agents face a borrowing constraint. This section relates to a literature on fiscal policy and rule-of-thumb agents developed by Mankiw (2000) and Galí, López-Salido, and Valles (2007), and a literature examining the effects of monetary policy when agents face borrowing constraints such as Iacoviello (2005) and Monacelli (2009). As this section illustrates, the credit spread model considered in this paper can easily be related to rule-of-thumb or borrowing constraint models and, therefore, the policy implications are likely to carry over to a broader class of DSGE models.

Rule-of-Thumb Agents

Rule-of-thumb agents face a static optimization problem and choose hours period-by-period facing a simple budget constraint with consumption equal to current disposable income:

$$U_c\left(C_t^y, N_t^y\right)W_t = -U_h\left(C_t^y, N_t^y\right) \tag{1}$$

$$C_t^y = W_t N_t^y - T_t. \tag{2}$$

Log-linearizing these equilibrium conditions and combining with the equilibrium conditions for the firms and saver households discussed earlier delivers a closed-form solution for output in terms of government purchases and taxes:

$$y_t = \frac{\alpha}{\left(\alpha + s_c\overline{\sigma}(1-\alpha) + s_c\frac{1-s_y}{1-l_y}\frac{\sigma_s}{\varphi_s} - s_c(1-\alpha)\phi\upsilon\right)}g_t$$

$$- \frac{\alpha\phi\overline{Y}/\sigma_y\overline{c}_y}{\left(\alpha + s_c\overline{\sigma}(1-\alpha) + s_c\frac{1-s_y}{1-l_y}\frac{\sigma_s}{\varphi_s} - s_c(1-\alpha)\phi\upsilon\right)} tax_t$$

$$\phi = s_y\frac{\sigma_y}{\varphi_y} - (1-s_y)\frac{\sigma_s}{\varphi_s}\frac{l_y}{1-l_y}$$

$$\upsilon = \left(1 - \frac{\overline{wn}_y}{\sigma_y\overline{c}_y}\right) / \left(\frac{1}{\varphi_b} + \frac{\overline{wn}_y}{\sigma_y\overline{c}_y}\right).$$

The multiplier on government spending has several terms similar to the multiplier derived in section 5, with the parameters ϕ and υ as the new terms. For Frisch elasticities less than unity, $\upsilon < 1$, and for households with sufficient symmetry, $\phi \approx 0$. Therefore, a tax reduction for the borrower household has negligible effect on output for plausible calibrations, and the government spending multiplier remains below unity, consistent with the numerical experiments shown in figure 1. Intuitively, a transfer from one household to the other has offsetting effects on the labor supply of each household, leaving total labor supply relatively unchanged and, therefore, output unchanged. To the extent that $\phi > 0$, tax rebates will be expansionary and the government spending multiplier will be larger than in the representative-agent benchmark.

Under sticky prices, analytical solutions with rule-of-thumb agents can be obtained under the assumption of GHH (Greenwood, Hercowitz, and Huffman 1988) preferences or wage rigidity that eliminates a labor supply effect. For simplicity and comparability to the rest of the paper, I consider the case of rigid wages. The Phillips curve from section 4.2 obtains along with an intertemporal IS curve of the form

$$\begin{aligned} y_t - g_t = {} & E_t\left(y_{t+1} - g_{t+1}\right) - s_c\sigma_s\left(1-s_y\right)\left(i_t^d - E_t\pi_{t+1}\right) \\ & - s_c s_y\left(E_t c_{t+1}^y - c_t^y\right) \end{aligned}$$

$$c_t^y = \frac{\overline{wn}_y}{\overline{c}_y}\frac{1}{\alpha}y_t - \frac{\overline{Y}}{\overline{c}_y}tax_t$$

$$\pi_t = \frac{\kappa}{\alpha}\left(1-\alpha\right)y_t + \beta E_t\pi_{t+1}.$$

The last term in the IS equation can be treated as equivalent to the credit spread ω_t, and responds to changes in both income and transfers. A temporary increase in transfers that is gradually withdrawn, as in the case of a debt-financed tax rebate, is equivalent to a fall in the credit spread that eventually becomes positive as the transfer turns negative when taxes are raised to return the public debt to its steady state. Relative to the credit spread model, transfers appear directly in the intertemporal IS equation instead of operating indirectly through private-sector debt. As before, when monetary policy is unconstrained, the Phillips curve is unchanged and monetary policy is free to target any combination of inflation and output subject to the Phillips-curve tradeoff.

When monetary policy is constrained by the zero lower bound, both purchases and transfers may be used for stabilization and an explicit tax rebate multiplier can be derived when there is a constant probability that the shock causing the zero lower bound to bind disappears. While a financial shock no longer appears because of the absence of intermediation, any of the shocks that cause the zero lower bound to bind in representative-agent models—such as a discount rate shock—would suffice here.[1] The solution for output at the zero lower bound is similar to the solution derived in section 6, with the addition of a multiplier on the tax rebate:

$$y_{zlb} = \nu_g^* \left(g_{zlb} - tax_{zlb}\right) - \zeta$$

$$\nu_g^* = \frac{(1-\rho)(1-\beta\rho)}{(1-\rho)(1-\beta\rho)\left(1 - inc_y \frac{1}{\alpha}\right) - s_c(1-s_b)\,\sigma_s \frac{\kappa}{\alpha}(1-\alpha)\,\rho},$$

where inc_y is the rule-of-thumb agents' share of wage income in national income. Comparison to the multiplier derived in section 6 reveals that the multiplier ν_g may be higher or lower; the effect of higher inflation reducing real interest rates (the last term in the denominator) is attenuated relative to the saver/borrower model,

[1] We can easily reintroduce the financial shock and credit spread by simply adding a measure of rule-of-thumb consumers to the existing saver/borrower model. Goods market clearing then implies that $Y_t = \eta_s C_t^s + \eta_b C_t^b + (1-\eta_s-\eta_b)\,C_t^y$. As before, under the assumption of zero debt elasticity of the credit spread, the log-linearized economy at the zero lower bound is summarized by an aggregate intertemporal IS curve and the standard Phillips curve. Moreover, in a life-cycle model with distinct borrowing and credit spreads, the stochastic steady state would be characterized by saver households, borrower households, and households living in autarky.

while the presence of rule-of-thumb agents raises the direct effect of government spending on the consumption of rule-of-thumb agents (the inc_y term) and the multiplier. Unlike an old-style Keynesian model, the government spending multiplier and tax rebate multiplier are the same, and the balanced-budget multiplier is zero.

The reason the multiplier is the same for both government spending and tax rebates is that both affect the savers' consumption in the same way. A rise in government spending or an equivalent fall in tax rebates raises aggregate demand by the same amount, and equilibrium in the goods market requires either a rise in output or a fall in the savers' consumption induced by a rise in the real interest rate. With the nominal rate held constant and no direct effect of either policy on the Phillips curve, the savers' consumption response is the same and, therefore, the output multiplier is the same for each policy. When the government's budget is balanced, the aggregate demand effects cancel out and the savers' consumption decision is unchanged.

Finally, it's worth relating this equilibrium analysis of the zero lower bound with rule-of-thumb agents to the extensive literature on the determinants of consumption and the aggregate consumption function where the real interest rate is taken as fixed and exogenous.[2] The multipliers attached to any particular fiscal policy are heavily dependent on the behavior of the real interest rate, and therefore conclusions regarding fiscal multipliers are inherently general equilibrium questions. In the same way that the credit spread—absent wealth effects—does not alter the Phillips curve, a more complex (and realistic) theory of consumption is unlikely to alter the effects of fiscal policy away from the zero lower bound. Unless fiscal stabilization has large effects on the production side of the economy—that is, incentives to supply labor and capital—monetary policy can achieve the same aggregate demand objectives of fiscal policy away from the zero lower bound. The nature of the aggregate consumption function will only become relevant at the zero lower bound where fiscal policies that have larger affect on desired consumption will be preferred to policies with a smaller effect.

[2]See, for example, and Kaplan and Violante (2014).

Borrowing-Constrained Agents

A broad range of models consider a class of agents that are constrained either by an exogenous or endogenous borrowing constraint but assume a single rate for lending and borrowing funds. These model often assume that the borrowing constraint binds at all times and solve for the dynamics of the model by log-linearizing around a binding constraint. Relative to the rule-of-thumb model in the previous section and assuming an exogenous borrowing constraint, the equilibrium conditions become

$$U_c\left(C_t^b, N_t^b\right) W_t = -U_h\left(C_t^b, N_t^b\right)$$
$$C_t^b + \frac{1+i_{t-1}^d}{\Pi_t} B_{t-1} = W_t N_t^b + B_t$$
$$B_t \geq \overline{B}.$$

To a log-linear approximation, the borrower's budget constraint differs from the rule-of-thumb budget constraint only by including the lagged interest rate. If steady-state interest payments are small, this term can be safely disregarded and the fiscal multipliers obtained in section 7.1 remain a good approximation in the case of exogenous constraints. Without further assumptions on the model, a general characterization of fiscal multipliers with an endogenous borrowing constraint is difficult.

Under sticky prices and a demand-driven labor market, a similar Phillips curve and intertemporal IS curve determine output and inflation. When borrowers are constrained by an endogenous or exogenous constraint, their optimal choice of borrowing is governed by a Euler equation with a non-zero Lagrange multiplier on the binding constraint:

$$\lambda_t^b = \gamma E_t \lambda_{t+1}^b \frac{1+i_t^d}{\Pi_{t+1}} + \Theta_t,$$

where the Lagrange multiplier represents the shadow price of the constraint. Since the constraint is assumed to be always binding for sufficiently small shocks, the borrower's Euler equation can be log-linearized and summed with the saver's Euler equation to obtain an intertemporal IS curve of the form

$$y_t - g_t = E_t\left(y_{t+1} - g_{t+1}\right) - s_c\overline{\sigma}\left(i_t^d - E_t\pi_{t+1}\right) - s_c s_b \sigma_b \overline{\theta}\theta_t.$$

As before, the last term can be regarded as the credit spread, and changes in fiscal policy will shift the credit spread depending on the nature of the borrowing constraint. Importantly, the multiplier is likely to be changed by policy given that any change in income, wages, or taxes will affect the shadow price of the borrowing constraint. Though the mapping of a borrowing constraint model into the credit spread model will depend on further assumptions, the insights on fiscal policy from the credit spreads model should carry over to alternative models of borrowing and lending.

Housing and Credit Spreads

I maintain the assumption of patient and impatient households, but I now assume a single market interest rate for savers and households. Instead of a credit spread, impatient households are constrained to borrow only a possibly time-varying fraction of the value of their residence. The impatient households choose

$$\max_{\left\{C_t^b, N_t^b, B_t, H_t^b\right\}} E \sum_{t=0}^{\infty} \beta^t U\left(C_t^b, N_t^b, H_t^b\right)$$

$$\text{subject to: } C_t^b = W_t N_t^b - \frac{1+i_{t-1}^d}{\Pi_t} B_{t-1} + B_t - T_t + Q_t\left(H_{t-1}^b - H_t^b\right)$$

$$B_t \leq \chi_t Q_t H_t^b.$$

Relative to the equilibrium conditions in section 3, the Euler equation changes and a housing Euler equation is introduced:

$$\lambda_t^b = \gamma E_t \lambda_{t+1}^b\left(\frac{1+i_t^d}{\Pi_t} + \Theta_t\right)$$

$$\lambda_t^b Q_t = \gamma E_t \lambda_{t+1}^b\left(r_{h,t+1}^b + Q_{t+1}\right) + \Theta_t \chi_t Q_t.$$

Furthermore, if impatient households are the only agents that demand housing services and the supply of housing is fixed, the housing Euler equation will determine the market-clearing price of housing. We can log-linearize the model around a steady state assuming that the collateral constraint is always binding. Under these assumptions, an aggregate IS equation of the same form as the rule-of-thumb case emerges. To a log-linear approximation:

$$\begin{aligned} y_t - g_t &= E_t \left(y_{t+1} - g_{t+1} \right) - s_c \sigma_s \left(1 - s_b \right) \left(i_t^d - E_t \pi_{t+1} \right) \\ &\quad - s_c s_b \left(E_t c_{t+1}^b - c_t^b \right) \\ c_t^b &= \frac{\overline{wn}_b}{\overline{c}_b} \frac{1}{\alpha} y_t - \frac{\overline{Y}}{\overline{c}_b} tax_t + \frac{\overline{B}}{\overline{c}_b} \left(\chi_t + q_t \right) - \frac{\left(1 + \overline{r} \right) \overline{B}}{\overline{c}_b} \left(\chi_{t-1} + q_{t-1} \right) \\ \pi_t &= \frac{\kappa}{\alpha} \left(1 - \alpha \right) y_t + \beta E_t \pi_{t+1}. \end{aligned}$$

The preceding equations, along with a monetary policy rule, do not fully specify the equilibrium of the economy; the borrower household's Euler equation and housing Euler equation are needed to determine the dynamics of housing prices and the Lagrange multiplier on the borrowing constraint.

The growth rate of borrowers' consumption takes the place of the credit spread in the aggregate IS equation just as in the case of rule-of-thumb households:

$$\begin{aligned} E_t \left(c_{t+1}^b - c_t^b \right) &= \gamma_y E_t \left(y_{t+1} - y_t \right) - \gamma_{tax} E_t \left(tax_{t+1} - tax_t \right) \\ &\quad + \gamma_b \left(E_t \chi_{t+1} - \left(2 + \overline{r} \right) \chi_t + \left(1 + \overline{r} \right) \chi_{t-1} \right) \\ &\quad + \gamma_b \left(E_t q_{t+1} - \left(2 + \overline{r} \right) q_t + \left(1 + \overline{r} \right) q_{t-1} \right), \end{aligned}$$

where γ_y, γ_{tax}, and γ_b are the appropriate constants. An exogenous tightening of the collateral constraint can be represented as a fall in χ_t and, ignoring the equilibrium dynamics of housing prices, will act like an increase in the credit spread so long as the stochastic process for χ_t is dominated by the middle term for some period of time. In particular, an AR(3) process of χ_t could generate an AR(1) process for the "interest rate" shock represented by borrower consumption growth in the aggregate IS equation.

The inclusion of housing dynamics further complicates matters since simply a fall in housing prices does not guarantee a rise in borrower consumption growth beyond the initial period. Nevertheless, it appears plausible that a collateral shock could act in the same manner as a credit spread shock in the aggregate IS equation even with endogenous house prices. Stronger conclusions require greater structure placed on the saver household's demand for housing and residential investment, both of which will determine the market clearing housing price.

Equivalence with Overlapping-Generations (OLG) Model

In this section, I show that the steady state of the model with infinitely lived agents with differing degrees of time preference is isomorphic to the steady state of a model with finitely lived agents who share the same rate of time preference but differ in effective labor over the life cycle.

Households live T periods with variation in the disutility of labor supply over the life cycle, and each generation that dies in a period is replaced by a generation of equal measure in the next period so that the total population is constant. Households choose consumption, hours worked, and whether to borrow or save in each period. Formally, for each generation $i \epsilon \{0, 1, \ldots T\}$, the household's optimization problem is

$$max \quad E_0 \sum_{t=0}^{T-i} \beta^t \{u(C_t(i)) - \theta_i\}$$

$$C_t(i) = W_t N_t(i) + B_t(i) - \frac{1 + i^b_{t-1}}{\Pi_t} B_{t-1}(i) - D_t(i)$$
$$+ \frac{1 + i^d_{t-1}}{\Pi_t} D_{t-1}(i) + \Pi^f_t - T_t$$
$$D_t(i) \geq 0$$
$$B_t(i) \geq 0$$
$$B_{T-i}(i) = 0,$$

where θ_i is an exogenous process for effective labor supply that captures the hump-shaped profile of earnings over the life cycle. The household is prohibited from borrowing in the final period of life. The first-order conditions characterizing the household's optimal consumption and savings decisions are given below:

$$u_c(C_t(i), N_t(i)) = \lambda_t(i) \tag{3}$$

$$-u_n(C_t(i), N_t(i)) = \lambda_t(i) W_t \theta_i \tag{4}$$

$$\lambda_t(i) = \beta E_t \lambda_{t+1}(i) \frac{\left(1+i_t^d\right)\left(1+\omega_t\right)}{\Pi_{t+1}} - \phi_t^b(i) \tag{5}$$

$$\lambda_t(i) = \beta E_t \lambda_{t+1}(i) \frac{1+i_t^d}{\Pi_{t+1}} + \phi_t^d(i) \tag{6}$$

$$\lambda_{T-i}(i) = -\phi_{T-i}^b(i) \tag{7}$$

$$\lambda_{T-i}(i) = \phi_{T-i}^d(i) \tag{8}$$

$$\phi_t^b(i) B_t(i) = 0 \tag{9}$$

$$\phi_t^d(i) D_t(i) = 0. \tag{10}$$

Household optimality requires that households do not borrow or save in the final period. Subtracting the Euler equation for borrowing from the Euler equation for deposits shows that households never simultaneously borrow and save, but may find it optimal to live in autarky:

$$0 = \beta E_t \frac{\lambda_{t+1}(i)}{\Pi_{t+1}} \omega_t - \left(\phi_t^d + \phi_t^b\right).$$

I consider a steady allocation of consumption, borrowing, and labor supply across generations where wages, interest rates, and the price level are constant, and assume that the utility functions and distribution of θ_i over the generations are sufficient to guarantee that a steady state exists.

The firm's problem, the intermediaries problem, fiscal policy, and monetary policy are unchanged from the discussion in section 3. Market clearing requires

$$Y_t = \sum_{i=0}^{T} C_t(i) + G_t \tag{11}$$

$$N_t = \sum_{i=0}^{T} N_t(i). \tag{12}$$

A steady state of the overlapping-generations model with credit frictions is a set of aggregate quantities $\left\{\overline{Y}, \overline{N}, \overline{C}, \overline{F}, \overline{K}, \overline{\Pi}_f\right\}$; a distribution of consumption, labor supply, deposits, and borrowings over generations $\left\{\overline{C}_i, \overline{N}_i, \overline{D}_i, \overline{B}_i, \overline{\lambda}_i, \overline{\phi}_i^d, \overline{\phi}_i^b\right\}_{i=0}^{T}$; a set of prices $\{\overline{W}, \overline{\Pi}, \bar{i}_d, \overline{\omega}, \overline{MC}\}$; and a fiscal policy $\{\overline{B}_g, \overline{T}, \overline{G}, \overline{reb}\}$ that jointly satisfy the steady-state versions of

(i) household optimality conditions (27)–(34),

(ii) household budget constraints,

(iii) firm optimality conditions in (15),

(iv) government budget constraint, fiscal rule, and solvency condition (10),

(v) monetary policy rule (13), and

(vi) market clearing conditions.

Given a definition for the steady state of the overlapping-generations model, for suitable choices of the distribution of θ_i and other model parameters, the steady state of the infinite-horizon model is equivalent to the steady state of the overlapping-generations model.

Proposition 1. *Consider a steady state of the overlapping-generations model. There exists a set of discount rates and functions for household utility that provide the steady state in the infinite-horizon model.*

Proof. Since the firms' problem, intermediaries' problem, and fiscal and monetary policy are unchanged in the overlapping-generations model, a steady state in the OLG model satisfies parts (iii)–(v) of the steady-state version of the definition of an equilibrium in the infinite-horizon model. It remains to show that household optimality conditions and market clearing conditions may be satisfied.

Let Ω denote the set of borrowers in $i \epsilon \{0, 1, \ldots T\}$. Savers' and borrowers' consumption and labor supply can be defined in the OLG model and will satisfy the corresponding market clearing conditions (12)–(13) in the infinite-horizon model:

$$\overline{C}_s = \frac{1}{1-\pi_b}\sum_{i\epsilon\Omega^c}\overline{C}_i$$
$$\overline{C}_b = \frac{1}{\pi_b}\sum_{i\epsilon\Omega}\overline{C}_i$$
$$\overline{N}_s = \frac{1}{1-\pi_b}\sum_{i\epsilon\Omega^c}\overline{N}_i$$
$$\overline{N}_b = \frac{1}{\pi_b}\sum_{i\epsilon\Omega}\overline{N}_i$$

For suitable definitions of the utility functions for each household, household's labor supply conditions hold in steady state:

$$U_c^s\left(\overline{C}_s, \overline{N}_s\right)\overline{W} = -U_h^s\left(\overline{C}_s, \overline{N}_s\right)$$
$$U_c^b\left(\overline{C}_b, \overline{N}_b\right)\overline{W} = -U_h^b\left(\overline{C}_b, \overline{N}_b\right).$$

Under the assumption that firm profits are only paid to savers and the assumption that θ_i implies only one switch from borrowing to saving midway through the life cycle, summing the budget constraints of borrower household:

$$\sum_{i\epsilon\Omega^c}\overline{C}_i = \overline{W}\sum_{i\epsilon\Omega^c}\overline{N}_i + \sum_{i\epsilon\Omega^c}\overline{B}_i\left(1-\frac{1+\overline{i}_b}{\overline{\Pi}}\right) - \overline{T}\sum_{i\epsilon\Omega^c}1\left[i\epsilon\Omega^c\right]$$
$$\Rightarrow \overline{B} = \frac{1}{\pi_b}\sum_{i\epsilon\Omega^c}\overline{B}_i.$$

Finally, the interest rate and borrowing rate from the OLG model determine the discount rates in the infinite horizon model:

$$\beta = \frac{1}{1+\overline{i}_d}$$
$$\gamma = \frac{1}{\left(1+\overline{i}_d\right)\left(1+\overline{\omega}\right)}.$$

Equilibrium Conditions

Household equilibrium conditions and relevant transversality conditions for $i\epsilon\{s,b\}$:

$$\lambda_t^i = u_c\left(C_t^i, N_t^i\right)$$
$$\lambda_t^i W_t = -u_n\left(C_t^i, N_t^i\right)$$
$$\lambda_t^s = \beta E_t \lambda_{t+1}^s \frac{1+i_t^d}{\Pi_{t+1}}$$
$$\lambda_t^b = \gamma E_t \lambda_{t+1}^b \frac{\left(1+i_t^d\right)\left(1+\omega_t\right)}{\Pi_{t+1}}.$$

Law of motion for public-sector and private-sector debt:

$$B_t = C_t^b - W_t N_t^b + \frac{1+i_{t-1}^b}{\Pi_t} B_{t-1} + T_t$$
$$B_t^g = G_t + \frac{1+i_{t-1}^d}{\Pi_t} B_{t-1}^g - T_t.$$

Firm production, cost minimization, price setting, and price level determination:

$$Y_t = N_t^\alpha$$
$$W_t = \alpha \frac{Y_t}{N_t} MC_t$$
$$1 = \theta \Pi_t^{\nu-1} + (1-\theta)\left(\frac{K_t}{F_t}\right)^{\nu-1}$$
$$F_t = \frac{\nu}{\nu-1} \lambda_t^s MC_t Y_t + \theta\beta E_t \Pi_{t+1}^{\nu} F_{t+1}$$
$$K_t = \lambda_t^s Y_t + \theta\beta E_t \Pi_{t+1}^{\nu-1} K_{t+1}.$$

Monetary and fiscal policy rules and solvency condition:

$$\left(\frac{i_t^d}{\bar{r}_d}\right) = \left(\Pi_t\right)^{\phi_\pi}\left(\frac{Y_t}{Y_t^n}\right)^{\phi_y}$$

$$T_t = \phi_b \left(B_{t-1}^g - \overline{B}_g \right) - reb_t$$
$$0 = lim_{T \to \infty} E_t \frac{P_t}{P_T} \frac{B_T^g}{\prod_t^T \left(1 + i_{t-1}^d \right)}.$$

Credit spread determination:

$$1 + \omega_t = E_t \Gamma \left(B_t, W_{t+1} N_{t+1}^b, Z_t \right).$$

Market-clearing conditions:

$$Y_t = \eta C_t^b + (1 - \eta) C_t^s + G_t$$
$$N_t = \eta N_t^b + (1 - \eta) N_t^s.$$

Exogenous processes:

$$log \left(\frac{G_t}{\overline{G}} \right) = \rho_g \left(\frac{G_{t-1}}{\overline{G}} \right) + \epsilon_t^g$$
$$reb_t - \overline{reb} = \rho_{reb} \left(reb_{t-1} - \overline{reb} \right) + \epsilon_t^{reb}.$$

Analytical Solutions

As noted in section 4, we presented solutions for the borrower-lender model if wages are rigid, $\chi_n = 0$, and prices are flexible or monetary policy keeps inflation stable at all times. Here, we show how the model can be simplified and solved analytically under these conditions via the method of undetermined coefficients.

Consumption of borrower and lender households are determined in equilibrium by the Euler equations determining optimal saving behavior for each type of household. Along with these expressions, the budget constraint of the borrower household, the credit spread equation, and the resource constraint are also needed to solve for the tax multiplier.

$$c_t^b = E_t c_{t+1}^b - \sigma_b \left(i_t + \omega_t - E_t \pi_{t+1} \right) \tag{13}$$
$$c_t^s = E_t c_{t+1}^s - \sigma_s \left(i_t - E_t \pi_{t+1} \right) \tag{14}$$
$$y_t = s_c \left(s_b c_t^y + (1 - s_b) c_t^s \right) + g_t \tag{15}$$
$$\omega_t = \chi_b b_t \tag{16}$$

$$c_t^b = \frac{\bar{w}\bar{n}_b}{\bar{c}_b}(w_t + y_t) + \frac{\bar{b}}{\bar{c}_b}b_t - \frac{1+\bar{\imath}}{\bar{\pi}}\frac{\bar{b}}{\bar{c}_b}(b_{t-1} + i_{t-1} + \omega_{t-1} - \pi_t)$$
$$-\frac{\bar{y}}{\bar{c}_b}tax_t \tag{17}$$

Rigid real wages and flexible prices/inflation stabilization ensure that $y_t = w_t = \pi_t = 0$. Combining the savers' and borrowers' Euler equations along with the aggregate resource constraint, the following expression for the nominal rate results:

$$i_t = -\frac{s_b\sigma_b}{\tilde{\sigma}}\omega_t, \tag{18}$$

where $\tilde{\sigma} = s_b\sigma_b + (1 - s_b)\,\sigma_s$ is the average intertemporal elasticity of substitution.

We can define a new variable $a_t = b_t + i_t + \omega_t$. Using (42) and (40), we obtain the following expression relating a_t to b_t:

$$a_t = \left(1 - \chi_b\left(1 + \frac{s_b\sigma_b}{\tilde{\sigma}}\right)\right)b_t. \tag{19}$$

Using the expression for the nominal rate obtained above, we can reduce this system to a system of equation in c_t^b and a_t:

$$c_t^b = \frac{\bar{b}}{\bar{c}_b}\frac{1}{1 + \chi_b\left(1 - \frac{s_b\sigma_b}{\tilde{\sigma}}\right)}a_t - \frac{1+\bar{\imath}}{\bar{\pi}}\frac{\bar{b}}{\bar{c}_b}a_{t-1} - \frac{\bar{y}}{\bar{c}_b}tax_t \tag{20}$$

$$c_t^b = E_t c_{t+1}^b - \sigma_b\frac{\left(1 - \frac{s_b\sigma_b}{\tilde{\sigma}}\right)\chi_b}{1 + \chi_b\left(1 - \frac{s_b\sigma_b}{\tilde{\sigma}}\right)}a_t. \tag{21}$$

A solution to this system of equation takes the following form, and we can compute the unknown coefficients using the method of undetermined coefficients:

$$a_t = \alpha_b a_{t-1} + \alpha_\tau tax_t \tag{22}$$

$$c_t^b = \beta_b a_{t-1} + \beta_\tau tax_t. \tag{23}$$

If $\chi_b = 0$, then the credit spread is invariant to the level of debt and private-sector debt follows a random walk. The coefficients are given below:

$$\beta_b = -\bar{\imath}\frac{\bar{b}}{\bar{c}_b} \tag{24}$$

$$\beta_\tau = -\frac{\bar{y}}{\bar{c}_b}\frac{\bar{\imath}}{1-\rho+\bar{\imath}} \tag{25}$$

$$\alpha_b = 1 \tag{26}$$

$$\alpha_\tau = \frac{\bar{y}}{\bar{b}}\frac{1-\rho}{1-\rho+\bar{\imath}}. \tag{27}$$

We can also compute the solution in the case $\lim_{\chi_b \to \infty}$. As can be seen, the response of borrower consumption to the tax rebate is the same as in the case of a rule-of-thumb household. The remaining coefficients are given below:

$$\beta_b = -\left(1+\bar{\imath}\right)\frac{\bar{b}}{\bar{c}_b} \tag{28}$$

$$\beta_\tau = -\frac{\bar{y}}{\bar{c}_b} \tag{29}$$

$$\alpha_b = \frac{1+\bar{\imath}}{1+\bar{\imath}+\sigma_b\frac{\bar{c}_b}{\bar{b}}} \tag{30}$$

$$\alpha_\tau = \frac{\left(1-\rho\right)\frac{\bar{y}}{\bar{c}_b}}{\left(1+\bar{\imath}\right)\frac{\bar{b}}{\bar{c}_b}+\sigma_b}. \tag{31}$$

At the zero lower bound, output is no longer fixed at zero. Instead, the nominal interest rate does not respond to changes in fiscal policy. With the additional assumption of a labor share equal to 1 ($\alpha = 1$), we can express the system of equations determining output and consumption as follows:

$$y_t - g_t = E_t\left(y_{t+1} - g_{t+1}\right) - s_c s_b \sigma_b \frac{\chi_b}{1+\chi_b} a_t \tag{32}$$

$$c_t^b = E_t c_{t+1}^b - \sigma_b \frac{\chi_b}{1+\chi_b} a_t \tag{33}$$

$$c_t^b = \frac{\bar{w}\bar{n}_b}{\bar{c}_b} y_t + \frac{\bar{b}}{\bar{c}_b}\frac{1}{1+\chi_b} a_t - \left(1+\bar{\imath}\right)\frac{\bar{b}}{\bar{c}_b} a_{t-1} - \frac{\bar{y}}{\bar{c}_b} tax_t, \tag{34}$$

where $a_t = b_t + \omega_t$ and $i_t^d = 0$. If $\chi_b = 0$, then equation (56) determines output while the zero lower bound is binding. Output in this equation is a function only of government spending and, therefore, tax rebates have no effect on output.

In the case when $\chi_{b\rightarrow\infty}$, the coefficient on a_t in equation (58) is zero and consumption by the borrower household is determined by current income and the tax rebate. We can solve for a_t and find an expression for y_t. Since this expression contains the endogenous state variable a_{t-1}, an explicit analytical expression cannot be found.

$$\left(\sigma_b\left(1-\eta\right)+\left(1+\bar{\imath}\right)\frac{\bar{b}}{\bar{y}}\right)a_t$$

$$= -g_t + E_t g_{t+1} + tax_t - E_t tax_{t+1} + \left(1+\bar{\imath}\right)\frac{\bar{b}}{\bar{y}}a_{t-1} \qquad (35)$$

$$y_t - g_t = E_t\left(y_{t+1} - g_{t+1}\right) - s_c s_b \sigma_b a_t \qquad (36)$$

References

Galí, J., D. López-Salido, and J. Valles. 2007. “Understanding the Effects of Government Spending on Consumption.” *Journal of the European Economic Association* 5 (1): 227–70.

Greenwood, J., Z. Hercowitz, and G. W. Huffman. 1988. “Investment, Capacity Utilization, and the Real Business Cycle.” *American Economic Review* 78 (3): 402–17.

Iacoviello, M. 2005. “House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle.” *American Economic Review* 95 (3): 739–64.

Kaplan, G., and G. Violante. 2014. “A Model of the Consumption Response to Fiscal Stimulus Payments.” *Econometrica* 82 (4): 1199–1239.

Mankiw, N. G. 2000. “The Savers-Spenders Theory of Fiscal Policy.” *American Economic Review: Papers and Proceedings* 90 (2): 120–25.

Monacelli, T. 2009. “New Keynesian Models, Durable Goods, and Collateral Constraints.” *Journal of Monetary Economics* 56 (2): 242–54.