Optimality of the Friedman Rule in Economies with Money Demand by Firms*

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Motivated by the facts that firms hold about two thirds of M1 in industrialized economies, I study the optimality of the Friedman rule in economies with money demand by firms and different types of fiscal instruments. Money demand by firms is introduced in two ways: a cash-in-advance (CIA) constraint on wages and money in the production (MIP) function. In the case of a CIA constraint on wages, I show that the Friedman rule is not optimal when the government has access to only one distortionary tax to finance its expenditures. This is the case both in an economy with labor only and in one with labor and capital. Once a sufficiently rich menu of taxes is available to the fiscal authorities, the Friedman rule, then, may or may not be optimal. In contrast, in the case of MIP function, the optimal policy is to satiate the economy with real balances, even in the presence of distorting taxes on consumption and capital. That is, the Friedman rule is optimal. I also show that adding a working capital constraint on wages to a cash-and-credit goods model has no impact on the optimality of the Friedman rule and that it still remains optimal if preferences satisfy homotheticity and separability assumptions.

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1. Introduction

"Our final rule for the optimum quantity of money is that it will be attained by a rate of price deflation that makes the nominal rate of interest equal to zero," Friedman (1969) wrote in "The Optimum Quantity of Money." By setting money growth at a rate that causes a deflation equal in magnitude to the real rate of return to physical assets, the central bank can make the return to holding money equal to the return to holding bonds. With that rate of deflation, the nominal interest rate is zero. The intuition for the Friedman rule is that because the marginal cost of creating additional money is zero, the opportunity cost of holding money, the nominal interest rate, faced by private agents should be zero. The main criticism to the Friedman rule is attributed to Phelps (1973). Phelps noted that analyzing inflation tax without consumption and labor supply functions is as if "professor Friedman has given us Hamlet without a prince." In a public finance context and a general equilibrium model where inflation tax is optimally treated like other distortionary taxes, Phelps counterargued the optimality of the Friedman rule in favor of a positive rate of inflation. Following Phelps, a large literature that tried to analyze the optimal inflation rate in different models from different points of view emerged.

In most of this body of work that studied the optimality of the Friedman rule, one motivates the existence of a liquidity premium by supposing that households obtain a service flow from cash balances. Realistically, it is not only households that have motivation to hold cash balances, but firms as well. Important to emphasize is that in these frameworks where the attention is restricted to the case in which money is held only by households, variations in the nominal interest rate affect real variables solely through their effect on aggregate demand. This paper departs from the existing literature by studying the optimality of the Friedman rule in the context of models with demand for money by firms only and by both firms and households and with different types of distorting taxes. Considering a money demand by firms is motivated by the facts that a substantial part of M1 is held by firms. For example, the Federal Reserve's (1988) Demand Deposit Ownership Survey (DDOS) surveyed changes over the 1980s in the shares of demand deposits. According to the DDOS, consumers held one-third of demand deposits at all insured commercial banks in 1980. This share had declined to about one-quarter by 1987, while holdings of demand deposits by financial and non-financial businesses rose from three-fifths to about two-thirds of total demand deposits.[1] Also, Mulligan (1997) estimates a model of the demand for money by firms using longitudinal data on sales, money holdings, and other variables at the firm level. He shows, using Compustat data on 12,000 firms for the years 1961–92, that in industrialized economies about two-thirds of M1 are held by firms. Also, a large literature in finance documented a very significant increase in cash holdings by firms during the recent years.[2] It is therefore natural to motivate a demand for money by firms and study the optimality of the Friedman rule under this motivation. My purpose in this paper is to pursue this line of reasoning. The focus is to investigate whether the policy conclusion arrived at by the existing literature regarding the optimality of the Friedman rule is robust with respect to a more realistic specification of the economic environment, that is, where firms have motivations to hold money.

To do so, I consider monetary economies models, somehow similar to the models described in Chari, Christiano, and Kehoe (1996) and Schmitt-Grohé and Uribe (2004). Specifically, I build models to study the optimality of the Friedman rule in a stochastic, flexible-price, production economy without and with capital, where sources of inefficiency in the models stem from the nominal frictions of the demand for cash balances by firms and households and distortionary taxes. I first study the optimality of the Friedman rule in economies with money demand by firms only and then study this optimality in the cash-and-credit goods model of Chari, Christiano, and Kehoe (1996), with an added demand for real balances by firms. I rationalize money demand by firms by assuming that wage payments are subject to a cash-in-advance constraint and by assuming money in the production function. The first one is a very common assumption in the literature (see, e.g., Schmitt-Grohé and Uribe 2005, 2007

[1] The Federal Reserve surveys for years prior to this period also show that a substantial part of M1 was held by firms. The Federal Reserve discontinued the DDOS in 1990.

[2] See, for example, Almeida, Campello, and Weisbach (2004), Ferreira and Vilela (2004), Han and Qiu (2007), Bates, Kahle, and Stulz (2009), and Gao, Harford, and Li (2013), among many others.

and, for a review, Christiano, Trabandt, and Walentin 2010). For the second one, I follow Benhabib, Schmitt-Grohé, and Uribe (2001, 2002) and assume that money affects production possibilities.

In a monetary economy where a fraction of wage payments is subject to a cash-in-advance constraint, I show that the Friedman rule is not optimal in the presence of distortionary taxes on consumption or labor income. This non-optimality of the Friedman rule recalls Phelps's criticism that Friedman's first-best argument ignores the second-best fact that inflation produces seigniorage incomes for the fiscal authority and all forms of taxation produce distortions of some kind. More precisely, it is no longer optimal to put the nominal interest rate equal to zero when the government has no access to lump-sum taxes and only to one distorting tax. Put another way, the optimal way to deal with a distortion depends on other existing distortions. When the government has access to taxes on both consumption and labor income, I show that the optimal policy is not unique and in fact there are a variety of policies that can implement the optimal allocation. Some of these have the optimality of the Friedman rule, but others do not. I also show that adding capital to the model and imposing taxes on it do not change the main result on the optimality of the Friedman rule. Again, as long as the fiscal authority doesn't have access to a rich menu of taxes and can only impose distortionary taxes on capital and labor income or capital and consumption, the Friedman rule ceases to be optimal. In the presence of a rich menu of taxes—that is, the distortionary taxes on capital, consumption, and labor income—the Friedman rule may or may not be optimal.

I then analyze the optimality of the Friedman rule in the context of a model where the demand for money is motivated by assuming that real balances facilitate firms' production. Specifically, I follow Benhabib, Schmitt-Grohé, and Uribe (2001, 2002) in assuming that money enters in the production function and affects production possibilities. In this context, in sharp contrast to the case of a working capital constraint on wages, the Friedman rule, even with distorting taxes on consumption and capital, emerges as the only possible solution to the Ramsey allocation problem.

I also study the optimality of the Friedman rule in the case of money demand by both households and firms. I consider a cash- and credit-goods model where cash goods and wage bill both are subject to a cash-in-advance constraint. Chari, Christiano, and Kehoe (1996) in the cash- and credit-goods economy of Lucas and Stokey (1983) show that if preferences are homothetic in cash and credit goods and separable in leisure, then the Friedman rule will be optimal even in the presence of distorting taxes. I show that their conclusion remains the case even if wage payments are also subject to a cash constraint. While the presence of the demand for money by households calls for the optimality of the Friedman rule, the presence of the working capital constraint calls for the optimality of deviating from it. However, this tradeoff that the Ramsey planner faces between using inflation as a tax and minimizing the opportunity cost of holding money (which requires setting the nominal interest rate equal to zero) is resolved in favor of the Friedman rule. The reason is that with homotheticity and separability conditions, as proved by Chari, Christiano, and Kehoe (1996), the monetary economy can be reinterpreted as a real intermediate-goods economy, where—based on the standard public finance result—the optimal tax rate on an intermediate good is zero. Therefore, a positive interest rate means that the cash good is effectively taxed at a higher rate than the credit good, which is not optimal. This will also be the case with a cash constraint on wage payments.

The only papers, to the best of my knowledge, that directly address the optimality of the Friedman rule in a model with a money demand by firms are Faig (1991) and Schmitt-Grohé and Uribe (2005). Faig (1991) studied the optimality of the Friedman rule in a simple one-period static model where the transactions activity is performed by firms. Schmitt-Grohé and Uribe (2005) study Ramsey-optimal monetary and fiscal policy in a medium-scale macroeconomic model with several real and nominal rigidities. They numerically find that under an income tax regime and in the presence of a cash-in-advance (CIA) constraint on wage payment, even in the case of fully flexible prices, the Friedman rule ceases to be optimal. The reason, in their context, is that the Ramsey planner would like to tax capital and labor income differently but the income tax does not allow that, hence the inflation tax can be used as an indirect tax on labor.

The rest of the paper is organized as follows. Section 2 considers the optimality of the Friedman rule in a model with a CIA constraint on wages and different tax instruments, and section 3 presents the same environment and the Ramsey problem with capital accumulation and capital taxation. Section 4 studies the optimality of the Friedman rule in an economy with money in the production function. Section 5 establishes that the Friedman rule in a cash-credit goods model with a CIA constraint on wages still remains optimal if the utility function is homothetic in consumption goods and separable in labor effort. Section 6 concludes.

2. A Model with a CIA Constraint on Wages, with Distorting Taxes, and without Capital

This section develops an infinite-horizon production economy with perfectly competitive product markets and flexible prices. A demand for money by firms is motivated by assuming that a fraction of wage payments is subject to a cash-in-advance constraint. The government finances an exogenous stream of purchases by printing money, imposing distorting taxes on labor income, or consumption, or both, and issuing one-period nominally risk-free bonds. I analyze the optimality of the Friedman rule in three different cases: first, the case that the only fiscal instrument is labor income tax; second, the case that the only fiscal instrument is consumption tax; and third, once the fiscal instruments are both consumption and income taxes.

2.1 Households

The economy is populated by a continuum of identical households that consume different varieties of goods, save, and work, and their preferences are defined over consumption, c_t, and labor effort, h_t. The representative household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t, h_t\right), \tag{1}$$

where E_0 denotes the mathematical expectations operator conditional on information available at time zero and β is the subjective discount factor. The utility function U is strictly concave and satisfies the Inada conditions.

The flow budget constraint of the household in period t reads as follows:

$$P_t c_t \left(1 + \tau_t^c\right) + B_t = P_t w_t h_t \left(1 - \tau_t^h\right) + R_{t-1} B_{t-1} + \Pi_t, \qquad (2)$$

where households are assumed to invest in non-state-contingent nominal bonds, B_t, which pay a nominal interest rate R_t one period later. τ_t^c is the distorting tax on consumption, τ_t^h is the distorting tax on labor income, w_t is the real wage rate, and Π_t are profits received from firms. Households are also assumed to be subject to a borrowing limit that prevents them from engaging in Ponzi schemes.

The representative household chooses the set of processes $\{c_t, h_t, B_t\}$, taking as given the set of processes $\left\{P_t, w_t, R_t, \tau_t^c, \tau_t^h\right\}$ so as to maximize utility (1) subject to the budget constraint (2). The first-order conditions are

$$U_c\left(c_t, h_t\right) = \lambda_t \left(1 + \tau_t^c\right) \qquad (3)$$

$$U_h\left(c_t, h_t\right) = -\lambda_t w_t \left(1 - \tau_t^h\right), \qquad (4)$$

$$\frac{\lambda_t}{P_t} = \beta R_t E_t \frac{\lambda_{t+1}}{P_{t+1}}, \qquad (5)$$

where λ_t refers to the Lagrange multiplier on equation (2).

The interpretation of these equilibrium conditions is as follows: equation (3) states that the distortionary consumption taxes distort the equality between the marginal utilities of consumption and wealth. Equation (4) shows the equality between marginal utility of leisure and the marginal utility of labor income that is distorted due to the presence of labor income taxes. Equation (5) is the Euler condition with respect to bonds.

2.2 Firms

Firms use labor to produce consumption goods. One unit of labor is assumed to produce one unit of good which is perishable. Firms' money holdings are motivated by assuming that a fraction of wage payments is subject to a cash-in-advance constraint of the form

$$\gamma w_t h_t \leq m_t, \qquad (6)$$

where $m_t = \frac{M_t}{P_t}$ is the demand for real money balances by firms in period t and M_t denotes nominal money holdings. γ is a parameter between zero and one, $0 < \gamma \leq 1$, and stands for the fraction of the wage bill that must be backed with cash.

The period-by-period budget constraint that the firm faces is

$$M_t = M_{t-1} + P_t h_t - P_t w_t h_t - P_t \Pi_t. \tag{7}$$

The firm chooses $\{h_t, M_t\}$ to maximize the present discounted value of profits, given by

$$\Pi_t = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \frac{1}{P_t} \{P_t h_t - P_t w_t h_t + M_{t-1} - M_t\} \tag{8}$$

subject to equation (6).

Here, $\beta^t \frac{\lambda_t}{\lambda_0}$ is the period-0 value to the representative household of period-t goods, which the firm uses to discount profit flows because households are the ultimate owners of firms.

This maximization gives the following first-order condition:

$$\frac{1}{w_t} = 1 + \gamma(1 - R_t^{-1}). \tag{9}$$

As is clear from equation (9), the presence of working capital constraint distorts the equality between the marginal product and the real wage of labor. When $R_t > 1$, the labor wage is less than its marginal product. In fact, the financial cost of labor is increasing in the opportunity cost of holding money, $1 + \gamma(1 - R_t^{-1})$, which is an increasing function of nominal interest rate.

2.3 The Government

The budget constraint that the government faces is given by

$$M_t + B_t + P_t \tau_t^c c_t + P_t \tau_t^h w_t h_t = R_{t-1} B_{t-1} + M_{t-1} + P_t g_t. \tag{10}$$

The consolidated government issues one-period nominally risk-free bonds, B_t; prints money, M_t; imposes distortionary taxes, τ_t^h and τ_t^c; and faces a stream of public consumption, denoted by g_t.

2.4 Competitive Equilibrium

Absent arbitrage opportunities in equilibrium implies that $R_t \geq 1$. This in turn implies that, based on equation (9), in equilibrium $1 \geq w_t$ must hold.

A competitive equilibrium is a set of plans $\{c_t,\, h_t,\, M_t,\, B_t,\, w_t,\, \lambda_t,\, R_t,\, P_t\}$ satisfying equations (3), (4), (5), (6), (9), (10), and

$$R_t \geq 1, \tag{11}$$

$$U_c\left(c_t, h_t\right)\left(1+\tau_t^c\right) \geq U_h\left(c_t, h_t\right)\left(1-\tau_t^h\right), \tag{12}$$

$$c_t + g_t = h_t, \tag{13}$$

given policies $\left\{R_t,\, \tau_t^c,\, \tau_t^h\right\}$ and the initial condition $R_{-1}B_{-1}+M_{-1}$.

Equation (12) results from the non-negativity of nominal interest rate and equation (13) shows the real resource constraint.

2.5 Non-optimality of the Friedman Rule with Distortionary Taxes on Labor Income

I first eliminate the consumption tax from the menu of taxes available to the government and study the optimality of the Friedman rule in the version of the model in which the menu of fiscal policies is restricted to only labor income taxes. I then consider the case of consumption taxes only and, after that, both taxes together.

With income taxes only, the Ramsey-optimal policy is the process $\left\{\tau_t^h,\, R_t\right\}$ associated with the competitive equilibrium that yields the highest level of utility to the representative household. We know that if the initial financial wealth held by consumers is positive, the welfare is maximized by increasing the initial price level to infinity. To avoid this feature, I restrict the initial price level to be given.

Now I begin by deriving the primal form of the competitive equilibrium and then I state the Ramsey problem and show that the Friedman rule is not optimal.

2.5.1 The Primal Form

The problem of determining the optimal structure of prices and taxes to finance a given level of expenditures is called the Ramsey problem, after the classic treatment of Ramsey (1928). In the representative-agent models, like the one studied here, the Ramsey problem is to maximize the utility of the representative agent subject to the government's revenue requirement.

There are two approaches to solve this problem. The first approach, often called the dual approach, employs the indirect utility function to express utility as a function of the government's control variables. The second approach, called the primal approach, which is considered here, involves the elimination of all prices and tax rates from the equilibrium conditions, so that the resulting reduced form involves only real variables. The primal form of the equilibrium conditions consists of two equations. One equation is a feasibility constraint, given by the resource constraint (13), which must hold at every date and under all contingencies. The other equation is a single, present-value constraint known as the implementability constraint (I do not present the derivation of the implementability constraint here, because it can be found in detail in both Chari, Christiano, and Kehoe 1991 and Schmitt-Grohé and Uribe 2004).

PROPOSITION 1. *Plans* $\{c_t, h_t\}_{t=0}^{\infty}$ *satisfying*

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U_c\left(c_t, h_t\right) c_t + U_h\left(c_t, h_t\right) h_t\right] = A_0 \frac{\lambda_0}{P_0}, \tag{14}$$

$$U_c\left(c_t, h_t\right) \geq U_h\left(c_t, h_t\right)\left(1 - \tau_t^h\right), \tag{15}$$

and equation (13), given $A_0 = (M_{-1} + R_{-1}B_{-1})$, $\lambda_0 = U_c\left(c_0, h_0\right)$, *and* P_0, *are the same as those satisfying equations (3), (4), (5), (6), (9), (10), (11), and (13).*[3]

The real variables that appear in the primal form are consumption and labor effort. Equation (14), which is the present-value implementability constraint, is the government budget constraint expressed in intertemporal form with all prices and taxes substituted out using equilibrium conditions.

[3]It is worth noting that the primal form derived above doesn't depend on the value of γ. This means that the analysis remains the same if we allow for 100 percent cash in advance on the wage bill, that is, $\gamma = 1$.

PROPOSITION 2. *The Friedman rule in a model without capital and with a cash-in-advance constraint on wage payments and distortionary taxes on income is not optimal.*

Proof. The Ramsey planner maximizes the representative household utility (1), subject to equations (13) and (14).[4]

Let ψ_t and φ denote, respectively, the Lagrange multipliers on the feasibility constraint, (13), and the implementability constraint, (14).

The first-order conditions of the Ramsey planner's problem are

$$\begin{aligned}\frac{\partial L}{\partial c_t} &= U_c(c_t,h_t) + \varphi U_{hc}(c_t,h_t)\,h_t + \varphi U_c(c_t,h_t) \\ &\quad + \varphi U_{cc}(c_t,h_t)\,c_t - \psi_t = 0, \qquad (16)\\ \frac{\partial L}{\partial h_t} &= U_h(c_t,h_t) - \varphi U_{hh}(c_t,h_t)\,h_t + \varphi U_h(c_t,h_t) \\ &\quad + \varphi U_{ch}(c_t,h_t)\,c_t + \psi_t = 0. \qquad (17)\end{aligned}$$

After rewriting equations (16) and (17), we get the following expressions:

$$(1+\varphi) + \varphi\left(\frac{U_{cc}(c_t,h_t)\,c_t + U_{hc}(c_t,h_t)\,h_t}{U_c(c_t,h_t)}\right) = \frac{\psi_t}{U_c(c_t,h_t)}, \quad (18)$$

$$(1+\varphi) + \varphi\left(\frac{U_{ch}(c_t,h_t)\,c_t + U_{hh}(c_t,h_t)\,h_t}{U_h(c_t,h_t)}\right) = \frac{-\psi_t}{U_h(c_t,h_t)}. \quad (19)$$

As is quite clear, the optimality conditions of the Ramsey problem do not imply the optimality of the Friedman rule. This is because, based on equations (3), (4), and (9), the Friedman rule $R_t = 1$ is optimal only if $U_c(c_t,h_t) = U_h(c_t,h_t)\left(1-\tau_t^h\right)$. This cannot, however, be concluded from the above Ramsey optimality conditions.

Thus we conclude that in a model with cash constraint for firms and in the presence of distortionary taxes on income, the Friedman

[4] I focus on the less-constrained problem, that is, on the case in which the constraint (15) is dropped. If the Friedman rule is not optimal in the less-constrained problem, it will not also be optimal in the more-constrained problem.

rule is not optimal. The basic intuition for this conclusion recalls the result due to Phelps (1973). Phelps, in a model with money in the utility function and distorting labor income taxes, shows that the Friedman rule is not optimal, because inflation produces seigniorage revenues for the government. More precisely, instead of just making one of the distortions equal to zero and only using the other one, it is optimal to equalize the marginal distortion caused by one unit of revenue collected with distortionary income tax with the marginal distortion caused by one unit of revenue collected with inflation tax. This can be shown to be the case if, instead of distorting income taxes, the government has access to only distortionary taxes on consumption. I turn to this task next.

2.6 Non-Optimality of the Friedman Rule with Distortionary Taxes on Consumption

Instead of assuming distortionary taxes on labor income, now suppose that the government can impose distortionary taxes only on consumption at rate τ_t^c. That is, we now set τ_t^h to zero and allow τ_t^c to vary. The primal form of the equilibrium conditions in this case is essentially the same as the one associated with the economy with distortionary income taxes, proposition 1, except that equation (12) now changes as follows:

$$U_c\left(c_t,h_t\right)\left(1+\tau_t^c\right) \geq U_h\left(c_t,h_t\right). \tag{20}$$

Since having access to distortionary consumption taxes doesn't change the primal form of the competitive equilibrium, the Ramsey problem remains the same. Thus, we can conclude that in this case also the Friedman rule ceases to be optimal. The reason is that with distortionary taxes on consumption, the Friedman rule would be optimal only if $U_c\left(c_t,h_t\right)\left(1+\tau_t^c\right) = U_h\left(c_t,h_t\right)$ holds. However, as in the case with labor income taxes, we cannot conclude this equality from equations (18) and (19).

2.7 The Friedman Rule with Distorting Taxes on Consumption and Labor Income

Suppose now that there is a sufficiently rich menu of taxes available to the fiscal authorities. In other words, instead of assuming only one distortionary tax, the fiscal authority now has access to both distortionary taxes on consumption and labor income. The primal form of the competitive equilibrium with taxes on both consumption and labor income is again the same as the primal form presented in proposition 1, with the exception that prices now should be expressed in terms of the consumer price gross of consumption taxes, $P_t^c = (1 + \tau_t^c)\, P_t$, where P_t, like before, is the producer price. In this case, the Ramsey allocation for each allocation $\{c_t,\, h_t\}_{t=0}^{\infty}$ is implemented with a unique path for $\left\{ \frac{\left(1-\tau_t^h\right)}{(1+\tau_t^c)\left(1+\gamma(1-R_t^{-1})\right)},\, w_t \right\}_{t=0}^{\infty}$. There is a unique path for $P_t^c = (1 + \tau_t^c)\, P_t$ given P_0^c, and a unique path for $\{M_t,\, B_t\}_{t=0}^{\infty}$, but for each implementable allocation there are multiple fiscal policies consistent with it. More precisely, the individual tax rates, τ_t^c and τ_t^h, and the nominal interest rate, R_t, are not uniquely determined by the Ramsey allocation, so there are multiple fiscal and monetary policies that can implement that allocation. Some of those policies support the optimality of the Friedman rule, $R_t = 1$, but others do not. The reason is that for an implementable allocation $\{c_t,\, h_t\}_{t=0}^{\infty}$ in the set defined by the resource constraint (13) and the implementability conditions, (12) and (14), the three variables τ_t^c, τ_t^h, and R_t are restricted by only two restrictions, $\frac{\left(1-\tau_t^h\right)}{(1+\tau_t^c)\left(1+\gamma(1-R_t^{-1})\right)}$ and $(1 + \tau_t^c)\, P_t$. Therefore among the set of optimal taxes there could be a subset where the nominal interest rate is zero. In terminology of public finance, the three taxes are equivalent. Thus with taxes on consumption and labor income, there is not a unique path for R_t, and so the Friedman rule may or may not be optimal.

3. The Model with Capital

In the previous section I used a monetary model without capital and with a cash-in-advance constraint on wages to argue that the Friedman rule is not optimal whenever the government, to finance its expenditures, raises revenue either by taxing consumption or by taxing labor income. Here, I show that this basic result extends to a generalized version of the previous model in which I allow for capital accumulation and capital taxation.

3.1 Households

As above, households draw utility from consumption goods, c_t, and disutility from labor, h_t, based on equation (1). The household's budget constraint with capital can be written as

$$(1+\tau_t^c)\,c_t + k_{t+1} + \frac{B_t}{P_t} = w_t h_t\left(1-\tau_t^h\right) + \left[(1-\delta)+(1-\tau_t^k)u_t\right]k_t + \frac{R_{t-1}B_{t-1}}{P_t} + \Pi_t, \tag{21}$$

where k_t is capital, u_t is the real rental rate of capital, δ is the depreciation rate, and τ_t^k is the rate of tax on capital income.

The household's problem is to maximize lifetime expected utility, given by equation (1), subject to the budget constraints given by equation (21), provided that the no-Ponzi-game borrowing limit is satisfied.

The first-order conditions of the household's problem are equations (3), (4), and (5), the same as in the previous section, and

$$\lambda_t = E_t\beta\lambda_{t+1}\left[(1-\delta)+(1-\tau_{t+1}^k)u_{t+1}\right], \tag{22}$$

where equation (22) is obtained from the first-order condition for capital.

3.2 Firms

To produce output, I now assume that firms use labor h_t and capital k_t as inputs. The production technology is given by $F(k_t, h_t)$, which is homogeneous of degree one, and factors are paid their marginal products. As in section 2, it is assumed that a fraction of wage payments is subject to a CIA constraint given by equation (6).

The firm's objective is to maximize the expected present discounted value of profits. In order to do so, it uses $\beta^t\frac{\lambda_t}{\lambda_0}$ to discount profit flows to maximize

$$E_0\sum_{t=0}^{\infty}\beta^t\frac{\lambda_t}{\lambda_0}\left\{F(k_t,h_t) - w_t h_t - u_t k_t - m_t + \frac{m_{t-1}}{\pi_t}\right\},$$

subject to equation (6). Here $\pi_t = \frac{P_t}{P_{t-1}}$ denotes the gross consumer price inflation.

The first-order conditions for $\{h_t, k_t, m_t\}$ give the following conditions:

$$F_k(k_t, h_t) = u_t, \tag{23}$$

$$F_h(k_t, h_t) = (1 + \gamma(1 - R_t^{-1}))w_t. \tag{24}$$

The optimality condition (23) states the equality between the marginal product of capital and the rental rate of capital, and equation (24) has the same interpretation as the condition (9) we derived above.

3.3 The Government

As in the previous section, I assume that the government prints money, issues nominal one-period bonds, and levies distorting taxes to finance an exogenous stream of public consumption. The only difference here is that the fiscal authority, in addition to having access to distorting taxes on consumption and labor, is also allowed to tax capital income at rate τ_t^k. Accordingly, the government's sequential budget constraint becomes

$$\begin{aligned} M_t + B_t + P_t\tau_t^c c_t + P_t\tau_t^h w_t h_t + P_t\tau_t^k u_t k_t \\ = R_{t-1}B_{t-1} + M_{t-1} + P_t g_t. \end{aligned} \tag{25}$$

3.4 Competitive Equilibrium

Before defining the competitive equilibrium, note that the aggregate resource constraint due to the presence of physical capital takes the following form:

$$c_t + g_t + k_{t+1} - (1-\delta)k_t = F(k_t, h_t). \tag{26}$$

A competitive equilibrium is now a set of plans $\{c_t, h_t, k_{t+1}, M_t, B_t, w_t, P_t, \lambda_t, R_t\}$ satisfying equations (3), (4), (5), (11), (12), (22), and (23), (24), (25), and (26), given policies $\left\{R_t, \tau_t^c, \tau_t^h, \tau_t^k\right\}$ and the initial conditions A_0 and k_0.

As in the previous section, I first restrict the menu of taxes available to the fiscal authorities and study the optimality of the Friedman rule without a consumption tax (with taxes on capital and labor income) and then discuss the case without labor taxes and with taxes on consumption. To this end, I begin by deriving the primal form of the competitive equilibrium and stating the Ramsey problem and then show that the Friedman rule is not optimal if fiscal instruments are restricted to taxes on labor income and capital or consumption and capital.

3.5 Non-Optimality of the Friedman Rule with Distortionary Income and Capital Taxes

In order to obtain the present-value implementability constraint, I do the same as before and iterate the government budget constraint forward and use the first-order conditions and also the fact that the production function exhibits constant returns to scale to eliminate prices, which leads to the same implementability condition obtained in the previous section in the model without capital:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U_c\left(c_t, h_t\right) c_t + U_h\left(c_t, h_t\right) h_t\right] = \lambda_0 \left\{B_0 + \frac{A_0}{P_0}\right\}, \tag{27}$$

given λ_0, A_0, P_0, and $B_0 = \left[(1-\delta) + \left(1-\tau_0^k\right) u_0\right] k_0$.

The second implementability condition required for non-negativity of the nominal interest rate is also the same as condition (15). The Ramsey allocation problem must also satisfy the resource constraint (26).

PROPOSITION 3. *The Friedman rule in a model with capital and a cash-in-advance constraint on wage payments and distortionary taxes on income and capital is not optimal.*

Proof. The Ramsey problem consists in choosing a set of strictly positive sequences $\{c_t,\, h_t,\, k_{t+1}\}_{t=0}^{\infty}$ to maximize the representative household's utility function (1) subject to the feasibility constraint (26) and the implementability conditions (15) and (27), given $\lambda_0 \left\{B_0 + \frac{A_0}{p_0}\right\}$.

This primal form of the equilibrium conditions is essentially the same as the one associated with the economy without capital except that the resource constraint is replaced by equation (26), which now includes capital. Therefore, letting $\varphi > 0$ denote the Lagrange multiplier on the implementability constraint and ψ_t denote the Lagrange multiplier on the resource constraint, the first-order conditions of the Ramsey problem with respect to c_t are the same as before (equation (16)) and lead to a similar solution. The first-order conditions with respect to h_t and k_{t+1} imply that

$$\frac{\partial L}{\partial h_t} = U_h(c_t, h_t) - \varphi U_{hh}(c_t, h_t) h_t + \varphi U_h(c_t, h_t) + \varphi U_{ch}(c_t, h_t) c_t + \psi_t F_h(k_t, h_t) = 0 \tag{28}$$

$$\frac{\partial L}{\partial k_{t+1}} = -\psi_t + \beta E_t \psi_{t+1} \left[F_k(k_{t+1}, h_{t+1}) + 1 - \delta\right] = 0. \tag{29}$$

If the Friedman rule holds, then $R_t = 1$. This requires that $U_c(c_t, h_t) = U_h(c_t, h_t)\left(1 - \tau_t^h\right)$. But, again, as the the first-order conditions of the Ramsey problem make clear, we cannot conclude that relationship; that is, the Friedman rule is not also optimal in this context.

As I showed in the previous section in the case without capital, eliminating labor taxes and allowing for consumption tax doesn't change this conclusion. This remains also the case in the version of the model with capital. Moreover, with a sufficiently rich menu of taxes available to the fiscal authorities, τ_t^c, τ_t^h, and τ_t^k, the nominal interest rate, R_t, along with these taxes are not uniquely determined by the Ramsey allocation, so there are multiple fiscal policies that can implement that allocation. Some of these fiscal policies imply the optimality of the Friedman rule, and some do not.

Therefore, adding capital and imposing distortionary taxes on it does not change the result that the Friedman rule ceases to be optimal in a monetary model with a cash-in-advance constraint on wages, so long as the menu of taxes available to the government is restricted to taxes on income and capital or consumption and capital.

4. A Model with Money in the Production Function

Thus far, I have studied monetary economies in which demand for money by firms is originated in a cash-in-advance constraint on the wage bill. In this section, I drop this assumption and replace it with—following Benhabib, Schmitt-Grohé, and Uribe (2001, 2002)—the assumption of money in the production function. For simplicity, I exclude labor from the model and assume that output is produced with money and capital. To finance its expenditure, the government is assumed to have access to distortionary taxes on consumption and capital. The main finding in this section is that the only solution to the Ramsey-optimal policy is to hold sufficient money to derive the marginal product of money to zero. That is, the Friedman rule emerges as the optimal policy even in the presence of distortionary consumption and capital taxes.

4.1 The Economic Environment

It is assumed that the representative household seeks to maximize the present value of utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_t\right), \tag{30}$$

subject to its budget constraint

$$\begin{aligned}\left(1+\tau_t^c\right) c_t + k_{t+1} + \frac{B_t}{P_t} = {} & \left[(1-\delta) + (1-\tau_t^k) u_t\right] k_t \\ & + \frac{R_{t-1} B_{t-1}}{P_t} + \Pi_t.\end{aligned} \tag{31}$$

The household chooses sequences $\{c_t,\ B_t,\ ,k_{t+1}\}_{t=0}^{\infty}$ so as to maximize (30) subject to the constraint (31), given that the no-Ponzi-game condition is assumed to be met. The optimality conditions associated with the household's problem are equations (5), (22), and

$$U_c\left(c_t\right) = \lambda_t\left(1+\tau_t^c\right). \tag{32}$$

Output is assumed to be produced during period t by the stock of capital k_t and real money balances $m_t = \frac{M_t}{P_t}$ at the beginning of the period using the available technology $F\left(k_t, m_t\right)$, which is a homogeneous technology of degree one. Following Benhabib, Schmitt-Grohé, and Uribe (2001, 2002), I assume that the production function, $F\left(k_t, m_t\right)$, is positive, increasing, and concave in m_t. I also assume that there is a point of satiation in real money balances $\bar{m}$ such that for $m \geq \bar{m}$, where $\bar{m}$ is a parameter, $F_m(k_t, m_t) = 0$.[5]

Firms seek to maximize the present value of their profits with respect to $\{k_t,\, m_t\}_{t=0}^{\infty}$, hence they maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{ F(k_t, m_t) - u_t k_t - m_t + \frac{m_{t-1}}{\pi_t} \right\}.$$

The first-order conditions are equation (23) and

$$F_m(k_t, m_t) = 1 - R_t^{-1}, \tag{33}$$

and since in any equilibrium $R_t \geq 1$, therefore $F_m \geq 0$ in equilibrium must hold.

The government budget constraint is essentially the same as the one given by the constraint (25) except that now $\tau_t^h = 0$.

The following proposition provides the proof for the optimality of the Friedman rule in this model.

Proposition 4. *The Friedman rule in a model with money in the production function and in the presence of distorting taxes on consumption and capital is optimal.*

Proof. The Ramsey problem is to maximize the expected lifetime utility (30) of the representative household by choosing $\{c_t,\, k_{t+1},\, m_t\}$ subject to the resource constraint

$$c_t + g_t + k_{t+1} - (1-\delta)\, k_t = F(k_t, m_t) \tag{34}$$

and the following single present-value implementability constraint:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_c(c_t)\, c_t = \lambda_0 \left\{ B_0 + \frac{A_0}{p_0} \right\}, \tag{35}$$

[5]Benhabib, Schmitt-Grohé, and Uribe (2001, 2002) do not make such an assumption. I make it, however, following Chari, Christiano, and Kehoe (1996), who made such an assumption regarding the utility function in order to get around the technical problem of non-existence of a solution to the Ramsey allocation problem in the case in which the value of real money balances is infinite. In other words, this assumption ensures that the Friedman rule, a zero nominal interest rate, need not be associated with an infinite demand for money.

given $\lambda_0 = U_c(c_0)$, A_0, p_0, and B_0. Let the Lagrange multiplier on the feasibility constraint (34) be denoted by ψ_t and the Lagrange multiplier on the implementability constraint (35) be denoted by ϕ. Then, the first-order conditions of the Ramsey problem are equation (29) and

$$U_c + \phi U_c + \phi c_t U_{cc} - \psi_t = 0, \tag{36}$$

and

$$\psi_t F_m(k_t, m_t) = 0. \tag{37}$$

The optimality condition for real money balances (37) is satisfied when either $\psi_t = 0$ or $F_m(k_t, m_t) = 0$. However, due to the insatiability of utility function, ψ_t is strictly positive. Therefore, the solution has to be $F_m(k_t, m_t) = 0$. In other words, despite the presence of distorting taxes on consumption and capital, it is optimal for the social planner to follow the Friedman rule and satiate the economy with money.

While, as I mentioned before, assuming a working capital constraint for firms is a very common assumption in the literature and recently has also received a big boost from the evidence that firms were hurt during the financial crisis, when they could not get access to credit to pay for working capital (see, e.g., Gilchrist et al. 2016), introducing money into the production function is not. However, it is interesting to see that the Friedman rule emerges as the optimal policy in this context and in the presence of distorting taxes without imposing any strong assumptions on preferences or the production function. This would also be the case if we include labor in the preferences and production function and allow for distorting taxes on labor income.

5. The Cash- and Credit-Goods Economy with a CIA Constraint on Wage Payments

Thus far, I have studied the optimality of the Friedman rule in monetary economies in which only firms have motivation to hold money. A more realistic monetary model would incorporate a money demand by households as well as firms. In sections 2 and 3 we saw that the Friedman rule is not optimal when we rationalize a demand for money by firms by imposing that wage payments be subject to a cash-in-advance constraint. On the other hand, in the context of the cash- and credit-goods model of Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1996) show that the Friedman rule in the presence of distortionary income taxes is optimal if utility is separable in leisure and homothetic in the cash and credit goods. Therefore, in this section, I study the Ramsey-optimal policy in a cash- and credit-goods model with distortionary income taxes and a working capital constraint on wages to see if the tradeoff between the calls for optimality of the Friedman rule (in a cash- and credit-goods model) and those deviating from the Friedman rule (in the monetary model with a CIA constraint on wages) is resolved and, if so, which model is favored. The model is essentially that of Chari, Christiano, and Kehoe (1996) with a CIA constraint on wages. The central result that emerges is that the optimality of the Friedman rule is unaltered by the introduction of a CIA constraint on wage payments.

5.1 The Model

Households are assumed to draw utility from cash goods c_{1t} and credit goods c_{2t} and disutility from labor effort h_t, according to the following function that satisfies the standard assumptions:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(c_{1t}, c_{2t}, h_t\right). \tag{38}$$

Expenditures on cash goods are subject to a cash-in-advance constraint of the form

$$\alpha c_{1t} \leq m_t^h, \tag{39}$$

where m_t^h stands for real money holdings by the household in period t and α is a constant that shows the fraction of consumption held in money.

The household's real budget constraint is given by

$$c_{1t} + c_{2t} + b_t + m_t^h = \frac{R_{t-1}B_{t-1} + M_{t-1}^h}{P_t} + \left(1 - \tau_t^h\right) w_t h_t + \Pi_t. \tag{40}$$

Assuming that households are subject to a borrowing limit that prevents them from engaging in Ponzi schemes, maximizing the utility function (38) subject to (39) and (40) gives (5) and the following first-order conditions:

$$U_{c_1}(c_{1t}, c_{2t}, h_t) = U_{c_2}(c_{1t}, c_{2t}, h_t)\left(1 + \alpha\left(1 - R_t^{-1}\right)\right), \tag{41}$$

$$U_h(c_{1t}, c_{2t}, h_t) = \lambda_t\left(1 - \tau_t^h\right) w_t, \tag{42}$$

where λ_t is the Lagrange multiplier associated with the budget constraint (40). It is apparent from the first-order condition (41) that the nominal interest rate distorts the equality between the marginal utilities of cash and credit goods.

As in sections 2 and 3, I introduce a demand for money by firms by assuming that wage payments are subject to a cash-in-advance constraint of the form

$$\gamma w_t h_t \leq m_t^f, \tag{43}$$

where m_t^f denotes the demand for real money balances by firms in period t. Solving the firm's optimization problem gives exactly the same optimality condition as the one we derived in section 2, namely, equation (9). The government budget constraint is also of exactly the same form as equation (10) except that now $\tau_t^c = 0$.

The set of equilibria is characterized by the conditions (5), (9)–(10), (41)–(42), the resource constraint

$$c_{1t} + c_{2t} + g_t = h_t, \tag{44}$$

and the equation that governs total aggregate real balances

$$m_t = \alpha c_{1t} + \gamma w_t h_t, \tag{45}$$

together with the non-negativity constraint on the nominal interest rates, which can be written as

$$U_{c_1}(c_{1t}, c_{2t}, h_t) \geq U_{c_2}(c_{1t}, c_{2t}, h_t). \tag{46}$$

Again, apart from the resource constraint (44), we can condense the equilibrium conditions of the economy into the following single present-value implementability constraint:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U_{c_1}(c_{1t}, c_{2t}, h_t)\, c_{1t} + U_{c_2}(c_{1t}, c_{2t}, h_t)\, c_{2t} \right.$$
$$\left. + \; U_h(c_{1t}, c_{2t}, h_t)\, h_t \right] = A_0 \frac{\lambda_0}{P_0}, \tag{47}$$

given $\lambda_0 = U_{c_1}(c_{10}, c_{20}, h_0)$, A_0, and P_0.

This primal form of the equilibrium conditions is essentially the same as the one associated with the Chari, Christiano, and Kehoe (1996) model, and introducing a CIA constraint on wages doesn't change it. Therefore, provided we maintain the homotheticity and weak separability assumptions about preferences, it can be shown, by virtually the same arguments as in Chari, Christiano, and Kehoe (1996), that the Friedman rule $R_t = 1$ holds in the presence of a working capital constraint on wage payments as well. The following proposition presents the proof of this statement.

PROPOSITION 5. *Suppose that preferences are homothetic in cash and credit goods and separable in labor effort. Then, under the Ramsey allocation, $R_t = 1$ for all $t > 0$.*

Proof. The Ramsey planner chooses processes $\{c_{1t},\, c_{2t},\, h_t\}$ in order to maximize lifetime discounted utility (38) subject to (44) and (47). Letting ψ_t and ϕ stand for the Lagrange multipliers associated with (44) and (47), the corresponding Ramsey first-order conditions for c_{1t} and c_{2t} are

$$(1+\phi)\, U_{c_1} + \phi \left[U_{c_1 c_1} c_{1t} + U_{c_2 c_1} c_{2t} + U_{h c_1} h_t \right] = \psi_t \tag{48}$$

$$(1+\phi)\, U_{c_2} + \phi \left[U_{c_1 c_2} c_{1t} + U_{c_2 c_2} c_{2t} + U_{h c_2} h_t \right] = \psi_t . \tag{49}$$

It can be shown that if the utility function is homothetic in c_{1t} and c_{2t} and separable in h_t, then the following condition holds:

$$\frac{U_{c_1 c_1} c_{1t} + U_{c_2 c_1} c_{2t}}{U_{c_1}} = \frac{U_{c_1 c_2} c_{1t} + U_{c_2 c_2} c_{2t}}{U_{c_2}}, \tag{50}$$

which, along with the Ramsey optimality conditions (48) and (49) and the consumer's first-order condition (41), shows that the Ramsey allocation gives the optimality of the Friedman rule.

Therefore, once the utility function is homothetic in c_{1t} and c_{2t} and separable in h_t, it is not optimal for the government to use inflation tax, even though it must rely on other distorting taxes. The reason is that under a positive nominal interest rate, an efficiency cost by distorting the consumer's optimal choice between credit good and cash good arises. While the presence of distortionary taxes and a cash-in-advance constraint on the wage bill of firms provides incentives for the Ramsey planner to deviate from the Friedman rule, the presence of a demand for money by households provides an incentive to set nominal interest rate at zero. Hence, we can conclude that if preferences are homothetic in cash and credit goods and separable in leisure, the tradeoff is resolved in favor of the Friedman rule. It is important, however, to note that dividing goods into cash and credit goods and then imposing homotheticity and separability conditions on preferences, which are very strong assumptions, are crucial and required for the optimality of the Friedman rule.

6. Conclusion

In this paper, I analyzed the optimality of the Friedman rule in different monetary economies without and with capital, where money enters into the models through a demand for money by firms motivated by either a working capital constraint on labor costs or money in the production function. Considering that a demand for money by firms is motivated by the fact that in industrialized countries, firms hold a substantial fraction of the cash balances. I first showed that the Friedman rule is not optimal, in the context of a model with and without capital, when firms are subject to a working capital constraint on wages and the government, in order to finance its expenditures, is restricted to impose distortionary taxes on either consumption or income. When the menu of taxes available to the fiscal authorities is sufficiently rich, I showed that the individual tax rates and the nominal interest rate are not uniquely determined by the Ramsey allocation, and therefore the Friedman rule may or may not be optimal. I then proved that the optimality of the Friedman rule is the only possible solution to the Ramsey allocation problem when output is produced with money held by the firms for production purposes. This is the case even with distorting taxes on consumption and capital. Furthermore, given the assumptions of homotheticity and separability on preferences, I showed that the Friedman rule remains optimal in the context of a cash- and credit-goods model with an added cash constraint on wages.

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