Discussion of "Lower-Bound Beliefs and Long-Term Interest Rates"

James D. Hamilton
University of California at San Diego

1. Introduction

Grisse, Krogstrup, and Schumacher (this issue) provide one of the first systematic evaluations of the effects of the negative interest rate policies recently adopted across various countries. The authors have made a welcome and useful contribution.

The authors use the Ruge-Murcia (2006) shadow-rate model as a theoretical framework for interpreting the evidence. I would first like to suggest a much more general way this could be done and then offer some comments on using event studies for empirical evaluation.

2. Shadow-Rate Models of the Lower Bound

Following the authors' notation, let r_t denote the short-term nominal interest rate, $\bar{r}$ the effective lower bound, and r_t^* the shadow rate. The latter is a theoretical construct that is allowed to be an arbitrarily large negative number. The idea is that when r_t^* is below $\bar{r}$, the observed short rate will be in the vicinity of $\bar{r}$:[1]

$$r_t = \max\{r_t^*, \bar{r}\}.$$

The authors employ Ruge-Murcia's (2006) description of the process followed by the shadow rate,

$$r_{t+1}^* = \alpha + \psi r_t + \varepsilon_{t+1}, \tag{1}$$

where $\varepsilon_{t+1} \sim N(0, \sigma^2)$. One problem with this specification is that it cannot explain a persistent episode with rates at the lower bound.

[1] Empirical models typically also incorporate the possibility of measurement error to allow some fluctuation of the observed rate around the constant $\bar{r}$.

Suppose for example that we're currently at a zero lower bound, with $r_t = \bar{r} = 0$. Then (1) implies that $E_t(r^*_{t+1}) = \alpha > 0$ and $E_t(r^*_{t+n}) > 0$ for all n. There is no way such a process could be consistent with interest rates that were observed to stay near zero for years.

A more popular specification in the literature (e.g., Krippner 2013, 2015; Christensen and Rudebusch 2015; and Wu and Xia 2016) takes the form

$$r^*_{t+1} = \alpha + \psi r^*_t + \varepsilon_{t+1}, \tag{2}$$

which implies $E_t(r^*_{t+n}) = \frac{\alpha(1-\psi^n)}{(1-\psi)} + \psi^n r^*_t$. If ψ is near unity and r^*_t is far below zero, then $E_t(r^*_{t+n})$ could remain negative for large n. News (in the form of realizations of ε) may change the expected date of "liftoff" above the lower bound $\bar{r}$ because it changes $E_t(r^*_{t+n})$.

To arrive at a general description of the term structure of interest rates for such a process, it's helpful to define a few additional terms. Let y_{nt} denote the continuously compounded yield on a zero-coupon bond maturing at $t+n$ (so that y_{1t} is just another symbol for r_t, the one-period rate). Let f_{nt} denote the n-period-ahead forward rate. This is a rate we could lock in at date t by selling an n-period bond and simultaneously buying an $(n+1)$-period bond:

$$f_{nt} = (n+1)y_{n+1,t} - ny_{nt}. \tag{3}$$

Note that from this definition, an n-period interest rate can be viewed as the average of the corresponding set of forward rates:

$$y_{nt} = n^{-1}(f_{n-1,t} + f_{n-2,t} + \cdots + f_{1t} + y_{1t}). \tag{4}$$

Note that (4) is true by the definition of a forward rate in (3), and does not make any assumptions whatever about investors' objectives or beliefs.

To calculate the predicted behavior of interest rates under a simple model of investors' preferences, consider first an economy that is currently far away from the lower bound, so that next period's short rate should equal the shadow rate: $r_{t+1} = r^*_{t+1}$. We could then buy a two-period bond at date t for price $\exp(-2y_{2t})$ and sell it at $t+1$ for $\exp(-r^*_{t+1})$ for a gross expected return of

$$E_t[\exp(2y_{2t})\exp(-r^*_{t+1})] = \exp(2y_{2t} - E_t r^*_{t+1} + \sigma^2/2),$$

where the term $\sigma^2/2$ is a consequence of Jensen's inequality.[2] The expected excess return from holding a two-period bond over a one-period bond is thus

$$2y_{2t} - E_t r^*_{t+1} + \sigma^2/2 - y_{1t} = f_{1t} - E_t r^*_{t+1} + \sigma^2/2,$$

with the last equality following from (3). We could define the expected excess return to be the term premium. Suppose that investors' tolerance for risk is correlated with the current shadow rate r^*_t and that the term premium could be written as $\lambda_0\sigma + \lambda_1\sigma r^*_t$ for some constants λ_0 and λ_1:[3]

$$f_{1t} - E_t r^*_{t+1} + \sigma^2/2 = \lambda_0\sigma + \lambda_1\sigma r^*_t. \tag{5}$$

Note that this specification includes the expectations hypothesis of the term structure as a special case when $\lambda_0 = \lambda_1 = 0$. The equilibrium condition (5) can alternatively be written

$$\begin{aligned} f_{1t} &= \alpha + \psi r^*_t - (\sigma^2/2) + \lambda_0\sigma + \lambda_1\sigma r^*_t \\ &= \alpha^Q + \psi^Q r^*_t - (\sigma^2/2), \end{aligned}$$

where $a^Q = \alpha + \lambda_0\sigma$ and $\psi^Q = \psi + \lambda_1\sigma$. In other words, investors could be viewed as if they take expectations of future r^*_{t+1} not using the objective process (2) but instead using the Q measure $r^*_{t+1} \sim N(\alpha^Q + \psi^Q r^*_t, \sigma^2)$:

$$f_{1t} = E^Q_t r^*_{t+1} - (\sigma^2/2).$$

We can likewise calculate a risk-adjusted expectation of the shadow rate n periods ahead:

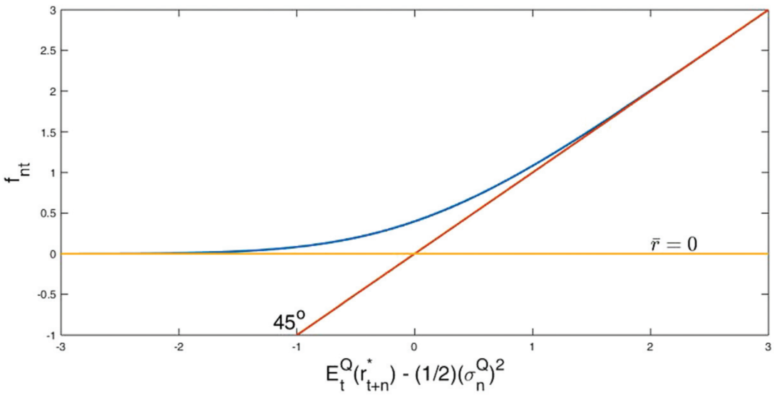
$$E^Q_t(r^*_{t+n}) = \frac{\alpha^Q[1 - (\psi^Q)^n]}{(1 - \psi^Q)} + (\psi^Q)^n r^*_t. \tag{6}$$

If we assume that investors care about risk-adjusted returns as calculated by the Q-measure parameterization (6), then Wu and

[2]Since $-r^*_{t+1}|r^*_t \sim N(-E_t r^*_{t+1}, \sigma^2)$, $E_t \exp(-r^*_{t+1}) = \exp(-E_t r^*_{t+1} + \sigma^2/2)$.

[3]Section 2.1 in Hamilton and Wu (2014) illustrates how such a functional dependence could arise.

Figure 1. The Equilibrium Forward Rate under the Shadow-Rate Model



Notes: Horizontal axis: Q-measure expectation of the shadow rate n periods in the future (with σ_n^Q normalized at 1). Vertical axis: n-period-ahead forward rate.

Xia (2016) demonstrated that in equilibrium, forward rates can be approximated as

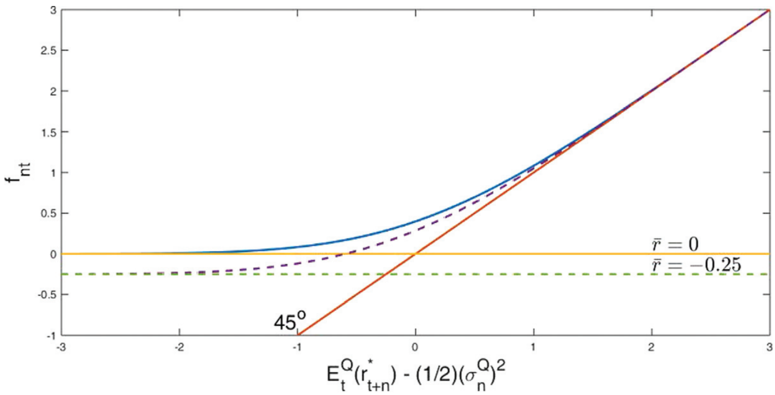
$$f_{nt} \simeq \bar{r} + \sigma_n^Q g\left(\frac{E_t^Q(r^*_{t+n}) - (1/2)(\sigma_n^Q)^2 - \bar{r}}{\sigma_n^Q}\right). \tag{7}$$

Here $g(z) = z\Phi(z) + \phi(z)$ for $\Phi(z)$ the cumulative distribution function for a standard normal variable and $\phi(z)$ the density, and

$$\begin{aligned}(\sigma_n^Q)^2 &= E_t^Q[r^*_{t+n} - E_t^Q(r^*_{t+n})]^2 \\ &= \frac{\sigma^2[1 - (\psi^Q)^{2n}]}{1 - (\psi^Q)^2}.\end{aligned}$$

To understand the intuition behind (7), suppose first that $g(.)$ was the identity function ($g(z) = z$). In this case (7) would simplify to $f_{nt} = E_t^Q(r^*_{t+n}) - (1/2)(\sigma_n^Q)^2$, corresponding to the case derived above when we are far from the lower bound. Figure 1 plots equation (7) for the general case when $g(z) = z\Phi(z) + \phi(z)$. When $E_t^Q(r^*_{t+n})$ is very far above the lower bound, $g(z)$ approaches the 45-degree line, and the forward rate is essentially the same as that

Figure 2. Effect of Changing the Lower Bound on the Forward Rate



predicted in the absence of a lower bound. When $E_t^Q(r^*_{t+n})$ is far below the lower bound, the value for f_{nt} is essentially the lower bound $\bar{r}$ itself. The $g(.)$ function thus provides a smooth pasting to generate a forward rate that is larger than $\bar{r}$ for all n and asymptotes to $E_t^Q(r^*_{t+n}) - (1/2)(\sigma_n^Q)^2$.

We can also see immediately from (7) the effects of a change in the lower bound:

$$\frac{\partial f_{nt}}{\partial \bar{r}} = 1 - g'.$$

Here g' denotes the derivative of the function $g(.)$, which analytically turns out to be given by $g'(z) = \Phi(z)$ and is bounded between 0 and 1 for all z. Thus

$$\frac{\partial f_{nt}}{\partial \bar{r}} = 1 - \Phi\left(\frac{E_t^Q(r^*_{t+n}) - (1/2)(\sigma_n^Q)^2 - \bar{r}}{\sigma_n^Q}\right).$$

As $E_t^Q(r^*_{t+n}) \to -\infty$, $\partial f_{nt}/\partial \bar{r} \to 1$, whereas when $E_t^Q(r^*_{t+n}) \to \infty$, $\partial f_{nt}/\partial \bar{r} \to 0$. The effect of changing $\bar{r}$ on the forward rate is illustrated in figure 2. Recalling (4), a decrease in the lower bound $\bar{r}$ should lower short-term yields nearly one-for-one but have a much more modest effect on long-term yields.

I have followed Grisse, Krogstrup, and Schumacher (this issue) up to this point in assuming that the shadow rate was described

Table 1. Changes in German Term Structure Associated with Changes in the ECB Lower Bound

Date	Old Rate	New Rate	Two Year	Five Year	Ten Year
June 5, 2014	0	−10	−1	−4	−1
September 4, 2014	−10	−20	−6	−6	1
December 2, 2015	−20	−30	14	18	20
March 10, 2016	−30	−40	8	10	6

Notes: Changes in yields (in basis points) of German government bonds of different maturities (GBBD02Y, GBBD05Y, GBBD10Y) on days of changes in ECB deposit rate cuts.

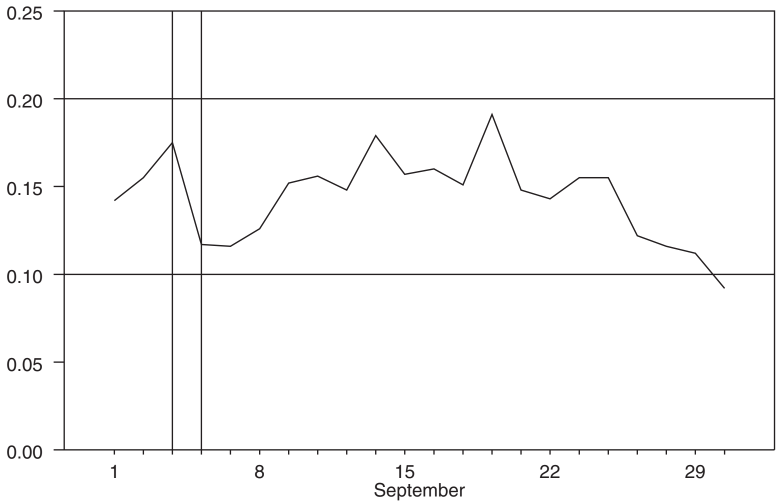
by a scalar AR(1) process. But it is straightforward to model it as part of a vector autoregression, as in Wu and Xia (2016). This generalization makes it possible to use the response of the entire term structure to individual announcements to infer parameters of the model. However, it would be necessary to augment the exercise with a description of how $\bar{r}_t$ could change over time and how investors form expectations about future values of $\bar{r}_{t+n}$. This exercise has recently been carried out by Wu and Xia (2017).

3. Empirical Evidence

Table 1 provides some tentative evidence extending the authors' analysis of changes in the European Central Bank's deposit rate into negative territory. Of these four episodes, only the drop in September 2014 looks much like the predicted theory which says that short rates should fall by less than the drop in the policy rate and long rates should fall less than short rates. For the last two episodes, yields on German government bonds actually rose on the days when the ECB cut the deposit rate. Here is the explanation from the *Wall Street Journal* for what happened on March 10, 2016:

> The ECB cut its deposit rate by 0.1 percentage point to minus 0.4% on Thursday, in line with investor expectations. But Mr. Draghi said the ECB had no imminent plans to cut rates further.

Figure 3. Interest Rates in September 2014



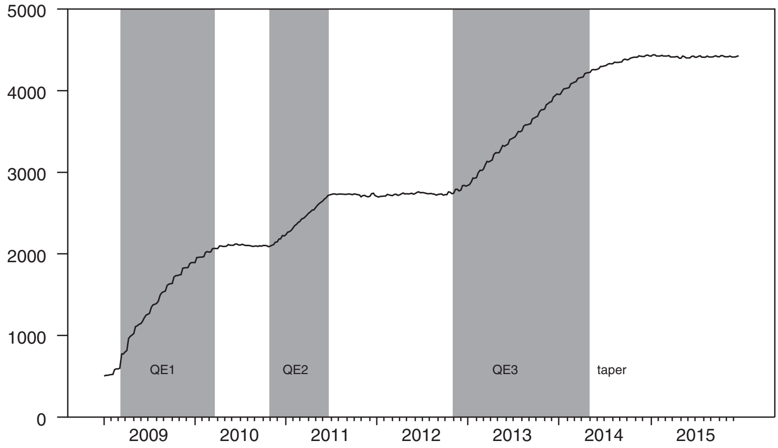
Note: Yield on five-year German bond (in percentage points) after ECB cut policy rate from –0.1 percent to –0.2 percent on September 4, 2014.

> That pushed up short-dated yields, which are particularly sensitive to interest rate moves. Two-year German bond yields, which are particularly sensitive to rate moves, rose sharply to −0.467 percent from −0.553 percent before the announcement.

Just as documented for normal times by Gürkaynak, Sack, and Swanson (2005), the news released to markets by a central bank announcement is more than a one-dimensional object. The market learns not just about the current level of the policy rate but also about its likely future trajectory. This highlights the need for an exercise as in Wu and Xia (2017), in which the shadow rate is part of a vector process and investors form expectations about possible future cuts in the lower bound.

It's also interesting to look in more detail at September 2014, the one episode among these four that seemed most consistent with the theory. As seen in figure 3, the drop in the five-year yield here proved to be temporary, and after a week the yield reached a higher level than it had been before the cut. One possible interpretation is that the cut in the deposit rate lowered the yield by 6 basis points, but subsequent shocks from other sources raised it back again. This is

Figure 4. Federal Reserve's Balance Sheet across Episodes of Quantitative Easing

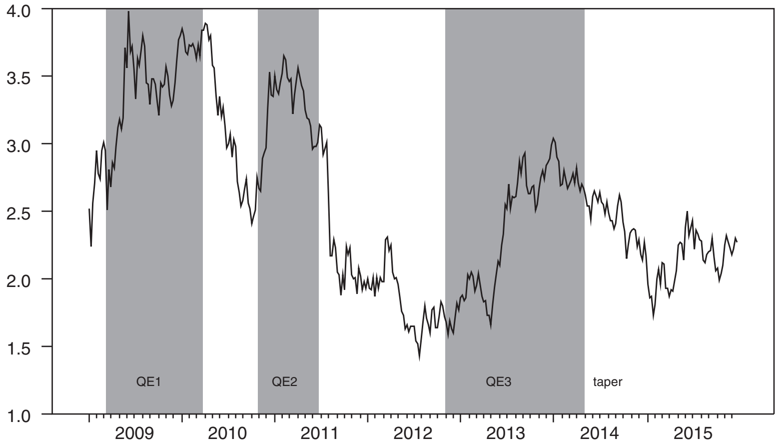


Notes: Federal Reserve holdings of Treasury, agency, and mortgage-backed securities, in billions of dollars, weekly, January 7, 2009 to December 23, 2015. Excludes inflation compensation and unamortized premiums or discounts. Shaded episodes denote March 18, 2009 to March 24, 2010; November 3, 2010 to June 2, 2011; and November 7, 2012 to April 30, 2014.

the implicit interpretation in the many hundreds of papers that have used event-study methodology to evaluate the effects of monetary shocks.

But all of this assumes that the market somehow knows within one day (or within fifteen minutes, for higher-frequency event studies) what the effect will be of a policy instrument that has never been used before. That's not necessarily always going to be the case. We saw a dramatic illustration on November 8, 2016, the day of the U.S. presidential elections. As election returns that night came in with the surprising news that Donald Trump would win the election, there was an immediate dramatic selloff in stock futures, giving an event-study estimate that the election outcome shaved 5 percent off the market value of capital. But the next morning this was all made up and then some. Should the reversal be interpreted as a new shock or as the result of the market continuing to absorb what the election outcome really meant? My view is that nobody knew Tuesday night, or Wednesday morning, and indeed may not know for years, the true implications of a Trump presidency.

Figure 5. Ten-Year Treasury Yields across Episodes of Quantitative Easing



Note: Yields on ten-year U.S. Treasury securities, weekly, January 7, 2009 to December 23, 2005.

This of course is an issue not just with the present paper, but with any studies relying on the event-study methodology. Consider for example the consequences of the three episodes of large-scale asset purchases conducted by the United States, popularly referred to as QE1, QE2, and QE3. These involved major additions to the Federal Reserve's balance sheet, as seen in figure 4. Several announcements were associated with significant drops in the ten-year yield on the day of the announcement, the most dramatic being a 50 basis point decline on March 18, 2009 when the first phase of QE1 was announced. Nevertheless, within a month the yield was back up to the level it had been before the announcement, and at the end of QE1 it stood nearly 100 basis points higher than its value before the program was announced (see figure 5). The event-studies conclusion of Krishnamurthy and Vissing-Jorgensen (2011) that QE1 lowered rates by over 100 basis points is thus implicitly assuming that there were other shocks that in the absence of the program would have raised rates by 200 basis points. Interest rates declined after QE1 ended, started to rise again under QE2, fell after the latter ended, rose under QE3, and fell after tapering of QE3 began, all exactly opposite the conclusions drawn from event studies.

None of this is to question the conventional wisdom that programs like large-scale asset purchases have the potential to lower interest rates in the presence of the zero lower bound, nor Grisse, Krogstrup, and Schumacher's conclusion that negative interest rates are an additional policy tool that could prove effective. Event studies are one of the best ways we have of trying to estimate the effects of these policies, and the current paper makes a useful contribution to the literature. But we should likewise not forget the limitations of this kind of empirical evidence.

References

Christensen, J. H. E., and G. D. Rudebusch. 2015. "Estimating Shadow-Rate Term Structure Models with Near-Zero Yields." *Journal of Financial Econometrics* 13 (2): 226–59.

Gürkaynak, R. S., B. Sack, and E. T. Swanson. 2005. "Do Actions Speak Louder Than Words? The Response of Asset Prices to Monetary Policy Actions and Statements." *International Journal of Central Banking* 1 (1): 55–93.

Hamilton, J. D., and J. C. Wu. 2014. "Risk Premia in Crude Oil Futures Prices." *Journal of International Money and Finance* 42: 9–37.

Krippner, L. 2013. "Measuring the Stance of Monetary Policy in Zero Lower Bound Environments." *Economics Letters* 118 (1): 135–38.

———. 2015. *Zero Lower Bound Term Structure Modeling: A Practitioner's Guide.* Palgrave Macmillan U.S.

Krishnamurthy, A., and A. Vissing-Jorgensen. 2011. "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy." *Brookings Papers on Economic Activity* (Fall): 215–65.

Ruge-Murcia, F. J. 2006. "The Expectations Hypothesis of the Term Structure When Interest Rates are Close to Zero." *Journal of Monetary Economics* 53 (7): 1409–24.

Wu, J. C., and F. D. Xia. 2016. "Measuring the Macroeconomic Impact of Monetary Policy at the Zero Lower Bound." *Journal of Money, Credit and Banking* 48 (2–3): 253–91.

———. 2017. "Time-varying Lower Bound of Interest Rates in Europe." Working Paper, University of Chicago.