Measuring Potential Growth with an Estimated DSGE Model of Japan’s Economy

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In this paper, we calculate the potential output and the output gap using a Bayesian-estimated DSGE model of Japan’s economy. For bridging the gap with conventional measures, we define our measure of potential output as a component of the efficient output generated only by persistent growth rate shocks. Our potential growth displays a high degree of smoothness and moves closely with conventional measures. Moreover, the output gap from our measure of potential output shows better forecasting performance for inflation—in particular, at short horizons—than other measures of output gap.

JEL Codes: E32, E37, O41, O47.

1. Introduction

It has been widely acknowledged that estimated dynamic stochastic general equilibrium (DSGE) models are able to fit the data as well as do reduced-form vector autoregression (VAR) models, as shown by Smets and Wouters (2003), Christiano, Eichenbaum, and Evans (2005), and Levin et al. (2006). A recent trend in developing DSGE

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models is to pursue their ability to “tell stories” in a policymaking context (Edge, Kiley, and Laforte 2008). For monetary and fiscal policy discussions, empirically plausible and theoretically coherent explanations for model-based estimates of potential output and output gap would be invaluable and essential.

Despite their conceptual importance in a policymaking context, measures of potential output and output gap from DSGE models are controversial. In general, model-based measures of potential output, which estimate an efficient level of output without pressure for inflation to either accelerate or decelerate, tend to be more volatile than conventional measures based on the production function approach or on statistical smoothing methods (e.g., the Hodrick-Prescott (HP) filter) which try to capture medium-term growth trends of output. This tendency reflects a significant difference in views between modelers and policymakers on which types of shocks drive the short-run macroeconomic fluctuations. While DSGE models attribute a substantial fraction of the fluctuations to fundamental shocks such as temporary technology shocks, policymakers’ traditional views implicitly assume that “animal spirit” expenditure shocks play a central role in the short-run fluctuations and that an efficient level of output is driven mainly by permanent technological changes.

The aim of this paper is to bridge the gap between the conventional and model-based measures of potential output using a Bayesian-estimated DSGE model of Japan’s economy. Our model shares many similar features with recent New Keynesian DSGE models in the literature and those practically used in central banks. A key feature of our model is that it takes into account persistent growth rate shocks, so that we can estimate directly the growth trend of output without detrending the data. Based on this model, we

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1For instance, Mishkin (2007) and Basu and Fernald (2009) discuss the characteristics of several measures and concepts of potential output.

2The model is a variant of the Medium-scale Japanese Economic Model (M-JEM), which was developed at the Bank of Japan’s Research and Statistics Department.

3Many DSGE models of Japan’s economy are estimated or calibrated using detrended data. For instance, Sugo and Ueda (2008) use the data detrended with kinked linear trends, and Ichise, Kurozumi, and Sunakawa (2013) use those detrended by potential output based on the production function approach. Hirose and Kurozumi (2012) consider technology growth rate shocks but use the output gap based on the production function approach in the estimation.
define our measure of potential output as a component of the efficient (flexible-price) level of output generated only by persistent growth rate shocks. This “long-run efficient output,” which corresponds to the long-run balanced growth path of the economy, displays a high degree of smoothness and moves closely with conventional measures of potential output, while the normally defined (short-run) efficient output in our model, which would be observed in the absence of nominal rigidities and shocks to price and wage markups, moves closely with the actual output and thus is more volatile than the conventional measures. We bridge the gap between the conventional and model-based measures of potential output by incorporating into our model-based measure the policymakers’ views that an efficient level of output is driven mainly by permanent technological changes.

Apart from its compatibility with the policymakers’ views, the long-run efficient output has some practical advantages over the short-run efficient output. First, the long-run efficient output may be more informative about short-term inflationary pressure than the short-run efficient output. We compare the predictability of inflation across several measures of output gap, which is defined as the deviation of the actual output from a measure of potential output. According to our results, the gap from the long-run efficient output shows better forecasting performance—in particular, at short forecast horizons—than the gap from the short-run efficient output and the conventional measures of output gaps. Second, the long-run efficient output is less sensitive to the specification or structural interpretation of the model. This feature is practically important because recent New Keynesian DSGE models are becoming more complex, and many aspects of them are controversial among researchers. Indeed, in those complex models, it is not necessarily straightforward to theoretically determine which measure of potential output represents a truly efficient level of output and which measure of output gap is most relevant to short-term inflationary pressure. Under these circumstances, the sensitivity to the details of models is a great concern for users of model-based measures.

In our working paper version (Fueki et al. 2010), we show that the long-run efficient output in our model is robust with respect to the specification of monetary policy rules and identifications of labor supply shocks, price and wage markup shocks, and measurement errors in prices and wages.
Moreover, our model has a two-sector production structure in which the final goods are explicitly divided into the consumption goods produced by a slow-growing sector and the investment goods produced by a fast-growing sector, so that the long-run efficient output in our model can be decomposed into the economy-wide and investment-specific technology growth rate shocks in addition to an exogenous population growth. This structure not only enhances the model’s empirical performance but also allows us to tell simple and plausible stories about the long-run growth trends of Japan’s economy: while the investment-specific technology growth rate shock has constantly raised the potential growth during the sample period since the 1980s, the economy-wide technology growth rate shock has reduced the potential growth since the 1990s.

A closely related analysis on DSGE model-based measures of potential output and output gap was conducted by Kiley (2013). He shows that the deviation of the U.S. actual output from its long-run stochastic trend (Beveridge-Nelson cycle) estimated from a DSGE model used at the Federal Reserve Board (the Estimated, Dynamic, Optimization-based, or EDO, model) is similar to the output gap based on the production function approach (the one calculated by the Congressional Budget Office). This result is similar to ours although the long-run efficient output in our model is a different concept from the Beveridge-Nelson stochastic trend. Meanwhile, some other recent studies investigate the cyclical properties and theoretical background of model-based measures of potential output and output gap. Sala, Söderström, and Trigari (2010) show that the output gap and the labor wedge are closely related in their estimated DSGE model, but the estimates are sensitive to the structural interpretation of shocks to the labor market. Justiniano, Primiceri, and Tambalotti (2013) show that the output gap in their model is procyclical and often quite large, but the policy trade-off between the stabilization of output gap and that of price and wage inflation is fairly weak on condition that the exogenous movements in the

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5 The two-sector production structure in our model reflects the secular trends in relative prices and different trends in categories of real expenditure apparent in the Japanese data. Moreover, this structure can generate empirically plausible co-movement between consumption and investment in response to investment-specific technology shocks.
competitiveness of the labor market are not a fundamental driver of macroeconomic fluctuations. Compared with their measures of potential output, the long-run efficient output in our model is less sensitive to the structural interpretation of labor market shocks.

The remainder of the paper is organized as follows. Section 2 describes our model. Section 3 explains the estimation procedures and shows the estimation results. In section 4, we calculate our measure of potential growth and compare it with alternative measures. In section 5, we calculate the several corresponding measures of output gap and compare the predictability of inflation across those measures. Section 6 concludes.

2. The Model

In this section, we provide an overview and a brief description of our model.

2.1 Overview

Our model is a two-sector growth model that takes into account persistent growth rate shocks including investment-specific technological progress. There are two final goods in the model: the consumption goods produced by the slow-growing sector and the investment goods produced by the fast-growing sector. We assume that the former goods are purchased by households and the government and that the latter goods are purchased by capital owners and foreign countries (net exports). The two-sector production structure with differential rates of technological progress across sectors induce different trends in categories of real expenditure and secular trends in relative prices, which are both apparent in the Japanese data.

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6 More details of the model, including the equilibrium conditions, stationary equilibrium conditions, and log-linearized system, are provided in appendix A of our working paper version (Fueki et al. 2010).

7 Our model closely follows the Federal Reserve Board’s EDO model (Edge, Kiley, and Laforte 2007; Chung, Kiley, and Laforte 2010). The two-sector representation of the investment-specific technological progress is also described by Whelan (2003), Ireland and Schuh (2008), and others.

8 In our data set from 1981 to 2009 (explained in section 3), the average annual growth rate of the real value added of the slow-growing sector is 1.86 percent, and
Meanwhile, our model shares many similar features with recent New Keynesian DSGE models in the literature, such as monopolistic competition, sticky prices and wages, adjustment costs, habit persistence, etc. The goods are produced in two stages by intermediate- and then final-goods-producing firms in each sector. The final-goods-producing firms aggregate differentiated sector-specific intermediate goods. The intermediate-goods-producing firms combine the aggregate labor inputs with utilized capital and set prices of their differentiated output. The capital owners rent their capital to the intermediate-goods-producing firms in both sectors. Households supply differentiated labor forces to the intermediate-goods-producing firms in both sectors. In what follows, we describe the decisions made by each of the agents in our economy.

\textbf{2.2 Final Goods Producers}

Final goods producers in the slow-growing sector (sector $c$) produce the consumption goods $X^c_t$, and those in the fast-growing sector (sector $k$) produce the investment goods $X^k_t$. They face competitive markets and produce the final goods, $X^s_t$, $s \in \{c, k\}$, by combining a continuum of $s$ sector-specific intermediate goods, $X^s_t(j)$, $j \in [0, 1]$, according to the following Dixit-Stiglitz type technology.

\begin{equation}
X^s_t = \left( \int_0^1 X^s_t(j) \frac{\Theta^x_{t,s} - 1}{\Theta^x_{t,s} - 1} \, dj \right) \frac{\Theta^x_{t,s}}{\Theta^x_{t,s} - 1}, \quad s = \{c, k\},
\end{equation}

where $\Theta^x_{t,s}$ is the elasticity of substitution between the differentiated intermediate goods input. Letting $\theta^x_{t,s}$ be the log-deviation from its steady-state value, we assume that $\theta^x_{t,s}$ follows an ARMA(1,1) process:

\begin{equation}
\theta^x_{t,s} = \rho \theta^x_{t,s} \theta^x_{t-1} + \epsilon^x_{t,s} - \rho \theta^x_{t,s}, \quad s = \{c, k\},
\end{equation}

that of the fast-growing sector is 2.81 percent, while the price of the investment goods relative to the consumption goods has declined at 1.75 percent per year on average.\footnote{Smets and Wouters (2007) assume that the price and wage markup shocks follow ARMA(1,1) processes to capture the high-frequency fluctuations in price and wage inflations.}

\begin{equation}
\theta^x_{t,s} = \rho \theta^x_{t,s} \theta^x_{t-1} + \epsilon^x_{t,s} - \rho \theta^x_{t,s}, \quad s = \{c, k\},
\end{equation}
where \( \epsilon^{\theta,x,s}_t \) is an i.i.d. shock process. This stochastic elasticity of substitution introduces transitory markup shocks into the pricing decisions of intermediate goods producers. Subject to the above aggregation technology, a final goods producer in each sector chooses the optimal level of each intermediate goods to minimize the cost of purchasing them, taking their prices as given.

### 2.3 Intermediate Goods Producers

Intermediate goods producers in both sectors face the monopolistically competitive market and produce the sector-specific intermediate goods \( X^s_t(j), s \in \{c, k\} \) with the following production function.

\[
X^s_t(j) = [K^{u,s}_t(j)]^\alpha [A^m_t Z^m_t A^s_t Z^s_t L^s_t(j)]^{1-\alpha},
\]

where \( K^{u,s}_t(j) \) and \( L^s_t(j) \) are the effective capital input and the labor input of a firm \( j \), respectively. Letting \( U^s_t(j) \) be the capital utilization rate in sector \( s \), the effective capital input is written as \( K^{u,s}_t(j) \equiv K^s_t(j) \times U^s_t(j) \). Further, the labor input of a firm \( j \) is the composite of the differentiated labor input, \( L^s_t(j) = \int_0^1 L^i_t(i,j)(\Theta^l_t-1)/\Theta^l_t di]^{\Theta^l_t/(\Theta^l_t-1)} \), where \( \Theta^l_t \) is the elasticity of substitution, and its log-deviation \( \theta^l_t \) follows an ARMA(1,1) process. This stochastic elasticity of substitution introduces transitory wage markup shocks into households’ labor supply decisions.

\( A^m_t Z^m_t \) is the economy-wide technology shock and \( A^k_t Z^k_t \) is the fast-growing (investment-goods-producing) sector-specific technology shock. In order to reduce the number of shocks in the model, we presume that the slow-growing (consumption-goods-producing) sector does not have the sector-specific shock \( A^c_t = Z^c_t = 1 \). We assume that each of the technology shocks contains two separate stochastic components: one \( (A^n_t) \) is stationary in levels and the other \( (Z^n_t) \) is stationary in growth rates, where \( n \in \{m, k\} \).

\[
\ln A^n_t = \ln A^n_* + \epsilon^a_{n,t} \quad (4)
\]

\[
\ln Z^n_t - \ln Z^n_{t-1} = \ln \Gamma_{n,t} = \ln (\Gamma_*^{z,n} \times \exp[\gamma_{n,t}^{z,n}]) = \ln \Gamma_*^{z,n} + \gamma_{n,t}^{z,n} \quad (5)
\]

\[
\gamma_{n,t}^{z,n} = \rho_{n}^{z,n} \gamma_{n,t-1}^{z,n} + \epsilon_{n,t}^{z,n}, \quad (6)
\]
where $\epsilon_{a,n}^t$ and $\epsilon_{z,n}^t$ are i.i.d. shock processes, and $A_n^*$ and $\Gamma_{z,n}^*$ are the constant technology level and growth rate, respectively. (Hereafter, variables with subscript * represent the variables at steady state.)

An intermediate goods producer $j$ in sector $s \in \{c, k\}$ maximizes the discounted future profit,

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_c^t}{P_c^t} \left\{ P_s^t(j) X_s^t(j) - MC_s^t(j) X_s^t(j) \right\} - \frac{100 \cdot \chi_p}{2} \left( \frac{P_s^t(j)}{P_{s-1}^t(j)} - \eta^p \Pi_{p,s}^t \right)^2 \right\} P_s^t X_s^t, \quad (7)$$

subject to the final goods producers’ demand schedule,

$$X_s^t(j) = \left( \frac{P_s^t(j)}{P_s^t} \right)^{-\Theta_{t,s}^x} X_s^t, \quad (8)$$

taking as given the marginal cost of production, $MC_s^t(j)$, the aggregate price level for its sector, $P_s^t = \left\{ \int_{1}^{1} [P_s^t(j)]^{(\Theta_{t-1}^s)/\Theta_t^s} dj \right\} \Theta_t^s / (\Theta_{t-1}^s)$, and households’ valuation of a unit nominal income in each period, $\Lambda_c^t / P_c^t$, where $\Lambda_c^t$ is the marginal utility of consumption. The second line in (7) represents the quadratic price adjustment cost as in Rotemberg (1982), where $\Pi_{p,s}^t = P_s^t / P_{s-1}^t$ and $\Pi_{p,s}^t$ is the time-invariant trend inflation. Since the cost is imposed on the deviation of the optimum price inflation from the past inflation, the equilibrium inflation as well as the price response to the marginal cost becomes sticky.

2.4 Capital Stock Owners

Capital stock owners provide the capital service to the intermediate goods producers in both sectors, receive the rental cost of capital in exchange, and accumulate the investment goods. Each capital stock owner $k$ chooses investment expenditure, $I_t^k(k)$, the amount and utilization of capital in both sectors, $K_c^t(k), U_c^t(k), K_k^t(k),$ and $U_k^t(k)$, to maximize its discounted profit,

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_c^t}{P_c^t} \left[ R_t^c U_c^t(k) K_c^t(k) + R_t^k U_k^t(k) K_k^t(k) - P_t^k I_t(k) \right], \quad (9)$$
subject to the capital evolution process with quadratic investment adjustment cost and the costs from higher utilization rates,

\[
K_{t+1}(k) = (1 - \delta)K_t(k) + I_t(k)
\]

\[
- \frac{100 \cdot \chi}{2} \left[ \frac{I_t(k)A_t^c - I_{t-1}(k)\Gamma_t^{z,m}\Gamma_t^{z,k}}{K_t} \right]^2 K_t
\]

\[
- \sum_{s=c,k} \kappa \left[ \frac{(Z_t^U U_t^s(k))^{1+\psi} - 1}{1 + \psi} \right] K_t^s,
\]

(10)

where \(K_t(k) = K_t^c(k) + K_t^k(k)\). The third term (in the second line) of the right-hand side is the quadratic investment adjustment cost, where \(A_t^c\) is a stochastic variation in the adjustment cost that is assumed to be unity at the steady state and to follow an AR(1) process. The last term of the right-hand side is the utilization cost, which presumes that higher capital utilization leads to faster capital depreciation, as in Greenwood, Hercowitz, and Krusell (1997). \(Z_t^U\) is a stochastic variation in the utilization cost that is assumed to be common in both sectors and to follow an AR(1) process. We set \(\kappa\) so that the utilization rate is unity at the steady state.

2.5 Households

Each household \(i\) chooses its purchase of consumption goods, \(C_t(i)\), its holdings of bonds, \(B_t(i)\), its wages for both sectors, \(W_t^c(i)\) and \(W_t^k(i)\), and supply of labor consistent with each wage, \(L_t^c(i)\) and \(L_t^k(i)\), given the demand schedule for the differentiated labor supply, \(L_t^c(i) = (W_t^c(i)/W_t^c)^{-\Theta_t} L_t^c\) and \(L_t^k(i) = (W_t^k(i)/W_t^k)^{-\Theta_t} L_t^k\), to maximize the utility function,

\[
E_0 \sum_{t=0}^{\infty} \beta^t \Xi_t \left\{ \zeta^c \ln (C_t(i) - hC_{t-1}(i)) - \zeta^i \left[ \frac{(L_t^c(i) + L_t^k(i)) / \Xi_t}{1 + \nu} \right]^{1+\nu} \right\},
\]

(11)
subject to its budget constraint,

\[ \frac{1}{R_t} B_{t+1}(i) = B_t(i) + \sum_{s=c,k} W^s_t(i)L^s_t(i) + \Omega_t(i) - P^c_tC_t(i) \]

\[ - \sum_{s=c,k} \frac{100 \cdot \chi^w}{2} \left\{ \frac{W^s_t(i)}{W^s_{t-1}(i)} - \eta^w_t \Pi^w,s_{t-1} - (1 - \eta^w_t) \Pi^w,s_\ast \right\}^2 \]

\[ \times W_t^s L_t^s \left( \frac{L^c_t}{L^c_\ast + L^k_t} W^c_t + \frac{L^k_t}{L^c_\ast + L^k_t} W^k_t \right) \]

\[ \times \left( \frac{L^c_t(i)}{L^c_t(i)} - \eta^l_t L^c_{t-1(i)} - (1 - \eta^l_t) \frac{L^c_\ast}{L^c_\ast} \right)^2 \frac{L^k_t}{L^c_t}. \] (12)

In the utility function, \( \Xi_t^\beta \) is the intertemporal preference shock, \( \Xi_t^l \) is the labor supply shock (intratemporal preference shock), \( \varsigma^c \) and \( \varsigma^l \) are scale parameters that determine the ratio between the household’s consumption and leisure, and \( h \) is the degree of the habit persistence of the household. We assume that the log-deviation of the intertemporal preference shock follows an AR(1) process and that the labor supply shock is non-stochastic (\( \Xi_t^l = 1 \)) to properly identify the wage markup shock. In the budget constraint, \( R_t \) is the nominal interest rate on the bonds and \( \Omega_t(i) \) is the household’s capital and profits income. The fifth term (in the second line) of the right-hand side is the quadratic wage adjustment cost imposed on the deviation of the wage growth from the past wage inflation, \( \Pi^w,s_{t-1} \), and from the trend wage inflation, \( \Pi^w,s_\ast \). With this formulation, the wage inflation as well as the wage level becomes sticky. The last term of the right-hand side is the labor reallocation cost, which helps to generate realistic sectoral co-movement of labor inputs during business cycles.

### 2.6 Real GDP Growth and GDP Deflator Inflation

Since the trend growth rate is different in each sector, we aggregate the real GDP as a divisia index, following Whelan (2003) and Edge, Kiley, and Laforte (2007). This divisia-index aggregation allows us

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10 We consider the case of stochastic labor supply shock in section 6.2 of our working paper version (Fueki et al. 2010).
to avoid the base-year bias of the deflator and to mimic the SNA data compiled with the chain-index aggregation. The growth rate of the real GDP is calculated as

\[ H_{t}^{dp} = \left[ \left( \frac{X_{t}^{c}}{X_{t-1}^{c}} \right) P_{t}^{c} X_{t}^{c} \left( \frac{X_{t}^{k}}{X_{t-1}^{k}} \right) P_{t}^{k} X_{t}^{k} \right]^{1} \frac{1}{P_{t}^{c} X_{t}^{c} + P_{t}^{k} X_{t}^{k}}. \] (13)

The inflation rate of the GDP deflator, \( \Pi_{t}^{p,dp} H_{t}^{dp} \), is implicitly defined by

\[ \Pi_{t}^{p,dp} H_{t}^{dp} = \frac{P_{t}^{dp} X_{t}^{dp}}{P_{t-1}^{dp} X_{t-1}^{dp}} = \frac{P_{t}^{c} X_{t}^{c} + P_{t}^{k} X_{t}^{k}}{P_{t-1}^{c} X_{t-1}^{c} + P_{t-1}^{k} X_{t-1}^{k}}. \] (14)

### 2.7 Monetary Authority

Following Chung, Kiley, and Laforte (2010), we assume that the monetary authority sets the short-term nominal interest rate following a Taylor-type feedback rule with interest rate smoothing.

\[ R_{t} = (R_{t-1})^{\phi^{r}} (\bar{R}_{t})^{1-\phi^{r}} \exp (\epsilon_{t}^{r}) \] (15)

\[ \bar{R}_{t} = R_{*} \left( \tilde{X}_{t} \phi^{h,dp} \left( \frac{X_{t}^{c}}{X_{t-1}^{c}} \right) \phi^{\Delta h,dp} \left( \frac{\Pi_{t}^{p,dp}}{\Pi_{t}^{p,dp}} \right) \phi^{\pi,dp} \right), \] (16)

where \( \phi^{r} \) is the degree of interest rate smoothing, \( \epsilon_{t}^{r} \) is the interest rate shock, and \( \phi^{h,dp}, \phi^{\Delta h,dp}, \) and \( \phi^{\pi,dp} \) are the degrees of responsiveness in the policy rule. \( \tilde{X}_{t} \) is the deviation of real GDP, which is calculated as a divisia index similarly to its growth rate (13), from its efficient level (the short-run efficient output defined in section 4).\[^{11}\]

### 2.8 Market Clearing

Before closing the model, we assume that government expenditure, \( G_{t} \), and net exports, \( F_{t} \), are produced by the slow-growing sector

\[^{11}\text{We consider the case of the monetary policy rule that responds to the output gap from the long-run efficient output in section 6.1 of our working paper version (Fueki et al. 2010).}\]
and fast-growing sector, respectively. Both factors are stochastic and obey AR(1) processes as follows.

\[
\ln G_t - \ln \{Z_t^m (Z_t^k)^{\alpha} (Z_t^c)^{1-\alpha}\} = \ln \tilde{G}_t = \rho^g \ln \tilde{G}_{t-1} + \epsilon^g_t \quad (17)
\]

\[
\ln F_t - \ln \{Z_t^m Z_t^k\} = \ln \tilde{F}_t = \rho^f \ln \tilde{F}_{t-1} + \epsilon^f_t \quad (18)
\]

At the symmetric equilibrium, each market clears.

\[
X^c_t = \int_0^1 C_t(i) di + G_t + \frac{100 \cdot \chi^w}{2} \left[ \Pi^w,c_t - \eta^w \Pi^w,c_{t-1} - (1 - \eta^w) \Pi^w_\ast \right]^2 \times W^c_t L^c_t + \frac{100 \cdot \chi^P}{2} \left[ \Pi^p,c_t - \eta^P \Pi^p,c_{t-1} - (1 - \eta^P) \Pi^p,c_\ast \right]^2 P^c_t X^c_t
\]

\[
+ \frac{100 \cdot \chi^l}{2} \left( \frac{L^c_t}{L^c_\ast + L^c_\ast} W^c_t + \frac{L^k_t}{L^k_\ast + L^k_\ast} W^k_t \right)
\]

\[
\times \left\{ \frac{L^c_t}{L^c_\ast} - \eta^l \frac{L^c_{t-1}}{L^c_{t-1}} - (1 - \eta^l) \frac{L^c_\ast}{L^c_\ast} \right\}^2 \frac{L^k_t}{L^k_\ast}
\]

\[
(19)
\]

\[
X^k_t = \int_0^1 I_t(k) dk + F_t + \frac{100 \cdot \chi^w}{2} \left[ \Pi^w,k_t - \eta^w \Pi^w,k_{t-1} - (1 - \eta^w) \Pi^w_\ast \right]^2 \times W^k_t L^k_t + \frac{100 \cdot \chi^P}{2} \left[ \Pi^p,k_t - \eta^P \Pi^p,k_{t-1} - (1 - \eta^P) \Pi^p,k_\ast \right]^2 P^k_t X^k_t
\]

\[
(20)
\]

\[
L^s_t(i) = \int_0^1 L^s_t(i, j) dj, \quad \forall i \in [0, 1], \ s \in \{c, k\}
\]

\[
(21)
\]

\[
\int_0^1 U^s_t(k) K^s_t(k) dk = \int_0^1 K^u,s_t(j) dj, \quad \forall i \in [0, 1], \ s \in \{c, k\}
\]

\[
(22)
\]

3. Model Estimation

3.1 Estimation Procedures

We solve the model and estimate its structural parameters using Bayesian methods. Since persistent growth rate shocks in the system make some of the endogenous variables non-stationary, we divide the non-stationary variables by the corresponding I(1) trends and stationarize the model. We then log-linearize the set of equilibrium
conditions, solve the linear rational expectations system, and obtain the transition dynamics of the whole system.

\[ \hat{\zeta}_t = G(\vartheta)\hat{\zeta}_{t-1} + H(\vartheta)\hat{\varepsilon}_t, \]

(23)

where \( \hat{\zeta}_t \) is a properly defined \( k \times 1 \) vector of stationarized and log-linearized endogenous variables, \( \hat{\varepsilon}_t \) is the \( n \times 1 \) vector of exogenous i.i.d. disturbances, and \( \vartheta \) is the \( p \times 1 \) vector of structural unknown coefficients. \( G(\vartheta) \) and \( H(\vartheta) \) are the conformable matrices of coefficients that depend on the structural parameters \( \vartheta \).

To estimate the model, we specify the observation equation,

\[ x_t = J\hat{\zeta}_t + \mu, \]

(24)

where \( x_t \) is the observed data described in the next subsection and \( \mu \) is the vector of constant terms. Since the variables in our model \( \hat{\zeta}_t \) include persistent growth rate shocks, we do not detrend or demean any data series, while some of the data are transformed into log-differences. Also, we do not incorporate measurement errors into the observation equation.\(^{12}\)

Letting \( x^T \) be a set of observable data, the likelihood function \( L(\vartheta, x^T) \) is evaluated by applying the Kalman filter. We combine the likelihood function \( L(\vartheta, x^T) \) with priors for the parameters to be estimated, \( p(\vartheta) \), to obtain the posterior distribution, which is proportional to \( L(\vartheta, x^T)p(\vartheta) \). Since we do not have a closed-form solution of the posterior, we rely on Markov chain Monte Carlo (MCMC) methods using Dynare. Draws from the posterior distribution are generated with the Metropolis-Hastings algorithm.\(^{13}\)

We obtain the posterior median estimates and posterior intervals

\(^{12}\)This is an important difference from the Federal Reserve Board’s EDO model, which incorporates measurement errors in most variables in the observation equation. As we discuss in section 6.3 of our working paper version (Fueki et al. 2010), where some alternative models with measurement errors in prices and wages are estimated, measurement errors can drastically affect the estimation results and complicate structural interpretation of shocks.

\(^{13}\)A sample of 800,000 draws was created (neglecting the first half of these draws). Our selected step size for the jumping distribution in the Metropolis-Hastings algorithm results in an acceptance ratio of 0.39. The resulting sample properties are not sensitive to the step size. To test the stability of the sample, we use the convergence diagnostic based on Brooks and Gelman (1998).
of unobservable model variables, including the efficient output, by applying the Kalman smoother.

3.2 Data

The model is estimated using ten key macroeconomic quarterly Japanese time series from 1981:Q1 to 2009:Q4 as observed data: nominal value added of the slow-growing sector, nominal value added of the fast-growing sector, nominal household consumption, nominal business investment, deflator of the slow-growing sector, deflator of the fast-growing sector, compensation of employees, total hours worked, short-term nominal interest rate (call rate) and capital utilization rate (operating ratio). All the variables, except for the last two, are transformed into log-differences. None of them, however, are detrended or demeaned.

The nominal value added of the slow-growing sector is the sum of nominal household consumption (including residential investment) and nominal government expenditure. The nominal value added of the fast-growing sector is the sum of nominal business (non-residential) investment and nominal net exports. Those GDP components are transformed into a per-capita base (divided by the population over fifteen years old).

Our model has two price indices for the slow-growing and the fast-growing sectors. In order to match the SNA data with our theoretical model, we construct the chain index of the real value added of each sector and calculate the implicit deflator, following Whelan (2003).

3.3 Estimation Results

The model’s calibrated parameters are presented in table 1, and the estimated parameters are reported in table 2. Referring to previous

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14 The GDP data are the second preliminary quarterly estimates released in March 2010.

15 The sample period includes the period after the short-term nominal interest rate effectively hit the zero lower bound. However, the estimation results using the data up to 1998:Q4, just before the Bank of Japan started the zero interest rate policy, are not much different from the baseline results using the full sample data.

16 More details of the data are summarized in appendix B of our working paper version (Fueki et al. 2010).
studies, we set six structural parameter values such as households’ subjective discount rate, capital share, capital depreciation, and elasticity of substitution. We also set the steady-state values based on historical averages of the data. Meanwhile, we estimate fourteen structural parameters as well as the parameters that characterize thirteen shock processes. In general, most of our posterior estimates of the structural parameters are consistent with previous studies.

Next we report the variance decompositions in table 3. It shows the posterior mean estimates of forecast error variance decompositions of output (real GDP) growth, consumption growth, investment growth, and GDP deflator inflation at forecast horizon $T = 1$ and $100$. The output fluctuations, both in the short and in the long run, are mainly caused by the technology shocks (economy wide and investment specific), the investment adjustment cost shocks, and the intertemporal preference shocks. The contributions of the investment-specific technology shocks are smaller than those of the economy-wide technology shocks, as in Hirose and Kurozumi (2012). The investment adjustment cost shocks contribute substantially to the investment fluctuations, and the intertemporal preference shocks contribute substantially to the consumption fluctuations. Meanwhile, the inflation fluctuations are mainly caused by the consumption goods price markup shocks. The investment-specific technology shocks and the intertemporal preference shocks also have large contributions to the long-run fluctuations in inflation.

Finally, we report the impulse responses of key variables to the economy-wide and investment-specific technology shocks in figure 1.

17 Although the GDP growth rate has declined during the sample period, we set the steady-state growth rates to its full sample average. The estimation results using the data from 1991:Q1, when Japan’s “lost decade” started, are not much different from the baseline results using the full sample data.

18 Prior and posterior distributions of the model parameters are shown in figure 1 of our working paper version (Fueki et al. 2010).
Table 2. Estimated Parameter Values (Prior and Posterior Distributions)

<table>
<thead>
<tr>
<th>Param.</th>
<th>Prior Distribution</th>
<th>Prior Mean</th>
<th>Prior S.D.</th>
<th>Posterior Distribution</th>
<th>Mean</th>
<th>5th Percentiles</th>
<th>95th Percentiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$</td>
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<td>0.49</td>
<td>0.39</td>
<td>0.59</td>
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<tr>
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<td>0.06</td>
<td>0.55</td>
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<tr>
<td>$\chi^p$</td>
<td>Gamma</td>
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<td>13.85</td>
<td>8.21</td>
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<td>0.16</td>
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<td>Gamma</td>
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<td>7.56</td>
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<td>0.96</td>
<td>0.99</td>
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<td>0.95</td>
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(continued)
Table 2. (Continued)

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<th>Prior S.D.</th>
<th>Posterior Distribution</th>
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<td>Mean</td>
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<td>Mean</td>
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<tr>
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<td>2</td>
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<td>Mean</td>
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<td>Mean</td>
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<tr>
<td></td>
<td>Output</td>
<td>Consumption</td>
<td>Investment</td>
<td>Inflation</td>
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<tr>
<td>------------------------------------------------------</td>
<td>--------</td>
<td>-------------</td>
<td>------------</td>
<td>-----------</td>
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<tr>
<td></td>
<td>$T = 1$</td>
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<tr>
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<td>7.03</td>
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<td>0.3</td>
</tr>
</tbody>
</table>

Note: Each table shows variance decompositions of the output growth rate, the consumption growth rate, the investment growth rate, and the inflation rate.
Figure 1. Responses to Structural Shocks

Notes: Each graph shows the impulse responses to a shock equal to one standard deviation. All impulse responses are reported as percentage deviations from non-stochastic steady state. EWT: the economy-wide technology shock (persistent growth rate shock); IST: the investment-specific technology shock (persistent growth rate shock); EWTL: the economy-wide technology shock (in level); ISTL: the investment-specific technology shock (in level).

The persistent technology growth rate shocks, either economy wide or investment specific, increase output and labor input, while the technology level shocks decrease labor input. This relates to the following result that our measure of potential output (long-run efficient output) driven by persistent growth rate shocks moves procyclically. Meanwhile, the economy-wide and investment-specific technology shocks, either in growth rate or in level, increase both

\[^{19}\text{Christiano, Trabandt, and Walentin (2010) discuss this point in detail.}\]
consumption and investment. The notable result here is that the investment-specific technology shocks increase consumption as well as investment, which is hard to obtain in a one-sector model. An important advantage of our two-sector model is that it generates empirically plausible co-movement between consumption and investment.

4. Potential Growth

In this section, we calculate our measure of potential growth and compare it with alternative measures.

4.1 Several Measures of Potential Output

First, we calculate the (short-run) efficient output, which is usually considered a DSGE model-based measure of potential output. It is defined as the level of output in an environment without nominal rigidities in goods and labor markets and without shocks to price and wage markups. Figure 2 shows the year-on-year growth rate of the above-defined efficient output, which moves closely with the actual output (real GDP). This implies that a substantial fraction of the actual economic fluctuations is viewed as efficient in our model.

As discussed in the introduction, many policymakers’ traditional views implicitly assume that the short-run fluctuations are inefficient and that an efficient level of output is driven by permanent

---

20 The responses of consumption and investment are generally consistent with those in the VAR results of Braun and Shioji (2007).
22 We calculate in this paper the “unconditional” efficient output based on the state variables in the counterfactually efficient allocation from the past to the future. In our working paper version (Fueki et al. 2010), we also show the “conditional” efficient output (Adolfson et al. 2011) calculated using the actual values of the state variables and assuming that the allocation becomes unexpectedly efficient (prices and wages become flexible) today and is expected to remain efficient in the future, which does not qualitatively affect our discussion below.
23 We show the total efficient output in the figure by multiplying the per-capita efficient output by the population over fifteen years old. The per-capita efficient output we calculate is the Kalman-smoothed posterior median. The year-on-year growth rate is calculated as the sum of the quarter-on-quarter growth rates for a year.
technological changes. In order to bridge the gap between model-based measures and conventional measures of potential output, we define our measure of potential output (long-run efficient output) as a component of the efficient output generated only by persistent growth rate shocks. The year-on-year growth rate of this long-run efficient output, which corresponds to the long-run balanced growth path of the economy, is also shown in figure 2 together with the short-run efficient output and the actual output. Compared with the growth rate of the short-run efficient output, our measure of potential growth displays a higher degree of smoothness. In the
In the lower panel of figure 2, we compare the year-on-year growth rate of the long-run efficient output with the HP-filtered output and the potential output based on the production function approach (PFA) by Hara et al. (2006). Our measure of potential growth moves closely with those conventional measures of potential growth.

4.2 Decomposition of Potential Growth

In figure 3 we decompose the year-on-year growth rate of the long-run efficient output into component parts generated by each source. Since the persistent growth rate shocks we consider in our model are the economy-wide and investment-specific technology shocks, our measure of potential growth can be decomposed into those two types of technology growth rate shocks in addition to the exogenous population growth. While the investment-specific technology growth rate shock has constantly raised the potential growth during the sample period, the economy-wide technology growth rate shock has reduced the potential growth since the 1990s, except in the early 2000s when information technology (IT) propagated.

\[24\] Braun and Shioji (2007) show that the investment-specific technological progress sustained the potential growth of Japan’s economy, even in the 1990s, by calibrating a neoclassical growth model.
through the economy. This widening of the difference in the pace of technological progress between the two sectors could result in sluggish reallocation or misallocation of resources in the labor and financial markets, which in turn could lead to further decline in the economy-wide technology growth. This decomposition makes different but somewhat related stories from those in the PFA-based “growth accounting,” in which the capital inputs and the total factor productivity have raised the potential growth while the labor inputs have reduced it.

5. Output Gap

Based on several measures of potential output discussed in the previous section, we can calculate the several corresponding measures of output gap, which is defined as the deviation of the actual output from a measure of potential output. In this section, we compare several measures of output gap.

5.1 Several Measures of Output Gap

The upper panel of figure 4 shows the output gaps from the short-run and long-run efficient outputs. The former is less volatile than the latter, because the short-run efficient output moves more closely with the actual output than the long-run efficient output as shown in figure 2. These model-based output gaps move procyclically in

---

25 Fueki and Kawamoto (2009) suggest the possibility that Japan experienced IT-driven pickup in productivity growth in the 2000s, which occurred not only in the investment-goods-producing sector but also in the consumption-goods-producing sector.

26 Hayashi and Prescott (2002) show that Japan’s “lost decade” in the 1990s can be explained by the fall in the growth rate of total factor productivity and by the reduction of the workweek length, using the growth accounting and a one-sector neoclassical growth model. They conjecture that the low productivity growth was the result of policy-induced misallocation in which inefficient firms and declining industries were subsidized. Caballero, Hoshi, and Kashyap (2008) discuss the possibility that Japanese banks’ lending to otherwise insolvent firms (“zombies”) had distorting effects on healthy firms and played an important role in the productivity slowdown in the lost decade.
The lower panel of figure 4 shows the output gap from the long-run efficient output together with the gaps from the HP-filtered output and the PFA-based potential output. The gaps from these conventional measures of potential outputs are as volatile as the gap from the long-run efficient output and they move closely with each other.

Note: Shaded regions show the ESRI recession dates.
5.2 Predictability of Inflation

The conventional measures of output gap discussed above have been widely used for forecasting future inflation. Meanwhile, within the New Keynesian theoretical framework, the gap from the short-run efficient output should be a relevant measure that indicates inflationary pressure. The gap from the long-run efficient output, which moves closely with the conventional measures, may also have some theoretical relevance to inflationary pressure and predictive ability for actual inflation. We compare the predictability of inflation across those several measures of output gap.

We evaluate the predictability of inflation by comparing bivariate models of output gap and inflation with an univariate autoregressive (AR) model of inflation, following Coenen, Smets, and Vetlov (2009). The general specification of the bivariate models is as follows:

$$\pi_{t+h} = a + b(L)\pi_t + c(L)x_t + \epsilon_{t+h},$$

(25)

where $\pi_{t+h}$ is the annualized $h$-period percent change in GDP or consumption goods deflator, $\pi_t$ is the annualized one-period inflation ($= \pi_{1t}$), $x_t$ is each measure of output gap, and $b(L)$ and $c(L)$ are finite polynomials of order $p$ and $q$ selected by the Schwarz information criteria. Parameters are estimated by ordinary least squares on rolling samples from 1985:Q1 to 1999:Q1 through 1985:Q1 to 2009:Q4. We then calculate the mean squared forecast errors (MSFE) of the bivariate models (25) and a univariate autoregressive model of inflation at forecast horizons ($h$) of one, four, and eight quarters ahead. The results are summarized in table 4. Judging from the MSFE of the bivariate models relative to that of the univariate AR model, the measures of output gap we consider generally have forecasting power for inflation when they are included in the bivariate models in addition to the lagged inflation. The comparison of the forecasting power across those measures of output gap reveals that our measure of output gap, the gap from the long-run efficient output, shows better performance—in particular, at short forecast horizons—than

\footnote{In a somewhat complex New Keynesian DSGE model including ours, however, the relationship between the output gap and inflation is not necessarily straightforward due to the existence of wage rigidity, capital accumulation, and so on.}
Table 4. Analysis of Forecast Accuracy

<table>
<thead>
<tr>
<th></th>
<th>GDP Deflator</th>
<th>Consumption Goods Deflator</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSFE</td>
<td>Relative to AR</td>
</tr>
<tr>
<td></td>
<td>Horizon 1q</td>
<td>Horizon 4q</td>
</tr>
<tr>
<td>GAP from the PFA-Based Output</td>
<td>2.98</td>
<td>0.74</td>
</tr>
<tr>
<td>GAP from the HP-Filtered Output</td>
<td>3.33</td>
<td>0.38</td>
</tr>
<tr>
<td>GAP from the Short-Run Efficient Output</td>
<td>2.73</td>
<td>0.69</td>
</tr>
<tr>
<td>GAP from the Long-Run Efficient Output</td>
<td>2.63</td>
<td>0.41</td>
</tr>
<tr>
<td>GAP from the Alternative Efficient Output 1</td>
<td>2.70</td>
<td>0.61</td>
</tr>
<tr>
<td>GAP from the Alternative Efficient Output 2</td>
<td>3.32</td>
<td>0.73</td>
</tr>
<tr>
<td>AR</td>
<td>3.81</td>
<td>0.77</td>
</tr>
</tbody>
</table>
other measures. For the GDP deflator, our measure of output gap gives the best performance at the one-quarter horizon, while the gap from the HP-filtered output gives the best performance beyond the four-quarter horizon. For the consumption goods deflator, our measure again gives the best performance at the one-quarter horizon, while the gaps from the PFA-based potential output and from the HP-filtered output give the best performance at the four-quarter and eight-quarter horizons, respectively. Meanwhile, the gap from the short-run efficient output shows consistently poorer performance than the gap from the long-run efficient output.

A disadvantage of the conventional measures of output gap, the gaps from the HP-filtered output and from the PFA-based potential output, in forecasting inflation is that they are calculated without using information on the actual inflation. The inflation data in Japan, either the GDP deflator or consumption goods deflator, have been persistently driven by many structural factors, including deregulation and import competition, and accordingly have a clear downward trend from the 1990s to the 2000s, but those factors cannot be considered in calculation of the conventional measures of output gap. Our measure of output gap is calculated using all information in the model and, as a result, moves more closely with the actual inflation than the conventional measures: for example, it was higher than the conventional measures in the 1990s while lower than the conventional measures in the 2000s, as shown in the lower panel of figure 4. Moreover, our measure also captures high-frequency movements in many temporary shocks, including price markup shocks that explain most of the short-run fluctuations in inflation according to the model’s variance decomposition shown in table 3, which is related to the better forecasting performance of our measure especially at short forecast horizons.

Meanwhile, the relatively poor performance of the gap from the short-run efficient output may imply some misspecification or misinterpretation in our model. In order to investigate the background

\[28\] At longer forecast horizons such as four quarters and eight quarters, the forecast performances of the univariate AR models are much better than those at the one-quarter horizon, which makes the advantages of our measure of output gap mentioned above relatively marginal when included in the bivariate models in addition to the lagged inflation.
of this result, we consider some alternative measures of efficient output. For instance, as a midpoint between the short-run and long-run efficient outputs, we can define a component of the efficient output generated by the technology shocks (both in level and growth rate), the investment adjustment cost shocks, the capital utilization adjustment cost shocks, and the intertemporal preference shocks as an alternative measure of efficient output. The gap from this variant of efficient output includes not only the inefficient response of output caused by nominal rigidity and the markup shocks but also the efficient (flexible-price) response of output to the monetary policy shocks, the government expenditure shocks, and the net export shocks. As shown in table 4, the gap from this “alternative efficient output 1” gives comparable forecasting performance to the short-run efficient output gap that includes only the inefficient response caused by nominal rigidity and the markup shocks. In the meantime, the long-run efficient output gap that includes the output response, either efficient or inefficient, to all kinds of temporary shocks shows in most cases substantially better forecasting performance than the above alternative gap as well as the short-run efficient output gap. These results may imply that our model has some misspecification or misinterpretation with respect to the efficient response to temporary shocks—in particular, the level technology shocks, the investment adjustment cost shocks, the capital utilization adjustment cost shocks, and the intertemporal preference shocks.

Moreover, the difference between the gaps from the short-run and long-run efficient output in the period from 2008 to 2009, when the global financial crisis affected Japan’s economy, may suggest that a possible candidate of the above-mentioned misspecification or misinterpretation in our model would be the efficient response of output to the investment adjustment cost shocks. As shown in the upper panel of figure 4, while the gap from the long-run efficient output declined sharply in the above period, the gap from the short-run efficient output did not decline so much. This large

\[29\]
The “alternative efficient output 2” in table 4 is the short-run efficient output in a model where the wage markup shock is replaced with a temporary labor supply shock. The gap from this alternative efficient output shows in most cases poorer forecasting performance than both the short-run and long-run efficient outputs in our benchmark model, which may imply that the wage markup shock in our model cannot be interpreted as the labor supply shock.
difference between the two gaps was mainly explained by the different contributions of the investment adjustment cost shocks. While the long-run efficient output gap includes the efficient as well as inefficient response of output to these shocks, the short-run efficient output gap includes only the inefficient response caused by nominal rigidity. The efficient response to the investment adjustment cost shocks in the above period, however, might actually capture some inefficiencies related to frictions other than nominal rigidity, such as financial frictions, which are not explicitly specified in our model. This possibility implies that the gap from the short-run efficient output would under-estimate the negative inflationary pressure in the period after the global financial crisis.

6. Concluding Remarks

In this paper, we have calculated the potential output and the output gap using a Bayesian-estimated DSGE model of Japan’s economy. For bridging the gap with conventional measures, we define our measure of potential output as a component of the efficient output generated only by persistent growth rate shocks. Our potential growth displays a high degree of smoothness and moves closely with conventional measures. Moreover, the output gap from our measure of potential output shows better forecasting performance for inflation—in particular, at short horizons—than other measures of output gap.

The short-run efficient output calculated from our model is more volatile and shows poorer forecasting performance for inflation than our measure, which may imply that a substantial fraction of the actual economic fluctuations is, somehow mistakenly, viewed as efficient in our model. Some recent DSGE models, however, consider various kinds of real frictions in the financial market, the labor market, and the open economy so that the models can generate

\[30\] While the long-run efficient output gap was lower than the short-run efficient output gap by 3.08 percentage points on average from 2008:Q3 to 2009:Q4 (the end of our sample period), the difference in the contributions of the investment adjustment cost shocks to the two gaps in the same period was 2.71 percentage points on average.
substantially inefficient fluctuations. Developing those kinds of models would be another way of bridging the gap with conventional measures of potential output. That will be an important future task.

References


