Liquidity Requirements: A Double-Edged Sword*

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This paper shows that bank liquidity regulation may be a "double-edged sword." Under certain conditions, it may hamper, rather than strengthen, a bank's resilience to financial stress. The reason is the existence of two opposing effects of liquidity regulation, a liquidity effect and a solvency effect. The liquidity effect arises because a bank mitigates its risk of illiquidity when it increases its liquidity buffer. The solvency effect arises because a larger liquidity buffer reduces the bank's returns and may therefore raise its insolvency risk. Liquidity regulation is effective in reducing a bank's overall default risk only if the former effect dominates the latter. The paper derives conditions under which this is the case and discusses the resulting relationship between capital and liquidity regulation.

JEL Codes: G21, G28.

1. Introduction

The recent financial crisis was associated with severe liquidity problems in the banking sector. This experience has led regulators to put greater emphasis on issues of bank liquidity risk than was the case before the crisis. In particular, in the course of the latest reform of the Basel Accords on banking regulation (Basel III), new liquidity standards have been proposed in order to strengthen liquidity risk measurement and supervision. Two regulatory standards on bank

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liquidity have been introduced which shall complement and institutionalize the Basel Committee's guidelines for liquidity risk management.[1] These newly introduced liquidity standards are the "net stable funding ratio" (NSFR) and the "liquidity coverage ratio" (LCR). The respective objectives of these measures differ but complement each other. The LCR is designed to enhance banking institutions' resilience to short-term funding stress. It shall ensure that banks hold sufficiently liquid assets to meet outflows of short-term liabilities under a stress scenario lasting for around one month.[2] The NSFR shall promote the funding stability of banks over a longer period (i.e., up to one year). It requires them to fund their assets using sufficiently stable sources, thereby limiting banks' exposure to short-term wholesale funding.[3]

The present paper uses the well-known banking model by Rochet and Vives (2004) in order to provide some analytical results on the functioning of short-term liquidity buffers (like the LCR). It discusses whether, and under which circumstances, they actually serve regulators' microprudential objective and improve an individual bank's resilience in periods of financial stress.

By refining a result of Rochet and Vives (2004, proposition 3), it is demonstrated that liquidity requirements strengthen a bank's resilience (as measured by its probability of default) if and only if the likelihood of a default is sufficiently low. This result is further translated into a threshold level on capital requirements. Essentially, liquidity requirements are effective whenever the bank satisfies a certain minimum capital requirement. This allows to derive conclusions concerning the interaction between a bank's liquidity and solvency.

In general, the reason why liquidity requirements may only work effectively under particular circumstances is the existence of two opposing effects, a *liquidity effect* and a *solvency effect*. The liquidity effect arises because it is costly for the bank to use an illiquid asset as a cushion against sudden funding roll-offs. Converting such an asset into cash at short notice involves a loss, and keeping a larger fraction of the balance sheet in the form of liquid assets thus allows the bank to withstand larger funding drains without engaging in costly fire

[1]See Basel Committee on Banking Supervision (2008).
[2]See Basel Committee on Banking Supervision (2013).
[3]See Basel Committee on Banking Supervision (2014).

sales. This in turn decreases the bank's risk of default due to illiquidity. The solvency effect occurs because liquid assets earn rather low returns on average and thus fail to generate the returns which the bank needs to service its interest-bearing deposit liabilities. Given that the bank manages to roll over its debt, the lower returns on liquid assets have to be compensated by sufficiently higher returns on more profitable (but less liquid) assets in order for the bank to remain solvent. This increases its risk of insolvency. Moreover, when creditors perceive the higher insolvency risk, their incentive to withdraw their funds may increase, thereby spurring the illiquidity of the bank. The existence of these competing effects implies that the use of liquidity requirements is a "double-edged sword" and that it is a priori not clear whether they make the bank more resilient to financial stress.

In their original analysis, Rochet and Vives (2004) neglect the solvency effect and focus only on the liquidity effect. However, their analysis is based on an implicitly assumed adjustment of the bank's balance sheet which, as will be explained below, happens to be mistaken. In what follows, I solve Rochet and Vives's model by making the underlying balance sheet adjustment explicit, thus taking insolvency risk into account.

The remainder of the paper is organized as follows. Section 2 provides a condensed description of Rochet and Vives's banking model.[4] Section 3 contains the relevant comparative statics exercises and a discussion of liquidity and solvency effects. Section 4 considers how balance sheet levels adjust under different assumptions concerning the determination of the bank's balance sheet scale. Section 5 contains two extensions of the model in order to study the robustness of the baseline results. Section 6 concludes. All proofs and most of the mathematical derivations are relegated to the appendix.

2. The Rochet-Vives Banking Model

The model studies a bank that operates at three dates, labeled $0, 1$, and 2. At date 0, the bank has equity E_0 and takes in wholesale deposits D_0. The deposit contract promises a repayment $D > D_0$

[4]For a full derivation of the model, the interested reader is referred to the original paper.

independent of whether the funds are withdrawn at date 1 or 2. The bank uses amount I to fund a risky asset and holds amount M in cash. Its balance sheet at date 0 reads

$$I + M = D_0 + E_0. \tag{1}$$

Cash holdings are non-interest bearing but they are perfectly liquid in the sense that one unit of cash can be used to cover one unit of liabilities at date 1 or 2.[5] The per-unit return on the risky asset is a normally distributed random variable $\tilde{R}$ with mean $\bar{R} > 1$ and precision (inverse variance) α. The asset pays out the realized return R at date 2. At date 1, the bank can sell the asset on a secondary market against a fire-sale price $R/(1 + \lambda)$ to obtain additional cash. The fire-sale discount rate $\lambda > 0$ renders the asset illiquid, which implies that the failure of the bank may depend on the fraction of deposits withdrawn early at date 1.

The bank fails at date 2 whenever

$$R < R_s + \frac{\lambda \max\{xD - M, 0\}}{I}, \tag{2}$$

where

$$R_s \equiv \frac{D - M}{I} \tag{3}$$

is henceforth called the bank's solvency point and $x \in [0, 1]$ denotes the proportion of deposits withdrawn. To gain the intuition behind equation (2), note that in case $xD < M$, the bank can cover all withdrawals out of its cash holdings and does not need to resort to fire sales of the illiquid asset on the secondary market. It can only fail due to *insolvency* at date 2, i.e., if $R < R_s$. In contrast, if $xD > M$, its cash holdings are not large enough and the bank has to fire-sell assets to cover early withdrawals. Compared with a situation where it would not have sold assets, the bank loses $\lambda R/(1+\lambda)$ per unit sold. Total losses rise with x and may deplete the bank's resources to such

[5]The assumption that the bank holds cash is not material for the results. Alternatively, the bank may hold other types of liquid assets. The only important assumption is that the returns on the liquid assets are lower than the interest rate on deposits.

an extent that it becomes unable to cover the remaining liabilities $(1-x)D$ at date 2 and defaults. This explains why the failure point after fire sales exceeds the solvency point by a factor proportional to the fire-sale discount rate. If the bank fails at date 2 even though it is solvent ($R > R_s$), it is said to fail due to *illiquidity*.[6]

It is further assumed that the management of deposits and the withdrawal decisions are made by fund managers on behalf of the original depositors. The bank has contracted with a large number of ex ante identical fund managers. Their total size is normalized to the unit interval, and each fund manager administers an amount D_0.[7]

Fund managers receive a remuneration B. However, when they withdraw early at date 1, they incur an additional cost C. Moreover, in case of bank default, a fund manager who decided to roll over is fired and receives a zero payoff.[8] The optimal decision of fund managers is to withdraw at date 1 whenever the expected payoff from withdrawing exceeds the expected payoff from rolling over. Formally, a typical fund manager $i \in [0, 1]$ withdraws if and only if

$$P_i > \gamma \equiv \frac{C}{B},$$

where P_i denotes the subjective probability that fund manager i attaches to the failure of the bank.

[6]The bank defaults already at date 1 if withdrawals fully exhaust its cash holdings and the liquidity obtained from fire-selling all its assets. This case will be taken up in section 5 below when the deposit interest rate is endogenized. However, for the determination of the equilibrium default probability, the relevant case to study is default at date 2 because it occurs for a larger range of return realizations than default at date 1; see Rochet and Vives (2004, p. 1126).

[7]The assumption that foreclosure decisions are carried out by fund managers instead of by fund owners themselves may reflect the fact that a major part of the short-term funding base of banks, especially in the United States, is managed by money-market mutual funds or collective investment funds. Experience during the recent crisis has shown that especially this part of bank funding is more volatile and evaporates quicker than the retail and customer deposit base. The assumption seems therefore reasonable for studying the short-term funding liquidity situation of a large bank that is subject to liquidity regulation.

[8]For a motivation of the modeling of fund managers' payoffs, see the explanations in Rochet and Vives (2004), who refer to the empirical studies of Chevalier and Ellison (1997, 1999).

Fund managers do not observe R but instead receive an idiosyncratic noisy signal about R at date 1.[9] The signal takes the form

$$s_i = R + \varepsilon_i,$$

where ε_i is i.i.d. normally distributed with zero mean and precision β. Fund managers then calculate their (posterior) belief P_i which equals the probability (conditional on s_i) that R is below the upper bound identified in condition (2).

The model satisfies the standard requirements needed to apply global game methods for the derivation of a unique equilibrium.[10] The unique equilibrium in this class of models is a symmetric monotone (or threshold) equilibrium. That is, there exists a threshold signal t^* such that a typical fund manager withdraws whenever $s_i < t^*$ and rolls over otherwise, as well as a critical threshold R^* such that the bank defaults whenever $R < R^*$ and survives otherwise. By the law of large numbers, the proportion of deposits withdrawn at date 1 can then be computed as the probability (conditional on R) that a single signal s_i is below t^*. The equilibrium is unique provided that the private signals are sufficiently precise. In the Rochet-Vives model, the equilibrium thresholds (t^*, R^*) solve the pair of equations

$$\Phi\left(\sqrt{\alpha+\beta}R^* - \frac{\alpha\bar{R}+\beta t^*}{\sqrt{\alpha+\beta}}\right) = \frac{C}{B} \tag{4}$$

and

$$R^* = R_s + \frac{\lambda \max\left\{\Phi(\sqrt{\beta}(t^* - R^*))D - M,\, 0\right\}}{I}, \tag{5}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function.[11]

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[9]Under common knowledge of R, the model would exhibit multiple self-fulfilling equilibria for $R \in [R_s, (1+\lambda)R_s)$. This outcome would, for example, be equivalent to the outcome in the well-known model by Diamond and Dybvig (1983). For a discussion of the influence of the assumption of common knowledge of R on the equilibrium multiplicity, see Morris and Shin (2001).

[10]See Morris and Shin (2003) or Heinemann (2005) for overviews of the global game methodology.

[11]See Rochet and Vives (2004, p. 1129).

Given the bank's failure point R^*, equation (4) is derived from the equilibrium requirement that a fund manager who uses a threshold strategy around t^* must be indifferent between rolling over and withdrawing if he observes a signal s_i exactly equal to the critical threshold t^*. Similarly, for a given t^* (determining the proportion of withdrawals x), equation (5) is derived from the requirement that at $R = R^*$, equation (2) must hold with equality. Furthermore, as shown by Rochet and Vives (2004, proposition 1), the equilibrium is unique provided that the precision of fund managers' signals is sufficiently large, i.e., if

$$\beta \geq \beta_0 \equiv \frac{\alpha^2}{2\pi}\left(\frac{\lambda D}{I}\right)^2. \tag{6}$$

Inequality (6) is the analogue to the standard uniqueness condition in global games with normally distributed fundamental and noise variables.[12] Whenever it fails to hold, equilibrium multiplicity may reemerge. Essentially, in this case, the weight on the publicly known expected return $\bar{R}$ in the formation of fund managers' beliefs is sufficiently large so that public information dominates private information and the outcome of the game approaches the outcome of a game where R is common knowledge and multiple self-fulfilling equilibria exist.

3. Analysis and Discussion of the Model

3.1 Comparative Statics

The ex ante default probability, given by

$$\mathbf{Pr}\left(\tilde{R} \leq R^*\right) = \Phi\left(\sqrt{\alpha}(R^* - \bar{R})\right),$$

is used as a measure of the bank's resilience against financial stress. Since $\Phi(\cdot)$ is a strictly monotone function, when studying the effects of regulatory ratios on the ex ante default risk, it suffices to study their effects on the failure point R^*. As the Basel Accords identify

[12] See Hellwig (2002) or Morris and Shin (2003).

the strengthening of banks' resilience as a major objective of microprudential regulation, the comparative statics properties of R^* may indicate to which extent regulatory requirements may or may not meet this objective.

Following Rochet and Vives, I focus on the effects of two regulatory ratios—the *liquidity ratio*, given by $m \equiv M/D$, and the *capital ratio*, given by $e \equiv E_0/I$. These ratios can be viewed as approximations to the Basel Accord's newly introduced LCR and the capital adequacy ratio.[13] Differently from Rochet and Vives, in the comparative statics below, the ratio of deposits to risky investments, $d \equiv D_0/I$, is treated as endogenous. To see its dependence on e, m, and the interest rate on deposits, $r \equiv D/D_0 - 1$, observe that the balance sheet constraint (1) can be rewritten as

$$d = \frac{1-e}{1-(1+r)m}. \tag{7}$$

Moreover, the solvency point can be expressed as

$$R_s = d(1+r)(1-m). \tag{8}$$

By substituting equation (7) into (8), the solvency point can be expressed as a function of m, e, and r,

$$R_s(m,\, e,\, r) = \frac{(1-e)(1+r)(1-m)}{1-(1+r)m}. \tag{9}$$

In a similar way, equation (5) can be written as

$$R^* = R_s(m,\, e,\, r) + \frac{\lambda(1-e)(1+r)\max\left\{\Phi(\sqrt{\beta}(t^*-R^*))-m,\, 0\right\}}{1-(1+r)m}. \tag{10}$$

Whenever $x > m$, the failure point R^* exceeds the solvency point R_s. In this case, R^* is given by the solution to the implicit function,

[13] Rochet and Vives label the ratio E_0/I as *solvency ratio,* which I found somewhat misleading, as this ratio affects not only the insolvency but also the illiquidity risk of the bank.

$$\begin{aligned}\psi(R^*, m, e, r) \equiv\ & (1 - m(1+r))R^* - (1-e)(1+r)(1-m) \\ & + \lambda(1-e)(1+r)m \\ & - \lambda(1-e)(1+r)\Phi\left(\frac{\alpha}{\sqrt{\beta}}(R^* - \bar{R}) \right. \\ & \left. - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma)\right) = 0. \qquad (11)\end{aligned}$$

This equation is derived by solving equation (4) for t^* and then substituting the result together with the expression for R_s from equation (9) into (10).

The comparative static properties of R^* are summarized in the following proposition.[14]

PROPOSITION 1. *The failure point R^* and the likelihood of bank default strictly decrease in the capital ratio e and increase in the interest rate on deposits r.*

If $R^* = R_s$, the failure point and the likelihood of default increase in the liquidity ratio m.

Otherwise, if $R^* > R_s$, they strictly decrease in m if and only if

$$R^* < (1-e)(1+\lambda). \qquad (12)$$

3.2 Liquidity and Solvency Effects

The description of the model in section 2 proceeded closely along the lines of Rochet and Vives (2004). But the approach to the comparative statics and the conclusion concerning the effects of the liquidity ratio on the failure point R^* differ. This is due to a different treatment of the bank's balance sheet constraint in the derivation of the failure point. While proposition 1 confirms Rochet and Vives's results that the failure point always decreases in the equity ratio e and increases in the interest rate r (see their proposition 3), the effect of the liquidity ratio m depends on the underlying model parameters. This result stands in contrast to Rochet and Vives (2004,

[14]As is common in the literature on global games, the comparative statics are conducted under the assumption that the uniqueness condition (6) is satisfied.

proposition 3), as they claim that an increase in the liquidity ratio unambiguously lowers the failure point and the default probability. The reason for this difference is that Rochet and Vives first use the balance sheet constraint to express R_s as a function of e and d, but then neglect the dependency of d on m when conducting the comparative statics with respect to m (recall equation (7)). Accordingly, in their analysis, for a given e, a change in m does not lead to an adjustment of R_s, and their assessment of the effects of the liquidity ratio on bank solvency becomes incorrect.[15]

By treating R_s as independent from m, Rochet and Vives suppress what I will henceforth call the *solvency effect*. To better appreciate the intuition behind it, note that, from equations (4) and (5), the failure point R^* and the switching signal t^* depend positively on the solvency point R_s.[16] Moreover, the solvency point R_s is strictly increasing in the liquidity ratio m: By definition, a higher liquidity ratio is equivalent to a higher level of non-interest-bearing assets relative to interest-bearing liabilities. Since the balance sheet constraint (1) holds, the bank needs to achieve higher net returns to service its deposit liabilities, i.e., without generating a higher rate of return on its profitable asset, the risk of becoming insolvent rises.

Whenever $R^* = R_s$, only the solvency effect is present and a higher liquidity ratio m cannot be used to effectively lower the bank's default risk. However, if $R^* > R_s$, an otherwise solvent bank may also fail due to illiquidity. In this case, a change in the liquidity ratio m is associated with a second effect which works in the opposite direction and which may be called the *liquidity effect*. Since illiquidity is essentially governed by the depletion of the bank's assets due to fire sales at date 1, a higher liquidity ratio m reduces the

[15] Keeping R_s constant upon a marginal change in m requires to adjust d by $\mathrm{d}d = (d/(1-m))\mathrm{d}m$ (see equation (8)). But from the balance sheet constraint (7), this implies $\mathrm{d}e = (rd/(1-m))\mathrm{d}m \neq 0$. Hence, it is impossible to adjust m for fixed e and R_s without violating the balance sheet identity.

[16] One may view this dependency as a formal restatement of Charles Goodhart's notion that illiquidity implies "at least a whiff of suspicion of insolvency" (1999, p. 346). If the insolvency risk increases (i.e., R_s increases), fund managers become more "suspicious" and attach a higher likelihood to the event that the bank fails. This increases the switching point t^* and therefore the failure point R^*.

depletion of profitable assets and thereby lowers the bank's exposure to illiquidity. Hence, the liquidity effect decreases the failure point R^*. Consequently, the use of liquidity requirements as a prudential measure to strengthen the bank's resilience crucially depends on whether the liquidity or the solvency effect dominates.

Condition (12) shows that the liquidity effect dominates the solvency effect whenever the failure point, i.e., the bank's default risk, is sufficiently low. However, since R^* is an endogenous object which itself depends on exogenous parameters—in particular on e and m—it is not immediately apparent how changes in these parameters affect condition (12). To derive a condition for the dominance of either liquidity or solvency effect purely in terms of exogenous parameters, equation (11) can be evaluated at $R^* = (1+\lambda)(1-e)$,

$$\psi\left((1+\lambda)(1-e),\, m,\, e,\, r\right) = 0,$$

thus yielding the particular combination of exogenous parameters for which liquidity and solvency effect cancel each other out completely so that R^* does not respond to m anymore. By solving the above equation for r, a threshold value for the interest rate can be derived,

$$r^m(e) = \frac{\lambda\left(1-\Phi\left(\omega(e)\right)\right)}{1+\lambda\Phi\left(\omega(e)\right)} \quad \text{with}$$

$$\omega(e) \equiv \frac{\alpha}{\sqrt{\beta}}((1+\lambda)(1-e)-\bar{R}) - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma), \quad (13)$$

such that $R^* = (1+\lambda)(1-e)$ if and only if $r = r^m(e)$. As a reduction of r decreases the failure point R^*, it follows that the liquidity effect dominates the solvency effect if and only if the interest rate on deposit liabilities is sufficiently low, i.e., $r < r^m(e)$. To appreciate the intuition behind this result, suppose that $r = 0$. The face value of the bank's liabilities equals the amount of initially obtained deposits. The bank does not need to make interest-bearing investments to pay back its deposits; rather, it may simply store them in cash. Moreover, from equation (9) follows that R_s ceases to depend on m. Thus, only the liquidity effect is present and the failure point R^* strictly decreases in m. But with positive interest rates, $r > 0$, the face value of liabilities exceeds the value of initial deposits. Per unit of deposits held, the bank needs to obtain positive net returns from its investments to become able to service its debt. This, however, is not possible by resorting to non-interest-bearing cash holdings. A fortiori, if we keep increasing r, the ability of the bank to cover withdrawals out of its cash holdings weakens while the solvency point R_s increases more and more in m. With r being above the critical value r^m, the solvency effect becomes too strong and eventually dominates the liquidity effect.

Another way to look at liquidity and solvency effect and to spell out the implied relationship between liquidity and capital regulation more clearly is by inverting the threshold rate $r^m(e)$ to obtain a critical capital ratio $e^m(r)$. As $r^m(e)$ strictly increases in e, the liquidity effect is dominant for $e > e^m$. In this case, liquidity and capital regulation are substitutes: A lower capital ratio can be compensated by a higher liquidity ratio without changing the risk level R^*. However, if the capital ratio falls short of e^m, this relationship suddenly reverses. While increases in the capital ratio still produce a lower default risk, increases in the liquidity ratio have the opposite effect. Consequently, in order to keep the bank's risk level fixed, increases in m have to be complemented by a higher level of e.

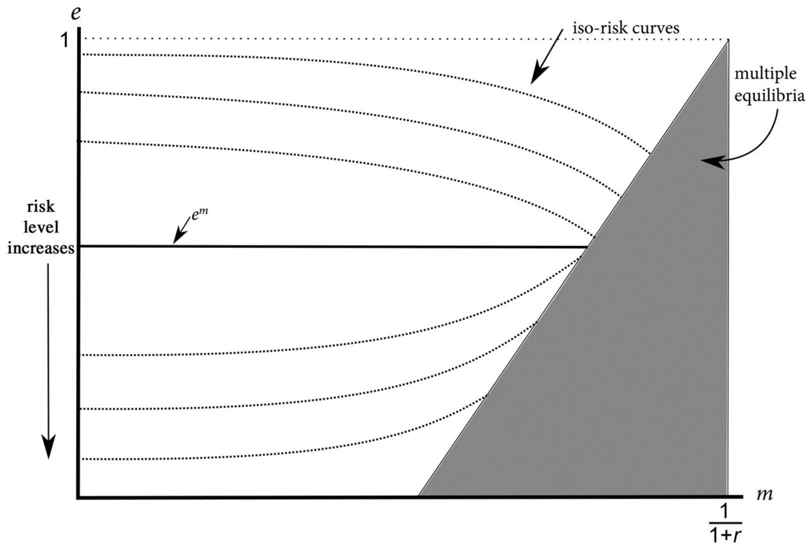
This relationship is also illustrated in figure 1, which plots iso-risk curves in m-e space.[17] Each curve shows the combinations of m and e needed to keep the risk level fixed. The risk level is lowest at the top curve and increases from top to bottom. For a given risk level—say, R_k^*—the iso-risk curve $e = e(m; R_k^*)$ is implicitly defined through equation (11).[18] Along the horizontal line at $e = e^m$, variations in m exert no effect on the risk level.[19] Above this line, the liquidity effect is dominant and iso-risk curves are downward sloping, thus reflecting the substitutability between liquidity and capital regulation. Conversely, below the line, iso-risk curves are upward sloping, reflecting the dominating solvency effect and the fact that increases in m need to be accompanied by increases in e to keep the risk level

[17]The domain of e is taken to be the unit interval and the domain of m is the interval $[0, 1/(1 + r)]$: Firstly, neither equity nor cash holdings can become negative, so e and m are bounded below by zero; secondly, given an ex ante expected return $\bar{R} > 1$, the bank would invest at least its equity into the risky asset, implying that $e \leq 1$ and $m \leq 1/(1 + r)$.

[18]The derivation of iso-risk curves is provided in the appendix.

[19]e^m is by definition the value of e where the derivative of R^* with respect to m vanishes. This implies that e^m is itself independent of m and the iso-risk curve at $e = e^m$ is a horizontal line.

Figure 1. Iso-Risk Curves in m-e Space



fixed. This shows that liquidity requirements can be a double-edged sword with respect to their effects on default risk. Moreover, the existence of structural differences across banks implies that liquidity regulation may have to be bank-specific as long as homogeneous capital standards are applied. That is, in order to stabilize banks' risk levels following an adverse shock, some banks would have to be advised to lower their liquidity ratios, while other banks would need to raise them. This may impede the use of general liquidity rules applying uniformly to all banks.

Finally, the shaded area in figure 1 shows combinations of e and m such that for given parameters β, α, r, and λ, multiple equilibria emerge. The boundary line of this area can be calculated by using equation (7), substituting $d(1+r) = D/I$ into condition (6), thus yielding (with equality)

$$\beta = \frac{\alpha^2}{2\pi}\left(\frac{\lambda(1+r)(1-e)}{1-(1+r)m}\right)^2 \tag{14}$$

and solving the latter for e in terms of m.

The point where any iso-risk curve touches the boundary line constitutes a bifurcation point from which essentially three equilibria emerge.[20] Since each combination of e and m is then associated with three different risk levels, iso-risk curves are only drawn until they touch the boundary.

For given parameter values, a higher capital ratio slackens condition (14) and implies that a unique equilibrium obtains for a larger range of values for β. In contrast, a higher liquidity ratio m has the opposite effect. In particular, if for given β the liquidity ratio is raised above a certain point, the unique equilibrium breaks down and multiple self-fulfilling equilibria occur. This can be seen by going back to condition (6) and noting that the condition becomes tighter when $\lambda D/I$ increases. A higher D/I raises the bank's dependency on short-term deposits and a higher λ increases potential fire-sale losses. Hence, the larger $\lambda D/I$, the higher the bank's vulnerability to a coordination failure and to self-fulfilling bank runs. In order to ensure uniqueness of the global game solution, this effect has to be balanced by more precise private signals of fund managers. As D/I depends negatively on e and positively on m, the boundary line is upward sloping.[21]

4. Balance Sheet Adjustment

The previous section showed that a bank's risk level is determined by the two ratios e and m. Yet, focusing only on ratios consisting of balance sheet components leaves open how their levels adjust to variations in these ratios. For example, when changing e and m while moving along an iso-risk curve, the underlying balance sheet may also change. In order to analyze these balance sheet adjustments in greater detail, I assume that e and m are completely determined by the regulatory authorities. This may reflect a situation where the

[20] This result was first explored in greater detail by Hellwig (2002). See also the discussion in Morris and Shin (2003).

[21] With respect to the impact of the liquidity ratio, it can be seen from equation (7) that D/I strictly increases in m. Intuitively, keeping D fixed, investments have to be lowered to raise cash holdings in order to increase m (thus D/I increases); alternatively, if I is fixed, m can be increased by raising new deposits and storing them in cash (again increasing D/I).

bank would choose too-low levels of e and m in the absence of regulatory intervention. For example, preferential tax treatment of debt due to the deductibility of interest payments from corporate taxes (debt tax shield), or the desire to avoid incurring large (opportunity) cost by holding cash may induce banks to minimize e and m. This, however, may conflict with the regulator's objectives of financial stability and more resilient banks. The regulator therefore enforces certain minimum levels of e and m. This way, the definitions of the two ratios and the balance sheet constraint provide three equations to determine the four balance sheet components E_0, D_0, I, and M. Thus, in order to close the model, one further assumption is needed. To this end, I distinguish between three border cases where either E_0, D_0, or I is fixed exogenously and then consider how the balance sheet adjusts to regulatory changes in e or m. The following three equations govern the balance sheet adjustment:

$$\frac{\mathrm{d}M}{M} - \frac{\mathrm{d}D_0}{D_0} = \frac{\mathrm{d}m}{m} \tag{15}$$

$$\frac{\mathrm{d}E_0}{E_0} - \frac{\mathrm{d}I}{I} = \frac{\mathrm{d}e}{e} \tag{16}$$

$$\mathrm{d}M - \mathrm{d}D_0 - \mathrm{d}E_0 + \mathrm{d}I = 0. \tag{17}$$

Exogenous Equity Level E_0. A fixed equity level may approximate a situation where the bank finds it overly costly to raise additional outside equity. Alternatively, as equity is a variable that needs relatively more time to be adjusted than, for example, debt, one may also think of this scenario as the short run where the equity level cannot be changed. In this case, the maximal investment scale is determined by the capital ratio. Consequently, if the bank always chooses the maximal investment scale, an inverse relationship between capital ratio e and investment I emerges. If the regulator raises, say, the capital ratio, the bank adjusts by scaling down its investments by $-Ie^{-1}\mathrm{d}e$. In addition, as the bank avoids holding too much idle cash and keeps the liquidity ratio constant at the regulatory minimum, the total balance sheet size shrinks. Given that I decreases while E_0 is constant, the lower level of investments requires a lower level of deposits and, to keep m constant, the level of cash holdings M can be lowered as well.

If the regulator enforces a higher liquidity ratio while keeping the capital ratio constant, I cannot be adjusted, and it follows from equations (15) and (17) that $\mathrm{d}D_0 = \mathrm{d}M = D \times \mathrm{d}m/(1-(1+r)m) > 0$. The bank's balance sheet lengthens and its indebtedness rises, but any new deposits are stored entirely in cash. Leaving a costly resource lying in idle cash may look like a bad decision from the point of view of portfolio management. But it may be the right decision from the regulator's point of view if the additional cash allows to reduce the bank's default risk by shielding it against fire-sale losses.

This example allows to discuss how regulations linking capital buffers with business or financial cycles in a countercyclical way (capital requirements which are loosened during downswings and tightened during upswings) may give rise to a trade-off between stabilizing an individual bank's default risk and the level of investment. On the one hand, if the capital ratio is lowered during business-cycle downswings, a bank can increase the level of its investments. This may help to stabilize the economy in the short run. On the other hand, a lower capital ratio increases the bank's default risk. What's more, whenever the bank's default risk negatively correlates with the business cycle—for example, due to fluctuations in the expected rate of return $\bar{R}$—countercyclical capital buffers amplify risk increases in the banking sector during business-cycle fluctuations. Is it possible to overcome this dilemma by resorting to additional liquidity regulation? It requires the regulator to know whether a bank is located on an iso-risk curve above or below the e^m line and how e^m itself reacts to the cycle. For example, the critical level e^m depends negatively on the expected return $\bar{R}$, which may positively correlate with business-cycle swings. A downswing therefore increases the area in figure 1 where the solvency effect dominates by shifting up the e^m line. If, following a decline in $\bar{R}$, the bank will be located on an iso-risk curve below the new e^m line, the regulator must advise a lower liquidity ratio to counter the higher risk level and vice versa if the bank will be above the new line. Hence, it may be possible to counter adverse effects of countercyclical capital buffers by adequate additional liquidity regulation. But for liquidity regulation to be indeed adequate, the regulator must know a bank's precise position in e-m space relative to the prevailing e^m line.

Exogenous Investment Scale *I*. Next, consider the case where the investment scale, rather than the level of equity, is exogenously given. One may think here of a smaller bank with a local monopoly in supplying loans or, alternatively, of a bank focusing on a particular clientele. The capital ratio now determines the necessary minimum amount of equity that the bank has to raise. Given that the bank would economize on the level of equity in order to maximize shareholders' rate of return, a positive relationship between the equity level and the capital ratio emerges.[22] If the bank tries also to economize on its cash balances, an increase in the capital ratio will be accompanied by a lower level of deposits and smaller cash holdings in order to keep the liquidity ratio constant. Adjustments in the liquidity ratio proceed in the same way as in the above case with exogenous E_0: investment and equity levels are not affected, and a higher liquidity ratio is achieved by taking in new deposits which are entirely kept in cash.

What happens if the bank faces new investment opportunities—i.e., $\mathrm{d}I > 0$—but has to maintain given levels of e and m? Firstly, it has to fund a fraction e of new investments by issuance of equity and the remaining fraction $(1-e)$ by new deposits. But secondly, as these new deposits would lower the liquidity ratio m, it has to raise an even larger amount of funds in order to fill up its cash buffer,

$$\mathrm{d}D_0 = (1-e)\mathrm{d}I/(1-(1+r)m) > (1-e)\mathrm{d}I.$$

This case may be used to shed some light on the situation in the banking sector in the absence of liquidity regulation. Prior to the crisis of 2008/2009, banking regulation almost exclusively focused on capital regulation. As observed by Goodhart (2009), banks shifted into less liquid assets and liquidity ratios deteriorated in the run-up to the crisis. In the model, under regulation focusing exclusively on capital, exogenous shifts in investment opportunities may lead to a deterioration of the liquidity ratio. If capital ratios are sufficiently high—i.e., above e^m—worsening liquidity ratios entail higher default risk by rendering banks more vulnerable to a sudden dry-up in funding markets. This counteracts, to some extent, the efforts made by

[22]In practice, the effectiveness of capital regulation in the short run may be impeded by the fact that adjustments of equity levels are more time-consuming and costly than adjustments of debt levels.

implementing high capital ratios. Put differently, in order to steer the risk level of banks and to weaken the link between variations in banks' portfolios and changes in their default risk, the regulator must pin down both capital and liquidity ratios.

Exogenous Debt Level D_0. Finally, consider the case where the deposit level is exogenously given. This case does not fully square with the interpretation of the model given above, where D_0 was interpreted as wholesale funds administered by fund managers. However, suppose for a moment that D_0 consists of uninsured retail funds that the bank cannot increase easily of its own accord. In this case, the investment level I is determined by the interplay between the regulatory ratios e and m. Using the balance sheet constraint given in equation (7), changes in the investment level can be computed as

$$\mathrm{d}I = \frac{-(1+r)D_0}{1-e}\mathrm{d}m + \frac{(1-m(1+r))D_0}{(1-e)^2}\mathrm{d}e.$$

A higher capital ratio e tends to induce an increase in I since the bank could use the newly issued equity to fund additional investments. Conversely, a higher liquidity ratio would lead to a reduction in the investment scale because, as $D = D_0(1+r)$ is fixed, the bank would need to swap investments for cash holdings to meet regulatory requirements. Then, moving from left to right along an iso-risk curve located above e^m, the investment level declines as the bank successively reallocates its funding into cash holdings. Below e^m, however, movements along the iso-risk curve are associated with an ambiguous effect since an increase in cash holdings is accompanied by additional issuances of equity and there exists a counterbalancing effect on the investment scale.

5. Extensions

This section considers two extensions of the model in order to examine the robustness of the liquidity and solvency effect with respect to the assumptions of exogenously given interest rate r and fire-sale rate λ. Treating these as parametric has facilitated the analysis of the model so far. Yet, it may be important to understand to which extent liquidity and solvency effects are modified once these variables are treated endogenously.

5.1 Endogenous Interest Rate

The baseline model in section 2 uses the rather extreme assumption that the interest rate does not depend on the bank's default risk and therefore does not react to changes in liquidity and capital ratios. To examine the importance of this assumption, I now turn to the opposite extreme and consider a variant of the model where the interest rate embodies a risk premium and perfectly reflects the bank's default risk. A straightforward way to capture this dependency is to require that, in expected terms, risk-neutral creditors have to be at least indifferent between investing with the bank or putting their funds into a risk-free two-period asset with exogenous interest rate $\bar{r}$. Thus, the fund managers take care not only of the interim withdrawal decision but also of the initial investment decision. Under the assumption of a perfectly competitive money market, parity in expected terms between the risk-free rate and the interest rate on deposits will hold in equilibrium.

In setting up the relevant interest parity condition, it becomes important to distinguish between default at date 1 (early default) and default at date 2, since the bank's liquidation value (and thus the amount paid out per creditor in case of default) differs between these cases.[23]

The bank defaults early at date 1 if it cannot obtain enough liquidity (cash plus secondary market liquidity from fire-selling assets) to meet withdrawals. This arises whenever

$$R < (1 + \lambda)\frac{xD - M}{I}. \tag{18}$$

Suppose that withdrawing creditors face a sequential service constraint, i.e., the bank pays creditors until its resources are exhausted. In case the bank defaults early at date 1, its liquid resources are insufficient to meet total withdrawals, and a withdrawing creditor

[23] In general, the creditor's cost of delegating the management of the funds to the fund manager, i.e., $B - C$ in the case of early withdrawal and B in the case of successful rollover, would also need to be considered. Since only the ratio $C/B \equiv \gamma$ matters for the analysis, I can assume that the costs of managing D_0 units of cash are small, so that neither B nor C affect the initial investment decision, while the ratio γ is still well defined.

receives his claim only with a certain probability.[24] Creditors whose claims are rolled over get nothing. However, if the bank defaults at date 2, withdrawals at date 1 are met in full, while those creditors who roll over receive a pro rata share of the remaining liquidation value of the bank.[25]

This modified version of the model then consists of three endogenous variables, the interest rate on deposits and the date 1 and date 2 failure points. The model can be solved by first determining the two default points from equations (11) and (18). The resulting solutions are functions of the interest rate and of the ratios m and e. Using these relationships, the interest rate can then be determined as the solution to the following interest parity condition:

$$\begin{aligned}(1+r)\left(1-F\left(R^*(r,m,e)\right)\right)\\ +\mathcal{L}(R^*(r,m,e),R^*_{EC}(r,m,e),r,m,e)=1+\bar{r},\end{aligned} \tag{19}$$

where R^*_{EC} solves equation (18) with equality and, for the sake of notational simplicity, the prior return distribution is denoted by $F(R)\equiv\Phi(\sqrt{\alpha}(R-\bar{R}))$ and the expected liquidation value of the bank (per unit of deposits) is expressed in compact notation by $\mathcal{L}(R^*,R^*_{EC},r,m,e)$.[26]

However, it is not obvious whether a solution r^* to equation (19) exists and, if it exists, whether it is unique. This is due to the positive dependency of the default probability on the interest rate. As creditors require a higher interest rate given an increase in default risk, the default risk increases further and induces an additional increase in the interest rate, and so on. This mechanism may, under certain conditions, induce a multiplicity of solutions. A thorough treatment of this possibility is beyond the scope of the present paper. I will focus here on the particular case where a unique solution exists in order to examine how the conclusions from the baseline model concerning the liquidity and solvency effects are altered. Intuitively, a sufficient condition for a unique solution is that in equilibrium

[24] Due to the nature of the sequential service constraint, the probability is given by the ratio of total resources to total claims.

[25] See Goldstein and Pauzner (2005) for a similar assumption on the payoff structure in their global game version of the Diamond-Dybvig model.

[26] A more detailed derivation of the condition and the explicit expression for $\mathcal{L}(\cdot)$ is provided in the appendix.

the creditor's marginal loss of an increase in the interest rate due to the higher default risk is smaller than the marginal benefit of such an increase stemming from the higher interest payments and the induced marginal change in the bank's liquidation value. That is, denoting the ratio of marginal loss to marginal benefit by ξ, equation (19) admits a unique solution if[27]

$$\xi(R^*, R^*_{EC}, r^*) \equiv \xi^* \leq 1. \tag{20}$$

Thus, if conditions (6) and (20) are satisfied, the model has a unique solution which can be written in terms of liquidity and capital ratios as $R^* = R^*(m, e), r^* = r^*(m, e)$, and $R^*_{EC} = R^*_{EC}(m, e)$.

Compared with the baseline model, changes in e and m are now associated with two additional effects due to the endogeneity of the interest rate. Firstly, the interdependency between default risk and interest rate induces a multiplier which magnifies the effects of liquidity and capital ratios. Secondly, changes in e and m affect the liquidation value of the bank and thereby the interest rate and the default risk. I refer to the marginal change in the liquidation value due to marginal changes in e or m as their respective *liquidation value effects*. Intuitively, the liquidation value effect of e has a clear-cut sign. A marginal increase in the capital ratio raises the liquidation value per unit of deposits. This allows to reduce the interest rate and lowers the default risk. But the liquidation value effect of m is ambiguous. Whether a marginal increase in m raises or lowers the liquidation value depends on whether the marginal return from holding more cash exceeds the foregone returns from holding less liquid assets. Whenever the profitable asset's expected return (conditional on the bank defaulting) is sufficiently large, an increase in the liquidity ratio reduces the liquidation value. This leads to a higher interest rate and raises the default risk. And conversely, whenever the expected return is sufficiently low, the liquidation value effect tends to lower both the interest rate and the default risk.

The following proposition summarizes the comparative statics of R^* and r^*.

[27] Again, the derivation of ξ is provided in the appendix.

Proposition 2.

(i) R^* *and* r^* *are strictly decreasing in the capital ratio* e*.*
(ii) *The derivatives of* R^* *and* r^* *with respect to the liquidity ratio* m *are given by*

$$\begin{aligned}\frac{\mathrm{d}R^*}{\mathrm{d}m} &= \frac{-\frac{\partial\psi/\partial m}{\partial\psi/\partial R^*} - \rho\left(-\frac{\partial\psi/\partial r}{\partial\psi/\partial R^*}\right)}{1-\xi^*} \quad \textit{and} \\ \frac{\mathrm{d}r^*}{\mathrm{d}m} &= \frac{\frac{\partial\psi/\partial m}{(\partial\psi/\partial r)}\xi^* - \rho}{1-\xi^*},\end{aligned} \tag{21}$$

which can be either increasing or decreasing, and where the liquidation value effect is reflected by the variable ρ*.*

Consider the derivatives in (21). The multiplier effect due to the feedback between default risk and interest rate can be seen from the term $1 - \xi^* \in (0, 1)$ in the denominator. Moreover, besides the partial effect on R^* (the first term in the numerator, which can be either positive or negative depending on whether the solvency or the liquidity effect dominates), the liquidation value effect, which is proportional to ρ, enters with a negative sign.[28] For example, when $\rho > 0$ (positive liquidation value effect), a marginal increase in m raises the liquidation value. As can be seen from (21), this tends to lower the deposit rate r^* and tends to decrease the default risk. Whenever $\rho < 0$, the effects are reversed. However, even though the sign of the liquidation value effect can be either positive or negative, there exists a close connection between liquidity, solvency, and liquidation value effect.

Proposition 3. *If the liquidity effect dominates the solvency effect,* $R^* < (1 + \lambda)(1 - e)$*, then the liquidation value effect is positive,* $\rho > 0$*. Yet, if the liquidation value effect is negative,* $\rho < 0$*, then the solvency effect dominates the liquidity effect,* $R^* > (1 + \lambda)(1 - e)$*.*

[28] ρ is proportional to the derivative of the liquidation value with respect to m. It is thus a function of endogenous variables and exogenous parameters. For notational simplicity I omit this dependency. The proof of proposition 2 contains the formal definition of ρ.

Propositions 2 and 3 together imply that the dominance of the liquidity effect is a sufficient condition for an increase in the liquidity ratio to effectively reduce the bank's default risk. Note that condition (12) is still the valid condition for assessing the dominance of the liquidity or solvency effect. Conditional on default, the expected return on the bank's asset is low when the liquidity effect dominates. A marginal increase in the liquidity ratio thus increases the liquidation value of the bank because in expected terms the additional cash holdings are worth more than their opportunity cost in terms of foregone investments. As indicated in proposition 3, the converse is not necessarily true. So even if the solvency effect dominates the liquidity effect, the liquidation value effect may still be negative and counteract the effect of a higher liquidity ratio on the likelihood of bank default.

With respect to the dominance of either solvency or liquidity effect, recall that in the baseline model, condition (12) was rewritten in terms of the interest rate/capital ratio to obtain a condition purely in terms of the exogenous parameters. Since the interest rate is now determined endogenously, one may therefore rewrite the condition in terms of the exogenous risk-free rate $\bar{r}$. The intuition remains broadly similar compared with the baseline model. If the risk-free rate is rather large, the interest rate on bank deposits must be large as well. This implies that the solvency point R_s increases strongly in m and pushes up the default risk. Thus, as shown in the proof of proposition 2, there is a critical value $\hat{r}^m(e)$ such that if $\bar{r} > \hat{r}^m(e)$, then $R^* > (1 - e)(1 + \lambda)$ and the solvency effect dominates the liquidity effect. Conversely, whenever $\bar{r} < \hat{r}^m(e)$, the liquidity effect is dominant and a higher liquidity ratio can be used to effectively reduce the bank's default risk. In practice, since the risk-free rate correlates positively with the interest rate set by the monetary authority, central bank actions may influence the effectiveness of bank regulatory measures. In particular, in low interest environments, a higher liquidity ratio may be effective in reducing bank default risk for a given capital ratio, whereas in high interest environments, additional increases in the capital ratio should accompany increases in the liquidity ratio to increase $\hat{r}^m(e)$ so that the default risk can be stabilized.

However, the relationship between liquidity and capital regulation that was visible from figure 1 becomes more complicated. By inverting $\hat{r}^m(e)$, one can still derive a critical capital ratio such that the liquidity effect dominates once the capital ratio exceeds this critical value. But it is not obvious how the liquidity ratio should be adjusted below this critical value. The existence of the liquidation value effect may in fact undermine the relationship between capital and liquidity regulation. There may be regions in e-m space where the liquidation value effect outweighs the solvency effect and conversely.

Hence, in the extreme case of no risk premium, liquidity regulation just has to take into account whether the capital ratio e is above or below the critical level e^m. But in the opposite extreme of a perfectly risk-reflecting interest rate, regulators have to take into account either the intricate relationship between liquidation value and solvency effects or rely only on the sufficient condition and impose a sufficiently high capital ratio to be sure to get around the double-edged-sword problem of liquidity regulation.

5.2 Endogenous Fire-Sale Rate

Next, return to the baseline model with exogenous interest rate r, but consider the effect of an endogenous fire-sale rate λ. With respect to the origin of the fire-sale rate, Rochet and Vives (2004, p. 1122) stress, in particular, the presence of asymmetric information. They point out that in a market plagued by informational asymmetries, a bank may have essentially two reasons for selling assets—an urgent liquidity need or the desire to offload bad assets. Having information inferior to that of the bank, market participants are unable to distinguish between the two reasons and offer to purchase assets only against a discount. Modeling this informational friction in greater detail would allow to express the fire-sale discount in terms of (exogenously given) parameters describing, for example, the fraction of good and bad assets in the economy or informational assumptions. While this would allow for more complex and detailed comparative statics, it would not necessarily render the fire-sale discount endogenously dependent on the failure point R^*. However, such a dependency generically occurs if the fire-sale rate is influenced by aggregate sales. This could, for example, reflect a situation where a bank's holdings of a particular asset class is large relative to the totally traded volume in the market. In line with the discussion in

Morris and Shin (2004), it may also proxy the situation of a certain group of financial institutions with similar asset positions operating in a market with a downward-sloping residual demand curve. As a consequence, the larger the amount withdrawn, the more assets have to be sold and the more deflated the asset price will be. A straightforward way to capture this feedback is to assume that the fire-sale discount rate depends negatively on the volume of sales,

$$\lambda = \zeta\left(\max\{x - m, 0\}\right).$$

I assume further that the price equals the fundamental price if no fire sales occur, $\zeta(0) = 0$. To ensure that the model behaves well, I additionally assume that prices, even though they may be quite small in the presence of maximal fire sales, are never zero, $\zeta(1) = \bar{\lambda} < \infty$, and that the rate of price change is bounded $\zeta' \in (0, \kappa]$. Finally, for computational simplicity, $\zeta^{''} = 0$.

Since a larger amount of fire sales lowers the asset's price by pushing up λ, the strategic complementarities in the withdrawal decisions of fund managers are strengthened. This requires imposing a more stringent condition than condition (6) to guarantee the uniqueness of the monotone global game equilibrium. The respective sufficient condition is derived in the appendix and is given by

$$\beta \geq \frac{\alpha^2}{2\pi}\left(\frac{\bar{\lambda}D + \kappa(D - M)}{I}\right)^2. \tag{22}$$

Given that condition (22) holds, the threshold value R^* can be used to compute the probability of a bank default. Moreover, as the following proposition makes clear, under certain conditions, the qualitative results from the baseline model may still hold and there may exist a critical interest rate/capital ratio where the relation between liquidity and solvency effect changes.

PROPOSITION 4. *If the fire-sale rate changes not too strongly,* $\kappa \leq \frac{\sqrt{2\beta\pi}}{2\alpha}$*, there exists a threshold function* $\ell(m, e)$ *with* $\partial\ell/\partial m < 0$ *and* $\partial\ell/\partial e < 0$ *such that the liquidity effect dominates the solvency effect if and only if*

$$R^* < \ell(m,e). \tag{23}$$

As in section 3, condition (23) can be used to derive a critical value $\hat{\hat{r}}(e)$ such that the liquidity effect dominates the solvency effect if $r \leq \hat{\hat{r}}(e)$. Liquidity and solvency effects are thus preserved also in the presence of an adverse feedback from withdrawals to prices.[29]

However, once the assumption in proposition 4 fails to hold and the fire-sale rate responds too strongly to the underlying withdrawal pressure, the monotone relationship between the failure point and the dominance of either solvency or liquidity effect may break down; a non-monotone relationship may emerge with more than one critical value demarcating regions in e-m space where the liquidity or the solvency effect dominates.

Given that the assumption in the foregoing proposition is met, the question arises to which extent regulatory measures exert influence over the expected fire-sale price? This would indicate the existence of an important effect of regulatory measures. Not only may regulation work to directly increase the liquidity of certain financial institutions, it may also improve funding terms ex ante, by affecting expected asset prices and thereby liquidity and risk premia that would have to be incurred when assets are used, for example, as collateral in other transactions.

Using equation (4), the expected fire-sale price can be written as

$$\begin{aligned}
&\mathbf{E}\left(\frac{R}{1+\zeta\left(\max\{x-m,0\}\right)}\right) \\
&\quad = \int_{-\infty}^{\frac{1}{\sqrt{\beta}}z(\Phi^{-1}(m)/\sqrt{\beta};R^*)} R\left(1+\zeta\left(\Phi\left(z(R;R^*)\right)-m\right)\right)^{-1}\mathrm{d}F(R) \\
&\qquad + \int_{\frac{1}{\sqrt{\beta}}z(\Phi^{-1}(m)/\sqrt{\beta};R^*)}^{\infty} R\,\mathrm{d}F(R),
\end{aligned}$$

where $z(R;R^*) \equiv \frac{\alpha}{\sqrt{\beta}}(R^*-\bar{R})+\sqrt{\beta}(R^*-R)-\sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma)$.

[29] Note that even though $\ell(m,e)$ depends on both liquidity and capital ratio, the critical value $\hat{\hat{r}}$ does not depend on m anymore since it is, by definition, the value of r where R^* ceases to depend on m.

By differentiating this expression with respect to e, it is straightforward to show that the expected fire-sale price strictly increases in the capital ratio. This implies that higher capital requirements lower ex ante expected losses in case fire sales become indeed necessary. The effects of the liquidity ratio, however, are ambiguous. Firstly, a higher liquidity ratio raises the fraction of withdrawals needed to trigger fire sales. Secondly, a higher liquidity ratio also affects the fraction of withdrawals directly, as it causes a change in the failure point. These two effects work in the same direction once the liquidity effect outweighs the solvency effect. In such a situation, a higher liquidity ratio depresses fire-sale discounts and helps to mitigate losses that may eventually drive the bank into default. But once the solvency effect dominates, the effects work in opposite directions. While higher cash holdings may help the bank to delay fire sales, they drive up the solvency risk which, in turn, raises fund managers' incentives to run to the bank. Thereby, the impact on the expected fire-sale discount becomes weaker and may eventually turn around, thereby driving up the expected fire-sale rate.

6. Conclusion

The paper revisits the well-known banking model by Rochet and Vives and analyzes complementarity and substitutability between liquidity and capital regulation. Importantly, the analysis fully takes into account the adjustments in the balance sheet needed to accommodate the specific variations in the regulatory ratios. It demonstrates that liquidity requirements, like the Basel Accord's newly proposed LCR, are only effective in strengthening the resilience of a bank once the liquidity effect outweighs the solvency effect. Albeit rather stylized, the model points to some important aspects in the design of regulatory rules. Firstly, besides strengthening the bank's ability to withstand sudden funding roll-offs, liquidity requirements may have a direct impact on the solvency of the bank. Secondly, whether liquidity and capital requirements are substitutes or complements depends on the level of the capital ratio. For higher liquidity ratios to effectively lower bank default risk, a sufficiently strong capital buffer above a certain critical level is required. Finally, the double-edged-sword nature of liquidity regulation obtains also in more general environments where interest rates carry risk premia or fire sales feed back into prices. Under such circumstances, however, the relationship between capital and liquidity regulation may become more complex.

The paper leaves several questions open that remain to be addressed in future research. Theoretically, since the Rochet-Vives model is a partial equilibrium model, a deeper understanding of the interactions between liquidity and capital regulation may be obtained by embedding the coordination problem of fund managers and creditors into a (general equilibrium) framework where the bank's composition of asset and liability sides are endogenous.[30] Moreover, the analysis in the present paper raises the empirical question whether the liquidity or the solvency effect dominates in practice and how large the critical capital ratio e^m is such that increases in liquidity ratios can reduce bank default risk.

Appendix

Proof of Proposition 1

If $R^* = R_s$, the failure point is given explicitly by equation (9) and it is straightforward to compute

$$\frac{\partial R_s}{\partial e} = -\frac{(1-m)(1+r)}{1-m(1+r)} < 0 \quad \text{and} \quad \frac{\partial R_s}{\partial m} = \frac{(1+r)(1-e)r}{(1-m(1+r))^2} \geq 0$$
$$\text{and} \quad \frac{\partial R_s}{\partial r} = \frac{(1-e)(1-m)}{(1-m(1+r))^2} \geq 0.$$

If $R^* > R_s$, the failure point is given by the solution to the implicit function (11). Since $\beta \geq \beta_0$ and the equilibrium is unique,

[30] Recent papers by Liu and Mello (2011) and Daniëls and König (2013) study financial institutions' choice of the asset side, whereas other work by Eisenbach (2013), Szkup (2013), or König and Pothier (2014) consider the determination of the liability side in global game bank-run models. What is missing so far from the literature is a treatment of an endogenous determination of both the asset and liability side in a coordination game setup.

$$\frac{\partial\,\psi(R^*,\,m,\,e,\,r)}{\partial\,R} = (1-m(1+r)) - \frac{\alpha\lambda(1-e)(1+r)}{\sqrt{\beta}}\phi \times \left(\frac{\alpha}{\sqrt{\beta}}(R^*-\bar{R}) - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma)\right) > 0.$$

For any $z \in \{m, e, r\}$, by the implicit function theorem,

$$\frac{\partial\,R^*}{\partial\,z} < 0 \Leftrightarrow -\frac{\frac{\partial\,\psi}{\partial\,z}}{\frac{\partial\,\psi}{\partial\,R}} < 0 \Leftrightarrow \frac{\partial\,\psi}{\partial\,z} > 0. \tag{24}$$

The partial derivative with respect to the e is given by

$$\begin{aligned}&\frac{\partial\,\psi(R^*,\,m,\,e,\,r)}{\partial\,e}\\ &= (1+r)(1-m) + \lambda(1+r)\\ &\quad\times\left(\Phi\left(\frac{\alpha}{\sqrt{\beta}}(R^*-\bar{R}) - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma)\right) - m\right) > 0\end{aligned}$$

and by equation (24), $\frac{\partial\,R^*}{\partial\,e} < 0$. The partial derivative with respect to r is given by

$$\begin{aligned}\frac{\partial\psi}{\partial r} &= -mR^* - (1-e)(1-m) - \lambda(1-e)\\ &\quad\times\left(\Phi\left(\frac{\alpha}{\sqrt{\beta}}((1-e)(1+\lambda)-\bar{R}) - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma)\right) - m\right) < 0,\end{aligned}$$

and it follows from equation (24) that $\frac{\partial\,R^*}{\partial\,r} > 0$. The partial derivative with respect to the m is given by

$$\frac{\partial\,\psi(R^*,\,m,\,e,\,r)}{\partial\,m} \lesseqgtr 0 \Leftrightarrow R^* \gtreqless (1-e)(1+\lambda).$$

Using (24), it follows that $\frac{\partial\,R^*}{\partial\,m} < 0$ if and only if $R^* < (1 - e)(1+\lambda)$.

Derivation of Iso-Risk Curves

The derivation of iso-risk curves proceeds in two steps. First, the critical level $e^m(r)$ and the corresponding risk level R^*_m are derived. Second, the properties of iso-risk curves for risk levels $R^*_k \neq R^*_m$ are shown.

(i) The critical level $e^m(r)$ and the associated risk level R^*_m constitute the simultaneous solution to the two equations

$$\psi(R^*_m, m, e^m, r) = 0 \quad \text{and} \quad R^*_m - (1 - e^m)(1 + \lambda) = 0.$$

The explicit solutions are given by

$$R^*_m = \bar{R} + \frac{\sqrt{\beta}}{\alpha}\Phi^{-1}\left(\frac{\lambda - r}{\lambda(1 + r)}\right) + \sqrt{\frac{\alpha + \beta}{\alpha^2}}\Phi^{-1}(\gamma)$$

and

$$e^m = 1 - \frac{\bar{R} + \frac{\sqrt{\beta}}{\alpha}\Phi^{-1}\left(\frac{\lambda - r}{\lambda(1+r)}\right) + \sqrt{\frac{\alpha+\beta}{\alpha^2}}\Phi^{-1}(\gamma)}{1 + \lambda}.$$

Since e^m is independent of m, it constitutes a horizontal line in e-m space. Along this line, variations in the liquidity ratio m do not exert any effect on the risk level R^*_m.

(ii) For fixed risk levels $R^*_k \neq R^*_m$, the iso-risk curves in e-m space are implicitly defined by $\psi(R^*_k, m, e, r) = 0$. Since $\partial\psi/\partial e > 0$, a capital ratio $e > e^m$ implies a risk level $R^*_k < R^*_m$ and vice versa for $e < e^m$. The slope of the iso-risk curve is given by

$$\left.\frac{\mathrm{d}e}{\mathrm{d}m}\right|_{R^* = R^*_k} = -\frac{\partial\psi/\partial m}{\partial\psi/\partial e},$$

which is positive for any $R^*_k > R^*_m$ ($e < e^m$) and negative for any $R^*_k < R^*_m$ ($e > e^m$). Moreover,

$$\left.\frac{\mathrm{d}^2 e}{\mathrm{d}m^2}\right|_{R^* = R^*_k} = -\frac{2\partial^2\psi/\partial m\partial e}{\partial\psi/\partial e}\frac{\mathrm{d}e}{\mathrm{d}m}.$$

Since $\frac{\partial^2\psi}{\partial e\partial m} < 0$, the iso-risk curves are concave for $R^*_k < R^*_m$ (i.e., $e > e^m$) and convex for $R^*_k < R^*_m$ (i.e., $e < e^m$).

Derivation of Interest Parity Condition (19)

With probability $1 - F(R^*)$, creditors obtain D independent of whether fund managers withdraw or not.

If the bank fails early, $R < R_{EC}$, its resources are insufficient to meet withdrawals. Creditors face a sequential service constraint, and the likelihood of receiving D is given by $(RI(1+\lambda)^{-1}+M)/xD$. By the law of large numbers, the probability of withdrawing—i.e., of getting a signal below t^* conditional on R—is given by

$$\Phi(\sqrt{\beta}(t^*(R^*) - R)) \equiv x(t^*(R^*), R) \equiv x(R^*, R).$$

Creditors whose funds are rolled over go away empty handed. Hence, if $R < R_{EC}$, a creditor expects to receive

$$\begin{aligned}&\int_{\underline{R}}^{R_{EC}} \frac{(RI(1+\lambda)^{-1}+M)}{x(R^*,R)D} x(R^*,R)D \,\mathrm{d}F(R) \\ &\quad = (1+r)D_0 \int_{-\underline{R}}^{R_{EC}} \left[\frac{R(1-m(1+r))}{(1+\lambda)(1-e)(1+r)} + m\right] \mathrm{d}F(R),\end{aligned}$$

where $\underline{R} \equiv -\frac{(1+\lambda)m(1+r)(1-e)}{1-(1+r)m}$ is the value of R such that the probability of a creditor getting served is at least zero.

Next, consider the case $R \in (R_{EC}, R^*)$. Conditional on R, a creditor withdraws with probability $x(R^*, R)$ and obtains D (available liquidity in this range is sufficient to meet withdrawals). The bank fails at date 2, so that creditors who roll over (with probability $(1-x(R^*, R))$ become residual claimants. The residual wealth of the bank (taking the losses due to early fire sales into account) is given by

$$RI + (1+\lambda)M - (1+\lambda)x(R^*, R)D.$$

This is divided on a pro rata basis among the remaining $(1 - x)D$ liabilities. So a creditor obtains a fraction

$$\frac{(RI + (1+\lambda)M - (1+\lambda)x(R^*, R)D)}{(1 - x(R^*, R))D}$$

of his original claim D.

That is, in case $R \in (R_{EC}, R^*)$, he expects to receive

$$
\begin{aligned}
&D \int_{R_{EC}}^{R^*} x(R^*, R)\mathrm{d}F(R) \\
&\quad + \int_{R_{EC}}^{R^*} \left[\frac{(RI + (1+\lambda)M - (1+\lambda)x(R^*, R)D)}{(1 - x(R^*, R))D} (1 - x(R^*, R))D \right] \\
&\quad\quad \times \mathrm{d}F(R),
\end{aligned}
$$

which can be written as

$$
\begin{aligned}
&(1+r)D_0 \int_{R_{EC}}^{R^*} x(R^*, R)\mathrm{d}F(R) \\
&\quad + (1+r)D_0 \int_{R_{EC}}^{R^*} \left[\frac{R(1 - m(1+r))}{(1-e)(1+r)} + (1+\lambda)(m - x(R^*, R)) \right] \\
&\quad\quad \times \mathrm{d}F(R).
\end{aligned}
$$

Putting everything together, the interest parity condition can be written as

$$
\begin{aligned}
&(1 - F(R^*))\,(1+r)D_0 \\
&\quad + (1+r)D_0 \int_{\underline{R}}^{R_{EC}} \left[\frac{R(1 - m(1+r))}{(1+\lambda)(1-e)(1+r)} + m \right] \mathrm{d}F(R) \\
&\quad + (1+r)D_0 \int_{R_{EC}}^{R^*} x(R^*, R)\mathrm{d}F(R) \\
&\quad + (1+r)D_0 \int_{R_{EC}}^{R^*} \left[\frac{R(1 - m(1+r))}{(1-e)(1+r)} + (1+\lambda)(m - x(R^*, R)) \right] \\
&\quad\quad \times \mathrm{d}F(R) \;=\; (1+\bar{r})D_0, \qquad (25)
\end{aligned}
$$

which can be expressed in compact form as in equation (19) by dividing both sides by D_0 and using the abbreviation

$$
\begin{aligned}
&\mathcal{L}(R^*, R_{EC}, r, m, e) \\
&\quad \equiv (1+r)\left[\int_{\underline{R}}^{R_{EC}} \left[\frac{R(1-m(1+r))}{(1+\lambda)(1-e)(1+r)} + m\right] \mathrm{d}F(R)\right] \\
&\quad + (1+r)\left[\int_{R_{EC}}^{R^*} x(R^*, R)\mathrm{d}F(R)\right. \\
&\quad + \left.\int_{R_{EC}}^{R^*} \left[\frac{R(1-m(1+r))}{(1-e)(1+r)} + (1+\lambda)(m - x(R^*, R))\right] \mathrm{d}F(R)\right].
\end{aligned}
$$

Derivation of Uniqueness Condition (20)

R^* and R^*_{EC} are determined as functions of r, m, and e from

$$\psi(R^*, r, m, e) = 0 \tag{26}$$

$$\frac{R^*_{EC}(1-m(1+r))}{(1+\lambda)(1-e)} + (1+r)(m - x(R^*, R^*_{EC})) = 0. \tag{27}$$

A sufficient condition for a stable/unique equilibrium is that the left-hand side of the interest parity condition strictly increases in r when evaluated at the equilibrium values r^*, R^*, and R^*_{EC}. Taking the derivative of equation (25) with respect to r and evaluating it at the equilibrium values, this condition becomes

$$
\begin{aligned}
&(1 - F(R^*)) + m\int_{\underline{R}}^{R^*_{EC}} \left[-\frac{R}{(1+\lambda)(1-e)} + 1\right] \mathrm{d}F(R) \\
&\quad + (1+\lambda)m\int_{R^*_{EC}}^{R^*} \left[-\frac{R}{(1+\lambda)(1-e)} + 1\right] \mathrm{d}F(R) \\
&\quad - \lambda\int_{R^*_{EC}}^{R^*} x(R^*, R)\mathrm{d}F(R) \\
&\quad - (1+\lambda)(1+r^*)\int_{R^*_{EC}}^{R^*} x_{R^*}(R^*, R)\left.\frac{\partial R^*}{\partial r}\right|_{r=r^*} \mathrm{d}F(R) > 0.
\end{aligned}
$$

The latter can be rewritten as

$$\xi(R^*, R^*_{EC}, r^*)$$

$$\equiv \frac{\overbrace{(1+\lambda)(1+r^*)\int_{R^*_{EC}}^{R^*} x_{R^*}(R^*,R)\left.\frac{\partial R^*}{\partial r}\right|_{r=r^*} \mathrm{d}F(R)}^{\text{marginal loss due to higher default risk}}}{\underbrace{1-F(R^*)-m\left[\begin{array}{l}\int_{\underline{R}}^{R^*_{EC}}\left[\frac{R}{(1+\lambda)(1-e)}-1\right]\mathrm{d}F(R)\\ +\lambda\int_{R^*_{EC}}^{R^*}\left[\frac{R}{(1+\lambda)(1-e)}-1+\frac{x(R^*,R)}{m}\right]\mathrm{d}F(R)\end{array}\right]}_{\text{marginal benefit due to higher interest and changed liquidation value}}} < 1.$$

Proof of Proposition 2

I use Cramer's rule to compute the derivatives of R^* and r^* with respect to e and m. Totally differentiating the system of equations consisting of (25), (27), and (28) and casting the results into matrix form gives

$$\mathbf{Jv} + \mathbf{Bu} = 0 \quad \text{where} \quad \mathbf{v}^T = \begin{pmatrix}\mathrm{d}r^* & \mathrm{d}R^* & \mathrm{d}R^*_{EC}\end{pmatrix}, \ \mathbf{u}^T = \begin{pmatrix}\mathrm{d}m & \mathrm{d}e\end{pmatrix}.$$

The elements of the 3×3 Jacobian matrix, evaluated at the equilibrium, are given by

$$\begin{aligned} j_{11} &= (1-F(R^*)) + m\int_{\underline{R}}^{R^*_{EC}}\left[-\frac{R}{(1+\lambda)(1-e)}+1\right]\mathrm{d}F(R) \\ &\quad + (1+\lambda)m\int_{R^*_{EC}}^{R^*}\left[-\frac{R}{(1+\lambda)(1-e)}+1\right]\mathrm{d}F(R) \\ &\quad - \lambda\int_{R^*_{EC}}^{R^*} x(R^*,R)\mathrm{d}F(R) \\ &= (1-F(R^*)) + \frac{m\left(F(R^*_{EC} - F(\underline{R})\right)}{(1+\lambda)(1-e)}\Big((1+\lambda)(1-e) \\ &\quad -\mathbb{E}\left(\tilde{R}\middle| R \in (\underline{R}, R^*_{EC})\right)\Big) \end{aligned}$$

$$
\begin{aligned}
&+\frac{m\left(F(R^*-F(R^*_{EC})\right)}{(1+\lambda)(1-e)}\Big((1+\lambda)(1-e)\\
&-\mathbb{E}\left(\tilde{R}\middle| R\in(R_{EC},R^*)\right)\Big)-\lambda\int_{R^*_{EC}}^{R^*}x(R^*,R)\mathrm{d}F(R),
\end{aligned}
$$

$$
\begin{aligned}
j_{12}&=-(1+\lambda)(1+r^*)\int_{R_{EC}}^{R^*}x_{R^*}(R^*,R)\mathrm{d}F(R)<0,\quad j_{13}=0,\\
j_{21}&=\left.\frac{\partial\psi}{\partial r}\right|_{R^*,r=r*}<0,\quad j_{22}=\left.\frac{\partial\psi}{\partial R^*}\right|_{R^*,r=r*}>0,\quad j_{23}=0,\\
j_{31}&=\frac{-mR^*_{EC}}{(1+\lambda)(1-e)}+m-x(R^*,R^*_{EC})<0,\\
j_{32}&=-(1+r^*)x_{R^*}(R^*,R^*_{EC})<0,\\
j_{33}&=\frac{1-m(1+r^*)}{(1+\lambda)(1-e)}-(1+r^*)x_{R^*_{EC}}(R^*,R^*_{EC})>0.
\end{aligned}
$$

Given the condition for a unique equilibrium and since $j_{12}<0$, it follows that $j_{11}>0$. The determinant of the Jacobian is given by $|\mathbf{J}|=j_{33}(j_{11}j_{22}-j_{21}j_{12})>0$, which can be written as $|\mathbf{J}|=j_{11}j_{22}j_{33}(1-\xi^*)$.

The elements of the 3×2 matrix B are given by

$$
\begin{aligned}
b_{11}=&\frac{(1+r^*)\left(F(R^*_{EC}-F(\underline{R})\right)}{(1+\lambda)(1-e)}\Big((1+\lambda)(1-e)\\
&-\mathbb{E}\left(\tilde{R}\middle| R\in(\underline{R},R^*_{EC})\right)\Big)\\
&+\frac{(1+r^*)\left(F(R^*-F(R^*_{EC})\right)}{(1-e)}\Big((1+\lambda)(1-e)\\
&-\mathbb{E}\left(\tilde{R}\middle| R\in(R^*_{EC},R^*)\right)\Big)\gtrless 0,
\end{aligned}
$$

$$
b_{21}=\left.\frac{\partial\psi}{\partial m}\right|_{R^*,r=r^*}\gtrless 0,\quad b_{31}=-(1+r^*)\left(\frac{R^*_{EC}}{(1+\lambda)(1-e)}-1\right)\gtrless 0,
$$

$$
\begin{aligned}
b_{12}=&\int_{\underline{R}}^{R^*_{EC}}\frac{R(1-m(1+r^*))}{(1+\lambda)(1-e)^2}\mathrm{d}F(R)\\
&+\int_{R^*_{EC}}^{R^*}\frac{R(1-m(1+r^*))}{(1-e)^2}\mathrm{d}F(R)>0
\end{aligned}
$$

$$b_{22} = \left.\frac{\partial \psi}{\partial e}\right|_{R^*, r=r^*} > 0, \quad b_{23} = \frac{(1+r^*)R^*_{EC}}{(1+\lambda)(1-e)^2} > 0.$$

Using Cramer's rule, it is straightforward to establish part (i) of the proposition.

$$\frac{\mathrm{d}R^*}{\mathrm{d}e} = \frac{\begin{vmatrix} j_{11} & -b_{12} & 0 \\ j_{21} & -b_{22} & 0 \\ j_{31} & -b_{32} & j_{33} \end{vmatrix}}{|\mathbf{J}|} < 0 \quad \text{and} \quad \frac{\mathrm{d}r^*}{\mathrm{d}e} = \frac{\begin{vmatrix} -b_{12} & j_{12} & 0 \\ -b_{22} & j_{22} & 0 \\ -b_{32} & j_{32} & j_{33} \end{vmatrix}}{|\mathbf{J}|} < 0$$

With respect to part (ii), define

$$\rho \equiv \frac{b_{11}}{j_{11}},$$

whose sign is determined by b_{11} as $j_{11} > 0$. b_{11} is the liquidation value effect of m since it is the marginal change in the liquidation value with respect to m. One obtains

$$\frac{\mathrm{d}R^*}{\mathrm{d}m} = \frac{\begin{vmatrix} j_{11} & -b_{11} & 0 \\ j_{21} & -b_{21} & 0 \\ j_{31} & -b_{31} & j_{33} \end{vmatrix}}{|\mathbf{J}|} \quad \text{and} \quad \frac{\mathrm{d}r^*}{\mathrm{d}m} = \frac{\begin{vmatrix} -b_{11} & j_{12} & 0 \\ -b_{21} & j_{22} & 0 \\ -b_{31} & j_{32} & j_{33} \end{vmatrix}}{|\mathbf{J}|},$$

which can be rewritten as

$$\frac{\mathrm{d}R^*}{\mathrm{d}m} = \frac{\frac{-b_{21}}{j_{22}} + \frac{j_{21}b_{11}}{j_{11}j_{22}}}{1-\xi^*} \quad \text{and} \quad \frac{\mathrm{d}r^*}{\mathrm{d}m} = \frac{\frac{-b_{11}}{j_{11}} + \frac{j_{21}b_{21}j_{12}}{j_{21}j_{11}j_{22}}}{1-\xi^*},$$

which, by using the respective expressions given above, yields the expressions in (21).

Recall from condition (12) and equation (13) that $R^* < (1+\lambda)(1-e)$ if and only if $r^* < r^m(e)$. Evaluating equation (25) at $r^m(e)$ and solving for $\bar{r}$ yields the value of the risk-free $\hat{r}^m$ that induces $r^* = r^m(e)$. As the interest rate and the default risk strictly increase in $\bar{r}$, if $\bar{r} < \hat{r}^m$, we have $r^* < r^m(e)$ and the liquidity effect dominates the solvency effect.

Proof of Proposition 3

Define $a_0 \equiv \frac{(1+r^*)(F(R_{EC})-F(\underline{R}))}{(1+\lambda)(1-e)j_{11}} > 0$ and $a_1 \equiv \frac{(1+r^*)(F(R^*)-F(R_{EC}))}{(1-e)j_{11}} > 0$.

Suppose first that the liquidity effect dominates the solvency effect. From proposition 1, this occurs if and only if $R^* < (1-e)(1+\lambda)$. This implies

$$\begin{aligned}0 &> (a_0 + a_1)(R^* - (1-e)(1+\lambda)) \\ &> a_0\left((F(R^*_{EC}) - F(\underline{R}))R^* - (1-e)(1+\lambda)\right) \\ &\quad + a_1\left((F(R^*) - F(R^*_{EC}))R^* - (1-e)(1+\lambda)\right) \\ &> a_0\left(\int_{\underline{R}}^{R^*_{EC}} R \mathrm{d}F(R) - (1-e)(1+\lambda)\right) \\ &\quad + a_1\left(\int_{R^*_{EC}}^{R^*} R \mathrm{d}F(R) - (1-e)(1+\lambda)\right) \\ &= -\rho.\end{aligned}$$

This establishes that if the liquidity effect dominates the solvency effect, then the liquidation value effect is positive. It follows immediately that the contrapositive of this claim—i.e., if the liquidation value effect is negative, then the solvency effect dominates the liquidity effect—is also true.

Derivation of Uniqueness Condition (22)

Since the dependency of the fire-sale price on aggregate withdrawals strengthens the strategic complementarities between the fund managers, the required uniqueness condition becomes more stringent. Inserting $\zeta(\cdot)$ into equation (11) and differentiating with respect to R^* gives

$$\begin{aligned}&\frac{\mathrm{d}\psi(R^*, m, e, r, \zeta(R^*, m))}{\mathrm{d}R^*} \\ &\quad = (1 - m(1+r)) - \frac{\lambda\alpha}{\sqrt{\beta}}\phi\left(\frac{\alpha}{\sqrt{\beta}}(R^* - \bar{R}) - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma)\right) \\ &\qquad \times (1-e)(1+r)\end{aligned}$$

$$
- \zeta'(1-e)(1+r)\phi\left(\frac{\alpha}{\sqrt{\beta}}(R^* - \bar{R}) - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma)\right)\frac{\alpha}{\sqrt{\beta}}
$$

$$
\times\left(\Phi\left(\frac{\alpha}{\sqrt{\beta}}(R^* - \bar{R}) - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma)\right) - m\right).
$$

Since $\zeta' > 0$, the sufficient condition for the uniqueness of the threshold equilibrium has to be modified by using the assumptions $\lambda < \bar{\lambda}$ and $\zeta' < \kappa$ to yield

$$
\sqrt{\beta} > \frac{\alpha}{\sqrt{2\pi}}\frac{(1-e)(1+r)(\bar{\lambda}+\kappa(1-m))}{1-m(1+r)}.
$$

Proof of Proposition 4

Inserting $\zeta(\cdot)$ into equation (11) and differentiating with respect to m yields

$$
\frac{\partial \psi}{\partial m} \gtreqless 0 \quad \Leftrightarrow \quad R^* \lesseqgtr \tilde{\ell}(R^*, e, m), \tag{28}
$$

where $\tilde{\ell}(R^*, e, m) \equiv (1-e)\big(1+\lambda+\zeta'\big(\Phi\big(\frac{\alpha}{\sqrt{\beta}}(R^* - \bar{R}) - \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(\gamma)\big) - m\big)\big)$. Equation (29) can be solved for a unique critical value R^* if $\kappa \leq \frac{\sqrt{2\beta\pi}}{2\alpha}$. As long as this condition holds, the left-hand side of equation (29) increases faster than the right-hand side and there exists a unique value $\ell(m,e)$ such that $\partial\psi/\partial m > 0$ (the liquidity effect dominates) if and only if $R^* < \ell(\cdot)$. The derivatives of ℓ with respect to m and e can be easily found by applying the implicit function theorem to $R^* - \tilde{\ell}(R^*, m, e) = 0$. Evaluating equation (11) at $R^* = \ell(m,e)$ yields

$$
\psi(\ell(m,e), m, e, r) = \tilde{\psi}(e, r) = 0,
$$

which is independent of m since ℓ is exactly equal to the value of R^* where the derivative of $\psi(\cdot)$ with respect to m vanishes. Equation $\tilde{\psi}(e,r) = 0$ can thus be solved for $r = \hat{\hat{r}}(e)$, and since $\psi(\cdot)$ is strictly decreasing in r, it follows that for any $r < \hat{\hat{r}}(e)$, $R^* < \ell$ and the liquidity effect dominates the solvency effect.

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