Monetary and Labor Interactions in a Monetary Union*

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A two-country general equilibrium model with large wage setters is developed to investigate the welfare implications of moving from a flexible exchange rate regime to a monetary union. The paper shows that the currency regime not only affects the central bank’s incentive to improve the terms of trade but also the labor unions’, generating different strategic interactions between monetary policy and wage setting. A switch from non-cooperation to monetary union does not necessarily lead to wage increases. However, a common central bank can be beneficial when a country is sufficiently open to trade, since the expected welfare gain due to the strategic effects at work more than offsets the welfare loss resulting from monetary policy’s inability to optimally stabilize the effects of asymmetric shocks.

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1. Introduction

European labor markets are characterized by centralized and collective wage bargaining (figure 1). In most of these countries negotiations are usually delegated to a few large labor unions, whose

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Figure 1. Centralization of Wage Setting and the Coverage of Workers by Collective Bargaining Provisions in OECD Countries for the Period 2000

Notes: Regression line: \( y = 16.58(\text{se} = 8.22) + 18.78(\text{se} = 3.31) x \). 1 = Company- and plant-level predominant; 2 = Combination of industry and company/plant level; 3 = Industry-level predominant; 4 = Predominantly industrial bargaining but also recurrent central-level agreements; 5 = Central-level agreements of overriding importance.

decisions affect aggregate wages at the national or sectoral level. In this context, wage setters can take into account the impact of their wage claims not only on the real cost of labor but also on domestic inflation.

The establishment of the Economic and Monetary Union (EMU) changed the way macroeconomic policy is conducted in Europe. According to macroeconomic models on strategic interactions between monetary policy and wage setting, a union-wide monetary policy is expected to reduce the extent to which each labor union internalizes the inflationary repercussions of wage demands, thereby curtailing wage restraints (e.g., Cukierman and Lippi 2001, Grüner and Hefeker 1999, and Soskice and Iversen 1998). Yet, there is evidence that the monetary unification has not led to more inflationary wage pressures (see figure 2, European Commission 2007, and Gould and Posen 2007).
This literature focuses on the role of centralized wage bargaining and central bank independence. Following Cubitt (1992, 1995) and Gylfason and Lindbeck (1994), the interaction between the labor unions and the central bank is usually modeled as a Stackelberg game. In this respect, two strategic mechanisms in a monetary union have been investigated. First, Cukierman and Lippi (2001) show that in a currency union wage setters internalize the impact of wage demands on union-average wages to a lesser extent. As a consequence, each union perceives wage claims as improving its relative wage. Second, Soskice and Iversen (1998) argue that the adoption of a common central bank leads unions to anticipate that wage hikes will have a lower impact on the union-average inflation and hence on the monetary policy response to their actions. Both channels predict that a switch from national monetary policies to a monetary union is detrimental to welfare, since unions’ incentive to restrain wage claims would be reduced.

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1See Acocella and Di Bartolomeo (2004) and Cukierman (2004) for a survey of studies where the conservatism (i.e., the degree of inflation aversion) of the central bank has a systematic effect on equilibrium employment in the presence of non-atomistic wage setters.
However, these arguments against the creation of a monetary union hinge on ad hoc objective functions and disregard the adverse “beggar-thy-neighbor” effect, typical of the open economy. As highlighted by Benigno (2002) and Corsetti and Pesenti (2001), in an open economy uncoordinated monetary authorities have an (ex post) incentive to contract their monetary stance in order to improve the terms of trade. Under flexible exchange rates, the expected level of output is lower than the efficient level because of this incentive. In a monetary union, instead, the common central bank internalizes the external distortion stemming from the terms of trade and eases its monetary stance up to the efficient level of output. Most of the findings on the impact of a monetary union with large unions overlook these externalities. This is due to the fact that previous studies either compare a monetary union regime with a national monetary policy operating in a closed-economy setup (e.g., Cukierman and Lippi 2001, Soskice and Iversen 1998) or simply hinge on employing ad hoc quadratic objective functions (e.g., Cavallari 2001b, Cuciniello 2011, Grüner and Hefeker 1999).

This paper assesses wage-setting behavior in a tractable new open-economy macroeconomic model in which the objectives of the policymakers are fully microfounded. I show that, besides involving monetary policy, the incentive to improve the terms of trade may affect wage setting as well. In a monetary union, large unions wish to exploit their domestic monopoly power and strategically improve the terms of trade through “excessive” wage demands so as to shift the burden of production to another country, since consumers purchase more foreign goods. On the one hand, improving the terms of trade is expected to raise utility through the reduction in the disutility of working. On the other hand, the output contraction, triggered by the reduction in the consumption of domestic goods, raises the elasticity of aggregate labor demand to wage hikes. This second channel, reducing the unions’ bargaining power, prompts wage restraints. It turns out that, whether the output effect prevails over the incentive to improve the terms of trade, a monetary union can actually reduce monopolistic distortions in the labor market.

The expected welfare effect of moving to a monetary union is, however, associated with the monetary authorities’ and labor unions’ incentives to improve the terms of trade. Specifically, the welfare gains of creating a monetary union are larger the higher the degree
of openness, namely the higher the external distortion associated with the terms of trade. Intuitively, the terms-of-trade externality tends to lower expected consumption under uncoordinated monetary policies and also lowers the expected disutility of labor in a monetary union through the wage markup. Although the main disadvantage of creating a monetary union is the impossibility of efficiently stabilizing asymmetric shocks, a numerical illustration indicates that the welfare losses generated by this inefficient monetary response to country-specific shocks are much smaller than the welfare gains generated by the internalization of the terms-of-trade effect.

This paper is organized as follows. The model is presented in section 2, and section 3 discusses the game equilibrium. Section 4 details the welfare results of the two monetary regimes. Section 5 concludes.

2. Economic Setup

I analyze strategic interactions between monetary policy and non-atomistic wage setters in a microfounded framework in line with the new open-economy macroeconomics literature (e.g., Corsetti and Pesenti 2001, Obstfeld and Rogoff 1995).

The world economy is composed of two equally sized countries, Home (H) and Foreign (F). Each country is specialized in the production of a traded good. The Home (Foreign) good requires a continuum of differentiated labor inputs indexed by \( j \in [0, 1] \) which are supplied by Home (Foreign) households. Wages are contractually fixed for one period (the contract period) and set by a finite number of labor unions. At the beginning of the contract period, these non-atomistic unions simultaneously set nominal wages in anticipation of the optimal monetary reaction to their wage hikes.

I consider two alternative monetary regimes: a non-cooperative monetary regime and a monetary union. In the non-cooperative monetary regime, each country’s monetary authority independently chooses the monetary stance under floating exchange rates. In the monetary union, instead, the two countries share the same currency and monetary policy is delegated to a common central bank. The optimal monetary stance under either regime is always known to unions when setting their wages.
2.1 Households

The representative household is composed of a continuum of members indexed by \( j \in [0, 1] \). There is full risk sharing within the household. As in Erceg, Henderson, and Levin (2000) and much of the subsequent literature, members of each household perfectly insure each other against variations in labor income so that they face the same budget constraint and make the same consumption choices even if they have different wages. Thus, the Home household’s period utility can be written as follows:

\[
U = \frac{C^{1-\rho}}{1-\rho} - \frac{K}{2} \int_0^1 L^2(j) dj, \tag{1}
\]

where \( L(j) \) expresses the total amount of time or effort that household members allocate to labor market activities and \( C \) is the consumption aggregator. The productivity shock is modeled as an exogenous shock to the disutility a household experiences by supplying labor, \( K \). I allow for home bias in preferences and denote as \( \phi \) the share of domestic goods in consumption expenditure in each country:

\[
C = \left( \frac{C_H}{\phi} \right)^\phi \left( \frac{C_F}{1-\phi} \right)^{1-\phi} \quad \text{and} \quad C^* = \left( \frac{C_H^*}{1-\phi} \right)^{1-\phi} \left( \frac{C_F^*}{\phi} \right)^\phi, \tag{2}
\]

where \( C \) and \( C^* \) are Home and Foreign aggregate consumption, respectively. Note that the home bias towards locally produced tradable goods is associated with the parameter restriction \( \phi > 1/2 \). The consumption-based price indexes expressed in domestic currency are

\[
P = P_H^\phi P_F^{1-\phi} \quad \text{and} \quad P^* = P_H^{1-\phi} P_F^\phi, \tag{3}
\]

where \( P_H \) and \( P_F \) are the Home-currency prices of the two goods, while \( P_H^* \) and \( P_F^* \) are the corresponding Foreign-currency prices.

\[\text{2Foreign households are modeled in an analogous way. Foreign variables are denoted with an asterisk.}\]

\[\text{3The results would be identical if I were to consider the productivity shock as a technological shift in the production function (see, e.g., Obstfeld and Rogoff 2000). But for the purpose of the paper, the formulation used in (1) is more convenient.}\]
At the beginning of each period, households enter with nominal balances $M_0$ and receive a lump-sum transfer, $(x - 1)M_0^s$, from the Home government, where $x$ is the gross growth rate of the Home money supply, i.e., $x = M^s/M_0^s$. Seigniorage revenue is thrown away at the end of each period. The representative household splits into a worker and a shopper. Trading takes place as follows. The shopper needs cash in advance to pay for nominal expenses and therefore faces the following constraint:

$$P_H C_H + P_F C_F \leq M_0 + (x - 1)M_0^s. \quad (4)$$

Each household is subject to the following budget constraint:

$$P_H C_H + P_F C_F + M = M_0 + (x - 1)M_0^s + \int_0^1 W(j)L(j)\,dj, \quad (5)$$

where $W(j)$ is the nominal wage for type $j$ labor. I assume that the wage for each labor type is set by a union representing that type of labor. For a given wage, each worker supplies the quantity of labor, $L(j)$, that is determined by the aggregation of firms’ labor demand decisions (and allocated uniformly across households). Both $W(j)$ and $L(j)$ are taken as given by each household. Nevertheless, an individual will be willing to work if the real wage for his labor type exceeds his disutility of labor, i.e.,

$$\frac{W(j)}{P} \geq KC^\rho L(j) \quad \text{and} \quad \frac{W^*(j^*)}{P^*} \geq K^*C^{*\rho}L^*(j^*). \quad (6)$$

### 2.2 Firms

Production of goods takes place under constant returns and is perfectly competitive.\(^4\) A representative firm produces the domestic good from labor using the following technology:

$$Y = \left[ \int_0^1 L(j)^{\sigma-1} \, dj \right]^{\frac{\sigma}{\sigma-1}}, \quad (7)$$

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\(^4\)Note that the assumption of Cobb-Douglas preferences over goods in conjunction with zero initial bond holdings entails the redundancy of the global securities market (see Corsetti and Pesenti 2001).

\(^5\)The main finding of the paper is not affected by assuming monopolistically competitive firms instead.
where $Y$ is total output of the firm and $L(j)$ is the demand for labor input $j$

$$L(j) = \left[ \frac{W(j)}{W} \right]^{-\sigma} Y. \quad (8)$$

Here $\sigma > 1$ indicates the elasticity of substitution across the different types of labor input, $W(j)$ denotes the nominal wage of labor type $j$, and $W$ is the nominal wage index defined as

$$W = \left[ \int_{0}^{1} W(j)^{1-\sigma} \, dj \right]^{\frac{1}{1-\sigma}}. \quad (9)$$

### 2.3 Timing

I assume a one-shot two-stage game. In the first stage of the game, each union sets the nominal wage in an uncoordinated way before the occurrence of the productivity shocks and the monetary stance. In this way, nominal wages are set one period in advance. Labor unions have full information about the firms’ optimal input decision as well as the optimal monetary policy, thereby taking them into account in the wage-setting process. Specifically, each union acts strategically as leader in the Stackelberg game against the central banks, and plays a Nash game against all the other unions. The equilibrium of this wage-setting game determines the nominal wages in the two countries.

In the second stage, the money supply is set after there has been a productivity shock under two alternative monetary regimes: a non-cooperative regime and a monetary union. The former is a Nash equilibrium, where each central bank sets the domestic monetary stance, taking the other central bank’s money supply as given. The monetary union differs from the non-cooperative regime because there is a common currency and also a common monetary authority that maximizes welfare union-wide. The class of policies described will yield a discretionary equilibrium in which the monetary authority reoptimizes at each date, taking as given previously set wages and current expectations.

Finally, production and trade take place through the firms’ and households’ decisions. After observing the nominal wage and money
balances, each firm decides the optimal labor input so as to maximize its profits, while households consume goods and supply the required labor services. The resulting string of first-order conditions simultaneously determines the general price level, consumption, and employment.

The choice of modeling the game between unions and monetary authorities as a Stackelberg game is in line with the literature on non-atomistic wage setting (e.g., Bratsiotis and Martin 1999; Lippi 2003; Soskice and Iversen 1998, 2000). Moreover, it reflects the fact that wage contracts are usually fixed for at least one year, while prices and money supplies can be adjusted more frequently. General equilibrium is characterized by solving the game using backward induction.

3. Equilibrium

3.1 Production and Trade

With Cobb-Douglas preference aggregators, the market clearing conditions for the two goods are given by

\[ Y = C_H + C^*_H = \phi \frac{PC}{P^*_H} + (1 - \phi) \frac{P^*C^*}{P^*_H} \]

and

\[ Y^* = C_F + C^*_F = (1 - \phi) \frac{PC}{P_F} + \phi \frac{P^*C^*}{P^*_F} \] (10)

Home (Foreign) producers receive the same price from sales at Home (Foreign) and abroad, thus the law of one price holds:

\[ P^*_H = \frac{P_H}{E} \quad \text{and} \quad P^*_F = EP^*_F, \] (11)

where \( E \) is the nominal exchange rate (domestic currency per unit of Foreign currency). Profit maximization implies that optimal prices are equal to domestic wages:

\[ P_H = W \quad \text{and} \quad P^*_F = W^*. \] (12)

I assume money-market equilibrium at the initial period, \( M_0 = M^*_0 \). Money-market clearing conditions require that money demand
equals money supply \( M = M^* \) at every point in time. Using this fact in the cash-in-advance and the resource constraints will yield

\[
M = PC \quad \text{and} \quad M^* = P^*C^*,
\]

and implies balanced trade, \( P_F^*C_F = P_H^*C_H^* \), or, alternatively,

\[
WY = PC = M \quad \text{and} \quad W^*Y^* = P^*C^* = M^*.
\]

Finally, combining the law of one price (11), trade balance (14), and market clearing (10) yields

\[
\mathcal{E} = M/M^*.
\]

3.2 Monetary Policy

As in Obstfeld and Rogoff (1995) and much of the subsequent literature, the monetary authority in each country operates under discretion, affecting the equilibrium allocation.

3.2.1 Non-Cooperative Monetary Regime

Under floating nominal exchange rates, monetary policy is conducted by an independent central bank in each country. The Home central bank controls the money supply, \( M \), and seeks to maximize the utility function of the representative household (1), taking the Foreign monetary policy, \( M^* \), and wages as given. Similarly, the Foreign central bank maximizes

\[
U^* = \frac{C^{*1-\rho}}{1-\rho} - \frac{K^*}{2} \int_0^1 L^*^2(j^*)dj^*,
\]

setting \( M^* \) and taking \( M \) and wages as given. The maximization problem is conditional on the decentralized equilibrium conditions derived in section 3.1 and the workers’ participation constraint (6) (see appendix 1).

In a Nash equilibrium the optimal monetary stance of the Home and Foreign monetary authorities yields the following expressions:

\[
1 = KY^2C^{\rho-1}/\phi
\]
and
\[
1 = K^* Y^* 2 C^*^{p - 1} / \phi,
\]
where \(1/\phi\) indicates the distance between the Nash allocation and the Pareto-efficient allocation, \(1 = KY^2C^{\rho-1}\), where the real wage expressed in terms of consumption is equal to the disutility of labor. Intuitively, the presence of monopolistic power in the labor market generates a wedge between the marginal rate of substitution between consumption and leisure and the real wage. Thus, in a closed economy (i.e., when \(\phi = 1\)), the monetary authority would expand output so as to eliminate any deviation of real economic activity from Pareto efficiency. This is the familiar Blanchard and Kiyotaki (1987) result. However, in an open economy (i.e., when \(\phi < 1\)), the monetary authority also has an incentive to contract output. A monetary tightening—appreciating the exchange rate—can improve the terms of trade. This shifts households demand towards the Foreign-produced good, thereby reducing the disutility of supplying labor services at Home more than utility from consumption (Corsetti and Pesenti 2001). This effect is larger as \(\phi\) diminishes. In other words, the improvement in the terms of trade increases the distance between the Nash allocation and the competitive allocation.

### 3.2.2 Monetary Union

Since members of a currency union adopt the same currency, I assume (without loss of generality) \(E = 1\). From (15) it turns out that \(M = M^*\). The union’s central bank chooses the global money supply \(M_W = M = M^*\) in order to maximize welfare union-wide,
\[
U_W \equiv \frac{U + U^*}{2},
\]
\(U_W\) taking wages as given. As in the case of non-cooperation, the maximization problem is subject to the Home and Foreign market clearing conditions, consistent with optimality conditions in the goods markets derived in section 3.1 (see appendix 1).

In a monetary union, the monetary authority internalizes the negative externalities originating from the terms of trade. It turns out that the global money supply is supplemented to the point
where both countries reach the competitive level. In fact, under this parameterization the participation constraint (6) is binding on both economies:

\[ 1 = KY^2C^{\rho-1}, \tag{20} \]
\[ 1 = K^*Y^*2C^{*\rho-1}. \tag{21} \]

The result that a switch from a non-cooperative regime to a monetary union entails more expansive monetary policy is in line with the Keynesian literature on international policy coordination. According to this strand of literature, monetary cooperation may remove the disincentive to inflate through the internalization of the terms-of-trade distortion (e.g., Benigno 2002, Rogoff 1985).

Having derived the optimal monetary policies, I turn to the implication of these policies for the wage setter’s decision.

### 3.3 Wage Setting

Each worker is unionized and specialized in a type of labor. In each country there are \( n > 1 \) labor unions. Without loss of generality, I assume that the nominal wage for each labor type \( j \in u \) is equal and set by a union representing each type. Therefore, each union \( u \) represents the mass of \( 1/n \) workers. In such a setup, both the level of centralization of wage setting and the union’s ability to internalize the macroeconomic consequences of wage hikes are proportional to the size of the union: the smaller the number of unions, the more they internalize the impact of their wage settlement on aggregate wages (see, e.g., Bratsiotis and Martin 1999, Calmfors and Driffill 1988, Lippi 2003, Soskice and Iversen 2000).

Labor unions set nominal wages a period in advance of the occurrence of a shock. Next, households supply the amount of labor that firms demand at the posted nominal wage. The representative union seeks to maximize the expected household utility by setting the wage \( W(u) \):

\[
\max_{W(u)} \mathbb{E}V_u = \mathbb{E} \left\{ \frac{C^{1-\rho}}{1 - \rho} - \frac{K}{2n} \int_{j \in u} L^2(j) \, dj \right\}, \tag{22}
\]

subject to the optimal monetary policy and a flow budget constraint as in (5) and labor demand schedules of the form (8). Here, \( \mathbb{E} \) denotes
the expectations operator, defined as \( \mathbb{E}(X) = \mathbb{E}(X_t \mid I_{t-1}) \), where \( X_t \) is a stochastic variable and \( I_{t-1} \) is the information set available to the union in period \( t-1 \). Notice that the above objective differs from the central bank’s objective in that the unions consider only a fraction of the agents in the economy.

Controlling the nominal wage, \( W(u) \), the \( u \)-th union anticipates in a symmetric equilibrium, \( W(u) = W \), that

\[
\frac{\partial W}{\partial W(u)} = \frac{1}{n} \tag{23}
\]

(see appendix 2). Equation (23) is key to understanding the model results. As long as \( n \) is finite, an increase in the union’s wage affects aggregate wages, which, in turn, raises domestic prices (12). In addition, each union anticipates that in a Nash equilibrium (\( NE \)) the optimal supplies of money, solving (17) and (18), are respectively given by

\[
M^{NE} = W K^{\frac{\phi - \rho \phi - 2}{2(1 + \rho)}} (K^*)^{\frac{(\rho - 1)(1 - \phi)}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)}} (1 + \rho)^{\frac{1}{1 + \rho}}, \tag{24}
\]

\[
M^{*NE} = W^* K^{\frac{\phi - \rho \phi - 2}{2(1 + \rho)}} (K^*)^{\frac{\phi - \rho \phi - 2}{2(1 + \rho)}} (1 + \rho)^{\frac{1}{1 + \rho}}, \tag{25}
\]

while in a monetary union (\( MU \)) the optimal level of money in the global economy, solving (20) and (21), is

\[
M^{MU}_W = K^{* - \frac{1}{2(1 + \rho)}} K^{\frac{1}{2(1 + \rho)}} (KW^*)^{\frac{1}{2}}. \tag{26}
\]

Plugging the above reaction functions into the expressions derived in section 3.1 yields the following (ex post) output and consumer price index:

\[
Y^{NE} = K^{* \frac{(\rho - 1)(1 - \phi)}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)}} K^{\frac{\phi - 2 - \rho \phi}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)}} (1 + \rho)^{\frac{1}{1 + \rho}}, \tag{27}
\]

\[
P^{NE} = W(K^*/K)^{\frac{1 - \phi}{3 - \rho + 2(\rho - 1)\phi}}, \tag{28}
\]

\[
Y^{MU} = (W^*/W)^{\frac{1}{2}} (KK^*)^{\frac{1}{2(1 + \rho)}}, \tag{29}
\]

\[
P^{MU} = W^{\phi}(W^*)^{1 - \phi}. \tag{30}
\]

Under uncoordinated monetary policies, pressure for higher wages leads to a proportional increase in the general level of prices so
that any attempt to modify the real wage is void. Domestic money supply is in fact increased to keep up with wage hikes (24). This implies that in a symmetric equilibrium (i.e., when \( W(u) = W \)), unions perceive the following elasticity of labor demand with respect to the nominal wage:

\[
\varepsilon_{NE}^h \equiv \frac{\partial \log L^{NE}(u)}{\partial \log W(u)} = -\sigma \left( 1 - \frac{1}{n} \right),
\]

(31)

where \( L(u) \) is the labor demand for all types of labor represented by a union. Intuitively, wage pressure has two effects on aggregate output: first, it leads directly to an increase in aggregate wages and so a decrease in aggregate labor demand; second, it induces an accommodative monetary response to wages through the monetary reaction function, hence leading to an increase in output. This effect of increasing output, however, offsets the effect of a decrease in aggregate labor demand. The monetary authority in fact injects money into the system to keep output at its optimal level (27). As a consequence, wage setters do not perceive wage settlements as affecting aggregate output, but only as increasing the union’s relative wage, \( W(u)/W \). Specifically, the higher the relative wage, the lower the impact on aggregate wages. I draw on Lippi’s (2003) terminology and label this mechanism the substitution effect.

Conversely, in a monetary union the common central bank increases the money supply so as to cope with a union-wide increase in wages, thereby yielding the following labor demand elasticity to a nominal wage increase:

\[
\varepsilon_{MU}^h \equiv \frac{\partial \log L^{MU}(u)}{\partial \log W(u)} = -\sigma \left( 1 - \frac{1}{n} \right) + \varepsilon_{MU}^H \frac{1}{n},
\]

(32)

where \( \varepsilon_{MU}^H \equiv \partial \log L^{MU}/\partial \log W = -1/2 \) denotes the elasticity of aggregate output with respect to aggregate wages. Equation (32) is

\[\text{6With a “conservative” monetary authority, namely with one that dislikes inflation (and deflation) more than society does, this would not be the case (see Cuciniello 2011). In this case, the inflationary consequences of wage demand are costly for monetary policy and generate a trade-off between inflation and output. Wage setters anticipate that they can affect this trade-off and reduce output in the wake of their decisions.}\]
a weighted average of the elasticity of substitution between labor types, $\sigma$, and the elasticity of aggregate employment to aggregate wages, which, in turn, is the sum of the direct impact of wages on aggregate employment and the indirect impact of wages on output through the monetary reaction function (26). Since wage pressure leads to a less than proportional increment in the global money supply, the direct impact on output is larger and wage demands cause output to decline. Drawing on Lippi (2003), I label this mechanism the output effect.

With atomistic wage setters (i.e., $n \to \infty$), equation (32) is simply given by the substitution effect. Instead, the role of the output effect, $\varepsilon_{MU}^H$, is increasing in the union’s size, $1/n$. With a single all-encompassing union (i.e., $n \to 1$), each labor type would receive the same wage, thereby preventing the substitution effect from coming into operation.

In a symmetric equilibrium, $W(u) = W$, the first-order condition associated with the wage-setting problem can be written as follows:

$$W = \frac{\eta}{\eta - 1} \mathbb{E} \left\{ KL^2 \right\} \mathbb{E} \left\{ \frac{L}{P} C^{-\rho} \right\}, \quad (33)$$

where $\eta \equiv -d \log L(u)/d \log (W(u)/P) > 1$ is the real consumption wage elasticity (in absolute terms) of the perceived demand for labor type $j \in u$. As wages are preset, equation (33) implies that the nominal wage is equal to a fixed markup $\eta/(\eta - 1)$ over the ratio of the expected disutility from supplying labor to the expected utility the household obtains from higher wage revenue. Without uncertainty, equation (33) would yield the familiar condition, namely the real wage equals a fixed markup over the (intratemporal) marginal rate of substitution between consumption and leisure.

4. Welfare Comparisons

As most of the literature on large labor unions deals with the relationship between monetary unions and structural (deterministic equilibrium) outcomes, I first concentrate on steady-state welfare analysis. I then assess the conventional wisdom whereby in the presence of asymmetric shocks a monetary union is more costly than flexible exchange rate regimes.
4.1 Deterministic Analysis

Deterministic welfare corresponds to a solution for the flex-wage model, combined with the assumption that shocks to labor disutility are given and constant at $K = K^* = 1$:

$$\bar{U}_r = \frac{(1 - 1/\eta r)^{1+\rho} - 1}{1 - \rho} - \frac{(1 - 1/\eta r)^{1+\rho}}{2} \quad r \in \{NE, MU\}. \quad (34)$$

A change in the currency regime, altering the response of monetary stance to wage claims, affects the perceived elasticity of labor demand to real wage. These elasticities, under non-cooperative monetary policies and in a monetary union, are respectively given by

$$\eta^{NE} = \sigma \quad (35)$$

and

$$\eta^{MU} = \begin{bmatrix} 1 - \frac{1}{n} + \frac{1 - \phi}{n} \text{ terms-of-trade effect} \\ \frac{1}{2n} \text{ output effect} \end{bmatrix}^{-1} \begin{bmatrix} \sigma \left(1 - \frac{1}{n}\right) + \frac{1}{2n} \text{ substitution effect} \end{bmatrix} > 1. \quad (36)$$

To provide intuition for the mechanisms at play behind the above elasticities, notice that labor unions internalize the monetary policy conditional on their wage. As explained in the previous section, they anticipate that their wage decisions cannot affect output under non-cooperative monetary policies. In fact, in the absence of a trade-off between output and inflation for monetary policy, large unions understand that the optimal monetary policy is to target the output level (27) (see footnote 6). Wage hikes reduce output and improve the terms of trade, but trigger a proportional expansion of money supply which, in turn, keeps output and terms of trade high. As a result, nominal wage inflation of union $u$ only decreases the real wage of the others, so that the labor demand elasticity is composed of the elasticity of substitution across labor types, $\sigma$.

In a monetary union, instead, the monetary response to domestic wages is less than proportional, which implies that unions expect
a decrease in output in the wake of wage hikes. Moreover, since the nominal exchange rate cannot vary, the terms of trade improve. This channel is captured by the terms-of-trade effect in (36). Now, while the output effect—increasing labor demand elasticity $\eta^{MU}$—restrains wage claims, the terms-of-trade effect reduces the labor demand elasticity and so increases the wage pressure. Intuitively, an improvement in the terms of trade drives up the relative real wage of domestic workers and produces a “beggar-thy-neighbor” welfare spillover: the burden of labor input into production switches from Home workers to Foreign workers, and consumption also switches from the Home-produced good to the Foreign-produced good.\footnote{In a monetary union Cavallari (2001b) highlights similar strategic effects on monopoly distortions in labor markets, but those effects are triggered by the presence of a conservative central bank.}

Compare the two elasticities (35) and (36). If the output and terms-of-trade effects are more significant than the substitution effect, i.e.,

$$\frac{1}{2(1 - \phi)} > \sigma, \quad (37)$$

the creation of a monetary union decreases the monopolistic distortions in the labor markets, since $\eta^{MU} > \eta^{NE}$. As (34) is increasing in $\eta$, a shift from uncoordinated monetary policies to a monetary union leads to a steady-state welfare improvement when the distortions in the labor market and/or the size of the domestic economy are substantial. This finding is in contrast with most of the literature, where a move from a national monetary policy to a monetary union is expected to yield a lower disciplining effect on wage setting in equilibrium (e.g., Cukierman and Lippi 2001, Soskice and Iversen 1998). Two considerations are to be made at this point.

First, most of the literature on non-atomistic wage setters hinges on ad hoc monetary payoffs. Theoretical explanations have argued that monetary union membership tends to weaken the output effect (Soskice and Iversen 1998) or the substitution effect (Cukierman and Lippi 2001), since it reduces the extent to which each union internalizes the inflationary repercussions of its wage demands. Specifically, Soskice and Iversen (1998) claim that the impact of any one labor union on the union-wide monetary stance would be very limited, thereby reducing the output effect. It is possible to show that
Table 1. Contemporaneous Correlations of Quarterly Log Changes

<table>
<thead>
<tr>
<th>Period</th>
<th>Real Wage Terms of Trade</th>
<th>Real Wage Employment Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970:Q1–1998:Q4</td>
<td>−0.02</td>
<td>0.05</td>
</tr>
<tr>
<td>1999:Q1–2008:Q4</td>
<td>−0.57</td>
<td>−0.26</td>
</tr>
</tbody>
</table>

the assumption that the central bank simply reacts to endogenous variations in inflation would entail both an output and terms-of-trade effect operating under flexible exchange rates (see Cuciniello 2011). However, it is also possible to show that with a sufficiently conservative central bank, the creation of a monetary union would still produce a larger decrease in wage demands, since the output and terms-of-trade effects would be more substantial than under uncoordinated monetary policies.

Second, the impact of monetary union membership obviously depends on the alternative monetary regime. Drawing on Cukierman and Lippi (2001), for instance, one can compare a two-country monetary union with a closed-economy national monetary policy. In this case, the formation of a monetary union always reduces monopolistic distortions. Intuitively, when the terms-of-trade effect—operating only in an open economy—vanishes, the elasticity of labor demand to real wages is always larger because of the presence of the output effect. In a monetary union, each union anticipates a less accommodating monetary response to wage claims, lessening aggregate output. In general, the lower $1 - \phi$, the larger the output effect relative to the terms-of-trade effect, and so $\eta^M_U$ relative to $\eta^N_E$ is also larger.

This result may account for the empirical evidence of a positive relation between openness and inflation under centralized wage bargaining (e.g., Cavallari 2001a; Daniels, Nourzad, and VanHoose 2006). Moreover, to provide a rough idea whether the formation of the EMU has modified the linkage between inflationary wage pressure and aggregate labor demand, I report in table 1 the

---

8A previous version of this work, Cuciniello (2009), analyzed a model with a conservative central bank.
(contemporaneous) correlation between the growth rate of real wages and the terms of trade, and the correlation between the growth rate of real wages and the employment rate.\(^9\) Consistent with the theoretical prediction, both the terms of trade and the employment rate are more negatively correlated with real wages in the wake of the establishment of the EMU.

### 4.2 Stochastic Analysis

I assume log-normally distributed productivity shocks, i.e., \( k \equiv \log K \sim \mathcal{N}(0, \sigma_k^2) \) and \( k^* \equiv \log K \sim \mathcal{N}(0, \sigma_{k^*}^2) \). In what follows, I will denote the natural logarithm of any variable \( X \) by the corresponding lowercase letter; thus \( x \equiv \log X \). Without loss of generality, I also assume that (log) productivity shocks have identical variances, so that \( \sigma_k^2 = \sigma_{k^*}^2 \).

Plugging equation (33) into (1), a Home resident’s expected utility under non-cooperation and in a monetary union can be rewritten as follows:

\[
\mathbb{E}U^r = \left( \frac{1}{1 - \rho} - \frac{\eta^r - 1}{2\eta^r} \right) \times \exp \left\{ \left(1 - \rho\right)\mathbb{E}c^r + \frac{(1 - \rho)^2}{2}\sigma_{c^r}^2 \right\} \quad r \in \{NE, MU\}, \tag{38}
\]

where the expected log of consumption, \( \mathbb{E}c^r \), and the variance of log consumption, \( \sigma_{c^r}^2 \), are derived in appendix 3. In contrast to the ad hoc linear-quadratic formulations used in standard non-atomistic wage-setter models, changes in monetary policy affect not only the levels at which nominal wages are preset through the wage markup but also the risks that workers are forced to bear (exhibited in \( \mathbb{E}c^r \) and \( \sigma_{c^r}^2 \)).

Let us start with the degree of risk aversion \( \rho = 1 \) (implying log-utility of consumption). In this case the ex post marginal utility of one good is not affected by the consumption of the other good. The

\(^9\)The time series used are from the area-wide model database (Fagan, Henry, and Mestre 2005) and include wages per head, the exports of goods and services deflator, the imports of goods and services deflator, the consumption deflator, and total employment. I also make use of the OECD Economic Outlook series on the euro-area working-age population to calculate the employment rate.
expected utility under non-cooperation and in a monetary union are, respectively,

\[
\mathbb{E}U^{NE} = \mathbb{E}c^{NE} - \frac{\eta^{NE} - 1}{2\eta^{NE}}
\]
\[
= \frac{1}{2} \left\{ \log[\phi] - \frac{\eta^{NE} - 1}{\eta^{NE}} \right\}
\]  
(39)

and

\[
\mathbb{E}U^{MU} = \mathbb{E}c^{MU} - \frac{\eta^{MU} - 1}{2\eta^{MU}}
\]
\[
= \frac{1}{2} \left\{ -\frac{\eta^{MU} - 1}{\eta^{MU}} \right\}.
\]  
(40)

The levels of expected utility under flexible wages, denoted by a tilde, (˜), are instead given by

\[
\mathbb{E}\tilde{U}^r = \frac{1}{2} \left\{ \log \left[ \frac{\eta^r - 1}{\eta^r} \right] - \frac{\eta^r - 1}{\eta^r} \right\} \quad r \in \{NE, MU\}.
\]  
(41)

It turns out that non-cooperative monetary policies replicate the flexible-wage allocation when \( \phi = (\eta^{NE} - 1)/\eta^{NE} \). As noted in Benigno and Benigno (2003), the strategy of flexible-wage allocation is time consistent only in the presence of a positive degree of monopolistic distortion. Specifically, when \( \phi = (\eta^{NE} - 1)/\eta^{NE} \), the deflationary incentive analyzed in section 3.2 corresponds to the monopolistic distortion in the labor market, and the expected level of consumption is equal to the flexible-wage consumption level. This is due to the fact that the expected level of consumption is equal to the Pareto-efficient level.

Comparing equation (39) and (40), expected utility is higher in a monetary union than under flexible exchange rates when

\[
- \log \phi > \frac{\eta^{MU} - 1}{\eta^{MU}} - \frac{\eta^{NE} - 1}{\eta^{NE}}.
\]  
(42)
Table 2. Welfare Gains of Moving to a Monetary Union  
(percentage of output)

<table>
<thead>
<tr>
<th>Degree of Openness</th>
<th>$\rho = 0.5$</th>
<th>$\rho = 1$</th>
<th>$\rho = 2$</th>
<th>$\rho = 4$</th>
<th>$\rho = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - \phi = 0.5$</td>
<td>33.91</td>
<td>36.13</td>
<td>38.63</td>
<td>40.84</td>
<td>42.43</td>
</tr>
<tr>
<td>$1 - \phi = 0.4$</td>
<td>25.75</td>
<td>26.67</td>
<td>27.67</td>
<td>28.54</td>
<td>29.17</td>
</tr>
<tr>
<td>$1 - \phi = 0.3$</td>
<td>18.40</td>
<td>18.62</td>
<td>18.86</td>
<td>19.07</td>
<td>19.24</td>
</tr>
<tr>
<td>$1 - \phi = 0.2$</td>
<td>11.69</td>
<td>11.60</td>
<td>11.52</td>
<td>11.47</td>
<td>11.46</td>
</tr>
</tbody>
</table>

Intuitively, as $\eta^{NE} \neq \eta^{MU}$, the steady-state level of output is different in either monetary regime, as is the expected disutility of labor. In a flexible exchange rate regime, however, expected consumption is always lower than in a monetary union, since the monetary authorities have an incentive to improve the terms of trade. It turns out that the expected utility in a monetary union is greater if the external distortion stemming from the terms-of-trade incentive more than offsets the difference between labor market distortions under the two regimes.

Condition (42) reveals that there are possible gains from the formation of a monetary union. Table 2 reports, in consumption equivalence, the steady-state welfare gains from a monetary union regime associated with different degrees of openness. To simulate the model numerically, I set the elasticity of labor types at $\sigma = 7$, the number of labor unions at $n = 3$, and the variance of shocks at $\sigma^2_k = 0.02$.

The expected welfare gains of moving to a monetary union are greater the higher the degree of openness, i.e., $1 - \phi$. In a monetary union the external distortion associated with the terms of trade only affects labor markets. Specifically, as expected consumption is at the efficient level, the labor unions’ incentive to set higher wages to improve the terms of trade is increasing with $1 - \phi$, thereby reducing the disutility of labor through the wage markup. Conversely, under non-coordination the monetary authorities have the incentive to improve the terms of trade, which induces them to contract the money supply, reducing the expected level of consumption. When the economy is relatively closed, this incentive is dampened.
While the discussion so far on the expected welfare has focused on two types of distortions, namely monopolistic competition in the labor market and monopoly power over the terms of trade, I now consider the stabilization gains under either currency regime. It is worth noticing that there are two further sources of distortion in the model: predetermined wages and imperfect international risk sharing. All these distortions can lead optimal monetary policies to deviate from the replication of flexible wages.

Consider the impact of the anticipated monetary policy on the volatility of economic variables. Nash monetary policies mimic the flexible-wage consumption variance, thereby mitigating the risk-sharing distortion through exchange rate adjustments to external shocks. These movements change the terms of trade, leading to an efficient adjustment of the economy. A negative shock, in fact, produces an increase in the disutility of work that requires a decrease in the terms of trade, i.e., a real appreciation of the domestic currency, in order to yield a production switch away from Home-traded goods and toward Foreign-traded goods. A fall in $\mathcal{E}$ induces Home workers to produce fewer of their exports and Foreign workers more of theirs.

As the union’s monetary authority cannot affect the terms of trade, this regime generates larger stochastic losses than under flexible exchange rates when $\rho \neq 1$ or $\phi \neq 1/2$. Intuitively, when $\phi = 1/2$, the optimal monetary policy in a monetary union replicates the flexible-wage consumption variance. It turns out that under this parameterization there are no differences between the expected volatility of consumption under either monetary regime, and consumption risk sharing is perfect.\footnote{Note that $C/C^* = (\mathcal{E}W^*/W)^{2\phi - 1}$.} When $\rho = 1$, instead, the marginal utility of consuming one good does not depend on the consumption of the other good. As a result, the volatility of the terms of trade does not trigger externality and the optimal monetary policies under either currency regime coincide—in terms of generated volatility—since they are isomorphic to the one in a closed economy. However, comparing table 2 and 3, it turns out that the deterministic gains of moving to a monetary union more than offset the stabilization gains generated by flexible exchange rates.
Table 3. Stabilization Gains of Moving to a Monetary Union (percentage of output)

<table>
<thead>
<tr>
<th>Degree of Openness</th>
<th>$\rho = 0.5$</th>
<th>$\rho = 1$</th>
<th>$\rho = 2$</th>
<th>$\rho = 4$</th>
<th>$\rho = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - \phi = 0.5$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$1 - \phi = 0.4$</td>
<td>-0.001</td>
<td>0</td>
<td>-0.004</td>
<td>-0.027</td>
<td>-0.085</td>
</tr>
<tr>
<td>$1 - \phi = 0.3$</td>
<td>-0.006</td>
<td>0</td>
<td>-0.014</td>
<td>-0.070</td>
<td>-0.170</td>
</tr>
<tr>
<td>$1 - \phi = 0.2$</td>
<td>-0.016</td>
<td>0</td>
<td>-0.027</td>
<td>-0.112</td>
<td>-0.229</td>
</tr>
</tbody>
</table>

5. Conclusions

This paper has studied the welfare implications of introducing non-atomistic wage setting under different international monetary policy arrangements in a simple and tractable stochastic model.

As discretionary monetary policies have an incentive to improve the terms of trade under a Nash equilibrium, expected consumption is smaller than in a monetary union. However, the impact of wage demands on the terms of trade is also perceived by unions as being substantially different across the two monetary regimes. Specifically, under non-cooperative monetary policies, the responses of monetary authorities to wage hikes lead unions to anticipate no impact on the terms of trade, and reproduce the atomistic wage-setting result. In a monetary union, instead, wage claims are encouraged by the possibility of improving the terms of trade. It turns out—in contrast with most of the literature—that in an open economy the strategic use of the terms of trade may trigger welfare gains in a monetary union, since the expected utility the household obtains from higher wage revenues is greater than the expected disutility from supplying labor. Moreover, these gains offset the stochastic losses associated with the impossibility of changing the exchange rate.

Although the framework is useful for analyzing the strategic mechanisms of international macroeconomic policy coordination in a stochastic setting, I am aware that this finding hinges on the assumption of one-period wage contracts. Under a staggered wage-setting mechanism, there would be a further cost connected with the volatility of wage inflation, which causes an inefficient distribution of employment across households. I leave this as a subject for future research.
Appendix 1. Derivation of Optimal Monetary Policies

Using the decentralized equilibrium conditions derived in section 3.1, I can rewrite consumption and total output in each country as follows:

\[ C = \left( \frac{M}{W} \right)^\phi \left( \frac{M^*}{W^*} \right)^{1-\phi} \quad \text{and} \quad Y = \frac{M}{W}, \tag{43} \]

\[ C^* = \left( \frac{M}{W} \right)^{1-\phi} \left( \frac{M^*}{W^*} \right)^\phi \quad \text{and} \quad Y^* = \frac{M^*}{W^*}. \tag{44} \]

Plugging the definition of price index (3) into the participation constraints (6) and aggregating over households yields

\[ C^{1-\rho} \geq KY^2 \quad \text{and} \quad C^{*1-\rho} \geq K^*Y^{*2}. \tag{45} \]

A Nash equilibrium in monetary policies is defined as the set \( \{ M^{NE}, M^{*NE} \} \) which satisfies the following problem:

\[
\begin{align*}
\max_{M} U(M, M^{*NE}; W, W^*) & \quad \text{s. to (45) and (43)} \\
\max_{M^*} U^*(M^{NE}, M^*; W, W^*) & \quad \text{s. to (45) and (44)}.
\end{align*}
\]

The solution to the above problem yields expressions (17) and (18) in the text. The reaction functions (24) and (25) are obtained using (43) and (44) and solving (17) and (18) for money supplies.

In a monetary union, the common central bank solves the following problem:

\[
\max_{M_W} \frac{U(M_W; W, W^*) + U^*(M_W; W, W^*)}{2} \quad \text{s. to (45), (43), (44),} \\
\mathcal{E} = 1, \quad \text{and} \quad M_W \equiv M^{1/2}M^{*1/2}.
\]

The solution to the problem of the union’s central bank yields the competitive allocation in both countries and the reaction function (26) in the text.
Appendix 2. Derivation of Equation (23)

Consider that each union takes as given the wage set by other unions
and that the wage is the same for all the workers of union \( u \). From (9) I have that

\[
\frac{\partial W}{\partial W(u)} = \frac{\partial}{\partial W(u)} \left[ \int_0^1 W(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}
\]

\[
= \frac{\partial}{\partial W(u)} \left[ \int_{j \in u} W(j)^{1-\sigma} dj + \int_{j \notin u} W(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}
\]

\[
= \frac{1}{n} \left[ \frac{W(u)}{W} \right]^{-\sigma}.
\]

Appendix 3. Covariances and Expected Values

Define the terms of trade as

\[
T \equiv E W^*/W.
\] (46)

Using the optimal monetary policies derived in appendix 1, it is possible to rewrite the ex post consumption, output, and terms of trade as follows:

\[
c^{NE} = \frac{\rho - 1 - 2\rho \phi}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)} k^* - \frac{2(1 - \phi)}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)} k^* + \frac{\log \phi}{1 + \rho},
\] (47)

\[
y^{NE} = \frac{\rho - 1 + \phi - \rho \phi}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)} k^* - \frac{2 - \phi + \rho \phi}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)} k^* + \frac{\log \phi}{1 + \rho},
\] (48)

\[
\tau^{NE} = e + w^* - w = \frac{k^* - k}{3 - \rho + 2(\rho - 1)\phi},
\] (49)

\[
c^{MU} = -\frac{k^* + k}{2(1 + \rho)} - \frac{1}{2} (2\phi - 1)(w^{MU} - w^{*MU}),
\] (50)
\[
\begin{align*}
y^{MU} &= -\frac{k^* + k}{2(1 + \rho)} + \frac{1}{2}(w^{*MU} - w^{MU}), \\
\tau^{MU} &= w^{*MU} - w^{MU}.
\end{align*}
\] (51)

In order to solve for the nominal wage levels, I combine the log of (43), (15), and (46) and take expectations as follows:

\[
\begin{align*}
w^{NE} &= -Ec^{NE} + Em - (1 - \phi)E\tau^{NE}, \\
w^{*NE} &= -Ec^{NE} + Em^* + \phi E\tau^{NE}, \\
w^{MU} &= -Ec^{MU} + Em_W - (1 - \phi)E\tau^{MU}, \\
w^{*MU} &= -Ec^{MU} + Em_W + \phi E\tau^{MU}.
\end{align*}
\] (52)

Plugging optimal monetary policies (24), (25), and (26) into the above expressions yields

\[
\begin{align*}
Ec^{NE} &= \frac{1}{1 + \rho} \log \phi + \frac{1}{2}(2\phi - 1)E\tau^{NE}, \\
Ec^{MU} &= \frac{1}{2}(2\phi - 1)E\tau^{MU}.
\end{align*}
\] (53)

Because \(w\) and \(w^*\) are predetermined, the expressions obtained above fully describe the effects on \(c\) and \(\tau\) of unanticipated shocks. Allowing for optimal monetary policies, it turns out that the variance and covariance of \(c\) and \(\tau\) are given by

\[
\begin{align*}
\sigma^2_{c^{NE}} &= \left(\frac{\rho - 1 - 2\rho\phi}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)}\right)^2 \sigma^2_k + \left(\frac{2(1 - \phi)}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)}\right)^2 \sigma^2_{k^*}, \\
\sigma^2_{c^{NE}} &= \sigma^2_{\tau^{NE}} = \frac{\sigma^2_k + \sigma^2_{k^*}}{(3 - \rho + 2(\rho - 1)\phi)^2},
\end{align*}
\] (54)

\[
\begin{align*}
\sigma^2_{c^{NE}e^{NE}} &= \frac{1 - \rho + 2\rho\phi}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)^2} \sigma^2_k - \frac{2(1 - \phi)}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)^2} \sigma^2_{k^*}.
\end{align*}
\] (55)
\[
\sigma_{c^{NE}k^*} = -\frac{2(1 - \phi)}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)}\sigma_k^{2*}, \quad (62)
\]
\[
\sigma_{c^{NE}k} = \frac{\rho - 1 - 2\rho\phi}{(1 + \rho)(3 - \rho + 2(\rho - 1)\phi)}\sigma_k^{2}, \quad (63)
\]
\[
\sigma_{e^{NE}k} = -\frac{\sigma_k^2}{3 - \rho + 2(\rho - 1)\phi}, \quad (64)
\]
\[
\sigma_{e^{NE}k^*} = \frac{\sigma_k^{2*}}{3 - \rho + 2(\rho - 1)\phi}, \quad (65)
\]
\[
\sigma_{c^{MU}}^2 = \frac{\sigma_k^2 + \sigma_k^{2*}}{4(1 + \rho)^2}, \quad (66)
\]
\[
\sigma_{c^{MU}k} = -\frac{\sigma_k^2}{2(1 + \rho)}, \quad (67)
\]
\[
\sigma_{c^{MU}k^*} = -\frac{\sigma_k^{2*}}{2(1 + \rho)}. \quad (68)
\]

In a monetary union the nominal exchange rate cannot vary, thus all covariances involving the exchange rate are equal to zero.

Now, in order to derive the expected log of the terms of trade, and thus the expected consumption level through (57) and (58), it is convenient to rewrite the optimal wage-setting condition (33) and its corresponding condition in the Foreign country as follows:

\[
\left( \frac{W^{NE}}{W^{*NE}} \right)^{2(1 - \phi)} = \frac{\eta^{NE}}{\eta^{NE} - 1} \frac{\mathbb{E}\{K(C^{NE})^2\mathcal{E}^{2(1 - \phi)}\}}{\mathbb{E}\{(C^{NE})^{1 - \rho}\}}, \quad (69)
\]
\[
\left( \frac{W^{*NE}}{W^{NE}} \right)^{1 + \rho(2\phi - 1)} = \frac{\eta^{NE}}{\eta^{NE} - 1} \frac{\mathbb{E}\{K^*(C^{NE})^2\mathcal{E}^{-2\phi}\}}{\mathbb{E}\{(1 - \rho)(1 - 2\phi)(C^{NE})^{1 - \rho}\}}, \quad (70)
\]
\[
\left( \frac{W^{MU}}{W^{*MU}} \right)^{2(1 - \phi)} = \frac{\eta^{MU}}{\eta^{MU} - 1} \frac{\mathbb{E}\{K(C^{MU})^2\}}{\mathbb{E}\{(C^{MU})^{1 - \rho}\}}, \quad (71)
\]
\[
\left( \frac{W^{*MU}}{W^{MU}} \right)^{1 + \rho(2\phi - 1)} = \frac{\eta^{MU}}{\eta^{MU} - 1} \frac{\mathbb{E}\{K^*(C^{MU})^2\}}{\mathbb{E}\{(C^{MU})^{1 - \rho}\}}. \quad (72)
\]
Combining these expressions under either currency regime and taking logs yields the following expected terms of trade,

$$
\mathbb{E}_T^{NE} = \frac{2}{3 - \rho + 2(\rho - 1)\phi} \times (\sigma^* - \sigma_{k^*} - \phi\sigma^* - (1 - \phi)\sigma) \\
+ \left(\rho - 1 - \frac{2(1 + \rho)}{3 - \rho + 2(\rho - 1)\phi}\right)\sigma e^{NE} \\
+ \frac{(1 - 2\phi)(2\phi - 5 + (\rho - 2)\rho(2\phi - 1))}{6 - 4\phi + \rho(4\phi - 2)}\sigma e^{NE} = 0 \tag{73}
$$

$$
\mathbb{E}_T^{MU} = \frac{4\sigma^* - 4\sigma_{k^*}}{6 - 4\phi + \rho(4\phi - 2)} = 0, \tag{74}
$$

where the second equality directly follows after using the covariances derived above. Thus, from (57) and (58),

$$
\mathbb{E}_C^{NE} = \frac{1}{1 + \rho} \log \phi \quad \text{and} \quad \mathbb{E}_C^{MU} = 0. \tag{75}
$$

References


