The Risk Channel of Monetary Policy*

Oliver de Groot
Federal Reserve Board

This paper examines how monetary policy affects the riskiness of the financial sector's aggregate balance sheet, a mechanism referred to as the *risk channel of monetary policy.* I study the risk channel in a DSGE model with nominal frictions and a banking sector that can issue both outside equity and debt, making banks' exposure to risk an endogenous choice and dependent on the (monetary) policy environment. Banks' equilibrium portfolio choice is determined by solving the model around a *risk-adjusted* steady state. I find that banks reduce their reliance on debt finance and decrease leverage when monetary policy shocks are prevalent. A monetary policy reaction function that responds to movements in bank leverage or to movements in credit spreads can incentivize banks to increase their use of debt finance and increase leverage, ceteris paribus, increasing the riskiness of the financial sector for the real economy.

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1. Introduction

The recent financial crisis highlighted the importance of financial intermediaries' balance sheets, demonstrating that the extent to which financial intermediaries leverage themselves, and the extent

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to which financial intermediaries make use of debt finance, affects the probability of future financial crises occurring and the amount of damage a negative shock (either originating in the financial sector or not) does to the economy. This paper assesses whether the monetary policy environment can meaningfully affect financial intermediaries' (privately) optimal mix of outside equity and debt finance and, as a consequence, their balance sheets' resilience to shocks.

As the literature on the balance sheet channel has made clear, the financial accelerator is greatest when borrowers' leverage ratios and reliance on debt are high.[1,2] Investment banks' balance sheets in the United States in the run-up to the financial crisis displayed these key indicators of a powerful propagation channel, with historically high leverage ratios and heavy reliance on short-term debt. Quantitative macroeconomic models have, however, largely remained silent on the determination of the balance sheet of the financial sector, often calibrating the steady state of financial friction models to match long-run averages of leverage and short-term debt ratios in the data. In reality, a bank's balance sheet composition is the product of an optimizing decision by the bank's owner(s) in which they face a trade-off between risk and return.

In this paper, I explore a model in which banks face such an optimizing decision. In particular, this paper is concerned with the role that the design of monetary policy plays in the determinants of a bank's balance sheet size and composition.

The design and implementation of monetary policy (as well as regulatory policy) in the run-up to the crisis has received much criticism after the event. However, the link between monetary policy and the likelihood or severity of a financial crisis has been difficult to reconcile within standard macroeconomic models. This paper builds on the work of Gertler, Kiyotaki, and Queralto (2012), which

[1] The financial accelerator literature, which has emphasized the balance sheet channel, can be traced back to Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). The balance sheet channel for banks has been stressed by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011).

[2] Several different agency problems have been adopted in the macroeconomic literature to generate a balance sheet channel. These include costly state verification of borrowers (Townsend 1979 in Bernanke and Gertler 1989), a holdup problem for lenders (Hart and Moore 1994 in Kiyotaki and Moore 1997), and coordination failure (Goldstein and Pauzner 2005 in de Groot 2012).

explicitly develops a real business-cycle model in which banks' balance sheet decisions are endogenously determined. The key innovation of this paper is to augment their model with sticky prices to motivate standard monetary policy objectives. The question is then to ask whether there exists a trade-off between the standard monetary policy objectives of a New Keynesian model and the effect these objectives may generate on incentives for the endogenous structure of financial institutions' balance sheets.

To be precise, I present a quantitative New Keynesian business-cycle model in which banks intermediate funds between households and non-financial firms. The banks hold a representative asset, which arises from lending to fund physical capital purchases of the production sector. Importantly, these assets yield a risky return. The composition of the liability side of the balance sheet is the main interest in this paper. I assume that banks have three sources of funding available: inside equity (or internal net worth), which is the accumulation of retained earnings; external equity issuance (outside equity); and external debt finance (in this case, household deposits).

The Modigliani and Miller (1958) theorem tells us that in a frictionless market, the value of a firm is independent of its capital structure. To motivate a trade-off between outside equity and debt finance, I introduce a simple agency problem that supposes bankers have an incentive to abscond with bank assets, so that at the margin, it is easier for a banker to expropriate funds if outside equity accounts for a larger share of the bank's balance sheet. To prevent bankers from absconding with assets, households limit the ability of banks to leverage up their inside equity. However, there is also a benefit to the bank of issuing outside equity. If a bank is heavily reliant on debt, which is a non-state-contingent claim on the bank, then any fluctuations in the return on assets will have to be absorbed by the bank's net worth. Since the return on outside equity is state contingent and linked to the return on assets, it provides a valuable hedge for banks' net worth when uncertainty is high.

In this framework, the optimal balance sheet composition of the bank will depend on the stochastic nature of asset returns. And one of the determinants of the stochastic nature of the economy is the policy environment.[3] Banks would like to stabilize volatility in the shadow value of their net worth. If monetary policy acts to achieve this, banks have less incentive to resort to outside equity finance and will leverage up their balance sheets, thus partly offsetting the aims of the change in the monetary policy regime. It is this endogenous response of the banking system to take on more risk when the asset return risk decreases that I refer to as the *risk channel of monetary policy*.

Investigating the endogenous portfolio structure of banks within a quantitative DSGE model is, however, not without its technical challenges. The predominant use in the macroeconomic literature of a first-order approximation around the deterministic steady state is problematic for what this paper wants to achieve. As is well known, altering the monetary policy rule does not alter the deterministic steady state of a DSGE model. Nor will it capture banks' incentive to alter their steady-state balance sheet composition. To overcome this problem I solve the model around a *risk-adjusted steady state* (in the spirit of Coeurdacier, Rey, and Winant 2011 and Devereux and Sutherland 2011, and developed in de Groot 2013), which explicitly accounts for uncertainty. In a prototypical real business-cycle model, this amounts to capturing the effect of household precautionary savings on steady-state capital stocks. In the model presented in this paper, the risk-adjusted steady state also captures the effect of risk on banks' steady-state balance sheet composition, which has important implications for the strength of the financial accelerator mechanism. Computing the risk-adjusted steady state provides a challenge precisely because it requires the steady state and the dynamics of the model around the steady state to be determined jointly. It follows that because the design of monetary policy can alter the risk-adjusted steady state (because of monetary policy's effect on the second moments of variables in the model), and because the altered steady state itself affects the dynamic behavior of the model around that steady state, we are able to capture the risk channel.

[3]It is left to future research to incorporate into this framework issues regarding direct regulation of the financial sector. Focus here is given to the indirect effect standard monetary policy has on asset returns and the balance sheet decisions of banks.

The paper makes two contributions to the literature. First, it shows that exogenous uncertainty (i.e., increases in the standard deviation of monetary and other shocks) can significantly alter banks' (privately) optimal balance sheet composition. Increased uncertainty reduces the ability of the banking sector to intermediate credit. Banks' balance sheets are particularly sensitive to monetary and capital quality uncertainty because both these shocks have first-order effects on asset returns in the model. Second, the paper shows how the monetary policy regime can also alter the determination of banks' balance sheets. Within a restricted class of monetary policy reaction functions, I find that altering the aggressiveness with which nominal interest rates react to inflation and output deviations only weakly affects the composition of banks' balance sheet. However, a reaction function that responds to deviations of banks' leverage or credit spreads can generate significant shifts in the composition of banks' balance sheet and therefore changes in the dynamic responses to shocks.

Many commentators have put forward the assertion that the conduct of monetary policy in the late 1990s and early 2000s generated a low-risk environment that incentivized banks to take on more risk, make greater use of short-term debt, and leverage up their balance sheets. More recently, several papers have provided theoretical models for such a risk channel. Chari and Kehoe (2009), Diamond and Rajan (2009), and Farhi and Tirole (2012), among others, focus on the moral hazard consequences of bailouts and credit market instruments. Farhi and Tirole (2012)'s paper, for example, considers a three-period endowment economy with strategic complementarities between private leverage and monetary policy. When maturity transformation is prevalent, the central bank has little choice but to facilitate refinancing. Equally, reducing private leverage lowers the return on equity. The key insight of this literature is that banks choose to correlate their risk exposures, that optimal monetary policy can be time inconsistent, and that macroprudential policy can therefore be welfare enhancing. Diamond and Rajan (2009), using a similar three-period endowment environment, also show that monetary policy is time inconsistent. Lowering interest rates when households demand funds prevents a damaging run on illiquid assets, but encourages banks to increase leverage and fund more illiquid projects. Optimal monetary policy under commitment in their environment involves raising interest rates when there is no liquidity crisis in order to punish banks that have chosen to be illiquid.

There is also a growing literature on macroprudential policy including, among others, Lorenzoni (2008), Nikolov (2010), Bianchi (2011), Korinek (2011), and Stein (2012). Lorenzoni (2008), for example, is another three-period model with financial frictions, via limited commitment in financial contracts, which results in excessive borrowing ex ante and excessive volatility ex post. The friction generates a pecuniary externality that is not internalized in private contracts and provides a framework to evaluate policies to prevent financial crises. While providing important insights, most of these models are not rich enough to provide quantitative insights into the important trade-offs policymakers may face.

The key extension of this paper, therefore, is that it studies the risk taking of banks within a quantitative macroeconomic model with nominal frictions, allowing for the joint examination of monetary policy design and banks' balance sheet composition. The paper proceeds as follows: Section 2 presents the model. Section 3 sets out the parameterization and explains the solution technique. Section 4 presents the results of the numerical experiments, and section 5 concludes.

2. Model

The baseline model is a DSGE model with investment costs, nominal rigidities, and financial frictions. There are five types of agents: households, capital producers, manufacturers, retailers, and bankers. The banking sector follows Gertler, Kiyotaki, and Queralto (2012). In particular, banks intermediate funds between households and manufacturers by raising both debt and (outside) equity. An agency problem between households and banks, however, limits how much banks are able to leverage their (inside) equity. The model is closed with a monetary policy reaction function. Of central interest to this paper is the interaction between the monetary policy environment and banks' endogenous balance sheet composition.

2.1 Households

There is a unit measure of identical households. Each household consists of a fraction $1 - f$ of bankers and f of workers. Workers supply labor to manufacturers and bring home wages to their household. Bankers manage banks and bring home any earnings. Within each family, there is consumption insurance. Workers and bankers rotate over time, with a banker becoming a worker with fixed probability $1-\theta$. As $(1-\theta)f$ bankers become workers, a proportion $(1-\theta)f/(1-f)$ of workers become bankers, keeping the size of the two populations unchanged. The household provides its new bankers with a small startup fund.

Household preferences are given by

$$\max \mathbf{E}_t \sum_{i=0}^{\infty} \beta^i \frac{1}{1-\zeta} \left(C_{t+i} - hC_{t+i-1} - \frac{\varrho}{1+\vartheta} L_{t+i}^{1+\vartheta} \right)^{1-\zeta}, \qquad (1)$$

where $\mathbf{E}_t(.)$ denotes the rational expectations operator, conditional on the time t information set; $\beta \in (0,1)$ is the subjective discount factor; C_t is consumption; and L_t is labor supply. The following parameter restrictions ensure well-behaved preferences: $h \in [0,1)$, $\varrho, \vartheta > 0$.

Households have access to two financial assets: bank debt (deposits), D_t, and bank (outside) equity, E_t, at relative price $Q_{E,t-1}$. Bank debt pays the non-state-contingent (risk-free) gross real return R_t from $t-1$ to t while bank equity pays a state-contingent gross real return, denoted $R_{E,t}$. Let W_t be the real wage and Υ_t net payoffs to the household from ownership of financial and non-financial firms. The household budget constraint is given by

$$C_t = W_t L_t + \Upsilon_t + R_t D_t + Q_{E,t-1} R_{E,t} E_t - D_{t+1} - Q_{E,t} E_{t+1}. \qquad (2)$$

The household's first-order optimality conditions are given by

$$W_t = -\frac{U_{L,t}}{U_{C,t}}, \quad \mathbf{E}_t \left(\Lambda_{t,t+1} \right) R_{t+1} = 1 \quad \text{and} \quad \mathbf{E}_t \left(\Lambda_{t,t+1} R_{E,t+1} \right) = 1, \qquad (3)$$

where

$$\Lambda_{t-1,t} \equiv \beta \frac{U_{C,t}}{U_{C,t-1}}$$

denotes the stochastic discount factor between periods $t-1$ and t, and $U_{C,t}$ and $U_{L,t}$ denote the marginal utility of consumption and the marginal (dis)utility of labor, respectively.

2.2 Manufacturers

A representative, perfectly competitive manufacturer produces intermediate goods that are sold to retailers. At the end of period t, the manufacturer purchases capital, K_{t+1}, at price $Q_{K,t}$ for use in production in period $t+1$. The manufacturer purchases the capital using funds from banks. By assumption, there is no friction in the process of obtaining funds from banks, and the manufacturer is therefore able to offer the bank a state-contingent security. In this regard, the banks are like private equity funds. Let $\varepsilon_{A,t}$ and $\varepsilon_{K,t}$ denote total factor productivity and capital quality, respectively. At each time t, the manufacturer uses capital and labor to produce output, Y_t:

$$Y_t = \exp\left(\varepsilon_{A,t}\right)\left(\exp\left(\varepsilon_{K,t}\right)K_t\right)^{\alpha} L_t^{1-\alpha}, \tag{4}$$

where $\alpha \in (0,1)$. $\varepsilon_{A,t}$ and $\varepsilon_{K,t}$ are exogenous stochastic processes of the form $\varepsilon_{s,t+1} = \rho_s \varepsilon_{s,t} + \eta_s \epsilon_{s,t+1}$ for $s = (A,K)$ and $\epsilon_{s,t+1} \sim Niid(0,1)$. Let $X_t = \frac{P_{m,t}}{P_t}$ be the ratio of the price of intermediate goods, $P_{m,t}$, to the aggregate price level, P_t. The manufacturer's first-order optimality condition for labor demand is given by

$$W_t = X_t\left(1-\alpha\right)\frac{Y_t}{L_t}. \tag{5}$$

Since manufacturers are perfectly competitive, the gross real return on capital is

$$R_{K,t} = \exp\left(\varepsilon_{K,t}\right)\frac{X_t \alpha \frac{Y_t}{\exp(\varepsilon_{K,t})K_t} + \left(1-\delta\right)Q_{K,t}}{Q_{K,t-1}}. \tag{6}$$

2.3 Capital Producers

At the end of period t, competitive capital producers buy the entire capital stock from manufacturers, repair depreciated capital, and build new capital. Production of capital involves convex adjustment costs. The capital producers then sell both the repaired and new capital back to manufacturers. The objective of a capital producer is given by

$$\max \mathbf{E}_t \sum_{i=0}^{\infty} \Lambda_{t,t+i} \left(Q_{K,t+i} I_{t+i} - \left(1 + \frac{\varphi_I}{2} \left(\frac{I_{t+i}}{I_{t+i-1}} - 1 \right)^2 \right) I_{t+i} \right), \tag{7}$$

where I_t is investment and $\varphi_I \geq 0$ scales the adjustment costs. The capital producer's first-order optimality condition determines the price of capital:

$$\begin{aligned} Q_{K,t} = 1 + \frac{\varphi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 + \left(\frac{I_t}{I_{t-1}} \right) \varphi_I \left(\frac{I_t}{I_{t-1}} - 1 \right) \\ - \mathbf{E}_t \Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 \varphi_I \left(\frac{I_{t+1}}{I_t} - 1 \right). \end{aligned} \tag{8}$$

The aggregate capital stock in the economy evolves according to

$$K_{t+1} = (1 - \delta) \exp \left(\varepsilon_{K,t} \right) K_t + I_t. \tag{9}$$

2.4 Retailers

Final output, Y_t, is a CES aggregator of measure one of differentiated retailers

$$Y_t = \left(\int_0^1 Y_{r,t}^{\frac{\varepsilon - 1}{\varepsilon}} dr \right)^{\frac{\varepsilon}{\varepsilon - 1}}, \tag{10}$$

where $Y_{r,t}$ is the output of retailer r and $\varepsilon > 1$ denotes the intratemporal elasticity of substitution across different varieties of retail goods. From cost minimization of users of final output,

$$Y_{r,t} = \left(\frac{P_{r,t}}{P_t} \right)^{-\varepsilon} Y_t \quad \text{and} \quad P_t = \left(\int_0^1 P_{r,t}^{1-\varepsilon} dr \right)^{\frac{1}{\varepsilon - 1}}. \tag{11}$$

Retailers costlessly brand intermediate output: One unit of intermediate output is used for one unit of retail output. Retailers enjoy monopolistic pricing power but face a convex price adjustment cost (à la Rotemberg 1982), which generates nominal rigidities in the economy. The objective of retailers is given by

$$\max \mathbf{E}_t \sum_{i=0}^{\infty} \Lambda_{t,t+i} \times \left(\frac{P_{r,t+i}}{P_{t+i}} Y_{r,t+i} - X_{t+i} Y_{r,t+i} - \frac{\varphi_\Pi}{2} \left(\frac{P_{r,t+i}}{P_{r,t+i-1}\Pi} - 1 \right)^2 Y_{t+i} \right), \tag{12}$$

where Π is the steady-state gross inflation rate and $\varphi_\Pi \geq 0$ scales the adjustment costs. Noting that the equilibrium will be symmetric $(P_{r,t+i} = P_{t+i})$ for all r and i, the retailers' first-order optimality condition is given by

$$\varphi_\Pi \left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} = 1 - \varepsilon \left(1 - X_t\right) + \varphi_\Pi \mathbf{E}_t \left(\Lambda_{t,t+1} \left(\frac{\Pi_{t+1}}{\Pi} - 1 \right) \frac{\Pi_{t+1}}{\Pi} \frac{Y_{t+1}}{Y_t} \right), \tag{13}$$

where Π_t is the gross inflation rate from $t-1$ to t.

Since price adjustment and investment adjustment costs are paid in real units, the economy's aggregate resource constraint is given by

$$\left(1 - \frac{\varphi_\Pi}{2} \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 \right) Y_t = C_t + \left(1 + \frac{\varphi_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right) I_t. \tag{14}$$

2.5 Banks

The model thus far is a conventional DSGE model. Frictionless financial intermediation would ensure that the following arbitrage condition should hold:

$$\mathbf{E}_t \Lambda_{t,t+1} \left(R_{K,t+1} - R_{t+1} \right) = 0.$$

Instead, this section develops a model of banking with agency problems, driving a wedge between $\mathbf{E}_t \Lambda_{t,t+1} R_{K,t+1}$ and $\mathbf{E}_t \Lambda_{t,t+1} R_{t+1}$, and ensuring a non-trivial role for the composition of banks' balance sheets for economic outcomes.

Banks lend funds, obtained from households, to manufacturers. Bank j's balance sheet is

$$Q_{K,t}K_{j,t+1} = N_{j,t} + D_{j,t+1} + Q_{E,t}E_{j,t+1},$$

where $K_{j,t+1}$ is the quantity of financial claims on manufacturers' gross returns on capital. Since these claims are perfectly state contingent, it is possible to denominate one claim as one unit of capital, as I have done, implying that $Q_{K,t}$ is also the relative price of each claim. $N_{j,t}$ is the amount of net worth—or inside equity—that a bank has and $D_{j,t+1}$ the deposits that the bank obtains from households. $E_{j,t+1}$ is the quantity of outside equity that the bank issues to households and $Q_{E,t}$ is the relative price of each claim. If one unit of outside equity is the claim on one unit of capital, then the gross real return on outside equity is given by

$$R_{E,t} = \exp\left(\varepsilon_{K,t}\right) \frac{X_t \alpha \frac{Y_t}{\exp(\varepsilon_{K,t})K_t} + (1-\delta)\, Q_{E,t}}{Q_{E,t-1}}.$$

The bank's inside equity evolves as the difference between earnings on assets and payments on liabilities,

$$\begin{aligned} N_{j,t+1} = {} & \left(R_{K,t+1} - R_{E,t+1}B_{j,t} - R_{t+1}\left(1 - B_{j,t}\right)\right) Q_{K,t}K_{j,t+1} \\ & + R_t N_{j,t}, \end{aligned}$$

where $B_{j,t} \equiv \frac{Q_{E,t}E_{j,t+1}}{Q_{K,t}K_{j,t+1}}$. Bank assets earn the state-contingent real gross return $R_{K,t+1}$. Household deposits get paid the non-contingent real gross return R_{t+1} and outside equity is paid the state-contingent real gross return $R_{E,t+1}$. The bank's objective is given by

$$V_{j,t} = \max \mathbf{E}_t \sum_{i=0}^{\infty} \left(1-\theta\right) \theta^i \Lambda_{t,t+1+i} N_{j,t+1+i}. \tag{15}$$

To motivate a limit on a bank's ability to expand its balance sheet, I follow Gertler, Kiyotaki, and Queralto (2012) by introducing a moral hazard problem: Bankers are able to abscond with a fraction, Θ, of bank assets. This introduces an incentive compatibility constraint:

$$V_{j,t} \geq \Theta\left(B_{j,t}\right) Q_{K,t} K_{j,t+1}. \tag{16}$$

Households will only provide funds up to the point at which bankers are still marginally better off by not absconding with assets. To motivate a non-trivial choice for the composition of bank's liabilities, I assume that the fraction of bank assets that bankers can abscond with is convex in the share of assets funded by outside equity:

$$\Theta\left(B_{j,t}\right) = \kappa_0 \left(1 + \kappa_1 B_{j,t} + \frac{\kappa_1}{2} B_{j,t}^2\right). \tag{17}$$

The rationale, that it is more difficult to abscond with assets funded by debt than by equity, comes from Calomiris and Kahn (1991) and relies on the insight that debt requires the bank to meet a non-contingent payment every period, while dividend payments on equity are tied to the performance of the banks' assets and are therefore more difficult to monitor by outsiders.[4]

We can express $V_{j,t}$ as follows:

$$V_{j,t} = \left(\mu_{K,t} + B_{j,t}\mu_{E,t}\right) Q_{K,t} K_{j,t+1} + \mu_{N,t} N_{j,t} \tag{18}$$

with

$$\mu_{K,t} = \mathbf{E}_t\left(\Lambda_{t,t+1}\Omega_{t+1}\left(R_{K,t+1} - R_{t+1}\right)\right) \tag{19}$$

$$\mu_{E,t} = \mathbf{E}_t\left(\Lambda_{t,t+1}\Omega_{t+1}\left(R_{t+1} - R_{E,t+1}\right)\right) \tag{20}$$

$$\mu_{N,t} = \mathbf{E}_t\left(\Lambda_{t,t+1}\Omega_{t+1}\right) R_{t+1}, \tag{21}$$

where $\Omega_t = (1-\theta) + \theta B_t \phi_t$. I assume that, in equilibrium, the incentive compatibility constraint, equation (16), binds.[5] Rewriting

[4] If Θ was independent of B, banks would strictly prefer to issue outside equity over debt. In this case, with outside financing coming from only equity, banks' net worth would be completely shielded from movements in assets returns, thus rendering the financial accelerator obsolete. There would, however, remain a credit spread in steady state that cannot be arbitraged away.

[5] I choose parameter values such that, within the neighborhood of the steady state, the incentive compatibility constraint does, in fact, bind.

equation (16) gives an expression for the inverse of the ratio of inside equity to total assets

$$\phi_{j,t} = \frac{\mu_{N,t}}{\Theta\left(B_{j,t}\right) - \left(\mu_{S,t} + B_{j,t}\mu_{E,t}\right)},$$

where $\phi_{j,t} \equiv \frac{Q_{K,t}K_{j,t+1}}{N_{j,t}}$. Combining the first-order conditions of the bank's objection function, equation (15), subject to the incentive compatibility constraint, equation (16), for $K_{j,t+1}$ and $B_{j,t}$ gives the following equilibrium condition:

$$\frac{\mu_{E,t}}{\left(\mu_{K,t} + B_{j,t}\mu_{E,t}\right)} = \frac{\Theta'\left(B_{j,t}\right)}{\Theta\left(B_{j,t}\right)}. \tag{22}$$

Symmetry of the equilibrium ensures that $B_{j,t} = B_t$ and $\phi_{j,t} = \phi_t$ for all j. New banks receive a startup fund from households of $\omega Q_{K,t}K_t$. The evolution of aggregate net worth is therefore given by

$$\begin{aligned} N_{t+1} = \theta\left(\left(R_{K,t} - R_{E,t}B_{t-1} - R_t\left(1 - B_{t-1}\right)\right)\phi_{t-1} + R_t\right)N_t \\ + \omega Q_{K,t}K_t. \end{aligned}$$

2.6 Monetary Policy

Monetary policy is characterized by a simple reaction function

$$\begin{aligned} \left(\frac{R_{N,t}}{R_N}\right) = \left(\left(\frac{\Pi_t}{\Pi}\right)^{\chi_\Pi}\left(\frac{X_t}{X}\right)^{\chi_X}\left(\frac{\phi_t}{\phi}\right)^{\chi_\phi}\exp\left(\chi_S\left(S_t - S\right)\right)\right)^{1-\chi_{R_N}} \\ \times\left(\frac{R_{N,t-1}}{R_N}\right)^{\chi_{R_N}}\exp\left(\varepsilon_{M,t}\right) \end{aligned} \tag{23}$$

with the nominal interest rate, $R_{N,t}$, reacting only to deviations of observable variables from their respective steady states (denoted by variables without time subscripts). In this setup, the central bank uses X_t as an observable proxy of the output gap. The reaction function allows for the possibility that monetary policy reacts to two financial indicators, bank leverage and the credit spread, $S_t \equiv \mathbf{E}_t R_{K,t+1} - R_{t+1}$. Monetary policy shocks follow the exogenous stochastic process $\varepsilon_{M,t+1} = \rho_M\varepsilon_{M,t} + \eta_M\epsilon_{M,t+1}$ with $\epsilon_{M,t} \sim Niid(0,1)$.

Finally, the link between nominal and real interest rates is given by the Fisher relation:

$$R_{N,t} = R_{t+1}\mathbf{E}_t\left(\Pi_{t+1}\right). \tag{24}$$

2.7 Discussion of the Model

The banking sector—and, in particular, banks' balance sheet composition—in section 2 is of primary interest in this paper; the rest of the model is relatively standard.

To understand the relationship between the monetary policy environment and the composition of banks' balance sheets, consider an expansionary monetary policy shock. Nominal rigidities in the economy means that a fall in the nominal risk-free rate generates a fall in the real risk-free rate. In a model without financial frictions, arbitrage ensures that the required expected return on capital falls (to first order) one-for-one with a fall in the risk-free rate. Since there are diminishing marginal returns to capital, a fall in the required expected return on capital means that a larger set of investment projects have a positive net present value, generating a boom in investment.[6]

The existence of an agency problem limits the amount of credit households are willing to extend as a function of banks' net worth, as an overextension of credit could mean that bankers have an incentive to forgo their accumulated retained earnings and abscond with a fraction of the banks' assets instead. The limit on the creation of credit prevents arbitrage, thus driving a wedge between the expected required return on capital and the risk-free rate.

When banks' asset returns are below expectation, banks use their retained earnings to pay their creditors. The fall in banks' net worth therefore heightens the agency problem, causing the wedge between the expected required return on capital and the real risk-free rate to move countercyclically While in a frictionless financial sector the expected required return on capital and the risk-free rate move one-for-one, in the model with an agency problem the expected required return on capital moves by more than one-for-one. As a consequence,

[6]For simplicity in this discussion, I abstract from changes in other relative prices, like wages and the effect on the labor market outcomes.

in response to an expansionary monetary policy shock, an even larger set of investment projects have a positive net present value, generating an even greater boom in investment.

The extent to which banks are able to leverage themselves, and the extent to which banks' net worth is damaged by shocks, is crucial for the amplification and propagation of shocks through the financial system. In particular, a bank heavily funded with non-contingent liabilities (debt) will experience high volatility in its net worth, while for a bank that issues a lot of outside equity, a state-contingent claim on the bank, unexpected movements in asset returns are absorbed by the concomitant movement in the return paid on outside equity, thus damping fluctuations in net worth.

This begs the question, why don't banks issue only state-contingent claims? The insight from Calomiris and Kahn (1991), is that debt is a disciplining device for bankers. Banks, in choosing their (privately) optimal mix of short-term debt and outside equity finance, are therefore an endogenous source of the amplification and propagation of shocks in the economy.

The banker maximizes the value of his or her bank, V, subject to the incentive constraint binding. Given the net worth of the bank, the banker has two choice variables: the quantity of assets (capital) it invests in, $Q_{K,t}K_{t+1}$, and the share of those assets funded by issuing outside equity, B_t. The marginal benefit of an additional unit of outside equity is given by $\mu_{E,t}Q_{K,t}K_{t+1}$, while the marginal benefit of an additional asset is $\mu_{K,t} + \mu_{E,t}B_t$. Thus, the marginal rate of substitution between outside equity and expanding the size of the balance sheet is $-\frac{\mu_{E,t}Q_{K,t}K_{t+1}}{\mu_{K,t}+\mu_{E,t}B_t}$. The unconstrained optimum for the bank, all else equal, is to choose the highest feasible leverage and to raise external funds using only outside equity.

From the incentive compatibility constraint, the *price* of an additional unit of outside equity is $\left(\frac{d\Theta_t}{dB_t} - \mu_{E,t}\right)Q_{K,t}K_{t+1}$, while the price of an additional asset is $\Theta_t - (\mu_{K,t} + \mu_{E,t}B_t)$. Both these prices are assumed positive. The first says that substituting debt with an additional unit of outside equity causes the ratio of assets that can be expropriated to rise more than the value of the bank, causing the constraint to tighten. The second says something similar, that an additional asset will raise the marginal quantity of assets that can be expropriated more than it raises the value of the bank, again causing the constraint to tighten. The marginal rate of transformation between outside equity and an additional asset is therefore

$$\frac{d\phi}{dB} = -\frac{\Theta' - \mu_E}{\Theta - (\mu_K + \mu_E B)}\phi < 0, \tag{25}$$

where Θ' refers to the first derivatives of equation (17) with respect to B. The marginal rate of transformation is strictly negative. In other words, the banker faces a trade-off since an increase in outside equity issuance must result in a lower capital-to-net-worth ratio, all else equal. Equating the marginal rate of substitution and the marginal rate of transformation delivers the equilibrium relation of equation (22).

In a risk-free environment, the benefit of substituting outside equity for debt is zero ($\mu_E = 0$), since the return on debt and outside equity is identical ($R = R_E$). In addition, it follows that $\Theta' = 0$. We can show the following comparative statics in the neighborhood of the deterministic steady state:

$$\begin{array}{ll} \frac{dB}{d\mu_E} = \frac{\Theta - B\Theta'}{\Theta''(\mu_K + \mu_E B)} & \left.\frac{dB}{d\mu_E}\right|_{\mu_E=0} = \frac{\Theta}{\Theta''\mu_K} > 0 \\ \frac{dB}{d\mu_K} = \frac{\Theta'}{\mu_E\Theta' - \Theta''(\mu_K + \mu_E B)} & \left.\frac{dB}{d\mu_K}\right|_{\mu_E=0} = 0 \end{array}. \tag{26}$$

The relations in the first line hold because Θ is convex by assumption. Thus, a marginal increase in the value of outside equity leads to an increase in the share of outside equity issuance. The second line says that, at the margin, an increase in the excess value of assets over deposits does not generate any portfolio shifting.

$$\begin{array}{ll} \frac{d\phi}{d\mu_E} = \frac{B\phi}{\Theta - (\mu_K + \mu_E B)} - \frac{\Theta' - \mu_E}{\Theta - (\mu_K + \mu_E B)}\phi\frac{dx}{d\mu_E} & \left.\frac{d\phi}{d\mu_E}\right|_{\mu_E=0} = \frac{B\phi}{\Theta - (\mu_K + \mu_E B)} > 0 \\ \frac{d\phi}{d\mu_K} = \frac{\phi}{\Theta - (\mu_K + \mu_E B)} - \frac{\Theta' - \mu_E}{\Theta - (\mu_K + \mu_E B)}\phi\frac{dx}{d\mu_K} & \left.\frac{d\phi}{d\mu_K}\right|_{\mu_E=0} = \frac{\phi}{\Theta - (\mu_K + \mu_E B)} > 0 \\ \frac{d\phi}{d\mu_N} = \frac{1}{\Theta - (\mu_K + \mu_E B)} - \frac{\Theta' - \mu_E}{\Theta - (\mu_K + \mu_E B)}\phi\frac{dx}{d\mu_N} & \left.\frac{d\phi}{d\mu_N}\right|_{\mu_E=0} = \frac{1}{\Theta - (\mu_K + \mu_E B)} > 0 \end{array} \tag{27}$$

The above comparative statics make use of the binding incentive compatibility constraint. The effect on leverage of a change in μ_E, μ_K, or μ_N is the outcome of a direct and an indirect effect, as a result of a change in the liability mix of the bank. In the neighborhood of the deterministic steady state, the indirect effect is zero.

To understand how risk affects this equilibrium relationship, we need to interpret $\mu_{E,t}$ and $\mu_{K,t}$. $\mu_{E,t}$ is the excess value of substituting outside equity for deposits while $\mu_{K,t}$ is the excess value of assets over deposits and $\mu_{K,t} + \mu_{E,t}B_t$ is the excess value of assets over external finance:

$$\mu_{E,t} \equiv \mathbf{E}_t \left(\Lambda_{t,t+1}\Omega_{t+1} \left(R_{t+1} - R_{E,t+1}\right)\right) \tag{28}$$

$$\begin{aligned}\mu_{K,t} + \mu_{E,t}B_t \equiv \mathbf{E}_t \left(\Lambda_{t,t+1}\Omega_{t+1} \left(R_{K,t+1} - R_{t+1} \left(1 - B_t\right)\right.\right.\\ \left.\left.-R_{E,t+1}B_t\right)\right).\end{aligned} \tag{29}$$

Clearly, both $\mu_{E,t}$ and $\mu_{E,t} + \mu_{E,t}B_t$ need to be non-negative for the banker's problem to be well defined. Notice also that both equations look like asset-pricing equations, where the left-hand side is the price, $\Lambda_{t,t+1}\Omega_{t+1}$ is the stochastic discount factor, and the remainder of the right-hand side is the expected return. We can interpret $\widetilde{\Lambda}_{t+1} \equiv \Lambda_{t,t+1}\Omega_{t+1}$ as the banker's stochastic discount factor, which is different from the household's stochastic discount factor, $\Lambda_{t,t+1}$. Ω_{t+1} is the shadow marginal value of a unit of net worth. Ω_{t+1} varies countercyclically because the incentive constraint becomes more binding in a downturn. Thus, the bankers' stochastic discount factor is countercyclical and more volatile than the household's stochastic discount factor.

To be clear, I compare equation (28) to the first-order conditions of the household's problem (equations in (3)), which give $0 = \mathbf{E}_t(\Lambda_{t,t+1}(R_{t+1} - R_{E,t+1}))$. At this stage, it is useful to introduce the concept of the risk-adjusted steady state. As discussed earlier, at the deterministic steady state, bankers and households are indifferent between holding debt and outside equity. To ensure a determinate portfolio choice, we must consider a steady state that accounts for future uncertainty. The risk-adjusted steady state is defined as the portfolio that agents would hold if there was no risk today (the vector $\varepsilon_t = \mathbf{0}$) but they expected risk in the future ($\varepsilon_{t+\tau} = Niid(\mathbf{0}, \mathbf{I})$ for all $\tau > 0$). To compute the risk-adjusted steady state, it is possible to take a second-order approximation around the point $\varepsilon_{t+1} = 0$, which gives

$$0 = \Lambda \left(R - R_E\right) - \text{cov}_t \left(\Lambda_{t,t+1}, R_{E,t+1}\right) \tag{30}$$

$$\mu_E = \widetilde{\Lambda}(R - R_E) - \text{cov}_t(\widetilde{\Lambda}_{t+1}, R_{E,t+1}), \tag{31}$$

where variables without time subscripts denote the variables at their risk-adjusted steady state and $\mathrm{cov}_t(\Lambda_{t,t+1}, R_{E,t+1})$ is a covariance term conditional on time t information. Clearly, incorporating expected risk at the risk-adjusted steady state allows monetary policy to affect steady-state portfolio choices because the conduct of monetary policy can affect how variables co-move in the economy.

It follows that because $\mathrm{cov}_t(\Lambda_{t,t+1}, R_{E,t+1})$ and $\mathrm{cov}_t(\widetilde{\Lambda}_{t+1}, R_{E,t+1})$ are both negative, but with the second larger than the first, and with $\Omega > 0$, the excess marginal value of substituting outside equity for debt for the banker is positive, $\mu_E > 0$. In other words, increased volatility in the return on outside equity makes bankers more willing to issue outside equity, causing the excess marginal value of substituting outside equity for debt to rise. Moreover, while the credit spread may rise in a high-uncertainty state, banks' discounted value of that credit spread, $\mu_{K,t}$, falls. This is because the realized spread between the return on capital and the cost of borrowing is procyclical. When $\mu_{K,t}+\mu_{E,t}B_t$ falls, the value of the bank is lower, as is the maximum leverage ratio that it can attain.

In section 4 I validate this discussion by conducting numerical experiments to show how banks' balance sheet compositions are affected by the policy environment in which banks operate.

3. Parameterization and Solution Technique

3.1 Parameterization

Table 1 summarizes the structural parameter values used in the numerical experiments that follow. Many of the parameters are conventional in the literature—such as the income share of capital, α; the subjective discount factor, β; and the depreciation rate, δ—and all are set consistent with time periods denoting quarters. Household preferences display habit formation and abstract from wealth effects on labor supply. The habit parameter, $h = 0.75$, is set towards the upper end of the range of values seen in the literature. $\zeta = 2$ implies (holding hours constant) an intertemporal elasticity of substitution of 0.5. $\vartheta = 0.33$ implies a Frisch elasticity of labor supply of 3. The weighting term, $\varrho = 0.25$, ensures that households allocate 20 percent of their time to work (at the deterministic steady state).

Table 1. Parameterization

	Description	**Value**
Standard DSGE Parameters		
α	Income Share of Capital	0.33
β	Quarterly Subjective Discount Rate	0.99
$h, \zeta, \varrho, \vartheta$	Preferences $\frac{1}{1-\zeta}\left(C_t - hC_{t-1} - \frac{\varrho}{1+\vartheta}L_t^{1+\vartheta}\right)^{1-\zeta}$	$0.75, 2, 0.25, 0.33$
ε	Price Elasticity of Demand	4.17
δ	Quarterly Depreciation Rate	0.025
φ_Π	Adjustment Cost $\frac{\varphi_\Pi}{2}\left(\Pi_t/\Pi - 1\right)^2$	48.8
φ_I	Adjustment Cost $\frac{\varphi_I}{2}\left(I_t/I_{t-1} - 1\right)^2$	1
Banking Sector		
θ	Survival Probability	0.9685
$\kappa_0, \kappa_1, \kappa_2$	Agency Cost: $\kappa_0\left(1 + \kappa_1 B + \frac{\kappa_2}{2}B^2\right)$	$0.26, -0.75, 8.35$
ω	Transfer to New Bankers	0.0289
Exogenous Shock Processes		
ρ_A, $100\eta_A$	Technology Shock	$0.95, 0.45$
ρ_K, $100\eta_K$	Capital Quality Shock	$0, 1.5$
ρ_M, $100\eta_M$	Monetary Policy Shock	$0.15, 0.24$
Monetary Policy		
$\chi_\Pi, \chi_X, \chi_{R_N}$	Standard Monetary Policy Parameters	$1.5, 0.5/4, 0$
χ_ϕ, χ_S	Leverage, Credit Spread	$0, 0$

I follow Gertler and Karadi (2011) in choosing $\varepsilon = 4.17$ for the price elasticity of demand. Following Keen and Wang (2007), I choose the Rotemberg price adjustment parameter, φ_Π, to match a Calvo (1983)-style model in which firms face a 0.779 probability of keeping prices fixed every quarter. The investment adjustment parameter is set at 1.

The four parameters novel to the banking sector's agency problem are calibrated to match the same steady-state moments as those in Gertler, Kiyotaki, and Queralto (2012). In particular, they match a credit spread of 120 basis points, a ratio of total equity to assets, $B+1/\phi = 1/3$, and a ratio of inside to outside equity of $2/3$. Finally, they ensure that a rise in the standard deviation of capital quality innovations from 0.69 percent to 2.07 percent reduces the inverse of the total-equity-to-asset ratio by 1/3. My method of solving for the risk-adjusted steady state differs from the method preferred by Gertler, Kiyotaki, and Queralto (2012), thus implying a need for the parameters of the banking sector to be recalibrated from those in Gertler, Kiyotaki, and Queralto (2012). In particular, the risk adjustment in my solution method is smaller for a given standard deviation of the exogenous shock process. Thus, the key difference is that my $\kappa_2 = 8.35$ as opposed to 13.4.[7] The smaller value of this parameter means that the incentive compatibility constraint tightens less in my model for a given change in the outside-equity-to-asset ratio.

The model has three exogenous stochastic processes. The capital quality shock is calibrated in line with Gertler, Kiyotaki, and Queralto (2012). In the baseline, I set $\eta_K = 1.5$ percent and $\rho_K = 0$. The technology shock is considered to be highly persistent, with the $\rho_A = 0.95$. The monetary policy persistence parameter is lower, $\rho_M = 0.15$. In the baseline, the standard deviations of the two shocks are $\eta^A = 0.46$ percent and $\eta^M = 0.24$ percent, respectively.

The baseline reaction function is fairly conventional, with the feedback coefficient on inflation and output set at 1.5 and 0.5/4, respectively.

3.2 Solution Technique

The aim is to find a first-order approximation of the model's dynamics in the neighborhood of a risk-adjusted steady state. A formal statement of the solution technique is presented in appendix 1. Here I provide a description of the technique using a prototypical real business-cycle model. In so doing, I explain how the solution technique compares to others in the literature.

[7] κ_1 is negative in the calibration. This means that even in the deterministic steady state, banks issue a non-zero amount of outside equity. The economic rationale might be that there are some efficiency gains in monitoring the bank by having at least a small proportion of banks' funding coming from outside equity. More importantly, though, the share of outside equity to assets responds to shocks. Therefore, the calibration of κ_1 and the standard deviation of shocks are chosen to ensure that the first-order approximation of the model does not generate outside equity turning negative.

Consider the prototypical real business-cycle model with equilibrium conditions,[8]

$$\mathbf{E}_t\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\zeta}\alpha\exp\left(\eta_A\epsilon_{A,t+1}\right)K_t^{\alpha-1}=1 \tag{32}$$

$$\exp\left(\eta_A\epsilon_{A,t}\right)\left(K_t\right)^{\alpha}-K_{t+1}=C_t, \tag{33}$$

the exact solution of which is given by the decision rules, $C_t = g(K_t, \eta_A\epsilon_{A,t})$ and $K_{t+1} = h(K_t, \eta_A\epsilon_{A,t})$. Since g and h do not, in general, have a closed-form representation, we wish to find a first-order approximation of the functions g and h around the risk-adjusted steady state

$$C_t - C = g_K\left(K,0\right)\left(K_t-K\right)+g_\epsilon\left(K,0\right)\eta_A\epsilon_{A,t} \tag{34}$$

$$K_{t+1} - K = h_K\left(K,0\right)\left(K_t-K\right)+h_\epsilon\left(K,0\right)\eta_A\epsilon_{A,t}, \tag{35}$$

where C and K without time subscripts denote the risk-adjusted steady-state values of consumption and capital, respectively, and $g_K, g_\epsilon, h_K, h_\epsilon$ are, as yet, undetermined coefficients. Note, though, that the first-order coefficients are a function of the steady state, K.

Following Coeurdacier, Rey, and Winant (2011), the risk-adjusted steady state is defined as the "point where agents choose to stay at a given date if they expect future risk and if the realization of shocks is 0 at this date." This implies setting $C_t = C$, $K_{t+1} = K_t = K$, and $\epsilon_{A,t} = 0$ in (32) and (33). The realization of C_{t+1} and $\epsilon_{A,t+1}$ are unknown at date t. Substituting equation (34) into equation (32) leaves only the exogenous stochastic variable, $\epsilon_{A,t+1}$:

$$\alpha\beta\frac{1}{C^{-\zeta}}K^{\alpha-1}\mathbf{E}_t\left\{C+g_\epsilon\left(K,0\right)\eta_A\epsilon_{A,t+1}\right\}^{-\zeta}\exp\left(\eta_A\epsilon_{A,t+1}\right)=1.$$

We need to evaluate the object

$$\mathbf{E}_t\left\{C+g_\epsilon\left(K,0\right)\eta_A\epsilon_{A,t+1}\right\}^{-\zeta}\exp\left(\eta_A\epsilon_{A,t+1}\right),$$

[8]This model is nested within the full model presented in section 2. Financial and nominal frictions have been removed. Households are assumed to supply a fixed quantity of labor, normalized to 1. In addition, $h = 0$, $\delta = 1$, and $\rho_A = \eta_K = 0$.

but because this object is unlikely to have a closed-form solution, we take a second-order approximation around $\epsilon_{A,t+1} = 0$:

$$C^{-\zeta}\left(1+\left(\zeta\left(1+\zeta\right)C^{-2}\left(g_{\epsilon}\left(K,0\right)\right)^{2}-2\zeta C^{-1}g_{\epsilon}\left(K,0\right)+1\right)\frac{\eta_{A}^{2}}{2}\right).$$

Finally, the steady-state counterparts to equations (32) and (33) are as follows:

$$\alpha\beta K^{\alpha-1}\left(1+\underbrace{\left(\zeta\left(1+\zeta\right)C^{-2}g_{\epsilon}^{2}-2\zeta C^{-1}g_{\epsilon}+1\right)\frac{\eta_{A}^{2}}{2}}_{\text{risk-adjustment}}\right)=1$$

$$K^{\alpha}-K=C,$$

where I have dropped the notation showing the explicit dependence of g_ϵ on K. Notice that when the risk adjustment term is zero, the steady state coincides with the deterministic steady state. The risk adjustment term is a function of the conditional variance of consumption, $g_\epsilon^2\eta_A^2$; the conditional covariance between consumption and the productivity shock, $g_\epsilon\eta_A^2$; and the variance of the productivity shock, η_A^2. In a prototypical real business-cycle model, this risk adjustment raises K relative to the deterministic steady state due to precautionary savings. In the full model, in addition to taking into account precautionary saving, the risk adjustment will also alter banks' portfolio choice and, in consequence, the strength of the financial accelerator. Moreover, since changes in monetary policy alter the first-order approximated coefficients of the decision rules, $\mathbf{g}(.)$, monetary policy is able to affect banks' balance sheet composition in the steady state as well.

Since the steady state and the first-order dynamics are jointly determined, the model is solved iteratively to find a fixed point. With an initial guess for the steady state, one can solve for the first-order dynamics using standard methods. The first-order dynamics, however, are unlikely to be consistent with the initial steady-state guess, prompting the steady-state vector to be updated. This procedure continues until convergence is achieved.

The first-order solution of the model in the neighborhood of a risk-adjusted steady state is in the spirit of Coeurdacier, Rey, and Winant (2011) and Devereux and Sutherland (2011).[9] There are, however, two important points of note. First, the risk-adjusted steady state as I use it here is not exactly as Coeurdacier, Rey, and Winant (2011) and Devereux and Sutherland (2011) proposed it. The Devereux and Sutherland (2011) technique is to take an approximation around the deterministic steady state. However, at the deterministic steady state, the international portfolio choice problem that concerns them is indeterminate.[10] They therefore apply a risk adjustment to the equilibrium conditions pertaining to the portfolio choice problem only. I, however, apply the risk adjustment to all of the model's forward-looking equations. The main quantitative effect of applying the risk adjustment to the entire set of equations is that I capture both changes in bankers' optimal portfolio and changes in households' precautionary savings as a result of changes in exogenous uncertainty and changes in policy.

The Coeurdacier, Rey, and Winant (2011) method requires a second-order approximation of the steady state as well as evaluating the coefficients of the first-order approximation of the equilibrium conditions up to second order. This second step is crucial for Coeurdacier, Rey, and Winant (2011), as it ensures that the net foreign asset position in their small open-economy model is no longer a unit-root process. However, this additional step adds greatly to the computational burden yet alters the solution (in my case) only imperceptibly. I therefore solve the model with the steady state up to second order but evaluate the coefficients of the first-order conditions only up to a zeroth order.[11]

[9]The DSGE literature accounting for risk and portfolio choices in solution methods are closely linked. A non-exhaustive list of other important papers in the literature include Collard and Juillard (2001), Evans and Hnatkovska (2012), Benigno, Benigno, and Nisticò (2013), and Kliem and Meyer-Gohde (2013). One difference with many of these papers is that they incorporate time-varying risk, which my solution method does not.

[10]The bank's portfolio choice problem in this model is uniquely pinned down at the deterministic steady state: $B^{dss} = -\kappa_1/\kappa_2$. This is because the optimality condition reduces to solving $\Phi'(B) = 0$ as $\mu_{E,t} = 0$. And that, in turn, is because households become indifferent between holding a debt or an equity contract with the bank.

[11]The risk-adjusted steady state I use also differs from the risk adjustment employed in Gertler, Kiyotaki, and Queralto (2012).

Finally, appendix 1 details the calculation of welfare at the risk-adjusted steady state. The measure of welfare I present in the next section is conditional welfare, measured in consumption-equivalent units. In some cases, I present welfare relative to the baseline calibration; in other simulations I present welfare relative to the deterministic steady state. A positive consumption-equivalent value indicates that households need to be compensated for moving from the baseline calibration (or deterministic steady state) to the new environment.

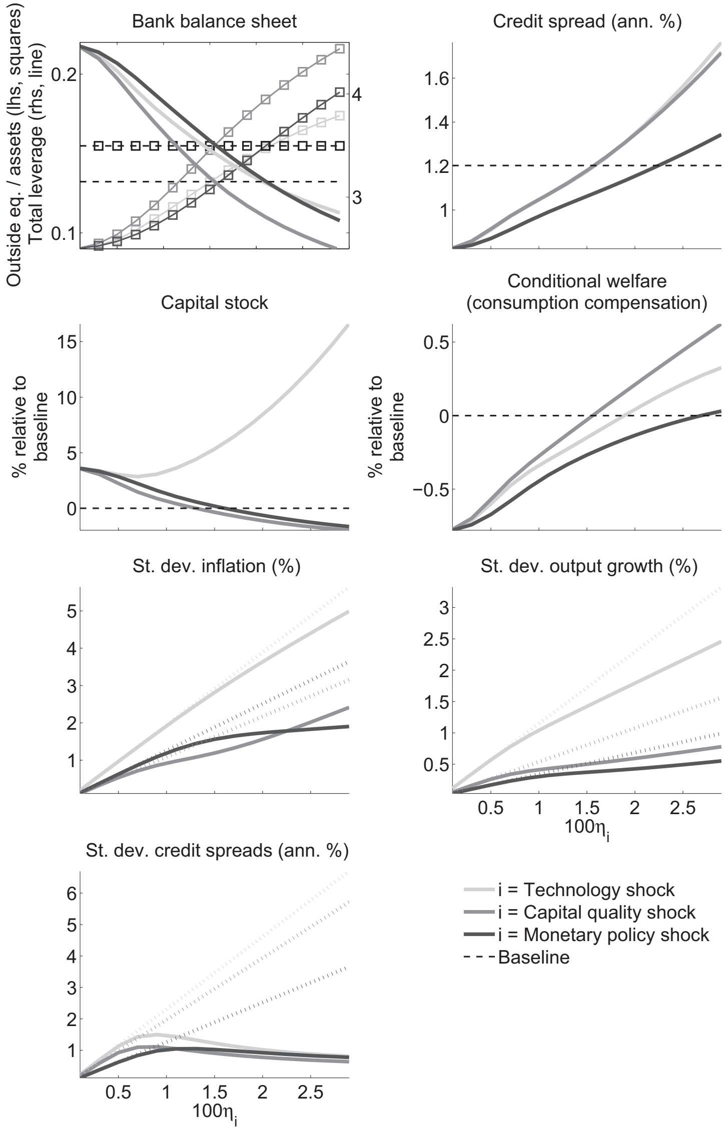
4. Results

In this section, I present several numerical experiments that shed light on the interaction between banks' balance sheet composition and the monetary policy environment. The monetary policy environment in this model is characterized by the nominal interest rate instrument that evolves as the product of an exogenous stochastic process and an endogenous reaction function to observable economic conditions. I first consider the effect on banks' balance sheet composition of changes in monetary and other exogenous uncertainty. I then consider the effect on bank balance sheets of changes to the endogenous reaction function that the policymaker employs.

4.1 Effect of Exogenous Uncertainty on Banks' Balance Sheets

Figure 1 presents the behavior of the economy in response to a change in monetary and other exogenous uncertainty. The horizontal axis in each plot indicates the standard deviation of exogenous innovations (in percent). The light gray, darker gray, and black lines plot the effect of technology, capital quality, and monetary policy shocks, respectively. In each case, we alter the value of η_i, holding $\eta_j = 0$ for $j \neq i$. The top four graphs show the effect of uncertainty on the risk-adjusted steady state of several variables. At the deterministic steady state, all these lines are flat. The bottom four graphs show the effect of uncertainty on the standard deviation of a set of variables on interest.

Figure 1. Effects of Exogenous Uncertainty



Notes: The horizontal axis measures changes in the standard deviation of the model's three exogenous innovations: technology, capital quality, and nominal interest rates. In each case the rest of the calibration is unchanged. The top four graphs plot risk-adjusted steady-state values of several key variables. The bottom three graphs plot standard deviations of several key variables. Black lines denote the risk-adjusted steady-state values at the baseline calibration. Dashed lines in the bottom three graphs denote the standard deviation of these variables with the steady state held fixed at its baseline value.

The top left graph shows the risk-adjusted steady state for two balance sheet indicators, the outside-equity-to-asset ratio and total leverage, respectively. In the face of increased uncertainty, banks issue more outside equity as a share of total assets (as well as in absolute terms). This is because there is greater value in hedging the asset return risk banks face relative to the benefit of higher leverage. Similarly, banks' inside-equity-to-asset ratio also increases in the face of increased exogenous uncertainty. This is in part because of the increased use of outside equity which exacerbated the agency problem between banks and households, and limits the amount of funds households are willing to provide. But inside equity (in absolute terms) also rises. This is because the credit spread, $\mathbf{E}_t(R_{K,t+1} - R_{t+1})$, has risen (as seen in the top right graph), leading to a higher steady-state accumulation of retained earnings. However, the risk-adjusted value, $\mathbf{E}_t \Lambda_{t,t+1} \Omega_{t+1}(R_{K,t+1} - R_{t+1})$, is lower for the bank in the more uncertain environment, which contributes to tightening the incentive compatibility constraint. As the standard deviation of capital quality shocks increases from zero to 3 percent, banks decrease total leverage from approximately $4\frac{1}{2}$ to 2, while the outside-equity-to-asset ratio rises from around 0.09 to 0.22. In parallel, credit spreads almost double, from 90 basis points to close to 180.

The second row of figure 1 shows the effect on the steady-state capital stock and on household welfare. The capital stock plot is interesting, as the effect of technology uncertainty is quite different from monetary and capital quality uncertainty. When technology uncertainty rises, the steady-state capital stock also rises. For the other two shocks, the capital stock falls. These differential effects result from two separate mechanisms. Most of the discussion in this paper has focused on the effect of uncertainty on a bank's balance sheet. As uncertainty rises, banks deleverage, resulting in a reduction of credit created for a given quantity of bank net worth and therefore a contraction in the capital stock, which is 100 percent financed by credit in the model. However, uncertainty has an important second effect in the model in that it generates precautionary savings by households as in a standard business-cycle model without financial frictions. Households wish to save more in an uncertain environment, resulting in a buildup of savings in the form of capital. Thus, with constrained banks, uncertainty has two offsetting effects on the capital stock in the economy. For technology shocks (at least at high levels of uncertainty), the precautionary saving motive dominates, while for monetary and capital quality uncertainty, the effect on credit constraints dominates.

Despite technology shocks generating a rise in steady-state capital (and consumption), the right-hand panel shows that uncertainty is unambiguously bad for household welfare, mostly because households are risk averse. The measure of welfare shown in the plot is the consumption-equivalent stream required for the household to be indifferent between the baseline level of uncertainty and the actual level of uncertainty as defined by the horizontal axis, conditional on the economy being initially at the baseline risk-adjusted steady state. (For the case of the technology shock, this means that households, in the transition, initially reduce consumption in order to build capital.) Even without the transitional dynamics (i.e., using an unconditional measure of welfare), higher uncertainty remains unambiguously bad for households. For a household to be indifferent between moving from the baseline calibration of the shocks to an environment in which $\eta_A = 3$ percent and $\eta_K = \eta_M = 0$, the household's stream of consumption in the initial calibration would need to be 0.3 percent lower every period.

The change in the composition of banks' balance sheets influences the transmission of shocks through the financial sector. Rows 3 and 4 plot the standard deviation of three key endogenous variables: inflation, output growth, and credit spreads. In each case, the dotted line shows the standard deviation, had the steady state been held unchanged. With a first-order approximation of the model around an unchanged steady state, the standard deviation of endogenous variables would increase linearly with the standard deviation of the exogenous shock process, as the dotted lines show. In all four plots, it is clear that the standard deviation of the endogenous variables changes with respect to the standard deviation of exogenous innovations at a slower rate than implied by a naive approximation of the model around an unaltered steady state (i.e., the dotted lines). This should not be a surprise. In the face of greater uncertainty, banks have shifted towards greater equity finance, severely dampening the financial accelerator channel. The effect of changes in banks' balance sheet on macroeconomic volatility can be quantitatively large. Suppose, for example, that the standard deviation of the capital quality shock innovation rose from 1.5 percent to 3 percent. If we believed that bank's balance sheet determination was orthogonal to this change in the environment, and total bank leverage remained at $3\frac{1}{2}$, the model would expect the standard deviation of output growth to increase from $\frac{3}{4}$ percent to $1\frac{1}{4}$ percent. However, when the model accounts for banks' endogenous balance sheet adjustment, reducing total leverage to $2\frac{1}{2}$, the standard deviation of output growth rises from $\frac{1}{2}$ percent to only 0.6 percent. The effect of changes in the stochastic environment through the balance sheet channel have the largest implications for the volatility of credit spreads. The standard deviation of credit spreads does not increase monotonically in the standard deviation of the exogenous innovations. Instead, the standard deviation of credit spreads rises initially but then falls again and flattens out at a standard deviation of around 1 percent.

We can use these plots to draw implications about the relationship between banks' balance sheets and exogenous shock processes in different periods of U.S. history, like the pre-Great Moderation, Great Moderation, and post-Great Moderation periods. The difficulty with calibrating the model to different periods of U.S. macroeconomic history using estimated shock processes from papers like Smets and Wouters (2007) is that these estimated shock processes are naturally a product of a linear model and detrended series. However, suppose we put these caveats aside and take the parameters from Smets and Wouters (2007) at face value. Then we would find little evidence of a strong risk channel. Smets and Wouters (2007) find estimates of η_A equal to 0.58 and 0.35 for the periods 1966–75 and 1984–2004, respectively.[12] As can be seen from figure 1, such a change induces a shift in total bank leverage of around one-tenth, which is not enough to account for the buildup of leverage during the Great Moderation.

An alternative, back-of-the-envelope exercise is to start with the (quarter-on-quarter) standard deviation of output growth for the United States before, during, and after the Great Moderation. These are 1.05 percent (1960–84), 0.49 percent (1985–2007), and 0.85 percent (2008–12), respectively, and are taken from the U.S. National Income and Product Accounts (NIPA) tables. Next, assume that the only structural change to have occurred in this time has been

[12] See table 5 on page 606 of Smets and Wouters (2007).

the standard deviation of technology innovations.[13] From the third row, the model implies a standard deviation of technology shock pre-Great Moderation of 1 percent and during the Great Moderation of 0.5 percent. In the absence of the risk channel, the estimate for the pre-Great Moderation period would have been 0.9 percent. Thus, ignoring the risk channel and banks' endogenous balance sheet choices can significantly bias the estimates of the standard deviation of shocks.

4.2 Effect of Monetary Reaction Function on Banks' Balance Sheets

Figures 2–5 present the behavior of the economy in response to changes in the monetary policy reaction function. The horizontal axis in each plot indicates the reaction-function parameter that is being adjusted. The remaining parameters are held at their respective baseline values. The information in the figures is similar to that in figure 1. However, there are three noteworthy differences. First, for each policy experiment, I show three different exogenous stochastic states—a low-, baseline, and high-risk state corresponding to $100\eta_K = 0$, 1.5, and 2.5, respectively—with the other shock processes held fixed at the baseline calibration. Second, the consumption-equivalent conditional welfare measure is relative to the deterministic steady state, an assumption made purely for clarity of the exposition that does not affect the welfare ranking of alternative policies. Third, the dotted lines in rows 3 and 4 have a different interpretation. In each case, the dotted lines show the standard deviation of the variables had the model been solved around a risk-adjusted steady state that responds to changes in the exogenous stochastic environment but does not respond to changes in the policy environment (i.e., the changes in the policy parameter values on the horizontal axis). The dotted lines can therefore be interpreted as the naive policymaker's prediction about the effect of a change in his or her reaction function, unaware of the risk channel.

[13]We can repeat this exercise in the next sub-section by backing out the balance sheet effect as a result of monetary policy changes pre-Volcker and thereafter using estimated values of Taylor-rule parameters from the literature. The risk channel, however, in terms of historical changes in monetary policy inflation activism, is again quantitatively small.

Figure 2 plots changes in the reaction function's responsiveness to deviation of inflation, $\chi_\Pi \in [1.01, 2.5]$. There are two unsurprising results. First, aggressively responding to deviations of inflation reduces the volatility of inflation, and second, aggressively responding to inflation is strictly welfare improving, in line with standard results in the literature. There are, however, three further effects of aggressive inflation targeting that are of interest, as they are the result of the risk channel of monetary policy. First, in a high-risk environment, aggressively responding to inflation reduces volatility in financial market credit spreads, while standard solution methods would have predicted a rise in the volatility of credit spreads. Along with this reduction in the volatility of credit spreads, the average credit spread also rises by about 5 basis points as the coefficient χ_Π increases from 1.5 to 2.5, a result of banks issuing modestly more outside equity and reducing their leverage.

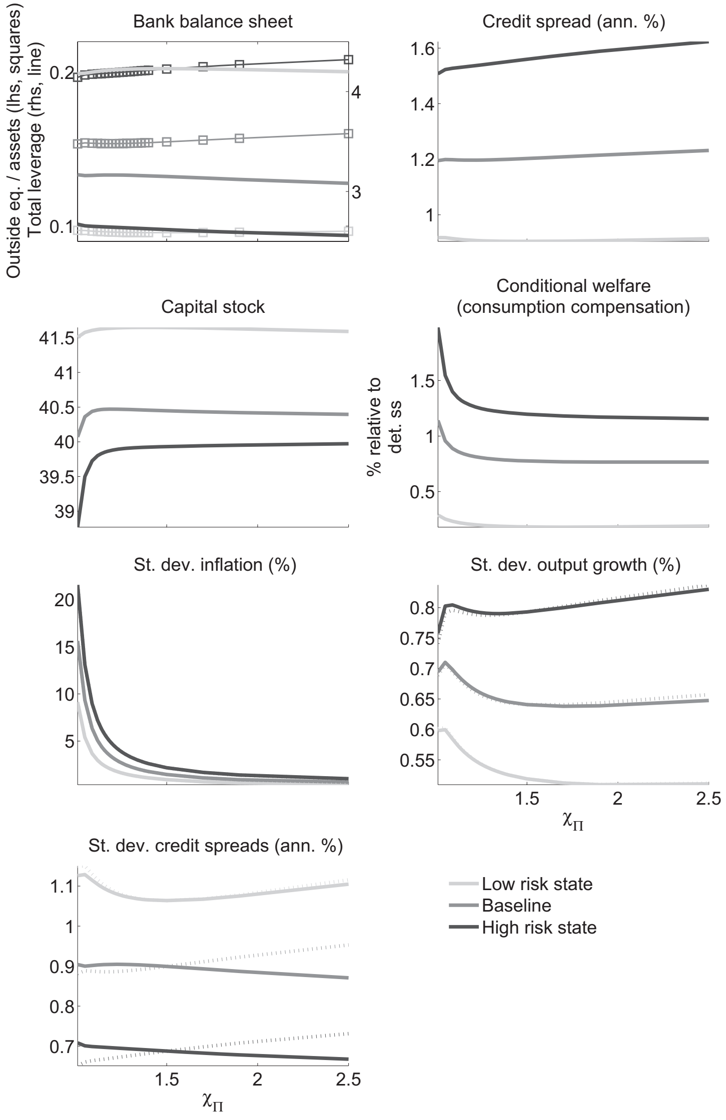
These effects are relatively small. The change in the risk environment from low to high risk has a several-orders-of-magnitude-larger effect on banks' balance sheet composition than does increasing the aggressiveness of inflation activism from 1.01 to 2.5.

Figure 3 repeats the same experiment for the coefficient $\chi_X \in [0, 0.25]$, the aggressiveness with which the policymaker responds to deviations of its proxy for the output gap. The flatness of the curves in rows 1 and 2 and the closeness of the solid and dotted lines in row 3 indicates that the risk channel is relatively negligible along this dimension of policy. The only significant differences appear in the volatility of financial-sector credit spreads in the bottom graph of the figure.

Figures 2 and 3 suggest that variation in the conventional arguments of the reaction function has little effect on banks' balance sheets. To the extent that the central bank has financial stability concerns, however, the effect can be much greater. And a discussion of whether central banks should use their monetary policy tool, the nominal interest rate, for financial stability objectives—leaning against assets bubbles, dampening credit cycles, and preventing the buildup of financial stability—is at the center of the current policy debate.

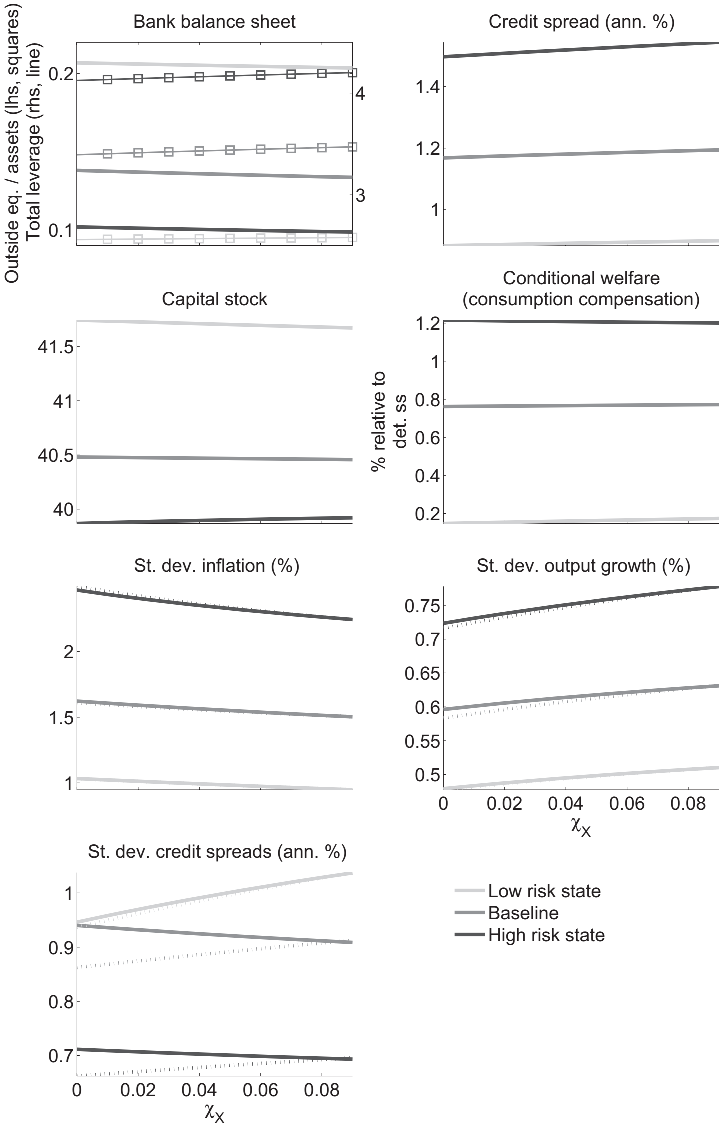
In this spirit, I consider the effect first of the central bank responding with its nominal interest rate to movements in bank

Figure 2. Effect of Variation in Monetary Policy Response to Inflation



Notes: The horizontal axis measures changes in a monetary policy parameter. In each case the monetary policy parameter is adjusted, holding the rest of the calibration unchanged. The top four graphs plot risk-adjusted steady-state value of several key variables. The bottom three graphs plot standard deviations of several key variables. The three different shades in each plot reflect three different levels of exogenous uncertainty. Low, baseline, and high refer to $100\eta_K = 0, 1.5, 2.5$, respectively. Dashed lines in the bottom three graphs denote the standard deviations with the steady state incorporating the state of exogenous uncertainty (low, baseline, or high) but not incorporating changes in the monetary policy parameter.

Figure 3. Effect of Variation in Monetary Policy Response to Output Gap



Notes: The horizontal axis measures changes in a monetary policy parameter. In each case the monetary policy parameter is adjusted, holding the rest of the calibration unchanged. The top four graphs plot risk-adjusted steady-state value of several key variables. The bottom three graphs plot standard deviations of several key variables. The three different shades in each plot reflect three different levels of exogenous uncertainty. Low, baseline, and high refer to $100\eta_K = 0, 1.5, 2.5$, respectively. Dashed lines in the bottom three graphs denote the standard deviations with the steady state incorporating the state of exogenous uncertainty (low, baseline, or high) but not incorporating changes in the monetary policy parameter.

leverage, shown in figure 4.[14] In figure 5, I consider the response of the central bank to movements in credit spreads. Since both leverage and spreads move countercyclically in the model, the policymaker would respond countercyclically, lowering the nominal interest rate when leverage rises and/or credit spreads rise. Thus χ_ϕ and χ_S are assumed to take negative values.

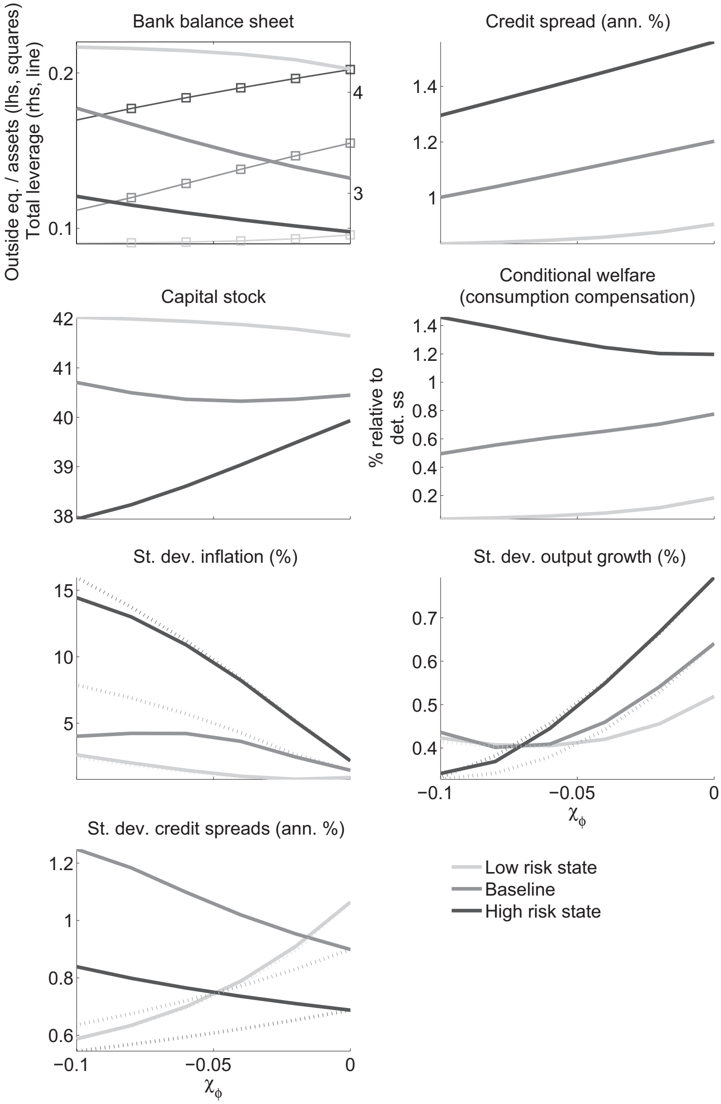
In figure 4, $\chi_\phi \in [-0.1, 0]$. Thus, at the extreme value of –0.1, a 10 percent rise in bank leverage is countered with a 1 percent reduction in the nominal interest rate.[15] Given the dominance of supply shocks in the calibration, responding to leverage dampens the volatility of output growth at the expense of greater inflation volatility. Consider the baseline calibration (darker gray line) and a switch of policy from $\chi_\phi = 0$ to $\chi_\phi = -0.1$. Banks, perceiving this policy change to dampen the volatility of their asset returns, decrease their share of funding from outside equity and expand their leverage. The outside-equity-to-asset ratio drops from 0.15 to 0.11, while total leverage rises from 3.1 to 3.9. In the absence of this balance sheet adjustment, the standard deviation of output would have halved, from 0.6 percent to 0.3 percent. However, due to banks' willingness to take on more balance sheet risk in response to the change in the policy environment, the standard deviation drops only to 0.45 percent. In this example, therefore, the risk channel of monetary policy is quantitatively important. When we look at the welfare implications of this policy choice, the model provides a mixed message. In benign or normal times, targeting leverage can be welfare enhancing. However, in a high-risk environment, targeting leverage is bad for welfare.

Figure 5 experiments with $\chi_S \in [-4, 0]$. This range implies that, at its most extreme, $\chi_S = -4$, a 10-basis-point rise in credit spreads results in a 40-basis-point reduction in the nominal interest rate. Again, the effects of the risk channel appear to be most powerful at the baseline calibration, with less-pronounced effects in high- and low-risk states. Targeting movements in credit spreads, like targeting leverage, increases banks' willingness to issue debt instead of outside equity and leverage up. Credit spreads also fall. Perversely, the volatility of credit spreads rises. In the counterfactual experiment

[14] The central bank is assumed to respond to the ratio of assets to inside equity. The results are little affected by changing the target variable to be total leverage.

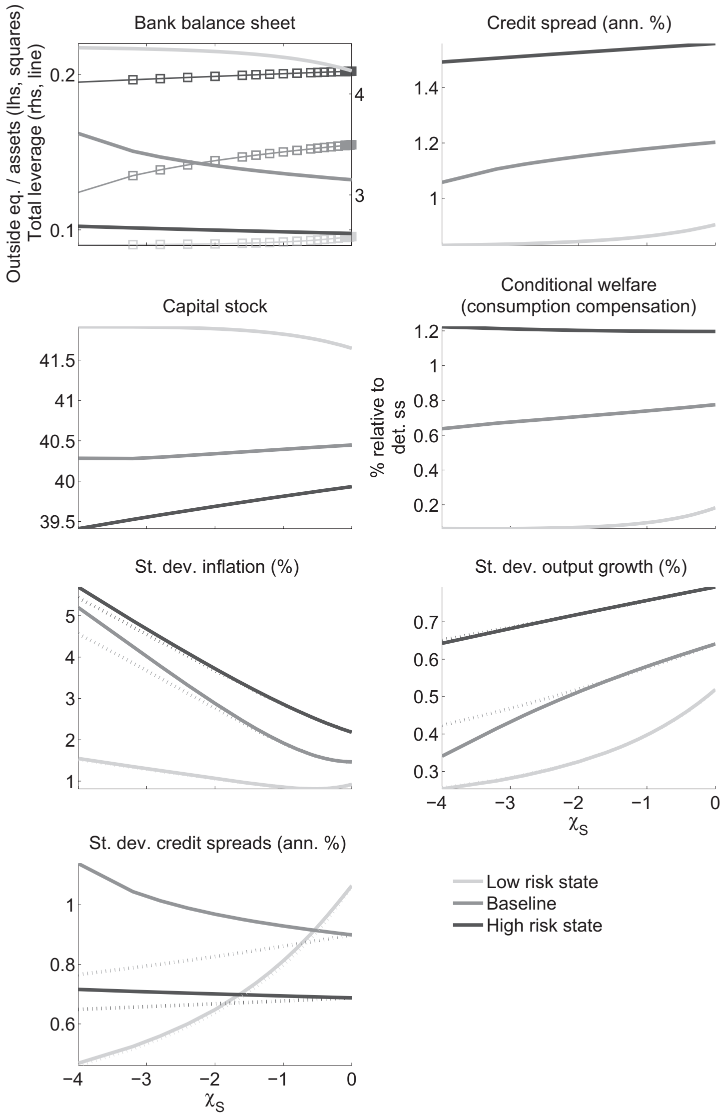
[15] Values of $\chi_\phi < -0.1$ generate convergence problems for the solution algorithm.

Figure 4. Effect of Variation in Monetary Policy Response to Leverage



Notes: The horizontal axis measures changes in a monetary policy parameter. In each case the monetary policy parameter is adjusted, holding the rest of the calibration unchanged. The top four graphs plot risk-adjusted steady-state value of several key variables. The bottom three graphs plot standard deviations of several key variables. The three different shades in each plot reflect three different levels of exogenous uncertainty. Low, baseline, and high refer to $100\eta_K = 0, 1.5, 2.5$, respectively. Dashed lines in the bottom three graphs denote the standard deviations with the steady state incorporating the state of exogenous uncertainty (low, baseline, or high) but not incorporating changes in the monetary policy parameter.

Figure 5. Effect of Variation in Monetary Policy Response to Credit Spread



Notes: The horizontal axis measures changes in a monetary policy parameter. In each case the monetary policy parameter is adjusted, holding the rest of the calibration unchanged. The top four graphs plot risk-adjusted steady-state value of several key variables. The bottom three graphs plot standard deviations of several key variables. The three different shades in each plot reflect three different levels of exogenous uncertainty. Low, baseline, and high refer to $100\eta_K = 0, 1.5, 2.5$, respectively. Dashed lines in the bottom three graphs denote the standard deviations with the steady state incorporating the state of exogenous uncertainty (low, baseline, or high) but not incorporating changes in the monetary policy parameter.

with no risk channel (the dotted line), a change in policy towards responding aggressively to movements in credit spreads would have yielded a reduction in the standard deviation of the credit spread from 0.9 percent to below 0.8 percent. However, with the risk channel present, the increase in bank leverage results in the standard deviation of credit spreads actually rising, to close to 1.1 percent. In contrast to targeting leverage, however, the risk channel amplifies the standard deviation of inflation while dampening the standard deviation of output growth, relative to the model solution without the risk channel.

The results presented in this section suggest that the risk channel of monetary policy, has, under certain policy prescriptions, meaningfully sized economic effects. Policymakers should be aware of this endogenous risk channel via endogenous changes in bank balance sheets, especially if they aim to redesign policy to target specific financial-sector indicators using the standard tools of monetary policy.

5. Conclusion

There is a popular view that the Great Moderation of the 1990s and early 2000s sowed the seeds of the global financial crisis in 2007. As macroeconomic outcomes became less uncertain, financial intermediaries built up leverage and took on more risk. In turn, another literature has tried to explain the causes of the Great Moderation, from which two main views have emerged. One is a good-luck story, that the global economy simply enjoyed a period in which the shocks hitting the economy were unusually modest. The other view is that central banks had a better design of monetary policy.

This paper explores the risk channel of monetary policy in a quantitative macroeconomic model by endogenizing the composition of banks' funding. I find that when central banks target financial variables such as cyclical leverage or credit spreads, policy can alter banks' balance composition in a quantitatively meaningful way, and affect how shocks are amplified and propagated through the financial sector. The numerical experiments in this paper suggest that central banks and financial-sector regulators should be vigilant of how periods of relative tranquility (like the Great Moderation) can generate a potential buildup of risks in the economy as financial institutions increase the size and leverage of their balance sheets and rely more heavily on debt financing.

One possible line of further investigation would be to consider macroprudential policy alongside standard monetary policy. I leave this for future research. My current avenue of research is to solve for optimal (monetary) policy in this model.

Appendix 1. Risk-Adjusted Steady-State and First-Order Dynamics: Theory

This section explains how to solve a model as a first-order approximation of the model around a second-order approximation of the model's risk-adjusted steady state.

Let the equilibrium conditions of the model be written as

$$\mathbf{E}_t\left[f\left(y_{t+1}, y_t, x_{t+1}, x_t, z_{t+1}, z_t\right)\right]=0 \tag{36}$$
$$z_{t+1}=\rho z_t+\eta\sigma\varepsilon_{t+1},$$

where y_t is an $n_y \times 1$ vector of endogenous non-predetermined variables, x_t is an $n_x \times 1$ vector of endogenous predetermined variables, z_t is an $n_z \times 1$ vector of exogenous variables, and ε_t is an $n_z \times 1$ vector of exogenous i.i.d. innovations with mean zero and unit standard deviations. The matrices ρ and η are of order $n_z \times n_z$ and σ is a scalar scaling the amount of uncertainty in the economy.

Next, let the (unknown) decision rules that solve the system of equations in (36) be $y_t = g(x_t, z_t)$ and $x_{t+1} = h(x_t, z_t)$. The risk-adjusted steady state, x^r, solves

$$x^r=h\left(x^r, 0\right) \text{ with } y^r=g\left(x^r, 0\right). \tag{37}$$

Substituting the decision rules into (36) and evaluating at the (also as yet unknown) risk-adjusted steady state gives

$$f\left(x^r, \sigma\right) \equiv \mathbf{E}_t\left[f\left(g\left(x^r, \eta\sigma\varepsilon_{t+1}\right), g\left(x^r, 0\right), x^r, x^r, \eta\sigma\varepsilon_{t+1}, 0\right)\right]=0.$$

Note that ε_{t+1} is not an argument but, instead, the variable of integration inside the expectations operator. Taking a second-order approximation of f around $\sigma = 0$ (but a first-order approximation of $g(.)$ and $h(.)$) gives

$$\left[f\left(x^r, \sigma\right)\right]^i \approx\left[f\left(x^r, 0\right)\right]^i+\frac{\sigma^2}{2}\left[f_{\sigma\sigma}\left(x^r, 0\right)\right]^i=0, \tag{38}$$

where

$$f\left(x^r,0\right)=f\left(y^r,y^r,x^r,x^r,0,0\right)$$

and

$$\begin{aligned}\left[f_{\sigma\sigma}\left(x^r,0\right)\right]^i &= \left[f_{y'y'}\right]^i_{\alpha\gamma}\left[g_z\eta\right]^\alpha_\phi\left[g_z\eta\right]^\gamma_\xi\left[I\right]^\phi_\xi+\left[f_{y'z'}\right]^i_{\alpha\delta}\left[g_z\eta\right]^\alpha_\phi\left[\eta\right]^\delta_\xi\left[I\right]^\phi_\xi\\ &\quad+\left[\Theta_{z'z'}\right]^i_{\beta\delta}\left[\eta\right]^\beta_\phi\left[\eta\right]^\delta_\xi\left[I\right]^\phi_\xi\end{aligned}$$

$$i=1,\ldots,n;\ \alpha,\gamma=1,\ldots,n_y;\ \ \beta,\delta=1,\ldots,n_z;\ \phi,\xi=1,\ldots,n_\varepsilon.$$

The notation follows that in Schmitt-Grohé and Uribe (2004). The first derivatives of the decision rules, g_x, are solved using standard methods of first-order approximation. The solution is found by iterating between a set of steady-state values (y,x) and a set of decision rule coefficients (g_z) until convergence is achieved.[16]

Appendix 2. Equilibrium Conditions

The model presented in section 2 has twenty-three endogenous variables, $\{C_t,I_t,K_t,N_t,\phi_t,Q_{K,t},Q_{E,t},R_{N,t},R_t,U_{C,t},B_t,Y_t,\Pi_t,L_t,\Lambda_{t-1,t},X_t,\mu_{S,t},\mu_{E,t},\mu_{N,t},\Omega_t,R_{E,t},R_{K,t},S_t\}$, and twenty-three equilibrium equations, listed below.

Aggregate resource constraint:

$$\left(1-\frac{\varphi_\Pi}{2}\left(\Pi_t-1\right)^2\right)Y_t=C_t+\left(1+\frac{\varphi_I}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right)I_t.$$

Capital accumulation:

$$K_{t+1}=\left(1-\delta\right)\exp\left(\varepsilon_{K,t}\right)K_t+I_t.$$

Price of capital:

$$\begin{aligned}Q_{K,t}&=1+\frac{\varphi_I}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2+\left(\frac{I_t}{I_{t-1}}\right)\varphi_I\left(\frac{I_t}{I_{t-1}}-1\right)\\ &\quad-\mathbf{E}_t\Lambda_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\varphi_I\left(\frac{I_{t+1}}{I_t}-1\right).\end{aligned}$$

[16] Code to implement this solution algorithm and replicate the results in the paper is available from the author on request.

Inverse of banking sector's inside-equity-to-asset ratio:

$$\phi_t = \frac{Q_{K,t} K_{t+1}}{N_t}.$$

Banking sector's inside-equity accumulation:

$$N_t = \theta \left(\left(R_{K,t} - R_{E,t} B_{t-1} - R_t \left(1 - B_{t-1} \right) \right) \phi_{t-1} + R_t \right) N_t \\ + \omega Q_{K,t} K_t.$$

Household Euler equation and arbitrage condition:

$$\mathbf{E}_t \Lambda_{t,t+1} R_{t+1} = 1 \quad \text{and} \quad \mathbf{E}_t \Lambda_{t,t+1} \left(R_{E,t+1} - R_{t+1} \right) = 1.$$

Household stochastic discount factor:

$$\Lambda_{t-1,t} = \beta \frac{U_{C,t}}{U_{C,t-1}}.$$

Banking sector's binding incentive compatibility constraint:

$$\phi_t = \frac{\mu_{N,t}}{\kappa_0 \left(1 + \kappa_1 B_t + \frac{\kappa_2}{2} B_t^2 \right) - \left(\mu_{S,t} + B_t \mu_{E,t} \right)}.$$

Banking sector's optimal leverage/outside equity trade-off:

$$\frac{\mu_{E,t}}{\mu_{S,t} + B_t \mu_{E,t}} = \frac{\kappa_1 + \kappa_2 B_t}{1 + \kappa_1 B_t + \frac{\kappa_2}{2} B_t^2}.$$

Return on capital and outside equity:

$$R_{K,t} = \exp\left(\varepsilon_{K,t} \right) \frac{X_t \alpha \frac{Y_t}{\exp(\varepsilon_{K,t}) K_t} + \left(1 - \delta \right) Q_{K,t}}{Q_{K,t-1}}$$

$$R_{E,t} = \exp\left(\varepsilon_{K,t} \right) \frac{X_t \alpha \frac{Y_t}{\exp(\varepsilon_{K,t}) K_t} + \left(1 - \delta \right) Q_{E,t}}{Q_{E,t-1}}.$$

Labor market equilibrium:

$$X_t \left(1 - \alpha \right) Y_t U_{C,t} = \varrho L_t^{1+\vartheta} \left(C_t - h C_{t-1} - \frac{\varrho}{1+\vartheta} L_t^{1+\vartheta} \right)^{-\zeta}.$$

Marginal utility of consumption:

$$U_{C,t} = \left(C_t - hC_{t-1} - \frac{\varrho}{1+\vartheta}L_t^{1+\vartheta}\right)^{-\zeta} - \xi_t \beta h \left(C_{t+1} - hC_t - \frac{\varrho}{1+\vartheta}L_{t+1}^{1+\vartheta}\right)^{-\zeta}.$$

Aggregate production function:

$$Y_t = \exp(\varepsilon_{A,t})(\exp(\varepsilon_{K,t})K_t)^{\alpha} L_t^{1-\alpha}.$$

Banking sector's marginal value of intermediating an additional unit of credit, of substituting one unit of debt for one unit of outside equity, and of an additional unit of net worth, respectively:

$$\begin{aligned}
\mu_{K,t} &= \mathbf{E}_t(\Lambda_{t,t+1}\Omega_{t+1}(R_{K,t+1} - R_{t+1})) \\
\mu_{E,t} &= \mathbf{E}_t(\Lambda_{t,t+1}\Omega_{t+1}(R_{t+1} - R_{E,t+1})) \\
\mu_{N,t} &= \mathbf{E}_t(\Lambda_{t,t+1}\Omega_{t+1})R_{t+1}.
\end{aligned}$$

Shadow value of net worth:

$$\Omega_t = (1-\theta) + \theta B_t \phi_t.$$

Phillips curve:

$$\varphi_\Pi \left(\frac{\Pi_t}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} = 1 - \varepsilon\left(1 - X_t\right) + \varphi_\Pi \mathbf{E}_t \left\{\Lambda_{t,t+1}\left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\frac{\Pi_{t+1}}{\Pi}\frac{Y_{t+1}}{Y_t}\right\}.$$

Monetary policy reaction function:

$$\left(\frac{R_{N,t}}{R_N}\right) = \left(\left(\frac{\Pi_t}{\Pi}\right)^{\chi_\Pi}\left(\frac{X_t}{X}\right)^{\chi_X}\left(\frac{\phi_t}{\phi}\right)^{\chi_\phi} \times \exp\left(\chi_S\left(S_t - S\right)\right)\left(\frac{R_{K,t}}{R_K}\right)^{\chi_{R_K}}\left(\frac{Q_{K,t}}{Q_K}\right)^{\chi_{Q_K}}\right)^{1-\chi_{R_N}} \times \left(\frac{R_{N,t-1}}{R_N}\right)^{\chi_{R_N}} \exp\left(\varepsilon_{M,t}\right)$$

Credit spread:

$$S_t = \mathbf{E}_t R_{K,t+1} - R_{t+1}.$$

Fisher relation:

$$R_{N,t} = R_{t+1} \mathbf{E}_t \left(\Pi_{t+1} \right).$$

Appendix 3. Steady State

We find a first-order approximation of the model in the neighborhood of the risk-adjusted steady state. With the exception of B_t, $\mu_{E,t}$, and S_t—which remain in levels—all the variables are expressed as log-deviations. The steady-state inflation rate is set at $\Pi = 1$ and $\Lambda = \beta$. This steady state is solved as described in appendix 2. The steady-state counterparts of the static equations simply involve removing the time subscripts. The forward-looking equilibrium equations are approximated to second order around $\sigma = 0$ (and the σ is normalized to 1). The risk-adjusted steady-state correction terms (the second-order terms) are given by the $\mathbf{M}$'s. At the deterministic steady state, the $\mathbf{M}$'s are zero. Equivalently, the deterministic steady state is a zeroth-order approximation of the steady-state equations around $\sigma = 0$.

Price of capital:

$$Q_K = 1 + \mathbf{M}_1.$$

Household Euler equations:

$$1 = \Lambda R + \mathbf{M}_2 \quad \text{and} \quad 1 = \Lambda \left(R_E - R \right) + \mathbf{M}_3.$$

Marginal utility of consumption:

$$U_C = (1 - \beta h) \left((1 - h) C - \frac{\varrho}{1 + \vartheta} L^{1+\vartheta} \right)^{-\zeta} + \mathbf{M}_4.$$

Credit spread:

$$S = R_K - R + \mathbf{M}_5.$$

Banking sector's marginal value of intermediating an additional unit of credit, of substituting one unit of debt for one unit of outside equity, and of an additional unit of net worth, respectively:

$$\mu_S = \Lambda\Omega\left(R_K - R\right) + \mathbf{M}_6$$
$$\mu_E = \Lambda\Omega\left(R - R_E\right) + \mathbf{M}_7$$
$$\mu_N = \Lambda\Omega R + \mathbf{M}_8.$$

Phillips curve:

$$X = \frac{\varepsilon - (1 + \mathbf{M}_9)}{\varepsilon}.$$

Fisher relation:

$$R_N = R + \mathbf{M}_{10}.$$

The risk-adjusted steady-state correction terms are as follows:

$$\mathbf{M}_1 \equiv -\frac{1}{2}\Lambda\varphi_I \sum_{i=1}^{n_\varepsilon}\left(5\left(g_i^I\right)^2 + 2g_i^\Lambda g_i^I\right)\eta_i^2$$

$$\mathbf{M}_2 \equiv \frac{1}{2}\Lambda R \sum_{i=1}^{n_\varepsilon}\left(g_i^\Lambda\right)^2 \eta_i^2$$

$$\mathbf{M}_3 \equiv \frac{1}{2}\Lambda \sum_{i=1}^{n_\varepsilon}\left(\left(R_E - R\right)\left(g_i^\Lambda\right)^2 + R_E\left(2g_i^\Lambda g_i^{R_E} + \left(g_i^{R_E}\right)^2\right)\right)\eta_i^2$$

$$\mathbf{M}_4 \equiv \frac{1}{2}\zeta\beta h\left((1-h)\,C - \frac{\varrho}{(1+\vartheta)}L_{t+1}^{(1+\vartheta)}\right)^{-\zeta-1}$$
$$\times \sum_{i=1}^{n_\varepsilon}\left(\begin{array}{l} C\left(1-(1+\zeta)C\left((1-h)C - \frac{\varrho}{(1+\vartheta)}L^{(1+\vartheta)}\right)^{-1}\right)\left(g_i^C\right)^2 \\ \quad + 2(1+\zeta)C\varrho L^{1+\vartheta}\left((1-h)C - \frac{\varrho}{(1+\vartheta)}L^{(1+\vartheta)}\right)^{-1} g_i^C g_i^L \\ \quad - \varrho L^{(1+\vartheta)}\left(\begin{array}{l}(1+\vartheta) + (1+\zeta)\,\varrho L^{(1+\vartheta)} \\ \quad \times\left((1-h)\,C - \frac{\varrho}{(1+\vartheta)}L^{(1+\vartheta)}\right)^{-1}\end{array}\right)\left(g_i^L\right)^2 \end{array}\right)\eta_i^2$$

$$\mathbf{M}_5 \equiv -\frac{1}{2}R_K \sum_{i=1}^{n_\varepsilon}\left(g_i^{R_K}\right)^2 \eta_i^2$$

$$\mathbf{M}_6 \equiv \frac{1}{2}\Lambda\Omega\left(R_K - R\right)\sum_{i=1}^{n_\varepsilon}\left(\left(g_i^\Lambda\right)^2 + 2g_i^\Lambda g_i^\Omega + \left(g_i^\Omega\right)^2\right)\eta_i^2$$

$$\mathbf{M}_7 \equiv -\frac{1}{2}\Lambda\Omega\Bigg(\left(R_E - R\right)\left(\left(g_i^\Lambda\right)^2 + 2g_i^\Lambda g_i^\Omega + \left(g_i^\Omega\right)^2\right)\left(R_E - R\right) + R_E\left(2g_i^\Lambda g_i^{R_E} + 2g_i^\Omega g_i^{R_E} + \left(g_i^{R_E}\right)^2\right)\Bigg)$$

$$\mathbf{M}_8 \equiv \frac{1}{2}\Lambda\Omega R\sum_{i=1}^{n_\varepsilon}\left(\left(g_i^\Lambda\right)^2 + 2g_i^\Lambda g_i^\Omega + \left(g_i^\Omega\right)^2\right)\eta_i^2$$

$$\mathbf{M}_9 \equiv \frac{1}{2}\Pi\Lambda\varphi_\Pi\sum_{i=1}^{n_\varepsilon}\left(3\left(g_i^\Pi\right)^2 + 2g_i^\Lambda g_i^\Pi + 2g_i^\Pi g_i^Y\right)\eta_i^2$$

$$\mathbf{M}_{10} \equiv \frac{1}{2}\Pi R\sum_{i=1}^{n_\varepsilon}\left(g_i^\Pi\right)^2\eta_i^2,$$

where n_ε is the number of exogenous shocks and $i = \{A, K, M\}$ indexes the shock. For example, g_A^C is the first-order response of consumption (on impact) of a shock to technology.

Appendix 4. Welfare

Welfare at time t is denoted

$$\mathbf{W}_t = \mathbf{E}_t\sum_{i=0}^{\infty}\beta^i U\left(C_{t+i}, L_{t+i}\right).$$

In recursive form, welfare can be written as

$$\mathbf{W}_t = U\left(C_t, L_t\right) + \beta\mathbf{E}_t\mathbf{W}_{t+1}.$$

At the risk-adjusted steady state, this becomes

$$\mathbf{W} = \frac{U\left(C, L\right) + \mathbf{M}_\mathbf{W}}{1-\beta},$$

where the risk adjustment takes the form

$$\mathbf{M}_\mathbf{W} \equiv \frac{\mathbf{W}}{2}\left(g_i^\mathbf{W}\right)^2\eta_i^2.$$

Given initial policy A and alternative policy B, the consumption-equivalent compensation required to be indifferent about switching from A to B is λ^u and is implicitly given by

$$\mathbf{W}^B = \frac{U\left((1-\lambda^u)\,C^A, L^A\right) + \mathbf{M}_{\mathbf{W}}^A}{1-\beta}.$$

Given the first-order approximation of the decision rule, $\mathbf{W}_t = g(x_t)$ around the risk-adjusted steady state with policy B,

$$\log \mathbf{W}_t - \log \mathbf{W}^B = g_x^{\mathbf{W}} \left(\log x_t - \log x^B\right),$$

the welfare of policy B, conditional on initially being at the state vector x^A (i.e., the risk-adjusted steady state with policy A), denoted $\mathbf{W}^{B|A}$, is

$$\log \mathbf{W}^{B|A} - \log \mathbf{W}^B = g_x^{\mathbf{W}} \left(\log x^A - \log x^B\right).$$

To rewrite this level of welfare in terms of consumption-equivalent units, λ^c, I calculate

$$\mathbf{W}^{B|A} = \frac{U\left((1-\lambda^c)\,C^A, L^A\right) + \mathbf{M}_{\mathbf{W}}^A}{1-\beta}.$$

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