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1. Introduction

The topic of Coenen and Warne (this issue) is of core importance to central bankers: the assessment of inflation risk. Their narrative of euro-area inflation risk from the end of 2008 through 2011 establishes a benchmark against which one may regard the monetary policy decisions made by the European Central Bank. The density forecasts for inflation are derived based on the NAWM’s (New Area Wide Model’s) interpretation of Consensus forecasts for endogenous variables in terms of the model’s structural shocks at each point in time. The density forecasts are made by simulations based on these derived structural shocks and the estimated covariance matrix of the NAWM, while taking into account the effective lower bound on interest rates when drawing shocks and simulating the model.

Referring to Kilian and Manganelli (2007), Coenen and Warne specify a particular loss function for the central banker and apply the Kilian-Manganelli concept of balance of risks (BR) when reporting the development of inflation risk through time.

My main comment to the paper is that the BR concept might seem more useful as a tool for the design of policy than as a device to communicate inflation risk. Coenen and Warne’s own assessment of appropriate forward guidance for monetary policy provides a nice example. But given this perspective, one might want to include more arguments than just inflation in the loss function. With only inflation in the loss function, I suggest a different interpretation of the loss function—it could be seen as an expression of a concern for the nominal anchor.
2. The Balance of Risks and Optimal Monetary Policy

Kilian and Manganelli (2008)\(^1\) show that the risk-management approach to monetary policy coincides with the more traditional loss-function approach (expected utility model) given the definitions of deflation risk (DR) and excessive inflation risk (EIR) that Coenen and Warne apply. Furthermore, the first-order and second-order stochastic dominance definitions (FOSD and SOSD) that Kilian and Manganelli (2003) establish\(^2\) relating to the central banker’s problem, and the associated proposition regarding preference rankings of inflation distributions, apply.\(^3\) In particular, all distributions that can be ranked according to Kilian and Manganelli’s (KM’s) FOSD criterion will have the same ranking, regardless of the central bank’s preferences (that is, for any \(\alpha, \beta\)), given some critical value for inflation \(U\) for which dislike for inflation kicks in (\(\pi\) in KM’s notation), and the critical value \(L\) (or \(\pi\)) for which dislike for low inflation or deflation kicks in. The FOSD criterion expresses that distributions that have both less mass below \(L\) and less mass above \(U\)—that is, mass is more centered within the target range—will be preferred.

Furthermore, densities that can be ranked according to SOSD will imply the same preference ranking for any \(\alpha, \beta\) such that the central banker is weakly risk averse (that is, for any \(\alpha, \beta \geq 1\)), given some range for inflation within which loss equals zero. The SOSD criterion expresses that distributions for which more of the mass in the tails are (at least weakly) closer to both \(L\) and \(U\), respectively, are preferred.

The BR measure is intended to summarize the central banker’s attitude towards risk and the underlying density forecast in a simple number. It adds information, since FOSD and SOSD provide far-from-complete rankings of distributions. Given the FOSD and SOSD results, though, we know that different BR numbers, scaled up or down by different \(\alpha\)’s and \(\beta\)’s, reflecting different calibrations

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\(^1\)See definition 4 in Kilian and Manganelli (2008). See also section 2.2 in Kilian and Manganelli (2003).

\(^2\)The standard first-order and second-order stochastic dominance (FOSD and SOSD) results are not directly applicable, because—unlike in the risky asset problem—a higher mean and stronger skewness to the right is not always preferred. Rather, in the central banker’s problem, utility is higher when the density is more centered around the target value or within the target interval.

\(^3\)See section 2.3 in Kilian and Manganelli (2003).
of the DR/EIR measures, may represent the same ranking of inflation distributions by the central banker.\footnote{FOSD and SOSD do not offer complete rankings of distributions. Hart (2011) establishes two new stochastic orders, which provide complete rankings of distributions. These measures apply to risky assets, like traditional stochastic dominance does. An extension of those stochastic orders to the central banker’s problem might be worth exploring in order to provide a more objective index measure of inflation and deflation risk.} Moreover, the same BR number may reflect different distributions of inflation, if the number is calculated using different $\alpha$’s and $\beta$’s. Hence, in order to interpret the BR, it seems necessary to know either the central banker’s preferences or the underlying density forecast. Or, like Coenen and Warne, one may solve this by highlighting the time-series properties of the balance of risks, rather than the absolute level.

Information about the time path of the BR will indicate the degree to which the central bank implements optimal policy: In Kilian and Manganelli (2003)\footnote{See section 2.4. See also Kilian and Manganelli (2008).} it is shown that the first-order condition for the central bank’s optimal policy problem is $BR = 0$, given the loss function that Coenen and Warne apply. This suggests that the BR measure may be more useful as a tool for the central banker when designing policy than it is for the public to draw conclusions about inflation risk in a more objective sense. In fact, the BR that Coenen and Warne apply is an example of Giannoni and Woodford (2012)’s target criterion\footnote{See also Giannoni and Woodford (2003).}: the target criterion is precisely the first-order condition for minimization of the central banker’s loss function. Like Kilian and Manganelli (2008), Giannoni and Woodford (2012) highlight that a nice property of the target criterion is that it may give guidance to monetary policy that is robust to model misspecification. But for the BR to provide guidance for stabilization aspects of monetary policy, it would seem natural to include more arguments than just inflation in the loss function.\footnote{As shown by Kilian and Manganelli (2008), the risk-management approach may more generally be applied to the problem of choosing among feasible joint distributions of inflation and other goal variables, e.g., unemployment. The same is the case with the target criterion approach. The monetary policy decision problem then concerns the choice among those joint distributions, where the distributions are conditional on monetary policy. Determinacy has to be established when implementing policy with this approach, as discussed by Giannoni and Woodford (2012).}
3. The Loss Function with Only One Argument: A Concern about the Nominal Anchor

The loss function that Coenen and Warne apply could be interpreted to express concern about the nominal anchor rather than a vehicle to assess optimal stabilization policy. A very simple model may serve as an illustration. Let the aggregate supply equation be

\[ y_t = a(\pi_t - E_{t-1}\pi_t) \]  

and the demand equation be

\[ y_t = b(i_t - E_{t}\pi_{t+1}) + u_t. \]  

Let monetary policy follow a rule, where the price level is pinned down when \( \lambda > \frac{1}{8} \):

\[ i_t = \lambda\pi_t + \lambda_u u_t + \lambda_{u-1}u_{t-1}. \]  

The solution to the model is

\[ y_t = \frac{a(1 + b\lambda_u + b\lambda_{u-1}/\lambda)}{(a - b\lambda)} \cdot u_t, \]  

\[ \pi_t = \left[\frac{(1 + b\lambda_u + b\lambda_{u-1}/\lambda)}{(a - b\lambda)}\right] \cdot u_t - \left(\frac{\lambda_{u-1}}{\lambda}\right) \cdot u_{t-1}. \]  

From equation (1), we know that inflation has to be predictable in order for output to be stabilized. From the solution for \( y_t \), we see that \( a(1 + b\lambda_u + b\lambda_{u-1}/\lambda)/(a - b\lambda) = 0 \) is required. Output may be stabilized by using \( \lambda_u \), \( \lambda \), or \( \lambda_{u-1} \):

CASE 1. STABILIZE USING \( \lambda_u \). One may let monetary policy absorb the contemporaneous shock and thereby stabilize expected inflation in order to make it predictable; \( \lambda_u = -1/b \) and \( \lambda_{u-1} = 0 \). This creates a variable nominal interest rate that absorbs the shock in equilibrium:

\[ i_t = -1/b \cdot u_t \] and \( \pi_t = 0. \]  

\(^8\lambda > 1 \) pins down the initial inflation rate \( \pi_0 \). With \( \pi_0 = \frac{P_0}{P_{-1}} \), \( P_0 \) is pinned down when \( \pi_0 \) is determined. But for stabilization of output, it is inflation going into the next period that matters, \( \pi_1 = \frac{P_1}{P_0} \). Therefore, one may close the output gap by stabilizing inflation without necessarily establishing determinacy at the same time.

\(^9\)The solution may be obtained by the method of undetermined coefficients.
Table 1. First and Second Moments of Inflation under Different Policies

<table>
<thead>
<tr>
<th>Policy</th>
<th>Stable Expected Inflation (Cases 1 and 2)</th>
<th>Inflation Rate Predictable, Nominal Rate Stable (Case 3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Nominal Anchor ($\lambda &gt; 1$)</td>
<td></td>
</tr>
<tr>
<td>$E(\pi)$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$E_t(\pi_{t+1})$</td>
<td>0</td>
<td>$1/b \cdot u_t$</td>
</tr>
<tr>
<td>$\sigma(\pi)$</td>
<td>0</td>
<td>$(1/b)^2 \sigma_u$</td>
</tr>
<tr>
<td>$\sigma_t(\pi_{t+1})$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Indeterminate Initial Price Level ($\lambda &lt; 1$)</td>
<td></td>
</tr>
<tr>
<td>$E(\pi)$</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>$E_t(\pi_{t+1})$</td>
<td>0</td>
<td>$1/b \cdot u_t$</td>
</tr>
<tr>
<td>$\sigma(\pi)$</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>$\sigma_t(\pi_{t+1})$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Case 2. Stabilize using $\lambda$.** One may alternatively respond strongly and directly to inflation in order to stabilize it: let $\lambda \rightarrow \infty$, while the other policy parameters are bounded but otherwise free to choose. This establishes the same equilibrium as above and, furthermore, provides a nominal anchor with $\lambda > 1$.

**Case 3. Stabilize using $\lambda u_{t-1}$.** Monetary authorities may make the inflation rate predictable by promising to respond to today’s shock in the next period; $\lambda_u = 0$ and $\lambda u_{t-1}/\lambda = -1/b$. The inflation rate is now variable but predictable and absorbs the shock, while the nominal interest rate is stable in equilibrium:

$$i_t = (\lambda/b) \cdot u_{t-1} - (\lambda/b) \cdot u_{t-1} = 0$$

The first and second moments for inflation under different policies summarized in table 1 illustrate that a stable conditionally expected inflation rate is neither necessary nor sufficient for a nominal anchor to be established.

Hence, in the case where the nominal anchor is secured and the central bank stabilizes output, the inflation rate may be variable but bounded. In the case where the nominal anchor is not secured,
the unconditional variance of inflation is unbounded—because we do not know where the initial price level will be. The KM/Coenen and Warne loss function reflects exactly a concern about inflation variance beyond certain limits.

4. Concluding Remarks

As KM and Coenen and Warne have stressed, one might want to provide more compact and accessible information about inflation and deflation risk than what is contained in density forecasts. But the BR concept describes deviations from optimal policy—it is a target criterion. Therefore, a search for a more objective way of providing compact information about inflation risk, which is more decoupled from preferences—perhaps inspired by Hart (2011), might be worthwhile.

More objectives than just inflation in the loss function might seem natural when applying the BR concept as a guide to the design of stabilization policy. With inflation as the only argument, the loss function—with tolerance for inflation variation within a range—instead seems to describe a concern about the existence of bounded first and second moments for the inflation rate, or a nominal anchor. It might then be useful to use density forecasts from surveys or density forecasts extracted from market data—instead of density forecasts from a model—as arguments in the loss function.\footnote{See also www.ecb.europa.eu/pub/pdf/scpwps/ecbwp1540.pdf.}

References

