

# Are Bayesian Fan Charts Useful? The Effect of Zero Lower Bound and Evaluation of Financial Stability Stress Tests\*

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We show how fan charts generated from Bayesian vector autoregression models can be useful for assessing (i) the effect of the zero-lower-bound constraint on forecasting uncertainty and (ii) the credibility of stress tests conducted to evaluate financial stability. To illustrate these issues, we use a data set for the Czech Republic and macroeconomic scenarios that are used by the Czech National Bank (CNB) in stress tests of the banking sector. Our results demonstrate how different modeling approaches to the zero lower bound affect the resulting fan charts. The pros and cons of the considered methods are discussed; ignoring the zero-lower-bound constraint represents the worst approach. Next, using our fan charts, we propose a method for evaluating whether the assumptions that are employed in the bank's stress tests regarding the macroeconomic outlook are sufficiently adverse and consistent with past cross-correlations observed in the data. We find that CNB stress tests are sufficiently conservative in this respect.

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## 1. Introduction

The recent financial crisis prompted a substantial increase in uncertainty regarding future economic developments. Nominal interest rates have reached historical lows and have converged to the zero-lower-bound constraint in many countries. Generating forecasts concerning the future economic environment has become a challenge. As a consequence, assessing and communicating forecast uncertainty has become of primary importance for inflation-targeting central banks. For these central banks, the release of macroeconomic forecasts is an integral part of the monetary policy framework, and the efficient communication of the forecast uncertainty is crucial to their credibility (Geraats 2010). Very recently, assessing macroeconomic forecast uncertainty also gained importance in the macroprudential policy area (Haldane 2009). Many central banks have become more focused on financial stability and have, consequently, begun to produce stress tests of financial sectors.<sup>1</sup> The macroeconomic scenarios that compose an important aspect of these stress tests substantially concern macroeconomic uncertainty.

While some inflation-targeting central banks describe the uncertainty relating to their forecasts verbally or with the aid of alternative scenarios (Šmídková 2005), the majority of these banks convey forecast uncertainty by publishing probability distributions known as fan charts. Fan charts were introduced by the Bank of England (BoE) in 1996 to improve communication and place greater emphasis on the risks in the inflation forecast and their direction (Britton, Fisher, and Whitley 1998). Over fifteen years later, approximately three-quarters of inflation targeters communicate their inflation forecasts with the aid of fan charts.

Inflation targeters use various methodologies to produce fan charts, from calibrations based on past forecasting errors (Czech National Bank 2008) to small-scale structural model simulations (Norges Bank 2005) and subjective assessments made by monetary policy decision makers (Britton, Fisher, and Whitley 1998).

Recently, researchers have proposed alternative methodologies. For example, Bayesian fan charts based on simulated density

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<sup>1</sup>Horváth and Vasko (2012) report that over forty central banks around the world published stress tests in 2011.

forecasts (Cogley, Morozov, and Sargent 2005) have been advanced as a potential improvement over current practice. This proposal seems especially attractive considering the recent crisis. Fan charts based on simulated density forecasts can carefully address the zero-lower-bound constraint in the nominal interest rate. As a result, they are more consistent with the nature of the forecasted variable than, for example, fan charts based on past forecast errors. This might be important for monetary policy because, according to our survey, the zero lower bound does not seem to be rigorously addressed in fan charts of interest rates published by central banks (more on this in the following section). Furthermore, density forecasts can be used to assess the probability of a particular forecasted path. In this work, to evaluate macroeconomic stress-test scenarios, we exploit the ability of density forecasts to assess the probability of particular forecasted paths.

As an empirical illustration of the issues discussed above, we use a small-scale Bayesian vector autoregression model (BVAR) to produce fan charts for the Czech economy. The Bayesian approach allows prior information to be incorporated into the system of equations, thus mitigating the problem of estimation based on a short data time span, as is the case for Czech macroeconomic data.

There is an intense discussion among policymakers concerning whether the macroeconomic scenarios in the numerous financial stability stress tests conducted during the financial crisis were sufficiently adverse. For example, many commentators contend that the European stress tests were not sufficiently stringent to evaluate the resilience of the banking sector (see, for example, Onado 2011). Borio and Drehmann (2009) argue that stress tests without sufficiently severe scenarios may produce a false sense of security. In this respect, Goodhart (2006) raises the need for an analytical framework to evaluate whether the scenarios in stress tests are sufficiently adverse. We contribute to this discussion and propose a method based on Bayesian fan charts to evaluate the extent to which the macroeconomic scenarios in stress tests are sufficiently adverse. This method draws on the interpretation of a fan chart as a probability distribution of forecasts, i.e., the scenarios should have sufficiently low probabilities of being realized. Furthermore, the plausibility of the scenarios is addressed from the perspective of their consistency with correlations that are estimated from the data. We use Czech

data in this exercise, but we use a general approach that is applicable to any organization that conducts stress tests. In this regard, our results demonstrate that macroeconomic scenarios employed in the regular stress tests of banking-sector stability conducted by the Czech National Bank (CNB) are sufficiently severe but are not fully consistent with the correlations estimated from our model.

In addition, we show how different means of addressing (or ignoring) the zero lower bound affect the resulting fan charts for the interest rate and other macroeconomic variables. We present three approaches: (i) “soft conditioning,” as in Waggoner and Zha (1999), allowing the future values of a variable to be restricted within an interval; (ii) the “non-linearity” approach,<sup>2</sup> in which the interest rate is set to zero for all draws generating negative values for this variable; and (iii) ignoring the zero lower bound. Our results indicate that (iii) delivers the least accurate overall forecasts. This finding suggests that institutions that generate fan charts should not ignore the zero lower bound if they wish to accurately communicate their forecasts and forecast uncertainty to the public.

The paper is organized as follows. Section 2 provides a brief description of the various fan chart methodologies employed by central banks and the methodologies proposed by researchers. Section 3 describes our BVAR model and discusses the basic properties of the model. Section 4 addresses the modeling of the zero lower bound employed to generate the fan charts. Section 5 illustrates how BVAR fan charts could be used to assess the stringency of the assumptions concerning the macroeconomic outlook used in stress testing. Section 6 concludes.

## 2. Inflation Targeters and Fan Charts

Since the BoE introduced fan charts in 1996, they have become widely popular among inflation-targeting central banks. This section provides a brief overview of the practices of inflation targeters in terms of publishing forecasts and the attendant fan charts. It also reviews researchers’ proposals for alternative methodologies for producing fan charts.

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<sup>2</sup>We also label this approach as the “passive monetary policy” approach further in the text.

Table 1 summarizes which forecasts are published by inflation-targeting central banks. It also summarizes whether fan charts are generated for these forecasts and whether they are made publicly available. Table 1 shows that all inflation targeters publish inflation forecasts, many of them publish GDP growth forecasts, some publish interest rate forecasts, and only the CNB publishes exchange rate forecasts. Central banks, which publish fan charts for interest rate projections, do not address the issue of the zero lower bound in a model-based framework. They either present fan charts where the interest rate can take negative values with non-zero probability or graphically trim the part of the fan chart with the negative interest rate values.

Regarding the fan charts, we survey central banks' official publications, primarily their inflation reports. According to our survey, most central banks currently publish fan charts for inflation, and quite a few publish them for GDP growth. Publishing interest rate fan charts is much less common, and again, only the CNB produces a publicly available fan chart for the exchange rate.

To illustrate the practice of inflation targeters in publishing fan charts, we select central banks in countries that score high in terms of transparency (Dincer and Eichengreen 2009): the United Kingdom, Sweden, Canada, the Czech Republic, and Hungary. In addition, we added Norway to our sample, as it publishes fan charts for inflation, GDP, and the interest rate.

The CNB fan charts are largely based on past forecast errors for inflation, GDP growth, the exchange rate, and the interest rate path from the CNB g3 model (CNB 2008). The g3 model is a dynamic stochastic general equilibrium (DSGE) model and has been used as the core forecasting tool since 2008 (Šmídková 2008). Therefore, only a few years of forecasting errors are available. As a consequence, the resulting fan chart is smoothed linearly over the individual forecast horizons. The fan charts are available for horizons covering one to seven quarters for CPI inflation, monetary-policy-relevant inflation, GDP growth, the 3M PRIBOR, and the exchange rate (see table 1). The confidence intervals for the relevant quantiles are generated using the normal distribution. To address the possibility of uncertainty changing over time, the forecast errors and fan charts are updated on a yearly basis.

**Table 1. Use of Fan Charts by Inflation-Targeting Central Banks: Are Forecasts and Fan Charts Publicly Available?**

Country	Inflation	GDP Growth	Interest Rate	Exchange Rate	Zero Lower Bound
Armenia	FC	FC	—	—	—
Australia	F	F	—	—	—
Brazil	FC	FC	—	—	—
Canada	FC	F	—	—	—
Chile	F	F	—	—	—
Colombia	FC	F	—	—	—
Czech Republic	FC	FC	FC	FC	—
Ghana	FC	F	—	—	—
Guatemala	F	—	—	—	—
Hungary	FC	FC	—	—	—
Iceland	F	—	—	—	—
Indonesia	F	F	—	—	—
Israel	FC	F	FC	—	—
Mexico	FC	FC	—	—	—
New Zealand	F	F	F	—	—
Norway	FC	FC	FC	—	—
Peru	FC	FC	—	—	—
Philippines	FC	—	—	—	—
Poland	FC	FC	—	—	—
Romania	F	F	—	—	—
Serbia	FC	—	—	—	—
South Africa	FC	FC	—	—	—
South Korea	FC	FC	—	—	—
Sweden	FC	FC	FC	—	—
Thailand	FC	FC	—	—	—
Turkey	FC	F	—	—	—
United Kingdom	FC	FC	—	—	—

**Notes:** The table indicates whether forecasts and fan charts for inflation, GDP growth (output gap), the interest rate, and the exchange rate are publicly available. FC denotes that both the forecast and the fan chart are available, F denotes that only the forecast is available, and — denotes that neither the forecast nor the fan chart is available. As for the zero-lower-bound constraint, + denotes that the constraint is addressed in a model-based approach, and — denotes that it is not. Information as of July 2011. Hammond (2011) is the source of the data on which macroeconomic variables in the forecasts are publicly available; the remaining data (fan charts and zero lower bound) were collected by the authors.

The Swedish Riksbank produces fan charts in a manner similar to that applied by the CNB. The uncertainty in the inflation and GDP forecasts is related to past forecast errors. For the interest rate (repo rate) forecast, the uncertainty reflects the past forecast errors for implied forward rates, which are adjusted to take into account the corresponding risk premium (Sveriges Riksbank 2007).

The BoE employs a very different approach. The BoE's fan charts for inflation and output are based on a subjective assessment of the overall uncertainty outlook and the directions of the risks to the forecast as perceived by the members of the Monetary Policy Committee (Britton, Fisher, and Whitley 1998). The BoE introduced the two-piece normal distribution to represent uncertainty to allow for asymmetry. The three moments of this distribution (mode, standard deviation, and skewness) are determined by a combination of model-based and expert-judgment methods. The mode represents the single most likely outcome based on current knowledge and judgment. Uncertainties relating to input variables are aggregated into fan charts for inflation and GDP growth. Experts and policymakers focus on discussing the balance of risks, which is defined as the difference between the mean and the mode. The final fan chart is created using a "top-down" approach in which policymakers make the final decision. Clements (2004) and Gneiting and Ranjan (2011) evaluate BoE fan charts based on probability integral transform and threshold- and quantile-weighted proper scoring rules, respectively.

The National Bank of Hungary also generates its fan charts for inflation and GDP growth using the two-piece normal distribution (Magyar Nemzeti Bank 2004). The mode is represented by the baseline forecast as the most probable scenario. The standard deviation is calculated from past forecast errors. The skewness is determined by experts according to the alternative scenarios considered when the baseline forecast is generated.

The Norwegian central bank initially produced model-based fan charts with an emphasis on historical uncertainty (Norges Bank 2005). The uncertainties relating to important variables are assessed on the basis of past performance. Next, they are combined with the aid of a small macroeconomic model that has been calibrated to obtain the desired aggregate system properties (Husebo et al. 2004). The performance of the fan charts has been evaluated several times. For example, in 2007, the fan charts were subjected to informal tests.

These tests demonstrated that a significant portion of the observed inflation data points were located outside the fan chart (Norges Bank 2007), which raised the question of how the model should be calibrated to obtain the appropriate fan chart width. Currently, Norges Bank uses a medium-scale DSGE model called NEMO for its fan charts, which are based in part on the model's density forecast and in part on judgment (Gerdrup and Nicolaisen 2011).

The Bank of Canada also produces model-based fan charts to communicate its inflation forecasts (Bank of Canada 2009). The Canadian fan charts are based on stochastic simulations of a core forecasting model called ToTEM (Murchison and Rennison 2006). For the first two quarters, the fan charts combine these stochastic simulations with past staff forecast errors to reflect the fact that the short-term forecasts are produced by experts, not by the core model. The fan charts also incorporate the uncertainty relating to external variables, as captured by a complementary model.

In summary, inflation targeters employ two main approaches to generate fan charts. First, some central banks, such as the CNB, rely on past forecast errors, assuming a normal distribution of risks and generating the central path using their core forecasting model. Second, other central banks, such as the BoE, subjectively assess the uncertainty of future economic developments with the aid of a two-piece normal distribution, assuming that the central forecast represents the mode of this distribution. In the first case, the fan charts are used solely as communication tools to inform the general public about the uncertainty relating to the forecast. In the second case, the fan charts have two roles. Internally, their preparation facilitates discussion among policymakers and experts concerning the distribution of forecast risks. Externally, fan charts both inform the general public about uncertainty (standard deviation) and send a potential signal regarding future policy moves (skewness).

Researchers have proposed various modifications to these current practices. Some researchers propose modifications to the fan charts based on two-piece normal distributions. Some claim that the aggregation of input variables should be modified to consider joint distributions and avoid the omission of such important information as trade-offs between forecasts (Pinheiro and Esteves 2012). Others contend that it is important to aggregate the density forecasts of individual policymakers into a single forecast (Osterholm 2009) or

argue that incorporating market information is important (Elekdag and Kannan 2009).

An important stream of literature focuses on BVAR models, which could be used to produce fan charts similar to those produced using expert judgment, but they would be model based. The first proposal in this area illustrates how to produce time-varying BVAR fan charts for the BoE (Cogley, Morozov, and Sargent 2005). More recently, a large structural BVAR, benefiting from a more than twenty-year-long data series for Sweden, was advanced to formalize forecast uncertainty (Osterholm 2008) and produce model-based fan charts. Using a large BVAR model, Giannone et al. (2010) produce fan charts for the euro-area inflation rate that explicitly address the interactions in the developments of the components underlying the aggregate price index.

### 3. Model for Fan Charts

We consider a vector autoregression model of order  $p$ :

$$y_t = C + \sum_{j=1}^p A_j y_{t-j} + \varepsilon_t, \quad (1)$$

where  $y_t$  for  $t = 1, \dots, T$  is an  $M \times 1$  vector of endogenous variables,  $C$  denotes an  $M \times 1$  vector of intercepts,  $A_j$  is an  $M \times M$  matrix of autoregressive (AR) coefficients, and  $\varepsilon_t$  is an  $M \times M$  vector of innovations. We assume that the innovations are i.i.d.  $N(0, \Sigma)$ .

The model is estimated using four endogenous variables ordered in the following way: real output (the year-on-year growth rate), monetary-policy-relevant inflation (the year-on-year growth rate of the CPI adjusted for the first-round effect of indirect taxes), the short-term interest rate (3M PRIBOR), and the CZK/EUR exchange rate (year-on-year growth rate). The growth rate of the exchange rate is chosen to account for the appreciating trend exhibited by the Czech koruna (see Babecký, Bulíř, and Šmídková 2010 for a discussion of why many Central European currencies exhibit appreciating trends). Furthermore, to account for changes in the inflation target during the sample period, the distance of monetary-policy-relevant inflation from the target is employed rather than the simple level of monetary-policy-relevant inflation. The data are

seasonally adjusted and cover the 1998:Q1–2012:Q2 period.<sup>3</sup> This period coincides with the period during which inflation targeting was applied in the Czech Republic.

We use the Bayesian approach to estimate the model in equation (1). We employ the normal-diffuse prior, allowing us to shrink the coefficients at lags of the dependent variables differently from lags of the other variables. This type of prior requires Gibbs sampling based on conditional distributions for AR coefficients and the error covariance matrix. The formulas for the conditional distributions can be found in Kadiyala and Karlsson (1997, table 1).

The prior mean of the vector of autoregressive coefficients is set to

$$A_1^{PR} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } A_j^{PR} = 0_M, j = 2, \dots, p.$$

The prior mean for the first lag of inflation (0.5) reflects the observed persistence of inflation (Franta, Saxa, and Šmídková 2010), and the prior mean for the first lag of the nominal interest rate (0.9) relates to our belief that the interest rate has followed a highly persistent process, as inflation targeting has been introduced as a disinflation strategy (Horváth and Mateju 2011). The specification of prior variance in the vector of AR coefficients and the values of hyperparameters are taken from Canova (2007).

For the estimation, we use five lags of the endogenous variables ( $p = 5$ ). The number of lags is determined with respect to the autocorrelation of the residuals.<sup>4</sup> The autocorrelation of the model residuals implied by the respective draws of model coefficients can be found in the online appendix.<sup>5</sup>

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<sup>3</sup>Note that output growth is published with a lag of one quarter relative to the other endogenous variables. To estimate the model based on the most recent quarter, we use the estimate of output growth for 2012:Q2, as used by the core CNB forecasting model.

<sup>4</sup>Note that Litterman (1986, pp. 27–28) suggests using the maximum number of lags feasible. Five lags could be viewed as the maximum feasible number, given the small data sample.

<sup>5</sup>The online appendix is available at <http://ies.fsv.cuni.cz/en/staff/horvath>.

To simulate the posterior distributions of the model parameters, 10,000 draws as provided by the Gibbs sampler are used.<sup>6</sup> A burn-in period of 10,000 draws is employed. The post-estimation diagnostics and the convergence diagnostics are presented in the online appendix.

In the next step, we use the BVAR model to generate density forecasts for GDP growth, monetary-policy-relevant inflation (MPR inflation), the 3M PRIBOR, and the exchange rate for horizons covering one to seven quarters  $h = 1, 2, \dots, 7$ .

To generate a density forecast, we use the sample from the posterior distribution of the model parameters obtained during the estimation procedure. For each draw, an iterative forecast up to seven quarters ahead is computed for all endogenous variables using draws from the distribution assumed for disturbances  $\varepsilon_t$ . The simulated density forecast is then described by the percentiles of the distribution constituted by the forecasts and is denoted as a fan chart.

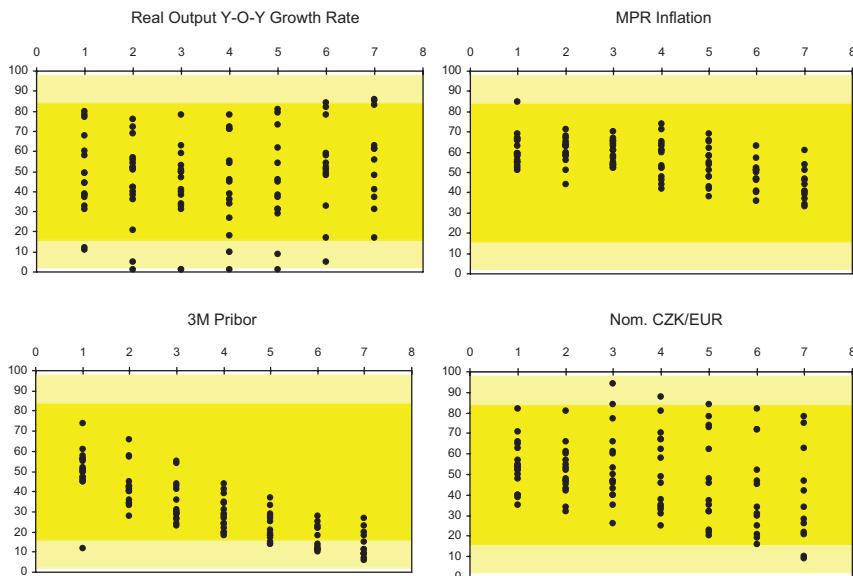
The forecasting performance of fan charts can be formally assessed by testing whether the ex post observed time series are drawn from the simulated distribution forecast (see Diebold, Tay, and Wallis 1999 and Elder 2005). However, the small sample size implies low test power. For example, only eighteen observations are available for a one-quarter forecast horizon for fan charts beginning in 2008:Q1. The small-sample problem is exacerbated because even fewer observations are available for longer forecasting horizons. Therefore, formal statistical density tests are not conducted in this study. However, an informal assessment of the BVAR fan charts is presented in figure 1.<sup>7</sup> The four panels show ex post observed values at a particular horizon (x-axis) together with the respective value of the distribution forecast (y-axis). The density forecast evaluation

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<sup>6</sup>When some draws are discarded (e.g., in the zero-lower-bound exercise), the number of draws is such that the number of accepted draws equals 10,000. For the application of BVAR fan charts to evaluate the macroeconomic scenarios, a sample of 50,000 draws is used to ensure that the accurate path forecast that is “close” to a scenario is obtained.

<sup>7</sup>The Bank of England (2005) conducts the same informal analysis for their fan charts, where it is also noted that even this informal analysis can suffer from the serial correlation of the relative positions of the distribution forecasts and ex post realized values, as two adjacent fan charts have nearly identical observed values. Data points concentrated in some areas can then result from serial correlation.

**Figure 1. Distribution Forecasts and Ex Post Observed Values at a Particular Horizon (x-axis)**



**Notes:** The dark belt indicates the centered 68 percent of a distribution forecast; the light color indicates the centered 98 percent of a distribution forecast. The density forecast evaluation period is from 2008:Q1 to 2012:Q3.

period starts in 2008:Q1. It can be observed that the centered 68 percent of the distribution forecasts (dark color) includes the vast majority of the observed data points. Moreover, with a few exceptions of real output growth related to the unprecedented decline of output during the recent economic crisis in 2008/2009,<sup>8</sup> all observed data are covered by the centered 98 percent of distribution forecasts (light color).

#### 4. Zero Lower Bound

As noted in section 2, central banks do not rigorously address the zero lower bound on the nominal interest rate when constructing fan

<sup>8</sup>A decline from pre-crisis growth of approximately 5 percent annually to about -4 percent in the first year of the crisis was observed.

charts. As discussed earlier, Bayesian fan charts represent a natural framework for accounting for the zero lower bound.

There are several possible approaches to the problem of density forecasting at the zero lower bound. First, individual forecasts constituting the sample from the density forecast can be conditioned on the shocks that lead to the non-negative nominal interest rate. Second, forecasts can be conditioned on the interest rate itself regardless of the shocks affecting the economy. Lastly, another potential approach would be to ignore the zero lower bound completely. We discuss these three approaches in detail.

In the first approach, the zero lower bound is implemented by means of “soft conditioning,” as introduced in Waggoner and Zha (1999). “Soft conditioning” allows the future values of a variable to be restricted to a given interval. Essentially, Waggoner and Zha (1999) suggest discarding the forecasts (i.e., the draws from the conditional posterior distributions of parameters  $A_j^{Draw}$  and reduced-form shocks  $\varepsilon_{t+h}^{Draw}$ ) that yield negative values of the nominal interest rate for at least one forecasting horizon. The resulting density forecasts thus exclude the negative values of interest rate and the corresponding values of the other variables.

The procedure of discarding some draws of reduced-form shocks affects as a consequence structural shocks that are included in the simulated density forecast.<sup>9</sup> Clearly, an expansionary monetary policy shock, which implies a decrease in the interest rate, is discarded at the zero lower bound. However, an expansionary monetary policy shock does not have to be the only type of structural shock that is discarded from the simulated density forecast. For example, a negative demand shock that, *ceteris paribus*, leads to a decline in output, inflation, and the interest rate can also be discarded. A negative demand shock is included only when a corresponding reduced-form residual represents a combination of the demand shock and another structural shock that positively affects the interest rate to overbalance the negative effect of the demand shock.

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<sup>9</sup>There are many possible vectors of structural shocks that correspond to a given vector of reduced-form residuals. A unique link is constituted by an assumed identification scheme. Note that we do not need to assume any particular identification, as density forecasting is based on the model in its reduced form. However, assuming some form of identification scheme is useful if one wants to interpret which structural shocks affect the shape of a fan chart.

The interpretation of the “soft-conditioning” approach can be based on the composition of structural shocks in the density forecast. Discarding the shocks that negatively affect the interest rate is equivalent to adding shocks that positively affect the interest rate. These additional “compensating” structural shocks can be interpreted as policy shocks related to the zero lower bound. For instance, depreciation exchange rate shocks that compensate for an interest rate effect of the negative demand shock can be interpreted as foreign exchange interventions by a central bank at the zero lower bound. Similarly, a positive demand shock that balances a decline in the interest rate can capture a fiscal stimulus.

Next, the second approach imposes values of zero on all negative parts of the interest rate paths throughout the sampling density forecasts. More specifically, for each forecast, the zero is imposed whenever the interest rate becomes negative. As forecasts are computed iteratively, subsequent horizons of the forecast are then computed with the imposed zero. In contrast to “soft conditioning,” the second approach uses *all* draws of reduced-form shocks to construct the density forecast. In addition, it restricts the effects of shocks on the interest rate because of the zero lower bound and, in turn, on all other variables.<sup>10</sup>

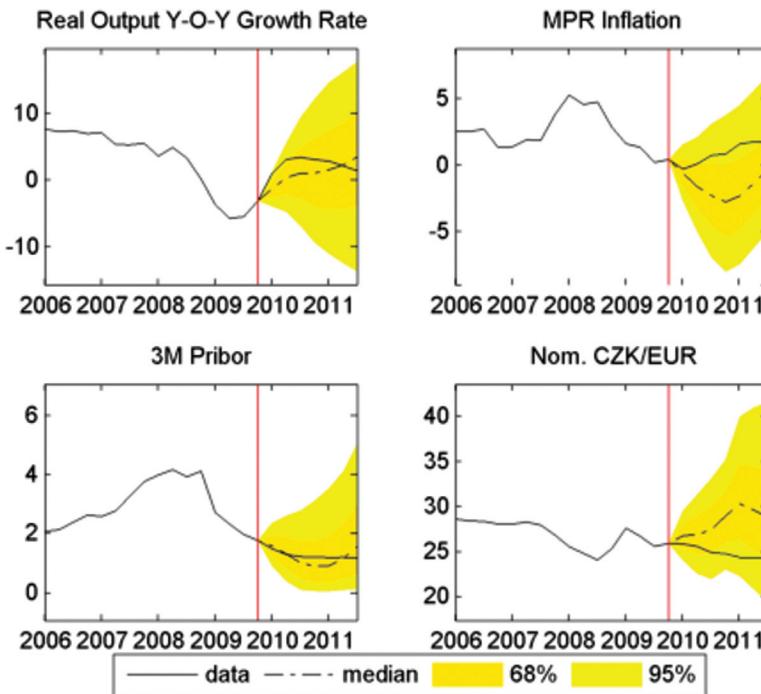
This approach could be denoted as “passive monetary policy.” Monetary policy is passive at the zero lower bound and does not react to shocks that negatively affect the interest rate. The resulting density forecasts are very close to the case when the zero lower bound is accounted for by graphically trimming the part of the fan chart for the interest rate that is below zero. The only difference is that imposing zeros for the interest rate also affects other variables, which is not the case when the area of negative interest rates is simply ignored. This second approach could also be viewed as directly imposing non-linearity on the interest rate rule, i.e., the zero lower bound is not introduced by changes in the distributions of shocks but rather by the change in the interest rate rule.

Lastly, in the third approach, which ignores the zero lower bound, density forecasts are constructed from *all* draws of the reduced-form

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<sup>10</sup>Clearly, not only the shocks but also other variables may drive the interest rate below zero and, in such a case, the restriction would also apply to the effect of other variables on the interest rate.

**Figure 2. Fan Charts Beginning in 2010:Q1, with the Zero Lower Bound Imposed by “Soft Conditioning”**

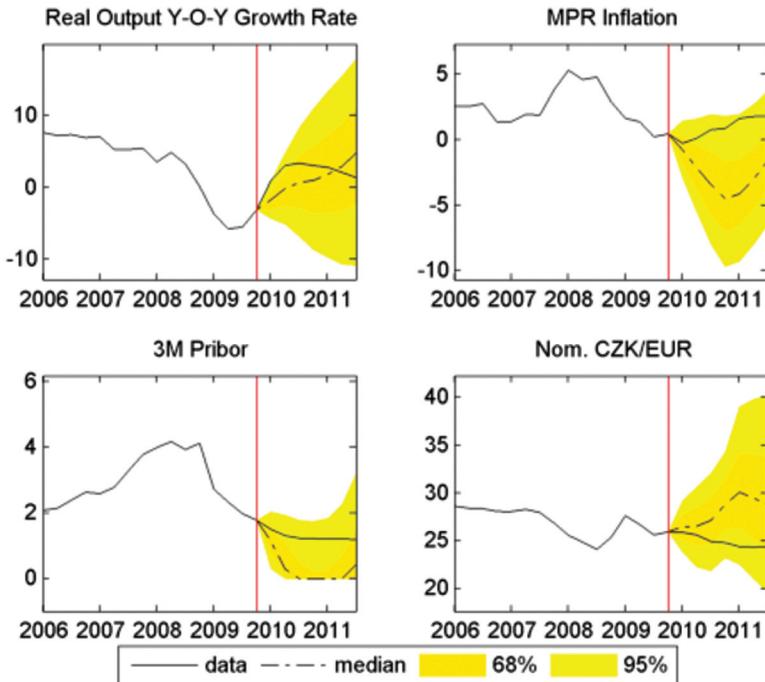


**Notes:** The vertical line denotes the period of the last observed values used in the estimation. The density forecasts are characterized by the centered 95 percent (light color), the centered 68 percent (dark color), and the median of the marginalized joint distribution.

residuals and without *any* restriction on the effects of such shocks. As a consequence, density forecasts incorporate shocks that cannot appear at the zero lower bound (e.g., an expansionary monetary policy shock manifested by an unexpected decrease in the interest rate) and do not exclude negative values of the nominal interest rate.

Figures 2, 3, and 4 present the changes in the fan charts due to the application of the three approaches to the zero lower bound on the nominal interest rate. The focus is on the case of forecasts starting in 2010:Q1. Clearly, in an environment of low interest rates, the forecasts ignoring the zero lower bound completely deliver the

**Figure 3. Fan Charts Beginning in 2010:Q1, with the Zero Lower Bound Directly Imposed on the Nominal Interest Rate**

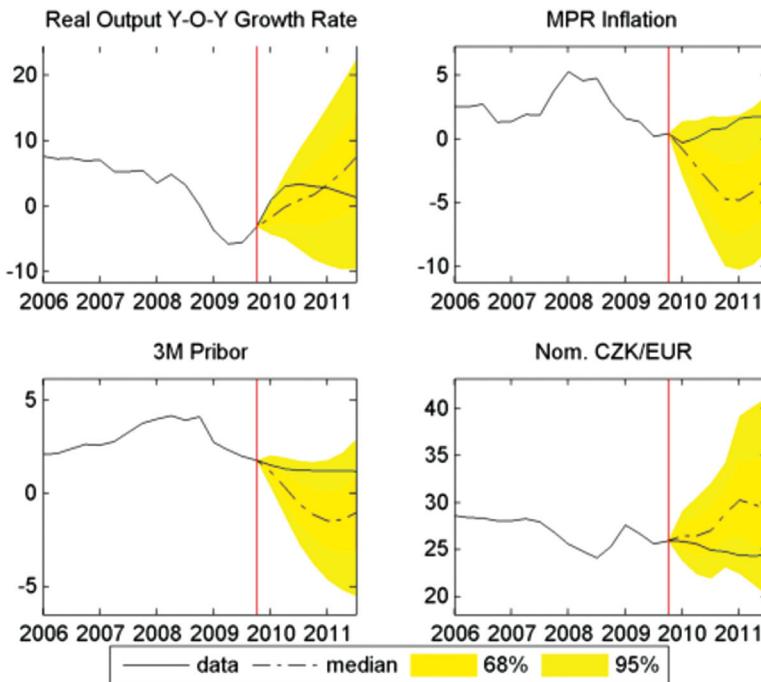


**Notes:** The vertical line denotes the period of the last observed values used in the estimation. The density forecasts are characterized by the centered 95 percent (light color), the centered 68 percent (dark color), and the median of the marginalized joint distribution.

least accurate results (see figure 4). For the “passive monetary policy” approach (figure 3), the median of interest rate is forecasted at zero for several periods. The “soft-conditioning” approach (figure 2) seems to produce forecasts that are the closest to the ex post observed values. Obviously, the changes in the interest rate fan charts are a straightforward consequence of the method employed.

A direct comparison of the “soft-conditioning” approach and the approach that ignores the zero-lower-bound constraint is available in figure 5. Ignoring the zero-lower-bound constraint leads to a greater probability of deflation and an overoptimistic outlook regarding output. Moreover, failing to account for the zero lower bound leads to ex

**Figure 4. Fan Charts Beginning in 2010:Q1, with No Zero Lower Bound Imposed**

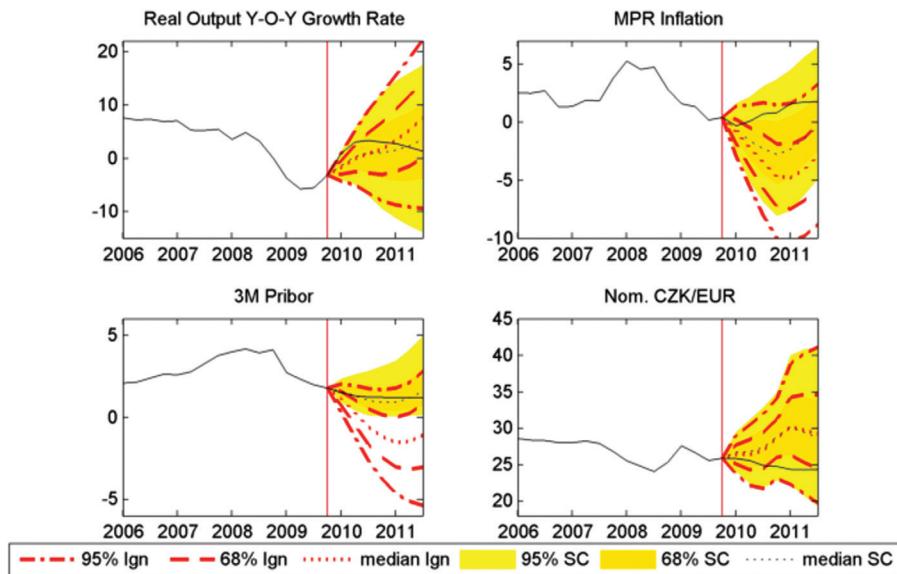


**Notes:** The vertical line denotes the period of the last observed values used in the estimation. The density forecasts are characterized by the centered 95 percent (light color), the centered 68 percent (dark color), and the median of the marginalized joint distribution.

post observed MPR inflation on the edge of the centered 95 percent of the respective fan chart.

The “passive monetary policy” approach (figure 3) yields fan charts covering slightly stronger currency relative to the approach of ignoring the zero lower bound (figure 4). This is a consequence of ignoring the effects of negative interest rates. A lower interest rate implies currency depreciation; thus, imposing zeros rather than negative values corresponds to currency appreciation. This is an inherent feature of the two approaches to the zero lower bound. However, for the “soft-conditioning” approach (figure 2), fan charts

**Figure 5. A Comparison of Fan Charts Imposing the Zero Lower Bound by “Soft Conditioning” (SC) and by Ignoring the Bound (Ign)**



**Notes:** The vertical line denotes the period of the last observed values used in the estimation. The density forecasts are characterized by the centered 95 percent, the centered 68 percent, and the median of the marginalized joint distribution.

for the exchange rate cover more depreciated currency than fan charts produced using the “passive monetary policy” approach. This greater currency depreciation results from the composition and size of shocks implied by data, not the methodology of accounting for the zero lower bound.

To summarize, intuitively, the “soft-conditioning” approach excludes too many structural shocks from the resulting fan chart. However, the “passive monetary policy” approach includes some effects of shocks that cannot appear at the zero lower bound.<sup>11</sup> Clearly, the third approach cannot address the zero lower bound. Forecasting performance exercises can serve as some guidance in

<sup>11</sup>For example, expansionary monetary policy shock at the zero lower bound can affect the exchange rate.

deciding which approach to use (see, for example, Mitchell and Hall 2005 for a methodology for comparing density forecasts based on the Kullback-Leibler information criterion, or KLIC).<sup>12</sup>

From the economic perspective, the most appropriate strategy is to identify structural shocks accurately and accept only those draws of reduced-form residuals that are related to monetary policy shocks that do not lead the interest rate to decrease below zero. For other types of structural shocks, which imply a falling interest rate, zero would be imposed on the interest rate, as in the second approach. The first two approaches—“soft conditioning” and “passive monetary policy”—in a sense, represent the two extremes of this approach for accounting for the zero lower bound. However, it is important to realize that combining these two approaches would only yield credible results if structural shocks are accurately identified in the model.

Furthermore, it is also important to account for how frequently a central bank must address the zero-lower-bound constraint. For example, in our case, we consider density forecasts for up to seven quarters, beginning with quarters in the 2008:Q1–2012:Q3 period. Only five of nineteen quarters are characterized by a ratio of accepted draws less than 0.5 (if “soft conditioning” is applied). The lowest value of the ratio is 0.05. Over half the draws are used to construct the forecast density in the majority of cases. It is therefore highly important to account for the zero lower bound in a few periods, but it is not important to do so in the majority of the periods. The following text considers fan charts with an imposed zero lower bound using the first approach (Waggoner and Zha 1999).

## 5. Bayesian Fan Charts and Stress Testing in the Banking Sector

The recent financial crisis reminded us of the important role that banks play in the economy. The stability of the banking sector is regularly evaluated by many central banks, international organizations, and the private sector. The stress-test scenarios need to be

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<sup>12</sup>The comparison is not conducted because of the small data sample. The main aim here is to illustrate and discuss various approaches to the zero lower bound.

sufficiently adverse to provide a true test of stability in the banking sector (Goodhart 2006, Alfaro and Drehmann 2009, or Borio and Drehmann 2009). Moreover, it has been argued that a proper analytical framework needs “to assess the probability, virulence and speed of occurrence of potential shocks” (Goodhart 2006). We take a step in this direction and illustrate how the adverseness and internal consistency of a macroeconomic stress-test scenario can be evaluated through BVAR fan charts. As an illustration, we consider the macroeconomic scenarios that are used by the CNB when testing the stability of the financial sector.<sup>13</sup>

### *5.1 How Adverse Are Macroeconomic Scenarios?*

To assess the aduerseness of a macroeconomic scenario, we draw on the fact that fan charts can be interpreted as the density of path forecasts. Therefore, any particular forecasted path can be assessed in terms of the percentiles of the simulated density forecast. A related stream of the literature proposes a formal evaluation of the severity of stress tests based on the risk-factor distribution (Breuer et al. 2009 or Breuer et al. 2012).

Alternative scenarios for the macroeconomic outlook serve as an input to the CNB’s banking-sector stress-test framework. All four endogenous variables of the BVAR model are included in the stress-test scenarios. We take the first seven quarters of the macroeconomic scenarios to ensure that they are comparable with our fan charts.

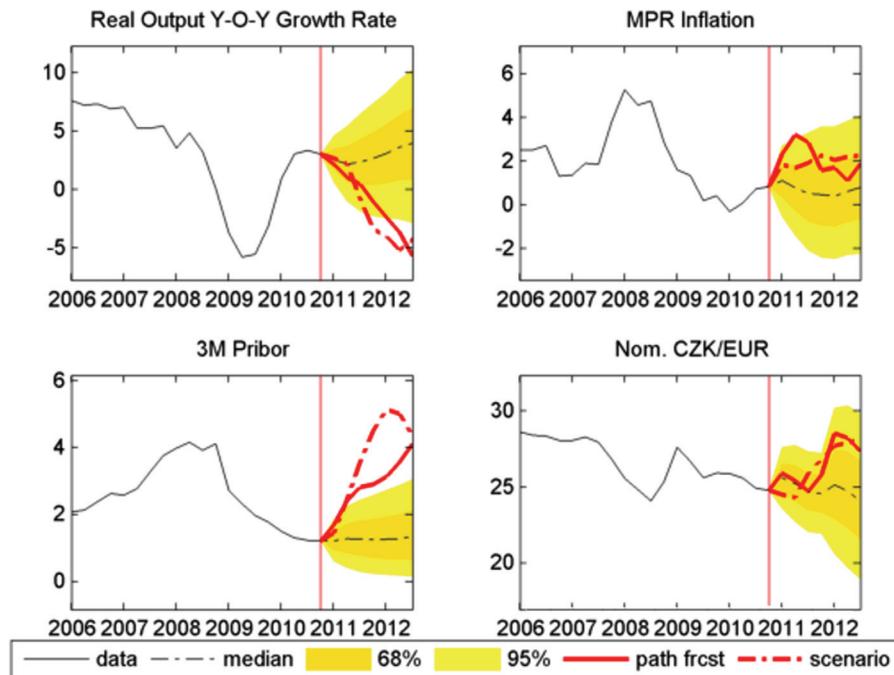
The problem of assessing the probability of a macroeconomic scenario is addressed as follows. The scenario does not have to necessarily constitute a path forecast. We therefore choose the path forecast that is the minimum distance from the scenario. Then, the probability assessment of this “nearest” path forecast can be conducted in several ways. We present the probabilities of each forecasting horizon and variable separately and discuss the probabilities of the scenarios based on the joint density of the path forecasts.

The construction of the minimum-distance path forecast follows Fry and Pagan (2005), who perform a similar exercise for the distance between the set of impulse responses identified by sign

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<sup>13</sup>The macroeconomic scenarios for the financial-sector stress tests can be found at [http://www.cnb.cz/en/financial\\_stability/stress\\_testing/index.html](http://www.cnb.cz/en/financial_stability/stress_testing/index.html).

**Figure 6. The “Unexpected Recession” Scenario and the Minimum-Distance Path Forecast**



**Notes:** The vertical line denotes the period of the last observed values that are used in the estimation. The density forecasts are characterized by the centered 95 percent (light color), the centered 68 percent (dark color), and the median of the marginalized joint distribution.

restrictions and their median. First, the set of path forecasts generated by the BVAR model is normalized to be unit free, i.e., for each endogenous variable and forecasting horizon, the values of the scenario are subtracted, and the result is divided by the standard error of the forecasted paths. Then, the forecasted path characterized by the lowest normalized distance over the whole forecasting horizon and over all four endogenous variables is drawn.

Figure 6 presents the BVAR fan charts, the stress-test scenario, and the minimum-distance path forecasts for the macroeconomic scenario from the CNB's stress test of February 2011. The scenario that is denoted the “Unexpected Recession” scenario is characterized by a significant decline in economic activity in the second half of

2011, which is caused by an external shock, exchange rate depreciation, and, in reaction to inflation pressures, a marked increase in the monetary policy rate, reflected in a rise in short-term interest rates. The scenario is based on the data set prior to 2010:Q4, and we thus use a BVAR model with an initial forecasting period of 2011:Q1.

Figure 6 shows that the “Unexpected Recession” scenario indeed represents extreme stress for the banking sector. First, the scenario and, to some extent, the “nearest” path forecast are not included in the centered 95 percent of the density forecast for nearly all horizons considered for the interest rate and real output growth. Therefore, the “Unexpected Recession” scenario is likely to represent a sufficiently adverse scenario. Second, the “nearest” path forecast significantly diverges from the scenario, especially at certain interest rate horizons, i.e., the scenario does not seem to be consistent with the model and the estimated shock variances. Even a BVAR model with a very unlikely set of coefficients and an unprecedented set of shocks (i.e., a path forecast consistent with a very low posterior probability of the parameter vector and a low-probability set of draws from the error covariance matrix) does not yield a path forecast that is close to the scenario. Therefore, the plausibility of the scenario could be questioned and is discussed in the next sub-section.

The overall probabilistic assessment of scenarios that are stated in terms of a simultaneous outlook for all variables must be based on the joint forecasting density. Note that the set of path forecasts constitutes a sample from the joint distribution over all variables and forecasting horizons. The various marginal distributions that suggest scenario likelihoods can be computed easily. Table 2 reports the value of the marginalized path forecasts distribution (for the particular horizon and variable) for the minimum-distance path forecast. The value is approximated by the normalized number of path forecasts with the value of a particular variable at a given horizon lower than or equal to the minimum-distance path forecast. For example, the probability that output growth at the horizon of a quarter is less than or equal to the path forecasts closest to the scenario is 0.34. Next, the probability of the minimum-distance path forecast for the interest rate exceeds 0.9 for all horizons.

Approximations of the marginal distributions for various subsets of variables and horizons can be similarly computed. Therefore, for example, the probability that the path forecast of output growth

**Table 2.** Marginalized Distribution of Path Forecasts

Horizon	“Unexpected Recession” Scenario			
	Output Growth	MPR Inflation	Interest Rate	Exchange Rate
1	0.34	0.94	0.93	0.62
2	0.24	0.99	0.99	0.56
3	0.17	0.96	1.00	0.51
4	0.06	0.78	0.99	0.83
5	0.03	0.81	0.99	0.98
6	0.01	0.63	1.00	0.99
7	0.01	0.77	1.00	0.99

**Note:** The table presents the probability that the forecasted variable is below the stress-test scenario path, approximated by the “nearest” BVAR path forecast.

and inflation is below the path forecast at the minimum distance from the scenario for the first two quarters is 0.16. The probability of the path forecast of output growth and inflation being above this forecast path is 0.004.<sup>14</sup>

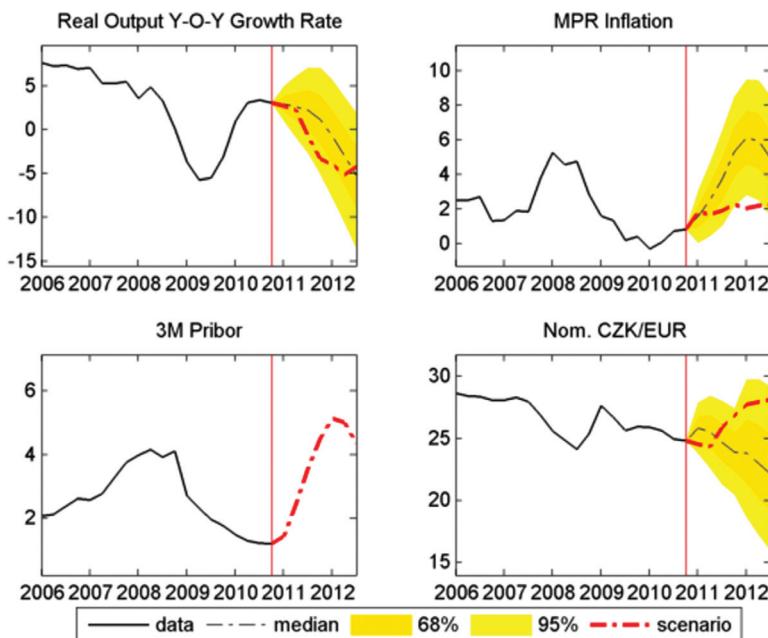
## 5.2 How Consistent Are Macroeconomic Scenarios?

The consistency of a scenario is examined in the following manner: provided that a variable achieves particular values in the future, how will uncertainty related to the forecasts of the other variables change? The consistency check draws on “hard conditioning” from Waggoner and Zha (1999). Waggoner and Zha (1999) provide a Gibbs sampling algorithm that incorporates the “hard condition” expressed in terms of the prescribed values for certain endogenous variables. In our case, the condition contains the values of the interest rate prescribed by the “Unexpected Recession” scenario.

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<sup>14</sup>In this manner, we can assess the probabilities of all possible combinations of horizons and variables. The low probability of both path forecasts being either above or below the scenario from the example is due to the high uncertainty of the future development reflected in the high shock variance estimates. There is, therefore, a high probability that the path forecast switches from below to above the scenario (and vice versa).

**Figure 7. The Consistency of the “Unexpected Recession” Scenario**



**Notes:** The vertical line denotes the period of the last observed values used in the estimation. The density forecasts are characterized by the centered 95 percent (light color), the centered 68 percent (dark color), and the median of the marginalized joint distribution.

Figure 7 suggests that if the interest rate outlook follows the scenario, the scenario's probability with respect to output growth, MPR inflation, and the nominal exchange rate is neither low nor high—the path described by the scenario lies almost completely within the centered 95 percent of the conditional distribution forecast; it frequently remains, however, outside the centered 68 percent of the conditional distribution forecast. Therefore, the scenario is not particularly consistent with the correlations captured by our small-scale BVAR model. The detected internal inconsistency may be a consequence of the fact that the scenario was originally constructed using a medium-scale DSGE model with variables that are exogenous to our BVAR model.

Note that the consistency of the scenario could be expressed in terms of probabilities, as is the case for the “nearest” path forecast in sub-section 5.1. The conclusions related to the consistency of a scenario are not affected by the choice of a particular variable to analyze the scenario. Therefore, if the scenario is found to be inconsistent when uncertainty is “hard conditioned” on the outlook for the interest rate, then it is also inconsistent when uncertainty is conditioned on any other variable, because the same correlations within the time series constituting our BVAR model apply for all possible choices of conditioning variables.

## 6. Concluding Remarks

Fan charts have become important communication tools for inflation-targeting central banks around the world. According to our survey, approximately three-quarters of inflation targeters publish fan charts for inflation, and some publish them for other important macroeconomic variables. However, less attention is devoted to modeling the effect of the zero lower bound when generating fan charts. Moreover, fan charts are not used for macroprudential policy purposes even though stress-testing exercises could benefit from cross-checks of the underlying macroeconomic scenarios.

In this study, we focus on two issues. We first examine how different strategies for addressing the zero lower bound affect the resulting fan charts and the consequences of ignoring the zero lower bound. Second, we propose a method to evaluate the conservativeness of macroeconomic scenarios that are used to assess the stability of banks through stress tests.

We used Czech data for this exercise, but the methods and findings are applicable to any other inflation-targeting central bank. We used Czech data because of the transparency of the Czech central bank, which publishes fan charts for inflation, GDP growth, the interest rate, and the exchange rate. A detailed description of the stress-test scenarios is also publicly available, and the severity of these scenarios—using our method—can therefore be evaluated. In addition, it is noteworthy that previous research has largely focused on the assessment of fan charts published by the central banks of Norway, Sweden, and the United Kingdom, but systematic evaluations for those of other countries are, to our knowledge, missing.

We compare the fan charts produced using two approaches that address the zero lower bound with one that completely ignores it in order to evaluate the usefulness of BVAR fan charts. In simple terms, the first approach discards draws of shocks that lead to negative interest rates, as in Waggoner and Zha (1999). The second approach sets the values for the interest rate to zero whenever the forecast suggests that the interest rate takes a negative value. Clearly, given the correlations among the variables, the fan charts for all variables—and not just those for the interest rate—are affected. The fan charts for the model that ignores the zero lower bound provide the least accurate assessment of future risks.

With respect to the usefulness of BVAR fan charts for assessing financial stability, we contribute to the literature by proposing a method to evaluate whether the scenarios in stress tests are sufficiently adverse to avoid signaling a false sense of security (Borio and Drehmann 2009). It is noteworthy that, in practice, central banks typically base their scenarios on the expert judgments of a group of financial stability experts who interact with board members. Using our BVAR fan charts, we propose a method based on the minimum-distance path forecast, allowing the severity of expert-judgment-based scenarios to be assessed. We find that the CNB scenarios concerning macroeconomic shocks are conservative and that they pass our “severity test.”

## References

- Alfaro, R. and M. Drehmann. 2009. “Macro Stress Tests and Crises: What Can We Learn?” *BIS Quarterly Review* (December): 29–41.
- Babecky, J., A. Bulir, and K. Šmídková. 2010. “Sustainable Real Exchange Rates in the New EU Member States: What Did the Great Recession Change?” IMF Working Paper No. 10/198.
- Bank of Canada. 2009. “Methodology Used to Construct Fan Charts in the April 2009 Monetary Policy Report.” Methodological attachment to the April *Monetary Policy Report*.
- Bank of England. 2005. “Assessing the MPC’s Fan Charts.” *Quarterly Bulletin* (Bank of England) 45 (3).

- Borio, C., and M. Drehmann. 2009. "Towards an Operational Framework for Financial Stability: 'Fuzzy' Measurement and Its Consequences." BIS Working Paper No. 284.
- Breuer, T., M. Jandacka, J. Mencia, and M. Summer. 2012. "A Systematic Approach to Multi-Period Stress Testing of Portfolio Credit Risk." *Journal of Banking and Finance* 36 (2): 332–40.
- Breuer, T., M. Jandacka, K. Rheinberger, and M. Summer. 2009. "How to Find Plausible, Severe, and Useful Stress Scenarios." *International Journal of Central Banking* 5 (3): 205–24.
- Britton, E., P. Fisher, and J. Whitley. 1998. "The Inflation Report Projections: Understanding the Fan Chart." *Quarterly Bulletin* (Bank of England) (February): 30–37.
- Canova, F. 2007. *Methods for Applied Macroeconomic Research*. Princeton, NJ: Princeton University Press.
- Clements, M. 2004. "Evaluating the Bank of England Density Forecasts of Inflation." *Economic Journal* 114 (498): 844–66.
- Cogley, T., S. Morozov, and T. Sargent. 2005. "Bayesian Fan Charts for U.K. Inflation: Forecasting and Sources of Uncertainty in an Evolving Monetary System." *Journal of Economic Dynamics and Control* 29 (1): 1893–1925.
- Czech National Bank. 2008. "Disclosure of Interest Rate Forecasts and Use of Fan Charts in Czech National Bank Communications." Downloaded from [http://www.cnb.cz/miranda2/export/sites/www.cnb.cz/en/monetary\\_policy/forecast/prognoza\\_sazeb\\_vejirove\\_grafy\\_en.pdf](http://www.cnb.cz/miranda2/export/sites/www.cnb.cz/en/monetary_policy/forecast/prognoza_sazeb_vejirove_grafy_en.pdf) on July 3, 2011.
- Diebold, F. X., K. Tay, and K. F. Wallis. 1999. "Evaluating Density Forecasts of Inflation: The Survey of Professional Forecasters." In *Cointegration, Causality and Forecasting: A Festschrift in Honor of Clive W. J. Granger*, ed. R. F. Engle and H. White. Oxford: Oxford University Press.
- Dincer, N., and B. Eichengreen. 2009. "Central Bank Transparency: Causes, Consequences and Updates." NBER Working Paper No. 14791.
- Elder, R. 2005. "Assessing the MPC's Fan Charts." *Quarterly Bulletin* (Bank of England) 45 (3): 326–48. Available at <http://ssrn.com/abstract=813845>.
- Elekdag, S., and P. Kannan. 2009. "Incorporating Market Information into the Construction of the Fan Chart." IMF Working Paper No. 09/178.

- Franta, M., B. Saxa, and K. Šmídková. 2010. "The Role of Inflation Persistence in the Inflation Process in the New EU Member States." *Czech Journal of Economics and Finance* 60 (6): 480–500.
- Fry, R., and A. Pagan. 2005. "Some Issues in Using VARs for Macroeconometric Research." CAMA Working Paper No. 19/2005.
- Geraats, P. M. 2010. "Talking Numbers: Central Bank Communications on Monetary Policy and Financial Stability." Paper presented at the 5th ECB Statistics Conference "Central Bank Statistics: What Did the Financial Crisis Change?" October 19–20.
- Gerdrup, K. R., and J. Nicolaisen. 2011. "On the Purpose of Models—The Norges Bank Experience." Norges Bank Staff Memo No. 6/2011.
- Giannone, D., M. Lenza, D., Momferatu, and L. Onorante. 2010. "Short-Term Inflation Projections: A Bayesian Vector Autoregressive Approach." CEPR Discussion Paper No. 7746.
- Gneiting, T., and R. Ranjan. 2011. "Comparing Density Forecasts Using Threshold- and Quantile-Weighted Scoring Rules." *Journal of Business and Economic Statistics* 29 (3): 411–22.
- Goodhart, C. A. E. 2006. "A Framework for Assessing Financial Stability?" *Journal of Banking and Finance* 30 (12): 3415–22.
- Haldane A. 2009. "Why Banks Failed the Stress Test." Speech given at the Marcus-Evans Conference on Stress-Testing, London, February 9–10. Available at <http://www.bankofengland.co.uk/archive/Documents/historicpubs/speeches/2009/speech374.pdf>.
- Hammond, G. 2011. *State of the Art of Inflation Targeting*. CCBS Handbook No. 29, February 2011 version. Bank of England.
- Horváth, R., and J. Mateju. 2011. "How Are Inflation Targets Set?" *International Finance* 14 (2): 265–300.
- Horváth, R., and D. Vasko. 2012. "Central Bank Transparency and Financial Stability: Measurement, Determinants and Effects." IES FSV UK Working Paper No. 25.
- Husebo, T., S. McCaw, K. Olsen, and O. Roisland. 2004. "A Small, Calibrated Macromodel to Support Inflation Targeting at Norges Bank." Norges Bank Staff Memo No. 2004/3.
- Kadiyala, K. R., and S. Karlsson. 1997. "Numerical Methods for Estimation and Inference in Bayesian VAR-Models." *Journal of Applied Econometrics* 12 (2): 99–132.

- Litterman, R. 1986. "Forecasting with Bayesian Vector Autoregressions—Five Years of Experience." *Journal of Business and Economic Statistics* 4 (1): 25–38.
- Magyar Nemzeti Bank. 2004. *Quarterly Report on Inflation*, May, 108.
- Mitchell, J., and S. G. Hall. 2005. "Evaluating, Comparing and Combining Density Forecasts Using the KLIC with an Application to the Bank of England and NIESR 'Fan' Charts of Inflation." *Oxford Bulletin of Economics and Statistics* 64 (s1): 995–1033.
- Murchison, S., and A. Rennison. 2006. "ToTEM: The Bank of Canada's New Quarterly Projection Model." Technical Report No. 97, Bank of Canada.
- Norges Bank. 2005. "Uncertainty Surrounding Future Interest Rate Developments." *Inflation Report* November (3): 19–21.
- . 2007. "The Fan Charts for the CPI-ATE Projections." *Economic Bulletin* (Norges Bank) 2/2007: 85–87.
- Onado, M. 2011. "European Stress Tests: Good or Bad News?" VoxEU.org, August 16.
- Osterholm, P. 2008. "A Structural Bayesian VAR for Model-Based Fan Charts." *Applied Economics* 40 (12): 1557–69.
- . 2009. "Incorporating Judgment in Fan Charts." *Scandinavian Journal of Economics* 111 (2): 387–415.
- Pinheiro, M., and P. S. Esteves. 2012. "On the Uncertainty and Risks of Macroeconomic Forecasts: Combining Judgments with Sample and Model Information." *Empirical Economics* 42 (3): 639–65.
- Šmídková, K. 2005. "How Inflation Targeters (Can) Deal with Uncertainty." *Czech Journal of Economics and Finance* 55 (7–8): 316–32.
- , ed. 2008. *Evaluation of the Fulfilment of the CNB's Inflation Targets 1998–2007*. Czech National Bank.
- Sveriges Riksbank. 2007. "Calculation Method for Uncertainty Bands." *Monetary Policy Report* 2007:1.
- Waggoner, F. D., and T. Zha. 1999. "Conditional Forecasts in Dynamic Multivariate Models." *Review of Economics and Statistics* 81 (4): 639–51.