DSGE Model Restrictions for Structural VAR Identification*

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The identification of reduced-form VAR models has been the subject of numerous debates in the literature. Different sets of identifying assumptions can lead to very different conclusions regarding the effects of shocks. This paper proposes a theoretically consistent identification strategy using restrictions implied by a DSGE model. Monte Carlo simulations suggest that both quantitative and qualitative restrictions work well together, where they act as complements to each other, in minimizing errors in finding the correct VAR identification. When using misspecified model restrictions, the data tend to push the identified VAR responses away from the misspecified model and closer to the true data-generating process.

JEL Codes: F31, E52.

1. Introduction

Since the pioneer work of Sims (1980), vector autoregressive (VAR) models have been used extensively by applied researchers, forecasters, and policymakers to address a range of economic issues. Although VAR models have been very successful in capturing the dynamic properties of the macroeconomic time-series data, the

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decomposition of these statistical relationships back to coherent economic stories is still under large debate. The key source of this disagreement arises from the difficulty in identifying structural disturbances from a set of reduced-form residuals. The sampling information in the data is not sufficient, and several assumptions are needed in order to recover the mapping between the structural and the reduced-form errors.[1] However, the outcome of the VAR analysis depends crucially on these assumptions, and the various competing identification restrictions cannot be easily tested against the data.

The literature has proposed a number of different exact-identification strategies. First, and most popular, is the Choleski short-run restriction on the VAR's reduced-form covariance matrix. Under the Choleski scheme, the ordering of the variables is particularly important for the structural economic interpretation of the VAR (see Lutkepohl 1993 and Hamilton 1994). Furthermore, as Canova (2005) explains, the Choleski decomposition implies "zero-type" restrictions that are rarely consistent with dynamic stochastic general equilibrium (DSGE) models. A similar procedure was introduced by Blanchard and Quah (1989) by imposing long-run relationships that are consistent with economic theory. However, a number of studies such as Chari, Kehoe, and McGrattan (2005), Erceg, Guerrieri, and Gust (2005), Christiano, Eichenbaum, and Vigfusson (2006), and Ravenna (2007) have concluded that long-run restrictions are inadequate in recovering the true structural disturbances. The main reason is that it is often difficult to obtain an accurate estimate of the long-run impacts because of the truncation bias associated with VAR models. Therefore, imposing long-run restrictions based on these bias estimates can lead to misleading conclusions.

More recently, Faust (1998), Canova and De Nicolo (2002), and Uhlig (2005) propose an identification scheme that imposes "sign" or "qualitative" restrictions on the structural responses. The strategy recognizes there are an infinite number of observationally equivalent mappings between the structural and the reduced-form errors, and the idea of the sign restriction is to select a subset of these mappings that are consistent with certain qualitative features. An attractive feature of this procedure is that it makes VAR and DSGE models

[1] The discussions here focus on exact VAR identification.

more comparable than other identification strategies. Researchers can then use qualitative information in the form of sign restrictions implied by DSGE models to help identify structural VAR shocks; for example, Peersman and Straub (2009) and Liu (2010) use this approach. Although attractive to applied researchers, as highlighted by Uhlig (2005) and explicitly illustrated by Fry and Pagan (2011), this type of identification scheme fails to deliver a unique identification mapping. There can be a range of impulse responses that are consistent with the sign restrictions. This leads to large uncertainty around the model's estimates (see Paustian 2007) and makes policy inference less informative.

Uhlig (2005) also discusses an alternative procedure known as the "penalty-function" approach. The idea is to find the set of orthogonal shocks that minimize a specific penalty function. This is certainly a less agnostic approach relative to the pure sign-restriction method. Nevertheless, the procedure produces a unique set of structural shocks and therefore reduces the degree of uncertainty related to the identification procedure. However, the choice of the penalty function remains arbitrary and difficult to motivate from an economic perspective.

An alternative procedure was proposed by Del Negro and Schorfheide (2004), who developed a methodology to generate a prior distribution using a DSGE model for a time-series VAR. The DSGE-VAR approach relaxes the tight theoretical restrictions of the DSGE model by making use of the model and data, summarized by the likelihood of the model. While the DSGE-VAR has been a very useful tool for model comparison and forecasting, as Sims (2008) pointed out, it remains difficult to use the DSGE-VAR for policy analysis—for example, impulse response analysis. The main reason is that the DSGE-VAR still faces the same identification problems as standard VAR models. Del Negro and Schorfheide (2004) suggest using the identification matrix from the DSGE model as an approximation; however, the resultant variance-covariance matrix will no longer be the same as the estimated DSGE-VAR (see Del Negro and Schorfheide 2004, footnote 17).

This paper proposes an identification strategy that extends Uhlig's (2005) penalty-function approach to a more formal setting. In particular, we construct a penalty function that is based on quantitative and qualitative restrictions implied by a DSGE model.

To assess the usefulness of the proposed identification strategy, we present a series of Monte Carlo experiments. First, we investigate whether the proposed algorithm can recover the true set of structural shocks given the correct set of restrictions; second, we assess how the proposed identification strategy performs using quantitative restrictions from a misspecified model; third, we investigate using only a partial set of qualitative restrictions. Lastly, we present an application using a seven-variable VAR model estimated on U.S. data.

A number of interesting results emerge from the analysis. First, the proposed identification strategy systematically gives a smaller bias compared with other identification schemes such as the Choleski decomposition and pure sign restrictions. Second, despite using restrictions implied by a misspecified model, the data (summarized by the reduced-form covariance matrix) tend to push the VAR responses away from the misspecified model and closer to the true data-generating process (DGP). Third, when we only impose a partial set of qualitative sign restrictions, the proposed method consistently yields a smaller bias relative to the pure sign-restriction approach. Our results suggest that both quantitative and qualitative restrictions work well together, where they act as complements to each other, in minimizing errors in finding the correct VAR identification.

The paper is organized as follows. Section 2 discusses the advantages of the proposed methodology. Section 3 outlines the methodology of the proposed identification strategy. Section 4 briefly describes the Monte Carlo experiments and outlines the medium-scale DSGE model used for the data-generating process. Section 5 reports the results from the Monte Carlo experiments. Section 6 presents an application of the proposed identification strategy using a seven-variable VAR model estimated on U.S. data. Section 7 contains concluding remarks and proposes direction for future research.

2. The Usefulness of the Proposed Methodology

Our starting point is the work of Pagan (2003), who argues that there is an inherent trade-off between theoretical and empirical coherence for macroeconomic models.[2] DSGE models are perhaps the most

[2]Pagan (2003) illustrates this by placing various types of models along a concave "modeling frontier" with the degree of theoretical coherence and the degree of empirical coherence along each axis.

transparent example of this negative trade-off between theoretical consistency and data coherence. These models often place a large amount of restrictions based on simplified economic theory, which could limit its ability to fit the data well. The literature has progressed significantly along this dimension following the pioneered work of Smets and Wouters (2007)—who illustrated that DSGE models can be designed to fit the data well, at the same time keeping its attractive theoretical coherence structure. However, the issue of model misspecification still remains (see discussions in Del Negro and Schorfheide 2009).

Our proposed approach offers an additional angle to this debate by creating a structural VAR model that mimics the DSGE model as long as the restrictions implied by the latter regarding the mapping between the reduced-form and structural errors do not significantly violate the estimated variance-covariance matrix of the VAR residuals. One can think of the approach presented here as bringing the statistical VAR analysis closer to the structural DSGE models along the "Pagan curve" but without sacrificing empirical coherence. Despite applying more restrictions on the behavior of the impulse response functions of the VAR, the proposed method does not change either the empirical fit or the forecasting performance—see proposition 1 of Waggoner and Zha (1999)—of the VAR model. Rather, it simply selects a unique identification mapping from the infinite number of observationally equivalent ones. This is a subtle but important difference from Del Negro and Schorfheide's (2004) DSGE-VAR, where the approach here maintains the same variance-covariance matrix as the reduced-form representation of the VAR for structural identification. Our identified VAR can also be a useful cross-check against the structural model to evaluate the simplified assumptions of the model.

3. Methodology

3.1 A Review of the Identification Problem

The use of VAR models to address key macroeconomic policy questions depends crucially on the identification of the reduced-form residuals. Even though several procedures have been proposed in the literature, shock identification remains a highly controversial issue. To illustrate the identification problem, consider the following stylized structural model:[3]

$$A_0 Y_t = A(L)^h Y_t + \eta_t \tag{1}$$

$$Y_t = A_0^{-1} A(L)^h Y_t + A_0^{-1} \eta_t, \tag{2}$$

where Y_t is an $(n \times 1)$ vector of endogenous variables, A_0 is an $(n \times n)$ matrix of coefficients, $A(L)^h = A_1 L + \cdots + A_h L^h$ is an h^{th}-order lag polynomial, and $E(\eta_t \eta_t') = I$ gives the variance-covariance matrix of the structural innovations. Equation (1) is the structural model and equation (2) is the corresponding reduced-form representation. The key parameters of interest are A_0 and $A(L)$. However, the sampling information in the data is not sufficient to identify both A_0 and $A(L)$ separately without employing further identifying restrictions. There are infinite combinations of A_0 and $A(L)$ that all imply exactly the same probability distribution for the observed data. To see this, premultiply the model in (1) by a full rank matrix Q, which leads to the following new model:

$$QA_0 Y_t = QA(L) Y_t + Q\eta_t \tag{3}$$

$$Y_t = A_0^{-1} Q^{-1} QA(L) Y_t + A_0^{-1} Q^{-1} Q\eta_t. \tag{4}$$

The reduced-form representation of the two models in equations (2) and (4) are exactly the same. That implies both models in (1) and (3) are observationally equivalent. Without additional assumptions—identifying restrictions—no conclusions regarding the structural behavior of the "true" model can be drawn from the data.

As explained by Canova (2005, chapter 4), popular identification schemes such as the Choleski decomposition and long-run restrictions impose "zero-type" restrictions that cannot be easily justified by a large class of DSGE models. In particular, DSGE models hardly display the type of recursive structures that are typically assumed by Choleski or long-run identification schemes. This raises the question of whether these identification schemes are the appropriate choices in relation to the economic theory. Furthermore, from the work Carlstrom, Fuerst, and Paustian (2009), it is known that

[3]For the moment, we remain agnostic as to the form of the structural model or where it comes from. More details are provided in the next sub-section.

these restrictions can severely distort the impulse response function if they are not consistent with the true DGP.

The identification scheme proposed by Faust (1998), Canova and De Nicolo (2002), and Uhlig (2005) seem to overcome some of these difficulties by imposing sign and/or shape (pure sign) restrictions on the structural responses. Although attractive to applied researchers, the procedure fails to deliver a unique identification mapping as it is highlighted by Uhlig (2005) and Fry and Pagan (2011). In other words, there can be a range of impulse responses that are consistent with the sign restrictions. This can lead to large uncertainty around the model's estimates and less reliable inference.

3.2 The Mapping between the DSGE and the VAR Model

To see the links between the structural and the reduced-form VAR model, it is useful to explore the mapping between the two. To be more specific, we are going to consider the class of structural models that are usually based on agents' optimization behavior and rational expectation formation (DSGE models). Generally, the solution of a linearized DSGE model can be summarized by the following state-space representation:[4]

$$X_t = B(\theta) X_{t-1} + \Gamma(\theta) \eta_t \tag{5}$$

$$Y_t = A(\theta) X_t, \tag{6}$$

where X_t is an $n \times 1$ vector of state variables, Y_t is an $m \times 1$ vector of variables observed by an econometrician, and η_t represents an $k \times 1$ vector of economic shocks such that $E(\eta_t) = 0$ and $E(\eta_t \eta_t') = I$.[5] The matrices $A(\theta)$, $B(\theta)$, and $\Gamma(\theta)$ are functions of the underlying structural parameters of the DSGE model. Equation (6) is usually referred to as the state equation (or policy function) that describes the evolution of the underlying economy, and equation (5) is the observation equation that relates the state of the economy with the set of observable variables. For notational convenience, we will drop

[4]The solution of the model can be obtained by using either Blanchard and Kahn (1980) or Sims's (2002) type algorithms.

[5]In the notation here, x_t also includes the current values of exogenous shock processes.

the indication that the matrices A, B, and Γ are functions of the structural parameters θ.

From the work of Christiano, Eichenbaum, and Vigfusson (2006), Fernandez-Villaverde et al. (2007), and Ravenna (2007), the state-space representation of the DSGE model described above has an infinite-order VAR process representation, VAR(∞), if and only if the eigenvalues of the following matrix,

$$M = \left[I_n - \Gamma \left(A\Gamma \right)^{-1} A \right] B, \tag{7}$$

are less than one in absolute terms and the number of the shocks coincides with the number of observed variables, i.e., $m = k$. This is known as the "poor man's invertibility condition" or simply the "invertibility condition" as in Fernandez-Villaverde et al. (2007). If this condition holds, the set of observable variables Y_t can be written as a VAR(∞) such that

$$Y_t = \sum_{j=1}^{\infty} \Delta_j Y_{t-j} + A\Gamma\eta_t, \tag{8}$$

where

$$\Delta_j = ABM^{j-1}\Gamma \left(A\Gamma \right)^{-1}.$$

On the other hand, a reduced-form VAR(h) model can be estimated on a set of stationary macroeconomic time series Y_t to provide a summary of its statistical properties,

$$y_t = \sum_{j=1}^{h} A_j y_{t-j} + v_t, \tag{9}$$

where v_t is normally distributed with zero mean and variance-covariance matrix Σ_v. Assuming the DSGE model in equation (9) is the true data-generating process (DGP) for Y_t, the reduced-form VAR in equation (9) can provide a reasonably good approximation of the process Y_t as the number of lags (h) tends to infinity. In such a case, the mapping between the reduced-form and structural shocks can be uniquely defined as (Christiano, Eichenbaum, and Vigfusson 2006, proposition 1)

$$v_t = A\Gamma\eta_t \tag{10}$$

or

$$\Sigma_v \equiv E\left(v_t v_t'\right) = A\Gamma\Gamma'A'. \tag{11}$$

It is this unique mapping that this paper exploits to help identify reduced-form VAR shocks.

3.3 DSGE Restrictions for Structural VARs

In addition to the pure sign-restriction approach, Uhlig (2005) also discusses an alternative procedure known as the "penalty function." The idea behind the procedure is to find a set of orthogonal shocks that minimizes a specific penalty function. However, the choice of the penalty function remains arbitrary and difficult to motivate from an economic perspective. The identification strategy described here essentially extends the penalty-function approach to a more formal setting. In particular, we exploit the mapping between the DSGE and the VAR model as shown earlier to construct the penalty function. This is attractive because it provides a theoretically consistent way of identifying structural VAR shocks, and the identifying assumptions are motivated from restrictions implied by DSGE models. Furthermore, the procedure can help bring together the two distinct approaches of macroeconomic modeling.

Assuming the DSGE model is the true DGP with variance-covariance matrix $A\Gamma\Gamma'A'$, Lutkepohl and Poskitt (1991) show that the estimated variance-covariance of a VAR(h) model converges to the true variance-covariance when the number of lags tends to infinity ($h \to \infty$) as the sample size tends to infinity ($T \to \infty$). The rate at which the sample size tends to infinity must be faster than the rate at which h^3 tends to infinity, that is,

$$\widehat{\Sigma}_v \to A\Gamma\Gamma'A' \quad \text{as} \quad \frac{h^3}{T} \to 0, \tag{12}$$

where $\widehat{\Sigma}_v$ is the estimated VAR variance-covariance or the reduced-form covariance. In practice, the two key assumptions underlying the above condition undoubtedly break down. First, mostly DSGE models are tools designed to explain certain subsets of stylized facts. Despite the recent success in improving its empirical performance, misspecification remains a concern (Del Negro et al. 2007). Second, samples of macroeconomic time-series data are limited, so the number of lags that can be included in the VAR is quite restrictive. Consequently, the estimated VAR variance-covariance can be quite different from the one implied by the DSGE model.

In the pure sign-restriction case, $\widehat{\Sigma}_v$ is decomposed into $\widehat{\Sigma}_v = \hat{C}P(\omega)P(\omega)'\hat{C}'$, where $\hat{C}$ is the Choleski factor of $\widehat{\Sigma}_v$, $P(\omega)$ is an orthonormal matrix such that $P(\omega)P(\omega)' = I$, and ω denotes a vector of rotation angles—each element of this vector lies between 0 and π. The matrix $P(\omega)$ is selected in such a way to meet the researcher's belief regarding the qualitative properties of the impulse responses. As discussed earlier, the selection of $P(\omega)$ is non-unique. The proposed identification strategy here essentially selects a unique matrix $P(\omega)$ to minimize the "distance" between the contemporaneous response of the VAR and the one implied by the DSGE model. The procedure can be summarized as the following minimization problem:

$$\omega^* = \arg\min_{\omega} \left\{ \left\| vec\left(\widehat{C}P\left(\omega\right) - A\Gamma\right)\right\|_2 + \sum_{j=1}^{k}\sum_{i=1}^{m} \delta_{ij} I\left(sign_{ij}\right) \right\} \tag{13}$$

subject to

$$P\left(\omega\right)P\left(\omega\right)' - I = 0, \tag{14}$$

where $\|\cdot\|_2$ stands for the Euclidian norm, $I(sign_{ij})$ is an indicator function for variable i in response to shock j that takes values 0 if the sign restrictions are satisfied and 1 otherwise, and δ_{ij} is a positive number. A few remarks are worth noting. The first part of equation (13) resembles Uhlig's (2005) penalty function, although here the function is based on restrictions from a DSGE model. The second part of equation (13) is analogous to the pure sign restrictions. The parameter δ_{ij} controls for a set of sign restrictions we want to impose on the model.[6] The key difference is that by imposing additional restrictions from a DSGE model, it will ensure a unique identification matrix $\hat{C}P(\omega^*)$ for the VAR. The difference between the identified VAR responses relative to the DSGE model will depend on how plausible the theoretical restrictions are in the face of the data summarized by $\hat{A}(L)^h$ and $\widehat{\Sigma}_v$. If these restrictions are deemed far away from the empirical evidence, then the difference can be quite large, and vice versa. There is no closed-form solution readily available for the above minimization problem, so we resort to numerical methods for the simulation experiments. However, Judd (1998, theorem 4.7.1) shows that under some regularity conditions, the vector ω^* is unique, implying that $P(\omega^*)$ is also unique.

4. Monte Carlo Experiments

This section sets out a series of Monte Carlo experiments designed to evaluate the usefulness of the proposed identification procedure.

4.1 The Model for the Data-Generating Process

The model used for the Monte Carlo study is based on the work by Smets and Wouters (2007).[7] This is an estimated medium-scale DSGE model that incorporates various sources of nominal and real frictions to match U.S. post-war business-cycle fluctuations. In this model, the steady state of the economy follows a deterministic trend according to the rate of labor-augmenting technological progress. Households select consumption and labor effort to maximize their non-separable utility preferences. Agents' consumption behavior exhibits habit formation, and households are assumed to supply differentiated labor services to firms. This gives households

[6] The first part of equation (13) penalizes positive and negative deviations from the DSGE responses in exactly the same manner. However, it is of greater economic interest for the VAR to deliver the same signs for the impulse responses compared with finding the smallest absolute deviation. The algorithm achieves this by attaching some positive weight to δ_{ij}. Note that the optimization algorithm is not sensitive to the value of δ_{ij}, and in our implementation we set $\delta_{ij} = 1$.

[7] Smets and Wouters' (2007) model is based on the earlier work of Smets and Wouters (2003) and Christiano, Eichenbaum, and Evans (2005).

monopoly power over wage negotiations, and therefore aggregate wages are sticky. In addition, households, who face capital adjustment costs, optimally decide how much capital to rent to firms and how much capital to accumulate.

On the production side, firms minimize the cost of production by optimally selecting the amount of labor and capital inputs subject to capital utilization costs and the wage rate set by households. Given demands for their product, firms reoptimize prices infrequently in a Calvo-type fashion. Finally, wages and prices that are not reoptimized every period are partially indexed to the past inflation. The appendix summarizes the key linearized equation of the model. Readers who are interested in the agents' decision problems are advised to consult the references mentioned above directly. The model's key parameters' values are taken directly from Smets and Wouters' (2007) study and summarized in table 7.

In the original model, Smets and Wouters assume seven exogenous driving processes or shocks. These are required in order to match the seven observable variables used in the estimation. Here, we simplify the model to contain only four shocks, namely a government spending shock, a price markup shock, a wage markup shock, and a monetary policy shock.[8]

4.2 The Monte Carlo Design

To investigate the properties of the identification strategy described in section 3.3, we set up two Monte Carlo experiments. In the first part of the experiments, we test the proposed identification strategy using the true model specification as well as restrictions implied by the following series of misspecified models:

- M_0: Benchmark
- M_1: Model with no habit formation ($h = 0$)
- M_2: Model with no price and wage indexation ($i_p = i_w = 0$)
- M_3: Model with no moving-average terms for price and wage markup shocks ($\mu_p = \mu_w = 0$)

[8]The original model also includes a net worth shock, a technology shock, and an investment-specific shock. In principle, it is possible to include all seven shocks for the Monte Carlo experiments, but this would increase the computation burden for the Monte Carlo experiment significantly.

- M_4: Model with no habit formation, price and wage indexation (M_1 and M_2)
- M_5: Model with no interest rate smoothing term in the Taylor rule ($\rho = 0$)

We compare the results using the proposed identification with the other identification strategies, such as Choleski ordering and the pure sign-restriction approach. Note that if we place no weight on the first part of the objective function (quantitative restrictions), our procedure collapses to the pure sign-restriction scheme, but this will not yield a unique $P(\omega)$ as discussed earlier. Next, we perform a similar set of experiments but using only a subset of sign restrictions implied by the model:

- Case I: No Government Spending Restrictions, $\delta_{i,1} = 0$ for $i = 1, \ldots, 4$
- Case II: No Interest Rate Restrictions, $\delta_{i,2} = 0$ for $i = 1, \ldots, 4$
- Case III: No Wage Markup Restrictions, $\delta_{i,3} = 0$ for $i = 1, \ldots, 4$
- Case IV: No Price Markup Restrictions, $\delta_{i,4} = 0$ for $i = 1, \ldots, 4$
- Case V: No Interest Rate and Wage Markup Restrictions, $\delta_{i,2} = \delta_{i,3} = 0 \forall i$
- Case VI: No Interest Rate and Government Spending Restrictions, $\delta_{i,1} = \delta_{i,2} = 0 \forall i$
- Case VII: Only Government Spending Restrictions, $\delta_{i,j} = 0$ for $i = 1, \ldots, 4$ and $j = 2, \ldots, 4$
- Case VIII: Only Interest Rate Restrictions, $\delta_{i,j} = 0$ for $i = 1, \ldots, 4$ and $j = 1, 2$ and 4
- Case IX: Only Price Markup Restrictions, $\delta_{i,j} = 0$ for $i = 1, \ldots, 4$ and $j = 1, \ldots, 3$

This allows us to investigate the role played by the pure sign-restrictions component of our proposed objective function (qualitative restrictions), $\sum_{j=1}^{k} \sum_{i=1}^{m} \delta_{ij} I(sign_{ij})$. In general, the simulation experiment follows these few steps:

(i) Simulate data—Y_t—of 200 observations of the observable vector (output growth, inflation, wage growth, and the nominal interest rate) using the model and parameters described in section 4.1.[9]

(ii) Estimate a DSGE model ($M_0, \ldots, M_5$) using maximum-likelihood estimation (MLE). The likelihood of the model is constructed via the Kalman filter and maximized using Sim's CSMINWEL algorithm. This gives the contemporaneous impact matrix of the DSGE model ($A(\theta_i)\Gamma(\theta_i)$) as functions of the model's structural parameters (θ_i)—where $i = M_0, \ldots, M_5$.

(iii) Estimate a reduced-form VAR(2) model using ordinary least squares (OLS) and compute the variance-covariance matrix ($\hat{\Sigma}_v$) of the reduced-form errors.

(iv) Decompose $\hat{\Sigma}_v$ into $\widehat{C}P(\omega_i)P(\omega_i)'\widehat{C}'$ and find an orthonormal matrix $P(\omega_i^*)$ such that it minimizes the loss function in equation (13). We use Matlab's *fminunc* function to find the minimum. To ensure that the minimization algorithm finds the unique global minimum, we repeat the minimization procedure 1,000 times using different random starting values.

(v) Construct impulse responses from the identified structural VAR (SVAR) using $\widehat{C}P(\omega_i^*)$.

(vi) Repeat the above steps 500 times.

5. Results from the Monte Carlo Study

5.1 Bias of the SVAR Impulse Response Functions

To assess the performance of our proposed methodology, we compute the bias of the impulse responses from the true DGP as

$$\text{bias}_T = 100 \sum_{t=1}^{T} \sum_{i=1}^{M} \sum_{j=1}^{K} \frac{|\Psi_{t,i,j} - \bar{\Psi}_{t,i,j}|}{\bar{\Psi}_{t,i,j}}, \tag{15}$$

where $\Psi_{t,i,j}$ is the t'th period's impulse response of the estimated benchmark or VAR model for variable i to shock j, $\bar{\Psi}_{t,i,j}$ is the DGP

[9] We simulate 10,000 observations and we keep only the 200.

equivalent, and the bias is calculated as the sum across all the M variables and K shocks up to periods T. Before turning to the discussion of the results, it is useful to identify where the SVAR bias may be coming from by reexpressing the SVAR in the form similar to equation (1):

$$A(L)Y_t = e_t = A_0^{-1}\eta_t \tag{16}$$

$$Y_t = A(L)^{-1}A_0^{-1}\eta_t = R(L)A_0^{-1}\eta_t. \tag{17}$$

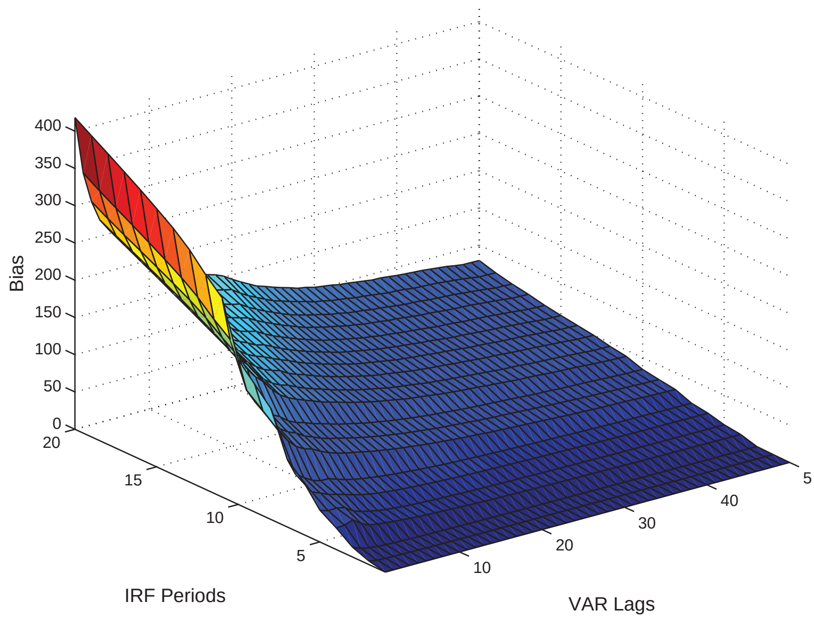
From equation (17), one can see that the response of Y_t to the underlying structural innovation, η_t, is influenced both by the reduced-form moving-average terms, $R(L)$, and by the identifying restrictions placed on A_0. Erceg, Guerrieri, and Gust (2005) usefully categorize this bias into three components:

$$\text{SVAR bias} = \text{R-bias} + \text{A-bias} + \text{Truncation bias}. \tag{18}$$

The first part, the “R-bias,” reflects the small-sample error in estimating the reduced-form moving-average terms, the $R(L)$ coefficients in equation (17). The second part, referred to as the “A-bias,” reflects the error associated with transforming the reduced form into its structural form by imposing certain identifying restrictions, the A_0 matrix. Lastly, the “truncation bias” arises because a finite-ordered VAR ($h < \infty$) is chosen to approximate the true dynamics implied by the model, a VARMA process. Kapetanios, Pagan, and Scott (2007) document that the truncation bias from medium- to large-scale models can be very large. However, it is important to recognize that the three types of biases are not necessary independent of each other; they can interact and exacerbate the overall bias of the SVAR responses. For example, using a fixed sample size, a larger truncation bias can increase the R-bias related to the estimation of the reduced-form coefficients. Similarly, for a fixed set of identifying assumptions, the imprecision in estimating $R(L)$ can exacerbate the A-bias associated with the identification of the structural shocks.

In contrast to Erceg, Guerrieri, and Gust (2005), we find that the truncation bias plays a dominant role in explaining the difference between the VAR and the DSGE model’s responses at longer

Figure 1. Truncation Bias (Accumulated) of the VAR Model



horizons. To illustrate this, figure 1 plots the truncation bias of an SVAR model along different lag lengths.[10] As one might expect, for a fixed number of lags, the bias is larger at longer horizons (see explanations in Ravenna 2007). On the other hand, the bias is a monotonic function decreasing with the number of lags included in the VAR. It is interesting to note that the bias decreases in a non-linear fashion. This is in line with the evidence provided by Kapetanios, Pagan, and Scott (2007) that in order to approximate a medium- to large-scale DSGE model, one would require a significantly large number of lags for the VAR.

The speed in which the truncation bias decreases with the number of lags will depend on the model's specification and parameters. More specifically, Fernandez-Villaverde et al. (2007) point out that

[10]The SVAR model is estimated based on a simulated data sample of 100,000 and the identification matrix is computed using the benchmark model restrictions (M_0).

the closer the largest (absolute value) eigenvalue of the matrix M in equation (7) is to one, the more lags are needed in order to approximate the true DGP. In our case, the largest eigenvalue of the matrix M is indeed very close to one and therefore it is not surprising that a large-order VAR is needed to approximate the dynamics of the model.[11] For discussions of the empirical results in the next subsections, we will focus on shorter horizons, where the truncation bias is less important.

5.2 SVAR Identification Using the Correct Model Restrictions

Table 1 reports the bias from our first set of simulation experiments. Columns 1, 4, 7, and 10 report the bias for all of the DSGE models described in section 4.2 at the one-, two-, eight-, and twelve-quarter horizon. Similarly, columns 2, 5, 8, and 11 present the same measures for the identified SVAR models. Finally, columns 3, 6, 9, and 12 display the ratio of these quantities.

Looking at the benchmark model (M_0) along the first row of table 1, we can see that the proposed SVAR closely matches the bias from the estimated DSGE at short horizons. In longer horizons, the truncation bias from the estimated VAR dominates and the DSGE model outperforms. This result is not that surprising given what we had discussed in the previous section, but nevertheless it is useful to know that our proposed algorithm is capable of recovering the true impulse responses if the correct set of model restrictions were applied.

To compare the proposed identification with other identification strategies, figure 2 plots the responses of the Choleski VAR model (solid-circle line), VAR model with sign restrictions (solid-cross line) implied by the model, the estimated DSGE model (solid line), and the true response (dashed line) against our proposed SVAR (dotted-dashed line).[12] In all cases, the Choleski and sign-restriction identification schemes produced impulse responses that

[11] We also experimented with a simple three-equation New Keynesian model where the eigenvalue of the matrix M is much smaller, in which case a VAR(2) together with the proposed identification strategy provides an excellent match with the DGP's impulse responses. These results are available on request.

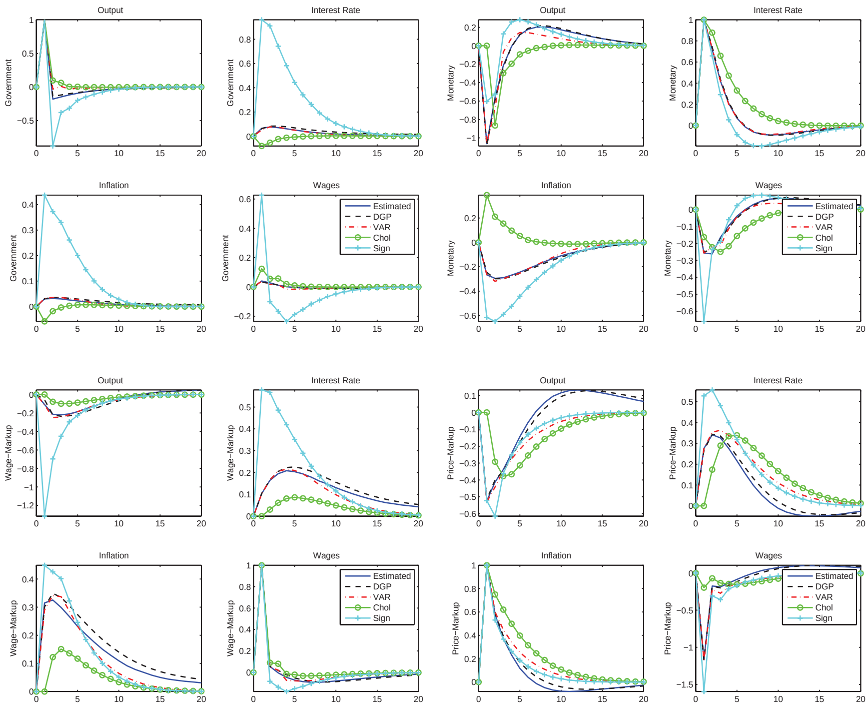
[12] For the Choleski decomposition, the variables are ordered as output, inflation, wage growth, and short-term interest rate.

Table 1. Accumulated Bias for Misspecified Models and the SVAR Model

	Period 1			Periods 2			Periods 8			Periods 12		
	DSGE	**SVAR**	**Ratio**	**DSGE**	**SVAR**	**Ratio**	**DSGE**	**SVAR**	**Ratio**	**DSGE**	**SVAR**	**Ratio**
M_0	1.5	1.6	0.9	2.5	3.7	0.7	39.2	98.4	0.4	68.4	190.2	0.4
M_1	6.1	5.9	1.0	10.3	9.5	1.1	61.9	131.8	0.5	85.3	230.0	0.4
M_2	2.1	1.9	1.1	6.3	4.4	1.4	74.1	109.7	0.7	97.2	213.4	0.5
M_3	4.2	5.0	0.8	9.8	7.9	1.2	72.2	117.4	0.6	109.8	203.3	0.5
M_4	5.7	5.4	1.0	11.3	9.3	1.2	68.7	128.9	0.5	104.1	235.3	0.4
M_5	12.8	12.9	1.0	19.8	19.6	1.0	117.5	190.0	0.6	164.0	263.4	0.6

Notes: The accumulated bias is calculated as the sum (across different number of periods) of the absolute percentage difference between the estimated DSGE model or the VAR's impulse responses with the DGP. The ratio measure is simply the bias of the estimated DSGE model relative to that of the SVAR model.

Figure 2. Median Impulse Response Functions of the Benchmark Model: 500 Samples of 200 Observations



are much further away from the DGP. The standard "price puzzle," for example, is evident in the Choleski scheme, and different recursive ordering structures produced very similar results. It is interesting to note that even though the sign restrictions deliver the right signs by construction, they consistently overestimate the impact of the shock.

The summary statistics in table 2, which shows the ratio of the bias relative to the proposed SVAR, reveal a similar conclusion. The size of the bias using the Choleski scheme is between eight and eleven times larger than the proposed identification strategy at shorter horizons, and the sign restriction scheme is thirty-one to forty-eight times larger. At longer horizons, the truncation bias dominates and the differences are much smaller.

Table 2. SVAR Bias Relative to Other Identification Methodologies

Horizon	$\frac{\text{SVAR}_C}{\text{SVAR}_D}$	$\frac{\text{SVAR}_S}{\text{SVAR}_D}$
1	11.0	48.5
2	8.3	31.9
3	3.0	10.2
4	3.0	10.2
8	2.8	5.5
12	2.2	3.3
16	1.9	2.7
20	1.8	2.4

Note: SVAR_D, SVAR_C, and SVAR_S correspond to the SVAR(2) model bias that arises using DSGE, Choleski, and sign restrictions, respectively.

5.3 Proposed Methodology Using Misspecified Model Restrictions

One natural question to ask is how the proposed identification scheme performs if the wrong set of quantitative restrictions were imposed, given that model misspecification was one of our initial motivations for the proposed identification strategy. Rows 2–6 in table 1 report the bias for the estimated misspecified DSGE models and the SVAR.[13] This is certainly not an exhaustive list of potential misspecifications one can consider, but it does provide a way of evaluating the usefulness of the proposed identification scheme when restrictions are derived from misspecified models.

The model with no interest rate smoothing term in the Taylor rule (M_5) gives the largest bias relative to the true DGP. Since the VAR identification also depends on the restrictions implied by the misspecified model, the bias for the SVAR model is also the largest for M_5. At shorter horizons (one and two quarters), where the comparison is more informative, the bias of the SVAR models is smaller than that of the misspecified models.[14]

[13] The quantitative restrictions of the SVAR were also based on the same misspecified model.

[14] The exception is model M_5, where the bias between the misspecified model and the VAR is very similar.

This is an interesting result: even though the identification bias (A-bias) is larger because we are using restrictions from a misspecified model, the data (summarized by the reduce-form VAR covariance matrix) tends to push the SVAR responses closer to the true DGP. Therefore, information from the data is useful in correcting some of the bias arising from using misspecified model restrictions. The results from this experiment show that despite imposing incorrect quantitative restrictions from a misspecified DSGE model, the proposed methodology is still useful in identifying the true underlying structural shocks.

5.4 Proposed Methodology Using Partial Sign Restrictions

Paustian (2007) demonstrates that if we identify all the shocks using sign restrictions, then the resulting rotation would be closer to that implied by the DGP. However, it is common for an applied economist to identify only a small subset of structural shocks derived from a VAR system—say, a supply, demand, and policy shock—or that the researcher only have strong priors on a subset of restrictions. In this sub-section, we look at the bias generated by the proposed procedure using only a partial set of sign restrictions. This allows us to investigate whether the proposed identification is sensitive to the set of assumed qualitative restrictions.

Tables 3–5 report the ratio of the bias of the proposed identification relative to the pure sign-restriction approach across the nine different cases and six DSGE models outlined earlier. A ratio less than one indicates that our proposed method yields a smaller bias compared with the pure sign-restriction approach. The results suggest that the improvement in using the proposed methodology can be substantial. In the case where we have the correct quantitative restrictions (M_0), the average improvement is more than 90 percent. More realistically, we would be working with a misspecified model. Looking at the results using the five misspecified models, M_1 to M_5, we still achieve significant improvements in all cases up to the four-quarter horizon. There is only one case (case V) where we see a small deterioration at the eight- and twelve-quarter horizon. On average, the proposed method improves the bias over the pure sign-restriction scheme by about 50 percent. In general, we find that by imposing more and more sign restrictions, this reduces the bias of the SVAR

Table 3. Results of Cases I, II, and III

	Case I				Case II				Case III			
	Q1	**Q4**	**Q8**	**Q12**	**Q1**	**Q4**	**Q8**	**Q12**	**Q1**	**Q4**	**Q8**	**Q12**
M_0	0.02	0.06	0.05	0.06	0.01	0.09	0.12	0.12	0.01	0.06	0.05	0.06
M_1	0.19	0.34	0.27	0.40	0.34	0.74	0.93	0.96	0.57	0.61	0.64	0.67
M_2	0.08	0.27	0.24	0.37	0.32	0.72	0.91	0.95	0.54	0.59	0.62	0.66
M_3	0.12	0.32	0.27	0.39	0.33	0.73	0.93	0.97	0.56	0.60	0.65	0.69
M_4	0.17	0.32	0.26	0.40	0.33	0.71	0.89	0.95	0.58	0.60	0.62	0.67
M_5	0.69	0.80	0.72	0.73	0.56	0.65	0.81	0.96	0.39	0.67	0.79	0.79

Note: Each table entry displays the accumulated bias of the DSGE-identified VAR model relative to the pure sign-restrictions SVAR accumulated bias.

Table 4. Results of Cases IV, V, and VI

	Case IV				Case V				Case VI			
	Q1	**Q4**	**Q8**	**Q12**	**Q1**	**Q4**	**Q8**	**Q12**	**Q1**	**Q4**	**Q8**	**Q12**
M_0	0.01	0.06	0.07	0.08	0.01	0.08	0.10	0.11	0.01	0.08	0.06	0.07
M_1	0.35	0.44	0.32	0.37	0.43	0.76	1.23	1.19	0.30	0.42	0.64	0.73
M_2	0.32	0.40	0.30	0.36	0.43	0.71	1.11	1.10	0.22	0.29	0.53	0.65
M_3	0.34	0.43	0.29	0.34	0.44	0.72	1.15	1.12	0.28	0.34	0.56	0.67
M_4	0.34	0.42	0.30	0.35	0.46	0.77	1.21	1.19	0.31	0.42	0.66	0.76
M_5	0.46	0.61	0.57	0.58	0.57	0.98	1.44	1.29	0.54	0.52	0.57	0.62

Note: Each table entry displays the accumulated bias of the DSGE-identified VAR model relative to the pure sign-restrictions SVAR accumulated bias.

with respect to the DGP—a result that is consistent with Paustian (2007) finding.[15]

Overall, our results suggest that both quantitative and qualitative restrictions work well together, where they act as complements to each other, in minimizing errors in finding the correct VAR identification.

[15] We perform a series of Monte Carlo experiments by imposing zero, one, two, three, and four sets of sign restrictions. The results are available upon request.

Table 5. Results of Cases VII, VIII, and IX

	Case VII				Case VIII				Case IX			
	Q1	**Q4**	**Q8**	**Q12**	**Q1**	**Q4**	**Q8**	**Q12**	**Q1**	**Q4**	**Q8**	**Q12**
M_0	0.01	0.07	0.08	0.09	0.02	0.06	0.06	0.07	0.01	0.07	0.07	0.07
M_1	0.41	0.60	0.85	0.94	0.61	0.59	0.64	0.74	0.23	0.35	0.62	0.72
M_2	0.41	0.58	0.82	0.91	0.55	0.61	0.63	0.71	0.22	0.35	0.66	0.78
M_3	0.44	0.62	0.88	0.96	0.56	0.63	0.68	0.76	0.24	0.36	0.64	0.75
M_4	0.42	0.56	0.77	0.88	0.60	0.58	0.64	0.73	0.22	0.37	0.66	0.76
M_5	0.61	0.80	0.91	0.95	0.72	0.77	0.85	0.88	0.33	0.34	0.42	0.50

Note: Each table entry displays the accumulated bias of the DSGE-identified VAR model relative to the pure sign-restrictions SVAR accumulated bias.

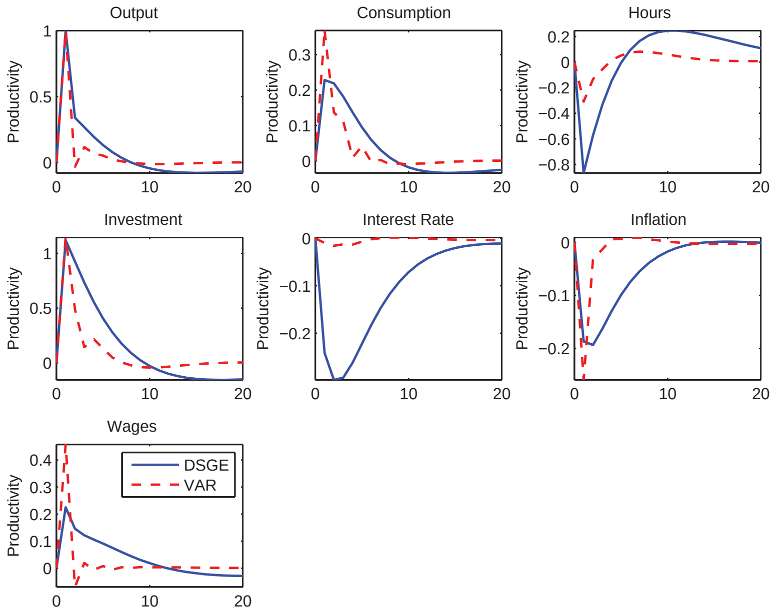
6. Application: Seven-Variable SVAR Model for the United States

To illustrate how the proposed identification scheme can be applied in practice, we estimate a seven-variable VAR using U.S. data from 1966:Q1 to 2004:Q4. The data set is taken from the Smets and Wouters (2007) paper which includes the log-difference of real GDP, real consumption, real investment and the real wage, log hours worked, the log-difference of the GDP deflator, and the federal funds rate. Although Smets and Wouters (2007) compare the estimated DSGE model's forecast performance with reduced-form VARs, they did not present any comparisons between the DSGE model's impulse response functions with a VAR model. This partly reflects the difficulty in finding the appropriate set of identifying restrictions for the VAR. From that perspective, our identification scheme is a natural candidate for comparison analysis. One can view this as a diagnostic tool for analyzing the dynamic behavior of the estimated DSGE model.

We follow the same procedure set out earlier in section 3.3, and the restrictions are based on Smets and Wouters' (2007) original model with all seven shocks.[16] First, we estimate the VAR

[16]The seven shocks include a government spending shock, a price markup shock, a wage markup shock, a monetary policy shock, a net worth shock, a technology shock, and an investment-specific shock.

Figure 3. U.S. Structural VAR: Productivity Shock



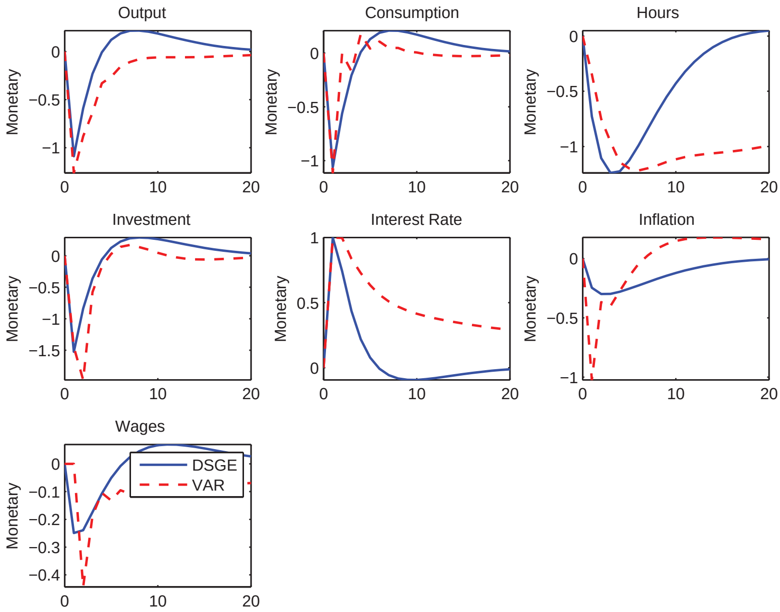
using simple OLS regression to obtain the reduced-form variance-covariance matrix ($\hat{\Sigma}_v$). Next, we find an orthogonal matrix $P(\omega^*)$ that minimized the distance in the first-period response between the DSGE and the SVAR model. The DSGE model's response is based on the parameter estimates obtained by Smets and Wouters (2007) as listed in table 7. For diagnostic comparison, the log-likelihood of the VAR is calculated to be –1,064 versus –1,256 for the DSGE model.

6.1 *Impulse Response Functions*

In figures 3 and 4, we present the impulse response functions for two of the most frequently analyzed shocks: an unexpected productivity shock and a monetary policy shock.[17] The solid line corresponds to the Smets and Wouters (2007) estimated model and the dashed line corresponds to the response of the SVAR model. It is

[17]The results of other shocks are available upon request to the authors.

Figure 4. U.S. Structural VAR: Monetary Policy Shock



worth highlighting that all the variables' responses have the same sign across the two models. However, there are some interesting differences in terms of the magnitudes and adjustment paths to the shocks.

For the productivity shock, the SVAR model tends to suggest a smaller impact on hours worked and the nominal interest rate. The impact on consumption, real wages, and inflation is slightly higher but less persistent. For the monetary policy shock, the SVAR model gives a larger but more temporary response for inflation. On the other hand, the SVAR displays a much more persistent behavior for hours worked, real wages, and the interest rate compared with the DSGE model. Interestingly, the response of output, consumption, and investment are very similar across the two models.

6.2 Forecast-Error-Variance Decomposition

In addition to the impulse response analysis, we also compute the forecast-error-variance (FEV) decomposition for both models—see table 6. For the SVAR, productivity and investment shocks are the

Table 6. Forecast-Variance Decomposition

	Productivity		**Preference**		**Government**		**Investment**		**Monetary**		**Price Markup**		**Wage Markup**	
Output	**VAR**	**DSGE**	**VAR**	**DSGE**	**VAR**	**DSGE**	**VAR**	**DSGE**	**VAR**	**DSGE**	**VAR**	**DSGE**	**VAR**	**DSGE**
Q1	35.4	13.5	4.8	26.0	3.4	38.9	46.6	13.8	0.0	5.3	2.4	2.4	7.4	0.1
Q4	32.2	13.7	7.6	23.3	4.6	33.7	45.8	15.3	1.0	5.9	2.9	4.7	6.0	3.5
Q8	29.9	13.2	7.8	22.8	4.7	32.2	42.4	15.3	2.1	6.0	5.8	4.7	7.3	5.7
Q12	29.6	13.0	7.8	22.4	4.7	31.6	41.8	15.9	2.4	6.3	6.2	4.9	7.4	5.9
∞	27.4	4.8	18.0	0.1	0.0	46.8	28.8	14.8	9.5	0.3	14.4	1.8	1.8	31.4
Consumption														
Q1	8.6	1.6	21.9	82.2	5.0	0.6	63.3	1.0	0.6	11.7	0.0	1.2	0.4	1.7
Q4	8.4	3.7	20.5	69.6	4.6	1.2	60.2	0.9	1.0	12.1	2.6	4.5	2.7	8.0
Q8	8.3	3.7	20.1	65.9	4.6	1.3	59.1	1.0	1.1	12.0	3.2	4.5	3.7	11.5
Q12	8.3	3.7	20.1	64.8	4.6	1.3	59.0	1.2	1.1	12.5	3.3	4.7	3.7	11.8
∞	21.9	12.0	19.3	0.2	1.2	2.1	14.8	4.9	2.7	0.5	25.9	1.2	14.3	79.0
Hours														
Q1	59.5	19.7	0.7	23.6	30.3	36.9	7.4	12.8	0.3	4.6	0.5	0.7	1.3	1.8
Q4	70.0	7.7	1.0	15.2	21.1	23.8	5.3	26.8	0.2	10.0	0.4	5.8	2.1	10.7
Q8	65.4	4.1	3.3	8.3	14.5	15.9	6.1	23.8	1.2	8.6	6.2	10.8	3.3	28.5
Q12	56.8	3.3	5.0	5.8	10.8	12.3	4.8	17.8	3.1	6.4	13.0	11.5	6.6	42.9
∞	42.7	0.1	9.2	0.4	4.0	20.2	3.0	0.1	7.7	0.7	25.4	2.3	7.9	76.2

(continued)

Table 6. (Continued)

	Productivity		Preference		Government		Investment		Monetary		Price Markup		Wage Markup	
	VAR	**DSGE**	**VAR**	**DSGE**	**VAR**	**DSGE**	**VAR**	**DSGE**	**VAR**	**DSGE**	**VAR**	**DSGE**	**VAR**	**DSGE**
Investment														
Q1	0.8	3.4	0.4	3.1	1.1	1.0	97.7	88.1	0.0	2.1	0.1	1.7	0.0	0.6
Q4	6.5	5.5	2.3	2.3	1.2	1.6	71.0	83.3	1.6	1.9	6.6	3.1	10.8	2.3
Q8	6.4	5.6	2.2	2.3	2.5	1.8	57.3	81.9	2.8	2.0	12.5	3.1	16.4	3.4
Q12	7.0	5.4	2.1	2.2	2.7	1.7	56.1	82.3	2.9	2.0	12.7	3.1	16.3	3.4
∞	0.3	21.6	3.2	0.0	1.6	16.5	22.0	20.7	11.6	0.2	42.2	2.4	19.2	38.4
Interest Rates														
Q1	22.4	9.6	17.5	20.6	1.7	1.9	29.6	1.7	14.1	54.7	14.8	8.0	0.0	3.4
Q4	19.7	14.7	18.6	15.4	0.7	3.4	21.3	12.5	16.7	28.8	21.9	12.9	1.2	12.3
Q8	19.8	13.6	17.7	11.2	0.7	3.5	16.3	22.2	16.7	19.3	26.3	11.1	2.6	19.2
Q12	21.8	12.5	16.8	10.0	0.9	3.4	14.1	24.8	15.9	17.4	27.2	9.9	3.3	21.9
∞	32.6	13.3	12.3	2.8	2.1	13.9	4.9	6.0	11.4	1.5	29.6	3.3	7.2	59.2
Inflation														
Q1	0.0	3.7	1.6	0.5	1.0	0.3	27.3	1.9	1.4	2.1	29.3	72.8	39.3	18.7
Q4	2.1	5.2	0.9	0.9	0.6	0.6	27.3	4.2	0.7	4.7	26.4	48.2	42.0	36.3
Q8	4.3	4.8	0.9	1.0	0.7	0.7	25.6	5.4	0.9	5.8	26.3	39.0	41.3	43.3
Q12	7.0	4.5	1.4	1.0	0.9	0.8	23.8	5.6	1.3	6.1	26.3	36.8	39.3	45.2
∞	30.9	4.1	6.2	0.5	1.8	4.4	0.0	1.1	6.3	3.9	34.8	5.4	19.9	80.6
Wages														
Q1	12.9	1.7	3.3	1.2	15.0	0.1	7.7	0.3	0.4	0.7	6.3	31.1	54.5	64.9
Q4	15.0	3.1	6.2	1.3	13.7	0.2	11.7	1.4	1.2	1.7	5.9	31.3	46.3	61.1
Q8	16.3	3.6	6.3	1.3	13.5	0.2	11.8	1.5	1.3	1.7	6.0	30.9	44.8	60.8
Q12	17.4	3.5	6.4	1.4	13.2	0.2	11.6	1.6	1.5	1.8	6.4	30.5	43.5	61.0
∞	36.8	13.4	11.7	0.1	1.7	10.1	3.9	11.2	9.6	0.7	34.9	3.2	1.2	61.3

main drivers of the FEV for output growth, whereas in the DSGE model, preference and government spending shocks play the dominant role. Similarly for consumption growth, investment shocks are more important than preference shocks in the SVAR model. For hours worked, productivity shocks explain over 50 percent of the FEV as opposed to the wage markup shock identified in the DSGE model. Investment shocks are the key factor in explaining investment growth across both models.

These observations highlight an important contrast across the two models: the SVAR tends to suggest that real shocks, such as investment and productivity shocks, play a relatively more important role than nominal shocks (government spending and wage markup shocks) for real economic variables. The sum of real shocks accounts for 80 percent of output, 87 percent of consumption, 70 percent of hours worked, and 65 percent of investment at the twelve-quarter horizon.[18] On the other hand, the DSGE model tends to suggest that both real and nominal shocks play an equally important role.

For the nominal interest rate, both models suggest that the contribution to the FEV is (roughly) equally divided among all seven shocks over the medium term. The DSGE model identifies both price and wage markup shocks to be the key drivers of the FEV for inflation, whereas the SVAR also attributes part of the FEV to investment shocks. For wage growth, while both models agree on the importance of wage markup shocks, the SVAR points to a much smaller role for price markup shocks over the medium term. Another interesting feature is that the DSGE model identifies wage markup shocks to be the dominant contributor of the unconditional variance for interest rate, inflation, and wage growth, whereas the SVAR highlights the importance of productivity and price markup shocks.

7. Conclusion

Issues relating to the identification of VAR models have been subject to numerous debates in the literature. The key source of

[18] We classify real shocks to include productivity, preference, and investment shocks. All other shocks are classified as nominal shocks.

this disagreement arises from finding a set of appropriate identifying assumptions to disentangle the reduced-form residuals back into structural disturbances. The sampling information in the data is often insufficient to distinguish between these different sets of assumptions. This paper proposes an identification strategy that extends Uhlig's (2005) penalty-function approach to a more formal setting. In particular, we construct a penalty function that is based on both quantitative and qualitative restrictions implied by a DSGE model. We present a series of Monte Carlo experiments to assess the usefulness of the proposed identification strategy. We also present an application using a seven-variable VAR model estimated on U.S. data and compare this with the results obtained from a medium-scale DSGE model by Smets and Wouters (2007).

By using the correct model restrictions, our proposed algorithm is successful in recovering the true structural identification matrix from the reduced-form VAR. In contrast to Erceg, Guerrieri, and Gust (2005), we find that the truncation bias is the dominant source of the bias in the estimated impulse response functions—particularly at longer horizons. Our result is consistent with the findings in Kapetanios, Pagan, and Scott (2007).

A number of interesting results emerge from the Monte Carlo analysis. First, the proposed identification strategy systematically gives a smaller bias compared with other identification schemes such as the Choleski decomposition and pure sign restrictions. Second, despite using restrictions implied by a misspecified model, the data (summarized by the reduced-form covariance matrix) tend to push the VAR responses away from the misspecified model and closer to the true DGP. Third, when we only impose a partial set of sign restrictions, the proposed method consistently yields a smaller bias relative to the pure sign-restriction approach. Our results suggest that both quantitative and qualitative restrictions work well together, where they act as complements to each other, in minimizing errors in finding the correct VAR identification.

The identification procedure proposed here is mainly applied to VAR models with a relatively small number of variables. But increasingly, the empirical literature emphasizes the importance of estimating statistical models based on a large information set. Examples include the large Bayesian VAR model put forward by Bandbura, Giannone, and Reichlin (2010) and the factor-augmented VAR model of Bernanke, Boivin, and Eliasz (2005). Future research could therefore be directed towards exploiting information contained in DSGE models to help identify VAR models with a large number of variables.

Appendix. The Model

This appendix briefly discusses some of the key linearized equilibrium conditions of Smets and Wouters' (2007) model. Readers who are interested in the agents' decision problems are advised to consult the references mentioned above directly. All the variables are expressed as log-deviations from their steady-state values, $\mathbb{E}_t$ denotes expectation formed at time t, "–" denotes the steady-state values, and all the shocks (η_t^i) are assumed to be normally distributed with zero mean and unit standard deviation.

The demand side of the economy consists of consumption (c_t), investment (i_t), capital utilization (z_t), and government spending ($\varepsilon_t^g = \rho_g \varepsilon_{t-1}^g + \sigma_g \eta_t^g$), which is assumed to be exogenous. The market clearing condition is given by

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g, \tag{19}$$

where y_t denotes the total output, and table 7 provides a full description of the model's parameters. The consumption Euler equation is given by

$$\begin{aligned} c_t = {} & \frac{\lambda/\gamma}{1+\lambda/\gamma} c_{t-1} + \left(1 - \frac{\lambda/\gamma}{1+\lambda/\gamma}\right) \mathbb{E}_t c_{t+1} \\ & + \frac{(\sigma_C - 1)\left(\bar{W}^h \bar{L}/\bar{C}\right)}{\sigma_C (1+\lambda/\gamma)} (l_t - \mathbb{E}_t l_{t+1}) \\ & - \frac{1-\lambda/\gamma}{\sigma_C (1+\lambda/\gamma)} (r_t - \mathbb{E}_t \pi_{t+1}), \end{aligned} \tag{20}$$

where l_t is the hours worked, r_t is the nominal interest rate, and π_t is the rate of inflation. If the degree of habits is zero ($\lambda = 0$), equation (20) reduces to the standard forward-looking consumption Euler equation. The linearized investment equation is given by

Table 7. Parameter Descriptions and Estimated Values from Smets and Wouters (2007)

Symbols	Description	M_0
γ	Steady-State Growth Rate	1.00
π	Steady-State Inflation	1.00
Φ	Fixed Cost	1.50
S"	Steady-State Capital Adjustment Cost Elasticity	5.74
α	Capital Production Share	0.19
σ	Intertemporal Substitution	1.38
h	Habit Persistence	0.71
ξ_w	Wages Calvo Parameter	0.70
σ_l	Labor Supply Elasticity	1.83
ξ_p	Prices Calvo Parameter	0.66
i_w	Wage Indexation	0.58
i_p	Price Indexation	0.24
z	Capital Utilization Adjustment Cost	0.27
ϕ_π	Taylor Inflation Parameter	2.04
ϕ_r	Taylor Inertia Parameter	0.81
ϕ_y	Taylor Output-Gap Parameter	0.08
ϕ_{dy}	Taylor Output-Gap Change Parameter	0.22
ρ_g	Government Spending Shock Persistence	0.97
ρ_{ms}	Policy Shock Persistence	0.15
ρ_p	Price Markup Shock Persistence	0.89
ρ_w	Wage Markup Shock Persistence	0.96
ma_p	Price Markup MA Term	0.69
ma_w	Wage Markup MA Term	0.84
σ_g	Government Spending Shock Uncertainty	0.53
σ_{ms}	Policy Shock Uncertainty	0.24
σ_p	Price Markup Shock Uncertainty	0.14
σ_w	Wage Markup Shock Uncertainty	0.24

$$i_t = \frac{1}{1+\beta\gamma^{1-\sigma_C}} i_{t-1} + \left(1 - \frac{1}{1+\beta\gamma^{1-\sigma_C}}\right) \mathbb{E}_t i_{t+1} + \frac{1}{\left(1+\beta\gamma^{1-\sigma_C}\right)\gamma^2\varphi} q_t, \tag{21}$$

where i_t denotes the investment and q_t is the real value of existing capital stock (Tobin's Q). The sensitivity of investment to real value of the existing capital stock depends on the parameter φ (see Christiano, Eichenbaum, and Evans 2005). The corresponding arbitrage equation for the value of capital is given by

$$q_t = \beta\gamma^{-\sigma_C} (1-\delta) \mathbb{E}_t q_{t+1} + \left(1 - \beta\gamma^{-\sigma_C} (1-\delta)\right) \mathbb{E}_t r^k_{t+1} - (r_t - \mathbb{E}_t \pi_{t+1}), \quad (22)$$

where $r^k_t = -(k_t - l_t) + w_t$ denotes the real rental rate of capital which is negatively related to the capital-labor ratio and positively related to the real wage.

On the supply side of the economy, the aggregate production function is defined as

$$y_t = \phi_p (\alpha k^s_t + (1-\alpha) l_t), \quad (23)$$

where k^s_t represents capital services, which is a linear function of lagged installed capital (k_{t-1}) and the degree of capital utilization, $k^s_t = k_{t-1} + z_t$. Capital utilization, on the other hand, is proportional to the real rental rate of capital, $z_t = \frac{1-\psi}{\psi} r^k_t$. The accumulation process of installed capital is simply described as

$$k_t = \frac{1-\delta}{\gamma} k_{t-1} + \frac{\gamma - 1 + \delta}{\gamma} i_t. \quad (24)$$

Monopolistic competition within the production sector and Calvo-pricing constraints gives the following New Keynesian Phillips curve for inflation:

$$\pi_t = \frac{i_p}{1+\beta\gamma^{1-\sigma_C} i_p} \pi_{t-1} + \frac{\beta\gamma^{1-\sigma_C}}{1+\beta\gamma^{1-\sigma_C} i_p} \mathbb{E}_t \pi_{t+1} - \frac{1}{(1+\beta\gamma^{1-\sigma_C} i_p)} \frac{\left(1-\beta\gamma^{1-\sigma_C}\xi_p\right)(1-\xi_p)}{(\xi_p ((\phi_p - 1)\varepsilon_p + 1))} \mu^p_t + \varepsilon^p_t, \quad (25)$$

where $\mu^p_t = \alpha(k^s_t - l_t) - w_t$ is the marginal cost of production and $\varepsilon^p_t = \rho_p \varepsilon^p_{t-1} + \sigma_p \eta^p_t - \mu_p \sigma_p \eta^p_{t-1}$ is the price markup price shock which is assumed to be an ARMA(1,1) process. Monopolistic competition in the labor market also gives rise to a similar wage New Keynesian Phillips curve,

$$
\begin{aligned}
w_t = {} & \frac{1}{1+\beta\gamma^{1-\sigma_C}} w_{t-1} + \frac{\beta\gamma^{1-\sigma_C}}{1+\beta\gamma^{1-\sigma_C}} \left(\mathbb{E}_t w_{t+1} + \mathbb{E}_t \pi_{t+1}\right) \\
& - \frac{1+\beta\gamma^{1-\sigma_C} i_w}{1+\beta\gamma^{1-\sigma_C}} \pi_t + \frac{i_w}{1+\beta\gamma^{1-\sigma_C}} \pi_{t-1} \\
& - \frac{1}{1+\beta\gamma^{1-\sigma_C}} \frac{\left(1-\beta\gamma^{1-\sigma_C}\xi_w\right)\left(1-\xi_w\right)}{\left(\xi_w\left(\left(\phi_w - 1\right)\varepsilon_w + 1\right)\right)} \mu_t^w + \varepsilon_t^w, \qquad (26)
\end{aligned}
$$

where $\mu_t^w = w_t - (\sigma_l l_t + \frac{1}{1-\lambda}(c_t - \lambda c_{t-1}))$ is the households' marginal benefit of supplying an extra unit of labor service and the wage markup shock $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \sigma_w \eta_t^w - \mu_w \sigma_w \eta_{t-1}^w$ is also assumed to be an ARMA(1,1) process.

Finally, the monetary policymaker is assumed to set the nominal interest rate according to the following Taylor-type rule:

$$
\begin{aligned}
r_t = {} & \rho r_{t-1} + (1-\rho)\left[r_\pi \pi_t + r_y\left(y_t - y_t^p\right)\right] \\
& + r_{\Delta y}\left[\left(y_t - y_t^p\right) + \left(y_{t-1} - y_{t-1}^p\right)\right] + \varepsilon_t^r, \qquad (27)
\end{aligned}
$$

where y_t^p is the flexible prices/wages and zero markup shocks level of output and $\varepsilon_t^r = \rho_r \varepsilon_{t-1}^r + \sigma_r \eta_t^r$ is the monetary policy shock.

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