Interest Rate Smoothing and "Calvo-Type" Interest Rate Rules: A Comment on Levine, McAdam, and Pearlman (2007)*

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In a recent paper, Levine, McAdam, and Pearlman (2007) propose a new type of interest rate rule, which they denote a "Calvo-type" rule. The Calvo-type interest rate responds to the discounted sum of current and future rates of inflation. We show that a Calvo-type rule can be derived from a very different assumption than the one used by Levine, McAdam, and Pearlman (2007), namely a preference for interest rate smoothing. In addition to giving an alternative rationale for the Calvo-type rule, we provide additional empirical support for the specification.

JEL Codes: E52, E37, E58.

1. Introduction

Monetary policy is commonly assumed to be forward looking. A popular way to specify the forward-lookingness in monetary policy is to let the interest rate respond to the inflation forecast, as in forward-looking Taylor rules. Levine, McAdam, and Pearlman (2007), henceforth LMP, note that such rules may have poor stabilization properties and often give real indeterminacy. They propose an alternative representation of monetary policy, which they refer

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to as a "Calvo-type" rule, where the interest rate responds to the discounted sum of all expected future rates of inflation. They show that this type of rule has better stabilization properties than traditional forward-looking Taylor rules. Moreover, Gabriel, Levine, and Spencer (2009) find empirical support for this kind of rule using data for the United States.

In this comment, we show that the rule suggested by LMP can be derived from a very different assumption than that made by LMP, namely a preference for *interest rate smoothing*. The current interest rate decision affects both the change in the interest rate from the previous period to the current one and the expected change from the current period to the next one. The interest rate decision today should therefore take into account both the lagged interest rate and the expected future interest rate. When the interest rate responds to current inflation, this gives rise to a Calvo-type rule. The Calvo-type rule thus has a more general foundation than previously believed.

In addition to providing an alternative rationale for the Calvo-type rule, we provide additional empirical support for this specification. Gabriel, Levine, and Spencer (2009) estimate the interest rate rule using GMM. Since GMM estimates are known to suffer from small-sample bias (see e.g., Hall 2005, chapter 6 and the references therein), we analyze the robustness of Gabriel, Levine, and Spencer (2009)'s results using maximum-likelihood system estimation and single-equation estimation where the implied forward interest rate is used as a proxy for the expected future interest rate. The results in Gabriel, Levine, and Spencer (2009) are generally confirmed and strengthen the case for the Calvo-type specification of interest rate rules. Since the Calvo-type specification has a general theoretical foundation and strong empirical support, we argue that future work on simple rules should consider Calvo-type rules as an alternative to the more common but less general forward-looking Taylor rules.

2. A Model of Interest Rate Smoothing

In the empirical literature on simple interest rate rules, it is common to specify the rule as a *partial-adjustment* equation, i.e.,

$$r_t = \rho r_{t-1} + (1 - \rho) r_t^*, \tag{1}$$

where r_t is the nominal interest rate and r_t^* is the target interest rate. The main motivation for the partial-adjustment specification is empirical fit, but it is often interpreted as evidence of central banks' preference for interest rate smoothing.[1] With partial adjustment, the central bank moves the interest rate gradually to the target rate. In the empirical literature, the target rate r_t^* is commonly specified as a non-inertial rule, such as the classic Taylor rule. There are, however, theoretical reasons for having an inertial target rate, as argued by Woodford (2003). But as noted by Rudebusch (2002), the partial-adjustment specification does not distinguish between inertia in the target rate itself or gradual adjustment toward a non-inertial target rate.

LMP derive optimal rules adding a lagged interest rate term. Even if the loss function considered by LMP does not include a preference for interest rate smoothing,[2] the authors still find that current policy should respond to the lagged interest rate. Indeed, they find that the optimal coefficient is one, thereby implying an integral (or difference) rule. This is a common result when the coefficients are optimized subject to the type of forward-looking models considered by LMP. Since LMP do not have an interest rate smoothing term in the loss function, their result on the optimal coefficients can be interpreted as finding an optimal target rate r_t^*.

Our aim is to show that the Calvo-type specification does not hinge on a specific model, as long as the central bank has a preference for interest rate smoothing. Following the traditional literature on empirical policy rules, we assume that the target interest rate is a standard (non-inertial) Taylor rule, i.e.,

$$r_t^* = a\pi_t + by_t, \tag{2}$$

where we for simplicity abstract from constant terms and assume that the neutral interest rate is zero. Even if it can be argued that such a simple, non-forward-looking target rule is sub-optimal, we deliberately choose this specification to show that the forward-looking nature of the rule with interest rate smoothing does not hinge on a forward-looking target interest rate.

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[1]See, e.g., Clarida, Galí, and Gertler (2000).

[2]They include a term with the interest rate *level*, but not the change in the interest rate.

Assume now that the central bank prefers to smooth the interest rate around the target rate. We model this by the following quadratic adjustment cost specification:

$$\Omega_t = \frac{1}{2} E_t \sum_{k=0}^{\infty} \delta^k \left[\left(r_{t+k} - r^*_{t+k} \right)^2 + \varphi (r_{t+k} - r_{t+k-1})^2 \right], \qquad (3)$$

where r^*_t is the target rate, δ is the discount factor, and φ is the cost of changing the interest rate. The first term represents the cost of deviating from the target interest rate, and the second term represents the cost of changing the interest rate. The first-order condition for minimization of (3) is

$$r_t - r^*_t + \varphi(r_t - r_{t-1}) - \delta\varphi(E_t r_{t+1} - r_t) - \sum_{k=0}^{\infty} \delta^k E_t \left(r_{t+k} - r^*_{t+k} \right) \frac{\partial E_t r^*_{t+k}}{\partial r_t} = 0. \qquad (4)$$

The term $-\sum_{k=0}^{\infty} \delta^k E_t (r_{t+k} - r^*_{t+k}) \frac{\partial E_t r^*_{t+k}}{\partial r_t}$ reflects that deviating from the target interest rate might affect the target rate itself, since the target rate depends on endogenous variables. We will, however, assume that interest rate smoothing has a negligible effect on the target interest rate in the near term. This is a reasonable assumption if the target rate depends on variables like inflation and the output gap that are affected by monetary policy with a time lag. Since the actual interest rate will only deviate significantly from the target rate in the first couple of periods, then for reasonable values of φ, one will tend to have that $|\frac{\partial E_t r^*_{t+k}}{\partial r_t}| \approx 0$ when $E_t(r_{t+k} - r^*_{t+k})$ is non-negligible and $E_t(r_{t+k} - r^*_{t+k}) \approx 0$ when $|\frac{\partial E_t r^*_{t+k}}{\partial r_t}|$ is non-negligible. The product $E_t(r_{t+k} - r^*_{t+k}) \frac{\partial E_t r^*_{t+k}}{\partial r_t} \approx 0$ for all $k = 1, 2, \ldots .T$, and the discounted sum of these products will be very small.[3] A close approximation to the optimal smoothing behavior given by (4) can then be written as

$$r_t = \gamma r_{t-1} + \gamma\delta E_t r_{t+1} + (1 - \gamma - \gamma\delta) r^*_t, \qquad (5)$$

[3] This is obviously not the case if the target interest rate depends on variables that display a significant contemporaneous response to changes in the interest rate such as, e.g., asset prices.

where $\gamma = \frac{\varphi}{1+\varphi(1+\delta)}$.[4] We see that optimal interest rate smoothing implies both forward- and backward-looking behavior, while partial adjustment implies only backward-looking behavior. More specifically, under partial adjustment the interest rate is set as a weighted average of the target rate and the lagged interest rate. Under optimal smoothing, the interest rate is a weighted average of the target rate, the lagged interest rate, and the expected next-period interest rate. Why does interest rate smoothing imply forward-looking behavior? The intuition is that a central bank that aims to smooth the interest rate is not only concerned about a smooth development in the interest rate from the previous period to the current period but also a smooth development from this period to the next. Since the interest rate set today has implications for both, a central bank with a preference for interest rate smoothing must be partly forward looking.

When the target rate r_t^* is given by (2), the rule with optimal interest rate smoothing can be written as

$$\begin{aligned} r_t &= \gamma r_{t-1} + \gamma\delta E_t r_{t+1} + (1-\gamma-\gamma\delta)(a\pi_t + by_t) \\ &= \hat{\rho} r_{t-1} + \varphi E_t \sum_{k=0}^{\infty} (\hat{\rho}\delta)^k (a\pi_{t+k} + by_{t+k}), \end{aligned} \tag{6}$$

where the last equality follows by solving the equation forward, which gives $\hat{\rho} = \frac{1}{2\delta\gamma}(\sqrt{1-4\delta\gamma^2}+1)$ and $\varphi = (1-(1+\delta)\gamma)\hat{\rho}\gamma^{-1}$. Note that for $b = 0$, the forward-solution specification is identical to the Calvo-type interest rate rule specified by equations (11) and (13) in LMP.[5] The key insight from our simple model is that a preference for interest rate smoothing is sufficient to make the central bank forward looking. This is in stark contrast to the sluggish backward-looking behavior implied by the standard partial-adjustment specifications in the empirical literature on interest rate rules. Rudebusch (2002) argued that the unreasonably high degree of inertia was due

[4] This specification is equal to the one in footnote 10 of LMP.

[5] Since our rule is derived from quadratic adjustment costs à la the Rotemberg (1982) approach of deriving the New Keynesian Phillips curve, it would perhaps be natural to call our specification a "Rotemberg-type" interest rate rule instead of a Calvo-type rule. But as with the New Keynesian Phillips curve, our Rotemberg foundation gives the same interest rate rule as LMP's Calvo type.

to omitted autocorrelated variables. Our model of optimal smoothing suggests that the omission of the expected future interest rate in (6) could be an important omitted variable.

3. Empirical Analysis

Gabriel, Levine, and Spencer (2009) find empirical support for the Calvo-type interest rate rule using single-equation GMM methods. Using U.S. data from 1960 to 2004, they report a positive and significant coefficient on the lagged interest rate term and the forward term in the interest rate rule.[6] It is well known that GMM estimators can exhibit substantial bias in small samples. In this section we examine the robustness of Gabriel, Levine, and Spencer (2009)'s results using two alternative approaches: maximum-likelihood system estimation and single-equation estimation where the implied forward interest rate is used as a proxy for the expected future interest rate.

We estimate the interest rule on quarterly U.S. data from 1987:Q3 to 2007:Q4.[7,8] Following the literature on estimated policy rules for the United States (e.g., Clarida, Galí, and Gertler 2000, Rudebusch 2002, and Jondeau, Le Bihan, and Galles 2004), we use the federal funds rate as the monetary policy instrument, r_t. Inflation is measured using the GDP deflator[9] (denoted P_t), so that $\pi_t = 400(\ln(P_t) - \ln(P_{t-1}))$.[10] The output gap is defined as the percentage deviation of real GDP from real potential GDP, i.e., $y_t = 100(\ln(GDP_t) - \ln(GDP_t^*)$, where real potential output is provided by the Congressional Budget Office.[11]

[6]Rewritten in comparable values, Gabriel, Levine, and Spencer (2009) find that (using the CBO output gap) $r_t = 0.56r_{t-1} + 0.4E_t r_{t+1} + 0.04(4.53E_t\overline{\pi}_{t+4} + 1.30E_t y_{t+1})$.

[7]The data series are obtained from the Federal Reserve Bank of St. Louis.

[8]The choice of estimation period is motivated by our desire to estimate the reaction function over a single monetary policy regime. Allowing for a structural break in the parameters of the reaction function in 1987:Q3, Jondeau, Le Bihan, and Galles (2004) strongly reject that the parameters are stable.

[9]The GDP deflator is seasonally adjusted.

[10]The results reported below are robust to using the GDP chain-weighted price index as the measure of inflation.

[11]The results are robust to replacing the output gap with a measure of the unemployment gap. Results are available upon request.

ML estimation requires that we specify an auxiliary model for the variables that determine the target rate (here, inflation and the output gap). We use a simple backward-looking model for inflation and output that has been shown to fit the data well. Specifically, we use a slightly modified version of the model proposed by Rudebusch and Svensson (1999). The model equations are

$$\pi_t = \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-2} + \alpha_3 \pi_{t-3} + (1 - \alpha_1 - \alpha_2 - \alpha_3)\pi_{t-4} + \alpha_y y_{t-1} + \varepsilon_{\pi,t}, \tag{7}$$

$$y_t = \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 y_{t-3} + \beta_4 y_{t-4} - \beta_r(\overline{r}_t - \overline{\pi}_t) + \varepsilon_{y,t}, \tag{8}$$

where variables with a bar are defined as $\overline{x}_t = \frac{1}{4}\sum_{j=0}^{3} x_{t-j}$. We demean the variables prior to estimation; hence the equations do not contain any constant terms.[12] The baseline reaction function is

$$\begin{aligned} r_t &= \rho_1 r_{t-1} + \rho_2 E_t r_{t+1} + (1 - \rho_1 - \rho_2)(\gamma_\pi \overline{\pi}_t + \gamma_y y_t) + \varepsilon_{r,t}, \\ \varepsilon_{r,t} &= \lambda_r \varepsilon_{r,t-1} + \xi_{r,t}. \end{aligned} \tag{9}$$

The motivation for allowing for autocorrelation in the disturbance term is to guard against misspecification of the target rule: Rudebusch (2002) argues that the significance of the lagged interest rate term in estimated reaction functions is due to the erroneous omission of serially correlated variables. However, English, Nelson, and Sack (2003) find that partial adjustment plays an important role in describing the behavior of the federal funds rate, even if one allows for serially correlated errors.

The estimates of the parameters in the reaction function are reported in table 1.[13,14] The estimates of the coefficients on the

[12] Compared with the specification in Rudebusch and Svensson (1999), the IS curve includes two extra lags of the output gap. The extra lags improve the empirical fit of the model and are needed to eliminate the autocorrelation in the residuals.

[13] The maximum-likelihood estimates are obtained using the Matlab routines provided by Jeffrey Fuhrer. The closed-form solution is derived using the Anderson-Moore algorithm (see Anderson and Moore 1985), and the likelihood function is maximized using Matlab's sequential quadratic programming algorithm `constr`. The estimation procedure does not impose any restrictions on the variance-covariance matrix of the (structural) shocks.

[14] The estimates of the parameters in the auxiliary models for inflation and output are documented in the appendix.

Table 1. ML Estimates, 1987:Q3–2007:Q4

Parameter	Estimate	SE
ρ_1	0.4092	0.04376
ρ_2	0.4993	0.09019
γ_π	2.3781	0.68540
γ_y	2.565	1.33497
λ_i	0.8798	0.07516
	Statistic	***p*-value**
Ljung-Box Test for Autocorrelation Q(12)	6.3658	0.89653
Value of Likelihood Function	−189.419	

lagged interest term and the forward term are both positive and statistically significant, thus confirming the results in Gabriel, Levine, and Spencer (2009).[15] The estimate of the autoregressive coefficient in the process for the disturbance term is 0.9 and is statistically significant. Thus, the significance of the coefficients on the interest rate term should not reflect the omission of serially correlated variables in the specification of the target rate.

Table 2 reports the estimates of the reaction function when the target rate is assumed to depend on average inflation four periods ahead and the output gap one period ahead—that is,

$$r_t = \rho_1 r_{t-1} + \rho_2 E_t r_{t+1} + (1 - \rho_1 - \rho_2)(\gamma_\pi E_t \overline{\pi}_{t+4} + \gamma_y E_t y_{t+1}) + \varepsilon_{r,t}. \qquad (10)$$

Following Rudebusch and Svensson (1999), we assume that the current (period t) state variables are included in the central bank's information set. As is evident from the table, the estimate of the coefficient on the forward term is now slightly smaller, but it is still statistically significant. The remaining parameters are not much affected.

[15] For comparison we also estimated the partial-adjustment version of the interest rate rule (i.e., excluding the forward interest rate term). The least-squares estimate of the coeffcient on the lagged interest rate is then 0.78.

Table 2. ML Estimates with Forward-Looking Target Rule, 1987:Q3–2007:Q4

Parameter	**Estimate**	**SE**
ρ_1	0.4582	0.0734
ρ_2	0.4234	0.1062
γ_π	2.3267	0.7405
γ_y	2.656	0.7349
λ_i	0.8603	0.0574
	Statistic	***p*-value**
Ljung-Box Test for Autocorrelation Q(12)	6.7673	0.8726
Value of Likelihood Function	−189.64	

We also estimated the reaction function using market expectations of the interest rate as a proxy for the expected policy rate interest rates one period ahead. We construct a measure of the expected future interest rate from the six-month and three-month LIBOR interest rates.[16] To guard against measurement error bias, we estimate the reaction function using GMM (see the discussion in Brissimis and Magginas 2008). The results are reported in table 3.[17,18] Again we find that the estimates of the coefficients on the interest rate terms are both positive and statistically significant. The J-statistic has a p-value of 0.71; hence, we cannot reject the validity of the over-identifying restrictions.

[16]We use the expectation hypothesis of the term structure of interest rates to compute the expected future interest rate: $(1+\frac{r_6}{100})^6 = (1+\frac{r_3}{100})^3(1+\frac{r_{impl,63}}{100})^{6-3}$, where r_3 and r_6 is three-month and six-month LIBOR, respectively, and $r_{impl,63}$ is the three-month forward rate to begin in three months.

[17]We use a heteroskedasticity and autocorrelation (HAC) consistent estimate of the variance-covariance matrix of the sample moments in the GMM estimator. The autocovariances are weighted using a Bartlett kernel with a bandwidth equal to 3 (selected using Newey West). The estimation results are obtained using EVIEWS.

[18]As discussed above, the monetary policy shock appears to be serially correlated, hence the first lag of the interest rate is not a valid instrument. Moreover, since forward rates are strongly correlated with the federal funds rate, we omit the first lag of forward rates from the instrument set.

Table 3. GMM Estimates with Forward Rates as Proxy for Expected Key Interest Rate, 1987:Q3–2007:Q4

Parameter	Estimate	SE
ρ_1	0.4625	0.0423
ρ_2	0.5000	0.0553
γ_π	1.9441	0.7742
γ_y	2.4927	0.8305
	Statistic	***p*-value**
J-test	6.2858	0.7110
Instrument set: $\{y_{t-j}, \pi_{t-j}\}_{j=0}^{3}\{r_{t-j}, r_{t-j}^{impl}\}_{j=2}^{3}$		

4. Conclusion

We show that the Calvo-type interest rate rule suggested by Levine, McAdam, and Pearlman (2007) can be derived from a preference for interest rate smoothing. Using both maximum-likelihood system estimation and single-equation estimation where the implied forward interest rate is used as a proxy for the expected future interest rate, we find additional empirical support for the Calvo-type rule.

Appendix. Estimated Auxiliary Models

Table 4. ML Estimates of Parameters in Auxiliary Model, 1987:Q3–2007:Q4 (Baseline Target Rate)

Phillips Curve			IS Curve		
Parameter	**Estimate**	**SE**	**Parameter**	**Estimate**	**SE**
α_1	0.2901	0.10625	β_1	1.1761	0.11660
α_2	0.1335	0.10634	β_2	0.0055	0.18072
α_3	0.1475	0.10655	β_3	−0.2445	0.14646
α_y	0.1221	0.05500	β_4	−0.0192	0.08687
			β_r	0.0108	0.02002

Table 5. ML Estimates of Auxiliary Model, 1987:Q3–2007:Q4 (Forward-Looking Target Rate)

Phillips Curve			IS Curve		
Parameter	**Estimate**	**SE**	**Parameter**	**Estimate**	**SE**
α_1	0.2735	0.1014	β_1	1.1358	0.1204
α_2	0.1303	0.1022	β_2	0.0738	0.1678
α_3	0.1733	0.1081	β_3	−0.2942	0.1377
α_y	0.1216	0.0561	β_4	−0.0010	0.0914
			β_r	0.0103	0.0182

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